IN3140 - Assignment 2

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Task 1

The Python code with the solutions to task 1 has been submitted alongside this PDF.
The code contains comments detailing the steps of the solution.
Filename: IN3140_oblig2_task1_majajma.py

Task 2

a) the forward kinematics from assignment I:

$$A_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & 0 \\ s\theta_1 & 0 & -c\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & a_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & a_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & a_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & a_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 A_2 = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 & a_2 c_1 c_2 \\ c_2 s_1 & -s_1 s_2 & -c_1 & a_2 c_2 s_1 \\ s_2 & c_2 & 0 & L_1 + a_2 s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^B_t = \begin{bmatrix} c_1 c_{23} & -c_1 s_{23} & s_1 & a_2 c_1 c_2 + a_3 c_1 c_{23} \\ c_{23} s_1 & -s_{23} s_1 & -c_1 & a_2 c_2 s_1 + a_3 s_1 c_{23} \\ s_{23} & c_{23} & 0 & L_1 + a_2 s_2 + a_3 s_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

find all of the components of the jacobian from the forward kinematics:

z_i - vector:

$$z_0 : \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1 : A_1[3] \rightarrow \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

$$z_2 : A_1 A_2 [3] \rightarrow \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}$$

O_i - vectors:

$$O_0 : \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$O_1 : A_1[4] \rightarrow \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$O_2 : A_1 A_2 [4] \rightarrow \begin{bmatrix} a_2 c_1 c_2 \\ a_2 c_2 s_1 \\ L_1 + a_2 s_2 \end{bmatrix}$$

$$O_3 : A_1 A_2 A_3 [4] = T^B_t [4] \rightarrow \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 c_2 s_1 + a_3 s_1 c_{23} \\ L_1 + a_2 s_2 + a_3 s_{23} \end{bmatrix}$$

Calculate J_v:

$$J_{v_i} = z_{i-1} \times (O_3 - O_{i-1})$$

$$z_{v_1} = z_0 \times (O_3 - O_0) = z_0 \times O_3$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 c_2 s_1 + a_3 s_1 c_{23} \\ L_1 + a_2 s_2 + a_3 s_{23} \end{bmatrix}$$

$$= \begin{array}{ccccc} x & y & z & x & y \\ 0 & 0 & 1 & 0 & 0 \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & a_2 c_2 s_1 + a_3 s_1 c_{23} & L_1 + a_2 s_2 + a_3 s_{23} & a_2 c_1 c_2 + a_3 c_1 c_{23} & a_2 c_2 s_1 + a_3 s_1 c_{23} \end{array}$$

$$= [-a_2 c_2 s_1 - a_3 s_1 c_{23},\ a_2 c_1 c_2 + a_3 c_1 c_{23},\ 0]$$

$$J_{v_2} = z_1 \times (O_3 - O_1) = z_1 \times O_3$$

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - 0 \\ a_2 c_2 s_1 + a_3 s_1 c_{23} - 0 \\ L_1 + a_2 s_2 + a_3 s_{23} - L_1 \end{bmatrix} = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} \\ a_2 c_2 s_1 + a_3 s_1 c_{23} \\ a_2 s_2 + a_3 s_{23} \end{bmatrix}$$

$$= \begin{array}{ccccc} x & y & z & x & y \\ s_1 & -c_1 & 0 & s_1 & -c_1 \\ a_2 c_1 c_2 + a_3 c_1 c_{23} & a_2 c_2 s_1 + a_3 s_1 c_{23} & a_2 s_2 + a_3 s_{23} & a_2 c_1 c_2 + a_3 c_1 c_{23} & a_2 c_2 s_1 + a_3 s_1 c_{23} \end{array}$$

$$= [-c_1 (a_2 s_2 + a_3 s_{23}),\ -s_1 (a_2 s_2 + a_3 s_{23}),\ a_2 c_2 s_1^2 + a_3 c_{23} s_1^2 + a_2 c_1^2 c_2 + a_3 c_1^2 c_{23}]$$

$$= [-c_1 (a_2 s_2 + a_3 s_{23}),\ -s_1 (a_2 s_2 + a_3 s_{23}),\ a_2 c_2 + a_3 c_{23}]$$

$$J_{v_3} = z_2 \times (O_3 - O_2) = z_2 \times O_3$$

$$\begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 c_1 c_2 + a_3 c_1 c_{23} - (a_2 c_1 c_2) \\ a_2 c_2 s_1 + a_3 s_1 c_{23} - (a_2 c_2 s_1) \\ L_1 + a_2 s_2 + a_3 s_{23} - (L_1 + a_2 s_2) \end{bmatrix} = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} \times \begin{bmatrix} a_3 c_1 c_{23} \\ a_3 s_1 c_{23} \\ a_3 s_{23} \end{bmatrix}$$

$$\begin{array}{ccccc} x & y & z & x & y \\ s_1 & -c_1 & 0 & s_1 & -c_1 \\ a_3 c_1 c_{23} & a_3 s_1 c_{23} & a_3 s_{23} & a_3 c_1 c_{23} & a_3 s_1 c_{23} \end{array}$$

$$= [-c_1 a_3 s_{23},\ -s_1 a_3 s_{23},\ a_3 s_1^2 c_{23} + a_3 c_1^2 c_{23}]$$

$$= [-c_1 a_3 s_{23},\ -s_1 a_3 s_{23},\ a_3 c_{23}]$$

Calculate $J\omega$:

$$J\omega_i = z_{i-1}$$

$$J\omega_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad J\omega_2 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix}, \quad J\omega_3 = \begin{bmatrix} S_1 \\ -C_1 \\ 0 \end{bmatrix}$$

Fill in the Jacobi matrix :

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$J_v = \begin{bmatrix} -a_2C_2S_1 - a_3S_1C_{23} & -C_1(a_2S_2 + a_3S_{23}) & -C_1a_3S_{23} \\ a_2C_1C_2 + a_3C_1C_{23} & -S_1(a_2S_2 + a_3S_{23}) & -S_1a_3S_{23} \\ 0 & a_2C_2 + a_3C_{23} & a_3C_{23} \end{bmatrix}, \quad J_\omega = \begin{bmatrix} 0 & S_1 & S_1 \\ 0 & -C_1 & -C_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J = \begin{bmatrix} -a_2C_2S_1 - a_3S_1C_{23} & -C_1(a_2S_2 + a_3S_{23}) & -C_1a_3S_{23} \\ a_2C_1C_2 + a_3C_1C_{23} & -S_1(a_2S_2 + a_3S_{23}) & -S_1a_3S_{23} \\ 0 & a_2C_2 + a_3C_{23} & a_3C_{23} \\ 0 & S_1 & S_1 \\ 0 & -C_1 & -C_1 \\ 1 & 0 & 0 \end{bmatrix}$$

b) Configurations for which the rank of Jacobian $J(q)$ is less than the maximum value are called <u>singular configurations</u>.

c) Calculate the determinant of J_v using MATLAB:

```
clc; clear; close all;
syms c1 s1 c2 s2 c3 s3 a2 a3 c23 s23

J_v = [-a2*c2*s1-a3*s1*c23, -c1*(a2*s2+a3*s23), -c1*a3*s23;
    a2*c1*c2+a3*c1*c23, -s1*(a2*s2+a3*s23), -s1*a3*s23;
    0,a2*c2+a3*c23,a3*c23];


% Compute determinant of J_v
det_Jv = det(J_v);


% Display result
disp('Determinant of J_v:');
disp(simplify(det_Jv));
```

→ I use syms to declare symbolic variables

→ I construct the J_v matrix manually using the variables

→ det (J_v) computes the symbolic determinant

→ simplify() cleans up the resulting expression

output:

```
Determinant of J_v:
-a2*a3*(a2*c2 + a3*c23)*(c2*s23 - c23*s2)*(c1^2 + s1^2)
```

$$\Rightarrow \det(J_v) = -a_2 a_3 (a_2 c_2 + a_3 c_{23}) \underbrace{(c_2 s_{23} - c_{23} s_2)}_{}$$

$\sin(\theta_2 + \theta_3 - \theta_2) = \sin\theta_3 = s_3$

$\Rightarrow c_2 s_{23} - c_{23} s_2 = s_3$

$$= -a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23})$$

Solve for $\det(J_v) = 0$:

$$-a_2 a_3 s_3 (a_2 c_2 + a_3 c_{23}) = 0$$

d)

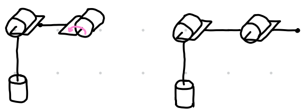
1. $a_2 = 0$
2. $a_3 = 0$

} in this case the link lengths are fixed, so these are not relevant

3. $s_3 = 0$

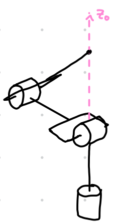
$\sin(\theta_3) = 0 \iff \theta_3 = \pi \wedge \theta_3 = 0$

→ the elbow is fully extended or retracted:

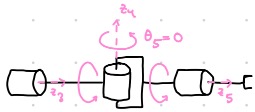


4. $a_2 c_2 + a_3 c_{23} = 0$

→ when the end effector intersects the z_0-axis:



e)



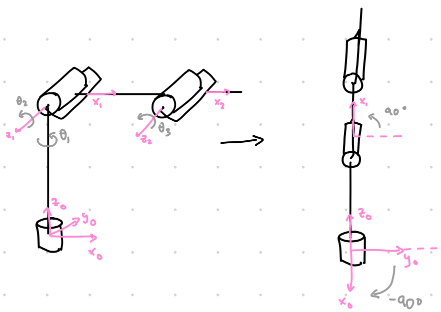
The singularity of the sperical wrist happens when the second wrist joint angle $\theta_5 = 0, \pi$.
When this happens, the z_3 and z_5 axes to align, and the wrist loses one degree of freedom.

f) If singolarities are not handled properly in a robot, they can cause problems like loss of control, sudden joint movements and instabillity. At a singolarity, the robot may lose one or more degrees of freedom, meaning it can't move the end-effector freely in all directions. This can lead to large or unpredictable joint motions, which may cause collisions or wear on the system. In some cases, joints can move too fast, which cause stress to the motors and joints.

Task 3

The python file with the solutions for task 3 has been submitted with comments. Filename: IN3140_oblig2_task3_majajma.py

b) To find the correct θ-angles I first find the angles needed to rotate the manipulator straight up, and then 270°, 60° and 45°.



$\theta_1 = -90° + 270° = \underline{180°}$

$\theta_2 = 90° - 60° = \underline{30°}$

$\theta_3 = 0 - 45° = \underline{-45°}$