Optimize datacenter

Problem statement for the 2015 hashcode qualification round. Link for the problem description and data could be found here.

Definitions

r=0…R, rows index
s=0…S, slot index
Unavailable = list of pairs u=(u_row, u_slot) containing the coordinates of the unavailable slots
p=0…P, pools index
m=0…M, server index
Servers = list of pairs (capacity, size) describing the servers

Decision variables

$x_{r,s,m,p} = \{0, 1\}$Whether the **leftmost** slot of 'm' is allocated to slot 's' from row 'r' and assigned to pool 'p'

Notation

$c_m = servers[m]_0$: server capacity

$s_m = servers[m]_1$: server size

Derived variables

$capacity_p = \sum_{r,s,m} x_{r,s,m,p} * c_m$:total pool capacity

$gc_p = capacity_p - max_{r,s,m}[x_{r,s,m,p} * c_m]$:guaranteed server capacity

Objective

$maximize[minimum_{p=0..P}(gc_p)]$

Constraints

C1: Each slot of the datacenter is occupied by at most one server
$\forall r, s, m, p: x_{r,s,m,p} = 1 \rightarrow \forall m' \neq m\, x_{r,s+i,m',p} = 0, i = 0... s_{m'}$
$\forall r, s, m, p: x_{r,s,m,p} = 1 \rightarrow x_{r,s+i,m,p} = 0, i = 1... s_m$
C2: No server can occupy the unavailable slots
$\forall u = (u_s, u_r), \forall m, p\, x_{u_r,u_s-i,m,p} = 0, i = 0... s_m - 1$

C3: No server extends beyond the slots of the row

$$\forall m, r, p, \; x_{r,S-i,m,p} = 0 \;, \; i = 0..\,(s_m - 1)$$

C4: Each server belong to no more than a single pool

$$\forall m, \; \sum_{r,s,p} x_{r,s,m,p} \leq 1$$

C5: Servers in a row cannot exceed total slots

$$\forall r, \; \sum_{s,m,p} x_{r,s,m,p} * s_m \leq R - \sum_{s} u_{r,s}$$