Deep Learning

Lecture 4: Perceptron

Dr. Mehrdad Maleki

Boolean Functions

Boolean variable, $X = 0$ or $X = 1$.

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3. NEGATION:

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$$\overline{X} = 1 \Leftrightarrow X = 0$$
$$\overline{X} = 0 \Leftrightarrow X = 1$$

4. XOR: $X \oplus Y = (\overline{X} \wedge Y) \vee (X \wedge \overline{Y})$

Some useful properties

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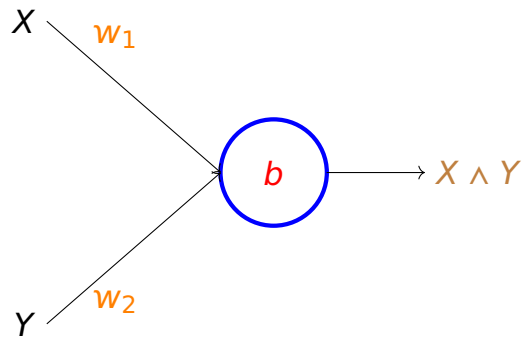

$$X \wedge Y = \begin{cases} 1 & \text{if } w_1 * X + w_2 * Y \geq b \\ 0 & \text{else} \end{cases}$$

$$1 \wedge 1 = 1 \Rightarrow w_1 + w_2 \geq b$$
$$1 \wedge 0 = 0 \Rightarrow w_1 < b$$
$$0 \wedge 1 = 0 \Rightarrow w_2 < b$$
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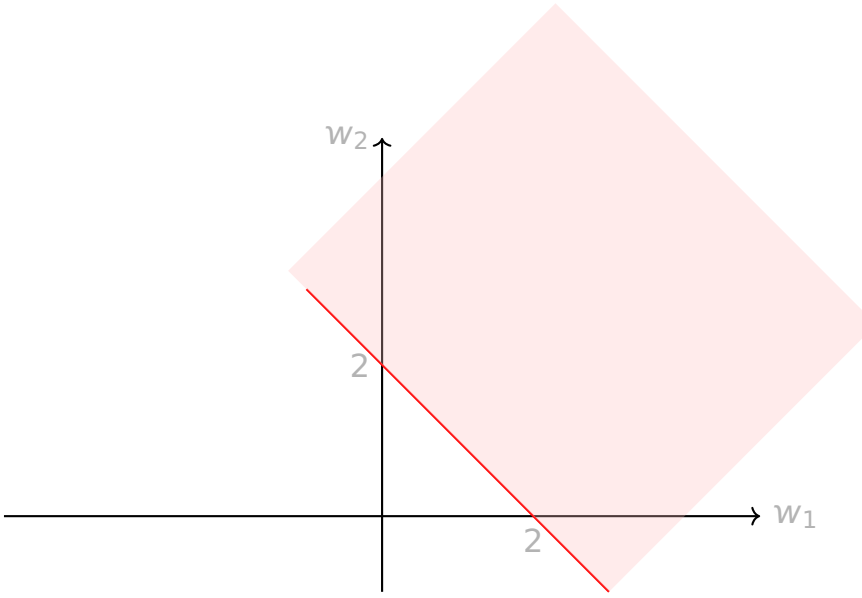
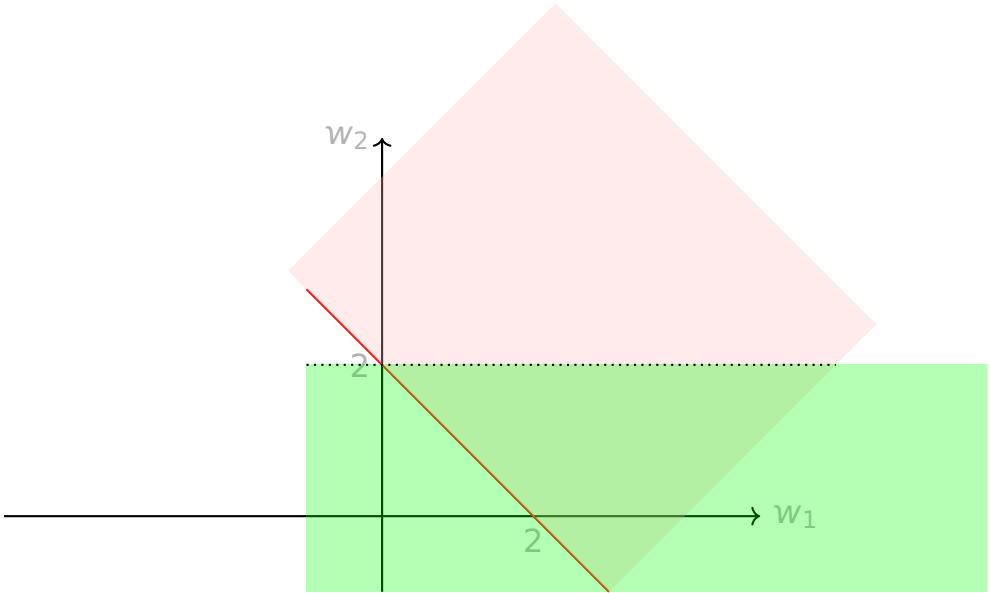
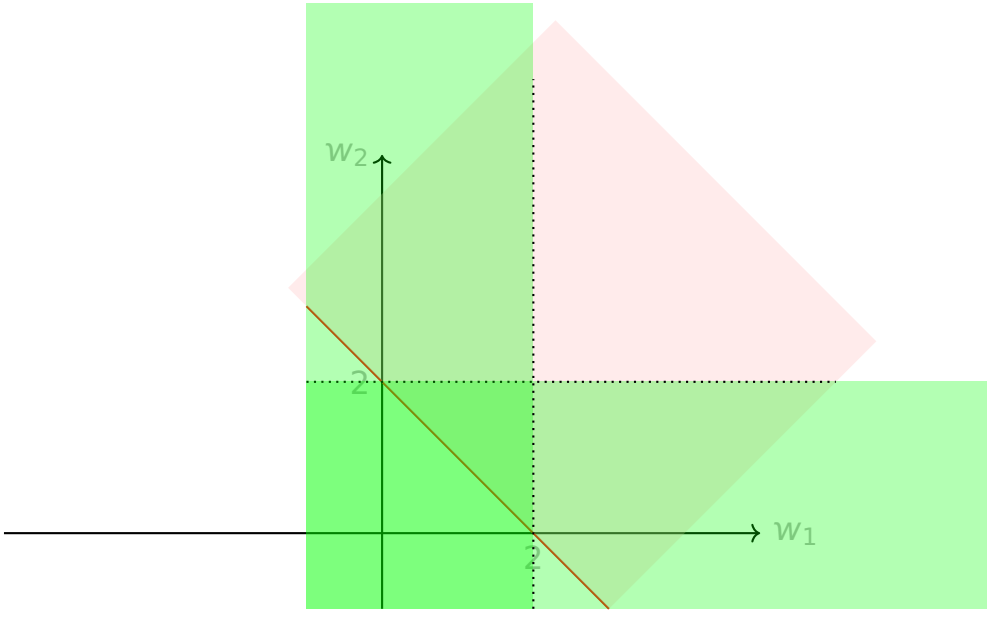
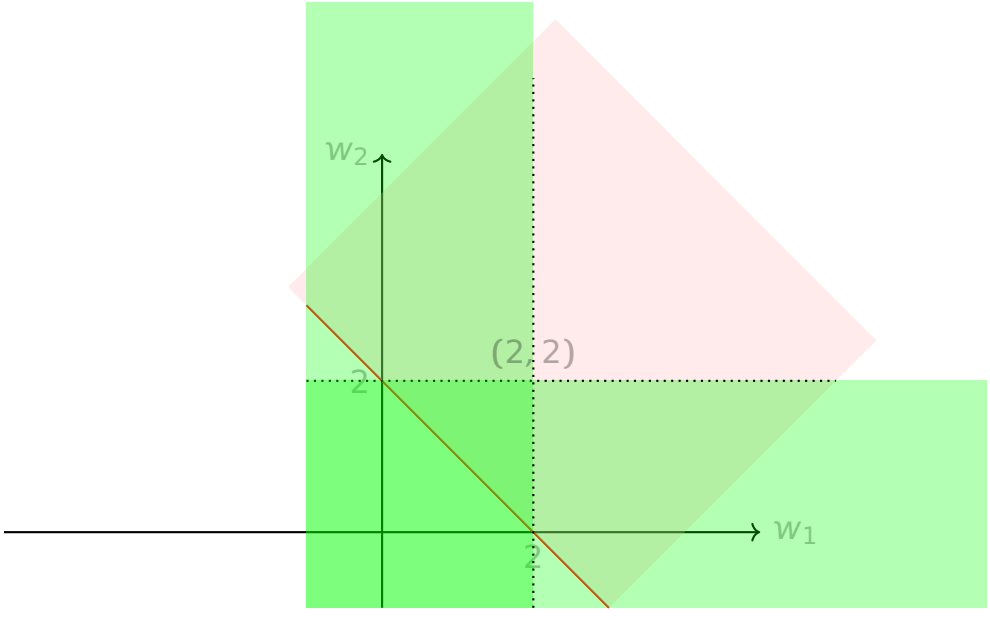
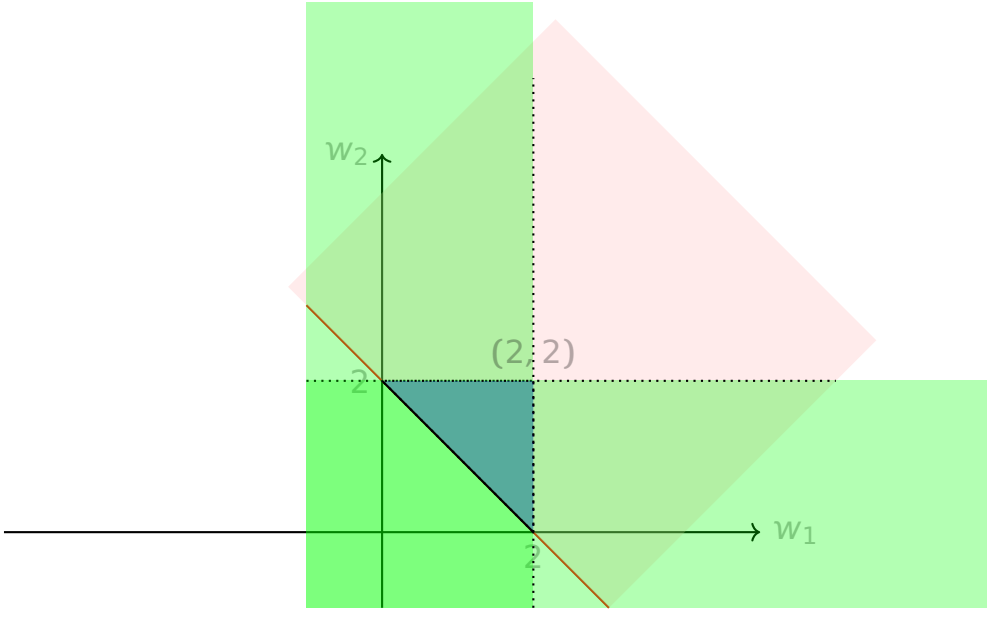
find integers w_1, w_2, b that satisfies all of these constraints.

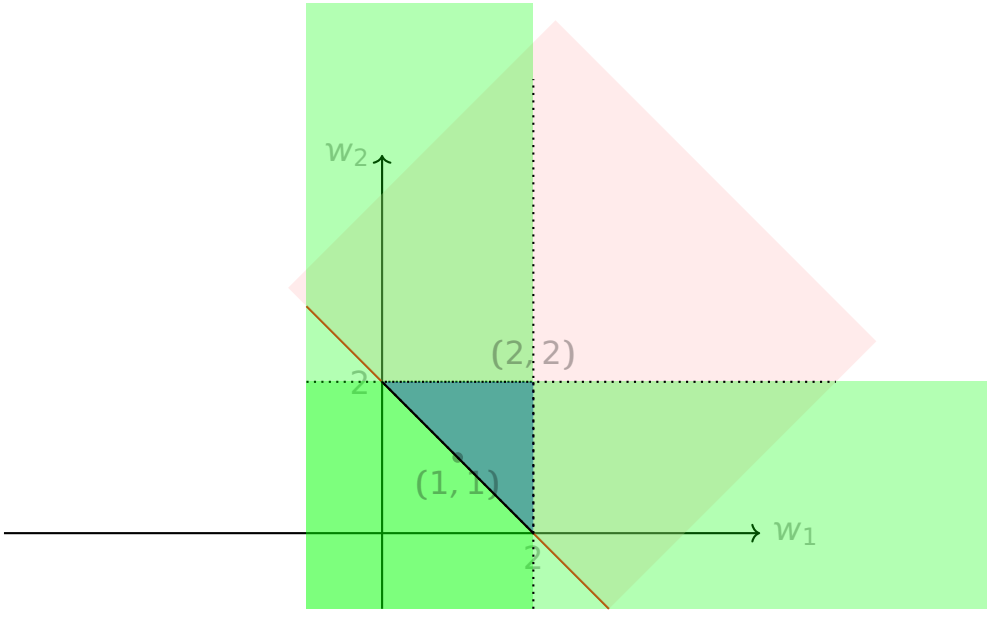

w_2
2
w_1
2

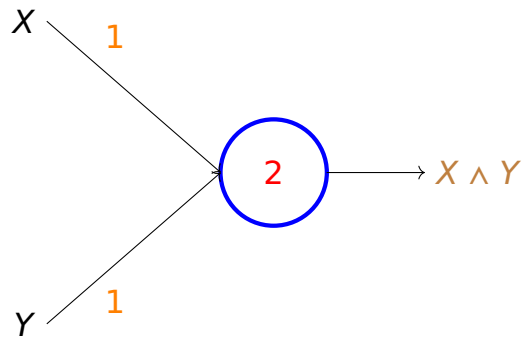

w_2
2
w_1
2


w_2
2
w_1
2

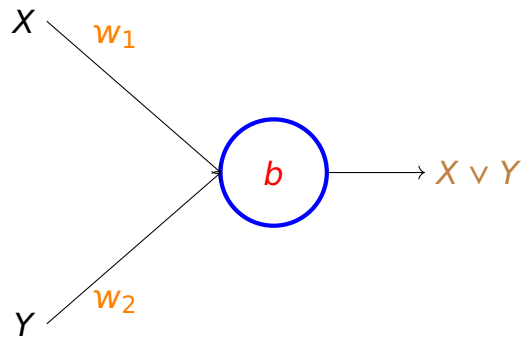

w_2
(2, 2)
2
w_1
2


w_2
(2, 2)
2
w_1
2

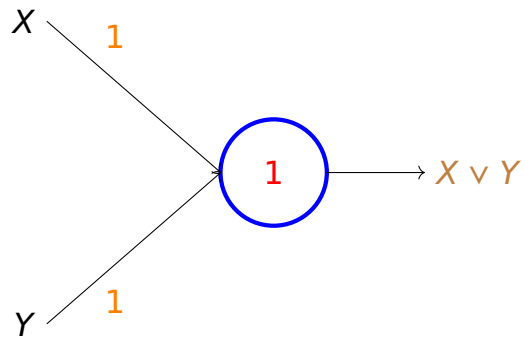

w_2
(2, 2)
2
(1, 1)
w_1
2



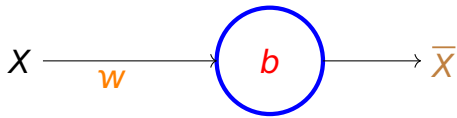
$$X \wedge Y = \begin{cases} 1 & \text{if } X + Y \geq 2 \\ 0 & \text{else} \end{cases}$$



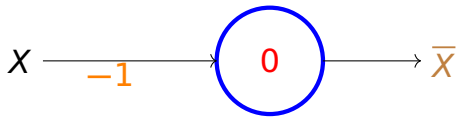
$$X \vee Y = \begin{cases} 1 & \text{if } w_1 X + w_2 Y \geq b \\ 0 & \text{else} \end{cases}$$



$$X \vee Y = \begin{cases} 1 & \text{if } X + Y \geq 1 \\ 0 & \text{else} \end{cases}$$

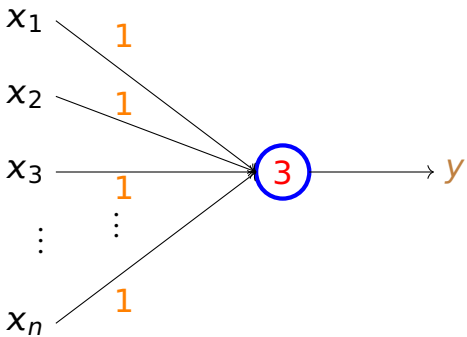


$$\overline{X} = \begin{cases} 1 & \text{if } wX \geq b \\ 0 & \text{else} \end{cases}$$



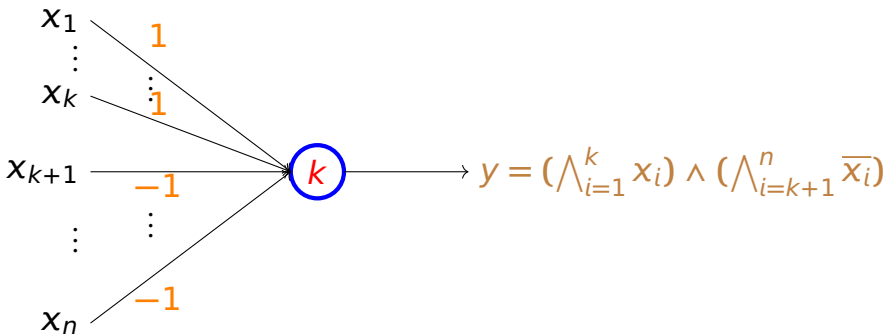
$$\overline{X} = \begin{cases} 1 & \text{if } -X \geq 0 \\ 0 & \text{else} \end{cases}$$

Majority Gate

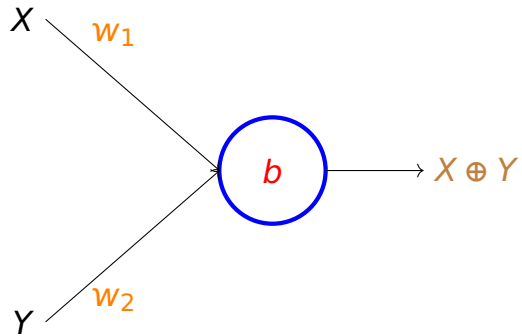


$$y = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i \geq 3 \\ 0 & \text{else} \end{cases}$$

Universal AND Gate



$$y = \begin{cases} 1 & \text{if } \sum_{i=1}^{k} x_i - \sum_{i=k+1}^{n} x_i \geq k \\ 0 & \text{else} \end{cases}$$



$$X \oplus Y = \begin{cases} 1 & \text{if } w_1 X + w_2 Y \geq b \\ 0 & \text{else} \end{cases}$$

$$1 \oplus 0 = 1 \Rightarrow w_1 \geq b$$
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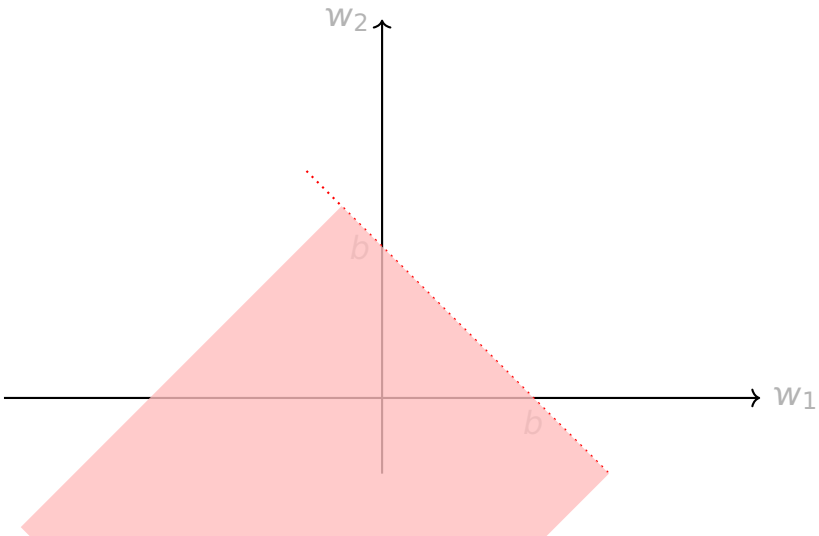
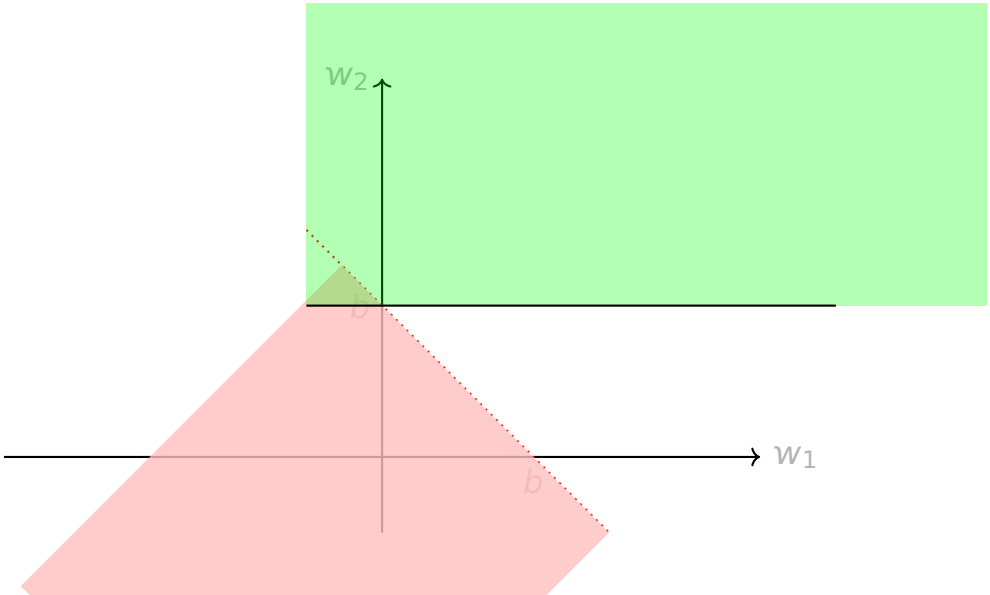
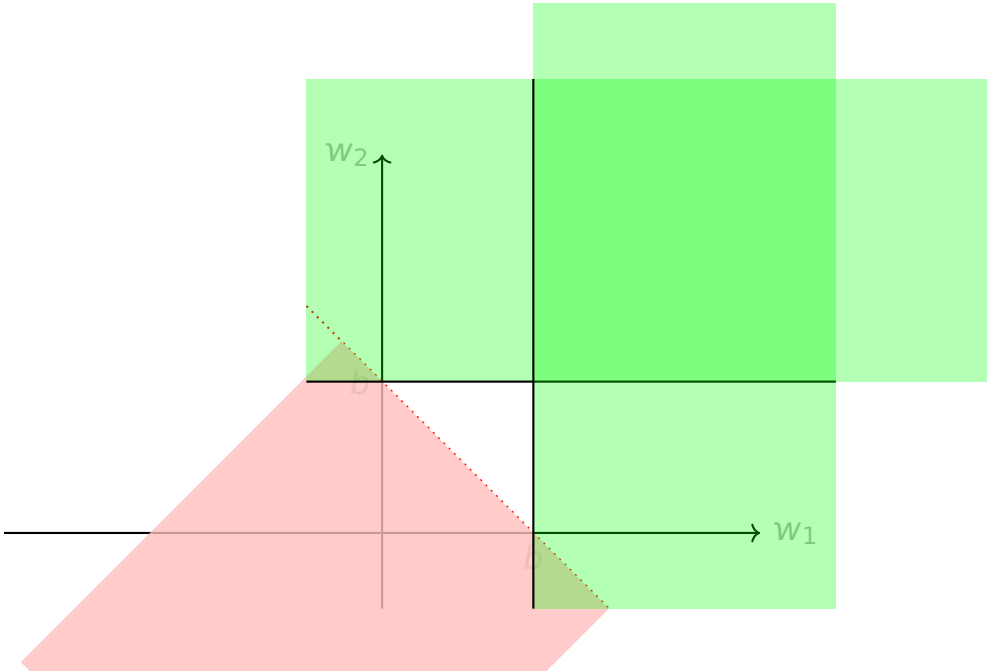
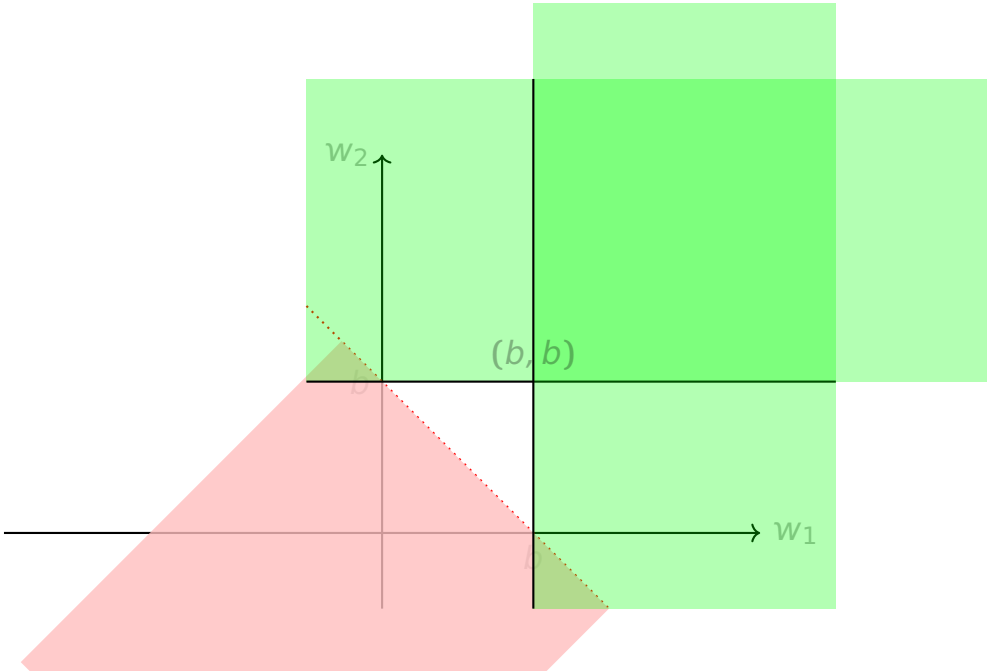
Find integers $w1, w_2, b$ that satisfies all of these constraints.


w_2
b
w_1
b

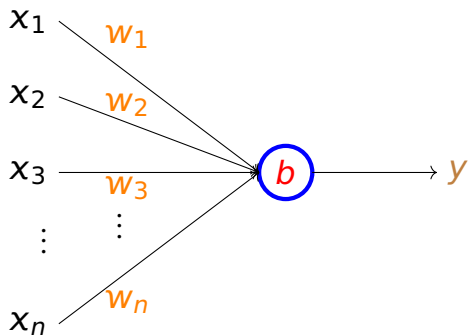

w_2
b
w_1
b


w_2
w_1


w_2
(b, b)
b
w_1
b

Perceptron

- Input $(x_1, \ldots, x_N)$
- Weights $(w_1, \ldots, w_N)$
- If $w_1 x_1 + \cdots + w_n x_n \geq b$ then output=1
- Else, i.e., $w_1 x_1 + \cdots + w_n x_n < b$, output=0



$$y = \begin{cases} 1 & \text{if } w_1 x_1 + \cdots + w_N x_N \geq b \\ 0 & \text{else} \end{cases}$$

Let,

$$\phi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{else} \end{cases}$$

then,

$$y = \phi(w_1 x_1 + \cdots + w_N x_N - b)$$

ϕ is called the **activation function**.

Activation Functions

- Identity: $\phi(z) = z$
- Rectified linear unit (ReLU): $\phi(z) = max\{0, z\}$
- Sigmoid: $\phi(z) = \frac{1}{1+e^{-z}}$
- Hyperbolic tangent: $\phi(z) = \frac{e^{z}-e^{-z}}{e^{z}+e^{-z}}$
- Binary step: $\phi(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{else} \end{cases}$

Thank You