

Beyond the Final Linear Layer: Enhancing Decision Boundaries

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Abstract

[TODO: (majurski) rewrite abstract once paper is done to fill in details] SSL leverages an abundance of unlabeled data to improve deep learning based model performance under limited training data regimes. This paper presents a novel extension to any image classification architecture which improves accuracy in low-label regimes. We extend the FixMatch [27] training scheme with our novel last layers and demonstrate test accuracy improvement. The novelty consists of 2 elements: first we replace the last linear layer with a GMM trained via backprop, and second, we impose class-wise constraints on the embedding space the GMM operates on. These methods match published SOTA 250 label CIFAR-10 [15] results and come close to matching SOTA in the 40 label regime without the significant model complexity of methods like SimMatchV2 [38]. Our method achieves 94.8% and 94.2% accuracy with 250 and 40 CIFAR-10 labels respectively. **[TODO: cleanup the repo]** Our code is available at: <https://github.com/mmajurski/ssl-gmm>

1. Introduction

SSL leverages an abundance of unlabeled data to improve deep learning based model performance under limited training data regimes [13, 18, 39]. Image classification has become a playground for exploring new SSL ideas. The early successes of deep learning based methods relied on large annotated datasets to enable models to learn the relevant features to perform the task, i.e. image classification build on top of ImageNet [9]. With data annotation becoming a significant bottleneck, especially in application domains outside of the standard benchmarks, another learning paradigm was needed.

There are several flavors of SSL. Contrastive learning methods leverage the intuition that similar instances should be close in the representation space, while different in-

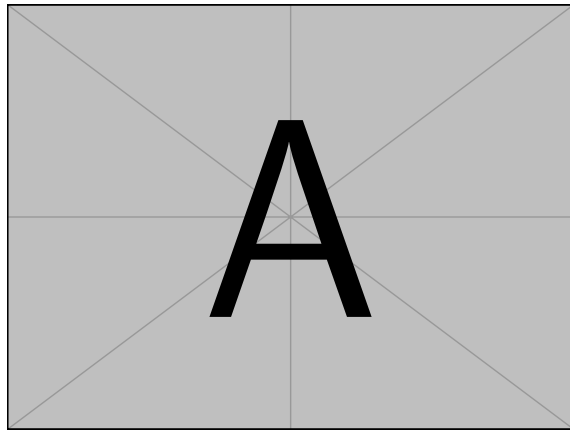


Figure 1. High level overview of our method. **[TODO: (majurski) Create this figure.]**

stances are farther apart [17, 33]. Consistency regularization borrows the intuition that modified views of the same instance should have similar representations and predictions [14, 16, 27, 36]. Pseudo-labeling methods like FixMatch [27] fall within the consistency regularization domain.

This work argues that pseudo-labeling methods can be improved with better calibration of the network logits used to filter the pseudo-labels into reliable and unreliable. Neural networks are known to be overconfident in their predictions [30], and this affects the pseudo-labeling process. Potentially allowing for the inclusion of more incorrect pseudo-labels any specific logit threshold would otherwise have. This work demonstrates the better calibrated replacements for a model's final linear layer can improve the final accuracy of pseudo-labeling based SSL algorithms in very label scarce regimes. This work proposes:

1. Replacing the final linear (fully connected) layer of the neural network with either kmeans [10] or axis-aligned differentiable Gaussian Mixture Model (GMM) trained via back prop, both of which have explicit modeling of class cluster centroids.

2. We explore various constraints on how the embedding space should be structured by adding penalties if the per-class clustering does not conform to between 0 and 4 of the first gaussian moments being identity/zero.
3. We demonstrate that increasing the specificity of how the embedding space should be structure negatively impacts model performance. **[TODO: (chapman) need citation for gaussian method of moments]**

This paper contributes a simple easy to implement improvement to pseudo-labeling methods where very few annotations are available, replace the last linear layer with a kmeans layer which explicitly models class cluster centers. We demonstrate this methodology using CIFAR-10 and CIFAR-100 [15] with 40 and 400 labels respectively. Additionally, we explore and demonstrate that high level prescriptive constraints on the embedding space produce significantly worse outcomes than allowing the embedding space to take on whatever emergent structure the training process produces. Finally, because the embedding constraint penalties are applied to all unlabeled data and not just the valid pseudo-labels, our method extracts training signal from every unlabeled data point, unlike FixMatch [27] and other methods which only learn from the valid pseudo-labels.

[TODO: (majurski) build t-SNE plots of the embedding spaces for the best models (1 per configuration)]

[TODO: (majurski) purge bibliography of arxiv pre-prints where possible, replacing with their peer reviewed equivalents]

2. Related Work

Semi-Supervised learning has recently show great progress in learning high quality models, in some cases match fully supervised performance, for a number of benchmarks [36]. The goal of SSL is to produce a trained model of equivalent accuracy to fully supervised training, with vastly reduced data annotation requirements.

2.1. Pseudo-Labeling

Self-supervised learning was among the initial approaches employed in the context of semi-supervised learning to annotate unlabeled images. This technique involves the initial training of a classifier with a limited set of labeled samples and incorporates pseudo-labels into the gradient descent process, exceeding a predefined threshold [8, 19, 20, 22, 26, 34, 35]. A closely related method to self-training is co-training, where a given dataset is represented as two distinct feature sets [4]. These independent sample sets are subsequently trained separately using two distinct models, and the sample predictions surpassing predetermined thresholds are utilized in the final model training process [4, 24]. A notably advanced approach to pseudo-

labeling is the Mean Teacher algorithm [28], which leverages exponential moving averages of model parameters to acquire a notably more stable target prediction. This refinement has a substantial impact on enhancing the convergence of the algorithm.

2.2. Consistency Regularization

Consistency regularization operates on the premise that when augmenting an unlabeled sample, its label should remain consistent. This approach implicitly enforces a smoothness assumption, promoting coherence between unlabeled samples and their basic augmentations [31]. In other words, the model should be able to predict the unlabeled sample x exactly the same way it predicts the class for $\text{Augmented}(x)$ [2, 3, 23, 27]. In addition to evaluating image-wise augmentations, recent research has demonstrated that incorporating class-wise and instance-based consistencies yields superior performance outcomes [17, 37]. Similarly, using consistencies between augmentations, of the predictions and low-dimensional embeddings of the strong and weak augmentations of the unlabeled images in a graph based setup has shown improvement over class-wise and instance-based consistencies [38]. Finally, pseudo-labeling filtering based on consistence between strongly augmented views, gaussian filtering and embedding based nearest neighbor filtering shows convergence improvement [14, 21].

2.3. Embedding Clustering/Constraints

Several papers have attempted to improve the quality of pseudo-labels to either improve the final model accuracy, improve the rate of convergence, or avoid confirmation bias [1]. Rizve et al. [25] explores how uncertainty aware pseudo-label selection/filtering can be used to reduce the label noise. Incorrect pseudo-labels can be viewed as a network calibration issue [25] where better network logit calibration might improve results [32]. Other work has attempted to improve the pseudo-labeling process by imposing curriculum [36] or by including a class-aware contrastive term [33]. Previous work has leveraged the concept of explicit class cluster centers for conditioning semantic similarity [37]. Recent work has extended purely clustering based methods like DINO [7] into semi-supervised methods [12].

3. Methodology

In this section, we explore our proposed final layer replacements and embedding space constraints. FixMatch [27] is a simple, well performing, SSL algorithm. As such it serves as a good comparison point for exploring the effect of our contributions. Our methodology is based upon the published FixMatch [27] algorithm, with identical hyperparameters unless otherwise stated. We add to FixMatch an

early-stopping condition when the model has not improved for 40 epochs (where epoch size is 1024 batches as in Fix-Match), with 2 learning rate reductions by a factor of 0.2 instead of configuring a fixed number of training epochs with a cosine learning rate decay. Additionally, we employ a cyclic learning rate scheduler to vary the learning rate by a factor of ± 2.0 within each epoch to make training less dependent upon exact learning rate value.

Both the linear layer replacements and the embedding constraints explored herein represent increasing levels of prescription about how the final embedding space should be arranged compared to a final linear layer. The idea of leveraging clusters in embedding space is not new [5, 6, 11], but we extend the core idea with a novel differentiable model of learned cluster centroids and GMMs.

3.1. Alternative Final Layers

A limitation of traditional final activation layers such as softmax is that they are fully discriminative; i.e. they estimate the posterior $p(Y|X)$, but do not attempt to model the sample distribution $p(X)$ or the joint probabilities $p(Y, X)$. To overcome this limitation, we present two semi-parametric final activation layers (a) the Axis Aligned GMM (AAGMM) layer, and (b) an equal variance version of AAGMM that we henceforth call the KMeans activation layer due to the similarity of the objective function with a gradient based KMeans.

These activation layers are fully differentiable and integrated into the neural network architecture as a module in the same way as a traditional final linear layer. As such, they do not require or depend on external training and do not use expectation maximization. They are drop in replacements for the final linear layer.

Importantly, these activation layers exhibit both discriminative and generative properties. The neural network model $F(X; \theta_F)$ transforms the data X into a latent space $Z = F(X; \theta_F)$, and the final activation layer estimates the probability densities $p(X)$, $p(Y; X)$ and $p(Y|X)$ by fitting a parametric model to the latent representation Z .

3.1.1 Axis Aligned Gaussian Mixture Model Layer

The AAGMM layer defines a set of K trainable clusters, one cluster per label category. Each cluster $k = 1 \dots K$ has a cluster center μ_k and cluster covariance Σ_k . The prior probability of any given sample X_i is defined by the mixture of cluster probability densities over the latent representation Z_i as follows,

$$p(X_i) = \sum_{k=1}^K \mathcal{N}(Z_i, \mu_k, \Sigma_k) \quad (1)$$

$$\text{where } Z_i = F(X_i, \theta_F)$$

Where $\mathcal{N}(Z_i, \mu_k, \Sigma_k)$ represents the multivariate gaussian pdf with centroid μ and covariance Σ_k . AAGMM is axis aligned because Σ_k is a diagonal matrix, as such the Normal pdf simplifies to the joint density of the pdfs along each of the D axes as follows,

$$\mathcal{N}(X_i, \mu_k, \Sigma_k) = \prod_{d=1}^D \frac{1}{\sigma_{k,d} \sqrt{2\pi}} \exp\left(-\frac{(Z_{i,d} - \mu_{k,d})^2}{\sigma_{k,d}^2}\right) \quad (2)$$

$$\text{where } \sigma_{k,d}^2 = \Sigma_{k,d,d}$$

As there is one cluster per label category, the joint probability for sample i with label assignment k , $p(Y_k, X_i)$ is the given by the normal pdf of the k^{th} cluster,

$$p(Y_{i,k}, X_i) = \mathcal{N}(Z_i, \mu_k, \Sigma_k) \quad (3)$$

By simple Bayesian identity, the posterior probability $\hat{Y}_k = p(Y_k|X_i)$ can therefore be inferred from eq 1 and 3 as follows,

$$\hat{Y}_{i,k} = p(Y_{i,k}|X_i) = \frac{p(Y_{i,k}, X_i)}{p(X_i)} \quad (4)$$

3.1.2 KMeans Layer

The KMeans final layer is a more restrictive form of the AAGMM layer, in the sense that we impose an additional constraint that the gaussian covariance matrix Σ_k for each cluster center k is the $[D \times D]$ identity matrix. This constraint yields spherical cluster centers similar to how the traditional KMeans algorithm also assumes spherical clusters.

3.2. Method of Moments Embedding Constraints

We introduce and evaluate a series of embedding constraints based on the Method of Moments (MoM). As our goal is semi-supervised classification, the primary goal is to minimize the expected difference between Y and \hat{Y} through cross entropy loss. However, if one were to train the model without any embedding constraints, then it is possible that the model could learn a good decision boundary for the posteriors $p(Y|X)$ but without actually modeling the probability of samples $p(X)$. The first four gaussian moments being identity/zero increasingly constrain the model latent embedding space.

The MoM relies on the use of *consistent estimators*, as these asymptotically share sample and population statistics. Assume that z is a finite sample of n elements drawn from infinite population Z , then a series of P well-behaved sample statistics g_p should very closely approximate their k population statistic as follows,

$$\forall p = 1 \dots P \quad \frac{1}{n} \sum_{i=1}^n g_p(z_i) \approx E(g_p(Z)) \quad (5)$$

We can therefore constrain the latent representation of our model to approximate an independent joint Gaussian distribution. In the univariate Gaussian case, the p^{th} order centralized moment constraint is the following.

$$E[(Z - \mu)^p] = \begin{cases} 0 & \text{if } p \text{ is odd} \\ \sigma^p(p-1)!! & \text{if } p \text{ is even} \end{cases} \quad (6)$$

By this formula, the univariate unit gaussian has mean 0, standard deviation 1, skew 0, and kurtosis 3.

In the joint multivariate case, each dimension is independent by definition. As such, if we redefine Z , μ , and p to be all D dimensional, then the centralized joint gaussian moment can be defined as follows,

$$E[g_p(Z - \mu)] = E\left[\prod_{d=1}^D (Z_d - \mu_d)^p\right] \quad (7)$$

Due to independence of the axes, this moment can be represented as a product of univariate moments of the individual gaussians as follows,

$$E\left[\prod_{d=1}^D (Z_d - \mu_d)^p\right] = \prod_{d=1}^D E[(Z_d - \mu_d)^p] \quad (8)$$

The error (loss) term associated with the embedding constraint for any moment p is equal to the L2 difference between the sample and population statistics as follows,

$$\varepsilon_p = \left(\frac{1}{n} \sum_{i=1}^n g_p(z_i) - E(g_p(Z))\right)^2 \quad (9)$$

Some moments are more important than others, and must be weighted more heavily. For example, first order moments are simply the sample mean, and should be given the greatest weight as an embedding constraint. The second order moments form a sample covariance matrix, which ideally should be equal to the identity matrix, but the diagonal terms should be given greater weight than the off-diagonal terms. This is because, in a $D \times D$ covariance matrix, there are $D(D-1)$ off diagonal terms, but only D , diagonal terms. The p^{th} order sample moments form a $p-1$ dimensional hyper-covariance matrix, with terms residing on the intersection of anywhere between 0 and $p-1$ hyper-diagonals. In order to prevent over-representation of off-diagonal terms, and encourage representation of on-diagonal terms, the loss function for any given moment term is inversely proportional to the number moment terms that share the same number of hyper-diagonals. This heuristic weighting scheme is reasonable as it ensures that the overall contribution of each moment order is not overly influenced by the off-diagonal terms, and that the error weighting is therefore diagonally dominant.

3.2.1 Moment1

This constraint encourages the learned cluster means to have zero distance to the observed centroids. Define $pred_k$ as the set of indices $i = 1 \dots K$ such that are predicted to be of category k as follows,

$$pred_k = \{i \mid \underset{j}{\operatorname{argmax}} \hat{Y}_{i,j} \text{ is equal to } k\} \quad (10)$$

This embedding constraint ensures that cluster centers μ_k approximate the sample distribution of latent points that are part of cluster k , $E(Z_i | \hat{Y}_{i,k} = 1)$. The $L2$ norm is used as a loss function to constrain cluster centers as follows,

$$\mathcal{L}_{L2} = \sum_{k=1}^K \left\| \mu_k - \bar{Z} \right\|_2 \quad (11)$$

where $\bar{Z} = E_{i \in pred_k} Z_i$

3.2.2 Moment2

Additionally, the L2 norm was also used to constraint the cluster covariance terms to the identity matrix. Although the AAGMM assumes a diagonal covariance matrix Σ_k , the mini-batch sample estimate of the covariance may exhibit off-diagonal terms, which represent

3.2.3 Moments 3 and 4

Moment3 encourages clusters to have zero skew.

Moment4 encourages cluster kurtosis to be the identity matrix.

[TODO: figure out how to talk about the moments 3 and 4, since all of our testing indicates that the GPU memory requirement is horrifying, and they are unstable as hell. As such they are not useful but as a baseline to compare the feasibility.]

4. Experiments

[TODO: (majurski) add rows for embedding size]

We evaluate enhancements replacements on common SSL datasets CIFAR-10/100 [15] under various label scarcities. When comparing against other CIFAR-10/100 SSL benchmark results (like SimMatch [37]) it is unclear how the labeled samples were selected. For our evaluation, for each model training run, the required number of labeled samples are drawn without replacement from the training population of the dataset. All data not in this labeled subset is used as unlabeled data (i.e. labels are not used). We evaluate our method against CIFAR-10 at 250 and 40 labeled data points, and CIFAR-100 at 400 and 2500 labels. This corresponds to 4 or 25 samples per class. As prior work [27] has noted, resulting model quality is highly variable when

CIFAR-10

Last Layer	Emb Dim	40 Labels				
Embedding Constraint		None	Moment1	Moment12	Moment123	Moment1234
FullyConnected	128	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	32	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	8	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
KMeans	128	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	32	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	8	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
AAGMM	128	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	32	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	8	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
Last Layer	Emb Dim	250 Labels				
Embedding Constraint		None	Moment1	Moment12	Moment123	Moment1234
FullyConnected	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
KMeans	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
AAGMM	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$
	???	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$

Table 1. Error rate % for CIFAR-10 SSL benchmark comparing various configurations of our method. Results are based on 6 runs.

only 4 samples are selected per class, as the quality and usefulness of the specific 4 samples can vary drastically.

4.1. CIFAR-10

[TODO: document hyper-parameters]

Method	CIFAR-10	
Label Count	40	250
Supervised	77.18 \pm 1.32	56.24 \pm 3.41
FixMatch	13.81 \pm 3.37	5.07 \pm 0.65
FlexMatch	5.29 \pm 0.29	4.97 \pm 0.07
SimMatch	5.38 \pm 0.01	5.36 \pm 0.08
SimMatchV2	4.90 \pm 0.16	5.04 \pm 0.09
Ours (TBD)	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$

Table 2. Error rate % for CIFAR-10 SSL benchmark comparing to state of the art results. Results are copied from USB [29] unless otherwise stated. USB results are based on 3 runs. FixMatch [27] results are based on 5 runs.

4.2. CIFAR-100

[TODO: document hyper-parameters]

Method	CIFAR-100	
Label Count	400	2500
Supervised	89.6 \pm 0.43	58.33 \pm 1.41
FixMatch	48.85 \pm 1.75	28.29 \pm 0.11
FlexMatch	40.73 \pm 1.44	26.17 \pm 0.18
SimMatch	39.32 \pm 0.72	26.21 \pm 0.37
SimMatchV2	36.68 \pm 0.86	26.66 \pm 0.38
Ours (TBD)	$xx.yy \pm z.zz$	$xx.yy \pm z.zz$

Table 3. Error rate % for CIFAR-100 SSL benchmark comparing to state of the art results. Results are copied from USB [29] unless otherwise stated. USB results are based on 3 runs. FixMatch [27] results are based on 5 runs.

4.3. Ablation Study

5. Conclusion

6. Acknowledgment

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Run Number	Our method on CIFAR-10
Label Count	40
Run 1	$xx.yy \pm z.zz$
Run 2	$xx.yy \pm z.zz$
Run 3	$xx.yy \pm z.zz$
Run 4	$xx.yy \pm z.zz$
Run 5	$xx.yy \pm z.zz$
Run 6	$xx.yy \pm z.zz$
Run 7	$xx.yy \pm z.zz$
Run 8	$xx.yy \pm z.zz$
Run 9	$xx.yy \pm z.zz$
Run 10	$xx.yy \pm z.zz$

Table 4. Error rate % for CIFAR-10 SSL benchmark showing the run-to-run variance depending on the quality of the 40 labels selected from the full population.

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