

We know that

$$y = T(x) \quad (1)$$

$$x = T^{-1}(y) \quad (2)$$

$$p_Y(y) = p_X(T^{-1}(y)) \left| \frac{d}{dy} T^{-1}(y) \right| \quad (3)$$

$$p_Y(y) = \frac{k}{l} * \left(\frac{y}{l}\right)^{k-1} * e^{-\left(\frac{y}{l}\right)^k} \quad (4)$$

$$P_Y(y) = 1 - e^{-\left(\frac{y}{l}\right)^k} \quad (5)$$

Combining (3) and (5) we have

$$1 - e^{-\left(\frac{y}{l}\right)^k} = \int_0^y p_X(T^{-1}(z)) \left| \frac{d}{dz} T^{-1}(z) \right| dz \quad (6)$$

Applying integration by substitution

$$1 - e^{-\left(\frac{y}{l}\right)^k} = \int_0^{T^{-1}(y)} p_X(z) dz \quad (7)$$

Using (1) and (2)

$$1 - e^{-\left(\frac{T(x)}{l}\right)^k} = \int_0^x p_X(z) dz \quad (8)$$

We approximate probability distribution with normalized histogram

$$\int_0^x p_X(z) dz \approx \tilde{H} \quad (9)$$

Which means that if  $N$  - number of pixels in the image

$$e^{-\left(\frac{T(x)}{l}\right)^k} = 1 - \frac{H(x)}{N} \quad (10)$$

Take logarithm of both sides

$$-\left(\frac{T(x)}{l}\right)^k = \ln\left(1 - \frac{H(x)}{N}\right) \quad (11)$$

Multiply both sides by  $-l^k$

$$T(x)^k = \ln\left(1 - \frac{H(x)}{N}\right) * -l^k \quad (12)$$

And finally take both sides to the power of  $1/k$

$$T(x) = \left[ \left| \ln\left(1 - \frac{H(x)}{N}\right) \right| \right]^{\frac{1}{k}} * l \quad (13)$$