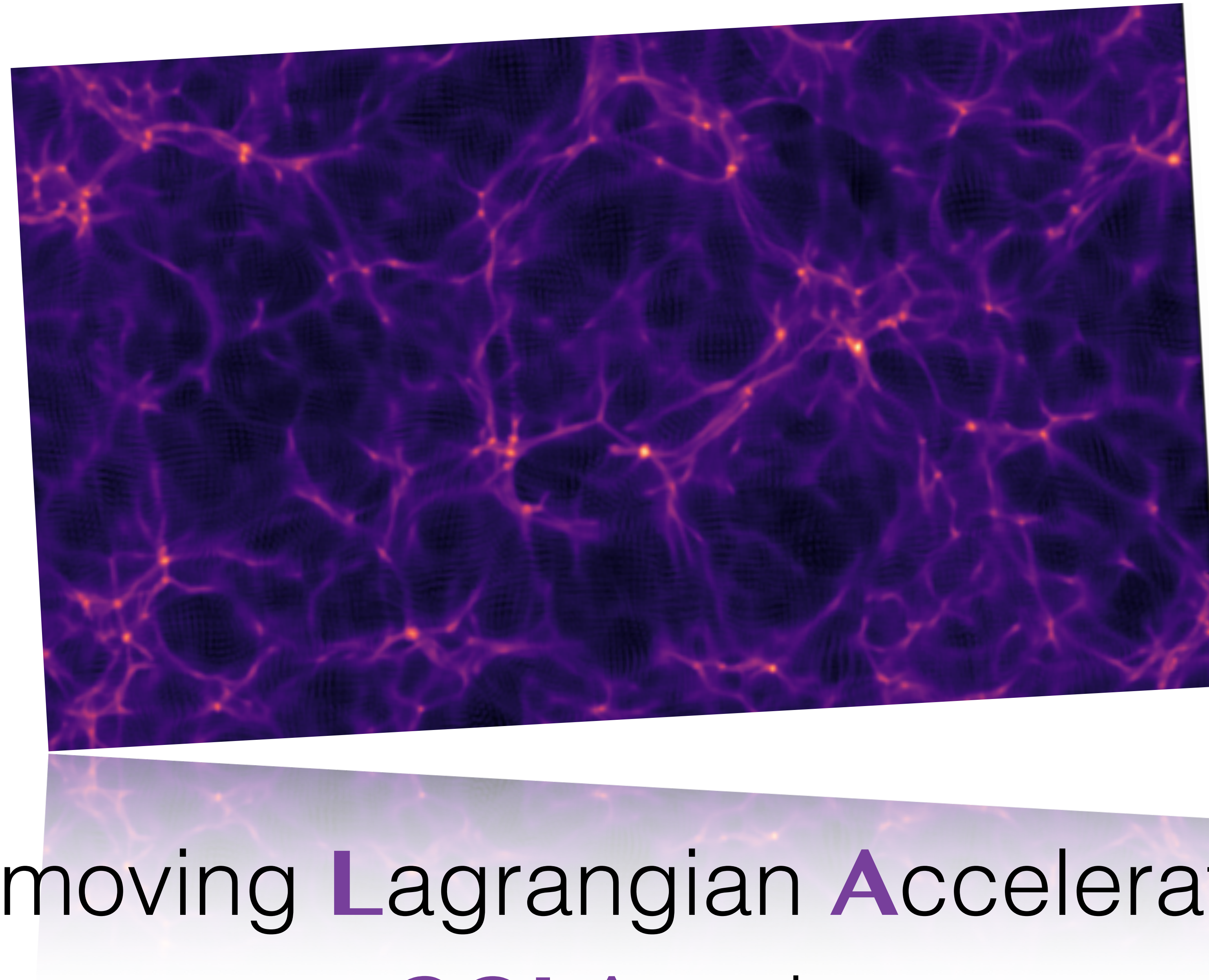


COmoving Lagrangian Acceleration : COLA code

Mariana Vargas Magaña (IF-UNAM)
Sébastien Fromenteau (ICF-UNAM)

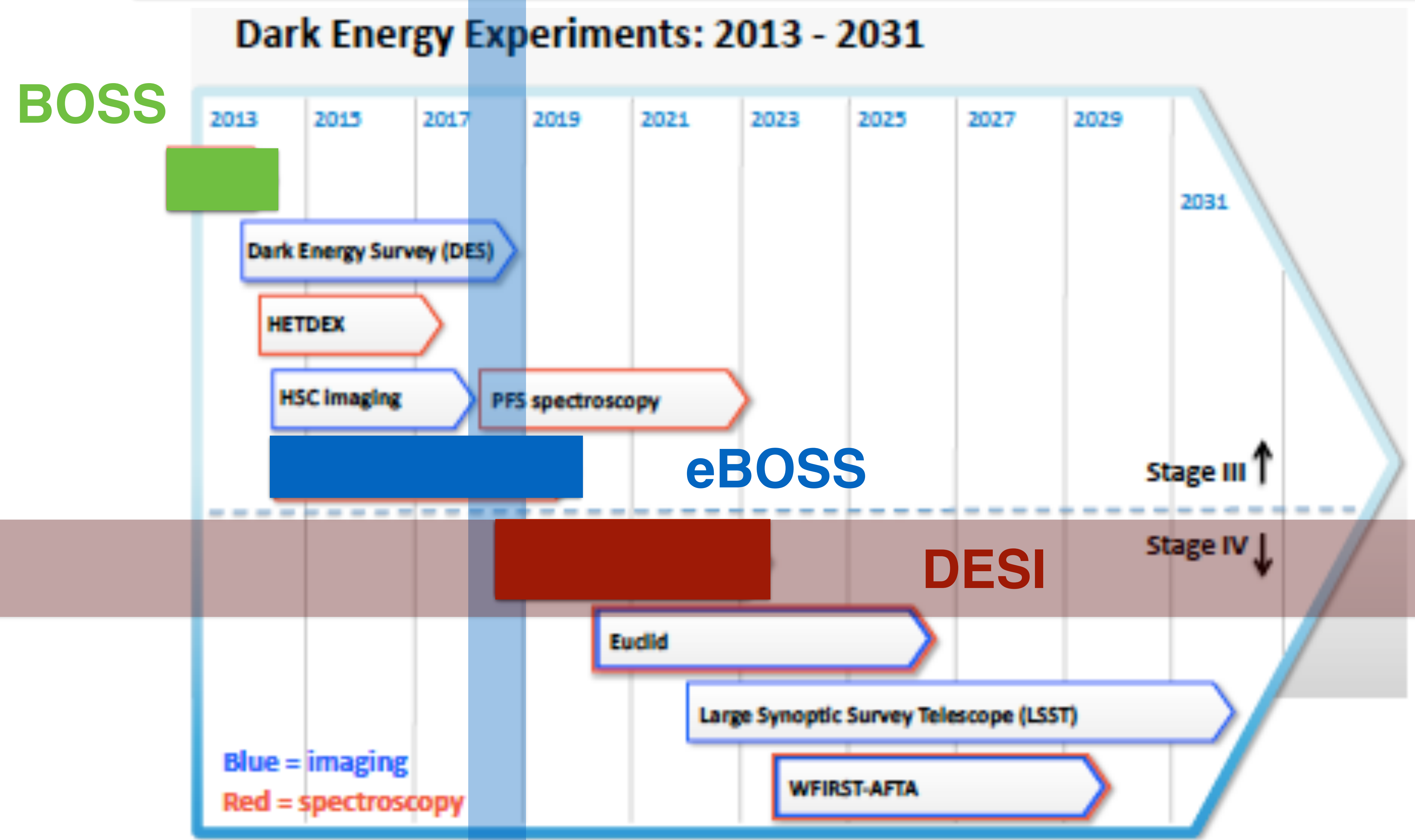
MEXPT-2 , Lunes 11 de Junio 2018

Outline

- **Cosmological simulations (for surveys)**
  - Simulations what for ?
  - Type of simulations
  - Fast Mocks: Specific case of covariance matrices

- **COmoving Lagrangian Acceleration code**
  - Standard leapfrog Kick-Drift-Kick
  - Modified leapfrog for COLA
  - Results and specifications

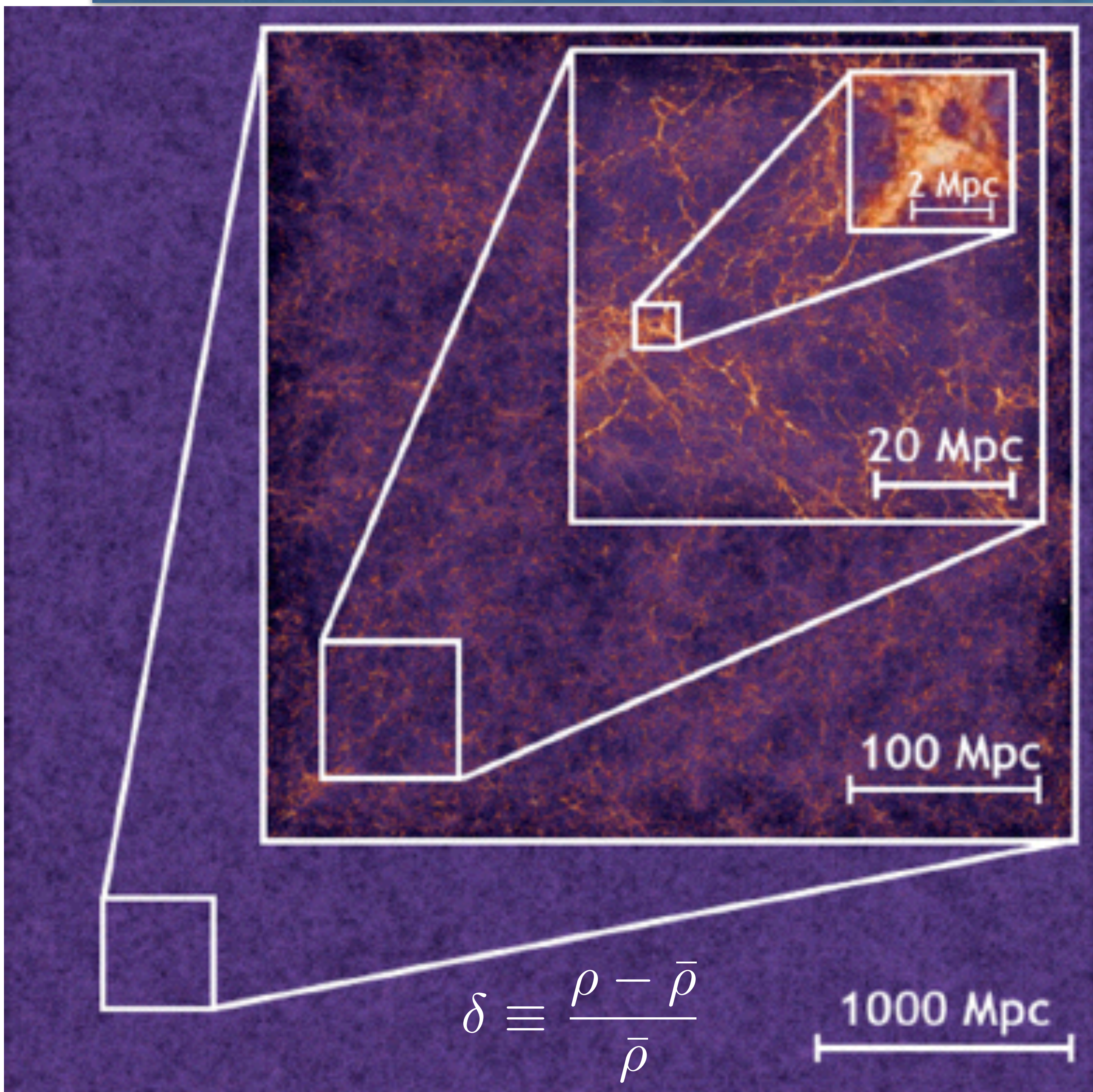
DE Experiments



Simulations what for?

- Testing Theory in the middly non linear regime
- Testing Methodology: Accounting Biases and Errors
- Testing Observational Sistematics
- Covariance Matrix

Scales in Cosmology



Millenium simulation

$\delta > 1$: Non-Linear

- **Simulations**

$\delta \sim 1$: Quasi-Linear

- **Simulations**
- **2-LPT**

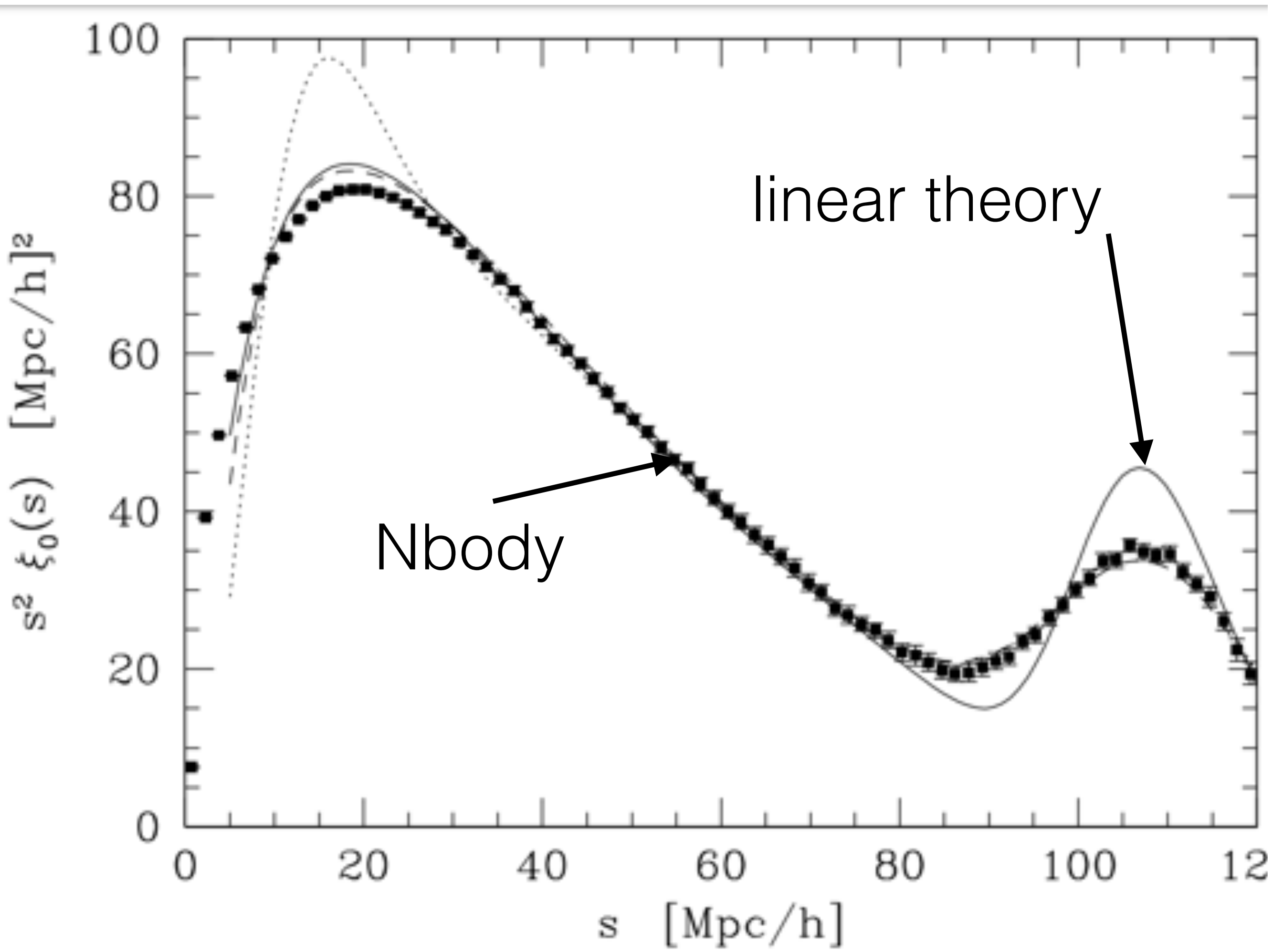
$\delta \ll 1$: Linear

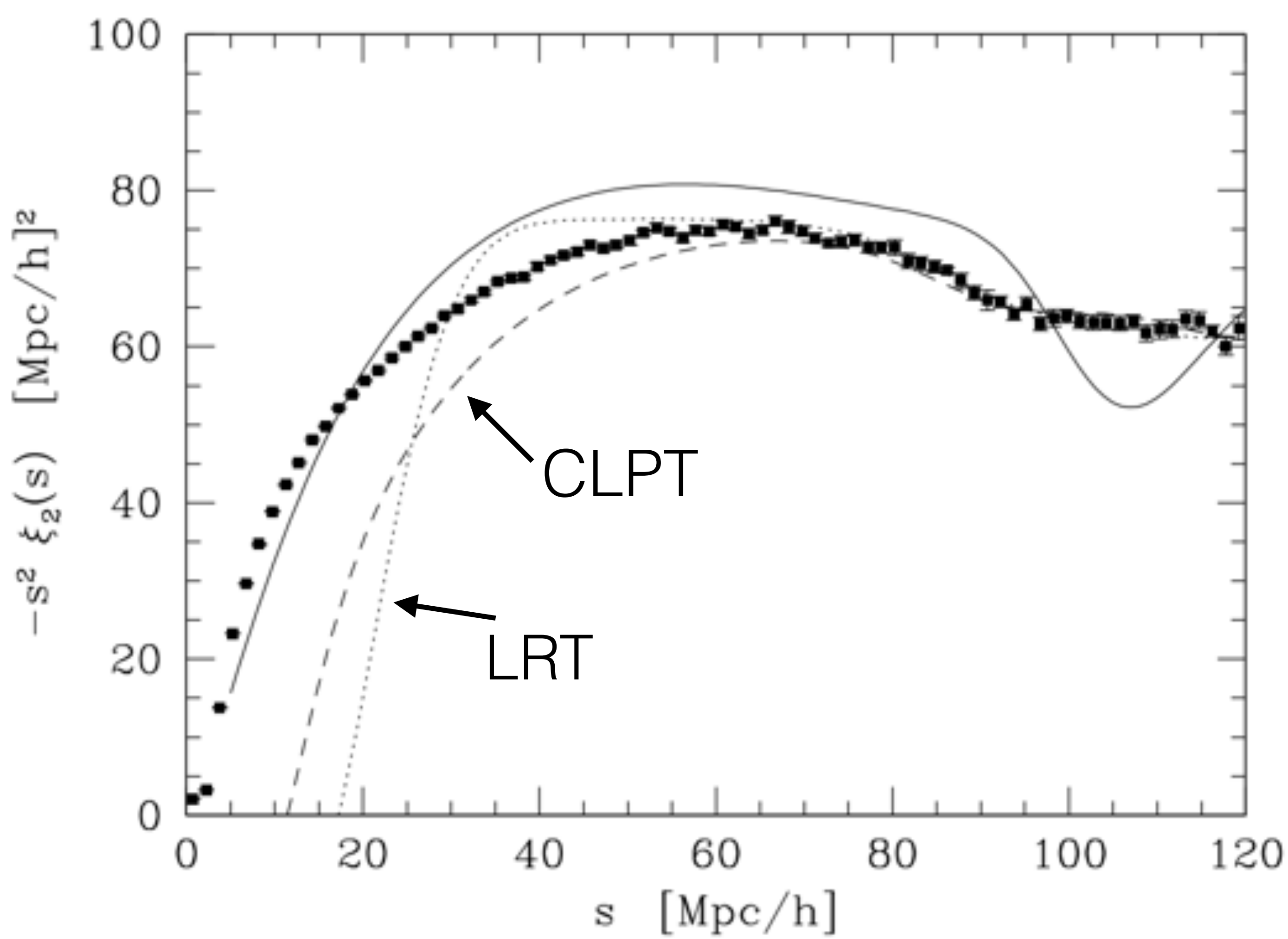
- **Simulations**
- **LPT** (Lagrangian Pert. Th)
- **Analytical approx.**

Simulations what for?
Testing Theory

Redshift space

z=0.5

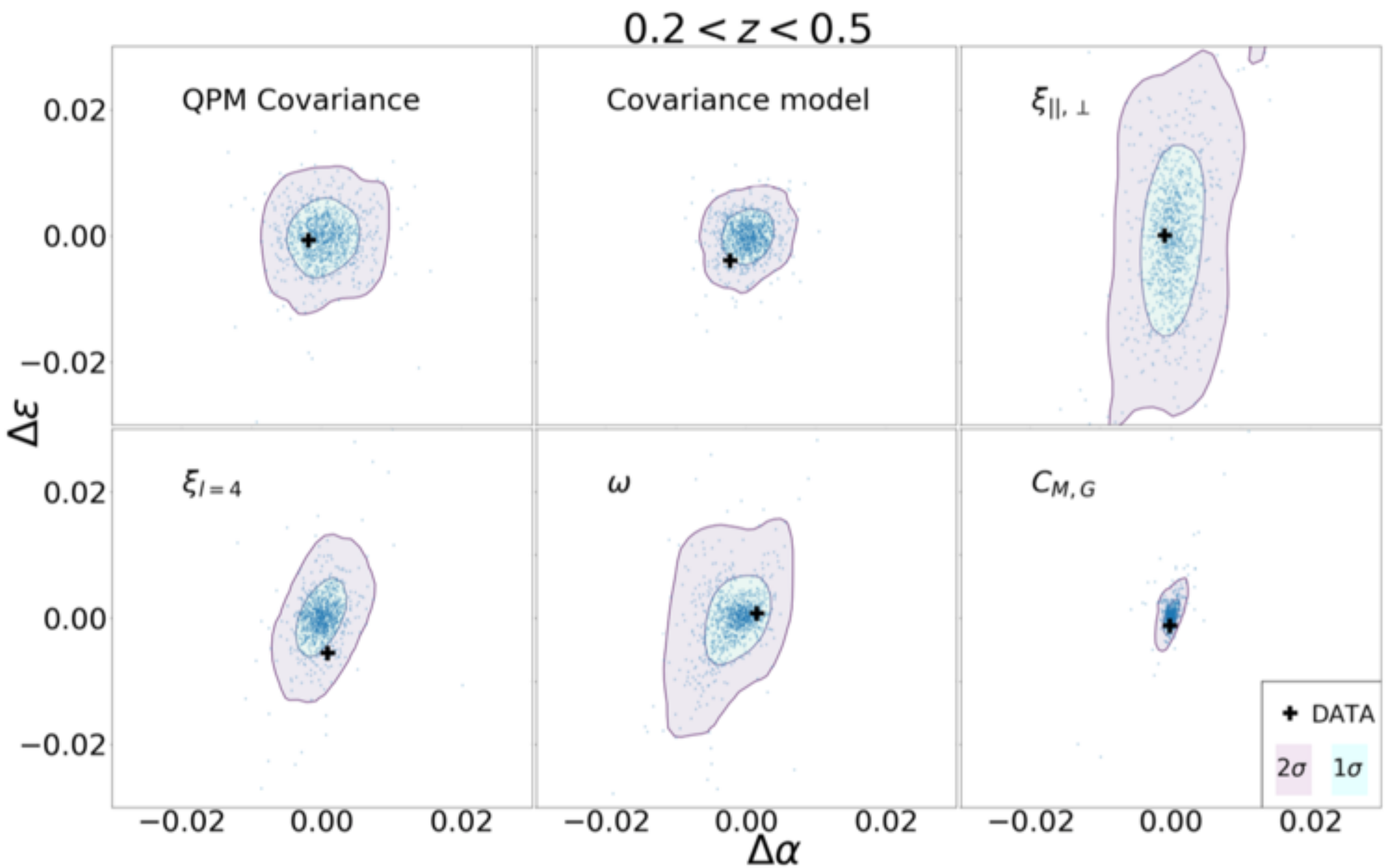




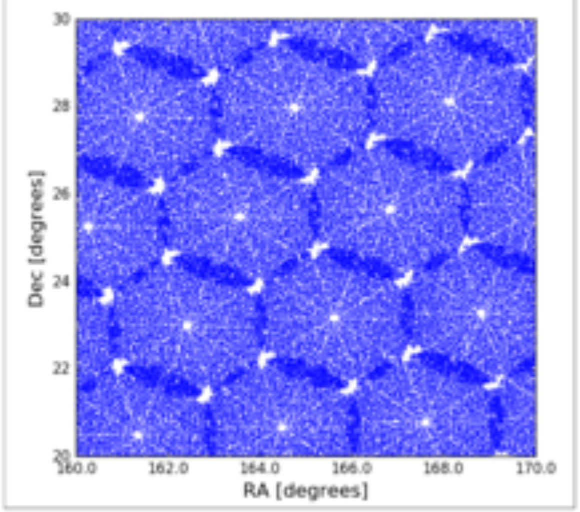
CLPT=Convolution Lagrangian Perturbation Theory, (Carlson 2012)

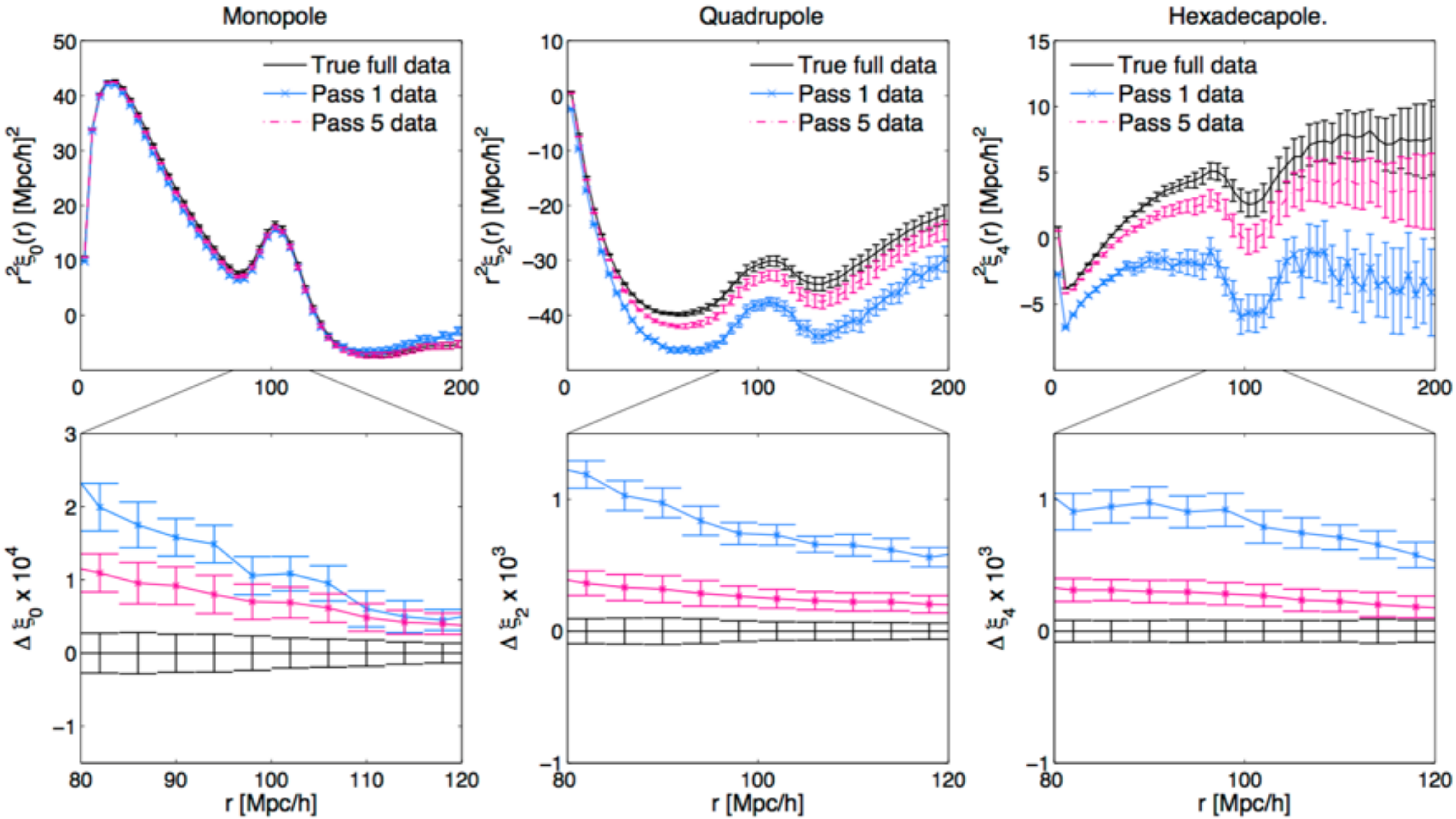
LRT=Lagrangian Resumption Theory, (Matsubara, 2008)

Simulations what for?
Testing Methodology (BAO,RSD,etc)



Simulations what for?
Testing Observational Systematics

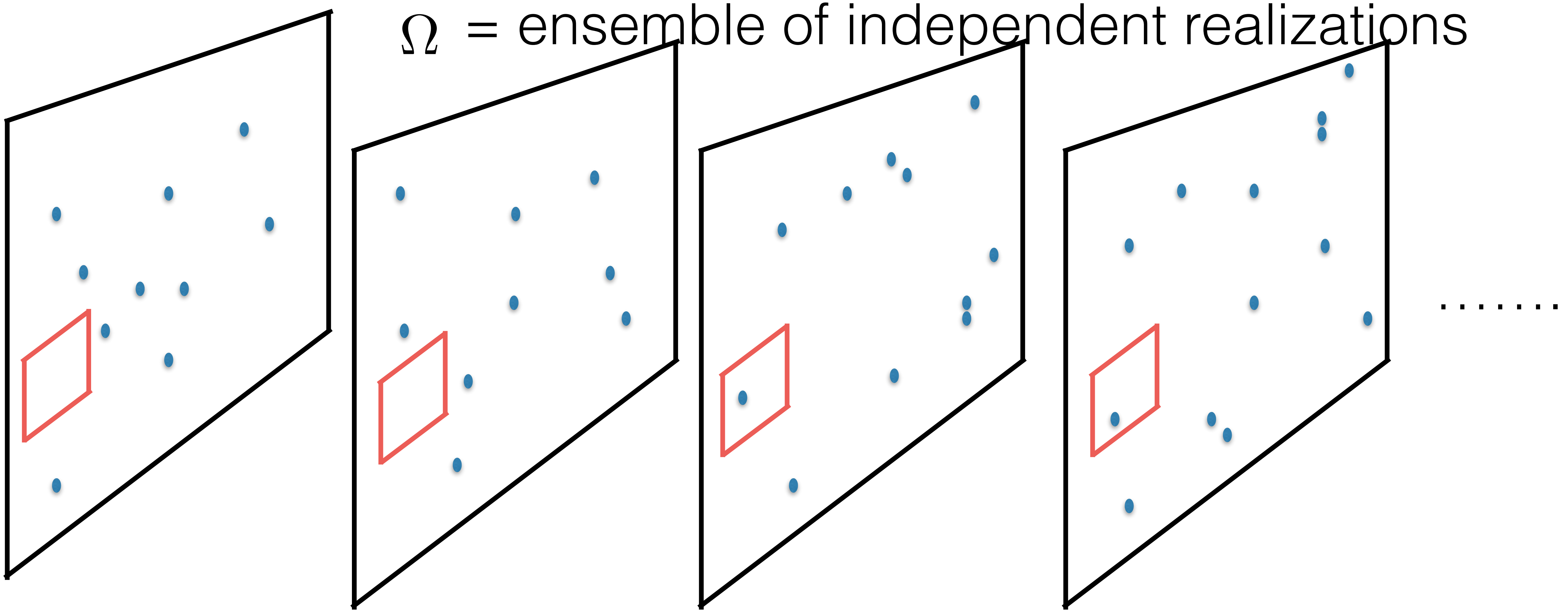




Simulations what for? Covariance Matrix

Statistically homogeneous and isotropic

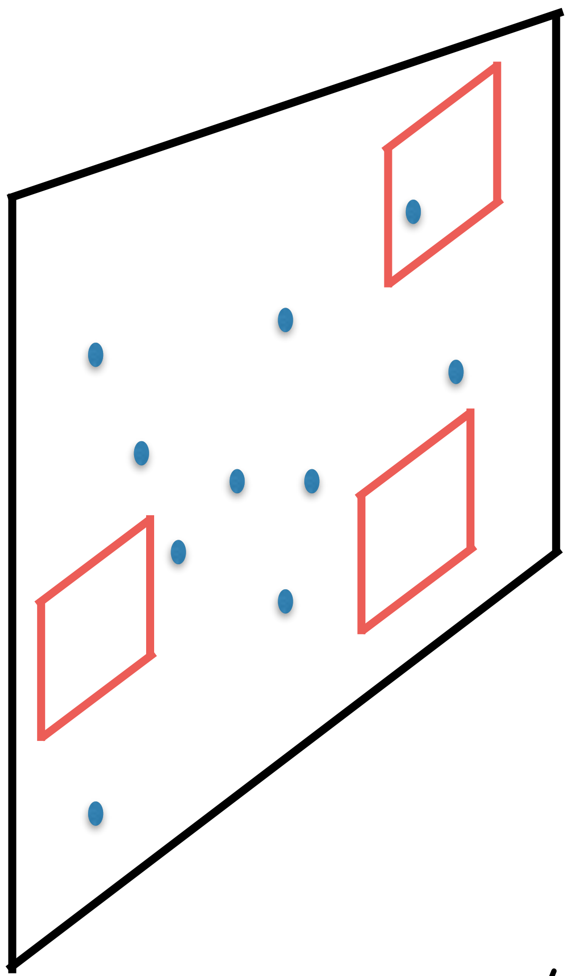
Ω = ensemble of independent realizations



$$\rho(\vec{x}, t) = \left\langle \frac{\#gal_i}{V} \right\rangle_\Omega$$ no depende de $\vec{x}$

Simulations what for?
Covariance Matrix

Statistically homogeneous and isotropic



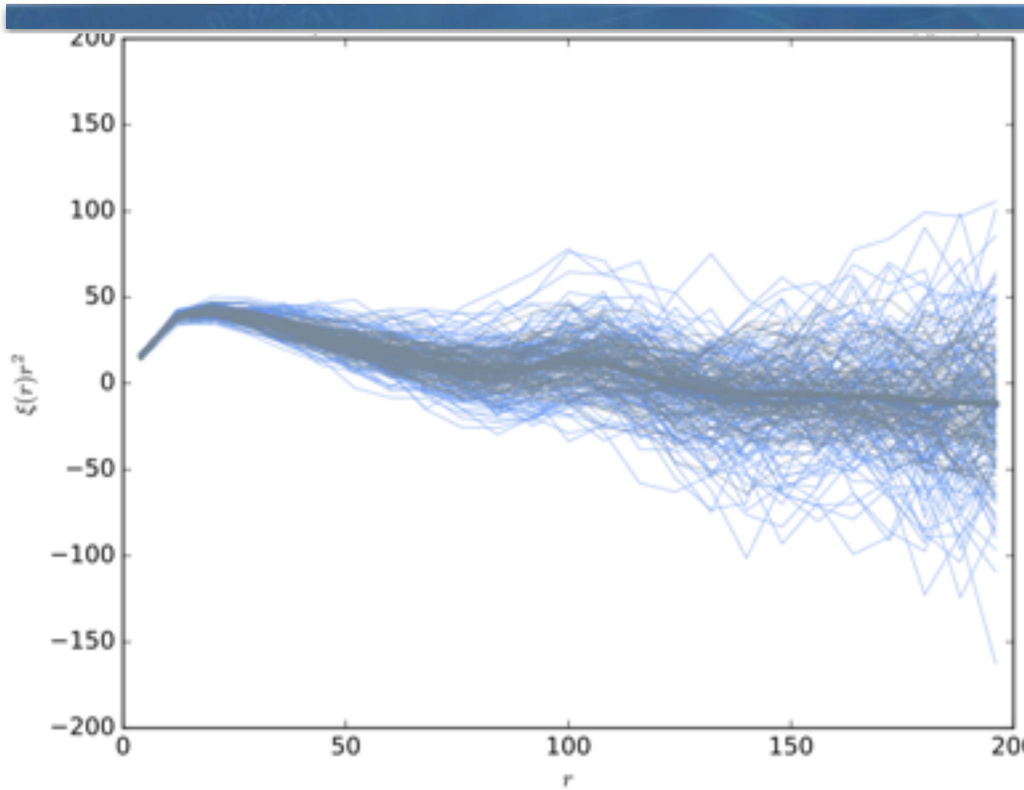
In practice, we have access to only one Universe realization

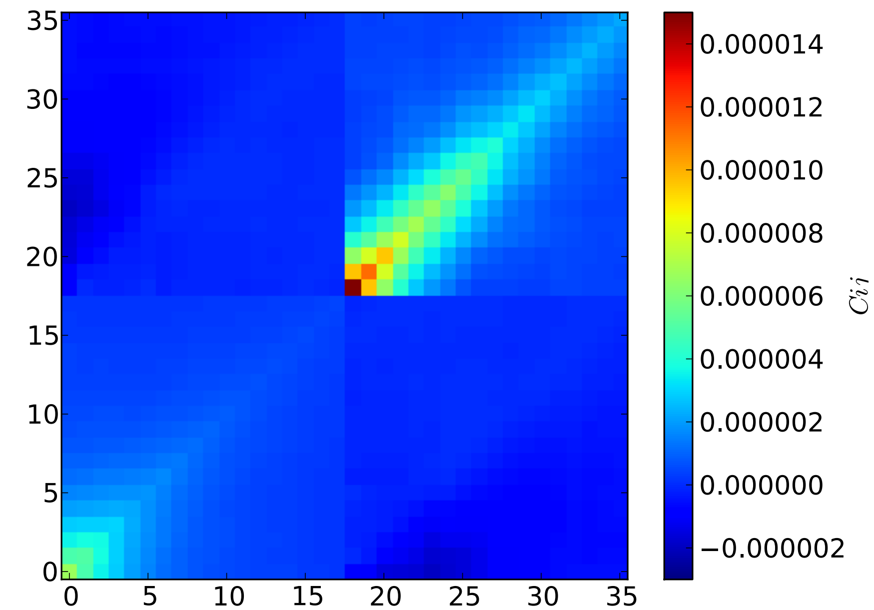
We considere distant enough points as independent realizations

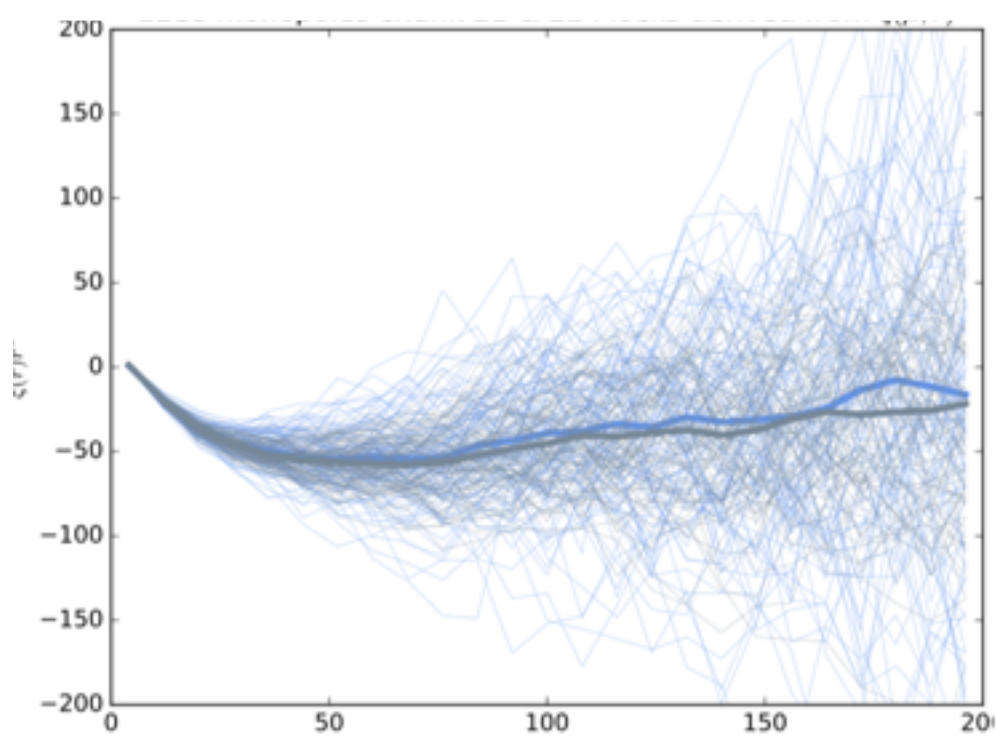
$$\rho(\vec{x}, t) = \left\langle \frac{\# gal_i}{V} \right\rangle_\Omega$$

La única forma de aplicar el principio cosmológico, y acceder a la información estadística, es a travez de la simulaciones….

Simulations what for? Covariance Matrix







La única forma de aplicar el principio cosmológico, y acceder a la información estadística, es a travez de la simulaciones….

Kinds of simulations

- **Hydrodynamic simulations** that in addition to gravitational instability and the expansion of the universe also account for the various baryonic effects.

- **N-body simulations:** provide the most accurate way of linking cosmological parameters (Ωm, wo, wa, etc.) to the clustering of matter, haloes, and galaxies at various redshifts, but are computationally expensive to produce in large numbers.

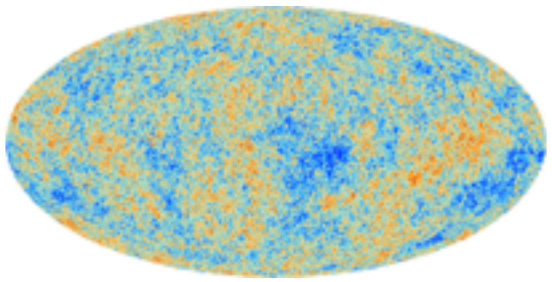
- **Fast simulations** can be produced in large numbers but are less accurate on small scales. They need matching N-body simulations to calibrate some of the simulation parameters.

- **Controlled input simulations** based on standard log-normal procedure can be made to mimic an arbitrary clustering signal to a very high precision. They are very inexpensive to produce in large numbers.

Códigos Híbridos

- **N-body.** In this approach, the forces on each particle are directly computed by accumulating the contributions of all remaining particles.
- **P-M codes.** Use a mesh for the density and potential. Solve the Poisson equation using the density field estimated with current particle positions, Advance momenta using the new potential, Update particle positions using new momenta. PM has been criticized for failing to reproduce the linear theory growth at large scale as the number of time steps is reduced.
- **Particle-Particle-Particle-Mesh (P3M Codes):** split the inter particle force in 2 components, a long range force calculated in a grid-based method; and a short range part summed directly from contributions of nearby particles.
- Others: AP3M scheme

Simulation Standard Scheme



2-LPT

CMB statistics

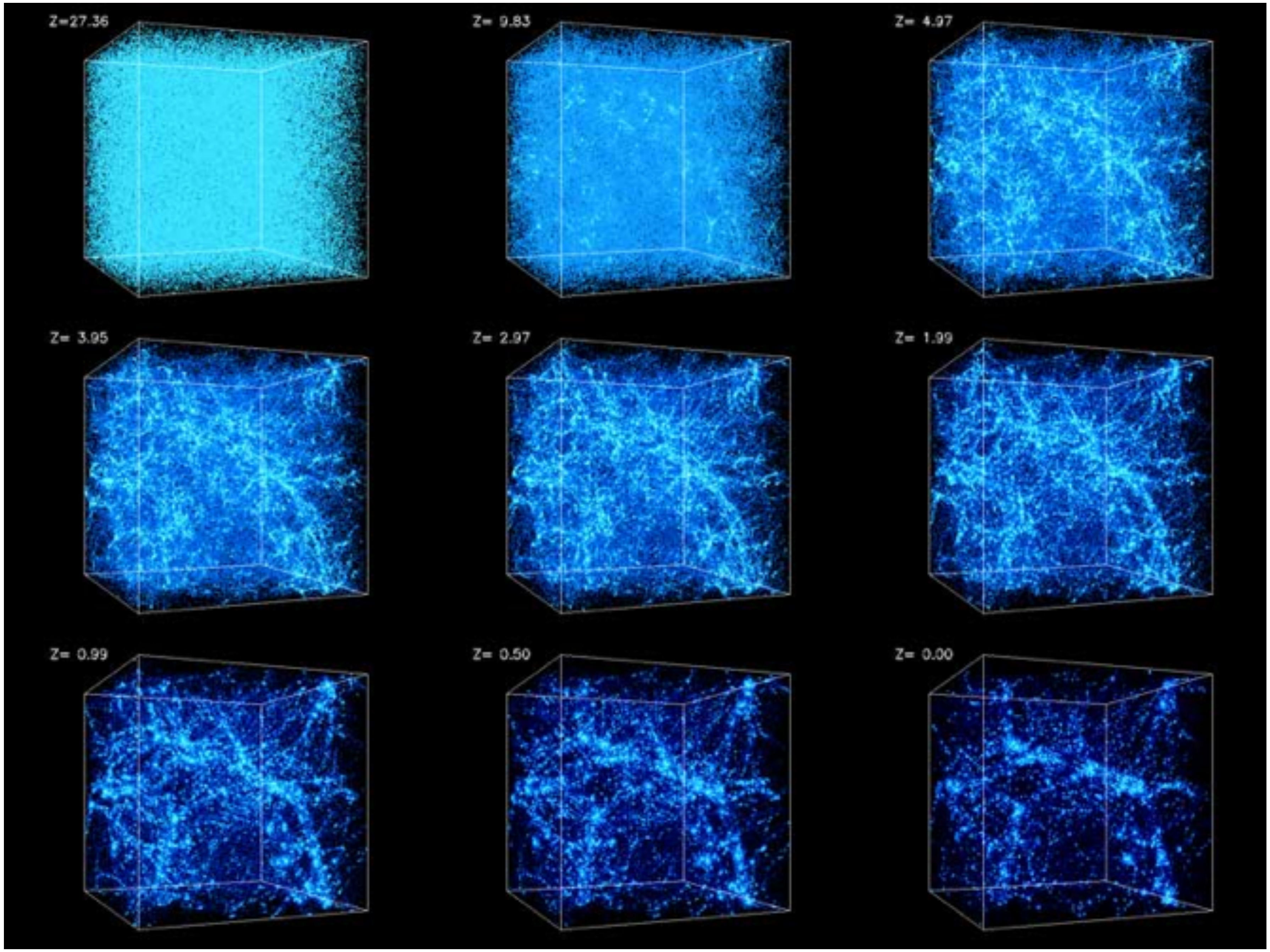
$$z \sim 1100$$

$$\delta \sim 10^{-5}$$

2-LPT = 2nd order Lagrangian Pert. Th.

P^3M = Particle-Particle Particle-Mesh

PM= Particle-Mesh



N-body / P^3M / P-M

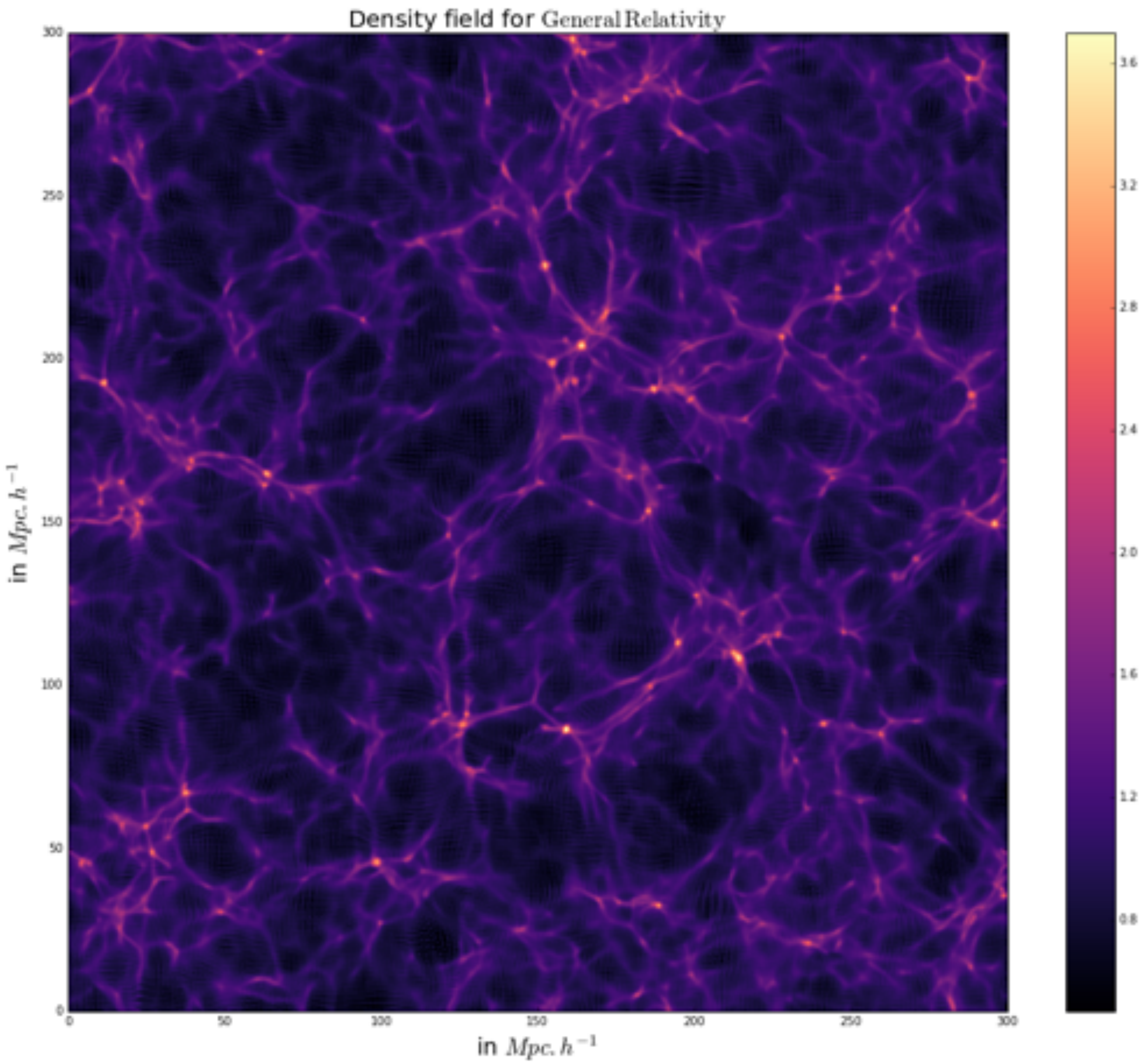
Populate DM Simulations

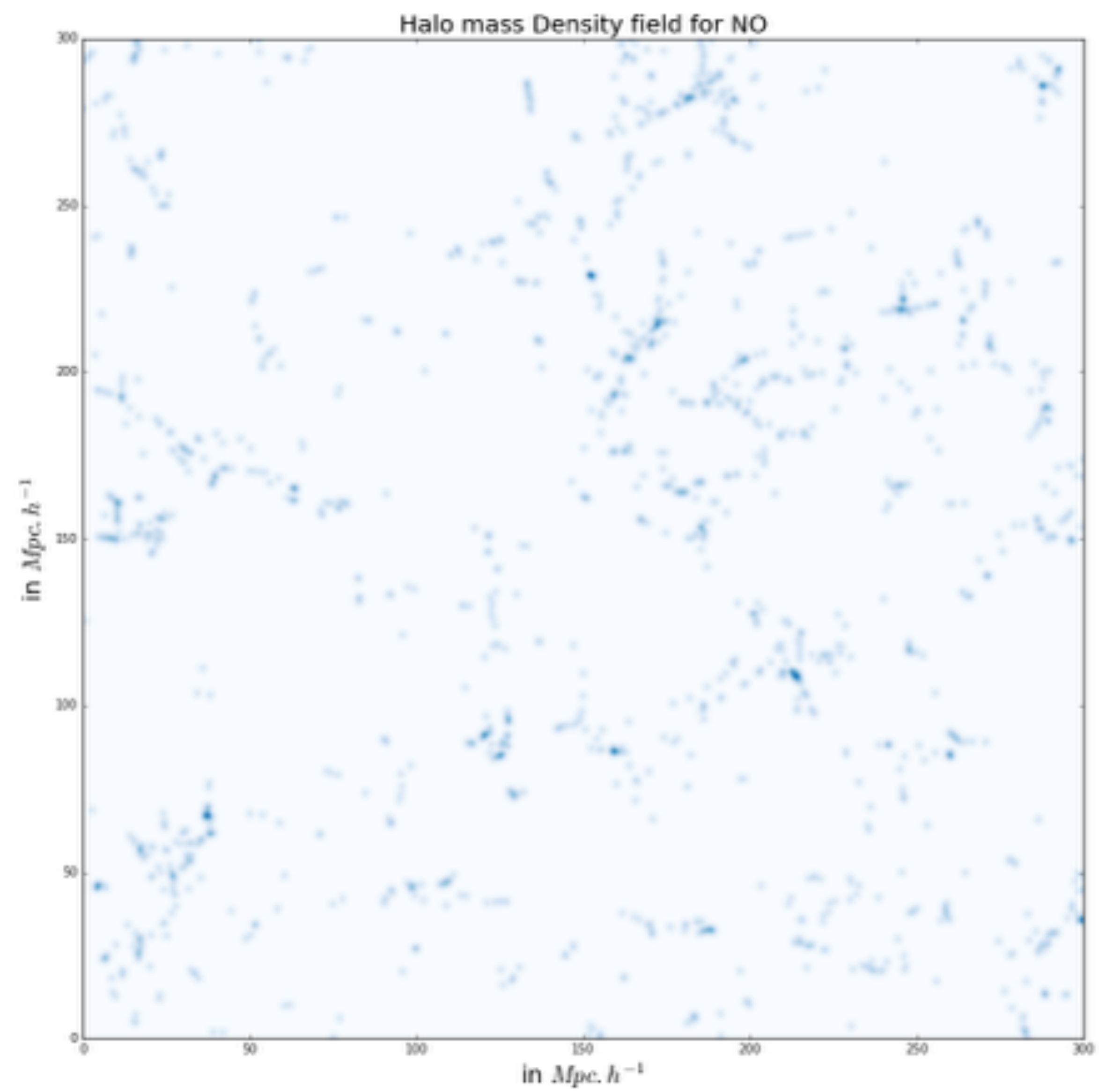
We do not spoke about hydro simulations which allows to take into account baryonic fluids and most of the physics effects

We generally run DM simulations because it is much simpler and much less CPU/memory consuming.

But we do not observe directly DM, so we need to populate the DM simulations with observable tracers.

From particles to halos





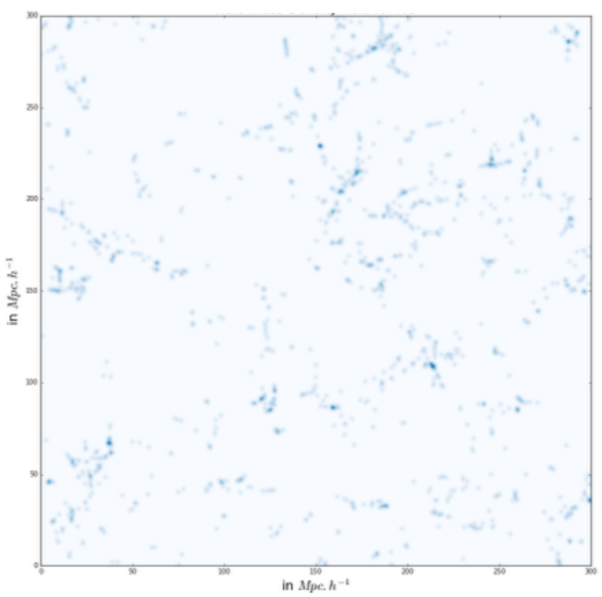
halos~regiones cuya densidad es 200 veces mayor que la densidad media

Halos to Galaxies

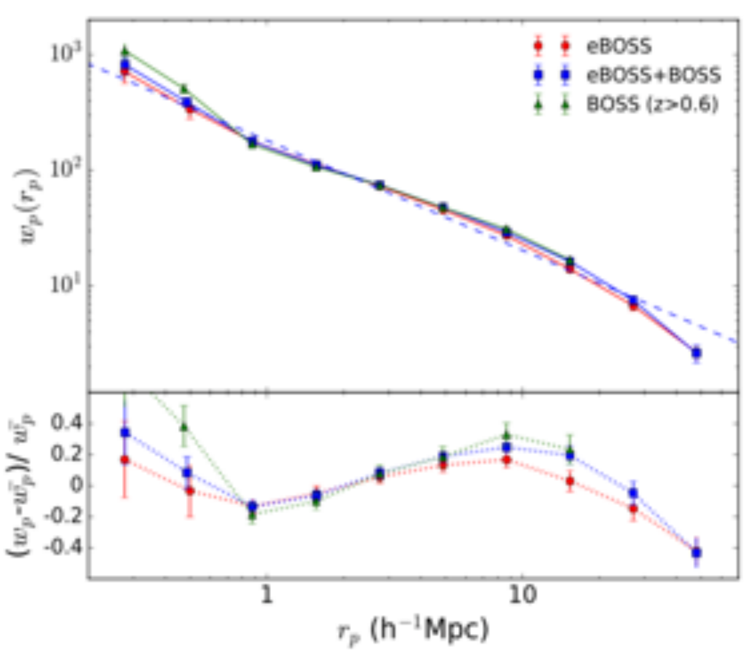
Halos

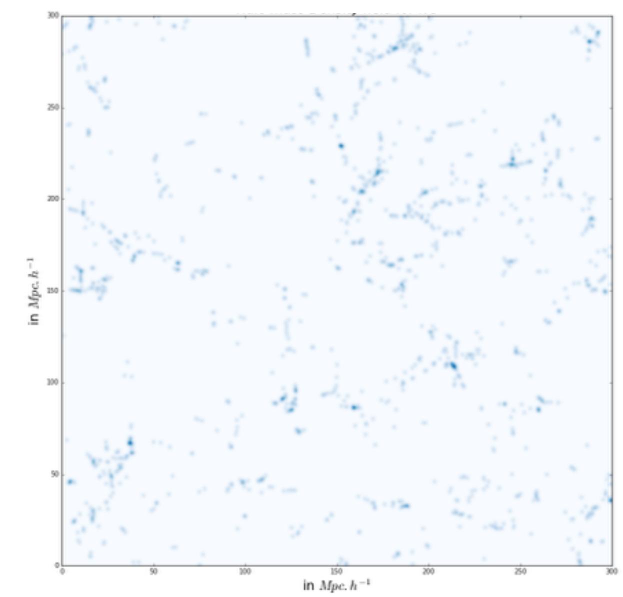
HOD

Galaxies



$$P(N|M)$$

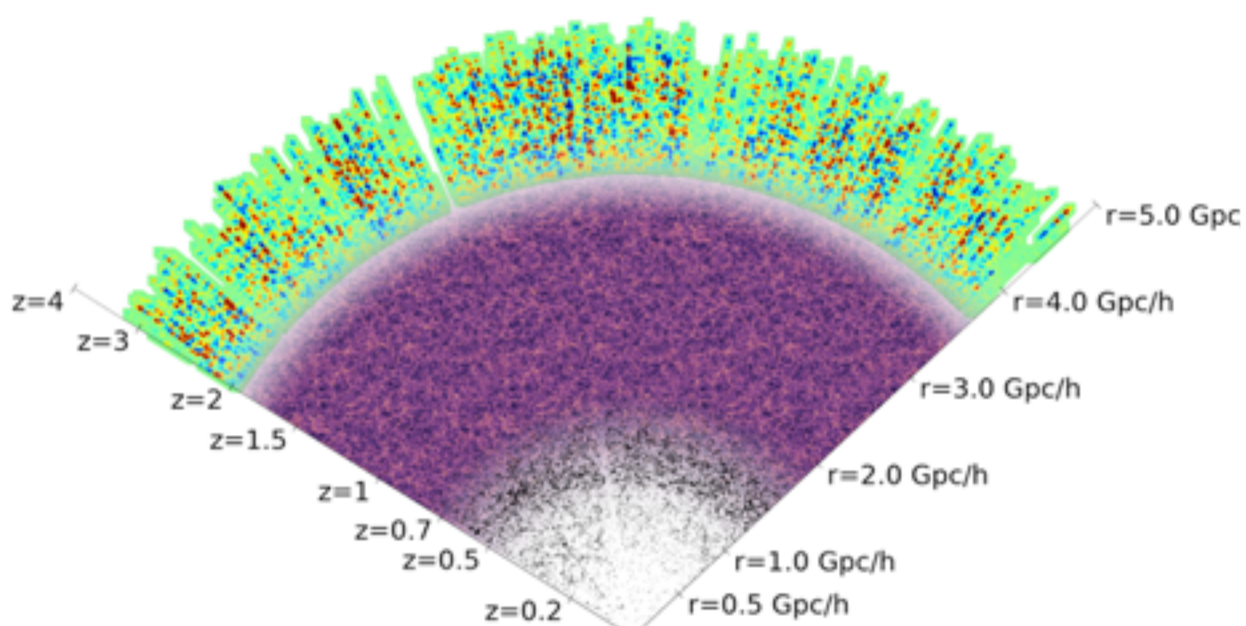




A cada survey (función de selección) corresponde una HOD

$$w_p(r_p) = 2\int_0^\infty d\pi \xi(r_p,\pi).$$

Important Parameters for surveys:



1) Volume

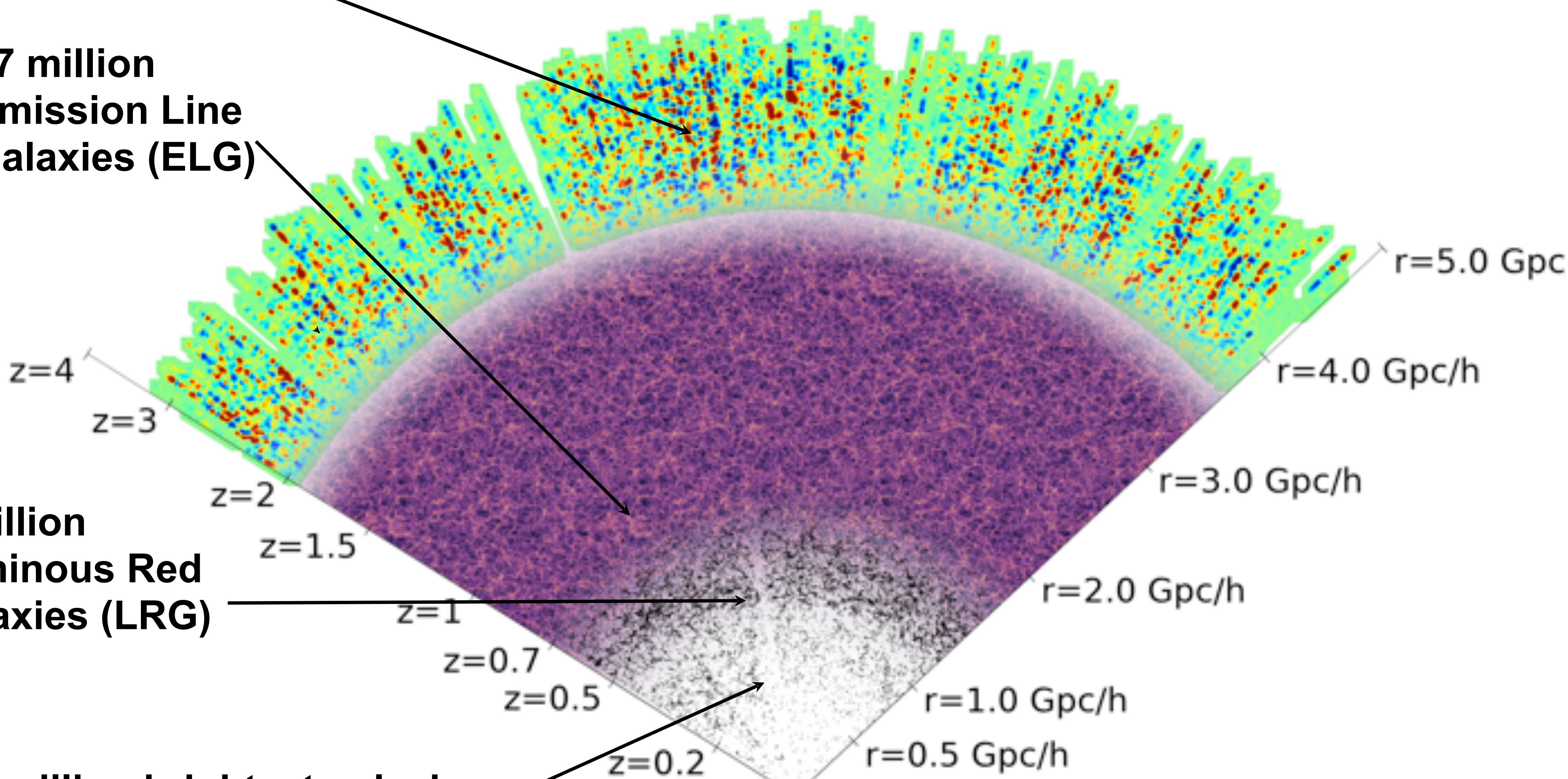
2)Mass Resolution

Survey Example: DESI

2.4 million QSO

17 million
Emission Line
Galaxies (ELG)

4 million
Luminous Red
Galaxies (LRG)



10 million brightest galaxies

Important Parameters for surveys: Volume

z	$nP_{0.2,0}$	$nP_{0.14,0.6}$	V $(Gpc/h)^3$	$\left(\frac{nP_{0.2,0}}{1+nP_{0.2,0}}\right)^2$	$\left(\frac{nP_{0.14,0.6}}{1+nP_{0.14,0.6}}\right)^2$
0.65	2.59	6.23	2.63	1.37	1.95
0.75	3.63	9.25	3.15	1.94	2.57
0.85	2.33	5.98	3.65	1.79	2.68
0.95	1.45	3.88	4.1	1.44	2.59
1.05	0.71	1.95	4.52	0.78	1.97
1.15	0.58	1.59	4.89	0.66	1.84
1.25	0.51	1.41	5.22	0.60	1.79
1.35	0.22	0.61	5.5	0.18	0.79
1.45	0.2	0.53	5.75	0.16	0.69
1.55	0.15	0.4	5.97	0.10	0.49
1.65	0.09	0.22	6.15	0.04	0.20
1.75	0.05	0.12	6.3	0.01	0.07
1.85	0.05	0.12	6.43	0.01	0.07
Total			64.26	9.07	17.71

Volume effective represents the comoving volume that if sampled with no shot noise

$$\left(\frac{nP}{1+nP}\right)^2$$

n is the comoving number density , P is the power spectrum.

Parameters

$$M_{TOT} = V \times \rho_m = V \times \rho_c \times \Omega_m$$

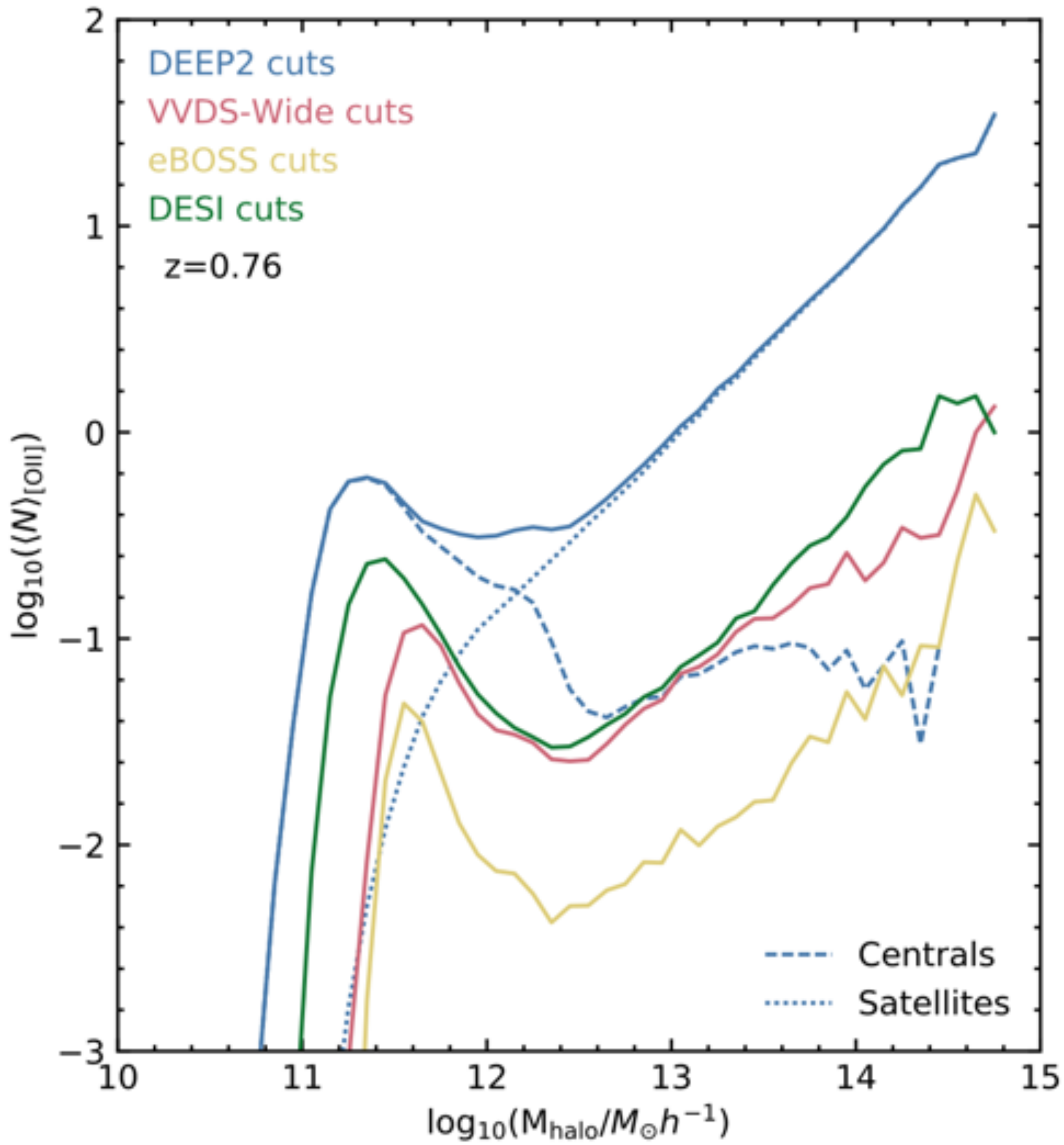
$$M_{TOT} = N_{part} \times m_{part}$$

$$\boxed{m_{part}} = \frac{M_{TOT}}{N_{part}} = \frac{\boxed{V} \times \rho_c \times \Omega_m}{N_{part}}$$

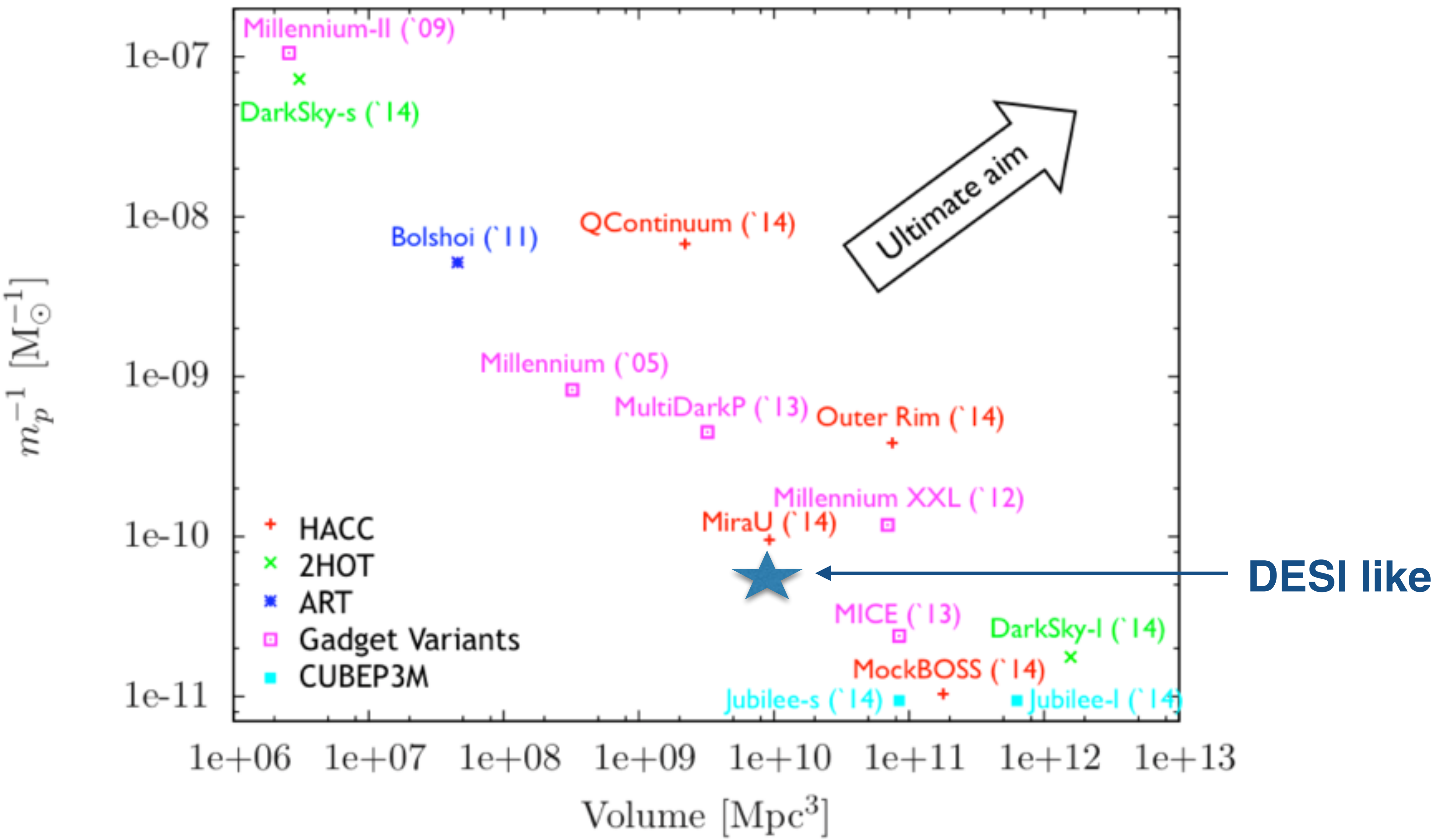
What you want is to simulate a volume with a given resolution in mass.

Important Parameters for surveys: Mass Resolution

- Example DESI ELGs are believed to have a minimum mass of approximately 1011 M⊙/h;
- As a minimum requirement, we require at least 20 particles to define the smallest ELG halo .



CPU/Memory



Millennium-II (`09)
DarkSky-s (`14)
Ultimate aim
Bolshoi (`11)
QContinuum (`14)
Millennium (`05)
MultiDarkP (`13)
Outer Rim (`14)
Millennium XXL (`12)
MiraU (`14)
MICE (`13)
DarkSky-l (`14)
MockBOSS (`14)
Jubilee-s (`14)
Jubilee-l (`14)
HACC
2HOT
ART
Gadget Variants
CUBEP3M
1e-07
1e-08
1e-09
1e-10
1e-11
m_p^{-1} [$M_\odot^{-1}$]
1e+06
1e+07
1e+08
1e+09
1e+10
1e+11
1e+12
1e+13
Volume [Mpc^3]
DESI like

Mass resolution Example: DESI

Box size (Gpc/h)	particle numbers	particle mass ($M_\odot/h$)
2	4096^3	1E10
1.6	4096^3	5E9
2	$\mathbf{5120^3}$	**5E9**
2	$\mathbf{5860^3}$	**3.3E9**
2	8192^3	1.2E9

Fast Mocks

Simulations what for?

- Testing Theory in the middle/and non linear regime
- Testing Methodology: Accounting Biases and Errors
- Testing Observational Sistematics
- Covariance Matrix

Simulations what for?
Covariance Matrix

Nbins=40

fractional increase in the uncertainty in the cosmological parameters

$$\mathcal{O}\left(1/\left(N_{\mathrm{mocks}} - N_{\mathrm{bins}}\right)\right)$$

Nmocks= 100* Nbins   1% error

Nmocks= 1000* Nbins   0.1% error

Comparative of simulations

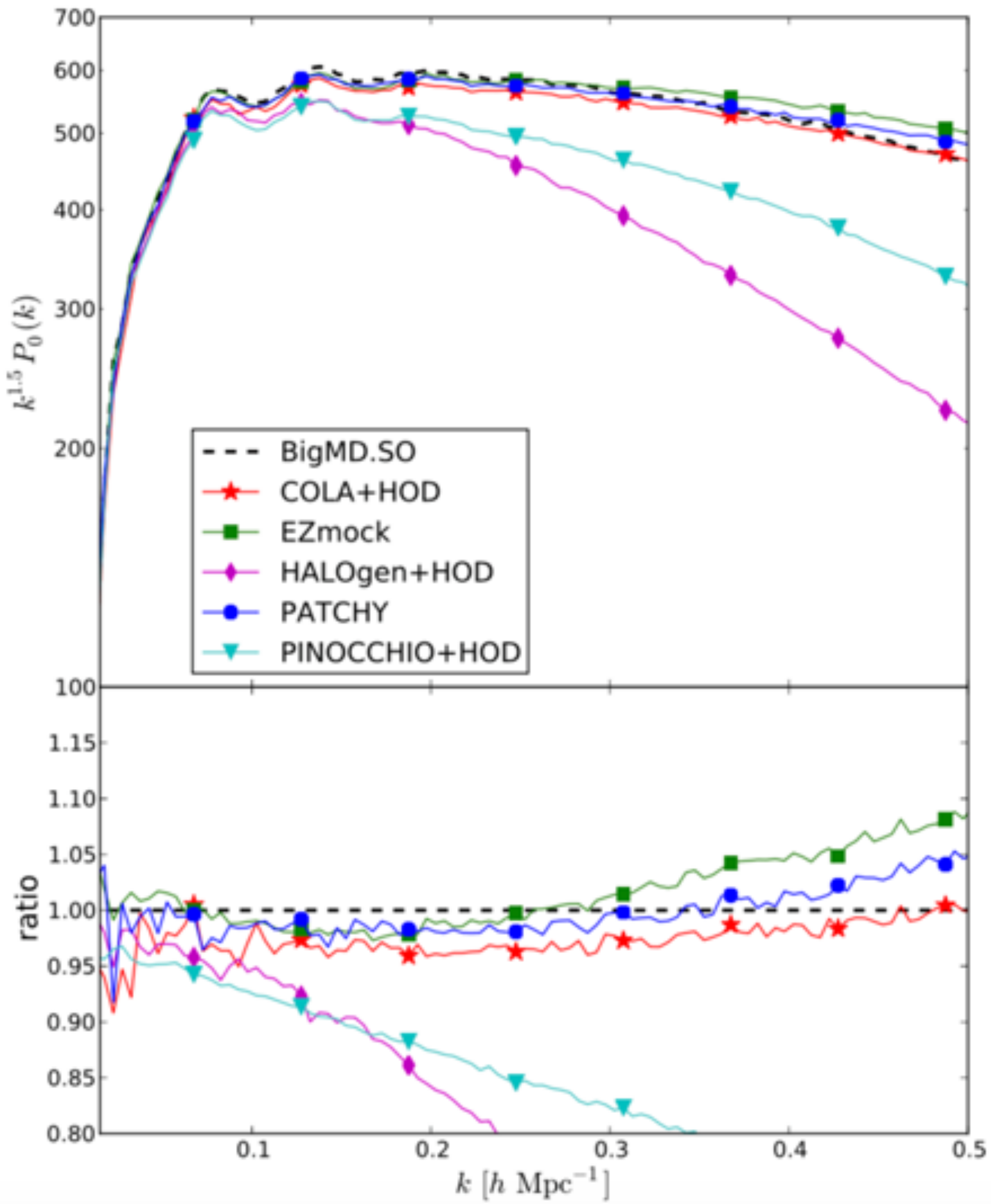
Methodology	reference
Log-Normal	Coles & Jones 1991
PTHalos	Manera et al. 2012, 2015
PINOCCHIO (PINpointing Orbit-Crossing Collapsed Hierarchical Objects)	Monaco et al. 2002, 2013
COLA (COmoving Lagrangian Acceleration simulation)	Tassev et al. 2013
PATCHY (PerturbAtion Theory Catalog generator of Halo and galaxY distributions)	Kitaura et al. 2014a,b
QPM (quick particle mesh)	White et al. 2013
EZmock (Effective Zel'dovich approximation mock catalogue)	Chuang et al. 2015
HALOgen	Avila et al. 2014

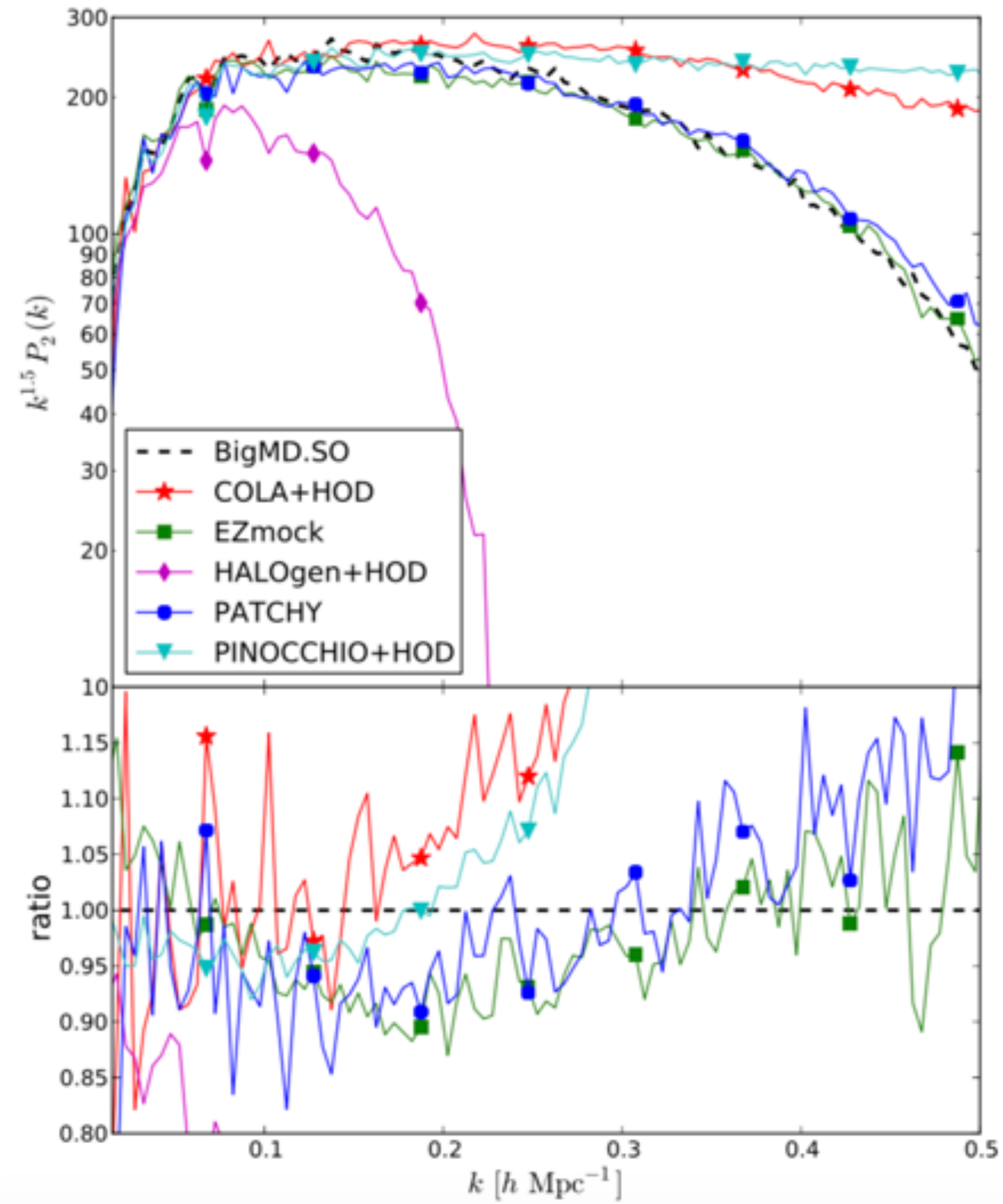
Table 1. The methodologies of generating mock halo/galaxy catalogues developed in the last years. The methodologies included in this study are highlighted using bold font.

Computing Performance in numbers

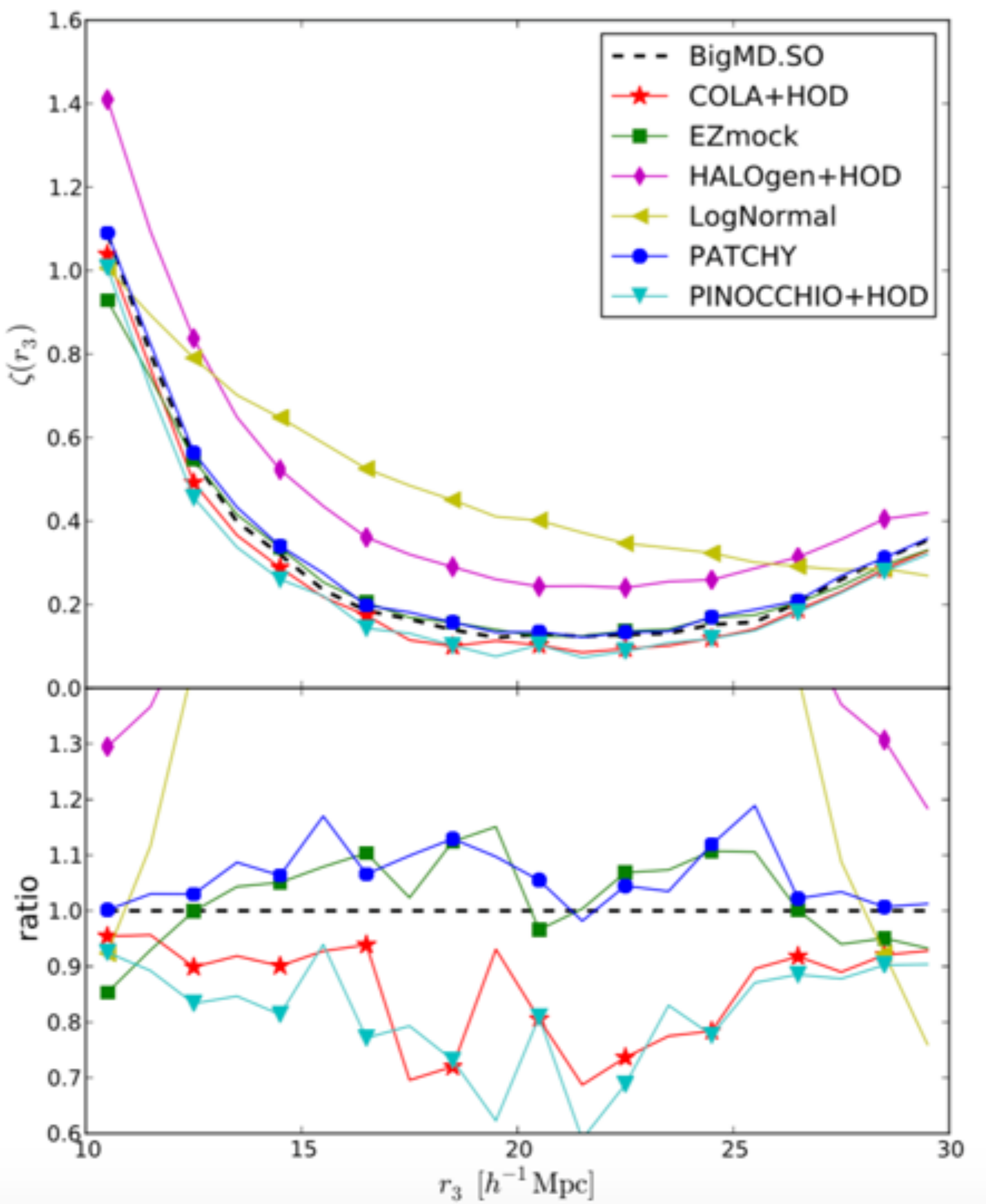
	BigMD	COLA	EZmock	HALOGEN	Log-normal	PATCHY	PINOCCHIO	PTHalos
Particle mesh size	3840^3	1280^3 (3840^3 for force)	960^3	1280^3	1280^3	960^3	1920^3	1280^3
Using white noise	YES	NO	YES	YES	NO	YES	YES	NO
CPU-hour	800,000	130	1.3	6.7	0.5	8	440	45
Memory	8Tb	550Gb	28Gb	130Gb	15Gb	24Gb	890Gb	112Gb

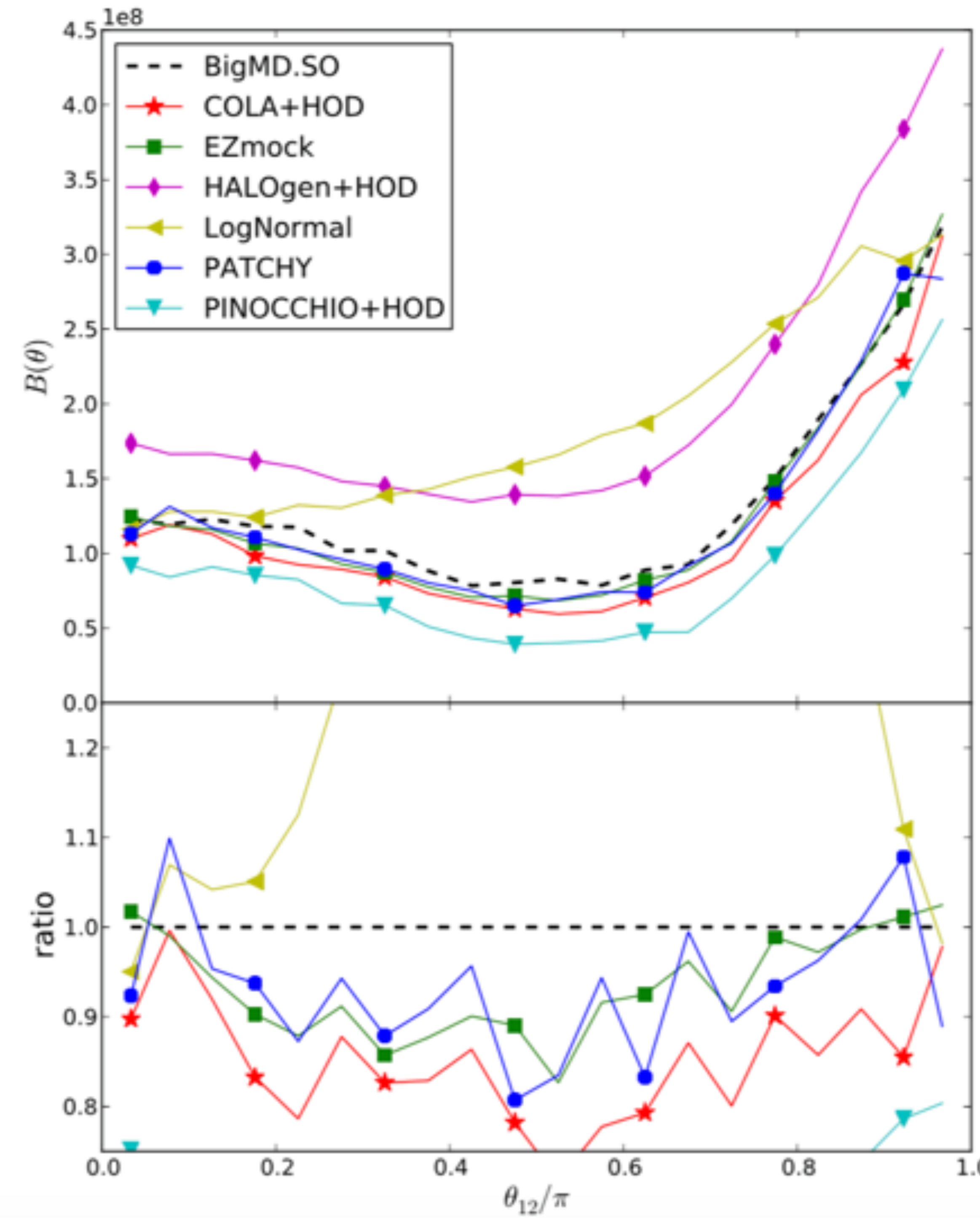
Comparative of simulations

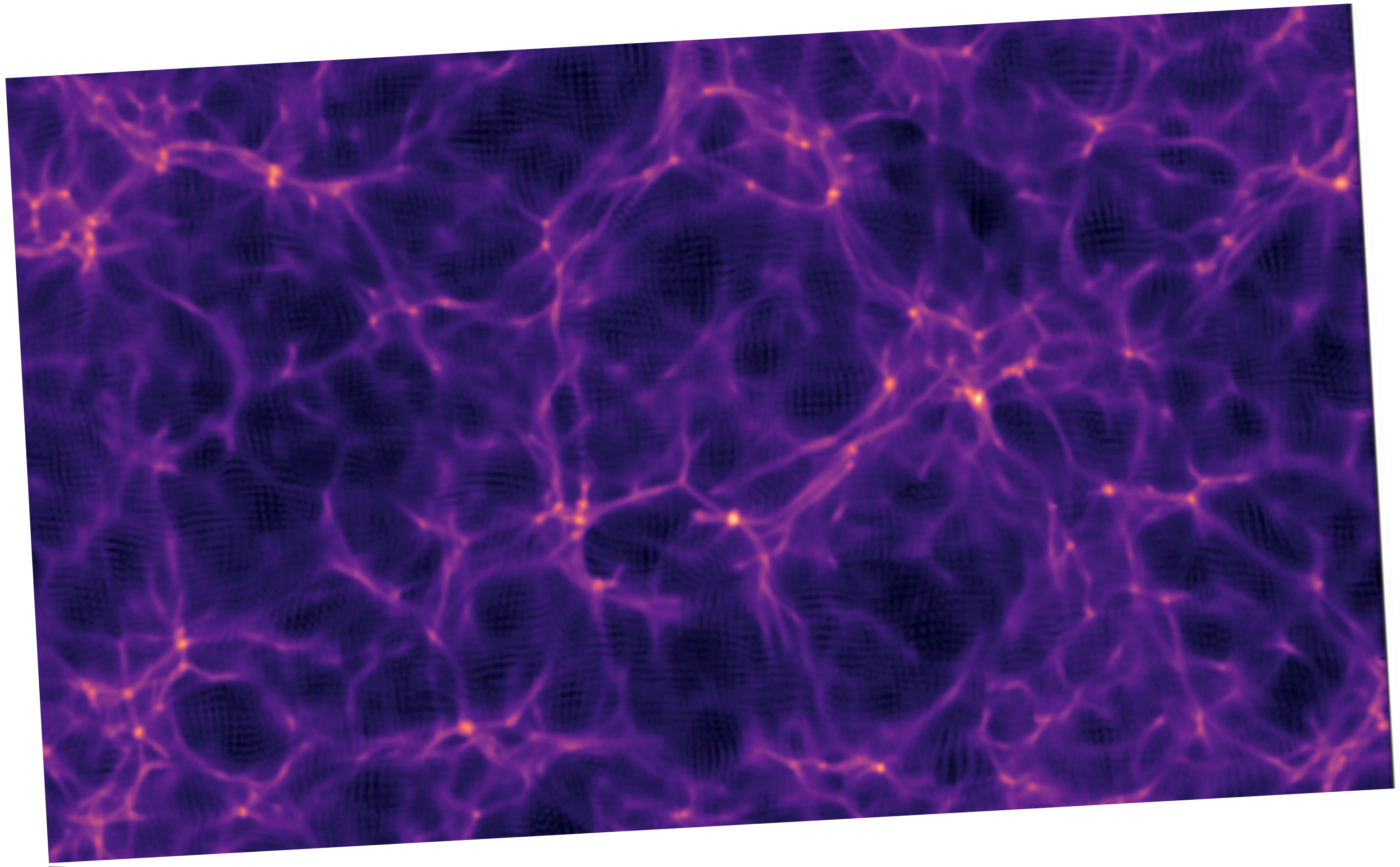




Comparative of simulations



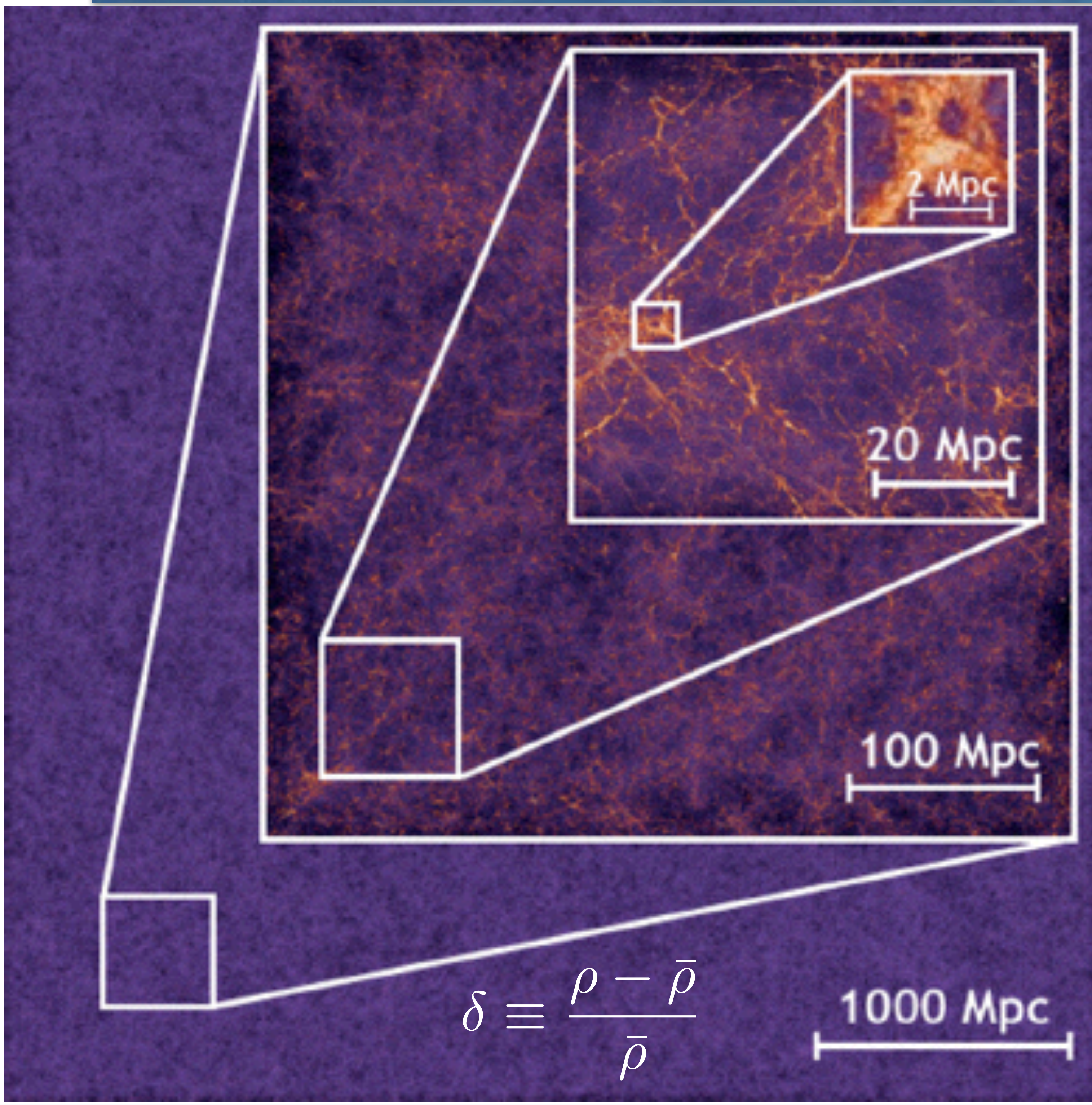




COmoving Lagrangian Acceleration :

COLA code

Scales in Cosmology



Millenium simulation

$\delta > 1$: Non-Linear

- **Simulations**

$\delta \sim 1$: Quasi-Linear

- **Simulations**
- **2-LPT**

$\delta \ll 1$: Linear

- **Simulations**
- **LPT** (Lagrangian Pert. Th)
- **Analytical approx.**

Linear evolution of Perturbations

$$\dot{\rho} + \nabla_{\vec{r}}.(\rho\vec{u}) = 0,$$

: Continuity equation

+

$$\rho\left[\dot{\vec{u}} + (\vec{u}.\nabla_{\vec{r}})\vec{u}\right] = -\nabla_{\vec{r}}p - \rho\nabla_{\vec{r}}\Phi,$$

: Euler's equation

+

$$\nabla^2_{\vec{r}}\Phi = 4\pi G(\rho + 3p) - \Lambda.$$

: Poisson's equation

+

$$\boxed{\delta = \frac{\rho - \bar{\rho}}{\bar{\rho}} << 1}$$

linearity condition

$$\boxed{\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho_b\delta}.$$

Linear Growth factor

$$\boxed{\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G \rho_b \delta}.$$

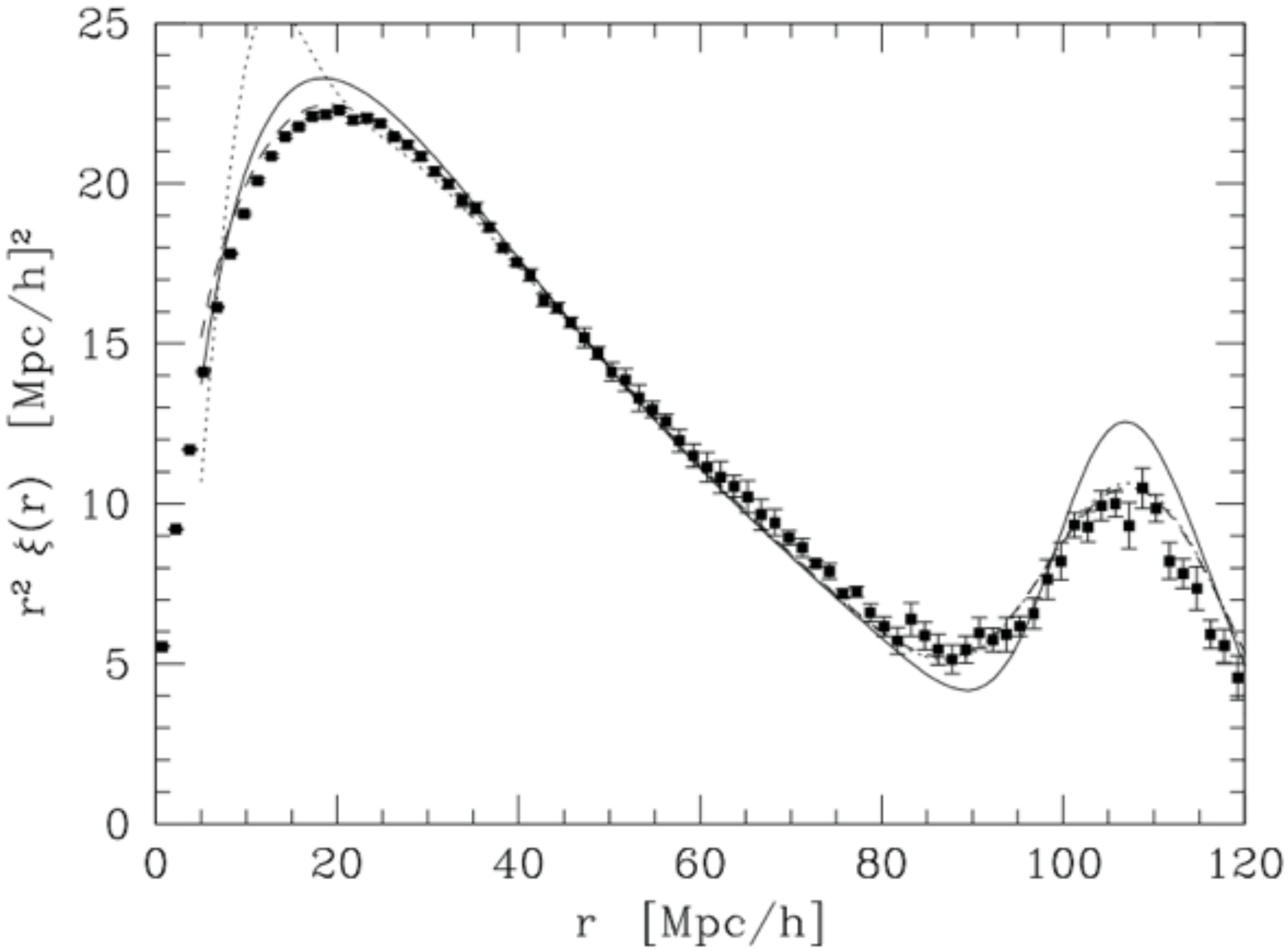
The growing solution of δ is noted D

The growth factor is defined as:

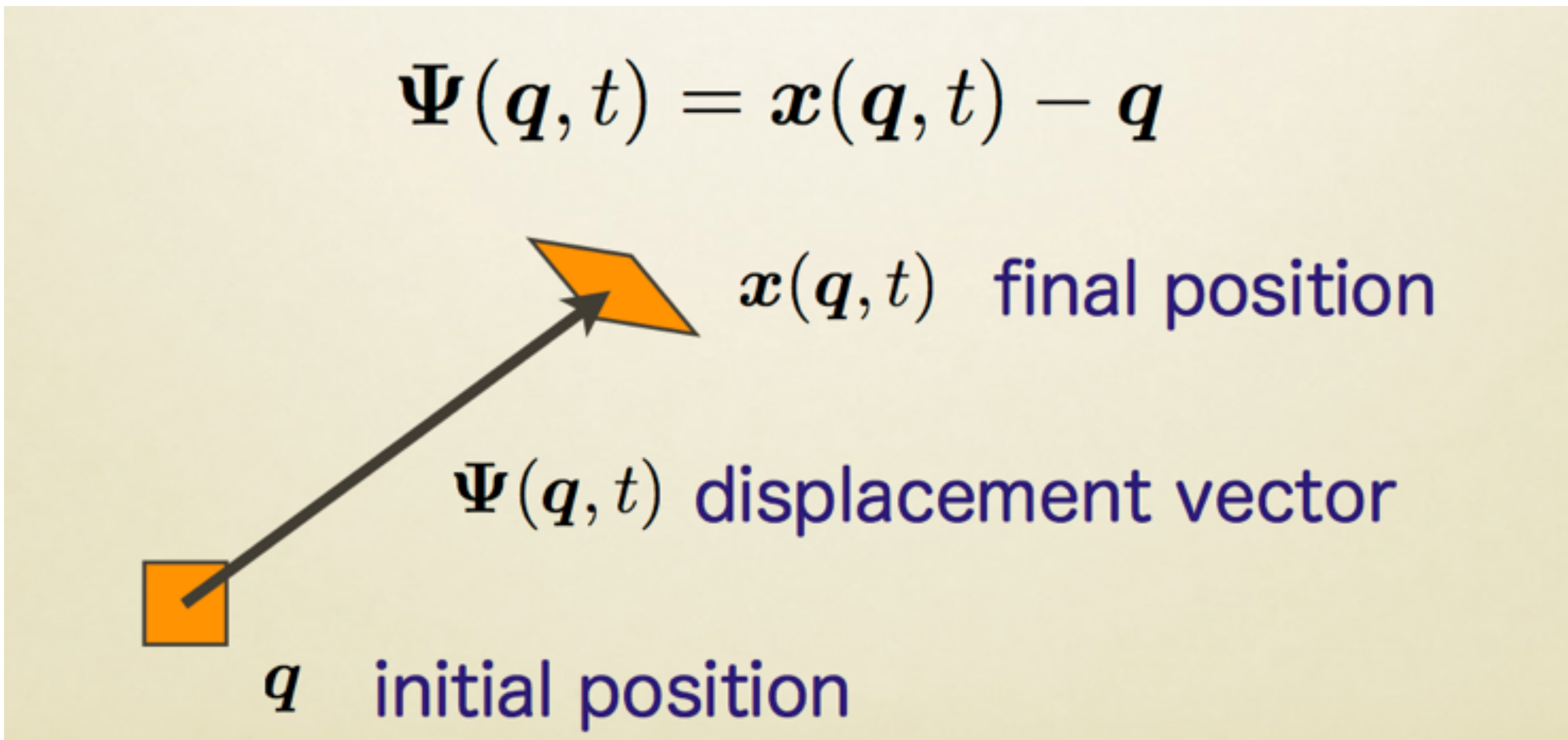
$$\boxed{f(a) = \frac{d\ln D}{d\ln a}}$$

Model for linear regime and parameter in Perturbation Theory for Quasi-linear regime

Linear Theory is not accurate at low z



Lagrangian Perturbation Theory



Credit to Matsubara's presentation

Lagrangian Perturbation Theory

Eulerian vs. Lagrangian

$$\mathbf{v}\ , \rho = \rho_0(1+\delta)\ ; \quad \text{vs.} \quad \mathbf{x} = \mathbf{q} + \mathbf{\Psi}\ .$$

- Density: $(1+\delta_m)\mathrm{d}^3x = \mathrm{d}^3q$

Lagrangian dynamics: Equation of motion of DM

$$\frac{\mathrm{d}^2\mathbf{\Psi}}{\mathrm{d}t^2} + 2H\frac{\mathrm{d}\mathbf{\Psi}}{\mathrm{d}t} = -\nabla_{\mathbf{x}}\phi\ , \quad \nabla^2_{\mathbf{x}}\phi = 4\pi G\rho_0 a^2\delta_m(\mathbf{x})\ .$$

- Local Lagrangian Bias: $1+\delta_X = F(\delta_m)$
- Correlation: $\xi(r) = \langle\delta_X(\mathbf{x})\delta_X(\mathbf{x}+\mathbf{r})\rangle_{\mathbf{x}}$

Credit to Lile Wang's presentation

Lagrangian Perturbation Theory

- **Lagrangian**
  - **field labels:** $\boldsymbol{q}$
  - **fundamental variables:**

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{x}(\boldsymbol{q},t) - \boldsymbol{q}$$

  - **density**

$$\rho(\boldsymbol{q},t) = \bar{\rho}\left[\det\left(\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{q}}\right)\right]^{-1}$$

$$= \bar{\rho}\left[\det\left(\boldsymbol{I} + \frac{\partial \boldsymbol{\Psi}}{\partial \boldsymbol{q}}\right)\right]^{-1}$$

  - **velocity**

$$\boldsymbol{v}(\boldsymbol{q},t) = a\dot{\boldsymbol{x}} = a\dot{\boldsymbol{\Psi}}(\boldsymbol{q},t)$$

Credit to Matsubara's presentation

Lagrangian Perturbation Theory

Equation of motion & Poisson's Equation

$$\ddot{\Psi} + \frac{\dot{a}}{a}\dot{\Psi} = -\frac{1}{a^2}\nabla_x \phi$$

$$\Delta_x \phi = 4\pi G \bar{\rho} a^2 \delta(x,t)$$

Linearization & Zeldovich Approximation

$$\delta(x,t) = \left[\det\left(I + \frac{\partial \Psi}{\partial q}\right)\right]^{-1} - 1 \approx -\nabla_q . \Psi$$

$$\Psi \approx -D(t)\nabla_q \rho_0(q)$$

Lagrangian Perturbation Theory

- **Taking into account the higher-order perturbations in the displacement**

$$\boldsymbol{\Psi} = \sum_{n=1}^{\infty} \boldsymbol{\Psi}^{(n)} = \boldsymbol{\Psi}^{(1)} + \boldsymbol{\Psi}^{(2)} + \boldsymbol{\Psi}^{(3)} + \cdots$$

$$\boldsymbol{\Psi}^{(1)} = -D(t)\nabla\varphi_0(\boldsymbol{q})$$

(First order: Zel'dovich approx.)

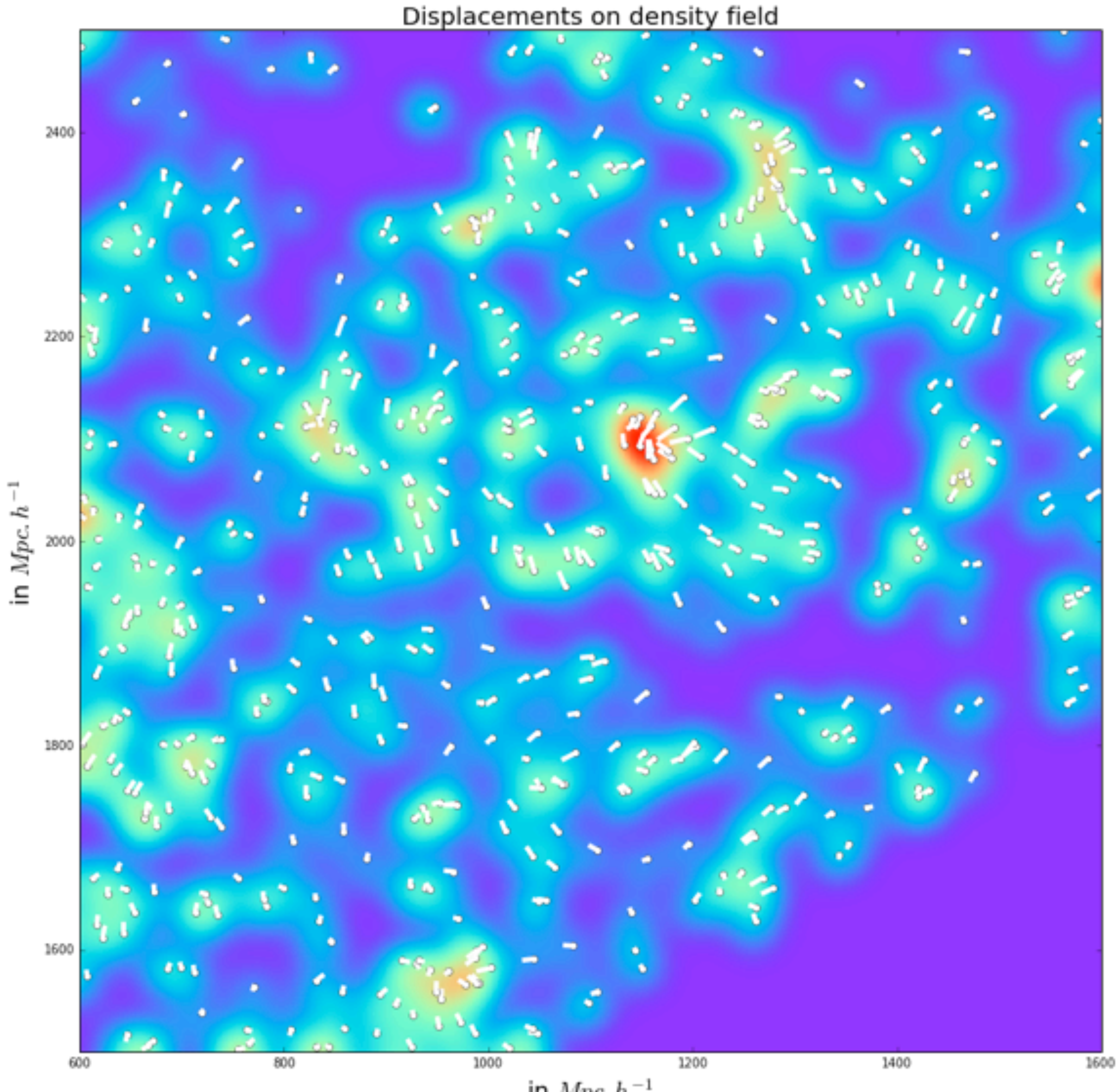
$$\boldsymbol{\Psi}^{(2)} = -\frac{1}{2}D_2(t)\nabla\triangle^{-1}\left[\Psi^{(1)}_{i,i}\Psi^{(1)}_{j,j} - \Psi^{(1)}_{i,j}\Psi^{(1)}_{i,j}\right]$$

$$\boldsymbol{\Psi}^{(3)} = -\frac{1}{3!}\left[D_{3a}(t)\nabla\triangle^{-1}\left(\Psi^{(1)}_{i,i}\Psi^{(2)}_{j,j} - \Psi^{(1)}_{i,j}\Psi^{(2)}_{i,j}\right) + D_{3b}(t)\nabla\triangle^{-1}\det\left(\Psi^{(1)}_{i,j}\right) + D_{3c}(t)\triangle^{-1}\left(\boldsymbol{\Psi}^{(1)}_{,j}\Psi^{(2)}_{i,j} - \Psi^{(1)}_{i,j}\boldsymbol{\Psi}^{(2)}_{,j}\right)_{,i}\right]$$

Recursive solutions

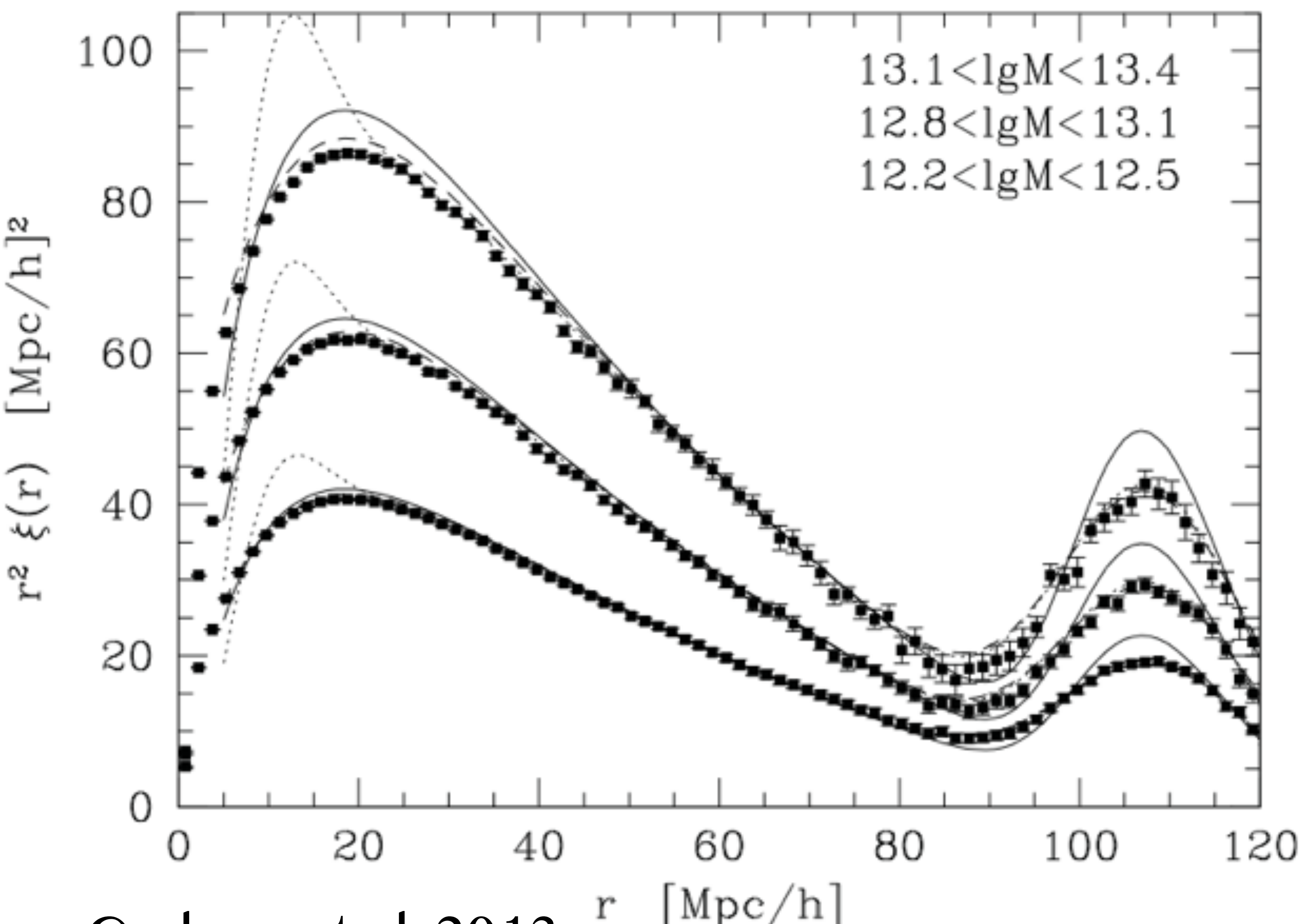
Credit to Matsubara's presentation

Displacements



Lagrangian Perturbation Theory

Bias is an expansion term. So higher the bias lower the precision



Carlson et al. 2013

…. LRT
--- CLPT
— Linear

The linear growth factor is part of the expansion

Basic Idea

Standard case is to solve :

$$\partial_t^2 \vec{x} = -\vec{\nabla}\phi$$

COLA idea is to solve :

$$\vec{x} = \vec{x}_{LPT} + \vec{x}_{res}$$

$\mathcal{O}(\delta^3)$

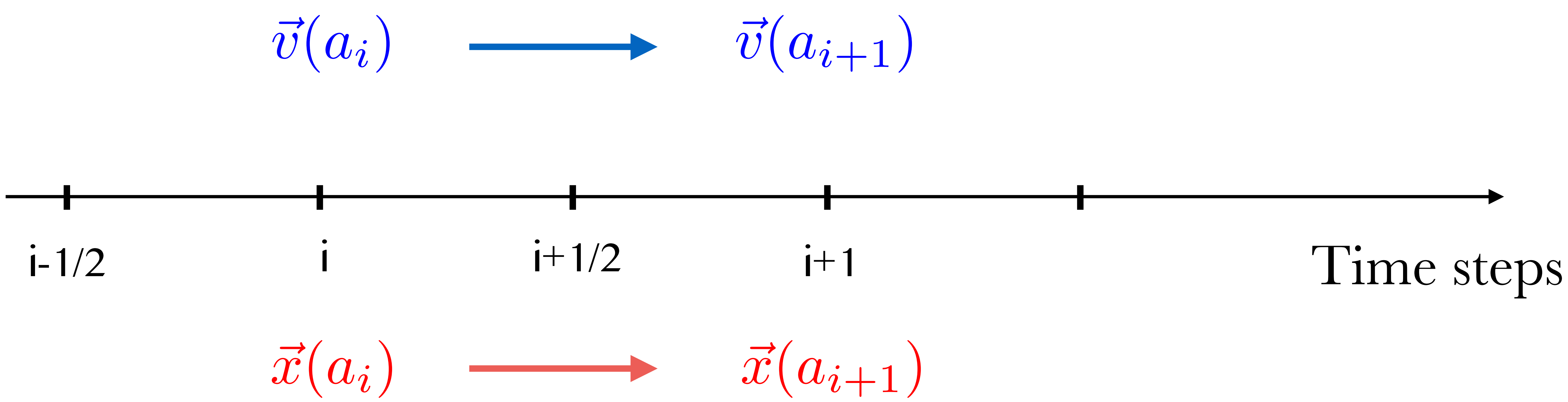
$$\partial_t^2 \vec{x}_{res} = -\vec{\nabla}\phi - \partial_t^2 \vec{x}_{LPT}$$

Numerical discretization in a **PM** code

Analytic

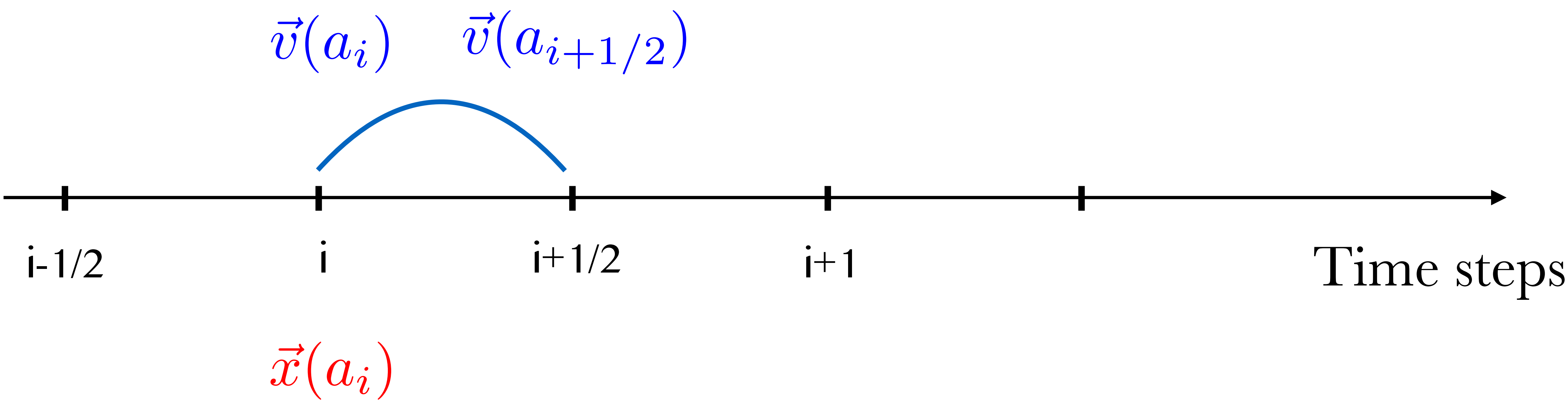
Standard Leapfrog KDK

The goal is to solve the time evolution from one step to the next one

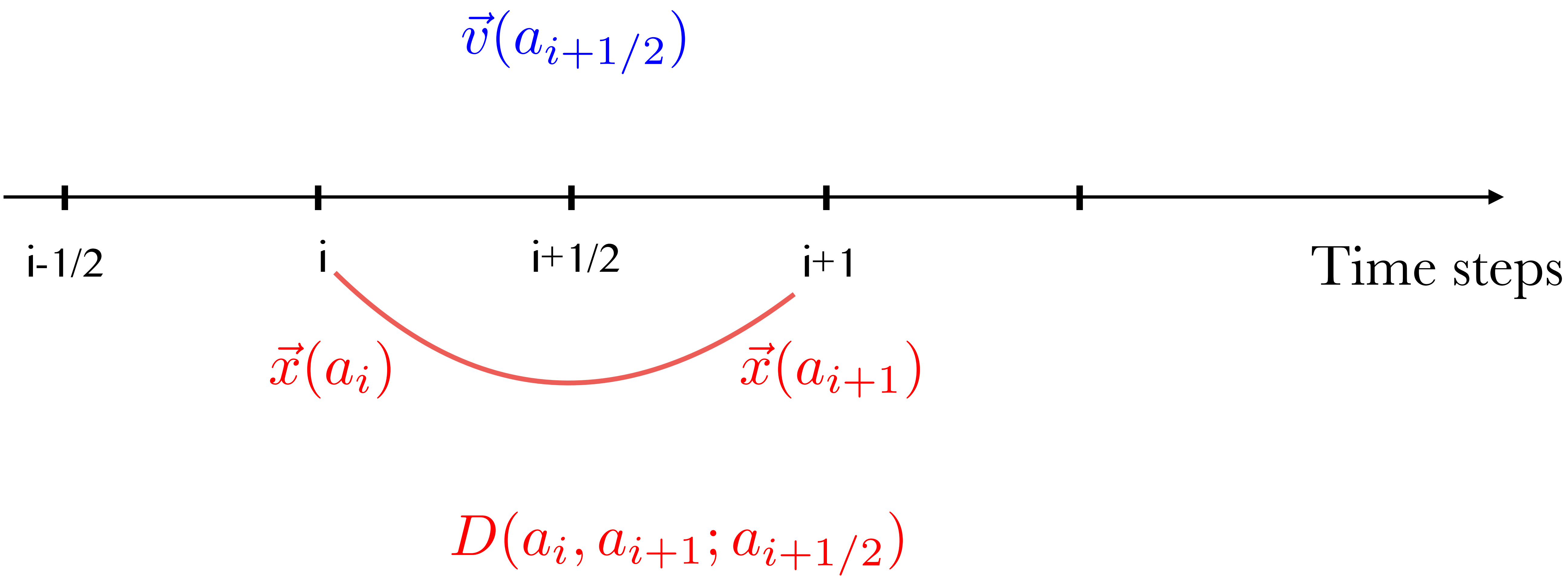


Standard Leapfrog KDK

$$K(a_i, a_{i+1/2}; a_i)$$

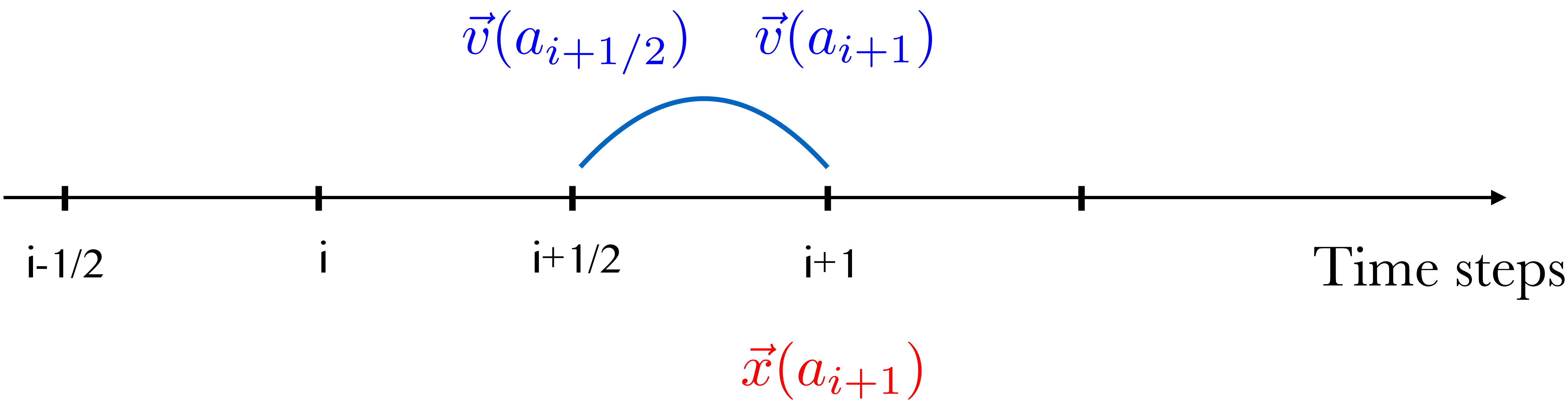


Standard Leapfrog KDK



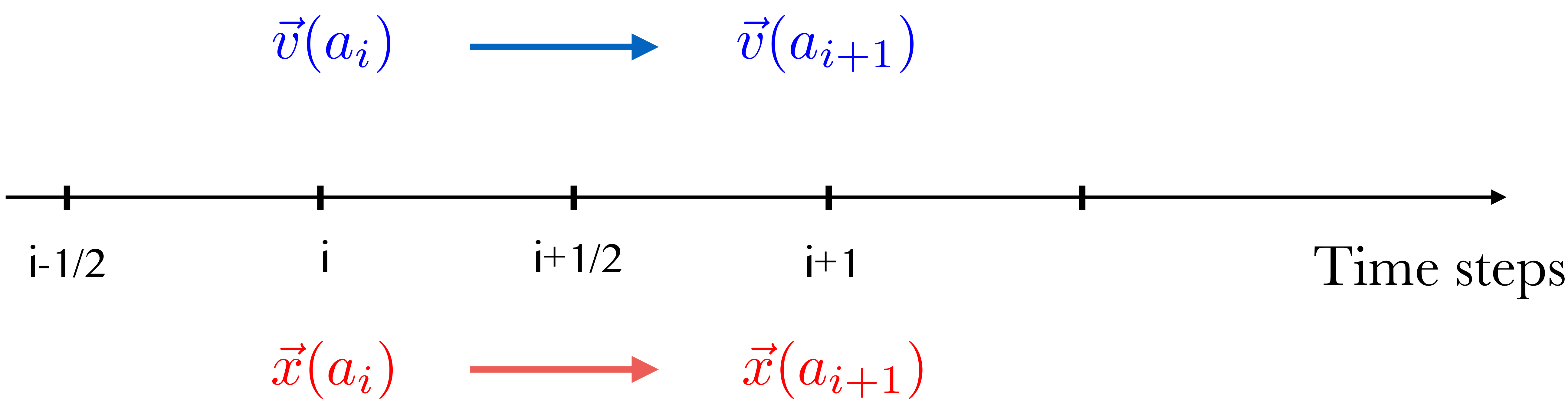
Standard Leapfrog KDK

$$K(a_{i+1/2}, a_{i+1}; a_{i+1})$$



Standard Leapfrog KDK

$$K(a_{i+1/2}, a_{i+1}; a_{i+1}) \quad D(a_i, a_{i+1}; a_{i+1/2}) \quad K(a_i, a_{i+1/2}; a_i)$$



Standard case

$$\mathrm{D}(a_i, a_f; a_c): \quad \boldsymbol{x}(a_i) \mapsto \boldsymbol{x}(a_f) = \boldsymbol{x}(a_i) + \boldsymbol{v}(a_c) \int_{a_i}^{a_f} \frac{d\tilde{a}}{Q(\tilde{a})}$$

$$\mathrm{K}(a_i, a_f; a_c): \quad \boldsymbol{v}(a_i) \mapsto \boldsymbol{v}(a_f) = \boldsymbol{v}(a_i) - \frac{3}{2}\Omega_M \partial_{\boldsymbol{x}} \partial_{\boldsymbol{x}}^2 \delta(\boldsymbol{x}, a_c) \int_{a_i}^{a_f} \frac{\tilde{a}}{Q(\tilde{a})} d\tilde{a}$$

$$Q(a) = \frac{a^3 H(a)}{H_0}$$

COLA case

$$\mathrm{D}(a_i, a_f; a_c): \quad \boldsymbol{x}(a_i) \mapsto \boldsymbol{x}(a_f) = \boldsymbol{x}(a_i) + \boldsymbol{v}(a_c) \int_{a_i}^{a_f} \frac{d\tilde{a}}{Q(\tilde{a})} + \\ + (D_1(a_f) - D_1(a_i))\boldsymbol{s}_1 + (D_2(a_f) - D_2(a_i))\boldsymbol{s}_2,$$

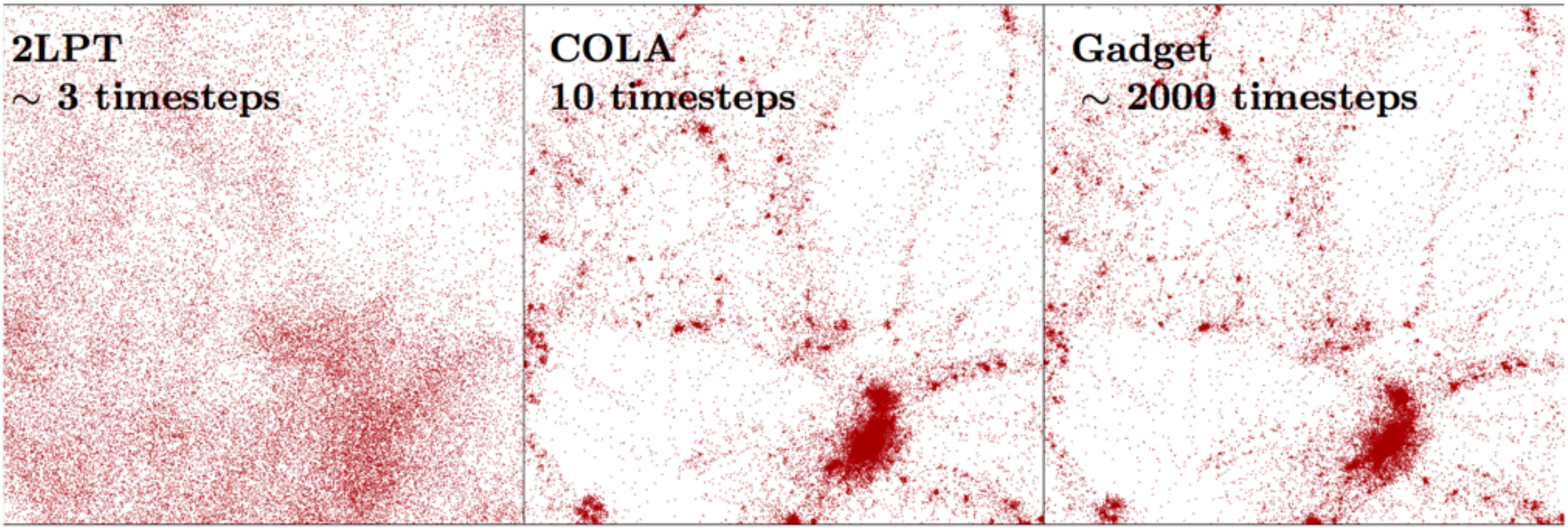
$$\mathrm{K}(a_i, a_f; a_c): \quad \boldsymbol{v}(a_i) \mapsto \boldsymbol{v}(a_f) = \boldsymbol{v}(a_i) - \left[\int_{a_i}^{a_f} \frac{\tilde{a}/a_c}{Q(\tilde{a})} d\tilde{a}\right] \times \\ \left(-\frac{3}{2}\Omega_M a_c \partial_{\boldsymbol{x}} \partial_{\boldsymbol{x}}^{-2} \delta(\boldsymbol{x}, a_c) - T^2[D_1](a_c)\boldsymbol{s}_1 - T^2[D_2](a_c)\boldsymbol{s}_2\right)$$

$$\vec{x}(\vec{q}, a) = \vec{q} + \vec{s}(\vec{q}, a) \qquad T[\vec{s}] = \vec{v} \qquad \vec{s} = \vec{s}_{LPT} + \vec{s}_{res}$$

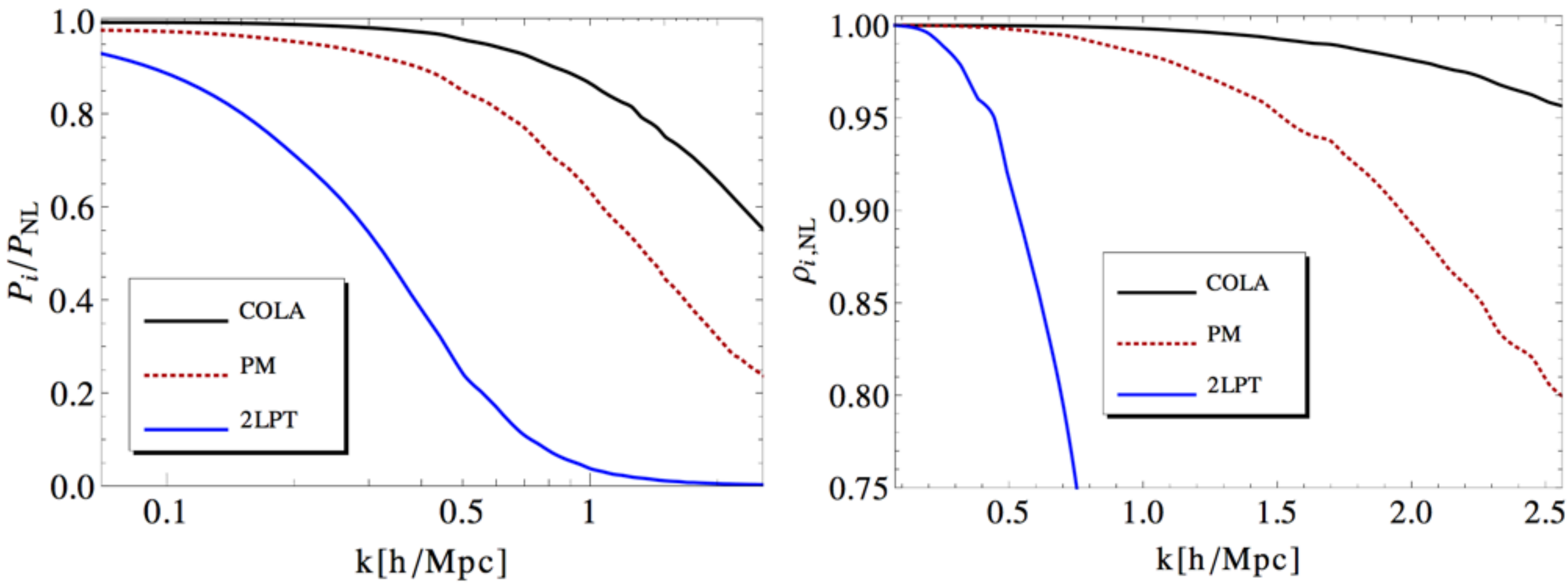
starting point

displacement

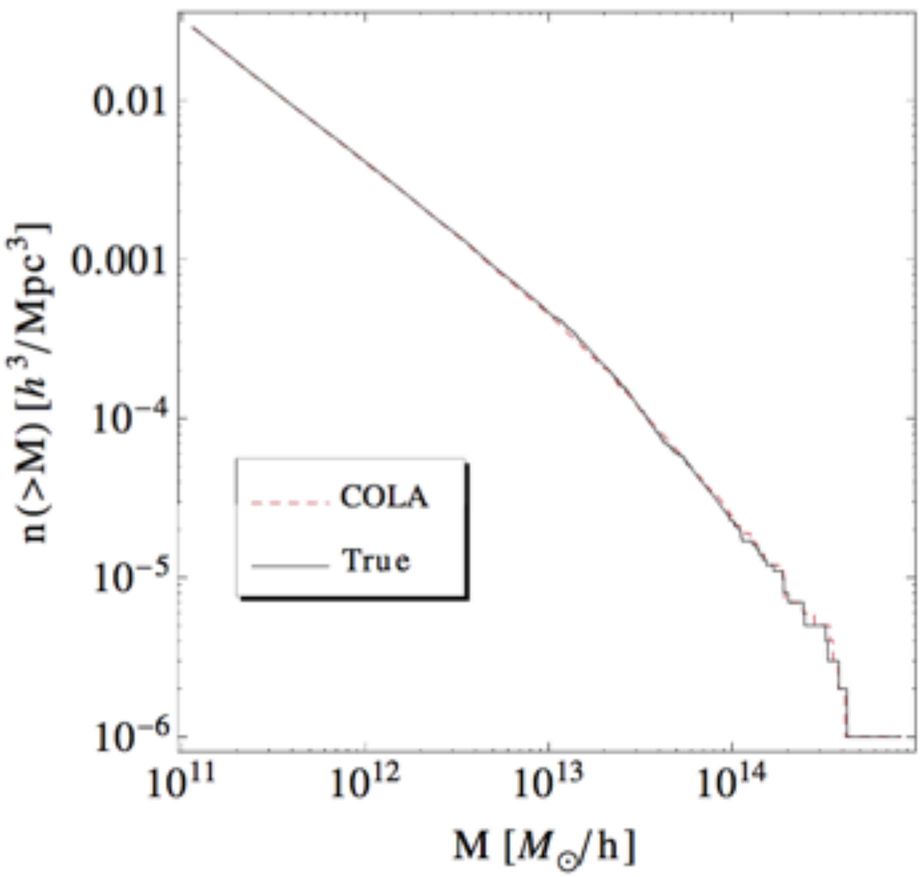
Performances (Time)



Performance 2-pt stat



Performance (Halos)



100Mpc/h

$$N_{mesh} = 768 \quad \Rightarrow \quad \sim 130\text{kpc/h}$$

$$256^3 \text{ particles} \quad \Rightarrow \quad m_{part} \sim 2.5\, 10^9 M_\odot$$

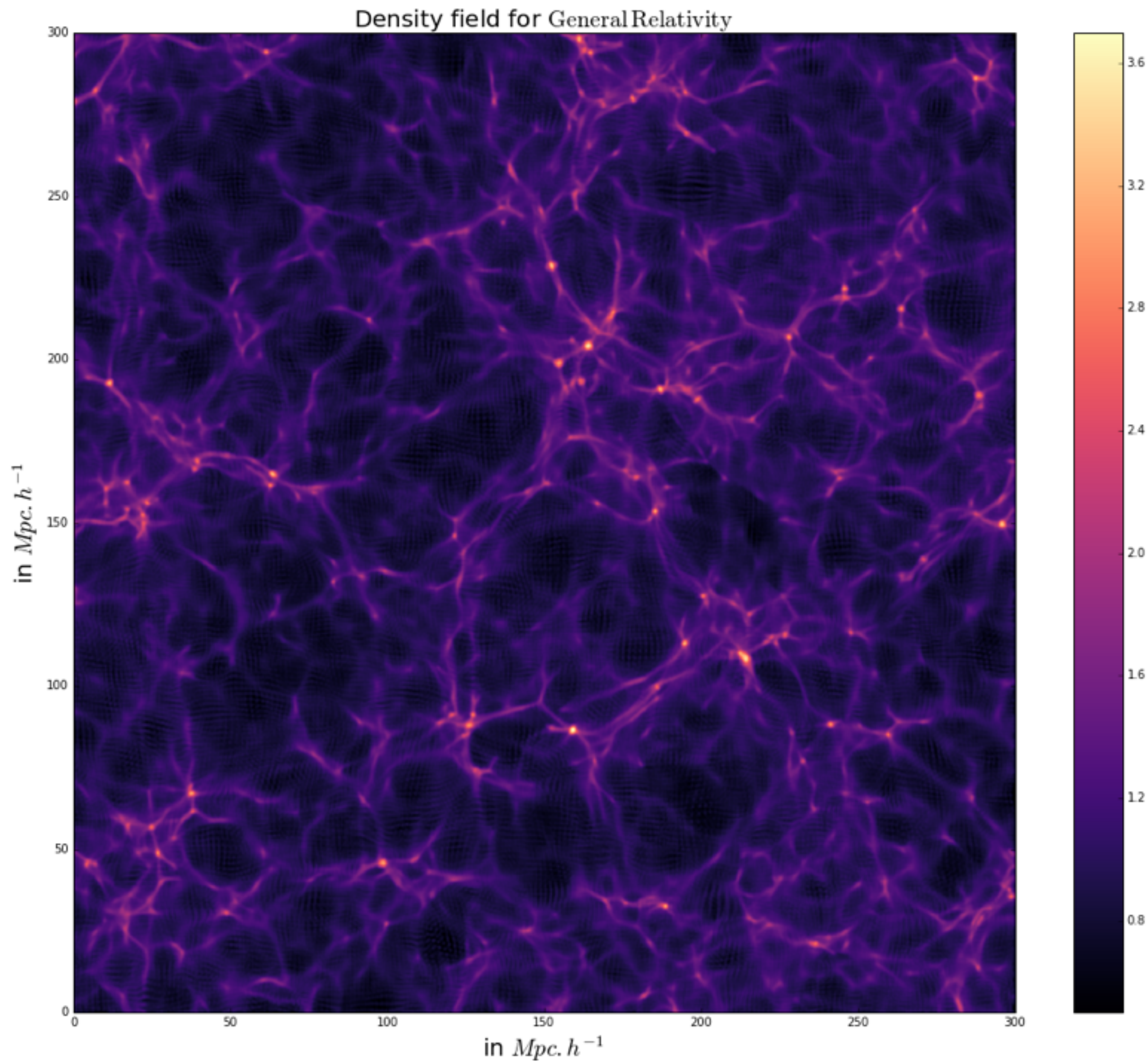
Parameters

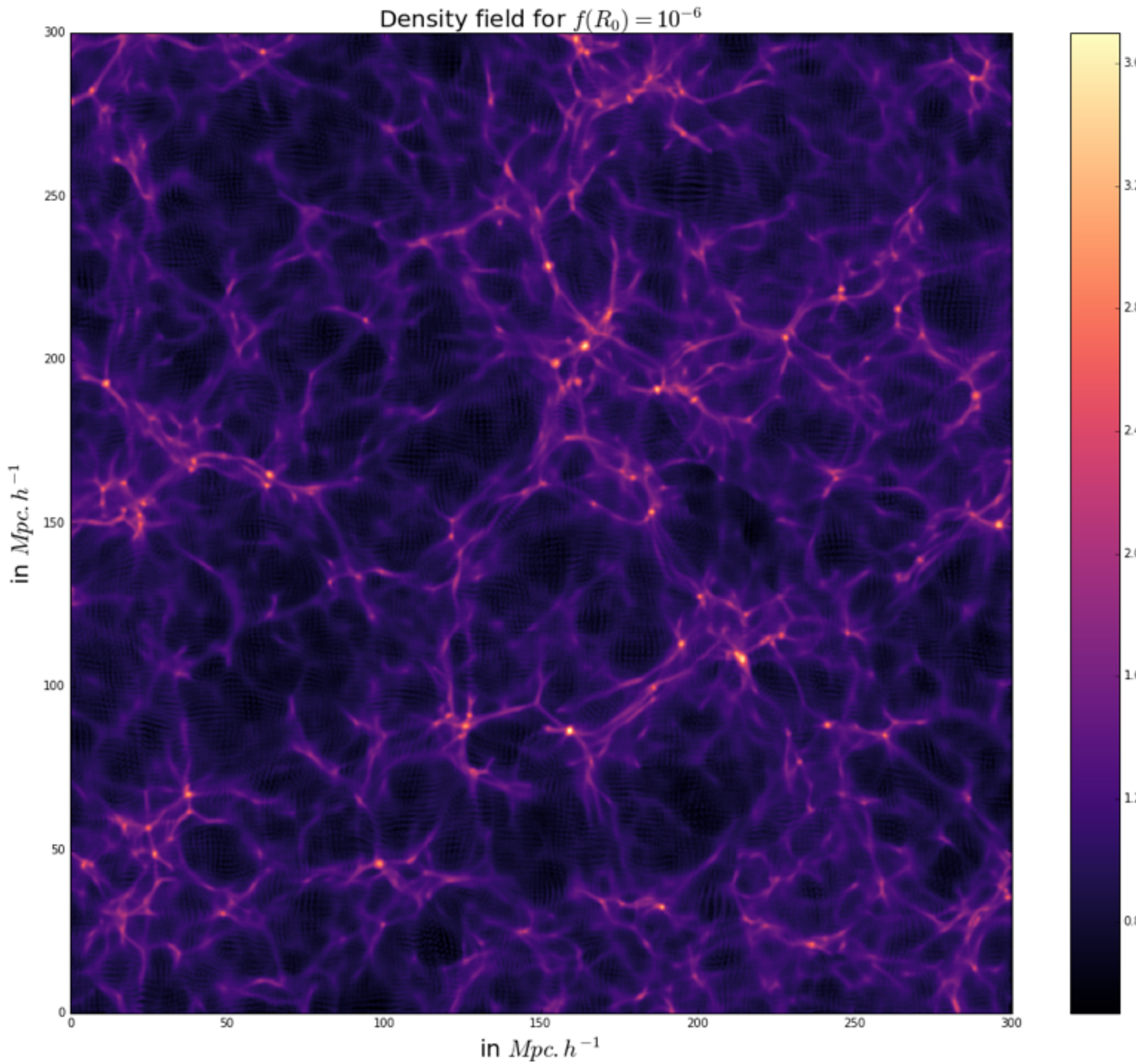
Volume + Num particles => Mass particles

Mesh precision => scale Accuracy parameter

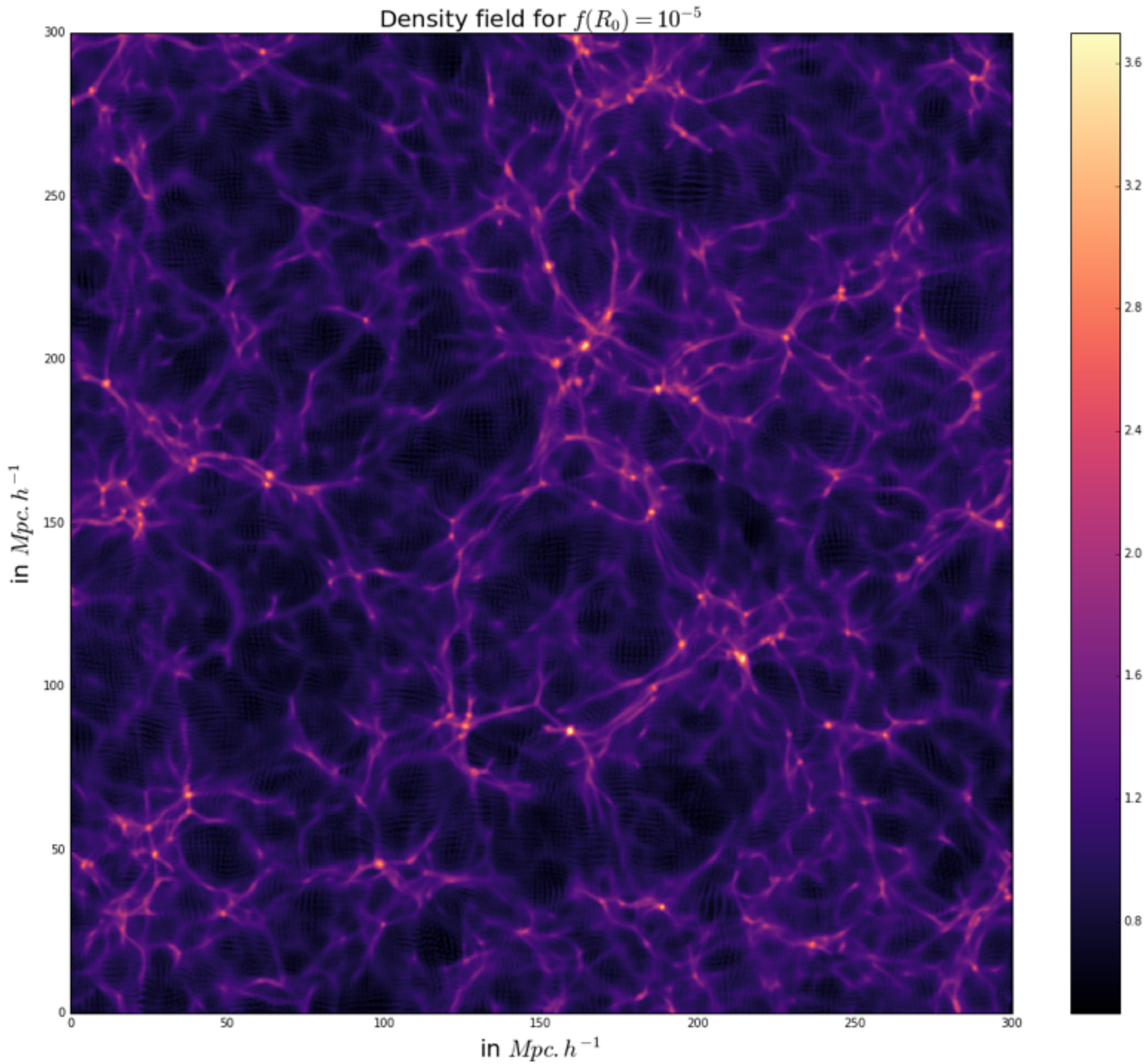
Time steps => Accuracy at small scale

MG-COLA Simulations for DESI MG white paper

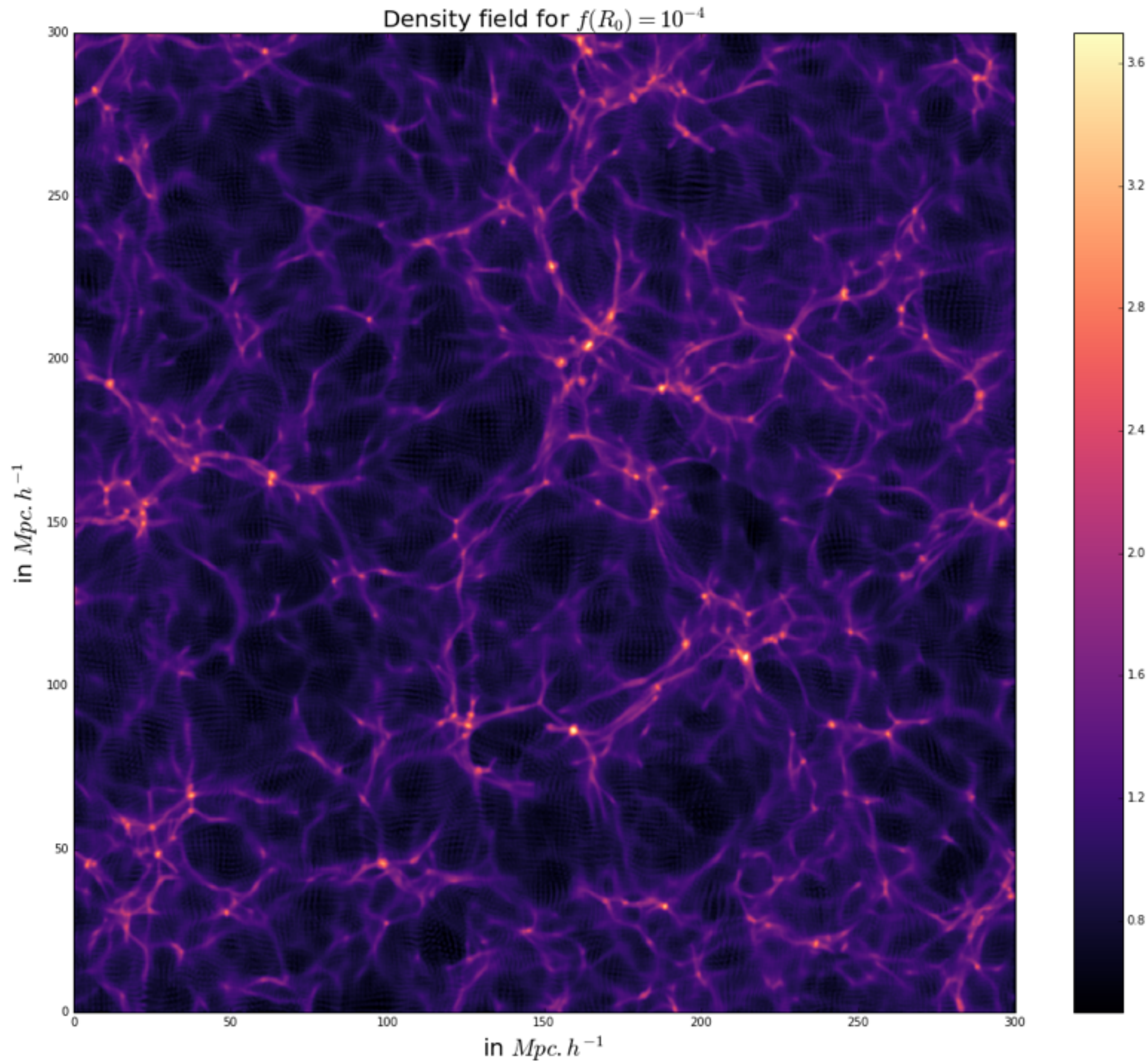

Density field for General Relativity
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6



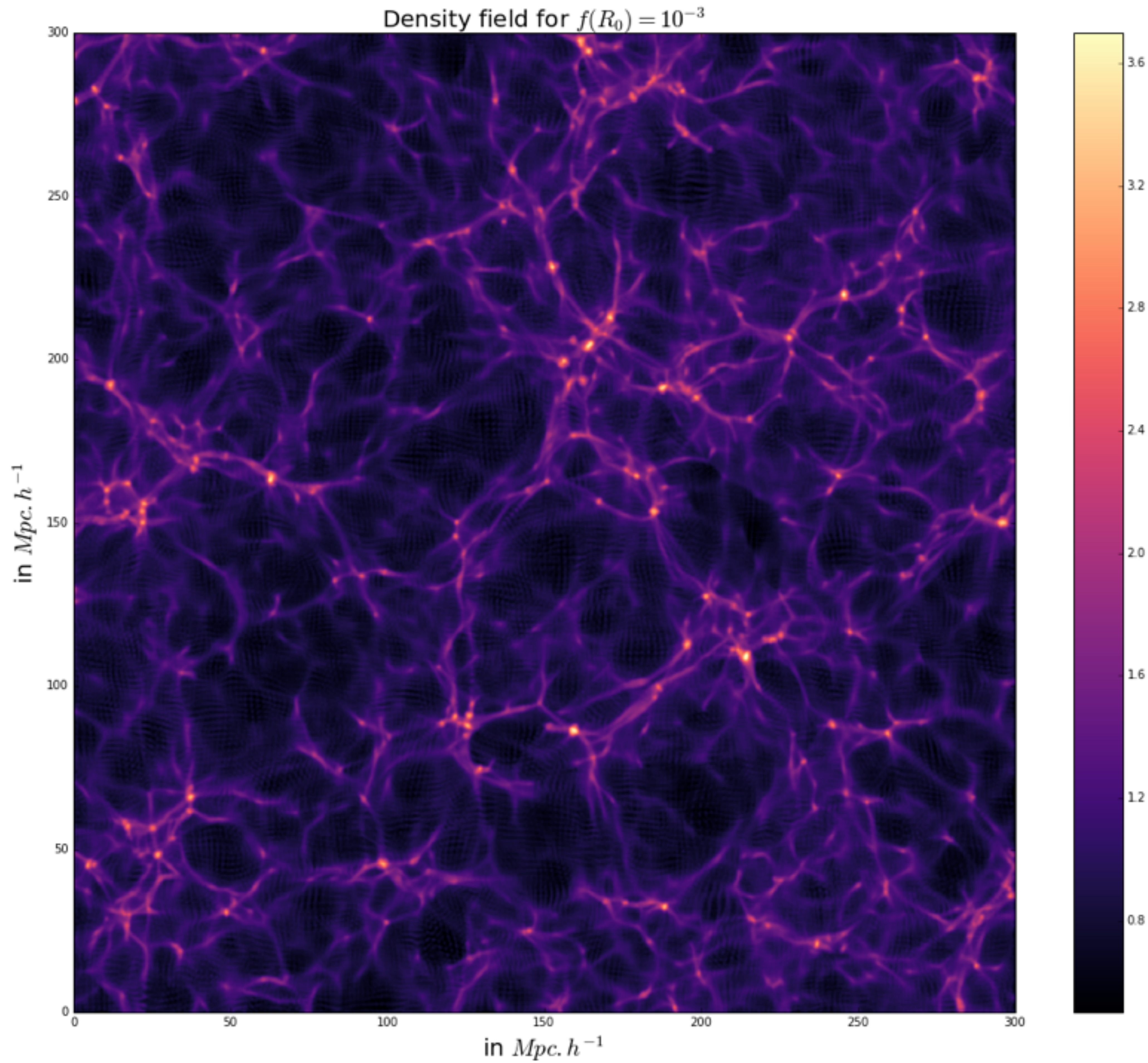
Density field for $f(R_0) = 10^{-6}$
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6



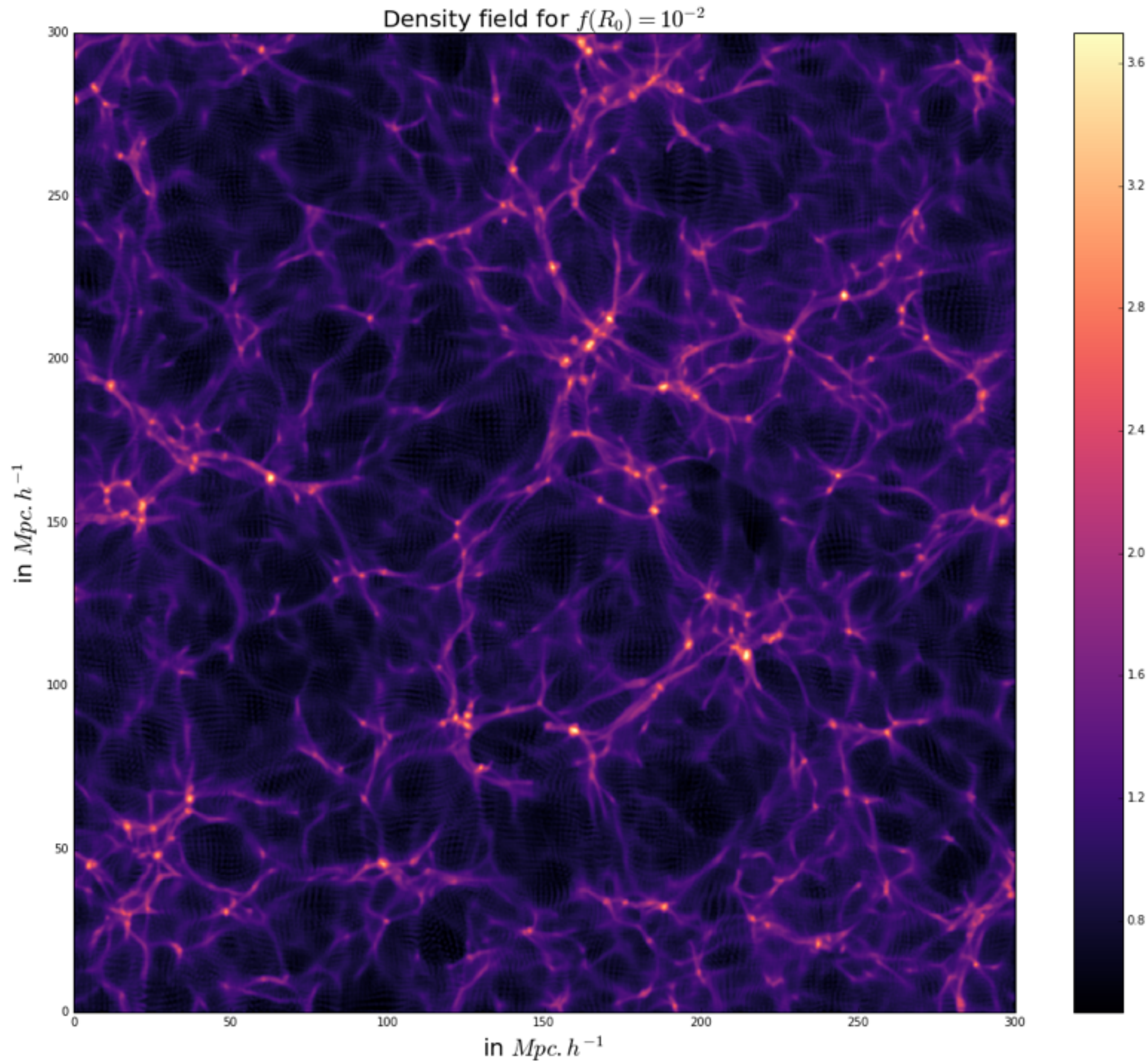
Density field for $f(R_0) = 10^{-5}$
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6



Density field for $f(R_0) = 10^{-4}$
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6



Density field for $f(R_0) = 10^{-3}$
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
0.8
1.2
1.6
2.0
2.4
2.8
3.2
3.6

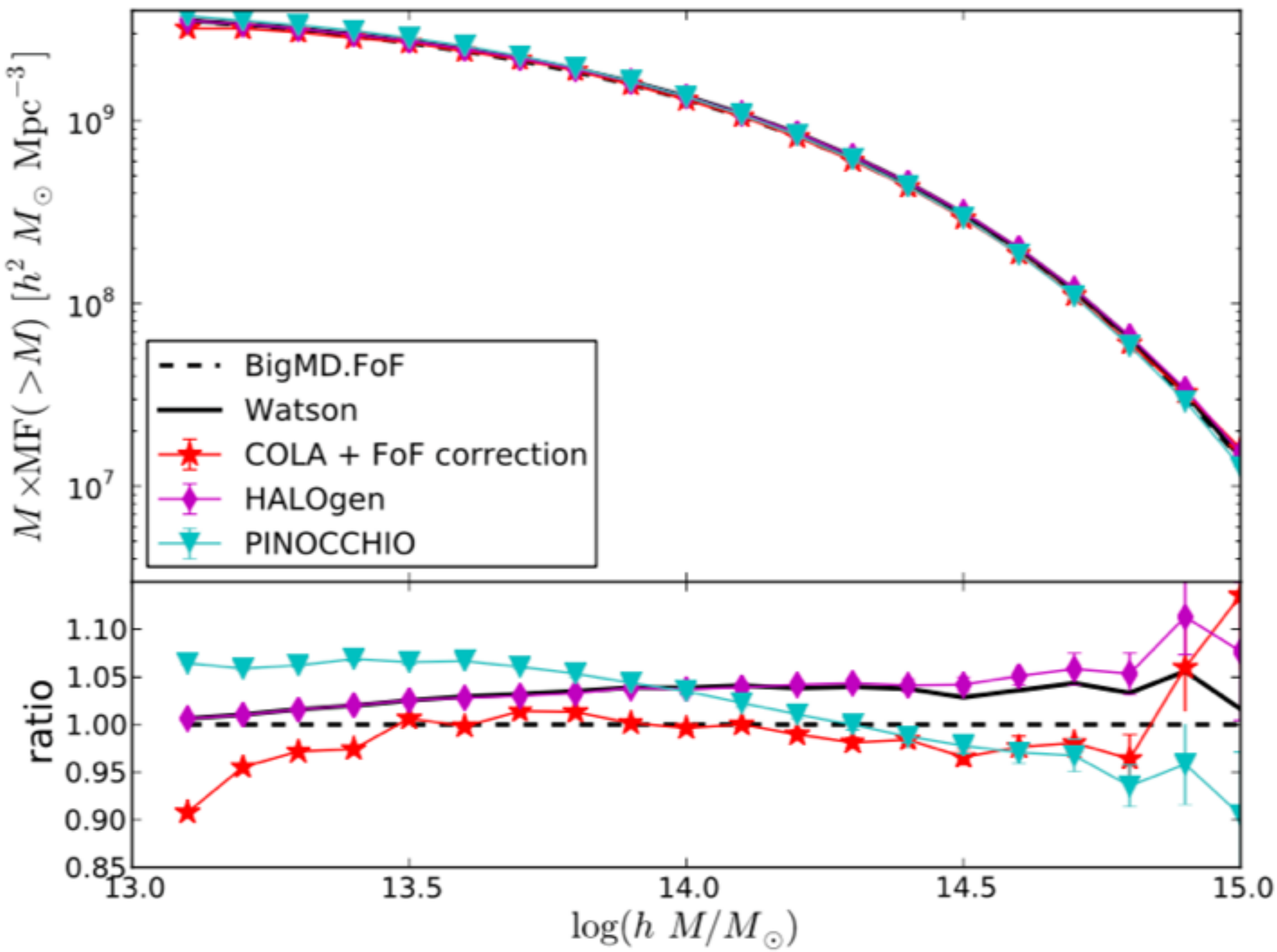

Density field for $f(R_0) = 10^{-2}$
in $Mpc.h^{-1}$
in $Mpc.h^{-1}$
0
50
100
150
200
250
300
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8

Lessons to take away

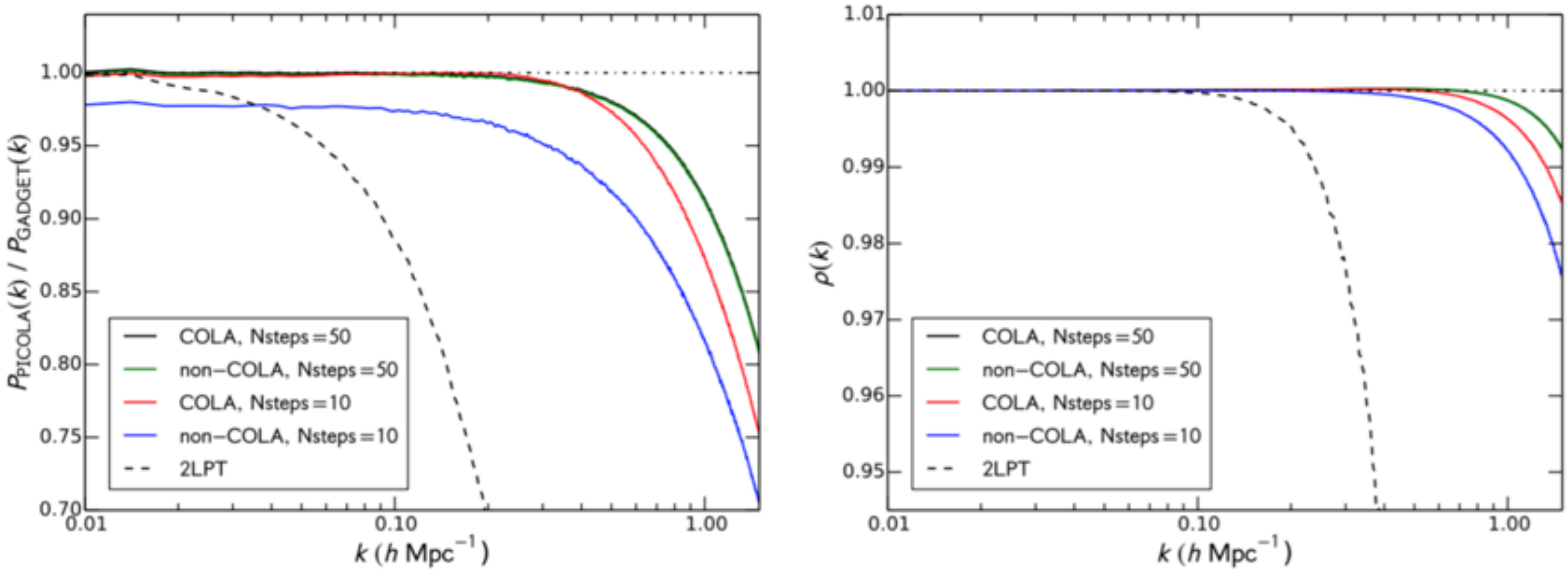
- Simulations are necessary for analyzing LSS.
- Future surveys requirements discussion is now happening for preparing future surveys.
- Number of options in the panorama. Work need to be done to optimize current techniques/codes to achieve the requirements for analyzing future surveys.

Backup

Comparative of simulations



K-max and Number of steps



$$\rho(k) = \frac{\langle \delta(\boldsymbol{k}) \delta^*_{NL}(\boldsymbol{k}) \rangle}{\langle |\delta(\boldsymbol{k})|^2 \rangle \langle |\delta_{NL}(\boldsymbol{k})|^2 \rangle}$$

K-max and Ngrid