Team Notebook

January 13, 2020

Contents

1 Geometry

1.1 3D Rotation

```
//From "You Know Izad?" team cheat sheet
Where c = cos (theta), s = sin(theta), t = 1-cos(theta), and
      <X,Y,Z> is the unit vector representing the arbitary
     axis
1. Left handed about arbitrary axis:
 tX^2+c  tXY-sZ  tXZ+sY 0
 tXY+sZ  tY^2+c  tYZ-sX 0
 tXZ-sY  tYZ+sX  tZ^2+c 0
 0  0  0 1

2. Right handed about arbitrary axis:
 tX^2+c  tXY+sZ  tXZ-sY 0
 tXY-sZ  tY^2+c  tYZ+sX 0
 tXZ+sY  tYZ-sX  tZ^2+c 0
 0  0  0 1

3. About X Axis
 1 0 0 0
 0 c -s 0
 0 s c 0
 0 0 0 1


4. About Y Axis
 c 0 s 0
 0 1 0 0
 -s 0 c 0
 0 0 0 1

5. About Z Axis
 c -s 0 0
 s c 0 0
 0 0 1 0
 0 0 0 1
```

1.2 Angle Bisector

```
// angle bisector
int bcenter( PT p1, PT p2, PT p3, PT& r ){
 if( triarea( p1, p2, p3 ) < EPS ) return -1;
 double s1, s2, s3;
 s1 = dist( p2, p3 );
 s2 = dist( p1, p3 );
 s3 = dist( p1, p2 );
 double rt = s2/(s2+s3);
 PT a1,a2;
 a1 = p2*rt+p3*(1.0-rt);
 rt = s1/(s1+s3);
 a2 = p1*rt+p3*(1.0-rt);
 intersection( a1,p1, a2,p2, r );
 return 0;
}
```

1.3 Circle Circle Intersection

```
// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p)  { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
  return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// compute intersection of circle centered at a with radius
     r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r,
     double R) {
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r + R || d + min(r, R) < max(r, R)) return ret;
  double x = (d * d - R * R + r * r) / (2 * d);
  double y = sqrt(r * r - x * x);
  PT v = (b - a) / d;
  ret.push_back(a + v * x + RotateCCW90(v) * y);
  if (y > 0)
    ret.push_back(a + v * x - RotateCCW90(v) * y);
  return ret;
}
```

1.4 Circle Line Intersection

```
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r
     ) {
  vector<PT> ret;
  b = b-a;
  a = a-c;
  double A = dot(b, b);
  double B = dot(a, b);
  double C = dot(a, a) - r*r;
  double D = B*B - A*C;
  if (D < -EPS) return ret;
  ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
  if (D > EPS)
    ret.push_back(c+a+b*(-B-sqrt(D))/A);
  return ret;
}
```

1.5 Circle from Three Points

```
Point center_from(double bx, double by, double cx, double cy
     ) {
  double B=bx*bx+by*by, C=cx*cx+cy*cy, D=bx*cy-by*cx;
  return Point((cy*B-by*C)/(2*D), (bx*C-cx*B)/(2*D));
}

Point circle_from(Point A, Point B, Point C) {
  Point I = center_from(B.X-A.X, B.Y-A.Y, C.X-A.X, C.Y-A.Y);
  return Point(I.X + A.X, I.Y + A.Y);
}
```

1.6 Closest Pair of Points

```
struct point {
  double x, y;
  int id;
  point() {}
  point (double a, double b) : x(a), y(b) {}
};

double dist(const point &o, const point &p) {
  double a = p.x - o.x, b = p.y - o.y;
  return sqrt(a * a + b * b);
}

double cp(vector<point> &p, vector<point> &x, vector<point>
     &y) {
  if (p.size() < 4) {
    double best = 1e100;
    for (int i = 0; i < p.size(); ++i)
      for (int j = i + 1; j < p.size(); ++j)
        best = min(best, dist(p[i], p[j]));
    return best;
  }

  int ls = (p.size() + 1) >> 1;
  double l = (p[ls - 1].x + p[ls].x) * 0.5;
```

```
  vector<point> xl(ls), xr(p.size() - ls);
  unordered_set<int> left;
  for (int i = 0; i < ls; ++i) {
    xl[i] = x[i];
    left.insert(x[i].id);
  }
  for (int i = ls; i < p.size(); ++i) {
    xr[i - ls] = x[i];
  }

  vector<point> yl, yr;
  vector<point> pl, pr;
  yl.reserve(ls); yr.reserve(p.size() - ls);
  pl.reserve(ls); pr.reserve(p.size() - ls);
  for (int i = 0; i < p.size(); ++i) {
    if (left.count(y[i].id))
      yl.push_back(y[i]);
    else
      yr.push_back(y[i]);

    if (left.count(p[i].id))
      pl.push_back(p[i]);
    else
      pr.push_back(p[i]);
  }

  double dl = cp(pl, xl, yl);
  double dr = cp(pr, xr, yr);
  double d = min(dl, dr);
  vector<point> yp; yp.reserve(p.size());
  for (int i = 0; i < p.size(); ++i) {
    if (fabs(y[i].x - l) < d)
      yp.push_back(y[i]);
  }
  for (int i = 0; i < yp.size(); ++i) {
    for (int j = i + 1; j < yp.size() && j < i + 7; ++j) {
      d = min(d, dist(yp[i], yp[j]));
    }
  }
  return d;
}

double closest_pair(vector<point> &p) {
  vector<point> x(p.begin(), p.end());
  sort(x.begin(), x.end(), [](const point &a, const point &b
      ) {
    return a.x < b.x;
  });
  vector<point> y(p.begin(), p.end());
  sort(y.begin(), y.end(), [](const point &a, const point &b
      ) {
    return a.y < b.y;
  });
  return cp(p, x, y);
}
```

1.7 Closest Point on Line

```
//From In 1010101 We Trust cheatsheet:
//the closest point on the line p1->p2 to p3
void closestpt( PT p1, PT p2, PT p3, PT &r ){
 if(fabs(triarea(p1, p2, p3)) < EPS){ r = p3; return; }
 PT v = p2-p1; v.normalize();
 double pr; // inner product
 pr = (p3.y-p1.y)*v.y + (p3.x-p1.x)*v.x;
 r = p1+v*pr;
}
```

1.8 Delaunay Triangulation

```
// Slow but simple Delaunay triangulation. Does not handle
// degenerate cases (from O'Rourke, Computational Geometry
     in C)
//
// Running time: O(n^4)
//
// INPUT:   x[] = x-coordinates
//          y[] = y-coordinates
//
// OUTPUT:  triples = a vector containing m triples of
     indices
//                    corresponding to triangle vertices

typedef double T;

struct triple {
    int i, j, k;
    triple() {}
    triple(int i, int j, int k) : i(i), j(j), k(k) {}
};

vector<triple> delaunayTriangulation(vector<T>& x, vector<T
     >& y) {
 int n = x.size();
 vector<T> z(n);
 vector<triple> ret;

 for (int i = 0; i < n; i++)
     z[i] = x[i] * x[i] + y[i] * y[i];

 for (int i = 0; i < n-2; i++) {
     for (int j = i+1; j < n; j++) {
  for (int k = i+1; k < n; k++) {
      if (j == k) continue;
      double xn = (y[j]-y[i])*(z[k]-z[i]) - (y[k]-y[i])*(z[j
          ]-z[i]);
      double yn = (x[k]-x[i])*(z[j]-z[i]) - (x[j]-x[i])*(z[k
          ]-z[i]);
      double zn = (x[j]-x[i])*(y[k]-y[i]) - (x[k]-x[i])*(y[j
          ]-y[i]);
      bool flag = zn < 0;
      for (int m = 0; flag && m < n; m++)
   flag = flag && ((x[m]-x[i])*xn +
     (y[m]-y[i])*yn +
     (z[m]-z[i])*zn <= 0);
      if (flag) ret.push_back(triple(i, j, k));
  }
     }
 }
 return ret;
}

int main()
{
    T xs[]={0, 0, 1, 0.9};
    T ys[]={0, 1, 0, 0.9};
    vector<T> x(&xs[0], &xs[4]), y(&ys[0], &ys[4]);
    vector<triple> tri = delaunayTriangulation(x, y);

    //expected: 0 1 3
    //          0 3 2

    int i;
    for(i = 0; i < tri.size(); i++)
        printf("%d %d %d\n", tri[i].i, tri[i].j, tri[i].k);
    return 0;
}
```

1.9 Latitude and Longitude

```
/*
Converts from rectangular coordinates to latitude/longitude
     and vice
versa. Uses degrees (not radians).
*/
```

```
using namespace std;

struct ll
{
  double r, lat, lon;
};

struct rect
{
  double x, y, z;
};

ll convert(rect& P)
{
  ll Q;
  Q.r = sqrt(P.x*P.x+P.y*P.y+P.z*P.z);
  Q.lat = 180/M_PI*asin(P.z/Q.r);
  Q.lon = 180/M_PI*acos(P.x/sqrt(P.x*P.x+P.y*P.y));

  return Q;
}

rect convert(ll& Q)
{
  rect P;
  P.x = Q.r*cos(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.y = Q.r*sin(Q.lon*M_PI/180)*cos(Q.lat*M_PI/180);
  P.z = Q.r*sin(Q.lat*M_PI/180);

  return P;
}

int main()
{
  rect A;
  ll B;

  A.x = -1.0; A.y = 2.0; A.z = -3.0;

  B = convert(A);
  cout << B.r << " " << B.lat << " " << B.lon << endl;

  A = convert(B);
  cout << A.x << " " << A.y << " " << A.z << endl;
}
```

1.10 Line Intersection

```
// Ax + By = C
A = y2 - y1
B = x1 - x2
C = A*x1 + B*y1
double det = A1*B2 - A2*B1
double x = (B2*C1 - B1*C2)/det
double y = (A1*C2 - A2*C1)/det

typedef pair<double, double> pointd;
#define X first
#define Y second
bool eqf(double a, double b) {
    return fabs(b - a) < 1e-6;
}
int crossVecs(pointd a, pointd b) {
    return a.X * b.Y - a.Y*b.X;
}
int cross(pointd o, pointd a, pointd b){
    return crossVecs(make_pair(a.X - o.X, a.Y - o.Y),
         make_pair(b.X - o.X, b.Y - o.Y));
}
int dotVecs(pointd a, pointd b) {
    return a.X * b.X + a.Y * b.Y;
}
int dot(pointd o, pointd a, pointd b) {
    return dotVecs(make_pair(a.X - o.X, a.Y - o.Y), make_pair
         (b.X - o.X, b.Y - o.Y));
}
bool onTheLine(const pointd& a, const pointd& p, const
     pointd& b) {
    return eqf(cross(p, a, b), 0) && dot(p, a, b) < 0 ;
}
class LineSegment {
    public:
    double A, B, C;
    pointd from, to;
    LineSegment(const pointd& a, const pointd& b) {
        A = b.Y - a.Y;
        B = a.X - b.X;
        C = A*a.X + B*a.Y;
        from = a;
        to = b;
    }

    bool between(double l, double a, double r) const {
        if(l > r) {
            swap(l, r);
        }
        return l <= a && a <= r;
    }

    bool pointOnSegment(const pointd& p) const {
        return eqf(A*p.X + B*p.Y, C) && between(from.X, p.X,
             to.X) && between(from.Y, p.Y, to.Y);
    }

    pair<bool, pointd> segmentsIntersect(const LineSegment& l
         ) const {
        double det = A * l.B - B * l.A;
        pair<bool, pointd> ret;
        ret.first = false;
        if(det != 0) {
            pointd inter((l.B*C - B*l.C)/det, (A*l.C - l.A*C)
                 /det);
            if(l.pointOnSegment(inter) && pointOnSegment(
                 inter)) {
                ret.first = true;
                ret.second = inter;
            }
        }
        return ret;
    }
};
```

1.11 Point in Polygon

```
// determine if point is in a possibly non-convex polygon (
     by William
// Randolph Franklin); returns 1 for strictly interior
     points, 0 for
// strictly exterior points, and 0 or 1 for the remaining
     points.
// Note that it is possible to convert this into an *exact*
     test using
// integer arithmetic by taking care of the division
     appropriately
// (making sure to deal with signs properly) and then by
     writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
  bool c = 0;
  for (int i = 0; i < p.size(); i++){
    int j = (i+1)%p.size();
    if ((p[i].y <= q.y && q.y < p[j].y ||
      p[j].y <= q.y && q.y < p[i].y) &&
      q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[
           j].y - p[i].y))
      c = !c;
  }
```

```
  return c;
}
```

1.12 Polygon Centroid

```
// This code computes the area or centroid of a (possibly
     nonconvex)
// polygon, assuming that the coordinates are listed in a
     clockwise or
// counterclockwise fashion. Note that the centroid is often
      known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
  double area = 0;
  for(int i = 0; i < p.size(); i++) {
    int j = (i+1) % p.size();
    area += p[i].x*p[j].y - p[j].x*p[i].y;
  }
  return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
  return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
  PT c(0,0);
  double scale = 6.0 * ComputeSignedArea(p);
  for (int i = 0; i < p.size(); i++){
    int j = (i+1) % p.size();
    c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
  }
  return c / scale;
}
```

1.13 Rotation Around Origin by t

```
 x   = x.Cos(t) - y.Sin(t)
 y   = x.Sin(t) + y.Cos(t)
```

1.14 Two Point and Radius Circle

```
vector<point> find_center(point a, point b, long double r) {
  point d = (a - b) * 0.5;
  if (d.dot(d) > r * r) {
    return vector<point> ();
  }
  point e = b + d;
  long double fac = sqrt(r * r - d.dot(d));
  vector<point> ans;
  point x = point(-d.y, d.x);
  long double l = sqrt(x.dot(x));
  x = x * (fac / l);
  ans.push_back(e + x);
  x =  point(d.y, -d.x);
  x = x * (fac / l);
  ans.push_back(e + x);
  return ans;
}
```

2 Graph

2.1 2-SAT

```
//From "You Know Izad?" team cheat sheet
//fill the v array
//e.g. to push (p v !q) use the following code:
//  v[VAR(p)].push_back( NOT( VAR(q) ) )
//  v[NOT( VAR(q) )].push_back( VAR(p) )
//the result will be in color array
#define VAR(X) (X << 1)
#define NOT(X) (X ^ 1)
#define CVAR(X,Y) (VAR(X) | (Y))z
#define COL(X) (X & 1)
#define NVAR 400
int n;
vector<int> v[2 * NVAR];
int color[2 * NVAR];
int bc[2 * NVAR];
bool dfs( int a, int col ) {
    color[a] = col;
    int num = CVAR( a, col );
    for( int i = 0; i < v[num].size(); i++ ) {
        int adj = v[num][i] >> 1;
        int ncol = NOT( COL( v[num][i] ) );
        if( ( color[adj] == -1 && !dfs( adj, ncol ) ) ||
            ( color[adj] != -1 && color[adj] != ncol ) ) {
            color[a] = -1;
            return false;
        }
    }
    return true;
}
bool twosat() {
    memset( color, -1, sizeof color );
    for( int i = 0; i < n; i++ ){
        if( color[i] == -1 ){
            memcpy(bc, color, sizeof color);
            if( !dfs( i, 0 )){
                memcpy(color, bc, sizeof color);
                if(!dfs( i, 1 ))
                    return false;
            }
        }
    }
    return true;
}
```

2.2 Bridge and Articulate Point Finding

```
typedef struct {
  int deg;
  int adj[MAX_N];
} Node;

Node alist[MAX_N];
bool art[MAX_N];
int df_num[MAX_N], low[MAX_N], father[MAX_N], count;
int bridge[MAX_N*MAX_N][2], bridges;

void add_bridge(int v1, int v2) {
  bridge[bridges][0] = v1;
  bridge[bridges][1] = v2;
  ++bridges;
}

void search(int v, bool root) {
  int w, child = 0;

  low[v] = df_num[v] = count++;

  for (int i = 0; i < alist[v].deg; ++i) {
    w = alist[v].adj[i];

    if (df_num[w] == -1) {
      father[w] = v;
      ++child;
      search(w, false);
      if (low[w] > df_num[v]) add_bridge(v, w);
      if (low[w] >= df_num[v] && !root) art[v] = true;
      low[v] = min(low[v], low[w]);
    }
```

```
    else if (w != father[v]) {
      low[v] = min(low[v], df_num[w]);
    }
  }

  if (root && child > 1) art[v] = true;
}

void articulate(int n) {
  int child = 0;

  for (int i = 0; i < n; ++i) {
    art[i] = false;
    df_num[i] = -1;
    father[i] = -1;
  }

  count = bridges = 0;

  search(0, true);
}
```

2.3 Eulerian Path

```
// Taken from https://github.com/lbv/pc-code/blob/master/
     code/graph.cpp
// Eulerian Trail

struct Euler {
  ELV adj; IV t;
  Euler(ELV Adj) : adj(Adj) {}
  void build(int u) {
    while(! adj[u].empty()) {
      int v = adj[u].front().v;
      adj[u].erase(adj[u].begin());
      build(v);
    }
    t.push_back(u);
  }
};
bool eulerian_trail(IV &trail) {
  Euler e(adj);
  int odd = 0, s = 0;
  /*
     for (int v = 0; v < n; v++) {
     int diff = abs(in[v] - out[v]);
     if (diff > 1) return false;
     if (diff == 1) {
     if (++odd > 2) return false;
     if (out[v] > in[v]) start = v;
     }
     }
     */
  e.build(s);
  reverse(e.t.begin(), e.t.end());
  trail = e.t;
  return true;
}
```

2.4 Euler Tour

```
// DirectedEulerTour0 ( E )
void visit (Graph& g, int a , vector<int>& path) {
 while (!g[a].empty()){
  int b = g[a].back().dst;
  g[a].pop_back();
  visit (g, b, path);
 }
 path.push_back (a);
}
bool eulerPath (Graph g, int s , vector<int> &path) {
 int n = g.size(), m = 0;
 vector<int> deg (n);
 REP (u , n) {
  m += g[u].size();
  FOR (e , g[u]) --deg[e->dst]; // in-deg
  deg[u] += g[u].size();   // out-deg
}
int k = n - count (ALL (deg), 0);
if (k == 0 || (k == 2 && deg[s] == 1)) {
 path.clear();
 visit (g, s , path);
 reverse (ALL (path));
 return path.size () == m + 1;
}
return false;
}
// UndirectedEulerTour0 ( E )
void visit(const Graph &g, vector< vector<int> > &adj, int s
     , vector<int> &path) {
 FOR (e , g[s])
 if (adj[e->src][e->dst]) {
  --adj[e->src][e->dst];
  --adj[e->dst][e->src];
  visit(g, adj, e->dst , path);
 }
 path.push_back(s);
}
```

```
bool eulerPath (const Graph &g, int s , vector<int> &path)
{
 int n = g.size();
 int odd = 0, m = 0;
 REP (i, n) {
  if (g[i].size() % 2 == 1)
   ++odd;
  m += g[i].size();
 }
 m/= 2;
 if (odd == 0 || (odd == 2 && g[s].size() % 2 == 0))
 {
  vector< vector<int> > adj (n , vector<int> (n));

  REP (u , n) FOR (e , g[u]) ++adj[e->src][e->dst];
  path.clear ();
  visit (g, adj, s, path);
  reverse (ALL (path));
  return path.size() == m + 1;
 }
 return false;
}
```

2.5 LCA

```
void dfsLCA(int u , int p){
    tin[u] = ++timer;
    up[u][0] = p;
    fore(i,1,l){
        up[u][i] = up[up[u][i-1]][i-1];
    }
    for(int v : tree[u]){
        if (v == p)
            continue;
        dfsLCA(v,u);
    }
    tout[u]=++timer;
}
bool isAnsector(int u , int v)
{
    return tin[u] <= tin[v] && tout[u] >= tout[v];
}

int lca(int u , int v){
    if (isAnsector(u , v))
        return u;
    if (isAnsector(v , u))
        return v;
    forn(i , l , 0){
```

```
        if (!isAnsector(up[u][i] , v))
            u = up[u][i];
    }
    return up[u][0];
}

void findLca(){
    memset(visited , false ,sizeof visited);
    memset(lev , 0 , sizeof lev);
    dfs(1 , 0);
    memset(tin , 0 , sizeof tin);
    memset(tout , 0 ,sizeof tout);
    timer = 0;
    l = ceil(log2(n));
    memset(up , 0 ,sizeof up);
    dfsLCA(1,1);
}
```

2.6 assignment

```
int assignment() {
    int n = a.size();
    int m = n * 2 + 2;
    vector<vector<int>> f(m, vector<int>(m));
    int s = m - 2, t = m - 1;
    int cost = 0;
    while (true) {
        vector<int> dist(m, INF);
        vector<int> p(m);
        vector<int> type(m, 2);
        deque<int> q;
        dist[s] = 0;
        p[s] = -1;
        type[s] = 1;
        q.push_back(s);
        while (!q.empty()) {
            int v = q.front();
            q.pop_front();
            type[v] = 0;
            if (v == s) {
                for (int i = 0; i < n; ++i) {
                    if (f[s][i] == 0) {
                        dist[i] = 0;
                        p[i] = s;
                        type[i] = 1;
                        q.push_back(i);
                    }
                }
            } else {
                if (v < n) {
                    for (int j = n; j < n + n; ++j) {
                        if (f[v][j] < 1 && dist[j] > dist[v] +
                             a[v][j - n]) {
                            dist[j] = dist[v] + a[v][j - n];
                            p[j] = v;
                            if (type[j] == 0)
                                q.push_front(j);
                            else if (type[j] == 2)
                                q.push_back(j);
                            type[j] = 1;
                        }
                    }
                } else {
                    for (int j = 0; j < n; ++j) {
                        if (f[v][j] < 0 && dist[j] > dist[v] -
                             a[j][v - n]) {
                            dist[j] = dist[v] - a[j][v - n];
                            p[j] = v;
                            if (type[j] == 0)
                                q.push_front(j);
                            else if (type[j] == 2)
                                q.push_back(j);
                            type[j] = 1;
                        }
                    }
                }
            }
        }

        int curcost = INF;
        for (int i = n; i < n + n; ++i) {
            if (f[i][t] == 0 && dist[i] < curcost) {
                curcost = dist[i];
                p[t] = i;
            }
        }
        if (curcost == INF)
            break;
        cost += curcost;
        for (int cur = t; cur != -1; cur = p[cur]) {
            int prev = p[cur];
            if (prev != -1)
                f[cur][prev] = -(f[prev][cur] = 1);
        }
    }

    // vector<int> answer(n);
    int answer = 0;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            if (f[i][j + n] == 1)
                answer+=a[i][j];
        }
    }
    return answer;
}
```

2.7 bipartie$_m$cmf

```
vector<edge> g[maxn];
int h[maxn], dst[maxn], prevv[maxn ], preve[maxn];
inline void add_edge(int f, int t, int cap, int cost)
{
 g[f].emplace_back(t, cap, cost, g[t].size());
 g[t].emplace_back(f, 0, -cost, g[f].size() - 1);
}

int mcmf(int s, int t , int maxFlow)
{
 int res = 0;
 int c = INT_MAX;
 memset(h, 0, sizeof(h));
  int f = 0;
 while (f < maxFlow) {
  priority_queue<ii, vector<ii>, greater<ii> > que;
  fill(dst, dst + n , inf);
  dst[s] = 0;
  que.push(mp(0, s));
  while (!que.empty()) {
   ii p = que.top(); que.pop();
   int v = p.second;
   if (dst[v] < p.first) continue;
   fore(i , 0 , g[v].size() - 1) {
    edge &e = g[v][i];
    int nd = dst[v] + e.cost + h[v] - h[e.to];
    if (e.cap > 0 && dst[e.to] > nd){
     dst[e.to] = nd;
     prevv[e.to] = v;
     preve[e.to] = i;
     que.push(mp(dst[e.to], e.to));
    }
   }
  }

  if (dst[t] == inf) return c;
  fore(i, 0 , n - 1) h[i] += dst[i];
```

```
  int d = inf;
  for(int v = t; v != s; v = prevv[v])
   d = min(d, g[prevv[v]][preve[v]].cap);
  f += d;
  res += d * h[t];
  c = min(c, res);
  if (res >= 0) break;

  for(int v = t; v != s; v = prevv[v]){
   edge &e = g[prevv[v]][preve[v]];
   e.cap -= d;
   g[v][e.rev].cap += d;
  }
 }

 return c;
}
```

2.8 flow

```
struct FlowEdge {
    int v, u;
    long long cap, flow = 0;
    FlowEdge(int v, int u, long long cap) : v(v), u(u), cap(
        cap) {}
};

struct Dinic {
    const long long flow_inf = 1e18;
    vector<FlowEdge> edges;
    vector<vector<int>> adj;
    int n, m = 0;
    int s, t;
    vector<int> level, ptr;
    queue<int> q;

    Dinic(int n, int s, int t) : n(n), s(s), t(t) {
        adj.resize(n);
        level.resize(n);
        ptr.resize(n);
    }

    void add_edge(int v, int u, long long cap) {
        // TRACE(v _ u _ cap);
        edges.emplace_back(v, u, cap);
        edges.emplace_back(u, v, 0);
        adj[v].push_back(m);
        adj[u].push_back(m + 1);
        m += 2;
    }

    bool bfs() {
        while (!q.empty()) {
            int v = q.front();
            q.pop();
            for (int id : adj[v]) {
                if (edges[id].cap - edges[id].flow < 1)
                    continue;
                if (level[edges[id].u] != -1)
                    continue;
                level[edges[id].u] = level[v] + 1;
                q.push(edges[id].u);
            }
        }
        return level[t] != -1;
    }

    long long dfs(int v, long long pushed) {
        if (pushed == 0)
            return 0;
        if (v == t)
            return pushed;
        for (int& cid = ptr[v]; cid < (int)adj[v].size(); cid
             ++) {
            int id = adj[v][cid];
            int u = edges[id].u;
            if (level[v] + 1 != level[u] || edges[id].cap -
                 edges[id].flow < 1)
                continue;
            long long tr = dfs(u, min(pushed, edges[id].cap -
                  edges[id].flow));
            if (tr == 0)
                continue;
            edges[id].flow += tr;
            edges[id ^ 1].flow -= tr;
            return tr;
        }
        return 0;
    }

    long long flow() {
        long long f = 0;
        while (true) {
            fill(level.begin(), level.end(), -1);
            level[s] = 0;
            q.push(s);
            if (!bfs())
                break;
            fill(ptr.begin(), ptr.end(), 0);
            while (long long pushed = dfs(s, flow_inf)) {
                f += pushed;
            }
        }
        return f;
    }
};
```

3 Math

3.1 Binary Gaussian Elimination

```
//Amin Anvari's solution to Shortest XOR Path problem
#include <bits/stdc++.h>
using namespace std;
typedef pair <int,int> pii;
#define L first
#define R second
const int maxn = 1e5, maxl = 31;
bool mark[maxn];
vector <pii> adj[maxn];
vector <int> all;
int n, s, w[maxn], pat[maxn], b[maxn];
void dfs(int v,int par = -1) {
    mark[v] = true;
    for (int i = 0; i < adj[v].size(); i++) {
        int u = adj[v][i].L, e = adj[v][i].R, W = w[e];
        if (!mark[u]) {
            pat[u] = pat[v] ^ W;
            dfs(u, e);
        }
        else if (e != par)
            all.push_back(pat[v] ^ pat[u] ^ W);
    }
}
int get(int x) {
    for (int i = maxl - 1; i >= 0; i--)
        if (x & (1 << i))
            return i;
    return -1;
}
void add(int x) {
    for (int i = 0; i < s; i++)
        if (get(b[i]) != -1 && (x & (1 << get(b[i]))))
            x ^= b[i];
    if (x == 0)
        return;
    for (int i = 0; i < s; i++)
```

```
        if (b[i] < x)
            swap(x, b[i]);
    b[s++] = x;
}
int GET(int x) {
    for (int i = 0; i < s; i++)
        if (get(b[i]) != -1 && (x & (1 << get(b[i]))))
            x ^= b[i];
    return x;
}
int main() {
    ios_base::sync_with_stdio(false);
    int m;
    cin >> n >> m;
    for (int i = 0; i < m; i++) {
        int v, u;
        cin >> v >> u >> w[i];
        v--, u--;
        adj[v].push_back(pii(u, i));
        adj[u].push_back(pii(v, i));
    }
    dfs(0);
    for (int i = 0; i < all.size(); i++)
        add(all[i]);
    cout << GET(pat[n - 1]) << endl;
    return 0;
}
```

3.2 Discrete Logarithm Solver

```
// discrete-logarithm, finding y for equation k = x^y % mod
int discrete_logarithm(int x, int mod, int k) {
  if (mod == 1) return 0;
  int s = 1, g;
  for (int i = 0; i < 64; ++i) {
    if (s == k) return i;
    s = (1ll * s * x) % mod;
  }
  while ((g = gcd(x, mod)) != 1) {
    if (k % g) return -1;
    mod /= g;
  }
  static unordered_map<int, int> M; M.clear();
  int q = int(sqrt(double(euler(mod)))) + 1; // mod-1 is
      also okay
  for (int i = 0, b = 1; i < q; ++i) {
    if (M.find(b) == M.end()) M[b] = i;
    b = (1ll * b * x) % mod;
  }
  int p = fpow(x, q, mod);
  for (int i = 0, b = 1; i <= q; ++i) {
    int v = (1ll * k * inverse(b, mod)) % mod;
    if (M.find(v) != M.end()) {
      int y = i * q + M[v];
      if (y >= 64) return y;
    }
    b = (1ll * b * p) % mod;
  }
  return -1;
}
```

3.3 Euler Totient Function

```
/* Returns the number of positive integers that are
 * relatively prime to n. As efficient as factor().
 * REQUIRES: factor()
 * REQUIRES: sqrt() must work on Int.
 * REQUIRES: the constructor Int::Int( double ).
 **/
int phi( int n ) {
 vector< int > p;
 factor( n, p );
 for( int i = 0; i < ( int )p.size(); i++ ) {
  if( i && p[i] == p[i - 1] ) continue;
  n /= p[i];
  n *= p[i] - 1;
 }
 return n;
}
```

3.4 Extended GCD

```
template< class Int >
struct Triple
{
 Int d, x, y;
 Triple( Int q, Int w, Int e ) : d( q ), x( w ), y( e ) {}
};

/* Given nonnegative a and b, computes d = gcd( a, b )
 * along with integers x and y, such that d = ax + by
 * and returns the triple (d, x, y).
 * WARNING: needs a small modification to work on
 * negative integers (operator% fails).
 **/
template< class Int >
Triple< Int > egcd( Int a, Int b )
{
 if( !b ) return Triple< Int >( a, Int( 1 ), Int( 0 ) );
 Triple< Int > q = egcd( b, a % b );
 return Triple< Int >( q.d, q.y, q.x - a / b * q.y );
}
```

3.5 Fibonacci Numbers Properties

Let A, B and n be integer numbers.

$$k = A - B \tag{1}$$

$$F_A F_B = F_{k+1} F_A^2 + F_k F_A F_{A-1} \tag{2}$$

$$\sum_{i=0}^{n} F_i^2 = F_{n+1} F_n \tag{3}$$

$ev(n) =$ returns 1 if n is even.

$$\sum_{i=0}^{n} F_i F_{i+1} = F_{n+1}^2 - ev(n) \tag{4}$$

$$\sum_{i=0}^{n} F_i F_{i-1} = \sum_{i=0}^{n-1} F_i F_{i+1} \tag{5}$$

3.6 Linear Diophantine Equation Solver

```
/* Solves integer equations of the form ax + by = c
 * for integers x and y. Returns a triple containing
 * the answer (in .x and .y) and a flag (in .d).
 * If the returned flag is zero, then there are no
 * solutions. Otherwise, there is an infinite number
 * of solutions of the form
 * x = t.x + k * b / t.d,
 * y = t.y - k * a / t.d;
 * where t is the returned triple, and k is any
 * integer.
 * REQUIRES: struct Triple, egcd
 **/
template< class Int >
Triple< Int > ldioph( Int a, Int b, Int c ) {
```

```
 Triple< Int > t = egcd( a, b );
 if( c % t.d ) return Triple< Int >( 0, 0, 0 );
 t.x *= c / t.d; t.y *= c / t.d;
 return t;
}
```

3.7 Maximum XOR (SGU 275)

```
int n;
long long x, ans;
vector<long long> st;
int main() {
 cin >> n;
 for (int i = 0; i < n; i++) {
  cin >> x;
  st.push_back(x);
 }
 for (int k = 0; k < n; k++)
  for (int i = 0; i < st.size(); i++)
   for (int j = i + 1; j < st.size(); j++)
    if (__builtin_clzll(st[j]) == __builtin_clzll(st[i]))
     st[j] ^= st[i];
 sort(st.begin(), st.end());
 reverse(st.begin(), st.end());
 for (auto e: st)
  ans = max(ans, ans ^ e);
 cout << ans << endl;
 return 0;
}
```

3.8 Modular Linear Equation Solver

```
/* Given a, b and n, solves the equation ax = b (mod n)
 * for x. Returns the vector of solutions, all smaller
 * than n and sorted in increasing order. The vector is
 * empty if there are no solutions.
 * REQUIRES: struct Triple, egcd
 **/
template< class Int >
vector< Int > msolve( Int a, Int b, Int n ) {
 if( n < 0 ) n = -n;
 Triple< Int > t = egcd( a, n );
 vector< Int > r;
 if( b % t.d ) return r;
 Int x = ( b / t.d * t.x ) % n;
 if( x < Int( 0 ) ) x += n;
 for( Int i = 0; i < t.d; i++ )
 r.push_back( ( x + i * n / t.d ) % n );
 return r;
}
```

3.9 Number of Divisors

```
/* Returns the number of positive divisors of n.
 * Complexity: about O(sqrt(n)).
 * REQUIRES: factor()
 * REQUIRES: sqrt() must work on Int.
 * REQUIRES: the constructor Int::Int( double ).
 **/
template< class Int >
Int divisors( Int n ) {
 vector< Int > f;
 factor( n, f );
 int k = f.size();
 vector< Int > table( k + 1, Int( 0 ) );
 table[k] = Int( 1 );

 for( int i = k - 1; i >= 0; i-- ) {
  table[i] = table[i + 1];
  for( int j = i + 1; ; j++ )
   if( j == k || f[j] != f[i] )
   { table[i] += table[j]; break; }
 }

 return table[0];
}
```

3.10 Prime Factors in n Factorial

```
using namespace std;
typedef long long ll;
typedef pair<ll ,int> pii;
vector <pii> v;
////////////// bozorgtarin i b shekli k N!%k^i==0
void fact(ll n) {
 ll x = 2;
 while (x * x <= n)
 {
  ll num = 0;
  while (n % x == 0) {
   num++;
   n /= x;
  }
  if (num) v.push_back(MP(x, num));
  x++;
  if (n == 1ll) break;
 }
 if(n > 1) v.push_back(MP(n, 1));
}

ll getfact(ll n) {
 ll ret = n;
 Rep(i, v.size()) {
  ll k = v[i].first;
  ll cnt = 0;
  ll t = n;
  while (k <= n) {
  cnt += n / k;
  n /= k;
  }
  n = t;
  ret = min(ret, cnt / v[i].second);
 }
 return ret;
}

int main() {
 int tc;
 ll n, k;
 cin >> tc;
 while (tc--) {
  v.clear();
  cin >> n >> k;
  fact(k);
  cout << getfact(n) << endl;
 }
 return 0;
}
```

3.11 Reduced Row Echelon Form

```
// Reduced row echelon form via Gauss-Jordan elimination
// with partial pivoting. This can be used for computing
// the rank of a matrix.
//
// Running time: O(n^3)
//
// INPUT:   a[][] = an nxm matrix
//
// OUTPUT:  rref[][] = an nxm matrix (stored in a[][])
//          returns rank of a[][]

#include <iostream>
```

```cpp
#include <vector>
#include <cmath>

using namespace std;

const double EPSILON = 1e-10;

typedef double T;
typedef vector<T> VT;
typedef vector<VT> VVT;

int rref(VVT &a) {
  int n = a.size();
  int m = a[0].size();
  int r = 0;
  for (int c = 0; c < m && r < n; c++) {
    int j = r;
    for (int i = r + 1; i < n; i++)
      if (fabs(a[i][c]) > fabs(a[j][c])) j = i;
    if (fabs(a[j][c]) < EPSILON) continue;
    swap(a[j], a[r]);

    T s = 1.0 / a[r][c];
    for (int j = 0; j < m; j++) a[r][j] *= s;
    for (int i = 0; i < n; i++) if (i != r) {
      T t = a[i][c];
      for (int j = 0; j < m; j++) a[i][j] -= t * a[r][j];
    }
    r++;
  }
  return r;
}

int main() {
  const int n = 5, m = 4;
  double A[n][m] = {
    {16,  2,  3, 13},
    { 5, 11, 10, 8},
    { 9,  7,  6, 12},
    { 4, 14, 15, 1},
    {13, 21, 21, 13}};
  VVT a(n);
  for (int i = 0; i < n; i++)
    a[i] = VT(A[i], A[i] + m);

  int rank = rref(a);

  // expected: 3
  cout << "Rank: " << rank << endl;
  // expected: 1 0 0 1
  //           0 1 0 3
  //           0 0 1 -3
  //           0 0 0 3.10862e-15
  //           0 0 0 2.22045e-15
  cout << "rref: " << endl;
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 4; j++)
      cout << a[i][j] << ' ';
    cout << endl;
  }
}
```

3.12 Solving Recursive Functions

```cpp
//From "You Know Izad?" team cheat sheet
/*
a[i] = b[i] (for i <= k)
a[i] = c[1]*a[i-1] + c[2]a[i-2] + ... + c[k]a[i-k] (for i >
    k)
Given:
b[1], b[2], ..., b[k]
c[1], c[2], ..., c[k]
a[N]=?
*/
typedef vector<vector<ll> > matrix;
int K;
matrix mul(matrix A, matrix B){
    matrix C(K+1, vector<ll>(K+1));
    REP(i, K) REP(j, K) REP(k, K)
        C[i][j] = (C[i][j] + A[i][k] * B[k][j]) % INF32;
    return C;
}
matrix pow(matrix A, ll p){
    if (p == 1) return A;
    if (p % 2) return mul(A, pow(A, p-1));
    matrix X = pow(A, p/2);
    return mul(X, X);
}
ll solve() {
    // base (initial) values
    vector<ll> F1(K+1);
    REP (i, K)
        cin >> F1[i];
    matrix T(K+1, vector<ll>(K+1));
    REP(i, K) {
        REP(j, K) {
            if(j == i + 1) T[i][j] = 1;
            else if(i == K) cin >> T[i][K - j + 1]; //
                multipliers
            else T[i][j] = 0;
        }
    }
    ll N;
    cin >> N;
    if (N == 1) return 1;
    T = pow(T, N-1);
    ll res = 0;
    REP(i, K)
        res = (res + T[1][i] * F1[i]) % INF32; // Mod Value
    return res;
}
int main() {
    cin >> K;
    cout << solve() << endl;
}
```

4 Others

4.1 Convex

```cpp
ll cross(Point a , Point b){
    return a.x * b.y - a.y *b.x;
}

void convex(){
 sort(points.begin(), points.end());
 int m = 0;
 fore(i,0,points.size()-1){
       while (m > 1 && cross(CH[m-1] - CH[m-2] , points[i] -
            CH[m-2]) <= 0){
           CH.pop_back();
           m--;
       }
       CH.push_back(points[i]);
       m++;
    }

    int k = m;
    forn(i,points.size()-2 , 0){
       while (m > k && cross(CH[m-1] - CH[m-2] , points[i] -
            CH[m-2]) <= 0){
           CH.pop_back();
           m--;
       }
       CH.push_back(points[i]);
```

```
        m++;
    }
}

ld area() {
    ld sum = 0;
    int i;
    fore(i,0,CH.size()-2){
        sum += (CH[i].x*CH[i+1].y - CH[i].y*CH[i+1].x);
    }
    return fabs(sum/2);
}
```

4.2 Convex$_{T}$*rick*

```
struct Line {
    ll k, b;

    Line() {
        k = b = 0ll;
    }

    Line(ll k, ll b) : k(k), b(b) {}

    ll get(ll x) {
        return k * x + b;
    }
};

ld interLine(Line a, Line b) {
    return (ld)(a.b - b.b) / (ld)(b.k - a.k);
}

struct CHT {
    vector<Line> V;

    CHT() {
        V.clear();
    }

    void addLine(Line l) {
        while (V.size() >= 2 && interLine(V[V.size() - 2], l)
              < interLine(V[V.size() - 2], V.back())) {
            V.pop_back();
        }

        V.push_back(l);
    }

    ll get(ll x) {
        int l = 0, r = (int)V.size() - 2, idx = (int)V.size()
             - 1;

        while (l <= r) {
            int mid = (l + r) >> 1;

            if (interLine(V[mid], V[mid + 1]) <= x) {
                l = mid + 1;
            }
            else {
                r = mid - 1;
                idx = mid;
            }
        }

        return V[idx].get(x);
    }
};

struct Hull_Static{
    /**
        all m need to be decreasing order
        if m is in increasing order then negate the m ( like
             , add_line(-m,c) ),
        remember in query you have to negate the x also
    **/

    const ll inf = 1000000000000000000;
    int min_or_max; ///if min then 0 otherwise 1
    int pointer; /// keep track for the best line for
         previous query, requires all insert first;
    vector < ll > M, C; ///y = m * x + c;

    inline void clear(){
        min_or_max = 0; ///initially with minimum trick
        pointer = 0;
        M.clear();
        C.clear();
    }

    Hull_Static(){
        clear();
    }
    Hull_Static(int _min_or_max){
        clear();
        this->min_or_max = _min_or_max;
    }
    bool bad_min(int idx1, int idx2, int idx3){
        //return (C[idx3] - C[idx1]) * (M[idx1] - M[idx2]) <
             (C[idx2] - C[idx1]) * (M[idx1] - M[idx3]);
        return 1.0 * (C[idx3] - C[idx1]) * (M[idx1] - M[idx2
             ]) <= 1.0 *(C[idx2] - C[idx1]) * (M[idx1] - M[
             idx3]); /// for overflow
    }

    bool bad_max(int idx1, int idx2, int idx3){
        //return (C[idx3] - C[idx1]) * (M[idx1] - M[idx2]) >
             (C[idx2] - C[idx1]) * (M[idx1] - M[idx3]);
        return 1.0 * (C[idx3] - C[idx1]) * (M[idx1] - M[idx2
             ]) >= 1.0 *(C[idx2] - C[idx1]) * (M[idx1] - M[
             idx3]); /// for overflow
    }

    bool bad(int idx1, int idx2, int idx3){ /// for removing
         line, which isn't necessary
        if(!min_or_max) return bad_min(idx1, idx2, idx3);
        else return bad_max(idx1, idx2, idx3);
    }

    void add_line(ll m, ll c){ /// add line where m is given
         in decreasing order
        //if(M.size() > 0 and M.back() == m) return; /// same
             gradient, no need to add
        M.push_back(m);
        C.push_back(c);

        while(M.size() >= 3 and bad((int)M.size() - 3, (int)M
             .size() - 2, (int)M.size() - 1)){
            M.erase(M.end() - 2);
            C.erase(C.end() - 2);
        }
    }
    ll getval(ll idx, ll x){ /// get the y coordinate of a
         specific line
        return M[idx] * x + C[idx];
    }

    ll getminval(ll x){ /// if queries are sorted, make sure
         all insertion first.
        while(pointer < (int)M.size() - 1 and getval(pointer
             + 1, x) < getval(pointer, x)) pointer++;
        return M[pointer] * x + C[pointer];
    }
    ll getmaxval(ll x){ /// if queries are sorted, make sure
         all insertion first.
        while(pointer < (int)M.size() - 1 and getval(pointer
             + 1, x) > getval(pointer, x)) pointer++;
        return M[pointer] * x + C[pointer];
```

```
    }
    ll getminvalternary(ll x){ /// minimum value with ternary
        search
        ll lo = 0;
        ll hi = (ll)M.size() - 1;
        ll ans = inf;
        while(lo <= hi){
            ll mid1 = lo + (hi - lo) / 3;
            ll mid2 = hi - (hi - lo) / 3;
            ll val1 = getval(mid1, x);
            ll val2 = getval(mid2, x);
            if(val1 < val2){
                ans = min(ans, val2);
                hi = mid2 - 1;
            }
            else{
                ans = min(ans, val1);
                lo = mid1 + 1;
            }
        }
        return ans;
    }

    ll getmaxvalternary(ll x){ /// maximum value with ternary
        search
//        cout<<M.size()<<endl;
        ll lo = 0;
        ll hi = (ll)M.size() - 1;
        ll ans = -inf;
        while(lo <= hi){
            ll mid1 = lo + (hi - lo) / 3;
            ll mid2 = hi - (hi - lo) / 3;
            ll val1 = getval(mid1, x);
            ll val2 = getval(mid2, x);
            if(val1 < val2){
                ans = max(ans, val2);
                lo = mid1 + 1;
            }
            else{
                ans = max(ans, val1);
                hi = mid2 - 1;
            }
        }
        return ans;
    }

};
```

4.3 FFT and Multiplication

```
//From "You Know Izad?" team cheatsheet
#define base complex<double>
void fft (vector<base> & a, bool invert){
    if (L(a) == 1) return;
    int n = L(a);
    vector <base> a0(n / 2), a1(n / 2);
    for (int i = 0, j = 0; i < n; i += 2, ++j){
        a0[j] = a[i];
        a1[j] = a[i + 1];
    }
    fft (a0, invert);
    fft (a1, invert);
    double ang = 2 * PI / n * (invert ? -1 : 1);
    base w(1), wn(cos(ang), sin(ang));
    fore(i, 0, n / 2) {
        a[i] = a0[i] + w * a1[i];
        3
        a[i + n / 2] = a0[i] - w * a1[i];
        if (invert)
            a[i] /= 2, a[i + n / 2] /= 2;
        w *= wn;
    }
}
void multiply (const vector<int> &a, const vector<int> & b,
     vector<int> &res){
    vector <base> fa(all(a)), fb(all(b));
    size_t n = 1;
    while (n < max(L(a), (L(b)))) n <<= 1;
    n <<= 1;
    fa.resize(n), fb.resize(n);
    fft(fa, false), fft(fb, false);
    fore(i, 0, n)
    fa[i] *= fb[i];
    fft (fa, true);
    res.resize (n);
    fore(i, 0, n)
    res[i] = int (fa[i].real() + 0.5);
}
```

4.4 FFT

```
const double PI = acos(-1);
#define base complex<double>
int lg_n;
int rev [maxn * 20];
vector<base> polies[maxn];
int reverse(int num ,int lll) {
    return rev[num];
}

void fft(vector<base> & a, bool invert) {
    int n = a.size();


    for (int i = 0; i < n; i++) {
        if (i < reverse(i, lg_n))
            swap(a[i], a[reverse(i, lg_n)]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        double ang = 2 * PI / len * (invert ? -1 : 1);
        base wlen(cos(ang), sin(ang));
        for (int i = 0; i < n; i += len) {
            base w(1);
            for (int j = 0; j < len / 2; j++) {
                base u = a[i+j], v = a[i+j+len/2] * w;
                a[i+j] = u + v;
                a[i+j+len/2] = u - v;
                w *= wlen;
            }
        }
    }

    if (invert) {
        for (base & x : a)
            x /= n;
    }
}


void multiply (int u , int v){
 int n = 1;
    while (n < max(L(a) , L(b))) {
     n <<= 1;
    }
    n <<= 1;
    lg_n = 0;
    while ((1 << lg_n) < n)
        lg_n++;

    for (int i=0; i<n; ++i) {
  rev[i] = 0;
  for (int j=0; j<lg_n; ++j)
   if (i & (1<<j))
    rev[i] |= 1<<(lg_n-1-j);
 }
 a.resize(n);
```

```
 b.resize(n);
    fft(a, false), fft(b, false);
    fore(i, 0, n - 1){
     a[i] *= b[i];
    }
    fft (a, true);
    res.resize (n);

    fore(i, 0, n - 1) {
     res[i] = round(a[i].real());
    }
}
```

4.5 Fermat's Theory

```
if a is a natural number and p is a prime number then

(a ^ p) % p = a

#define nbit(x) __builtin_popcount(x)
#define fore(i, l, r) for(int i = (int)l; i <= (int)r; i++)
#define TRACE(x) cout << #x << " : " << x << endl
#define _ << " : " <<
```

4.6 GEO

```
Line(Point p1 , Point p2){
 a = p2.y - p1.y;
 b = p1.x - p2.x;
 c = a * p1.x + b * p1.y;
 c = -c;

}

Point intersection(Line l1 , Line l2){
 ld a1 = l1.a;
 ld b1 = l1.b;
 ld c1 = -l1.c;
 ld a2 = l2.a;
 ld b2 = l2.b;
 ld c2 = -l2.c;
 ld determinant = a1*b2 - a2*b1;
 ld x = (b2*c1 - b1*c2)/determinant;
 ld y = (a1*c2 - a2*c1)/determinant;
 return Point(x, y);
}

Point mirrorImage(Point p , Line l)
{
 ld a = l.a;
 ld b = l.b;
 ld c = l.c;
 ld x1 = p.x;
 ld y1 = p.y;
 ld temp = -2 * (a * x1 + b * y1 + c) /
 (a * a + b * b);
 ld x = temp * a + x1;
 ld y = temp * b + y1;
 return Point(x, y);
}

ld pointToLine(Point p0, Point p1, Point p2){
 //p0 to (p1 , p2)
 ll x0 = p0.x;
 ll y0 = p0.y;
 ll x1 = p1.x;
 ll y1 = p1.y;
 ll x2 = p2.x;
 ll y2 = p2.y;
 ld a = ((y2 - y1)*x0 - (x2 - x1)*y0 + x2 * y1 - y2 * x1);
 ld b = (y2 - y1)*(y2 - y1) + (x2 - x1)*(x2 - x1);
 return a * a / b;
}

inline p3d rotate(const p3d& p /*pt*/, const p3d& u /*axis*/
     , const ld& angle) {
//p center u
ld c = cos(angle), s = sin(angle), t = 1 - cos(angle);
     return {
 p.x*(t*u.x*u.x + c) + p.y*(t*u.x*u.y - s*u.z) + p.z*(t*u.x*
      u.z + s*u.y),
 p.x*(t*u.x*u.y + s*u.z) + p.y*(t*u.y*u.y + c) + p.z*(t*u.y*
      u.z - s*u.x),
 p.x*(t*u.x*u.z - s*u.y) + p.y*(t*u.y*u.z + s*u.x) + p.z*(t*
      u.z*u.z + c) };
}


int cmp(ld x){
 if (fabs(x) < eps)
  return 0;
 return ((x < 0 ) ? -1 : 1);
}

ld Dot( const Vec2& a, const Vec2& b )
{
    return a.x * b.x + a.y * b.y;
}

int orientation(Point p, Point q, Point r)
{
 ld val = (q.y - p.y) * (r.x - q.x) -
 (q.x - p.x) * (r.y - q.y);

 if (cmp(val) == 0) return 0;
 return (cmp(val) > 0)? 1: 2;
}

bool onSegment(Point p, Point q, Point r)
{
 // (p , r) point q
    if (cmp(q.x - max(p.x, r.x)) >= 0 && cmp(q.x - min(p.x, r
         .x)) <= 0 &&
           cmp(q.y - max(p.y, r.y)) >= 0 && cmp(q.y - min(p.
                y, r.y)) <= 0 )
       return true;
    return false;
}

bool doIntersect(Point p1, Point q1, Point p2, Point q2)
{
 // (p1 , q1) intersect (p2 , q2)
 int o1 = orientation(p1, q1, p2);
 int o2 = orientation(p1, q1, q2);
 int o3 = orientation(p2, q2, p1);
 int o4 = orientation(p2, q2, q1);
 if (o1 != o2 && o3 != o4)
  return true;
 if (o1 == 0 && onSegment(p1, p2, q1)) return true;
 if (o2 == 0 && onSegment(p1, q2, q1)) return true;
 if (o3 == 0 && onSegment(p2, p1, q2)) return true;
 if (o4 == 0 && onSegment(p2, q1, q2)) return true;
    return false; // Doesn't fall in any of the above cases
}

bool isInside(Point p)
{
 if (n < 3) return false;
 Point extreme = {1e18, p.y};
 int count = 0, i = 0;
 do
 {
  int next = (i+1)%n;
  if (doIntersect(polygon[i], polygon[next], p, extreme))
  {
   if (orientation(polygon[i], p, polygon[next]) == 0)
    return onSegment(polygon[i], p, polygon[next]);
```

```
   count++;
  }
  i = next;
 } while (i != 0);
 return count&1;
}
ld cross(Vec2 a , Vec2 b){
 return a.x * b.y - a.y * b.x;
}
ld len(Vec2 a){
 return hypotl(a.x , a.y);
}
ld SqDistancePtSegment( Vec2 a, Vec2 b, Vec2 p )
{
 Vec2 v1 = b - a;
 Vec2 v2 = p - a;
 Vec2 v3 = p - b;
 if (cmp(Dot(v1 , v2)) < 0)return len(v2);
 if (cmp(Dot(v1 , v3)) > 0) return len(v3);
 return fabs(cross(v1 , v2)) /len(v1);
}
Point F( int i ,int j , int k){
 Vec2 a , b , c;
 a.x = polygon[i].x;
 a.y = polygon[i].y;
 b.x = polygon[j].x;
 b.y = polygon[j].y;
 c.x = polygon[k].x;
 c.y = polygon[k].y;
 Vec2 v1 = b - a;
 Vec2 v2 = c - a;
 Vec2 nimsaz = (v2 * len(v1)) + (v1 * len(v2)) ;
 ld sz = len(nimsaz);
 nimsaz.x /= sz;
 nimsaz.y /= sz;
 ld costeta = Dot(nimsaz , v2) / (len(nimsaz) * len(v2));
 ld sinteta = sqrt(1.0 - costeta * costeta);
 ld d = R / sinteta;
 Vec2 point = (nimsaz * d) + a;
 return {point.x , point.y};

}

Point rotate(Point c , Point p , ld angle)
{
    ld sn = sin(angle);
    ld cs = cos(angle);
    Point q(cs*(p.x-c.x) - sn *(p.y-c.y) + c.x , sn *(p.x-c.x
        ) + cs * (p.y-c.y) + c.y);
    return q;
}
```

4.7 Segment Tree Lazy Propagation

```
#include <iostream>

#include <vector>

using namespace std;

#define Long long long int

const int N = 200 * 1000;
const Long inf = (Long)1000 * 1000 * 1000 * 1000;
int n;
int ary[N];
Long tree[N << 2];
Long lazy[N << 2];
vector <Long> ans;

void getArray();
void build(int, int, int);
inline int lc(int);
inline int rc(int);
vector <string> split(string, char);
int toInteger(string);
Long minimum(int, int, int, int, int);
void add(int, int, int, int, int, int);
void propagate(int, int, int);
void update(int, int);
void print();

int main(){
    getArray();
    build(1, 0, n - 1);
    int q;
    string line;
    cin >> q;
    getline(cin, line);
    for(int i = 0; i < q; i++){
        getline(cin, line);
        vector <string> command = split(line, ' ');
        int lf = toInteger(command[0]);
        int rg = toInteger(command[1]);
        int l1, l2, r1, r2;
        if(lf > rg){
            l1 = lf;
            r1 = n - 1;
            l2 = 0;
            r2 = rg;
        }
        else{
            l1 = lf;
            r1 = rg;
            l2 = -1;
            r2 = -1;
        }
        if(command.size() == 2){
            Long min1 = minimum(1, 0, n - 1, l1, r1);
            Long min2 = minimum(1, 0, n - 1, l2, r2);
            ans.push_back(min(min1, min2));
        }
        else{
            int val = toInteger(command[2]);
            add(1, 0, n - 1, l1, r1, val);
            add(1, 0, n - 1, l2, r2, val);
        }
    }
    print();
    return 0;
}

void getArray(){
    ios_base :: sync_with_stdio(false);
    cin.tie(0);
    cin >> n;
    for(int i = 0; i < n; i++)
        cin >> ary[i];
}

void build(int node, int l, int r){
    if(l == r)
        tree[node] = ary[l];
    else{
        int mid = (l + r) >> 1;
        build(lc(node), l, mid);
        build(rc(node), mid + 1, r);
        tree[node] = min(tree[lc(node)], tree[rc(node)]);
    }
}

inline int lc(int node){
    return node << 1;
}

inline int rc(int node){
    return node << 1 | 1;
}
```

```
vector <string> split(string str, char character){
    vector <string> res;
    string s = "";
    for(int i = 0; i < str.size(); i++){
        char c = str[i];
        if(c == character){
            res.push_back(s);
            s = "";
        }
        else
            s += c;
    }
    res.push_back(s);
    return res;
}

int toInteger(string str){
    int res = 0;
    bool positive = true;
    char zero = '0';
    for(int i = 0; i < str.size(); i++){
        char c = str[i];
        if(c == '-')
            positive = false;
        else{
            int d = int(c) - int(zero);
            res = res * 10 + d;
        }
    }
    if(!positive)
        res *= -1;
    return res;
}

Long minimum(int node, int l, int r, int beg, int end){
    if(l > end || r < beg)
        return inf;
    else if(l >= beg && r <= end)
        return tree[node];
    else{
        propagate(node, l, r);
        int mid = (l + r) >> 1;
        Long min1 = minimum(lc(node), l, mid, beg, end);
        Long min2 = minimum(rc(node), mid + 1, r, beg, end);
        return min(min1, min2);
    }
}

void add(int node, int l, int r, int beg, int end, int val){
    if(l > end || r < beg)
        return;
    else if(l >= beg && r <= end)
        update(node, val);
    else{
        propagate(node, l, r);
        int mid = (l + r) >> 1;
        add(lc(node), l, mid, beg, end, val);
        add(rc(node), mid + 1, r, beg, end, val);
        tree[node] = min(tree[lc(node)], tree[rc(node)]);
    }
}

void propagate(int node, int l, int r){
    if(l < r){
        int mid = (l + r) >> 1;
        update(lc(node), lazy[node]);
        update(rc(node), lazy[node]);
    }
    lazy[node] = 0;
}

void update(int node, int val){
    lazy[node] += val;
    tree[node] += val;
}

void print(){
    for(int i = 0; i < ans.size(); i++)
        cout << ans[i] << endl;
}
```

4.8 order$_set$

```
// C++ program to demonstrate the
// ordered set in GNU C++
#include <iostream>
using namespace std;

// Header files, namespaces,
// macros as defined above
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

#define ordered_set tree<int, null_type,less<int>,
     rb_tree_tag,tree_order_statistics_node_update>

// Driver program to test above functions
int main()
{
    // Ordered set declared with name o_set
    ordered_set o_set;

    // insert function to insert in
    // ordered set same as SET STL
    o_set.insert(5);
    o_set.insert(1);
    o_set.insert(2);

    // Finding the second smallest element
    // in the set using * because
    // find_by_order returns an iterator
    cout << *(o_set.find_by_order(1))
        << endl;

    // Finding the number of elements
    // strictly less than k=4
    cout << o_set.order_of_key(4)
        << endl;

    // Finding the count of elements less
    // than or equal to 4 i.e. strictly less
    // than 5 if integers are present
    cout << o_set.order_of_key(5)
        << endl;

    // Deleting 2 from the set if it exists
    if (o_set.find(2) != o_set.end())
        o_set.erase(o_set.find(2));

    // Now after deleting 2 from the set
    // Finding the second smallest element in the set
    cout << *(o_set.find_by_order(1))
        << endl;

    // Finding the number of
    // elements strictly less than k=4
    cout << o_set.order_of_key(4)
        << endl;

    return 0;
}
```

5 String

5.1 AHO

```
struct Node
{
    char c;
    int parent;
    int isWord;
    int suffLink;
    vi children;
    int len;
    Node(){
        parent = -1;
        isWord = false;
        suffLink = -1;
        children.clear();
        len = 0;
    }
};
struct Aho
{
    vector<Node> nodes;

    Aho(){
        nodes.pub(Node());
        nodes[0].suffLink=0;
    }
    void addString(string s){
        int cur = 0;
        fore(i, 0 , s.size()-1){
            int nxt = -1;
            if (cur < nodes.size())
                fore(j , 0 , nodes[cur].children.size()-1){
                    if (nodes[nodes[cur].children[j]].c == s[i
                        ])
                    {
                        nxt = nodes[cur].children[j];
                        break;
                    }
                }
            if (~nxt)
            {
                cur = nxt;
                continue;
            }
            nodes[cur].children.pub(nodes.size());
            nodes.pub(Node());
            nodes[nodes.size()-1].parent = cur;
            nodes[nodes.size()-1].c = s[i];
            cur = nodes.size()-1;
        }
        nodes[cur].isWord = true;
        nodes[cur].len = s.size();
    }

    int calc(int cur){
        if (nodes[cur].suffLink == -1)
        {
            if (nodes[cur].parent == 0) return 0;
            return nodes[cur].suffLink = trans(calc(nodes[cur
                ].parent) , nodes[cur].c);
        }
        return nodes[cur].suffLink;
    }
    int res = inf;
    int pos;
    int trans(int cur , char c ){
        if (nodes[cur].isWord){
            res = min(res , pos - nodes[cur].len + 1);
        }
        fore(i,0 , nodes[cur].children.size()-1){
            if (nodes[nodes[cur].children[i]].c == c)
                return nodes[cur].children[i];
        }
        if (cur == 0) return 0;
        return trans(calc(cur) , c);
    }
    int find(string s){

        int cur = 0;
        fore(i , 0 , s.size()-1){
            pos = i;
            cur = trans(cur , s[i]);
            if (nodes[cur].isWord){
                res = min(res , i - nodes[cur].len +2);
            }
            calc(cur);
        }
        return res;
    }
};
```

5.2 Aho Corasick

```
#include <bits/stdc++.h>
#define FOR(i, n) for (int i = 0; i < (n); ++i)
#define REP(i, n) for (int i = 1; i <= (n); ++i)
using namespace std;

struct AC_trie {
  int N, P;
  vector<map<char, int>> next; // trie
  vector<int> link, out_link;
  vector<vector<int>> out;
  AC_trie(): N(0), P(0) { node(); }
  int node() {
    next.emplace_back(); // trie
    link.emplace_back(0);
    out_link.emplace_back(0);
    out.emplace_back(0);
    return N++;
  }
  int add_pattern(const string T) {
    int u = 0;
    for (auto c : T) {
      if (!next[u][c]) next[u][c] = node();
      u = next[u][c];
    }
    out[u].push_back(P);
    return P++;
  }
  void compute() {
    queue<int> q;
    for (q.push(0); !q.empty(); ) {
      int u = q.front(); q.pop();
      // trie:
      for (auto e : next[u]) {
        int v = e.second;
        link[v] = u ? advance(link[u], e.first) : 0;
        out_link[v] = out[link[v]].empty() ? out_link[link[v
            ]] : link[v];
        q.push(e.second);
      }
    }
  }
  int advance(int u, char c) {
    // trie:
    while (u && next[u].find(c) == next[u].end())
      u = link[u];
    if (next[u].find(c) != next[u].end())
      u = next[u][c];
    return u;
  }
  void match(const string S) {
    int u = 0;
    for (auto c : S) {
      u = advance(u, c);
```

```
    for (int v = u; v; v = out_link[v])
      for (auto p : out[v])
        cout << "match " << p << endl;
  }
 }
};
struct AC_automaton {
 int N, P;
 vector<vector<int>> next; // automaton
 vector<int> link, out_link;
 vector<vector<int>> out;
 AC_automaton(): N(0), P(0) { node(); }
 int node() {
  next.emplace_back(26, 0); // automaton
  link.emplace_back();
  out_link.emplace_back();
  out.emplace_back();
  return N++;
 }
 int add_pattern(const string T) {
  int u = 0;
  for (auto c : T) {
   if (!next[u][c - 'a']) next[u][c - 'a'] = node();
   u = next[u][c - 'a'];
  }
  out[u].push_back(P);
  return P++;
 }
 void compute() {
  queue<int> q;
  for (q.push(0); !q.empty(); ) {
   int u = q.front(); q.pop();
   // automaton:
   for (int c = 0; c < 26; ++c) {
    int v = next[u][c];
    if (!v) next[u][c] = next[link[u]][c];
    else {
     link[v] = u ? next[link[u]][c] : 0;
     out_link[v] = out[link[v]].empty() ? out_link[link[
          v]] : link[v];
     q.push(v);
    }
   }
  }
 }
 int advance(int u, char c) {
  // automaton:
  while (u && !next[u][c - 'a']) u = link[u];
  u = next[u][c - 'a'];
  return u;
 }
};
int main() {
 int P;
 string T;
 cin >> P;

 AC_trie match1;
 AC_automaton match2;
 REP (i, P) {
  cin >> T;
  match1.add_pattern(T); match2.add_pattern(T);
 }
 match1.compute();
 match2.compute();
 cin >> T;
 match1.match(T);
 match2.match(T);
 return 0;
}
```

5.3 KMP

```
//From "You Know Izad?" team cheat sheet
int fail[100005];
void build(const string &key){
    fail[0] = 0;
    fail[1] = 0;
    fore(i, 2, L(key)) {
        int j = fail[i - 1];
        while (true) {
            if (key[j] == key[i - 1]) {
                fail[i] = j + 1;
                break;
            }
            else if (j == 0) break;
            j = fail[j];
        }
    }
}
int KMP(const string &text, const string &key) {
    build(key);
    int i = 0, j = 0;
    while (true) {
        if (j == L(text)) return -1;
        if (text[j] == key[i]) {
            i++;
            j++;
            if (i == L(key)) return j - i;
        }
        else if (i > 0) i = fail[i];
        else j++;
    }
}
```

5.4 SuffixTree

```
#define pci pair< char, int >
#define NV N[v]
string s;
struct node {
    int p, b, e, link;
    /*map< char, int > children;*/
    vector< pci > children;
    node( int _p, int _b, int _e ) { p = _p, b = _b, e = _e,
         link = -1; }
    void addChild( pci a ) {
        children.push_back( a );
    }
    void changeChild( pci a ) {
 //children[a.first] = a.second;
        for( int i = 0; i < children.size(); i++ ) {
            if( children[i].first == a.first ) {
                children[i].second = a.second;
                return;
            }
        }
    }
    int length() { return e - b + 1; }
    bool gotoNext( char c, int &nv, int &nd) {
        if( nd < e - b ) {
            if( s[b + nd + 1] == c ) {
                nd++;
                return true;
            }
        } else {
            for( int i = 0; i < children.size(); i++ ) {
                if( children[i].first == c ) {
                    nv = children[i].second, nd = 0;
```

```
                return true;
            }
        }
    }
    return false;
  }
};
vector< node > N;
void add2Tree( ) {
    N.clear();
    N.push_back( node( -1, -1, -1 ) );
    N[0].link = 0;
    int j = 0, pp = -1, v = 0, d = 0;
    for( int i = 0; i < s.length(); i++ ) {
        pp = -1;
        for( ; j <= i; j++ ) {
            if( NV.gotoNext( s[i], v, d ) ) {
                if( pp != -1 ) N[pp].link = NV.p;
                break;
            } else {
               int id = N.size();
                if( d < NV.e - NV.b ) {
                    if( pp != -1 ) N[pp].link = id;
                    N.push_back( node( NV.p, NV.b, NV.b + d )
                        );
                    N[NV.p].changeChild( pci( s[NV.b], id ) );
                    NV.b += d + 1;
                    NV.p = pp = id;
                    N[id].addChild( pci( s[NV.b], v ) );
                    int len = N[id].p ? d + 1 : d;
                    v = N[N[id].p].link;
                    d = NV.length() - 1;
                    while( len ) {
                        int temp = v;
                        N[temp].gotoNext( s[i - len] , v, d );
                        int l = NV.length();
                        if( len <= l ) {
                            d = len - 1;
                            break;
                        }
                        d = l - 1;
                        len -= l;
                    }
                    id++;
                } else {
                    if( pp != -1 ) N[pp].link = v;
                    pp = v;
                    v = NV.link;
                    d = NV.length() - 1;
                }
                N[pp].addChild( pci( s[i], id ) );
                N.push_back( node( pp, i, s.length() - 1 ) );
            }
        }
    }
}
```

5.5 Trie

```
int ind(char c)
{
 return c - 'a';
}
int n, m;
int node[maxn][26];
int cnt = 1;
vector<int> edges[maxn];
int val [maxn];
string s;
vector<int> v;
void insert(){
 int nd = 0;
 int idx = 0;
 int last = -1;
 while(~node[nd][ind(s[idx])] && idx<s.size()){
  last = nd;
  nd = node[nd][ind(s[idx])];
  idx++;
 }
 v.clear();
 while(idx < s.size())
 {
  v.push_back(nd);
  node[nd][ind(s[idx])] = cnt;
  idx++;
  nd = cnt++;
 }
 v.push_back(nd);
 if (v.size() < 2)
  return;
 fore(i,0,v.size()-2){
  edges[v[i]].push_back(v[i+1]);
  edges[v[i+1]].push_back(v[i]);
 }
 if(v.size()>2)
  edges[v[1]].push_back(nd);
}
```

5.6 rope

```
#include <bits/stdc++.h>
#include <ext/rope> //header with rope
using namespace std;
using namespace __gnu_cxx; //namespace with rope and some
     additional stuff
int main()
{

    rope <int> v; //use as usual STL container
    int n, m;
    cin >> n >> m;
    for(int i = 1; i <= n; ++i)
        v.push_back(i); //initialization
    string p ;
    int idx;
    cin>>p>>idx;
    fore(i,1,p.size()){
        s.insert(i + idx -1 , p[i-1]);
    }
    int l, r;
    for(int i = 0; i < m; ++i)
    {
        cin >> l >> r;
        --l, --r;
        rope <int> cur = v.substr(l, r - l + 1);
        v.erase(l, r - l + 1);
        v.insert(v.mutable_begin(), cur);

    }
    for(rope <int>::iterator it = v.mutable_begin(); it != v.
         mutable_end(); ++it)
        cout << *it << " ";
    return 0;
}
```