Hashcode: Book scan problem model formulation

Intro:

Usage of integer optimization model to solve the problem

Definitions:

- D days
- B books:
  - $beta_b$: book score
  - Id_b: book identifier
- L libraries:
  - t_l: library integration time
  - n_l: max number of books that can be shipped per day
  - $books_l$: vector of book identifiers in the library

Decision variables:

$x_{d,b,l} = \{0, 1\}$: Whether book “b” from library “l” gets scanned on day “d”

$y_{d,l} = \{0, 1\}$: whether library “l” gets selected on day “d”

$y_l = \{0, 1\}$: Whether library “l” gets integrated or not

Derived useful variables:

$x_b = \sum_{d,l} x_{d,b,l}$: $whether\ book\ b\ gets\ scanned$

$x_{d,l} = \sum_{b} x_{d,b,l}$: $total\ amount\ of\ books\ shipped\ from\ library\ l\ at\ day\ d$

Objective function

$max \sum_{b=1}^{B} x_b * beta_b$: maximize total scanned books score

Constraints

C1: A library can only ship books on its list of books

$\forall l \in <1, L>, \forall b \notin books_l, \forall d \in <1, D>, x_{d,b,l} = 0$ (1)

C2: A book is scanned no more than once

$$\forall b \in <1, B>, x_b \leq 1 \quad (2)$$

C3: at any given day no more than a single library is integrated

$$\forall d \in <1, D>, \sum_d y_{d,l} \leq 1 \quad (3)$$

C4: A library is integrated no more than once

$$\forall l \in <1, L>, \sum_d y_{d,l} = y_l t_l \quad (4)$$

C5: No more than n_l books per day can be shipped from library l

$$\forall d \in <1, D>, \forall l \in <1, L>, x_{d,l} \leq n_l \quad (5)$$

C6: Books can only be shipped once integration is finished

$$\forall d > t_l, \forall l \in <1, L>, x_{d,l} \leq y_l \quad (6)$$

$$\forall d \leq t_l, \forall l \in <1, L> \; x_{d,l} = 0 \quad (7)$$

C7: Library is integrated in consecutive days

$$\forall l \in <1, L>, y_l = \sum_{d>n_l} \prod_{i=1}^{n_l} y_{l,d-i}$$