Optimize datacenter

Problem statement for the 2015 hashcode qualification round. Link for the problem description and data could be found here.

Definitions

r=0…R, rows index
s=0…S, slot index
Unavailable = list of pairs u=(u_row, u_slot) containing the coordinates of the unavailable slots
p=0…P, pools index
m=0…M, server index
Servers = list of pairs (capacity, size) describing the servers

Decision variables

$x_{r,s,m,p} = \{0, 1\}$Whether the **leftmost** slot of 'm' is allocated to slot 's' from row 'r' and assigned to pool 'p'

Notation

$c_m = servers[m]_0$: server capacity
$s_m = servers[m]_1$: server size

Derived variables

$capacity_p = \sum_{r,s,m} x_{r,s,m,p} * c_m$:total pool capacity

$poolRowCapacity_{p,r} = capacity_p - \sum_{s,m} x_{r,s,m,p} * c_m$: pool 'p' capacity when row 'r' drops

$gc_p = minimum_r(poolRowCapacity_{p,r})$:guaranteed pool 'p' capacity

Objective

$maximize[minimum_{p=0..P}(gc_p)]$

Constraints

C1: Each slot of the datacenter is occupied by at most one server
$\forall r, s, m, p\colon x_{r,s,m,p} = 1 \rightarrow \forall m', x_{r,s+i,m',p} = 0,\ i = s_{m'} - 1 \ldots s_m$
C2: No server can occupy the unavailable slots
$\forall u = (u_s, u_r), \forall m, p\ x_{u_r,u_s-i,m,p} = 0, i = 0 \ldots s_m$

C3: No server extends beyond the slots of the row

$\forall m, r, p,\ x_{r,S-i,m,p} = 0\ ,\ i = 0..(s_m - 1)$

C4: Each server can be used at most once

$\forall m,\ \sum_{r,s,p} x_{r,s,m,p} \leq 1$