Estimation of Interactive Fixed Effects in Panel Data

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Abstract

This extract is taken from a term paper for my econometrics research module and considers the interactive-effects estimator of Bai (2009) in panel data with large unit and time dimensions. The paper discusses the limitations of the questionable assumption imposed in additive effects model that unit- and time-specific fixed effects stay constant, even when the panel dimensions become large. In contrast to the additive effects model, the interactive-effects model relaxes this assumption and allows for a more general form of unobserved heterogeneity by interacting unit- and time-specific fixed effects. In the whole paper, particular emphasis is put on the assumptions underlying the interactive-effects estimator, asymptotic properties and the proposed recursive estimation procedure. The sections contained in this extract show the finite sample properties of the interactive-effects estimator via Monte-Carlo simulations as well as an implementation of the estimator in a real-world data context by replicating earlier results on intersectoral linkages in U.S. manufacturing by Voigtländer (2014). Due to the limited scope of this extract, only the most important Tables and Figures are included, although additional ones are mentioned in the text. The latter, as well as the R code implementing the estimator, the simulation process, and the empirical results, are available on request.

Keywords: Additive effects, interactive effects, factor error structure, time-invariant regressors, common regressors.

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1 Introduction

Panel data models are regularly used to capture unobserved time- or unit-invariant heterogeneity through the use of fixed effects. Crucially, this approach relies on the often untested assumption that these unobserved parts enter the regression framework additively and are constant within units or time periods. In cases of large panel dimensions, this assumption often appears questionable. If the assumption is not justifiable, though, unobserved heterogeneity is not fully accounted for and might be correlated with the observed regressors. Resulting estimates will be inconsistent accordingly due to omitted variable bias.

The interactive-effects estimator (hereafter IFE) proposed by Bai (2009) overcomes this issue by allowing the individual and time fixed effects to enter the model multiplicatively by assuming an encompassing factor structure to capture unobserved heterogeneity. Recently factor structures have seen increasing use in a variety of panel data model settings since the multiplicative structure allows each cross-sectional unit to react heterogeneously to each common factor at each time period. In macroeconomic applications, one common factor might be financial crises, while the factor loading represents the degree to which each country is affected by these crises depending on how open the economy of a said country is. In contrast to the additive-effects model, the IFE framework allows each country to have a heterogeneous reaction to each crisis, which is a much more realistic assumption given a large time dimension.

To be specific, let N be the number of cross-sectional units, T the number of time periods, p the number of regressors and R the number of factors. Then the IFE model reads as follows

$$Y_{it} = X_{it}'\beta + v_{it}, \quad v_{it} = \lambda_i' F_t + \varepsilon_{it}, \tag{1}$$

where Y_{it} is the dependent variable, X_{it} is a $p \times 1$ vector of regressors, β a $p \times 1$ vector of unknown parameters, ε_{it} the stochastic error and $\lambda_i' F_t$ the assumed factor structure. In particular, λ_i is an $R \times 1$ vector of factor loadings, F_t is an $R \times 1$ vector of common factors such that $\lambda_i' F_t = \lambda_{1,i} F_{1,t} + \cdots + \lambda_{R,i} F_{R,t}$. For $\lambda_i = (\alpha_i, 1)'$ and $F_t = (1, \xi_t)'$ the IFE model reduces to the additive fixed-effects model.

The applied and theoretical research in econometrics employing factor structures is extensive and has been present in the literature for some time. The presence of a common factor structure in panel data goes back to Goldberger (1972), Jöreskog and Goldberger (1975), Chamberlain and Griliches (1975) and Chamberlain (1977), who used large N, small T panel data. MaCurdy (1982) considers the somewhat easier case when the errors display a factor structure not correlated with the observed regressors. Ahn et al. (2001) utilize generalized method of moments estimation in the context of panel data models with IFE. Carneiro et al. (2003) incorporate a common factor structure into a dynamic treatment effects model in the context of the returns to schooling. Ahn et al. (2007) use IFE to estimate individual firms' technical inefficiency, which is unobservable and time-varying. Recent developments in the literature provide theoretical guidance in accommodating IFE when both N and T are large. Pesaran (2006) augments the regression equation with the cross-sectional averages of the dependent and independent variables. In contrast, the method in Bai (2009) is based on the principal component analysis. The estimator in Pesaran (2006) is root N consistent while the one in Bai (2009) is root NT consistent.

A variety of authors have built on the IFE estimator proposed by Bai (2009). Moon and Weidner (2015) extend the framework of Bai (2009) by allowing an unknown number of factors, while Kneip et al. (2012) apply the IFE structure to non-stationary factors. Further, Moon and Weidner (2017) consider the use of lagged regressors in the IFE model, while Lu and Su (2016) propose to use the adaptive group LASSO to determine the number of factors and select proper regressors. Chudik et al. (2011) explore the applicability of the IFE framework with weak, semi-weak, semi-strong, and strong common factors.

This paper makes three contributions. First, I give a thorough discussion of the assumptions on which the IFE estimator builds and introduce the proposed estimation method. Second, I provide a comprehensive overview of finite sample properties of the IFE estimator via Monte-Carlo-Simulations. Noteworthy, I show preliminary evidence that the asymptotic property of the IFE estimator can potentially be extended to the case of non-stationary factors. Finally, I consider the effect of using the IFE estimator in a real-world data context by replicating a study by Voigtländer (2014), who analyses intersectoral linkages and white-collar labor demand in U.S. manufacturing by relying on an additive-effects framework. I find that the use of the IFE model leads too much more consistent results across the inclusion of various control variables through the utilization of left-over information in the residuals and show that the original estimates most likely understate the causal effect.

The rest of the paper is organized as follows. In Section 2, I thoroughly derive the additive fixed-effects model. Section 3 introduces the IFE model including issues of identification and the proposed recursive estimation method. In Section 4, the underlying assumptions of the IFE are discussed and put into context. Deliberations on the asymptotic theory are given in Section 5. Section 6 shows the finite sample properties of the IFE estimator via Monte-Carlo-Simulations. Section 7 gives an implementation of the estimation method, as well as a comparison to the corresponding additive fixed-effects model. Section 8 concludes.

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6 Monte Carlo Simulation

To investigate the finite sample properties of the IFE estimator, I report Monte Carlo results for various model specifications. I compute the IFE estimator according to the iteration scheme described in Section 3. As starting values, I rely on the simple OLS estimate or the principal component estimate of F depending on which yields a smaller sum of squared residuals. Throughout the simulations I run 1000 replications for each specification and assume the true factor structure to be of dimension $R^0 = 2$, while taking combinations of the panel data identifiers N and T from the set $\{3, 5, 10, 20, 50, 100\}$.

The data generating process relies on an inherent correlation between regressors and the factor structure, such that the naive OLS estimator is by construction even asymptotically biased. In particular, I assume the regressors to be generated in the following way

$$X_{it,1} = \mu_1 + \lambda_i' F_t + \iota' \lambda_i + \iota' F_t + \eta_{it,1}$$

$$X_{it,2} = \mu_2 + \lambda_i' F_t + \iota' \lambda_i + \iota' F_t + \eta_{it,2}$$

where $\lambda_i = (\lambda_{1,i}, \lambda_{2,i})'$, $F_t = (F_{1,t}, F_{2,t})'$ and $\iota' = (1, 1)$. I assume $\lambda_{i,j}$, $F_{t,j}$ as well as $\eta_{it,j}$ to be $i.i.d.N(0, 1)$ for $j = 1, 2$, while setting $\mu_1 = \mu_2 = 1$. The parameters of interest are $(\beta_1, \beta_2) = (1, 3)$.

I start by considering the most general model in this framework, only imposing the errors to be i.i.d $N(0, 4)$ and $\alpha = 1$, leading to the model

$$Y_{it} = X_{it,1}\beta_1 + X_{it,2}\beta_2 + \alpha\lambda_i' F_t + \varepsilon_{it}. \tag{2}$$

To establish comparability, I report results for the IFE, OLS, Within and infeasible estimator, where the latter assumes F_t to be known. From Table 1 one can infer that both the infeasible and the IFE estimator can recover the parameters, while the Within estimator is biased even for large N and T. In contrast to this even in cases of small N and T the IFE and infeasible estimator yield unbiased results. Remarkably, the naive OLS estimator leads to less biased estimates compared to the Within estimator, implying that assuming no factor structure leads to less biased estimates than assuming a factor dimension R lower than the true one, $1 \leq R < R^0$.[1]

Further, as is shown in Appendix-Table A.1 the bias for the OLS and Within estimator increases linearly in α, doubling each time α duplicates. This implies that the initial advantage of the OLS over the Within estimator continuously grows for larger α, e.g. while $\beta^{Within}_{bias} - \beta^{OLS}_{bias} > 0.2$ for $\alpha = 1$ this extends to $\beta^{Within}_{bias} - \beta^{OLS}_{bias} > 2$ in the case of $\alpha = 8$.[2] The IFE estimator still yields unbiased and consistent estimates, even getting more efficient for larger values of α.

Next, I investigate an additive-effects model, which is nested in the foregoing framework for $\lambda_i = (\alpha_i, 1)'$ and $F_t = (1, \xi_t)'$, such that

$$Y_{it} = X_{it,1}\beta_1 + X_{it,2}\beta_2 + \alpha_i + \xi_t + \varepsilon_{it}.$$

The results reported in Table 2 indicate that while the naive OLS estimator again fails to recover the parameters, both the Within and IFE estimator yield unbiased and consistent estimates. The inefficiency of the IFE compared to the Within estimator results from the estimation of a too general model that encompasses the underlying additive model.

Crucially, Bai (2009) assumes the factor dimension to be known. As discussed in Section 4 this assumption is hardly if ever fulfilled in empirical settings, implying the need to estimate the underlying factor dimension. In Table 3 I show the performance of the IFE estimators for $R \in \{0, 1, 2, 3, 4, 5\}$, again assuming the true factor dimension to be $R^0 = 2$ and considering the setup in Equation (2). In particular, note that for $R = 0$ the estimator reduces to the naive OLS estimator. Underestimating the true factor dimension, i.e., $R < R^0 = 2$, leads to biased and inconsistent results across all N, T combinations. Intuitively, this result stems from the fact that consistency relies on controlling the space spanned by Λ and F, which is no longer the case when $R < R^0$. While the estimator is unbiased for $R > R^0 = 2$ it is less efficient then using the true factor dimensions, this inefficiency increases in R. Therefore, the practitioner might be well-advised to choose a bigger, and therefore less efficient factor dimension than trying to use the true factor dimension with the risk of obtaining biased and inconsistent estimates.

Appendix Tables A.2 and A.3 show the properties of the IFE estimator for cross and serially-correlated errors. In both cases the correlation is induced by an AR(1) process according to

[1] This implication only holds for the data generating considered in this section and is not necessarily generalizable to other specifications possessing a factor structure.

[2] Again, this result is not generalizable and may only hold for the data generating process at hand.

$\varepsilon_{it} = \rho\varepsilon_{i,t-1} + e_{it}$ and analogously for the case of cross-correlated errors. By choosing $\rho = 0.7$ and $\sigma_e^2 = 2$ I get $E[\varepsilon_{it}^2] = \sigma_e^2/(1-\rho^2) \approx 4$ and thus ensure comparability to the error variance used in the preceding tables. In the Appendix-Tables A.4 and A.5, I further show that the IFE estimator also remains consistent in the case of heteroskedasticity and simultaneous heteroskedasticity and correlation.

In the last step, I consider the finite sample properties of the IFE estimator in the case of a single non-stationary factor. Therefore, I adjust the initial data generating process by modeling $F_{1,t}$ to follow a unit-root process, i.e. $F_{1,t} = F_{1,t-1} + \eta_t$ with $\eta_t \sim N(0,1)$. Table 4 indicates that the IFE estimator is still able to recover the estimates, although the rate of convergence is slower compared to the case of stationary factors described in Table 1. As expected both the OLS and Within estimator are inconsistent. This result indicates that the asymptotic theory of the IFE estimator derived in Bai (2009) might be extended to allow for the presence of non-stationary factors.

7 Real World Data Application

In the following, I illustrate the implementation and relevance of the IFE in the context of real-world data. In essence, I build on the results of Voigtländer (2014), who analyses skill bias of technological change in U.S. manufacturing using a two-way fixed-effects approach, by additionally assuming the presence of an underlying factor structure. The setup seems especially appealing for applying the IFE estimator due to the presence of a balanced and large panel, covering 358 industry sectors across 48 years, therefore fulfilling the large N, T requirements described in Section 4.

I focus on the model introduced in Section 3 of Voigtländer (2014)

$$h_{it} = \alpha_i + \alpha_t + \beta\sigma_{it} + \gamma Z_{it} + \varepsilon_{it} \tag{3}$$

linking final production skill shares h_{it} and input skill intensity σ_{it} in sector i at time t. Further, Z_{it} is a vector of controls variables and ε_{it} the stochastic error. Due to the inclusion of sector α_i and time α_t fixed effects, β is identified by within sector and year variation. Standard errors are clustered on the sector level to account for within-group across time dependence in the errors. Table 5 replicates the main results of Table 3 in Voigtländer (2014), showing highly positive and significant effect of input skill intensity σ_{it} on high-skill labor share h_{it}, ranging from 0.539 for the most conservative specification to 1.360 for the pooled OLS model. To trace and visualize changes after applying the IFE framework, I plot the residuals resulting from specification (4) in Table 5 by sector and year in Figure 1a, which I consider the most mature models.[3]

I set up the IFE framework by assuming an additional underlying factor structure in Equation (3). This leads to the following model

$$h_{it} = \beta\sigma_{it} + \gamma Z_{it} + \lambda_i' F_t + \varepsilon_{it}. \tag{4}$$

While h_{it}, σ_{it}, Z_{it} and ε_{it} are defined in the same way as in Specification 3, $\lambda_i' F_t$ represents the newly added factor structure, which additionally needs to be estimated. Again, I cluster standard errors on the sector level to account for within-group dependence in the errors.[4] Since sector and time fixed effects are nested within the more general IFE approach, I start by estimating the IFE model in its most general form without explicitly assuming an additive fixed-effects structure.

As mentioned in Section 4, the true number of factors is usually not observable in real-world data applications, requiring the factor dimension to be estimated by appropriate criteria. While Bai (2009) only considers the use of stochastically bounded factors, various authors have extended the IFE framework to the case of non-stationary factors, which in turn require a new class of criteria. I start by reporting the estimated factor dimension for a range of criteria and values of the maximal factor dimension R_{max}, which is required for the computation of some criteria. As can be seen in Table 6, the estimated number of factors crucially hinges on R_{max} and the stationarity assumption, resulting in up to fifteen factors for some of the stationary criteria, while leading to just two in the case of the non-stationarity IPC3 criterion.

In a first step, I exclude the class of IC and PC criteria from consideration, which merely grow in R_{max} and therefore seem to be a poor choice. This leaves me to choose from factor dimension estimates ranging from $\hat{R} = 2$ to $\hat{R} = 8$, although most criteria hint at $R^0 = 2$ or $R^0 = 3$. In line with the simulation results in Table 3 the choice of $\hat{R}$ should not matter too much, as long as $\hat{R}$ is chosen big enough to cover all relevant factors. In line with this argument I report results for my preferred specifications in column (4) for $\hat{R} \in \{0, \dots, 8\}$ in Table 7. For $\hat{R} \geq 2$ the estimates for the parameter of interest are basically unchanged, following the pattern shown for $R > R^0$ in Table 3 of the simulation results and thereby indicating a factor dimension of $R^0 = 2$ or $R^0 = 3$. This

[3] Although the specification corresponding to columns (5) and (6) control for a broader range of control variables, the exclusion of 29 sectors could distort estimates in a serious way. However, for the final IFE model I again report results for all specifications considered in Table 3 of Voigtländer (2014).

[4] In lack of an analytical formula to derive clustered standard errors in the IFE framework I follow the suggestion of Cameron et al. (2008) to apply bootstrap procedures to derive the correct parameter standard errors.

guess is supported by the corresponding screeplots in Figure 2, displaying the sorted eigenvalues of the residuals $\hat{u}'_{\hat{R}}\hat{u}_{\hat{R}}$. For $R \geq 3$ it is no longer obvious how to decompose the eigenvalues into a few larger eigenvalues, stemming from factors and smaller eigenvalues relatable to the error term. I settle on the more conservative estimate of $R^* = 3$, embracing the possibility of inefficient estimates in case of $R^0 = 2$.

From Table 8, one can infer that the estimation results change modestly for columns (1)-(3) and (4), while leading to substantially higher estimates for columns (5) and (6), implying that the original model underestimates the effect of interest.[5][6] Also, as can be seen in Figure 1b, the IFE specification clearly outperforms the original model in utilizing additional information from the data, resulting in much more balanced and white noise residuals.

To get a better grasp of the underlying factor structure, I plot the factors corresponding to Model (4) with $R^* = 3$ in Figure 3a, where the factor numeration corresponds to the size of the respective eigenvalues. Noteworthy, the third factor (displayed in gray) exhibits a non-stationary pattern, which I verify through various stationarity-tests in Table 9. This could lead to the violation of stationary Assumption B discussed in Section 4 and thereby to the inapplicability of IFE estimator. However, as is indicated by the simulation results in Table 4 in Section 6, using the IFE framework proposed by Bai (2009) in the presence of an unit-root factor still leads to consistent parameter estimates. Although I refrain from interpreting this result as generalizable to the whole class of non-stationary factors, I consider it as an indication that the asymptotic theory described in Bai (2009) can potentially be extended to encompass non-stationary factors. To further provide evidence for this interpretation I redo the preceding model specification in Appendix-Section C using the estimator of Kneip et al. (2012), which explicitly allows for the presence of non-stationary factors. The corresponding estimates in Table 8 are of the same magnitude as those derived in Appendix-Table A.9, confirming the applicability of the IFE estimator in this context.

Further, the first factor (displayed in blue) in Figure 3a clearly seems to indicate the presence of sector fixed effects, staying almost perfectly constant across the observation period. In this case, it is more efficient to explicitly model sector fixed effects α_i in Specification (4), while accordingly reducing the factor dimension to $R^* = 2$. This leads to the final specification

[5] Due to missing values for the Outsourcing variables in specifications (5) and (6), I drop the respective sectors, 29 in total, to ensure a balanced panel as mentioned in 4. In particular, I am not able to reconstruct those observations since all occurring gaps last for at least 14 consecutive years.

[6] The fact that some control variable parameters become insignificant after modeling the factor structure results from a high correlation between those controls and the resulting factor structure as is shown in Appendix-Table A.11.

$$h_{it} = \alpha_i + \beta\sigma_{it} + \gamma Z_{it} + \lambda_i' F_t + \varepsilon_{it}$$

combining sector fixed effects α_i with an additional factor structure $\lambda_i' F_t$ of $R^* = 2$. The corresponding estimation results are displayed in Table 10. As expected the explicit use of sector fixed effects does not alter the estimates of the slope parameters in Table 8 in a serious way, but leads to more precise estimates.

While the preceding analysis justifies the use of an IFE model from the standpoint of an econometrician, I argue that the model specification is also underpinned by economic intuition. The claim that unobserved fixed effects stay the same for the observed time period of 48 years seems hardly justifiable, in particular, if one keeps in mind the technological changes that took place across the manufacturing industries from 1958 to 2005. As can be seen in Figure 3d, the factor structure shows a declining pattern for food, stone, and transportation sectors, while especially electronic-sectors exhibit an inverted pattern, translating to higher skill shares in the latter.[7] This means that unobserved heterogeneity led to a higher share of skilled labor in electronics compared to the other sectors, e.g, through reputational gains, which makes intuitive and economic sense.

In sum, I am able to show that the original specification of Voigtländer (2014) leads to an underestimation of the actual effect of input skill intensity on high-skill labor share when compared to the IFE framework. Additionally, the use of the IFE model allows for much more robust estimation of the parameter of interest, while at the same time harnessing more data information, as indicated by the corresponding residuals comparison in Figure 1.

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[7] I aggregate all 358 sectors to the more general two-digit SIC industry definition for easier identification of patterns in the factor structure.

References

Ahn, S. C., Lee, Y. H., and Schmidt, P. (2001). Gmm estimation of linear panel data models with time-varying individual effects. *Journal of Econometrics*, 101(2):219–255.

Ahn, S. C., Lee, Y. H., and Schmidt, P. (2007). Stochastic frontier models with multiple time-varying individual effects. *Journal of Productivity Analysis*, 27(1):1–12.

Ahn, S. C., Lee, Y. H., and Schmidt, P. (2013). Panel data models with multiple time-varying individual effects. *Journal of Econometrics*, 174:1–14.

Bai, J. (2004). Estimating cross-section common stochastic trends in nonstationary panel data. *Journal of Econometrics*, 122(1):137–183.

Bai, J. (2009). Panel Data Models With Interactive Fixed Effects. *Econometrica*, 77(4):1229–1279.

Bai, J. and Ng, S. (2002). Determining the number of factors in approximate factor models. *Econometrica*, 70(1):191–221.

Cameron, A. C., Gelbach, J. B., and Miller, D. L. (2008). Bootstrap-based improvements for inference with clustered errors. *The Review of Economics and Statistics*, 90(3):414–427.

Carneiro, P., Hansen, K. T., and Heckman, J. J. (2003). Estimating distributions of treatment effects with an application to the returns to schooling and measurement of the effects of uncertainty on college choice. *International Economic Review*, 44(2):361–422.

Chamberlain, G. (1977). Education, income, and ability revisited. *Journal of Econometrics*, 5(2):241–257.

Chamberlain, G. and Griliches, Z. (1975). Unobservables with a variance-components structure: Ability, schooling, and the economic success of brothers. *International Economic Review*, pages 422–449.

Chudik, A., Pesaran, M. H., and Tosetti, E. (2011). Weak and strong cross-section dependence and estimation of large panels. *The Econometrics Journal*, 14(1):C45–C90.

Goldberger, A. S. (1972). Structural equation methods in the social sciences. *Econometrica: Journal of the Econometric Society*, pages 979–1001.

Jöreskog, K. G. and Goldberger, A. S. (1975). Estimation of a model with multiple indicators and multiple causes of a single latent variable. *Journal of the American Statistical Association*, 70(351a):631–639.

Kneip, A., Sickles, R. C., and Song, W. (2012). A new panel data treatment for heterogeneity in time trends. *Econometric Theory*, 28(3):590–628.

Lu, X. and Su, L. (2016). Shrinkage estimation of dynamic panel data models with interactive fixed effects. *Journal of Econometrics*, 190(1):148–175.

MaCurdy, T. E. (1982). The use of time series processes to model the error structure of earnings in a longitudinal data analysis. *Journal of Econometrics*, 18(1):83–114.

Moon, H. R. and Weidner, M. (2015). Linear regression for panel with unknown number of factors as interactive fixed effects. *Econometrica*, 83(4):1543–1579.

Moon, H. R. and Weidner, M. (2017). Dynamic linear panel regression models with interactive fixed effects. *Econometric Theory*, 33(1):158–195.

Onatski, A. (2010). Determining the number of factors from empirical distribution of eigenvalues. *The Review of Economics and Statistics*, 92(4):1004–1016.

Pesaran, M. H. (2006). Estimation and inference in large heterogeneous panels with a multifactor error structure. *Econometrica*, 74(4):967–1012.

Voigtländer, N. (2014). Skill bias magnified: Intersectoral linkages and white-collar labor demand in us manufacturing. *Review of Economics and Statistics*, 96(3):495–513.

Tables

Table 1: Various Estimators; IFE model with i.i.d. Errors

		OLS Estimator				Within Estimator				Infeasible Estimator				Interactive Estimator			
N	T	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD
100	3	1.126	0.114	3.119	0.116	1.364	0.135	3.369	0.142	1.000	0.162	3.001	0.161	0.982	0.225	2.977	0.221
100	5	1.126	0.083	3.129	0.087	1.380	0.095	3.383	0.097	1.001	0.087	2.998	0.088	1.015	0.122	3.013	0.126
100	10	1.133	0.058	3.130	0.057	1.392	0.060	3.390	0.061	1.001	0.051	2.999	0.051	1.009	0.063	3.007	0.063
100	20	1.133	0.043	3.131	0.042	1.397	0.044	3.395	0.044	1.001	0.037	3.000	0.037	1.005	0.042	3.004	0.043
100	50	1.134	0.026	3.133	0.027	1.399	0.026	3.398	0.027	1.000	0.021	3.000	0.022	1.002	0.024	3.002	0.025
100	100	1.132	0.019	3.133	0.020	1.398	0.020	3.399	0.020	1.000	0.015	3.000	0.014	1.001	0.016	3.001	0.016
3	100	1.125	0.119	3.125	0.115	1.370	0.144	3.368	0.136	1.003	0.093	3.000	0.096	0.980	0.238	2.985	0.238
5	100	1.133	0.085	3.124	0.082	1.385	0.097	3.377	0.095	1.006	0.067	2.996	0.067	1.020	0.127	3.003	0.124
10	100	1.131	0.061	3.130	0.059	1.392	0.064	3.393	0.059	0.999	0.047	3.000	0.047	1.009	0.064	3.007	0.066
20	100	1.130	0.043	3.133	0.042	1.393	0.045	3.396	0.043	0.999	0.034	3.002	0.035	1.004	0.041	3.006	0.041
50	100	1.131	0.027	3.134	0.027	1.397	0.027	3.399	0.027	0.999	0.022	3.001	0.022	1.000	0.025	3.003	0.025

Table 2: Various Estimators; Additive Model with i.i.d. Errors

		OLS Estimator				Within Estimator				Infeasible Estimator				Interactive Estimator			
N	T	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD
100	3	1.107	0.101	3.109	0.102	0.997	0.148	3.007	0.146	0.993	0.208	3.003	0.204	1.008	0.234	3.016	0.230
100	5	1.109	0.078	3.112	0.079	1.002	0.098	3.005	0.101	1.002	0.116	3.005	0.119	1.093	0.133	3.101	0.134
100	10	1.111	0.051	3.112	0.052	0.999	0.068	3.004	0.065	1.001	0.072	3.003	0.069	1.105	0.100	3.110	0.100
100	20	1.112	0.038	3.114	0.038	0.999	0.047	3.002	0.048	0.998	0.047	3.001	0.049	1.095	0.090	3.098	0.090
100	50	1.114	0.024	3.113	0.024	1.002	0.028	3.000	0.030	1.002	0.028	3.000	0.030	1.050	0.071	3.047	0.074
100	100	1.113	0.017	3.115	0.018	0.999	0.019	3.001	0.020	0.999	0.019	3.001	0.020	1.007	0.027	3.009	0.028
3	100	1.111	0.097	3.109	0.096	1.002	0.145	2.998	0.144	1.003	0.116	3.000	0.116	1.011	0.232	3.007	0.228
5	100	1.110	0.076	3.112	0.074	1.000	0.102	3.002	0.100	0.999	0.090	3.002	0.091	1.093	0.134	3.096	0.138
10	100	1.116	0.053	3.107	0.053	1.004	0.066	3.001	0.068	1.005	0.062	3.000	0.064	1.114	0.097	3.109	0.096
20	100	1.113	0.037	3.113	0.038	1.002	0.047	3.002	0.045	1.002	0.045	3.002	0.045	1.101	0.089	3.101	0.086
50	100	1.112	0.024	3.115	0.024	0.999	0.029	3.001	0.028	0.999	0.029	3.001	0.028	1.045	0.073	3.047	0.071

Table 3: Interactive-Effects Estimator; Various Factor Dimensions

		R = 0				R = 1				R = 2			
N	T	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD
100	5	1.135	0.087	3.124	0.085	1.085	0.105	3.080	0.111	1.035	0.129	3.031	0.134
100	10	1.130	0.060	3.129	0.060	1.083	0.068	3.083	0.067	1.011	0.071	3.011	0.071
100	20	1.131	0.042	3.133	0.043	1.090	0.049	3.092	0.047	1.003	0.041	3.007	0.041
100	50	1.132	0.027	3.133	0.027	1.096	0.033	3.097	0.033	1.000	0.024	3.002	0.024
100	100	1.133	0.020	3.133	0.020	1.100	0.027	3.100	0.028	1.001	0.017	3.001	0.017
5	100	1.130	0.088	3.124	0.087	1.079	0.106	3.075	0.103	1.027	0.134	3.030	0.127
10	100	1.133	0.060	3.131	0.060	1.088	0.071	3.087	0.070	1.014	0.070	3.016	0.070
20	100	1.131	0.042	3.132	0.042	1.088	0.046	3.091	0.046	1.002	0.040	3.006	0.041
50	100	1.132	0.028	3.133	0.028	1.096	0.034	3.097	0.034	1.002	0.025	3.001	0.024
		R = 3				R = 4				R = 5			
N	T	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD
100	5	1.031	0.167	3.036	0.169	1.046	0.225	3.042	0.227	1.135	0.087	3.124	0.085
100	10	1.009	0.072	3.008	0.075	1.010	0.081	3.011	0.085	1.009	0.094	3.014	0.100
100	20	1.003	0.043	3.007	0.044	1.003	0.046	3.008	0.046	1.003	0.049	3.007	0.049
100	50	1.000	0.024	3.002	0.024	1.000	0.025	3.002	0.025	1.000	0.025	3.001	0.026
100	100	1.001	0.017	3.001	0.017	1.001	0.018	3.001	0.018	1.001	0.018	3.001	0.018
5	100	1.031	0.173	3.029	0.162	1.035	0.232	3.035	0.229	1.130	0.088	3.124	0.087
10	100	1.012	0.080	3.015	0.077	1.011	0.088	3.016	0.085	1.011	0.099	3.018	0.096
20	100	1.002	0.042	3.006	0.043	1.001	0.046	3.006	0.046	1.001	0.048	3.005	0.048
50	100	1.002	0.025	3.001	0.025	1.002	0.026	3.001	0.026	1.002	0.027	3.001	0.026

Table 4: Various Estimators; Single Unit Root Factor

		OLS Estimator				Within Estimator				Interactive Estimator			
N	T	β_1	SD	β_2	SD	β_1	SD	β_2	SD	β_1	SD	β_2	SD
100	3	1.252	0.306	3.218	0.305	1.355	0.146	3.345	0.143	1.177	0.220	3.171	0.223
100	5	1.238	0.243	3.226	0.246	1.378	0.102	3.374	0.095	1.140	0.150	3.137	0.150
100	10	1.238	0.168	3.231	0.173	1.409	0.065	3.405	0.067	1.089	0.103	3.085	0.107
100	20	1.229	0.130	3.245	0.132	1.435	0.048	3.435	0.048	1.060	0.080	3.060	0.083
100	50	1.239	0.088	3.241	0.089	1.462	0.030	3.463	0.029	1.055	0.078	3.055	0.076
100	100	1.242	0.073	3.243	0.074	1.477	0.021	3.479	0.021	1.056	0.098	3.058	0.095
3	100	1.224	0.362	3.181	0.353	1.450	0.121	3.462	0.121	1.137	0.249	3.136	0.241
5	100	1.225	0.292	3.213	0.300	1.471	0.083	3.467	0.082	1.122	0.172	3.121	0.171
10	100	1.233	0.203	3.235	0.210	1.474	0.054	3.472	0.054	1.087	0.133	3.089	0.131
20	100	1.247	0.149	3.227	0.147	1.477	0.039	3.475	0.039	1.066	0.104	3.065	0.104
50	100	1.241	0.101	3.242	0.100	1.479	0.025	3.478	0.027	1.055	0.097	3.055	0.098

Table 5: Replication of Table 3 from Voigtländer (2014)[ab]

Input skill measure	σ_{it}					σ_{it}^{2d}
	(1)	(2)	(3)	(4)	(5)	(6)
Input skill intensity	1.360***	1.094***	0.982***	0.883***	0.662***	0.539***
	(0.100)	(0.099)	(0.167)	(0.163)	(0.122)	(0.173)
Equipment per worker				−0.066	−0.050	−0.044
				(0.052)	(0.047)	(0.050)
Office equipment				0.274	0.145	0.143
				(0.165)	(0.161)	(0.159)
High-Tech capital				0.483**	0.497**	0.571***
				(0.193)	(0.191)	(0.182)
R&D intensity					0.650***	0.742***
					(0.200)	(0.207)
Outsourcing					0.089*	0.107**
					(0.047)	(0.051)
Outsourcing (broad)					0.189***	0.206***
					(0.062)	(0.064)
Sector fixed effects	no	yes	yes	yes	yes	yes
Time fixed effects	no	no	yes	yes	yes	yes
R^2 (within)	0.854	0.379	0.392	0.429	0.476	0.459
Adjusted R^2	0.854	0.366	0.377	0.415	0.463	0.446
Observations	17,184	17,184	17,184	17,184	16,633	16,633

[a] Regression results of final production skill on input skill intensity and various controls. The observation period consists of annual data, 1958-2005, for 358 U.S. manufacturing sectors. All regressions are weighted by sectors' average share in manufacturing employment. Keys: *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$.

[b] Standard errors are displayed in parentheses and clustered on the sector level, which are derived by bootstrap replications. While clustered standard errors could also be derived by using explicit formula, I use the same bootstrap method as for the interactive regressions to ensure comparability between estimates.

Table 6: Estimated Factor Dimension for Various Criteria and R_{max}

Criterion	$R_{max} = 5$	$R_{max} = 10$	$R_{max} = 15$
PC1	5	10	15
PC2	5	10	15
PC3	5	10	15
BIC3	4	6	8
IC1	5	10	12
IC2	5	10	11
IC3	5	10	15
IPC1	2	3	3
IPC2	2	3	3
IPC3	2	2	3
ED	3	3	3
ER	3	3	3
GR	2	2	2

Notes: The stationary criteria PC1, PC2, PC3, BIC3, IC1, IC2 and IC3 trace back to Bai and Ng (2002). In recent years a number of criteria were proposed to deal with both stationary and non-stationary factors. Here I consider IPC1, IPC2, IPC3 developed by Bai (2004), ED by Onatski (2010) and ER, GR from Ahn et al. (2013).

Table 7: Various Factor Dimensions Column (4)

Factor Dimension	$R = 0$	$R = 1$	$R = 2$	$R = 3$	$R = 4$	$R = 5$	$R = 6$	$R = 7$	$R = 8$
Input skill intensity	1.060	0.866	0.813	0.836	0.858	0.907	0.874	0.824	0.851
	(0.003***)	(0.017***)	(0.017***)	(0.016***)	(0.016***)	(0.016***)	(0.017***)	(0.019***)	(0.019***)
Equipment per worker	-0.383	-0.075	-0.062	0.136	0.134	0.141	0.132	0.122	0.085
	(0.006***)	(0.008***)	(0.011***)	(0.014***)	(0.017***)	(0.018***)	(0.021***)	(0.023***)	(0.023***)
Office equipment	1.300	0.242	0.141	-0.135	-0.125	-0.149	0.156	0.859	0.924
	(0.019***)	(0.022***)	(0.023***)	(0.029***)	(0.029***)	(0.034***)	(0.039)	(0.059)	(0.059***)
High-Tech capital	0.834	0.239	0.035	-0.098	-0.041	-0.016	0.024	0.130	0.134
	(0.023***)	(0.028***)	(0.029)	(0.032**)	(0.031)	(0.033)	(0.031)	(0.038***)	(0.044**)

Table 8: Results from IFE using $R^0 = 3$[a]

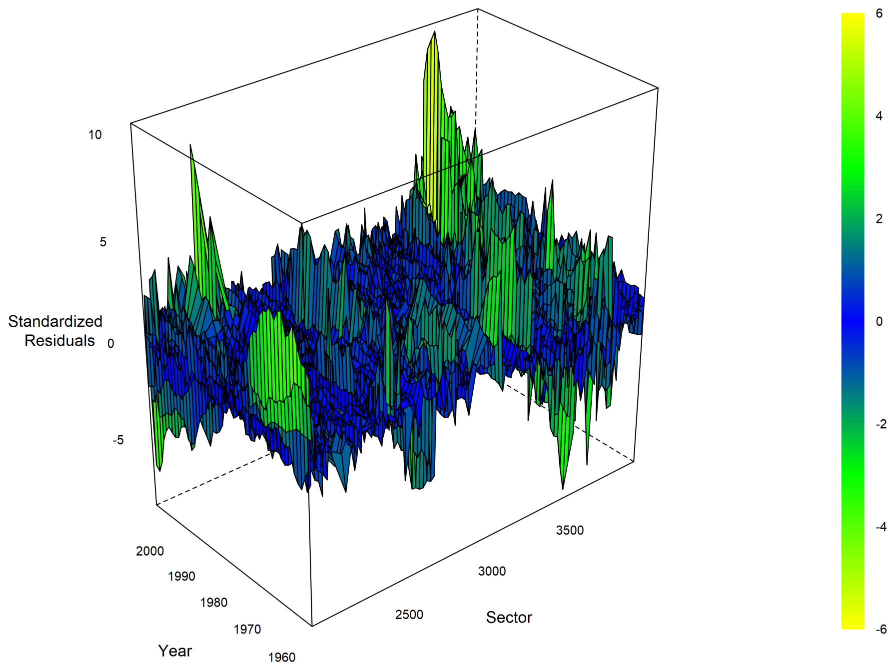
Input skill measure	σ_{it}			σ_{it}^{2d}
	(1) - (3)[b]	(4)	(5)	(6)
Input skill intensity	0.833***	0.836***	0.853***	0.809***
	(0.051)	(0.053)	(0.053)	(0.059)
Equipment per worker		0.136**	0.112**	0.116*
		(0.059)	(0.062)	(0.064)
Office equipment		−0.135	−0.121	−0.197
		(0.101)	(0.096)	(0.103)
High-Tech capital		−0.098	−0.100	−0.121
		(0.088)	(0.092)	(0.094)
R&D intensity			0.095	0.082
			(0.072)	(0.074)
Outsourcing			0.005	−0.010
			(0.030)	(0.032)
Outsourcing (broad)			0.077**	0.059
			(0.038)	(0.037)
Factor dimension	3	3	3	3
Fixed Effects	none	none	none	none
Observations	17,184	17,184	15,792	15,792

[a] Notes: Regression results of final production skill on input skill intensity and various controls using an interactive effects estimator. The observation period consists of annual data, 1958-2005, for 358 U.S. manufacturing sectors. Standard errors are clustered on the sector level and displayed in parentheses. All regressions are weighted by sectors' average share in manufacturing employment. Keys: $*p < 0.10$, $**p < 0.05$, $***p < 0.01$.

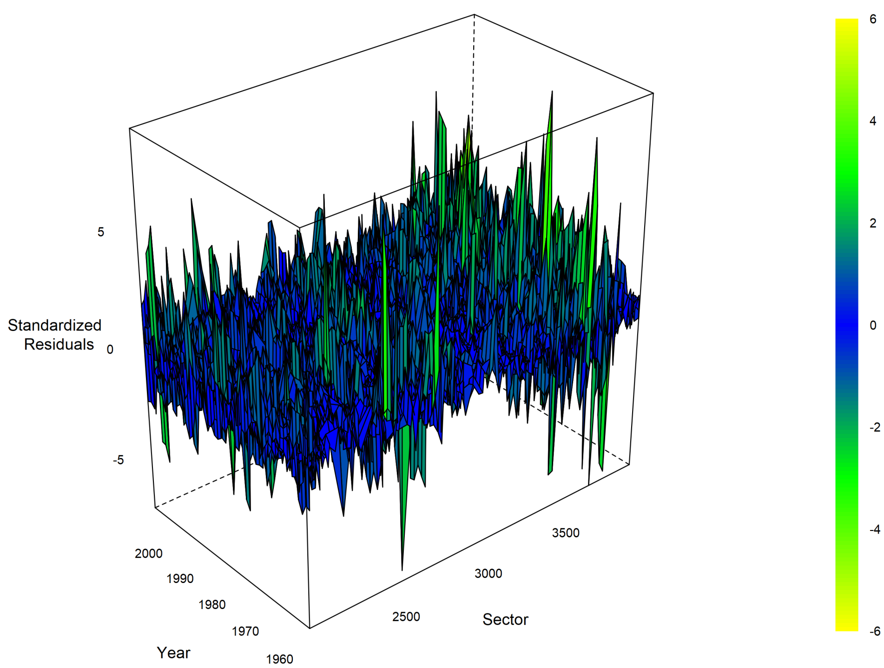
[b] Since I do not explicitly include sector or time fixed effects, specification (1) - (3) of the original estimtes lead to the same model when using an interactive effects framework. However, to ensure comparability across both models I proceed with same enumeration of specifications.

Figures

Figure 1: Residual plots Corresponding to Specification (4) in Voigtländer (2014)



(a) Residuals resulting from using original specification



(b) Residuals resulting from using IFE specification

Figure 2: Screeplots for specification (4) and various factor dimensions R

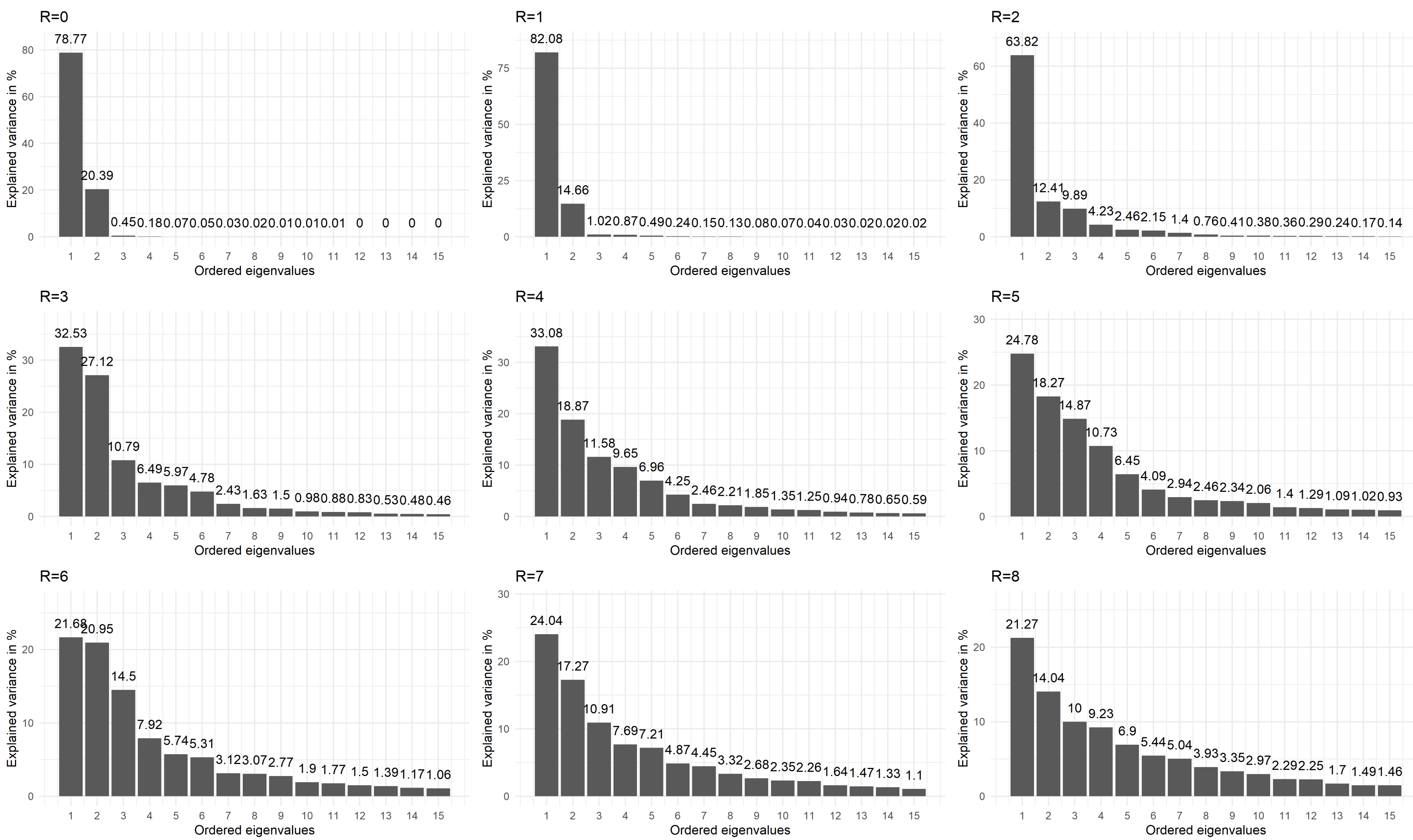
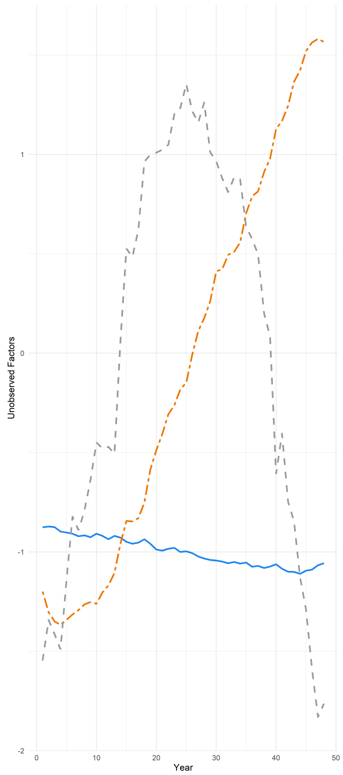
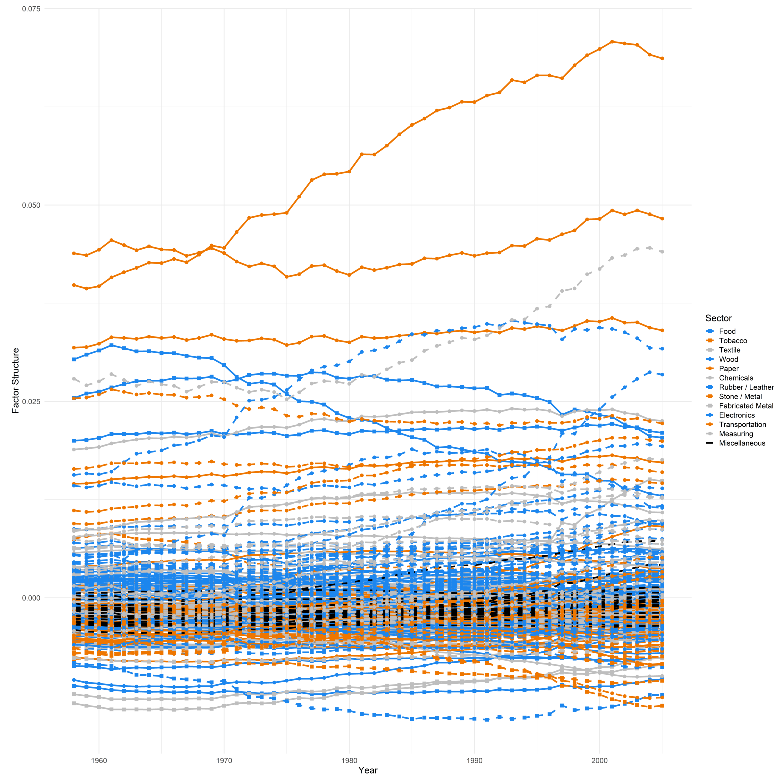


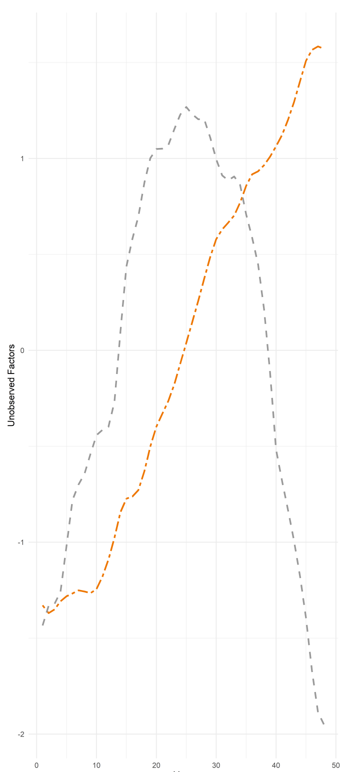
Figure 3: Factor and Factor-Structure Plots (by sector) for $R^* = 3$ and $R^* = 2$



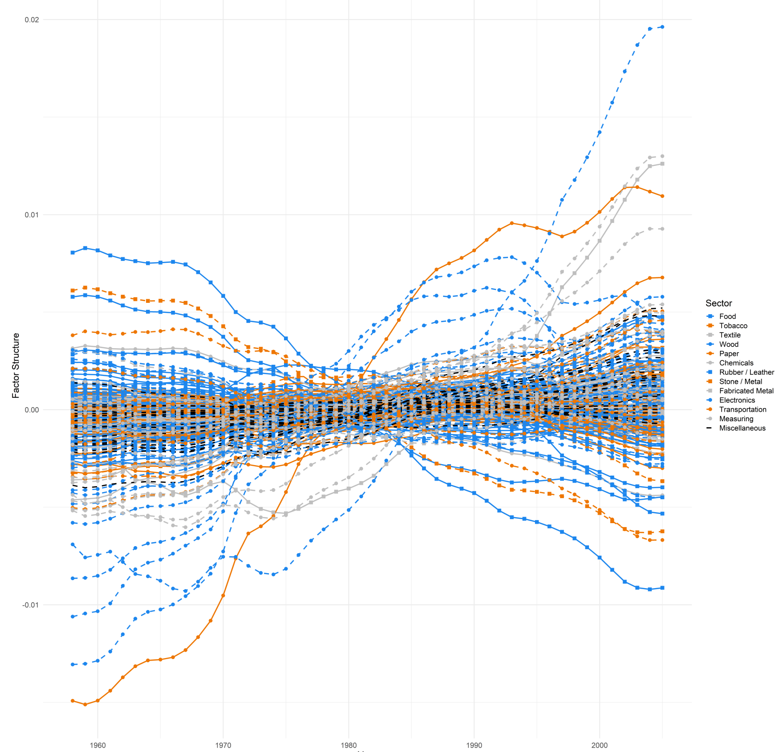
(a) Factors F_t for $R^* = 3$



(b) Factor-Structure $\lambda_i' F_t$ for $R^* = 3$



(c) Factors F_t for $R^* = 2$



(d) Factor Structure $\lambda_i' F_t$ for $R^* = 2$