

$$Z: \begin{pmatrix} 1/2 & 1 & 2 & 3/2 & 4 & 9 \\ 0.1 & 0.15 & 0.35 & 0.2 & 0.05 & 0.15 \end{pmatrix}$$

Exercice - calcul coef de corrélation

$$b) \rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X) \cdot \text{var}(Y)}}$$

$$\text{cov}(X, Y) = E(X \cdot Y) - E(X) \cdot E(Y)$$

$$E(X) = 0.25 + 2 \cdot 0.4 + 3 \cdot 0.35 = 2.1$$

$$E(Y) = 0.35 + 2 \cdot 0.65 = 1.65$$

$$E(X \cdot Y) =$$

$$X \cdot Y: \begin{pmatrix} 1 & 2 & 2 & 4 & 3 & 6 \\ 0.15 & 0.1 & 0.05 & 0.35 & 0.15 & 0.2 \end{pmatrix}$$

$$X \cdot Y: \begin{pmatrix} 1 & 2 & 3 & 4 & 6 \\ 0.15 & 0.15 & 0.15 & 0.35 & 0.2 \end{pmatrix}$$

$$E(X \cdot Y) = 0.15 + 2 \cdot 0.3 + 0.45 + 4 \cdot 0.35 + 6 \cdot 0.2 = 3.5$$

$$\text{cov}(X, Y) = E(X \cdot Y) - E(X) \cdot E(Y) = 3.5 - 2.1 \cdot 1.65 = 0.035$$

$$\text{var}(X) = E(X^2) - (E(X))^2$$

$$X^2: \begin{pmatrix} 1 & 4 & 9 \\ 0.25 & 0.4 & 0.35 \end{pmatrix}$$

$$\text{var}(Y) = E(Y^2) - (E(Y))^2$$

$$Y^2: \begin{pmatrix} 1 & 4 \\ 0.35 & 0.65 \end{pmatrix}$$

$$E(X^2) = 0.25 + 4 \cdot 0.4 + 9 \cdot 0.35 = 5$$

$$E(Y^2) = 0.35 + 4 \cdot 0.65 = 2.95$$

$$\text{var}(X) = 5 - (2.1)^2 = 0.59$$

$$\text{var}(Y) = 2.95 - (1.65)^2 = 0.2275$$

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X) \cdot \text{var}(Y)}} = \frac{0.035}{\sqrt{0.59 \cdot 0.2275}} = \frac{0.035}{\sqrt{0.134225}} = \frac{0.035}{0.3663} \approx 0.095$$