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Отчет об исследовательском проекте

на тему: Приближение магнитудной функции метрического графа

(промежуточный, этап 1)

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1 Introduction

Metric graph is a topological space which, simply put, represents a glued set of closed intervals. Unlike combinatorical weighened graph it is not discrete, but continuous, containing not only vertexes but also all points on edges.

For a given metric graph we can calculate a magnitude function, objects originally coming from category theory. Being a generalized notion of objects size, magnitude function is defined for enriched categories, which contain metric spaces as a subset, which in their turn contain metric graphs.

The aim of the following work is to study relation between metric graphs and magnitude function by generating metric graphs and approximating their magnitude function using symbolic computations. After the results of the experiments are ready the final step would be to make a suggestion of its value for a general case of a metric graph.

2 Background

2.1 Definitions

2.1.1 Metric graph

As already metioned in introduction we can intuitevely understand metric graphs as weighened graphs which besides vertexes also contain all the points on the edges. The other intuitive understanding would be a set of intervals bounded in the ends, therefore forming a graph.

A stricter definition, originally coming from Mugnolo [1], is given below.

$$\mathcal{E} := \bigsqcup [0, l_e] \mid e \in E,\ l_e \in (0, \infty), \text{ where } E \text{ is a countable set and } \bigsqcup \text{ is a disjoined topological union}$$

$$\text{The notation for elements of } \mathcal{E} \text{ is: } (x, e) \in \mathcal{E} \iff e \in E \wedge x \in [0, l_e]$$

$$\mathcal{V} := \bigsqcup \{(0, e), (l_e, e)\}$$

$$\text{On } \mathcal{V} \text{ we define an equivalency relation } \sim_{\mathcal{V}} \text{ with classes of equivalency } \{V_i\}$$

$$\text{On } \mathcal{E} \text{ we define an equivalency relation } \sim_{\mathcal{E}} \mid (x_1, e_1) \sim_{\mathcal{E}} (x_2, e_2) \iff x_1 = x_2; e_1 = e_2 \text{ or } (x_1, e_1) \sim_{\mathcal{V}} (x_2, e_2)$$

Definition 1:

2.1.2 Magnitude

2.1.3 Magnitude function

2.1.4 Spread

2.2 Metric graph as a compact metric space

2.3 Magnitude function calculation for compact metric spaces

2.3.1 Example for interval of length l

3 Methodology

4 Current results and future work plan

References

[1] Delio Mugnolo. *What is actually a metric graph?* 2019. DOI: 10.48550/ARXIV.1912.07549. URL: https://arxiv.org/abs/1912.07549.