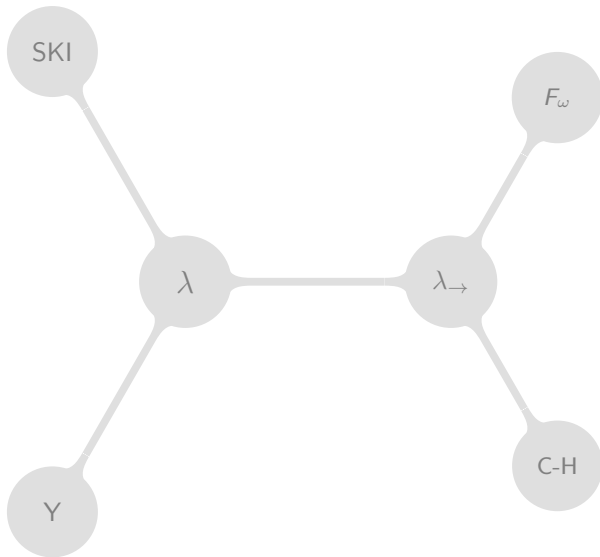# Introduction to Lambda Calculus
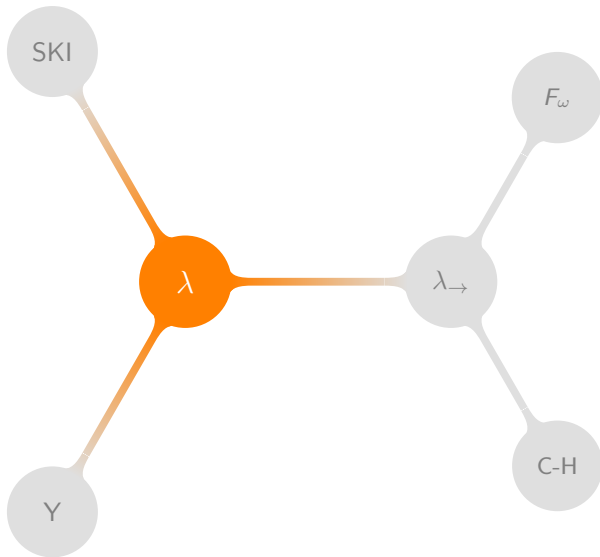
Maciek Makowski (@mmakowski)

12th August 2014

The Plan

Basic Lambda Calculus

Syntax

$$\langle term \rangle ::= x \qquad \qquad \text{(variable)}$$
$$| \quad (\lambda x.\langle term \rangle) \qquad \text{(abstraction)}$$
$$| \quad (\langle term \rangle \ \langle term \rangle) \qquad \text{(application)}$$

where $x \in \mathbb{X}$ – the set of variables

Syntax

v_1

Syntax

v_1 var v_1

Syntax

x y

Syntax

$x\ y$

```
         app
        /    \
    var x    var y
```
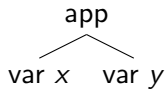
Syntax

$\lambda a.b$

Syntax

$\lambda a.b$

```
abs a
 |
var b
```

Syntax

$(\lambda a.\lambda b.a)\ c\ (\lambda b.b)$

Syntax

$$\langle term \rangle ::= x \qquad\qquad\qquad \text{(variable)}$$
$$\mid \quad (\lambda x.\langle term \rangle) \qquad\qquad \text{(abstraction)}$$
$$\mid \quad (\langle term \rangle \ \langle term \rangle) \qquad \text{(application)}$$

Syntax

$(\lambda a.\lambda b.a)\ c\ (\lambda b.b)$

```
                              app
                          ╱          ╲
                       app            abs b
                     ╱      ╲            │
                 abs a     var c       var b
                   │
                 abs b
                   │
                 var a
```
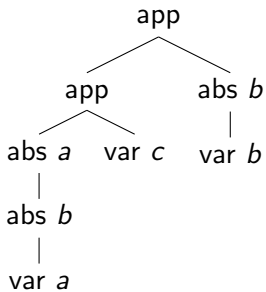
Rewriting

$$(\lambda x.x\,y)\,(\lambda x.x) \longleftrightarrow_\alpha (\lambda a.a\,y)\,(\lambda b.b)$$
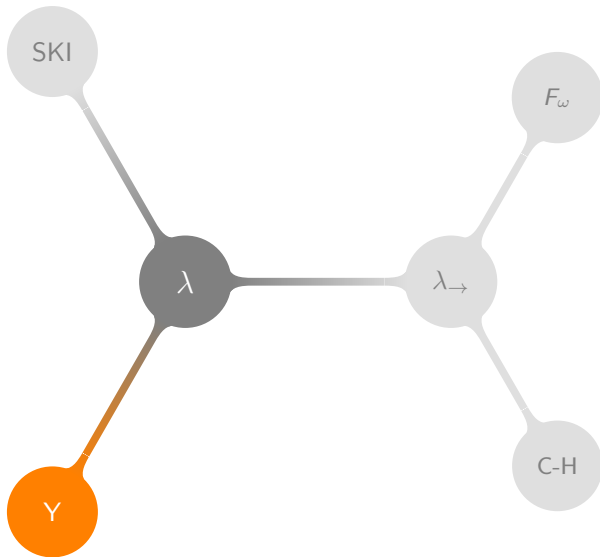
Rewriting

$$(\lambda x.x\,y)\,(\lambda z.z) \longrightarrow_\beta (\lambda z.z)\,y \longrightarrow_\beta y$$
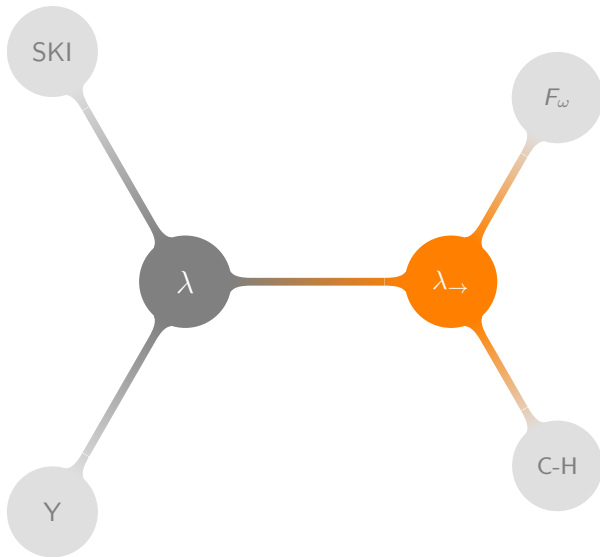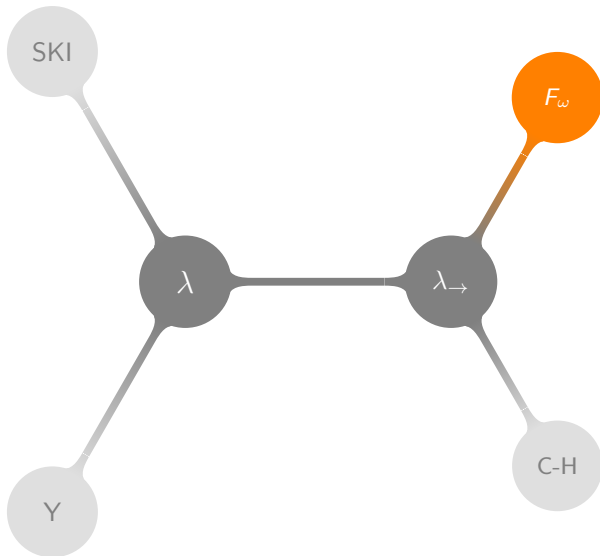
Rewriting

TODO: multiple redexes

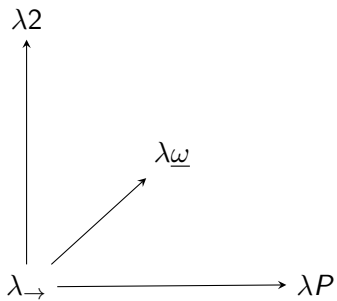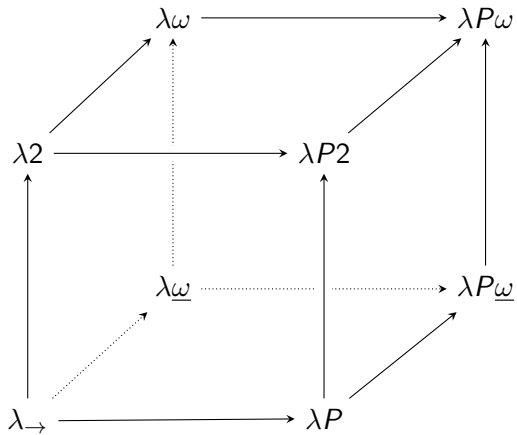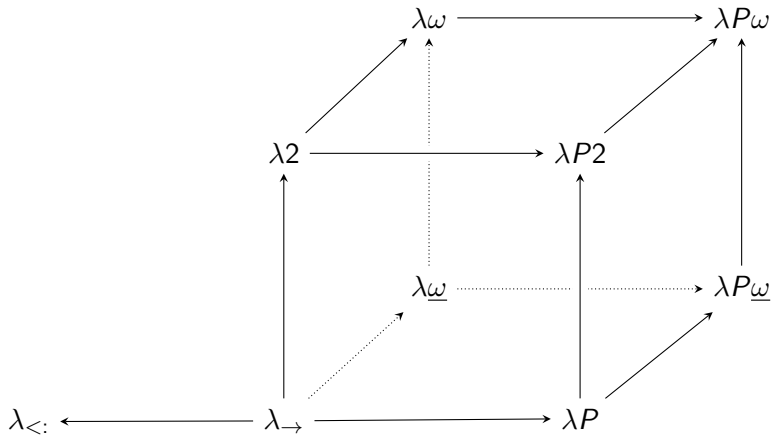# Programming in Lambda Calculus

Simple Types

More Types

More Types

$$\lambda 2$$

$$\lambda \underline{\omega}$$

$$\lambda_{\to} \longrightarrow \lambda P$$

More Types

More Types

More Types