

# Beyond the Final Linear Layer: Enhancing Decision Boundaries

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## Abstract

*SSL leverages an abundance of unlabeled data to improve deep learning based model performance under limited training data regimes. This paper presents a novel extension to any image classification architecture which improves accuracy in low-label regimes. We extend the FixMatch [1] training scheme with our novel last layers and demonstrate test accuracy improvement. The novelty consists of 2 elements: first we replace the last linear layer with a GMM trained via backprop, and second, we impose class-wise constraints on the embedding space the GMM operates on. These methods match published SOTA 250 label Cifar10 [2] results and come close to matching SOTA in the 40 label regime without the significant model complexity of methods like SimMatchV2 [3]. Our method achieves 94.8% and 94.2% accuracy with 250 and 40 Cifar10 labels respectively.*

## 1. Introduction

SSL leverages an abundance of unlabeled data to improve deep learning based model performance under limited training data regimes [4, 5, 6]. Image classification has become a playground for exploring new SSL ideas. The early successes of deep learning based methods relied on large annotated datasets to enable models to learn the relevant features to perform the task, i.e. image classification build on top of ImageNet [7]. With data annotation becoming a significant bottleneck, especially in application domains outside of the standard benchmarks, another learning paradigm was needed.

There are several flavors of SSL. Contrastive learning methods leverage the intuition that similar instances should be close in the representation space, while different instances are farther apart [8, 9]. Consistency regularization borrows the intuition that modified views of the same instance should have similar representations and predictions

[1, 10, 11, 12]. Pseudo-labeling methods like FixMatch [1] fall within the consistency regularization domain.

## 2. Related Work

### 2.1. Pseudo-Labeling

Self-supervised learning was among the initial approaches employed in the context of semi-supervised learning to annotate unlabeled images. This technique involves the initial training of a classifier with a limited set of labeled samples and incorporates pseudolabels into the gradient descent process, exceeding a predefined threshold [13, 14, 15, 16, 17, 18]. A closely related method to self-training is co-training, where a given dataset is represented as two distinct feature sets. These independent sample sets are subsequently trained separately using two distinct models, and the sample predictions surpassing predetermined thresholds are utilized in the final model training process [19, 20]. A notably advanced approach to pseudolabeling is the Mean Teacher algorithm [21], which leverages exponential moving averages of model parameters to acquire a notably more stable target prediction. This refinement has a substantial impact on enhancing the convergence of the algorithm.

### 2.2. Consistency Regularization

Consistency regularization operates on the premise that when augmenting an unlabeled sample, its label should remain consistent. This approach implicitly enforces a smoothness assumption, promoting coherence between unlabeled samples and their basic augmentations. In other words, the model should be able to predict the unlabeled sample  $x$  exactly the same way it predicts the class for Augmented( $x$ ) [1, 22, 23, 24]. In addition to evaluating image-wise augmentations, recent research has demonstrated that incorporating class-wise and instance-based consistencies yields superior performance outcomes [9, 25].

## 2.3. Contrastive Learning

## 3. Methodology

### 3.1. KMeans

### 3.2. Axis Aligned Differentiable Gaussian Mixture Model

### 3.3. Embedding Constraints

## 4. Experiments

### 4.1. Ablation Study

## 5. Conclusions

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