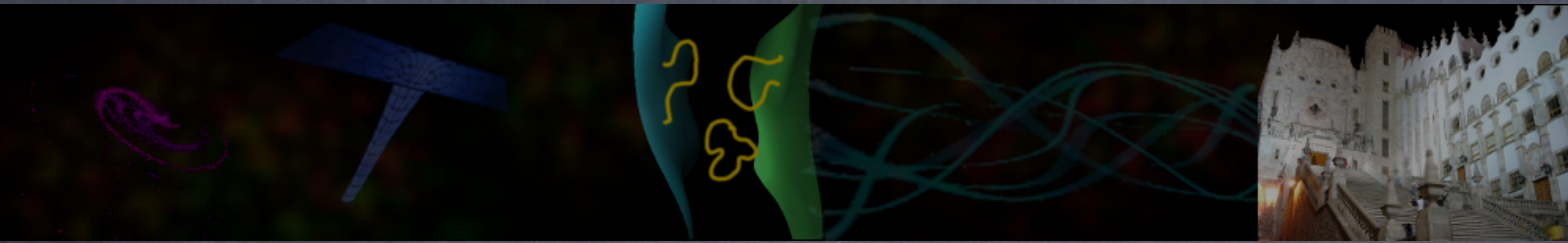

Perturbation of the Scalar Field Dark Matter

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In collaboration with Stefany Medellin, Francisco Linares, Oleg Burgueño and Luis Ureña. ArXiv 1511.08195, 1703.10180, and two works in prep.

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Scalar Field Dark Matter

a.ka. Axion like Dark Matter, BECDM, FuzzyDM, wave DM, ULA-DM, etc…

- Dark Matter is described by a scalar field. Can be coupled to the SM: ej. Axion for QCD.
- Or it can be only coupled gravitationally. We'll work with a real scalar field.

$$\mathcal{L}_\phi = -(1/2)(\partial\phi)^2 - (1/2)m^2\phi^2$$

$$T^\mu_\nu = g^{\mu\alpha}\phi_\alpha\phi_{,\nu} - \delta^\mu_\nu(V(\phi) + 1/2g^{\kappa\lambda}\phi_\kappa\phi_{,\lambda})$$

Hu et. al 2000, Matos & Ureña 2002, P. Sikivie and Yang, 2009. Marsh &Silk 2013, Shive et. al 2014, and many others.

Most recent review on the subject: H. Lam, J. Ostriker, S. Tremaine, Edward Witten arXiv: 1610.08297

It is a field representation, not a particle one.

Scalar Field Dark Matter

Mass $m_a \gtrsim 10^{-23} eV$ to be consistent with CMB and LSS bounds.

The mass of the scalar field sets a cut-off in the mass power spectrum in the small scales (large k's).

Self-gravitating objects could form of galactic size. But in general a Halo will be composed of a inner "soliton" and an external cloud.

SFDM halos always have a core density profile, due to the presence of the soliton. The more massive the Axion, the less extended the "soliton"

Clear Predictions/differences with respect to CDM parentheses for the motivation to work on SFDM

Missing Satellite problem since 2000's

- Observational problem: Determine the precise number of satellite galaxies. Luminosity below detection threshold, non-complete samples, etc.

- Theoretical problem: What makes a halo not to host/produce stars so that they are undetectable. Or else, what inhibits the creation of small halos?

"Is theres a missing satellite problem with CDM ? The answer is likely to be not in the era of DES and LSST" Hargis et. al 2014

There is no missing satellite problem. Kim et al. 2017

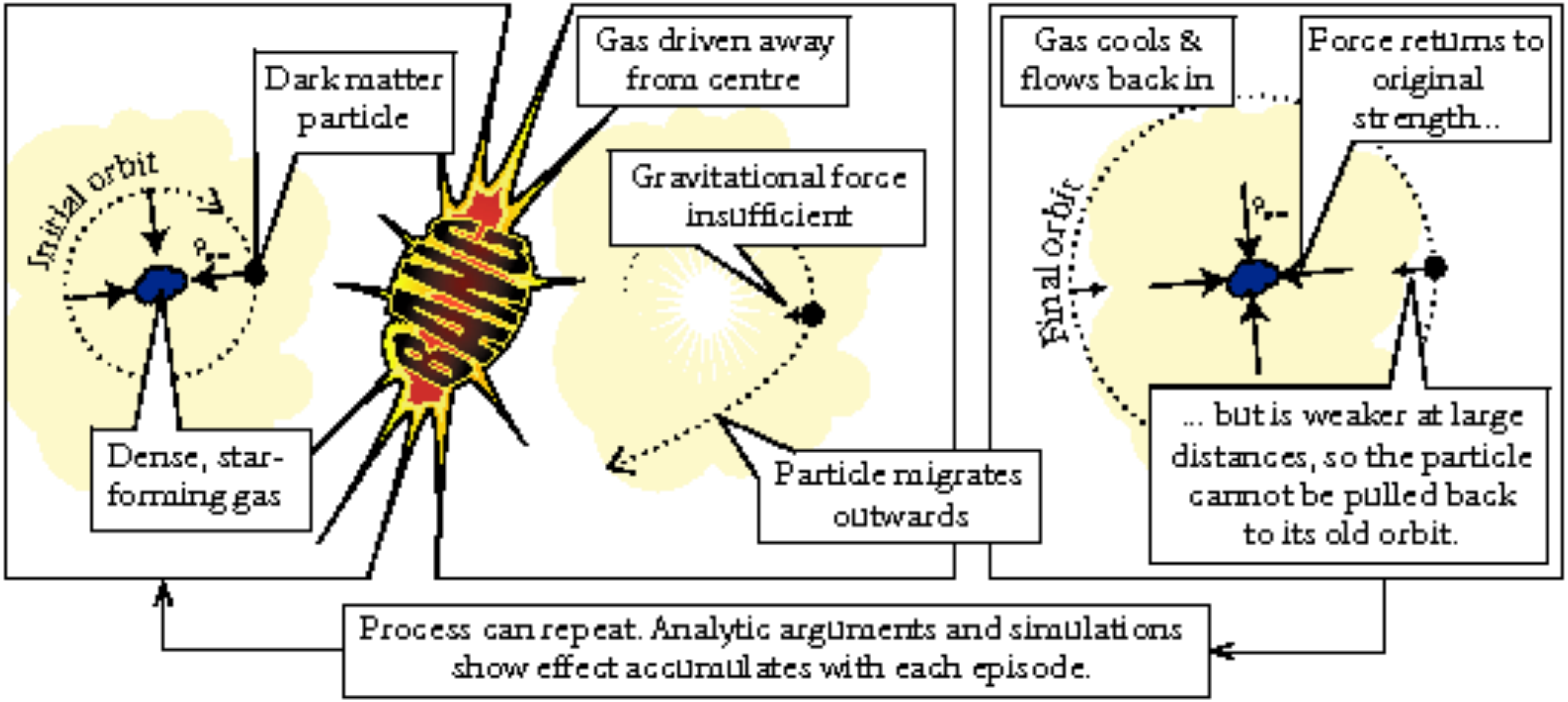
Cusp Vs Core status since 2000's

- N-Body Simulations of only CDM (cold, neutral, collisionles) predicts cusp density profiles (down to simulation resolution)
- Baryonic effects are very important, two effects compite: Contraction Vs Feedback.
- Some DM candidates predicts core profiles (ignoring baryonic effects).

Cusp Vs Core status: No consensus

Theoretical Problem: Not so easy to include baryons on simulations to determine how DM properties+baryons shape the final DM density profile.

Contraction Vs Feedback



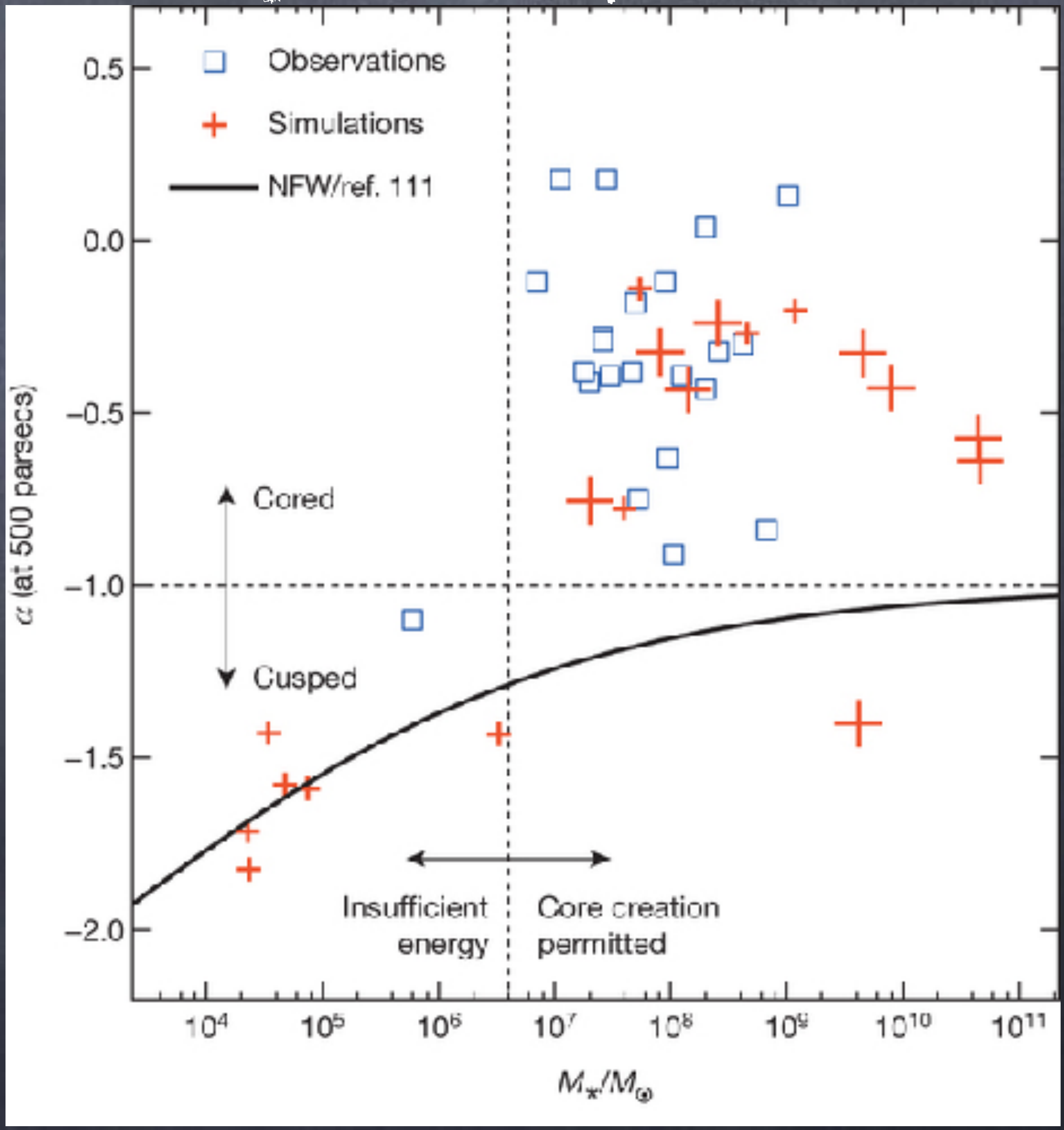
Pontzen & Governato, 2010

Cusp Vs Core status: No consensus

Problem:

Theoretical: No so easy to include baryons on simulations to determine how DM properties+baryons shape the final DM density profile.

In dSph's
different groups seems
to find different
conclusions. Possibly due
to different resolution

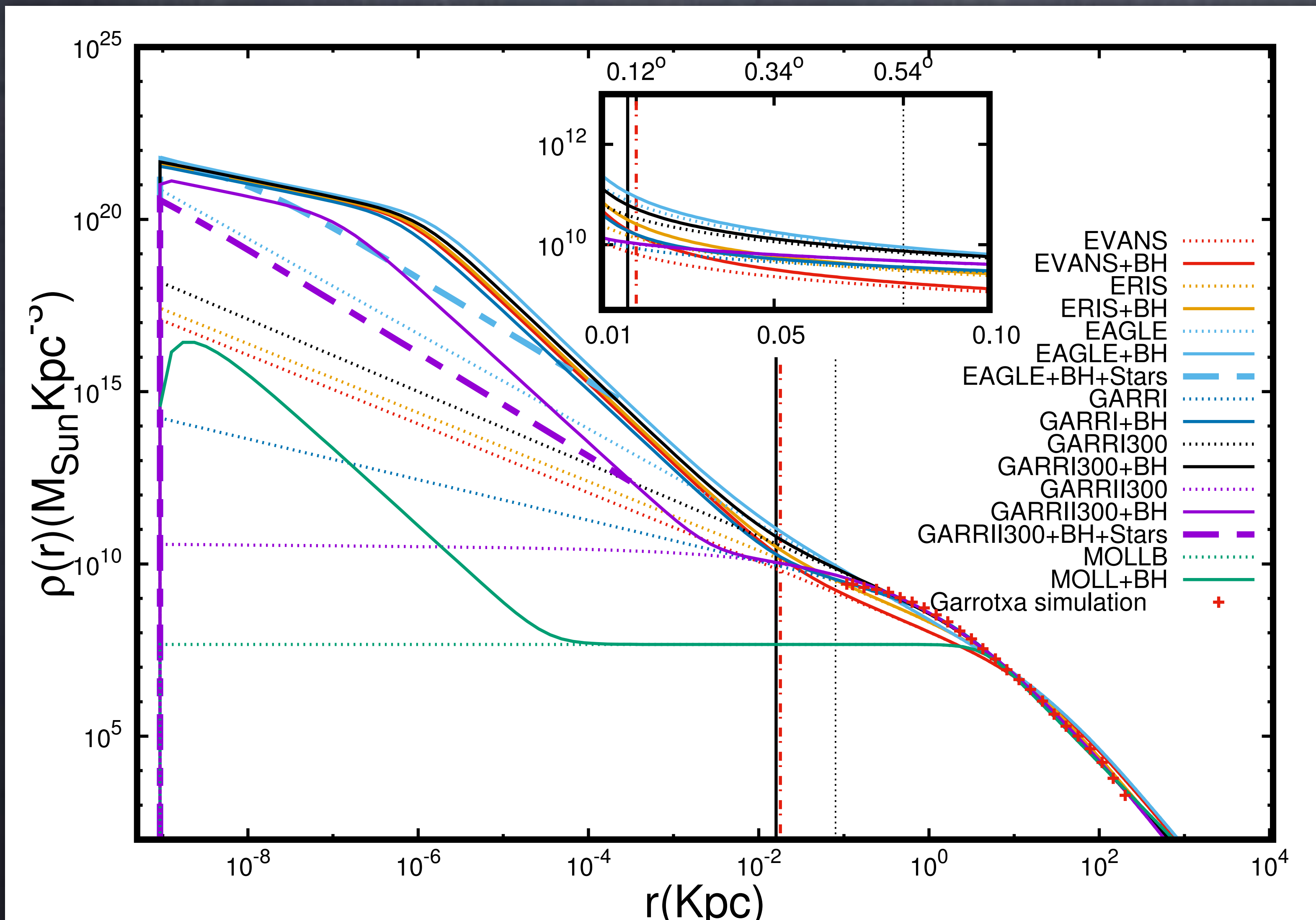


Pontzen & Governato, Nature, 2014

Peñarrubia et al. 2012 , Read et al. 2016
Sawala et al. 2016; Zhu et al. 2016

Cusp Vs Core status: No consensus

Theoretical Problem: No so easy to include baryons on simulations to determine how DM properties+baryons shape the final DM density profile.



¿What is the best model for the DM Halo?

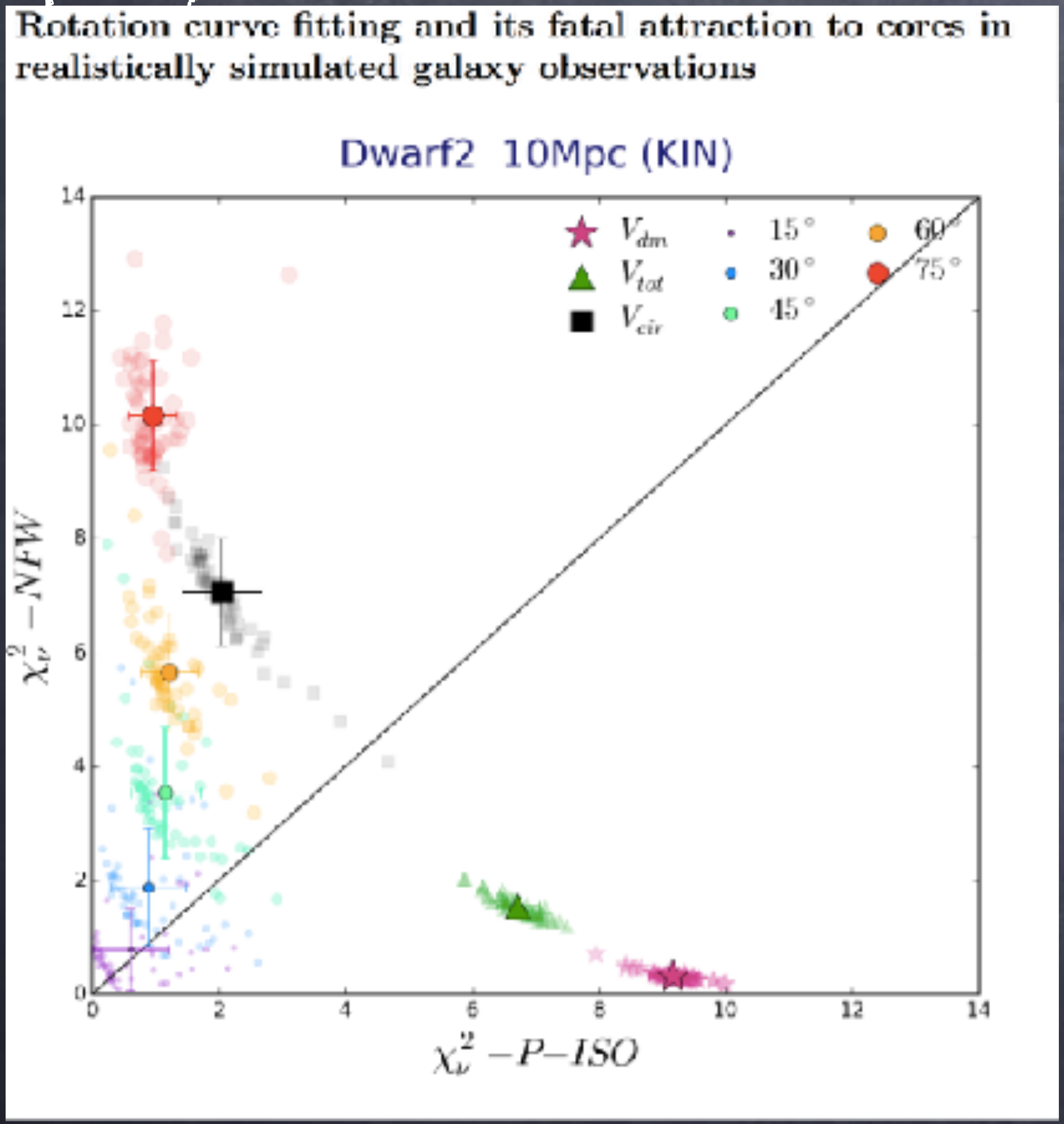
¿What are the implications for DM properties?

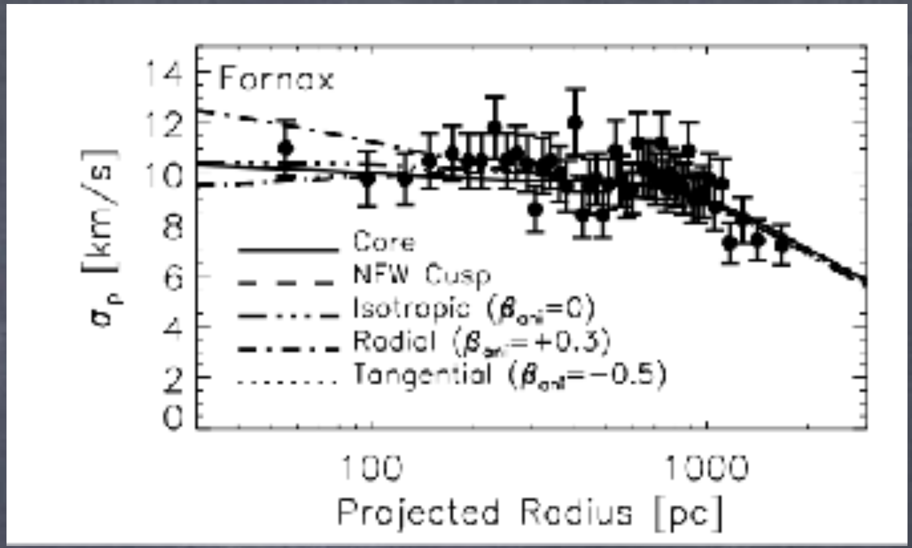
V.Gammaldi, A. Avila, O.Valenzuela & A. Gonzalez-Morales, 2016

Cusp Vs Core status: No consensus

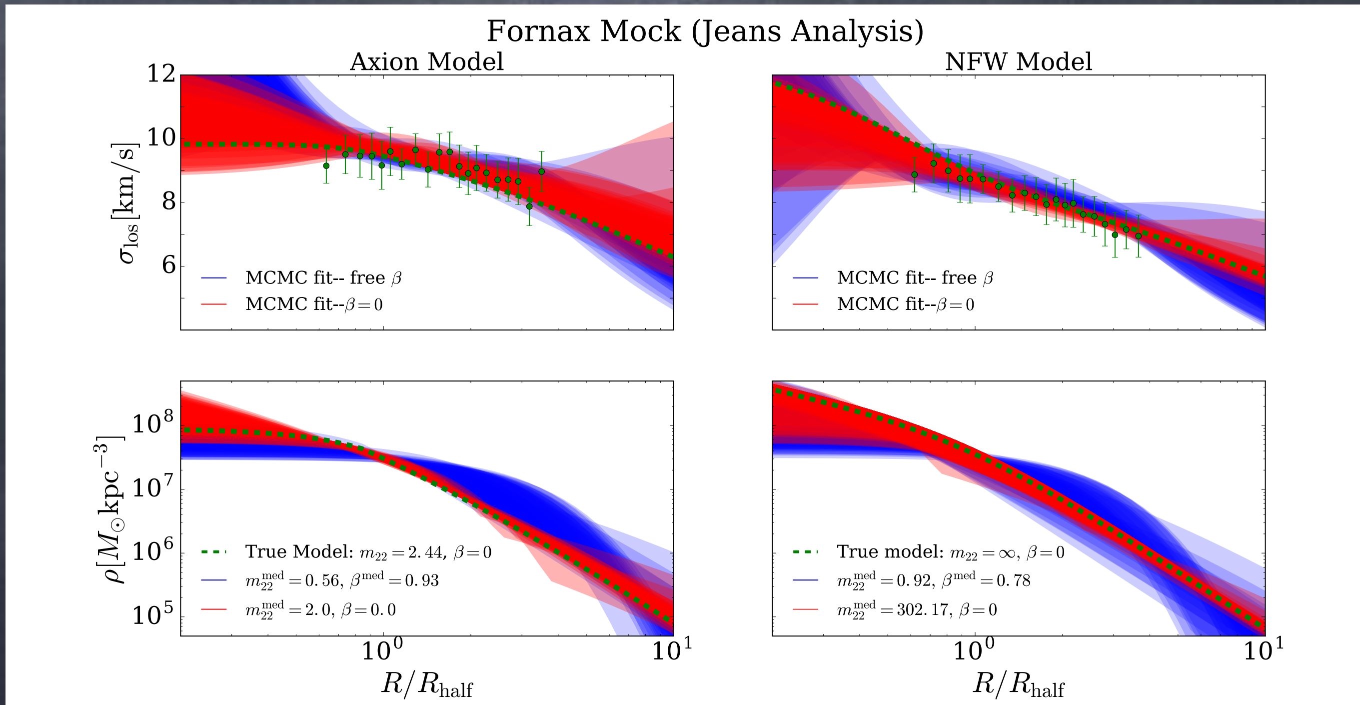
Observational problem: Degeneracies between different effects makes not trivial to recover the "true" density profile.

In dSph's, a strong degeneracy with stellar orbital anisotropy.





arXiv: 1108.2404v3

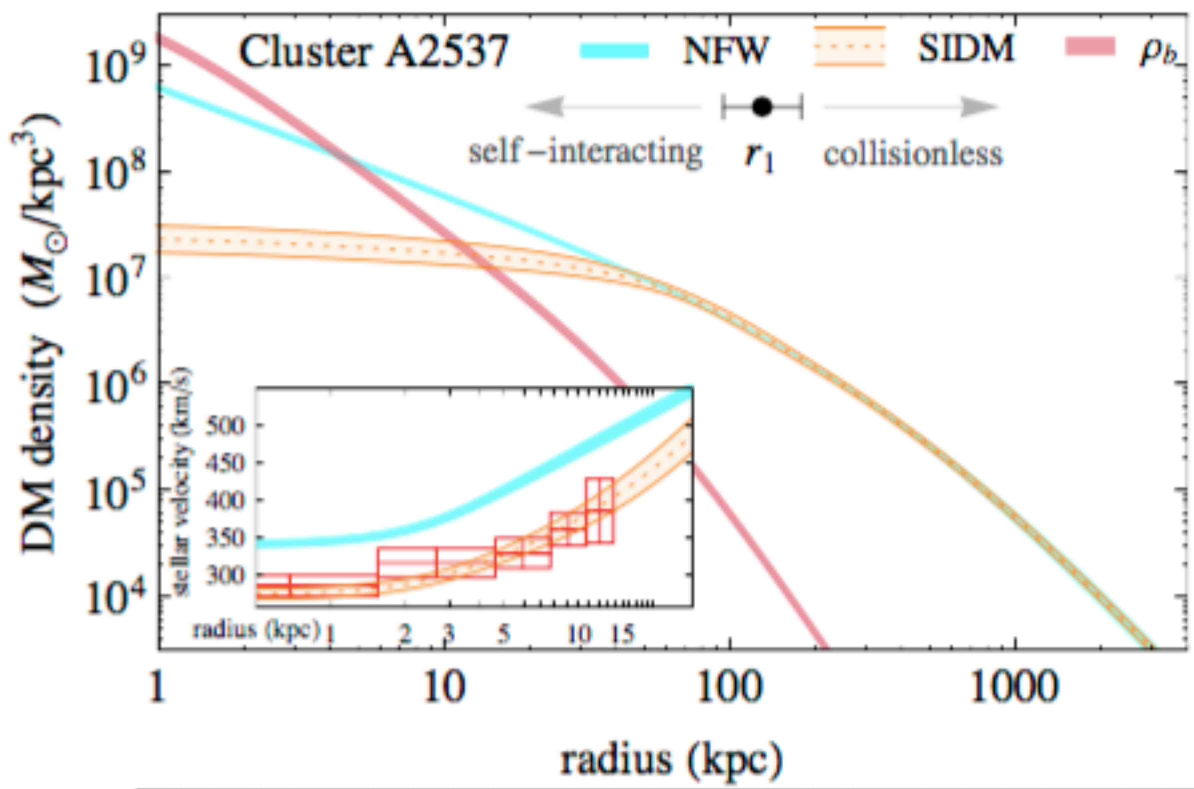


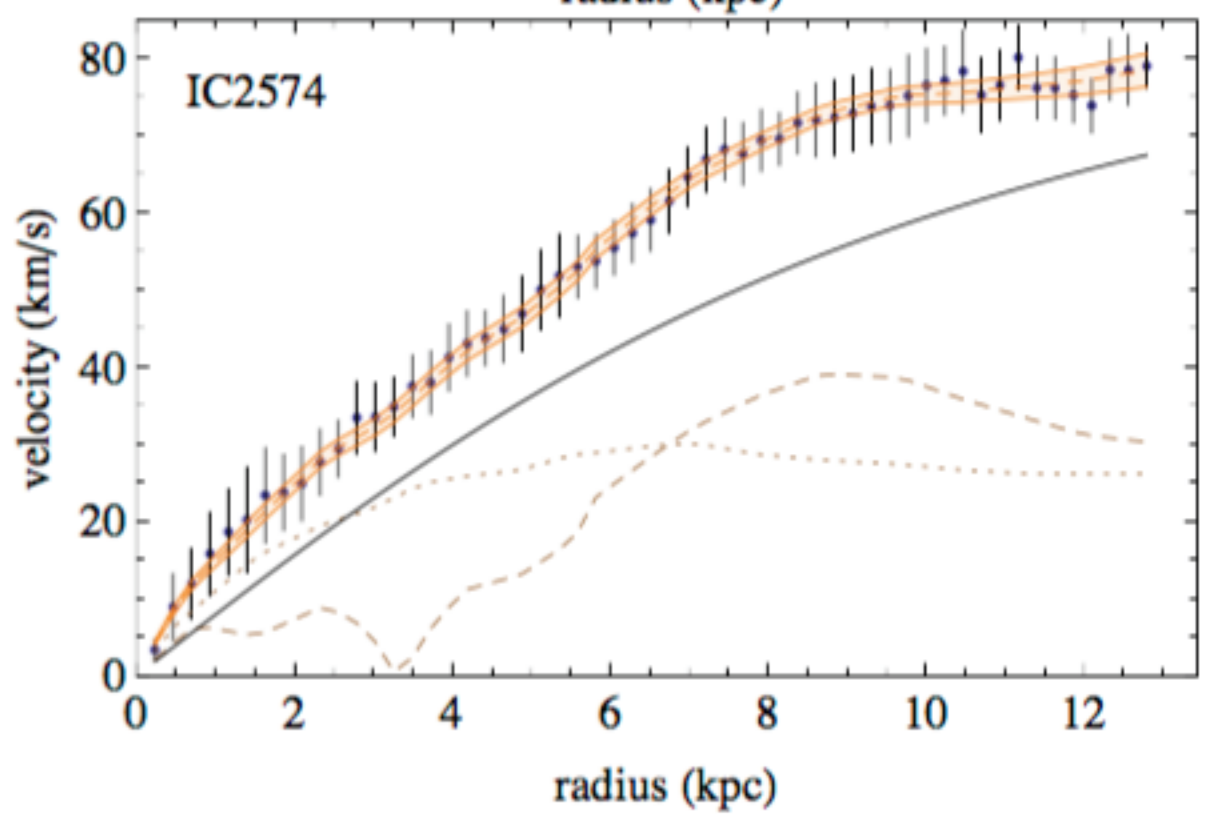
- In disk/irregular Galaxies:
  - Gas pressure effect in the center.
  - Projection effects
  - Finite Spatial

arXiv: 1602.07690v1

arXiv: 1609.05856

Problema Cusp Vs Core: Properties of the DM, like Self-Ineracting, WDM, Axion DM, etc. can modify the shape of the density profile. Example:





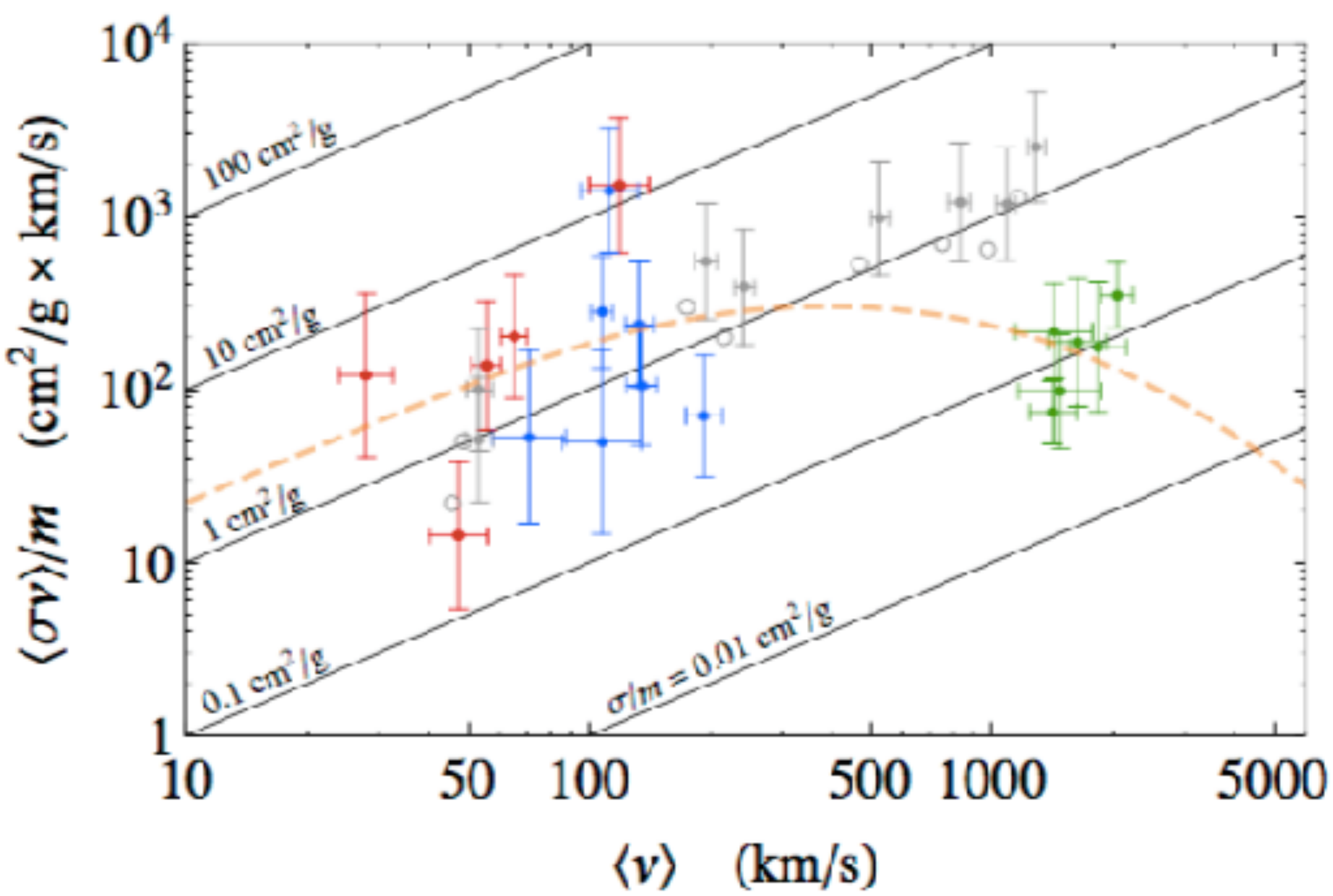


FIG. 1: Self-interaction cross section measured from astrophysical data, given as the velocity-weighted cross section per unit mass as a function of mean collision velocity. Data includes dwarfs (red), LSBs (blue) and clusters (green), as well as halos from SIDM N-body simulations with $\sigma/m = 1$ cm^2/g (gray). Diagonal lines are contours of constant σ/m and the dashed curve is the velocity-dependent cross section from our best-fit dark photon model (Sec. V).

Kaplinghat 2015

END of parentheses.

GO BACK TO TODAY's topic

Background Cosmology

$$T^\mu_\nu = g^{\mu\alpha}\phi_\alpha\phi_{,\nu} - \delta^\mu_\nu(V(\phi) + 1/2 g^{\kappa\lambda}\phi_\kappa\phi_{,\lambda})$$

$$H^2 = \frac{\kappa^2}{3}\left(\sum_I \rho_I + \rho_\phi\right) \qquad \dot{H} = -\frac{\kappa^2}{2}\left[\sum_I (\rho_I + p_I) + (\rho_\phi + p_\phi)\right]$$

$$\dot{\rho}_I = -3H(\rho_I + p_I)\,, \qquad \ddot{\phi} = -3H\dot{\phi} - \frac{dV(\phi)}{d\phi}$$

Klein-Gordon equatio

For the SF we can identify:

$$\kappa^2 = 8\pi G$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Not zero in general

We'll talk about two cases

$$V(\phi) = (1/2)m^2\phi^2 \qquad V(\phi) = m^2 f^2\left[1 + cos\left(\frac{\phi}{f}\right)\right]$$

Free scalar field

Axion field or self interacting $\lambda = 3/\kappa^2 f^2$

Some convenient variable transformation to write KG equation.

$$\Omega_\phi^{1/2}\sin(\theta/2) \equiv \frac{\kappa\dot{\phi}}{\sqrt{6}H}, \quad \Omega_\phi^{1/2}\cos(\theta/2) \equiv \frac{\kappa V^{1/2}}{\sqrt{3}H}, \; y_1 \equiv -\frac{2\sqrt{2}}{H}\partial_\phi V^{1/2}$$

$$\theta' = -3\sin\theta + y_1$$

$$y_1' = \frac{3}{2}\left(1 + w_{tot}\right) y_1 + \frac{\lambda}{2}\Omega_\phi \sin\theta$$

$$\Omega_\phi' = 3(w_{tot} - w_\phi)\Omega_\phi$$

The free case is recovered for

$$\lambda \to 0$$

$$w_\phi \equiv \frac{p_\phi}{\rho_\phi} = \frac{x^2 - y^2}{x^2 + y^2} = -\cos\theta$$

$$\Omega_I \equiv \frac{\kappa^2 \rho_I}{3H^2}, \quad w_{tot} \equiv \frac{p_{tot}}{\rho_{tot}} = \sum_I \Omega_I w_I + \Omega_\phi w_\phi$$

Initial conditions set at radiation domination epoch (Background cosmology)

To first order

$$\theta' \simeq -3\theta + y_1\,, \quad y_1' \simeq 2y_1\,, \quad \Omega_\phi' \simeq 4\Omega_\phi$$

$$\theta = (1/5)y_1 + C(a/a_i)^{-3}$$

$$y_1 = y_{1i}(a/a_i)^2$$

$$\Omega_\phi = \Omega_{\phi i}(a/a_i)^4$$

The scalar field behaves as DM if it is oscillating around the minimum of the potential .

$$a_{osc} < a_{eq}$$

$$H^{-1} > m^{-1}$$

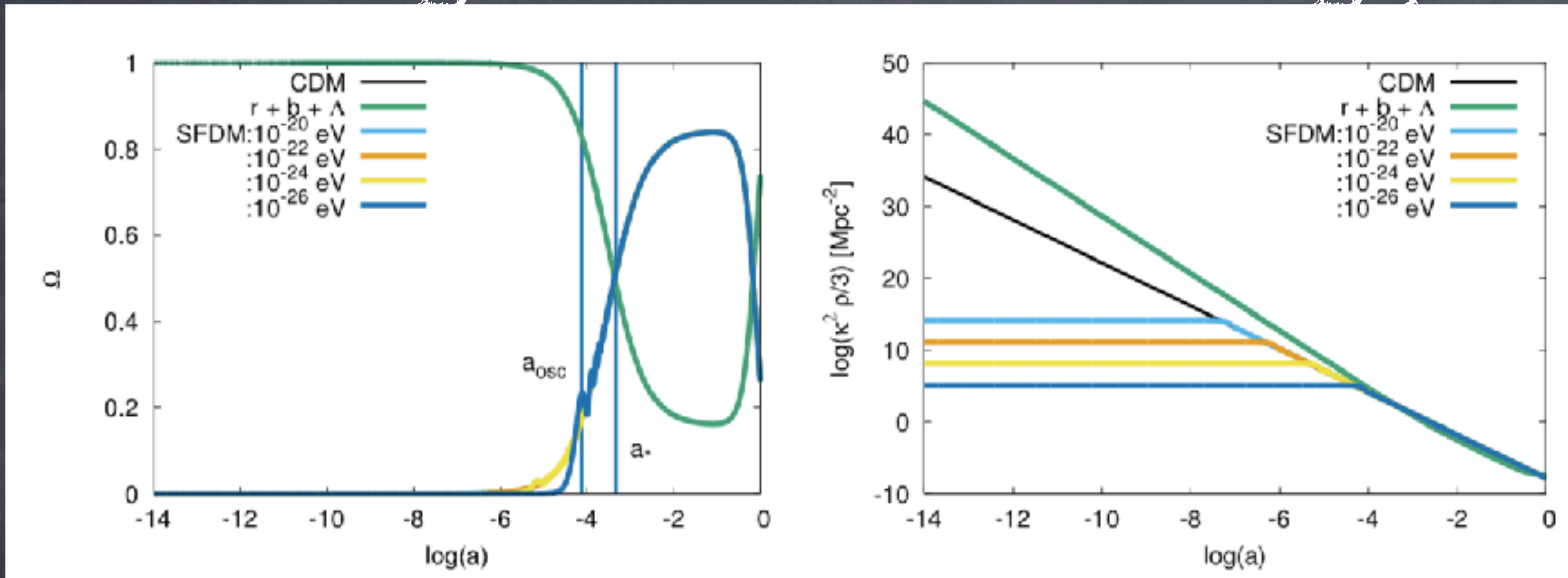
$$a_{\rm osc}^2\left(1 + \frac{\lambda}{72}\frac{\Omega_{\phi 0}}{\Omega_{r0}}a_{\rm osc}\right) = \frac{\pi\,\theta_i^{-1} a_i^2}{2\sqrt{1+\pi^2/36}}$$

$$4\frac{m^2}{H_i^2} = y_{1i}^2 + 4\lambda\Omega_{\phi i}$$

$$y_{1i} = 5\theta_i\left(1 + \frac{\lambda}{40}\Omega_{\phi i}\right)\,, \quad \Omega_{\phi i} = \frac{a_i^4}{a_{\rm osc}^3}\frac{\Omega_{\phi 0}}{\Omega_{r0}}$$

We solve this inside CLASS code with a shooting parameter

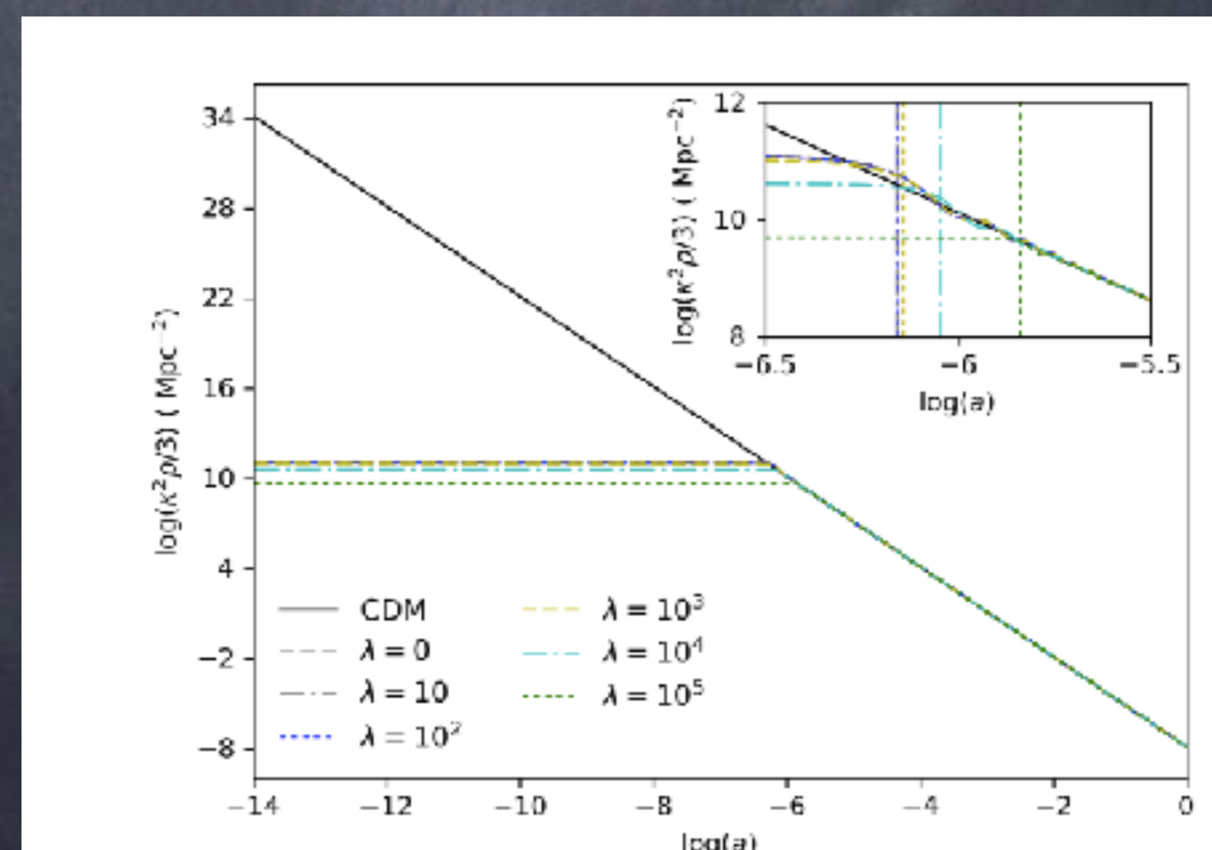
Background Cosmology



*Technical complication about the oscillations. So we have to cut them at some point.

$$\{\cos_\star\gamma, \sin_\star\gamma\} \equiv$$
$$(1/2)\left[1-\tanh(\gamma^2-\gamma_\star^2)\right]\{\cos\gamma, \sin\gamma\}$$

$$\{\cos_\star\gamma, \sin_\star\gamma\} \to 0$$
$$\gamma > \gamma_\star$$



Linear Perturbation Theory

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

Syncronous gauge

$$\ddot{\varphi} = -3H\dot{\varphi} - \left(\frac{k^2}{a^2} + \frac{\partial^2 V(\phi)}{\partial \phi^2}\right)\varphi - \frac{1}{2}\dot{\phi}\dot{\bar{h}}$$

Perturbed KG eq. to linear order, and for a k-mode

$$\phi(x,t) = \phi(t) + \varphi(x,t)$$

$$\delta_\phi = \frac{\dot{\phi}\dot{\varphi} + \partial_\phi V\,\varphi}{\dot{\phi}^2/2 + V(\phi)}, \quad \delta_{\delta p_\phi} = \frac{\dot{\phi}\dot{\varphi} - \partial_\phi V\,\varphi}{\dot{\phi}^2/2 + V(\phi)}, \quad (\rho_\phi + p_\phi)\theta_\phi = (k^2/a)\dot{\phi}\varphi$$

$$\delta_\phi \equiv \delta\rho_\phi/\rho_\phi$$

$$\delta_{p_\phi} \equiv \delta p_\phi/p_\phi$$

Various approaches to solve this. Ours tries to keep information about the oscillations, both in the background and the perturbations for as long possible (numerical stiffness)

After some variable changes, as we did with the background...

$$\delta_0' = \left[-3\sin\theta - \frac{k^2}{k_J^2}(1-\cos\theta)\right]\delta_1 + \frac{k^2}{k_J^2}\sin\theta\delta_0 - \frac{\bar{h}'}{2}(1-\cos\theta)$$

$$\delta_1' = \left[-3\cos\theta - \frac{k^2}{k_J^2}\sin\theta + \Omega_\phi^{1/2}\sin\left(\frac{\theta}{2}\right)\frac{y_2}{y_1}\right]\delta_1 + \left[\frac{k^2}{k_J^2}(1+\cos\theta) - \Omega_\phi^{1/2}\cos\left(\frac{\theta}{2}\right)\frac{y_2}{y_1}\right]\delta_0$$

$$k_J^2 = H^2 a^2 y_1$$

$$\delta_\phi = \delta_0\,, \quad \delta_{p_\phi} = \sin\theta\delta_1 - \cos\theta\delta_0\,,$$

$$-\frac{\bar{h}'}{2}\sin\theta$$

$$(\rho_\phi + p_\phi)\theta_\phi = \frac{k^2}{2am}\rho_\phi\left[(1+\omega_\phi)\delta_1 - \sin\theta\delta_0\right]$$

For the axion potential

$$\delta_0' = \left[-3\sin\theta - \frac{k^2}{k_J^2}(1-\cos\theta)\right]\delta_1 + \frac{k^2}{k_J^2}\sin\theta\delta_0 - \frac{\bar{h}'}{2}(1-\cos\theta)$$

$$\delta_1' = \left[-3\cos\theta - \frac{k_{eff}^2}{k_J^2}\sin\theta\right]\delta_1 + \frac{k_{eff}^2}{k_J^2}(1+\cos\theta)\,\delta_0 - \frac{\bar{h}'}{2}\sin\theta$$

$$k_{eff}^2 \equiv k^2 - \lambda a^2 H^2 \Omega_\phi/2$$

$$k_J = a\sqrt{2Hm}$$

The free case is recovered for

$$\lambda \to 0$$

can be positive or negative

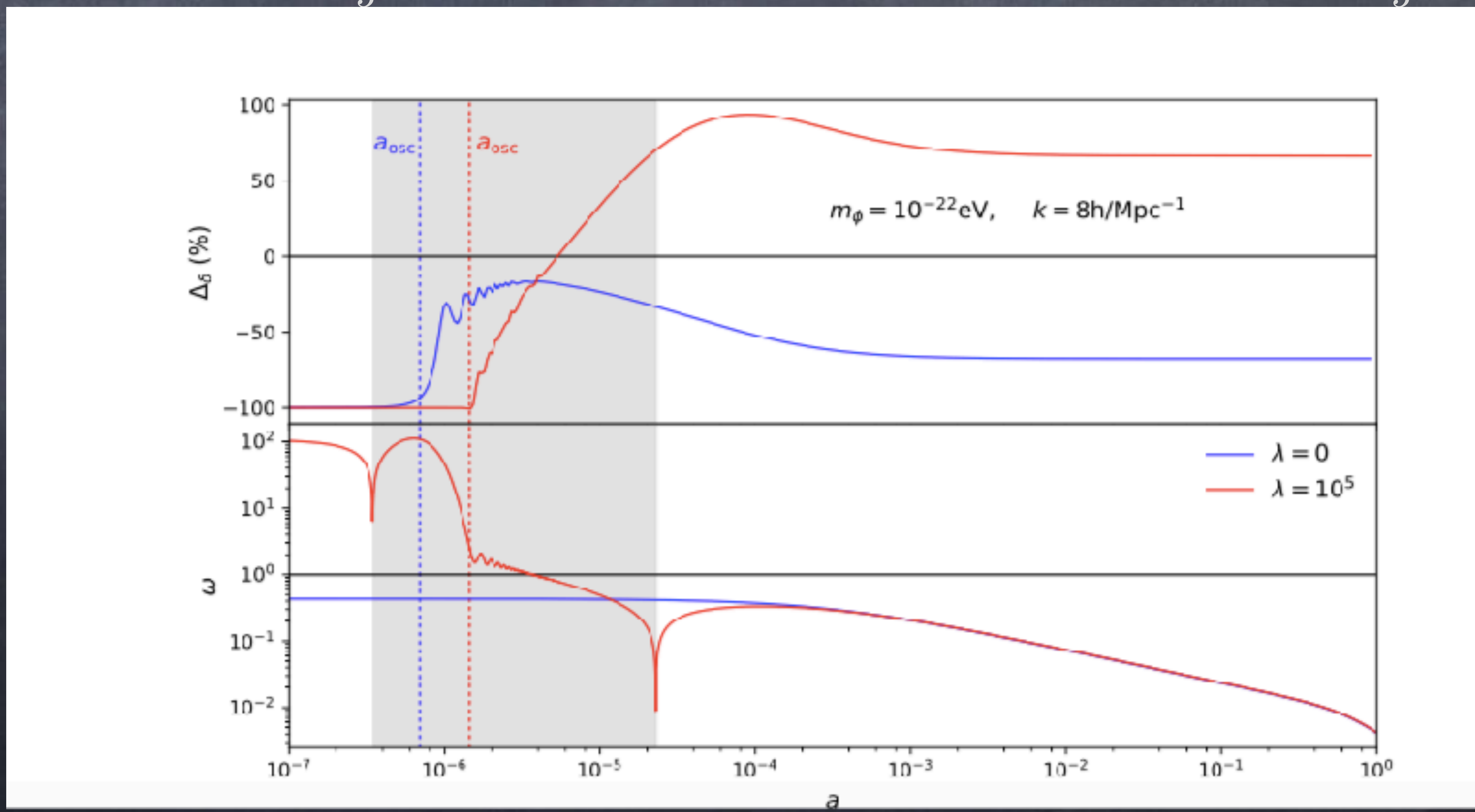
For $\lambda > 0$ an interesting thing happen.

- Tachionich instability. The perturbations can grow lager for some scales (wavenumbers).

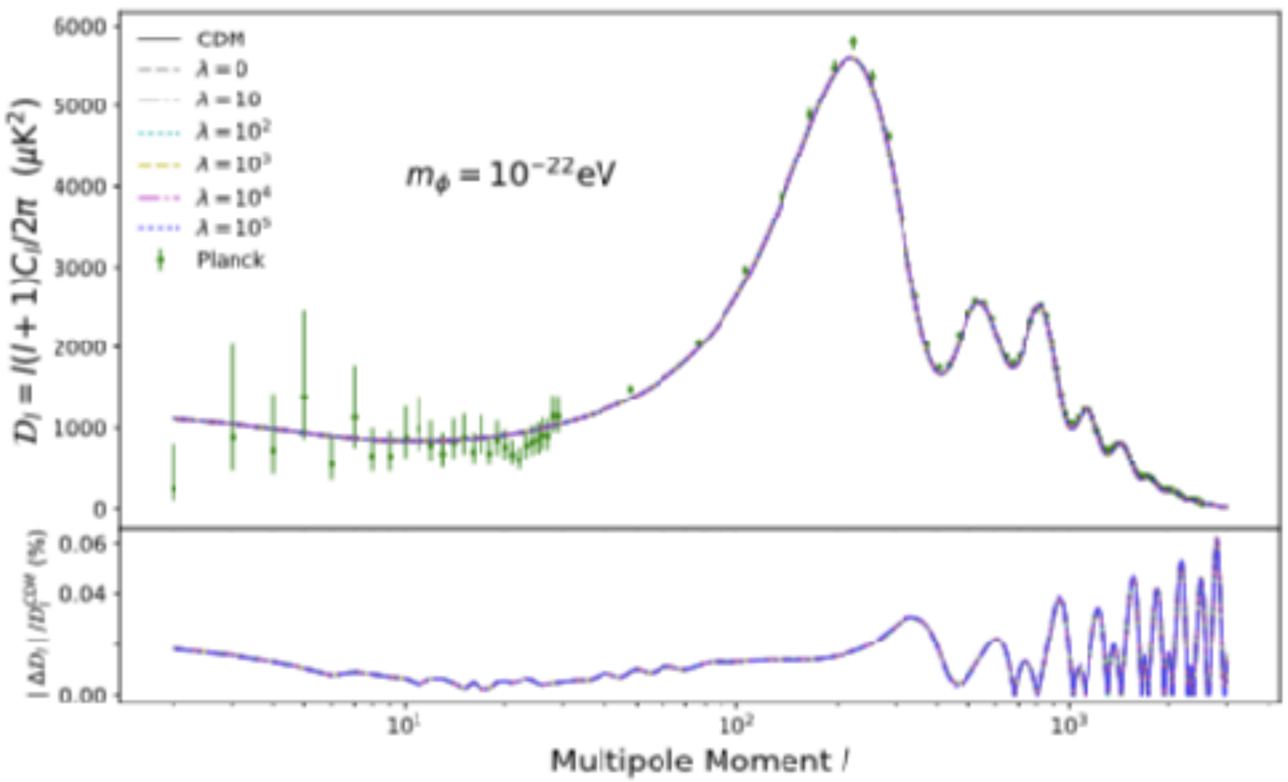
Qualitatively, once oscillations started:

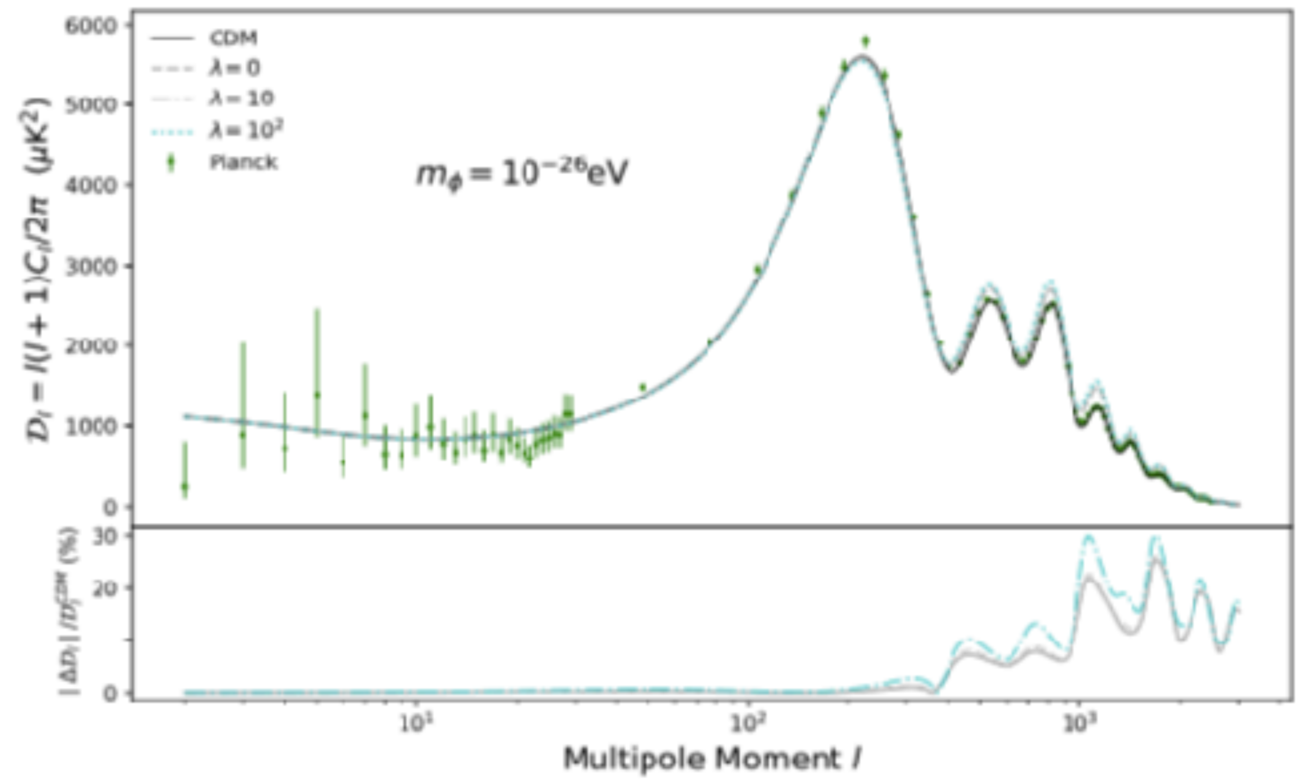
$$\delta_0' = -\frac{k^2}{k_J^2}\delta_1 - \frac{\bar{h}'}{2}, \quad \delta_1' = \frac{k_{eff}^2}{k_J^2}\delta_0 . \qquad \delta_0'' + \omega^2\delta_0 = -\frac{\bar{h}''}{2}, \quad \omega^2 \equiv \frac{k^2 k_{eff}^2}{k_J^4}$$

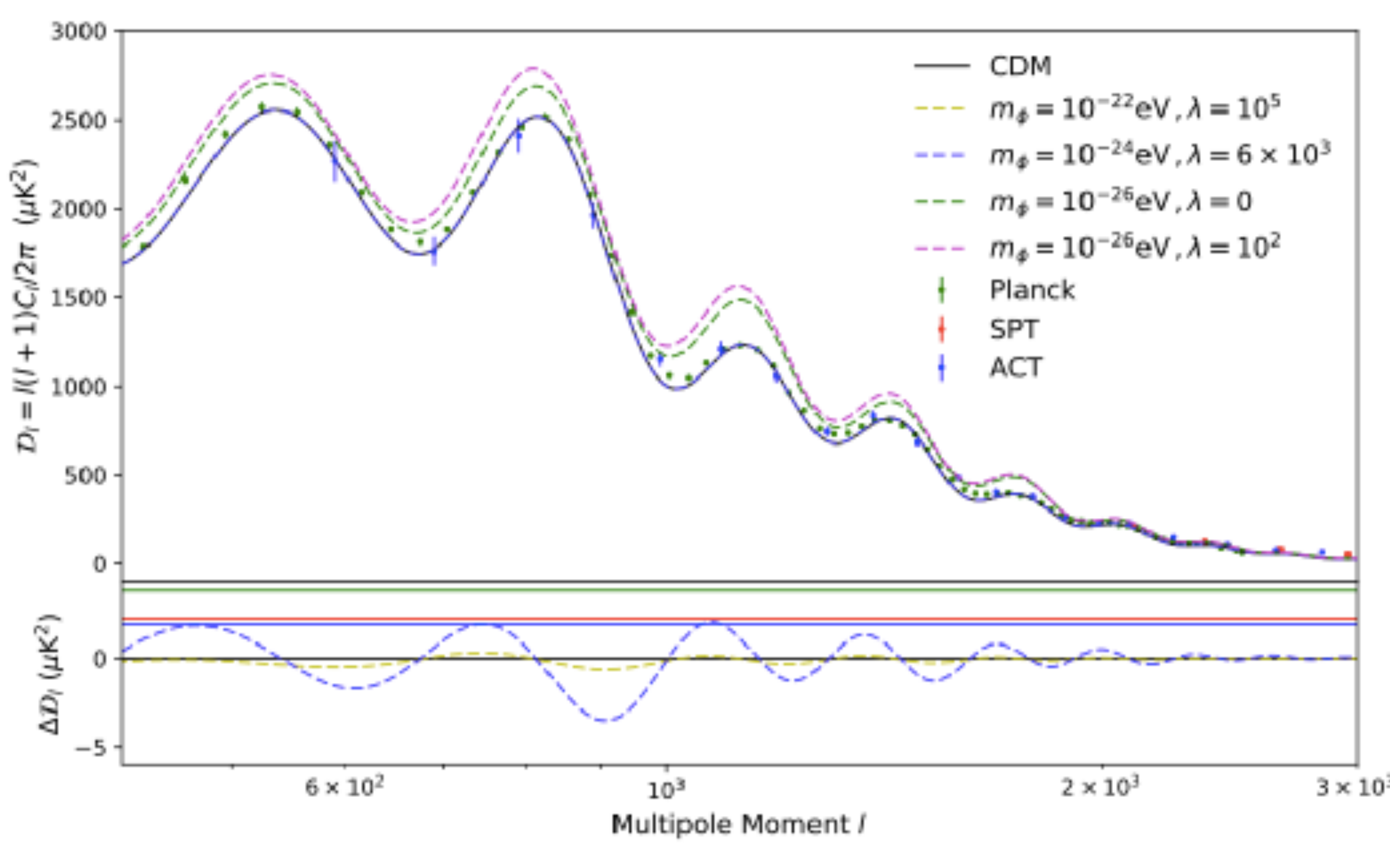
$\Delta_\delta \equiv (\delta_\phi - \delta_{CDM})/\delta_{CDM}$



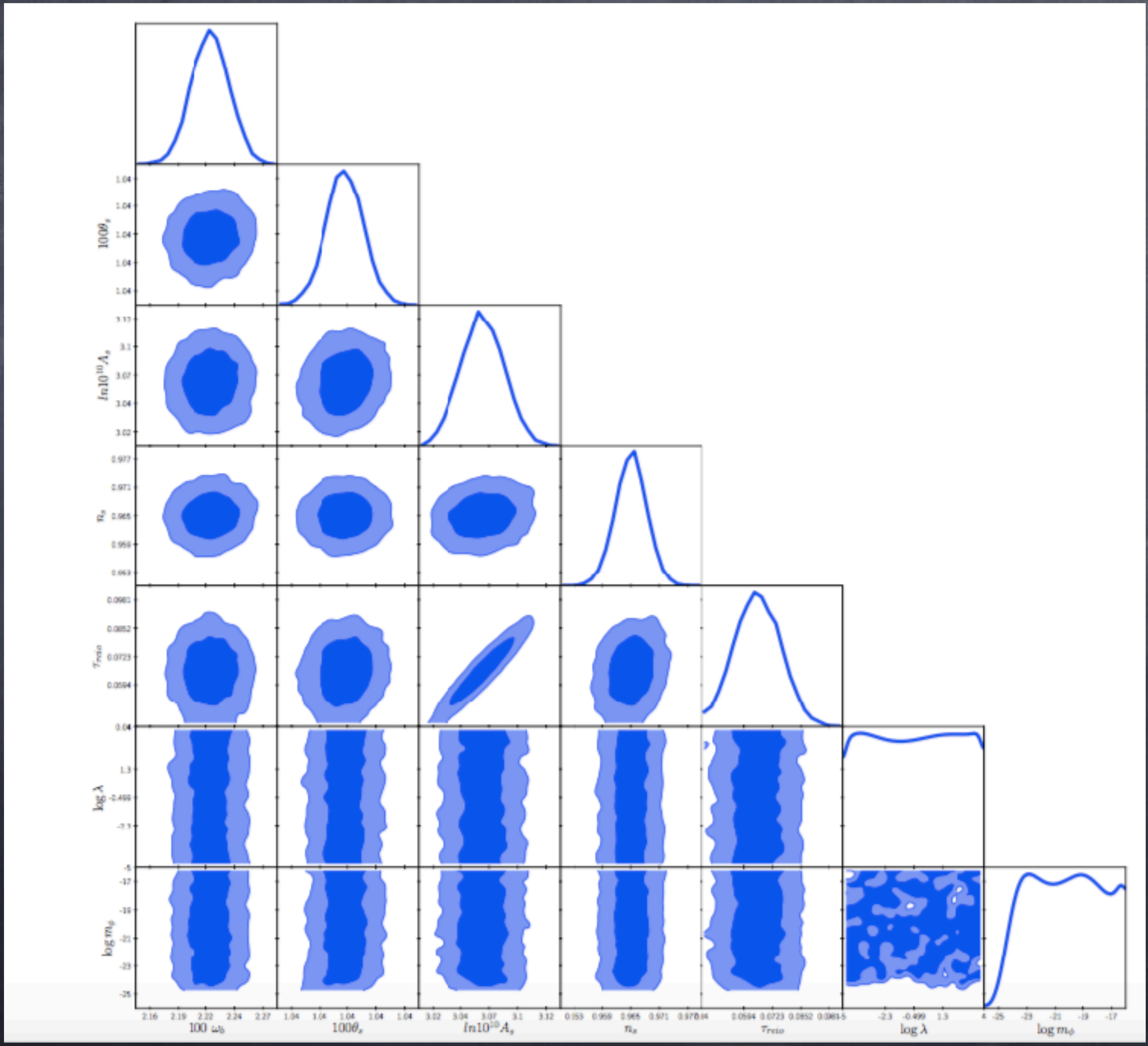
Observables. CMB



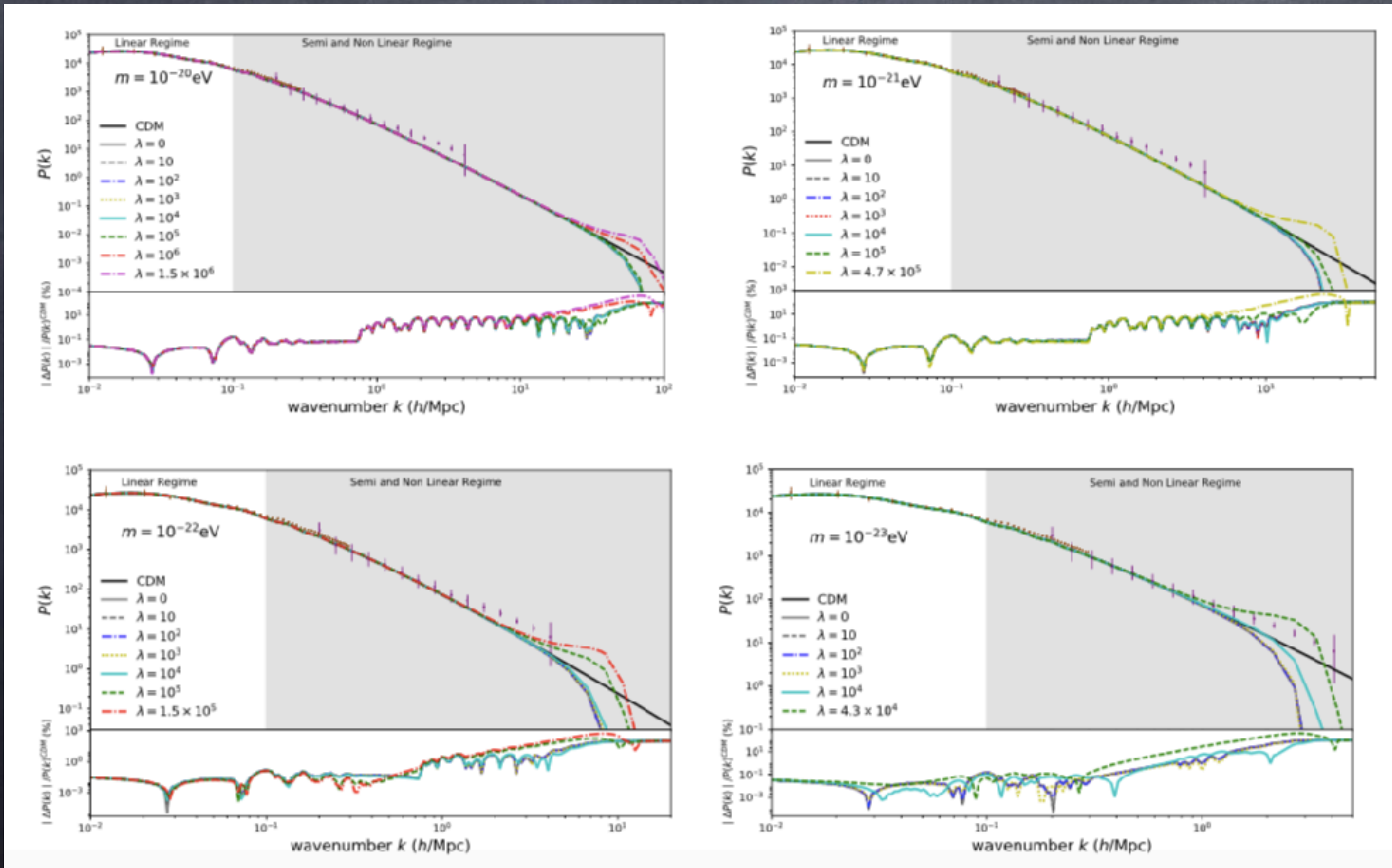




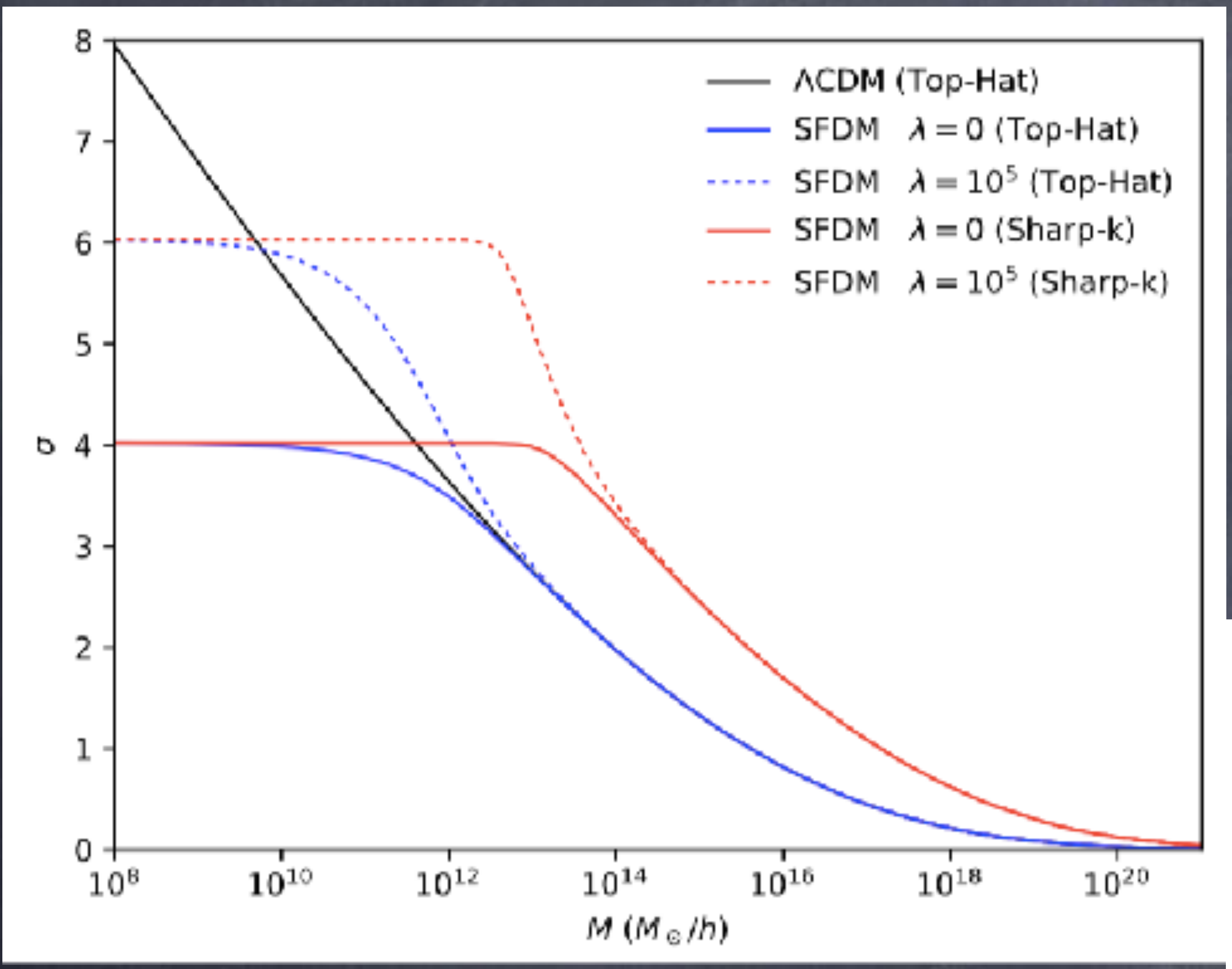
CMB Constraints



Mass Power Spectrum



Approximate Mass Function

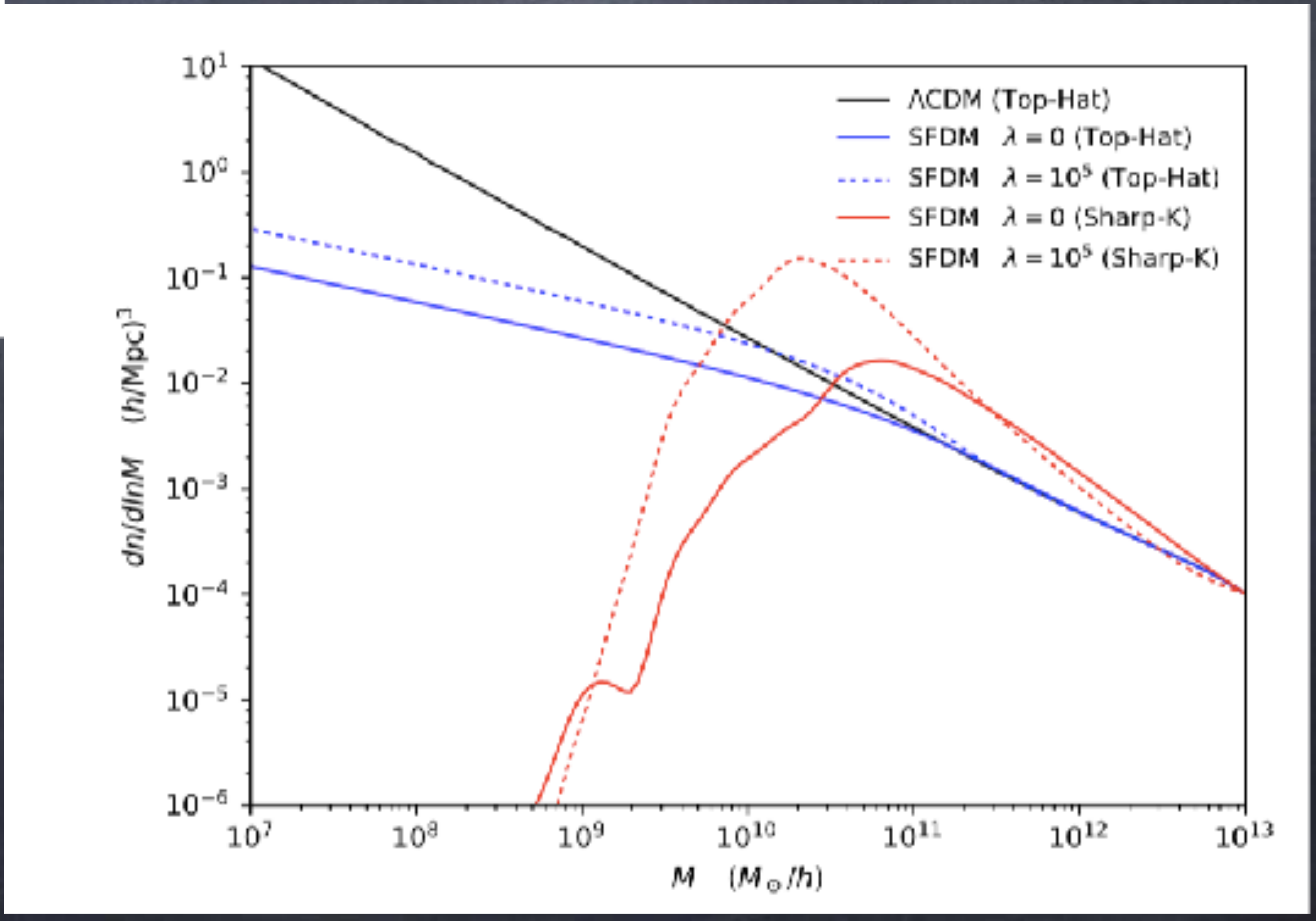


$$\sigma^2(r) = \int \frac{d^3\vec{k}}{(2\pi)^3} P(k) W^2(kr) \,.$$

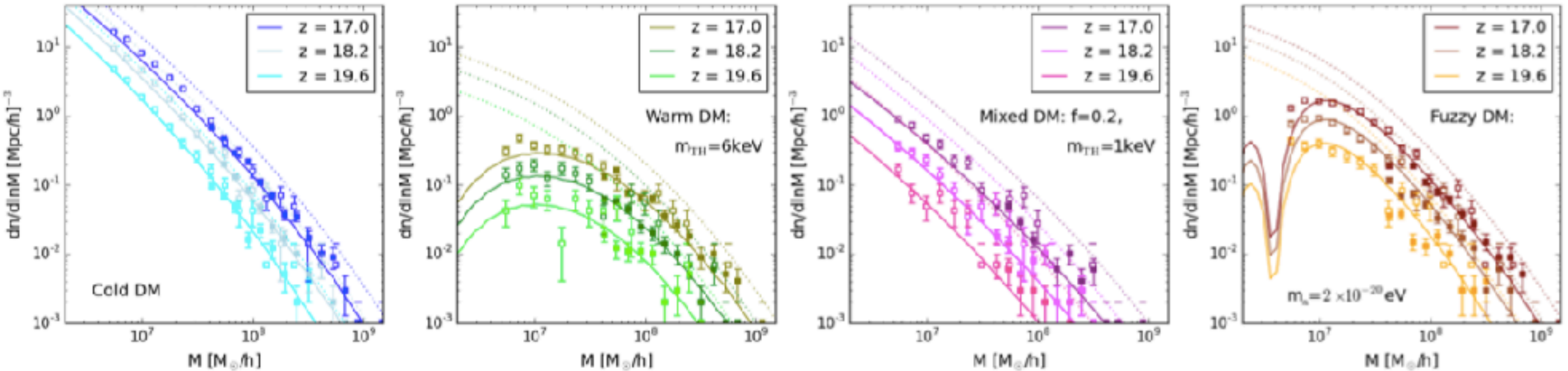
$$\frac{dn}{d\ln M} = -\frac{1}{2}\frac{\bar{\rho}}{M} f(\nu) \frac{d\ln\sigma^2}{d\ln M}$$

$$W_{TH}(kr) = \frac{3}{(kr)^3}\left[\sin(kr) - kr\cos(kr)\right]$$

$$W_{SK}(kr) = \Theta(2\pi - kr)$$

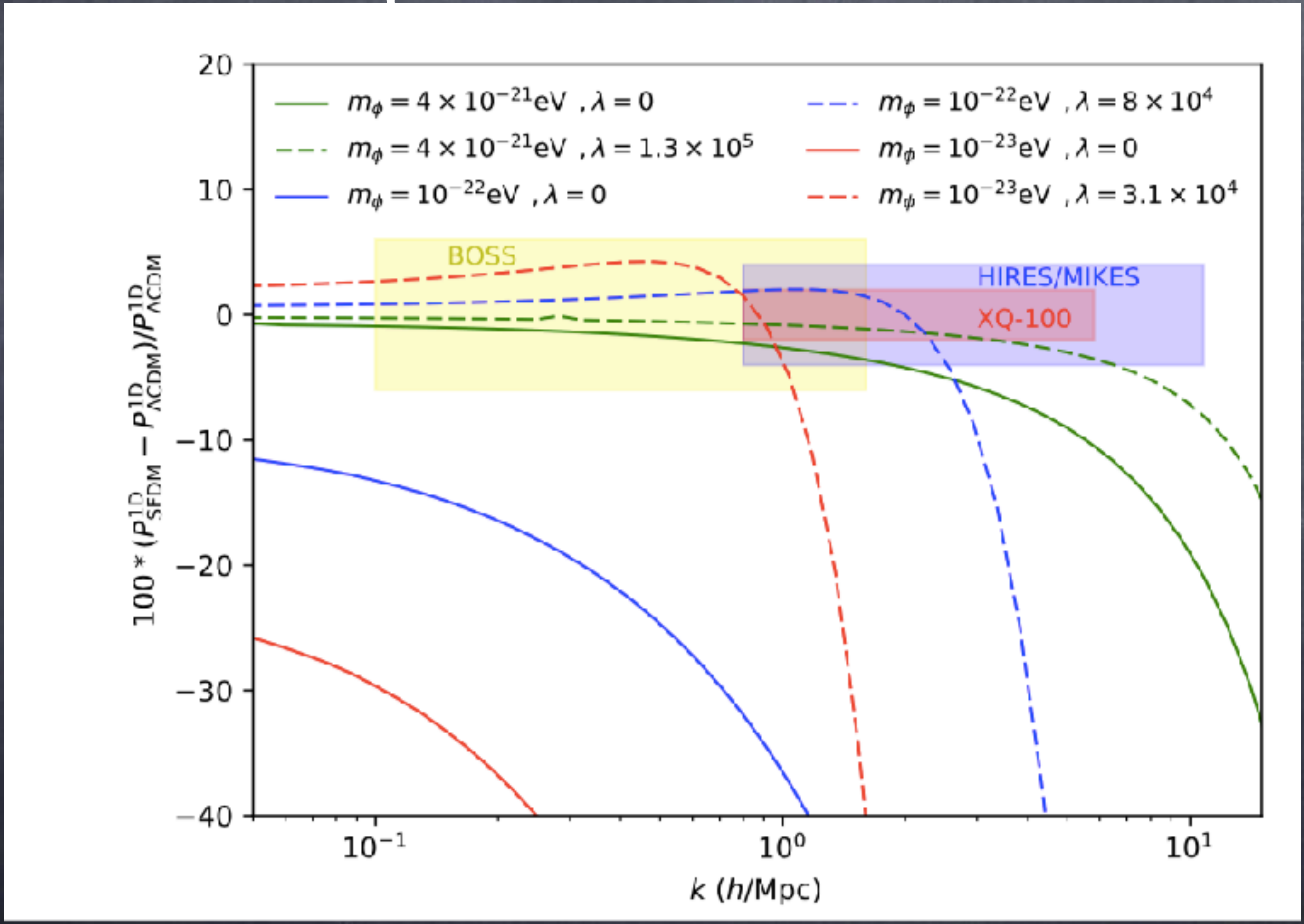


Sharp-K WF seems to work better for Pk's with cut-off



Schneider 2018 & 2015

Some naive prospects for Lyman-alpha constraints

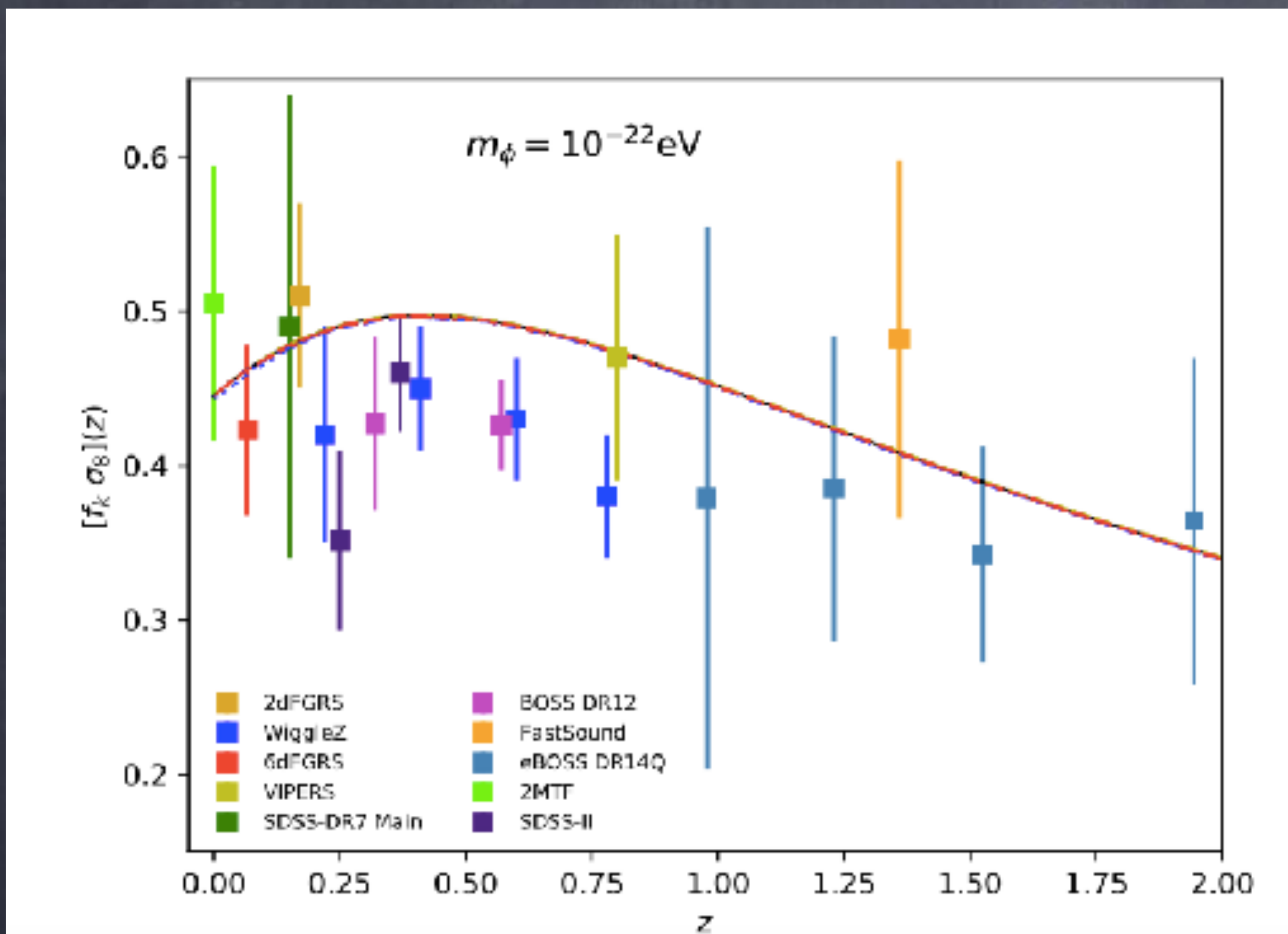


For \lambda=0, strong constraints have been set.

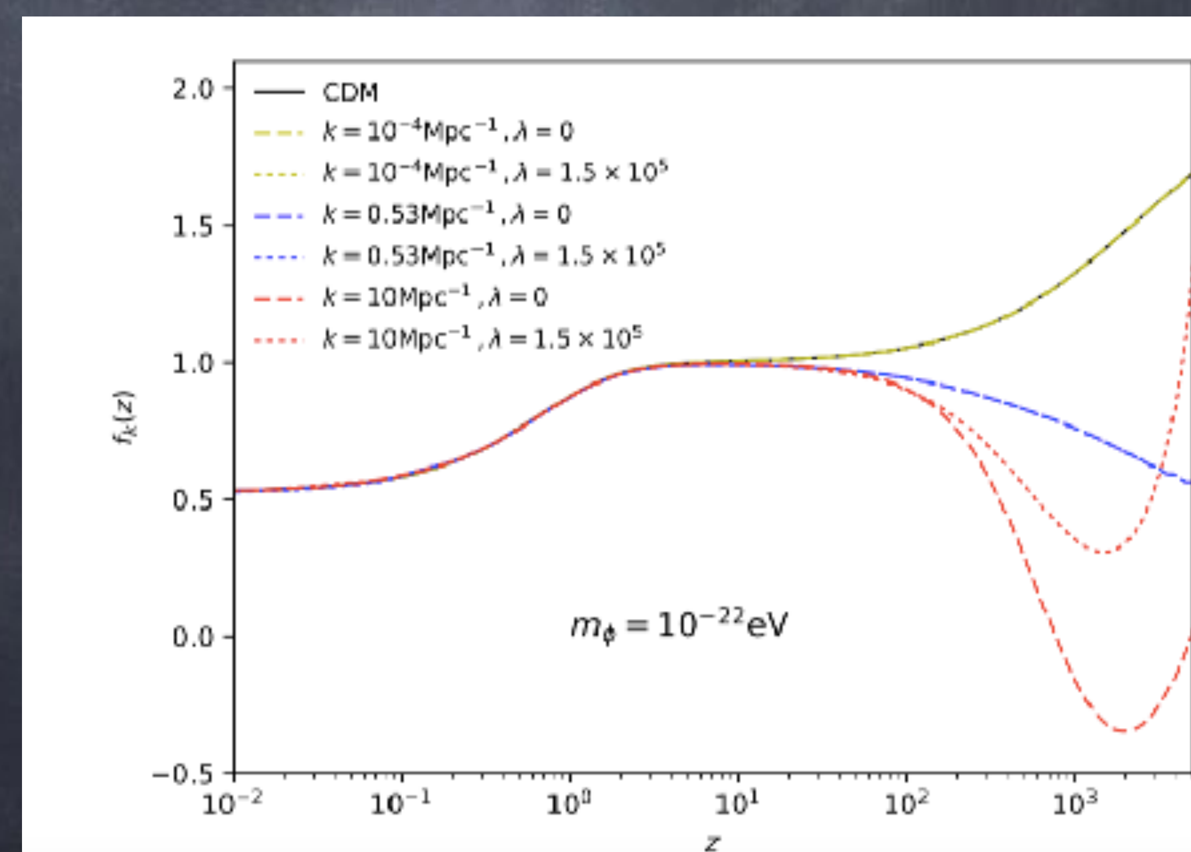
Armengaud et. al 2017 & Iršič et. al 2017 $m > 3\times10^{-21} eV$

$f\sigma_8$ Observable at low redshift is unaffected

*might be obvious...



$$D_k(N) \equiv \frac{\delta_0(N,k)}{\delta_0(N_{\rm late},k)}$$

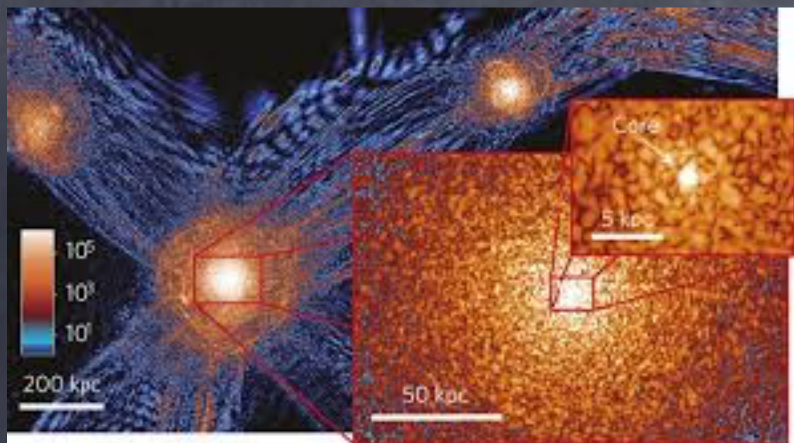


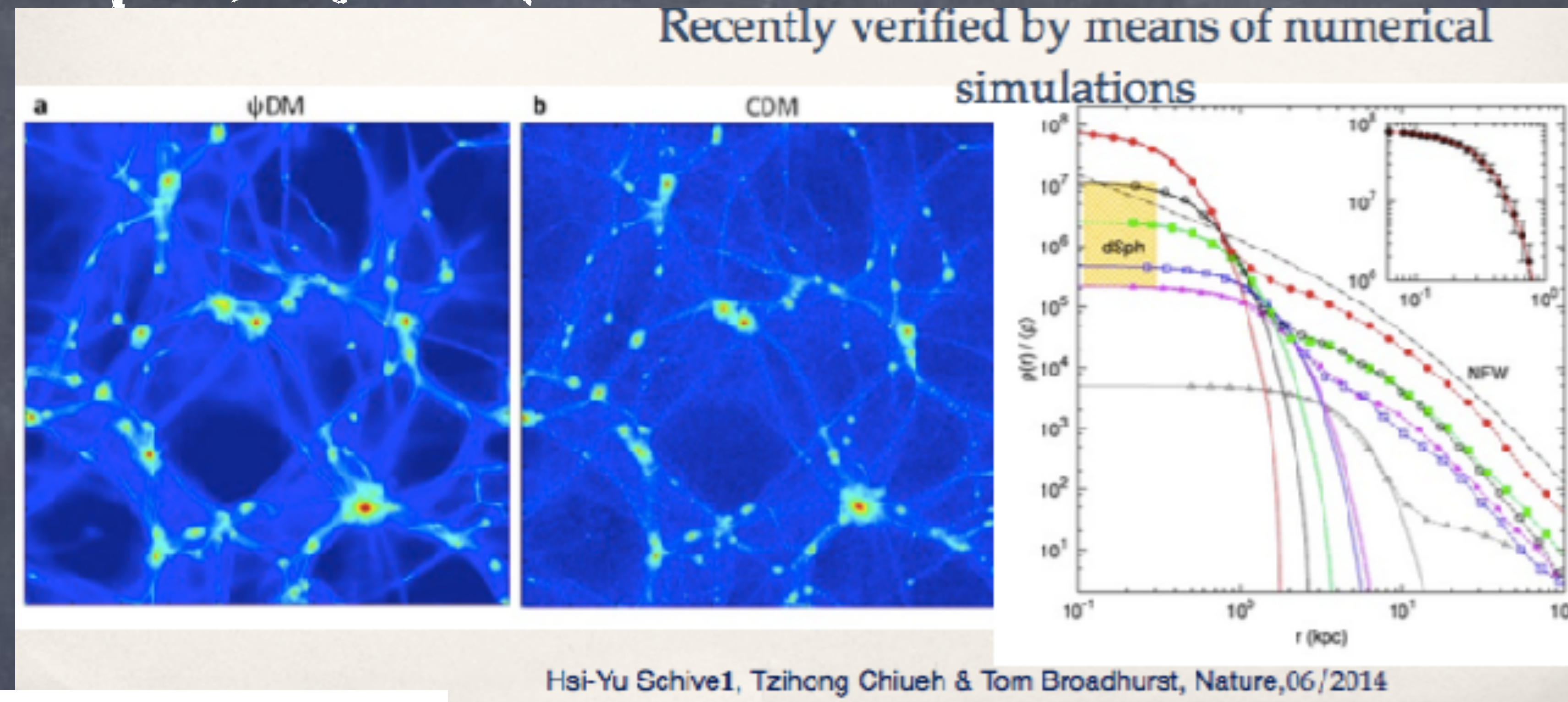
$$f_k(N) = \frac{d\log D_k(N)}{dN} = \frac{\delta_0'(N,k)}{\delta_0(N,k)}$$

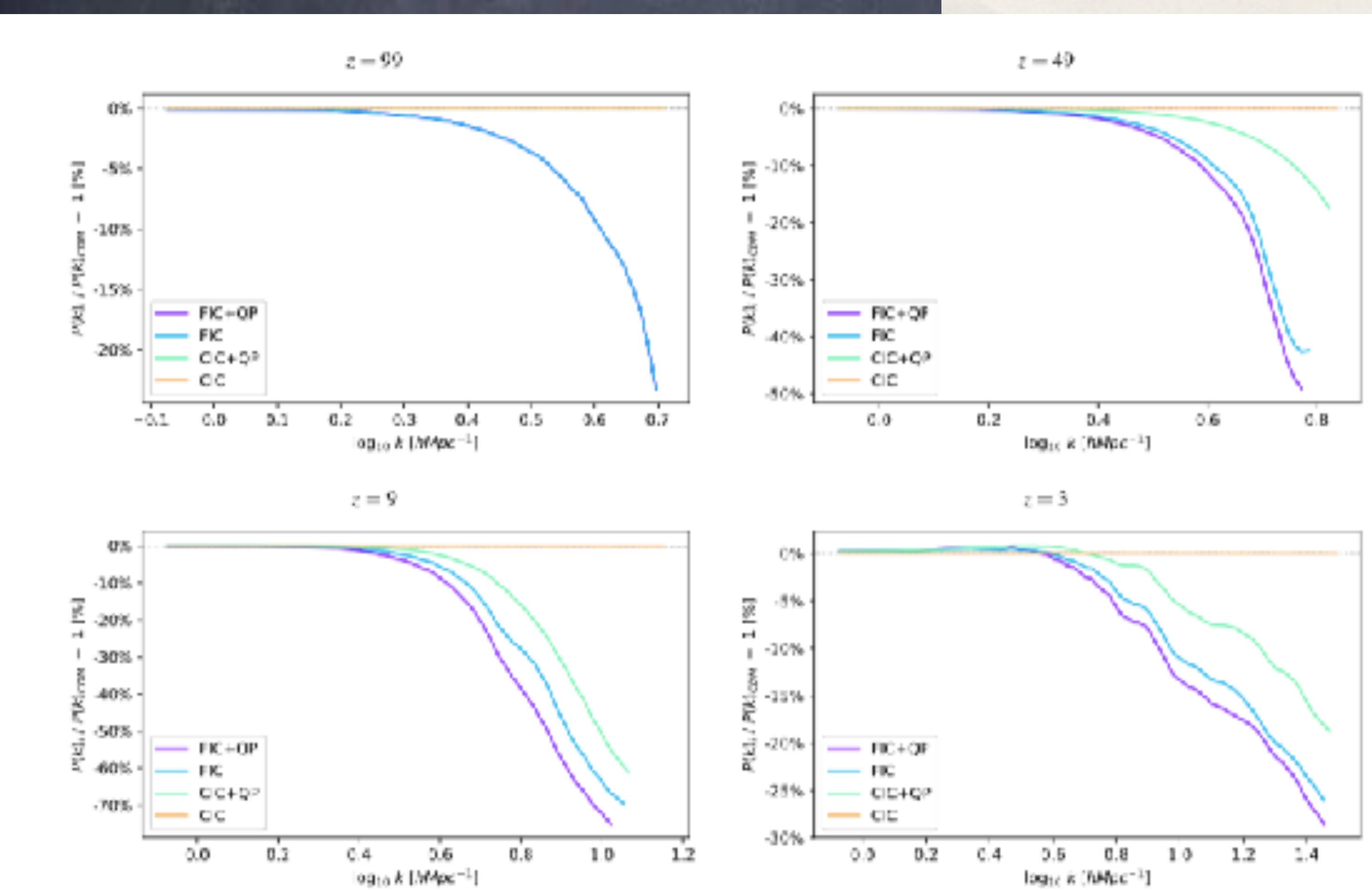
This is how far we can go with linear theory

Next: Going non-linear

Structure Formation Simulations for SFDM







Recently: AX-GADGET(Nori 2018) follows the FDM evolution with SPH techniques (To take into account the "Quantum Presure", and improves on the work of AXION-GADGET (Liam 2017). Other work by P. Mocz, Veltmann and Niemeyer

Other works run standard Nbody simulations with modified IC's, but neglecting QP. Good approximation for some cases

Our approach to generate rapid simulations with COLA (Based on MG-PICOLA)

In general one wants to solve an effective Poisson equation.

$$\nabla^2\tilde{\Psi} = 4\pi G A^2 \rho_b a^4 \delta + \left(\tilde{\lambda} c^4 \rho_b a^2/\tilde{m}^4\right)\nabla^2\delta$$

to linear order

$$-(c^2/2\tilde{m}^2)\nabla^2\left[\left(\nabla^2\sqrt{1+\delta}\right)/\sqrt{1+\delta}\right]$$

QP:"Quantum Pressure"

Not going in detail but we can write down an equation for the evolution of the SF overdensity

$$\ddot{\delta}_k = 4\pi G\rho_b a^4 \mu(a,k,\tilde{\lambda})\delta_k$$

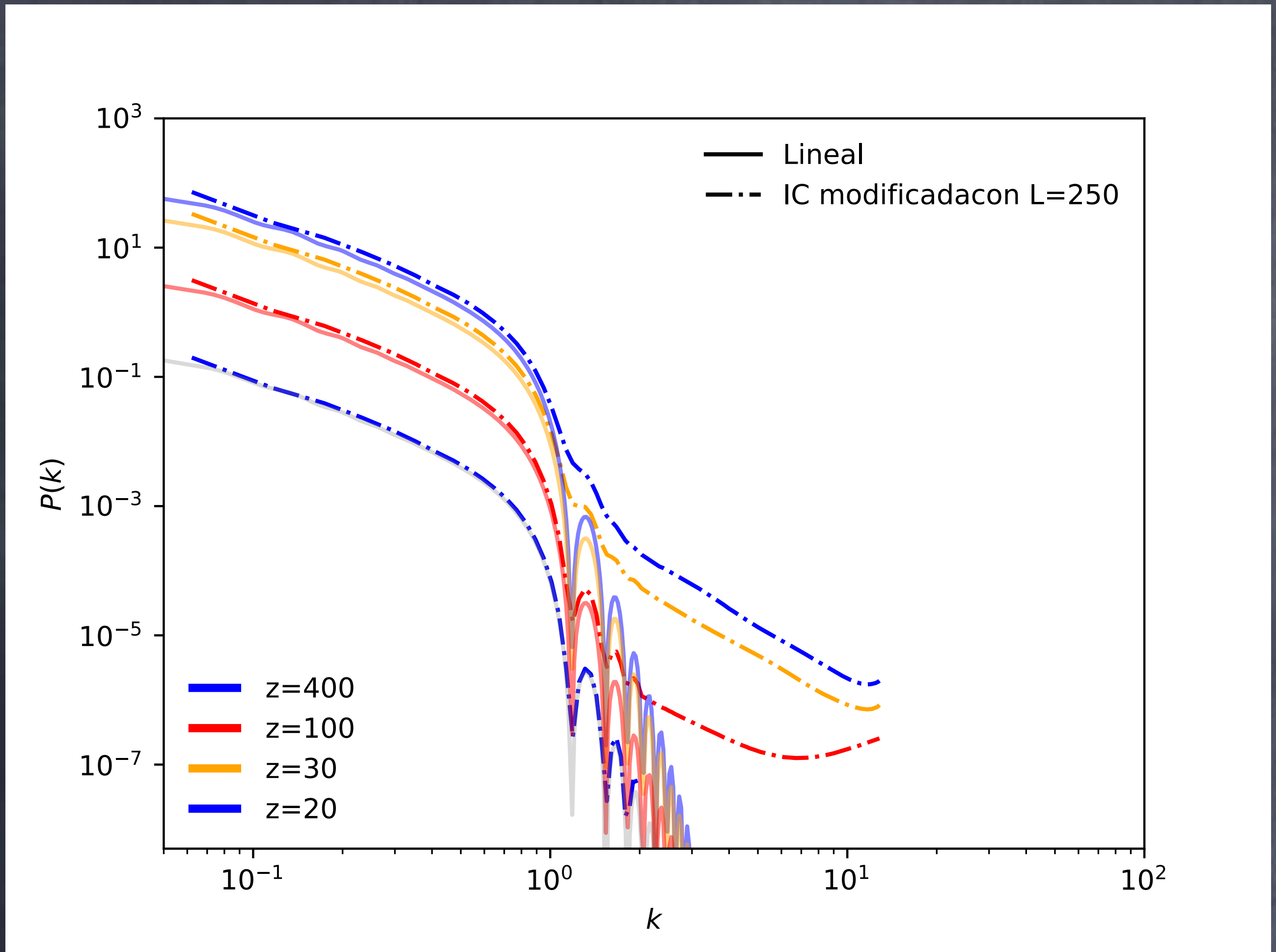
$$\mu(a,k,\tilde{\lambda}) = A^2\left(1 - k^2/k_{J1}^2 - k^4/k_{J0}^4\right)$$

$$k_J^2 = k_{J0}^2\left(\sqrt{1 + k_{J0}^4/2k_{J1}^4} - k_{J0}^2/2k_{J1}^2\right)$$

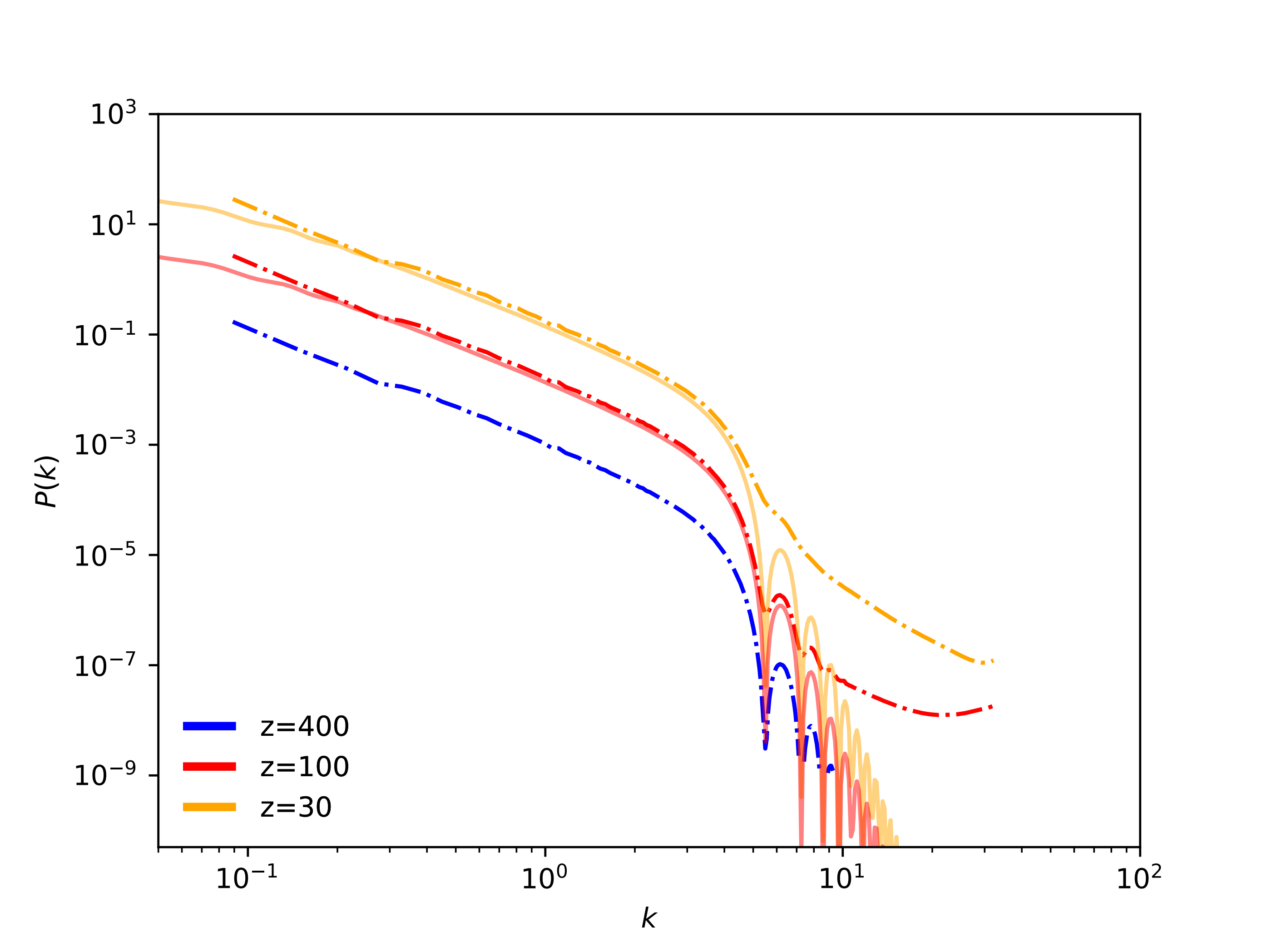
$$\ddot{D}_1 = 4\pi G\rho_b a^4 \mu(a,k,\tilde{\lambda}) D_1$$

An equation solved by MG-PICOLA

Still work in progress



$m = 1 \times 10^{-24} eV$



$m = 3 \times 10^{-23} eV$

Npart=1024, Lbox=1Gpc. ~200 steps.

Simulations are being done in our local machine (Thanks to Oleg for maintenance and most of the

Conclusion

- We are in a good track to get to the non-linear regime.
- Lots of things to do, and observables to compare with in the near future.

FIG FESTIVAL, León, November

Thanks