Deep Learning

Lecture 2: Probability Theory

Dr. Mehrdad Maleki

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Probability

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If S be a finite set then probability of event $A \subseteq S$ is define as follow,

$$P(A) = P(\mathbf{X} \in A) = \frac{|A|}{|S|}$$

$$0 \leq P(A) \leq 1$$

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$$\textbf{if } A \cap B = \emptyset \Rightarrow P(A \cup B) = P(A) + P(B)$$

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Probability mass function (pmf): $f(i) = P[\mathbf{X} = i]$ for descrete random variables.

Continious Distribution

In general, quantities such as pressure, height, mass, weight, density, volume, temperature, and distance are examples of continuous random variables.

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Probability density function (pdf): Is a continuous positive function $f_{\mathbf{X}}(x) : \mathbb{R} \rightarrow \mathbb{R}^{\geq 0}$ such that,

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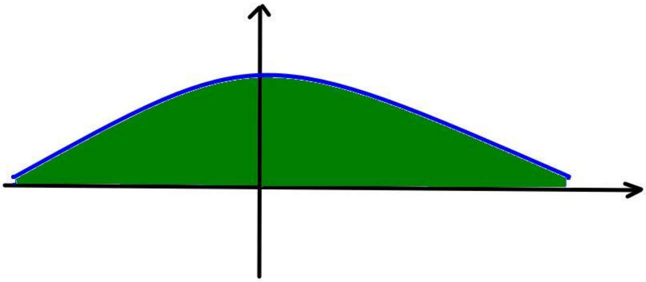

Cumulative distribution function: is the function that evaluating the probability of $\mathbf{X} \leq x$,

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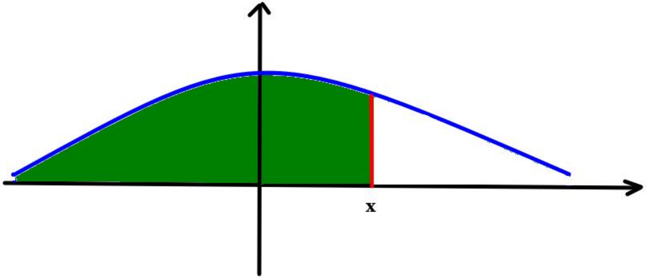

Bernoulli Trial

A **Bernoulli trial** is a random experiment with exactly two possible outcomes, "success" and "failure".

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$$p = P[\mathbf{X} = "success"]$$
$$q = P[\mathbf{X} = "failure]$$
$$p + q = 1$$

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for $x \in \{0, 1\}$.

Categorical (generalized Bernoulli) Distribution

Categorical distribution is a discrete probability distribution that describes the possible results of a random variable that can take on one of K possible categories, with the probability of each category separately specified. So $\mathbf{X} \in \{1, 2, \ldots, K\}$.

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where

$$\mathbf{I}_{[x=i]} = \begin{cases} 1 & \text{if } x = i \\ 0 & \text{else} \end{cases}$$

for $x \in \{0, 1\}$.

Conditional Probability

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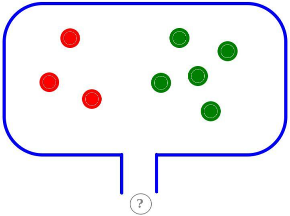
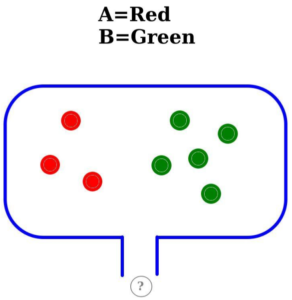
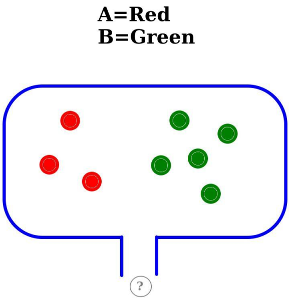
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A=Red
B=Green





We choose one ball randomly. What is the probability that this ball is red?



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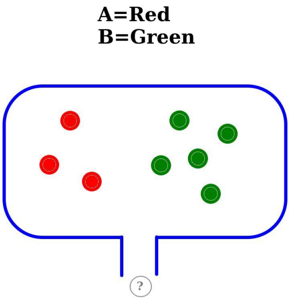
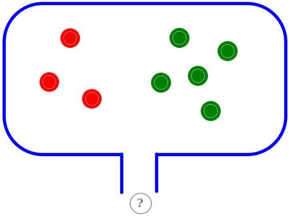
$P(A) = \frac{3}{8}$,



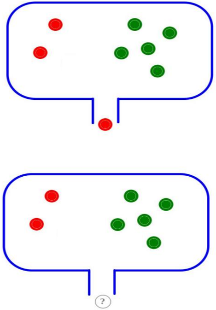
We choose one ball randomly. What is the probability that this ball is red?
$P(A) = \frac{3}{8}$, What about the probability that this ball is green?

A=Red
B=Green



We choose one ball randomly. What is the probability that this ball is red? $P(A) = \frac{3}{8}$, What about the probability that this ball is green? $P(B) = \frac{5}{8}$.

A=second ball green
B=first ball red



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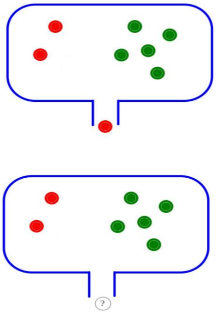

We choose one ball randomly and through it away. We choose another ball randomly. What is the probability that the second ball is green such that the fist ball is red?

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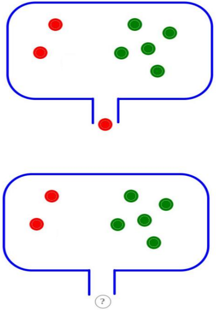
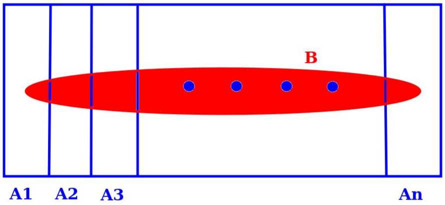
B=first ball red



We choose one ball randomly and through it away. We choose another ball randomly. What is the probability that the second ball is green such that the fist ball is red? $P(A|B) = \frac{5}{7}$

Law of Total Probability



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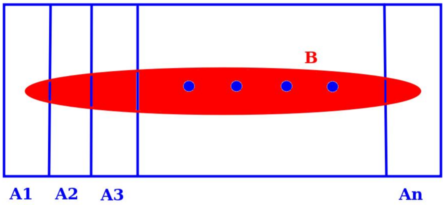

$$P(B) = P(A_1 \cap B) + \cdots + P(A_n \cap B)$$

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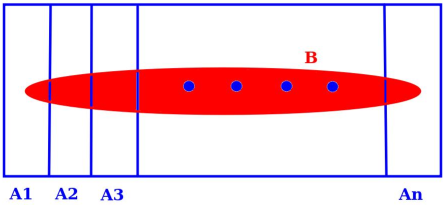
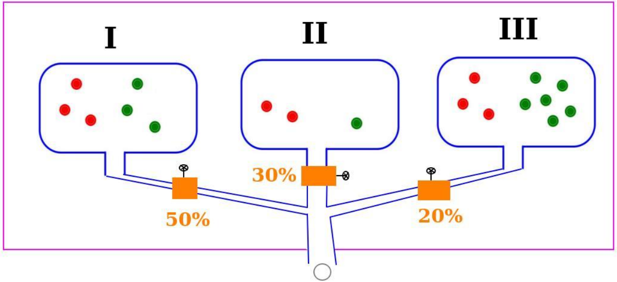
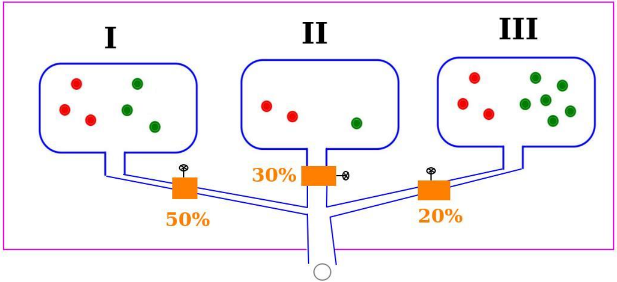


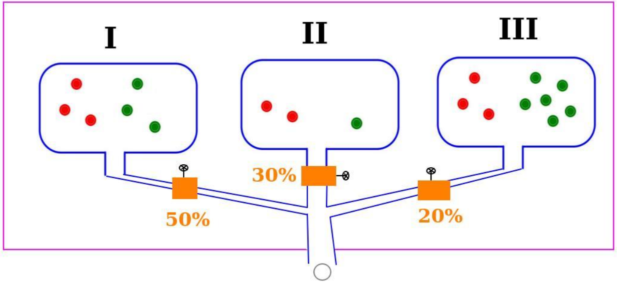
$$
\begin{aligned}
P(B) &= P(A_1 \cap B) + \cdots + P(A_n \cap B) \\
&= P(A_1)P(B|A_1) + \cdots + P(A_n)P(B|A_n)
\end{aligned}
$$



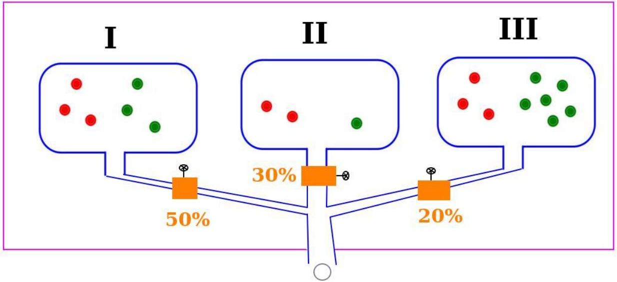
$$P(R) =$$



$$P(R) = P(I)P(R|I) + P(II)P(R|II) + P(III)P(R|III)$$



$$P(R) = P(I)P(R|I) + P(II)P(R|II) + P(III)P(R|III)$$
$$= 0.5\frac{3}{6} + 0.3\frac{2}{3} + 0.2\frac{3}{9}$$



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&\approx 0.52
\end{aligned}
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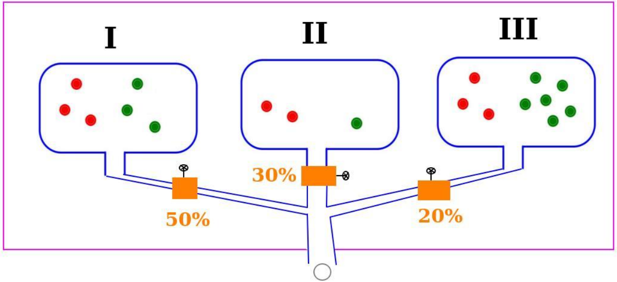
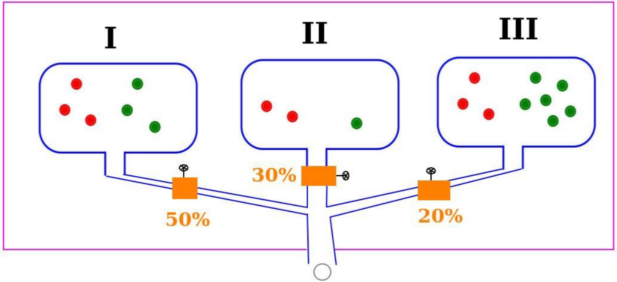


$P(G) =$



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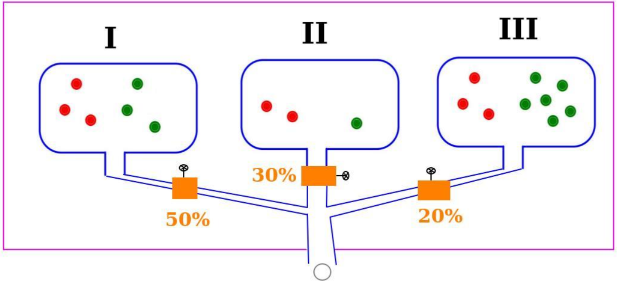

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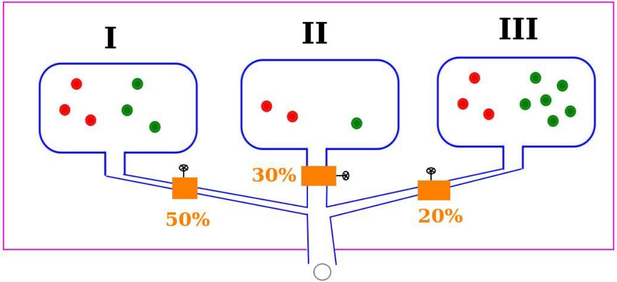

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Bayes Rule

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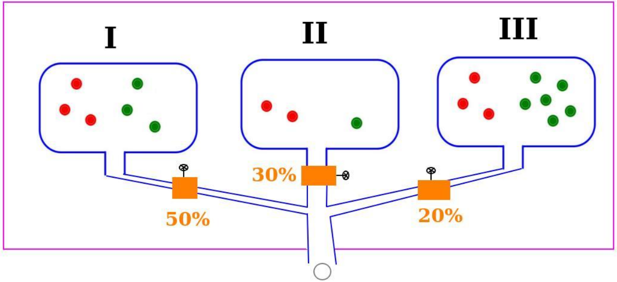
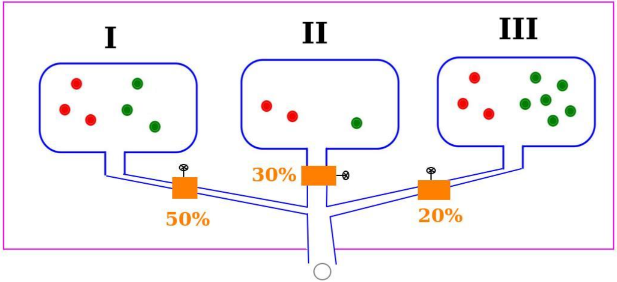
- $P(A|B) =$ **the posterior**, is the degree of belief after incorporating news that B is true.
- $P(B|A) =$ **the likelihood**, that can be estimated from the training data.



$$P(I|R) =$$



$$P(I|R) = \frac{P(R|I)P(I)}{P(R)}$$
$$= \frac{\frac{3}{6}0.5}{0.52}$$
$$\approx 0.48$$

$$P(II|R) =$$

$$
\begin{aligned}
P(II|R) &= \frac{P(R|II)P(II)}{P(R)} \\
&= \frac{\frac{2}{3}0.3}{0.52} \\
&\approx 0.38
\end{aligned}
$$

$$
P(III|R) =
$$

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\end{aligned}
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$$
\begin{aligned}
P(III|R) &= \frac{P(R|III)P(III)}{P(R)} \\
&= \frac{\frac{3}{9}0.2}{0.52} \\
&\approx 0.12
\end{aligned}
$$

$$P(I|G) =$$

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P(I|G) &= \frac{P(G|I)P(I)}{P(G)} \\
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P(II|G) &= \frac{P(G|II)P(II)}{P(G)} \\
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&= \frac{\frac{1}{3}0.3}{0.48} \\
&\approx 0.2 \\
P(III|G) &= \frac{P(G|III)P(III)}{P(G)} \\
&= \frac{\frac{6}{9}0.2}{0.48} \\
&\approx 0.27
\end{aligned}
$$

Chain rule for conditional probability

$$P(A_n \cap \cdots \cap A_1) =$$

Chain rule for conditional probability

$$P(A_n \cap \cdots \cap A_1) = P(A_n|A_{n-1} \cap \cdots \cap A_1) \cdot P(A_{n-1}|A_{n-2} \cap \cdots \cap A_1) \cdot \cdots \cdot P(A_1)$$

Chain rule for conditional probability

$$\begin{aligned}P(A_n \cap \cdots \cap A_1) &= P(A_n | A_{n-1} \cap \cdots \cap A_1) \cdot P(A_{n-1} | A_{n-2} \cap \cdots \cap A_1) \cdot \cdots \cdot P(A_1) \\ &= \prod_{k=1}^{n} P(A_k | A_{k-1} \cap \cdots \cap A_1)\end{aligned}$$

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\end{aligned}
$$

$$
P(I\ like\ you) = P(you | I\ ,\ like) \cdot P(like | I) \cdot P(I)
$$

Naive Bayes Classifier

- Suppose you have a binary classifier with two classes, C_1, C_2.

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- The bigger probability determine the class of $\mathbf{x}$.

- By the Bayes formula,

$$p(C_k|\mathbf{x}) = \frac{p(C_k)p(\mathbf{x}|C_k)}{p(\mathbf{x})}$$

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- **Naive Bayes Assumption**: all features in $\mathbf{x}$ are mutually independent, i.e.,

$$p(x_i|x_{i+1}, \ldots, x_n, C_k) = p(x_i|C_k)$$

- So,

$$p(C_k|x_1, \ldots, x_n) = \frac{p(C_k)}{p(\mathbf{x})} p(x_1|C_k) \times \cdots \times p(x_n|C_k)$$

Example

- If $C_1 = 0$, $C_2 = 4$ and $\mathbf{x} = (I, like, you)$ then the probability that this sentence has the tag 0 is,

$$p(C_1|I, like, you) = \frac{p(C_1)}{p(\mathbf{x})} p(I|C_1) \times p(like|C_2) \times p(you|C_1)$$

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- $p(C_1) = \frac{\text{number of sentence with tag 0}}{\text{total number of sentences}}$
- $p(\mathbf{x})$ is constant.
- $p(like|C_1) = \frac{\text{(how many times "like" appears in sentences with tag 0)+1}}{\text{number of words in the dictionary}}$

Expectation

$$\mathbb{E}[\mathbf{X}] = \sum_{i=1}^{n} i\, P[\mathbf{X} = i]$$

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If $\mathbf{X} \in \{1, 2, 3, 4, 5, 6\}$ be the random variable of tossing a fair dice then,

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$$\begin{aligned}\mathbb{E}[\mathbf{X}] &= 1\, P[\mathbf{X} = 1] + 2\, P[\mathbf{X} = 2] + \cdots + 6\, P[\mathbf{X} = 6] \\ &= \frac{1+2+3+4+5+6}{6} \\ &= 3.5\end{aligned}$$

Variance

$$\mathbb{V}[\mathbf{X}] =$$

Variance

$$\mathbb{V}[\mathbf{X}] = \mathbb{E}[(\mathbf{X} - \mathbb{E}[\mathbf{X}])^2]$$

Variance

$$
\begin{aligned}
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Tossing a coin,

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Tossing a coin,

$$\mathbb{V}[\mathbf{X}] = (0^2\, P[\mathbf{X} = 0] + 1^2\, P[\mathbf{X} = 1]) - (0\, P[\mathbf{X} = 0] + 1\, P[\mathbf{X} = 1])^2$$

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Tossing a coin,

$$\begin{aligned}\mathbb{V}[\mathbf{X}] &= (0^2\, P[\mathbf{X} = 0] + 1^2\, P[\mathbf{X} = 1]) - (0\, P[\mathbf{X} = 0] + 1\, P[\mathbf{X} = 1])^2 \\ &= \frac{1}{2} - (\frac{1}{2})^2 \\ &= \frac{1}{4}\end{aligned}$$

Normal (Gaussian) Distribution

Why Should I care about Normal Distribution?

- It is the most important distribution in nature.

- Examples: weight, height, blood pressure, etc.

- It is a fundamental concept in data science.

Probability Density Function (PDF)

```python
import numpy as np
import matplotlib.pyplot as plt
```

```python
def normal(x,mu,sigma):
    return (1/(np.sqrt(2*np.pi*sigma**2))*np.exp(-(x-mu)**2/(2*sigma**2)))
```

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

mean

standard deviation

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

If $\mu = 0$, $\sigma = 1$ it is called standard normal distribution.

Code

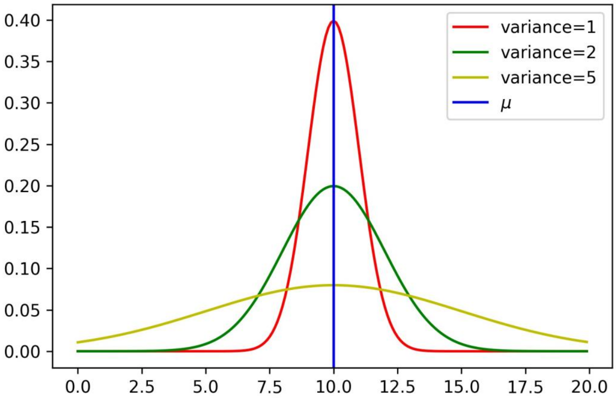
```
x=np.arange(0,20,0.1)
v1=1
v2=2
v3=5
y1=normal(x,10,v1)
y2=normal(x,10,v2)
y3=normal(x,10,v3)
```

Generating three different normal distribution with same mean but different variance

```
plt.plot(x,y1,'r',label='variance={}'.format(v1))
plt.plot(x,y2,'g',label='variance={}'.format(v2))
plt.plot(x,y3,'y',label='variance={}'.format(v3))
plt.axvline(x=10, c='b',label='$\mu$')
plt.legend()
```

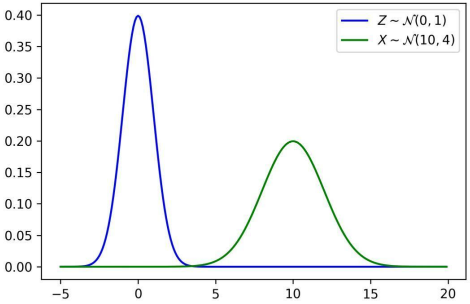
Draw vertical line x=10

Increasing Variance Reduce the Height of Curve

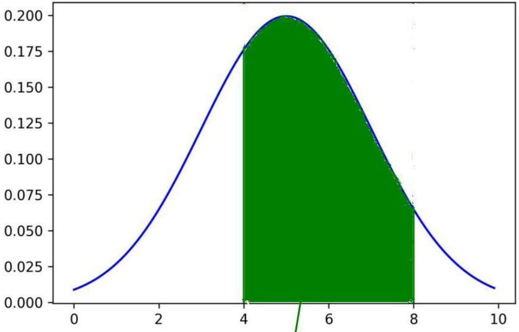


Converting to Standard Normal Distribution

If $X \sim \mathcal{N}(\mu, \sigma^2) \longrightarrow$ then $Z = \frac{X-\mu}{\sigma}$ is a standard random variable, i.e., $Z \sim \mathcal{N}(0, 1)$



Area Under Normal Distribution



$\mathbb{P}[4 < X < 8]$

Probability of X to be between 4 and 8

scipy

```
import math
from scipy import stats
A = stats.norm(3, math.sqrt(16)) # Declare A to be a normal random variable
print A.pdf(4)       # f(3), the probability density at 3
print A.cdf(2)       # F(2), which is also P(Y < 2)
print A.rvs()        # Get a random sample from A
```

What is N-day moving average?

In a time series it compute the average of past N days as the present value.

If f represent the time series of Bitcoin then a 7-day moving average of f is another time series, say g, such that today's value of g is the average price of past 7 days.

How to compute the moving average?

Use Convolution of original series with a kernel of average weight to obtain moving average.

$$(f \star g)[n] = \sum_{i=-\infty}^{\infty} f[i] \cdot g[n-i]$$

Don't worry about the formula of the convolution, Numpy will compute it for you!

Example

Let f be a time series and we want to compute the 10-day moving average.

Let g=$[0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1]$ be the kernel

np.convolve(f,g)=convolution of f and g by numpy

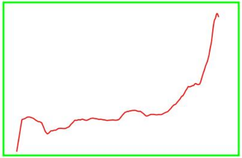
Convolution is act like a filter.

original series $\longrightarrow$ $\boxed{\star\, g}$ $\longrightarrow$ smooth time series

$$\boxed{f} \xrightarrow{g = [0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]} \boxed{f \star g}$$

input

output



Python Code

```python
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

```python
bitcoin=pd.read_csv("BTC-USD.csv", index_col="Date")
```

```python
kernel=[0.1 for i in range(10)]
```

```python
kernel
```

```
[0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1]
```

```python
smooth=np.convolve(bitcoin['Open'],kernel)
```

convolution of bitcoin['Open'] and kernel

Thank You