Ice Engine Paradox

Mason Mault and Michael Patterson
(Dated: April 30, 2024)

I. INTRODUCTION

Machines make tasks more efficient and less labor intensive, an important example being the heat engine. The first documented heat engine is accredited to Hero of Alexander in AD 50 [1]. In 1824 Sadi Carnot developed a theoretical understanding of an ideal heat engine still taught today, the Carnot Cycle [2]. The necessity to understand heat engines was responsible for the development of pockets of thermodynamics. For example, the quantification of heat engine properties lead to ideas such as entropy and engine efficiency. We now introduce the workings of a typical heat engine.

A heat engine absorbs heat Q_h from a hot reservoir of temperature T_h, does some work W with this heat, and deposits the leftover heat Q_c to a cold reservoir of temperature T_c.

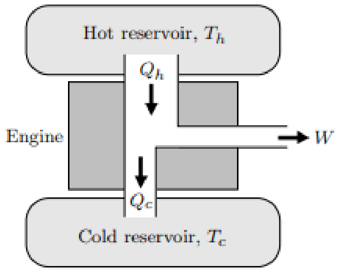

FIG. 1: (Figure 4.1 Schroeder, Page 123)

The working substance is whatever actually *does* the work. For example, a steam engine is a heat engine where steam is the working substance. Different working substances may be used in different scenarios to achieve various outcomes. In this report we examine an inventors proposed "ice engine", in which water/ice is used as the working substance. The inventor claims their engine performs unlimited work using finite energy. Our analysis shows this claim is false, and that the ice engine adheres to the same physical restrictions as a heat engine.

II. BACKGROUND AND THEORY

The efficiency e of a heat engine is defined as the benefit to cost ratio, and is given in eq (1). The benefit is the work done and cost the total heat supplied.

$$e = \frac{W}{Q_h} \tag{1}$$

The maximum efficiency of a heat engine is given $1 - T_c/T_h$. However, the engine of concern in this report claims unlimited efficiency via clever use of phase transitions in water/ice. Phase (PT) diagrams demonstrate which phase of a substance is stable at a given temperature and pressure. The PT diagram for water is given in Figure 2. The phase boundary lines represent states where phases may coexist in diffusive equilibrium, for example all three phases are stable at the marked triple point. Two phases are in diffusive equilibrium if they share a chemical potential μ. The Gibbs free energy G *is* the chemical potential scaled by the number of particles N

$$G = N\mu \tag{2}$$

That is, two phases of the same substance are in diffusive equilibrium if they share a G, occurring along all phase boundary lines. The Clausius-Clapeyron relation describes the slope of a phase boundary line and is derived from the thermodynamic identity for G given in eq (3).

$$dG = -SdT + VdP + \mu dN \tag{3}$$

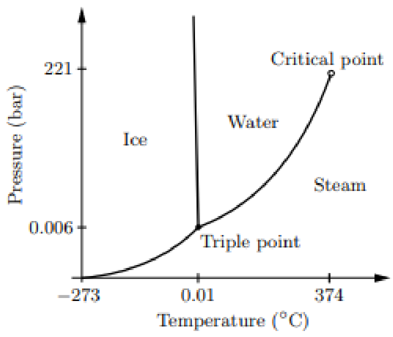

FIG. 2: (Figure 5.12 Schroeder, Page 167)

By considering a small change dG in both phases such that their Gibbs free energy remains equal, the thermodynamic identity can be substituted to obtain the Clausius-Clapeyron relation, eq (4).

$$\frac{dP}{dT} = \frac{L}{T\Delta V} \tag{4}$$

Where L is the total latent heat used to convert the material between phases, ΔV the phases difference in volumes. The Clausius-Clapeyron relation will be used to analyze the ice engines efficiency.

III. ICE ENGINE DESIGN

The ice engine uses water/ice as the working substance, and the inventor claims it functions as follows. The weight to be lifted is placed on a piston over a cylinder of water at 1° C. The system is then attached to a cold reservoir at −1° C, causing the ice to freeze and expand (as ice is less dense than water), lifting the weight. The weight is removed and the system is placed in contact with a high temperature reservoir at 1° C, melting the ice and returning to the original state. This process is summarized in Figure 3.

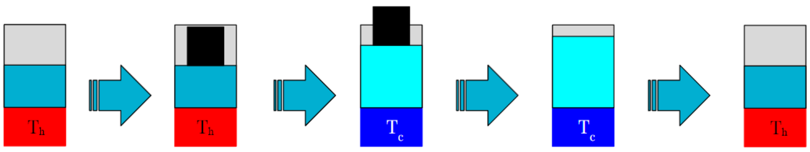

FIG. 3: Ice engine schematic

Figure 4 demonstrates the process of a normal heat engine.

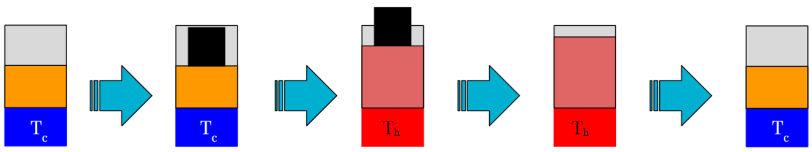

FIG. 4: Heat engine schematic

Notice the ice engine flips T_c and T_h of the heat engine process.

IV. RESULTS

The inventor claims the ice engine performs an unlimited amount of work on a finite amount of heat. We show this to be false, and claim the inventors misconception stems from a misunderstanding of energy flow. As shown in Figure 1 and Figure 4, the heat supplying the work in a heat engine comes from the hot reservoir. Some work is done with this heat while the extraneous heat is deposited to a cold reservoir. In the ice engine, the inventor swaps the hot and cold reservoirs as a means to increase efficiency. What they fail to realize is energy does not spontaneously flow from cold to hot objects, but instead from hot to cold. For example, if hot food is left at room temperature and checked on an hour later, it will be cooler. If the goal is to keep the food hot, additional heat must continuously be supplied. In analogy, while the inventor believes they are increasing efficiency by swapping the reservoirs, they are doing the opposite. With this reservoir swap, there is additional heat/work needed as the ice engine goes from a high temperature (1° C) to a low temperature (-1° C).

To perform the analysis we use $Q_h \approx L$, only considering the latent heat during the phase change, as the additional heat needed to get from $-1°$ C to $0°$ C or $0°$ C to $1°$ C is small in comparison. Beginning with the Clausius-Clapeyron relation given in eq (4), integrate by separating the variables to respective sides of the equation. The bounds of integration on the LHS are 0 to P. The bounds of integration on the RHS are T_h to T_c, as the engine starts hot (the state of the working substance is water), and is then attached to a cold reservoir (the state of the working substance is ice).

$$\int_0^P dP = \frac{L}{\Delta V}\int_{T_h}^{T_c}\frac{1}{T}dT \tag{5}$$

$$P = \frac{L}{\Delta V}\ln\left(\frac{T_c}{T_h}\right) \tag{6}$$

$$P\Delta V = L\ln\left(\frac{T_c}{T_h}\right) \tag{7}$$

Notice $P\Delta V$ on the left hand side is the definition of negative work under the assumption of quasi-static processes.

$$-W = L\ln\left(\frac{T_c}{T_h}\right) \tag{8}$$

Performing a Taylor expansion of $\ln\left(\frac{T_c}{T_h}\right)$ and keeping the first non zero term results in

$$\frac{-W}{L} = \frac{T_c}{T_h} - 1 \tag{9}$$

$$\frac{W}{L} = 1 - \frac{T_c}{T_h} \tag{10}$$

We keep only the first non zero term as our interest is the maximum efficiency of the engine, keeping additional terms would only reduce this efficiency. The approximation $Q \approx L$ means the left hand side of eq (9) is nothing other than eq (1), the efficiency of a heat engine. This demonstrates by use of the Clausius-Clapeyron relation that the maximum efficiency of the ice engine is

$$e = 1 - \frac{T_c}{T_h} \tag{11}$$

Exactly the maximum efficiency of an ideal heat engine, contrasting the inventors thoughts.

V. CONCLUSIONS AND DISCUSSIONS

The inventor of the ice engine claims it produces unlimited work using finite heat by cleverly melting/freezing ice/water to lift a weight. A heat engine does work while attached to a hot reservoir, and deposits waste heat to the cold reservoir. The ice engine performs work on the weight *while* attached to a cold reservoir. We claim the inventor fails to understand energy spontaneously flows from hot to cold objects. While they believe flipping the reservoirs made the ice engine (unlimitedly) more efficient, we show by use of the Clausius-Clapeyron relation that the ice engine has a maximum efficiency of $1 - T_c/T_h$. This directly agrees with the maximum efficiency of a heat engine. It also illustrates the ice engine still operates within the well established thermodynamic principles and cannot do an unlimited amount of work with a finite amount of energy.

VI. REFERENCES

[1] Smoot, G. Heat Engines, Professor George Smoot's Physics 10 Class.
[2] Carnot, S. Reflections on the Motive Power of Fire. Dover.
Schroeder numerous times in the report.