

Release Notes 2022-12-24 : : Its the end of the year and I wanted to boost my publication count :) This is very quick analysis of a potentially big problem but in the rush I may have made significant blunders. I have some hope this is correct because their subgroup stats show lower relative risk than overall which is inline with this analysis. Any proofreading appreciated. My algebra has not gotten any better lately and I need the practice.

The release may use an experimental bibliography code that is not designed to achieve a particular format but to allow multiple links to reference works with modifications to the query string to allow identification of the citing work for tracking purposes. This may be useful for a bill-of-materials and purchases later.

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I am not a veterinarian or a doctor or health care professional and this is not particular advice for any given situation. Read the disclaimers in the appendicies or text, take them seriously and take prudent steps to evaluate this information.

This work addresses a controversial topic and likely advances one or more viewpoints that are not well accepted in an attempt to resolve confusion. The reader is assumed familiar with the related literature and controversial issues and in any case should seek additional input from sources the reader trusts likely with differing opinions. For information and thought only not intended for any particular purpose. Caveat Emptor

An Interesting Issue with Inhomogeneous Population Averages

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(Dated: December 24, 2022)

This is a comment on one particular work that purports to find increased risk of traffic accidents among those avoiding covid-19 vaccines. Their data are considered as reflecting uninformative correlations in the age distribution of traffic events compared to the covid-19 vaccination age distribution. This type of correlation may be an interesting case of a larger problem. The authors conclude something about psychological factors but the analysis just suggests that the correlation of event and vaccination rates could be due to many factors. To reach their conclusion, and explore the changes in psychological factors such as risk avoidance as a function of age, event rates as a function of age would be more useful for hypothesis generation but still not prove anything about causality. There is no indication from the published analysis that covid-19 vaccination will improve driving outcomes or increase subjects' age.

This is a comment on a recent work

"COVID Vaccine Hesitancy and Risk of a Traffic Crash" , Redelmeier , Donald A.[...] Thiruchelvam , Deva; The American Journal of Medicine. 2022
from which the abstract reads,

We conducted a population-based longitudinal cohort analysis of adults and determined COVID vaccination status through linkages to individual electronic medical records. Traffic crashes requiring emergency medical care were subsequently identified by multicenter outcome ascertainment of all hospitals in the region over a 1-month follow-up interval (178 separate centers). [...] CONCLUSIONS: These data suggest that COVID vaccine hesitancy is associated with significant increased risks of a traffic crash. An awareness of these risks might help to encourage more COVID vaccination. [3]

that was brought to my attention by a post on LinkedIn¹ The authors find a significant robust difference in car crash rate between vaccinated and unvaccinated people with a large sample size including millions of participants, thousands of events, and a wide age range. They give an overall odds ratio of 1.72 favoring the vaccinated and

*Electronic address: marchywka@hotmail.com; to cite or credit this work, see bibtex in Appendix G. Note also that machine readable citation information should be in the extended information of this file.

¹ https://www.linkedin.com/posts/erwinloh_covid19-vaccine-hesitancy-activity-7008399556962902016-d5B6?utm_source=share&utm_medium=member_desktop

consider coarse subgroups of 18-39, 40-64, and 65 years and older. The baseline characteristic (from their Table 1) and outcomes are as follows (the final column is calculated based on a ratio of their "risks " from figure 2),

Age	pct vax	pct-nonvax	risk ratio*
18-39	32.3	50.8	1.71
40-64	42.3	37.1	1.38
65 or older	25.4	12.0	.947
Overall*	84	16	1.72

TABLE I: A summary of sample characteristics and computed outcomes. Starred figures were calculated rather than simply copied from the subject work. Note that the overall risk is higher than any subgroups as predicted by this analysis rather than being some weighted average as may be expected otherwise. The result is clearly dominated by the young group which may be expected to have the steepest event-age slope.

The authors state only 16 percent had not received the vaccine.

If I understand their analysis approach, it may help to consider a null hypothesis where event rates are merely a function of age without regard to vaccination status. Their conclusion that event rates differ suggests a rejection of this or a similar null hypothesis.

Define some variables and functions,

1. t : age or "time" of the subjects in question
2. $e(t)$: the event rate as function of age assumed independent of vaccination status
3. $p(t)$: the study sample age distribution probability function
4. $w(t)$: a generalized weighting function
5. $v(t)$: the probability of being vaccinated as a function of age

The overall event rate then should be,

$$\langle e \rangle = \int e(t)p(t)dt \quad (1)$$

with vaccinated and unvaccinated expectation values (with the same $e(t)$) being as follows, for the vaccinated,

$$\langle e_v \rangle = \frac{\int e(t)p(t)v(t)dt}{I_v}; I_v = \int p(t)v(t)dt \quad (2)$$

and the unvaccinated,

$$\langle e_{nv} \rangle = \frac{\int e(t)p(t)(1-v(t))dt}{I_{nv}}; I_{nv} = \int p(t)(1-v(t))dt \quad (3)$$

For simplicity, $p(t)$ will be ignored but w will be used instead of v just to denote it is not quite right. Consider the expectation of e over some weighting function ,

$$\langle e \rangle = \frac{\int e(t)w(t)dt}{\int w(t)dt} \quad (4)$$

over some interval around t_0 ,

$$x(t) = \sum_n \frac{1}{n!} x^{(n)}(t_0) \Delta t^n \quad (5)$$

Define the expansion coefficients to avoid writing the Taylor expression,

$$x_i = \frac{x^{(i)}(t_0)}{i!} \quad (6)$$

$$\int dt x(t) = \sum_n \frac{1}{(n+1)!} x^{(n)}(t_0) \Delta t^{n+1} = \sum_n \frac{x_n}{(n+1)} (t_0) \Delta t^{n+1} \quad (7)$$

$$< e > = \frac{\int dt \sum_i e_i \Delta t^i \sum_j w_j \Delta t^j}{\sum_i \frac{w_i}{(i+1)} \Delta t^{i+1}} \quad (8)$$

Evaluating the integrals in some interval between plus and minus Δt , taking a ratio with the non-vaccinated weighting function $w_{nv} = 1 - w_v$, and retaining only linear terms for w and briefly some second order terms for e, with only zero and linear w terms,

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{(e_0 \Delta t - e_0 w_0 \Delta t - \frac{\Delta t^3}{3} (e_1 w_1 + e_2 w_0)) (w_0 \Delta t)}{(\Delta t - w_0 \Delta t) (e_0 w_0 \Delta t + \frac{\Delta t^3}{3} (e_1 w_1 + e_2 w_0))} \quad (9)$$

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{(e_0 - e_0 w_0 - \frac{\Delta t^2}{3} (e_1 w_1 + e_2 w_0)) (w_0)}{(1 - w_0) (e_0 w_0 + \frac{\Delta t^2}{3} (e_1 w_1 + e_2 w_0))} \quad (10)$$

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{(-\frac{\Delta t^2}{3} (e_1 w_1 + e_2 w_0)) (w_0)}{(1 - w_0) (e_0 w_0 + \frac{\Delta t^2}{3} (e_1 w_1 + e_2 w_0))} + \frac{e_0 w_0}{(e_0 w_0 + \frac{\Delta t^2}{3} (e_1 w_1 + e_2 w_0))} \quad (11)$$

As the interval goes to zero, the expected result is obtained and the ratio is unity. There are constraints on the weighting function expansion but e(t) is simply non-negative without an upper bound. In the limit of large time interval (within this approximation), dropping e_2 and noting this is the case of opposite slopes,

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{e_0 w_0}{(e_0 w_0 + \frac{\Delta t^2}{3} (e_1 w_1))} - \frac{(\frac{\Delta t^2}{3} (e_1 w_1)) (w_0)}{(1 - w_0) (e_0 w_0 + \frac{\Delta t^2}{3} (e_1 w_1))} \quad (12)$$

Approximate ratio under null hypotheses

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{1}{1 + \frac{\Delta t^2}{3} \frac{(e_1 w_1)}{(e_0 w_0)}} - \frac{(\frac{\Delta t^2}{3} (e_1 w_1)) w_0}{(1 - w_0) (\frac{e_0 w_0}{e_1 w_1} + \frac{\Delta t^2}{3})} \quad (13)$$

Interestingly, only one time and slope parameter exists. Define

$$\eta = \frac{\Delta t^2}{3} \frac{(e_1 w_1)}{(e_0 w_0)} \quad (14)$$

and the expression reduces to,

$$\frac{< e_{nv} >}{< e_v >} \approx \frac{1}{1 + \eta} - \frac{(\eta) w_0}{(1 - w_0) (1 + \eta)} = \frac{1 - (1 + \eta) w_0}{(1 - w_0) (1 + \eta)} \quad (15)$$

For small time intervals,

$$\frac{< e_{nv} >}{< e_v >} \approx 1 - \frac{(\frac{\Delta t^2}{3} (e_1 w_1))}{(1 - w_0) e_0} \quad (16)$$

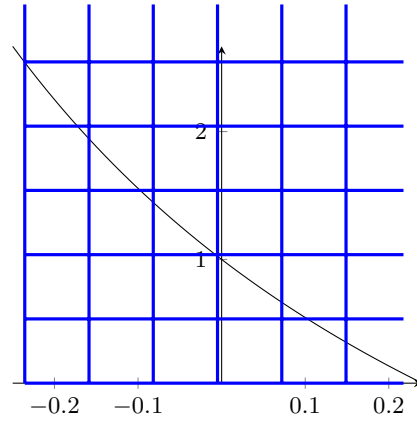
η is a dimensionless quantity that normalized the time interval squared to the the fractional slopes (log) of e and w. w is constrained between 1 and zero while e can be arbitrarily high.

Some values of the risk ratio under the null hypothesis or tabulated from Eqn 13 in the following table. The range of applicability of this approximation is not clear but the order of magnitude appears to be important where the event curve is steep in the younger population.

Thinking aloud

my algebra and computer code may have errors, needs a good verification

η	e_1/e_0 risk fraction/year	w_1/w_0 vax fraction/year	w_0 vax rate	Δt (years) half-interval	risk ratio	risk using eta	diff
0.03	0.1	0.025	0.8	6	0.8543	0.8543	0
0.06	0.1	0.05	0.8	6	0.7169	0.7169	0
0.09	0.1	0.075	0.8	6	0.5871	0.5871	0
0.021	0.07	0.025	0.8	6	0.8971	0.8971	0
0.042	0.07	0.05	0.8	6	0.7984	0.7984	0
0.063	0.07	0.075	0.8	6	0.7036	0.7036	0
0.012	0.04	0.025	0.8	6	0.9407	0.9407	0
0.024	0.04	0.05	0.8	6	0.8828125	0.8828125	0
0.036	0.04	0.075	0.8	6	0.8262	0.8262	0
0.003	0.01	0.025	0.8	6	0.9850	0.9850	0
0.006000	0.01	0.05	0.8	6	0.9701	0.9701	0
0.009000	0.01	0.075	0.8	6	0.9554	0.9554	0
-0.006	-0.02	0.025	0.8	6	1.030	1.030	0
-0.012	-0.02	0.05	0.8	6	1.060	1.060	0
-0.018	-0.02	0.075	0.8	6	1.091	1.091	0
-0.015	-0.05	0.025	0.8	6	1.07	1.07	0
-0.03	-0.05	0.05	0.8	6	1.154	1.154	0
-0.045	-0.05	0.075	0.8	6	1.235	1.235	0
-0.024	-0.08	0.025	0.8	6	1.122	1.122	0
-0.048	-0.08	0.05	0.8	6	1.252	1.252	0
-0.072	-0.08	0.075	0.8	6	1.387	1.387	0
-0.033	-0.11	0.025	0.8	6	1.170	1.170	0
-0.066	-0.11	0.05	0.8	6	1.353	1.353	0
-0.099	-0.11	0.075	0.8	6	1.549	1.549	0
-0.042	-0.14	0.025	0.8	6	1.219	1.219	0
-0.084	-0.14	0.05	0.8	6	1.458	1.458	0
-0.126	-0.14	0.075	0.8	6	1.7	1.7	0
-0.051	-0.17	0.025	0.8	6	1.268	1.268	0
-0.102	-0.17	0.05	0.8	6	1.567	1.567	0
-0.153	-0.17	0.075	0.8	6	1.903	1.903	0
-0.06	-0.2	0.025	0.8	6	1.319	1.319	0
-0.12	-0.2	0.05	0.8	6	1.681	1.681	0
-0.18	-0.2	0.075	0.8	6	2.097	2.097	0
-0.069	-0.23	0.025	0.8	6	1.370	1.370	0
-0.138	-0.23	0.05	0.8	6	1.800	1.800	0
-0.207	-0.23	0.075	0.8	6	2.305	2.305	0
-0.078	-0.26	0.025	0.8	6	1.422	1.422	0
-0.156	-0.26	0.05	0.8	6	1.924	1.924	0
-0.234	-0.26	0.075	0.8	6	2.527	2.527	0
-0.087	-0.29	0.025	0.8	6	1.476	1.476	0
-0.174	-0.29	0.05	0.8	6	2.053	2.053	0
-0.261	-0.29	0.075	0.8	6	2.7	2.7	0

FIG. 1: A plot of the risk ratio as a function of η for a vaccination rate of .8

As long as the slopes are of different signs, the ratio will be inflated as the time interval expands although how much is a matter of debate. As the event rate is only limited by zero, it could be significant.

The authors' original analysis reflect a weighted average of event rates over two populations- the vaccinated and unvaccinated- but they suggest an intervention to reduce the risk of the unvaccinated that does not involve a time machine. Under the null hypothesis presented here, large integrated risk ratios may be due to diverging event and vaccination rates but they will shrink with smaller age bins. Their analysis then does not seem to produce a result that provides much insight into causality. Given the author's conclusion about psychological factors, a better analysis would use the large number of data points to simply create two event-age curves for vaccinated and unvaccinated people and analyze differences with appropriate statistics. Smaller bins involved in making a curve should diminish this correlation effect or artifact leaving any real differences. With finite size bins of 1-2 years the analysis presented here may be able to correct results even further.

Association or observational studies are plagued by many hidden correlations and population inhomogeneities. It is rare to find one so compelling however and if this is accurate it makes a good example of a larger problem. The authors invoke several quality checks such as negative and positive controls, have complicated diagrams in their appendix, but they do not appear to address this problem. With such a large number of events, finer bins should be informative and even corrected using this formula.

While covid-19 had become politicized and controversies over vaccines an even bigger issue, it is tempting to think that effort is being made to stigmatize the non-vaccinated. Artifacts of this type may be more overt in this topic but always exist and need to be recognized. I discussed some of this regarding vitamin D dose-response curves [2] and it has come up recently with Alzheimer's Disease clinical trials [1]. Generally the point of statistics is to try to "average out" the unknown factors or noise but that can be difficult as they are not known and indeed blinds and randomization try to assist in these goals. Perhaps its particularly important with inverse problems but one consideration is to try to measure exactly what is needed but have sanity checks. It is always possible to measure "something" but normally the goal is to synthesize and prove an intervention produces some benefit. In the present case, without being able to modify subject age which correlates with vaccination status and accident rate, controlling for it may help measure something more useful. Driving habits would arguably fall under "cause and effect" or the psychology the original authors wanted to explore. Issues like miles driven, judgement, experience could be considered. With covid-19 having a steep age-severity curve, there may be cognitive issues even within certain age groups. Or, there may be no difference in vaccinated and unvaccinated event curves. In any case, synthesizing a successful intervention would still be a better hypothesis test.

1. SUPPLEMENTAL INFORMATION

1.1. Computer Code

File rr.R to list rr with various parameters

```
big<-function(e1e0,w1w0,w0,dt)
{
# clutsy implementation
x<- dt*dt/3*w0/(1-w0)/(dt*dt/3 +1/e1e0/w1w0)
```

```

y <-1/(1+dt*dt/3*e1e0*w1w0)
# cleaner approach
eta=dt*dt*e1e0*w1w0/3
r2=(1-(1+eta)*w0)/(1-w0)/(1+eta)
#print(x)
#print(y)
d=y-x
if (d!=1) if (d<3) print(paste(eta," ",e1e0," ",w1w0," ",w0," ",dt," ",(y-x)," ",r2," ",floor((abs(y-x-r2))
*1000)/1000))
  y-x
}

e=.1;
w0=.8
while (e> -.3)
{
w=0;
while (w<.1)
{
big(e,w,w0,6)
w=w+.025
}
e=e-.03
}

```

Extract R output to table,

```

cat xxx | sed -e 's/^[~"]*"/g' | sed -e 's/"//g' | sed -e 's/[0-9]\{10\}$//g' | sed -e 's/ */ \&/g' | awk '{
print $0 "\\\""}' | sed -e 's/[0-9][0-9]\{10\} //g'

```

2. BIBLIOGRAPHY

-
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Acknowledgments

1. Pubmed eutils facilities and the basic research it provides.
2. Free software including Linux, R, LaTeX etc.
3. Thanks everyone who contributed incidental support.
4. Individual cites from LinkedIn for bringing this work to my attention.

Appendix A: Statement of Conflicts

No specific funding was used in this effort and there are no relationships with others that could create a conflict of interest. I would like to develop these ideas further and have obvious bias towards making them appear successful. Barbara Cade, the dog owner, has worked in the pet food industry but this does not likely create a conflict. We have no interest in the makers of any of the products named in this work.

Appendix B: About the Authors and Facility

This work was performed at a dog rescue run by Barbara Cade and housed in rural Georgia. The author of this report ,Mike Marchywka, has a background in electrical engineering and has done extensive research using free online literature sources. I hope to find additional people interested in critically examining the results and verify that they can be reproduced effectively to treat other dogs.

Appendix C: Symbols, Abbreviations and Colloquialisms

TERM definition and meaning

Appendix D: Old Math

Scraps of algebra for later failure analysis if result is wrong,

Thinking aloud

do this as a Taylor series and include integration limits that remove even terms lol

This may be easier to see if the event and vaccination functions are expanded as polynomials,

$$v(t) = \sum v_i t^i; e(t) = \sum e_i t^i \quad (D1)$$

and assume that $p(t)$ is uniform and just drop it.

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum \frac{1}{i+j+1} e_i \left(\frac{v_j - \delta(j=0)I_v}{I_v(1-I_v)} \right) t^{i+j+1} \quad (D2)$$

the first few terms,

$$\langle e_v \rangle - \langle e_{nv} \rangle \approx A \left(e_0(v_0 - I_v)t + \frac{1}{2}e_1(v_0 - I_v)t^2 + \frac{1}{2}e_0v_1t^2 + \frac{1}{3}e_1v_1t^3 \right) \quad (D3)$$

Note first that this may be a little misleading too- A and I and $p(t)$ need some care with changes in time interval and the zero result when the only e term is e_0 is not apparent. There are several constraints on the polynomial expansions - the event rate can not become negative and the vaccination fraction has to be between 1 and 0 for example. However, there is no obvious constraint to force this difference to be zero under the null hypothesis. Over the age distribution relevant to these data, the event rate is probably decreasing while the vaccination rate is increasing leading to a contribution of $e_1 * v_1$ making the difference negative or making the non-vaccinated event rate appear to be higher (under the null hypothesis).

$$\langle e \rangle = \frac{\int dt \sum_p \Delta t^p \sum_q w_q e_{p-q}}{\sum_i \frac{1}{(i+1)} w_i \Delta t^{i+1}} \quad (D4)$$

$$\langle e \rangle = \frac{\sum_p \frac{1}{((p+1))} \Delta t^{p+1} \sum_q w_q e_{p-q}}{\sum_i \frac{1}{(i+1)} w^{(i)}(t_0) \Delta t^{i+1}} \quad (D5)$$

Evaluating at half interval delta t around t_0 ,

$$\langle e \rangle = \frac{\sum_{p=0,2,4,\dots} \frac{2}{((p+1))} \Delta t^{p+1} \sum_q w_{(q)} e_{(p-q)}}{\sum_{i=0,2,4,\dots} \frac{2}{(i+1)} w_{(i)} \Delta t^{i+1}} \quad (D6)$$

and the first few terms are then, cancelling the 2,

$$\langle e \rangle = \frac{e_0 w_0 \Delta t + \frac{\Delta t^3}{3} (e_0 w_2 + e_1 w_1 + e_2 w_0)}{w_0 \Delta t + \frac{1}{3} w_2 \Delta t^3 + \frac{1}{5} w_4 \Delta t^5} \quad (D7)$$

$$\frac{\langle e_{nv} \rangle}{\langle e_v \rangle} = \frac{(e_0 \Delta t - e_0 w_0 \Delta t - \frac{\Delta t^3}{3} (e_0 w_2 + e_1 w_1 + e_2 w_0)) (w_0 \Delta t + \frac{1}{3} w_2 \Delta t^3 + \frac{1}{5} w_4 \Delta t^5)}{(\Delta t - w_0 \Delta t - \frac{1}{3} w_2 \Delta t^3 - \frac{1}{5} w_4 \Delta t^5) (e_0 w_0 \Delta t + \frac{\Delta t^3}{3} (e_0 w_2 + e_1 w_1 + e_2 w_0))} \quad (D8)$$

$$\frac{\langle e_{nv} \rangle}{\langle e_v \rangle} = \frac{(e_0 \Delta t - e_0 w_0 \Delta t - \frac{\Delta t^3}{3} (e_0 w_2 + e_1 w_1 + e_2 w_0)) (I_v)}{(e_0 w_0 \Delta t + \frac{\Delta t^3}{3} (e_0 w_2 + e_1 w_1 + e_2 w_0)) (1 - I_v)} \quad (D9)$$

Appendix E: More Old Math

Left over algebra for later math checking and failed approaches,

An unweighted average rate could be defined as,

$$\langle e_v \rangle + \langle e_{nv} \rangle = \int e(t) p(t) \left(\frac{1}{I_{nv}} - \frac{v(t)}{I_{nv}} + \frac{v(t)}{I_v} \right) dt \quad (E1)$$

which apparently only reproduces the overall rate when the samples are equally divided,

$$\langle e_v \rangle + \langle e_{nv} \rangle = \frac{\langle e \rangle}{I_{nv}} - \int e(t) p(t) v(t) \left(\frac{1}{I_{nv}} - \frac{1}{I_v} \right) dt \quad (E2)$$

$$\langle e_v \rangle + \langle e_{nv} \rangle = \frac{\langle e \rangle}{I_{nv}} - \langle e_v \rangle \left(\frac{1}{I_{nv}} - \frac{1}{I_v} \right) \quad (E3)$$

If the I denominators are equal the expected result is obtained but not otherwise.

The difference also helps show that these are just age weighed events rates with the age weighting is the vaccination rate as a function of age,

$$\langle e_v \rangle - \langle e_{nv} \rangle = \int e(t) p(t) \left(\frac{v(t)}{I_v} - \frac{1 - v(t)}{I_{nv}} \right) dt \quad (E4)$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \int e(t) p(t) \left(v(t) \left(\frac{1}{I_v} + \frac{1}{I_{nv}} \right) - \frac{1}{I_{nv}} \right) dt \quad (E5)$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \int e(t) p(t) \left(v(t) \left(\frac{1}{I_v I_{nv}} \right) - \frac{1}{I_{nv}} \right) dt \quad (E6)$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \frac{\langle e_v \rangle}{I_{nv}} - \frac{\langle e \rangle}{I_{nv}} \quad (E7)$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \frac{\langle e_v \rangle - \langle e \rangle}{I_{nv}} \quad (\text{E8})$$

This difference is not obviously zero but should be in the limit of "small" age ranges. Inserting delta functions at t_0 and normalizing $\langle e \rangle$,

$$\langle e_v \rangle = e(t_0) \text{ and } \langle e \rangle = e(t_0) \text{ with } I_{nv} = p(t_0)(1 - v(t_0)) \quad (\text{E9})$$

Dividing the difference by the sum,

$$\frac{\langle e_v \rangle - \langle e_{nv} \rangle}{\langle e_v \rangle + \langle e_{nv} \rangle} = \frac{\frac{\langle e_v \rangle - \langle e \rangle}{I_{nv}}}{\frac{\langle e \rangle}{I_{nv}} - \langle e_v \rangle \left(\frac{1}{I_{nv}} - \frac{1}{I_v} \right)} \quad (\text{E10})$$

$$\frac{\langle e_v \rangle - \langle e_{nv} \rangle}{\langle e_v \rangle + \langle e_{nv} \rangle} = \frac{\langle e_v \rangle - \langle e \rangle}{\langle e \rangle - \langle e_v \rangle \left(1 - \frac{I_{nv}}{I_v} \right)} \quad (\text{E11})$$

The authors results are just vaccination age weight event rates and should go away uniformly with much smaller bin sizes. There may be a real age dependent effect but it is not clear from this analysis. Assuming the null hypothesis is wrong and there are two event rates based on vaccination status, $e_v(t)$ and $e_{nv}(t)$, they need to be computed as functions of time first. The mistake is pulling the normalization out rather than cancelling it in the integrand. Simply put, the vaccinated group has fewer high event members.

This may be easier to see if the event and vax functions are expanded as polynomials,

$$v(t) = \sum v_i t^i; e(t) = \sum e_i t^i \quad (\text{E12})$$

$$\langle e_v \rangle = \frac{\int \sum e_i v_j t^{i+j} p(t) dt}{I_v}; I_v = \int p(t) \sum v_i t^i dt \quad (\text{E13})$$

Ignoring p for simplicity,

$$\langle e_v \rangle = \frac{\sum \frac{1}{i+j+1} e_i v_j t^{i+j+1}}{I_v}; I_v = \sum \frac{1}{i+1} v_i t^{i+1} dt \quad (\text{E14})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum \frac{1}{i+j+1} e_i \left(\frac{v_j}{I_v} - \frac{x_j}{1 - I_v} \right) t^{i+j+1} \quad (\text{E15})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum \frac{1}{i+j+1} e_i \left(\frac{v_j(1 - I_v) - x_j I_v}{I_v(1 - I_v)} \right) t^{i+j+1} \quad (\text{E16})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum_{j \neq 0} \frac{1}{i+j+1} e_i \left(\frac{v_j}{I_v(1 - I_v)} \right) t^{i+j+1} + \sum \frac{1}{i+1} e_i \left(\frac{v_0 - I_v}{I_v(1 - I_v)} \right) t^{i+1} \quad (\text{E17})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum \frac{1}{i+j+1} e_i \left(\frac{v_j - \delta(j=0)I_v}{I_v(1 - I_v)} \right) t^{i+j+1} \quad (\text{E18})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum_{j \neq 0} \frac{1}{i+j+1} e_i \left(\frac{v_j(I_v + I_{nv})}{I_v(I_{nv})} \right) t^{i+j+1} + \sum \frac{1}{i+1} e_i \left(\frac{-1}{(I_{nv})} \right) t^{i+1} \quad (\text{E19})$$

$$\langle e_v \rangle - \langle e_{nv} \rangle = \sum \frac{1}{i+j+1} e_i \left(\frac{v_j(I_v + I_{nv}) - \delta(j=0)I_v}{I_v I_{nv}} \right) t^{i+j+1} \quad (\text{E20})$$

Appendix F: General caveats and disclaimer

This document was created in the hope it will be interesting to someone including me by providing information about some topic that may include personal experience or a literature review or description of a speculative theory or idea. There is no assurance that the content of this work will be useful for any particular purpose.

All statements in this document were true to the best of my knowledge at the time they were made and every attempt is made to assure they are not misleading or confusing. However, information provided by others and observations that can be manipulated by unknown causes ("gaslighting") may be misleading. Any use of this information should be preceded by validation including replication where feasible. Errors may enter into the final work at every step from conception and research to final editing.

Documents labelled "NOTES" or "not public" contain substantial informal or speculative content that may be terse and poorly edited or even sarcastic or profane. Documents labelled as "public" have generally been edited to be more coherent but probably have not been reviewed or proof read.

Generally non-public documents are labelled as such to avoid confusion and embarrassment and should be read with that understanding.

Appendix G: Citing this as a tech report or white paper

Note: This is mostly manually entered and not assured to be error free.

This is tech report MJM-2022-016.

Version	Date	Comments
0.01	2022-12-14	Create from empty.tex template
0.01	2022-12-15	post draft on LI and cite on relevant post
0.02	2022-12-17	cleaner Taylor series, clean up text
0.03	2022-12-24	cleaner add η plot
-	December 24, 2022	version 0.03 MJM-2022-016
1.0	20xx-xx-xx	First revision for distribution

Released versions,
build script needs to include empty releases.tex

Version	Date	URL
.01	2022-12-15	https://www.linkedin.com/posts/marchywka_comment-on-a-work-recently-cited-in-li-activity-70090886678
.02	2022-12-17	https://www.researchgate.net/publication/366389694_An_Interesting_Issue_with_Inhomogeneous_Population
.03	2022-12-24	https://zenodo.org/record/7480634#.Y6dwcHXMJCU

```
@techreport{marchywka-MJM-2022-016-0.03,
filename="vaxacc" ,
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title=" An Interesting Issue with Inhomogeneous Population Averages" ,
author="Mike J Marchywka " ,
type="techreport" ,
name="marchywka-MJM-2022-016-0.03" ,
number="MJM-2022-016" ,
version="0.03" ,
institution="not institutionalized, independent " ,
address=" 44 Crosscreek Trail, Jasper GA 30143" ,
date="December 24, 2022" ,
startdate="2022-12-14" ,
day="24" ,
month="12" ,
year="2022" ,
author1email="marchywka@hotmail.com" ,
contact="marchywka@hotmail.com" ,
doi="10.13140/RG.2.2.30725.22249" ,
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pages="12"
}
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Supporting files. Note that some dates,sizes, and md5's will change as this is rebuilt.

This really needs to include the data analysis code but right now it is auto generated picking up things from prior build in many cases

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