Commuting signatures and verifiable encryption

Georg Fuchsbauer

University of Bristol

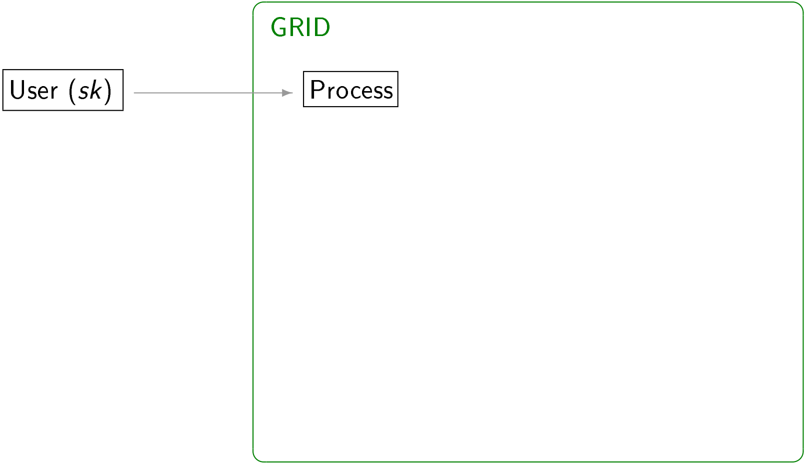
Darmstadt, 21.11.2011

Outline of this talk

User (sk)

GRID

Motivation : GRID Computing



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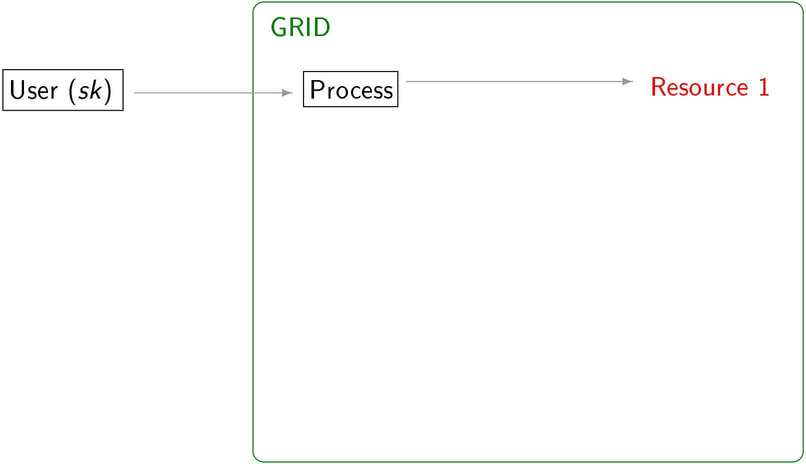
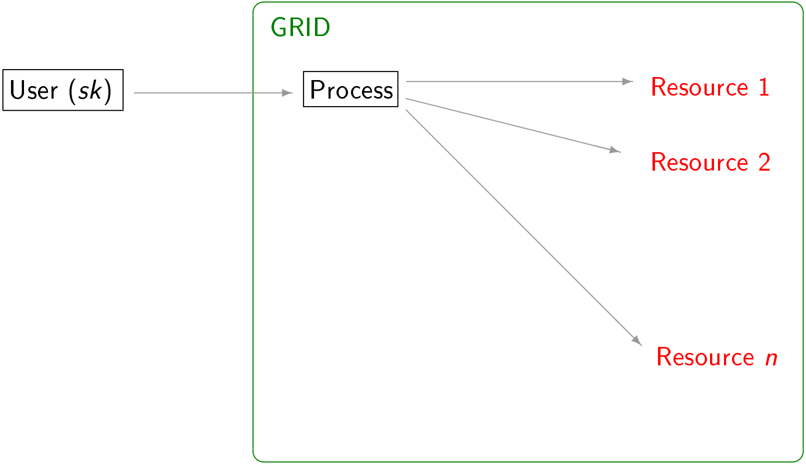
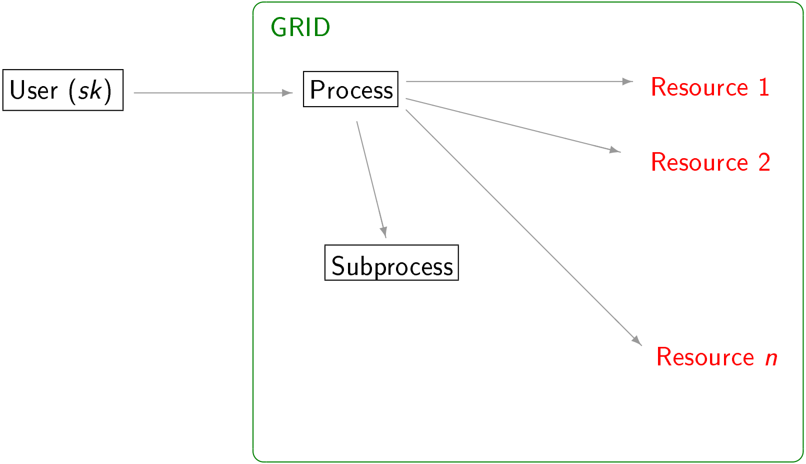
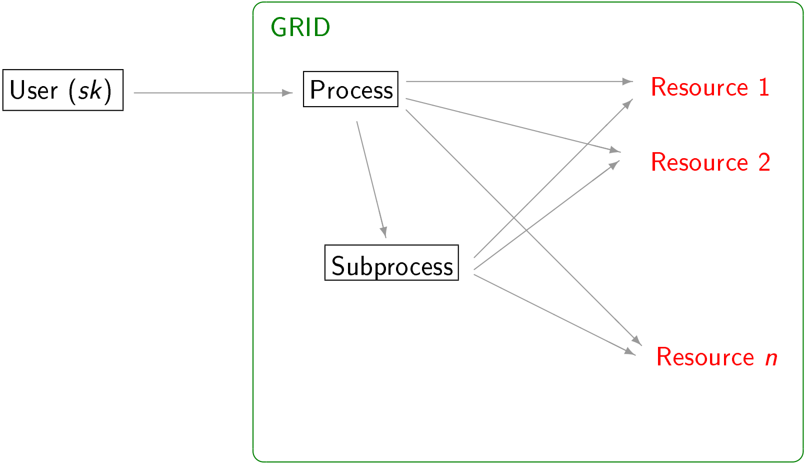



GRID
User (sk)
Process
Resource 1
Resource 2
Resource n


GRID
User (sk)
Process
Subprocess
Resource 1
Resource 2
Resource n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GRID
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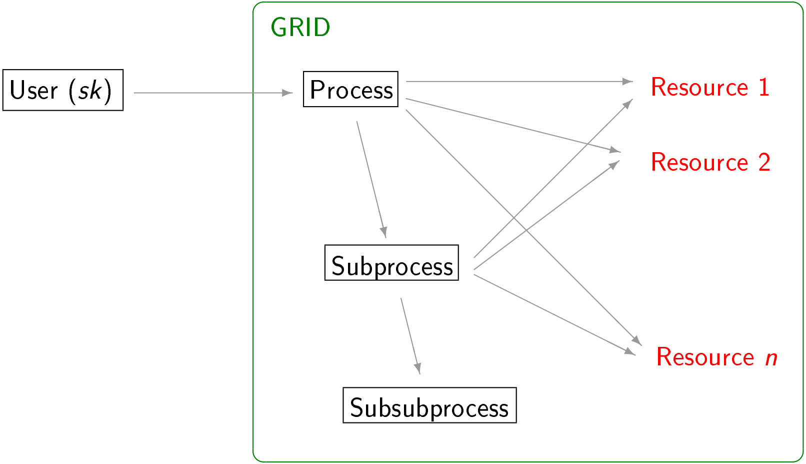
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Resource 2
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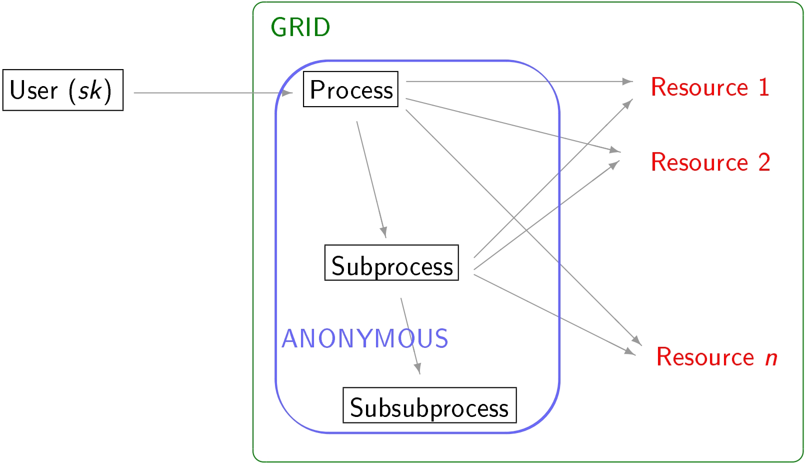
GRID
User (sk)
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Subprocess
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ANONYMOUS
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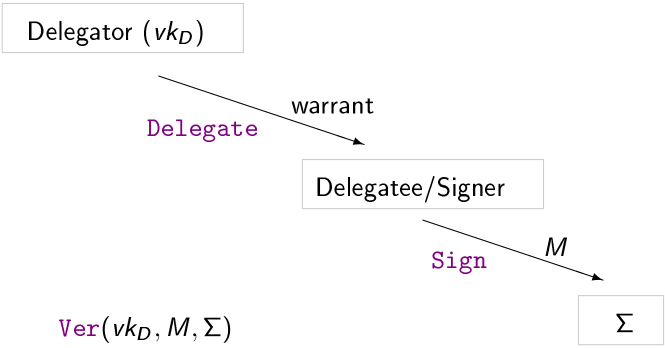
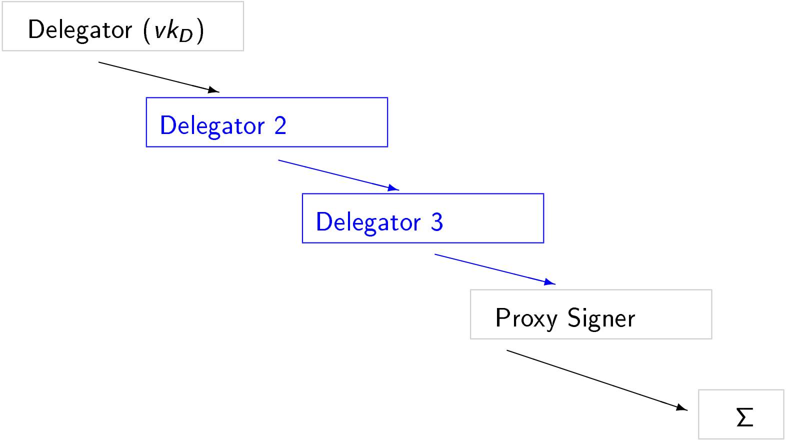
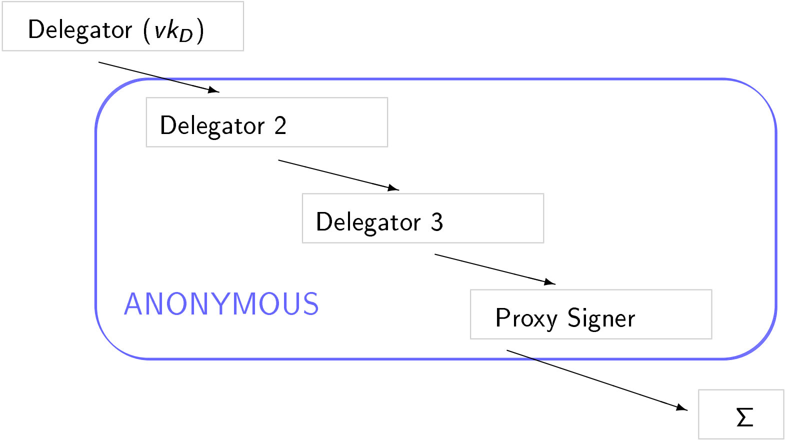

Delegator (vk_D)
warrant
Delegate
Delegatee/Signer
M
Sign
Σ
$\mathtt{Ver}(vk_D, M, \Sigma)$


Delegator (vk_D)
Delegator 2
Delegator 3
Proxy Signer
Σ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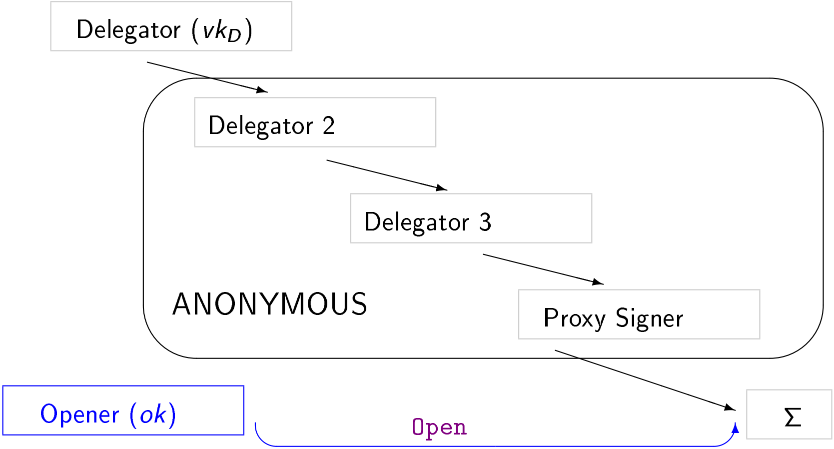
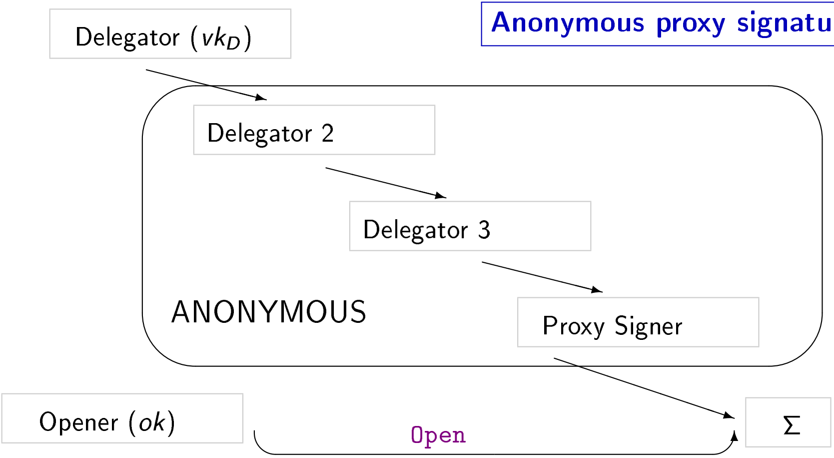

Delegator (vk_D)
Delegator 2
Delegator 3
ANONYMOUS
Proxy Signer
Opener (ok)
Open
Σ

Anonymous proxy signatures



Ingredients

- Digital signatures
- Public-key encryption

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- Public-key encryption
- Non-interactive zero-knowledge proofs (NIZK)

  ...allow us to prove validity of a statement

  without revealing anything else

[simplified version]

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System parameters : [. . .]

Anonymous proxy signatures : Generic construction II

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System parameters : [. . .]

Delegate Sign delegatee's verification key → warrant
Re-delegate Additionally forward received warrant(s)

$$vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \bullet\bullet\bullet \xrightarrow{\Sigma_n} vk_n$$

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Encrypt

- delegators' verification keys
- warrants
- signature on message

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- ciphertexts
- proofs

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Open Decrypt ciphertext → verification keys

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Showed theoretic feasability
but can we instantiate them practically ?

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Open Decrypt ciphertext → verification keys

Bilinear group : $(p, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e, G, H)$ with

Groups : $\mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T$ cyclic groups of prime order p

Pairing : $e \colon \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ bilinear, ie

$e(X^a, Y^b) = e(X, Y)^{ab}$ for all $X \in \mathbb{G}_1; Y \in \mathbb{G}_2; a, b \in \mathbb{Z}$

Generators : $\mathbb{G}_1 = \langle G \rangle$, $\mathbb{G}_2 = \langle H \rangle$, $\mathbb{G}_T = \langle e(G, H) \rangle$

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Pairing-product equation (PPE)

over variables $X_1, \ldots, X_m \in \mathbb{G}_1$, $Y_1, \ldots, Y_n \in \mathbb{G}_2$

$$\prod_{i=j}^{n} e(A_j, Y_j) \prod_{i=1}^{m} e(X_i, B_i) \prod_{i=1}^{m} \prod_{j=1}^{n} e(X_i, Y_j)^{\gamma_{i,j}} = \mathbf{t} \ , \qquad \text{(E)}$$

defined by $A_i \in \mathbb{G}_1, B_i \in \mathbb{G}_2$, $\gamma_{i,j} \in \mathbb{Z}_p$ and $\mathbf{t} \in \mathbb{G}_T$

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Groth–Sahai proofs [GS08]
Efficient non-interactive zero-knowledge proofs :

1. Encrypt X_i's and Y_j's
2. Make proof π that encrypted values satisfy E w/o revealing anything else

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Have a *proof system* for very specific language
but can we combine it with signatures ?

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- unforgeable (under chosen-message attacks)

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Automorphic signatures

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Combined with Groth-Sahai proofs :

- encrypt keys, messages, and signatures
- prove validity of encryptions

  $\Rightarrow$ verifiably encrypt certificate chain

Applications of automorphic signatures

- Efficient *anonymous proxy signatures*
- *Non-frameable group signatures* with concurrent join
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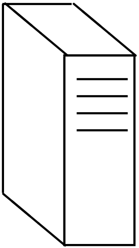
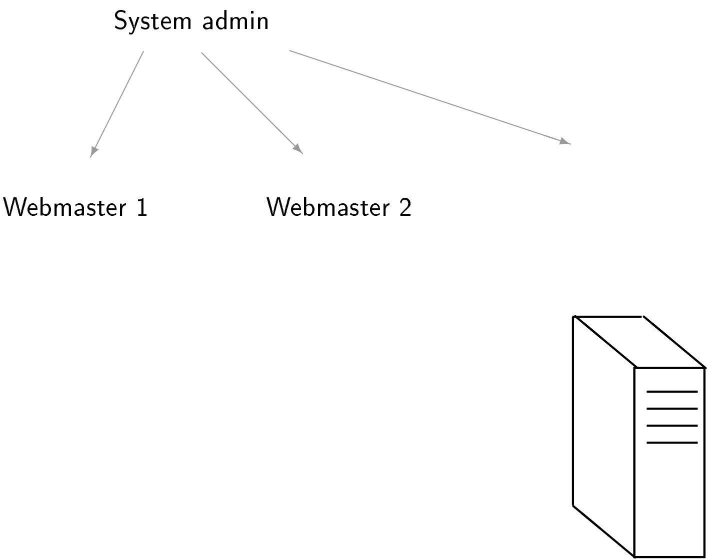
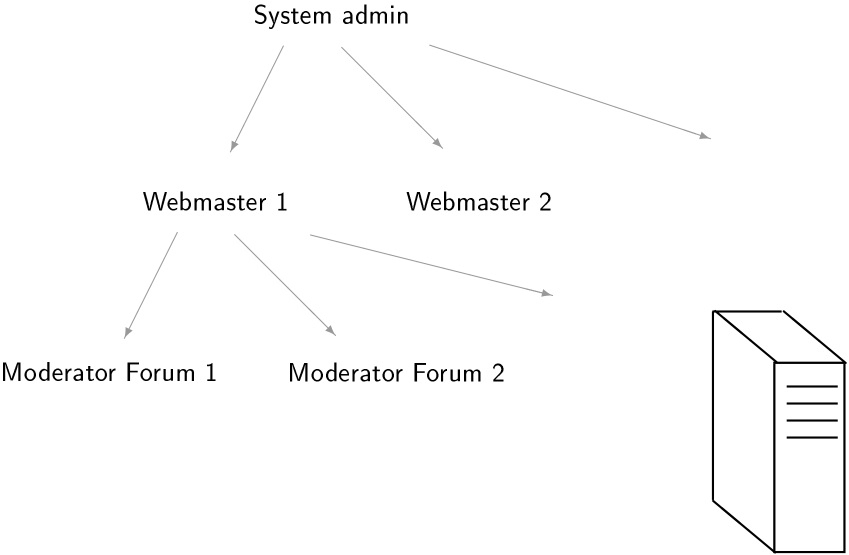
System admin




System admin
Webmaster 1
Webmaster 2

Motivation : Delegatable credentials



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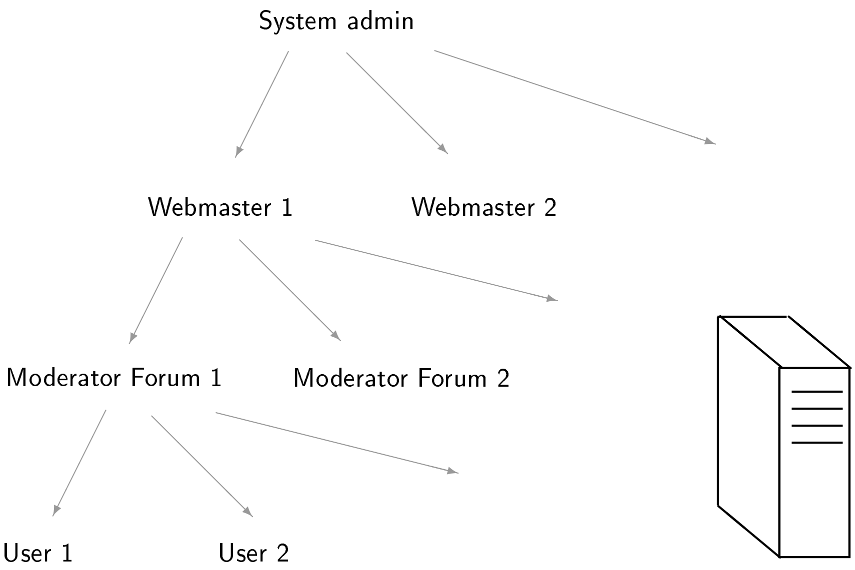

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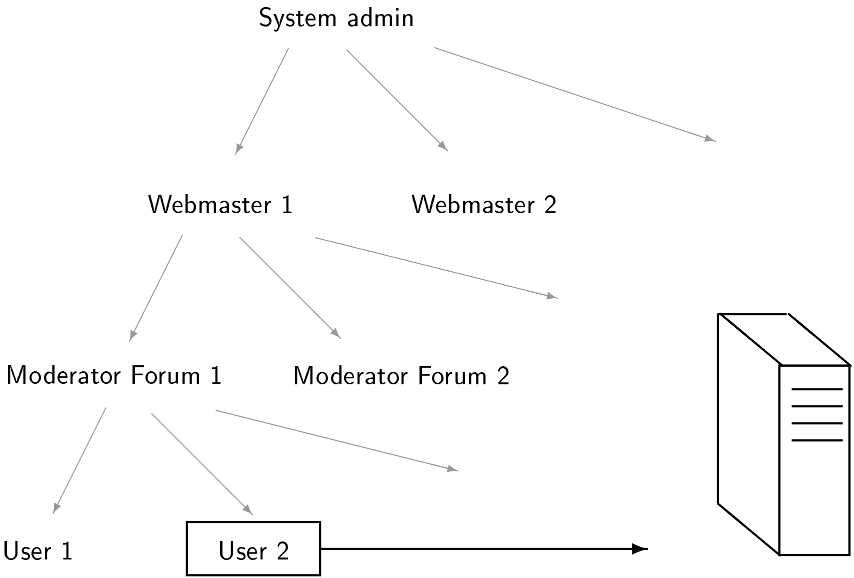

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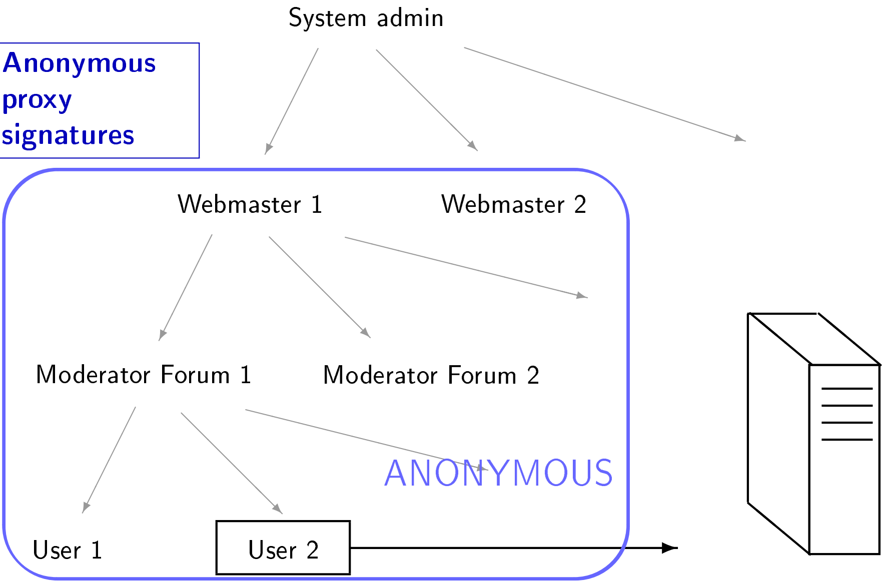
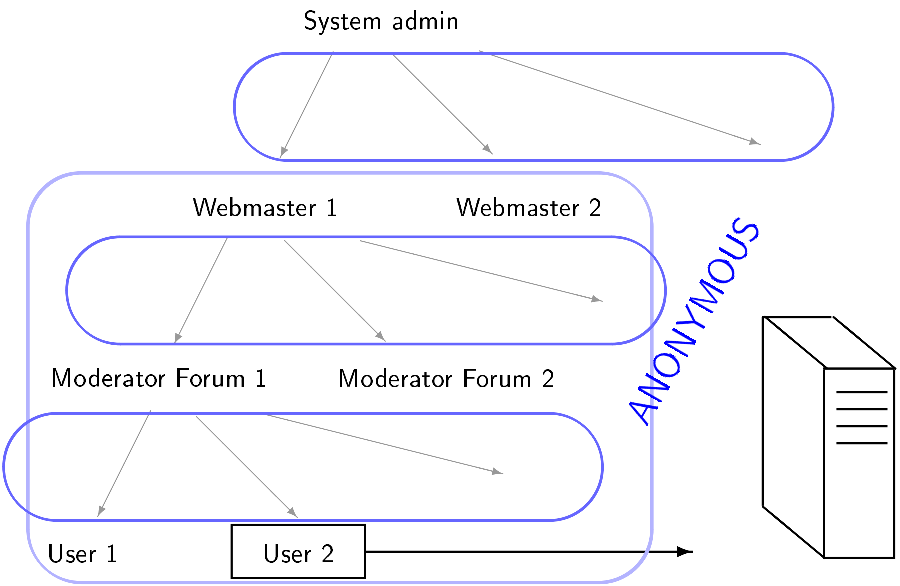
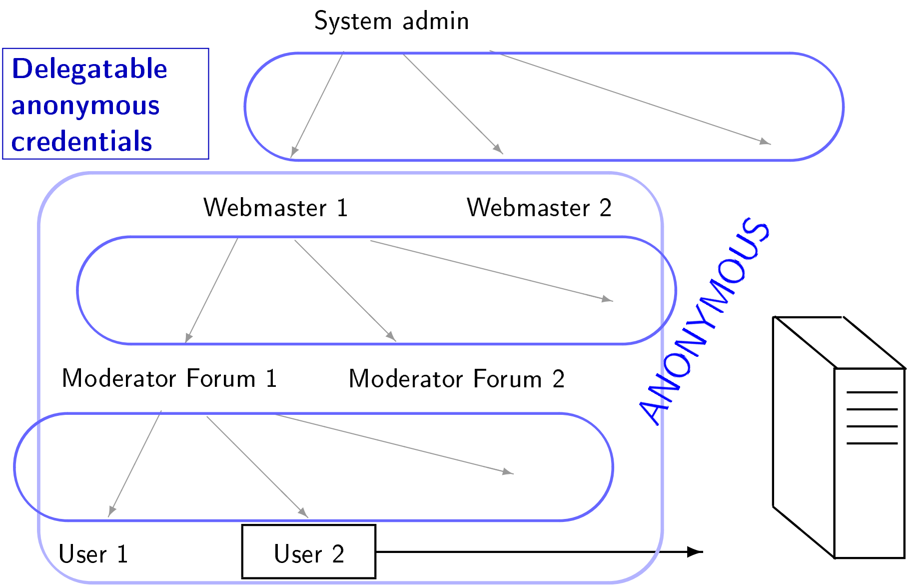



System admin
Webmaster 1
Webmaster 2
Moderator Forum 1
Moderator Forum 2
User 1
User 2
ANONYMOUS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Delegatable anonymous credentials
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Webmaster 2
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Delegatable anonymous credentials [BCCKLS09]

- Users can *prove* to hold credential w/o revealing their identity
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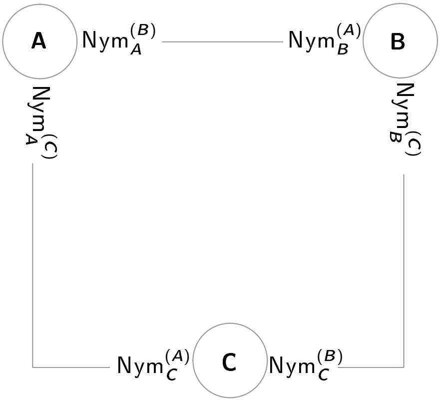
Our scheme : *Non-interactive* issuing & delegation


A
$\mathrm{Nym}_A^{(B)}$
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B
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$\mathrm{Nym}_B^{(C)}$
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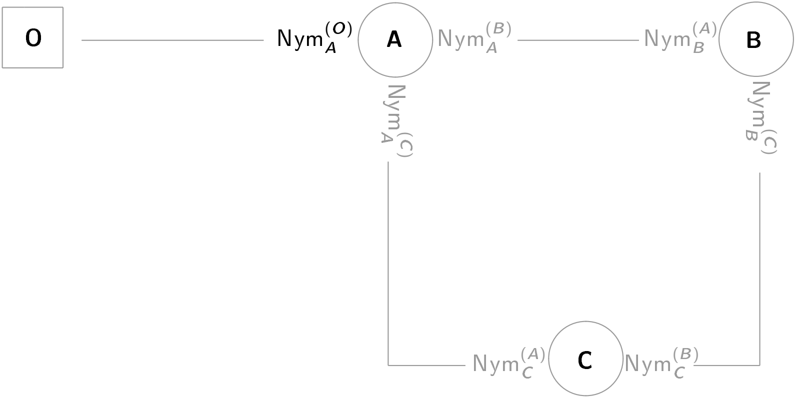
O
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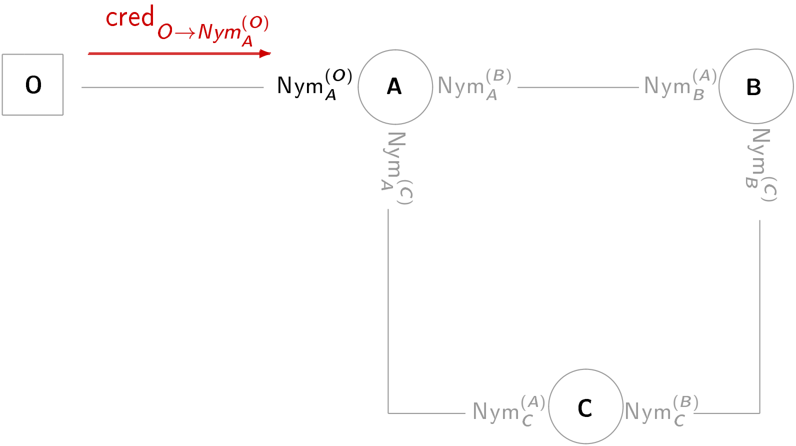
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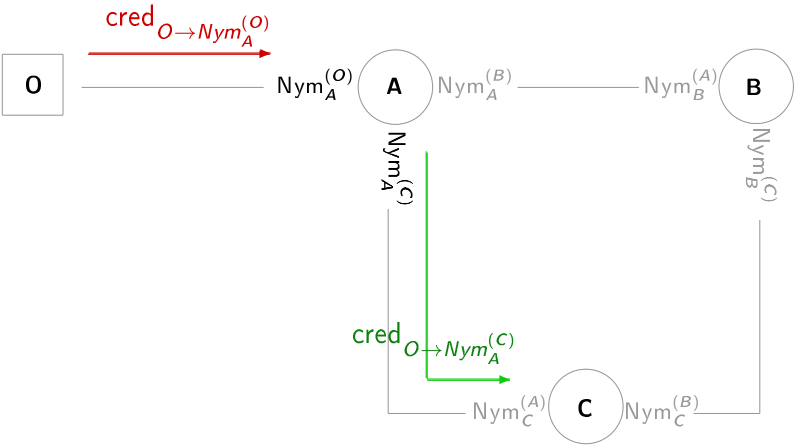

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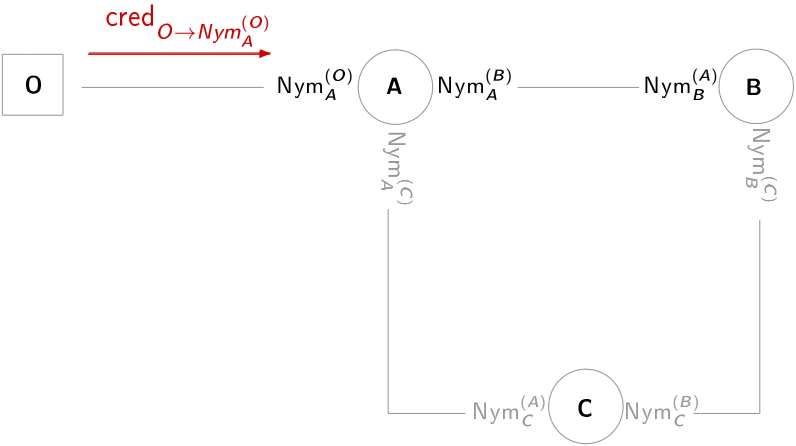
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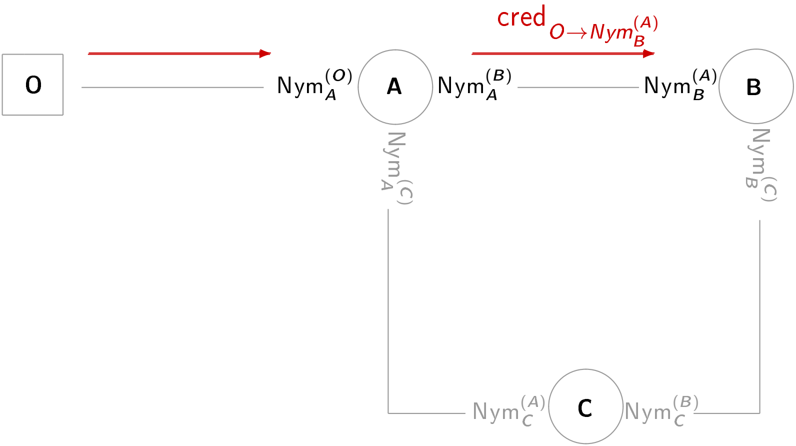
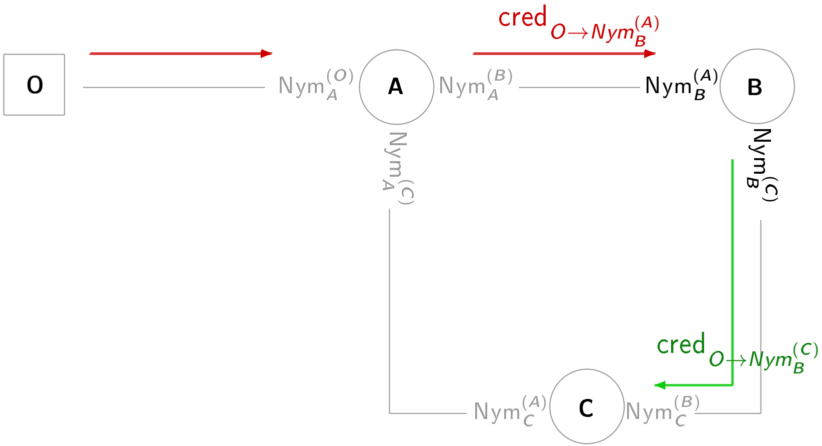
B
$\mathsf{Nym}_A^{(C)}$
$\mathsf{Nym}_B^{(C)}$
$\mathsf{Nym}_C^{(A)}$
C
$\mathsf{Nym}_C^{(B)}$

Non-interactively delegatable anonymous credentials





$\mathsf{cred}_{O \to Nym_B^{(A)}}$
O
$\mathsf{Nym}_A^{(O)}$
A
$\mathsf{Nym}_A^{(B)}$
$\mathsf{Nym}_B^{(A)}$
B
$\mathsf{Nym}_A^{(C)}$
$\mathsf{Nym}_B^{(C)}$
$\mathsf{cred}_{O \to Nym_B^{(C)}}$
$\mathsf{Nym}_C^{(A)}$
C
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Commuting signatures and verifiable encryption I

- Signature $\quad M \xrightarrow{sk} \Sigma \quad$ Verification : vk, M, Σ

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Verification : $vk,\ M,\ \boxed{\Sigma}, \widetilde{\pi}$

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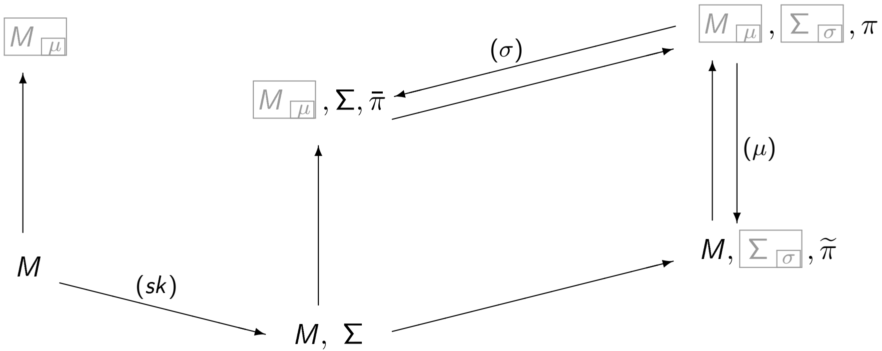
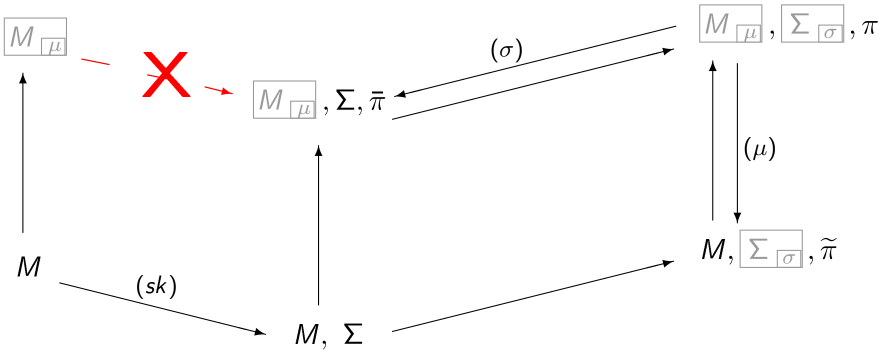
Sign plaintext then encrypt $\Longleftrightarrow$ encrypt then sign plaintext

Sign M given $\boxed{M}$: $\quad \boxed{M} \xrightarrow{sk} \boxed{\Sigma}, \pi \qquad$ Verification : $\boxed{vk, \boxed{M}, \boxed{\Sigma}, \pi}$

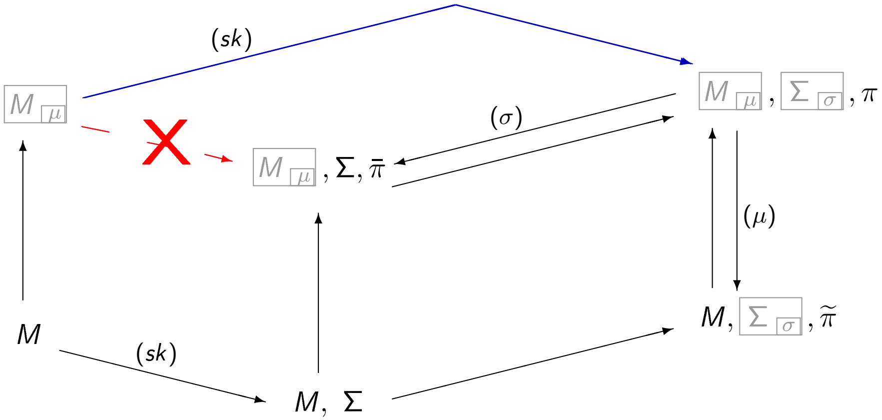
$$
\begin{tikzcd}
& \boxed{M_{\boxed{\mu}}}, \boxed{\Sigma_{\boxed{\sigma}}}, \pi \arrow[dl, shift right, "(\sigma)"'] \arrow[d, shift left, "(\mu)"] \\
\boxed{M_{\boxed{\mu}}}, \Sigma, \bar{\pi} \arrow[ur, shift right] & M, \boxed{\Sigma_{\boxed{\sigma}}}, \widetilde{\pi} \arrow[u, shift left] \\
M, \Sigma \arrow[u] \arrow[ur] &
\end{tikzcd}
$$


$\boxed{M_{\boxed{\mu}}}$
$\boxed{M_{\boxed{\mu}}}, \Sigma, \bar{\pi}$
$\boxed{M_{\boxed{\mu}}}, \boxed{\Sigma_{\boxed{\sigma}}}, \pi$
(σ)
(μ)
M
(sk)
M, Σ
$M, \boxed{\Sigma_{\boxed{\sigma}}}, \widetilde{\pi}$

Commuting signatures and verifiable encryption II



Commuting signatures and verifiable encryption II



In a nutshell

- Pseudonym : encryption of user verification key
- Credential : verifiably encrypted signature
- Non-interactive delegation : commuting signature

- Delegation of signing rights

Black-box instantiation of NIDAC

- Delegation of signing rights $\quad vk_0 \xrightarrow{\Sigma_1} vk_1$

Black-box instantiation of NIDAC

- Delegation of signing rights $\quad vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \cdots \xrightarrow{\Sigma_n} vk_n$

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$$vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \cdots \xrightarrow{\Sigma_n} vk_n$$

Signatures

credential

- Delegation of signing rights

  Signatures

$$vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \cdots \xrightarrow{\Sigma_n} vk_n$$

credential

- Anonymous show

$$vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \cdots \xrightarrow{\Sigma_n} vk_n$$

- Delegation of signing rights

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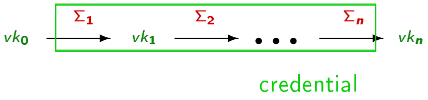

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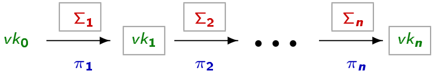

Black-box instantiation of NIDAC

- Delegation of signing rights

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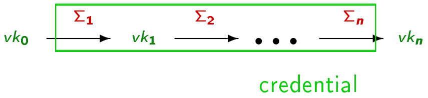

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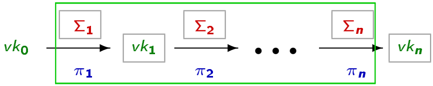
  Verifiable encryption



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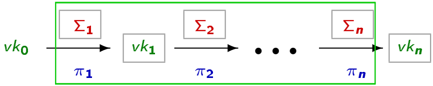

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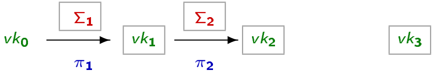

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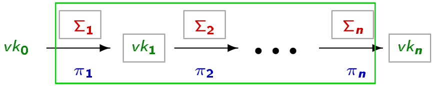

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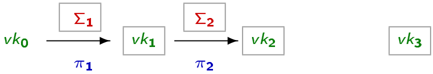

Commuting signatures

- Sign encrypted value $\boxed{vk_3}$ $\Rightarrow$ (vk_2, $\boxed{\Sigma_3}$, $\boxed{vk_3}$, π'_3)

- Delegation of signing rights

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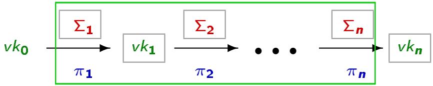

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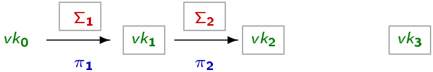

- Sign encrypted value $\boxed{vk_3}$ $\Rightarrow$ $(vk_2, \boxed{\Sigma_3}, \boxed{vk_3}, \pi'_3)$
- Adapt proof for $\boxed{vk_2}$ $\Rightarrow$ $(\boxed{vk_2}, \boxed{\Sigma_3}, \boxed{vk_3}, \pi_3)$

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  Signatures

$$vk_0 \xrightarrow{\Sigma_1} vk_1 \xrightarrow{\Sigma_2} \bullet\bullet\bullet \xrightarrow{\Sigma_n} vk_n$$

credential

- Anonymous show

  Verifiable encryption

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Send credential $\left(\underset{\pi_1}{\boxed{\Sigma_1}}, \boxed{vk_1}, \underset{\pi_2}{\boxed{\Sigma_2}}, \boxed{vk_2}, \underset{\pi_3}{\boxed{\Sigma_3}} \right)$

- Groth-Sahai (GS) proofs
  (shown to be randomizable [BCCKLS09])
- Automorphic signatures
  (allow signing of verification keys)

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Algebraic properties of GS proofs $\Longrightarrow$ instantiation of new functionalities

- Proof adaptation $\left.\begin{matrix}\widetilde{\pi}\\ \bar{\pi}\end{matrix}\right\} \longleftrightarrow \pi \longleftrightarrow \widehat{\pi}$
- Sign encrypted messages : $\boxed{M} \xrightarrow{sk} \boxed{\Sigma}, \pi$ (where Σ is signature on M)

Properties of Groth-Sahai proofs

$$\pi : \quad e(A_1, \boxed{Y_1}) \cdots e(\, A_n, \boxed{Y_n}) \cdot \cdots \cdots e(\boxed{X_i}, B_i) \cdot \cdots \cdots e(\boxed{X_m}, \boxed{Y_n})^{\gamma_{m,n}} \quad = \quad \mathbf{t}$$

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Proofs do not depend on $\mathbf{t}$

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Proofs can be adapted when constants are turned into variables or vice versa

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$$\pi': \quad e(A'_1, \boxed{Y'_1}) \cdots e(A'_n, \boxed{Y'_n}) \cdots\cdots e(\boxed{X'_i}, B'_i) \cdots\cdots e(\boxed{X'_m}, \boxed{Y'_n})^{\gamma'_{m,n}} = \mathbf{t}'$$

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Homomorphic

The product of 2 proofs is a proof for the product of the 2 equations

$$\begin{array}{lllr}
\pi & e(A_1, \boxed{Y_1}) \cdots e(A_n, \boxed{Y_n}) \cdots\cdots e(\boxed{X_i}, B_i) \cdots\cdots e(\boxed{X_m}, \boxed{Y_n})^{\gamma_{m,n}} & & \mathbf{t} \\
\bullet\ : & \bullet & = & \bullet \\
\pi' & e(A'_1, \boxed{Y'_1}) \cdots e(A'_n, \boxed{Y'_n}) \cdots\cdots e(\boxed{X'_i}, B'_i) \cdots\cdots e(\boxed{X'_m}, \boxed{Y'_n})^{\gamma'_{m,n}} & & \mathbf{t}'
\end{array}$$

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Conclusion

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& no more complex 2-party protocols ; size halved

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- Receipt-free e-voting [BFPV11]
- Fully anonymous transferable e-cash [BCFGST11]

Thank you ! ☺