Gaussian beam ray-equivalent modeling and optical design

Robert Herloski, Sidney Marshall, and Ronald Antos

It is shown that the propagation and transformation of a simply astigmatic Gaussian beam by an optical system with a characteristic *ABCD* matrix can be modeled by relatively simple equations whose terms consist solely of the heights and slopes of two paraxial rays. These equations are derived from the *ABCD* law of Gaussian beam transformation. They can be used in conjunction with a conventional automatic optical design program to design and optimize Gaussian beam optical systems. Several design examples are given using the CODE-V optical design package.

I. Introduction

Optical engineers designing optical systems involving use of laser beams need to be familiar with Gaussian beam propagation concepts. The standard Gaussian beam equations are given in the much referenced Kogelnik and Li articles.[1,2] Kogelnik and Li[1] show that the complex beam parameter q (whose real part is a phase curvature and whose imaginary part represents the off-axis Gaussian intensity profile) is transformed through an (orthogonal) optical system by the characteristic *ABCD* law. One can rewrite this law into separate equations for the waist size, waist location, and spot size of the output beam as a function of the *ABCD* system parameters and the input beam parameters.

In addition, Arnaud and Kogelnik have published two papers[3,4] which give the (more complex) equations for the propagation of a "generally astigmatic" Gaussian beam through a general optical system. A generally astigmatic beam can be produced, for example, by sending a circular Gaussian beam through two crossed cylinder lenses. This type of optical system is nonorthogonal and is not the subject of this paper.

There are several papers in the literature that consider geometric ray constructs to model the propagation and transformation of Gaussian beams while giving intuitive insight into these processes. Steier[5] derives a "ray packet equivalent" of a Gaussian beam and shows that this packet, propagated geometrically [see Ref. 5, Eq. (10)] through a paraxial optical system, will predict the output Gaussian beam characteristics. Arnaud[6,7] discusses at length the concept of a complex ray representing a Gaussian beam. He presents a "... convenient beam tracing method ..."[7] that represents this complex rays as two real rays that can be traced by ordinary ray tracing methods through an optical system and shows that the spot size at any plane of the system is given simply by the square root of the sum of the squares of the traced ray heights.

In the following section of this paper the authors will review the Kogelnik Gaussian beam formulas and also the first-order optics concepts necessary to derive the *ABCD* parameters. Then formulas will be derived from Kogelnik's *ABCD* law of transformation for the waist size, spot size, and waist location of a Gaussian beam transformed by an optical system in terms of the heights and slopes of two arbitrarily traced paraxial rays. These equations simplify dramatically if the paraxial rays traced are specially chosen; these are the special rays Arnaud uses in his beam tracing method.[7] It should be noted here that the paraxial rays do not in themselves represent a new theory of the propagation of a Gaussian beam but rather are used to effectively obtain the *ABCD* parameters of the optical system for use in the Kogelnik equations. These equations can then be implemented for use in conventional lens design programs.

In the third section of this paper, these formulas will be applied to the problem of the optical design of Gaussian beam systems. In particular, they will be used in conjunction with the optimization/constraint handler portions of a geometrical optical design program CODE-V[8] to directly design and optimize lens systems having specifically intended Gaussian beam transformation properties.

The authors are with Xerox Corporation, Joseph C. Wilson Center for Technology, Webster, New York, 14580.

Received 19 November 1982.
0003-6935/83/081168-07$01.00/0.

II. Analysis

A. Gaussian Beams

The formulas for the propagation and transformation of a Gaussian beam by an orthogonal paraxial optical system are well known.[1,2] The ($1/e^2$ intensity) spot size $\omega(z)$ and the radius of curvature $R(z)$ are related to the waist size ω_0 and the distance from the waist z by

$$\omega(z) = \omega_0[1 + (\lambda z/\pi\omega_0^2)^2]^{1/2}, \tag{1}$$

$$\omega^2(z) = \omega_0^2 + (\lambda z/\pi\omega_0)^2, \tag{2}$$

$$R(z) = z[1 + (\pi\omega_0^2/\lambda z)^2]^{1/2}, \tag{3}$$

where λ is the wavelength in the medium. The spot size and radius of curvature can be combined into a single q parameter defined as

$$1/q(z) = 1/R(z) - j\lambda/\pi\omega^2(z) \tag{4}$$

or

$$q(z) = z + j\pi\omega_0^2/\lambda. \tag{5}$$

The $ABCD$ law of propagation of the q parameter is

$$q' = (Aq + B)/(Cq + D) \tag{6}$$

or

$$1/q' = (C + D/q)/(A + B/q) \tag{7}$$

$$= (Cq + D)/(Aq + B), \tag{8}$$

where q and q' are the complex beam parameters at the input and output planes, respectively, and A, B, C, and D are the characteristic constants of the $ABCD$ matrix between the input and output planes.

Figure 1 gives a sketch of the transformation of a Gaussian beam by a thin lens. [This drawing is similar to Fig. (9a) of Ref. 7.] In general (as Fig. 1 illustrates), the waist locations in the object and image space are not conjugate planes in the geometric sense. However, it can be shown that in any optical system the ratio of the spot sizes at any two conjugate planes of the system is exactly the conventional geometric magnification between the conjugate planes. (Note that a beam waist can be located at one of the planes, but only under very special conditions will a beam waist also be located at the second plane.)

The characteristics of a general optical system between any two planes in the object and image spaces can be represented by an $ABCD$ matrix. (The object space has a refractive index n, and the image space has a refractive index n'.) If the chosen planes of this system are conjugate, $B = 0$. In addition, $A = m$ (the magnification), and, because $AD - BC = n/n'$, $D = n/mn'$. If Eq. (4) is substituted into Eq. (7), with $A = m$, $B = 0$, and $D = n/mn'$,

$$1/q' = C/m + n/n'm^2R - j\lambda'/\pi(m\omega)^2. \tag{9}$$

From the imaginary part of this expression and Eq. (4), it is easily seen that the spot size in the image plane is just m times the spot size in the object plane.

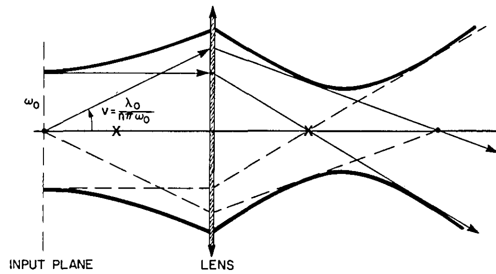


Fig. 1. Schematic representation of the transformation of a Gaussian beam by a thin lens.

Equations (6)–(8) require that the $ABCD$ parameters of the optical system be calculated before the output Gaussian beam parameters can be derived. This is most easily performed, especially for complicated optical systems, by paraxial ray tracing. The next part of this section presents some first-order optics theory that shows the significance of the $ABCD$ parameters and the mechanics of paraxial ray tracing.

B. General First-Order Theory

A general optical system with homogeneous input and output spaces defines, according to Lagrangian optics,[9] four functions that describe the transformation of the (x,y,u,v) coordinates of an input ray into the (x',y',u',v') coordinates of an output ray, where (x,x') and (y,y') are the transverse coordinates, and (u,u') and (v,v') are the direction tangents with respect to the z axis:

$$x' = F_1(x,y,u,v), \tag{10}$$

$$y' = F_2(x,y,u,v), \tag{11}$$

$$u' = F_3(x,y,u,v), \tag{12}$$

$$v' = F_4(x,y,u,v). \tag{13}$$

Generally the functions $F_1 - F_4$ can be expressed as a power series in terms of x, y, $= u$, and v; i.e.,

$$x' = M_{11}x + M_{12}y + M_{13}u + M_{14}v + O(2), \tag{14}$$

$$y' = M_{21}x + M_{22}y + M_{23}u + M_{24}v + O(2), \tag{15}$$

$$u' = M_{31}x + M_{32}y + M_{33}u + M_{34}v + O(2), \tag{16}$$

$$v' = M_{41}x + M_{42}y + M_{43}u + M_{44}v + O(2), \tag{17}$$

where $O(2)$ represents all the higher-order terms of the expansion. Only ten of these sixteen constants M_{11} to M_{44} are arbitrary.[10] These constants define the first-order properties of a general optical system expanded about an arbitrarily known ray. The linear portion of Eqs. (14)–(17) can be written in a matrix form, which Arnaud[4] calls the **M** matrix, or the generalized **ABCD** matrix, where **A**, **B**, **C**, and **D** are 2×2 real matrices. The linear portion of Eqs. (14)–(17) can also be written

$$x' = A_{11}x + B_{11}u + A_{12}y + B_{12}v, \tag{18}$$

$$u' = C_{11}x + D_{11}u + C_{12}y + D_{12}v, \tag{19}$$

$$y' = A_{21}x + B_{21}u + A_{22}y + B_{22}v, \tag{20}$$

$$v' = C_{21}x + D_{21}u + C_{22}y + D_{22}v. \tag{21}$$

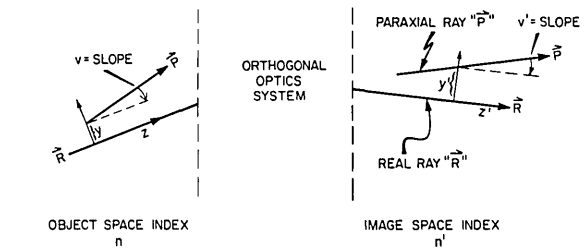


Fig. 2. Diagram of one equivalent plane of an orthogonal optics system.

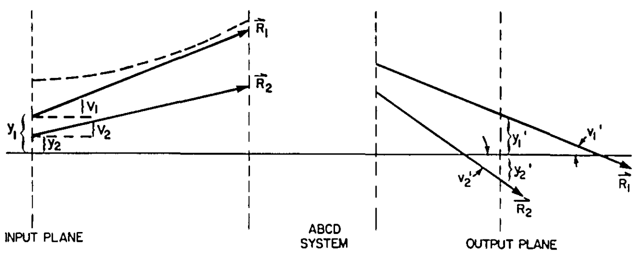


Fig. 3. Diagram of the ray trace of two arbitrary paraxial rays.

From Eqs. (18)–(21), one can easily see that the formal definitions for the constants A_{11} to D_{22} are, e.g.,

$$A_{11} = \partial x'/\partial x,$$

$$B_{21} = \partial y'/\partial u,$$

$$D_{22} = \partial v'/\partial v, \text{ etc.,}$$

all evaluated at the central ray ($x = y = u = v = 0$).

If the cross-term coefficients (i.e., A_{12},D_{21}) are identically zero or if the object and/or image space coordinates can be rotated about the z and z' axes to produce null cross-term coefficients, this optical system is equivalent (in the paraxial regime) to two one-(transverse) dimensional systems. In Luneburg's terminology, this system is orthogonal. Equations (18)–(21) can then be separated into the x and y portions of the familiar 1-D paraxial or $ABCD$ law:

$$x' = A_x x + B_x u, \tag{22}$$

$$u' = C_x x + D_x u, \tag{23}$$

$$y' = A_y y + B_y v, \tag{24}$$

$$v' = C_y y + D_y v, \tag{25}$$

where $A_x \ldots D_y$ are constants, and $(A_xD_x - B_xC_x)$ [as well as $(A_yD_y - B_yC_y)$] equals n/n'. A diagram of one plane of this system is shown in Fig. 2. Note that the center ray of the system does not necessarily lie along a line in a global coordinate system. More complete discussions of first-order optics are given by Buchdahl,[9] Luneburg,[10] and Sands.[11,12]

Only the 1-D case will be considered in the following sections, since it has been shown that a 2-D orthogonal system is formally equivalent to the 1-D case. As stated previously, the nonorthogonal case will not be treated here.

C. Ray-Equivalent Gaussian Beam Representation

Assume that there exists an optical system with paraxial constants A, B, C, and D defining the system between an input and output plane and a Gaussian beam with waist size ω_0 (and corresponding complex beam parameter q) located at the input plane is incident on the system. To calculate the complex beam parameter at the output plane, one substitutes Eq. (5) into Eq. (8). Rationalizing the complex fraction and collecting real and imaginary terms, one obtains

$$1/q' = [BD + AC(\pi\omega_0^2/\lambda)^2]/[B^2 + (A\pi\omega_0^2/\lambda)^2] - j(AD - BC)(\pi\omega_0^2/\lambda)/[B^2 + (A\pi\omega_0^2/\lambda)^2]. \tag{26}$$

If one substitutes Eq. (5) directly into Eq. (6), one obtains

$$q' = [BD + AC(\pi\omega_0^2/\lambda)^2]/[D^2 + (C\pi\omega_0^2/\lambda)^2] + j(AD - BC)(\pi\omega_0^2/\lambda)/[D^2 + (C\pi\omega_0^2/\lambda)^2]. \tag{27}$$

Equations (26)–(27) can be separated [using Eqs. (5) and (6)] so that the beam parameters at the output plane are

$$\omega'^2 = [B^2 + (A\pi\omega_0^2/\lambda)^2][\lambda\lambda'/(\pi\omega_0)^2]/(AD - BC), \tag{28}$$

$$z' = [BD + AC(\pi\omega_0^2/\lambda)^2]/[D^2 + (C\pi\omega_0^2/\lambda)^2], \tag{29}$$

$$\omega_0'^2 = (AD - BC)^2[\omega_0^2]/[D^2 + (C\pi\omega_0^2/\lambda)^2]. \tag{30}$$

[During the derivation for $\omega_0'^2$, one uses the relations $AD - BC = n/n' = \lambda'/\lambda = (AD - BC)^2\,\lambda/\lambda'$.]

Consider now the tracing of two arbitrary paraxial rays from the input plane, as shown in Fig. 3. Ray 1 has a height y_1 and a slope v_1. Ray 2 has a height y_2 and a slope v_2. From the $ABCD$ law of paraxial ray tracing [Eqs. (24) and (25)],

$$y_1' = Ay_1 + Bv_1, \tag{31}$$

$$v_1' = Cy_1 + Dv_1, \tag{32}$$

$$y_2' = Ay_2 + Bv_2, \tag{33}$$

$$v_2' = Cy_2 + Dv_2. \tag{34}$$

Solving Eqs. (31)–(34) for A to D,

$$A = (y_1'v_2 - y_2'v_1)/(y_1v_2 - y_2v_1), \tag{35}$$

$$B = (y_1y_2' - y_2y_1')/(y_1v_2 - y_2v_1), \tag{36}$$

$$C = (v_1'v_2 - v_2'v_1)/(y_1v_2 - y_2v_1), \tag{37}$$

$$D = (y_1v_2' - y_2v_1')/(y_1v_2 - y_2v_1), \tag{38}$$

and then substituting these equations into Eqs. (28)–(30), one obtains

$$\omega'^2 = [(y_1y_2' - y_2y_1')^2 + (y_1'v_2 - y_2'v_1)^2(\pi\omega_0^2/\lambda)^2][\lambda\lambda'/(\pi\omega_0)^2]/ [(y_1'v_2 - y_2'v_1)(y_1v_2' - y_2v_1') - (y_1y_2' - y_2y_1')(v_1'v_2 - v_2'v_1)], \tag{39}$$

$$z' = [(y_1y_2' - y_2y_1')(y_1v_2' - y_2v_1') + (y_1'v_2 - y_2'v_1) \times (v_1'v_2 - v_2'v_1)(\pi\omega_0^2/\lambda)^2]/ [(y_1v_2' - y_2v_1')^2 + (v_1'v_2 - v_2'v_1)^2(\pi\omega_0^2/\lambda)^2], \tag{40}$$

$$\omega_0'^2 = [(y_1'v_2 - y_2'v_1)(y_1v_2' - y_2v_1') - (y_1y_2' - y_2y_1') \times (v_1'v_2 - v_2'v_1)]^2(\omega_0^2)/ \{[(y_1v_2' - y_2v_1')^2 + (v_1'v_2 - v_2'v_1)^2(\pi\omega_0^2/\lambda)^2](y_1v_2 - y_2v_1)^2\}, \tag{41}$$

which express the output beam parameters in terms of

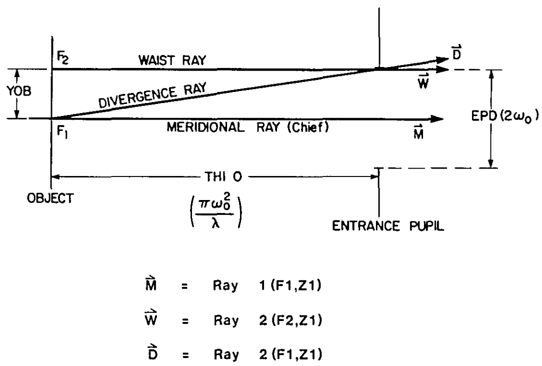


Fig. 4. Input beam representation technique for CODE-V.

the traced ray heights, slopes, λ, and ω_0. These equations are certainly very complex and do not immediately lend insight into beam transformation. They are general, however, and show that Gaussian beam propagation and transformation can be modeled in a sense by the tracing of two arbitrary paraxial rays. If one chooses these two rays very carefully, Eqs. (39)–(41) simplify immensely. Following Arnaud's[7] model, let

$$y_w = y_1 = \omega_0, \qquad y_d = y_2 = 0,$$

$$v_w = v_1 = 0, \qquad v_d = v_2 = \lambda/\pi\omega_0.$$

These rays can be called the waist ray (tangent to the input beam at the waist) and the divergence ray (tangent to the input beam at infinity). If one substitutes these initial conditions into Eqs. (39)–(41), one obtains

$$\omega' = (y_d'^2 + y_w'^2)^{1/2}, \tag{42}$$

$$z_0' = (y_d' v_d' + y_w' v_w')/(v_d'^2 + v_w'^2), \tag{43}$$

$$\omega_0' = (y_w' v_d' - v_w' y_d')/(v_d'^2 + v_w'^2)^{1/2}. \tag{44}$$

Equations (42)–(44) are the basis of our beam propagation method. Note that the equations do not explicitly contain λ or ω_0; that information is carried implicitly due to the initial conditions given to the paraxial rays. One can see that the equations are very simple.

As Arnaud states, the spot size at any plane in the system is equal to the square root of the sum of the squares of the ray heights at that plane. The numerator in Eq. (44) is merely the Lagrange invariant of the system divided by n'. By the property of the Lagrange invariant,[13] the numerator of Eq. (44) equals $[(y_w v_d - v_w y_d)n/n']$ or λ'/π. Thus the far-field divergence of the beam at any plane in the system is equal to the square root of the sum of the squares of the ray slopes at that plane. Equation (43), which gives the waist location, is the only equation containing explicitly all four terms of the ray trace.

The above derivation shows that a Gaussian beam can be easily represented by three rays: (1) the center ray (a real meridional ray about which the characteristic functions of the system are expanded); (2) the waist ray; and (3) the divergence ray (both the waist ray and divergence rays being paraxial with respect to the meridional ray). The word meridional is used because this is the situation most often found in practice.

III. Design Examples

In Sec. II, it was shown that the propagation of a specified input Gaussian beam through an orthogonal optical system can be represented by the tracing of two paraxial rays. Since the parameters of paraxial rays are made available to a designer using an automatic optical design program, one should be able to design a system of lenses to transform a given input Gaussian beam in a desired fashion, using only the geometric properties of the lenses.

To use this technique with a commercially available design program for Gaussian beam optical design, that program must have the following characteristics: (1) the ability to optimize on user-defined functions (or constraints); and (2) the ability to use ray trace data as input to the user-defined function. The author's experience was with the CODE-V optical design program. CODE-V met the above criteria, and, in addition, the user-defined constraints were defined using an easy-to-program high-level FORTRAN-like language.

The following sections describe implementation of the Gaussian beam formulas [Eqs. (42)–(44)] on CODE-V and demonstrate the design of two simple lens systems using this technique: (1) a beam converter; and (2) a focusing element.

A. Input Beam Representation

To simulate a specified input Gaussian beam, one must be able to specify to the program the representing rays and to have the ray intercepts and slopes of those rays available during optimization. As of Jan. 1983, the CODE-V program allows the definition of four arbitrary rays for use during optimization. An alternate way to generate the appropriate rays is to define the entrance pupil, object size, and location carefully so that the reference rays traced are the desired rays. This technique is described below.

The object for the system is specified to be at the input Gaussian beam waist location (see Fig. 4). An entrance pupil is specified to be one Rayleigh length from the waist (in CODE-V terminology, THI O = $\pi\omega_0^2/\lambda$). Its diameter (EPD) is $2\omega_0$. The object height (YOB) is ω_0, with field position 1 on-axis and field position 2 at the specified object height. The object height, pupil distance, and pupil vignetting (VUY and VLY) values are zoomed, if needed, to account for an anamorphic input beam. For example, zoom position 1, with appropriate values for VUY and VLY, could represent the y direction, and zoom position 2, with appropriate values for VUX and VLX, could represent the x direction. Then, in CODE-V terminology, the center ray of the Gaussian beam would be ray1(F1,Z1) the y-waist ray would be ray2(F2,Z1), and the y-divergence ray would be ray2(F1,Z1). The x-direction rays would be specified equivalently by the appropriate rays in zoom position 2.

```
: RAY HEIGHTS
:
DEF YME  =  Y1(F1.Z1.SI)
DEF YDV  =  Y2(F1.Z1.SI)
DEF YWA  =  Y2(F2.Z1.SI)
:
: RAY SLOPES
:
DEF VME  =  M1(F1.Z1.SI)/N1(F1.Z1.SI)
DEF VWA  =  M2(F2.Z1.SI)/N2(F2.Z1.SI)
DEF VDV  =  M2(F1.Z1.SI)/N2(F1.Z1.SI)
:
: DENOMINATOR
:
DEF DNY  =  (VWA-VME)**2 + (VDV-VME)**2
:
: SPOT SIZE
:
DEF SSY  =  SQRT((YWA-YME)**2 + (YDV-YME)**2)
:
: WAIST LOCATION
:
DEF WLY  =  ((VDV-VME)*(YDV-YME)+(VWA-VME)*(YWA-YME))/DNY
:
: WAIST SIZE
:
DEF WSY  =  SQRT((((VDV-VME)*(YWA-YME)-(VWA-VME)
               *(YDV-YME))**2)/DNY)
```

Fig. 5. CODE-V user-defined constraint code for Gaussian beam representation.

Table I. Beam Converter Constraints and Variables

Input Beam Parameters	ω_{0x}	=	0.193 mm
	ω_{0y}	=	0.193 mm
Constraints	ω_{0x}'	=	0.032 mm
	ω_{0y}'	=	0.083 mm
	z_x'	=	0.0 mm
	z_y'	=	0.0 mm
	OAL	=	100.0 mm
Lens Variables	t_1, t_2, t_3		
	$c_1\,(y),\ c_2\,(x)$		

B. User-Defined Constraints

Construction of the user-defined constraints required for optimization is straightforward. For clarity, portions of Eqs. (42)–(44) were programmed as separate constraints, since previously defined user constraints can be used as input functions to successive constraints. The CODE-V user-defined constraint source code is given in Fig. 5.

Note that in this code, the values for height and slope of the center ray are subtracted from the heights and slopes of the waist and divergence rays. This is required because the center ray might not propagate along the z axis of the local coordinate system (e.g., a beam propagating along a meridional ray in a symmetrical optical system). The parameters obtained using the above code are the projections of the beam parameters along the center ray onto the local coordinate system. The other change from Eqs. (42)–(44) is that the numerator of Eq. (44) is squared, then the square root is extracted (to calculate the absolute value of the numerator). This is done because it is possible for the waist size to take on a negative value (corresponding to a reflection in the system), and the optimization routine makes a distinction between positive and negative values.

One may raise the objection that in Sec. II it was required for the waist and divergence rays to be paraxial. Yet, in the above source code, all three rays are real rays. Indeed all three rays are traced as real rays, but the waist and divergence rays are assumed to be paraxial to the center ray. If the difference between a real ray trace and a paraxial ray trace is significant, there is significant aberration in the system over the region of the assumed Gaussian beam. One then has to propagate an aberrated Gaussian beam, and the above formulas (that assume an unaberrated Gaussian beam) will indeed give incorrect results. This caveat is, of course, true for all simple Gaussian beam propagation formulas (the requirement of negligible aberration over the beam); the tracing of real rays in this code and the interpretation of the results by the designer help emphasize this requirement.

C. Beam Converter

The first example is the design of a two-element anamorphic beam converter. The function of this lens is to convert a given circular input Gaussian beam into a specified simply astigmatic output Gaussian beam with the x and y waist locations coincident and at a given distance from the input beam waist. Both lenses are to be biconvex cylinders, each with a thickness of 6.35 mm, made of BK7 (the index at 632.8 nm is 1.515089), and with power in one direction only. A summary of the actual values of the assumed constraints and allowable variables of the system are given in Table I.

The initial condition of the design was all-zeroes; i.e., the optimization was started with unseparated plane-parallel plates with zero object and image distances. This initial condition was chosen to see how well the program could optimize with these nonlinear constraint functions, starting with a very unreasonable first-order set of parameters, far from the final solution. The constraint values put into the lens deck were:

SSY, 0.08306	: set spot size;
WTC, 122	: relative weight of above constraint (in the merit function);
WLY, 0.0	: set waist location;
WTC, 4	: relative weight of above constraint (in the merit function).

The user-defined constraints were put into the error function (via the WTC card) because it was found that the program was not able to converge solving for the constraints absolutely. This is probably due to the difference in the program's handling of absolute constraints vs its handling of error function (aberrational) constraints in the optimization process. The WTC values were each set so that the allowable deviation (tolerance) of the parameter from the nominal value would result in an error function contribution of about ten times the expected geometric contribution to the error function for each constraint. (One could also set all the WTC values of the program's internal error function to zero, so that there was no geometric contribution at all.)

Table II. CODE-V Optimization: Selected Intermediate Cycle Results

Cycle	t_1	c_1	t_2	c_2	t_3	Constraint function
0	0.0	0.0	0.0	0.0	0.0	4521308.1
2	0.229	0.0825	1.506	0.0004	1.507	3146978.2
6	−0.247*	0.1485	1.740	0.0017	1.742	1681322.3
15	2.156	0.6634	0.910	0.0454	0.371	51085.7
25	1.017	0.3331	2.042	1.5639	0.378	1835.4
30	0.814	0.2965	2.194	1.4884	0.430	0.0

Table II summarizes the convergence conditions of the optimization run, giving the intermediate variables and values of the constraint function for several cycles. From the table, one can see that the program took ~30 cycles to come to the (unique) solution (with all thicknesses positive). The asterisk after t_1 of cycle 6 indicates that the solution temporarily passed through a forbidden region on its way to the final solution. If an additional constraint had been added that all thicknesses should be positive (MNA, 0.0 in CODE-V terminology), the program would have stopped after cycle 6 or so, remaining far from the actual solution. Otherwise, the program was remarkably adept at converging to the only solution in this very nonlinear parameter space. The only hint that was given to the program was that the first lens was specified (and fixed) to be a y-direction cylinder. This was specified knowing that the final solution would put the y cylinder before the x cylinder.

The final solution is diagrammed in Fig. 6, which gives the lens parameters as calculated by the program and the final error function contributions. The geometric error function, as calculated by subtracting the constraint error function from the composite error function, is ~1.4. This residual aberration is defocus, because the specified image plane is not a geometric image plane for either the y or x direction. To check the solution, a beam trace was performed on the lens after optimization, using CODE-V's BEAM option. The results calculated in BEAM agreed exactly with those given by the user-defined constraints.

D. Focusing Element

The second problem was chosen to be the design of a focusing element. In other words, given a beam size a certain distance from a desired image plane, calculate the focal length of the lens located at the input beam that minimizes the spot size at the image plane and calculate that minimum spot size.

This problem is discussed in detail by Gaskill,[14] and he demonstrates that the minimum spot size does not occur with a beam waist located at the image plane but rather with a beam waist located slightly before the image plane. A diagram of the problem is shown in Fig. 7. The quantity ω_1 is the input beam waist radius with curvature R_1; ω_2 is the beam radius on the image plane (located a distance z_t from the input plane), with the corresponding waist ω_0 located a distance z_1 from the input plane containing the lens (of focal length F_1) and ω_1. Rewriting the formulas given by Gaskill, the equations relating ω_2, ω_0, z_1 and F_1 to ω_1, R_1, and z_t are

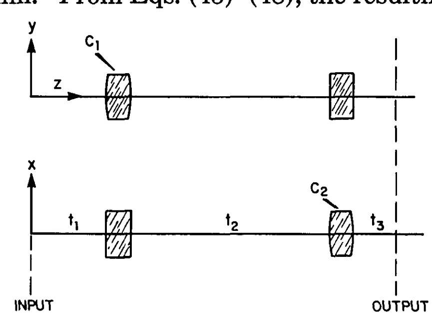
$$\omega_2 = (\lambda/\pi\omega_1)z_t, \tag{45}$$

$$\omega_0 = [1 + (\pi\omega_1^2/\lambda z_t)^2]^{-1/2}\omega_1, \tag{46}$$

$$z_1 = (\pi\omega_1\omega_0/\lambda)^2/z_t, \tag{47}$$

$$F_1^{-1} = R_1^{-1} + z_t^{-1}. \tag{48}$$

The sample problem given by Gaskill has the following input parameters: $\lambda = 0.5\ \mu$m; $z_t = 500$ mm; and $\omega_1 = 0.56419$ mm $[=1/\sqrt{(\pi)}]$. In addition, it was assumed that the beam waist for the input beam was located 250 mm in front of the input plane. This gives an input waist radius of 0.07109 mm and a value for R_1 of 254.03 mm. From Eqs. (45)–(48), the resulting output



t_1	= 0.813588"		ω_{0x}'	=	0.00126"
c_1	= 0.29653930 in^{-1}		ω_{0y}'	=	0.00327"
t_2	= 2.193859"		z_x'	=	2.76 E-7"
c_2	= 1.48843099 in^{-1}		z_y'	=	-1.3 E-8"
t_3	= 0.429553"		OAL	=	3.937"

ERROR FUNCTION = 1.44390119

CONSTRAINT FUNCTION = 0.00001130

Fig. 6. Sketch of the designed anamorphic beam converter giving the system parameters and error functions.

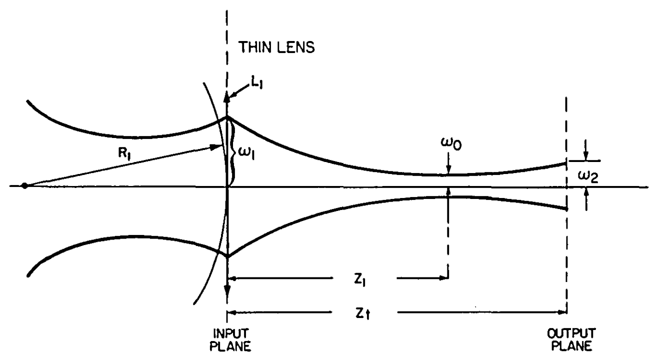


Fig. 7. Diagram of the spot size minimization problem.

parameters are $\omega_2 = 0.14105$ mm, $\omega_0 = 0.13684$ mm, $z_1 = 470.59$ mm (i.e., the waist is located 29.41 mm from the image plane), and $F_1 = 168.45$ mm. Assuming a thin plano-spherical lens of BK7 glass (index = 1.521415), the required curvature would be -0.01138535 mm^{-1}.

For the CODE-V constraints, the following constraints were specified: SSY, 0.0 and WTC, 10. These would force the program to try to minimize the spot size on the image plane; however, the program should not be able to minimize it past the theoretical value. Thus the theoretical minimum constraint error function to which the program should converge is $\sim 1.98 \times 10^6$.

CODE-V took only two cycles to converge to the correct solution, starting with a plane-parallel plate for the lens. The values it calculated were: WSY = 0.13682 mm, SSY = 0.14104 mm; and WLY = 29.441 mm. The calculated, curvature was -0.01138543 mm^{-1}. The resulting constraint error function was 1,989,333.3. It can be seen that these values were extremely close to the theoretical values. The differences were due to the tiny residual effect of the geometric error function on the convergence to a minimum composite error function; i.e., the composite error function was minimized at the slight expense of the constraint error function.

IV. Conclusions

It has been demonstrated that the propagation of simply astigmatic Gaussian beams can be represented by relatively simple formulas, whose terms are composed only of heights and slopes of appropriately traced rays. These formulas can be used with an automatic optical design program that allows the definition of user constraints to optimize a system for Gaussian beam optics. Two design examples have been given that illustrate use of this technique with the CODE-V optical design program.

As mentioned previously, this method is obviously applicable only to the case in which: (1) the aberrations of the system are negligible around the region of the propagating beam; i.e., the ray trace functions as defined in Sec. II [Eqs. (10)–(13)] are adequately described by their first-order expansions about the center meridional ray (the paraxial, parabasal, or paraprincipal region about the ray includes the region of the propagating beam); and (2) the first-order expansions implicitly have no cross-dependencies in x and y.

In the cases that do not satisfy condition (1), one has an aberrated Gaussian beam, whose propagation will not be adequately described by the traditional Gaussian beam formulas. In the cases that do not satisfy condition (2), one has a twisted astigmatic geometry, and the formulas for the propagation and transformation of generally astigmatic Gaussian beams as given by Arnaud and Kogelnik[3,4] are appropriate.

Excerpts from this paper were presented at the 1982 Spring Conference on Applied Optics in Rochester, N.Y. under the title "Gaussian Beam Optical Design and Optimization Using a Conventional Optical Design Program."[15]

References

1. H. W. Kogelnik and T. Li, Appl. Opt. **5**, 1550 (1966).
2. H. W. Kogelnik, Bell Syst. Tech. J. **44**, 455 (1965).
3. J. A. Arnaud and H. Kogelnik, Appl. Opt. **8**, 1687 (1969).
4. J. A. Arnaud, Bell Syst. Tech. J. **49**, 2311 (1970).
5. W. H. Steier, Appl. Opt. **5**, 1229 (1966).
6. J. A. Arnaud, Appl. Opt. **8**, 1909 (1969).
7. J. A. Arnaud, "Hamiltonian Theory of Beam Mode Propagation," in *Progress in Optics, Vol. 11*, E. Wolf, Ed. (North-Holland, Amsterdam, 1973).
8. CODE-V is a product of Optical Research Associates.
9. H. Buchdahl, *An Introduction to Hamiltonian Optics* (Cambridge U.P. London, 1970).
10. R. K. Luneburg, *Mathematical Theory of Optics* (U. California Press, Berkeley, 1964), pp. 216–243.
11. P. J. Sands, J. Opt. Soc. Am. **62**, 369 (1972).
12. P. J. Sands, J. Opt. Soc. Am. **58**, 1365 (1968).
13. W. J. Smith, *Modern Optical Engineering* (McGraw-Hill, New York, 1966), p. 43.
14. J. D. Gaskill, *Linear Systems, Fourier Transforms, and Optics* (Wiley, New York, 1978), pp. 435–438.
15. R. P. Herloski, S. Marshall, and R. L. Antos, J. Opt. Soc. Am. **72**, 1106 (1982).