Job shop problem model formulation

Intro

Usage of integer optimization model to solve the problem. The problem is described in the following link https://developers.google.com/optimization/scheduling/job_shop

Definitions

- t= 1…T, task index where T is the longest task per a job
- j=1…J, Jobs index
- m=1…M machines index
- M(j,t, m): whether machine “m” is assigned to task “t” from job “j”
- D(j,t): duration of task “t” from job “j”

Decision variables

$x_{j,t,d,m} = \{0, 1\}$: Whether task “t” from job “j” runs on day “d” on machine “m”

Derived useful variables

$x_t = argmax_{d,x\neq 0} x_{j,t,d}$: $task\ makespan$

Objective function

Minimize (max x_t, $t = 0 \ldots T$)

Constraints

C1: Foreach job “j”, each task “t” should run in D(t,j) time in consecutive days

$$\forall j, t, m: \sum_d x_{j,t,d,m} = D(j,t) * \sum_{d=0}^{D-D(t,j)} \prod_{i=d}^{d+D(t,j)} x_{j,t,i,m} \quad (1)$$

C2: Foreach job “j”, tasks start sequentially

$$\forall j, \forall t \geq 1, \forall d \leq D(j,t),\ x_{j,t,d} = 0 \quad (2)$$

$$\forall j, \forall t \geq 1, \forall d > D(j,t), \sum_{i<d} x_{j,t-1,d,m} < D(j,t) \rightarrow x_{j,t,d,m} == 0 \quad (3)$$

C3: Each task runs only on its defined machine

$$\forall d, j, t, m,\ x_{t,j,d,m} = M(j,t,m) \quad (4)$$

C4: Each machine runs at most one task at any given day

$$\forall d, m \sum_{t,j} x_{j,t,d,m} \leq 1 \quad (5)$$