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Analytical structure of an apertured vector Gaussian beam in the far field

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Abstract

Based on the angular spectrum representation of the Maxwell's equations and the complex Gaussian expansion of the aperture function, the structure of an apertured vector Gaussian beam in the far field is presented in the integral form. By means of the method of stationary phase, the analytical vectorial structures are obtained. According to the analytical expressions, the characteristics of vectorial structure of an apertured Gaussian beam are investigated in the far field. The influence of a linearly polarized angle on the vectorial structure is also studied in the far field. This research provides a novel approach to further comprehend the vectorial property of an apertured Gaussian beam.
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1. Introduction

Gaussian beams are the fundamental modes of laser cavities with spherical mirrors [1]. However, there are usually apertures in the practical optical systems. Therefore, the apertured Gaussian beams have attracted considerable attention and have been studied using many different methods. Moreover, the characteristics of an apertured Gaussian beam are well revealed and understood. In the far field, a Gaussian beam weakly diffracted by a circular aperture can be approximately viewed as another Gaussian beam with slightly different characteristics, and the characteristics of the diffracted beam may appreciably differ from those of the incident beam [2]. Within the non-paraxial framework, the far field properties of an apertured Gaussian beam have been analyzed using the scalar angular spectrum method [3]. The analysis shows that the f-parameter and the truncation parameter affect the far field properties of an apertured Gaussian beam. On the basis of the vectorial Rayleigh diffraction integral, the approximate analytical expression for the propagation equation of an apertured vector Gaussian beam has been derived, and the focusing effect has also been discussed [4]. Recently, the vectorial structures of Laguerre–Gaussian beam and Ince–Gaussian beam have been revealed in the far field [5,6]. A Gaussian beam is a particular case of Laguerre–Gaussian and Ince–Gaussian beams. The influence of the presence of a circular aperture in the vectorial structure of a Gaussian beam is deserved investigation.

Here, the description of a Gaussian beam through a circular aperture is directly derived from the Maxwell's equations. The vector angular spectrum method is a useful tool to resolve the Maxwell's equations [7–10]. Furthermore, the aperture function is expanded as a sum of finite-term complex Gaussian functions. As the vector angular spectrum can be separated into two terms in the frequency domain, the apertured Gaussian beam is decomposed into the TE and TM terms. The TE term denotes the electric field transverse to the propagation axis, and the TM term

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the associated magnetic field transverse to the propagation axis [10–12]. Expressed in angular spectrum representation, the vectorial structure of the apertured Gaussian beam appears as the integral form. By means of the method of stationary phase, the analytical expressions of the TE and TM terms are presented in the far field. As Gaussian beams are treated as linearly polarized fields in both theoretical and practical applications [7,13], the influence of a linearly polarized angle on the vectorial structure of an apertured Gaussian beam is also investigated in the far field.

2. Vectorial structure of an apertured vector Gaussian beam in the far field

A Cartesian coordinate system is constructed as follows. The circular aperture plane is selected as the x–y plane and coincides with the beam waist plane of a Gaussian beam. The center of the circular aperture is the origin. The z-axis is taken to be the propagation axis. The half space $z \geqslant 0$ is a free space with electric permittivity ε_0 and the magnetic permeability μ_0. Considering that the radius of the circular aperture R is much larger than the wavelength of incident Gaussian beam λ, the vector Gaussian beam just behind the aperture reads as

$$\begin{pmatrix} E_x(x_0,y_0,0) \\ E_y(x_0,y_0,0) \end{pmatrix} = \exp\left(-\frac{\rho_0^2}{w_0^2}\right)\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}\mathrm{circ}(\zeta), \tag{1}$$

where w_0 is the Gaussian beam waist, $\zeta = \rho_0/R$, $\rho_0 = (x_0^2+y_0^2)^{1/2}$, Jones vector $\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}$ describes the linearly polarized state, and α is the linearly polarized angle. The aperture function is given by

$$\mathrm{circ}(\zeta) = \begin{cases} 1 & 0 \leqslant \zeta < 1 \\ 0 & \zeta \geqslant 1 \end{cases}. \tag{2}$$

The time dependent factor $\exp(-\mathrm{i}\omega t)$ is omitted in Eq. (1), and ω is the circular frequency. In the present paper, the description of an apertured Gaussian beam is directly derived from the Maxwell's equations:

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) - \mathrm{i}k\boldsymbol{H}(\boldsymbol{r}) = 0, \tag{3}$$

$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) + \mathrm{i}k\boldsymbol{E}(\boldsymbol{r}) = 0, \tag{4}$$

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \nabla \cdot \boldsymbol{H}(\boldsymbol{r}) = 0, \tag{5}$$

where $\boldsymbol{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, and $k = 2\pi/\lambda$. $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r})$ are the propagating electromagnetic fields of the apertured Gaussian beam. If the Maxwell's equations are transformed from the space domain into the frequency domain, Eqs. (3)–(5) become

$$\boldsymbol{L} \times \widetilde{\boldsymbol{E}}(p,q,z) - \mathrm{i}k\widetilde{\boldsymbol{H}}(p,q,z) = 0, \tag{6}$$

$$\boldsymbol{L} \times \widetilde{\boldsymbol{H}}(p,q,z) + \mathrm{i}k\widetilde{\boldsymbol{E}}(p,q,z) = 0, \tag{7}$$

$$\boldsymbol{L} \cdot \widetilde{\boldsymbol{E}}(p,q,z) = \boldsymbol{L} \cdot \widetilde{\boldsymbol{H}}(p,q,z) = 0, \tag{8}$$

where $\boldsymbol{L} = \mathrm{i}kp\mathbf{i} + \mathrm{i}kq\mathbf{j} + \partial/\partial z\mathbf{k}$. p/λ and q/λ are the transversal frequencies. $\widetilde{\boldsymbol{E}}(p,q,z)$ and $\widetilde{\boldsymbol{H}}(p,q,z)$ denote the spatial Fourier transforms of $\boldsymbol{E}(\boldsymbol{r})$ and $\boldsymbol{H}(\boldsymbol{r})$, respectively. The solutions of Eqs. (6)–(8) can be expressed in the form as

$$\widetilde{\boldsymbol{E}}(p,q,z) = \boldsymbol{A}(p,q,\gamma)\exp(\mathrm{i}k\gamma z), \tag{9}$$

$$\widetilde{\boldsymbol{H}}(p,q,z) = [\boldsymbol{s} \times \boldsymbol{A}(p,q,\gamma)]\exp(\mathrm{i}k\gamma z), \tag{10}$$

where $\boldsymbol{s} = p\mathbf{i} + q\mathbf{j} + \gamma\mathbf{k}$, and $\gamma = (1-p^2-q^2)^{1/2}$. The vector angular spectrum $\boldsymbol{A}(p,q,\gamma)$ reads as

$$\boldsymbol{A}(p,q,\gamma) = A_x(p,q,\gamma)\mathbf{i} + A_y(p,q,\gamma)\mathbf{j} + A_z(p,q,\gamma)\mathbf{k}. \tag{11}$$

The propagating electric field of an apertured Gaussian beam yields [7–10]

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \widetilde{\boldsymbol{E}}(p,q,z)\exp[\mathrm{i}k(px+qy)]\mathrm{d}p\,\mathrm{d}q \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boldsymbol{A}(p,q,\gamma)\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\,\mathrm{d}q.\end{aligned} \tag{12}$$

The transverse components of the vector angular spectrum is given by the Fourier transform of the boundary condition

$$\begin{aligned}A_x(p,q,\gamma) &= \left(\frac{k}{2\pi}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_x(x_0,y_0,0) \\ &\quad \times \exp[-\mathrm{i}k(px_0+qy_0)]\mathrm{d}x_0\mathrm{d}y_0 \\ &= \frac{k\cos\alpha}{\lambda}\int_0^{\infty}\exp\left(-\frac{\rho_0^2}{w_0^2}\right)\mathrm{circ}(\zeta)J_0(k\rho_0 b)\rho_0\mathrm{d}\rho_0,\end{aligned} \tag{13}$$

$$A_y(p,q,\gamma) = \frac{k\sin\alpha}{\lambda}\int_0^{\infty}\exp\left(-\frac{\rho_0^2}{w_0^2}\right)\mathrm{circ}(\zeta)J_0(k\rho_0 b)\rho_0\mathrm{d}\rho_0, \tag{14}$$

where $b = (p^2+q^2)^{1/2}$. The longitudinal component of the vector angular spectrum is given by the orthogonal relation $\boldsymbol{s} \cdot \boldsymbol{A}(p,q,\gamma) = 0$ and turns out to be

$$A_z(p,q,\gamma) = -[pA_x(p,q,\gamma) + qA_y(p,q,\gamma)]/\gamma. \tag{15}$$

To obtain the analytical vector angular spectrum, the aperture function should be expanded as the linear superposition of the complex Gaussian function [14]

$$\mathrm{circ}(\zeta) = \sum_{n=1}^{15} B_n\exp(-C_n\zeta^2), \quad \zeta \in [0,\infty), \tag{16}$$

where the complex expanded and Gaussian coefficients B_n and C_n can be obtained by optimization computation [14]. Substituting Eq. (16) into Eqs. (13) and (14), one can obtain

$$A_x(p,q,\gamma) = \frac{\cos\alpha z_r}{\lambda}\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\exp\left(\frac{-b^2}{4f^2(1+C_n/C_0)}\right), \tag{17}$$

$$A_y(p,q,\gamma) = \frac{\sin\alpha z_r}{\lambda}\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\exp\left(\frac{-b^2}{4f^2(1+C_n/C_0)}\right), \tag{18}$$

where $z_r = kw_0^2/2$, $f = 1/kw_0$, and $C_0 = R^2/w_0^2$.

According to the vectorial structure of the electromagnetic beam [10–12], two unit vectors $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ can be defined as follows

$$\boldsymbol{e}_1 = (q\mathbf{i} - p\mathbf{j})/b, \quad \boldsymbol{e}_2 = \gamma(p\mathbf{i} + q\mathbf{j})/b - b\mathbf{k}. \tag{19}$$

The three unit vectors $\boldsymbol{s}$, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ form a mutually perpendicular right-handed system

$$\boldsymbol{s} \times \boldsymbol{e}_1 = \boldsymbol{e}_2, \quad \boldsymbol{e}_1 \times \boldsymbol{e}_2 = \boldsymbol{s}, \quad \boldsymbol{e}_2 \times \boldsymbol{s} = \boldsymbol{e}_1. \tag{20}$$

In this system, the vector angular spectrum $\boldsymbol{A}(p,q,\gamma)$ can be decomposed into two terms

$$\boldsymbol{A}(p,q,\gamma) = [\boldsymbol{A}(p,q,\gamma) \cdot \boldsymbol{e}_1]\boldsymbol{e}_1 + [\boldsymbol{A}(p,q,\gamma) \cdot \boldsymbol{e}_2]\boldsymbol{e}_2. \tag{21}$$

As a result, the propagating apertured Gaussian beam can be decomposed into the TE and TM terms

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r}) + \boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}), \tag{22}$$

with $\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r})$ and $\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r})$ given by

$$\begin{aligned}\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r}) = \frac{z_r}{\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0} \\ \times \exp\left(\frac{-b^2}{4f^2(1+C_n/C_0)}\right)\frac{q\cos\alpha - p\sin\alpha}{b^2} \\ \times (q\mathbf{i} - p\mathbf{j})\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\,\mathrm{d}q,\end{aligned} \tag{23}$$

$$\begin{aligned}\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}) = \frac{z_r}{\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0} \\ \times \exp\left(\frac{-b^2}{4f^2(1+C_n/C_0)}\right)\frac{p\cos\alpha + q\sin\alpha}{b^2\gamma} \\ \times (p\gamma\mathbf{i} + q\gamma\mathbf{j} - b^2\mathbf{k})\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\,\mathrm{d}q.\end{aligned} \tag{24}$$

As the divergence condition of the electric field should be satisfied and the polarized direction of every plane wave component must be perpendicular to its own wave vector, the TE and TM terms of the apertured Gaussian beam are unique. By taking the curl of Eq. (24), the corresponding magnetic field turns out to be

$$\begin{aligned}\boldsymbol{H}_{\mathrm{TM}}(\boldsymbol{r}) = -\sqrt{\frac{\varepsilon_0}{\mu_0}}\frac{z_r}{\lambda}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0} \\ \times \exp\left(\frac{-b^2}{4f^2(1+C_n/C_0)}\right)\frac{p\cos\alpha + q\sin\alpha}{\gamma b^2} \\ \times (q\mathbf{i} - p\mathbf{j})\exp[\mathrm{i}k(px+qy+\gamma z)]\mathrm{d}p\,\mathrm{d}q.\end{aligned} \tag{25}$$

Here, the TE and TM terms denote that the longitudinal component of the electric and magnetic fields is equal to zero, respectively.

As the TE and TM terms are expressed in integral forms, it is un-intuitive and difficult to analyze their physical meaning. However, the condition $kr = k(x^2+y^2+z^2)^{1/2} \to \infty$ is satisfied in the far field regime. Therefore, the analytical TE and TM terms of the apertured Gaussian beam in the far field can be presented by means of the method of stationary phase [15,16]. According to the method of stationary phase, Eq. (12) can be rewritten in the analytical form as

$$\boldsymbol{E}(\boldsymbol{r}) = -\frac{\mathrm{i}\lambda z}{r^2}\boldsymbol{A}\left(\frac{x}{r},\frac{y}{r},\frac{z}{r}\right)\exp(\mathrm{i}kr) \tag{26}$$

Therefore, the analytical TE and TM terms in the far field are found to be

$$\begin{aligned}\boldsymbol{E}_{\mathrm{TE}}(\boldsymbol{r}) = -\frac{\mathrm{i}z_r z(y\cos\alpha - x\sin\alpha)}{r^2\rho^2}(y\mathbf{i} - x\mathbf{j})\exp(\mathrm{i}kr) \\ \times \sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right),\end{aligned} \tag{27}$$

$$\begin{aligned}\boldsymbol{E}_{\mathrm{TM}}(\boldsymbol{r}) = -\frac{iz_r(x\cos\alpha + y\sin\alpha)}{r^2\rho^2}(xz\mathbf{i} + yz\mathbf{j} - \rho^2\mathbf{k}) \\ \times \exp(\mathrm{i}kr)\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right),\end{aligned} \tag{28}$$

where $\rho = (x^2+y^2)^{1/2}$. Apparently, the TE and TM terms are orthogonal to each other in the far field. The propagating apertured Gaussian beam is given by the sum of the TE and TM terms

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) = -\frac{\mathrm{i}z_r z}{r^2}\left(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j} - \frac{x\cos\alpha + y\sin\alpha}{z}\mathbf{k}\right)\exp(\mathrm{i}kr) \\ \times \sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right).\end{aligned} \tag{29}$$

Considering the paraxial case and the far field conditions, the above equation reduces to be

$$\begin{aligned}\boldsymbol{E}(\boldsymbol{r}) = -\frac{\mathrm{i}z_r z}{r^2}(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})\exp(\mathrm{i}kr) \\ \times \sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right).\end{aligned} \tag{30}$$

The light intensity of the TE term turns out to be

$$\begin{aligned}I_{\mathrm{TE}}(\boldsymbol{r}) = I_{\mathrm{TE}_x}(\boldsymbol{r}) + I_{\mathrm{TE}_y}(\boldsymbol{r}) = \frac{z_r^2 z^2\sin^2(\varphi-\alpha)}{r^4} \\ \times \left|\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2,\end{aligned} \tag{31}$$

where

$$\begin{aligned}I_{\mathrm{TE}_x}(\boldsymbol{r}) = \frac{z_r^2 z^2\sin^2\varphi\sin^2(\varphi-\alpha)}{r^4} \\ \times \left|\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2,\end{aligned} \tag{32}$$

$$\begin{aligned}I_{\mathrm{TE}_y}(\boldsymbol{r}) = \frac{z_r^2 z^2\cos^2\varphi\sin^2(\varphi-\alpha)}{r^4} \\ \times \left|\sum_{n=1}^{15}\frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2,\end{aligned} \tag{33}$$

and $\varphi = \mathrm{atan}(y/x)$. Similarly, the light intensity of the TM term is given by

$$I_{\mathrm{TM}}(\boldsymbol{r}) = I_{\mathrm{TM}x}(\boldsymbol{r}) + I_{\mathrm{TM}y}(\boldsymbol{r}) = \frac{z_r^2 z^2 \cos^2(\varphi-\alpha)}{r^4} \times \left|\sum_{n=1}^{15} \frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2, \tag{34}$$

where

$$I_{\mathrm{TM}_x}(\boldsymbol{r}) = \frac{z_r^2 z^2 \cos^2\varphi\cos^2(\varphi-\alpha)}{r^4} \times \left|\sum_{n=1}^{15} \frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2, \tag{35}$$

$$I_{\mathrm{TM}_y}(\boldsymbol{r}) = \frac{z_r^2 z^2 \sin^2\varphi\cos^2(\varphi-\alpha)}{r^4} \times \left|\sum_{n=1}^{15} \frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2. \tag{36}$$

The longitudinal component of the light intensity of the TM term is omitted because

$$I_{\mathrm{TM}_z}(\boldsymbol{r}) = \frac{\rho^2}{z^2}\frac{z_r^2 z^2 \cos^2(\varphi-\alpha)}{r^4} \times \left|\sum_{n=1}^{15} \frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2 = \frac{\rho^2}{z^2}[I_{\mathrm{TM}_x}(\boldsymbol{r}) + I_{\mathrm{TM}_y}(\boldsymbol{r})] \approx 0. \tag{37}$$

Accordingly, the light intensity of the apertured Gaussian beam yields

$$I(\boldsymbol{r}) = I_{\mathrm{TE}}(\boldsymbol{r}) + I_{\mathrm{TM}}(\boldsymbol{r}) = \frac{z_r^2 z^2}{r^4}\left|\sum_{n=1}^{15} \frac{B_n}{1+C_n/C_0}\left(\frac{-\rho^2}{4f^2r^2(1+C_n/C_0)}\right)\right|^2. \tag{38}$$

As C_0 is determined by the ratio of R^2/w_0^2, the aperture radius affects the light intensity distributions of the TE term, the TM term and the apertured Gaussian beam.

When the aperture radius is far larger than the value of w_0 and tends to infinity, the light intensity of the TE term reduces to be

$$I_{\mathrm{TE}}(\boldsymbol{r}) = \frac{z_r^2 z^2 \sin^2(\varphi-\alpha)}{r^4}\exp\left(-\frac{\rho^2}{2f^2r^2}\right)\left|\sum_{n=1}^{15} B_n\right|^2 = \frac{z_r^2 z^2 \sin^2(\varphi-\alpha)}{r^4}\exp\left(-\frac{\rho^2}{2f^2r^2}\right). \tag{39}$$

The last step is stemmed from that $\left|\sum_{n=1}^{15} B_n\right|^2 = |\mathrm{circ}(0)|^2 = 1$. Similarly, the light intensity of the TM term reads as

$$I_{\mathrm{TM}}(\boldsymbol{r}) = \frac{z_r^2 z^2 \cos^2(\varphi-\alpha)}{r^4}\exp\left(-\frac{\rho^2}{2f^2r^2}\right). \tag{40}$$

Accordingly, the light intensity of the un-apertured Gaussian beam is found to be

$$I(\boldsymbol{r}) = \frac{z_r^2 z^2}{r^4}\exp\left(-\frac{\rho^2}{2f^2r^2}\right). \tag{41}$$

The vectorial structure of a un-apertured Gaussian beam with $\alpha = 0$ is same as the results of Refs. [5,6]. The traditional propagating scalar Gaussian beam is given by

$$E_{\mathrm{G}}(\boldsymbol{r}) = \frac{w_0}{w(z)}\exp\left(-\frac{\rho^2}{w^2(z)}\right) \times \exp\times\left[\mathrm{i}\left(kz - \mathrm{atan}(z/z_r) + \frac{z\rho^2}{z_r w^2(z)}\right)\right], \tag{42}$$

where $w(z) = w_0(1+z^2/z_r^2)^{1/2}$ is the beam radius. In the far field, Eq. (42) is simplified to be

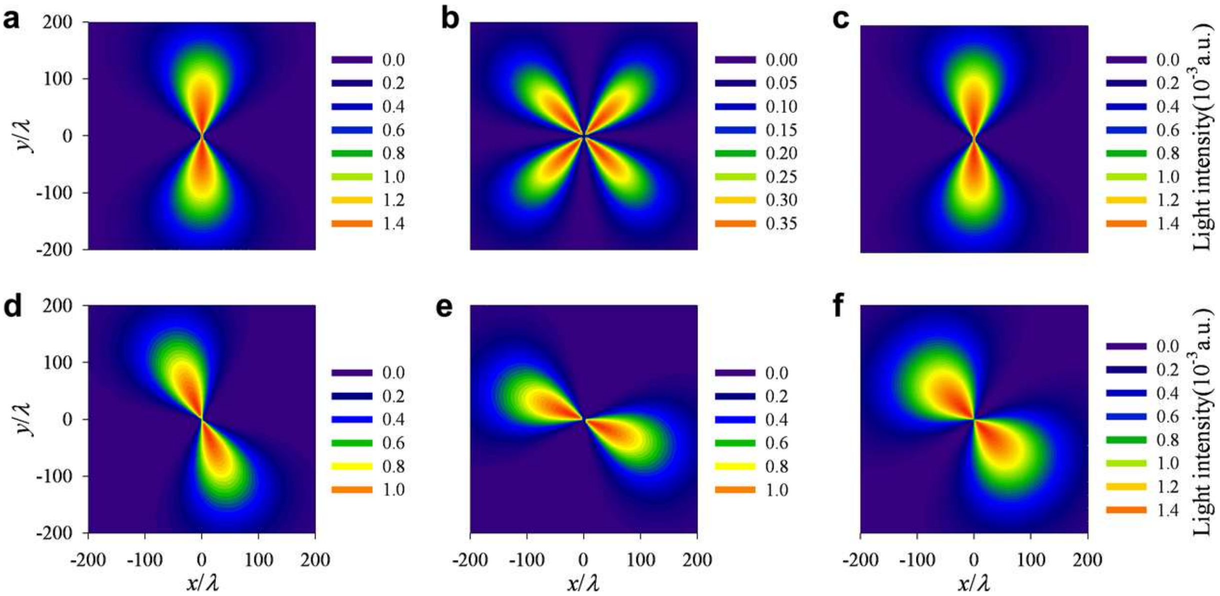

Fig. 1. Light intensity distributions of the TE term and its component. $w_0 = 10\lambda$, $R = 10\lambda$ and $z = 5000\ \lambda$. The top row denotes $\alpha = 0°$, and the bottom row $\alpha = 45°$: (a, d) I_{TE_x}, (b, e) I_{TE_y}, and (c, f) I_{TE}.

$$E_G(\boldsymbol{r}) = -\frac{\mathrm{i}z_r}{z}\exp\left(-\frac{\rho^2}{4f^2z^2}\right)\exp(\mathrm{i}kz). \tag{43}$$

As to the paraxial case, $r = (z^2+\rho^2)^{1/2} = z + \frac{\rho^2}{2z} \approx z$ is valid in the far field. Compared with Eqs. (43) and (41), the unapertured Gaussian beam obtained here is approximately equal to the traditional scalar Gaussian beam in the far field.

Now, we analyze the influence of a linearly polarized angle on the light intensity distributions of TE and TM terms. When φ is equal to $\alpha + 90°$, the light intensity of TE term appears maximum value. Therefore, the beam distribution of the TE term is located along the orientation perpendicular to the direction of linearly polarized angle. When φ is equal to α, the light intensity of TM term reaches maximum. The beam distribution of the TM term is located along the direction of linearly polarized angle. Due to the circular symmetry of circular aperture, the light intensity of apertured Gaussian beam is independent of the linearly polarized angle. For the sake of intuition, the light intensity distributions of the TE and TM terms with different linearly polarized angles are depicted in Figs. 1 and 2. The radius of the aperture is $R = 10\lambda$, and the reference plane is $z = 5000\lambda$. The beam waist is $w_0 = 10\lambda$. α is set to be 0° and 45°, respectively. The light intensity distribution of TE term is relatively centralized in the direction of linearly polarized angle, while that of TM term is relatively centralized in the direction perpendicular to the linearly polarized angle. When $\alpha = 0°$, the y component of the light intensities of the TE and TM terms is smaller than their corresponding x component. Moreover, the y component is composed of four lobes, whose orientations are 45°, 135°, 225° and 315° with respect to the positive direction of the x-axis, respectively. The sum of Figs. 1 and 2 is just the light intensity distribution of an apertured Gaussian beam, which is shown in Fig. 3a and b. Fig. 3c is the light intensity distribution of a traditional propagating scalar Gaussian beam in the same reference plane. Apparently, the presence of a circular aperture influences the light intensity distribution in the far field.

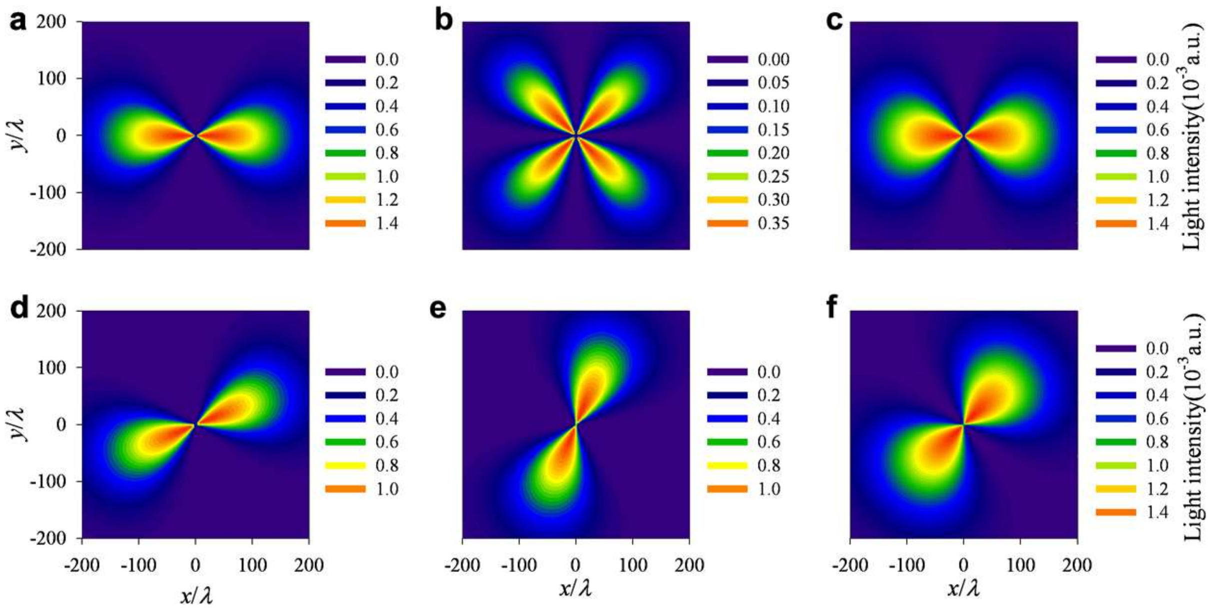

Fig. 2. Light intensity distributions of the TM term and its component. $w_0 = 10\lambda$, $R = 10\lambda$ and $z = 5000\lambda$. The top row denotes $\alpha = 0°$, and the bottom row $\alpha = 45°$: (a, d) I_{TM_x}, (b, e) I_{TM_y}, and (c, f) I_{TM}.

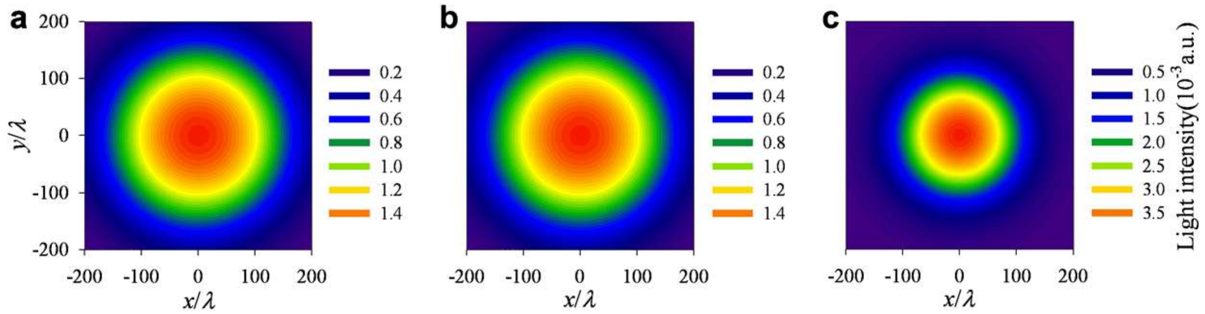

Fig. 3. Light intensity distributions of an apertured Gaussian beam and a traditional scalar Gaussian beam. $w_0 = 10\lambda$ and $z = 5000\lambda$, (a) $\alpha = 0°$ and $R = 10\lambda$, (b) $\alpha = 45°$ and $R = 10\lambda$, and (c) is a traditional scalar Gaussian beam.

3. Conclusions

Based on the angular spectrum representation of the Maxwell's equations and the complex Gaussian expansion of the aperture function, the vectorial structure of an apertured vector Gaussian beam has been obtained in the far field. By means of the method of stationary phase, the analytical TE and TM terms are presented. The influences of a linearly polarized angle on the light intensity distributions of an apertured Gaussian beam, its TE and TM terms are also analyzed. The beam distribution of the TE term is located along the orientation perpendicular to the direction of linearly polarized angle, and that of the TM term is located along the direction of linearly polarized angle. However, the light intensity of an apertured Gaussian beam is independent of the linearly polarized angle. The light intensity distributions of an apertured Gaussian beam, the TE term and the TM term with different linearly polarized angles are depicted in the far field reference plane, which distinctly reveals the decomposition of the apertured Gaussian beam. Comparison between the apertured vector Gaussian beam and the traditional scalar Gaussian beam shows that the presence of a circular aperture influences the light intensity distribution in the far field. This research provides a new approach to further comprehend the vectorial property of an apertured Gaussian beam.

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References

[1] H. Kogelnik, T. Li, Appl. Opt. 5 (1966) 1550.
[2] P. Belland, J.P. Crenn, Appl. Opt. 21 (1982) 522.
[3] K. Duan, B. Lü, Opt. Express 11 (2003) 1474.
[4] C.W. Zheng, Y.J. Zhang, L. Wang, Opt. Laser Technol. 39 (2007) 598.
[5] G. Zhou, Opt. Lett. 31 (2006) 2616.
[6] G. Zhou, K. Zhu, F. Liu, J. Mod. Opt. (2007), doi:10.1080/09500340701243616.
[7] C.G. Chen, P.T. Konkola, J. Ferrera, R.K. Heilmann, M.L. Schattenburg, J. Opt. Soc. Am. A 19 (2002) 404.
[8] G. Zhou, Opt. Commun. 265 (2006) 39.
[9] G. Zhou, X. Chu, L. Zhao, Opt. Laser Technol. 37 (2005) 470.
[10] R. Martínez-Herrero, P.M. Mejías, S. Bosch, A. Carnicer, J. Opt. Soc. Am. A 18 (2001) 1678.
[11] P.M. Mejías, R. Martínez-Herrero, G. Piquero, J.M. Movilla, Prog. Quantum Electron. 26 (2002) 65.
[12] H. Guo, J. Chen, S. Zhuang, Opt. Express 14 (2006) 2095.
[13] P. Varga, P. Török, Opt. Lett. 21 (1996) 1523.
[14] J.J. Wen, M.A. Breazeale, Scatter, Gaussian beam, and Aeroacoust. 2 (1990) 181.
[15] W.H. Carter, J. Opt. Soc. Am. 62 (1972) 1195.
[16] L. Mandel, E. Wolf, Optical Coherence and Quantum Optics, Cambridge U. Press, Cambridge, 1995.