# Measuring Cultural Relatedness Using Multiple Seriation Ordering Algorithms

Mark E. Madsen and Carl P. Lipo

*Manuscript version: 2016-03-15: bce5ad4– draft for Electronic Symposium, "Evolutionary Archaeologies: New Approaches, Methods, And Empirical Sufficiency" at the Society for American Archaeology conference, April 2016*

**Abstract** Seriation is a long-standing archaeological method for relative dating that has proven effective in probing regional-scale patterns of inheritance, social networks, and cultural contact in their full spatiotemporal context. The orderings produced by seriation are produced by the continuity of class distributions and unimodality of class frequencies, properties that are related to social learning and transmission models studied by evolutionary archaeologists. Linking seriation to social learning and transmission enables one to consider ordering principles beyond the classic unimodal curve. Unimodality is a highly visible property that can be used to probe and measure the relationships between assemblages, and it was especially useful when seriation was accomplished with simple algorithms and manual effort. With modern algorithms and computing power, multiple ordering principles can be employed to better understand the spatiotemporal relations between assemblages. Ultimately, the expansion of seriation to additional ordering algorithms allows us an ability to more thoroughly explore underlying models of cultural contact, social networks, and modes of social learning. In this paper, we review our progress to date in extending seriation to multiple ordering algorithms, with examples from Eastern North America and Oceania.

Mark E. Madsen
Dept. of Anthropology, University of Washington, Box 353100, Seattle, WA 98195 e-mail: mark@madsenlab.org

Carl P. Lipo
Environmental Studies Program and Dept. of Anthropology, Binghamton University, 4400 Vestal Parkway East Binghamton, NY 13902-6000 e-mail: clipo@binghamton.edu

## 1 Introduction

Seriation is a set of methods which uses patterns in the occurrence or abundance of historical classes to construct an ordering among otherwise unordered assemblages or objects (Dunnell, 1970). Traditionally, the orders constructed by seriation were intended to be chronological, since seriation was intended for use as a relative dating method by its early 20th century developers (O'Brien and Lyman, 2000, 1998; Lyman and O'Brien, 2006; O'Brien and Lyman, 1999; Lyman et al., 1997). But seriation techniques also create orderings which incorporate the effects of spatial variation in addition to temporal change, as James Ford pointed out (Ford, 1938; Phillips et al., 1951; Ford, 1935).

Despite the success of seriation in understanding the large-scale structure of the archaeological record in the New World (Beals et al., 1945; Bluhm, 1951; Evans, 1955; Ford, 1949; Kidder, 1917; Mayer-Oakes, 1955; Meggers and Evans, 1957; Phillips et al., 1951; Rouse, 1939; Smith, 1950), the method has largely been ignored since the advent of radiocarbon dating given its primary association as a relative dating method. But seriation is only a dating method in the sense that chronology is one possible inference from mapping the spatiotemporal pattern of change in cultural variants. Other inferences are possible, and in particular, there is a growing understanding that seriation is one of several methods for inferring historical and heritable continuity and thus documenting the evolutionary history of past populations (e.g., O'Brien and Lyman, 1999, Ch. 3).

Similarity between classes of artifacts constitutes heritable continuity when it arises from information being passed between populations over time; that is, from cultural transmission processes. Although the fact that seriation, in some sense, measures cultural transmission has been implicit since the earliest discussions of the method, the connection remained a common sense generalization until the mid 1990's. Fraser Neiman, in his dissertation and later his seminal 1995 article, noted that the unimodal patterns that form the core of the traditional frequency seriation technique are regularly seen in the trajectories seen when simulating unbiased transmission (Neiman, 1995). In order to make this connection both rigorous and useful in empirical work, we began a research program aimed at exploring the connection between cultural transmission models and seriation methods (Lipo et al., 1997), which has resulted in numerous publications, new seriation software algorithms, and many conference papers (Hunt et al., 1995; Lipo et al., 1995; Lipo and Eerkens, 2008; Lipo and Madsen, 2001; Lipo, 2001, 2005; Lipo and Madsen, 1997; Lipo et al., 2015; Madsen and Lipo, 2014, 2015; Madsen et al., 2008; O'Brien et al., 2015).

The core of the all seriation techniques are a set of "ordering principles" which describe how the data points making up each assemblage or object are rearranged in order to achieve a valid seriation solution. Traditionally, there are two (Dunnell, 1970; Rouse, 1967; Whitlam, 1981). The "occurrence principle" states that a valid ordering leaves no temporal gaps in the distribution of the historical classes used, and thus that temporal orders are continuous (Dempsey and Baumhoff, 1963; Rowe, 1959). The "frequency principle" states that in a valid ordering, the frequencies

making up the continuous distribution of each historical type will be unimodal, possessing a single peak of "popularity" (Nelson, 1916).

Both principles work, as demonstrated by the robustness and continued utility of the basic chronological frameworks erected by culture historians in the first half of the 20th century using seriation along with stratigraphy and marker types (Lyman et al., 1997). The frequency principle remains, however, an empirical generalization which is only suggested by the behavior of cultural transmission models, rather than being a necessary consequence. This suggests to us that seriation as a method requires further methodological development, especially if it is to be one of our major tools in tracing historical and heritable continuity in the archaeological record.[1]

INTRODUCTION INCOMPLETE UNTIL LATER SECTIONS DRAFTED

## 2 Seriation and the Frequency Principle

Seriation, in the Americanist sense, was initially developed by Alfred Kroeber (Kroeber, 1916) in the Southwest, on the basis of changes in ceramic decorations from Zuni Pueblo. The primitive seriation proposed by Kroeber was quickly amended by Leslie Spier, Alfred V. Kidder and Nels C. Nelson all of whom were conducting stratigraphic excavations in the American Southwest (Kidder, 1917; Nelson, 1916; Spier, 1917). This group of researchers all noticed that when ceramics were described in a particular way – called "stylistic" by Kidder (1917) – the temporal distribution of the types took the form of "normal curves." Using such types, it was apparent that a series of assemblages collected from the surface or otherwise undated could be arranged in chronological order by rearranging them so that all type distributions approximated "normal curves" simultaneously. The orders constructed in this way could also be tested by finding stratified deposits and were found to be correct.

As powerful as seriation proved to be, these early formulations were entirely intuitive and based on the generalization that greater temporal differences between assemblages caused larger differences between frequencies of decorated types, and that properly constructed historical types displayed a clear pattern of change (Phillips et al., 1951, p. 220):

> If our pottery types are successful measuring units for a continuous stream of changing cultural ideas, it follows that when the relative popularity of these types is graphed through time, a more or less long, single-peak curve will usually result. Put in another way, a type will first appear in very small percentages, will gradually increase to its maximum popularity, and then, as it is replaced by its succeeding type, will gradually decrease and disappear.

This compactly describes the "popularity principle," originally articulated by Nelson (1916) and Wissler (1916). A key word in the above is "usually," since not all

[1] Cladistics and phylogenetic methods, especially those which take into account temporal differences in the samples being studied (stratocladistics) and which are capable of yielding phylogenetic networks in addition to trees, are the other major tools by which we can measure heritable and historical continuity.

types display the unimodal distribution described, even when the attributes chosen are explicitly stylistic and decorative. Types suitable for frequency seriation were a subset of stylistic variation, comprising those which displayed spatial and temporal contiguity, a long enough duration that the types overlapped in their representation among sites and assemblages, and those whose distribution through time displayed the characteristic unimodal form which allowed the analyst to arrange them by eye. The process of constructing and testing such types became known, after Krieger (1944), as applying the "test of historical significance."

## *2.1 Unimodality and Cultural Transmission Processes*

In most cases (such as the above quote from Phillips, Ford, and Griffin), the popularity principle is simply assumed to hold in culture-historical applications. It is clear that culture historians assumed that what generates heritable continuity, and thus allows the tracing of chronological relations, is cultural transmission. As Lyman (2008) documents in careful detail, early 20th century anthropology and archaeology understood and discussed a variety of transmission processes informally, as generating the patterns they studied, even if they used different terms and did not form quantitative models for it. Rouse (1939), for example, explicitly discussed the diffusion of cultural traits, in terms that we now recognize as a spatiotemporal model of transmission. Kroeber, the father of frequency seriation, clearly understood the connection between his previous work and trait diffusion (Kroeber, 1937). There are many more examples (Lyman, 2008).

It was not until archaeologists began working with stochastic models of cultural transmission, however, that we could easily visualize the sheer variety of patterns that cultural transmission processes can, and do, generate. Archaeologists had long had quantitative models of diffusion (e.g., Ammerman and Cavalli-Sforza, 1971), but Dunnell's (1978) exposition of style as neutral variation led to adoption of stochastic models of drift from population genetics as the main tool for exploring cultural transmission dynamics. Many previous models of diffusion tended to be deterministic, especially those stemming from the interdisciplinary literature on the diffusion of innovations (e.g., Rogers, 2003). Neiman (1995) simulated drift in cultural variants as an unbiased transmission process, as shown in Figure 1. Immediately apparent is the fact that some variants do display unimodal patterns, but most variants are multimodal or display violations of unimodality at small scales even if the macroscopic shape seems to conform to the popularity principle.

The lesson of Figure 1 is that there is nothing necessary about unimodality given cultural transmission, but that it can occur. But culture historical types used in seriation were **constructed** to yield unimodal distributions, and a key element in such construction is ensuring that types are composed of multiple dimensions of variation which co-occur on artifacts identified to that type. We can imagine selecting the traits shown in Figure 1 and intersecting combinations of them to form multidimensional classes. In doing so, it is likely that unique combinations of those variants

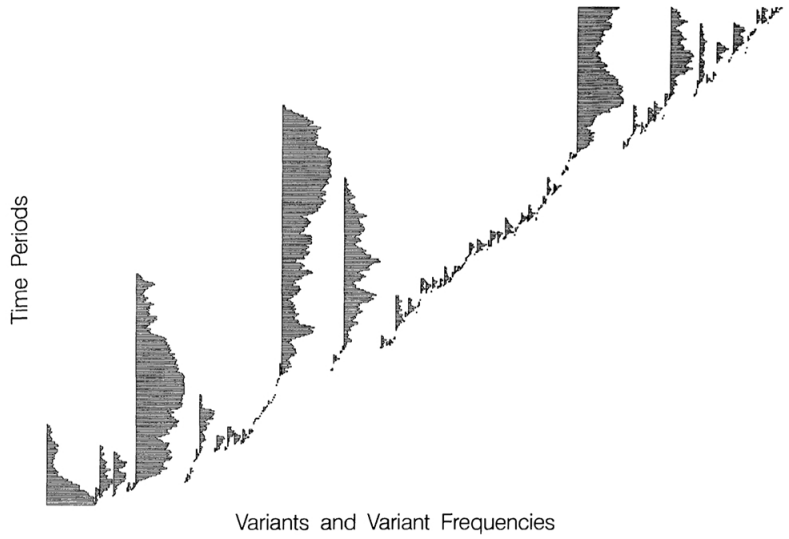


**Fig. 1** Neiman's simulation of drift in cultural variant frequencies under unbiased cultural transmission (reproduction of Figure 2a from Neiman 1995.)

would not recur and might show unimodal distributions more often. It is also likely that time averaging (ubiquitous in the archaeological record) will smooth out some of the minor variation in variant frequencies, as will the vagaries of sampling archaeological deposits.

Taken together, these factors seem to explain why the intuitive construction of historical types, from the continuous flow of the products of cultural transmission processes, worked to produce chronology through application of the common-sense popularity principle, and why not all artifact classes constructed from otherwise "stylistic" dimensions of variation, are suitable for frequency seriation using unimodality as the ordering criterion.

## *2.2 Continuity: An Alternative to Unimodality*

There are several reasons why we should explore alternatives to unimodality as an ordering algorithm for frequency seriation. First, from a performance perspective, searching for unimodal orders is computationally expensive, even for relatively small data sets (Madsen and Lipo, 2014). Even with the iterative, agglomerative method that we introduced recently (Lipo et al., 2015), the computation time can diverge for data sets as small as 30. This is a large number of assemblages by most archaeological standards, but with good techniques and ordering principles seriation

may scale to much larger problems, and even be applicable to the flood of data seen in modern day life.

Second, and more importantly from a theoretical perspective, it is important to be able to trace heritable continuity even if does not display a particular type of temporal frequency distribution. Using traditional type construction methods and the test of historical significance, culture historians were able to find **enough** conforming types and classes to construct regional chronologies. But there is a strong relationship between the number of classes in a seriation, and our ability to map differences across space and time. The use of seriation as a method for tracing evolutionary relationships is a more demanding task than establishing rough chronology in a region. Thus, it is worth searching for additional ordering principles that may be useful for seriating more classes of cultural variants.

A theoretically sound ordering principle for seriation should be derivable from characteristics of the underlying cultural transmission processes that we believe drive the spatiotemporal variation seriation measures. Formal models of cultural transmission, such as those formulated by Boyd and Richerson, Cavalli-Sforza and Feldman, and borrowed from population genetics (Boyd and Richerson, 1985; Cavalli-Sforza and Feldman, 1981; Neiman, 1995) are stochastic autoregressive processes, in the sense that the probability distribution of outcomes at a given time are dependent upon the outcomes from the immediate past. Mathematically, we usually formulate cultural transmission models as Markov processes, usually of first order (i.e., without dependencies on states previous to the immediate past state). Such models are certainly capable of making large changes in state over short time intervals, but large jumps are rare compared to small changes in state, especially in large populations.This is the reason why we (and culture historians) often have an expectation that cultural transmission has a "gradual" character to it.

The probabilistic gradualism of change over small time periods in our cultural transmission processes suggests a "continuity" principle, strongly related to notions of continuous functions in mathematics: samples which originate close together in time, space, or both will be close in type frequency and the presence/absence of types, especially compared to samples which are further apart. This continuity principle immediately leads to considering ordering algorithms based upon minimizing a suitable distance metric, with assemblages represented by points in a multidimensional space of type frequencies or counts.

## *2.3 Statistical Seriation Methods*

The earliest statistical techniques for seriation were also built upon using interassemblage distance metrics. Brainerd and Robinson (Brainerd, 1951; Robinson, 1951) pioneered a method for seriation based upon the similarity between assemblages, measured as a scaled version of the Manhattan (or city-block) distance between assemblage frequencies. When these scaled distances (which became known as Brainerd-Robinson coefficients) are arranged in a matrix with the largest values

nearest the diagonal and the lowest values in the corners and away from the diagonal, the order of assemblages by row or column provides the seriation solution. In practice, most real data matrices cannot be put in perfect Robinson form without violations.

What followed Brainerd and Robinson's pioneering work was a minor industry in methods for matrix ordering, in the face of the practical difficulties in coercing most data sets into a perfect linear ordering (e.g., Dempsey and Baumhoff, 1963; Kendall, 1963; Matthews, 1963; Bordaz and Bordaz, 1970; Gardin, 1970; Kendall, 1970, 1971). As access to computers by researchers in the social sciences increased, computerized algorithms for examining permutations quickly proliferated (Ascher and Ascher, 1963; **?**; Kuzara et al., 1966). Kendall (1969) and others attacked the ordering problem through the use of multidimensional scaling, and later correspondence analysis would be used with success in determining probabilistic seriation orders, and just as importantly, quantifying the degree of departure from the ideal seriation model (Smith and Neiman, 2005). For a detailed review of the many variants on this type of probabilistic seriation solution, see (Marquardt, 1978).

Not all of the similarity measures used in this literature are true distance metrics, but many are, and there were calls to simplify the problem by directly minimizing inter-assemblage distance, and thus the total "path length" of a candidate seriation solution. Kadane (1971) describes this approach, and it was adopted later by Shepardson (2006) in his construction of the "Optipath" seriation algorithm, which has distance minimization at its core.

Where existing distance/similarity methods encounter a problem is the assumption that a seriation solution must be a single linear order. In an earlier paper, we describe a seriation algorithm (iterative deterministic seriation solutions, or IDSS) that finds all of the possible orders in a set of data that conform to an ordering principle, and where those orders have overlap in assemblages, IDSS constructs a graph with branches that recognizes that the best solutions may not be linear (Lipo et al., 2015). Departures from linear seriation solutions have always been treated as "stress" or "error," especially when statistical methods such as MDS or correspondence analysis are employed. Practitioners usually recognize that such departures arise from coercing data which naturally sit in a larger number of dimensions – because of spatial variation and other factors – into a one-dimensional order. In essence, methods which attempt to coerce a complex spatiotemporal pattern into a linear ordering tend to treat departures from linearity as noise, which is then ignored.

But the departure from linearity is not "noise," in the statistical sense. Especially if one accounts for sampling error in constructing seriation orders (as we do in IDSS by using the bootstrap to construct confidence intervals around the empirical frequencies), then departures from a linear ordering are **signal**, not noise. Such solutions reflect the fact that an assemblage at time $T_1$, for example, may be the closest match to two different assemblages at later times $T_2$ and $T_3$ for example, given slightly different areas of overlap in their type frequencies. This can occur because the seriation method is inherently spatiotemporal, instead of simply measuring time (as culture historians have always known), and it can also reflect the splitting of populations into separate lineages (or their merger).

### *2.4 Exact Distance Minimization Ordering: "Continuity" Seriation*

Instead of the "approximate" distance minimization algorithms employed in multidimensional scaling and Shepardson's OptiPath seriation software, we explore exact solutions using our IDSS software. For simplicity in the configuration of the software, we summarize our approach by calling it "continuity" seriation, to emphasize that we want solutions that have the smoothest, most continous transition of type frequencies when we consider pairs of assemblages. We achieve this by locally minimizing the inter-assemblage distance within the solution graph, which automatically yields the minimum total "path length" for a seriation solution.

Our algorithm makes no use of the unimodality criterion, and yet produces equivalent results in almost all cases, as we show in the next section. The algorithm currently employs the Euclidean distance between assemblage counts or frequencies, although it can use any distance metric. Given a table of inter-assemblage distance metrics, we first construct pairs of two-vertex graphs which represent the "closest" assemblage for each assemblage in the data set (mirrored pairs are filtered out since they are isomorphic). The edge weight given to each edge is the Euclidean distance between the assemblages represented by vertices. For each of the minimal graphs in this initial set, we then find the assemblage with the shortest distance to each of the two ends, and continue iterating. Crucially, if there are equal-distance options, both possible solutions are retained. The result of this iteration is a collection of graphs which represent partial minimum-distance paths through the set of assemblages. This collection of partial graphs are then overlaid to form a single solution using a "minmax" approach as described in our paper on the IDSS algorithm in general (Lipo et al., 2015). The general approach is the same one we take to frequency seriation, what differs here with "continuity" seriation is how we form the set of candidate partial solutions. Instead of enforcing unimodality within each partial solution, we minimize Euclidean inter-assemblage distance. The resulting minmax graph is linear only if all of the candidate partial solutions perfectly overlay themselves into a linear solution, and otherwise will have a tree structure with branches.

## 3 Comparing Frequency and Continuity Seriation

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum viverra est est. Proin eget tellus metus. Aenean ac tortor pharetra libero ultricies sagittis. Nulla facilisi. Cras tincidunt interdum tellus, quis consectetur nunc facilisis nec. Sed fermentum erat a ligula posuere quis semper risus ullamcorper. Morbi vel tincidunt augue. Nam dolor ipsum, sagittis quis dignissim eu, pulvinar sed magna. In interdum magna eu orci facilisis congue. Cras a tellus et lorem sagittis viverra. Donec risus lectus, mollis at dignissim viverra, dapibus a nulla. Vivamus porttitor scelerisque turpis, eget lobortis orci auctor eget. Donec ultricies enim ac augue porttitor convallis. Pellentesque nisl lorem, consequat a facilisis in, ornare sed lorem. In luctus, elit ac mattis dapibus, lacus elit varius tortor, vel sollicitudin massa nisl id massa.

Ut sit amet nibh a sem egestas sollicitudin. Vestibulum scelerisque, dui at tincidunt accumsan, ipsum enim feugiat neque, vel interdum turpis lectus sed nisi. Nullam ultrices sodales sem, et placerat nunc euismod eu. Duis leo lacus, semper quis eleifend vitae, viverra ut nisl. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin rutrum eleifend est, id tempor velit viverra sed.

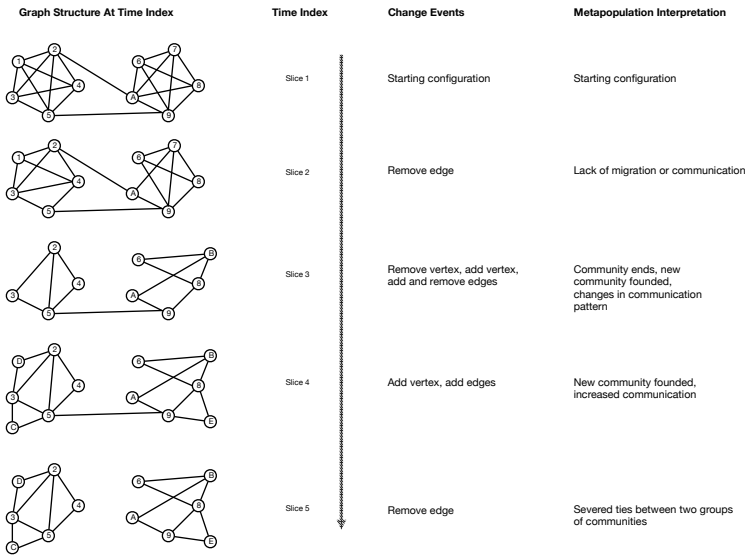


**Fig. 2** Foobar

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum viverra est est. Proin eget tellus metus. Aenean ac tortor pharetra libero ultricies sagittis. Nulla facilisi. Cras tincidunt interdum tellus, quis consectetur nunc facilisis nec. Sed fermentum erat a ligula posuere quis semper risus ullamcorper. Morbi vel tincidunt augue. Nam dolor ipsum, sagittis quis dignissim eu, pulvinar sed magna. In interdum magna eu orci facilisis congue. Cras a tellus et lorem sagittis viverra. Donec risus lectus, mollis at dignissim viverra, dapibus a nulla. Vivamus porttitor scelerisque turpis, eget lobortis orci auctor eget. Donec ultricies enim ac augue porttitor convallis. Pellentesque nisl lorem, consequat a facilisis in, ornare sed lorem. In luctus, elit ac mattis dapibus, lacus elit varius tortor, vel sollicitudin massa nisl id massa. Ut sit amet nibh a sem egestas sollicitudin. Vestibulum scelerisque, dui at tincidunt accumsan, ipsum enim feugiat neque, vel interdum turpis lectus sed nisi. Nullam ultrices sodales sem, et placerat nunc euismod eu. Duis leo lacus, semper quis eleifend vitae, viverra ut nisl. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin rutrum eleifend est, id tempor velit viverra sed.

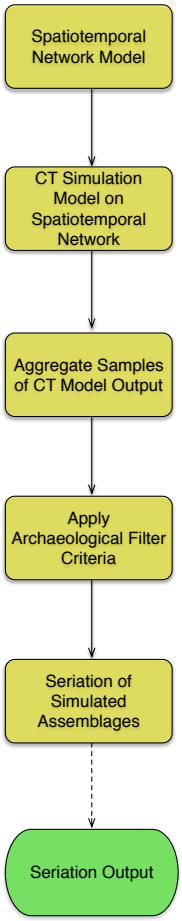


**Fig. 3** Foobar

## 4 Discussion

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum viverra est est. Proin eget tellus metus. Aenean ac tortor pharetra libero ultricies sagittis. Nulla facilisi. Cras tincidunt interdum tellus, quis consectetur nunc facilisis nec. Sed fermentum erat a ligula posuere quis semper risus ullamcorper. Morbi vel tincidunt augue. Nam dolor ipsum, sagittis quis dignissim eu, pulvinar sed magna. In interdum magna eu orci facilisis congue. Cras a tellus et lorem sagittis viverra. Donec risus lectus, mollis at dignissim viverra, dapibus a nulla. Vivamus porttitor scelerisque

turpis, eget lobortis orci auctor eget. Donec ultricies enim ac augue porttitor convallis. Pellentesque nisl lorem, consequat a facilisis in, ornare sed lorem. In luctus, elit ac mattis dapibus, lacus elit varius tortor, vel sollicitudin massa nisl id massa. Ut sit amet nibh a sem egestas sollicitudin. Vestibulum scelerisque, dui at tincidunt accumsan, ipsum enim feugiat neque, vel interdum turpis lectus sed nisi. Nullam ultrices sodales sem, et placerat nunc euismod eu. Duis leo lacus, semper quis eleifend vitae, viverra ut nisl. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin rutrum eleifend est, id tempor velit viverra sed.

## References


Ammerman, A.J., Cavalli-Sforza, L.L., 1971. Measuring the rate of spread of early farming in europe. Man , 674–688.

Ascher, M., Ascher, R., 1963. Chronological ordering by computer. American Anthropologist 65, 1045–1052.

Beals, R.L., Brainerd, G.W., Smith, W., 1945. Archaeological studies in northeast arizona. University of California Publications in American Archaeology and Ethnology 44.

Bluhm, E., 1951. Ceramic sequence in central basin and hopewell sites in central illinois. American Antiquity 16, 301–312.

Bordaz, V.v.H., Bordaz, J., 1970. A computer pattern recognition method of classification and seriation applied to archaeological material, in: Gardin, J.C. (Ed.), Archéologie et Calculateurs. Centre National de la Recherche Scientifique, pp. 229–244.

Boyd, R., Richerson, P., 1985. Culture and the Evolutionary Process. University of Chicago Press, Chicago.

Brainerd, G.W., 1951. The place of chronological ordering in archaeological analyis. American Antiquity 16, 301–312.

Cavalli-Sforza, L., Feldman, M.W., 1981. Cultural Transmission and Evolution: A Quantitative Approach. Princeton University Press, Princeton.

Dempsey, P., Baumhoff, M., 1963. The statistical use of artifact distributions to establish chronological sequence. American Antiquity , 496–509.

Dunnell, R.C., 1970. Seriation method and its evaluation. American Antiquity 35, 305–319.

Dunnell, R.C., 1978. Style and function: a fundamental dichotomy. American Antiquity 43, 192–202.

Evans, C., 1955. A ceramic study of Virginia Archaeology. BAE Bulletin 160, Washington.

Ford, J.A., 1935. Ceramic Decoration Sequence at an Old Indian Village Site near Sicily Island, Louisiana. Dept. Conservation, Louisiana Geological Survey, New Orleans.

Ford, J.A., 1938. A chronological method applicable to the southeast. American Antiquity 3, 260–264.

Ford, J.A., 1949. Cultural dating of prehistoric sites in Viru Valley, Peru. volume 43 of *Anthropological Papers*. American Museum of Natural History, New York.

Gardin, J.C., 1970. A computer pattern recognition method of classification and seriation applied to archaeological material Archchaeologie et Calculateurs, 229–244.

Hunt, T.D., Madsen, M.E., Lipo, C.P., 1995. Examining cultural transmission using frequency seriation, in: Poster presented at the 60th SAA Annual Meeting, Minneapolis MN.

Kadane, J.B., 1971. Chronological ordering of archeological deposits by the minimum path length method. Center for Naval Analyses, Arlington, VA.

Kendall, D.G., 1963. A statistical approach to flinders petrie's sequence dating. Bulletin of the International Statistical Institute 40, 657–680.

Kendall, D.G., 1969. Some problems and methods in statistical archaeology. World Archaeology 1, 68–76.

Kendall, D.G., 1970. A mathematical approach to seriation. Philosophical Transactions of the Royal Society, Series A, Mathematical and Physical Sciences 269, 125–135.

Kendall, D.G.a., 1971. Seriation from abundance matrices. Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete , 214–252.

Kidder, A.V., 1917. A design sequence from new mexico. Proceedings of the National Academy of Sciences 3, 369–370.

Krieger, A.D., 1944. The typological concept. American Antiquity 3, 271–288.

Kroeber, A., 1937. Diffusion. The Encyclopedia of Social Science, II , 137–142.

Kroeber, A.L., 1916. Zuni potsherds. American Museum of Natural History Anthropological Papers 18, 1–37.

Kuzara, R.S., Mead, G.R., Dixon, K.A., 1966. Seriation of anthropological data: A computer program for matrix-ordering. American Anthropologist 68, 1442–1455.

Lipo, C.P., 2001. Science, Style and the Study of Community Structure: An Example from the Central Mississippi River Valley. British Archaeological Reports, International Series, no. 918, Oxford.

Lipo, C.P., 2005. The resolution of cultural phylogenies using graphs, in: Mapping Our Ancestors: Phylogenetic Methods in Anthropology and Prehistory. Aldine Transaction Press, New York, pp. 89–107.

Lipo, C.P., Eerkens, J.W., 2008. Culture history, cultural transmission, and explanation of seriation in the southeastern united states, in: Cultural Transmission and Archaeology: Issues and Case Studies. Society for American Archaeology Press, Washington, DC, pp. 120–131.

Lipo, C.P., Madsen, M.E., 1997. The method seriation: Explaining the variability in the frequencies of types, in: 62nd Annual Meeting for the Society for American Archaeology.

Lipo, C.P., Madsen, M.E., 2001. Neutrality, "style," and drift: building methods for studying cultural transmission in the archaeological record, in: Hurt, T.D., Rakita, G.F.M. (Eds.), Style and Function: Conceptual Issues in Evolutionary Archaeology. Bergin and Garvey, Westport, Connecticut, pp. 91–118.

Lipo, C.P., Madsen, M.E., Dunnell, R.C., 2015. A theoretically-sufficient and computationally-practical technique for deterministic frequency seriation. PLoS ONE 10, e0124942.

Lipo, C.P., Madsen, M.E., Dunnell, R.C., Hunt, T., 1997. Population structure, cultural transmission, and frequency seriation. Journal of Anthropological Archaeology 16, 301 – 333.

Lipo, C.P., Madsen, M.E., Hunt, T.D., 1995. Artifact style dynamics ii: Deriving seriation from a network model of transmission, in: Paper presented at the 60th Annual Meeting for the Society for American Archaeology, Minneapolis MN.

Lyman, R., O'Brien, M., Dunnell, R., 1997. The rise and fall of culture history. Springer.

Lyman, R.L., 2008. Cultural transmission in north american anthropology and archaeology, ca. 1895-1965, in: O'Brien, M. (Ed.), Cultural Transmsision and Archaeology: Issues and Case Studies. SAA Press, pp. 10–20.

Lyman, R.L., O'Brien, M., 2006. Measuring Time with Artifacts. University of Nebraska, Lincoln.

Madsen, M., Lipo, C.P., 2015. An approach to fitting transmission models to seriations for regional-scale analysis, in: Paper presented at the 80th Annual Meeting of the Society for American Archaeology, San Francisco, CA.

Madsen, M.E., Lipo, C.P., 2014. Combinatorial structure of the deterministic seriation method with multiple subset solutions. Http://arxiv.org/abs/1412.6060.

Madsen, M.E., Lipo, C.P., Bentley, R.A., 2008. Explaining seriation patterns through network-structured cultural transmission models, in: Poster presented at the 73rd Annual Meeting of the Society for American Archaeology.

Marquardt, W.H., 1978. Advances in archaeological seriation. Advances in Archaeological Seriation 1, 257–314.

Matthews, J., 1963. Application of matrix analysis to archaeological problems. Nature 198, 930–934.

Mayer-Oakes, W.J., 1955. Prehistory of the Upper Ohio Valley: A Introductory Study. Carnegie Museum, Annals Vo. 34, Pittsburgh.

Meggers, B.J., Evans, C., 1957. Archaeological investigation in the mouth of the Amazon. Bureau of American Ethnology, Bulletin 167, Washington.

Neiman, F.D., 1995. Stylistic variation in evolutionary perspective: inferences from decorative diversity and interassemblage disstance in illinois woodland ceramic assemblages. American Antiquity 60, 7.

Nelson, N.C., 1916. Chronology of the tano ruins, new mexico. American Anthropologist 18, 159–180.

O'Brien, M., Lyman, R., 1998. James A. Ford and the growth of Americanist archaeology. Univ of Missouri Pr.

O'Brien, M.J., Boulanger, M.T., Buchanan, B., Bentley, R.A., Lyman, R.L., Lipo, C.P., Madsen, M.E., Eren, M.I., 2015. Design space and cultural transmission: Case studies from paleoindian eastern north america. Journal of Archaeological Method and Theory , 1–49.

O'Brien, M.J., Lyman, R.L., 1999. Seriation, Stratigraphy, and Index Fossils. The Backbone of Archaeological Dating. Kluwer Academic/Plenum, New York.

O'Brien, M.J., Lyman, R.L., 2000. Applying evolutionary archaeology: A systematic approach. Springer.

Phillips, P., Ford, J.A., Griffin, J.B., 1951. Archaeological Survey in the Lower Mississippi Alluvial Valley, 1940-1947. volume 25. Peabody Museum, Harvard University, Cambridge.

Robinson, W.S., 1951. A method for chronologically ordering archaeological deposits. American Antiquity 16, 293–301.

Rogers, E., 2003. The Diffusion of Innovations, 5th edition. Free Press, New York.

Rouse, I., 1967. Seriation in archaeology, in: Riley, C., Taylor, W. (Eds.), American Historical Anthropology. Southern Illinois University Press, pp. 153–195.

Rouse, I.B., 1939. Prehistory in Haiti: A Study in Method. Yale University Publications in Anthropology, No. 21, New Haven.

Rowe, J.H., 1959. Archaeological dating and cultural process. Southwestern Journal of Anthropology , 317–324.

Shepardson, B.L., 2006. Explaining Spatial and Temporal Patterns of Energy Investment In The Prehistoric Statuary of Rapa Nui (Easter Island). Ph.D. thesis. University of Hawaií.

Smith, C.S., 1950. The archaeology of coastal New York. American Museum of Natural History, Anthropological Papers 43(2), New York.

Smith, K., Neiman, F.D., 2005. Frequency seriation, correspondence analysis, and woodland period ceramic assemblage variation in the deep south. Southeastern Archaeology 26, 49–72.

Spier, L., 1917. An outline for a chronology of zuni ruins. Anthropological Papers of the American Museum of Natural History 18, 209–331.

Whitlam, R.G., 1981. PROBLEMS IN CERAMIC CLASSIFICATION AND CHRONOLOGY: AN EXAMPLE FROM THE MOBILE BAY AREA, ALABAMA. Midcontinental Journal of Archaeology 6, 179–206.

Wissler, C., 1916. The application of statistical methods to the data on the trenton argillite culture. American Anthropologist 18, 190–197.