Calculus 1 Quiz 1 Warm up

Calculus 1 Spring 2025

I. REVIEW

A. Tangent line as the limit of secant lines

The slope of the secant line between two points $P = (p_x, p_y), Q = (q_x, q_y)$ is

$$m_{PQ} = \frac{p_y - q_y}{p_x - q_x} \tag{1}$$

and represents the "rise over run" of a given function. As the distance between p_x and q_x shrinks, the secant line approaches the tangent line. In other words, the tangent line is the limit of the secant line as the distance between p_x and q_x approaches 0.

B. Building the Tangent Line

To estimate the slope of the tangent line at a point P, take multiple values of q_x and approach p_x from both sides. If the two sides approach the same finite value, this will be the estimation of the slope at P, call it m_P.

$$m_P \approx \lim_{q_x \to p_x^-} \frac{p_y - q_y}{p_x - q_x} = \lim_{q_x \to p_x^+} \frac{p_y - q_y}{p_x - q_x} \tag{2}$$

With m_P in hand, use the point slope formula

$$y - p_y = m_P(x - p_x) \tag{3}$$

and we have an estimation of the tangent line at P.

C. Limits

A limit refers to the behavior of a function as the input (x) approaches a certain value (c). The functionâs value $(f(x))$ approaches a particular number, x never gets to the point c, just arbitrarily close. Because of this, the function does not need to be defined at $f(c)$, and if it is defined, it does not have to be the same as the limit, for the limit to exist.

$$\lim_{x \to c} f(x) = L \qquad \Longleftrightarrow \qquad \lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L \tag{4}$$

II. PROBLEMS

1) The point $P = (2, -1)$ lies on the curve $y = 1/(1 - x)$.

a) For point $Q = (x, 1/(1 - x))$, find the slope of the secant line m_{PQ} for the following values of x: (1.9, 1.99, 1.999, 2.001, 2.01, 2.1).

b) Using the values in part a, estimate the slope of the tangent line at point P.

c) Using the estimation in part b, find an equation for the tangent line at point P.

2) Explain what it means to say that $\lim_{x \to 1^-} f(x) = 3$ and $\lim_{x \to 1^+} f(x) = 7$. In this situation, what can be said about $\lim_{x \to 1} f(x)$?

3) For the function f whose graph is given (Fig. 1), state the value of each quantity if it exists. If it does not exist explain why.
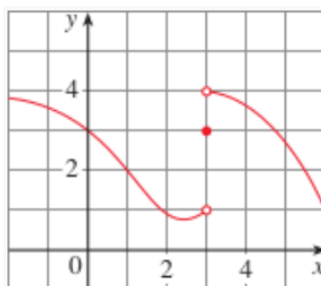

FIG. 1

a) $\lim_{x \to 1} f(x) =$

b) $\lim_{x \to 3^-} f(x) =$

c) $\lim_{x \to 3^+} f(x) =$

d) $\lim_{x \to 3} f(x) =$

e) $f(3) =$