

Exercise 1

Deep Learning

Dr. Mehrdad Maleki

1. Let $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$ and $C = \{1, 4, 7, 8\}$. Write a Python code for $A \cap (B \cup C)$ and $(A \cap B) \cup (A \cap C)$.
2. A 4-dimensional sphere is the locus of the points in \mathbb{R}^4 such that their distance from the origin is constant. A 4-dimensional sphere with radius 1 is denoted by \mathbf{S}^3 , so elements of \mathbf{S}^3 are all vectors,

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

such that $\|\mathbf{x}\| = 1$. Write a Python code to recognize that is $(0.5, 0.5, 0.2, 0.7)$ inside \mathbf{S}^3 or not?

3. Let,

$$\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

What is the angle between \mathbf{u} and \mathbf{v} ? Use Python!

4. Calculate $\sin(\mathbf{W}\mathbf{x})$ where,

$$\mathbf{W} = \begin{bmatrix} 0.5 & 0.2 & 0.1 \\ 0.8 & 5 & 4.5 \\ -3 & 0.7 & 0.01 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} 1.5 \\ 2 \\ 0.7 \end{bmatrix}$$

5. What is the solution of the following equation?(Python!)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 + x_1 \\ 4 + x_2 \\ 1 + x_3 \end{bmatrix}$$

6. What are the eigenvalues and eigenvectors of \mathbf{A}^2 where,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

7. Calculate $2\mathbf{I}_3 \otimes \mathbf{1}_{4 \times 4}$, where,

$$\mathbf{1}_{4 \times 4} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$