Simulation and numerical analysis

Basics

To simulate paths of USDPLN exchange rate and GOLD price in USD we used Box-Muller transform. The historical data was gained from stooq.com portal. To estimate all parameters data from the period of one year was taken.

The risk free interest rate used for PLN and USD are WIBOR overnight and LIBORUSD overnight, respectively.

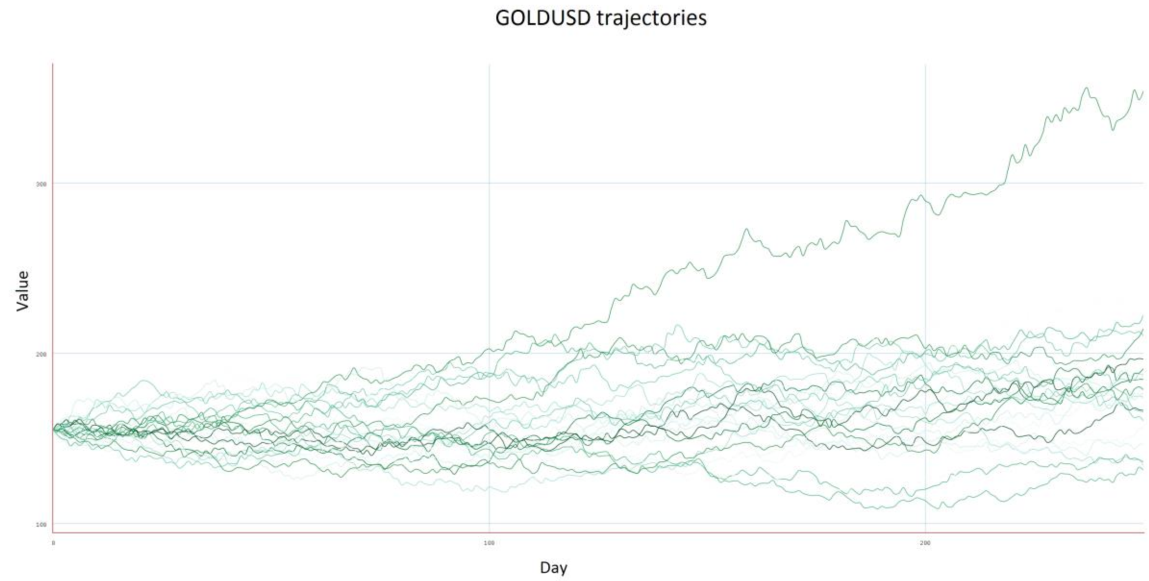

Figure 1

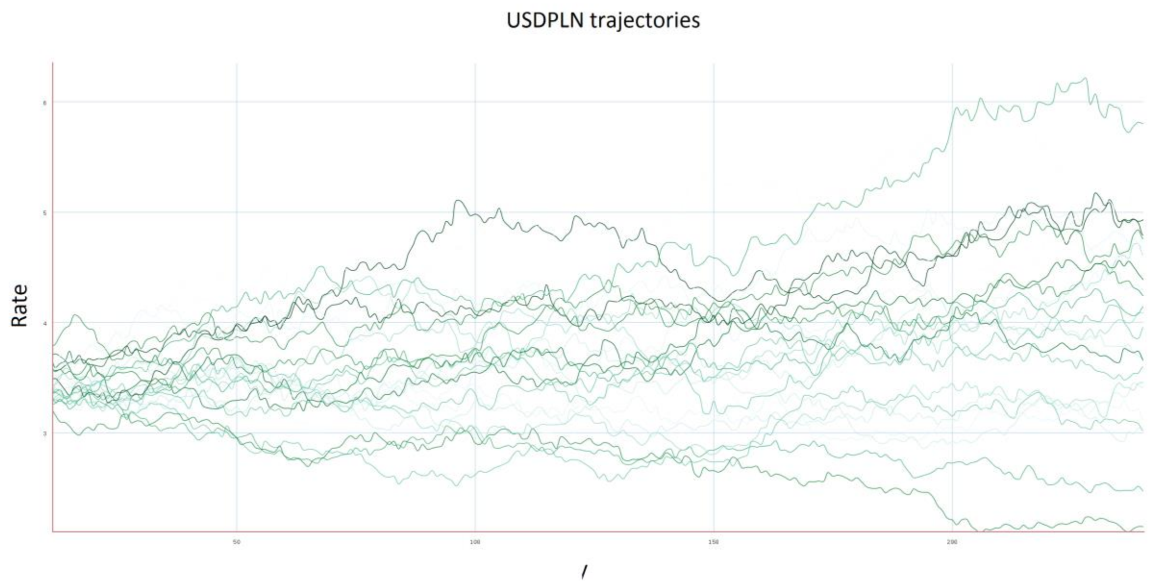

Figure 2

Fig. 1 & 2 shows 25 paths of each underlying, just to have the overall imagination how the price and rate look like. The GOLDPLN asset is made by multiplication of the two.

As introduced earlier, there are two possibly ways of hedging quanto option. First of it is calibrated by USDPLN ex-rate and GOLD in USD, second is calibrated by USDPLN and GOLD in PLN. Theoretically they are exactly the same, but we will see, that in real world they usually gives us different results. This is because, at the very beginning we assume that paths of underlying meets the Brownian motion definition, what, obviously, is not always true.

With following parameters:

- sigma of USDPLN underlying $\sigma_{USDPLN} = 0.1929997$,
- sigma of GOLD in USD $\sigma_{GOLDUSD} = 0.2176335$,
- sigma of GOLD in PLN $\sigma_{GOLDUSD} = 0.2696640$,
- PLN risk free rate $r_{domestic} = 4.90$,
- USD risk free rate $r_{foreign} = 0.16950$,
- correlation between underlyings $\rho_{GOLDUSD} = -0.2449385$, $\rho_{GOLDPLN} = 0.7449756$,
- time horizon $T = 1$,
- number of simulations $n = 10^4$,

the prices are:

- GOLD in USD calibration: 96.97324,
- GOLD in PLN calibration: 95.83456.

The main difference in prices come from of course from difference in sigmas and different formulas. Also, the prices are not

Histograms

On following pages the histograms of Profit/Loss will be presented. The X axis are exactly the same of every plot. The main difference in the calibration methods is the shift of the mass and mean. The first method usually gives us more losses than profits, the second method is exactly opposite. Unfortunately the purpose of it isn't really obvious. Unfortunately, that could be an error in implementation, but there also came out different hypothesis: in the first method, algorithm "buys" gold on American market, so the American risk-free rate was used for discounting, in the second, the Polish risk-free rate was used. The second is around 30 times higher than the first one. The test for this observation was made, but it doesn't change much – still there were more losses that profits, even if $r_{foreign} > r_{domestic}$.

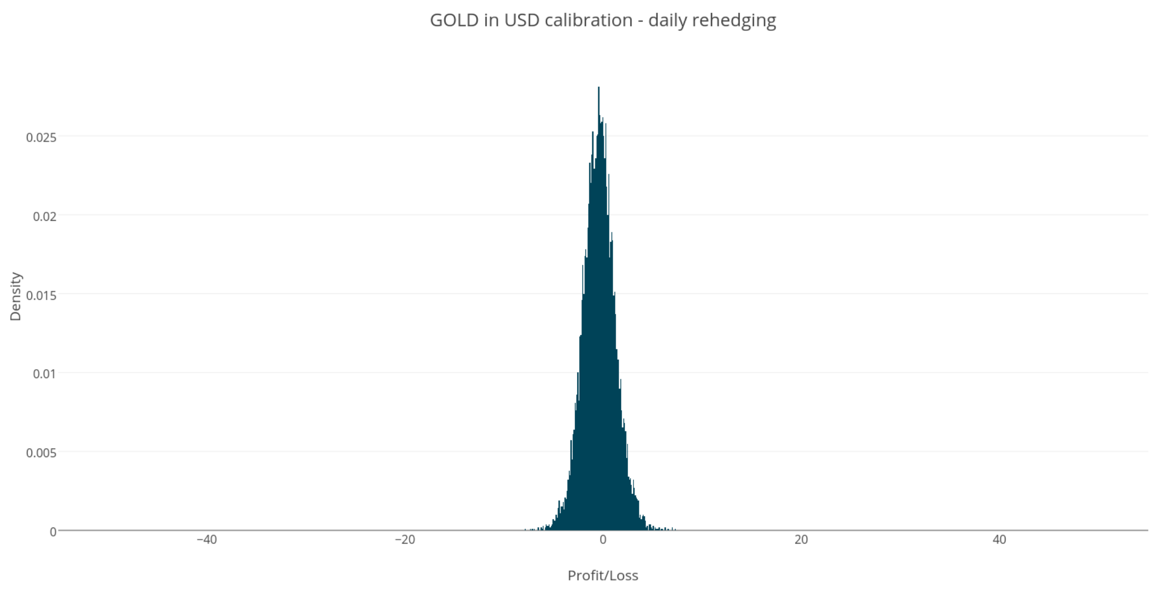

Figure 3

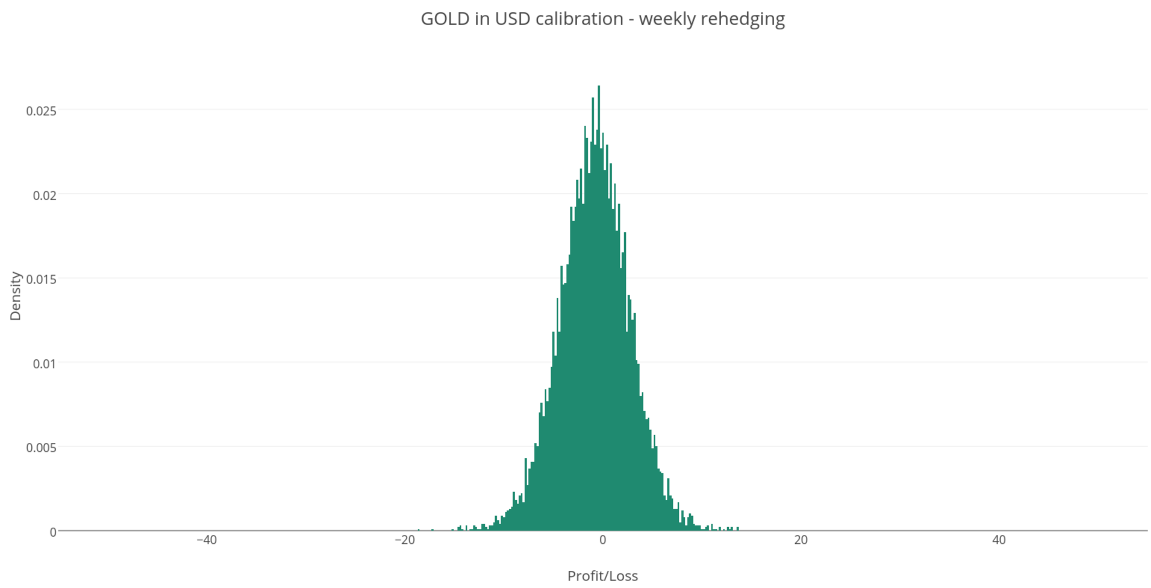

Figure 4

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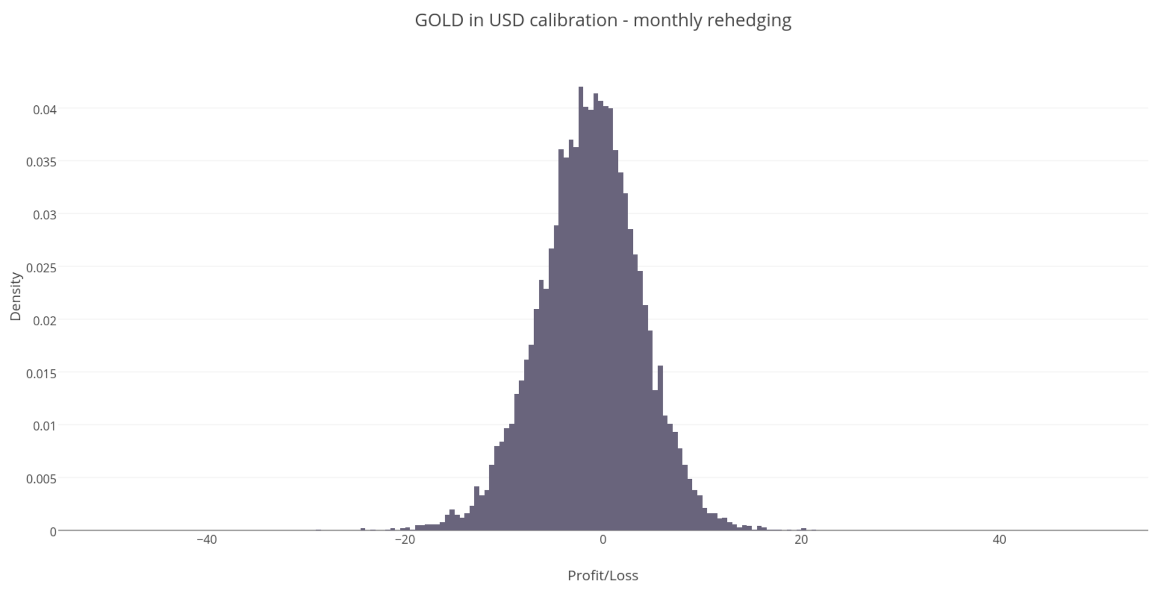

Figure 5

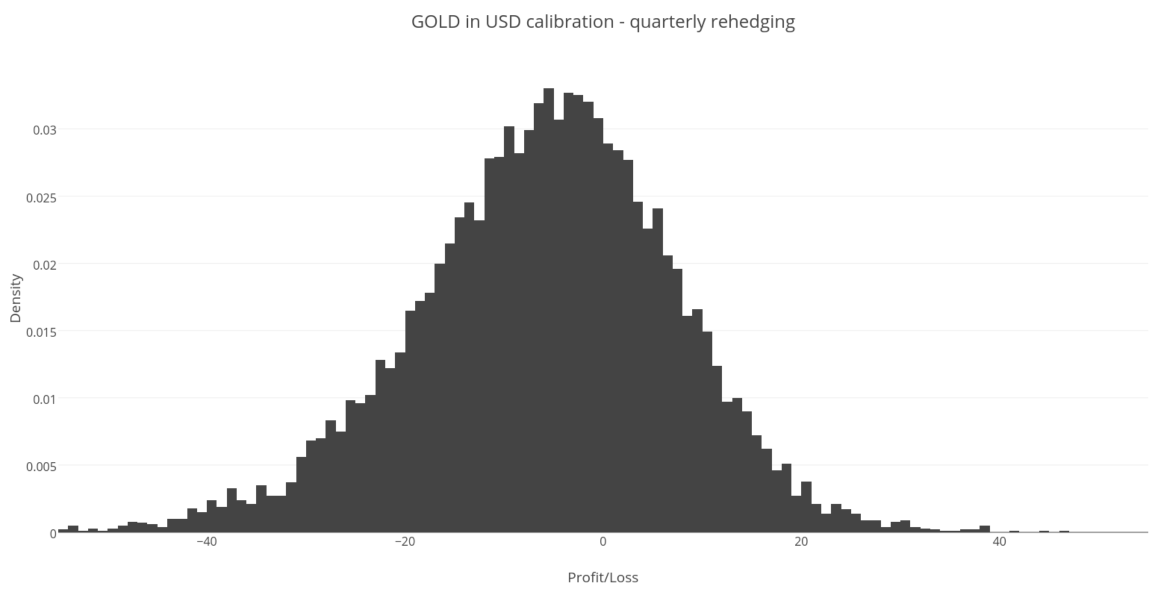

Figure 6

As introduced earlier, in every case, the mean of hedging cost is below zero. The obvious observation is that, the more hedges we take, we less variance we get. The X-axis range is [-55,55] in every case. What is interesting is that, the number of observations close(relatively) to the mean is isn't increasing much. The mean for every hedging density is following:

- Daily: -0.42,
- Weekly: -0.85,
- Monthly: -1.36,
- Quarterly: -6.01.

This assure us, that the hedging algorithm isn't working good, but it is not useless, the means are shifting towards zero when we are decreasing the period between hedges.

Following histograms are showing the profit/loss distribution in case of second type of calibration.

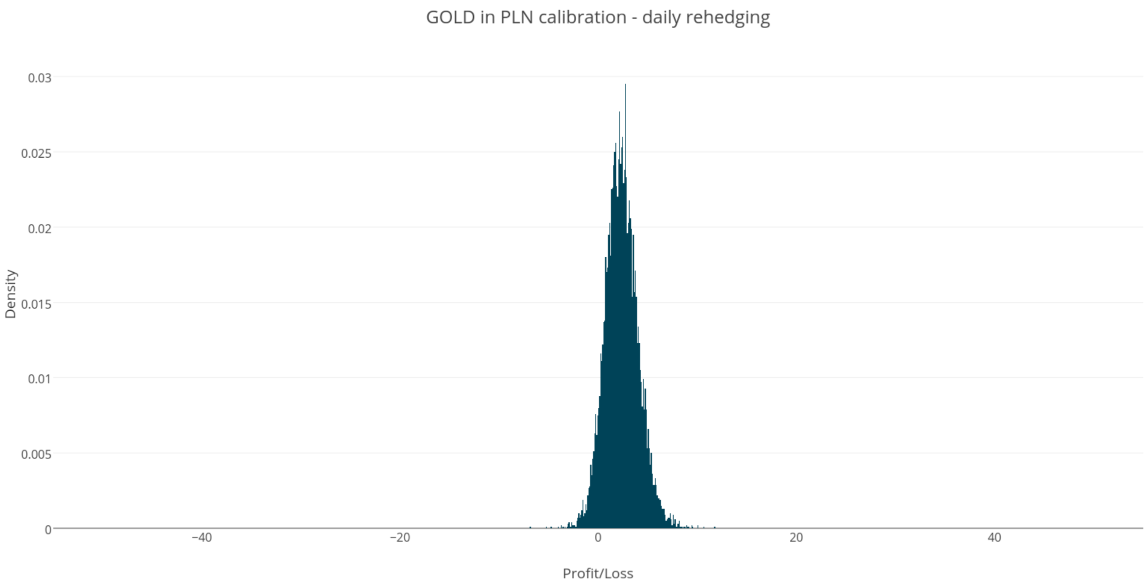

Figure 7

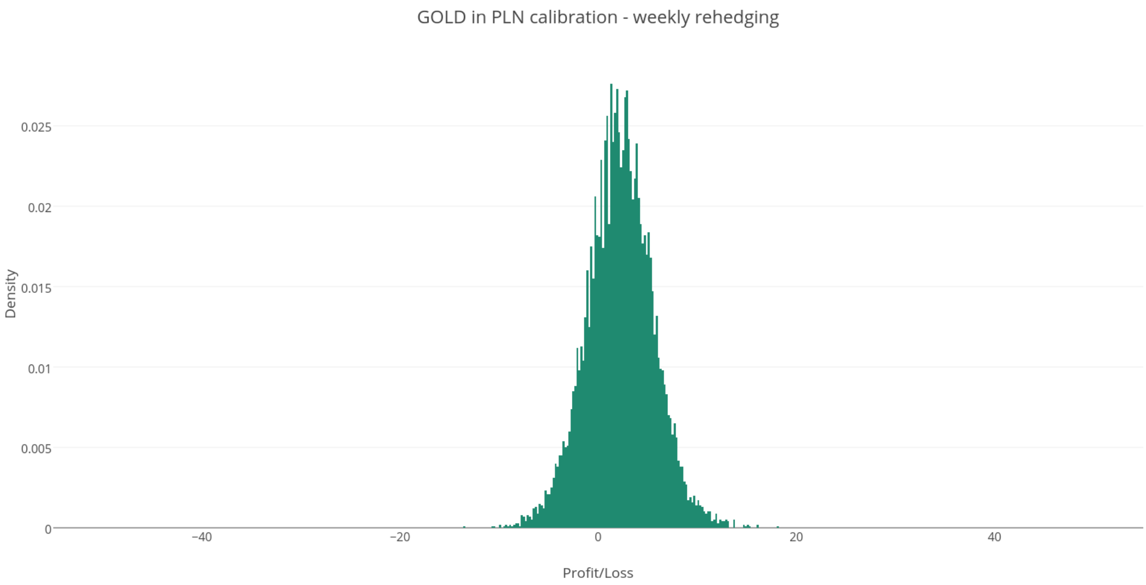

Figure 8

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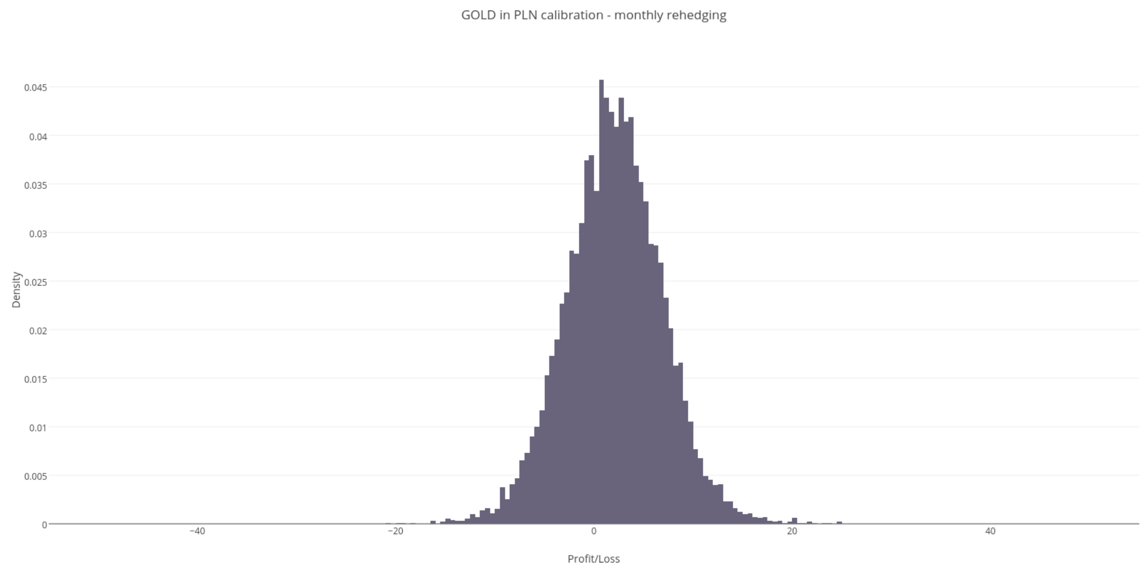

Figure 9

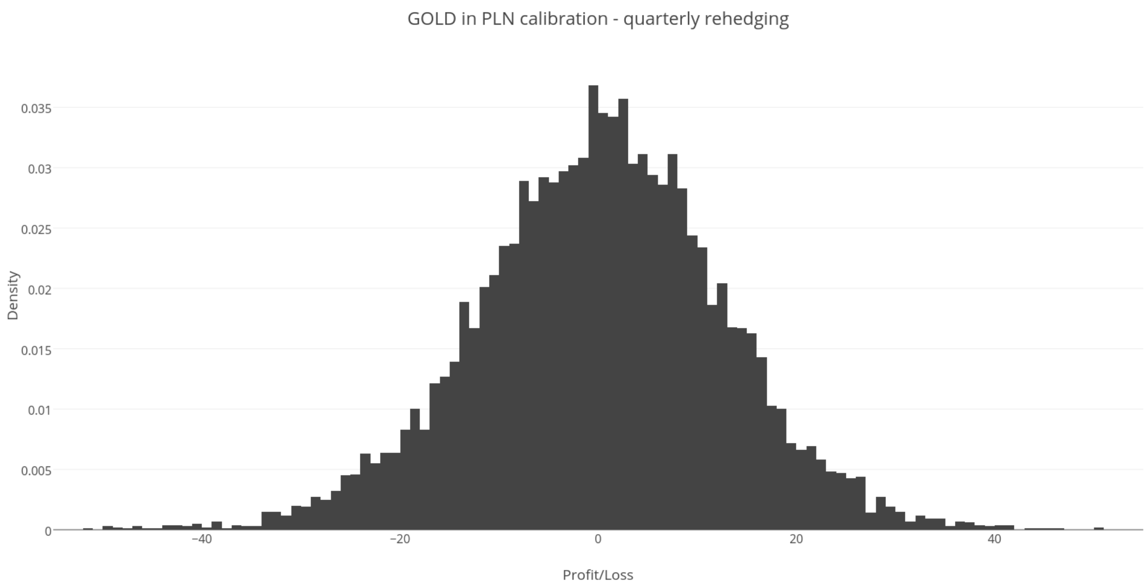

Figure 10

As said at the begging of the paragraph, we can clearly see, that the mean of hedging cost is above zero in all situations. It is opposite to the first method of calibration/hedging. The means are as following:

- Daily: 2.67
- Weekly: 2.21,
- Monthly: 2.04,
- Quarterly: 1.18.

In this case it is visible that hedging algorithm work properly, the more hedgings we take, the closer to zero we get. Interesting fact is that the mean is always above zero. It common in the analysis which was done by other groups (what does not mean it is right).

Real data

Now we will proceed to the analysis the difference in two calibration methods on real data. We assume that today is 30th July 2012. The data comes from the stooq.com website.

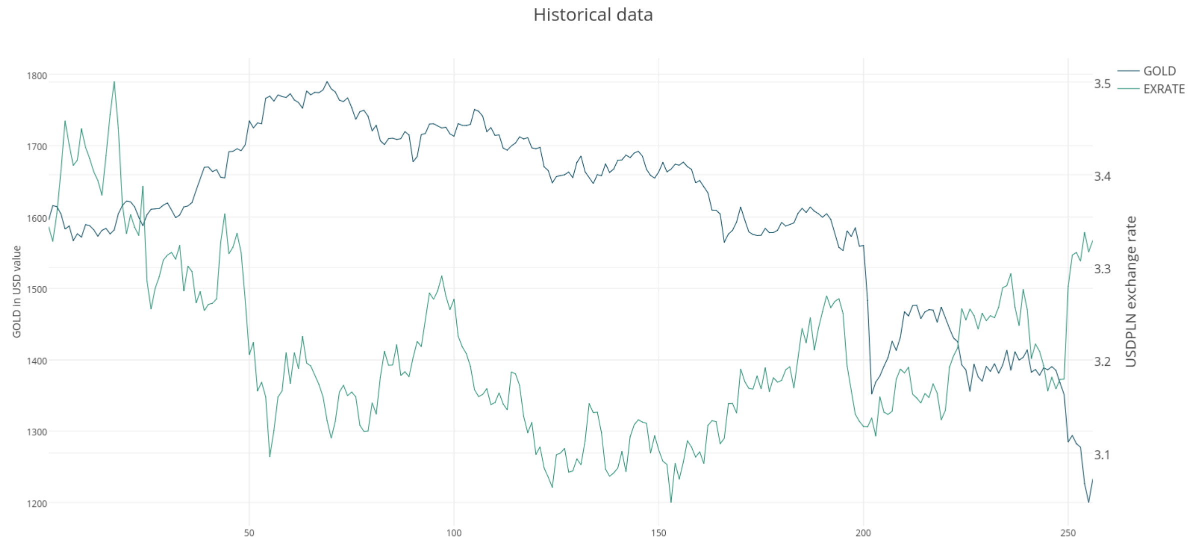

Figure 11

Both underlyings have had a tendency to decrease, but in micro scale it looks like they are negative correlated. It is only an visual comparison, but this gives us hope, that our hedging will give satisfying results (in calibration USDGOLD they actually was negative correlated).

Figure 12 shows the change in option price during its life. It is visible that the price at the very beginning was different, but after time they coincide to each other. The payoff of the option depends on the gold price, so it is not surprising that the price of it looks like the actual price of gold multiplied by a parameter.

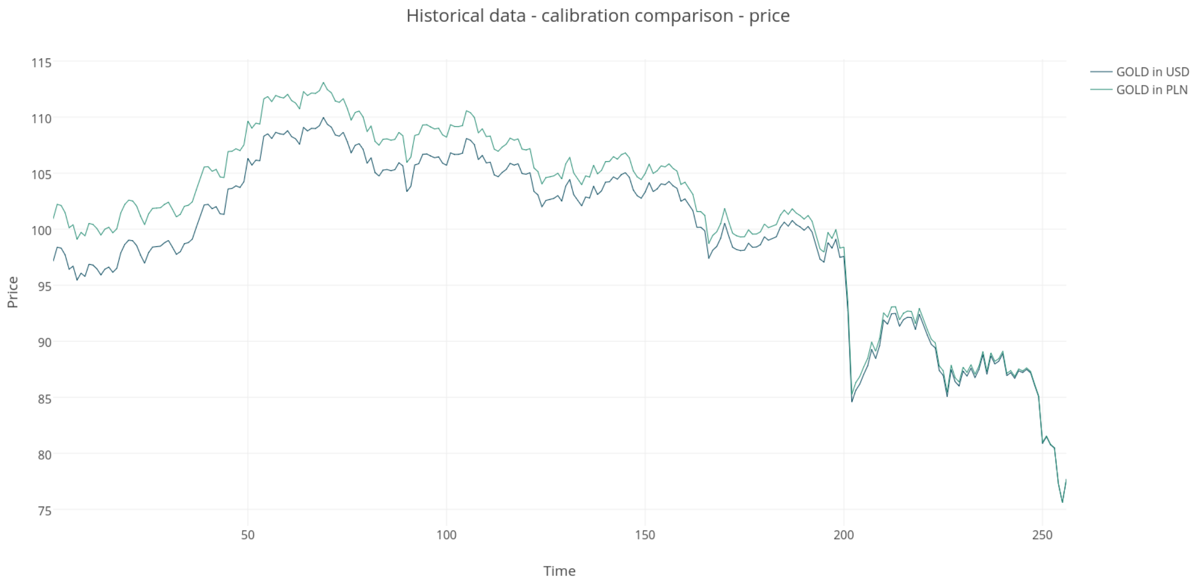

Figure 12

Figure 13 shows the hedging costs in time (Y axis on the left) and an absolute value of difference between them(Y axis on the right). Interesting is that, the difference is looks familiar to us – it somehow resemble the path of gold during the life of the option. The value is not high, around 4, but is is enough to bring our hedging strategy below zero. The total costs are as following:

GOLD in PLN: 1.35 PLN,

GOLD in USD: -2.65PLN.

This is the main, ant the most import difference. In this particular case, calibration with GOLD in PLN gives us better results that the with GOLD in USD. It does not mean it always will be better, but in this case it was (assuming that, the hedging algorithm works well). What is import, the absolute difference does not coverage to zero (as it was with difference in price, and will be in next parameters), it could be the hint for looking where our hedging in first case is not working. We may not properly valuating quanto option or using wrong parameters for hedging.

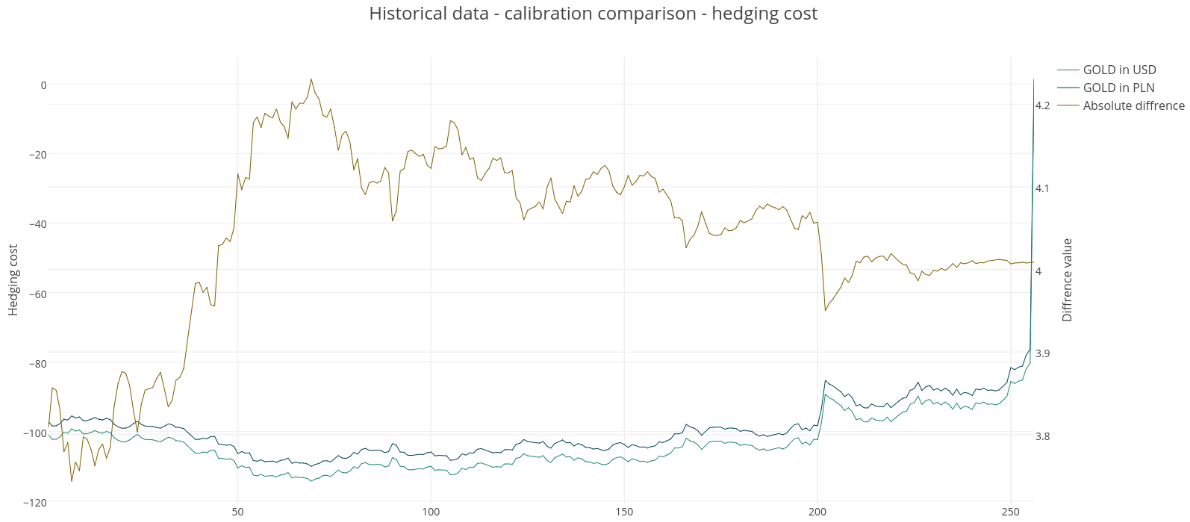

Figure 13

The following two figures (Figure 14 & 15) shows how does the deltas looks like. We can observe, that they are somehow mirrored. After analysing formulas for two deltas we could easy deduct it. The minus sign is always before dollar delta. We could also observe the property of converging – the closer maturity we go, the lower difference we get.

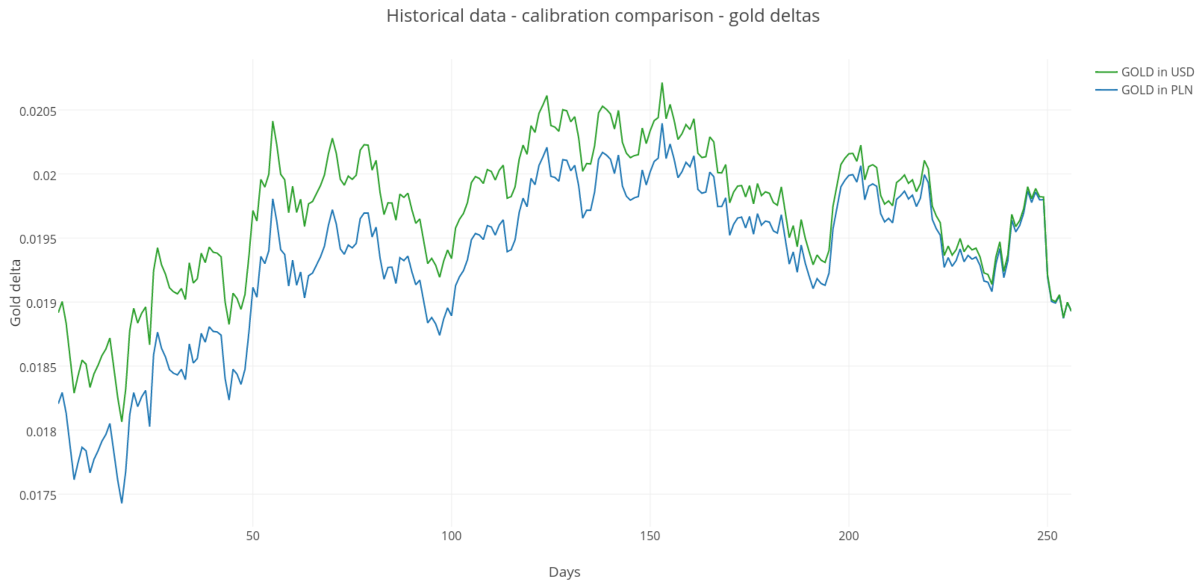

Figure 14

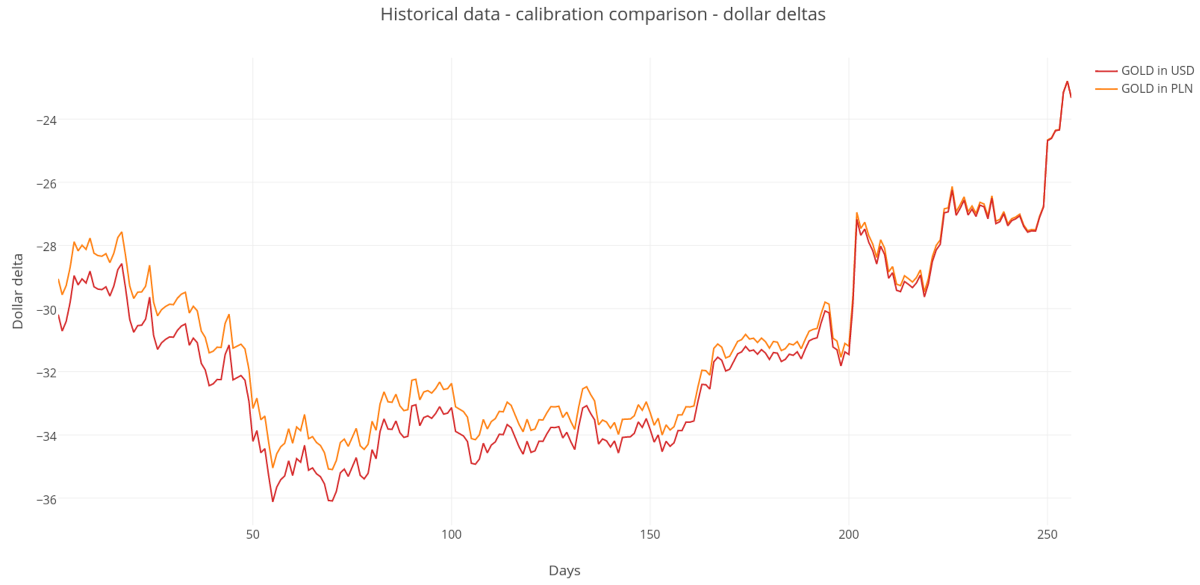

Figure 15

The interesting is the analysis of cost of buying underlying in every step. It is visible, that in every step we are hedging our portfolio using almost the same value in both assets. Following two plots (Figures 16 & 17) are showing how much we spent to hedge on every step. The two calibrating methods looks almost the same.

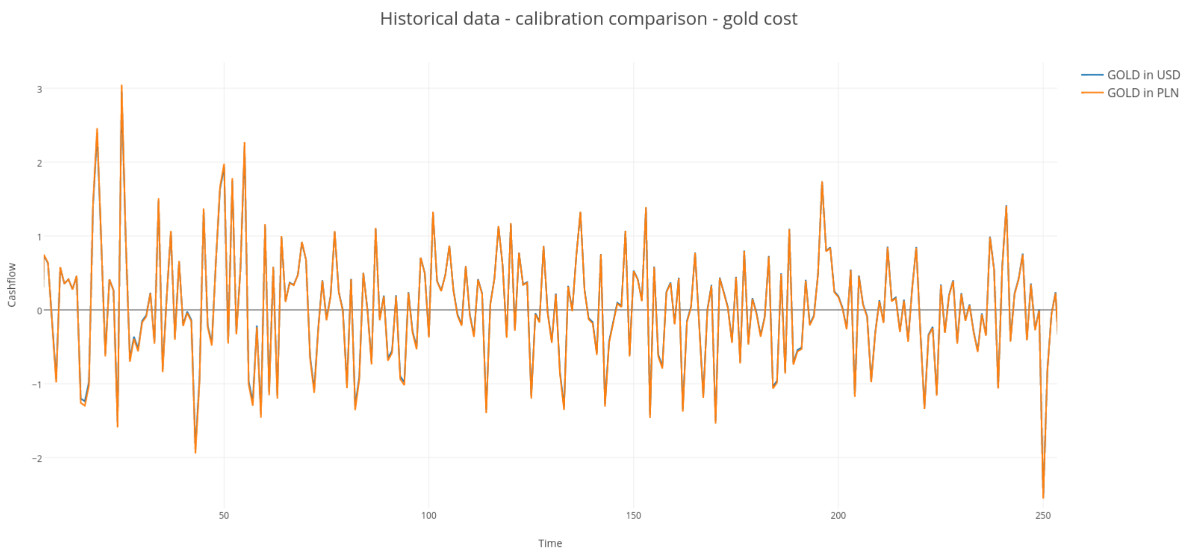

Figure 16

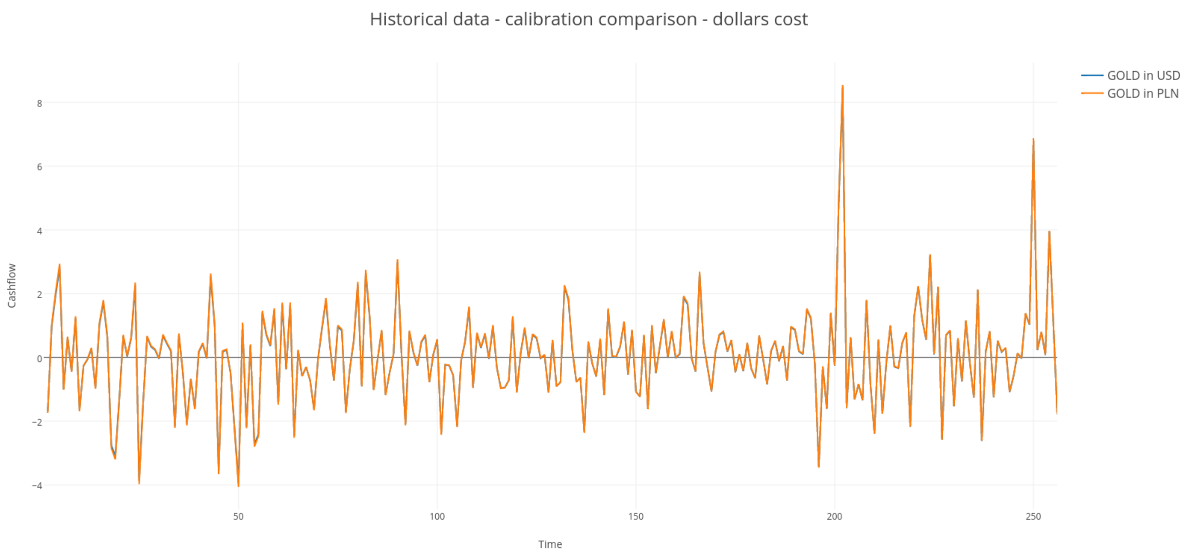

Figure 17

Sensitivity analysis

Next paragraph will cover sigma sensitivity analysis of formulas. We know that formula for calibration with GOLD in USD is a function of several parameters, where sigmas are always a linear compound. We could see it a Fig. 18, where we have a 3D surface of price function, where variables are $\sigma_{USDPLN} = \sigma_X$ and $\sigma_{GOLDUSD} = \sigma_Y$. The plat is flat, the edges looks like linear function.

Situation is different in case on second calibration. In this case, the $\sigma_{GOLDUSD}$ variable occurs with a power of two. And this is what we can observe on a plot. When the $\sigma_{GOLDUSD}$ increase, the price is increasing like a binomial function(Figure 19).

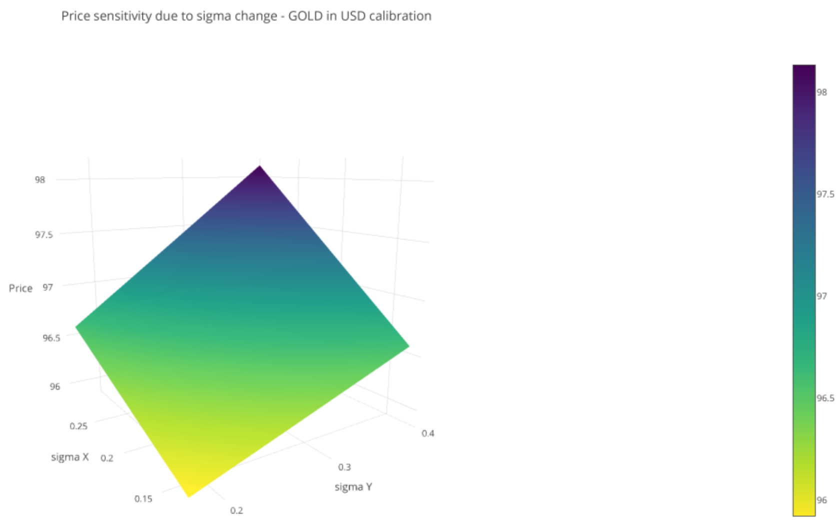

Figure 18

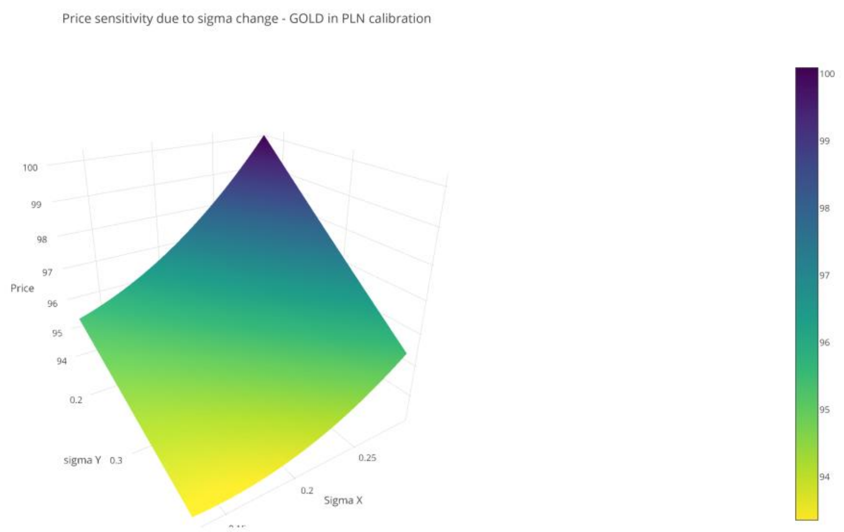

Figure 19