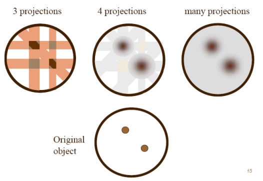
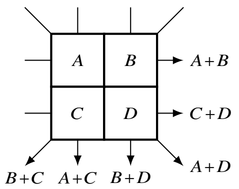
Contents

10 Radon Transform





$$\begin{bmatrix} 1&1&0&0\\ 1&0&1&0\\ 1&0&0&1\\ 0&1&1&0\\ 0&1&0&1\\ 0&0&1&1\\ 1&1&0&0 \end{bmatrix} \begin{bmatrix} A\\B\\C\\D \end{bmatrix} = \begin{bmatrix} A+B\\A+C\\A+D\\B+C\\B+D\\C+D \end{bmatrix}$$

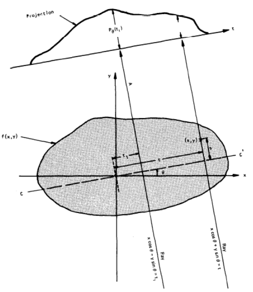
$$\Leftrightarrow K\mathbf{x} = \mathbf{b}$$

$$\rightsquigarrow K^T K\mathbf{x} = K^T\mathbf{b}$$

$$\rightsquigarrow \left[K^T K\right]^{-1} K^T\mathbf{b} = \mathbf{x}$$

Over-determined non-square matrix K. Larger problems must be solved iteratively using standard methods for solving large matrix operation problems.

10.1 Definition



We will use teh coordinate system defined in the figure above to describe line integrals and projections. In this example the object is represented by a two-dimensional function $f(x, y)$ and each line integral by the (θ, t) parameters. The equation of line AB in the figure above is

$$x\cos\theta + y\sin\theta = t$$

and we will use this relationship to define line integral P_θ as

$$P_\theta(t) = \int_{(\theta,t)\text{line}} \mathrm{d}s\, f(x, y)$$

Using a delta function, this can be rewritten as

$$P_\theta(t) = \int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y\, f(x, y) \cdot \delta(x\cos\theta + y\sin\theta - t)$$

10.2 Fourier Slice Theorem

Two-dimensional Fourier transform of the object function

$$F(u, v) = \int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y\, f(x, y) \cdot e^{-j2\pi(ux+vy)}$$

Likewise, define a projection at an angle θ, $P_\theta(t)$, and its fourier transform by

$$S_\theta(w) = \int_{\mathbb{R}^2} \mathrm{d}t\, P_\theta(t) e^{-j2\pi wt}$$

The simplest example of the Fourier Slice Theorem is given for a projection at $\theta = 0$. First, consider the Fourier transform of the object along the line in the frequency domain given by $v = 0$. The Fourier transform integral now simplifies to $F(u, 0)$

$$= \int_{\mathbb{R}^2} \mathrm{d}x\mathrm{d}y\, f(x, y) e^{-j2\pi ux}$$

$$= \int_{-\infty}^{\infty} \mathrm{d}x\, P_{\theta=0} e^{-j2\pi ux}.$$

This represents the 1D FT of the projection $P_{\theta=0}$; thus we have the following relationship between the vertical projection and the 2D transform of the object function:

$$F(u, 0) = S_{\theta=0}(u)$$

This is the simplest form of the Fourier Slice Theorem.

11 Questions

- $$I_{\text{comp}} = I_\alpha I_a + (1 - I_\alpha) I_b$$
- MAP, Maximum a posteriori detector.
- graph cuts
- Solve MRFs with graph cuts
- impulse response $t(-x, -y)$
- Canny nonmaxima suppression
- Entropy Coding (Huffman code)
- Aperture problem: normal flow
- Lucas-Kanade: Iterative refinement/local gradient method
- Coarse-to-fine-estimation
- SNR scalability EI, EP frame
- MPEG Structure

A Big equations

$$\mathcal{F}[h](u, v) = \frac{1}{2\ell}\int_{-\ell}^{\ell} \mathrm{d}x_1\, \exp(-i2\pi u x_1) \cdot \underbrace{\int_{-\infty}^{\infty} \mathrm{d}x_2\, \delta(x_2)\exp(-i2\pi v x_2)}_{=1} = \operatorname{sinc}(2\pi u\ell) \tag{A.1}$$

$$E = \iint \mathrm{d}x\mathrm{d}y \left[\left(\frac{\partial I}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial I}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial I}{\partial t}\right)^2 + \alpha^2(\|\nabla\dot{x}\| + \|\nabla\dot{y}\|)^2\right] \tag{A.2}$$

$$\mathbf{v} = \begin{pmatrix} \sum_i w_i I_x(q_i)^2 & \sum_i w_i I_x(q_i) I_y(q_i) \\ \sum_i w_i I_x(q_i) I_y(q_i) & \sum_i w_i I_y(q_i)^2 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -\sum_i w_i I_x(q_i) I_t(q_i) \\ -\sum_i w_i I_y(q_i) I_t(q_i) \end{pmatrix} \tag{A.3}$$