Deep Learning

Lecture 3: Calculus

Dr. Mehrdad Maleki

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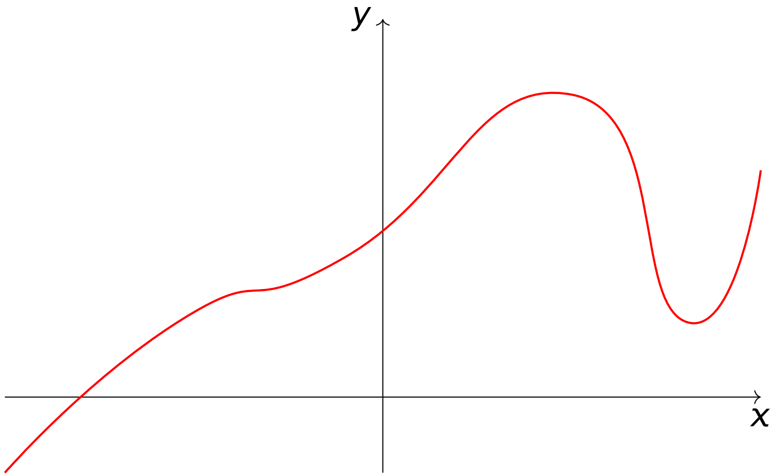
In one dimensional case when we have a scalar function of one variable $y = f(x)$ we can plot this on the $x - y$ coordinate and the slop of to the graph of the function at point x_0 is the derivative of f at x_0 and it is denoted by $f'(x_0)$ or $\frac{dy}{dx}\Big|_{x=x_0}$.



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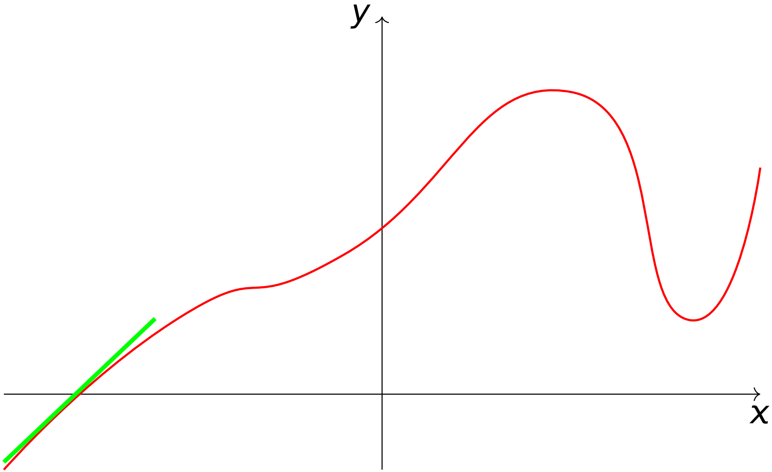

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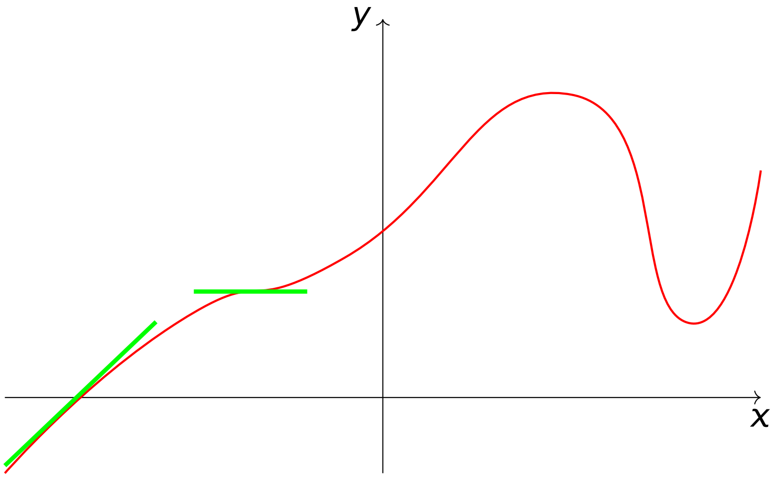

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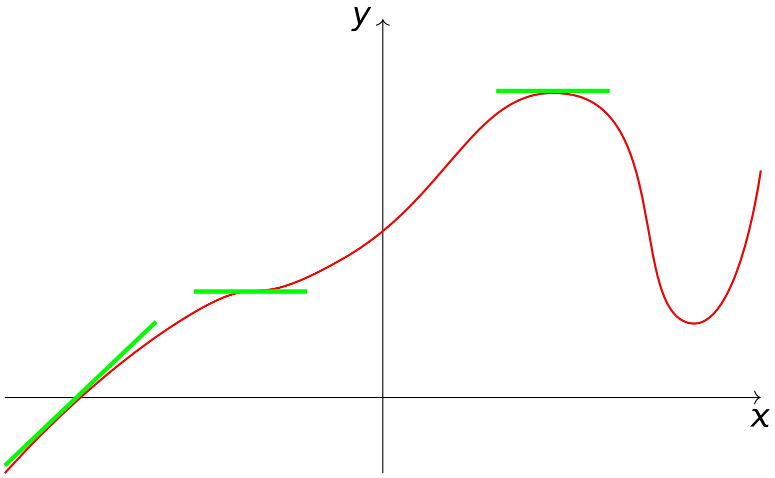

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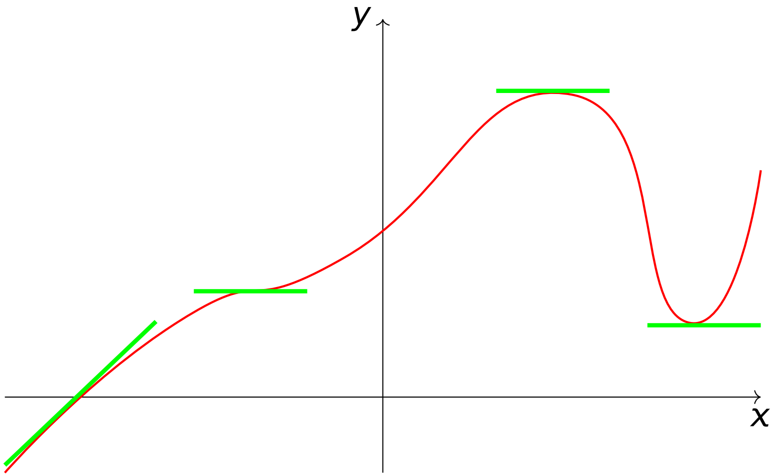

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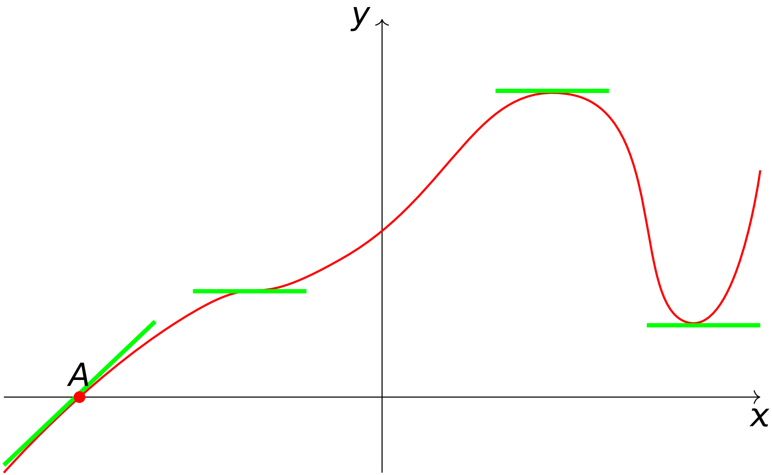

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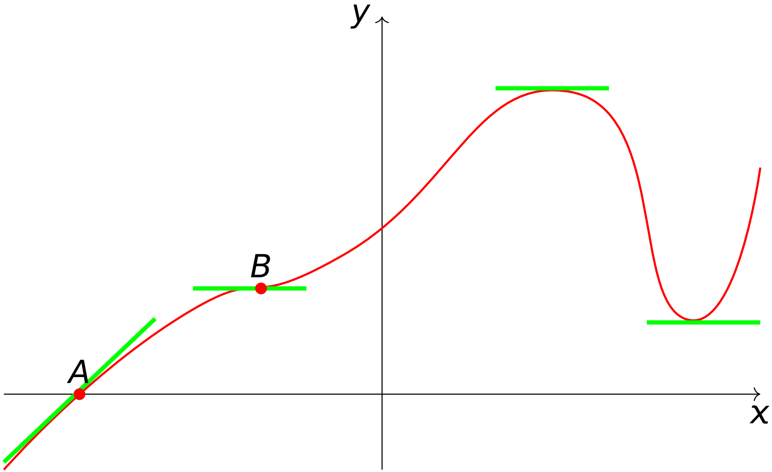

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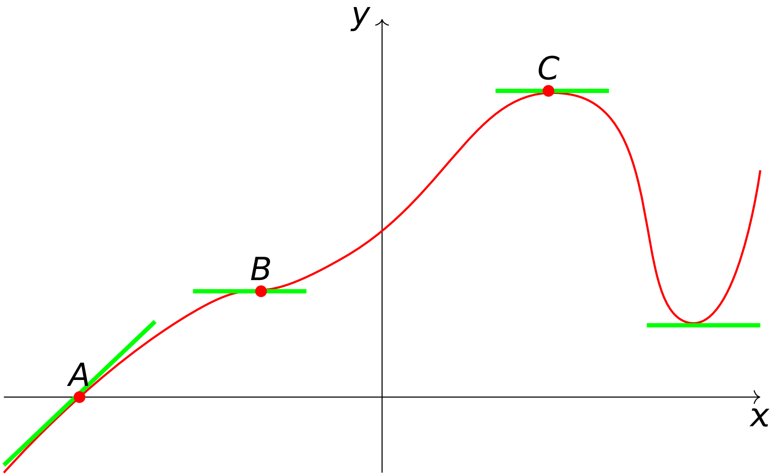

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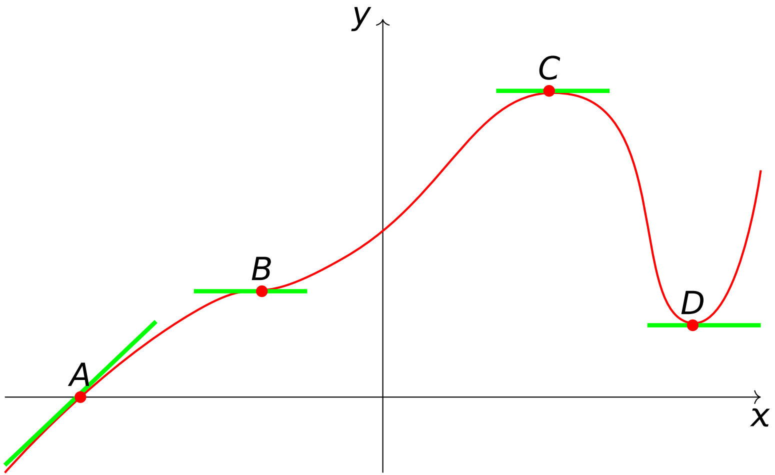

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&= sin(2) \\
&\approx 0.034899497
\end{aligned}$$

If $f : \mathbb{R}^n \rightarrow \mathbb{R}$, i.e., $y = f(x_1, \ldots, x_n)$, i.e., scalar y depends on reals $x_1, \ldots, x_n$, then the **Gradient** of f at $\mathbf{x_0}$ is

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$$\nabla f(\mathbf{x_0}) = \begin{bmatrix} \left.\frac{\partial y}{\partial x_1}\right|_{\mathbf{x_0}} \\ \vdots \\ \left.\frac{\partial y}{\partial x_n}\right|_{\mathbf{x_0}} \end{bmatrix}$$

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Directional Derivative

Let $\mathbf{u} \in \mathbb{R}^n$ and $\|\mathbf{u}\| = 1$, then the **Directional Derivative** of f in the direction of $\|\mathbf{u}\|$ in $\mathbf{x_0}$ is denoted by $\mathcal{D}_{\mathbf{u}} f(\mathbf{x_0})$ and is define by,

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In which direction, we have maximum Directional Derivative? If we choose $\mathbf{u} = \frac{\nabla f(\mathbf{x_0})}{\|\nabla f(\mathbf{x_0})\|}$ then we have maximum directional derivative and in the direction of $-\mathbf{u}$ we have minimum directional derivative.

What is the directional derivative of the funcion $f(x_1, x_2) = x_1^2 + x_2^2$ in the direction of $\mathbf{u} = \frac{1}{25}(3, 4)$ in $\mathbf{x_0} = (1, 1)$?

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$$
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\mathcal{D}_{\mathbf{u}} f(x_1, x_2) &= \nabla f(x_1, x_2) \cdot \mathbf{u} \\
&= \begin{bmatrix} 2 \\ 2 \end{bmatrix} \cdot \frac{1}{25} \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\
&= \frac{14}{25} \\
&= 0.56
\end{aligned}
$$

while $\mathcal{D}_{\frac{\nabla f(\mathbf{x_0})}{\|\nabla f(\mathbf{x_0})\|}} f(x_1, x_2) = \|\nabla f(\mathbf{x_0})\| \approx 2.82$

Jacobian

For a function $f : \mathbb{R}^n \to \mathbb{R}^m$, such that $(y_1, \ldots, y_m) = f(x_1, \ldots, x_n)$, the **Jacobian** of f at $\mathbf{x_0}$ is a $m \times n$ matrix and denoted by $\mathbf{J}_f(\mathbf{x_0})$ and is defined as follow:

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$$\mathbf{J}_f(\mathbf{x_0}) = \frac{\partial(y_1, \ldots, y_m)}{\partial(x_1, \ldots, x_n)}\Big|_{\mathbf{x_0}}$$

When $m = 1$ then the Jacobian of $f : \mathbb{R}^n \to \mathbb{R}$ is a row vector and its transpos is the gradient of f, i.e., When $m = 1$ then the Jacobian of $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a row vector and its transpos is the gradient of f, i.e.,

$$\mathbf{J}_f(\mathbf{x}) = (\nabla f(\mathbf{x}))^T$$

Let $f(x_1, x_2) = (x1x2, sin(x_1), cos(x_2))$. What is $\mathbf{J}_f(2, 3)$?

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$$= \begin{bmatrix} 3 & 2 \\ & \\ & \end{bmatrix}$$

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$$= \begin{bmatrix} x_2\Big|_{(2,3)} & x_1\Big|_{(2,3)} \\ cos(x_1)\Big|_{(2,3)} & 0\Big|_{(2,3)} \\ 0\Big|_{(2,3)} & -sin(x_2)\Big|_{(2,3)} \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 \\ 0.99 & 0 \\ 0 & -0.05 \end{bmatrix}$$

Let define $f(\mathbf{x}) = \mathbf{x}^T \mathbf{a}$, where $\mathbf{a} : \mathbb{R}^n$ is a constant vector. Then $f : \mathbb{R}^n \rightarrow \mathbb{R}$. What is the Jacobian of f at $\mathbf{x_0}$?

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$$
\begin{aligned}
f(\mathbf{x_0}) &= \begin{bmatrix} x_1 & \dots & x_n \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} \\
&= x_1 a_1 + \cdots + x_n a_n
\end{aligned}
$$

$$\mathbf{J}_f(\mathbf{x_0}) = \begin{bmatrix} \frac{\partial(x_1 a_1 + \cdots + x_n a_n)}{\partial x_1} & \ldots & \frac{\partial(x_1 a_1 + \cdots + x_n a_n)}{\partial x_n} \end{bmatrix}$$

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\begin{aligned}
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&= \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix} \\
&= \mathbf{a}^T
\end{aligned}
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\begin{aligned}
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&= x_1 a_1 + \cdots + x_n a_n
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$$
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\end{aligned}
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Let $\mathbf{W}_{m \times n}$ be a constant matrix and let define $f(\mathbf{x}) = \mathbf{W}\mathbf{x}$. Then $f : \mathbb{R}^n \to \mathbb{R}^m$. What is the Jacobian of f at $\mathbf{x_0}$?

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$$f(\mathbf{x}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ & & \\ & & \end{bmatrix}$$

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&= \begin{bmatrix} w_{11}x_1 + \cdots + w_{1n}x_n \\ \\ \\ \end{bmatrix}
\end{aligned}
$$

Let $\mathbf{W}_{m\times n}$ be a constant matrix and let define $f(\mathbf{x}) = \mathbf{W}\mathbf{x}$. Then $f : \mathbb{R}^n \to \mathbb{R}^m$. What is the Jacobian of f at $\mathbf{x_0}$?

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&= \left[\begin{matrix} w_{11}x_1 + \cdots + w_{1n}x_n \\ \vdots \\ \\ \end{matrix} \right.
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\end{aligned}
$$

$$\mathbf{J}_f(\mathbf{x_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_1} & \cdots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_n} \\ & & \\ & & \end{bmatrix}$$

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$$
\begin{aligned}
\mathbf{J}_f(\mathbf{x_0}) &= \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_1} & \dots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_1} & \dots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_n} \end{bmatrix} \\
&= \left[\begin{matrix} w_{11} & \dots & w_{1n} \\ & & \\ & & \end{matrix} \right.
\end{aligned}
$$

$$
\mathbf{J}_f(\mathbf{x_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1 + \cdots + w_{1n}x_n)}{\partial x_1} & \cdots & \frac{\partial(w_{11}x_1 + \cdots + w_{1n}x_n)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1 + \cdots + w_{mn}x_n)}{\partial x_1} & \cdots & \frac{\partial(w_{m1}x_1 + \cdots + w_{mn}x_n)}{\partial x_n} \end{bmatrix}
$$

$$
= \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ & & \end{bmatrix}
$$

$$
\mathbf{J}_f(\mathbf{x_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_1} & \dots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_1} & \dots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_n} \end{bmatrix}
$$

$$
= \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{J}_f(\mathbf{x_0}) &= \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_1} & \cdots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_1} & \cdots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial x_n} \end{bmatrix} \\
&= \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \\
&= \mathbf{W}
\end{aligned}
$$

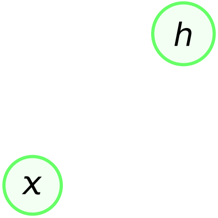
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a quadratic map defined by $f(\mathbf{x}) = \mathbf{x}^T \mathbf{A}\mathbf{x}$, where A is a symmetric matrix of dimension $(n \times n)$.

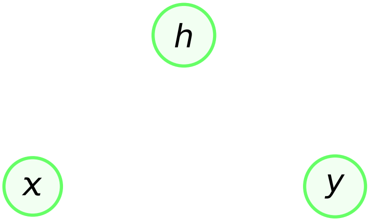
Let $f : \mathbb{R}^n \longrightarrow \mathbb{R}$ be a quadratic map defined by $f(\mathbf{x}) = \mathbf{x}^T\mathbf{A}\mathbf{x}$, where A is a symmetric matrix of dimension $(n \times n)$. Since matrix multiplication is associative we can see f as the multiplication of two matrix $\mathbf{x}^T$ and $\mathbf{y}(\mathbf{x}) = \mathbf{A}\mathbf{x}$.

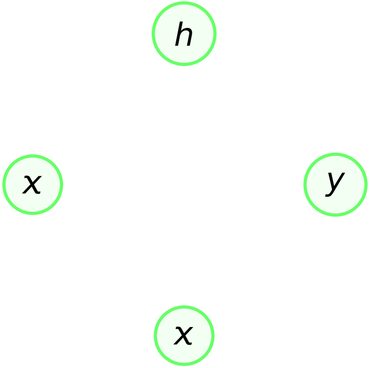
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a quadratic map defined by $f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x}$, where A is a symmetric matrix of dimension $(n \times n)$. Since matrix multiplication is associative we can see f as the multiplication of two matrix $\mathbf{x}^T$ and $\mathbf{y}(\mathbf{x}) = \mathbf{A}\mathbf{x}$. Note that the matrix multiplication is exactly the function composition!

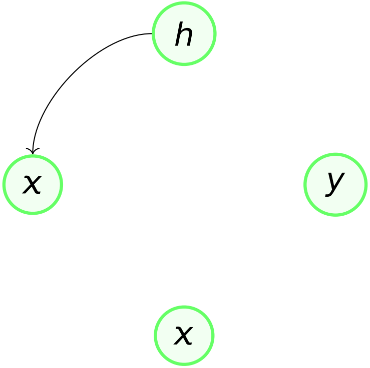
Let $f : \mathbb{R}^n \to \mathbb{R}$ be a quadratic map defined by $f(\mathbf{x}) = \mathbf{x}^T \mathbf{A}\mathbf{x}$, where A is a symmetric matrix of dimension $(n \times n)$. Since matrix multiplication is associative we can see f as the multiplication of two matrix $\mathbf{x}^T$ and $\mathbf{y}(\mathbf{x}) = \mathbf{A}\mathbf{x}$. Note that the matrix multiplication is exactly the function composition! So $f(\mathbf{x}) = h(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \mathbf{y}$ and if we apply the chaine rule to h we have:

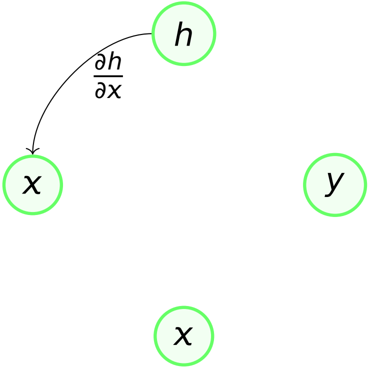

h

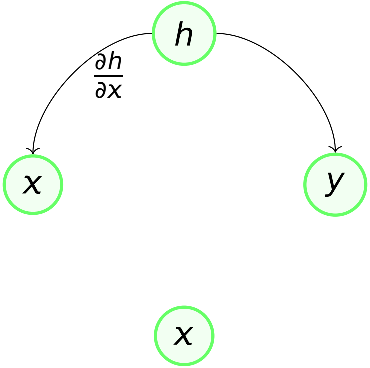

h
x


h
x
y

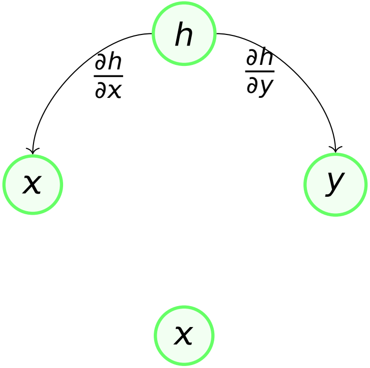

h
x
y
x

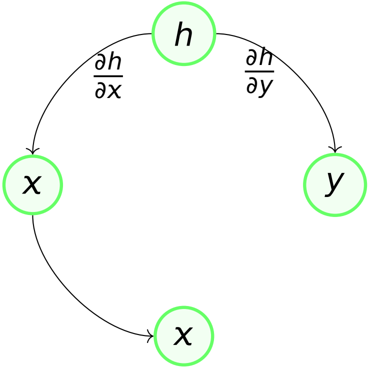

h
x
y
x


h
$\frac{\partial h}{\partial x}$
x
y
x

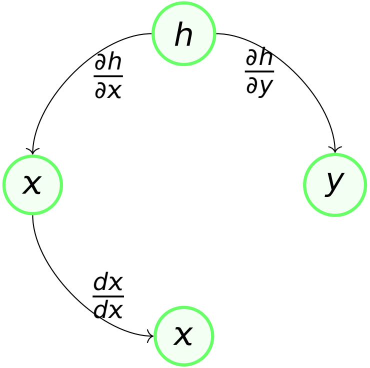


h
∂h/∂x
x
y
x

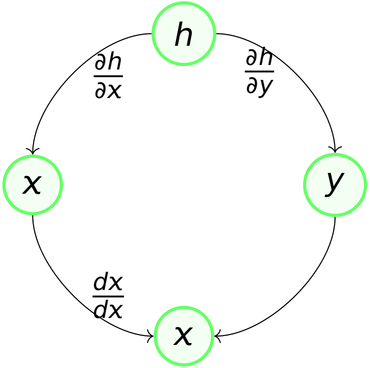

h
∂h/∂x
∂h/∂y
x
y
x



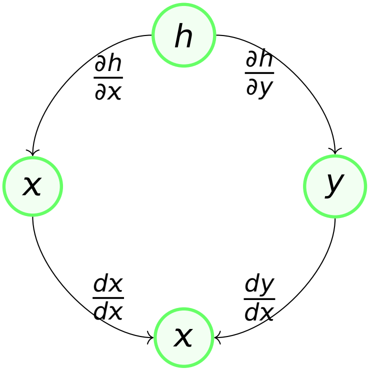
h
$\frac{\partial h}{\partial x}$
$\frac{\partial h}{\partial y}$
x
y
x



h
x
y
x
∂h/∂x
∂h/∂y
dx/dx



h
$\frac{\partial h}{\partial x}$
$\frac{\partial h}{\partial y}$
x
y
$\frac{dx}{dx}$
x


h
x
y
x
$\frac{\partial h}{\partial x}$
$\frac{\partial h}{\partial y}$
$\frac{dx}{dx}$
$\frac{dy}{dx}$

$$\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}}$$

$$
\begin{aligned}
\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\
&= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}}
\end{aligned}
$$

$$\begin{aligned}\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\ &= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \\ &= \mathbf{y}^T + \mathbf{x}^T \mathbf{A}\end{aligned}$$

$$
\begin{aligned}
\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\
&= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \\
&= \mathbf{y}^T + \mathbf{x}^T \mathbf{A} \\
&= (\mathbf{A}\mathbf{x})^T + \mathbf{x}^T \mathbf{A}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\
&= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \\
&= \mathbf{y}^T + \mathbf{x}^T \mathbf{A} \\
&= (\mathbf{A}\mathbf{x})^T + \mathbf{x}^T \mathbf{A} \\
&= \mathbf{x}^T \mathbf{A}^T + \mathbf{x}^T \mathbf{A}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\
&= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \\
&= \mathbf{y}^T + \mathbf{x}^T \mathbf{A} \\
&= (\mathbf{A}\mathbf{x})^T + \mathbf{x}^T \mathbf{A} \\
&= \mathbf{x}^T \mathbf{A}^T + \mathbf{x}^T \mathbf{A} \\
&= \mathbf{x}^T (\mathbf{A}^T + \mathbf{A})
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} &= \frac{\partial h(\mathbf{x}, \mathbf{y})}{\partial \mathbf{x}} \\
&= \frac{\partial h}{\partial \mathbf{x}} \frac{\partial \mathbf{x}}{\partial \mathbf{x}} + \frac{\partial h}{\partial \mathbf{y}} \frac{\partial \mathbf{y}}{\partial \mathbf{x}} \\
&= \mathbf{y}^T + \mathbf{x}^T \mathbf{A} \\
&= (\mathbf{A}\mathbf{x})^T + \mathbf{x}^T \mathbf{A} \\
&= \mathbf{x}^T \mathbf{A}^T + \mathbf{x}^T \mathbf{A} \\
&= \mathbf{x}^T (\mathbf{A}^T + \mathbf{A}) \\
&= 2\mathbf{x}^T \mathbf{A}
\end{aligned}
$$

The last equality holds because $\mathbf{A}$ is symmetric, i.e., $\mathbf{A}^T = \mathbf{A}$

Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{W}\mathbf{x}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$f(\mathbf{W}) =$$

Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{W}\mathbf{x}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ & & \\ & & \end{bmatrix}$$

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Let $f : \mathbb{R}^{m \times n} \rightarrow \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{Wx}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix}$$

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$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \\ \\ \end{bmatrix}$$

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$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \left[\begin{matrix} x_1 \\ \vdots \\ \end{matrix} \right.$$

Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{W}\mathbf{x}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

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$$
\begin{aligned}
f(\mathbf{W}) &= \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \\
&= \left[\begin{matrix} w_{11}x_1 + \cdots + w_{1n}x_n \\ \\ \\ \end{matrix} \right.
\end{aligned}
$$

Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{Wx}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$f(\mathbf{W}) = \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \left[\begin{matrix} w_{11}x_1 + \dots + w_{1n}x_n \\ \vdots \\ \end{matrix} \right.$$

Let $f : \mathbb{R}^{m \times n} \to \mathbb{R}^n$ with $f(\mathbf{W}) = \mathbf{W}\mathbf{x}$ for a fixed $\mathbf{x} : \mathbb{R}^n$. Then,

$$
\begin{aligned}
f(\mathbf{W}) &= \begin{bmatrix} w_{11} & \dots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{m1} & \dots & w_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \\
&= \begin{bmatrix} w_{11}x_1 + \cdots + w_{1n}x_n \\ \vdots \\ w_{m1}x_1 + \cdots + w_{mn}x_n \end{bmatrix}
\end{aligned}
$$

$$\mathbf{J}_f(\mathbf{W_0}) = \left[\begin{matrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{11}} & \cdots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{mn}} \\ & & \\ & & \end{matrix} \right.$$

$$\mathbf{J}_f(\mathbf{W_0}) = \left[\begin{matrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{11}} & \cdots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ & & \end{matrix} \right.$$

$$\mathbf{J}_f(\mathbf{W_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{11}} & \cdots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{11}} & \cdots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix}$$

$$\mathbf{J}_f(\mathbf{W_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & \dots & x_n & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ & & & & & & & & & \\ & & & & & & & & & \end{bmatrix}$$

$$\mathbf{J}_f(\mathbf{W_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix}$$

$$= \begin{bmatrix} x_1 & \dots & x_n & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & x_1 & \dots & x_n & \dots & 0 & \dots & 0 \\ & & & & & & & & & \\ & & & & & & & & & \end{bmatrix}$$

$$
\mathbf{J}_f(\mathbf{W_0}) = \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix}
$$

$$
= \begin{bmatrix} x_1 & \dots & x_n & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & x_1 & \dots & x_n & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & x_1 & \dots & x_n \end{bmatrix}
$$

$$
\begin{aligned}
\mathbf{J}_f(\mathbf{W_0}) &= \begin{bmatrix} \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{11}x_1+\dots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{m1}x_1+\dots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix} \\
&= \begin{bmatrix} x_1 & \dots & x_n & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & x_1 & \dots & x_n & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & x_1 & \dots & x_n \end{bmatrix} \\
&= \begin{bmatrix} x_1 & \dots x_n \end{bmatrix} \otimes \mathbf{I}_m
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{J}_f(\mathbf{W_0}) &= \begin{bmatrix} \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{11}x_1+\cdots+w_{1n}x_n)}{\partial w_{mn}} \\ \vdots & \ddots & \vdots \\ \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{11}} & \dots & \frac{\partial(w_{m1}x_1+\cdots+w_{mn}x_n)}{\partial w_{mn}} \end{bmatrix} \\
&= \begin{bmatrix} x_1 & \dots & x_n & 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & x_1 & \dots & x_n & \dots & 0 & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & x_1 & \dots & x_n \end{bmatrix} \\
&= \begin{bmatrix} x_1 & \dots x_n \end{bmatrix} \otimes \mathbf{I}_m \\
&= \mathbf{x} \otimes \mathbf{I}_m
\end{aligned}
$$

Thank You