Quiz 3 Warm Up

Calculus 1 Spring 2025

I. THE DERIVATIVE

$$\frac{d}{dx}[\quad] \tag{1}$$

means take the derivative with respect to x of whatever lies in $[\quad]$. If it is $f(x)$, then

$$\frac{d}{dx}f(x) = f'(x) \tag{2}$$

and $f'(x)$ is called the derivative of $f(x)$. Geometrically, the derivative is the slope of the function (as shown later). It is useful: The derivative of position is velocity, the derivative of charge flow is current, many computer algorithms rely on taking derivatives to navigate towards minimum values, etc. There are different rules for taking derivatives depending on the form of the function. The fundamental derivative rules wont go away, as all rules continue to rely on the fundamentals. A good grasp of these topics will help tremendously for the rest of the course.

II. 3.1 DERIVATIVES OF POLYNOMIALS AND EXPONENTIAL FUNCTIONS

A. Derivative of a Constant Function

If $f(x) = c$, where c is some constant, then

$$\frac{d}{dx}f(x) = \frac{d}{dx}c = 0 \tag{3}$$

The graph of $f(x) = c$ is the line $y = c$ shown below. Asking "what is the derivative of $f(x)$?" is the same question as "what is the slope of $f(x)$?". The slope of a constant function is 0, so $f'(x) = 0$.

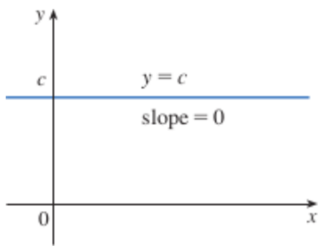

FIG. 1

B. Power Rule

If $f(x) = x^n$, where n is a real number, then

$$\frac{d}{dx}x^n = nx^{n-1} \tag{4}$$

Using the power rule, if $f(x) = x$, then $f'(x) = 1$. From a geometric perspective, $f(x) = x$ is the line with slope 1 everywhere, therefore its derivative is 1, as shown in the figure below.

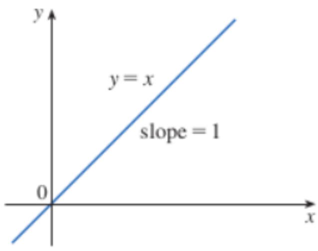

FIG. 2

C. Constant Multiple Rule

If c is some constant and f is differentiable, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x) \tag{5}$$

Take $g(x) = 3x^{13}$, then

$$g'(x) = 3\frac{d}{dx}x^{13} = 39x^{12} \tag{6}$$

D. The Sum and Difference Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) \tag{7}$$

Take $g(x) = 2x^3 + 14x + 99999999$, then

$$\frac{d}{dx}[2x^3 + 14x + 99999999] = \frac{d}{dx}2x^3 + \frac{d}{dx}14x + \frac{d}{dx}99999999 = 6x^2 + 14 \tag{8}$$

E. Exponential Functions

The exponential function, e^x, is the function who is its own derivative. If $f(x) = e^x$, then $f'(x) = e^x$. However, if $f(x) = e^{cx}$, where c is some constant, then

$$\frac{d}{dx}e^{cx} = \frac{d}{dx}[cx]e^{cx} = ce^{cx} \tag{9}$$

I give a spiel about the use of e^x on pages 2 and 14 of my exam 2 review in the Canvas files. If $g(x) = 3e^{-x}$, then by the constant multiple rule and the derivative of exponential functions,

$$\frac{d}{dx}3e^{-x} = 3\frac{d}{dx}[-x]e^{-x} = -3e^{-x} \tag{10}$$

III. 3.2 THE PRODUCT AND QUOTIENT RULES

A. Product Rule

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x) \tag{11}$$

The functions $\sqrt{x}e^x$, $x^2g(x)$, etc need the product rule. For the second,

$$\frac{d}{dx}x^2g(x) = 2xg(x) + x^2g'(x) \tag{12}$$

B. Quotient Rule

If f and g are both differentiable, then

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \tag{13}$$

The function $4x/e^x$ needs the quotient rule

$$\frac{d}{dx}\frac{4x}{e^x} = \frac{4e^x - 4xe^x}{e^{2x}} = \frac{4e^x(1-x)}{e^{2x}} = 4e^{-x}(1-x) \tag{14}$$

IV. 3.3 DERIVATIVES OF TRIG FUNCTIONS

$\frac{d}{dx}\sin(x) = \cos(x)$,

$\frac{d}{dx}\cos(x) = -\sin(x)$,

$\frac{d}{dx}\tan(x) = \sec^2(x)$,

$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$,

$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$,

and last but not least $\frac{d}{dx}\cot(x) = -\csc^2(x)$. Heres a visual representation of $\frac{d}{dx}\sin(x) = \cos(x)$ for fun.

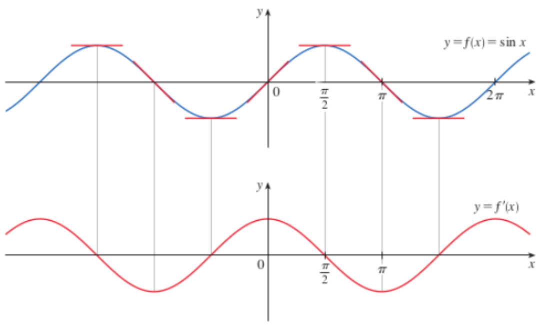

FIG. 3

V. 3.4 THE CHAIN RULE

If g is differentiable at x and f is differentiable at $g(x)$, then the composition $f(g(x))$ is differentiable, and

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x) \tag{15}$$

The function $h(x) = e^{\sin(\sqrt{x})}$ needs the chain rule. Let $h(x) = f(g(x))$ where $f(x) = e^x$ and $g(x) = \sin(\sqrt{x})$.

$$\frac{d}{dx}e^{\sin(\sqrt{x})} = e^{\sin(\sqrt{x})}\frac{d}{dx}\sin(\sqrt{x}) \tag{16}$$

We need the chain rule again. Then $\sin(\sqrt{x}) = r(t(x))$, where $r(x) = \sin(x)$ and $t(x) = \sqrt{x} = x^{1/2}$. So

$$\frac{d}{dx}\sin(\sqrt{x}) = \cos(\sqrt{x})\frac{1}{2}x^{-1/2} \tag{17}$$

and the final answer is

$$\frac{d}{dx}e^{\sin\left(\sqrt{x}\right)} = e^{\sin\left(\sqrt{x}\right)}\frac{d}{dx}\sin\left(\sqrt{x}\right) = e^{\sin\left(\sqrt{x}\right)}\cos\left(\sqrt{x}\right)\frac{1}{2}x^{-1/2} = \frac{e^{\sin\left(\sqrt{x}\right)}\cos\left(\sqrt{x}\right)}{2\sqrt{x}} \tag{18}$$

VI. DERIVATIVES OF LOGARITHMS

These assumptions are more complicated, so I'll just state the rule

$$\frac{d}{dx}b^x = b^x\ln(b) \tag{19}$$

If $b = e$, then $\frac{d}{dx}e^x = e^x\ln(e) = e^x$, since ln is the inverse of e. Though this rule looks similar to the power rule, where the functions are of form x^n, the variables are in different places, so the rules are very different. If $g(x) = 6^x$, then

$$\frac{d}{dx} = 6^x\ln(6) \tag{20}$$

VII. PRACTICE PROBLEMS

The solution to most of the problems are worked out in my exam 2 review (files->unfiled->exam material->exam 2), the page number indicated in the question if so.

Find the derivative of the following functions with respect to x

1) $y = 2e^{-x} + \sin(x)$ page 1

2) $y = (x^3 + 4)/9x$ page 3

3) $y = \sec(4\pi x)$

4) $y = e^{23} + \sin(13x) + e^x$

6) $y = (1 + \sqrt{x})^3$ page 7 and 8, found both y' and y''

7) Find the equation of the tangent line to $y = \sin(x) + \cos(x)$ at the point $P = (0, 1)$. Page 5.