Тема — решение

1) а) $X: \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & 4/5 \end{pmatrix}$        $y: \begin{pmatrix} -3 & -2 \\ 4/5 & 1/5 \end{pmatrix}$

$3X = \begin{pmatrix} 6 & 9 \\ 1/3 & 4/5 \end{pmatrix}$        $X^{-1} = \begin{pmatrix} 1/6 & 1/9 \\ 1/5 & 4/5 \end{pmatrix}$

$\cos\left(\frac{\pi}{2} \cdot X\right) = \cos\begin{pmatrix} \frac{4\pi}{2} & \frac{3\pi}{2} \\ 1/5 & 4/5 \end{pmatrix} = \cos\begin{pmatrix} \pi & \frac{3\pi}{2} \\ 1/5 & 4/5 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 1/5 & 4/5 \end{pmatrix}$

$y^2 = \begin{pmatrix} 9 & 4 \\ 4/5 & 1/5 \end{pmatrix}$        $y+3 = \begin{pmatrix} 0 & 1 \\ 4/5 & 1/5 \end{pmatrix}$

b) $X: \begin{pmatrix} 0 & 9 \\ 1/2 & 1/2 \end{pmatrix}$  $Y: \begin{pmatrix} -3 & 1 \\ 1/7 & 6/7 \end{pmatrix}$

$X-1 = \begin{pmatrix} -1 & 8 \\ 1/2 & 1/2 \end{pmatrix}$  $X^{-2} = \begin{pmatrix} 0 & \frac{1}{9^2} \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & 81^{-1} \\ 1/2 & 1/2 \end{pmatrix}$

$\sin\left(\frac{\pi}{4} \cdot X\right) = \sin\begin{pmatrix} 0 & 9\pi/4 \\ 1/2 & 1/2 \end{pmatrix} = \begin{pmatrix} 0 & \sin\left(\frac{9\pi}{4}\right) \\ 1/2 & 1/2 \end{pmatrix}$

$Y \cdot 5 = \begin{pmatrix} -15 & 5 \\ 1/7 & 6/7 \end{pmatrix}$  $e^Y = \begin{pmatrix} e^{-3} & e^1 \\ 1/7 & 6/7 \end{pmatrix} = \begin{pmatrix} \frac{1}{e^3} & e \\ 1/7 & 6/7 \end{pmatrix}$

c) $X: \begin{pmatrix} 5 & 8 \\ 1/3 & 2/3 \end{pmatrix}$  $Y: \begin{pmatrix} -1 & 1 \\ 1/6 & 5/6 \end{pmatrix}$

$2X = \begin{pmatrix} 10 & 16 \\ 1/3 & 2/3 \end{pmatrix}$ ; $X^{-3} = \begin{pmatrix} \frac{1}{5^3} & \frac{1}{8^3} \\ 1/3 & 2/3 \end{pmatrix}$ ; $tg\left(\pi \cdot X\right) = tg\begin{pmatrix} 5\pi & 8\pi \\ 1/3 & 2/3 \end{pmatrix}$

$Y - 2 = \begin{pmatrix} -3 & -1 \\ 1/6 & 5/6 \end{pmatrix}$  $|Y| = \begin{pmatrix} 1 & 1 \\ 1/6 & 5/6 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

d) $X: \begin{pmatrix} -3 & 6 \\ 1/8 & 7/8 \end{pmatrix}$  $Y: \begin{pmatrix} e & e^3 \\ 1/4 & 3/4 \end{pmatrix}$

$2 - X = \begin{pmatrix} 5 & -4 \\ 1/8 & 7/8 \end{pmatrix}$  $X^3 = \begin{pmatrix} -27 & 216 \\ 1/8 & 7/8 \end{pmatrix}$ ; $\cos\left(\frac{\pi}{6}X\right) = \cos\begin{pmatrix} -\frac{\pi}{2} & \pi \\ 1/8 & 7/8 \end{pmatrix} =$

$= \begin{pmatrix} 0 & -1 \\ 1/8 & 7/8 \end{pmatrix}$

$Y^{-1} = \begin{pmatrix} \frac{1}{e} & \frac{1}{e^3} \\ 1/4 & 3/4 \end{pmatrix}$  $\ln Y = \begin{pmatrix} 1 & 3 \\ 1/4 & 3/4 \end{pmatrix}$

3) a) $2X + 3y = \begin{pmatrix} 4 & 6 \\ 1/5 & 4/5 \end{pmatrix} + \begin{pmatrix} -9 & -6 \\ 4/5 & 1/5 \end{pmatrix}$

$= \begin{pmatrix} -5 & -2 & -3 & \frac{0}{6} \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$

$3X - Y = \begin{pmatrix} 6 & 9 \\ 1/5 & 4/5 \end{pmatrix} + \begin{pmatrix} +3 & +2 \\ 4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 9 & 8 & 12 & 11 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$

$X^2 \cdot Y^3 = \begin{pmatrix} 4 & 9 \\ 1/5 & 4/5 \end{pmatrix} \cdot \begin{pmatrix} -27 & -8 \\ 4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} -108 & -32 & -243 & -72 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$

b) $X - Y = \begin{pmatrix} 5 & 4 & 6 & 5 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 6 \\ 1/25 & 8/25 & 16/25 \end{pmatrix}$

$\cos(\pi \cdot X \cdot Y) = \cos\left(\pi \cdot \begin{pmatrix} -6 & -4 & -9 & -6 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}\right) =$

$= \cos\left(\pi \begin{pmatrix} -4 & -6 & -9 \\ 1/25 & 8/25 & 16/25 \end{pmatrix}\right) = \cos\begin{pmatrix} -4\pi & -6\pi & -9\pi \\ 1/25 & 8/25 & 16/25 \end{pmatrix} =$

$= \begin{pmatrix} \cos -4\pi & \cos -6\pi & \cos -9\pi \\ 1/25 & 8/25 & 16/25 \end{pmatrix}$

$X^2 + 3Y = \begin{pmatrix} 4 & 9 \\ 1/5 & 4/5 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 4 & 5 & 9 & 10 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$

c) $X + Y = \begin{pmatrix} -1 & 0 & 0 & 1 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 \\ 4/25 & 17/25 & 4/25 \end{pmatrix}$

$\sin\left(\frac{\pi}{2} \cdot X \cdot Y\right) = \sin\begin{pmatrix} -4\frac{\pi}{2} & -6\frac{\pi}{2} & -9\frac{\pi}{2} \\ 1/25 & 8/25 & 16/25 \end{pmatrix} = \begin{pmatrix} \sin -2\pi & \sin -3\pi & \sin -9\pi/2 \\ 1/25 & 8/25 & 16/25 \end{pmatrix}$

$X^{-1} + Y^{-1} = \begin{pmatrix} 1/2 & 1/3 \\ 1/5 & 4/5 \end{pmatrix} + \begin{pmatrix} -1/3 & -1/2 \\ 4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} 1/2 - 1/3 & 0 & 0 & 1/3 - 1/2 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$

$= \begin{pmatrix} 1/6 & 0 & -1/6 \\ 4/25 & 17/25 & 4/25 \end{pmatrix}$

d) $x \cdot y$ - calculat pentru $\cos(\pi \cdot x \cdot y)$ și $\sin\left(\frac{\pi}{2} \cdot x \cdot y\right)$

$$\frac{x}{y} = x \cdot y^{-1} = \begin{pmatrix} 2 & 3 \\ 1/5 & 4/5 \end{pmatrix} \cdot \begin{pmatrix} -1/3 & -1/2 \\ 4/5 & 1/5 \end{pmatrix} = \begin{pmatrix} -2/3 & -1 & -1 & -3/2 \\ 4/25 & 1/25 & 16/125 & 4/25 \end{pmatrix}$$

$$= \begin{pmatrix} -2/3 & -1 & -3/2 \\ 4/25 & 17/25 & 4/25 \end{pmatrix}$$

$$|x - y^2| = \left| \begin{pmatrix} 2 & 3 \\ 1/5 & 4/5 \end{pmatrix} + \begin{pmatrix} -9 & -4 \\ 4/5 & 1/5 \end{pmatrix} \right| = \left| \begin{pmatrix} -7 & -2 & -27 & -12 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix} \right|$$

$$= \begin{pmatrix} 7 & 2 & 27 & 12 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$$

3) $p, q$ a.că

a) sunt v.a. bine definite

$$X: \begin{pmatrix} 1 & 2 \\ p & q \end{pmatrix} \qquad y: \begin{pmatrix} 3 & 9 \\ 0,1 & \frac{p^2 + 0.02}{2} \end{pmatrix}$$

$$\begin{cases} p + q = 1 \\ 0,1 + \dfrac{q^2 + 0.02}{2} = 1 \end{cases} \iff \begin{cases} p + q = 1 \\ \dfrac{q^2 + 0.02}{2} = 0.9 \Rightarrow q^2 + 0.02 = 1,8 \end{cases}$$

$$q^2 = 1.78$$

$$q = \sqrt{1.78} \Rightarrow$$

$$\Rightarrow q = \sqrt{1.78} \Rightarrow q = 1 - \sqrt{1.78}$$

b) bine definite

$$X: \begin{pmatrix} 1 & 2 & 3 \\ 1/3 & p & q^2 \end{pmatrix} \qquad x^2: \begin{pmatrix} 1 & 4 & 9 \\ p & p & q^2 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} 1 & 4 & 9 \\ 1/3 & p & q^2 \end{pmatrix}$$

$$\Rightarrow p = 1/3$$
$$p^2 = q^2 \quad \Rightarrow q^2 = \frac{1}{9} \Rightarrow$$
$$\Rightarrow q = \frac{1}{3}$$

c) sunt bine definite

$$x : \begin{pmatrix} -1 & 0 & 1 \\ q & q^2 & 2 \end{pmatrix} \qquad x^4 : \begin{pmatrix} 0 & 1 \\ \frac{9}{25} & \frac{16}{25} \end{pmatrix} \Bigg/ \Rightarrow$$

$$x^4 = \begin{pmatrix} 1 & 0 & 1 \\ p & p^2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ p+q & p^2 \end{pmatrix} \Bigg/ \Rightarrow$$

$$\Rightarrow \begin{cases} p^2 = \dfrac{9}{25} \Rightarrow p = \dfrac{3}{5} \\ p+q = \dfrac{16}{25} \end{cases} \Bigg/ \Rightarrow q = \dfrac{16}{25} - \dfrac{15}{25} = \dfrac{1}{25}$$

$$\frac{7}{16} + \frac{7}{8}$$

d) $x : \begin{pmatrix} -1 & 1 \\ 2p & 2 \end{pmatrix} \qquad y : \begin{pmatrix} 0 & 1 \\ q & 7q \end{pmatrix}$

$$\begin{cases} 2p+q = 1 \\ q+7q = 1 \end{cases} \Leftrightarrow \begin{cases} 2p+q = 1 \\ 8q = 1 \end{cases} \Leftrightarrow \begin{cases} 2p+q = 1 \\ q = \dfrac{1}{8} \end{cases} \Leftrightarrow \begin{cases} 2p = 1 - \dfrac{1}{8} = \dfrac{7}{8} \\ q = \dfrac{1}{8} \end{cases}$$

$$\Leftrightarrow \begin{cases} p = \dfrac{7}{16} \\ q = \dfrac{1}{8} \end{cases} \quad ? \qquad p + q \neq 1$$