Introduction to Lambda Calculus

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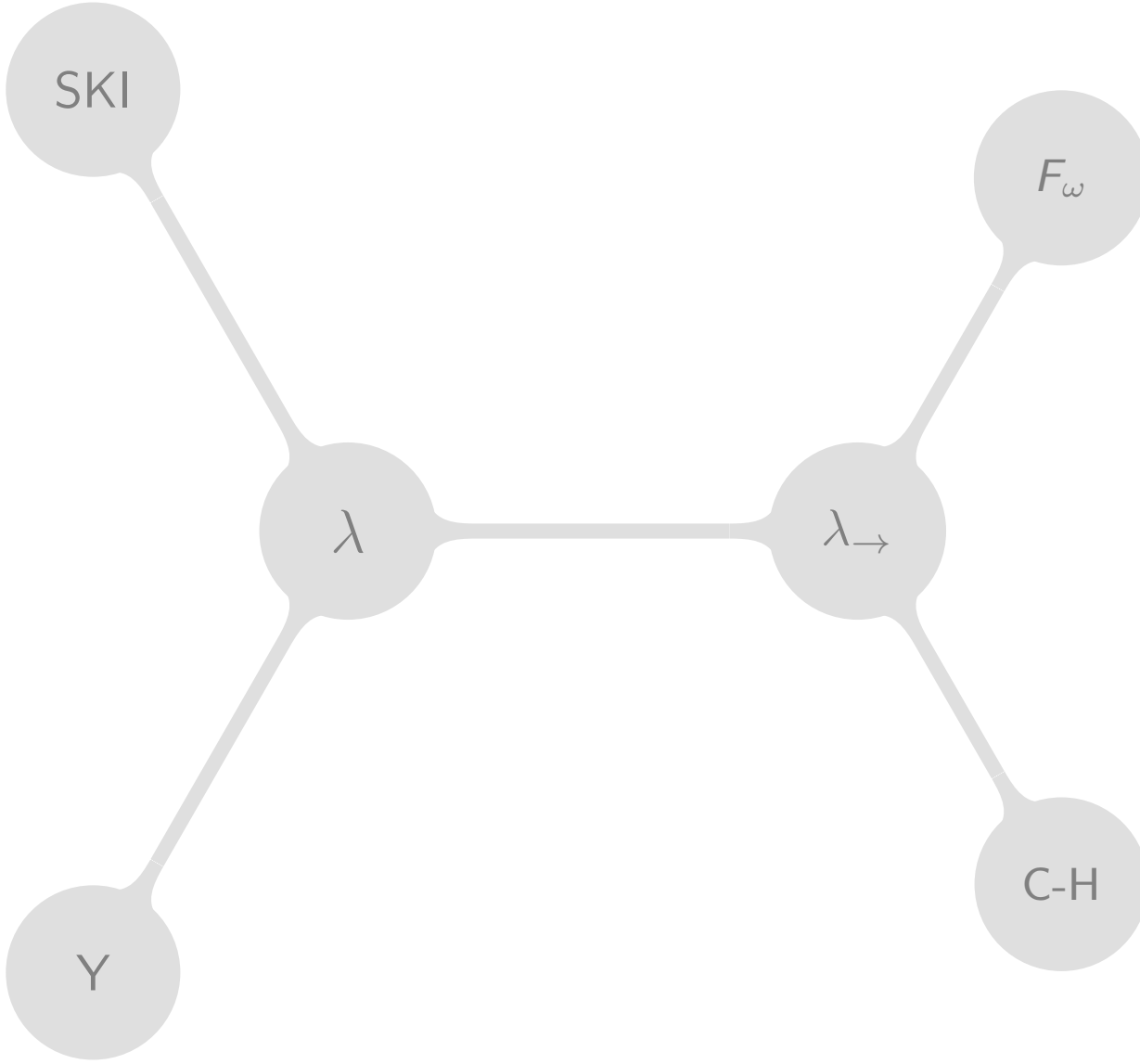
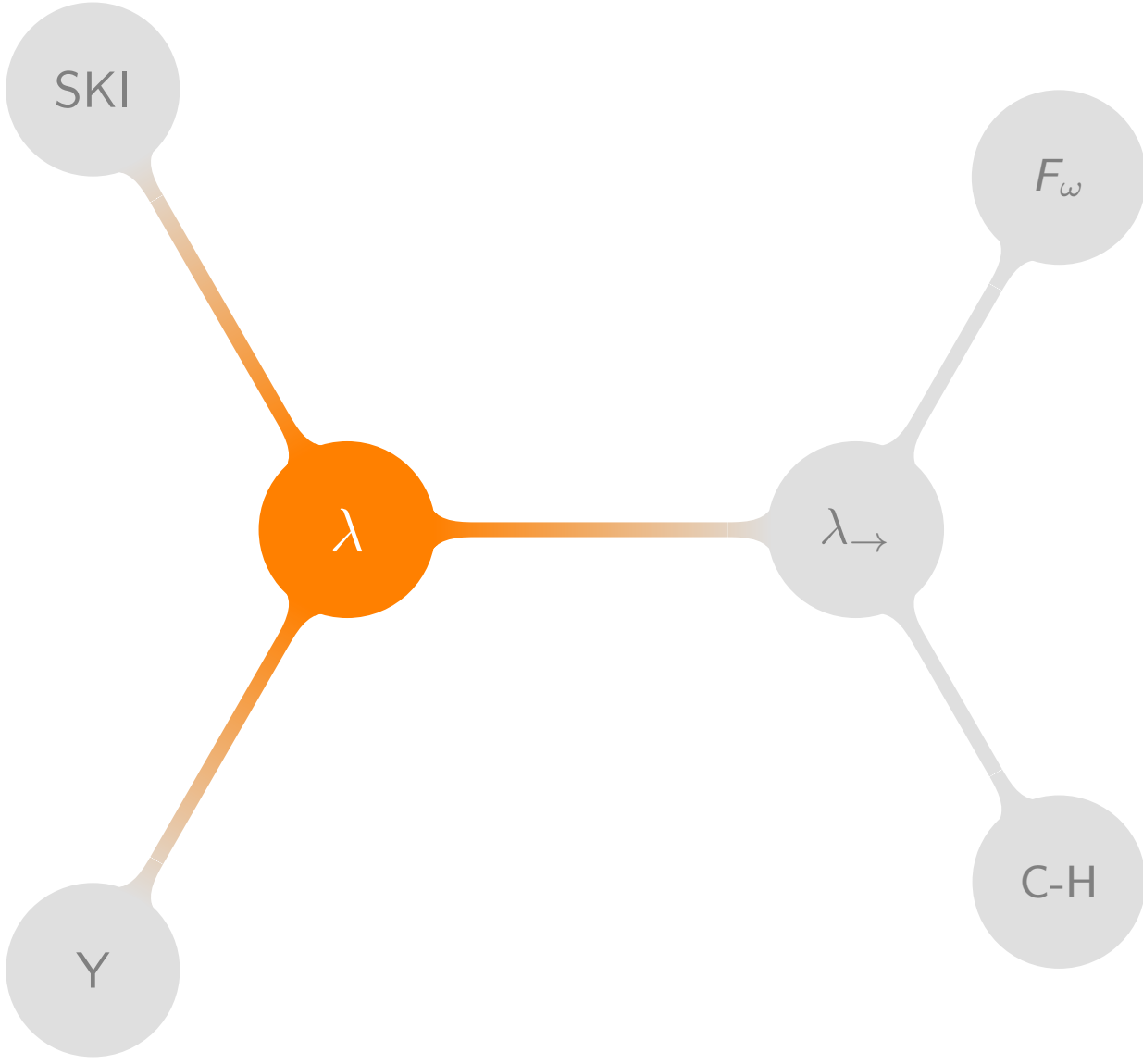
28th September 2014

The Plan



Basic Lambda Calculus



Syntax

$$
\begin{array}{lll}
\langle \mathit{term} \rangle ::= x & & \text{(variable)} \\
\quad | \quad (\lambda x.\langle \mathit{term} \rangle) & & \text{(abstraction)} \\
\quad | \quad (\langle \mathit{term} \rangle \ \langle \mathit{term} \rangle) & & \text{(application)}
\end{array}
$$

where $x \in \mathbb{X}$ – the set of variables

Syntax

v_1

Syntax

v_1

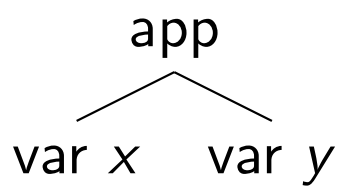
var v_1

Syntax

$x\ y$

Syntax

$x\ y$

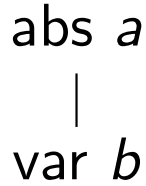


Syntax

$\lambda a.b$

Syntax

$\lambda a.b$



Syntax

$$(\lambda a.\lambda b.a)\ c\ (\lambda b.b)$$

Syntax

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\mid & (\langle \mathit{term} \rangle\ \langle \mathit{term} \rangle) & \text{(application)}
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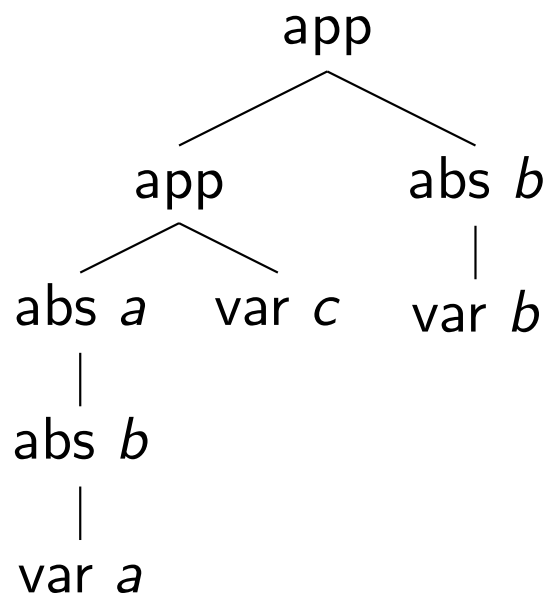
Syntax

$(\lambda a.\lambda b.a)\ c\ (\lambda b.b)$



Rewriting

$$(\lambda x.x\, y)\ (\lambda x.x) \longleftrightarrow_{\alpha} (\lambda a.a\, y)\ (\lambda b.b)$$

Rewriting

$$(\lambda x.x\,y)\ (\lambda z.z) \longrightarrow_\beta (\lambda z.z)\ y \longrightarrow_\beta y$$

Rewriting

TODO: multiple redexes, Church-Rosser

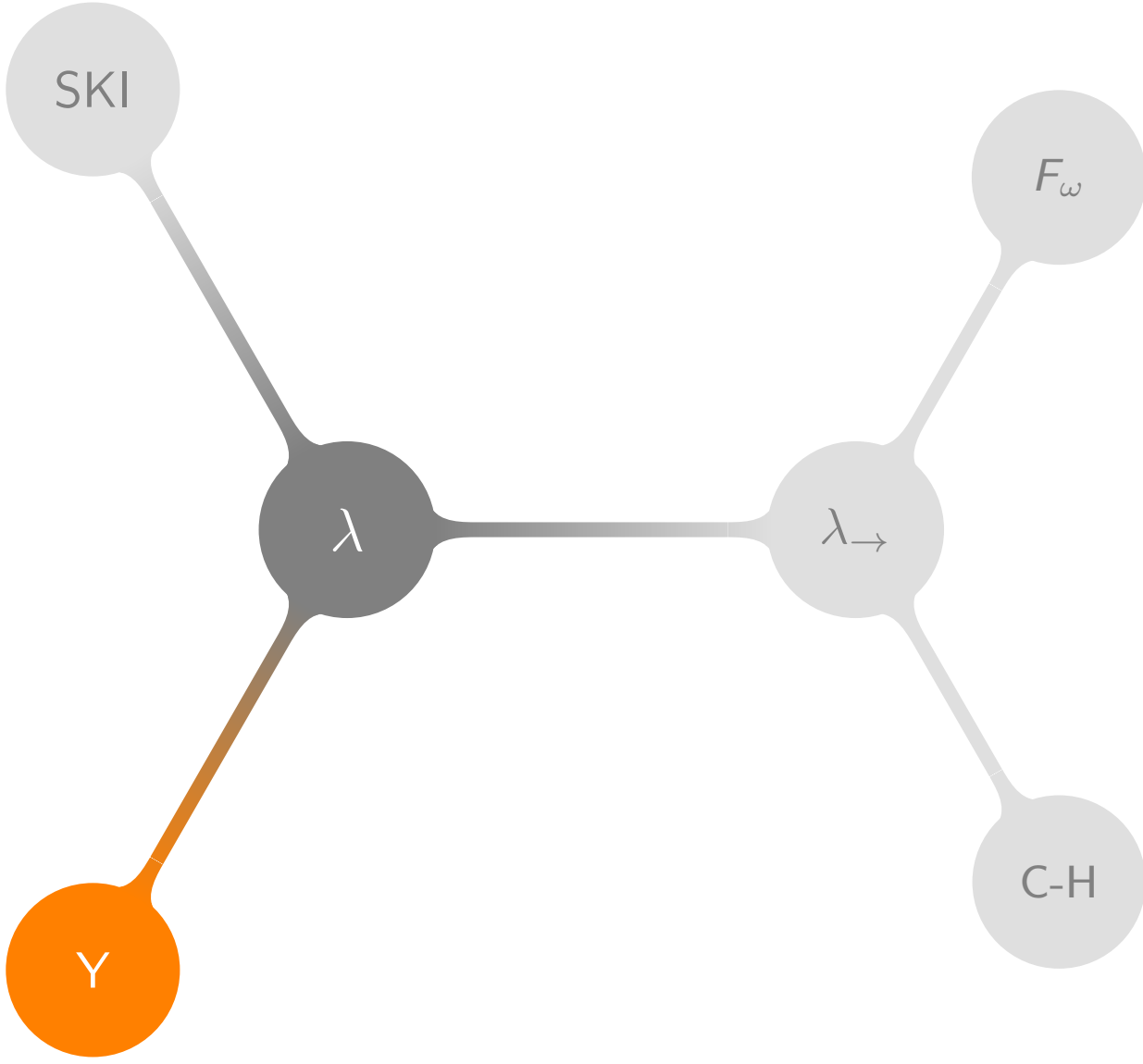
Rewriting

TODO: Ω

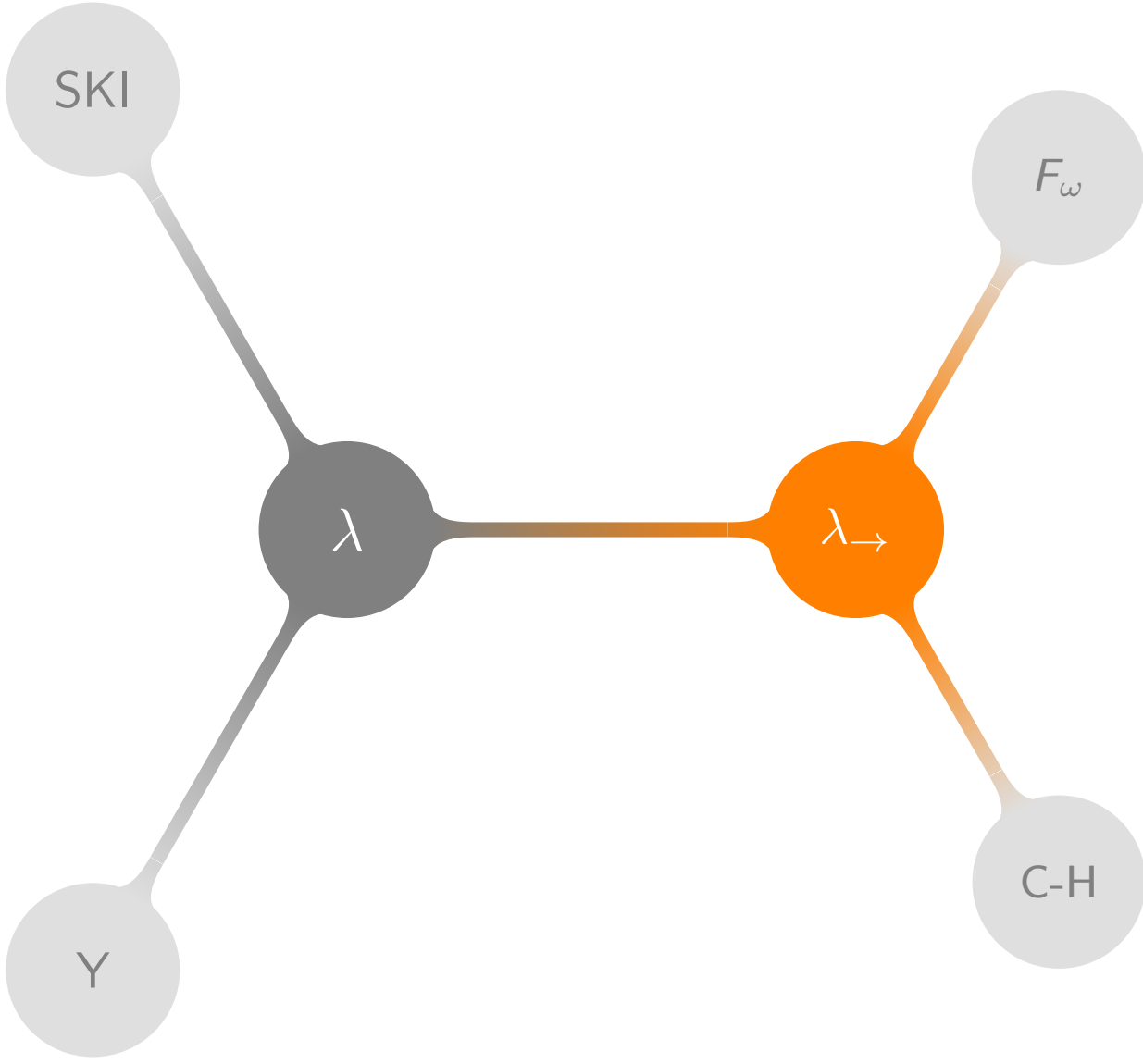
Semantics

TODO: functions

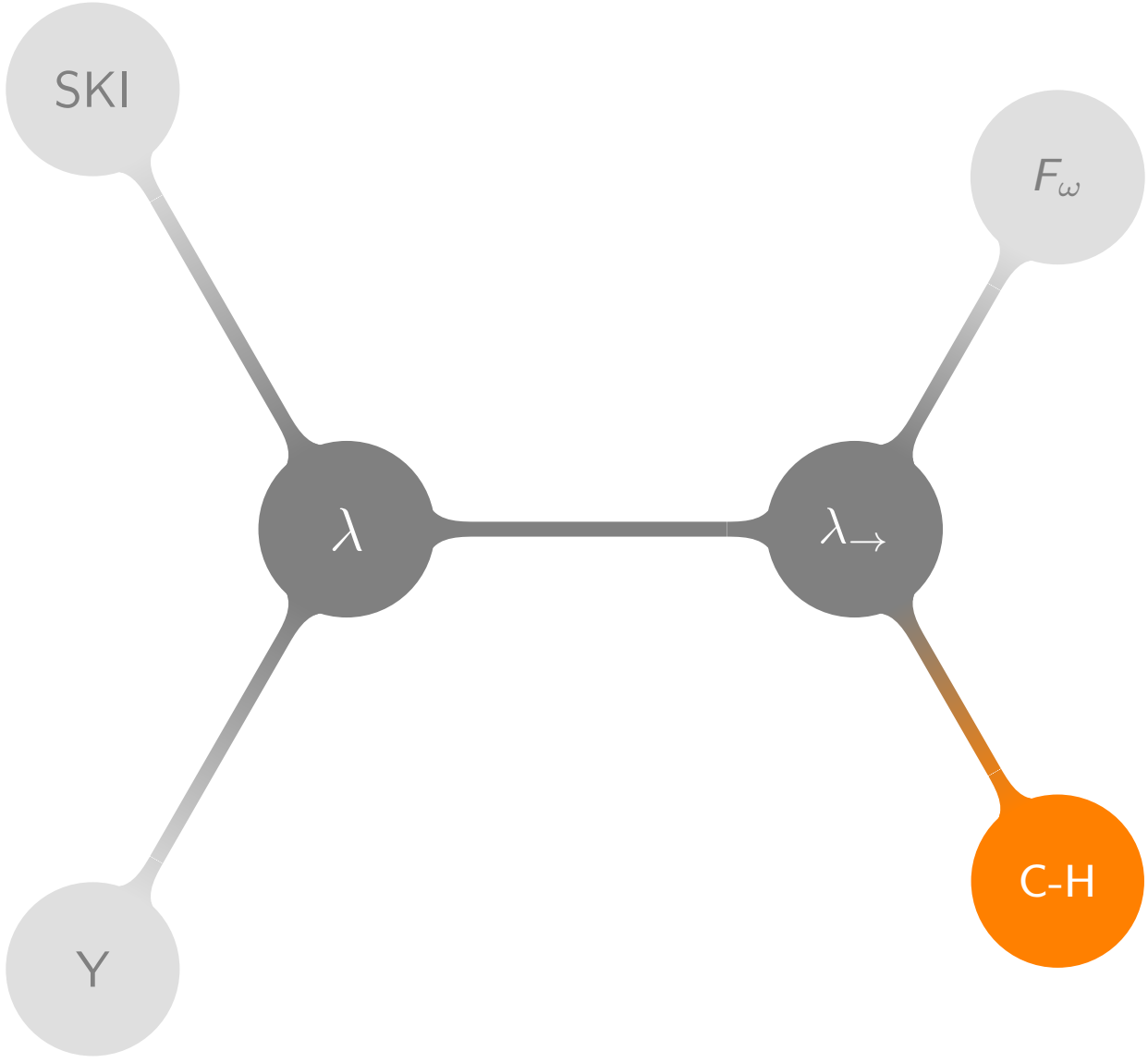
Programming in Lambda Calculus



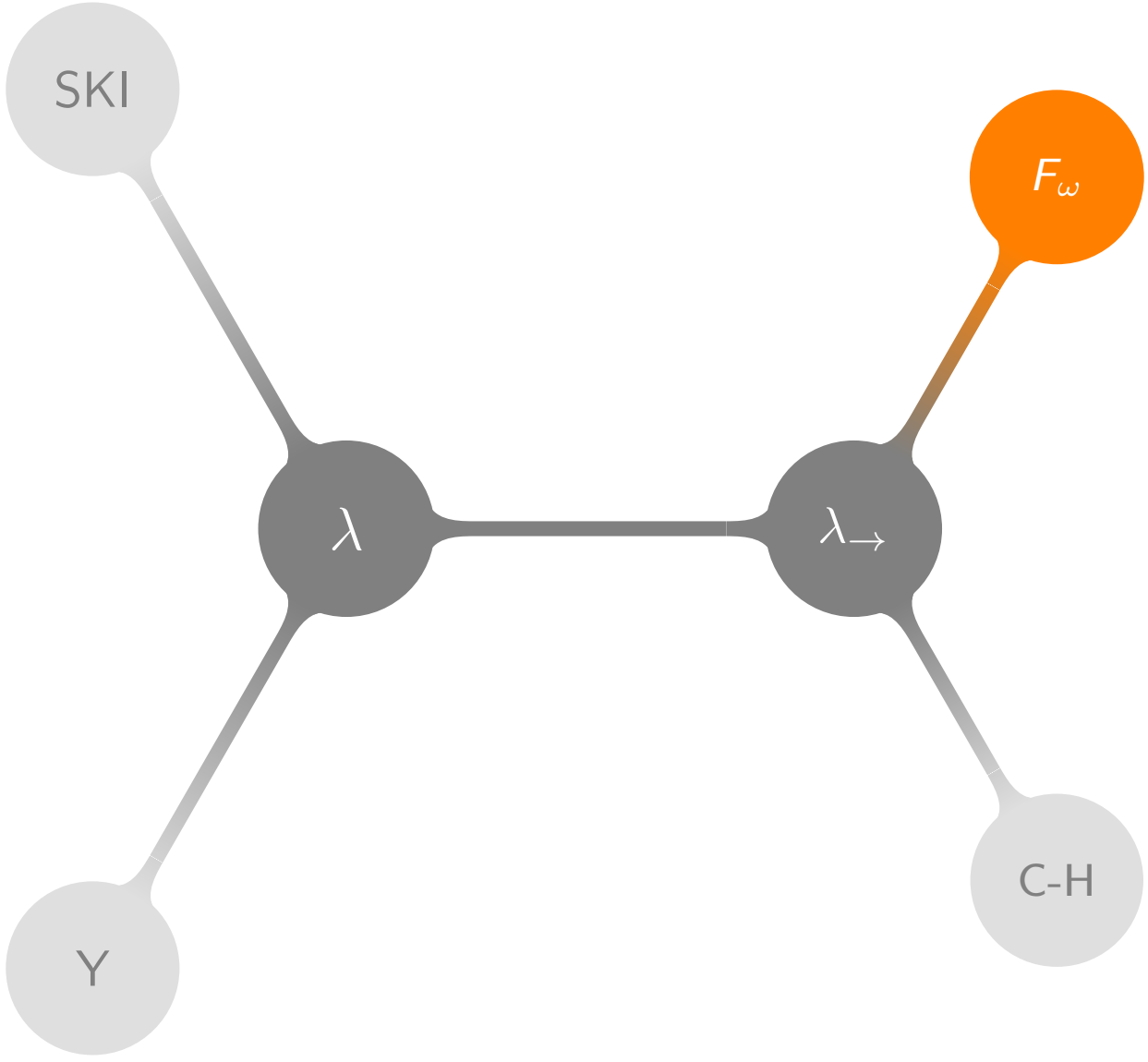
Simple Types



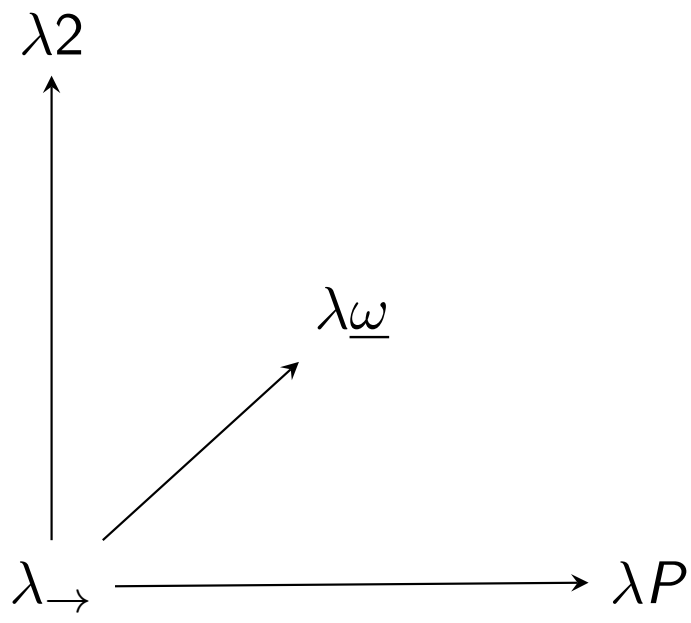
Curry-Howard Correspondence



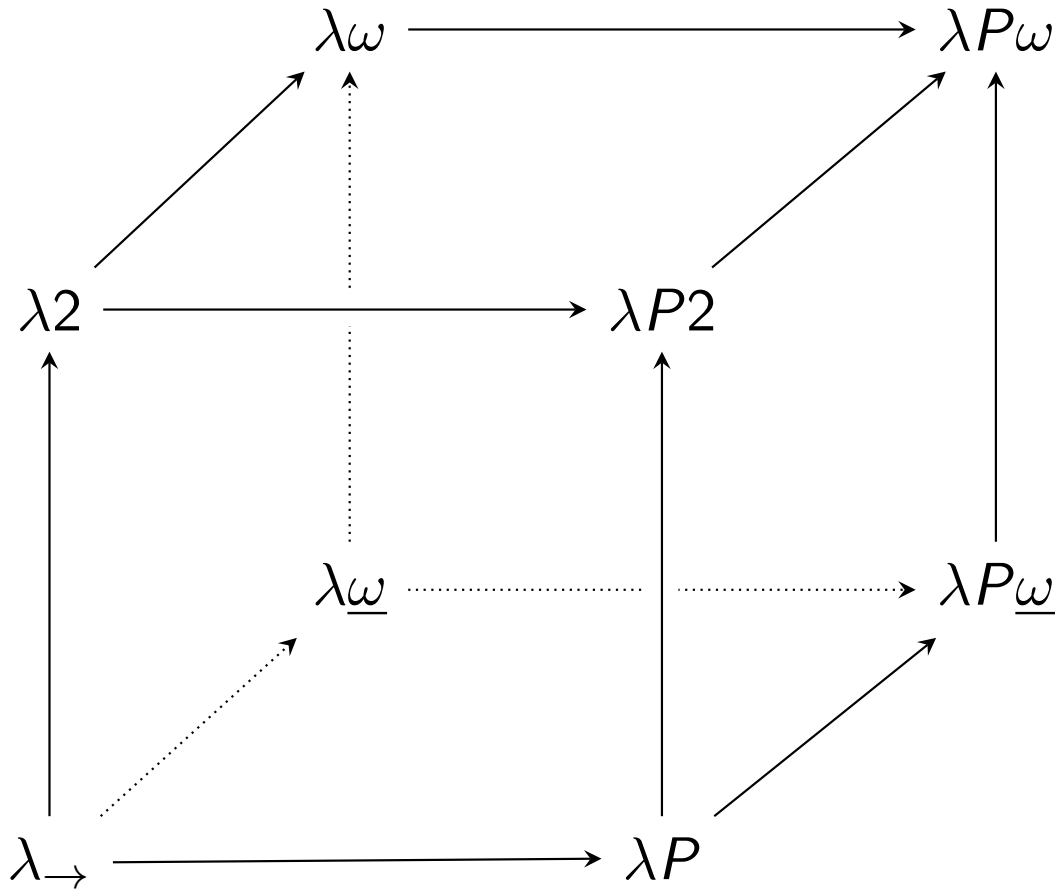
More Types



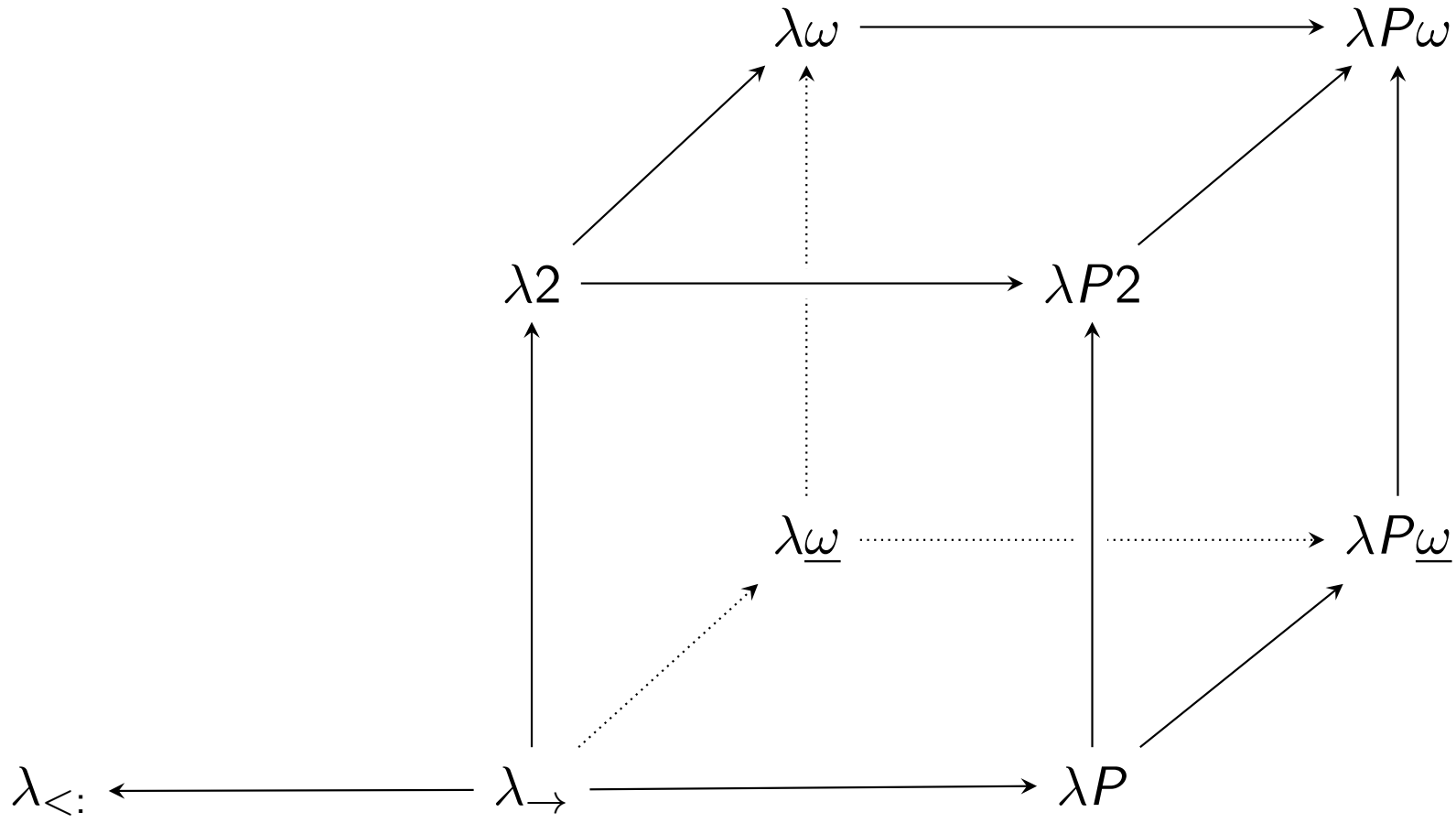
The Lambda Cube



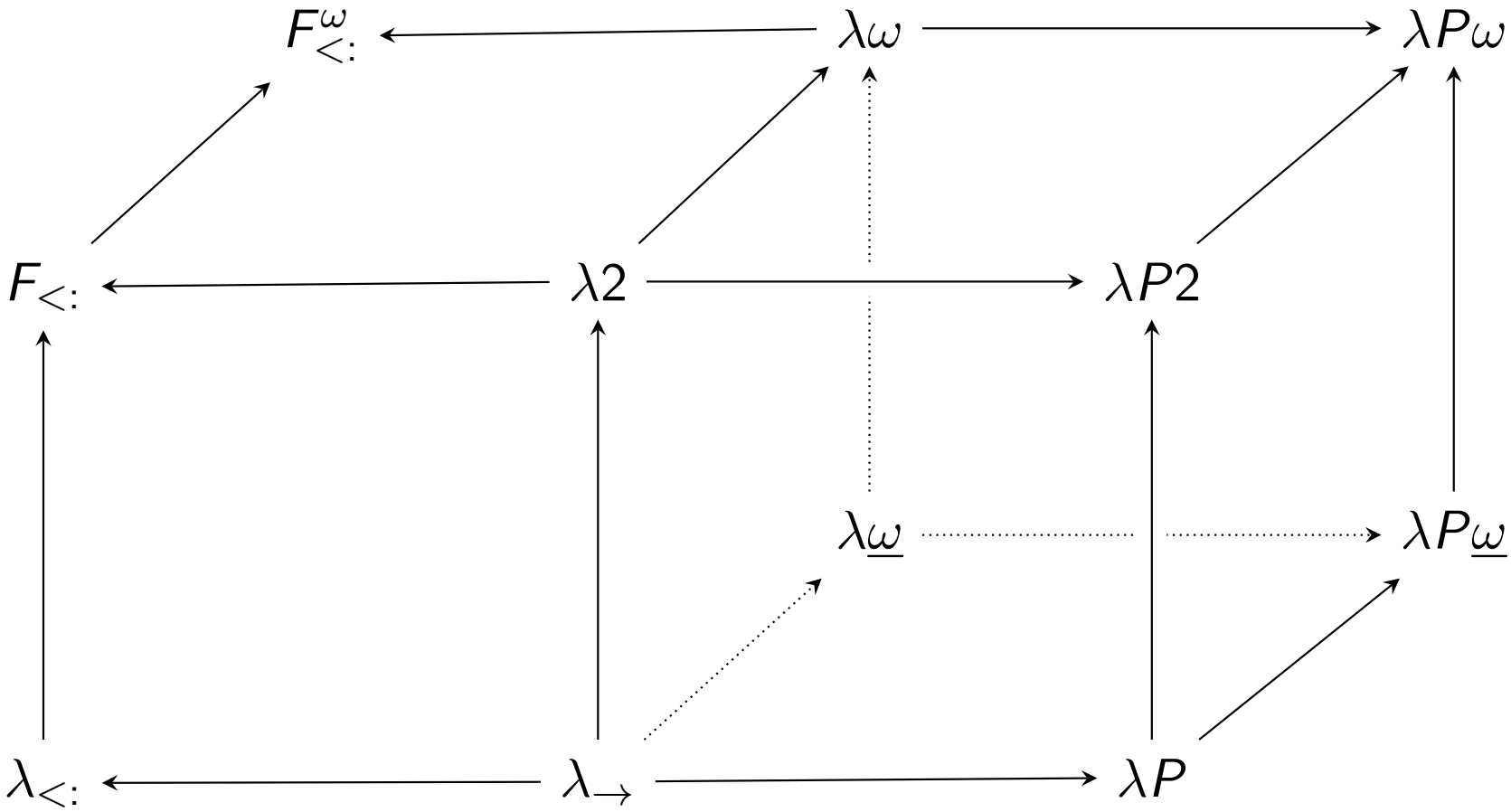
The Lambda Cube



Subtyping


$\lambda\omega$
$\lambda P\omega$
$\lambda 2$
$\lambda P2$
$\lambda\underline{\omega}$
$\lambda P\underline{\omega}$
$\lambda_{<:}$
$\lambda_{\rightarrow}$
λP

Subtyping


$F^{\omega}_{<:}$
$\lambda\omega$
$\lambda P\omega$
$F_{<:}$
$\lambda 2$
$\lambda P2$
$\lambda\underline{\omega}$
$\lambda P\underline{\omega}$
$\lambda_{<:}$
$\lambda_{\to}$
λP