The three point correlation function: predictions and observables

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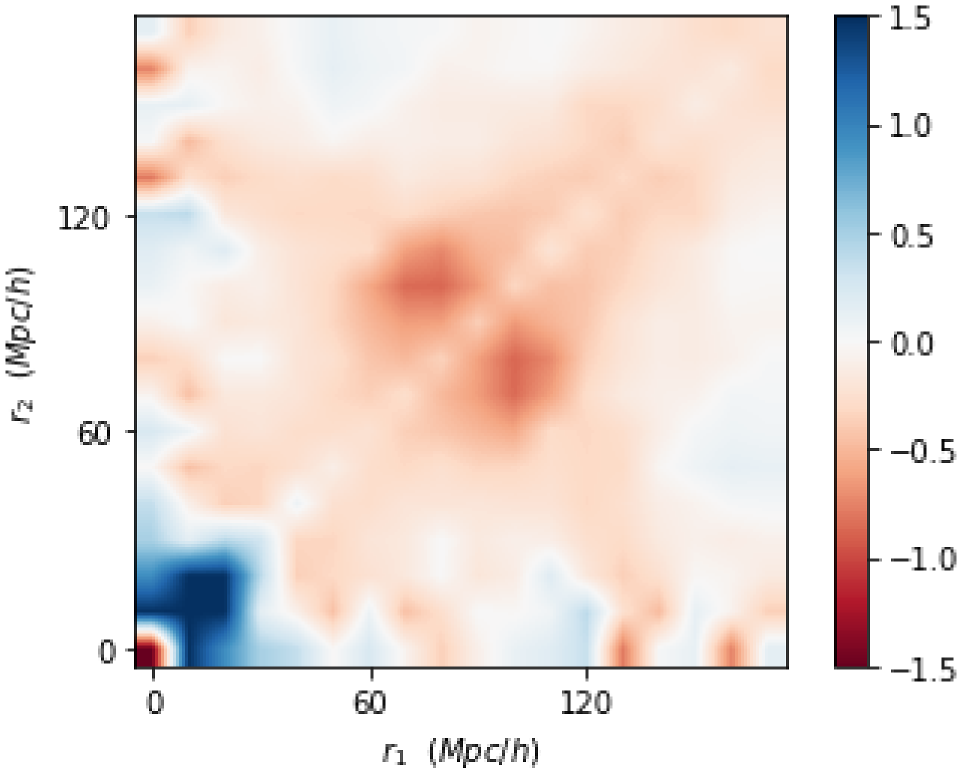
(Guanajuato)

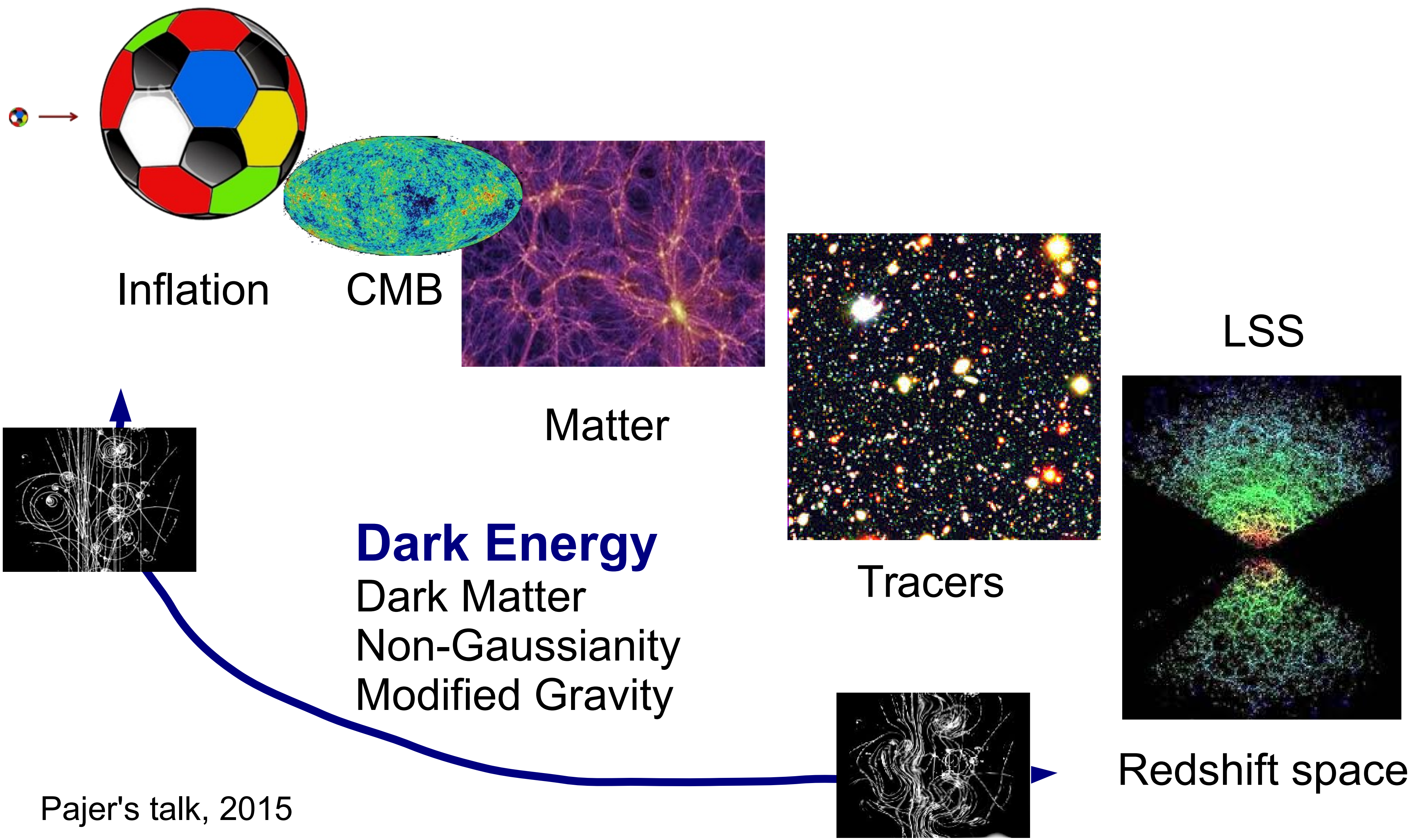


Physics in the “cosmic collider”

The Perturbed Universe

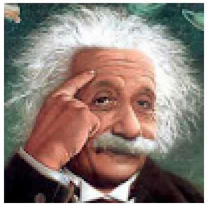
From Quantum Fluctuations to Human Beings...

Cosmic collider



Simple model

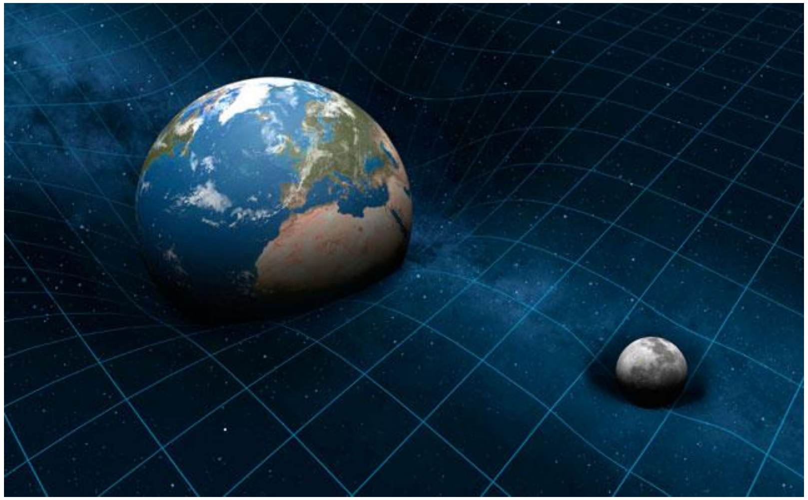
Einstein (1915)



$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Space-time

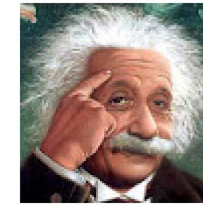
Geometry



Matter/energy

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Simple model



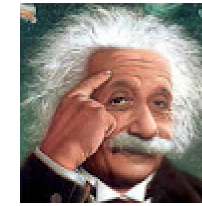
$\mu = \nu = 3$

$$
\begin{aligned}
&\frac{1}{4}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\mathrm{g22}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial y}\,\frac{\partial\,\mathrm{g11}}{\partial y} + \frac{1}{4}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g11}}{\partial t}\,\frac{\partial\,\mathrm{g33}}{\partial t} + \frac{1}{4}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial x}\,\frac{\partial\,\mathrm{g33}}{\partial x} + \\
&\frac{1}{4}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial z}\,\frac{\partial\,\mathrm{g11}}{\partial z} + \frac{1}{2}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\frac{\partial^2\,\mathrm{g11}}{\partial t^2} - \frac{1}{4}\,(\mathrm{g00}^{-1})^2\,\mathrm{g11}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial t}\,\frac{\partial\,\mathrm{g11}}{\partial t} - \\
&\frac{1}{4}\,\mathrm{g00}^{-1}\,(\mathrm{g11}^{-1})^2\left(\frac{\partial\,\mathrm{g11}}{\partial t}\right)^2 + \frac{1}{2}\,\mathrm{g00}^{-1}\,\mathrm{g11}^{-1}\,\frac{\partial^2\,\mathrm{g00}}{\partial x^2} - \frac{1}{4}\,(\mathrm{g00}^{-1})^2\,\mathrm{g11}^{-1}\left(\frac{\partial\,\mathrm{g00}}{\partial x}\right)^2 - \\
&\frac{1}{4}\,\mathrm{g00}^{-1}\,(\mathrm{g11}^{-1})^2\,\frac{\partial\,\mathrm{g00}}{\partial x}\,\frac{\partial\,\mathrm{g11}}{\partial x} + \frac{1}{4}\,\mathrm{g00}^{-1}\,\mathrm{g22}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial y}\,\frac{\partial\,\mathrm{g33}}{\partial y} + \frac{1}{2}\,\mathrm{g00}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial^2\,\mathrm{g33}}{\partial t^2} - \\
&\frac{1}{4}\,(\mathrm{g00}^{-1})^2\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g00}}{\partial t}\,\frac{\partial\,\mathrm{g33}}{\partial t} - \frac{1}{4}\,\mathrm{g00}^{-1}\,(\mathrm{g33}^{-1})^2\left(\frac{\partial\,\mathrm{g33}}{\partial t}\right)^2 + \frac{1}{2}\,\mathrm{g00}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial^2\,\mathrm{g00}}{\partial z^2} - \\
&\frac{1}{4}\,(\mathrm{g00}^{-1})^2\,\mathrm{g33}^{-1}\left(\frac{\partial\,\mathrm{g00}}{\partial z}\right)^2 - \frac{1}{4}\,\mathrm{g00}^{-1}\,(\mathrm{g33}^{-1})^2\,\frac{\partial\,\mathrm{g00}}{\partial z}\,\frac{\partial\,\mathrm{g33}}{\partial z} + \frac{1}{4}\,\mathrm{g11}^{-1}\,\mathrm{g22}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g11}}{\partial y}\,\frac{\partial\,\mathrm{g33}}{\partial y} + \\
&\frac{1}{2}\,\mathrm{g11}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial^2\,\mathrm{g33}}{\partial x^2} - \frac{1}{4}\,\mathrm{g11}^{-1}\,(\mathrm{g33}^{-1})^2\left(\frac{\partial\,\mathrm{g33}}{\partial x}\right)^2 - \frac{1}{4}\,(\mathrm{g11}^{-1})^2\,\mathrm{g33}^{-1}\,\frac{\partial\,\mathrm{g11}}{\partial x}\,\frac{\partial\,\mathrm{g33}}{\partial x} + \\
&\frac{1}{2}\,\mathrm{g11}^{-1}\,\mathrm{g33}^{-1}\,\frac{\partial^2\,\mathrm{g11}}{\partial z^2} - \frac{1}{4}\,\mathrm{g11}^{-1}\,(\mathrm{g33}^{-1})^2\,\frac{\partial\,\mathrm{g11}}{\partial z}\,\frac{\partial\,\mathrm{g33}}{\partial z} - \frac{1}{4}\,(\mathrm{g11}^{-1})^2\,\mathrm{g33}^{-1}\left(\frac{\partial\,\mathrm{g11}}{\partial z}\right)^2
\end{aligned}
$$

Non-linear theory

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Simple model



$\mu = \nu = 3$

$$\frac{1}{4}\, g00^{-1}\, g11^{-1}\, g22^{-1} \frac{\partial g00}{\partial y} \frac{\partial g11}{\partial y} + \frac{1}{4}\, g00^{-1}\, g11^{-1}\, g33^{-1} \frac{\partial g11}{\partial t} \frac{\partial g33}{\partial t} + \frac{1}{4}\, g00^{-1}\, g11^{-1}\, g33^{-1} \frac{\partial g00}{\partial x} \frac{\partial g33}{\partial x} +$$

$$\frac{1}{4}\, g00^{-1}\, g11^{-1}\, g33^{-1} \frac{\partial g00}{\partial z} \frac{\partial g11}{\cdots} \cdots \frac{1}{4} (g00^{-1})^2\, g11^{-1} \frac{\partial g00}{\partial t} \frac{\partial g11}{\partial t} -$$

$$\frac{1}{4}\, g00^{-1} (g11^{-1})^2 \left(\frac{\partial g11}{\partial t}\right)^2 + \cdots (g00^{-1})^2\, g11^{-1} \left(\frac{\partial g00}{\partial x}\right)^2 -$$

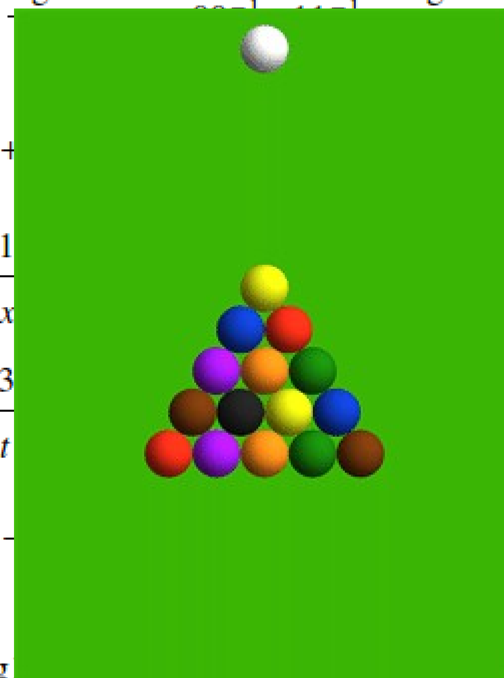
$$\frac{1}{4}\, g00^{-1} (g11^{-1})^2 \frac{\partial g00}{\partial x} \cdots \frac{\partial g33}{\partial y} + \frac{1}{2}\, g00^{-1}\, g33^{-1} \frac{\partial^2 g33}{\partial t^2} -$$

$$\frac{1}{4} (g00^{-1})^2\, g33^{-1} \frac{\partial g00}{\partial t} \cdots + \frac{1}{2}\, g00^{-1}\, g33^{-1} \frac{\partial^2 g00}{\partial z^2} -$$

$$\frac{1}{4} (g00^{-1})^2\, g33^{-1} \left(\frac{\partial g00}{\partial z}\right)^2 \cdots + \frac{1}{4}\, g11^{-1}\, g22^{-1}\, g33^{-1} \frac{\partial g11}{\partial y} \frac{\partial g33}{\partial y} +$$

$$\frac{1}{2}\, g11^{-1}\, g33^{-1} \frac{\partial^2 g33}{\partial x^2} - \frac{1}{4} \cdots (g11^{-1})^2\, g33^{-1} \frac{\partial g11}{\partial x} \frac{\partial g33}{\partial x} +$$

$$\frac{1}{2}\, g11^{-1}\, g33^{-1} \frac{\partial^2 g11}{\partial z^2} - \frac{1}{4}\, g11^{-1} (g33^{-1})^2 \frac{\partial g11}{\partial z} \frac{\partial g33}{\partial z} - \frac{1}{4} (g11^{-1})^2\, g33^{-1} \left(\frac{\partial g11}{\partial z}\right)^2$$

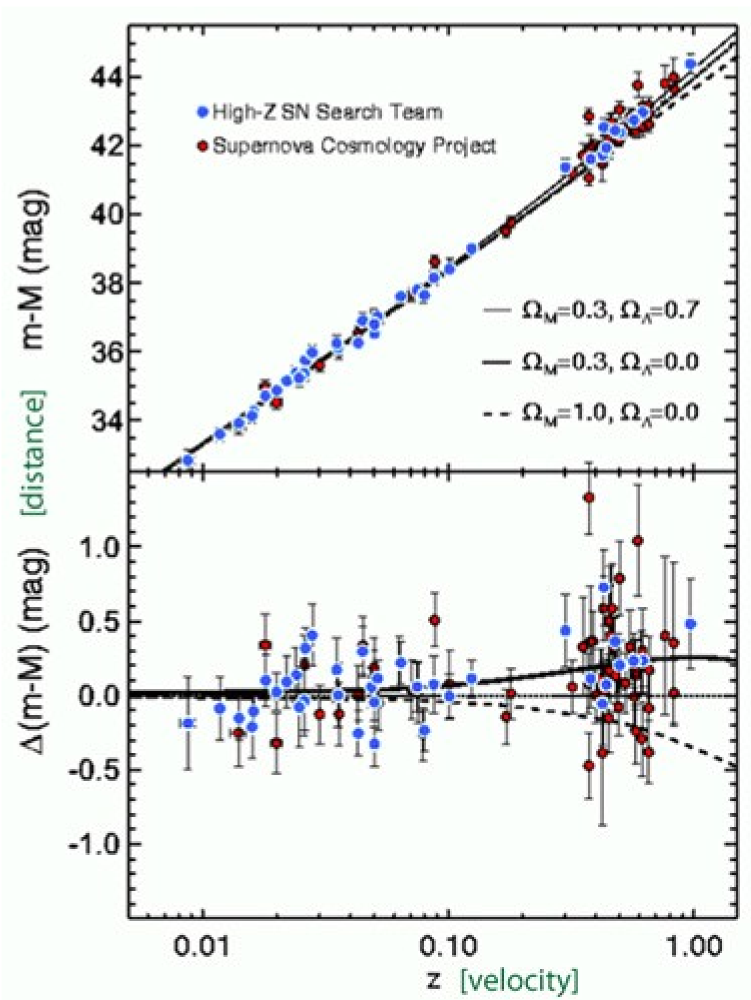


Describes interactions!

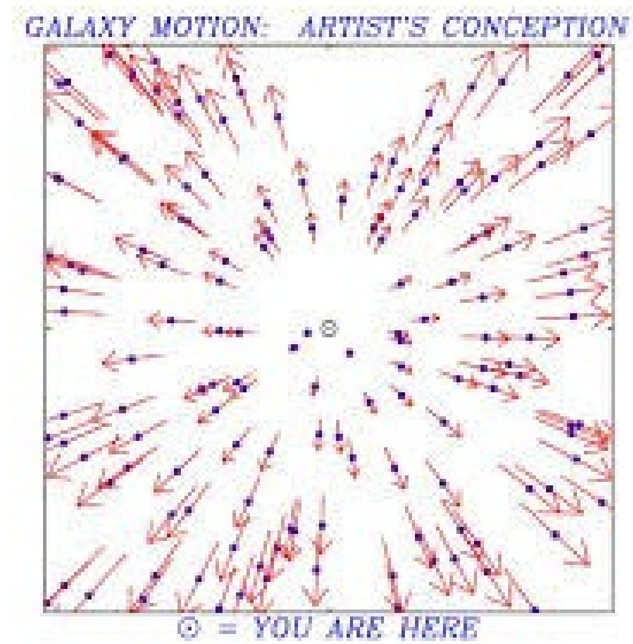
Simple model

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Cosmological principle:
Isotropy and homogeneity

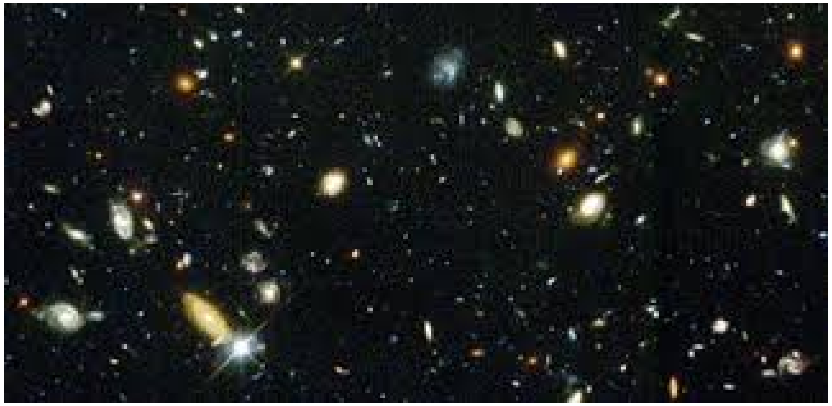


FRWL



Describes well the
AVERAGED Universe

Beyond average



Universe is not fully homogeneous and isotropic

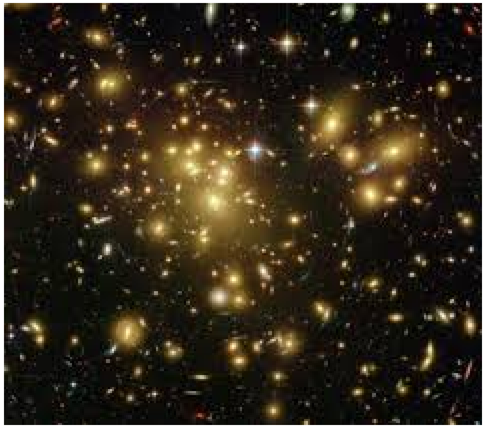
Trick: Use perturbation theory

$$\rho = \bar{\rho}(1+\delta) \qquad \delta \ll 1$$

Beyond average

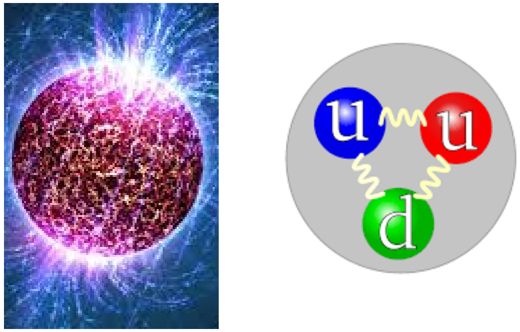
$$\rho = \bar{\rho}(1+\delta) \qquad\qquad \bar{\rho} \sim 10^{-26}\mathrm{Kg/m^3}$$

Note that



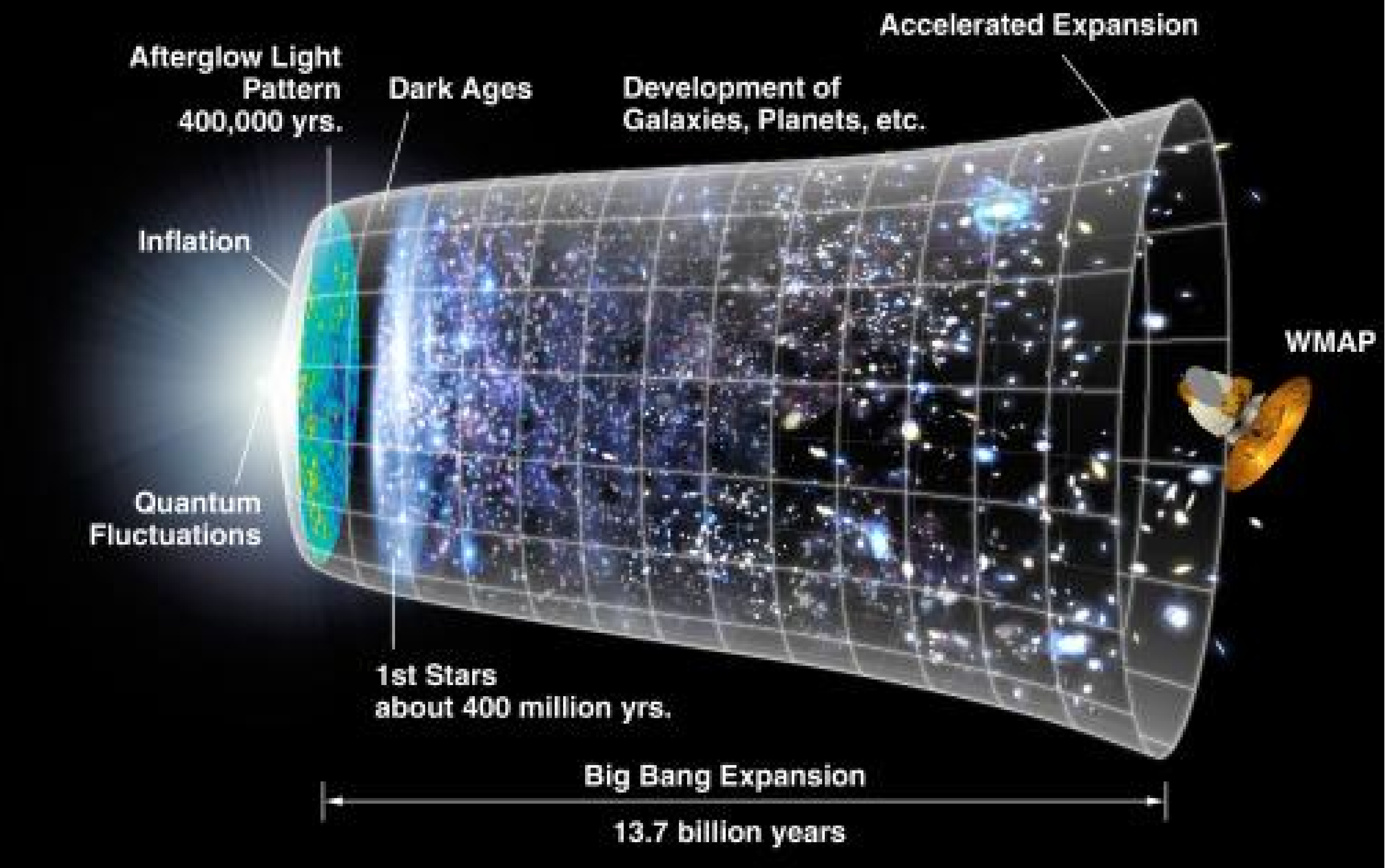
$$\delta \sim 100$$

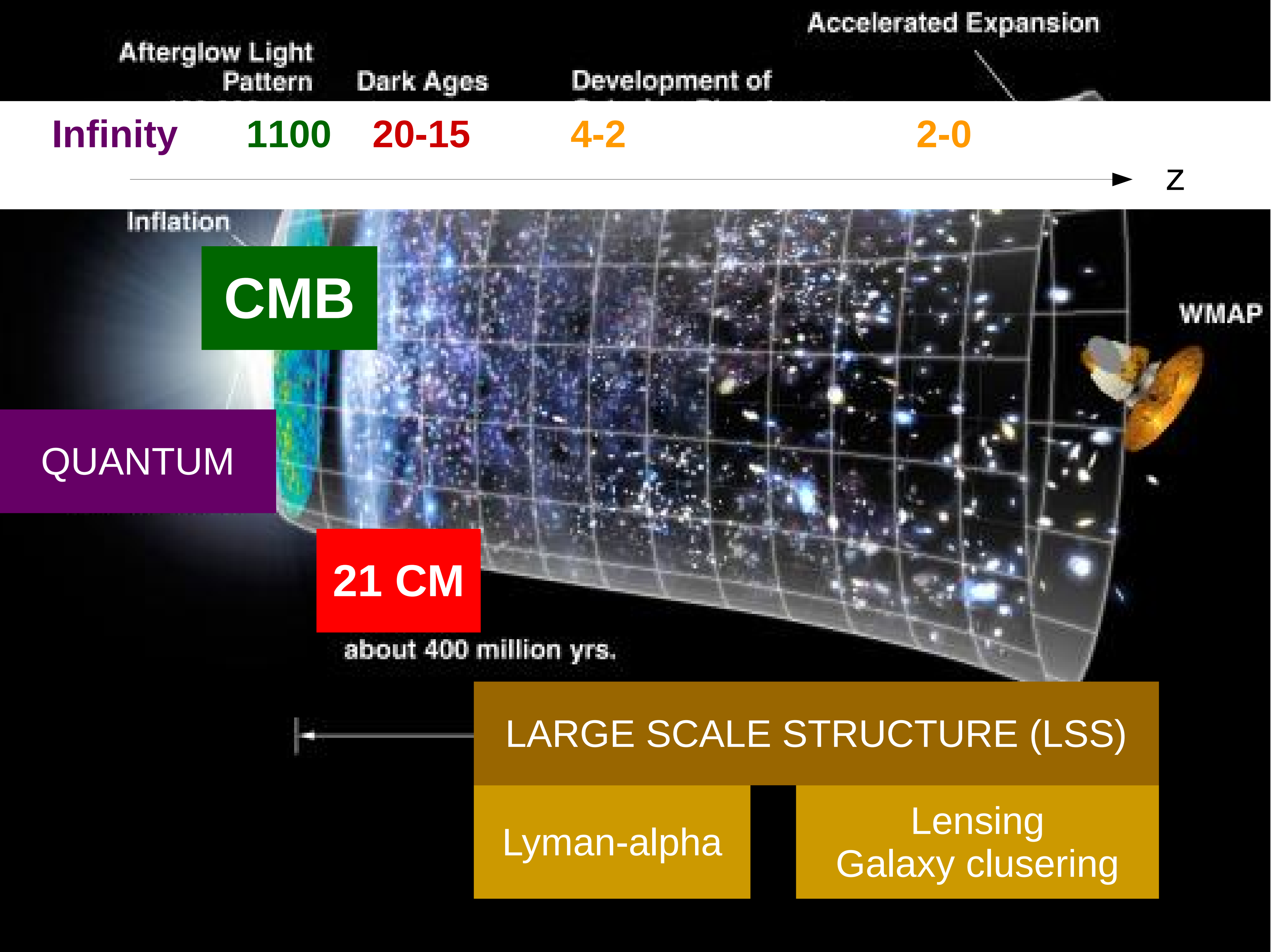
$$\delta \sim 10^{29}$$



$$\delta \sim 10^{43}$$

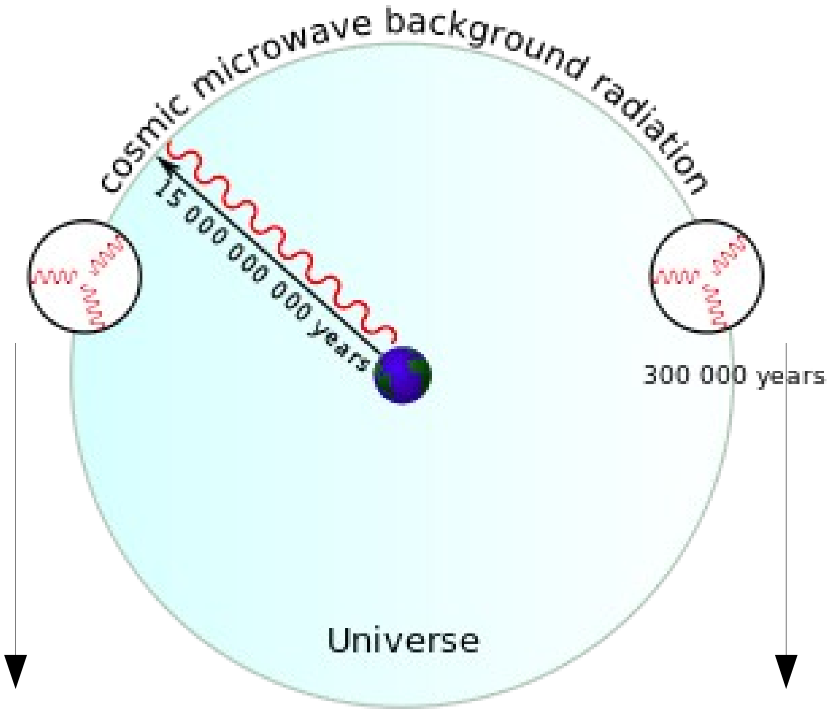
Could it really describe our Universe?


Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

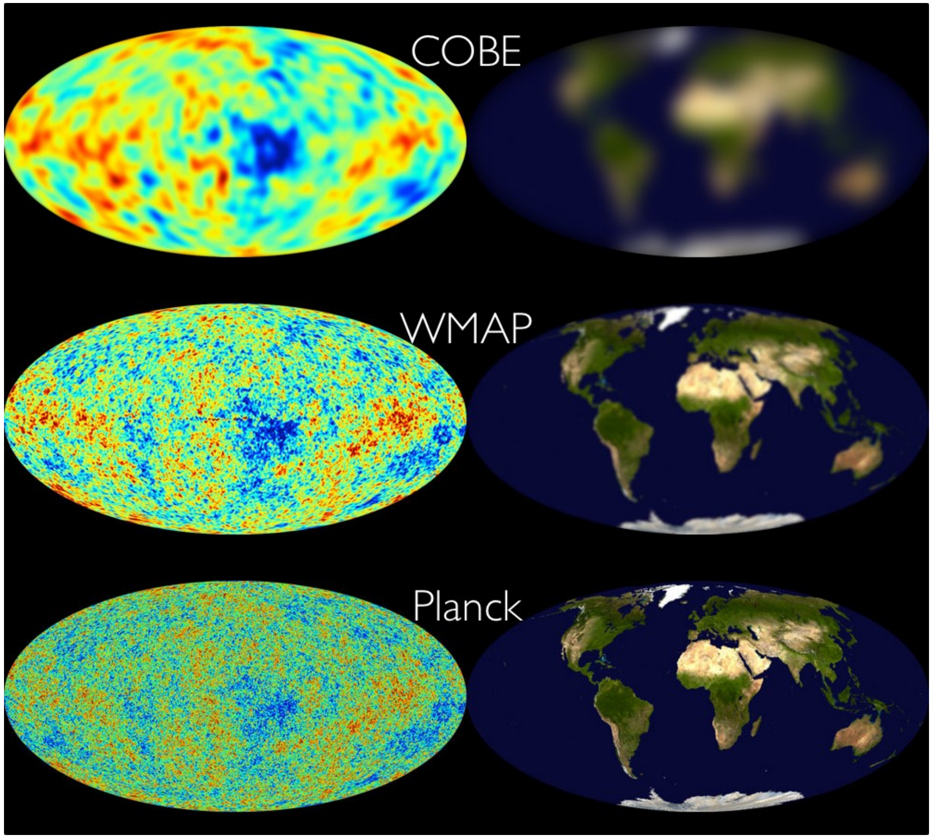


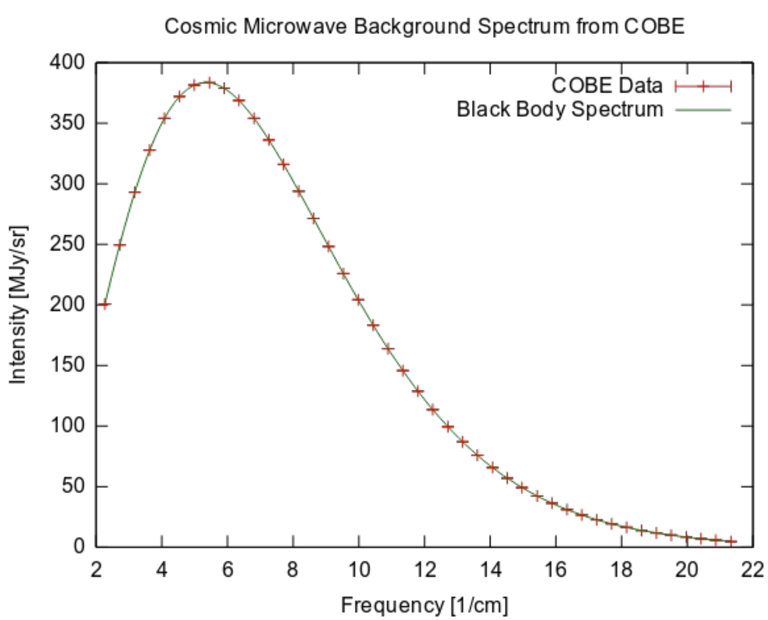
Accelerated Expansion
Afterglow Light Pattern
Dark Ages
Development of
Infinity
1100
20-15
4-2
2-0
z
Inflation
CMB
WMAP
QUANTUM
21 CM
about 400 million yrs.
LARGE SCALE STRUCTURE (LSS)
Lyman-alpha
Lensing
Galaxy clusering

CMB



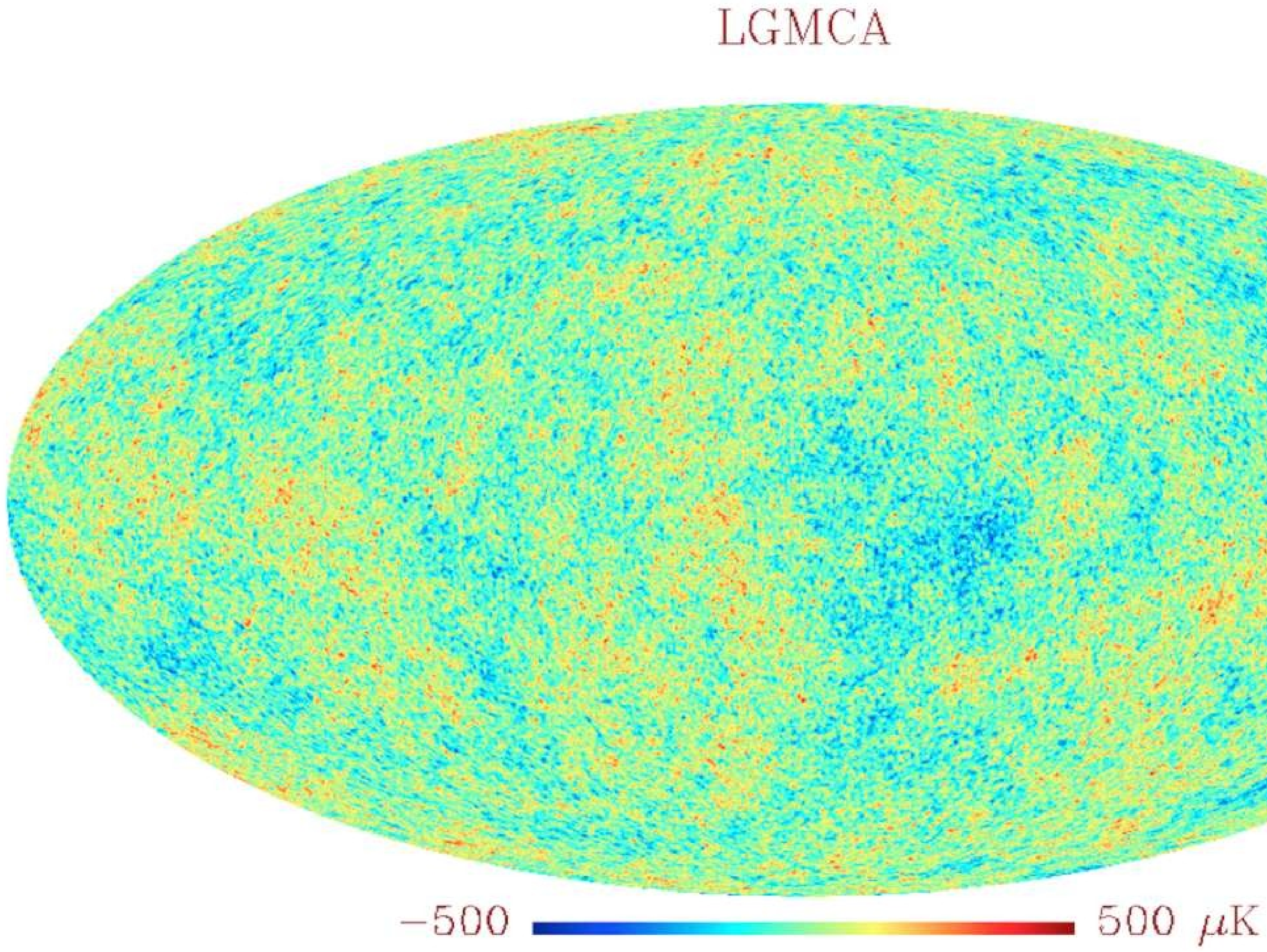
T=2.72517 K   T=2.72513 K

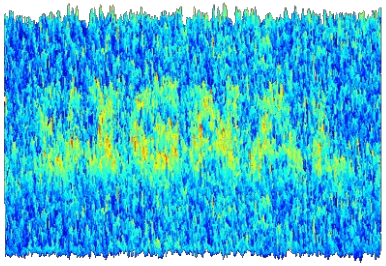




Amplitude $A \sim \mathcal{O}(10^{-5})$

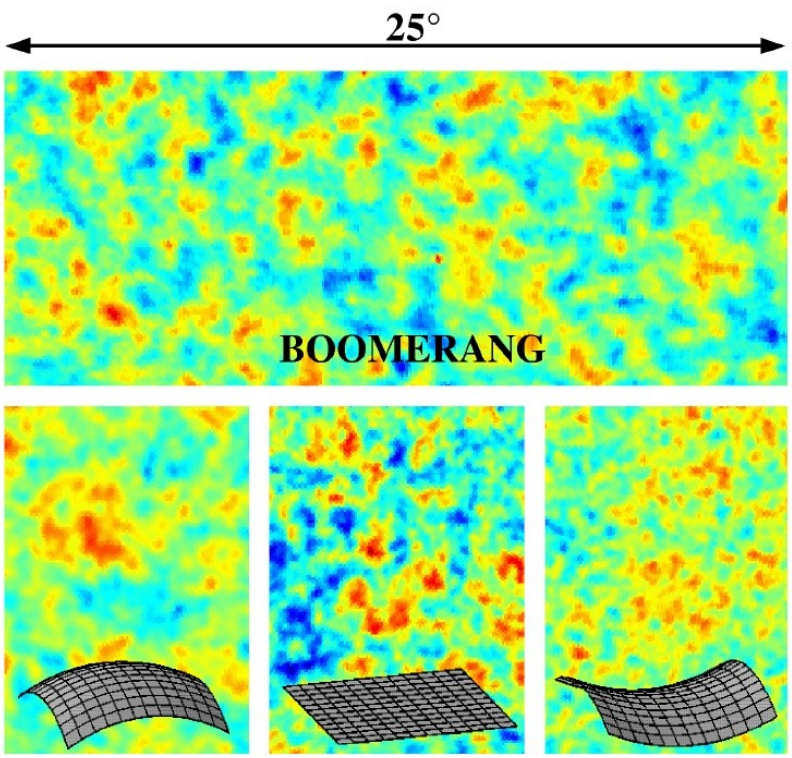
CMB perturbations

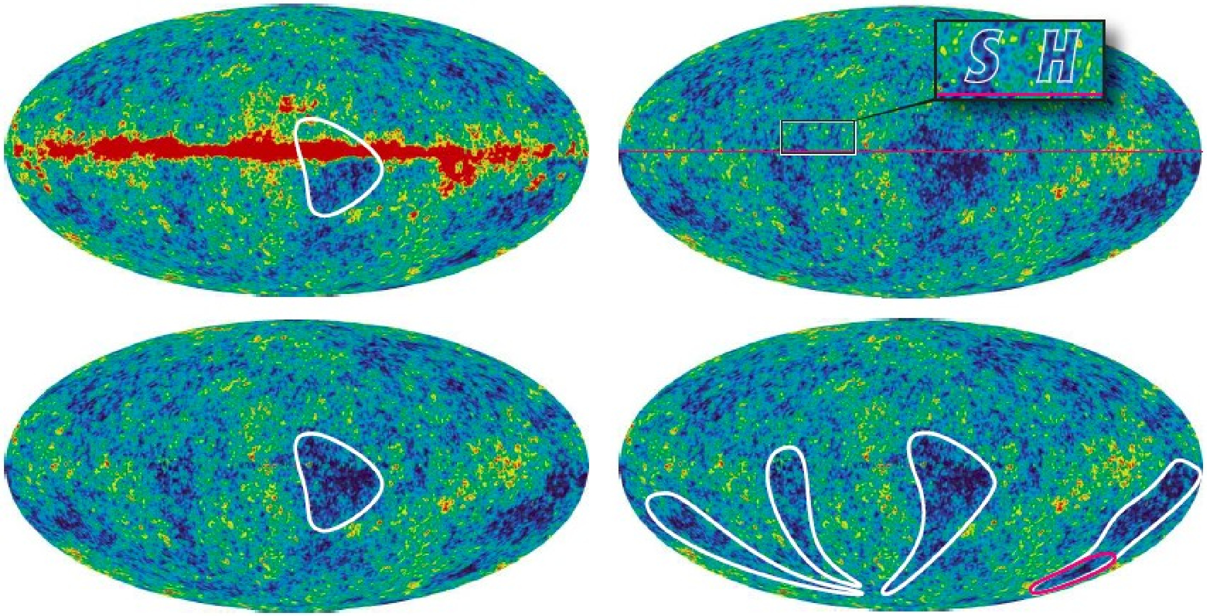




CMB perturbations

- How do we extract info from a map like this?
- Are then any hidden patterns in this map?

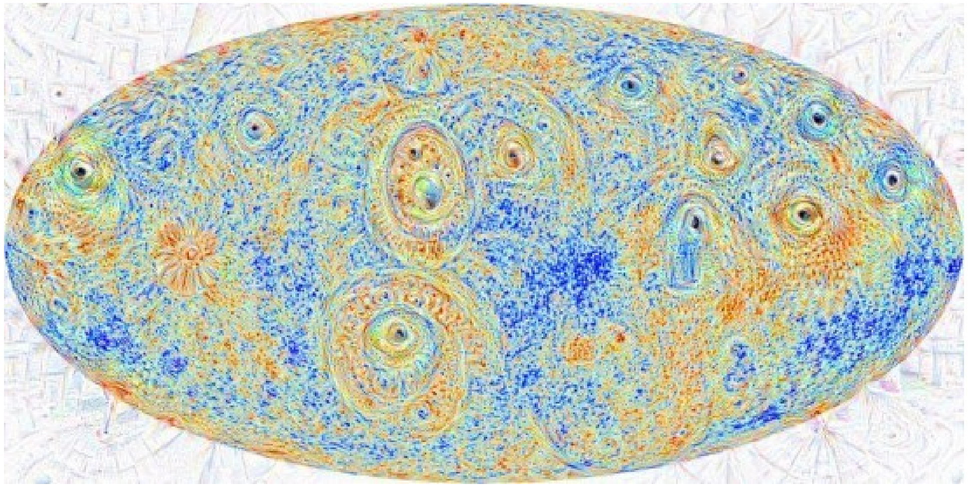




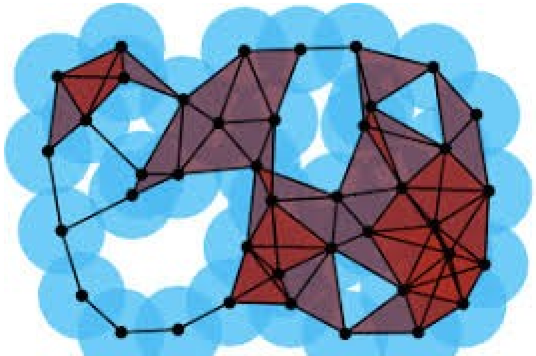
Characterising distributions

Complex analysis

e.g. Deep Dreaming – Google

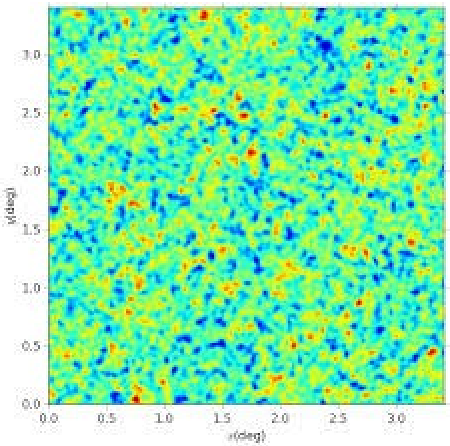


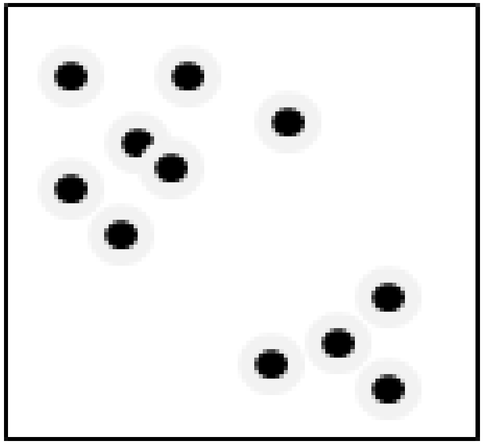
or topological data analysis



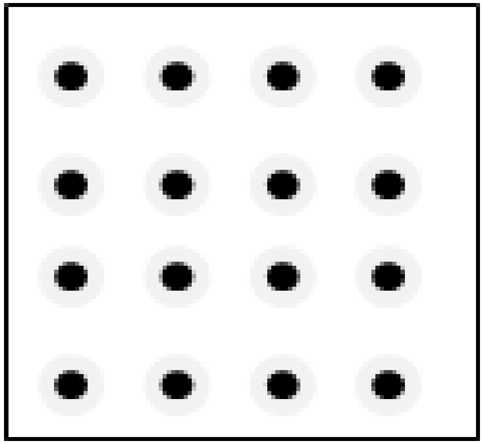
Characterising distributions

Simpler study

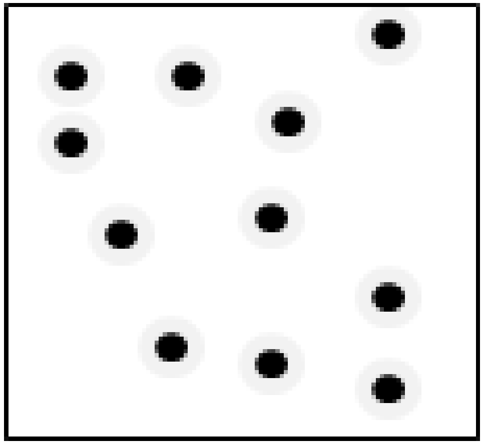




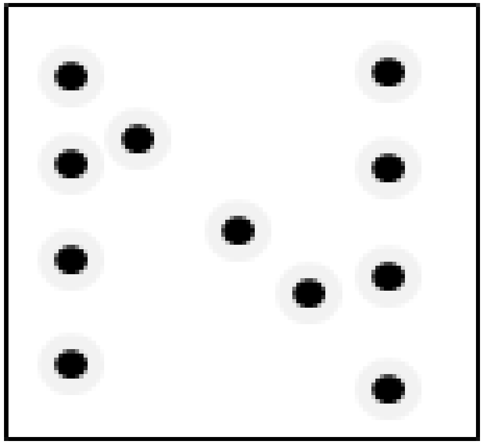
Clusters



Ordered



Random

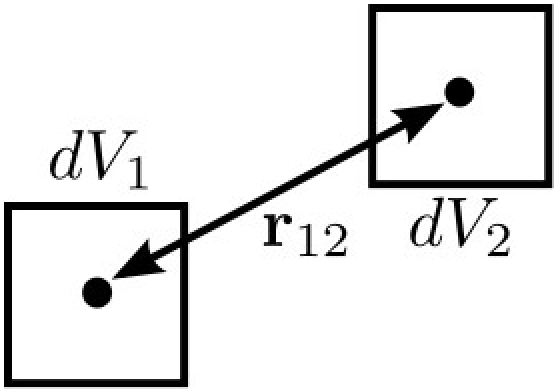


Patterns

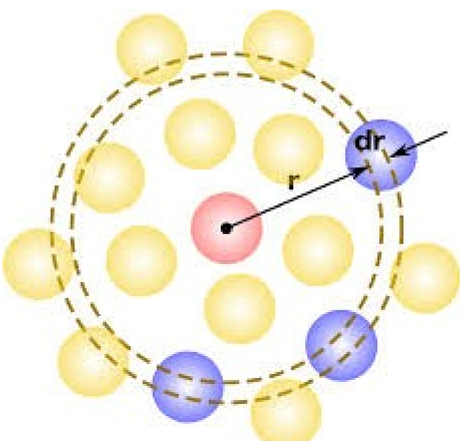
Characterising distributions

Usual to consider N-point correlation functions

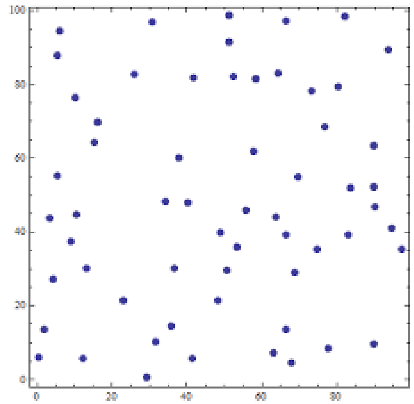
- N=2 (2pcf)



$$dP = \bar{n}^2(1 + \xi^{(2)}(\mathbf{r_{12}}))dV_1 dV_2$$

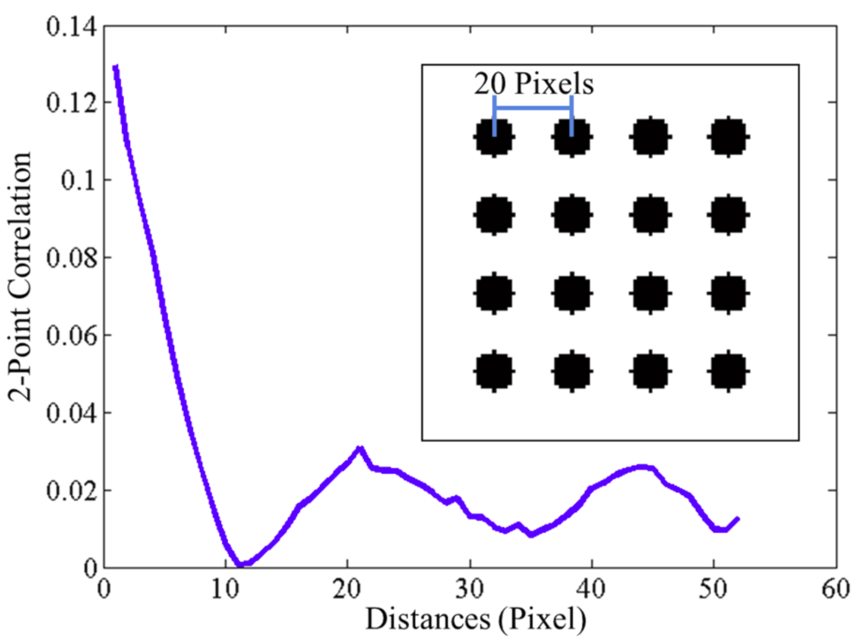


Excess correlation over the random pairs

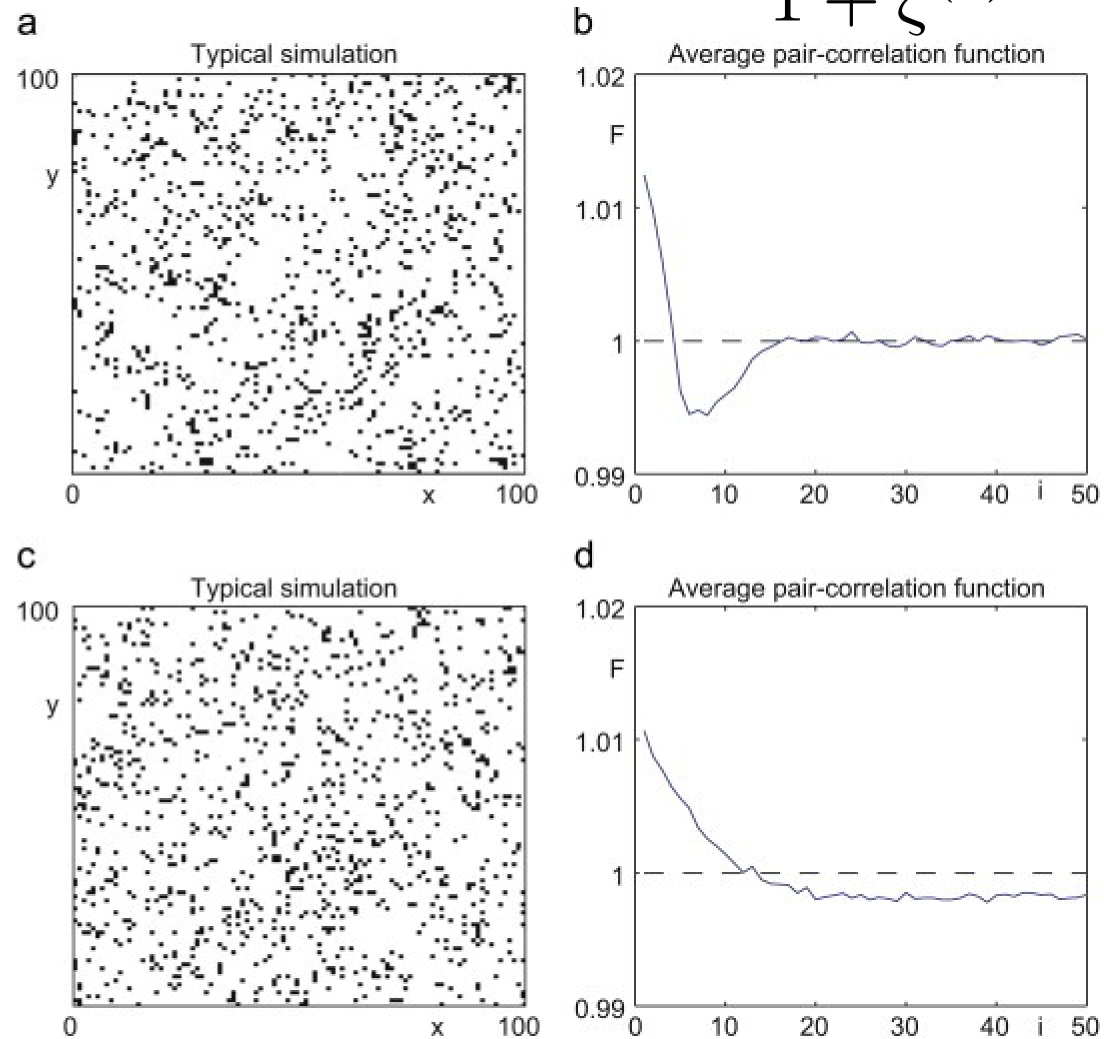


2PCF

- examples

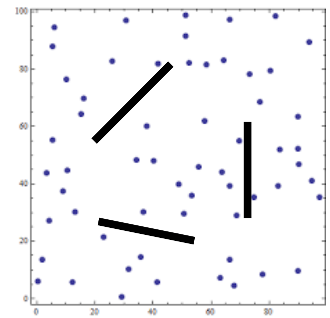


$$1+\xi^{(2)}$$

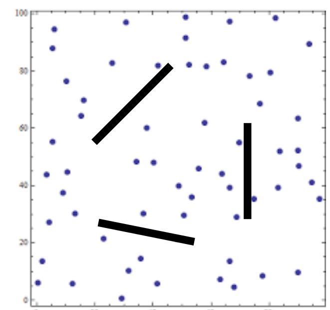


2PCF

Practial way to do it over sample data
(Estimator of Peebles-Hauser)



D



R

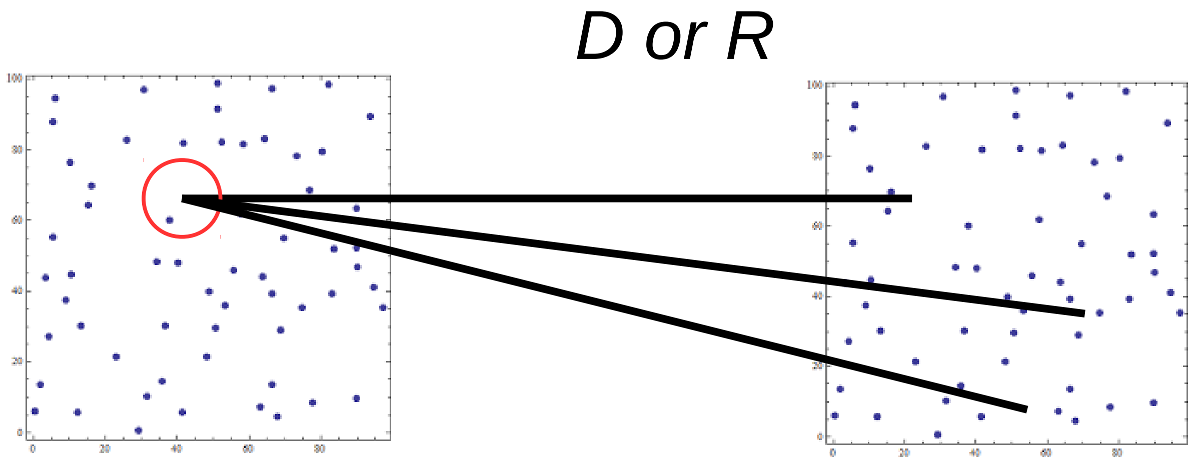
$$\xi^{(2)}(r) = \frac{DD(r)}{RR(r)} - 1 = \frac{DD(r) - RR(r)}{RR(r)}$$

If no. of particles in R and D are not equal:

$$\xi^{(2)} = \frac{N_D(N_D - 1)}{N_R(N_R - 1)} \frac{DD}{RR} - 1$$

2PCF

Computing time



For i = 1 to n

For j = 1 to n

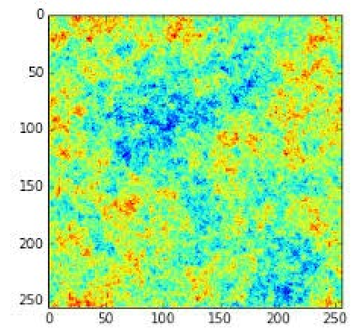
count(r)

scales as

$$\mathcal{O}(n^2)$$

Can do better (eg. kd tree) $\mathcal{O}(n\log(n))$

2PCF



Biased estimator!

$$\delta = \frac{n - \bar{n}}{\bar{n}}$$

Sample is volume finite

$$\langle , \rangle = \int dV$$

- *W(r)*: indicator function $\Rightarrow$ $$no.\ points = \langle W(r) n(r) \rangle$$
- Uncertainity in mean density $$\bar{\delta} = \langle W(r) \delta(r) \rangle$$

$$\xi^{(2)}_{PH} = \frac{\xi^{(2)}_{true} - 2\bar{\delta} - \delta^2}{(1 + \delta^2)}$$

2PCF

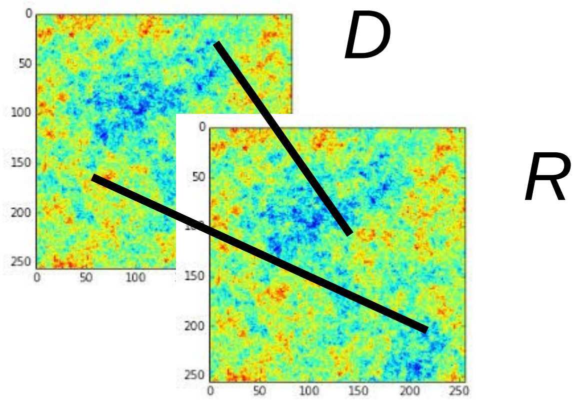
Better estimators

Hamilton

$$\xi_H^{(2)}(r) = \frac{DD(r)RR(r)}{DR(r)^2}$$

Landy-Szalay

$$\xi_{LZ}^{(2)}(r) = \frac{DD(r) + RR(r) - 2DR(r)}{RR(r)}$$



Correction is second order in $\bar{\delta}$ in the numerator for both!

Pair correlations in other spaces

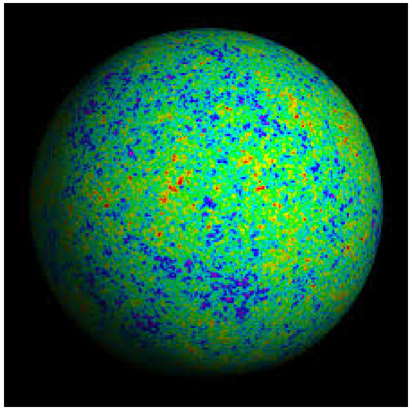
Real space $\xleftrightarrow{e^{ik\cdot x}}$ Fourier space

$$\xi^{(2)}(r) = \langle \delta(r)\delta(r') \rangle$$

$$\langle \delta(k)\delta(k') \rangle = \delta(k-k')P(k)$$

Power Spectrum

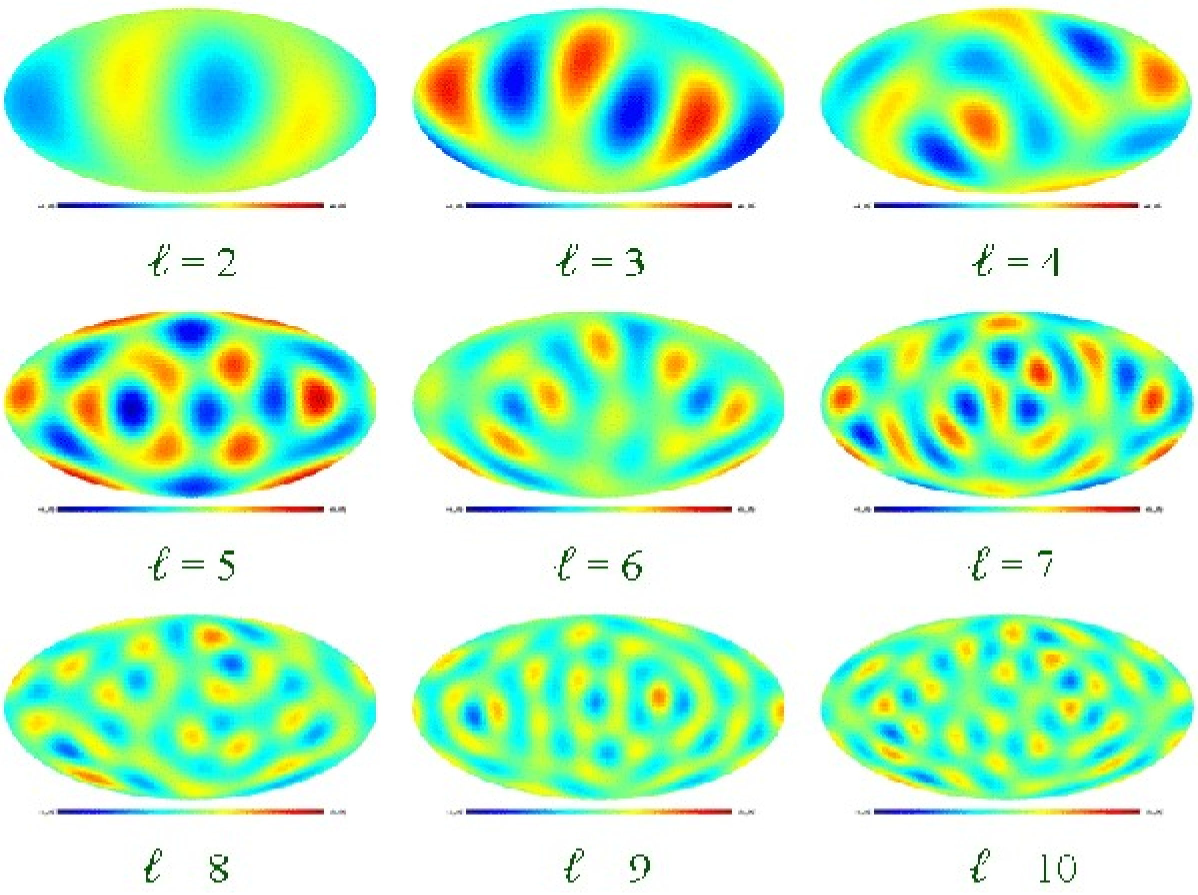
Other basis: e.g. spherical harmonics for CMB

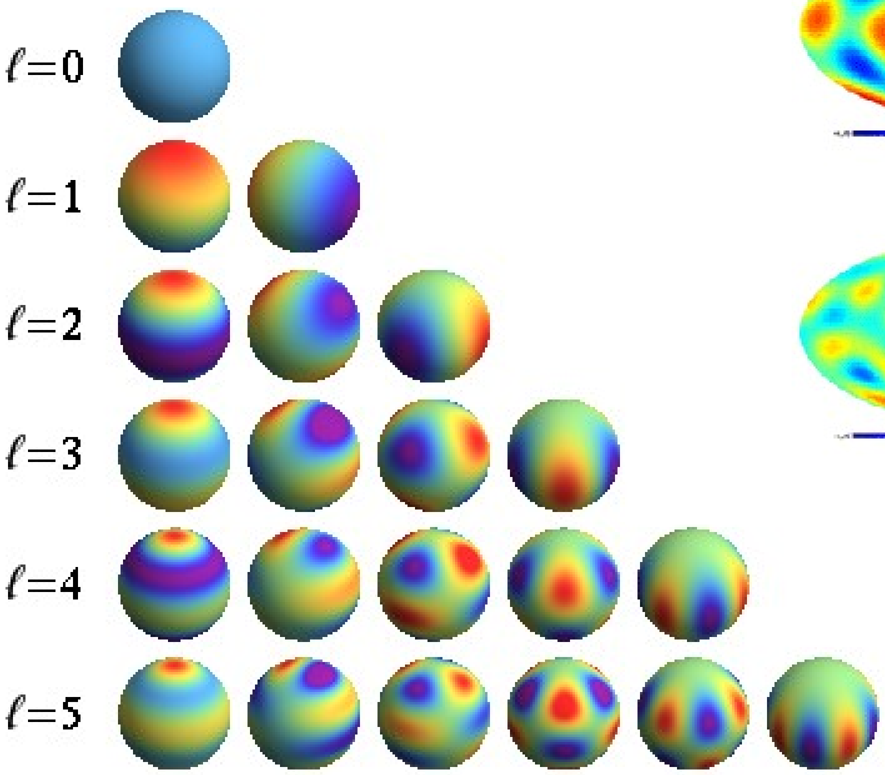


$$\delta = \frac{\Delta T}{\bar{T}} = \sum_{lm} T_{lm} Y_{lm}$$

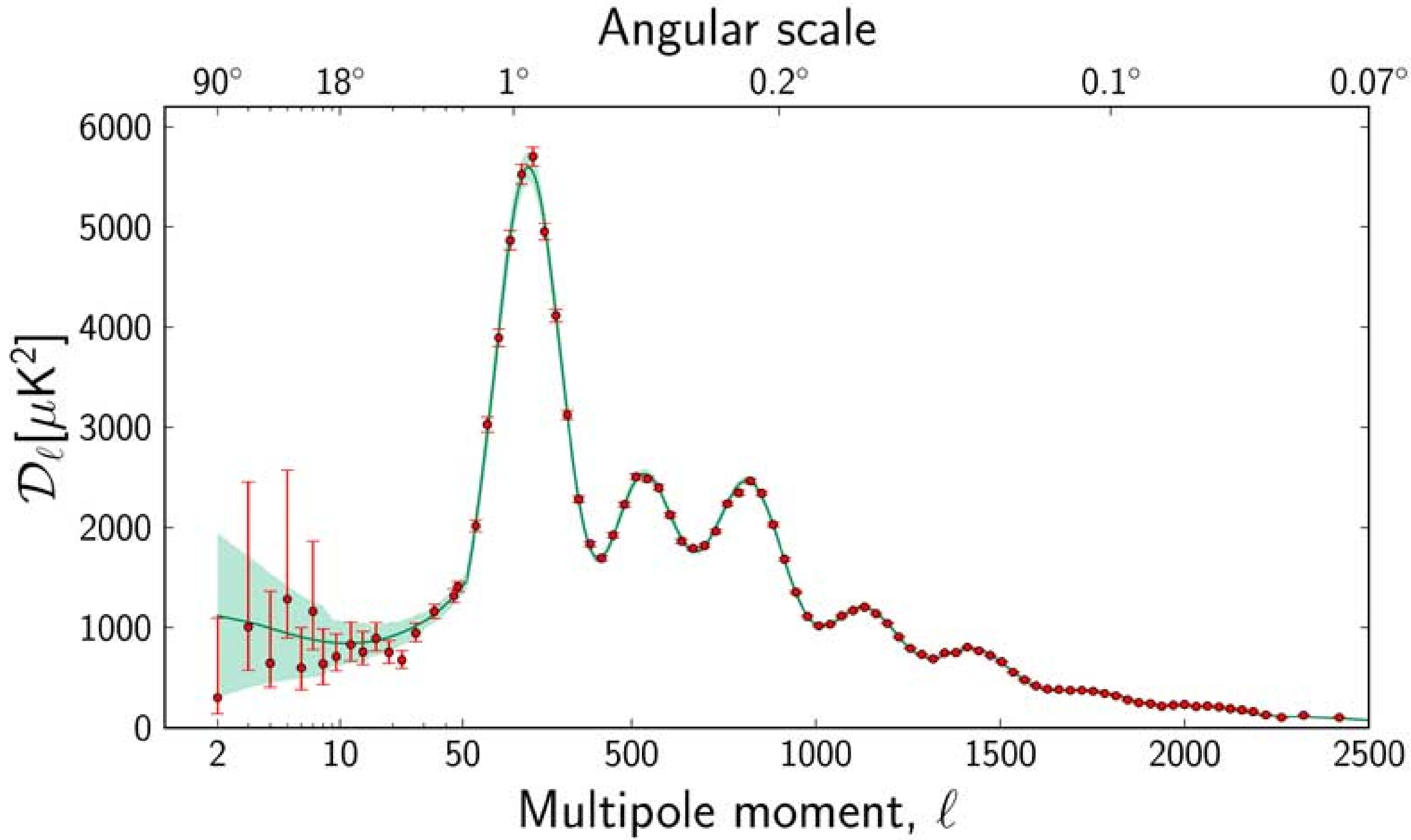
$$\langle T_{lm} T_{l'm'} \rangle = 2\pi \mathcal{D}_l \; \delta_{ll'} \delta_{mm'}$$

Returning to the CMB



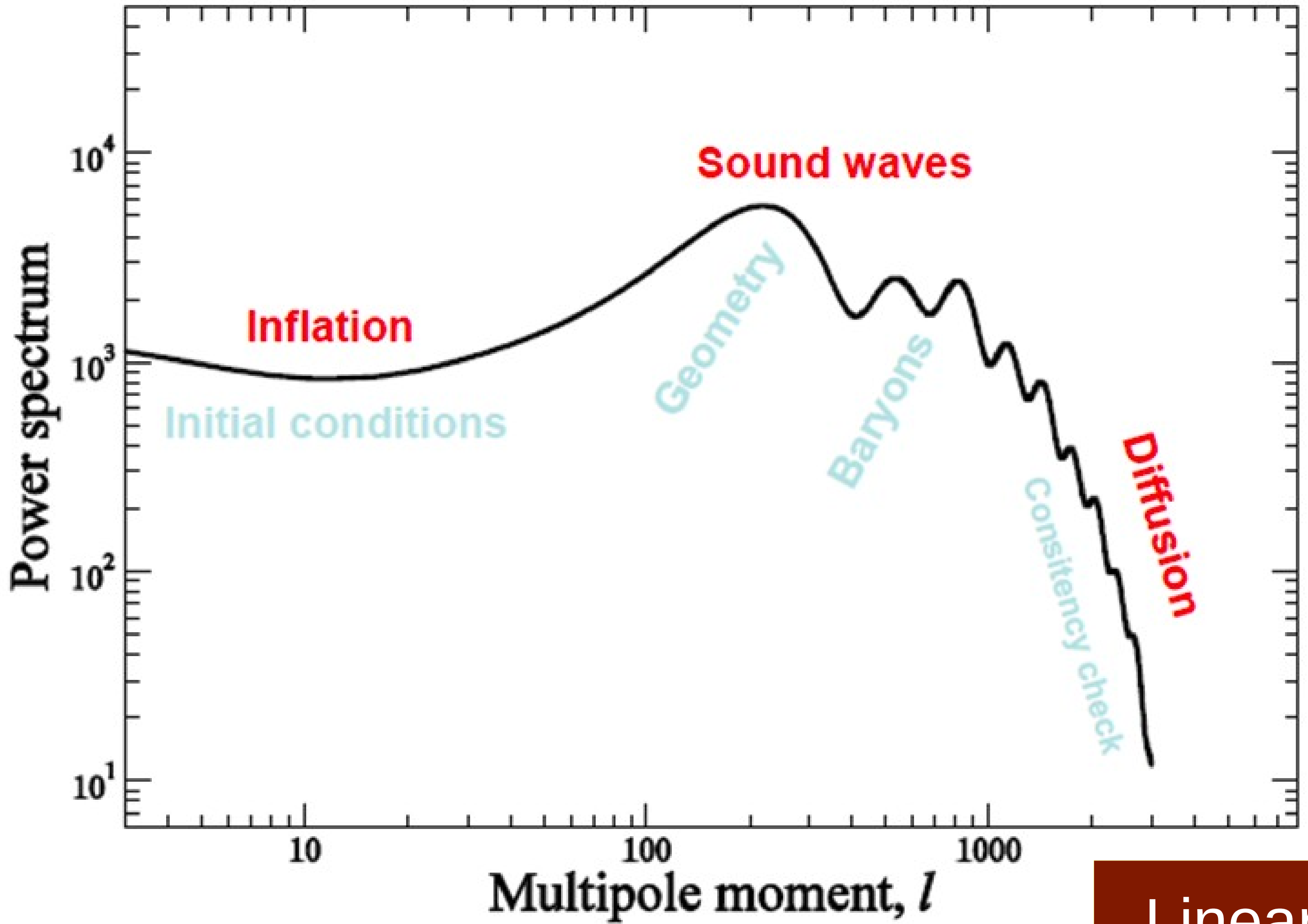


CMB power spectrum



$$\langle T_{lm} T_{l'm'} \rangle = 2\pi \mathcal{D}_l \; \delta_{ll'} \delta_{mm'}$$

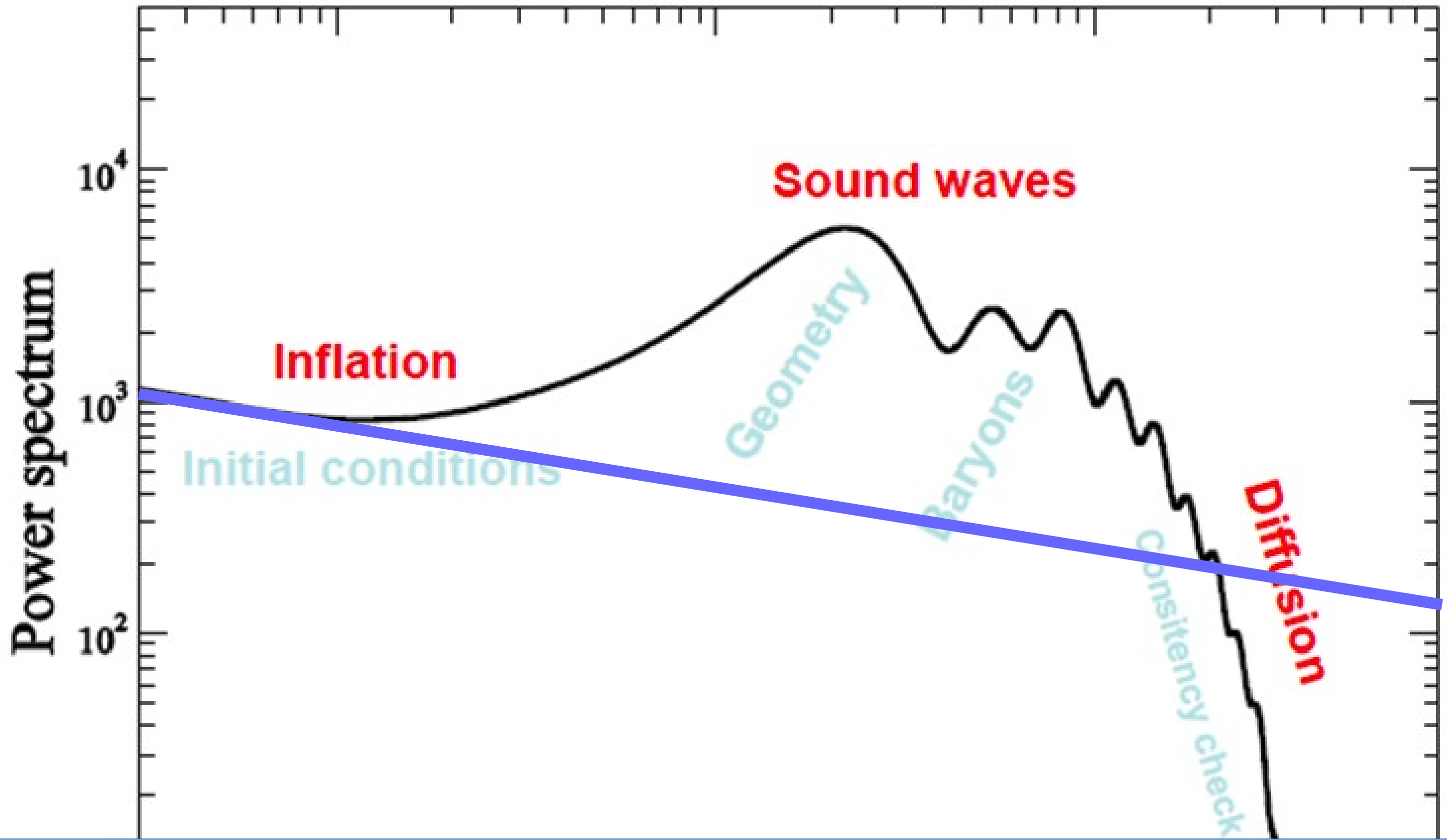
Overview of the CMB spectrum



From a presentation by Hans Kristian Eriksen (2011)

Linear theory
Works
(see A. Vazquez)

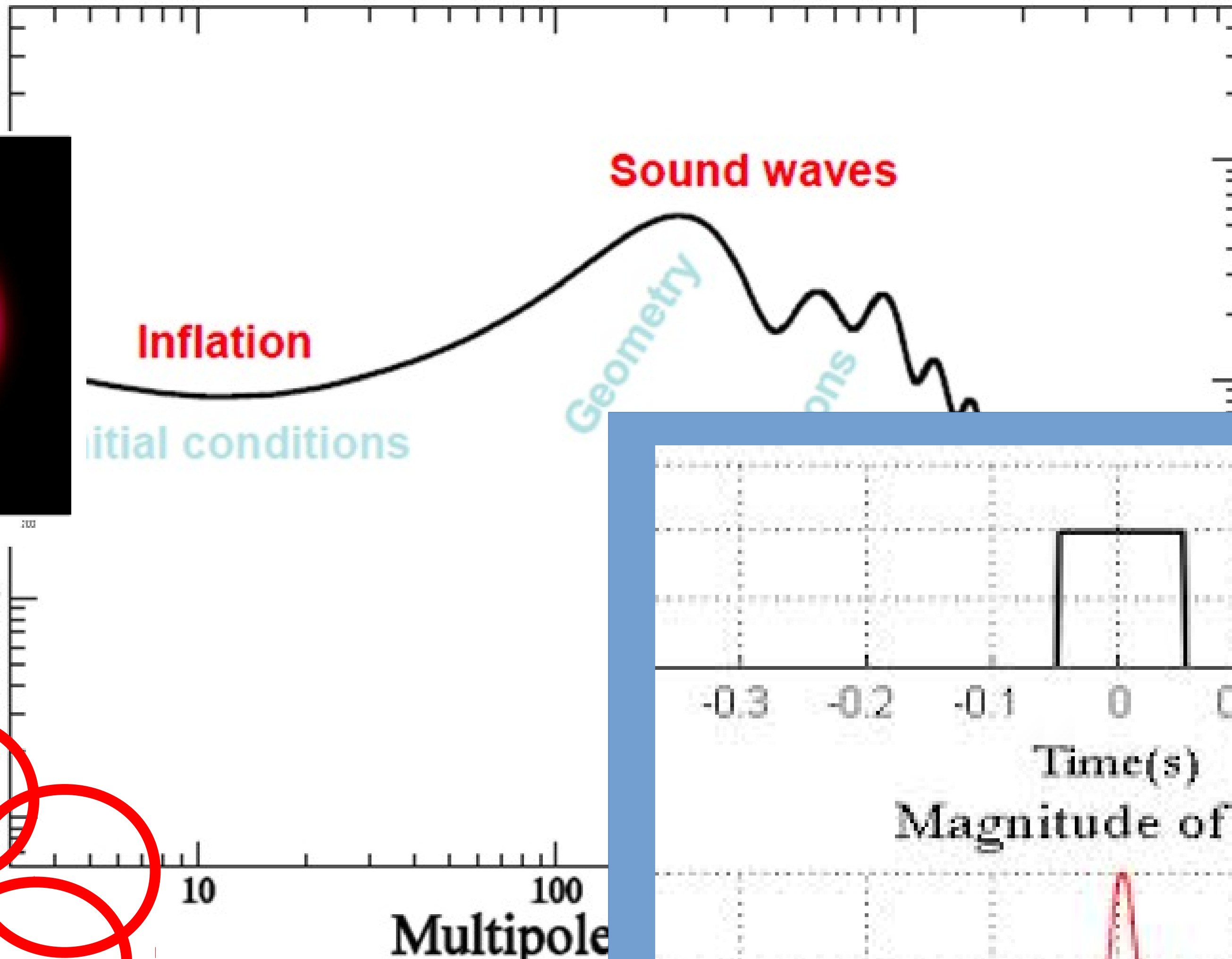
Overview of the CMB spectrum

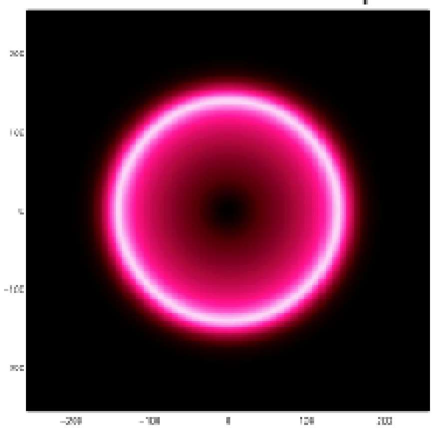


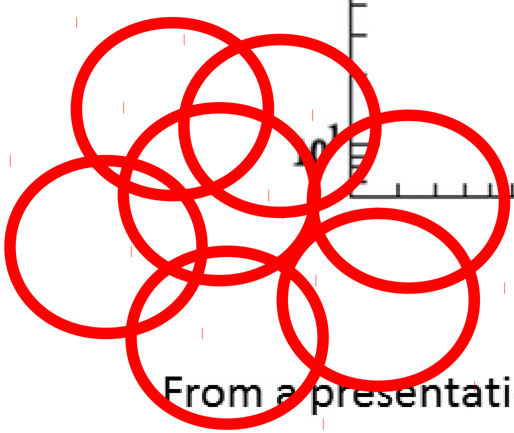
$$P(k) \sim (k)^{1-n_s} \qquad n_s \simeq 0.96$$

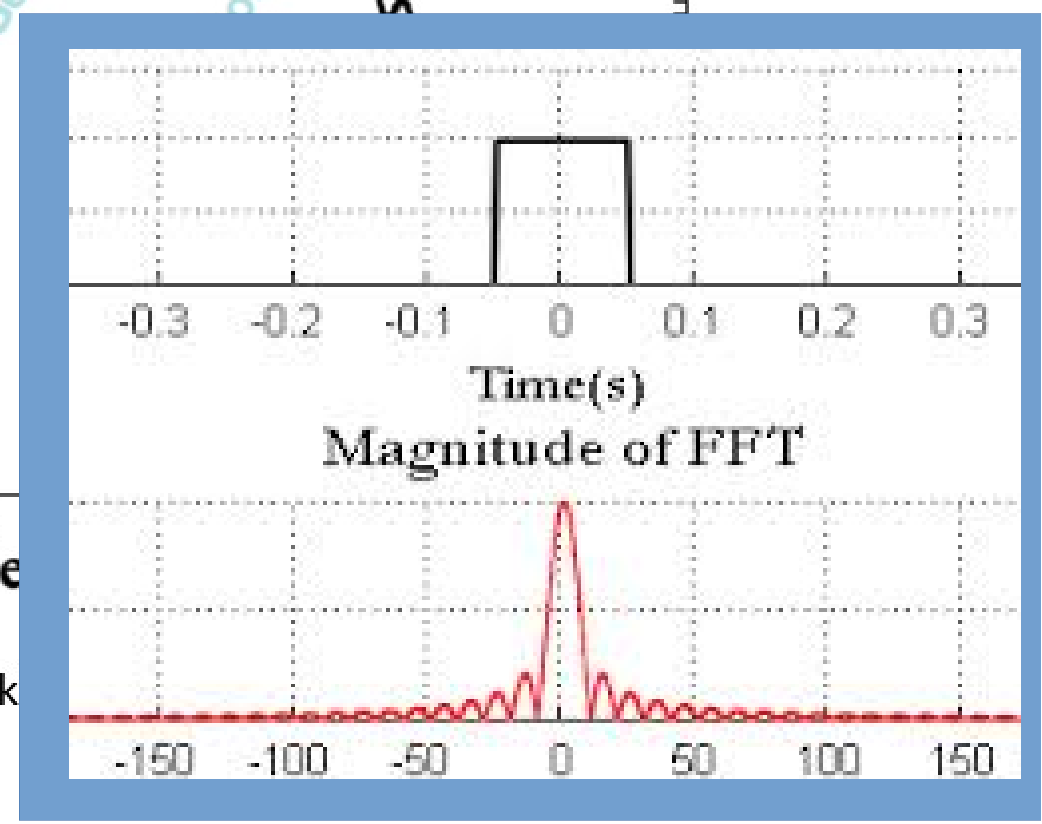
Fr

Overview of the CMB spectrum



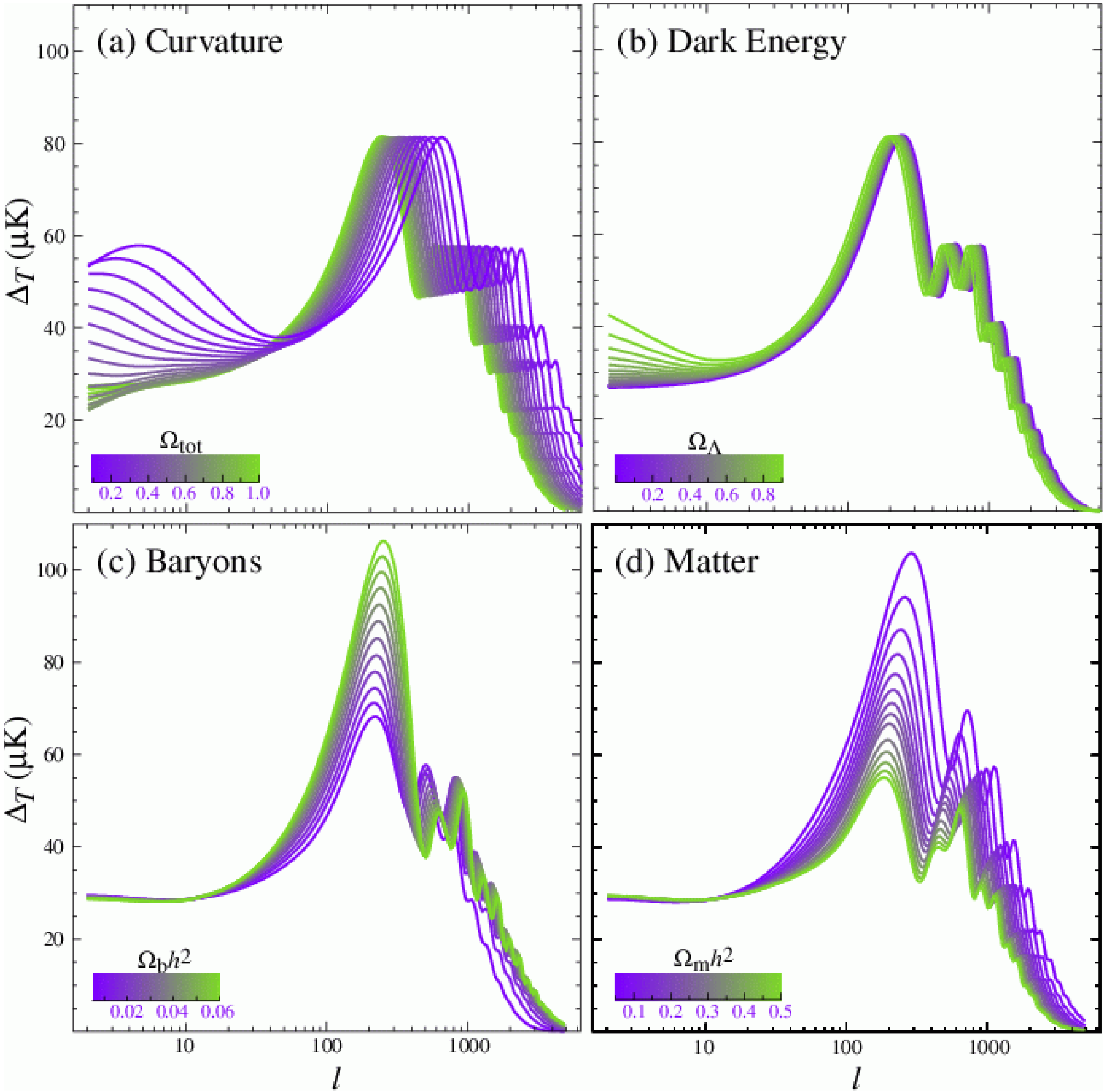






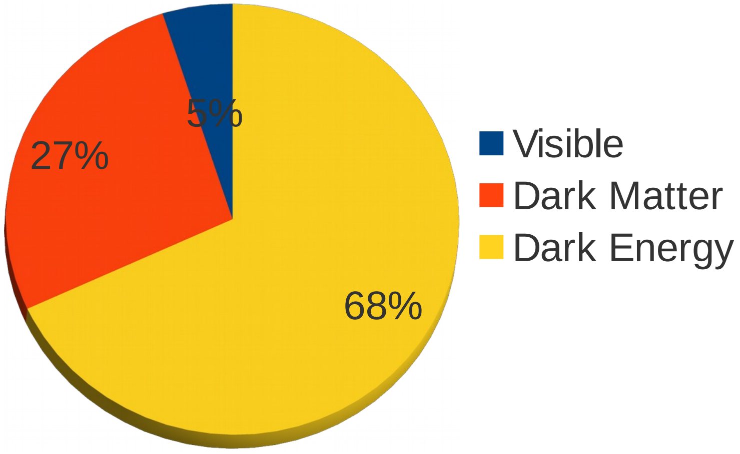
From a presentation by Hans Kristian Erik

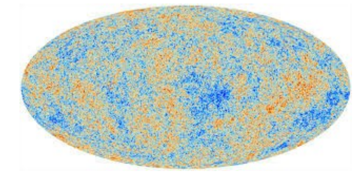
CMB power spectrum



Planck satellite

Flat Universe to 1%

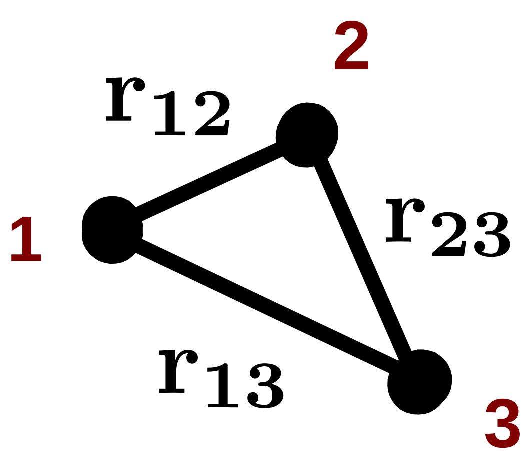


Is there more information in  ?

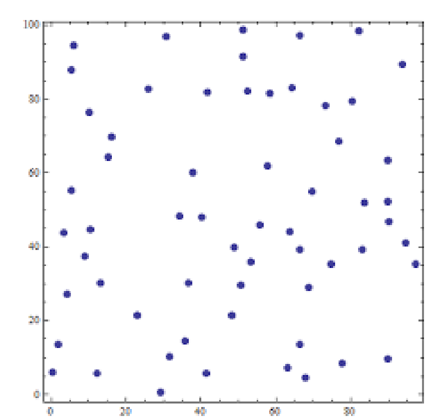
Higher N-point correlation functions

- N=3 (3pcf)

$$dP = \bar{n}^2(1 + \xi^{(3)}(\mathbf{r_{12}}, \mathbf{r_{13}}, \mathbf{r_{23}}))dV_1 dV_2 dV_3$$



Excess correlation
over the random triples



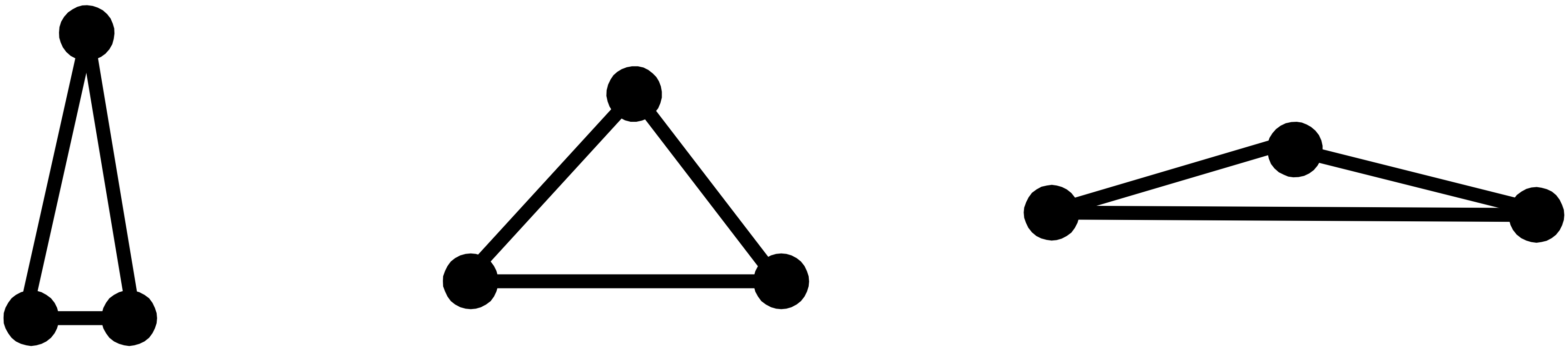
3PCF

- Isotropy

$$\xi^{(3)}(\mathbf{r_1}, \mathbf{r_2}, \mathbf{r_3}) = \xi^{(3)}(r_1, r_2, r_3)$$

  - 3 parameter function → 4d plot!

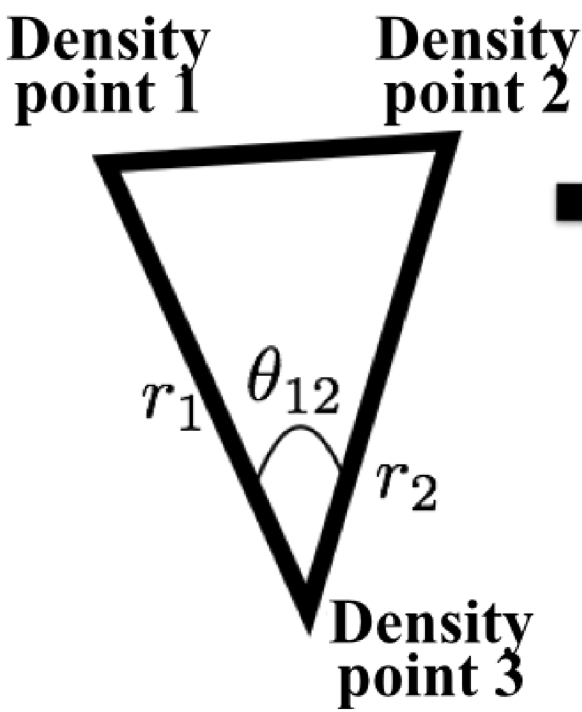
Options A) Consider particular shapes



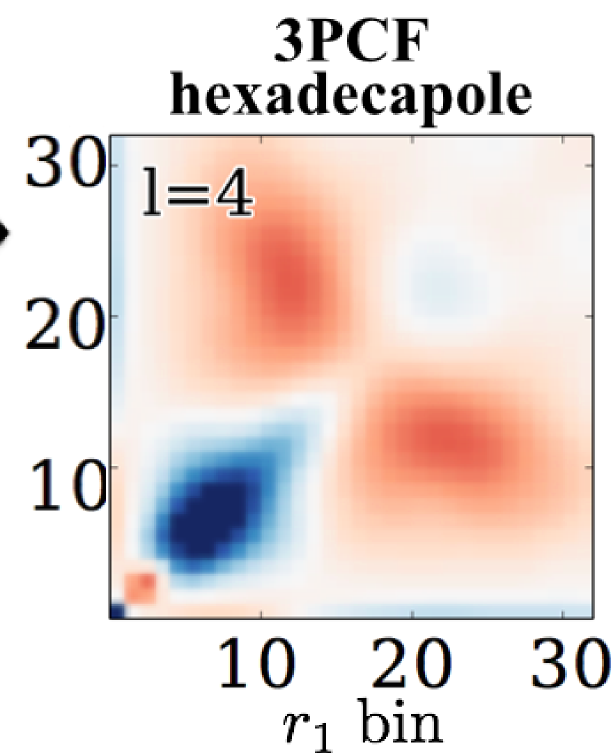
3PCF

B) Use a basis: ej. Szapudi

$$\xi^{(3)}(r_1, r_2, \theta_{12})) = \sum_{\ell} \xi_{\ell}^{(3)}(r_1, r_2) P_{\ell}(\cos(\theta_{12}))$$

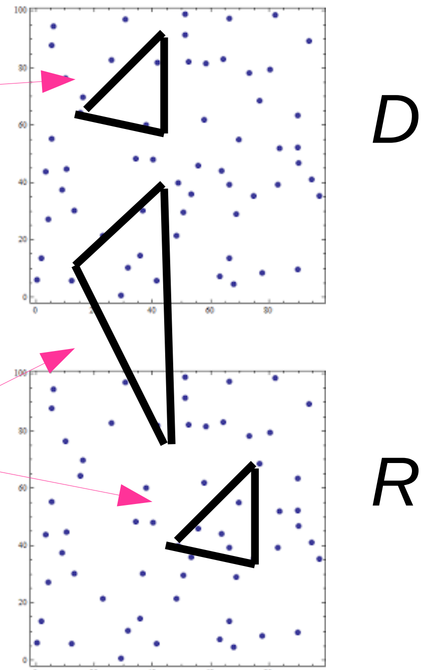


red = excess
blue = deficit



3PCF

Practial way to do it over sample data (Biased and unbiased estimators)



$$\xi^{(2)}(r) = \frac{DDD(r) - RRR(r)}{RRR(r)}$$

$$\xi^{(2)}_{SS}(r) = \frac{NNN(r) - RRR(r)}{RRR(r)}$$

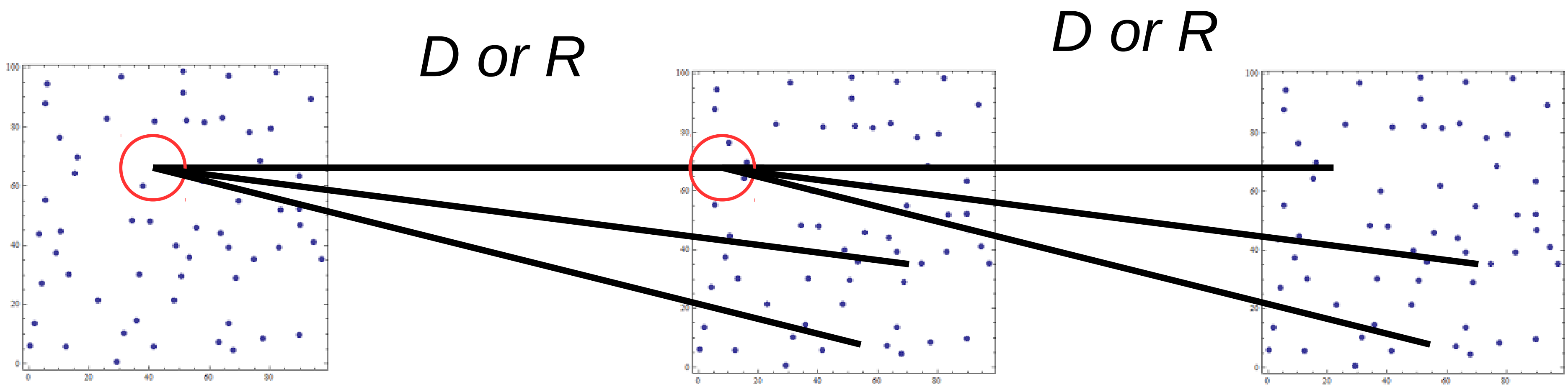
Salay-Szapudi

$$= \frac{DDD(r) - 3DDR(r) + 3DRR(r) - RRR(r)}{RRR(r)}$$

There is an equivalent to Hamilton’s estimator for 3pcf by Jing & Borner

3PCF

Computing time



For i = 1 to n

For j = 1 to n

For k = 1 to n

count(r)

scales as

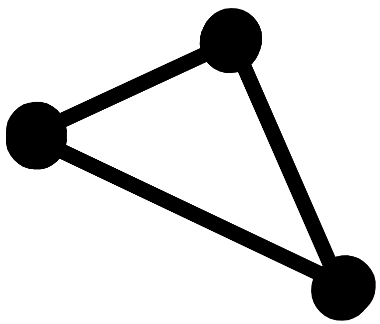
$$\mathcal{O}(n^3)$$

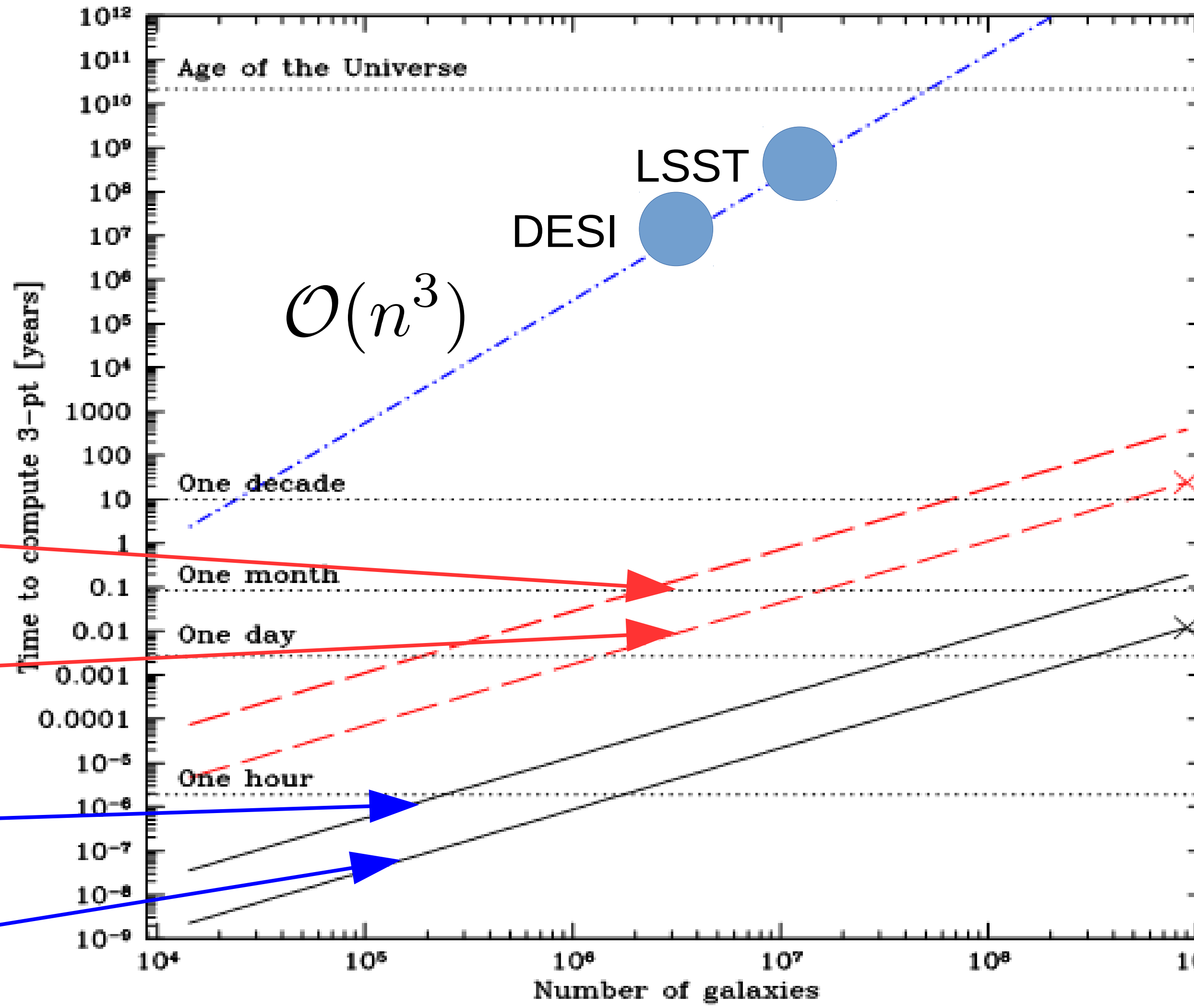
Using kd trees or FFTs

$$\mathcal{O}(n^3 \log(n))$$

3PCF

Computing time





$\mathcal{O}(n^3 \log(n))$

Parallel

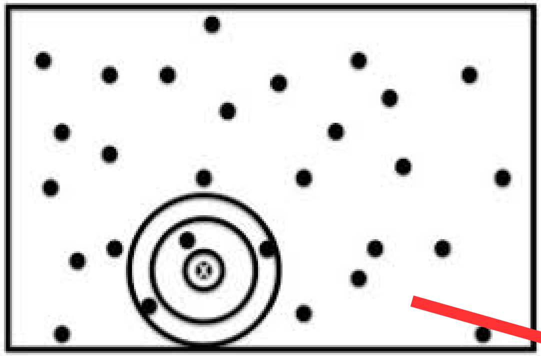
2PCF: $\mathcal{O}(n^2)$

Parallel

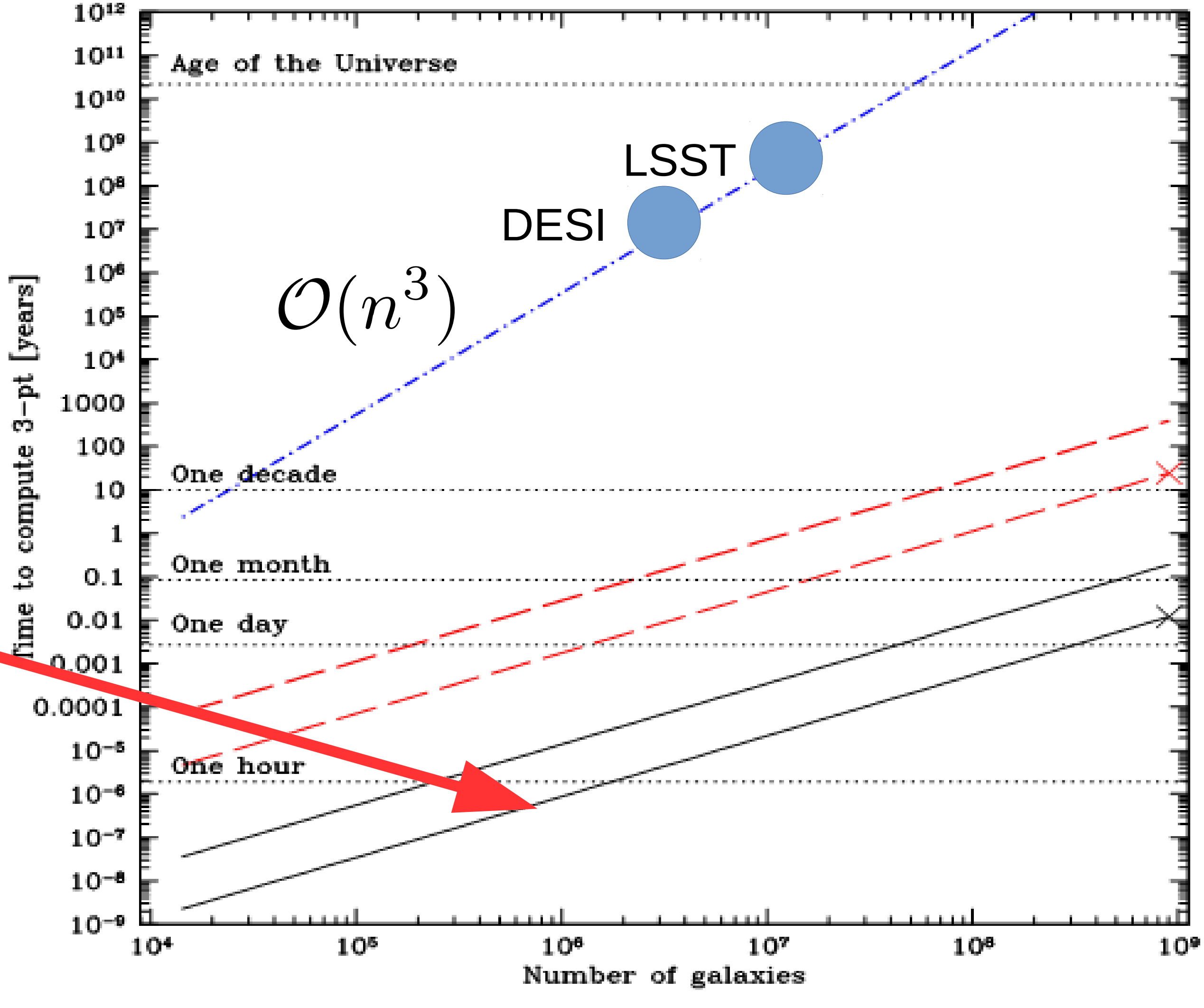
3PCF

Use Legendre basis to save time (Slepian & Eisenstein)

Decompose in multipole



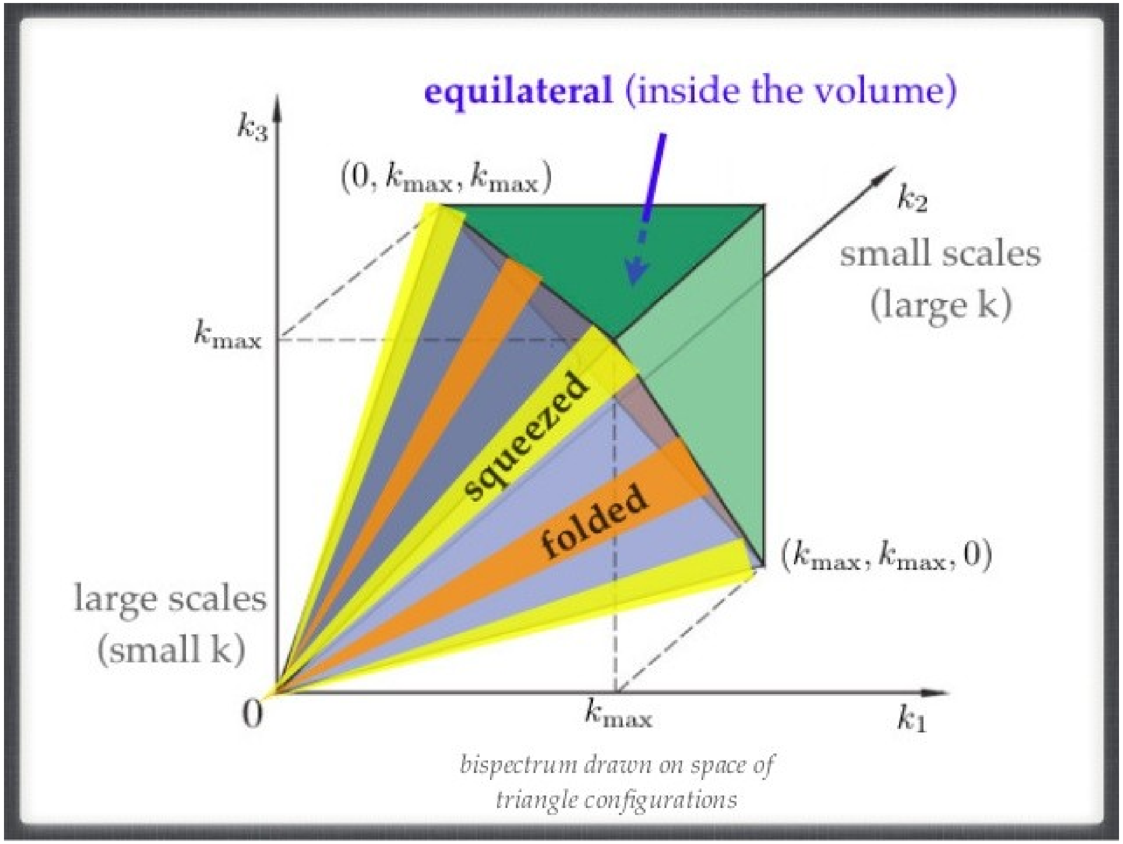
$$\mathcal{O}(n^2)$$



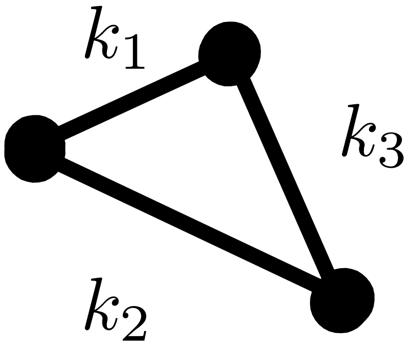
Triple's correlations in other spaces

Real space $$\xi^{(3)} = \langle \delta\delta\delta \rangle$$

Fourier $$\langle \delta(k_1)\delta(k_2)\delta(k_3) \rangle = \delta(\sum_i k_i) B(k_1, k_2, k_3)$$



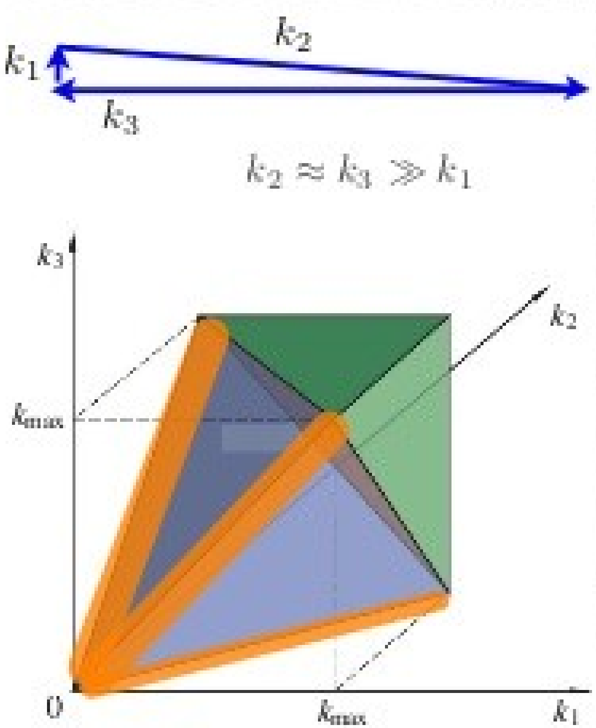
Bi-spectrum



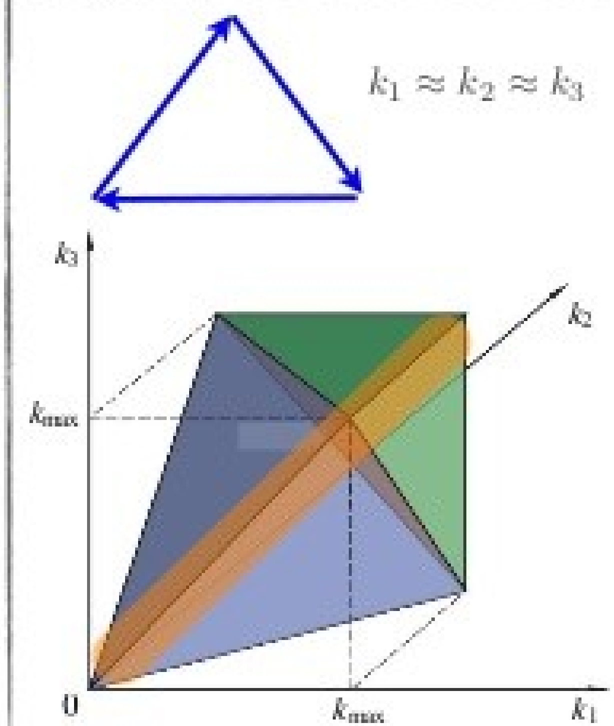
BISPECTRUM SHAPES

Different inflation models induce different momentum dependencies (shapes) of $B_\Phi(k_1, k_2, k_3)$

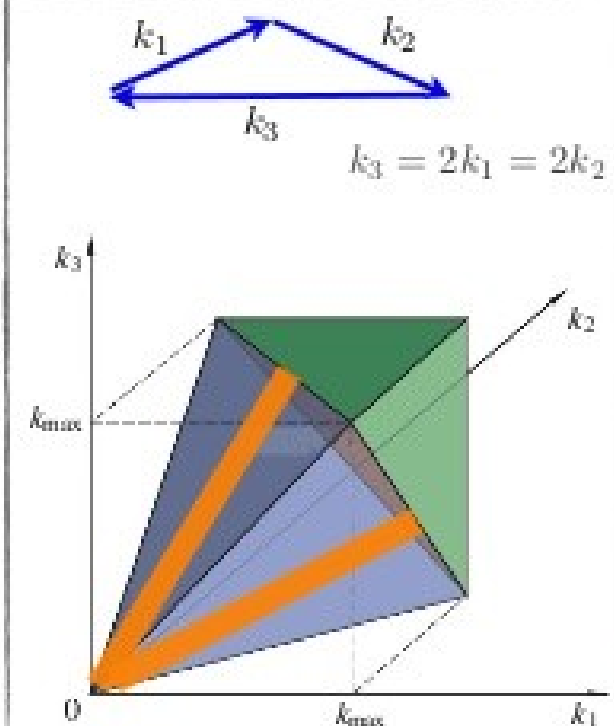
Squeezed **triangles (local shape)**	***Equilateral*** **triangles**	***Folded*** **triangles**



Arises in multifield inflation; detection would rule out all single field models!



Typically higher derivative kinetic terms, e.g. DBI inflation



E.g. non-Bunch-Davies vacuum

Schmittfull Talk

Higher point correlations

- Easy to generalise to

$$\xi^{(n)} = \langle \delta_1 ... \delta_n \rangle$$

- Each correlation function may have additional information about data distributions
  - However, in some cases they are related

Guassian Random Field

- Completely characterised by 2PCF

$$< \delta_{k_1} \ldots \delta_{k_{2n+1}} > \sim 0$$

$$< \delta_{k_1} \ldots \delta_{k_{2n}} > \sim F(P(k_i))$$

Wick's theorem

- It is the same for *free theories* → With Gaussian Initial conditions

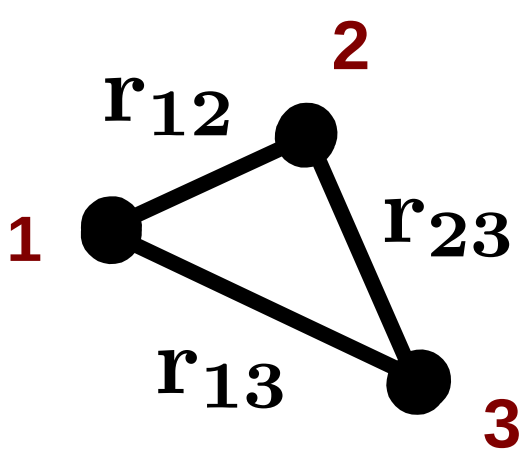
i.e. No interactions!

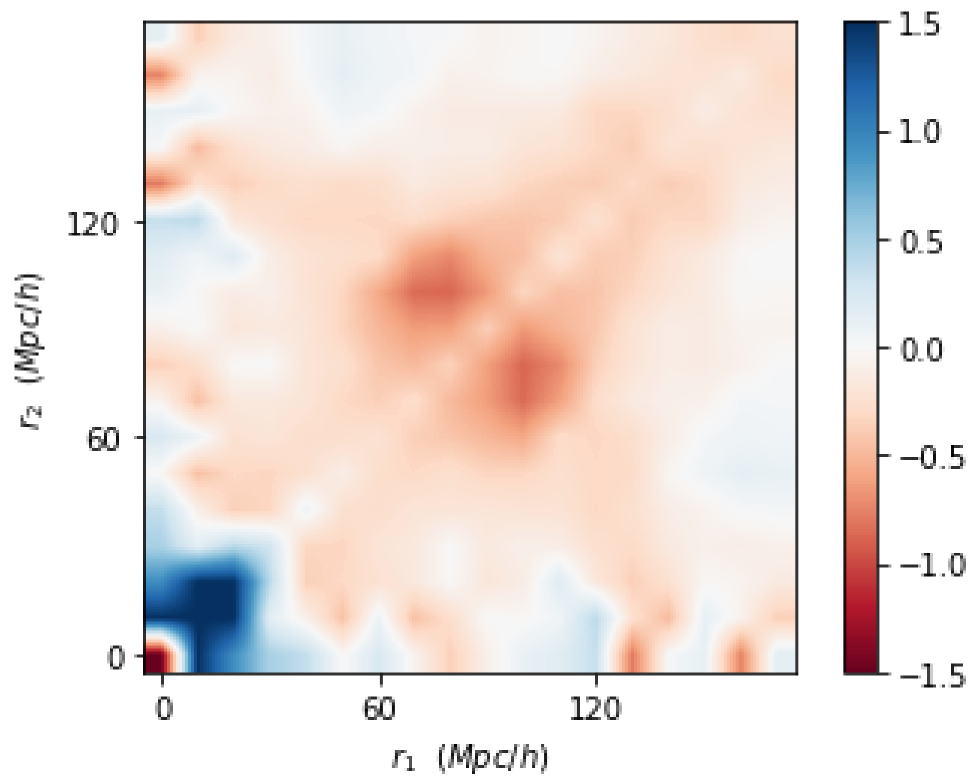
Examples: ϕ^2 inflation (except for graviton couplings)

Galaxies in the Hubble flow (with no interactions)

Converse: Non trivial 3PCF implies interactions!

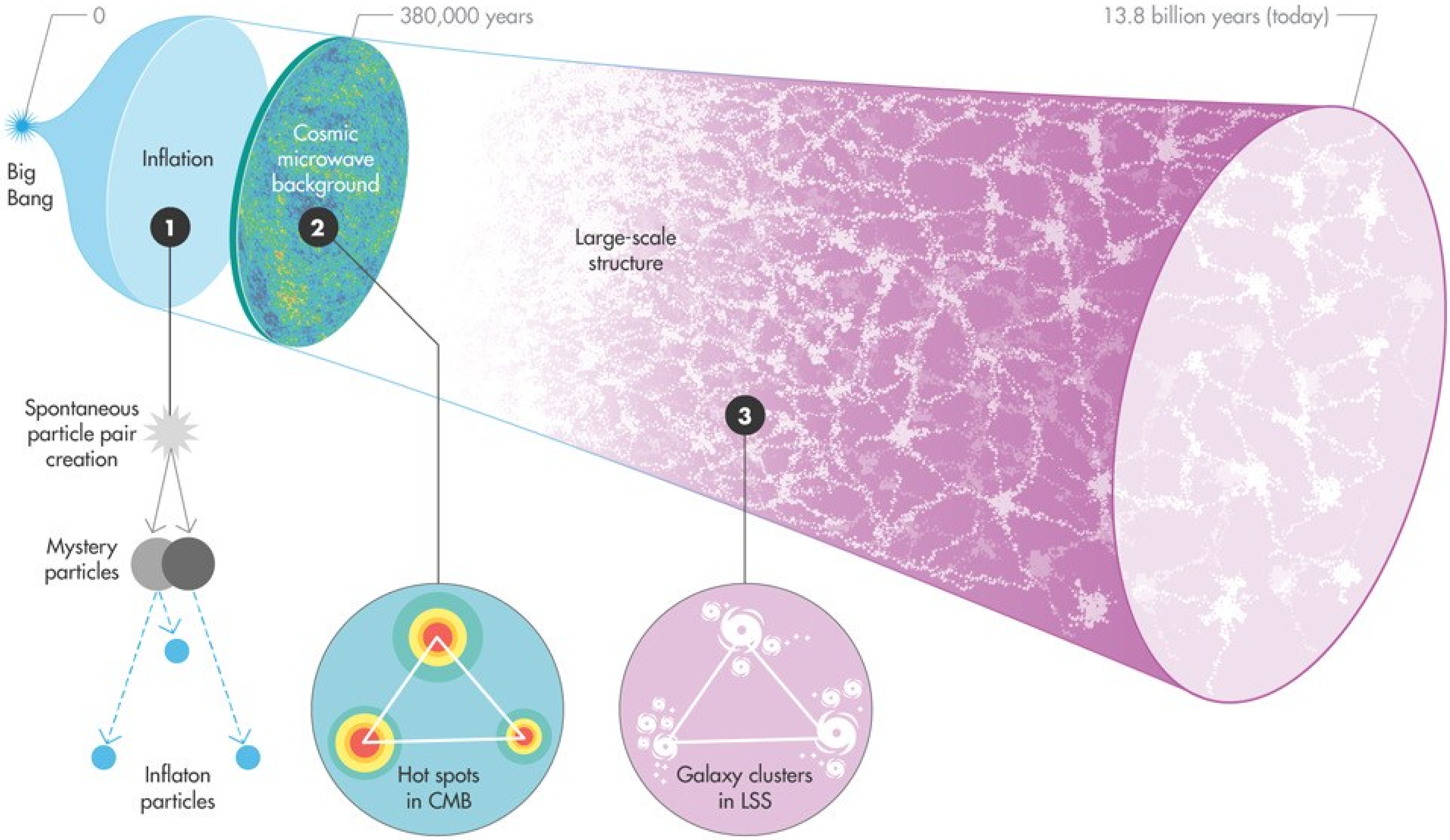
Using the 3PCF
in the Cosmic Collider





TRIANGLES IN THE SKY

According to the theory of cosmic inflation, pairs of particles spontaneously surfaced throughout the primordial universe. Some pairs decayed into three "inflaton" particles, producing triangular configurations that expanded into arrangements of cosmological structures that are visible today. Triangles may appear as correlations between three hot spots in the 2-D cosmic microwave background (CMB), or between three galaxy clusters in the 3-D large-scale structure (LSS). These triangles and other shapes reveal the types and relationships of particles that existed during inflation.

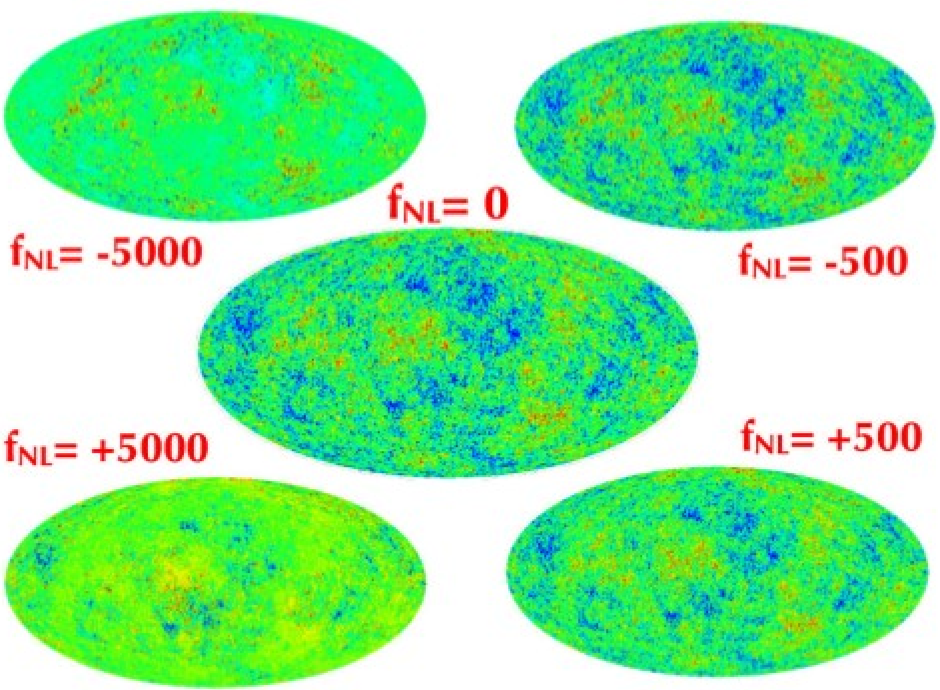


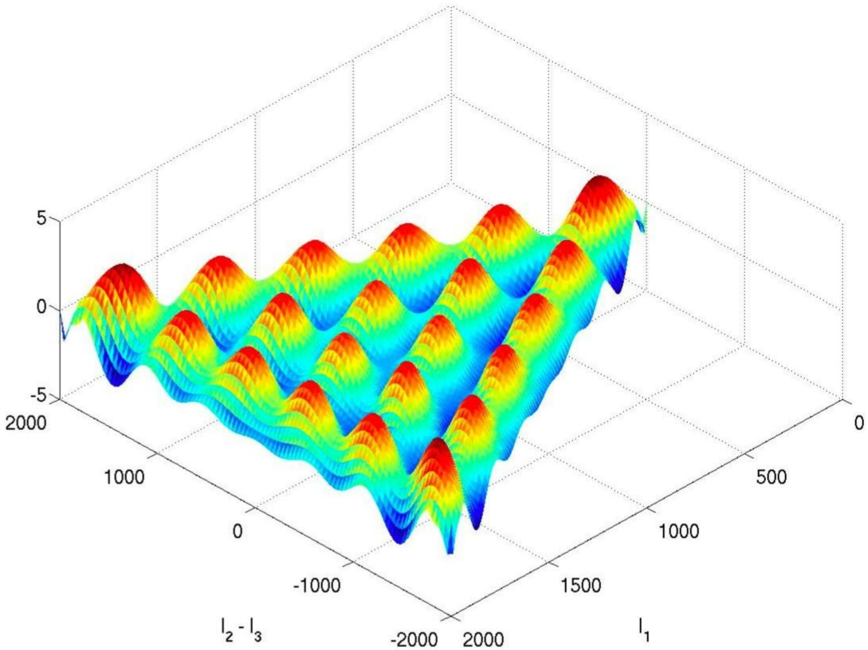
Non-Gaussianity in inflation

$$\phi = \phi_G + \phi_{NG}$$

Inflaton | Guassian Part | Non-Gaussian Part

Simplest case: non-local NG $\phi = \phi_G + f_{NL}(\phi_G^2 - \langle \phi_G \rangle^2)$





Test inflation NG with CMB

- Not much about interactions (Planck 2015)

  $< \delta^n >$ consistent with zero for *n>2*

- Consistent with EFT of inflation $\quad f_{NL} \sim \mathcal{O}(\epsilon, \eta)$

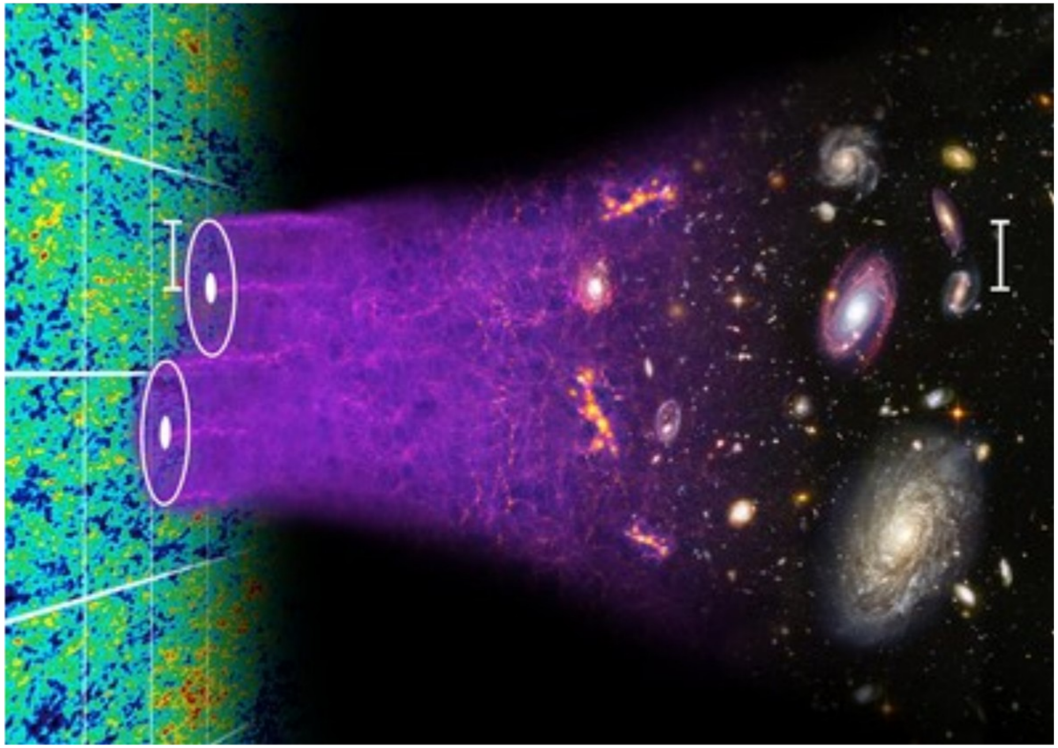
$$S = \int d^4x \sqrt{-g} \quad \Big[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2 \left(3H^2(t+\pi) + \dot{H}(t+\pi)\right) +$$
$$+ M_{\rm Pl}^2 \dot{H}(t+\pi)\left((1+\dot\pi)^2 g^{00} + 2(1+\dot\pi)\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi\right) +$$
$$\frac{M_2(t+\pi)^4}{2!}\left((1+\dot\pi)^2 g^{00} + 2(1+\dot\pi)\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^2 +$$
$$\frac{M_3(t+\pi)^4}{3!}\left((1+\dot\pi)^2 g^{00} + 2(1+\dot\pi)\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi + 1\right)^3 + \ldots\Big]$$

Cheung et al, ...

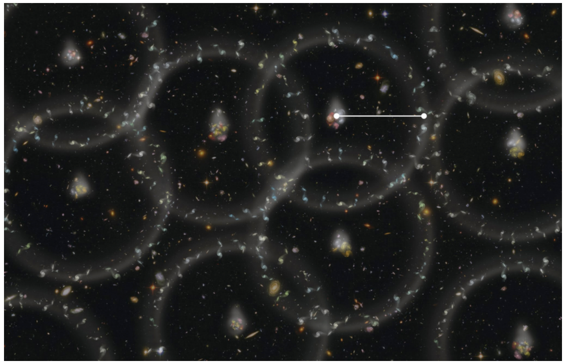
- Cannot gain more mode statistics to achieve better accuracy than with Planck

Option: explore the LSS

Large Scale Structure (LSS)

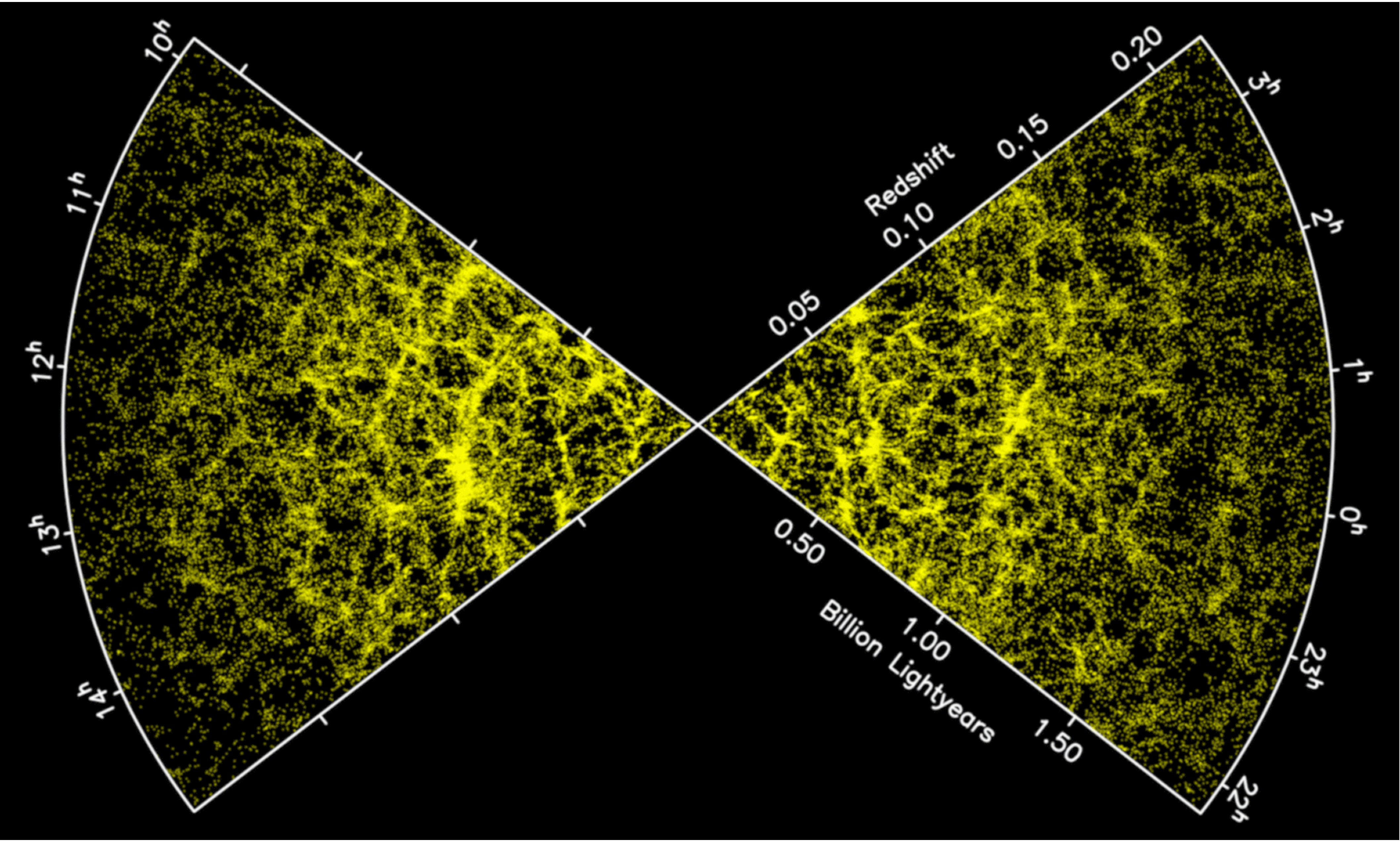


Baryonic
Acoustic
Oscillations



CMB seeds the structure at large scales

LSS



Sloan Digital Sky Survey, 2013

LSS

Benefits

1) Volume Vs area (CMB)

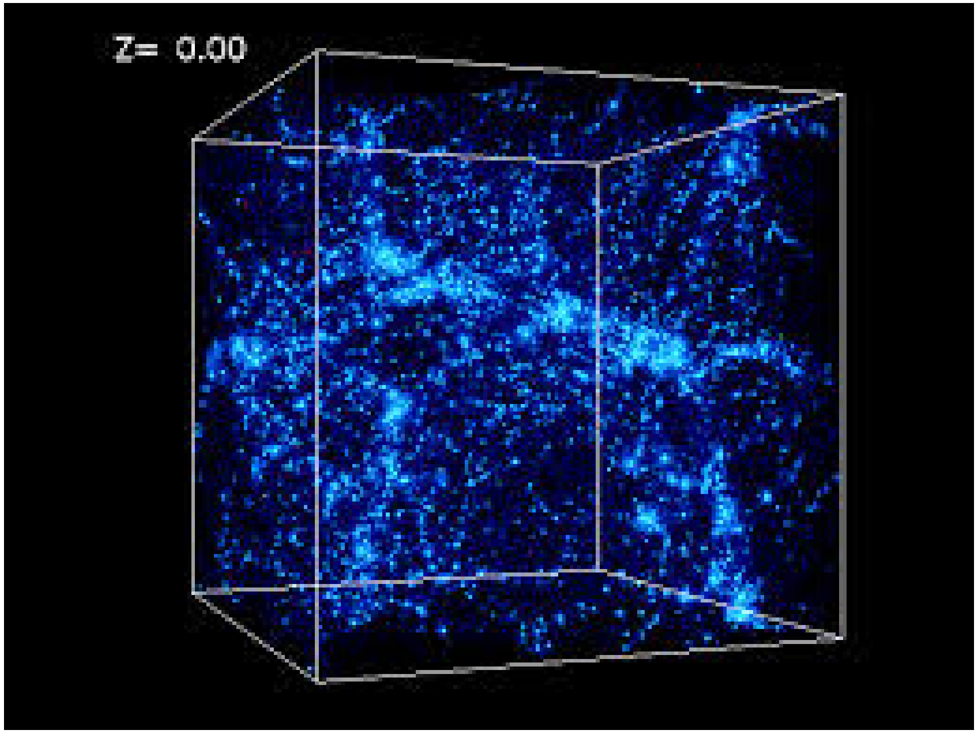
*More Resolution!!!

2) Non-linear physics

*More difficult

*Richness

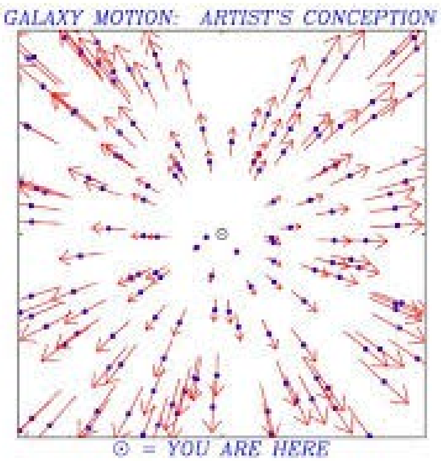
*Non-zero 3PCF!!!



LSS: Perturbation theory and nPCF

Matter density field

(see Aviles, Hidalgo & Santiago courses)

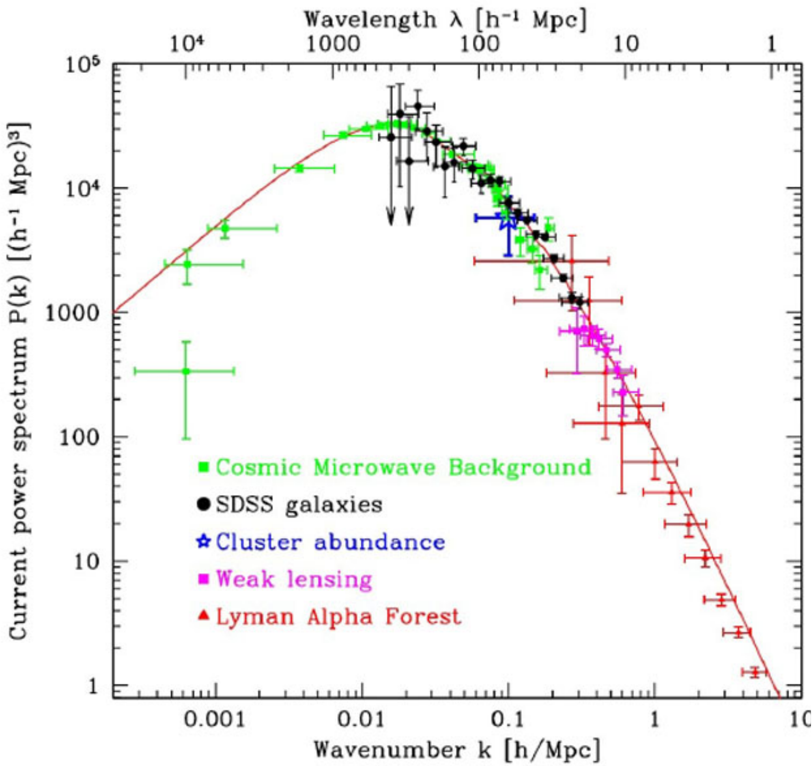


$$\delta = \delta^{(1)} + \delta^{(2)} + \ldots$$

- Affected by *H* only
- If vanishing primordial NG

  2PCF only

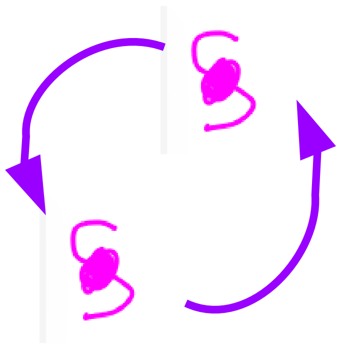
- 3PCF or higher nPCF contain primordial NG



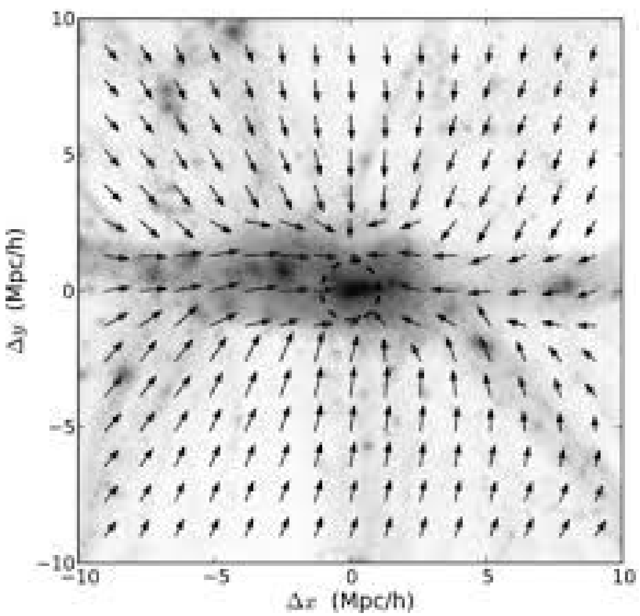
LSS: Perturbation theory and nPCF

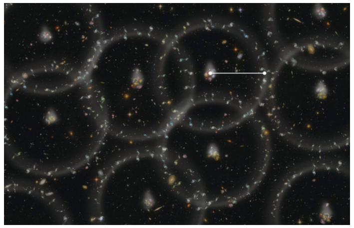
Matter density field

$$\delta = \delta^{(1)} + \delta^{(2)} + \ldots$$



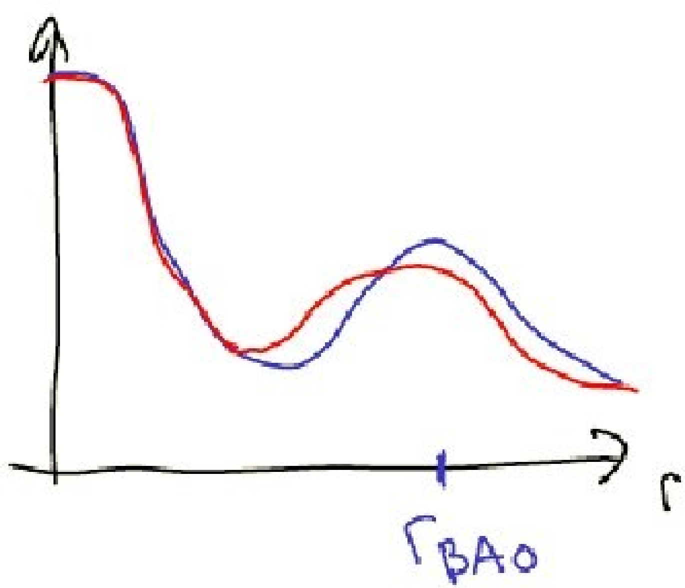
- Corrects 2PCF
- Correlates to other galaxies in the Hubble flow to give 3PCF
- Mixes with primordial NG

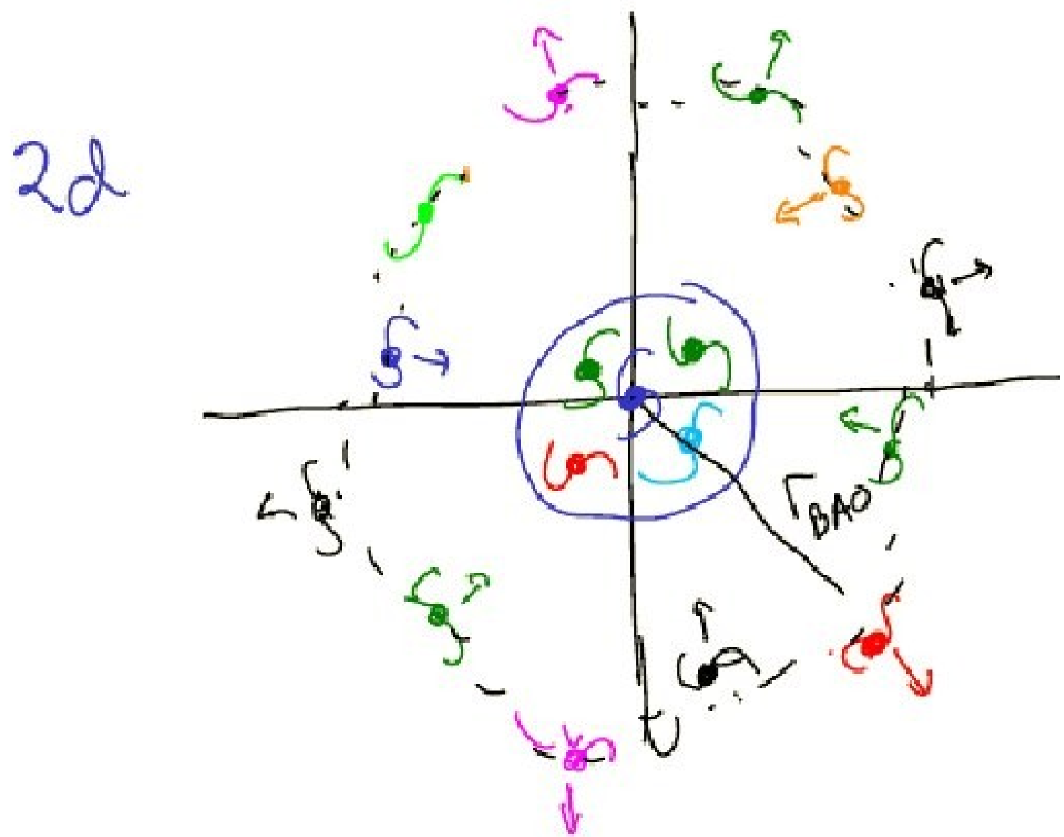




BAO signal in LSS

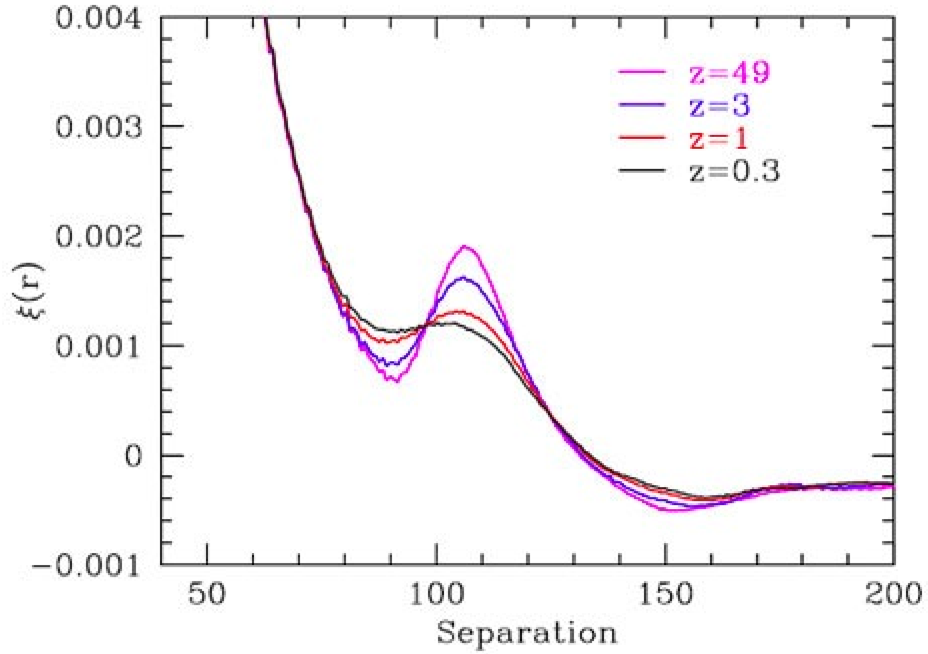
- 2PCF ➡ Non-linear mode evolution

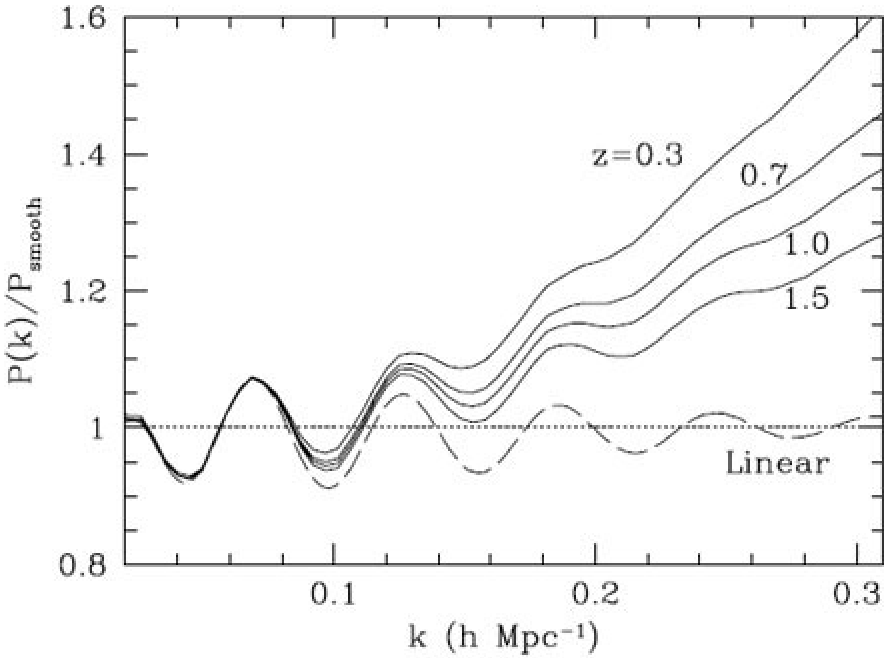




BAO signal in LSS

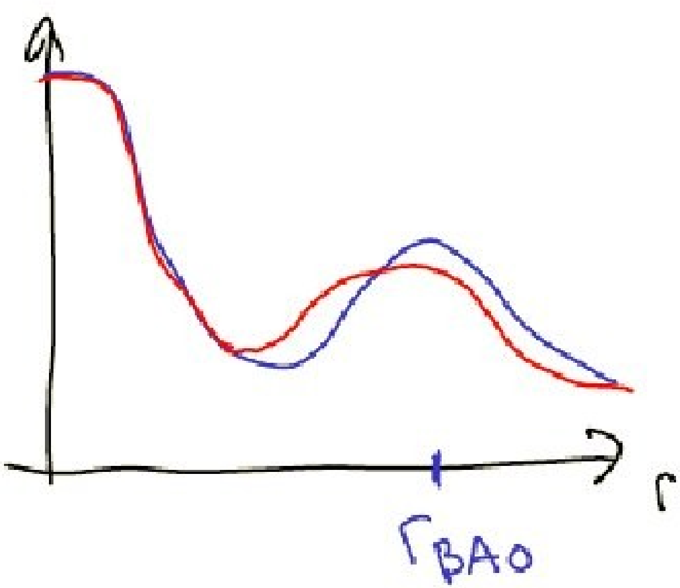
- 2PCF → Non-linear mode evolution

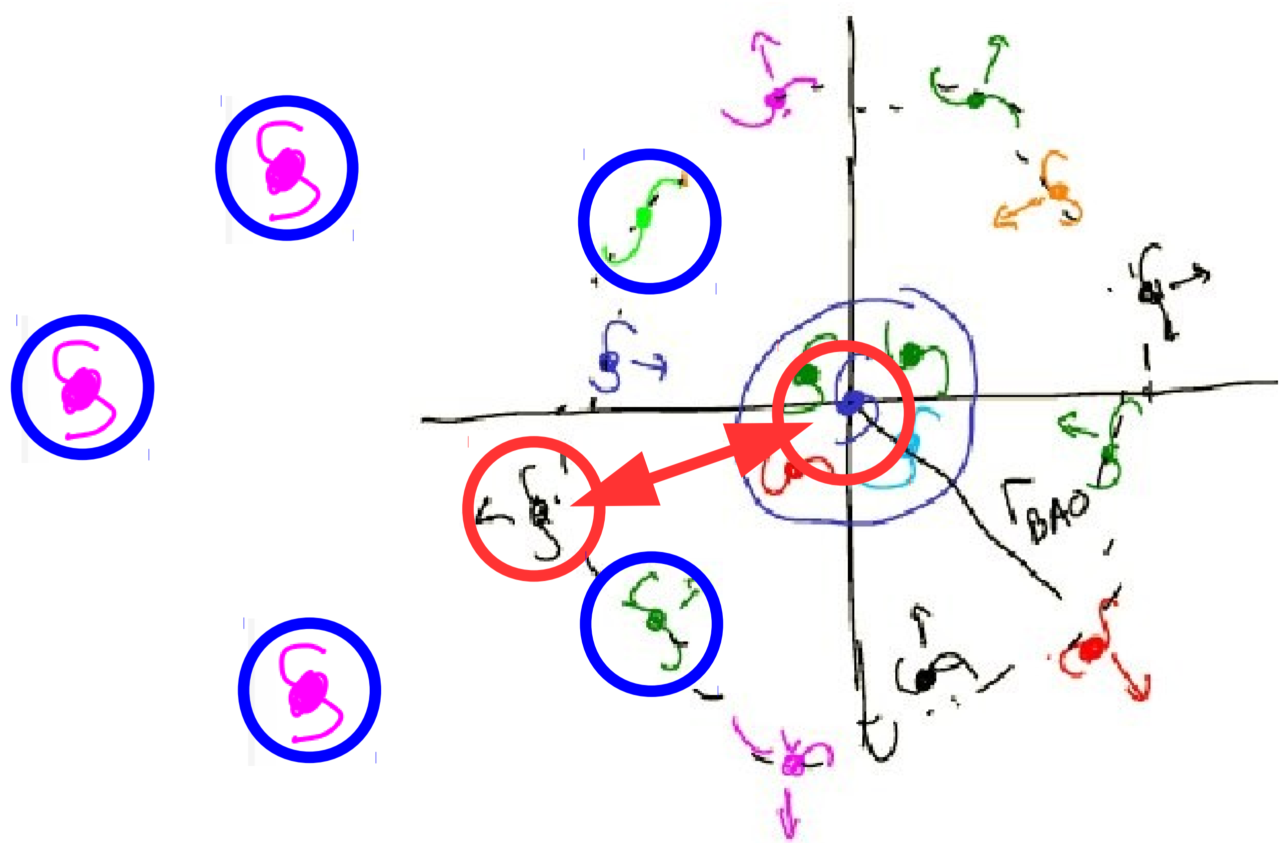




BAO signal in LSS

- 2PCF ➡ Non-linear mode evolution
  - It does contain the 3PCF, but *averaged*



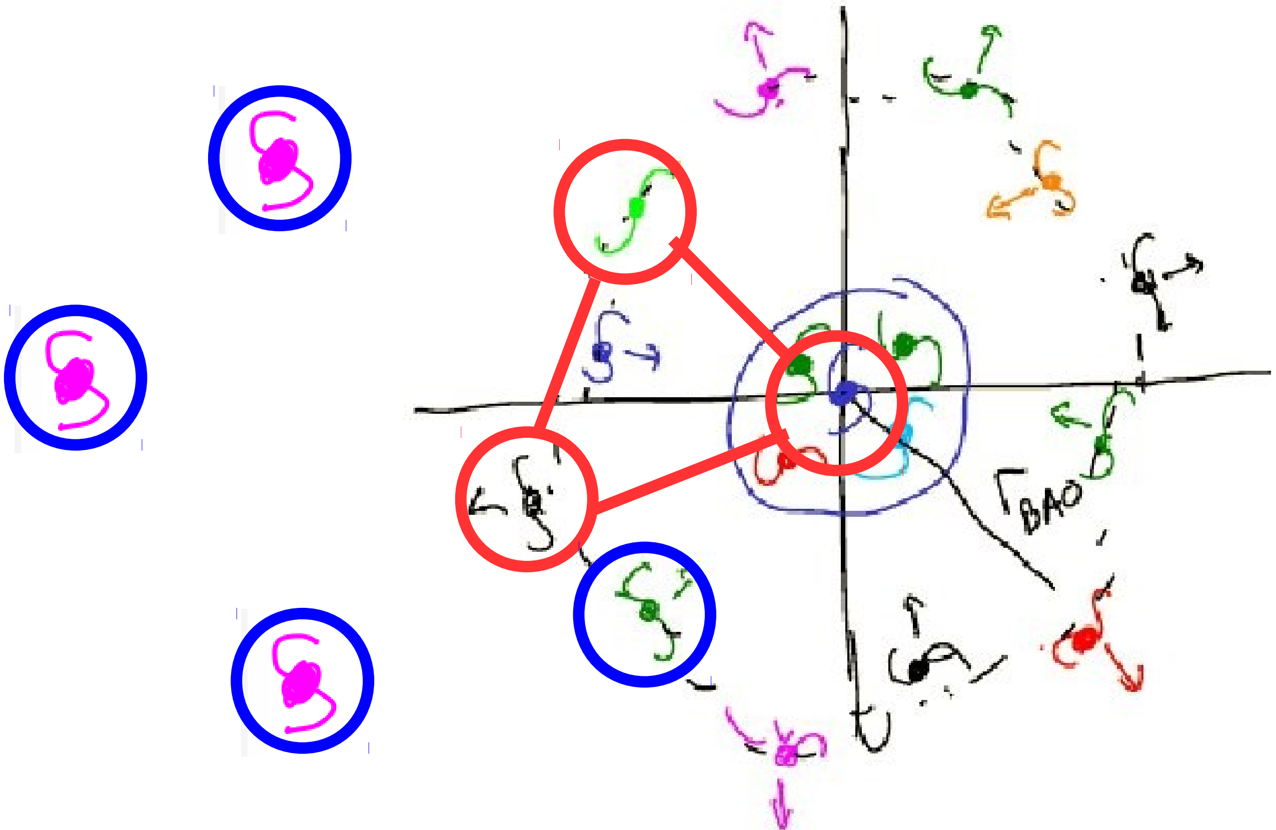


BAO signal in LSS

- 3PCF ➡ Explicit Non-linear evolution

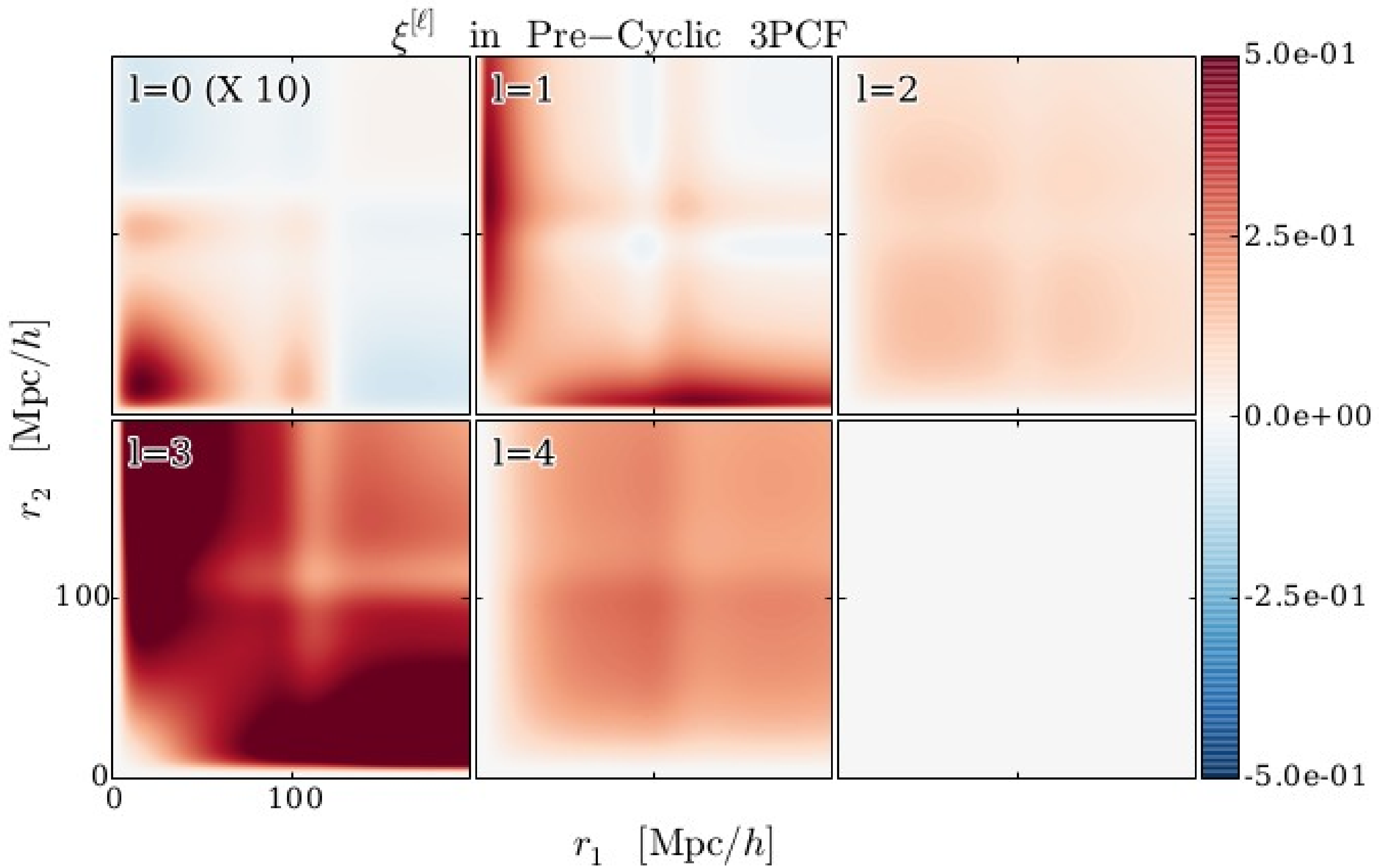
  Ask about the third galaxy!

  Averaged over higher point correlations!



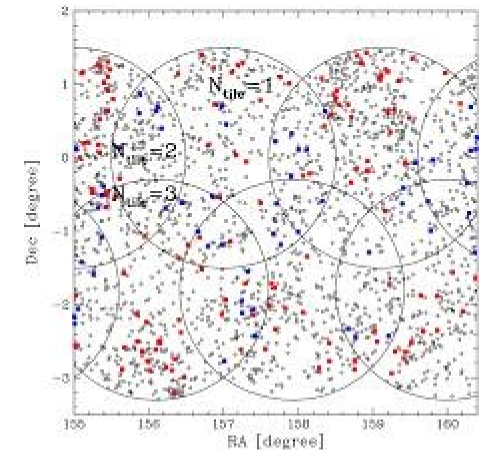
BAO signal in LSS

- 3PCF: how does it look? Legendre basis



Utility of 3PCF

- Break degeneracies
  - Observational systematics (fiber assignments DESI)
  - Galaxy-Dark Matter Bias
  - Galaxy-Dark Matter Velocity Bias
  - Primordial Non-Gaussianity
- Error for 2PCF
- Consistency conditions
- Cross correlation with other data (voids, lensing, etc.)
- Test gravity



$$< n+1 > \sim \mathcal{O} < n >$$

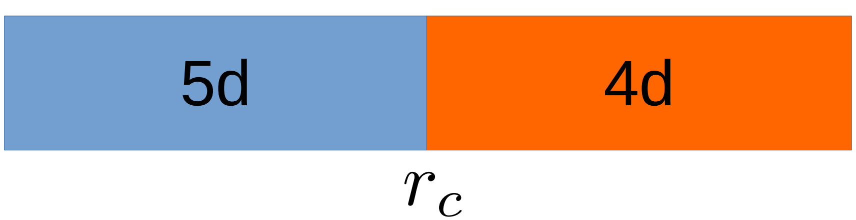
Challenges of 3PCF

- Systematics
- Computation time
- Signal to noise
- Modelling

Testing gravity

- What is not ΛCDM (or GR)?
  - Focus on subsample based on GR + 1 extra DOF
- Non-linear dynamics → clustering effect
- Hard to model analytically → simulations (see Vargas & Fromenteau)
- Choose two representative screening realizations

Vainshtein (nDGP)



$r_c H_0 = 1, 5$ → N1, N5

Chameleon (*f(R)*)

Hu-Sawicky

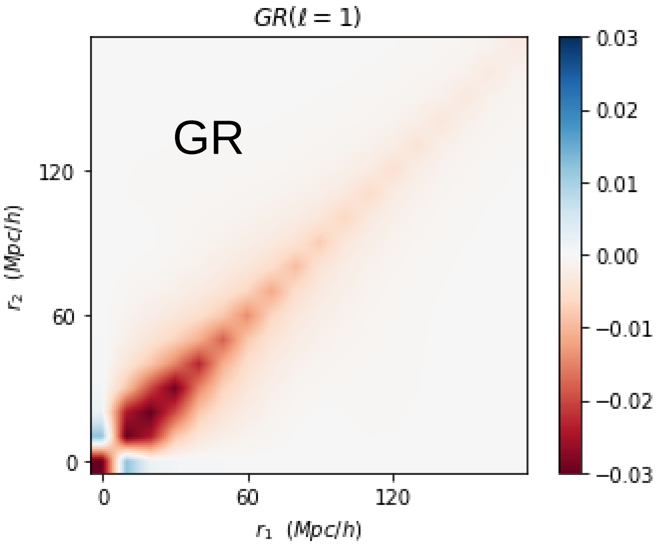
$$f(R) = -6\Omega_\Lambda H_0^2 + |f_{R0}| \frac{\bar{R}^2}{R},$$

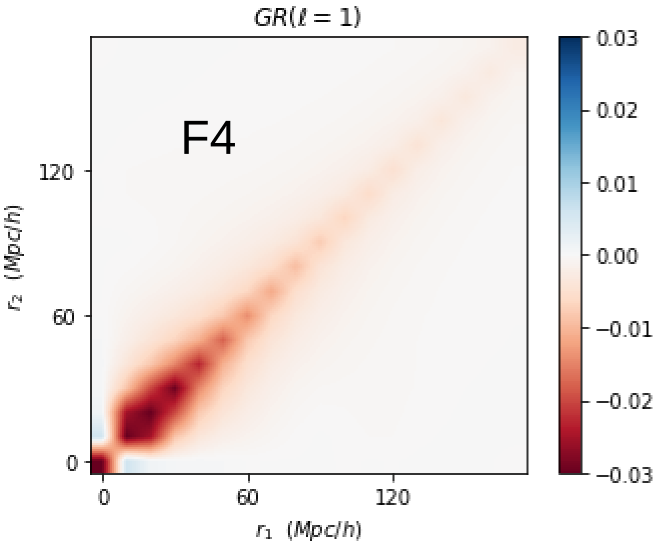
$$|f_{R0}| = 10^{-4}, 10^{-5}, 10^{-6}$$

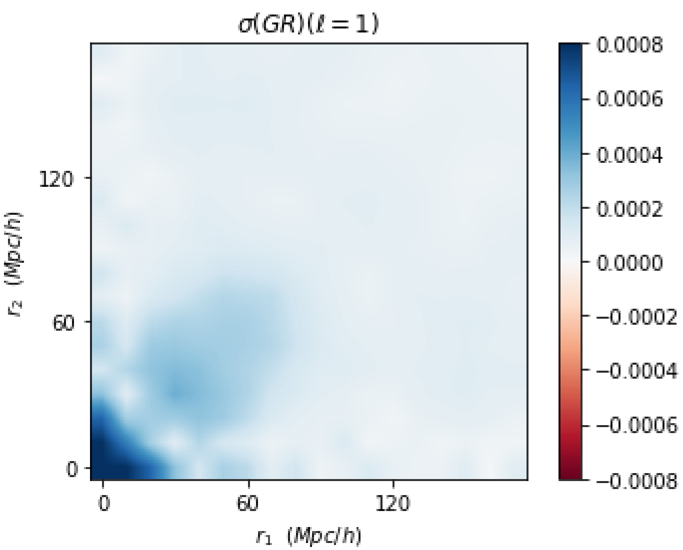
F4 F5 F6

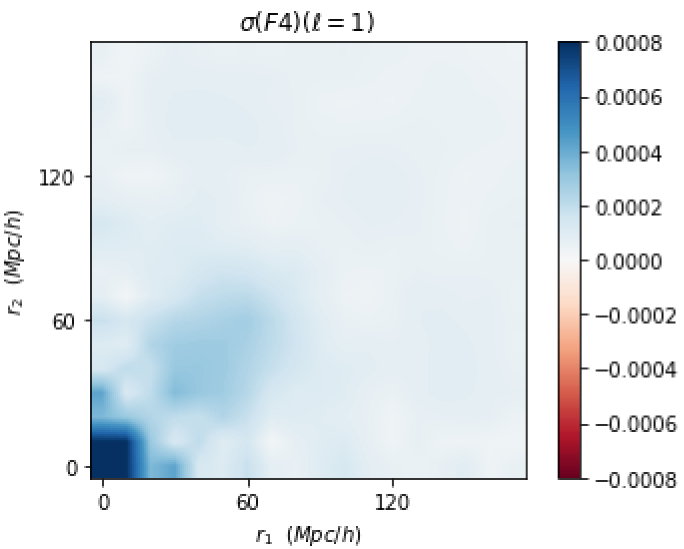
Isotropic 3PCF

- Mean + std over realisations



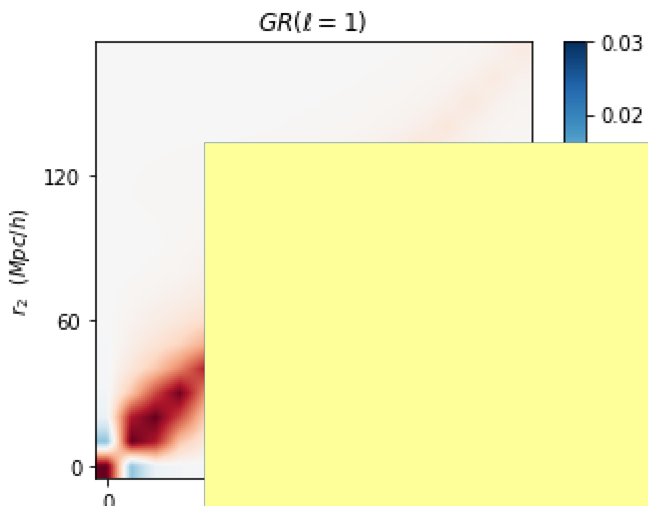


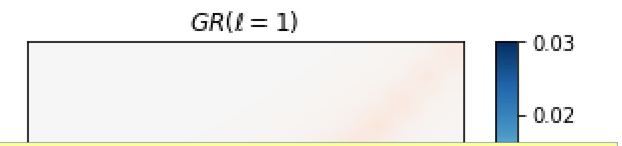


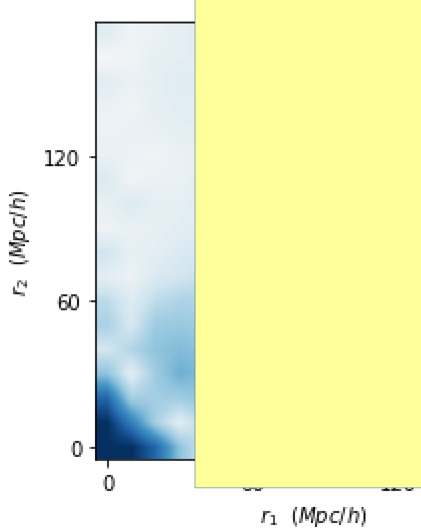


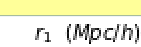
Isotropic 3PCF

- Mean + std over realisations

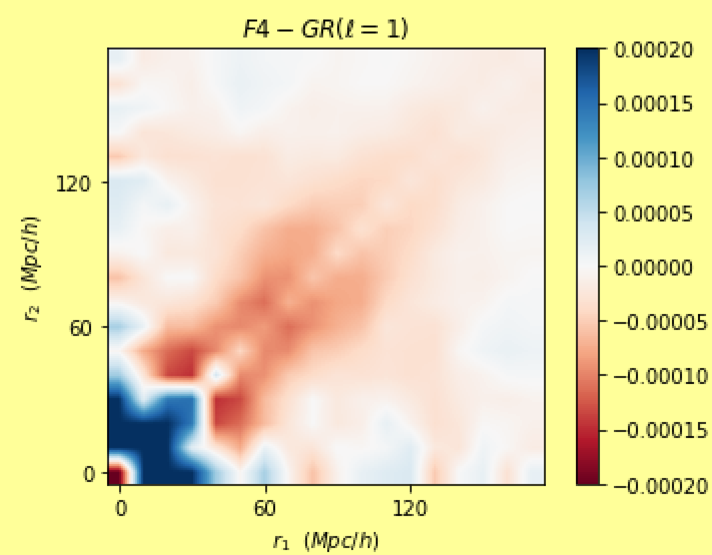








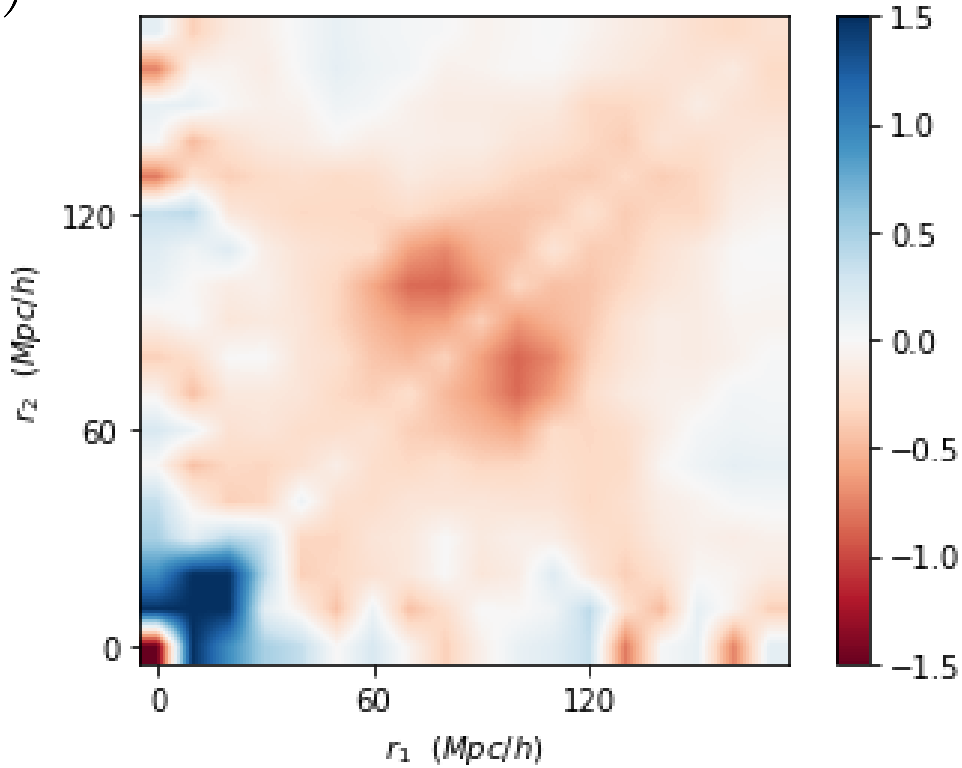
Relative difference

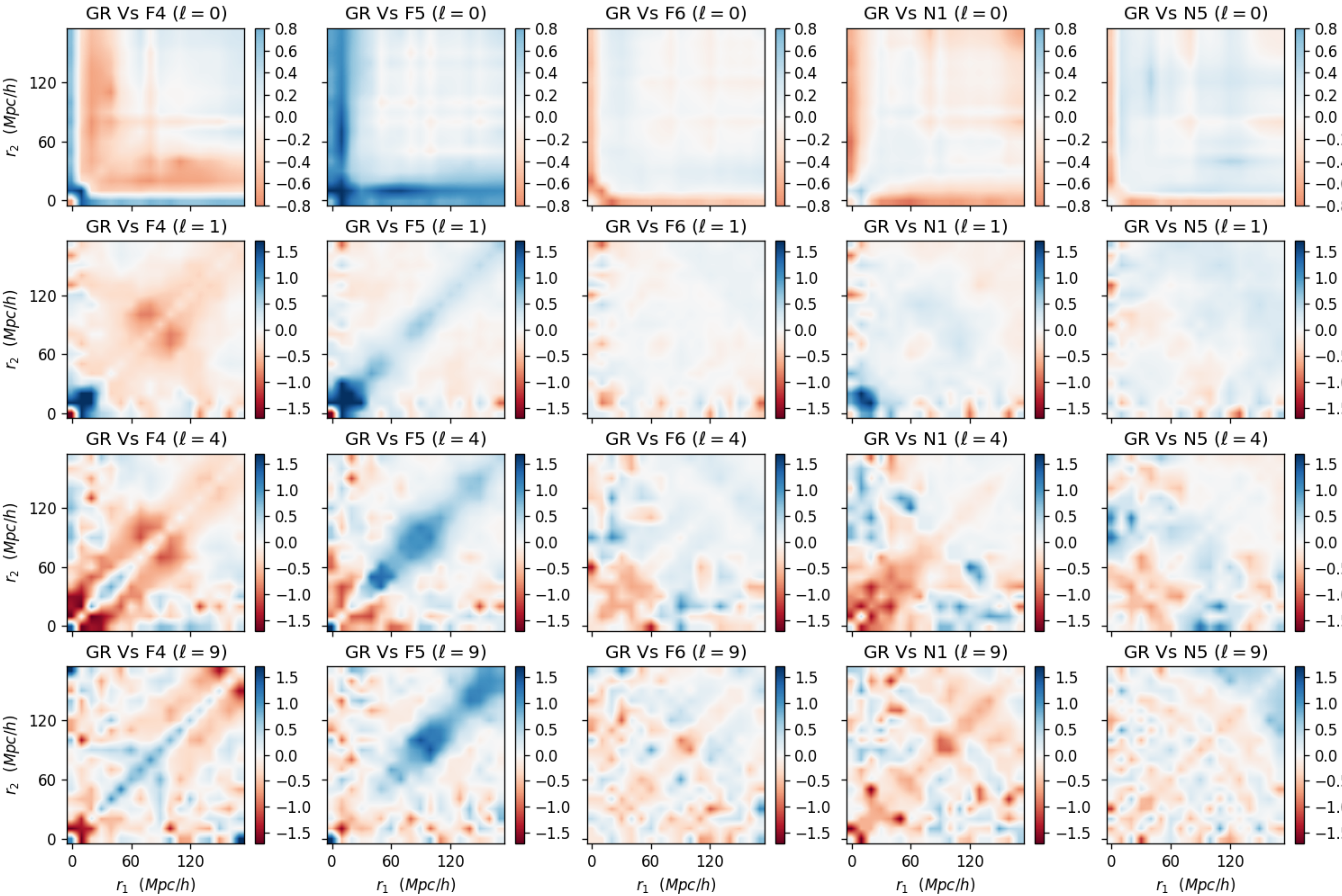


Isotropic 3PCF

- Considering realisation's dispersion

$$\Delta_\sigma \xi_\ell^{(3)} = \frac{\xi_\ell^{(3)}(X) - \xi_\ell^{(3)}(GR)}{\sqrt{\sigma_\ell^2(X) + \sigma_\ell^2(GR)}}$$




GR Vs F4 ($\ell = 0$)
GR Vs F5 ($\ell = 0$)
GR Vs F6 ($\ell = 0$)
GR Vs N1 ($\ell = 0$)
GR Vs N5 ($\ell = 0$)
GR Vs F4 ($\ell = 1$)
GR Vs F5 ($\ell = 1$)
GR Vs F6 ($\ell = 1$)
GR Vs N1 ($\ell = 1$)
GR Vs N5 ($\ell = 1$)
GR Vs F4 ($\ell = 4$)
GR Vs F5 ($\ell = 4$)
GR Vs F6 ($\ell = 4$)
GR Vs N1 ($\ell = 4$)
GR Vs N5 ($\ell = 4$)
GR Vs F4 ($\ell = 9$)
GR Vs F5 ($\ell = 9$)
GR Vs F6 ($\ell = 9$)
GR Vs N1 ($\ell = 9$)
GR Vs N5 ($\ell = 9$)
r_2 (Mpc/h)
r_1 (Mpc/h)
0
60
120
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
1.5
1.0
0.5
−0.5
−1.0
−1.5

DESI fiber assignments

- Martin White's ELG catalogs with fiber assignments