

Modelling maximum daily yearly rainfall in northern Algeria using generalized extreme value distributions from 1936 to 2009

Naima Boudrissa,^{a*} Hassen Cheraitia^b and Lotfi Halimi^c

^a *Department of Applied Statistics and Econometrics, ENSSEA, National High School of Statistics and Applied Economics, Kolea, Algeria*

^b *Department of Economics, Jijel University, Algeria*

^c *National Office of Meteorology, Algiers, Algeria*

ABSTRACT: The generalized extreme value distribution is generally used to model annual maximum daily precipitation, and it then allows the calculation of the return values of this phenomenon. The distribution was fitted for selected stations in the north of Algeria; it was found that the generalized extreme value distribution of type I or the Gumbel distribution is more suitable for the Algiers and Miliana stations and the Fréchet distribution is more appropriate for the Oran station. The parameters were estimated using the maximum likelihood method, and the return levels were calculated at selected return periods T ; for instance, about 100 years must elapse to record a level of 181.9 mm of rainfall *per day* in Algiers, 173 mm in Miliana and 109.54 mm in Oran.

KEY WORDS generalized extreme value distribution; daily precipitations; annual daily maxima; return level

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1. Introduction

Climate change has become a clearly perceptible reality; it is measured by changes in temperature, precipitation, wind, snow and other indicators. In recent decades a number of indicators and studies have shown that the climate is warming across the globe, but the climate changes go beyond a mere warming trend. Algeria, as a Mediterranean country, has undergone severe climatic changes during the last decades in terms of rainfall. The media coverage of such events as the floods of Bab El Oued in 2001 and Ghardaia in 2008 has led to a perceived increase in extreme rainfall in Algeria. However, no coherent signal has been detected in the variables describing these phenomena (floods). This can be explained by the fact that the rainfall series are by nature subject to high natural variability.

One of the fundamental problems encountered in climatology is the need to establish an assessment of climate risks resulting from extreme precipitation to avoid human and material damage, and therefore to provide for the occurrence of disasters and unforeseen events (floods) and if possible their intensity. The modern theory of extreme values developed between 1920 and 1940 due to Fréchet (1927), Fisher and Tippet (1928), Gnedenko (1943) and Gumbel (1958) finds application in many fields; for instance, it has been used for estimating safety by modelling crash risk and its frequency (Songchitruksa and Tarko, 2006), measuring financial risk (Gilli and Këllezi, 2006), modelling annual maximum temperature over Florida (Waylen *et al.*, 2012), Sweden (Rydén, 2011) and Belgrade (Unkašerić and Tošić, 2009) to detect possible trends, and modelling air pollution problems to assess the impact of high air pollution because air quality standards are formulated in terms of highest level of permitted emissions (Singpurwalla, 1972; Roberts, 1979; Horowitz, 1980;

Smith, 1989; Sharma *et al.*, 1999; Tobias and Scotto, 2005). The extreme levels of a river causing floods in hydrology were also introduced in the literature by Coles and Tawn (2005). Concerning precipitation, and the risk of floods caused by this phenomenon, a considerable number of studies aiming to model such events can be found in the literature: Coles (2001) provided a detailed discussion about the methods used to model such events, and extreme precipitation was modelled by Friederichs (2010) in Germany, Benestad (2010) in Norway, Kim *et al.* (2009) in the south of Korea and Deka *et al.* (2011) in India.

When the maximum is modelled by a set of random variables, then, under certain conditions, the distribution can only belong to one of the three laws Weibull, Gumbel and Fréchet. Different methods can be used to estimate the parameters of the extreme laws such as the method of maximum likelihood (Coles, 2001), the method of weighted moments (Hosking, 1990) and the Bayesian method (Smith and Naylor, 1987).

The aim of the present work was to model the annual maximum of daily precipitation in different regions of northern Algeria using the generalized extreme value (GEV) distribution, in order to understand the behaviour of maximum rainfall and to establish an adequate forecasting model that helps meteorologists, insurers and authorities to understand these exceptional events and thus prevent climate risks. The R package *Extremes* was used to perform the analyses (Gilleland and Katz, 2005). The paper is organized in the following manner, additional to this introduction: the GEV distribution, the maximum likelihood estimates of its parameters and the return level are presented in Section 2; then the theoretical model is applied to data in Section 3; finally some conclusions are given in Section 4.

2. Generalized extreme value distribution

2.1. Definition and properties

The GEV distribution arises from the limit theorem of Fisher and Tippet (1928) and Gnedenko (1943) to model the maxima

* Correspondence: N. Boudrissa, Department of Applied Statistics and Economics, LAMOPS Laboratory, ENSSEA, Kolea University Pole, Tipaza, Algeria. E-mail: boudrina2005@yahoo.com

in sample data. It is a continuous family of distributions developed under the extreme value theory to nest three main distributions, Gumbel, Fréchet and Weibull, under a unique parametric representation as proposed by Jenkinson (1955). Its cumulative distribution function is given by:

$$F(x) = \begin{cases} \exp \left\{ - \left[1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} & \xi \neq 0 \\ \exp \left(- \exp \left(- \frac{x-\mu}{\sigma} \right) \right) & \xi = 0 \end{cases} \quad (1)$$

where μ is the location parameter, σ is the scale parameter and ξ is the shape parameter or the tail index.

The density function (pdf) obtained from the derivation of the distribution function specified in Equation (1) is given by:

$$f(x) = \begin{cases} \frac{1}{\sigma} \left[1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right]^{-\frac{1+\xi}{\xi}} \exp \left\{ - \left[1 + \xi \left(\frac{x-\mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} & \xi \neq 0 \\ \frac{1}{\sigma} \exp \left(- \left(\frac{x-\mu}{\sigma} + \exp \left(- \frac{x-\mu}{\sigma} \right) \right) \right) & \xi = 0 \end{cases} \quad (2)$$

As special cases the following are recognized: the Gumbel distribution for $\xi = 0$ including exponential tailed distributions such as exponential, normal, gamma and log-normal distributions; the Fréchet class of distributions with parameter $\alpha = 1/\xi$ if ξ is positive including fat tailed distributions such as Cauchy and Pareto; the last class is the Weibull distribution with parameter $\alpha = -1/\xi$ if ξ is negative for short tailed distributions such as uniform and beta distributions.

2.2. Parameter estimation

Several methods have been used in the literature to estimate the parameters of the GEV distribution; for example, the method of moments by Christopheit (1994), the L-moments method (Hosking, 1990; Hosking and Wallis, 1997) which is analogous to the ordinary method of moments but is less influenced by outliers and provides robust estimates (Von Storch and Zwiers, 1999; for more details about this method see Hosking, 1990); the Bayesian method by Smith and Naylor (1987), Lye *et al.* (1993), Coles and Tawn (2005); and the maximum likelihood method (Smith and Naylor, 1987; Unkašerić and Tošić, 2009) which is the most popular and has the advantage of allowing the addition to the fitting of co-variables (such as trends, cycles or physical variables) (Katz *et al.*, 2002). The last method was used to estimate the parameters of the GEV distribution as follows.

If X_1, \dots, X_n is a random sample of maxima from a random variable having a GEV distribution with pdf as defined in Equation (2), for $\xi \neq 0$ the likelihood function of the sample is given by:

$$L(\xi, \mu, \sigma; X) = \prod_{i=1}^n \frac{1}{\sigma} \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-\frac{1+\xi}{\xi}} \times \exp \left\{ - \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} \\ = \frac{1}{\sigma^n} \prod_{i=1}^n \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-\frac{1+\xi}{\xi}} \times \exp \left\{ - \sum_{i=1}^n \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \right\} \quad (3)$$

Taking the logarithm of the likelihood function:

$$l(\xi, \mu, \sigma; Y) = -n \ln \sigma - \left(1 + \frac{1}{\xi} \right) \sum_{i=1}^n \ln \left(1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right) - \sum_{i=1}^n \left[1 + \xi \left(\frac{x_i - \mu}{\sigma} \right) \right]^{-\frac{1}{\xi}} \quad (4)$$

This function is maximized under the constraints that $\sigma > 0$ and $1 + \xi \{ (x_i - \mu)/\sigma \} > 0$ and numerical methods are needed. The GEV support depends on unknown parameter values, and then the usual regularity conditions (underlying the asymptotic properties of maximum likelihood estimators) are not satisfied as stated by Smith (1989). In the case $\xi > -0.5$, the usual properties of consistency, asymptotic efficiency and asymptotic normality hold as $n \rightarrow +\infty$. In the same way, for $\xi = 0$, the logarithm of the likelihood function is given by:

$$l(\xi, \mu, \sigma; Y) = -n \ln \sigma - \sum_{i=1}^n \exp \left(- \frac{x_i - \mu}{\sigma} \right) - \sum_{i=1}^n \frac{x_i - \mu}{\sigma} \quad (5)$$

Differentiating this function with respect to the two parameters, the following system of equations is obtained:

$$\begin{cases} n - \sum_{i=1}^n \exp \left(- \frac{x_i - \mu}{\sigma} \right) = 0 \\ n + \sum_{i=1}^n \frac{x_i - \mu}{\sigma} \left[\exp \left(- \frac{x_i - \mu}{\sigma} \right) - 1 \right] = 0 \end{cases} \quad (6)$$

No closed form exists for the estimators, and numerical methods are needed to solve the above system of equations.

2.3. Return period

The return level is an interesting notion to determine the mean waiting time between extreme precipitations. The T year return is given as the solution of the equation:

$$F_X(z_T) = P(X \leq z_T) = 1 - 1/T \Rightarrow z_T = F^{-1}(1 - 1/T)$$

where z_T is the return period or the level that is expected to be exceeded by the annual maximum of daily precipitations once every T years, on average.

For a given return period T , the following quintiles are found according to Equations (1):

$$z_T = \begin{cases} \mu + \frac{\sigma}{\xi} \left\{ 1 - \ln \left(1 - \frac{1}{T} \right) \right\}^{\xi} & \xi \neq 0 \\ \mu - \sigma \ln \left[- \ln \left(1 - \frac{1}{T} \right) \right] & \xi = 0 \end{cases} \quad (7)$$

3. Application

3.1. Data description

The principal components method was applied to our data represented by the annual daily maxima of rainfall from 1936 to 2009 registered in the 18 Algerian stations, which were regrouped in three classes. The first class includes only one station which is Miliana; the second class includes the stations Algiers, Annaba, Skikda, Mostaganem and Constantine; and the third and last one includes Oran, Chleff, Saida, Djelfa, Batna, Tébessa, El Bayadh, Biskra, Tlemcen, Mascara, Bordj Bouriridj and Tiaret (Figure 1).

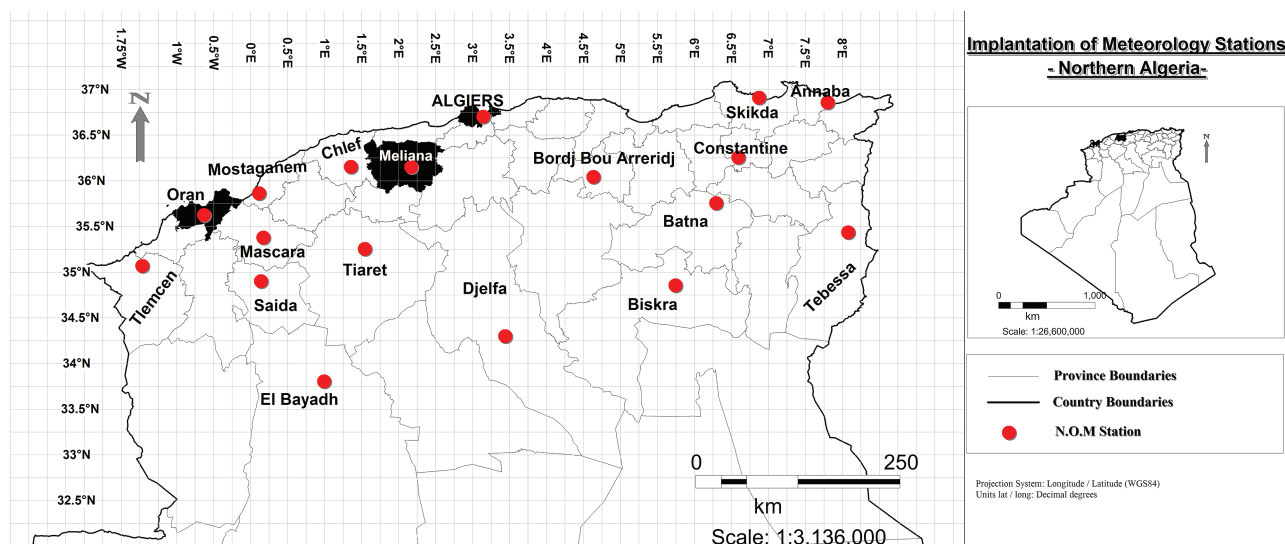


Figure 1. Location of meteorological stations in northern Algeria.

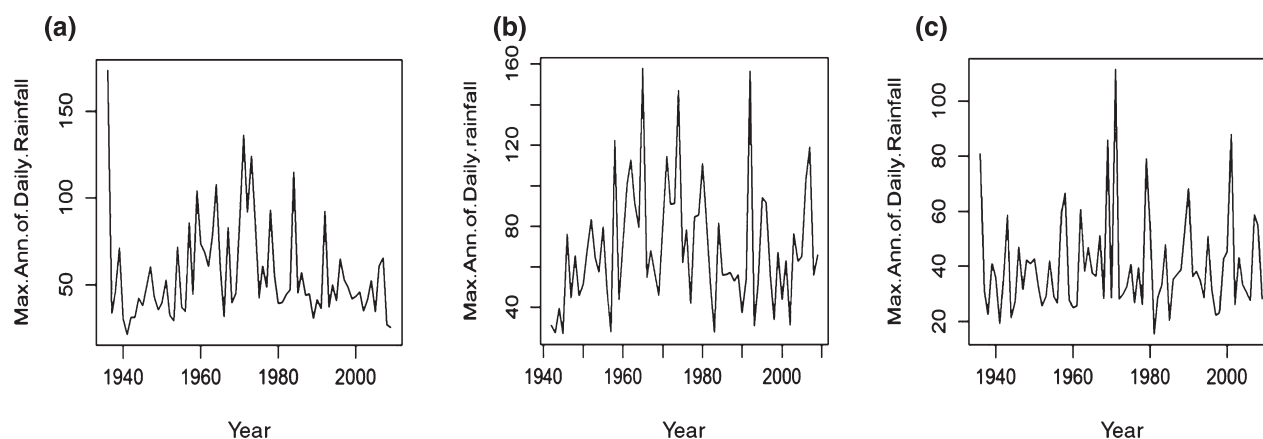


Figure 2. Maximum annual daily rainfall for the different stations: (a) Algiers 1936–2009; (b) Miliana 1942–2009; (c) Oran 1936–2009.

One station was then chosen from each class, and the others have the same behaviour since they are similar. These stations are Algiers, Miliana and Oran, for which a suitable model is fitted. The data show that all precipitation measurements made each day for the period 1936–2009 are available (Figure 2).

The statistical characteristics of each station are given in Table 1. From the table it is found that Miliana is characterized by a high annual maximum of average daily precipitation compared with the other stations; after that is Algiers in the second position and then Oran in the third position with the lowest rainfall average. These results show that the automatic classification applied to the data lends support to the geographical rule that rainfall is more important in eastern Algeria than in western Algeria. The rainfall recorded at the stations of the second and third classes is less dispersed compared with Miliana. That is, the cloud point of the first class is more condensed. Also the normality assumption is rejected for all the stations since the skewness for each one is greater than zero, and the same applies to the kurtosis which is different from 3 for all the stations.

3.2. Parameter estimation and model validation

The maximum likelihood method was used (as described in Section 2.2) to estimate the three parameters of the GEV

distribution; by differentiating Equation (4) with respect to the three parameters and solving the resulting system of equations numerically, the following results were found.

From Table 2 it is noted that the shape parameter ξ is positive for Algiers station implying that the GEV distribution is Fréchet type; it is also close to zero implying that the Gumbel distribution is a candidate, and the confidence interval confirms the latter. For Miliana station, the shape parameter of the distribution (0.04) is of positive sign, while the GEV distribution is Fréchet type, $\xi > 0$. But a Gumbel distribution must be considered since zero is included in the confidence interval. The shape parameter ξ is positive for Oran station meaning that the GEV distribution is Fréchet. The confidence interval confirms this conclusion.

The goodness of fit of the distributions was tested using the Kolmogorov–Smirnov test, and the results are given in Table 3 (for 0.05 significance level).

The calculated Kolmogorov–Smirnov statistic is less than the critical value 0.15 taken from the Kolmogorov–Smirnov table; thus a GEV distribution is suitable for all stations, and the same conclusion is drawn for the Gumbel distribution. In order to choose the best distribution (between GEV and Gumbel) the likelihood ratio test was applied too and it was found that the Gumbel distribution is more suitable for Algiers and Miliana

Table 1. Statistical properties of the annual maximum of daily precipitation (in mm) for the different stations (Algiers and Oran from 1936 to 2009, and Miliana from 1942 to 2009).

Station	<i>n</i>	Mean	Standard deviation	Maximum	Minimum	Skewness	Kurtosis
Algiers	74	56.17	28.16	176.8	21.1	1.79	3.88
Miliana	68	69.29	30.77	158.0	26.8	0.99	0.90
Oran	74	40.14	17.87	111.6	15.2	1.67	3.35

Table 2. Maximum likelihood estimation of the parameters of the GEV distribution.

Station	Location parameter			Scale parameter			Shape parameter		
	μ	SD	CI	σ	SD	CI	ξ	SD	CI
Algiers	42.31	1.98	(38.43, 46.19)	15.07	1.64	(11.84, 18.30)	0.10	0.09	(−0.007, 0.26)
Miliana	54.88	3.28	(61.31, 48.45)	22.89	2.46	(27.72, 18.07)	0.04	0.10	(−0.160, 0.26)
Oran	31.62	1.39	(28.88, 34.35)	10.67	1.09	(08.53, 12.80)	0.18	0.09	(0.003, 0.35)

GEV, generalized extreme value; CI, confidence interval.

Table 3. Kolmogorov–Smirnov test to determine whether the annual maxima of daily rainfall follow GEV and Gumbel distributions for the studied stations.

Station	Kolmogorov–Smirnov statistic	
	GEV	Gumbel
Algiers	0.12	0.13
Miliana	0.05	0.06
Oran	0.05	0.09

GEV, generalized extreme value.

Table 4. Testing the stationarity of the series of annual maxima of daily rainfall by the likelihood ratio test.

Station		GEV ₀ –GEV ₁	GEV ₀ –GEV ₂	GEV ₁ –GEV ₂
Algiers	Chi-squared	3.841	5.991	3.841
	λ	0.020	0.110	0.130
	Decision	GEV ₀	GEV ₀	GEV ₁
Miliana	λ	0.005	0.002	0.003
	Decision	GEV ₀	GEV ₀	GEV ₁
Oran	λ	0.010	0.050	0.040
	Decision	GEV ₀	GEV ₀	GEV ₁

GEV, generalized extreme value.

stations and the Fréchet distribution is more appropriate for Oran station.

Three models were fitted to the data: the stationary GEV₀ with constant parameters; the non-stationary model in mean GEV₁ which takes the form GEV₁(μ_t , σ , ϵ) with $\mu_t = \beta_0 + \beta_1 t$; and finally the GEV₂ which is non-stationary in mean and variance and is written as GEV₂(μ_t , σ_t , ϵ) with $\mu_t = \beta_1 + \beta_2 t$ and $\sigma_t = \alpha_1 + \alpha_2 t$. Since each model is a sub-model of the previous one and in order to determine the better fitting model the likelihood ratio test was used. If L_i , $i = 0, 1, 2$, is the maximum likelihood of GEV_i, under the simpler model the statistic $\lambda = -2 \log(L_i/L_j)$ for $i = 0, 1$, $j = 1, 2$ is distributed as a chi-squared random variable. The results are given in Table 4.

From the results given in Table 4, it is observed that for Algiers station $\lambda = 0.020 < \chi^2_{1,0.95} = 3.841$ and the best fit for this station is GEV₀; in the same way it can be concluded that the stationary Gumbel model (without trend) is the best model for modelling annual maximum rainfall in Algiers and Miliana stations. However, the stationary GEV model of the Fréchet type is the best for modelling the annual maximum of daily rainfall in Oran station.

To validate the above models the QQ plot and PP plot techniques were used. It can be seen that for all the stations the QQ plot and the PP plot are aligned, and they form a cloud of linear shape (Figure 3); then the annual maximum of daily rainfall for Algiers and Miliana are well adjusted by the Gumbel distribution and Oran is very well fitted by a GEV distribution of the Fréchet type.

After validation of the models, they were used to estimate the return levels of the annual maximum of daily precipitations for the different stations using Equation (7) and the maximum likelihood estimates of the parameters (Figure 4); the results are given in the Table 5.

It is concluded from Table 5 that 100 years must elapse to observe a level of maximum annual daily rainfall (i.e. a time-average every 100 years) of 173 mm for Miliana, 182 mm in Algiers and 109.54 in Oran.

4. Conclusion

Algeria is a Mediterranean country and it has a Mediterranean climate: summers are hot and dry, rainfall becomes scarce or non-existent and winters are mild and rainy and sometimes snowy. In the Aures (the wettest area of Algeria), the amount of rainfall indicates 100 mm average annually and annual rainfall in the highlands and in the Saharan Atlas does not exceed an amount of 200 mm of rain. However, heavy rains are recorded in several cities in the north of Algeria. In this study the annual maxima of daily rainfall in northern Algeria from 1936 to 2009 was modelled using the generalized extreme value (GEV) distribution to control and predict the behaviour of rainfall. The maximum likelihood method was used to estimate the parameters and it was found that the stationary Gumbel model (no trend) is more appropriate for the Algiers and Miliana stations and all the stations of the first and second classes. The Fréchet type GEV

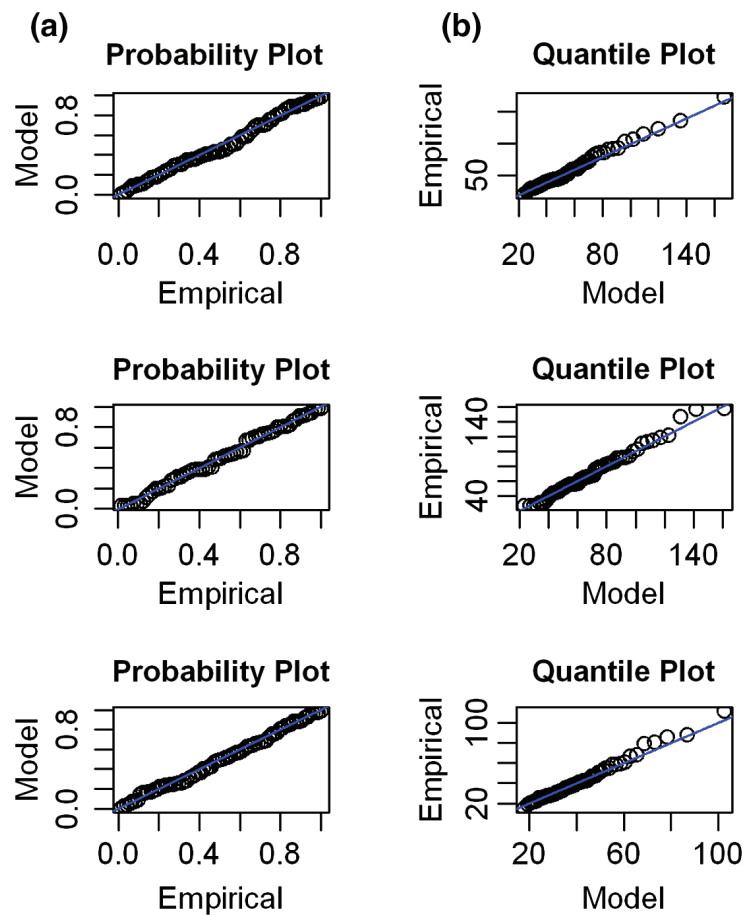


Figure 3. The PP plots (a) and QQ plots (b) for the stations Algiers (top), Miliana (middle) and Oran (bottom).

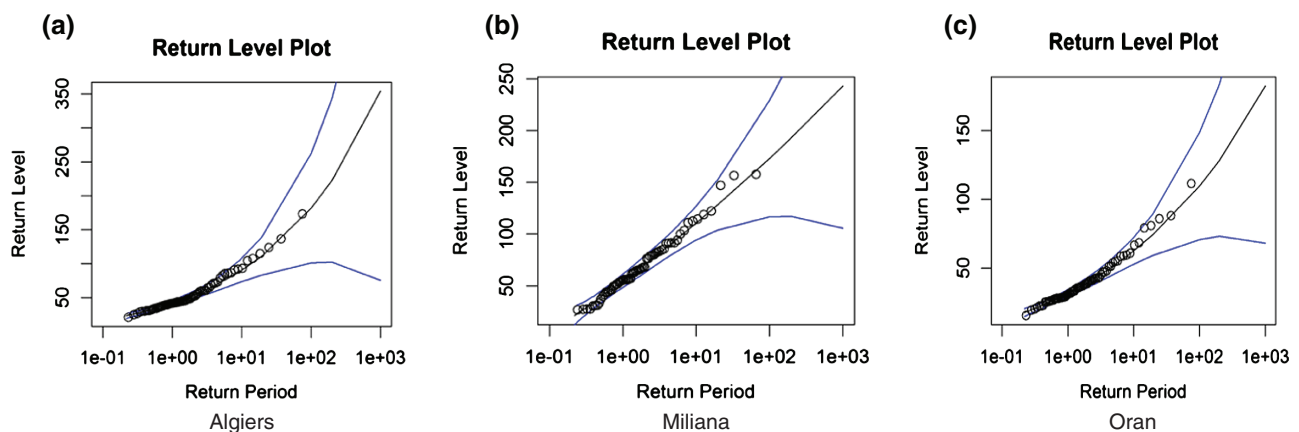


Figure 4. Time and return levels of the annual maximum of daily rainfall for the different stations (in millimetres): (a) Algiers; (b) Miliana; (c) Oran.

Table 5. Estimated return levels (in mm) at selected return periods (in years) of the annual maximum of daily rainfall for the studied stations with confidence intervals.

Station	$T=2$	CI	$T=20$	CI	$T=50$	CI	$T=100$	CI
Algiers	48.10	(43.7, 53.3)	111.60	(91.3, 154.0)	147.90	(112.3, 224.1)	181.90	(129.3, 321.7)
Miliana	63.40	(56.4, 71.1)	128.10	(110.5, 167.4)	153.30	(127.0, 226.0)	173.00	(138.0, 222.0)
Oran	35.86	(32.6, 39.1)	074.06	(062.8, 096.5)	093.01	(074.6, 102.4)	109.54	(083.6, 170.4)

GEV, generalized extreme value; CI, confidence interval.

model without trend is the best for the Oran station and all the stations of the third class. Return levels were estimated for several return time periods; for instance, about 50 years must elapse to record a level of 153 mm of rainfall *per* day in Miliana, 147.9 mm in Algiers and 93.01 mm in Oran.

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References

- Benestad R. 2010. Downscaling precipitation extremes: correction of analog models through PDF predictions. *Theor. Appl. Climatol.* **100**: 1–21.
- Christopeit N. 1994. Estimating parameters of an extreme value distribution by the method of moments. *J. Stat. Plann. Inference* **41**(2): 173–186.
- Coles SG. 2001. *An Introduction to Statistical Modeling of Extreme Values*. Springer-Verlag: New York, NY.
- Coles SG, Tawn J. 2005. Bayesian modeling of extreme surges on UK east coast. *Philos. Trans. R. Soc. A* **363**: 1387–1406.
- Deka S, Borah M, Kakaty SC. 2011. Statistical analysis of annual maximum rainfall in North-East India: an application of LH-moments. *Theor. Appl. Climatol.* **104**: 111–122.
- Fisher RA, Tippet LHC. 1928. Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proc. Cambridge Philos. Soc.* **24**: 180–190.
- Fréchet M. 1927. Sur la loi de probabilité de l'écart maximum. *Ann. Soc. Pol. Math.* **6**(3): 93–116.
- Friederichs P. 2010. Statistical downscaling of extreme precipitation events using extreme value theory. *Extremes* **13**: 109–132.
- Gilleland E, Katz R. 2005. Tutorial for the extremes toolkit: weather and climate applications of extreme value statistics. <http://www.assessment.ucar.edu=toolkit> (accessed 10 February 2012).
- Gilli M, Kellezi E. 2006. An application of extreme value theory for measuring financial risk. *Comput. Econ.* **27**(1): 1–23.
- Gnedenko BV. 1943. Sur la distribution limite du terme maximum d'une série aléatoire. *Ann. Math.* **44**: 423–453.
- Gumbel EJ. 1958. *Statistics of Extremes*. Columbia University Press: New York, NY.
- Horowitz J. 1980. Extreme values form a non-stationary stochastic process: an application to air quality analysis (with discussion). *Technometrics* **22**: 469–482.
- Hosking JRM. 1990. L-moments: analysis and estimation of distributions using linear combinations of order statistics. *J. R. Stat. Soc. Ser. B* **52**: 105–124.
- Hosking JRM, Wallis JR. 1997. *Regional Frequency Analysis. An Approach based on L-Moments*. Cambridge University Press: UK.
- Jenkinson AF. 1955. The frequency distribution of the annual maximum (or minimum) of meteorological elements. *Q. J. R. Meteorol. Soc.* **81**: 158–171.
- Katz RW, Parlange MB, Naveau P. 2002. Statistics of extremes in hydrology. *Adv. Water Resour.* **25**: 1287–1304.
- Kim C, Suh MS, Hang KO. 2009. Bayesian change-point analysis of the annual maximum of daily and sub-daily precipitation over South Korea. *J. Clim.* **22**: 6741–6757.
- Lye LM, Hapuarachchi KP, Ryan S. 1993. Bayes estimation of the extreme value reliability function. *IEEE Trans. Reliab.* **42**(4): 641–644.
- Roberts EM. 1979. Review of statistics of extreme values with applications to air quality data. Part II. *J. Air Pollut. Control Assoc.* **22**: 733–740.
- Rydén J. 2011. Statistical analysis of temperature extremes in long time series from Uppsala. *Theor. Appl. Climatol.* **105**: 193–197.
- Sharma P, Khare M, Chakrabarti SP. 1999. Application of extreme value theory for predicting violations of air quality standards of an urban road intersection. *Transp. Res. Part D4* **4**: 201–216.
- Singpurwalla ND. 1972. Extreme values from a lognormal law with application to air pollution problems. *Technometrics* **14**: 703–711.
- Smith RL. 1989. Extreme value analysis of environmental time series: an application to trend detection in ground-level ozone. *Stat. Sci.* **4**(4): 367–393.
- Smith RL, Naylor JC. 1987. A comparison of maximum likelihood and Bayesian estimators for the three parameter Weibull distribution. *J. R. Soc., Ser. C* **36**(3): 358–369.
- Songchitruksa P, Tarko AP. 2006. The extreme value theory approach to safety estimation. *Accid. Anal. Prev.* **38**: 811–822.
- Tobias A, Scotto MG. 2005. Prediction of extreme ozone levels in Barcelona, Spain. *Environ. Monit. Assess.* **100**: 23–32.
- Unkašerić M, Tošić I. 2009. Changes in extreme daily winter and summer temperatures in Belgrade. *Theor. Appl. Climatol.* **95**: 27–38.
- Von Storch H, Zwiers FW. 1999. *Statistical Analysis in Climate Research*. Cambridge University Press: Cambridge.
- Waylen P, Keellings D, Qiu Y. 2012. Climate and health in Florida: changes in risks of annual maximum temperatures in the second half of the twentieth century. *Appl. Geogr.* **33**: 73–81.