Vector plane wave spectrum of an arbitrary polarized electromagnetic wave

Hanming Guo, Jiabi Chen, Songlin Zhuang

Shanghai Key Laboratory of Contemporary Optics System, College of Optics and Electronics, University of Shanghai for Science and Technology, 516 Jungong Rd, 200093 Shanghai, China
slzhuangx@yahoo.com.cn, ghanming@vip.sina.com

Abstract: By using the method of modal expansions of the independent transverse fields, a formula of vector plane wave spectrum (VPWS) of an arbitrary polarized electromagnetic wave in a homogenous medium is derived. In this formula VPWS is composed of TM- and TE-mode plane wave spectrum, where the amplitude and unit polarized direction of every plane wave are separable, which has more obviously physical meaning and is more convenient to apply in some cases compared to previous formula of VPWS. As an example, the formula of VPWS is applied to the well-known radially and azimuthally polarized beam. In addition, vector Fourier-Bessel transform pairs of an arbitrary polarized electromagnetic wave with circular symmetry are also derived.

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1. Introduction

Any a complicated electromagnetic wave can be expressed as a superposition of lots and lots of plane waves by Fourier transform, which is called method of plane wave spectrum and has been widely used in the scalar diffraction theories. When the study of polarization is of major concern vector diffraction theory must be used. At this time, we will need to determine not only the amplitude of every plane wave but also its polarized direction for using method of plane wave spectrum. We refer to this plane wave spectrum as vector plane wave spectrum (VPWS). In addition, VPWS is also an effective method to derive exact solutions of Maxwell's equations. It is, therefore, of interest to derive a simple and conveniently applied formula of VPWS.

Doicu *et al.* [1] give a VPWS formula of Gaussian beams by using Davis [2] approximations for the Hertz vector. Varga *et al.* [3] and Seshadri [4] use scalar Fourier transform of Hertz vector instead of VPWS. The essence of their methods all must first determine the Hertz vector. Rosario *et al.* [5] present a VPWS formula in terms of the so-called closest field to a given a beam. In practical applications the initial fields of an arbitrary polarized electromagnetic wave may be obtained by theoretical analyses or experimental results. Obviously, if the VPWS may be directly determined by the initial electromagnetic field, it will be very convenient. Some authors [6-8], therefore, employ the scalar pane wave spectrum of each transverse component of initial electromagnetic field and divergence theorem to obtain a formula of VPWS. In this formula, the amplitude and polarization of every plane wave can not be separable.

In this paper a formula of VPWS of an arbitrary polarized electromagnetic wave in a homogenous medium is derived by the method of modal expansions of the independent transverse fields. In this formula VPWS is composed of TM- (the polarized direction of magnetic field perpendicular to the plane formed by wave vector and longitudinal direction) and TE-mode (the polarized direction of electric field perpendicular to the plane formed by wave vector and longitudinal direction) plane wave spectrum, where the amplitude and unit polarized direction of every plane wave are separable. To author's knowledge, such explicit expression of VPWS has never been derived. Furthermore, it should be stressed that, although mathematically equivalent to the formula of VPWS in Refs. 6-8 using suitable transformation, the formula of VPWS in this paper has more obviously physical meaning because of separation of the amplitude and polarized direction, and is more convenient to apply in some cases because of some particular characteristics of TM- and TE-mode plane waves.

In Section 2, methods for constructing VPWS of an arbitrary polarized electromagnetic wave in a homogenous medium are discussed in detail. As an example for showing the advantage of the formula of VPWS in this paper, the VPWS is applied to the well-known radially and azimuthally polarized beam in Section 3. Finally, some conclusions are drawn.

2. Vector plane wave spectrum

As we know, Maxwell's equations may be separated into transverse and longitudinal field equations. For unbounded and uniform cross sections, it is possible to seek modal expansions of the independent transverse electric fields $\boldsymbol{E}_t(\boldsymbol{r})$ and magnetic fields $\boldsymbol{H}_t(\boldsymbol{r})$ and to derive therefrom the dependent longitudinal components. These modes may be described by means of TM- and TE-mode decomposition. Here, let the coordinate z axis be longitudinal direction and the (x, y) plane be the cross section which is homogeneous and of infinite extent. So a possible complete transverse eigenvector set comprising TM-mode functions $\boldsymbol{e}'(\boldsymbol{\rho})$, $\boldsymbol{h}'(\boldsymbol{\rho})$ and TE-mode functions $\boldsymbol{e}''(\boldsymbol{\rho})$, $\boldsymbol{h}''(\boldsymbol{\rho})$ may be found easily, as follows [9]:

$$\boldsymbol{e}_i'(\boldsymbol{\rho}) = -\frac{\nabla_t \Phi_i(\boldsymbol{\rho})}{k_{ti}'}, \tag{1a}$$

$$\boldsymbol{e}_i''(\boldsymbol{\rho}) = -\frac{\nabla_t \Psi_i(\boldsymbol{\rho})}{k_{ti}''} \times \hat{z}, \tag{1b}$$

$$\boldsymbol{h}_i'(\boldsymbol{\rho}) = -\hat{z}\times\frac{\nabla_t \Phi_i(\boldsymbol{\rho})}{k_{ti}'}\,, \tag{2a}$$

$$\boldsymbol{h}_i''(\boldsymbol{\rho}) = -\frac{\nabla_t \Psi_i(\boldsymbol{\rho})}{k_{ti}''}\,, \tag{2b}$$

$$\Phi_i(\boldsymbol{\rho}) = \Psi_i(\boldsymbol{\rho}) = \frac{1}{2\pi}\exp[-\mathrm{j}(k_{xi}x + k_{yi}y)]\,, \tag{3}$$

where i is in general a double index , single and double quotation marks separately denote TM- and TE-mode, $\nabla_t = \nabla - (\partial/\partial z)\hat{z}$, $\boldsymbol{r}$ is location vector, $\boldsymbol{\rho} = \boldsymbol{r} - \hat{z}z$ is the projection of $\boldsymbol{r}$ on the (x, y) plane, and $\hat{z}$ is the unit vector in the positive z direction, and the ith-order transverse Cartesian components k_{xi} , k_{yi} and transverse component k_{ti} of the wave vector $\boldsymbol{k}$ satisfy the relation

$$k_{ti}^2 = k_{xi}^2 + k_{yi}^2\,,\ -\infty < k_{xi} < \infty\,,\ -\infty < k_{yi} < \infty\,. \tag{4}$$

As the cross sections are unbounded and uniform, mode $\boldsymbol{e}_i$ and $\boldsymbol{h}_i$ will become continuous. So, in terms of the indicated mode functions, representations of the independent transverse fields are given as

$$\boldsymbol{E}_t(\boldsymbol{r}) = \iint_\infty V'(z)\boldsymbol{e}'(\boldsymbol{\rho})dk_x dk_y + \iint_\infty V''(z)\boldsymbol{e}''(\boldsymbol{\rho})dk_x dk_y\,, \tag{5a}$$

$$\boldsymbol{H}_t(\boldsymbol{r}) = \iint_\infty I'(z)\boldsymbol{h}'(\boldsymbol{\rho})dk_x dk_y + \iint_\infty I''(z)\boldsymbol{h}''(\boldsymbol{\rho})dk_x dk_y \tag{5b}$$

$$\boldsymbol{E}_z(\boldsymbol{r}) = \hat{z}\frac{1}{\mathrm{j}2\pi\omega\varepsilon}\iint_\infty I'(z)k_t'\exp[-\mathrm{j}(k_x x + k_y y)]dk_x dk_y\,, \tag{5c}$$

where $V'(z)$, $V''(z)$, $I'(z)$ and $I''(z)$ refer to the TM- and TE-mode amplitudes of electric and magnetic field, respectively. ε is homogenous medium permittivity and ω is angular frequency. According to the orthogonality conditions of transverse eigenvector mode functions over the cross-sectional domain, the desired transmission-line equations for the TM- and TE-mode amplitudes may be obtained as [9]

$$-\frac{dV(z)}{dz} = \mathrm{j}k_z\eta I(z)\,, \tag{7a}$$

$$-\frac{dI(z)}{dz} = \mathrm{j}\frac{k_z}{\eta}V(z)\,, \tag{7b}$$

where the modal characteristic impedance η is defined as $\eta = k_z/\omega\varepsilon$ for TM-mode and $\eta = \omega\mu/k_z$ for TE-mode. The longitudinal component of wave vector is $k_z = (k^2 - k_t^2)^{1/2} = -\mathrm{j}(k_t^2 - k^2)^{1/2}$, where k is wave number in a homogenous medium. The choice of sign on the square roots in k_z must assure the damping of non-propagating modes (k_z imaginary) away from the source region for the harmonic time dependence $\exp(\mathrm{j}\omega t)$. In

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the Eqs. (7), subscript i, single and double quotation marks are omitted because they are applicable for every TM- and TE-mode. Let the medium be homogenous and an electromagnetic wave propagate along the positive z direction, so the solutions of Eqs. (7) may be written as

$$I'(z) = \tilde{E}_{\mathrm{TM}} \exp(-\mathrm{j}k_z z) , \tag{8a}$$

$$V'(z) = \frac{k_z}{\omega\varepsilon} \tilde{E}_{\mathrm{TM}} \exp(-\mathrm{j}k_z z) , \tag{8b}$$

$$V''(z) = \tilde{E}_{\mathrm{TE}} \exp(-\mathrm{j}k_z z) , \tag{8c}$$

where $\tilde{E}_{\mathrm{TM}}$ and $\tilde{E}_{\mathrm{TE}}$ are independent constant of position variable (x, y, z).

Substituting Eqs. (1) and (8) into Eqs. (5), transform pairs of VPWS of an arbitrary polarized electromagnetic wave may be derived, as follows:

$$\begin{aligned} \boldsymbol{E}(\boldsymbol{r}) = &-\frac{1}{4\pi^2}\iint_\infty [\frac{k_z}{kk_t}(\hat{x}k_x + \hat{y}k_y) - \hat{z}\frac{k_t}{k}]\tilde{E}_{\mathrm{TM}} \exp[-\mathrm{j}(k_x x + k_y y + k_z z)]dk_x dk_y \\ &-\frac{1}{4\pi^2}\iint_\infty (\hat{x}\frac{k_y}{k_t} - \hat{y}\frac{k_x}{k_t})\tilde{E}_{\mathrm{TE}} \exp[-\mathrm{j}(k_x x + k_y y + k_z z)]dk_x dk_y \end{aligned} , \tag{9}$$

$$\tilde{E}_{\mathrm{TM}} = \iint_\infty \frac{k}{k_t k_z}(k_x E_x + k_y E_y) \exp[\mathrm{j}(k_x x + k_y y + k_z z)]dxdy , \tag{10a}$$

$$\tilde{E}_{\mathrm{TE}} = \iint_\infty \frac{1}{k_t}(k_y E_x - k_x E_y) \exp[\mathrm{j}(k_x x + k_y y + k_z z)]dxdy . \tag{10b}$$

Equation (9) suggests that an arbitrary polarized electromagnetic wave can be expressed in terms of the sum of two terms, namely TM- [the first part of right of Eq. (9)] and TE-mode [the second part of right of Eq. (9)] plane wave spectrum. $\tilde{E}_{\mathrm{TM}}$ and $\tilde{E}_{\mathrm{TE}}$ are called the amplitude of TM- and TE-mode spectrum function, which are determined by the two transverse field components instead of one of those. The formula (9) of VPWS can be simply demonstrated that the divergence condition of the electric field is satisfied and the polarized direction of every plane wave component is perpendicular to its own wave vector. To author's knowledge, such explicit expression of VPWS has never been derived. Furthermore, compared with the formula of VPWS in Refs. 6-8, the formula of VPWS in this paper has more obviously physical meaning because of separation of the amplitude and polarized direction, and is more convenient to apply in some cases because of some particular characteristics of TM- and TE-mode plane wave.

In addition, equations (9) and (10) also represent exact solutions of Maxwell's equations. Comparing Eqs. (9) and (10) with exact solutions of Maxwell's equations expressed by Eqs. (22)~(25) in Ref. 3, they are actually identical. Differences between them are that the former is denoted by transverse electric field components and the latter is denoted by Hertz vector.

3. Examples

As an example for showing the advantage of the formula of VPWS in this paper, the VPWS is applied to the well-known radially and azimuthally polarized beam. We first introduce Cylindrical coordinates (ρ, φ, z) and (u, ϕ, k_z) in spatial domain and spectral domain, respectively. As both radially and azimuthally polarized beam are circular symmetry, namely they are independent of variable φ in Cylindrical coordinates [10,11], then according to the relations among the orthogonal coordinates [12], in the Cylindrical coordinates, equations (9) and (10) may be rewritten as

$$\begin{aligned} \boldsymbol{E}(\boldsymbol{r}) = & \frac{1}{2\pi}\int_0^k \frac{1}{k}[\hat{\rho}\mathrm{j}(k^2-u^2)^{1/2}J_1(u\rho)+\hat{z}uJ_0(u\rho)]u \\ & \times\tilde{E}_{\mathrm{TM}}(u)\exp[-\mathrm{j}(k^2-u^2)^{1/2}z]\mathrm{d}u \\ & -\hat{\varphi}\mathrm{j}\frac{1}{2\pi}\int_0^k \tilde{E}_{\mathrm{TE}}(u)J_1(u\rho)u\exp[-\mathrm{j}(k^2-u^2)^{1/2}z]\mathrm{d}u \end{aligned} \quad , \tag{11}$$

$$\tilde{E}_{\mathrm{TM}}(u) = \frac{\mathrm{j}2\pi k}{\sqrt{k^2-u^2}}\int_0^\infty E_\rho(\rho)J_1(u\rho)\rho\mathrm{d}\rho\,, \tag{12a}$$

$$\tilde{E}_{\mathrm{TE}}(u) = -\mathrm{j}2\pi\int_0^\infty E_\varphi(\rho)J_1(u\rho)\rho\mathrm{d}\rho\,, \tag{12b}$$

where $\hat{\varphi}$ and $\hat{\rho}$ are unit vectors in the azimuthal and radial directions, respectively. $J_n(u\rho)$ ($n=0,1$) is the Bessel function of the first kind and order *n*. Equations (11) and (12) are called vector Fourier-Bessel transform pairs of an arbitrary polarized electromagnetic wave with circular symmetry, where evanescent waves are neglectful.

For a radially polarized beam, we take $E_\varphi(\rho)=0$. At this time, the amplitude of TE-mode spectrum function $\tilde{E}_{\mathrm{TE}}$ and the second term of Eq. (11) are identically equal to zero when the beam propagates. Therefore, only TM-mode plane waves contribute the electric fields of a radially polarized beam. Furthermore, for on-axis illumination, as polarization of every TM-mode polarized light ray only rotates in the plane formed by wave vector and *z* axis when it propagates through a lens with principal axis being *z* axis [13], the polarization of every light ray will still be TM polarized, namely a radially polarized beam propagates as a purely radial polarization without azimuthal components through the entire image space [14].

For an azimuthally polarized beam, we take $E_\rho(\rho)=0$. At this time, the amplitude of TM-mode spectrum function $\tilde{E}_{\mathrm{TM}}$ and the first term of Eq. (11) are identically equal to zero when the beam propagates. Therefore, only TE-mode plane waves contribute the electric fields of an azimuthally polarized beam. Obviously, an azimuthally polarized beam propagates as a purely transverse polatization without radial and longitudinal components through the entire space. Moreover, for on-axis illumination, as polarization of every TE-mode polarized light ray remains unchanged when it propagates through a lens with principal axis being *z* axis [13], the polarization of every light ray located in image space will also remain unchanged, namely an azimuthally polarized beam propagates as a purely transverse polatization without radial and longitudinal components through the entire image space [14].

In view of the analyses above, we can draw a conclusion that a radially polarized beam is the exclusive beam composed of only TM-mode plane waves, and an azimuthally polarized beam is the exclusive beam composed of only TE-mode plane waves. Therefore, we may predict that, for on-axis illumination, when a radially polarized beam is coupled into a perfect planar waveguide by a lens, only TM-mode plane waves are excited in the waveguide, and when an azimuthally polarized beam is coupled into a perfect planar waveguide by lens, only TE-mode plane waves are excited in the waveguide.

Of course, the characteristics above of focusing of radially and azimuthally polarized beams can also be obtained in terms of an integral expression for the focal field distribution, where long vector operations will be necessary as introduced in Refs. 13 and 14. However, compared with the method of VPWS in this paper, physical interpretations of the method of the integral expression for the focal field distribution are not straightforward.

Apart from this typical example, one of the many possible applications of the method of VPWS in this paper is in the vector diffraction theories as the wide applications of the method of scalar plane wave spectrum in the scalar diffraction theories. For example, it may be effectively used to investigate the spatial characteristics of light beams [5] when polarization is of major concern and derive exact solutions of Maxwell's equations [3].

4. Conclusions

A formula of VPWS of an arbitrary polarized electromagnetic wave in a homogenous medium is presented, where VPWS is composed of TM- and TE-mode plane wave spectrum, and the amplitude and unit polarized direction of every plane wave are separable. To author's knowledge, such explicit expression of VPWS has never been derived. Although this foumula is mathematically equivalent to the formula of VPWS in Refs. 6-8 using suitable transformation, it has more obviously physical meaning because of separation of the amplitude and polarized direction, and is more convenient to apply in some cases because of some particular characteristics of TM- and TE-mode plane waves. A good example is its application to a radially or azimuthally polarized beam. In terms of particular characteristics of TM- and TE-mode plane waves, we can be easier to obtain and understand some characteristics of propagation and diffraction of a radially or azimuthally polarized beam, and predict that, for on-axis illumination, a radially or azimuthally polarized beam can excite only TM- or TE-mode plane waves in the waveguide when they are coupled into a perfect planar waveguide by a lens, respectively. In addition, vector Fourier-Bessel transform pairs of an arbitrary polarized electromagnetic wave with circular symmetry are also derived.

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