$B(n-1, p)$

# Temă ~ seminar PS ~

1. $X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix}, p \in \mathbb{R}$

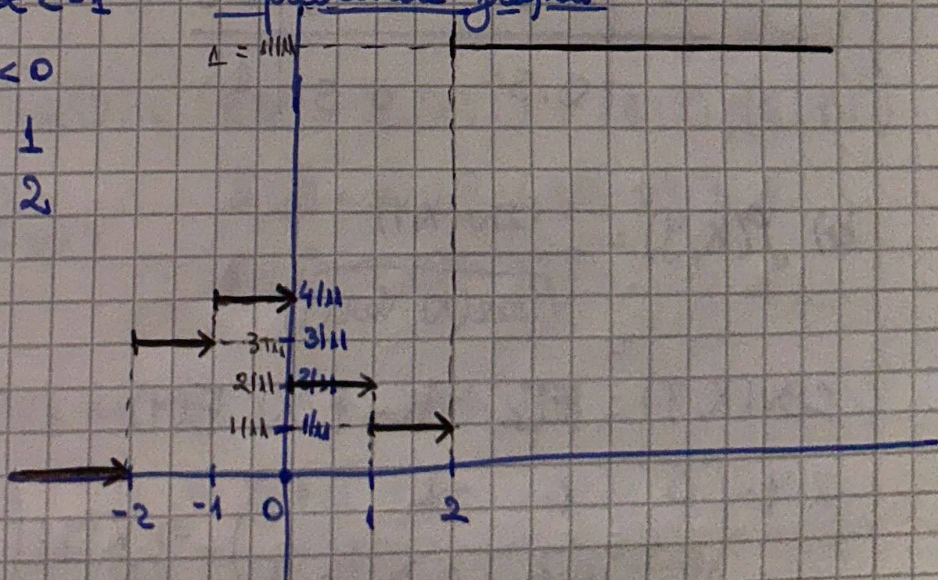
a) $p = ?$

X - bine definită $\Rightarrow 3p + 4p + 2p + p + p = 1 \Leftrightarrow 11p = 1 \Rightarrow p = 1/11$

b) $F(x) = ?$ $X : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3/11 & 4/11 & 2/11 & 1/11 & 1/11 \end{pmatrix}$

$$F(x) = \begin{cases} 0, & x < -2 \\ 3/11, & -2 \le x < -1 \\ 4/11, & -1 \le x < 0 \\ 2/11, & 0 \le x < 1 \\ 1/11, & 1 \le x < 2 \\ 1, & x > 2 \end{cases}$$

Reprezentare grafică :



c) media + dispersia $16X-23$ și $3X-2$

$E[16X-23] = (16\cdot(-2)-23)\cdot 3/11 + \cancel{\ }(16(-1)-23)\cdot 4/11 + 0 +$
$+ \cancel{\ }(16-23)\cdot 1/11 + \cancel{2\cdot(16-23)}(16\cdot 2-23)\cdot 1/11$
$= (-32-23)\cdot 0.27 + (-39)\cdot 0.36 + (-7)\cdot 0.09 + 9\cdot 0.09 \cong -28.71$

$Var(3X-2) = Var(3X) = 9Var(X) = 9(E[X^2] - (E[X])^2)$

$E[X^2] = 4\cdot 3/11 + (-1)^2\cdot 4/11 + 0 + 1^2\cdot 1/11 + 2^2\cdot 1/11$
$= 4\cdot 0.27 + 0.36 + 0.09 + 4\cdot 0.09$
$= 1.89$

$(E[X])^2 = (-2\cdot 0.27 + (-1)\cdot 0.36 + 0 + 0.09 + 2\cdot 0.09)^2$
$= (-0.63)^2 = 1.26$

2) $X: \begin{pmatrix} 0 & 1 \\ 0.4 & 0.6 \end{pmatrix}$ $Y: \begin{pmatrix} -1 & 1 \\ 0.5 & 0.5 \end{pmatrix}$

Fie $k = P(X=1, Y=-1)$.

a) repartiția comună

b) coef de corelație

c) k pt care X și Y sunt necorelate; să se testeze dc. X, Y sunt independente

Rezolvare

a)

| X\Y | -1 | 1 | |
|---|---|---|---|
| 0 | 0.2 | 0.2 | 0.4 |
| 1 | 0.3 | 0.3 | 0.6 |
| | 0.5 | 0.5 | 1 |

b) $\rho(X,Y) = \dfrac{cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}$

$cov(X,Y) = E[X\cdot Y] - E(X)\cdot E(Y)$

$X\cdot Y: \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0.2 & 0.2 & 0.3 & 0.3 \end{pmatrix} \Rightarrow X\cdot Y: \begin{pmatrix} 0 & 1 & -1 \\ 0.4 & 0.3 & 0.3 \end{pmatrix}$

$E(X\cdot Y) = 0 + 0.3 - 0.3 = 0$

$E(X) = 0 + 0.6 = 0.6$

$E(Y) = -0.5 + 0.5 = 0$

$cov(X,Y) = 0 - 0\cdot 0.6 = 0 \Rightarrow \rho(X,Y) = 0$

c) $k = P(X=1, Y=1)$

$k = 0.6 \cdot 0.5 = 0.3$

din b) $\Rightarrow$ ~~cov(X~~ $\rho(X,Y) = 0 \Leftrightarrow k - E(X)\cdot E(Y) = 0 \Rightarrow k = 0.3$

deci ca X, Y - necorelate $\Rightarrow k = 0.3$

Obs din tabel că $P(X=x_i, Y=y_i) = P(X=x_i)\cdot P(Y=y_i) \Rightarrow$ sunt independente

3) $X: \begin{pmatrix} a & 1 & 2 \\ \frac{1}{3} & p & q \end{pmatrix}$ $\quad Y: \begin{pmatrix} a+1 & 1 & 2 \\ \frac{1}{3} & \frac{2}{3}-q & p \end{pmatrix}$, $p, q, a \in \mathbb{R}$

V.a. independente

$a = ?$ a.î. $Var(X-Y) = 4/9$, influențează a $\rho(X,Y)$?

X, Y - bine definite $\Leftrightarrow \begin{cases} 1/3 + p + q = 1 \\ 1/3 + 2/3 - q + p = 1 \end{cases} \Leftrightarrow \begin{cases} p+q = 2/3 \\ 2/3 - q + p = 2/3 \end{cases}$

$\begin{cases} p + \cancel{q} = 2/3 - q \\ \cancel{p = 2/3 + q}\ p - q = 0 \end{cases} \Leftrightarrow \begin{cases} p = 2/3 - q \\ p = q \end{cases} \Leftrightarrow \begin{cases} q = 2/3 - p \\ p = q \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 2p = 2/3 \\ p = q \end{cases} \Leftrightarrow \begin{cases} p = 2/6 = 1/3 \\ p = q = 1/3 \end{cases}$

$X: \begin{pmatrix} a & 1 & 2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$ $\quad Y: \begin{pmatrix} a+1 & 1 & 2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$

$Var(X-Y) \overset{indep}{=} Var(X) + Var(Y) = (E[X^2] - (E[X])^2) + (E(Y) - (E(Y))^2)$

$E(X) = a\cdot 1/3 + 1/3 + 2/3$
$\quad = a/3 + 3/3 = (a+3)/3$

$E(X^2) = a^2\cdot 1/3 + 1/3 + 4/3$

$E(X^2) = (a^2+5)/3$

$(E(X))^2 = (a+3)^2/9$

$E(Y) = (a+1)/3 + 1/3 + 2/3 \Leftrightarrow (E(Y))^2 = (a+4)^2/9$

$E(Y^2) = (a+1)^2 \cdot 1/3 + 1/3 + 4/3 = \frac{(a+1)^2+5}{3}$

$Var(X-Y) = \left(\frac{(a^2+5)}{3} - \frac{(a+3)^2}{9}\right) + \left(\frac{(a+1)^2+5}{3} - \frac{(a+4)^2}{9}\right) =$

$$= \frac{3a^2+15-a^2-6a-9}{9} + \frac{3a^2+6a+3+15-a^2-8a-16}{9}$$

$$= \frac{4a^2-8a+33-25}{9} = \frac{4a^2-8a+8}{9} = \frac{4((a-1)^2+1)}{9} = \frac{4(a-1)^2+4}{9}$$

$Var(X,Y) = \frac{4}{9} \Leftrightarrow$

$\Leftrightarrow 4(a-1)^2+4 = 4$

$4(a-1)^2 = 0$

$(a-1)^2 = 0 \Leftrightarrow a^2-2a+1=0$

$\Delta = 0 \Rightarrow a = 1$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}$$

$cov(X,Y) = E[X\cdot Y] - E[X]\cdot E[Y]$

$$X\cdot Y: \begin{pmatrix} a^2+a & a & 2a & a+1 & 1 & 2 & 2a+2 & 2 & 4 \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{pmatrix}$$

având în vedere că $E[X\cdot Y]$, $E[X]$ și $E[Y]$ - depind de valoarea parametrului a, acestea vor influ. val $\rho(X,Y)$ - deci, depinde de a.

4) $X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ 6p & 2p & 9p & p \end{pmatrix}$ $\quad Y = aX+b$, $E[Y] = 57$

$Var(Y) = 575$

$F(x)$ + reprez, $a, b = ?$

Rezolvare:

$\to a, b = ?$

X - bine def $\Leftrightarrow 18p = 1 \Rightarrow p = 1/18 \Rightarrow$

$$\Rightarrow X: \begin{pmatrix} -2 & 3 & 4 & 6 \\ 1/3 & 1/9 & 1/2 & 1/18 \end{pmatrix}$$

$6/18 \quad 2/18 \quad 9/18$

$E(aX+b) = (-2a+b)\cdot 1/3 + (3a+b)\cdot 1/9 + (4a+b)\, 1/2 + (6a+b)\, 1/18$

$= -\cancel{2}/3a + 1/3b + 1/\cancel{3}a + 1/9 \cdot b + 2a + 1/2\, b + 1/\cancel{18}a + 1/18\, b$

$= 2a + 3/9\, b + 1/9\, b + 1/2\, b + 1/18\, b$

$= 2a + b\left(\dfrac{6+2+9+1}{18}\right)$

$= 2a + b$
$E(Y) = 57$ $\Rightarrow 2a + b = 57$

$Var(Y) = \left(E(Y^2) - (E(Y))^2\right)$

$E(Y^2) = E((aX+b)^2) = (-2 \cdot a + b)^2 \cdot 1/3 + (3 \cdot a + b)^2 \cdot 1/9 + (4a+b)^2 \cdot 1/2$
$+ (6a+b)^2 \cdot 1/18$

$= \dfrac{4a^2 - 4ab + b^2}{3} + \dfrac{9a^2 + 6ab + b^2}{9} + \dfrac{16a^2 + 8ab + b^2}{2} + \dfrac{36a^2 + 12ab + b^2}{18}$

$= \dfrac{\cancel{24a^2} - \cancel{24ab} + \cancel{6b^2} + \cancel{18a^2} + \cancel{12ab} + \cancel{2b^2} + \cancel{144a^2} + \cancel{72ab} + \cancel{9b^2} + \cancel{36a^2} + \cancel{12ab} + \cancel{b^2}}{18}$

$= \dfrac{198a^2 + 24a^2 + 72ab + 18b^2}{18}$ $= \cancel{11a^2 + 4ab + b^2}$

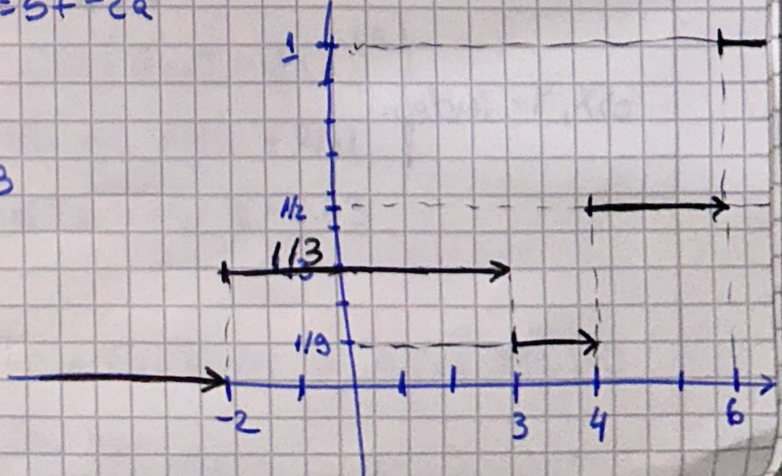
$= \dfrac{222}{18} a^2 + 4ab + b^2$

$= 12a^2 + 4ab + b^2 = 75$

$\Rightarrow \begin{cases} 8a^2 + (2a+b)^2 = 75 \\ 2a + b = 57 \end{cases} \Leftrightarrow \begin{cases} 8a^2 + (\cancel{2a} + 57 - \cancel{2a})^2 = 75 \\ b = 57 - 2a \end{cases}$

$\begin{cases} 8a^2 + 57^2 = 75 \\ b = 57 - 2a \end{cases} \Leftrightarrow \begin{cases} a^2 = -1587/4 \\ b = 57 - 2a \end{cases}$ ?

$F(x) = \begin{cases} 0, & x < -2 \\ 1/3, & -2 \le x < 3 \\ 1/9, & 3 \le x < 4 \\ 1/2, & 4 \le x < 6 \\ 1, & x \ge 6 \end{cases}$



5) $X:\begin{pmatrix} -2 & 1 \\ 0.4 & 0.6 \end{pmatrix}$ $Y:\begin{pmatrix} -1 & 3 \\ 0.3 & 0.7 \end{pmatrix}$

$k = P(X=-2, Y=3)$

a) rep. comună

| $X \backslash Y$ | $-1$ | $3$ | |
|---|---|---|---|
| $-2$ | $0.12$ | $0.28$ | $0.4$ |
| $1$ | $0.18$ | $0.42$ | $0.6$ |
| | $0.3$ | $0.7$ | |

b) $k$ a.î. $X, Y$ - necorelate $\Leftrightarrow cov = 0$

$cov(X,Y) = E[X \cdot Y] - E[X] \cdot E[Y]$

~~$E(X \cdot Y)$~~ $E(X) = -2 \cdot 0.4 + 0.6 = -0.2$

$E(Y) = -0.3 + 3 \cdot 0,7 = 1.8$

$X \cdot Y : \begin{pmatrix} 2 & -6 & -1 & 3 \\ 0.12 & 0.28 & 0.18 & 0.42 \end{pmatrix}$

$E(X \cdot Y) = 2 \cdot 0.12 - 6 \cdot 0.28 - 0.18 + 3 \cdot 0.42 = -0.36$

$cov(X,Y) = -0.36 + 0.36 \Rightarrow (\forall) k$ $X, Y$ - necorelate

c) conform tabelului: $P(X=-2, Y=-1) = P(X=-2) \cdot P(Y=-1)$

$P(X=-2, Y=3) = P(X=-2) \cdot P(Y=3)$

$P(X=1, Y=-1) = P(X=1) \cdot P(Y=-1)$ $\Rightarrow$

$P(X=1, Y=3) = P(X=1) \cdot P(Y=3)$

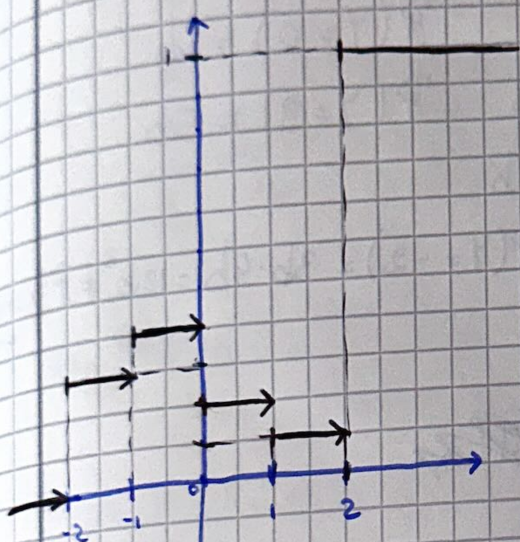
$\Rightarrow X, Y$ - indep.

6) $X: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix}, p \in \mathbb{R}$

a) $p = ?$

$11p = 1 \Leftrightarrow p = 1/11 \Rightarrow X: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3/11 & 4/11 & 2/11 & 1/11 & 1/11 \end{pmatrix}$

b) $F(x) = \begin{cases} 0, & x < -2 \\ 3/11, & -2 \leq x < -1 \\ 4/11, & -1 \leq x < 0 \\ 2/11, & 0 \leq x < 1 \\ 1/11, & 1 \leq x < 2 \\ 1, & x \geq 2 \end{cases}$




c) $E(3X-2)$, $Var(6X-3)$, $E(X+X^2)$

$E(3X-2) = (-6-2) \cdot 3/11 + (-3-2)\, 4/11 + (0-2) \cdot 2/11 + (3-2)\, 1/11$
$+ (6-2) \cdot 1/11$

$= -24/11 - 20/11 - \cancel{4/11} + 1/11 + \cancel{4/11}$

$= -43/11 = -3.9$

$Var(6X-3) = 36\, Var(X) = 36\left(E(X^2) - (E(X))^2\right)$

$E(X) = -2 \cdot 3/11 - 1 \cdot 4/11 + 0 + 1/11 + 2/11$

$= -6/11 - 4/11 + 3/11 = -7/11 \simeq -0.63$

$(E(X))^2 \simeq 0.39$

$E(X^2) = 4 \cdot 3/11 + 1 \cdot 4/11 + 0 + 1/11 + 4/11 = 21/11 \simeq 1.90$

$Var(6X-3) = 36 \cdot 1.51 = 54.36$

$E(X+X^2) = \cancel{E(\dots)}$

$= (-2+4)\cdot 3/11 + (-1+1)\cdot 4/11 + (0+0^2)\cdot 2/11 + (1+1^2)\cdot 1/11 + (2+2^2)\cdot$

$= 6/11 + 2/11 + 6/11 = 14/11 = 1.27$

6) $X: \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 3p & 4p & 2p & p & p \end{pmatrix}$

a) $\cancel{3p}\ 11p = 1 \Rightarrow p = 1/11$

b) identic cu ex 5

7)

| $X \backslash Y$ | $-2$ | $0$ | $9$ |
|---|---|---|---|
| $-1$ | $b$ | $2b$ | $0$ |
| $0$ | $3b$ | $4b$ | $5b$ |

$b \in \mathbb{R}$

a) rep. marginale: $P(X=-1) = 3b$ $\quad P(Y=0) = 6b$

$P(X=0) = 12b$ $\quad P(Y=9) = 5b$

$P(Y=-2) = 4b$

b) even ≠ indep Ex: $P(X=-1)\cdot P(Y=-2) = 3b\cdot 4b = 12b^2 \neq P(X=-1$

$P(X\cdot Y \neq 0) = P(X\neq 0 \wedge Y \neq 0)$

$= P(X=-1, Y=-2) + P(X=-1, Y=9)$

$= b + 0 = b$

c) $Var(3X-2Y)$ $\quad X, Y$ - nu sunt indep $\Rightarrow$

$\Rightarrow Var(3X-2Y) = Var(3X) + Var(2Y) - 2cov(3X, 2Y)$

$= 9Var(X) + 4Var(Y) - 12cov(X,Y)$

$Var(X) = (E(X^2) - (E(X))^2)$

$Var(Y) = (E(Y^2) - (E(Y))^2)$

$cov(X,Y) = E(X\cdot Y) - E(X)\cdot E(Y)$

$E(X) = -1\cdot 3b + 0 = -3b$

$(E(X))^2 = 9b^2$

$E(X^2) = 1\cdot 3b + 0 = 3b$

$Var(X) = 3b - 9b^2 = 3b(1-3b)$

$E(Y) = -2\cdot 4b + 0 + 9\cdot 5b = -8b + 45b = 37b$

$(\mathbb{E}(Y))^2 = 37^2 b^2 = 1369 b^2$

$\mathbb{E}(Y^2) = 4 \cdot 4b + 0 + 81 \cdot 5b = 16b + 405b = 411b$

$Var(Y) = 411b - 1369b^2 = b(411 - 1369b)$

$\mathbb{E}(X \cdot Y) = 2 \cdot b + 0 \cdot 2b - 9 \cdot 0 + 0 + 0 + 0$

$= 2b$

$cov(X,Y) = 2b - (-3b) \cdot (37b) = 2b + 111b^2 = b(2 + 111b)$

$Var(3X - 2Y) = 9 \cdot (3b(1-3b)) + 4(b(411 - 1369b)) - 12b(2 + 111b)$

8) a)

| X\Y | -2 | -1 | 0 | 1 | $p_i$ |
|---|---|---|---|---|---|
| -1 | 1/80 | 2/80 | 3/80 | 14/80 | 1/4 |
| 0 | 2/80 | 3/80 | 14/80 | 1/80 | 1/4 |
| 1 | 3/80 | 14/80 | 1/80 | 2/80 | 1/4 |
| 2 | 14/80 | 1/80 | 2/80 | 3/80 | 1/4 |
| $q_i$ | 1/4 | 1/4 | 1/4 | 1/4 | |

b) $\rho(X,Y) = \dfrac{cov(X,Y)}{\sqrt{Var(X) \cdot Var(Y)}}$

$cov(X,Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X] \cdot \mathbb{E}[Y]$

$\mathbb{E}[X \cdot Y] = 2 \cdot 1/80 + 1 \cdot 2/80 - 1 \cdot 14/80 - 2 \cdot 3/80 - 1 \cdot 14/80 + 2/80$

$- 4 \cdot 14/80 - 2/80 + 2 \cdot 3/80$

$= \dfrac{2 + 2 - 14 - \cancel{6} - 14 + \cancel{2} - 56 - \cancel{2} + \cancel{6}}{80}$

$= -1$

$\mathbb{E}[X] = -1 \cdot \cancel{1/4} + \cancel{1/4} + 2/4 = 2/4 = 1/2$

$\mathbb{E}[Y] = -2/4 - \cancel{1/4} + \cancel{1/4} = -2/4 = -1/2$

$cov(X,Y) = -1 + 1/4 = -3/4$

$Var(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

$Var(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2$

$(\mathbb{E}(X))^2 = (1/2)^2 = 1/4$

$(\mathbb{E}(Y))^2 = (-1/2)^2 = 1/4$

$\mathbb{E}(X^2) = 1 \cdot 1/4 + 1 \cdot 1/4 + 4 \cdot 1/4 = 6/4 = 3/2$

$\mathbb{E}(Y^2) = 4/4 + 1/4 + 1/4 = 6/4 = 3/2$

~~$\rho(X,Y) =$~~

$\rho(X,Y) = \dfrac{-3/4}{\sqrt{9/4}} = \dfrac{-3/4}{3/2}$

$-\dfrac{\cancel{3}}{\cancel{4}_2} \cdot \dfrac{\cancel{2}}{\cancel{3}} = -\dfrac{1}{2}$

c) $X|Y=0: \begin{pmatrix} -1 & 0 & 1 & 2 \\ 3/20 & 14/20 & 1/20 & 2/20 \end{pmatrix}$

$\mathbb{E}(X|Y=0) = -3/20 + 1/20 + 4/20$
$= 0$

$Y|X=2: \begin{pmatrix} -2 & -1 & 0 & 1 \\ 14/20 & 1/20 & 2/20 & 3/20 \end{pmatrix}$

$\mathbb{E}(Y|X=2) = -28/20 - 1/20 + 3/20$
$= -26/20$

d) $Var(-3Y+3) = 9\,Var(Y)$

$Var(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2$

$\mathbb{E}(Y) = -2/4 - 1/4 + 1/4 = -2/4 \Rightarrow (\mathbb{E}(Y))^2 = 4/16 = 1/4$

$\mathbb{E}(Y^2) = 4/4 + 1/4 + 1/4 = 6/4 = 3/2$

$Var(Y) = 6/4 - 1/4 = 5/4$

$Var(-3Y+3) = 9 \cdot \frac{5}{4} = \frac{45}{4}$

e) $\mathbb{P}(X<1, Y>-1) = 3/80 + 14/80 + 14/80 + 1/80$
$= 32/80$

g)

| X\Y | -2 | -1 | 0 | 1 | 2 | $p_i$ |
|---|---|---|---|---|---|---|
| -1 | 1/10 | 1/50 | 3/50 | 1/50 | 1/10 | 15/50 |
| 0 | | 3/25 | | 3/25 | | |
| 1 | 2/25 | 1/50 | 7/50 | 1/50 | 2/25 | |
| $q_i$ | 11/50 | | 6/25 | | 11/50 | |

a)

| X\Y | -2 | -1 | 0 | 1 | 2 | $p_i$ |
|---|---|---|---|---|---|---|
| -1 | 5/50 | 1/50 | 3/50 | 1/50 | 5/50 | 15/50 |
| 0 | 2/50 | 6/50 | 2/50 | 6/50 | 2/50 | 18/50 |
| 1 | 4/50 | 1/50 | 7/50 | 1/50 | 4/50 | 17/50 |
| $q_i$ | 11/50 | 8/50 | 12/50 | 8/30 | 11/50 | 1 |

b) $\rho(X,Y) = \dfrac{cov(X,Y)}{\sqrt{Var(X)\cdot Var(Y)}}$ $\quad cov(X\cdot Y) = E[X\cdot Y] - E[X]\cdot E[Y]$

$E[X\cdot Y] = \cancel{2/20 + 1/50 - 1/50 - 2/20}\ 10/50 + 1/50 - 1/50 - 10/50 - \cancel{8/50}$

$-1/50 + 1/50 + 8/50 = 0$

$E[X] = -15/50 + 0 + 17/50 = 2/50 = 1/25$

$E[Y] = -22/50 - 8/50 + 0 + 8/50 + 22/50 = 0$

$cov(X\cdot Y) = 0 - 1/25 \cdot 0 = 0 \Rightarrow \rho(X,Y) = 0$

c) $X|Y=0 : \begin{pmatrix} -1 & 0 & 1 \\ \frac{3}{50}\cdot\frac{50}{12} & \frac{2}{12} & \frac{7}{12} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 1/4 & 1/6 & 7/12 \end{pmatrix}$

$E(X|Y=0) = -1/4 + 0 + 7/12 = \dfrac{-3+7}{12} = \dfrac{4}{12} = 1/3$

$Y|X=1 : \begin{pmatrix} -2 & -1 & 0 & 1 & 2 \\ 4/17 & 1/17 & 7/17 & 1/17 & 4/17 \end{pmatrix}$

$E[Y|X=1] = -8/17 - 1/17 + 0 + 1/17 + 8/17 = 0$

d) $Var(3X+5) = 9Var(X) \quad Var(X) = E(X^2) - (E(X))^2$

$E(X^2) = 15/50 + 0 + 17/50 = 32/50$

$(E(X))^2 = (1/25)^2 = 1/625$

$Var(3X+5) = 9\cdot\left(\dfrac{32}{50} - \dfrac{1}{625}\right)$

e) $P(X<1, Y>0) = 1/50 + 5/50 + 6/50 + 2/50 = 18/50$