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Key Points:

- We model extremes as block maxima among a variable number of events
- A metastatistical description of extremes is developed based on the full distribution of ordinary events
- The metastatistical approach reduces high-quantile estimation uncertainty by up to 50% with respect to traditional methods

Supporting Information:

- Supporting Information S1
- Figure S1
- Figure S2
- Figure S3
- Figure S4
- Figure S5

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On the emergence of rainfall extremes from ordinary events

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Abstract The analysis and estimation of extreme event occurrences is a central problem in many fields of geoscience. Advancements in the study of extreme events have recently been limited, arguably in connection with asymptotic assumptions in the traditional extreme value theory (EVT) and with its focusing on a small fraction of the available observations representing the tail properties of the underlying event generation process. Here we develop a Metastatistical Extreme Value framework (MEV) which relaxes limiting assumptions at the basis of the traditional EVT and accounts for the full distribution of the underlying "ordinary" events. We apply this general approach to the relevant case of daily rainfall and find that the MEV approach reduces the uncertainty in the estimation of high-quantile extremes by up to 50% with respect to the classical EVT. The improved predictive power of the MEV framework is connected with its recognizing that extremes emerge from repeated sampling of ordinary events, thereby being able to use all available observations.

1. Introduction

Extreme value theory (EVT) [*Fischer and Tippett*, 1928; *Gnedenko*, 1943; *Gumbel*, 1958] is a fundamental tool in the study of many geophysical processes, such as the local and global hydrologic cycle [*Katz et al.*, 2002], wind velocities [*Cook and Harris*, 2004], earthquake magnitudes [*Pisarenko et al.*, 2014], ecological processes [*Katz et al.*, 2005], storm-surge marine levels [*Coles and Tawn*, 1990], pollutant dynamics in the environment [*Eastoe and Tawn*, 2009], and many others. In the classical EVT, extremes are defined as "block maxima," i.e., as the events with maximum magnitude x occurred over a period of fixed length (often 1 year). The n events occurring in each block are assumed to be independent, and their magnitude is assumed to follow the same parent cumulative distribution $F(x)$. Hence, block maxima have cumulative distribution $H_n(x) = F(x)^n$. This expression is not directly applicable as n is the value assumed by a random variable N. To obtain a closed-form expression for $H_n(x)$, the classical EVT makes one of two possible assumptions. A first approach is to assume the number of events per block to be "large enough" (i.e., $n \rightarrow \infty$), such that the succession $H_n(x)$, upon proper renormalization, tends to an asymptotic distribution, $H(x)$, which takes the form of the generalized extreme value (GEV) distribution [*Von Mises*, 1936]. It has been noted that in many applications the number of events from which the maximum value is selected is not nearly sufficient for this asymptotic hypothesis to be valid [*Cook and Harris*, 2004; *Koutsoyiannis*, 2004]. A second approach, termed peak over threshold (POT) method [*Balkema and Haan*, 1974; *Pickands*, 1975] (i) fixes a high intensity threshold, q; (ii) assumes a Poisson occurrence of events above the threshold; and (iii) models the excess values over q (assumed to be independent of the occurrence process) using a generalized Pareto distribution (GPD) [*Davison and Smith*, 1990]. Also, in this second approach, sometimes referred to as partial duration series [*Stedinger*, 1993] the resulting EV distribution is GEV. Both these classical EVT approaches lead to formulations which neglect a significant proportion of the observations, as they fit the block maxima distribution, $H(x)$, using only the block maxima themselves, or a relatively small number of exceedances over a high threshold. Effectively, both these approaches discard the information contained in the bulk of the parent distribution, $F(x)$, along with most of the observations. Here we refine and apply a statistical approach based on the assumption that the extreme events are block maxima among a finite and stochastically variable number of ordinary events. These are defined as the values obtained by the repeated sampling, in each block, from an underlying and possibly time-varying distribution (e.g., all rainfall occurrences in a given year in the case of daily rainfall). This simple consideration allows us to use the entire observational set to infer the distribution of extremes, by means of a Metastatistical Extreme Value framework (MEV), with obvious statistical advantages. This approach is here applied to the relevant case of daily rainfall events using a worldwide data set of long rainfall records and a Monte Carlo approach to comparatively assess MEV and GEV high-quantile estimation uncertainties.

2. Theoretical Framework

We propose the use of a Metastatistical Extreme Value (MEV) approach that relaxes the limiting assumptions of the classical EVT by considering as random variables the parameters defining the number of events and the probability distribution of event magnitudes [*Marani and Ignaccolo*, 2015]. This leads to a compound distribution [*Dubey*, 1968] or superstatistics [*Beck and Cohen*, 2003; *Porporato et al.*, 2006; *Botter et al.*, 2013] for the distribution of the block maxima. In the MEV approach the variability of these parameters accounts for (i) the random process of event occurrence, which generates a finite and varying number of events in each block, and (ii) the possibly changing probability distribution of event magnitudes across different blocks. The MEV approach accomplishes this by recognizing the number of events in each block, n, and the values of the parameters, $\vec{\theta}$, of the parent distribution $F(x;\vec{\theta})$ to be realizations of stochastic variables (N and $\vec{\Theta}$). The probability distribution of block maxima can now be defined, by the use of the total probability theorem and by considering all possible values N and $\vec{\Theta}$, thereby yielding a MEV cumulative distribution function:

$$\zeta(x) = \sum_{n=1}^{\infty} \int_{\Omega_{\vec{\Theta}}} F(x;\vec{\theta})^n \, g(n,\vec{\theta}) \mathrm{d}\vec{\theta} \tag{1}$$

where $g(n,\vec{\theta})$ is the joint probability distribution of N and $\vec{\Theta}$ (discrete in N and continuous in $\vec{\Theta}$) and $\Omega_{\vec{\Theta}}$ is the population of all possible values of the parameters. The probability distribution of the extremes thus arises from the full distribution of the ordinary events (not just from a predetermined part of the tail), which is sampled—each year—a variable number of times n. For this reason, the MEV approach exploits all the available observations defining the probability distributions of ordinary events in each block rather than censor the data set to only include values from the tail of $F(x)$. It is interesting to note that if one assumes (i) x to be the excess over a high-threshold q, (ii) $F(x;\vec{\theta})$ to be a generalized Pareto distribution (with fixed, deterministic parameters), and (iii) n to be generated by a Poisson distribution, then the GEV distribution is recovered as a particular case of the MEV distribution by means of the POT approach.

Rather than specifying the joint probability density function (pdf) $g(n,\vec{\theta})$, we obtain here an approximate expression for $\zeta(x)$ by substituting the expectations in equation (1) with sample averages. We illustrate this derivation with application to the relevant case of daily rainfall observed at a point. Following *Wilson and Tuomi* [2005] and *Marani and Ignaccolo* [2015], we adopt the Weibull, or stretched exponential distribution [*Laherrere and Sornette*, 1998], $F(x;C,w) = 1 - e^{-\left(\frac{x}{C}\right)^w}$ to model the nonzero daily rainfall amounts (C and w being, respectively, the Weibull scale and shape parameters). One can thus define the MEV-Weibull cumulative distribution function as

$$\zeta(x) = \sum_{n=1}^{\infty} \int_C \int_w g(n,C,w) \cdot \left[1 - e^{-\left(\frac{x}{C}\right)^w}\right]^n \mathrm{d}C\mathrm{d}w \tag{2}$$

The Weibull distribution, $F(x;C_j,w_j)$, is assumed to describe the observations in each year on record ($j = 1, 2, \dots, M$). A sample of yearly maxima distributions, $H_{n_j}(x) = F(x;C_j,w_j)^{n_j}$ (where n_j is the number of wet days in year j), can thus be defined, and a sample average approximation can be computed $\zeta(x) \cong \zeta_m(x) = 1/M \sum_j F(x;C_j,w_j)^{n_j}$ (Figure 1). The discrete expression of the MEV-Weibull distribution thus reads

$$\zeta_m(x) = \frac{1}{M} \sum_{j=1}^{M} \left[1 - e^{-\left(\frac{x}{C_j}\right)^{w_j}}\right]^{n_j} \tag{3}$$

Convergence of (3) to (2) is ensured provided that (C_j, w_j, N_j) are sampled $\forall j$ from their joint distribution $g(N,C,w)$; see supporting information and [*Weinzierl*, 2000] for additional details. We fit the Weibull distribution to observations in each single year by means of the probability weighted moments method (PWM), which, compared to other methods (e.g., maximum likelihood, ML), attributes a greater weight to the tail of the distribution. Moreover, the PWM method performs well for small samples and is not very sensitive to the presence of outliers [*Greenwood et al.*, 1979]. ML is, on the contrary, known to be a biased estimator of the Weibull parameters, especially the shape parameter, for small samples [*Sornette*, 2003].

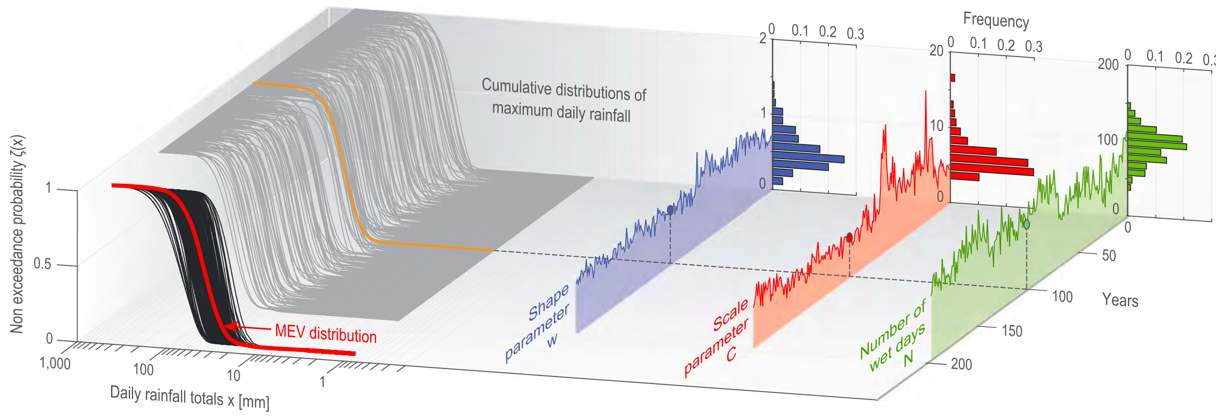

Figure 1. Conceptual representation of the MEV approach applied to daily rainfall EV analysis. Yearly values of the Weibull parameters (in blue and orange) and of the number of wet days (in green) define the cumulative distribution of maximum yearly rainfall as $H_n(x) = F(x; C, w)^n$ (grey distributions are on the left and their projections in black on the vertical *xy* plane in foreground). The MEV distribution (in red in the vertical *xy* plane in the foreground), accounting for the stochastic variability in *C*, *w*, and *n*, is obtained by averaging over the empirical frequency distribution of the parameters.

In the following we will often consider the event magnitude, $\hat{x}$, corresponding to a given value of the return period of interest, which we obtain by numerically solving $\zeta_m(\hat{x}) = (Tr - 1)/Tr$ ($\zeta_m(x)$ being given by equation (3)).

We describe below extensive comparisons of MEV high-quantile estimates with those obtained from the traditional generalized extreme value distribution. We estimate GEV parameters using the most efficient and most commonly used techniques: maximum likelihood (ML) [*Coles*, 2001; *Martins and Stedinger*, 2000; *Gençay et al.*, 2001], L-moments [*Hosking et al.*, 1985; *Hosking*, 1990], the peak over threshold approach [*Davison and Smith*, 1990], and mixed methods [*Morrison and Smith*, 2002]. The POT approach was applied by selecting threshold values such that an average of five excesses/year are used to fit the parameters. Overall, we find that, for our 37-station data set, the POT and L-moment methods yield the best estimates of GEV parameters (see supporting information for details), whereas ML estimators exhibit a larger error standard deviation, especially for smaller samples.

3. Data Sets

We gathered data from 37 rainfall records distributed globally and spanning different rainfall regimes. Many of the records were extracted from NOAA's Global Historical Climatology Network (GHCN) (ftp://ftp.ncdc.noaa.gov/pub/data/ghcn/daily/). Particularly long time series were gathered independently (e.g., the Padova time series, the longest daily rainfall record worldwide, consisting of 272 years of observations [*Marani and Zanetti*, 2015]). See Table S2 in the supporting information for a complete description of the data included in the analysis. The stations span different climatic conditions, thus allowing to test the ability of the MEV approach to capture observed extremes across a wide variety of precipitation regimes. We restricted our analysis to time series with length exceeding 100 years (mean length in the data set is 135 years). Furthermore, only years with less than 10% of missing daily data were considered, which implies that about 2.4% of the years in the global data set were excluded from the analysis. We tested the ability of the Weibull distribution to describe observed daily rainfall for all the stations considered using the Kolmogorov-Smirnov and Cramer Von Mises statistical tests. The positive outcomes of the statistical tests performed for the different stations are described in the supporting information (see Figure S3a in the supporting information).

4. Monte Carlo Analysis of Model Performance

The possible presence of nonstationarity or of periodicities in observed rainfall records adds an additional, and difficult to control, source of uncertainty in the comparative evaluation of extreme value analyses [*Serinaldi and Kilsby*, 2015]. Hence, we used a Monte Carlo approach which by construction removes possible nonstationarities in the observations while preserving the distribution of the rainfall accumulation values and number of events/year present in the observed data set. To this end, for each station in the data set,

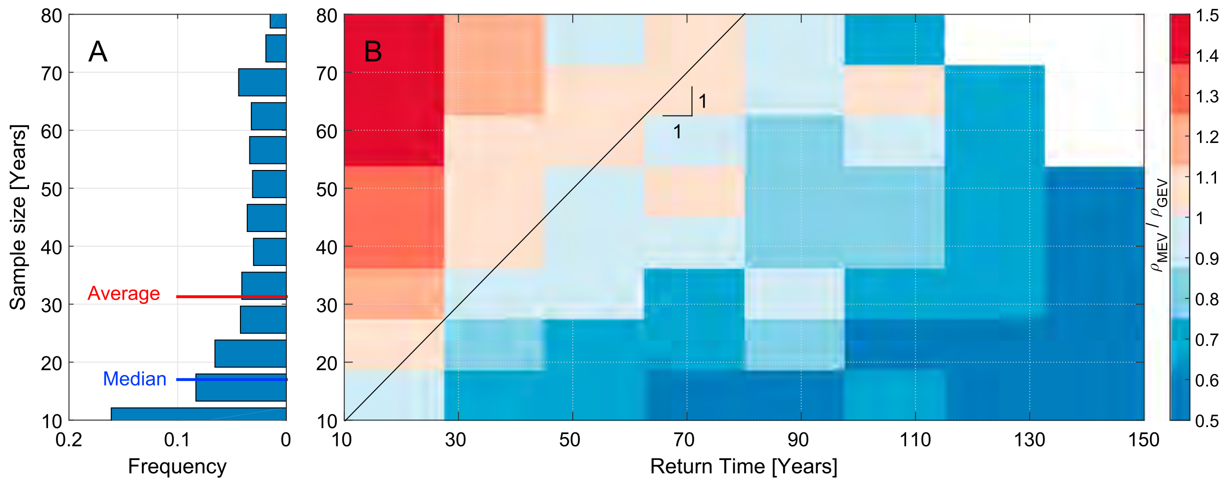

Figure 2. Comparative performance of MEV and GEV distributions. (a) Frequency distribution of sample sizes from the NOAA-NCDC global daily rainfall data set. (b) Ratio ρ_{MEV}/ρ_{GEV}, of the root-mean-square errors of quantile estimates from the MEV-Weibull and GEV-LMOM approaches as a function of return period and size of the sample in our data set. Individual ρ_{MEV}/ρ_{GEV} values from each site are pooled together and averaged over rectangular tiles on an uniform grid (with sides $\Delta Tr = 10$ years and $\Delta s = 20$ years). The $Tr/s = 1$ line is indicated as a reference. The MEV distribution outperforms the GEV distribution in the blue area. Areas in white contain no data.

we randomly reshuffle the observed numbers of wet days/year (n_j's), thereby preserving their original distribution but destroying any serial correlation that may be present. Subsequently, we construct an m year synthetic sample by randomly drawing (without resubstitution) n_j rainfall accumulation values ($j = 1, \dots, m$) from the original record. The resulting synthetic time series lacks any serial correlation and preserves the original frequency distribution of rainfall depths. From each rainfall sequence generated through the above procedure (with length, m years, equal to the original observed time series), we extract the first s years to be used as a sample to fit the EV distributions. The training sample size s is varied from 10 to 80 years with a 2 year step, to explore the range of commonly available sample sizes (see Figure 2a). The remainder of the time series ($m - s$ years) is then used to independently test the MEV and GEV models performances. The selection of observed time series with length exceeding 100 years allows us to use empirical frequencies as references for the exceedance probabilities inferred through the EV models. The sample frequency of an yearly maximum value, x_i, is computed using the Weibull plotting position formula as $F_i = i/(m - s + 1)$ and is assumed to be the best estimate of the actual exceedance probability $F(x_i)$. The i is the rank of x_i in the list, sorted in descending order, of the $s - m$ yearly maxima available in the validation subset. We finally compare $\hat{x}_i = F^{-1}(F_i)$ (where $F^{-1}(\cdot)$ denotes the inverse of one of the EV distributions to be tested) and x_i to determine the estimation error for the 20 largest events in each Monte Carlo generated data sets. We repeat this bootstrap/reshuffling procedure $n_r = 100$ times for each observed time series, in order to obtain a large number of realizations over which to average the root-mean-square error. The accuracy of the empirical frequency estimates of the underlying probability improves with the length of the time series and with the number of Monte Carlo realizations considered and decreases as the return period examined increases. For this reason we focus our attention on return times in the range 10–150 years. For every bootstrap realization, for every sample size and return time (s, Tr), theoretical quantiles, $\hat{x}$, were estimated from the EV distributions being compared. Using the observational quantiles x_{obs} relative to the same Tr_i, the nondimensional estimation error can be computed as $\epsilon = \left(\hat{x} - x_{obs}\right)/x_{obs}$. The values of ϵ_i obtained from each reshuffled series are then averaged over all the Monte Carlo realizations ($n_r = 100$) to obtain a global performance metric:

$$\rho\left(s, T_r\right) = \left[\frac{1}{n_r}\sum_{k=1}^{n_r}\left(\frac{\hat{x}_k\left(s, T_r\right) - x_{obs,k}\left(s, T_r\right)}{x_{obs,k}\left(s, T_r\right)}\right)^2\right]^{\frac{1}{2}} \tag{4}$$

Figure 2 plots the ratio of ρ_{MEV} to ρ_{GEV} as a function of s and Tr, in which data from all the stations have been pooled together. In order to obtain meaningful statistics, individual values of the ratio of the RMSE's computed from equation (4) are averaged over tiles in the plane (s, Tr) of size 20 years × 10 years.

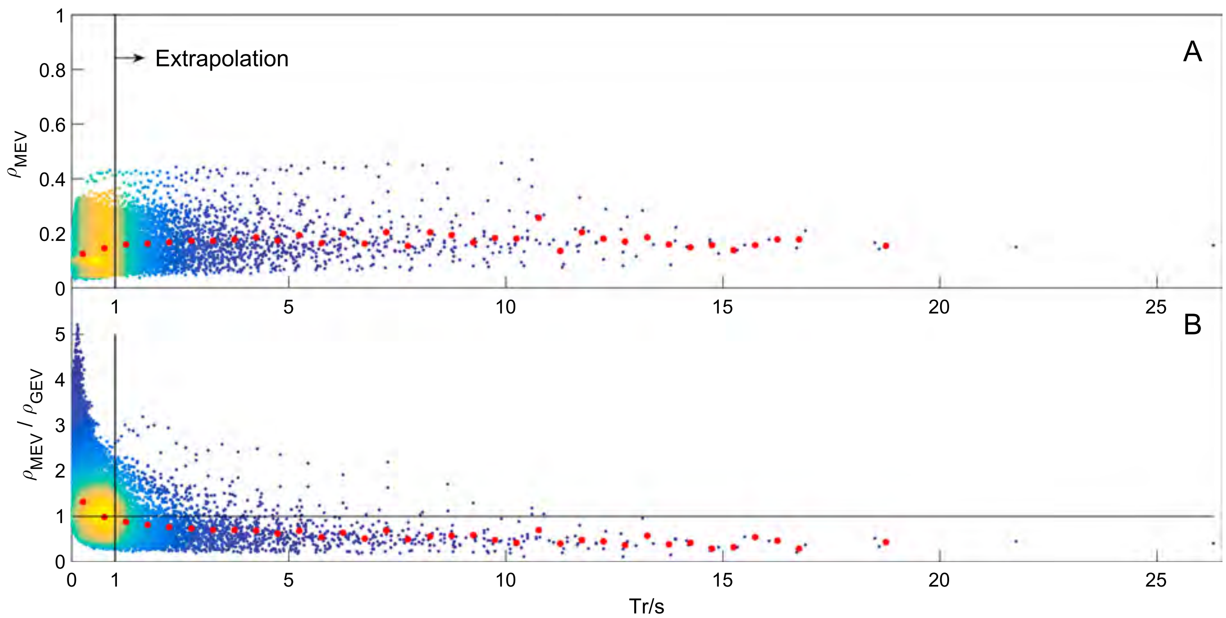

Figure 3. Performance of the MEV and GEV distributions as a function of the dimensionless parameter Tr/s. (a) Root-mean-square error ρ_{MEV}, obtained from 100 Monte Carlo generations. Points denote values from single realizations, while red closed circles represent averages over bins of width 0.5 units. Colors denote the density (points/unit area of the plot, computed over circles of fixed radius) of the values falling in each area of the scatter plot (blue indicating the lowest density and yellow the highest one). (b) Ratio ρ_{MEV}/ρ_{GEV} of the root-mean-square errors obtained with the MEV and GEV approaches.

5. Results

Figure 2b shows that MEV on average outperforms GEV when used to obtain estimates for return periods exceeding the length of the sample used to fit the distribution. For the largest return periods, often of greatest practical interest, the average MEV error is of the order of 50–60% the average GEV estimation error. This result has broad implications, as most of the time series globally available only span a few decades (Figure 2a), while return periods of common interest are greater than $Tr = 100$ years. The analysis of the ratio of the estimation errors as a function of the dimensionless number Tr/s (Figure 3b) clarifies this notion. While some scatter exists, the average of ρ_{MEV}/ρ_{GEV} over bins of Tr/s values clearly indicates that the MEV error tends to be smaller than the GEV error when Tr is greater than the sample size (i.e., $Tr/s > 1$), attaining a 50% improvement for Tr/s indicatively larger than 5. In absolute terms the average root-mean-square error for MEV and $Tr/s = 5$ is roughly 20% (Figure 4a). Figures S1 and S4 in the supporting information show similar results for the comparison with the POT and GEV-ML approaches. Additionally, the comparison of the full distributions of the estimation errors for a common return period and sample size confirms that the MEV approach leads to a significantly narrower error distribution with a mode close to zero (Figures 3 and S5 and Table S1 in the supporting information).

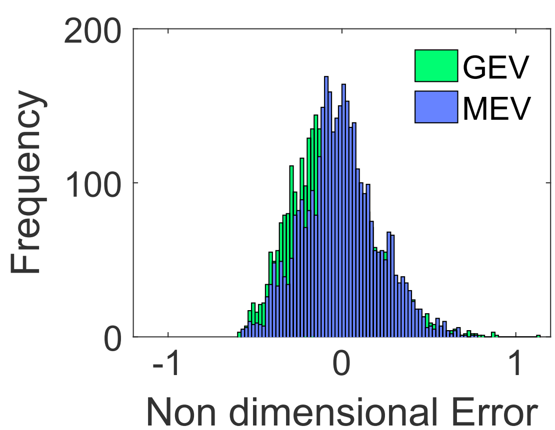

Figure 4. Distribution of the relative error $\epsilon = (\hat{x} - x_{obs})/x_{obs}$ for GEV-LMOM and MEV distributions. ϵ was computed over all the available stations and Monte Carlo realizations ($n_r = 100$). The return time is $T_r = 50$ years and the sample size $s = 30$ years (close to the mean length of the time series in the NOAA-NCDC global data set). The mode of the MEV error is nearly zero, and the error distribution exhibits a smaller spread compared to the frequency distribution of the GEV error.

6. Discussion

The MEV approach presents significant conceptual advantages with respect to traditional methods rooted in the EVT. It removes any asymptotic hypothesis and hence does not require that a sufficiently large number of events/year takes place (see supporting information and *Papalexiou and Koutsoyiannis* [2013] and *Serinaldi and Kilsby* [2014] for further details). The hypothesis of a Poisson occurrence of events is also removed in the MEV approach, the POT approach being retrieved as a special case. The use of a distribution with varying parameters to describe the ordinary event intensities embeds the interannual variability of the rainfall generation process and paves the way to the natural incorporation of trends or multiannual climatic oscillations. The MEV approach recognizes that annual maxima do not necessarily come just from the tail of the underlying parent distribution, a known limitation of the classical EVT [*Veneziano et al.*, 2009].

Classical EVT shows that extremes can only exhibit three types of tail behaviors (upper bounded, exponential, and power law tailed), which become manifested in the value of the GEV shape parameter [*Fischer and Tippett*, 1928; *Von Mises*, 1936; *Leadbetter et al.*, 1983], a conceptually important implication of the classical EVT requiring further discussion. This fat (power law) versus thin (exponential) tail asymptotic dichotomy is conceptually suggestive and practically relevant, such that one wonders if it is negated by the MEV-Weibull approach, which seems to invariably yield a thin-tailed behavior dictated by the exponential nature of the Weibull distribution. However, the Weibull distribution has been noted to exhibit a subexponential tail when $w < 1$, with a behavior which is intermediate between an exponential ($w = 1$) and a power law [*Laherrere and Sornette*, 1998; *Sornette*, 2003]. Furthermore, we note that the combination of exponential distributions with different decay parameters in a metastatistical framework can lead to power law tails [*Dubey*, 1968; *Beck and Cohen*, 2003; *Porporato et al.*, 2006; *Ganti et al.*, 2010]. For example, in the present MEV formulation one can show that (see supporting information for the details), when only the scale parameter of the Weibull distribution varies stochastically, the MEV distribution can assume a power law form, i.e., a heavier tail than the underlying Weibull distributions in equation (3). Hence, we conclude that the MEV-Weibull formulation, even though it is based on stretched exponential building blocks, can reproduce thin- and fat-tailed extreme value distributions. The adoption of a single Weibull distribution to describe all daily events within each year implies that seasonality and different rainfall generation mechanisms are not explicitly resolved. Recent work on flood frequency analysis [*Morrison and Smith*, 2002; *Villarini and Smith*, 2010] indeed suggests that power law tails may artificially emerge from a mixture of probability distributions associated to different rainfall-generating mechanisms. However, this interpretation is not fully in contrast with our approach, which explains thick-tailed extremes by the metastatistical mixing of distributions of the same type but with stochastic parameters.

7. Conclusions

Analysis of extremes in several ultra-centennial daily rainfall records shows that the MEV approach on average outperforms traditional GEV methods when the return period of interest is longer than the length of the observational time series available. The GEV distribution does provide accurate descriptions of the specific samples used to fit it, as shown by the high goodness of fit obtained when the performance is evaluated on the same data used for its calibration (see Figure S2 in the supporting information), but, compared to the MEV approach, it fails to properly generalize and capture the underlying statistical properties of the population. The MEV approach, on the contrary, uses information from the bulk of the distributions of ordinary values and is able to more effectively capture the characters of the population of extremes, such that the estimated high quantiles are less sensitive to the specific sample used for fitting. In conclusion, we argue that the MEV approach should be preferred to the GEV distribution, especially when small samples are available and high-quantile extremes are to be estimated.

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