Pizza model formulation

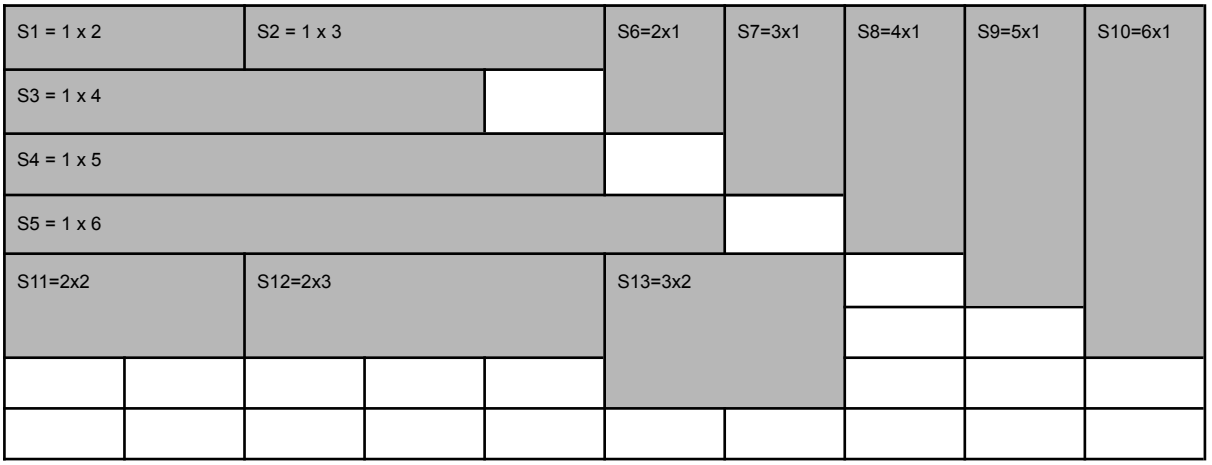
Problem statement can be accessed:
https://github.com/mmarouen/hascode/tree/master/pizza

Definitions:

r=0…R row index
c=0…C column index
Ingredients(r,c)=1 for mushroom, 0 for tomatoes
L: minimum number of each ingredient in each slice
H: maximum number of pizza cells in each slice

Approach

Given a max slice size **H** and a minimum size **2*L**, we can define the space of possible shapes S that satisfy a surface <= **H** and surface >= **2*L**.
Example, H = 6, L = 1 (demo file), there are in total **13** shapes (shown below):



In order to slice the pizza, some or all of the shapes need to be used at least once.
Lets denote by:
Shapes[s] = (n_row_s, n_col_s, size of shape “s”)

Decision variables

$x_{r,c,s} = \{0, 1\}$ Whether the top left of the shape s belongs to cell (r,c)

Derived functions

$y_s = \sum_{r,c} x_{r,c,s}$: total count of shape ‘s’ used

$slicedSurface_{total} = \sum_{s} y_s * shape_s$: total sliced surface

Objective function

$maximize(slicedSurface_{total})$

Constraints

C1: Slice surface should not exceed total pizza size

$slicedSurface_{total} \leq n_{col} * n_{row}$

C2: A cell belongs to one shape at most

$\forall r, c \sum_{s} x_{r,c,s} \leq 1$

C3: If a shape is active at cell (r,c) then no overlap should occur:

$\forall r, c, \forall s, s', x_{r,c,s} = 1 \rightarrow x_{r+i,c+j,s'} = 0, \ -shapes[s']_0 + 1 \leq i \leq shapes[s]_0, \ -shapes[s']_1 + 1 \leq j \leq shapes[s]_1$

C4: Border conditions

$\forall s, c, x_{R-r,c} = 0 for \ r < shape\ width$

$\forall s, r, x_{r,C-c} = 0 for \ c < shape\ height$

C5: Each slice must contain at least L cells from each ingredient

$\forall r, c, s \ x_{r,c,s} = 1 \rightarrow \sum_{shape} Ingredients_{r+i,\, c+j,s} \geq L$

$\forall r, c, s \ x_{r,c,s} = 1 \rightarrow (shapes_s - \sum_{shape} Ingredients_{r+i,\, c+j,s}) \geq L$