Quiz 7 Warm Up

Calculus 1 Spring 2025

I. NOTE

My chapter 4 notes, posted in files -> unfiled -> exam material -> exam 3, are more thorough.

II. L'HOPITALS RULE

If direct substitution of a limit problem fails (gives some indeterminate form) L'Hopitals Rule can help solve the problem. The rules is

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)} \tag{1}$$

A. Worked Example (page 11 chapter 4 notes)

Find $\lim_{x\to 1}\frac{\ln(x)}{x-1}$.

Direct substitution gives

$$\lim_{x\to 1}\frac{\ln(x)}{x-1} = \frac{\ln(1)}{1-1} = \frac{0}{0} \tag{2}$$

an indeterminate form. Apply L'Hopitals rule

$$\lim_{x\to 1}\frac{\ln(x)}{x-1} \quad\rightarrow\quad \lim_{x\to 1}\frac{(1/x)}{1} \quad\rightarrow\quad \lim_{x\to 1}\frac{1}{x} = 1 \tag{3}$$

III. 4.5 CURVE SKETCHING

Use the ideas learned so far to sketch the graph of some given function. The early ideas of the course, such as domains and asymptotes, play a role, as well as more recent ideas, like intervals of increasing/decreasing, and intervals of concavity. The only "new" part of this section is actually drawing the graph. (These are lengthy, and I think the worked example and practice problem are more labor intensive than the problem you will see on the quiz).

A. Worked Example

Use the guidelines of this section to sketch the curve of the following function

$$f(x) = \frac{2x^2}{x^2-1} \tag{4}$$

a) Domain: The domain is all real numbers where the denominator is not equal to 0, and the function is defined.

$$x^2 - 1 = 0 \rightarrow x^2 = 1 \rightarrow x = \pm 1 \tag{5}$$

So the domain is $(-\infty,-1)\bigcup(-1,1)\bigcup(1,\infty)$

b) Intercepts: The y intercept is found by evaluating $f(x)$ at $x = 0$, the x intercept found by setting $f(x) = 0$, and solving for x. The intercepts are both 0, so the intercept is the origin.

c) Asymptotes: Vertical asymptotes occur where the denominator is equal to 0, so there are vertical asymptotes at $x = \pm 1$. Horizontal asymptotes are found by taking the limit of the function as $x \to \pm\infty$.

$$\lim_{x\to\pm\infty} \frac{2x^2}{x^2-1} = \lim_{x\to\pm\infty} \frac{2}{1-\frac{1}{x^2}} = 2 \tag{6}$$

So there is a horizontal asymptote at $y = 2$.

d) Critical points: Take the derivative (quotient rule), set it equal to 0, solve for critical points.

$$f'(x) = \frac{-4x}{(x^2-1)^2} \quad \to \quad \frac{-4x}{(x^2-1)^2} = 0 \quad \to \quad -4x = 0 \quad \to \quad x = 0 \tag{7}$$

So $x = 0$ is a critical point. e) Increasing/Decreasing: Build a sign chart with the critical point $x = 0$, testing a value on the left of 0 gives a positive answer, and to the right of 0 gives a negative answer. So f increasing on $(-\infty, -1) \bigcup (-1, 0)$ and is decreasing on $(0, 1) \bigcup (1, \infty)$.

f) Local minimum/maximum: Using the sign chart, the function is increasing (moving up) until $x = 0$, and is decreasing (moving down) after $x = 0$, so $x = 0$ is a local max.

g) Concavity: Set the second derivative equal to zero to get inflection points, use a sign chart.

$$f''(x) = \frac{12x^2+4}{(x^2-1)^2} \quad \to \quad \frac{12x^2+4}{(x^2-1)^2} = 0 \quad \to \quad 12x^2+4 = 0 \quad \to \quad x = \sqrt{\frac{-1}{3}} \tag{8}$$

which has no real solutions, and there are no inflection points. We know there are asymptotes at $x = \pm 1$, so we should build a sign chart with these values on it to test the concavity. Doing so, you would find f is concave up on $(-\infty, 1) \bigcup (1, \infty)$, and concave down on $(-1, 1)$.

i) Sketch the Graph: Using all this info we get

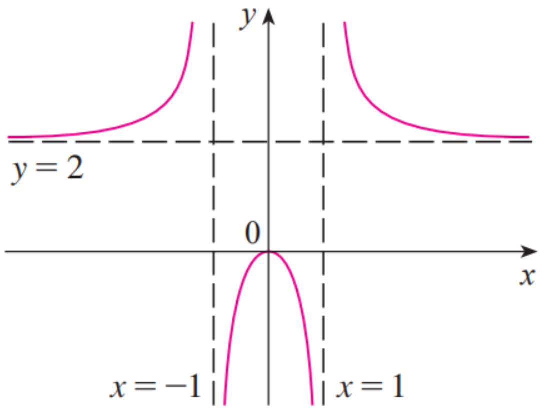

FIG. 1

IV. PRACTICE PROBLEMS

1)

$$\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[3]{x}} \tag{9}$$

2) Make sure your calculator is in radians if you use it

$$\lim_{x\to(\pi/2)^-} \sec(x) - \tan(x) \tag{10}$$

3)

$$\lim_{x \to 0} \frac{\sin(x)}{x} \tag{11}$$

4)

$$\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{e^x - x - 1} \tag{12}$$

5) Work the 4.5 steps for the function $f(x) = (x-3)\sqrt{x}$.