

To Professor Albert Cohen and Professor Felipe Cucker  
Managing Editors of Foundations of Computational Mathematics



Technische Universität München



Fakultät für Mathematik  
Lehrstuhl für Angewandte  
Numerische Analysis

Object: Submission of the paper “Inferring Interaction Rules from  
Observations of Evolutive Systems I: The Variational Approach”

Dear Professor Cohen and Professor Cucker,

with this letter and on behalf of my coauthors, I wish to submit the paper

“Inferring Interaction Rules from Observations of Evolutive Systems I:  
The Variational Approach”

a joint work with Mattia Bongini, Markus Hansen, and Mauro Maggioni,  
for publication on Foundations of Computational Mathematics.

Let me briefly describe the main content of the paper. We are concerned with the learnability of nonlocal interaction kernels for first order systems modeling certain social interactions, from observations of realizations of their dynamics. This paper is the first of a series on learnability of nonlocal interaction kernels and presents a variational approach to the problem. In particular, we assume here that the kernel to be learned is bounded and locally Lipschitz continuous and that the initial conditions of the systems are drawn identically and independently at random according to a given initial probability distribution. Then the minimization over a rather arbitrary sequence of (finite dimensional) subspaces of a least square functional measuring the discrepancy from observed trajectories produces uniform approximations to the kernel on compact sets. The convergence result is obtained by combining mean-field limits, transport methods, and a Gamma-convergence argument. A crucial condition for the learnability is a certain coercivity property of the least square functional, majoring an energy-norm discrepancy to the kernel with respect to a probability measure, depending on the given initial probability distribution by suitable push forwards and transport maps. We illustrate the convergence result by means of several numerical experiments.

As this paper addresses a very relevant learning problem, combining several tools from approximation theory, dynamical systems, mean-field equations, measure theory, and variational methods, we consider Foundations of Computational Mathematics a very suitable journal.

Sincerely yours

Massimo Fornasier

Prof. Dr.  
**Massimo Fornasier**

Boltzmannstraße 3  
85748 Garching  
Germany

Tel.: +49.89.289.17485  
Fax: +49.89.289.18435

[massimo.fornasier@ma.tum.de](mailto:massimo.fornasier@ma.tum.de)  
[w-m15.ma.tum.de](http://w-m15.ma.tum.de)

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