# Quiz 6 Warm Up

Calculus 1 Spring 2025

## I. NOTE

Most of this information comes from my chapter 4 notes, which are posted in files -> unfiled -> exam material -> exam 3, which are more thorough. For example, I don't mention the "first derivative test" or "second derivative test" here (though I use their ideas), but these are mentioned in the notes.

## II. 4.1 ABSOLUTE MIN AND MAX

### A. Critical Numbers/Points

A **critical number/point** of $f(x)$ is a number $c$ in the domain of $f(x)$ such that either $f'(c) = 0$ or $f'(c)$ DNE. Critical points represent the $x$ value where the function switches from increasing to decreasing, or vice versa. To find critical points of a function $f(x)$, take the derivative and set it equal to 0, and solve for $x$.

### B. Worked Example (from Lab 12)

Find the critical points of $f(x) = x^4 - 2x^2 + 4$.

First, take the derivative: $f'(x) = 4x^3 - 4x$. Next, set this equal to 0 and solve for $x$.

$$4x^3 - 4x = 0 \quad \rightarrow \quad 4x(x^2 - 1) = 0 \tag{1}$$

From this, we get

$$4x = 0 \quad \text{and} \quad x^2 - 1 = 0 \tag{2}$$

From $4x = 0$, we learn $x = 0$ is a critical point. From $x^2 - 1 = 0$, we learn $x = \pm 1$ are two more critical points. So, $x = -1, 0, 1$ are the critical points for the given $f(x)$.

### C. Min/Max on a Closed Interval

To find the absolute max/min of a continuous function $f(x)$ on the closed interval $[a, b]$:

1) Find the critical points of $f(x)$ in $(a, b)$
2) Find the values of $f(x)$ at its critical points
3) Find the values of $f(x)$ at its endpoints (at $f(a)$ and $f(b)$)
4) Largest of (1) and (2) is the absolute max
5) Smallest of (1) and (2) is the absolute min

The local min/max will be the values from (1) and (2) that are not the absolute min/max.

### D. Worked Example from Lab 12

Find the absolute min/max, as wall as local min/max of $f(x) = x^4 - 2x^2 + 4$ on the closed interval $[0, 3]$.

This is the same function as above, which has critical points $x = -1, 0, 1$. However, the critical point $x = -1$ lies outside of the closed interval, so we do not include it in the following steps. Next, evaluate $f(x)$ at all critical points in the closed interval and endpoints.

$f(0) = 4$, $f(1) = 3$, and $f(3) = 67$.
Absolute Max: $f(3) = 67$
Absolute Min: $f(1) = 3$
Local Max: $f(0) = 4$
Local Min: $f(1) = 3$ also counts as a local min because there are function values on the left and right of $x = 1$. If the interval were from $[1, 3]$, then $f(1)$ would not count as a local min, since there would be no values to check on the left (ask for clarification if needed).

## III. 4.3 DERIVATIVES AND GRAPH SHAPE

Use the first and/or second derivatives to learn information about a function. The sign (aka $\pm$) of the first derivative tells us whether the function is increasing or decreasing, the sign of the second derivative tells us whether the function is concave up or down.

### A. Intervals of Increasing and Decreasing

To find the intervals a function is increasing or decreasing, find its critical points. One approach is to use the table method that has been introduced in lecture. A second approach is to place the critical points on a number line, and test $x$ values in the intervals between critical points. The tested $x$ values will be plugged into $f'(x)$. Only one value is tested per interval, which is sufficient because a function can only change direction at critical points. The sign of the interval indicates its direction.

### B. Worked Example

Find the increasing/decreasing intervals of $f$, and its local min/max values, where $f(x) = x^4 - 2x^2 + 4$.

This is the same function as above, which has critical points $x = -1, 0, 1$. Method 1 is using the table method (which gets you the same result). Method 2 is placing the critical points on a number line, and testing values in each interval created by the critical points.
For the interval $(-\infty, -1)$, choose $x = -2$, For $(-1, 0)$, choose $x = -1/2$. For $(0, 1)$, choose $x = 1/2$. For $(1, \infty)$, choose $x = 2$. Evaluate at these 4 points: $f(-2) = -24$, $f(-1/2) = 3/2$, $f(1/2) = -3/2$, and $f(2) = 24$.
Using this information, the number line would look like Which tells us that $f(x)$ is

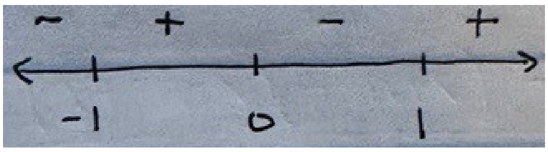


FIG. 1

Increasing: $(-1, 0) \bigcup (1, \infty)$
Decreasing: $(-\infty, -1) \bigcup (0, 1)$

To find local min/max, evaluate $f(x)$ at the critical points, and look at which values are biggest/smallest.
$f(-1) = 2$, $f(0) = 3$, and $f(1) = 2$. These values tell us
Local Min: $x = -1, 1$
Local Max: $x = 0$

### C. Inflection Points

An inflection points is a point where a function $f(x)$ changes concavity, which is another word for the curvature of the function. Finding inflection points is analogous to finding critical points, except $f''(x)$ is set equal to zero and solved for $x$.

### D. Intervals of Concavity

To find the intervals of concavity, find the functions inflections points, and repeat the same process used to find increasing and decreasing from the first derivative. This time, the only change will be when testing $x$ values in the intervals, they are plugged in to the second derivative. Examples shown in lab 12 solutions.

## IV. PRACTICE PROBLEMS

Find the absolute min/max of $f(x) = 4x^4 + x^3 + 4$ on $[-1, 1]$.

Lab 12 Worksheet. Recommend 2a and 2b.