Quiz 5 Warm Up

Calculus 1 Spring 2025

I. 3.7 RATES OF CHANGE IN NATURAL AND SOCIAL SCIENCES

There is no "new math" in this section, but applications of the derivative to scientific fields are explored. The book gives uses in physics, chemistry, biology, economics, as well as others.

A. Worked Example

Suppose a particle has position function $s = f(t) = t^2 - 6t + 10$, for $t \geq 0$.

1) Find the velocity function at time t of the particle.

Velocity is the time derivative of position,

$$v(t) = f'(t) = 2t - 6 \tag{1}$$

2) Determine where the particle is at rest:

The particle will be at rest when the velocity is 0. (If I throw a ball up, it has positive velocity. On its way down, it has negative velocity. At the point the ball turns around, it must have 0 velocity.) Then

$$v(t) = 0 \quad \rightarrow \quad 2t - 6 = 0 \quad \rightarrow \quad t = 3 \tag{2}$$

At time $t = 3$, the particle has 0 velocity.

3) On what interval(s) is the particle moving in the positive direction?:

The particle is moving in the positive direction when $v(t) > 0$.

$$v(t) > 0 \quad \rightarrow \quad 2t - 6 > 0 \quad \rightarrow \quad t > 3 \tag{3}$$

When $t > 3$, the particle is moving in the positive direction.

4) Determine the total distance traveled by the particle in the first 5 seconds:

The total distance traveled is the sum of the paths where the direction was unchanged. Here, the particle is moving negatively from $t = 0$ to $t = 3$, and positively from $t = 3$ to $t = 5$. The distance from $t = 0$ to $t = 3$ is

$$|f(0) - f(3)| = |10 - 1| = 9 \tag{4}$$

and the distance from $t = 3$ to $t = 5$ is

$$|f(3) - f(3)| = |1 - 5| = 4 \tag{5}$$

Then the total distance is $9 + 4 = 13$.

II. 3.9 RELATED RATES

Idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity, normally one quantity is easier to measure than the other. Next, find an equation to relate the two quantities of interest, then use implicit differentiation with respect to time. Lab 10 is a good study resource for these questions.

A. Worked Example

Two people on bikes are at the same place. One of the bikers starts riding directly north at a rate of 8 m/sec. The second stars to ride directly east at a rate of 5 m/sec. At what rate is the straight line distance between the two riders increasing 20 seconds after they started riding?

Our end goal is dz/dt. Start by drawing a picture of the given quantities, here it will be a triangle. From the problem statement, $dx/dt = 5$ and $dy/dt = 8$. Using these rates, at $t = 2$, $x = 100$ and $y = 160$. Using the Pythagorean theorem to solve for the length of the hypotenuse (call it z) at $t = 20$ gives

$$z^2 = x^2 + y^2 \quad \rightarrow \quad z = \sqrt{100^2 + 160^2} \approx 188.7 \tag{6}$$

Now, begin again with the Pythagorean theorem, and do implicit differentiation with respect to time

$$z^2 = x^2 + y^2 \quad \rightarrow \quad 2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} \quad \rightarrow \quad 2(188.7)\frac{dz}{dt} = 2(100)(5) + 2(160)(8) \tag{7}$$

Rearranging for dz/dt gives

$$\frac{dz}{dt} = 9.433 \tag{8}$$

III. PRACTICE PROBLEMS

The solution to the problem given for 3.7 is in the textbook, the solutions for the problems given for 3.9 are in files-> unfiled -> lab material -> solutions-> lab 10 solutions sec 260.

3.7) The position of a particle is given by the equation $s = f(t) = t^3 - 6t^2 + 9t$.

a) Find the velocity at time t.
b) What is the velocity after $2s$? After $4s$?
c) When is the particle at rest?
d) When is the particle moving forward?
e) Find the total distance traveled by the particle during the first five seconds.

3.9) Lab 10 Worksheet