Photometric Redshift

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The purpose of this project is to understand the photometric data (images and magnitudes), the use we can do to select the kind of objects and the redshift estimation using underlying model. The program is to understand different concepts as:

- Expanding Universe and redshift
- Distances in cosmology
- What are a flux and a luminosity .... in an expanding Universe
- How to convert a flux in magnitude a for given system of reference
- How to find a best model using data (minimum χ^2 method)

and use it all of them to:

- Find the best photometric redshift for real red galaxies and verified your result with the spectroscopic information.

1 Starter

The first session you need to explain the general idea of the practice. This note was created to help you to identify the key concepts, it is far from being complete and you are strongly encouraged to look for additional material.

You will find questions along the text that you are supoussed to investigate by yourself, these questions will help you to achieve a complete understanding of the physics behind the practice.

As you are not supoussed to know the cosmology part, the note is particularly focused on the cosmological concepts. But if you want to learn more consult the additional references or take any cosmology book, the distances and Hubble parameter should be discussed in any book.

The references I ask you to read as a minimum before next session are:

- This note.
- eBOSS paper. (https://arxiv.org/abs/1508.04473). Yo do not have to understand everything, but at least have an idea of the goal of the Dark energy experiment and the observables used for that purpose, as well as the data used for the cosmology science.

Material supplementary recommended for Cosmology:

- An introduction t Modern Cosmology by Andrew Liddle. (A very nice material to start)
- Modern Cosmololgy by Scott Dodelson (More advanced material)

Material supplementary recommended for Statistics:

- https://arxiv.org/abs/0911.3105 (A very nice note to start...)
- Statistics, Data Mining and Machine Learning in Astronomy (More advanced material)

Material supplementary for astronomical concepts.

- https://web.archive.org/web/20120915140659/http://www.observatorio.unal.edu.co/docentes/armando/archivosMAH

Do not hesitate to look in wikipedia or google!!!!

2 Equations for Cosmology

We define the scale factor of the Universe by "a" which is, by definition, equal to 1 today ($a(t=0)=1$). $a(t)$ defines the ratio you have to multiply with to obtain a distance today at the time t to compensate the expansion of the Universe. It does not concern the distances of collapsed objects, for which there is no expansion inside. The Hubble function, which express the rate of expansion of the Universe (expressed as a velocity by distance unit: km.s^{-1}/Mpc) at a given moment (which can be t, z or a) is given by:

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^4}\sum_i \rho_i - \frac{k}{a^2}, \tag{1}$$

where the ρ_i are the density of the different energetic component of the Universe (baryonic matter, dark matter, dark energy, radiation) and k the isotropic curvature of the Universe ($k=0$ if flat Universe which will be the case for the lecture). If we divide this equation by the value of $H(a)$ today ($H_0 \sim 70$km.s^{-1}/Mpc), and knowing that the critical density of the Universe ρ_{crit} today is defined as $\rho_{crit,0} = \frac{3H_0^2c^4}{8\pi G}$ we find that:

$$\frac{H^2(a)}{H_0^2} = \sum_i \frac{\rho_i(a)}{\rho_{crit,0}} - \frac{k}{a^2\rho_{crit,0}}. \tag{2}$$

We will see further (see appendix A) that the evolution with the scale factor can be wrote as:

$$\rho_i(a) = \rho_i(a=1) \times a^{-\alpha_i} \qquad \alpha_b = \alpha_{DM} = 3, \alpha_r = 4, \alpha_{DE} = 0. \tag{3}$$

This just mean that density of matter (baryons and Dark Matter) is diluted by the volume, the radiation density is diluted by the volume and a stretching effect more on the wavelength of the photons which reduce the energy by a scale factor too. The Dark Energy is not diluting with time. Considering a flat Universe (k=0), we can so write the Hubble function as:

$$H^2(a) = H_0^2\sum_i \frac{\rho_{i,0}}{\rho_{crit,0}} \times a^{-\alpha_i} = H_0^2\sum_i \frac{\rho_{i,0}}{\rho_{crit,0}} \times (1+z)^{\alpha_i}, \tag{4}$$

where the $\rho_{i,0}$ are the actual values for density energy of matter, radiation and dark energy we measure in the local Universe. We generally expressed the quantities in term of critical density that we not:

$$\Omega_i = \frac{\rho_{i,0}}{\rho_{crit,0}} \tag{5}$$

so the expression we generally use for the Hubble function is given by:

$$\boxed{H(z) = H_0\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}.} \tag{6}$$

One implication of the expansion of the Universe is the effect on the photons because the expansion stretch the wavelength λ which correspond to down the energy of them. The relation between the wavelength of an emitted photon at time t_{emit} far away and the one observed today is:

$$\frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a(t_{obs})}{a(t_{emit})}. \tag{7}$$

If we interpret this "redshift" z (stretch of the wavelength) as a Doppler effect, then we can rely the "recessing velocity" with the redshift as:

$$\lambda_{obs} = \lambda_{emit}(1+\frac{v}{c}) = \lambda_{emit}(1+z). \tag{8}$$

Considering that the scale factor today (t_{obs}) is equal to 1, it comes directly the relation between the redshift and the scale factor:

$$\lambda_{obs} = \lambda_{emit}\frac{a(t_{obs})}{a(t_{emit})}, \quad a(t_{obs}) = 1 \Rightarrow a(t) = \frac{1}{1+z(t)}. \tag{9}$$

Two important equations, known as FRW equations, allowing to calculate the evolution of the expansion of the Universe considering a dominating energetic component are

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}, \tag{10}$$

and

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \tag{11}$$

where p is the pressure. Because we consider generally the different energetic component as perfect fluids, we can simplify the last equation considering $p = w\rho$ where w is the equation of state of the considered component. For example, $w = 0$ for the dark matter and $w = 1/3$ for the baryonic matter.

3 Distances in Cosmology

3.1 Comoving distance

In order to calculate distances in cosmology, we refer to the distance done by the photon to move from one event (time and position) to another space-time event. The distance we are mostly interested with is the one between us ($a = 1$, $z = 0$ or $t = 0$) and an object far away. The metric considering an isotropic Universe without curvature is given by:

$$ds^2 = -c^2dt^2 + a^2(t)d\chi^2\,, \qquad \gamma \Rightarrow ds^2 = 0, \tag{12}$$

where $d\chi^2$ is the infinitesimal radial distance element. The second equation is verified only by a massless particle and allows us to rely the infinitesimal space and time elements :

$$c^2dt^2 = a^2(t)d\chi^2 \quad \Rightarrow \quad \frac{c^2}{a^2(t)}dt^2 = d\chi^2. \tag{13}$$

Thanks to this relation, we can calculate the distance integrating the inverse of the scale factor with during the time travel of the photon:

$$d\chi = \frac{c}{a(t)}dt \quad \Rightarrow \quad \chi(t_{emit}) = c\int_{t_{emit}}^{t_0}\frac{dt}{a(t)}. \tag{14}$$

Doing the substitution from t to a using the Hubble function, we obtain:

$$H(t) = \frac{\dot{a}(t)}{a(t)}\,;\, \dot{a} = \frac{da}{dt} \quad \Rightarrow \quad dt = \frac{da}{a(t)H(t)}\chi(a_{emit}) = c\int_{a_{emit}}^{a(t_0)=1}\frac{da}{a^2H(a)}. \tag{15}$$

Now, we do the substitution from the scale factor a to the redshift z:

$$a = \frac{1}{1+z} \quad \Rightarrow \quad da = \frac{-dz}{(1+z)^2}, \tag{16}$$

$$a = \frac{1}{1+z} \Rightarrow da = \frac{-dz}{(1+z)^2}, \tag{17}$$

and we obtain finally the following integral

$$\chi(z_{emit}) = -c\int_{z_{emit}}^{z(t_0)=0}\frac{dz(1+z)^2}{(1+z)^2H(z)}, \tag{18}$$

and inverting the integration limits, to integrate from us to the object, we obtain finally:

$$\chi(z_{emit}) = c\int_0^{z_{emit}}\frac{dz}{H(z)} \tag{19}$$

$$H(z) = H_0\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda} \tag{20}$$

$$\boxed{\chi(z_{emit}) = \frac{c}{H_0}\int_0^{z_{emit}}\frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}}} \tag{21}$$

3.2 Luminosity distance and Angular distance

We have to define two others distances which are the "Luminosity distance" ($D_L(z)$) and the "Angular distance" ($D_A(z)$) . The first one come from the relation between the luminosity and the flux of an object. They are expressed as following:

$$\boxed{D_l(z_{emit}) = (1+z_{emit}) \times \chi(z_{emit}) = \frac{c(1+z_{emit})}{H_0} \int_0^{z_{emit}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}},} \tag{22}$$

and

$$D_A(z_{emit}) = \frac{\chi(z_{emit})}{(1+z_{emit})} = \frac{c}{(1+z_{emit})H_0} \int_0^{z_{emit}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}}. \tag{23}$$

We will see in details on the next section why the Luminosity distance has this factor $(1+z)$.

4 Equations for the astronomical observations

In astronomy, the flux is defined as the energy from photons we receive during 1s on a surface of 1cm^2 at a given frequency (i.e wavelength). For this reason, the unit of a flux is generally expressed in $\text{erg.s}^{-1}.\text{cm}^{-2}.\text{Hz}^{-1}$ where the erg is an astronomical unit of energy.

The luminosity of an object is the integral of the photons emitted during the same second. So we have to integrate over all the surface of the shell. If we consider an isotropic emission, we just have to multiply the flux we measure by the the surface of the shell with radius equal to the distance between us and the object. This surface have to be expressed in cm^2 because of the definition of the flux.

So, the relation between the flux and the luminosity, for a given wavelength λ, without expansion (for example a star in our galaxy) is

$$f(\lambda) = \frac{L(\lambda)}{4\pi d^2}, \tag{24}$$

where d is the distance between us and the star expressed in cm.

However, we know that the Universe expands implying 2 things:

1. photons are redshifted
2. the distance has to be calculated in a similar way than the comoving distance

So, the frequency or the wavelength of a photon emitted from a far galaxy is different when we measure it :

$$\lambda_{obs} = (1+z)\lambda_{emit} \qquad \text{or} \qquad \nu_{obs} = \frac{\nu_{emit}}{(1+z)}, \tag{25}$$

which mean that the energy change of size by a factor $(1+z)$. The energy for a photon is given by:

$$E = h\nu = \frac{hc}{\lambda} \tag{26}$$

so the difference between the emitted energy and the one observed is

$$\boxed{E_{obs} = h\nu_{obs} = \frac{h\nu_{emit}}{(1+z)} = \frac{E_{emit}}{(1+z)}.} \tag{27}$$

We also have a difference for the number of photons we observe per second. Because of the distance between the photons increase by a factor $(1+z)$, the photons hosted in box with size of 1 light second are in a box ox $(1+z)$ light second. So, in one second we receive only $1/(1+z)$ of the photons. Another way to think about it is to say that we need (1+z) seconds to receive the photons emitted in a 1 second range. So, we loose another factor $(1+z)$ on the flux. For this reason, we measure :

$$f = \frac{L}{4\pi\chi^2(z)(1+z)^2}, \tag{28}$$

where we did not specified the wavelength on purpose and we can see immediately that

$$D_L^2(z) = (1+z)^2\chi^2(z) \qquad \Rightarrow \qquad \boxed{D_L(z) = (1+z)\chi(z).} \tag{29}$$

Because the wavelength of the emitted photon is different from the one received, we have to explicitly write it:

$$\boxed{f(\lambda) = \frac{L(\lambda/(1+z))}{4\pi D_L^2(z)},} \tag{30}$$

which one of the main equations you need to know for this lecture.

However, we generally do not use directly the flux to define the brightness of a star or a galaxy (why we need a reference? What are the definitions of flux and brightness). We generally use the magnitude in a given band of observation. Look for some examples of magnitud for astronomical objects, what it means larger magnitud?

5 Astronomical Magnitud

In astronomy, magnitude is a logarithmic measure of the brightness of an object in a defined passband. Magnitudes should be quoted for a specific wavelength range since real detectors are not sensitive to the entire EM spectrum, and the Earth?s atmosphere transmits radiation only over certain wavelength regions. Astronomers use two different definitions of magnitude: apparent magnitude and absolute magnitude. The apparent magnitude is the brightness of an object as it appears in the night sky from Earth, while the absolute magnitude describes the intrinsic brightness of an object as it would appear if it were placed at a certain distance from Earth, 10 parsecs for stars.

5.1 Apparent Magnitud

This quantity is a logarithmic comparison between the measured flux and another of reference in the same band of observation. During many years, the spectrum from the star Vega (Vega, also designated Alpha Lyrae (? Lyrae, abbreviated Alpha Lyr or ? Lyr), is the brightest star in the constellation of Lyra)was used as the reference (Take a look to the history of astronomical observations !), but we often prefer to use the AB system which use a constant flux independently of the frequency (What is the AB system?). The definition of a magnitude in a band b is given by:

$$mag_b = -2.5\log_{10}\left(\frac{f_b^{mes}}{f_b^{ref}}\right), \tag{31}$$

where f_b^{mes} and f_b^{ref} are the measured flux and the one of reference (Why there is a factor -2.5 in the equation?). For example, the AB system use a constant flux of 3.631×10^{-20}erg.s^{-1}cm^{-2}Hz^{-1}. We understand that we need to make corrections when we want to compare various magnitudes from different systems.
Because the flux is the sum of the stars, we can say that the flux is $f^{mes} = M_* \times f_b^{mes}(1M_\odot)$, where the M_* is the stellar mass and $f_b^{mes}(1M_\odot)$ is the flux for a galaxy of only 1 solar mass. For this reason, we can write the difference between to magnitudes as:

$$mag_{b1} - mag_{b2} = -2.5\left[\log_{10}\left(\frac{f_{b1}^{mes}}{f_{b1}^{ref}}\right) - \log_{10}\left(\frac{f_{b2}^{mes}}{f_{b2}^{ref}}\right)\right] \tag{32}$$

$$mag_{b1} - mag_{b2} = -2.5\left[\log_{10}\left(\frac{f_{b1}^{mes} f_{b2}^{ref}}{f_{b1}^{ref} f_{b2}^{mes}}\right)\right] \tag{33}$$

$$mag_{b1} - mag_{b2} = -2.5\left[\log_{10}\left(\frac{M_* \times f_{b1}^{mes}(1M_\odot) f_{b2}^{ref}}{M_* \times f_{b2}^{mes}(1M_\odot f_{b1}^{ref})}\right)\right] \tag{34}$$

so we can simplify the M_* and we find that the difference between two magnitudes do not depend on the stellar mass:

$$mag_{b1} - mag_{b2} = -2.5\left[\log_{10}\left(\frac{f_{b1}^{mes}(1M_\odot) f_{b2}^{ref}}{f_{b2}^{mes}(1M_\odot f_{b1}^{ref})}\right)\right] = mag_{b1}(1M_\odot) - mag_{b2}(1M_\odot) \tag{35}$$

The difference between 2 magnitudes is named a color.

We will see how we can generate synthetic spectra and magnitudes of galaxies taking into account all these effects on the next section.

6 SDSS-I-I-II -IV

SDSS has imaged about one-third of the night sky in five broad bands (ugriz). The resulting catalog includes photometry for almost half a billion unique objects. Understanding how to use SDSS imaging data requires some knowledge of how the data were collected. This page http://www.sdss.org/dr14/imaging/imaging_basics/ explains what you need to know about SDSS imaging data. What are the bands in SDSS

To learn more about current dark energy experiment read (https://arxiv.org/abs/1508.04473)

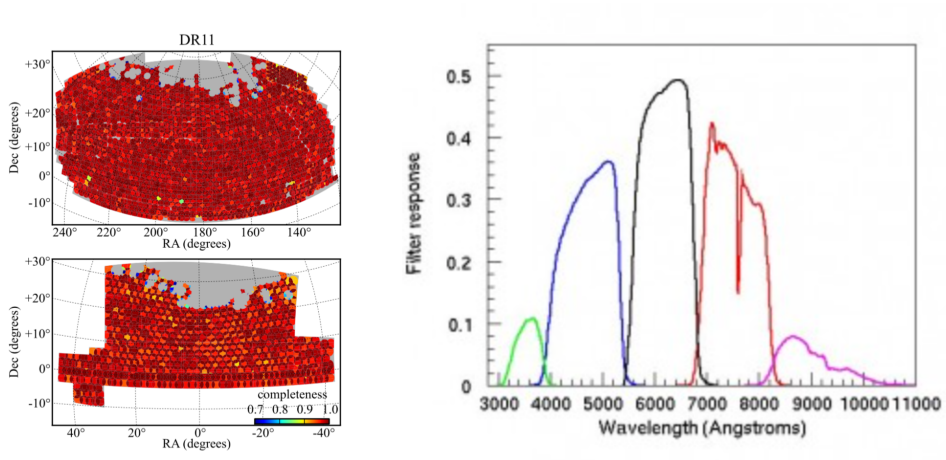

Figure 1: BOSS footprint.

7 Spectral Energy Density (SED)

In order to model the galaxy spectra (what is a galaxy?), it is necessary to take into account the sum of the stars spectra and dust effects. I propose to use the synthetic model from Bruzual & Charlot 2003 (?)

The SED of a galaxy is the sum of all the star spectra considering their mass and ages, adding if necessary the effect of the dust (which will not be for our case). The changes between different galaxies at a same redshift will depend exclusively on the composition in mass and ages of the stars inside the galaxy. (How the spectra varies as a function of time?) Thus for modeling the SED we need at least 2 ingredients: the distribution initial of stars per bin of mass called Initial Mass Function (IMF) and the distribution of ages.

7.1 Initial Mass Function (IMF)

It allows to choose the distribution on mass for the stars formed during a burst (what is a burst?). For our project, we propose to use the standard Chabrier IMF. (Add a reference here?)This prescription is given by:

$$\phi(\log(m)) \propto \begin{cases} \exp\left[\frac{-(\log(m)-\log(m_c))^2}{2\sigma^2}\right] & \text{for } m \leq 1M_\odot \\ m^{-1.3} & \text{for } m > 1M_\odot \end{cases} \tag{36}$$

where $m_c = 0.08M_\odot$ and $\sigma = 0.69$. The results provided by the files with name like "couleurs_yjhbk_zf_70.dat" are following this prescription.

7.2 Star Formation History (SFH)

One time we choose the distribution of star masses are formed during a burst, we have to decide how much stellar masses are formed and when. This is exactly the definition of the star formation history (SFH). In our project, we propose to use simple single-burst. This suppose that all the stars were formed at the same time (just one burst) at a given redshift of formation z_f.

The files "couleurs_yjhbk_zf_#.dat" provides the SDSS magnitudes (u,g,r,i,z) in the AB system for **stellar mass $\mathbf{M_* = 1M_\odot}$** at all redshift between 0 and 3 taking into account the distance, the redshift evolution and the star evolutions for $z_f = \#/10$. You will play with these files to find the best photometric redshift of red galaxies from the SDSS survey.

The SED is given in the rest-frame of the galaxy so, we have to take into account the evolution of the spectrum with the redshift (Distance and stretch) and consider that the age of the galaxy is defined by the time between the observed redshift and the formation redshift. Using a single burst model, all the stars formed at a formation redshift z_f. The age of these stars in a galaxy we observe at z_{obs} is the time passing between these 2 redshifts. This time is generally named look-back time and corresponds to the time travelling of the photon between the 2 redshift (so the comoving distance divided by c). So we have the age defined as:

$$age(z_{obs}) = t_{lb}(z_{obs}, z_f) = \int_{z_{obs}}^{z_f} \frac{dz}{H(z)} \tag{37}$$

7.3 Photometric redshift (Photo-z)

Using this simple model, you will see that you have a nice tool to determine the photometric redshift. But what is a photo-z? As you can see on the figure 3, the spectrum of a red galaxy present a break at 4000Å in the galaxy restframe. This break creates a difference in the magnitude of the bands which strongly depends on the redshift of observation. Using the colors, so the difference between magnitudes, we can determine which redshift allows to model the best the observed color.

You will play with the various files to create an algorithm which allows to determine this photometric redshift for SDSS galaxies.

8 Practice

We have 6 sessions for the practice. First session we will discuss the cosmology material in particular the distances. And if any time we will review briefly python.

8.1 Play with the distances and densities

The practice is designed for 2 sessions of work by yourself.

1. Using Python, right a code to plot the Hubble function depending on the redshift (up to z=1) for the values $\Omega_m = 0.3$, $\Omega_\lambda = 0.7$, $\Omega_R = 0$
2. Using this result, calculate the comoving distance $\chi(z)$, the luminosity distance $D_L(z)$ and the angular distance $D_A(z)$ for redshift between 0 and 1 with a step $\Delta z = 0.01$. Plot the 3 curves on a same figure
3. One time it works, do the same for redshift up to 3. What do you remark for the angular distance?
4. Compute the distance for a model with not cosmological constant for a flat Universe.
5. (BONUS) reproduce the figure 2 to find the period of domination of Dark Energy, Matter and Radiation.

8.2 Play with the data

Fourth session we will briefly review the data access. And the astronomical/astrophysical concepts. Then the practice is designed for 2 sessions.

1. Using the notebook examples (using astro_query.sdss method), get the magnitudes in u,g,r,i and z band of SDSS objects (doc for using tables astropy: http://docs.astropy.org/en/stable/table/)We want to get the spectroscopic redshift for these objects too. We will use a radius of 40 arcminutes in order to have enough galaxies. Use one (ra,dec) covered by the SDSS mask (See Figure 1 of the footprint in equatorial Coordinate System). (What is the equatorial reference system?).
2. Read the model magnitudes for $z_f = 0.7$ and $z_f = 0.5$ and plot the colors $g-u$, $g-r$, $g-i$ and $g-z$ for redshift between 0 and 1.
3. Overplot the colors for the objects you download from the server.
4. select the red galaxies using a simple cut $g-r > 0.5$ (You can put whatever you want in the y-axis). Verify this cut separates stars and galaxies. This is a simplified version, search for the criteria used in the target selection of SDSS for Luminous Red Galaxies. (https://arxiv.org/pdf/astro-ph/0108153.pdf) for separating stars and galaxies and verify as well.

5. Find the best redshift for all the galaxies using the 4 colors. Plot the χ^2 and mark the minimun with a vertical line and the $\Delta\chi^2$ as an horizontal line, and derive the confidence interval at 1σ (Take into account the d.o.f).

6. Compare them with the spectroscopic redshift get from the SDSS data in a scatter plot. Make a distribution of $z_{photo} - z_{spec}$ and derive the mean photometric error. Report this number. Is the distribution gaussian?

7. (BONUS) Up to now we used colors which are independent of the total stellar mass. Reason why the magnitudes from the model files are given for 1 solar mass. The stellar mass can be derived as $M_* = 10^{\left(\frac{mag(1M_\odot)-mag_{obs}}{2.5}\right)}$. Calculate the corresponding stellar mass for each galaxy using the band r and check if it is coherent with the typical stellar masses we find in the literature for this kind of galaxies Do you know where to look for papers, Check in Arxiv & ADS !.

9 Evaluation

The evaluation would be 50% oral and 50% written part. The oral should be prepared in an structured way as if you were in a conference. Please think in advance what you want to say, do not improvise. The written part should be prepared as an article. The deadline for send it by email is one day after the oral presentation. Follow the same instructions given by the coordinator of the Laboratory.

A Energy density evolution

Using the equations of Hubble and Friedmann-Robertson-Walker, we can develop the evolution of the scale factor of the Universe considering a dominant component of energy. We will first postulate that each component dominates at one time and we will develop the results. Then, using these results and using the actual values of the component, we will derive the time each of the component was the dominant one during the Universe history.

Let start with the FRW equations relying the pressure $p$and the energy density ρ with the scale factor for a dominating component:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad \Rightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = 0 \tag{38}$$

where we substitute the pressure with its relation to the density assuming a perfect fluid with equation of state $p = w\rho$. We can rewrite this equation as follow

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1+\omega) = 0 \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) + 3(1+\omega)\frac{d}{dt}ln(a) = 0, \tag{39}$$

where we can recognize the time derivative of the log of a and ρ. Using the log property between the multiplication and the power law of the argument we directly find:

$$\frac{d}{dt}ln(\rho) = -3(1+\omega)\frac{d}{dt}ln(a) \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) = \frac{d}{dt}ln(a^{-3(1+\omega)}), \tag{40}$$

which give th trivial result:

$$\rho \propto a^{-3(1+\omega)}, \qquad \rho = \rho_0 a^{-3(1+\omega)}. \tag{41}$$

Considering the different values of the equations of state for the matter, radiation and Dark Energy (specific case of it is $\omega = -1$ which correspond to the cosmological constant):

$$\omega_m = 0 \quad \Longleftrightarrow \quad \rho_m(a) = \rho_{m,0} \times a^{-3} \tag{42}$$

$$\omega_R = 1/3 \quad \Longleftrightarrow \quad \rho_R(a) = \rho_{R,0} \times a^{-4} \tag{43}$$

$$\omega_\Lambda = -1 \quad \Longleftrightarrow \quad \rho_\Lambda(a) = Cte \tag{44}$$

$$\omega_{DE} < -1/3 \tag{45}$$

We will see in the next appendix why the Dark Energy condition is $\omega_{DE} < -1/3$.

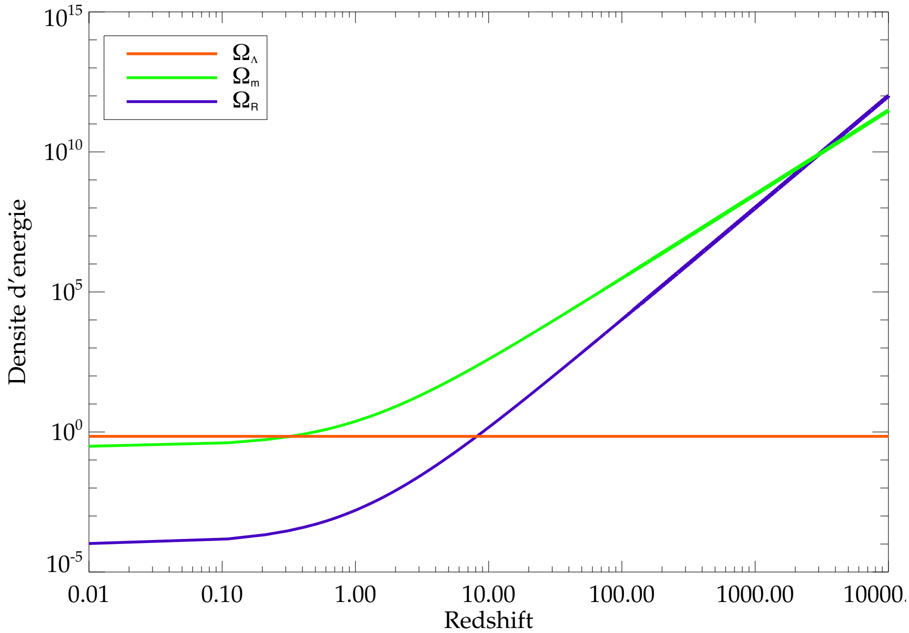

Figure 2: Evolution of the energy density starting with the local values. We can see that today the Universe is dominated by the Dark Energy/Cosmological constant. It was dominated by the matter before; and first by the radiation far away in the past.

B Scale factor evolution with the time

The goal of this appendix is to derive the time evolution of the scale factor of the Universe with the time over the different epoch of domination. Starting from the Hubble equation, in the case of a dominating component of energy we obtain (By definition, the sum disappear and we just keep one):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}. \tag{46}$$

in which we assume that the scale factor evolution follows a power law of time: Then we assume a

$$\begin{aligned} a &\propto t^n && (47)\\ \dot{a} &\propto t^{n-1} && (48)\\ \frac{\dot{a}}{a} &\propto t^{-1} && (49)\\ \left(\frac{\dot{a}}{a}\right)^2 &\propto t^{-2} && (50) \end{aligned}$$

Using the results from the last appendix and reintroducing on this result we finally obtain that:

$$\begin{aligned} \rho &\propto t^{-3(1+\omega)\times n} && (51)\\ -2 &= -3(1+\omega)\times n && (52)\\ n &= \frac{2}{3(1+\omega)} && (53)\\ a(t) &\propto t^{\frac{2}{3(1+\omega)}}. && (54) \end{aligned}$$

The specific cases of evolution are:

$$\begin{aligned} \omega_R = 1/3 &\quad\Rightarrow\quad a(t) \propto t^{1/2} && (55)\\ \omega_m = 0 &\quad\Rightarrow\quad a(t) \propto t^{2/3} && (56) \end{aligned}$$

Exercise: Show the condition over the value of ω to have an acceleration of the expansion of the Universe. Tip: it's equivalent to show the condition to have $\ddot{a} > 0$

However, in the case of $\omega = -1$ then we can not use this method because it is equivalent to devide by 0. But the solution is easier because we now since the previous appendix that it correspond to constant value of the energy density. Using this, we simply find that:

$$\begin{aligned} \rho_\Lambda = cte \Leftrightarrow \dot{\rho_\Lambda} = 0 && (57)\\ \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_\Lambda}{3} = cte && (58) \end{aligned}$$

$$\omega_\Lambda = -1 \quad\Rightarrow\quad \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho_\Lambda}{3}} \quad\Rightarrow\quad a(t) \propto \exp^{\sqrt{\frac{8\pi G\rho_\Lambda}{3}}} \tag{59}$$

which is an exponential evolution with the time!

C Spectra

In figure 3, we can see the evolution (without the dilution, just the strecth) of a typical spectrum of red galaxy formed a redshift=7 and observed at $z = 0.$, $z = 0.2$ and $z = 0.4$. Are represented the 5 bands of the Sloan Digital Sky Survey instrument (SDSS). We can see immediately that the difference of magnitudes in the bands G, R, I change with the redshift. So, we see that the colors $G - R$ and $R - I$ will be important to determine the photometric redshift of these galaxies.

The figure 4 shows the evolution of standard mix of stars spectra, given a Initial Mass Function (IMF), with the age which is indicated in billion years. The galaxy spectrum is the sum of the stars spectra, with different ages depending on the Star Formation History (SFH) and the presence or not of the dust.

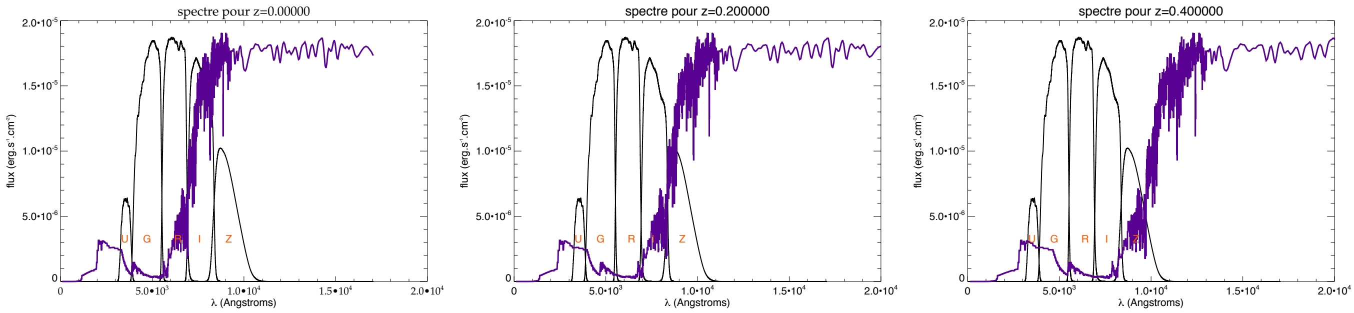

Figure 3: Typical spectrum of red galaxy formed a redshift=7 and observed at $z = 0.$, $z = 0.2$ and $z = 0.4$ with the SDSS bands. The spectrum here is just stretched by the redshift but non effect of dilution and evolution are take into account. Is just a sketch to understand the magnitude and color variations.

D AB System

The AB magnitude system is an astronomical magnitude system. Unlike many other magnitude systems, it is based on flux measurements that are calibrated in absolute units, namely spectral flux densities

E Definitions

- Luminosity, In astronomy, luminosity is the total amount of energy emitted by a star, galaxy, or other astronomical object per unit time.
- Brightness, which is the luminosity of an object in a given spectral region.

F Report

Include tittle, authors, affiliation, date and abstract, introduction to the topic, theory, data used, methodology, and results, discussion , conclusions and references. Appendices, include the jupyter notebooks.

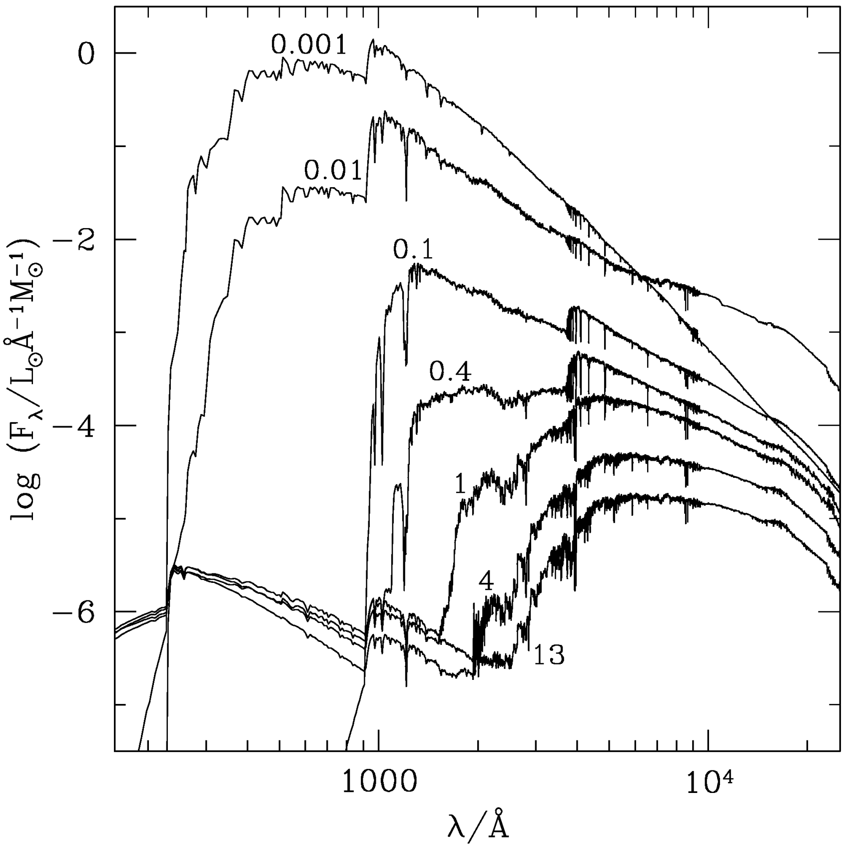

Figure 4: Age evolution of a single burst galaxy spectrum. The ages are given in billion years