Raios : $P + \hat{k} \cdot t$

Planos : $\hat{n} \cdot (X - O) = 0$

Incidência : $\hat{n} \cdot (P_1 + \hat{k}_1 \cdot t - O) = 0$

$\Rightarrow \hat{n} \cdot (P_1 - O) + \hat{n} \cdot \hat{k}_1 \cdot t = 0$

$\Rightarrow t = \dfrac{\hat{n} \cdot (O - P)}{\hat{n} \cdot \hat{k}_1} = \dfrac{n^T (O - P_1)}{n^T k_1}$

$\Rightarrow P_2 = P_1 + \dfrac{n^T (O - P_1)}{n^T k_1} k_1$

Reflexão : $\hat{k}_2 = \hat{k}_1 - 2(\hat{n} \cdot \hat{k}_1) \cdot \hat{n}$

$k_2 = k_1 - 2(n^T k_1) n$

Refração : $\vec{k}_i = n_i \hat{k}$ (* $\perp$ e $\parallel$ com rel. $\hat{n}$)

$\vec{k}_{2\perp} = \vec{k}_{1\perp} = n_1 \hat{k}_{1\perp} = n_2 \hat{k}_{2\perp}$

$\Rightarrow \hat{k}_{2\perp} = \dfrac{n_1}{n_2} \hat{k}_{1\perp} = \dfrac{n_1}{n_2} [\hat{k}_1 - (\hat{n} \cdot \hat{k}_1) \hat{n}]$

$\hat{k}_{2\parallel} = (1 - |k_{2\perp}|^2)^{1/2} \cdot \hat{n}$

$\Rightarrow \hat{k}_2 = \hat{k}_{2\perp} + k_{2\parallel}$

$k_{2\perp} = \dfrac{n_1}{n_2} [k_1 - (n^T k_1) \cdot n]$

$k_{2\parallel} = \sqrt{1 - k_{2\perp}^T k_{2\perp}} \cdot n$

Convenção : Reflexão : 

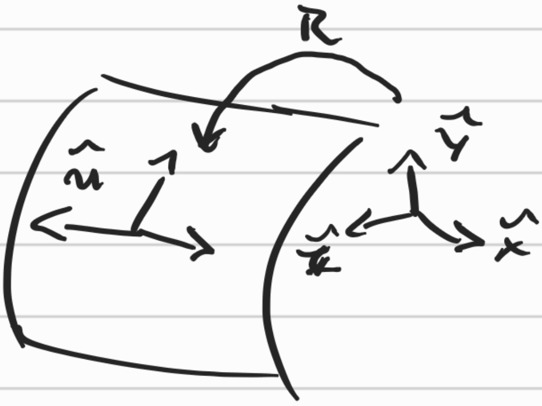
Refração : 

Ou seja :

se $\hat{n} \cdot \hat{k} < 0$: reflexão

se $\hat{n} \cdot \hat{k} > 0$: refração

Lente fina: $P_2 = P_1 + \dfrac{\hat{m}\cdot(O-P_1)}{\hat{m}\cdot\hat{k}_1}\cdot\hat{k}_1$



$$\hat{m} = R\cdot\hat{z}$$
$$P_2 = R\cdot P_2' + O$$
$$P_2' = R^T(P_2 - O)$$
$$\hat{k}_1' = R^T\hat{k}_1$$

$$\theta_{x1} = \sin^{-1}(k'_{1x}/k'_{1xz})$$
$$\theta_{y1} = \sin^{-1}(k'_{1y}/k'_{1yz})$$

$$k'_{1xz} = \sqrt{1-k'^2_{1y}}\ ;\quad k'_{1yz} = \sqrt{1-k'^2_{1x}}$$

$$\theta_{x2} = -\frac{P'_{2x}}{f_x} + \theta_{x1}$$
$$\theta_{y2} = -\frac{P'_{2x}}{f_y} + \theta_{y1}$$

$$k'_{2x} = \sin\theta_{x2}\cdot k'_{2xz} = \sin\theta_{x2}\sqrt{1-k'^2_{2y}}$$
$$k'_{2y} = \sin\theta_{y2}\cdot k'_{2yz} = \sin\theta_{y2}\sqrt{1-k'^2_{2x}}$$

$$k'^2_{2x} = \sin^2\theta_{x2}\cdot(1-k'^2_{2y})$$
$$k'^2_{2y} = \sin^2\theta_{y2}\cdot(1-k'^2_{2x})$$

$x = k'^2_{2x}$; $y = k'^2_{2y}$;
$a = \sin^2\theta_{x2}$; $b = \sin^2\theta_{y2}$

$$\Rightarrow \begin{cases} x = a(1-y) \\ y = b(1-y) \end{cases} \Rightarrow \begin{cases} x + ay = a \\ bx + y = b \end{cases}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1-ab}\begin{bmatrix} 1 & -a \\ -b & 1 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{1-ab}\begin{bmatrix} a-ab \\ b-ab \end{bmatrix}$$

$$\begin{cases} k_{zx}'^2 = \dfrac{\sin^2\theta_{x_2}(1-\sin^2\theta_{y_2})}{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}} = \dfrac{\sin^2\theta_{x_2}\cos^2\theta_{y_2}}{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}} \\ k_{zy}'^2 = \dfrac{\sin^2\theta_{y_2}(1-\sin^2\theta_{x_2})}{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}} = \dfrac{\sin^2\theta_{y_2}\cos^2\theta_{x_2}}{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}} \end{cases}$$

$$k_{zx}' = \frac{\sin\theta_{x_2}\cos\theta_{y_2}}{\sqrt{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}}}$$

$$k_{zy}' = \frac{\sin\theta_{y_2}\cos\theta_{x_2}}{\sqrt{1-\sin^2\theta_{x_2}\sin^2\theta_{y_2}}}$$

$$k_{zz}' = \sqrt{1-k_{zx}'^2-k_{zy}'^2}$$

$$\Rightarrow k_z' = k_{zx}'\,\hat{x} + k_{zy}'\,\hat{y} + k_{zz}'\,\hat{z}$$