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Dynamic and stochastic shortest path in transportation networks with two components of travel time uncertainty

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Abstract

The existing dynamic and stochastic shortest path problem (DSSPP) algorithms assume that the mean and variance of link travel time (or other specific random variable such as cost) are available. When they are used with observed data from previous time periods, this assumption is reasonable. However, when they are applied using forecast data for future time periods, which happens in the context of ATIS, the travel time uncertainty needs to be taken into account. There are two components of travel time uncertainty and these are the individual travel time variance and the mean travel time forecasting error.

The objectives of this study are to examine the characteristics of two components of travel time uncertainty, to develop mathematical models for determining the mean and variance of the forecast individual travel time in future time periods in the context of ATIS, and to validate the proposed models. First, this study examines the characteristics of the two components of uncertainty of the individual travel time forecasts for future time periods and then develops mathematical models for estimating the mean and variance of individual route travel time forecasts for future time periods. The proposed models are then implemented and the results are evaluated using the travel time data from a test bed located in Houston, Texas. The results show that the proposed DSSPP algorithms can be applied for both travel time estimation and travel time forecasting.

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1. Introduction

The dynamic and stochastic shortest path problem (DSSPP) has been the subject of extensive research in the transportation area for many years. In this problem the link travel time is assumed to be a time-dependent random variable. With the advent of Advanced Transportation Management Systems (ATMS), which are designed to improve transportation system performance, an opportunity exists for extending the DSSPP and implementing it on an actual transportation network. In ATMS, real-time travel time information is obtained directly from probe vehicles or estimated from inductive loop data. Probe vehicles are outfitted with special automatic vehicle identification (AVI) equipment or geographic positioning system (GPS) units. One of the important advantages of probe vehicles is that the travel time of the individual vehicles over each link in their route can be measured and recorded, which has been impossible with the inductive loop data. Recently there have been considerable research in using probe vehicles to estimate and/or predict travel time (Boyce et al., 1993; Eisele and Rilett, 2002; Hellinga, 2001; Park and Rilett, 1998, 1999; Sen et al., 1997; Srinivasan and Jovanis, 1996; Tarko and Rouphail, 1993; Turner and Holdner, 1995). The real-time link travel times are used as input to link travel time forecasting algorithms which provide forecast link and route travel times in future time periods.

With the advent of Advanced Traveler Information Systems (ATIS), and in particular Route Guidance Systems (RGS), the prediction of short-term link travel times has become increasingly important. Intuitively, the RGS's route selection algorithms should use link travel times that are based on the time at which the driver is expected to arrive at a given link rather than use link travel times that are based on current conditions. Because drivers implicitly base their routes on the anticipated link travel time, the RGS should have the same capabilities (Park et al., 1999; Rilett and Park, 2001). Given this requirement, most of the existing travel time forecasting models provide forecast mean link travel time for some future time using observed link travel times (Boyce et al., 1993; Park and Rilett, 1998, 1999; Park et al., 1999, 2002, in press; Rilett and Park, 2001; Sen et al., 1997; Tarko and Rouphail, 1993; Turner and Holdner, 1995; Van Arem et al., 1997). These models are typically "discrete" in that the travel time data are aggregated over predefined intervals (e.g. 5 min) and the travel time in any interval is modeled as a random variable with a mean and variance. The variance or uncertainty of the link travel time forecast for future time periods is the forecasting error associated with "mean" link travel time rather than the variance of the "individual" drivers' link travel times. In this sense, the overall uncertainty or variance of the individual driver's link travel time forecasts consists of two components: (i) the mean travel time forecasting error and (ii) the individual travel time differences among vehicles (referred to as *individual variance* in this paper).

All of the existing DSSPP algorithms and stochastic shortest path problem (SSPP) algorithms in the transportation engineering and operations research literature assume that mean and variance of link travel time (or other specific random variable such as cost) are available (Hall, 1986; Loui, 1983; Miller-Hooks and Mahmassani, 1998, 2000; Mirchandani, 1976; Murthy and Sarkar, 1996). When these models are applied to the previous time period (i.e. observed aggregated travel time), this assumption may be true because the travel time variance corresponds to the travel time differences among vehicles within an aggregation interval. However, when they are applied using forecast travel times and the travel time uncertainty includes the randomness or possible error of the individual travel time forecast, this is not the case. The assumption is wrong because the mean link travel times for the "discrete future time periods" are forecast, which implies that only the mean travel time forecasting error will be available. In other words, the variance associated with the individual drivers is ignored.

Fu and Rilett (1998) proposed approximation models which estimates route travel time mean and variance using the mean and variance of link travel time as a function of time of day. The route travel time variance is defined with respect to individual drivers and therefore, in practice, it is appropriate for estimating individual travel time only for "previous" time periods. Strictly speaking it is not applicable for a forecasting application unless the travel time uncertainty of the forecasting model explicitly considers the mean link travel time forecasting error and individual variance.

In this sense, there is a need to reformulate the DSSPP so that it can consider both individual travel time variance and the mean travel time forecasting error. The objectives of this paper are to examine the characteristics of two components of travel time uncertainty and to develop the mathematical models for determining the mean and variance of the forecast individual travel time in future time period in the context of ATIS.

This paper first introduces the notation and subsequently discusses the assumptions and general practices in travel time estimation for previous time periods in ATIS environment. Then, the characteristics of the two components of uncertainty of the individual travel time forecasts for future time periods are illustrated and their implications from a traffic flow perspective are discussed in Section 4. Following the reformulation of the DSSPP in Section 5, mathematical models for estimating mean and variance of individual route travel time forecast for future time period are proposed in Section 6. The proposed models are then implemented and the results are evaluated using the travel time data from Houston, Texas that had been collected as part of the AVI in Section 7. Finally a concluding discussion follows in Section 8 and includes a summary of the findings and recommendations for future extensions.

2. Notation

2.1. Previous time period (travel time estimation)

y_i individual arrival time at node i

$x_a^i(t)$ travel time of the ith vehicle entering link a at time of day t

$x_a(t)$ random variable of individual travel time on link a at time of day t

$E[x_a(t)] = \mu_{x_a}(t)$ expected value of individual travel time on link a at time of day t

$\mathrm{Var}[x_a(t)] = v_{x_a}(t)$ variance of individual travel time on link a at time of day t

$X_a(t)$ mean link travel time on link a at time of day t

$E[X_a(t)] = \mu_{X_a}(t)$ expected value of mean link travel time (in this paper it also represents the kernel estimate on link a at time of day t)

$\mathrm{Var}[X_a(t)] = V_{X_a}(t)$ variance of mean link travel time on link a at time of day t

h index for discrete time interval or period

$\mu_{x_a}(h)$ mean of individual travel time on link a for time period h
$v_{x_a}(h)$ variance of individual travel time on link a for time period h

2.2. Future time period (travel time forecasting)

$\hat{y}_i$ estimated individual arrival time at node i
$\widehat{E}[y_i]$ estimated expectation or mean of individual arrival time at node i
$\widehat{\mathrm{Var}}[y_i]$ estimated variance of individual arrival time at node i
$\hat{x}_a(t)$ random variable of predicted individual link travel time at link a at time of day t
$\widehat{E}[\hat{x}_a(t)] = \hat{\mu}_{\hat{x}_a}(t)$ estimated expectation or mean of individual link travel time forecasts at link a at time of day t
$\widehat{\mathrm{Var}}[\hat{x}_a(t)] = \hat{v}_{\hat{x}_a}(t)$ estimated variance of individual travel time forecasts at link a at time of day t
$\widehat{X}_a(t)$ mean travel time forecasts at link a at time of day t
$\widehat{E}[\widehat{X}_a(t)] = \hat{\mu}_{\widehat{X}_a}(t)$ estimated expectation or mean of mean link travel time forecasts at link a at time of day t
$\widehat{\mathrm{Var}}[\widehat{X}_a(t)] = \widehat{V}_{\widehat{X}_a}(t)$ estimated variance of mean link travel time forecasts at link a at time of day t
$\hat{\mu}_{\hat{x}_a}(h)$ estimated expectation or mean of individual travel time forecasts for time period h on link a
$\hat{v}_{\hat{x}_a}(h)$ estimated variance of individual travel time forecasts for time period h on link a

3. Assumption in travel time estimation

While the link travel time may be considered a continuous random variable, for practical purposes, the observed travel times are often aggregated and stored in discrete time segments (Boyce et al., 1993; Park and Rilett, 1998, 1999; Park et al., 1999, 2002, in press; Rilett and Park, 2001; Sen et al., 1997; Tarko and Rouphail, 1993; Turner and Holdner, 1995; Van Arem et al., 1997). Because of cost constraints only measures of central tendency such as the mean and standard deviation, and not the individual observations, for each time period are stored. The models and algorithms developed in the study will be based on the following assumptions:

(1) Because probe vehicle data is available the sample mean link travel time and associated variance for each discrete time interval are available;
(2) Link travel times are continuous and time-varying random variables;
(3) Forecast link travel times are available for each discrete future time period. The forecasting algorithms use the sample mean link travel times and/or sample variances from observed discrete time periods as input;
(4) Because both the estimated and forecast link travel times are discrete but the DSSPP is, by definition, continuous, a method for translating between the two is required. In this paper a three-point polynomial approximation developed in Fu and Rilett's (1998) study is used

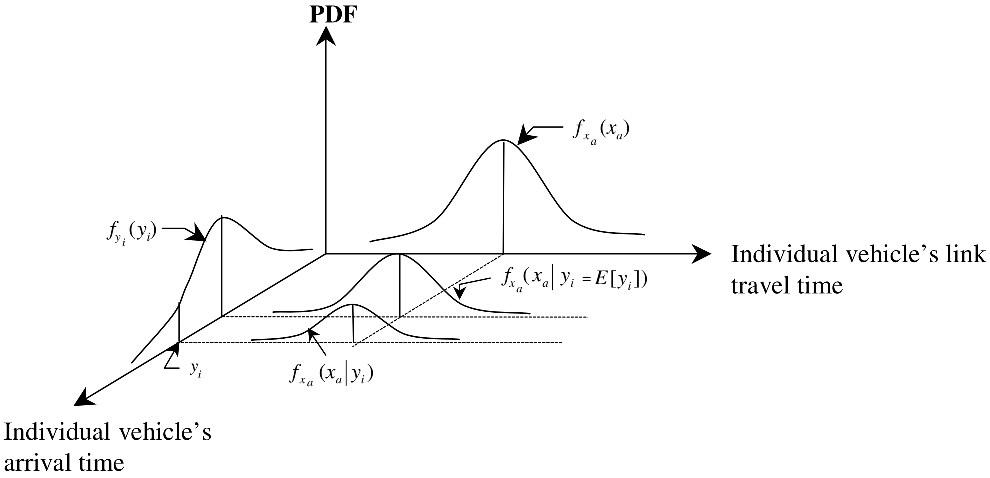

Fig. 1. Conditional density function of an individual vehicle's link travel time.

to estimate the true mean link travel time at particular time t given the discrete travel time estimates and forecasts.

When a vehicle enters link a at time t, the link travel time it experiences is denoted as $x_a^i(t)$. Note that the individual link travel time $x_a(t)$ is a dynamic random variable, where t is time of day, $t \in T$, $T = R_+ = [0, \infty)$. Consider a particular link a with upstream node j and downstream node $j+1$. If the ith vehicle on the network is assumed to enter at node j at time y_i and if it is assumed that there is no waiting time at node j, then the individual link travel time along link a is $x_a^i(y_i)$ and the random variable will be represented by $x_a(y_i)$. In essence, it is assumed that the individual link travel time depends on two random variables: the individual arrival time at the upstream node and the travel time on the link itself. In this study the conditional density function of x_a given on $y = y_i$ is $f_{x_a}(x_a|y_i)$ as shown in Fig. 1.

The sample mean and variance of the individual link travel time at time of day t are defined in Eqs. (1) and (2), respectively.

$$E[x_a^i(t)] = \frac{\sum_{i=1}^{n} x_a^i}{n} \tag{1}$$

$$\mathrm{Var}[x_a^i(t)] = v_{x_a}(t) = E[(x_a^i(t) - E[x_a^i(t)])^2] = \frac{\sum_{i=1}^{n} (x_a^i(t) - \mu_{x_a}(t))^2}{n-1} \tag{2}$$

where n is the number of vehicles traveling on *link a* at time t; $v_{x_a}(t)$ is the kernel variance.

Typically in ATMS, the sample travel time statistics, such as mean and variance, are available only over pre-defined discrete, rather than continuous, time intervals. The aggregated mean and variance for the link travel times are calculated using the observed link travel times of individual vehicles over a given time period of duration h using Eqs. (3)–(6).

$$E[x_a^i(h)] = E[x_a^i(t, t+\Delta t)] = \frac{\sum_{i=1}^{N} x_a^i(t, t+\Delta t)}{N} \tag{3}$$

$$\mu_{x_a}(h) = E[x_a(h)] = \int_t^{t+\Delta t} \int_{x_a|t} x_a \cdot f_{x_a}(x_a, t)\,\mathrm{d}x_a\,\mathrm{d}t \tag{4}$$

$$v_{x_a}(h) = \mathrm{Var}[x_a^i(t, t+\Delta t)] = E[(x_a^i(t) - \mu_{x_a}(h))^2] = \frac{\sum_{i=1}^{N}(x_a^i(t) - \mu_{x_a}(h))^2}{N-1} \tag{5}$$

$$v_{x_a}(h) = E[(x_a(t) - \mu_{x_a}(h))^2] = \int_t^{t+\Delta t} \int_{x_a|t} (x_a(t) - \mu_{x_a}(h))^2 \cdot f_{x_a}(x_a, t)\,\mathrm{d}x_a\,\mathrm{d}t \tag{6}$$

where N is the total number of vehicles that arrive link a within the time period h.

In this paper a Gaussian kernel is used to estimate the continuous mean travel time at a particular point in time t. Therefore, the expected mean of the individual link travel times at time t is assumed to be equal to the Gaussian kernel estimate. The mean and variance of the individual link travel times at a particular point in time t may be calculated using Eqs. (7) and (8), respectively, if the underlying probability density function is available.

$$\mu_{x_a}(t) = E[x_a(t)] = \int_0^{+\infty} x_a \cdot f_{x_a}(x_a, t)\,\mathrm{d}x_a \tag{7}$$

$$v_{x_a}(t) = E[(x_a(t) - \mu_{x_a}(t))^2] = \int_0^{+\infty} (x_a(t) - \mu_{x_a}(t))^2 \cdot f_{x_a}(x_a, t)\,\mathrm{d}x_a \tag{8}$$

4. Two components of travel time uncertainty in individual travel time forecasts

4.1. One component of travel time uncertainty in link travel time estimation

In dynamic and stochastic transportation networks, the mean link travel time is represented as a multinomial random variable. Because it is difficult to derive the PDF typically only the measures of central tendency, such as the mean and variance, are estimated. Note that only the observed mean and variance are available for discrete time periods, and therefore a Gaussian kernel, which is calculated from observed individual travel time, is used to estimate the true mean travel time at time t (see Park et al., in press, for details). Because the mean link travel times in previous time periods are estimated using the observed individual travel times, in general it is assumed that, besides the error associated with sampling, there is no other error associated with the mean link travel time estimation for discrete previous time period (i.e. mean link travel time "estimation" problem).

However, there may be considerable travel time differences among individual drivers. For example, an individual driver may drive faster or slower than the observed mean link travel time for each time of day t depending on their own desires and the amount of traffic around them. In this case the individual link travel time can be represented as a function of the mean link travel time and a residual "error" which represents the individual driver's behavior as shown in Eq. (9).

$$x_a(t) = X_a(t) + \varepsilon_a(t) \tag{9}$$

It is assumed that for the link travel time estimation problem the mean link travel time at a specific point in time t, which is represented by the first term in Eq. (9), is deterministic. The residual "error", which is represented by the last term in Eq. (9), is random. It has a mean equal to zero ($E[\varepsilon_a(t)] = 0$) and a variance ($\mathrm{Var}[\varepsilon_a(t)]$) which will be referred to as the individual variance (IV).

Therefore, the mean of the individual link travel time, which is derived by taking the expectation of Eq. (9), is shown in Eq. (10).

$$\mu_{x_a}(t) = E[X_a(t)] + E[\varepsilon_a(t)] = \mu_{X_a}(t) \tag{10}$$

Eq. (10) implies that at time of day t the mean of the individual link travel time is equal to the expected mean link travel time. Recall that the expected mean link travel time at a specific point in time t is obtained using a Gaussian kernel estimator. In this study, only $\mu_{x_a}(t)$ will be used and will refer to the mean of the individual link travel time on link a at time of day t.

The variance of the individual link travel time is obtained by summing the variance of the two terms in Eq. (9) as shown in Eq. (11).

$$v_{x_a}(t) = V_{X_a}(t) + \mathrm{Var}[\varepsilon_a(t)] + 2\mathrm{COV}(X_a(t), \varepsilon_a(t)) \tag{11}$$

The first term in Eq. (11) represents the variance of the mean link travel time at time t. Because the mean link travel time is assumed to have zero bias, both the first and third terms will equal to zero. Finally, the variance of individual link travel time of a given time of day t is equal to the IV as shown in Eq. (12).

$$v_{x_a}(t) = \mathrm{Var}[\varepsilon_a(t)] = E[(x_a(t) - \mu_{x_a}(t))^2] \tag{12}$$

There are two important points associated with the above formulation. First, the above equations only apply to estimation so forecasting errors have not been addressed. Secondly, while the IV is continuous, the sample variance is available for discrete time intervals.

Fig. 2 shows an idealized set of link travel times over a given discrete time interval h. The sample variance is the total variance calculated with respect to the sample mean over the entire discrete time period. Therefore, the sample variance of a discrete time interval h can be decomposed as shown in Eq. (13).

$$\begin{aligned} v_{x_a}(h) &= E[(x_a(t) - \mu_{x_a}(h))^2] = E[(x_a(t) - \mu_{x_a}(t) + \mu_{x_a}(t) - \mu_{x_a}(h))^2] \\ &= E[(x_a(t) - \mu_{x_a}(t))^2] + 2(x_a(t) - \mu_{x_a}(t)) \cdot (\mu_{x_a}(t) - \mu_{x_a}(h)) + E[(\mu_{x_a}(t) - \mu_{x_a}(h))^2] \end{aligned} \tag{13}$$

In Eq. (13), the cross term is assumed to be zero because the two components of it are independent of each other and the expectations of both components within a time interval are equal to zero (Park et al., 2002). This assumption will be validated with real-world probe data later in this

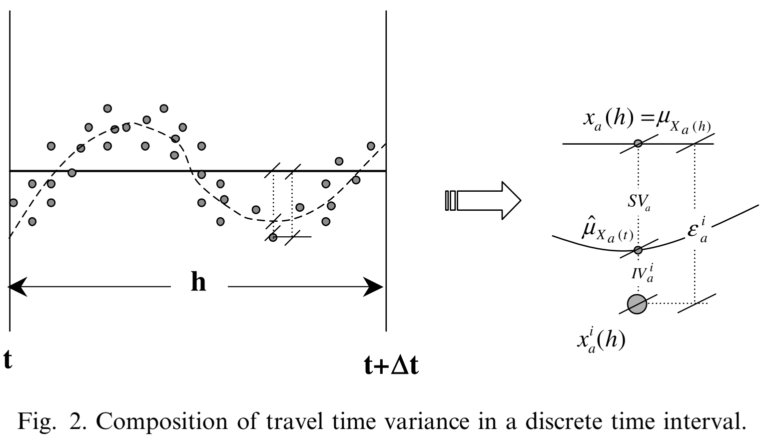

Fig. 2. Composition of travel time variance in a discrete time interval.

paper. The overall variance within the time interval h or sample variance, therefore, is assumed to consist of two terms as shown in Eq. (14).

$$v_{x_a}(h) \cong E[(x_a(t) - \mu_{x_a}(t))^2] + E[(\mu_{x_a}(t) - \mu_{x_a}(h))^2] = \mathrm{IV}(h) + \mathrm{SV}_a(h) \tag{14}$$

where $\mathrm{IV}(h) = E[(x_a(t) - \mu_{x_a}(t))^2]$ and $\mathrm{SV}_a(h) = E[(\mu_{x_a}(t) - \mu_{x_a}(h))^2]$.

Eq. (14) implies that the sample variance calculated over a discrete time interval consists of two components: one is the individual variance (IV) within the time interval, which is the non-systematic variance due to the different driving characteristics of drivers. The other is the systematic variance (SV), which is the variance caused by traffic fluctuations or travel speed/or time changes within a time interval. The systematic variance would be zero if the mean travel time for a particular time of day within a discrete interval does not change which would occur during steady state conditions.

4.2. Two components of travel time uncertainty in link travel time forecasting

Most of the link travel time forecasting models use observed link travel times of the previous time intervals as input and predict the mean link travel times for “discrete” future time intervals. In order to use the mean link travel time forecasts in the DSSPP, the link travel time forecast at a particular time rather than for an interval is required. To do this, an interpolation technique is needed. In this paper a three-point polynomial approximation is used to estimate the mean link travel time as a function of time of day (i.e. $\widehat{X}_a(t)$) as illustrated in Fig. 3. The forecast mean link travel times, which are random variables, are represented in this paper by their mean and variance (i.e. forecasting error). Note that Fu and Rilett’s (1998) model assumes that the mean link travel times for each time of day (and also for each discrete time interval) are available without any forecasting error. In other words, the variance or error term of the travel times in Fu and Rilett’s model corresponds to the travel time variance among drivers exclusive of the mean travel time forecasting error.

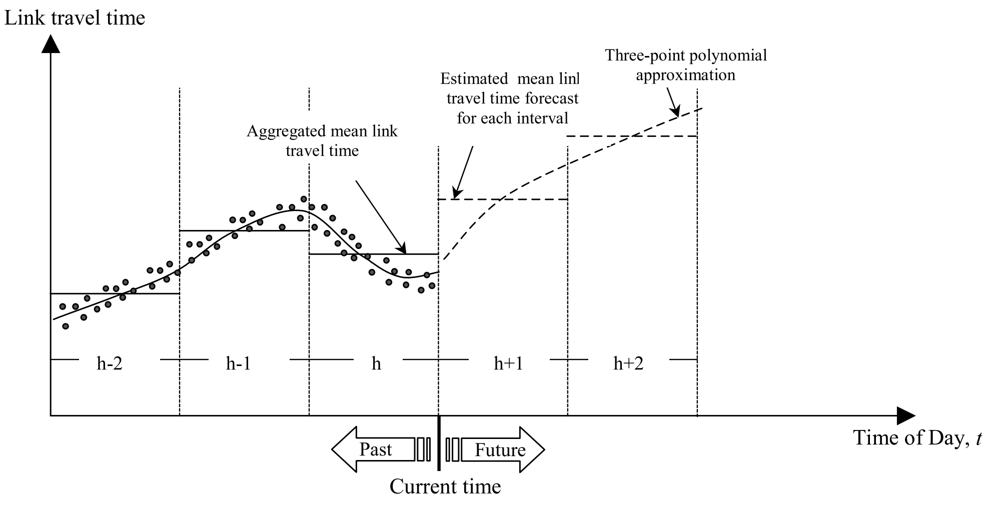

Fig. 3. Link travel time forecasting profile.

Table 1
Composition of mean and individual link travel times

Time period	Mean link travel time	Individual link travel time
Previous time of day	$X_a(t)\{\mu_{X_a}(t), 0\}$	$x_a(t)\{\mu_{x_a}(t), v_{x_a}(t)\}$
Future time of day	$\widehat{X}_a(t)\{\hat{\mu}_{\widehat{X}_a}(t), \widehat{V}_{\widehat{X}_a}(t)\}$	$\hat{x}_a(t)\{\hat{\mu}_{\hat{x}_a}(t), \hat{v}_{\hat{x}_a}(t)\}$

Note: A is estimated or true mean and B is estimated or true variance at {A,B}.

Similar to the travel time uncertainty for previous time periods, the forecast individual link travel time for a particular time of day t is decomposed into the forecast mean link travel time for a time of day t and some residual due to individual driver characteristics as shown in Eq. (15).

$$\hat{x}_a(t) = \widehat{X}_a(t) + \varepsilon_a(t) \tag{15}$$

In Eq. (15), the second term is the IV of individual travel time forecast. The overall variance of individual link travel time forecast for future time of day t can then be formulated as shown in Eq. (16).

$$\hat{v}_{\hat{x}_a}(t) = \widehat{V}_{\widehat{X}_a}(t) + \mathrm{Var}[\varepsilon_a(t)] + 2\mathrm{COV}(\widehat{X}_a(t), \varepsilon_a(t)) \tag{16}$$

It is assumed that the last term in Eq. (16) is zero because the two random variables are likely to be independent of each other and the expectations of both components are zero. Then, the total variance of the individual link travel time forecasts for future time of day t would be as follows:

$$\hat{v}_{\hat{x}_a}(t) \cong \widehat{V}_{\widehat{X}_a}(t) + \mathrm{Var}[\varepsilon_a(t)] \cong \widehat{V}_{\widehat{X}_a}(t) + v_{x_a}(t) \tag{17}$$

In summary, the uncertainty of the individual link travel time forecast at some point in the future, t, consists of two terms: the forecasting error and the individual variance. The forecasting error and IV are defined as Eqs. (18) and (19), respectively.

$$\widehat{V}_{\widehat{X}_a}(t) = \widehat{E}[(\hat{\mu}_{\hat{x}_a}(t) - \mu_{x_a}(t))^2] \tag{18}$$

$$v_{x_a}(t) = \widehat{E}[(\hat{x}_a(t) - \hat{\mu}_{\hat{x}_a}(t))^2] \tag{19}$$

The different components of the travel time uncertainty for the estimation and forecasting problems are summarized in Table 1.

5. Reformulating DSSPP for future time periods

Suppose that a driver will travel from an origin node to a destination node on a given path and wishes to know how long her journey will be. In addition, she will begin her journey at the present time or at some point in the near future. The route travel time is the summation of the travel times on all links on the route and the following recursive formula may be used to estimate the arrival time at the destination node:

$$\hat{y}_j = \hat{y}_i + \hat{x}_a|\hat{y}_i \tag{20}$$

Due to the stochastic nature of link travel time forecast, the estimated route travel time forecast is reformulated as shown in Eq. (21).

$$\widehat{E}[y_j] = \widehat{E}[y_i] + \widehat{E}[\hat{x}_a|y_i] \tag{21}$$

The second term of Eq. (21) is the conditional link travel time which results from the dynamic nature of link travel time. Similar to Fu and Rilett's (1998) model, it can be further transformed to $\widehat{E}[\widehat{E}[\hat{x}_a|y_i]]$, which is defined as $\widehat{E}[\hat{\mu}_{\hat{x}_a}(\hat{y}_i)]$. Accordingly, Eq. (21) can be rewritten as follows:

$$\widehat{E}[y_j] = \widehat{E}[y_i] + \widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)] \tag{22}$$

where

$$\widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)] = \int_0^{\infty} \hat{\mu}_{\hat{x}_a}(y_i) \cdot \hat{f}_{y_i}(y_i)\,\mathrm{d}y_i \tag{23}$$

Note that forecasting route travel time of a driver using Eq. (22) requires the identification of the probability density function of the forecast arrival time ($\hat{f}_{y_i}(y_i)$) at each node at some point in the future. It is impractical in realistic transportation networks to identify this function even for a previous time of day where observed travel times are available. In terms of future time periods, the only information available about individual link travel time are the forecast mean link travel times for each discrete time period, the mean travel time forecasting error, and the IV which is estimated from travel time data for previous time periods. Furthermore, even if the PDF of individual link travel time forecast as a function of the time of a day is derived or assumed, the derivation of the PDF of individual arrival time is mathematically impractical. The following section shows how to forecast the mean route travel time and its variance from an individual driver's perspective based on the forecast mean travel time for discrete future time intervals.

6. Estimating individual travel time forecast on a given path

Fig. 4 illustrates a two-link network where it is assumed that the travel time on link a follows a normal distribution. In Fig. 4(a), it is assumed that the mean link travel times are available over all time periods without bias (i.e. previous time period) and the travel time on link a has the mean of 5 and variance of 1 ($x_a = N\{5, 1\}$). In addition it is assumed that the travel time on link b is deterministic and dynamic.

Fig. 4(b) shows the same network for a travel time forecasting scenario. Assume that the forecast mean travel time on link a is 5 min with a standard deviation of $\sqrt{2}$ min due to the forecasting error. The individual variance of the future time period is assumed to be the same as observed in previous time periods. Therefore, if the individual travel time variance and forecasting error are assumed to be independent of each other, the individual travel time forecast random variable can be estimated as follows: $\hat{x}_a(y_i) = \widehat{X}_a(y_1) + \varepsilon_a(y_1) = N\{5, \sqrt{2}\} + N\{0, 1\} = N\{5, \sqrt{3}\}$. The distribution of the forecast individual travel time on link a is normal where its expected value is equal to the mean travel time forecast and the overall variance is the summation of the mean travel time forecasting error and the individual variance.

The travel times on link b in both Fig. 4(a) and (b) are deterministic and time-dependent. Therefore, the travel time on a downstream link (i.e. link b) is a function of the arrival time at

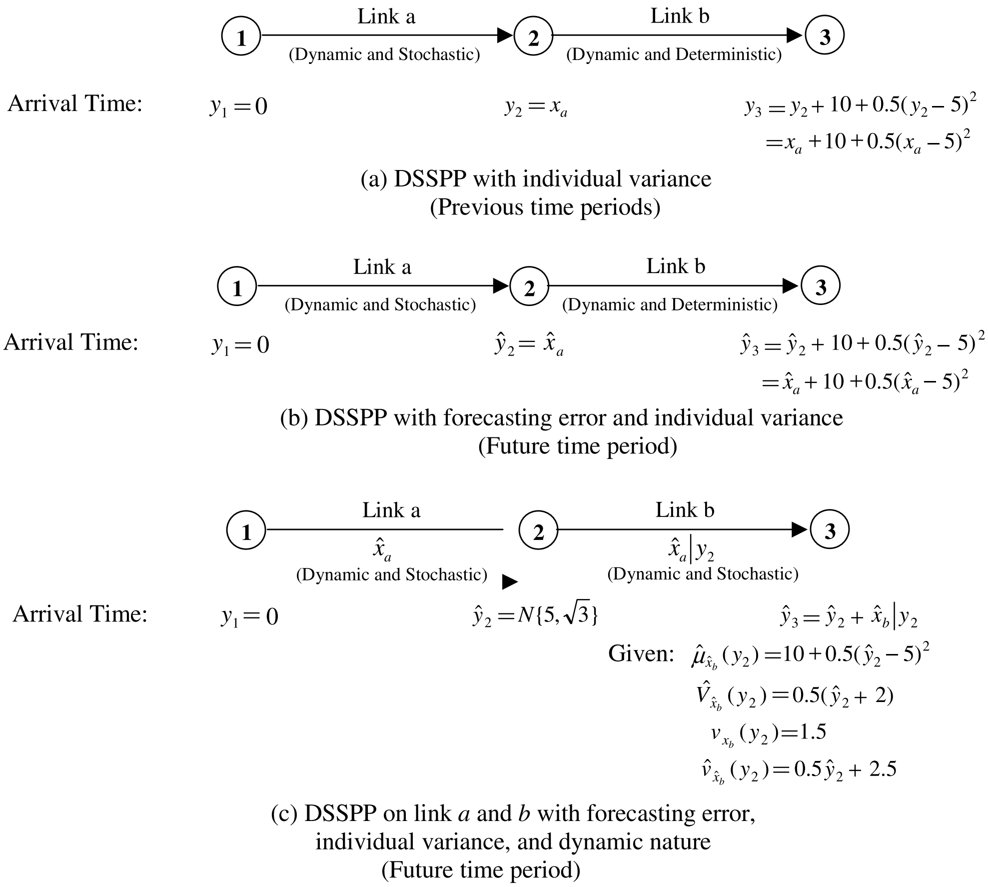

Fig. 4. A two-link network with two components of travel time uncertainty.

node 2 (i.e. y_2). In Fig. 4(b), if a vehicle departs node 1 at the present time (given $y_1 = 0$), the expected arrival time at node 2 will be $\hat{x}_a$. The arrival time at node 3 is therefore $\hat{y}_3 = \hat{y}_2 + 10 + 0.5(\hat{y}_2 - 5)^2$. In spite of the fact that $\hat{y}_3$ is a Gaussian random variable, its PDF is not easily obtainable. It is clear that identifying the PDF of the arrival time at downstream links will become quickly intractable if more complicated PDF's are used and this problem will occur even if simple functions are used.

Consider Fig. 4(c) where travel times on two consecutive links are dynamic and stochastic. To statistically derive the mean and variance of $\hat{x}_b|\hat{y}_2$, the joint probability density function $(\hat{f}(\hat{x}_b, \hat{y}_i))$ is required. However, from a practical standpoint it is impossible to estimate this function. The conditional terms, therefore, are usually transformed into $\hat{x}_b|y_2 = \hat{\mu}_{\hat{x}_b}(y_i)$ as discussed in the previous section. One possible way to calculate the conditional mean and variance would be to derive them from the mean link travel time forecast and the combination of mean travel time forecasting error and IV. By applying the three-point polynomial approximation approach (Fu and Rilett, 1998) to the mean travel time forecasts for future time intervals, the estimated continuous mean travel time forecast on link b $(\hat{\mu}_{\hat{x}_b}(y_2))$ can be determined. Let $\hat{\mu}_{\hat{x}_b}(y_2) = 10 + 0.5(\hat{y}_2 - 5)^2$. Then the problem is to identify when the driver will arrive node 3, if he departs node 1 at the present time (given $y_1 = 0$) using the recursive formula of $\hat{y}_3 = \hat{y}_2 + \hat{x}_b|y_2$.

6.1. Estimating mean of individual route travel time forecasts

Following Fu and Rilett (1998), the function of $\hat{\mu}_{\hat{x}_a}(y_i)$ is expanded with a Taylor's series around the point $t = \widehat{E}(y_i)$ in order to determine the second term of Eq. (22) as shown in Eq. (24).

$$\hat{\mu}_{\hat{x}_a}(t) = \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]) \cdot (t - \widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i]) \cdot (t - \widehat{E}[y_i])^2 + \cdots \tag{24}$$

Note that this step requires the differentiable function of $\hat{\mu}_{\hat{x}_a}(y_i)$ at point $t = \widehat{E}(y_i)$. As a first order approximation, the Taylor's series of individual route travel time forecasts can be truncated by assuming that the second and higher order derivatives are equal to zero. The truncated series is substituted into Eq. (22) and is shown below:

$$\begin{aligned}
\widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)] &\cong \int_0^{+\infty} \{\hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]) \cdot (y_i - \widehat{E}[y_i])\} \cdot \hat{f}_{y_i}(y_i)\,\mathrm{d}y_i \\
&\cong \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \int_0^{+\infty} \hat{f}_{y_i}(y_i)\,\mathrm{d}y_i + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \int_0^{+\infty} (y_i - \widehat{E}[y_i]) \cdot \hat{f}_{y_i}(y_i)\,\mathrm{d}y_i \\
&\cong \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i])
\end{aligned} \tag{25}$$

Therefore, the first order approximation model of the recursive formula is

$$\widehat{E}[y_j] \cong \widehat{E}[y_i] + \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) \tag{26}$$

In the first order approximation the forecast individual link travel time random variables are replaced with the forecast mean link travel time at the forecast arrival time at each link. Note that based on Eq. (16) all that would be required to implement the first order model are the mean of the individual link travel time forecast obtained from the mean travel time forecasting model and the three-point polynomial approximation approach.

In the second order approximation model, the third and higher order derivatives could be set equal to zero as follows:

$$\begin{aligned}
\widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)] &\cong \int_0^{+\infty} \left\{\hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]) \cdot (t - \widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i]) \cdot (y_i - \widehat{E}[y_i])^2\right\} \cdot \hat{f}_{y_i}(y_i)\,\mathrm{d}y_i \\
&= \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i]
\end{aligned} \tag{27}$$

Accordingly, the recursive formula of the second order approximation is shown as Eq. (28).

$$\widehat{E}[y_j] \cong \widehat{E}[y_i] + \hat{\mu}_{\hat{x}_a}(\widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i] \tag{28}$$

To estimate forecast mean route travel time using the second order approximation model, the second derivative of the forecast mean travel time for all links along the route is required. In addition, the total variance of the arrival time at upstream node, which consists of two components of uncertainty, is needed.

Table 2 shows the results of the estimated forecast mean travel time for the three problems in Fig. 4 using the first order approximation, the second order approximation, and direct calculation (see Pattanamekar, 2001, for details). From these hypothetical examples, three important impli-

Table 2
Summary of the estimated/forecast mean arrival times at node 3

Problem	Method	Estimated travel time (min)	
		Previous time period	Future time period
Fig. 4(a)	Direct calculation	15.5	–
	First order model	15.0	–
	Second order model	15.5	–
Fig. 4(b)	Direct calculation	–	16.5
	First order model	–	15.0
	Second order model	–	16.5
Fig. 4(c)	Direct calculation	–	16.5
	First order model	–	15.0
	Second order model	–	16.5

cations are observed. First, the first order approximation model in Eq. (26) implies that the mean of the forecast arrival time at the downstream node depends on both the arrival time at the starting node and the forecast mean link travel time at the time of day of arrival. In addition, it is observed that because the link travel time forecast, which is by definition stochastic, is replaced with an estimated mean value, the estimated individual link travel time forecasts become dynamic and deterministic. Intuitively, this model can be acceptable when the variance of arrival time is small relative to the mean. However, as discussed earlier, the variance of individual travel time forecasts are, in reality, a combination between the forecasting error and the individual variance. Therefore, it would be expected that the total variance will become larger relative to the mean. In this sense, the first order approximation model may not be acceptable when travel time forecasts are used and the second derivative of the mean link travel time forecasts is not zero. This hypothesis will be tested using real-world data.

Second, the estimated mean travel time forecasts which are obtained from the first order approximation model for both previous and future time condition are equal. This is due to the fact that the first order approximation model does not take into account any variance in travel time. Lastly, the second order approximation model in Eq. (28) takes into account both dynamic and stochastic effects of the travel time. From Table 2, it is observed that the estimated forecast mean travel time obtained from the second order approximation model is equal to that obtained from direct calculation. The latter result is a direct result of the fact that the underlying PDF's were Gaussian. However, it would be expected that the second order model would always provide better results. The magnitude of the improvement is a function of congestion and the accuracy of the forecasting techniques.

6.2. Estimating variance of individual route travel time forecasts

In order to estimate the mean of the individual arrival time for future time periods using the second order approximation model, the variance of the arrival time forecast at the upstream node is required. The recursive formula for estimating the variance of a route is shown in Eq. (29).

$$\widehat{\mathrm{Var}}[y_j] = \widehat{\mathrm{Var}}[y_i] + \widehat{\mathrm{Var}}[\hat{x}_a|y_i] + 2\mathrm{COV}(y_i, \hat{x}_a|y_i) \tag{29}$$

The middle term in Eq. (29) can be transformed further as shown in Eq. (30).

$$\widehat{\mathrm{Var}}[\hat{x}_a|y_i] = \widehat{E}[\widehat{\mathrm{Var}}[\hat{x}_a(y_i)]] + \widehat{\mathrm{Var}}[\widehat{E}[\hat{x}_a(y_i)]] = \widehat{E}[\hat{v}_{\hat{x}_a}(y_i)] + \widehat{\mathrm{Var}}[\hat{\mu}_{\hat{x}_a}(y_i)] \tag{30}$$

The last term in Eq. (29) can be further transformed as follows:

$$\begin{aligned}\mathrm{COV}(\hat{y}_i, \hat{x}_a|\hat{y}_i) &= \widehat{E}[y_i \cdot \hat{x}_a|y_i] - \widehat{E}[y_i] \cdot \widehat{E}[\hat{x}_a|y_i] = \widehat{E}[y_i \cdot \widehat{E}[\hat{x}_a|y_i]] - \widehat{E}[y_i] \cdot \widehat{E}[\hat{\mu}_{x_a}(y_i)] \\ &= \widehat{E}[y_i \cdot \hat{\mu}_{\hat{x}_a}(y_i)] - \widehat{E}[y_i] \cdot \widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)]\end{aligned} \tag{31}$$

Consequently, the variance of individual forecast arrival time at node j is given as Eq. (32).

$$\widehat{\mathrm{Var}}[y_j] = \widehat{\mathrm{Var}}[y_i] + \widehat{E}[\hat{v}_{\hat{x}_a}(y_i)] + \widehat{\mathrm{Var}}[\hat{\mu}_{\hat{x}_a}(y_i)] + 2\widehat{E}[y_i \cdot \hat{\mu}_{x_a}(y_i)] - 2\widehat{E}[y_i] \cdot \widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)] \tag{32}$$

In Eq. (32), the first term $\widehat{\mathrm{Var}}[y_i]$ is the variance of forecast individual arrival times at the starting node. The second term $\widehat{E}[\hat{v}_{\hat{x}_a}(y_i)]$ is the expected variance of estimated individual link travel time forecasts. Note that this term includes the mean travel time forecasting error and the individual variance. The third term $\widehat{\mathrm{Var}}[\hat{\mu}_{\hat{x}_a}(y_i)]$ is the estimated variance of the conditional mean link travel time forecast with respect to the variable arrival time due to forecasting error of the mean link travel time. Similar to using $\widehat{E}[\hat{\mu}_{\hat{x}_a}(y_i)]$ when forecasting mean individual travel time, in practice $\hat{\mu}_{\hat{x}_a}(y_i)$ and $\hat{v}_{\hat{x}_a}(y_i)$ are not available. Therefore, they are approximated by the Taylor series expansions about point $t = \widehat{E}(y_i)$. The first and the second order approximation models for Eq. (32) are shown in Eqs. (33) and (34), respectively.

$$\widehat{\mathrm{Var}}[\hat{y}_j] = \{1 + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i])\}^2 \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\hat{x}_a}(\widehat{E}[y_i]) \tag{33}$$

$$\begin{aligned}\widehat{\mathrm{Var}}[\hat{y}_j] = &\left\{[1 + \hat{\mu}'_{x_a}(\widehat{E}[y_i])]^2 + \tfrac{1}{2}\hat{v}''_{x_a}(\widehat{E}[y_i]) - \tfrac{1}{4} \cdot \hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i]\right\} \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\hat{x}_a}(\widehat{E}[y_i]) \\ &+ \hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{E}[(y_i - \widehat{E}[y_i])^3] + \tfrac{1}{4}\hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{E}[(y_i - \widehat{E}[y_i])^4]\end{aligned} \tag{34}$$

The second order model can be reformulated as Eq. (35) if the coefficient of skewness (θ_3) and coefficient of kurtosis (θ_4) are assumed to be available.

$$\begin{aligned}\widehat{\mathrm{Var}}[\hat{y}_j] = &\left\{(1 + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]))^2 + \tfrac{1}{2}\hat{v}''_{\hat{x}_a}(\widehat{E}[y_i]) - \tfrac{1}{4} \cdot \hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i]\right\} \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\hat{x}_a}(\widehat{E}[y_i]) \\ &+ \hat{\mu}''_{\hat{x}_a}(\widehat{E}[y_i])\theta_3 \cdot (\mathrm{Var}[y_i])^{3/2} + \tfrac{1}{4}\hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \theta_4 \mathrm{Var}^2[y_i]\end{aligned} \tag{35}$$

Assume that the forecast individual arrival time is symmetric ($\theta_3 = 0$) and its distribution is neither platykurtic or leptokuric ($\theta_4 = 3$). Then the second order approximation model can be formulated as shown in Eq. (36).

$$\begin{aligned}\widehat{\mathrm{Var}}[\hat{y}_j] \cong &\left\{(1 + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]))^2 + \tfrac{1}{2}\hat{v}''_{\hat{x}_a}(\widehat{E}[y_i]) - \tfrac{1}{4} \cdot \hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i]\right\} \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\hat{x}_a}(\widehat{E}[y_i]) \\ &+ 0 + \tfrac{3}{4}\hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \mathrm{Var}^2[y_i] \\ \cong &\left\{(1 + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]))^2 + \tfrac{1}{2}\left(\hat{v}''_{\hat{x}_a}(\widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i]) \cdot \widehat{\mathrm{Var}}[y_i]\right)\right\} \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\hat{x}_a}(\widehat{E}[y_i])\end{aligned} \tag{36}$$

Given Eq. (17), the first and second order approximation models of the variance of route travel times for individuals are given by Eqs. (37) and (38), respectively.

$$\widehat{\mathrm{Var}}[\hat{y}_j] \cong \{1 + \hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i])\}^2 \cdot \widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\widehat{X}_a}(\widehat{E}[y_i]) + v_{\hat{x}_a}(\widehat{E}[y_i]) \tag{37}$$

Table 3
Summary of the estimated/forecast variance of individual arrival time at node 3

Problem	Method	Estimated variance (min^2)	
		Previous time period	Future time period
Fig. 4(a)	Direct calculation	1.5	–
	First order model	1	–
	Second order model	1.5	–
Fig. 4(b)	Direct calculation	–	7.5
	First order model	–	3
	Second order model	–	7.5
Fig. 4(c)	Direct calculation	–	12.5
	First order model	–	8
	Second order model	–	12.5

$$\widehat{\mathrm{Var}}[\hat{y}_j] \cong \left\{(1+\hat{\mu}'_{\hat{x}_a}(\widehat{E}[y_i]))^2 + \tfrac{1}{2}(\hat{v}''_{\widehat{X}_a}(\widehat{E}[y_i]) + v''_{x_a}(\widehat{E}[y_i]) + \tfrac{1}{2}\hat{\mu}''^2_{\hat{x}_a}(\widehat{E}[y_i])\cdot\widehat{\mathrm{Var}}[y_i])\right\}\cdot\widehat{\mathrm{Var}}[\hat{y}_i] + \hat{v}_{\widehat{X}_a}(\widehat{E}[y_i]) + v_{x_a}(\widehat{E}[y_i]) \quad (38)$$

The variance of the arrival time forecasts at node 3 in the problems in Fig. 4(a)–(c) can be calculated directly and by using the above approximation models. The results are shown in Table 3 (see Pattanamekar, 2001, for details).

Based on the hypothetical examples, a number of important points regarding the variance approximation models are observed. First, the first order approximation model for estimating variance of individual travel time forecasts in Eq. (37) shows that the variance of individual travel time forecast depends not only on variance of the arrival time forecast but also on the variance associated with the individual travel time forecast. Second, similar to Fu and Rilett's (1998) conclusion, it can be noted that the difference between the first and the second order approximation models may be trivial. The second derivatives could be negligible because the two second derivative terms in Eq. (38) would be relatively small compared to the other terms. Third, as illustrated in Fig. 4(b) and Table 3, the variance of travel time due to dynamic impact is equal to 7.5 min^2. However, from the driver point of view, the link travel time forecast consists of two components of travel time uncertainty. In this case the overall variance of individual travel time forecasts in DSSPP should be comprised of three components: the dynamic impact, the mean link travel time forecasting error, and the individual variance. For example, the total variance of 12.5 min^2 can be decomposed into the dynamic impact of 7.5 min^2 and the combination of the two uncertainties in the forecast individual link travel time which is equal to 5 min^2.

7. Application of the proposed approach

7.1. Test bed freeway corridor and data collection

The test bed for this study was US-290 which is a radial six-lane urban freeway located in Houston, Texas. It has a barrier-separated HOV lane that runs along the centerline of the freeway

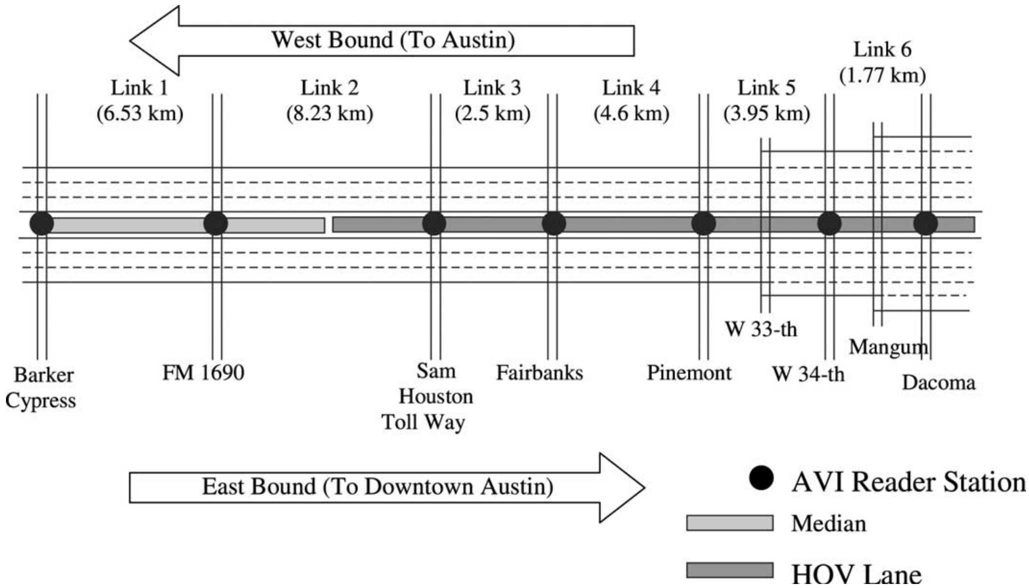

Fig. 5. Test bed freeway corridor: US-290 Houston, Texas.

for approximately 19 km, however, the data utilized was from the non-HOV section of the freeway.

Travel time data were collected over a 27.6 km stretch of US-290 from seven AVI reader stations (yielding six links) as shown in Fig. 5. The data were collected over a 24 h period each weekday in both directions of travel for 12 months in 1997 resulting in 251 weekdays. Only the Eastbound A.M. peak travel time data were employed in this study because these links experienced higher congestion levels than the westbound links. Based on a visual inspection of the travel time patterns, the study time period was defined as lasting from 6:00 to 11:00 A.M. and the data collected over this time period were used for this study. The mean speed on the corridor during the morning peak hour ranges proximately from 30 and 50 km/h. During this peak period, an average of approximately 17–35 vehicles per 5-min period traversed links 3, 4, 5, and 6 while an average of approximately 8–12 vehicles traversed links 1 and 2 per 5-min period. The travel time data from the AVI vehicles were subsequently aggregated for each 5-min period for each link. Among six links, links 2, 3, 4, and 5 were selected to examine the decomposition of the travel time variances (SV and IV) and links 3, 4, and 5 were used as the test bed route of the proposed approaches.

7.2. Experimental study design

7.2.1. Estimating individual variance for future time periods

The proposed models requires three inputs: individual variance, forecast mean link travel times, and the mean travel time forecasting error. By definition, the individual variance for future time periods cannot be observed. Therefore, this study estimates the individual variance from previous (or historical) travel time data. Table 4 shows the composition of the overall variance of the aggregated travel times for 5-min intervals under various levels of traffic congestion. It may be

Table 4
Combination of the individual variance and systematic variance

Link	Combination of variance (s^2)	V/C					
		0.5–1.0		1.0– 1.5		>1.5	
1	Sample Var	179	100%	520	100%	1329	100%
	IV	41	23%	72	14%	217	16%
	SV	123	69%	422	81%	1054	79%
	Covariance	15	8%	26	5%	58	4%
2	Sample Var	516	100%	1868	100%	8556	100%
	IV	128	25%	265	14%	1208	14%
	SV	351	68%	1471	79%	7348	86%
	Covariance	37	7%	132	7%	204	2%
3	Sample Var	32	100%	66	100%	1542	100%
	IV	11	36%	16	24%	140	9%
	SV	18	55%	45	68%	1361	88%
	Covariance	3	9%	5	8%	41	3%
4	Sample Var	83	100%	161	100%	747	100%
	IV	28	33%	42	26%	98	13%
	SV	48	58%	109	68%	623	83%
	Covariance	7	9%	9	6%	26	4%
5	Sample Var	63	100%	92	100%	1297	100%
	IV	21	34%	31	34%	262	20%
	SV	36	57%	54	58%	970	75%
	Covariance	6	9%	7	8%	65	5%

seen that, on average, the covariance between the IV and SV is approximately 7% and 4% of the overall variance for V/C values of 0.5–1.5 and greater than 1.5, respectively. In this sense, the hypothesis that was used to derive Eq. (17) is considered acceptable. Note that as the level of congestion increases, the relative size of the covariance decreases. Based on the results in Table 4, this study assumed the IV for each future time of day as shown in Table 5.

7.2.2. *Estimating mean travel time forecasting error and mean travel time forecasts for future discrete and continuous time periods*

In previous studies on forecasting mean travel times for discrete future time intervals (Park and Rilett, 1998, 1999; Park et al., 1999), it was found that the forecasting errors are a function of the

Table 5
Individual variance assumed for individual route travel time forecasting models

Link	Individual variance (s^2)	
	$V/C = 0$–1.5	$V/C > 1.5$
2	159	1208
3	14	140
4	34	98
5	25	264

Table 6
Mean travel time forecasting error (%)

V/C ratio	Forecasting range (min)					
	0–5	5–10	10–15	15–20	20–25	>25
0.00–1.00	4	5	7	9	11	12
1.00–1.50	7	9	12	13	15	18
>1.5	12	15	18	21	23	25

Source: Park and Rilett (1998).

level of congestion and the forecasting range. Table 6 shows the absolute percent error of travel time mean forecast found using spectral basis neural network models.

In this study, rather than using mean travel time forecasting models to forecast the mean travel time for each discrete 5-min time interval, the forecasting errors shown in Table 6 were used and the mean travel time forecasting errors were assumed to be uniformly distributed. That is, the forecast mean link travel times were estimated by Eq. (39), where the error is chosen by randomly selecting one value from the uniform distribution with extreme values shown in Table 6.

$$\hat{\mu}_{\hat{x}_a}(t) = \mu_{x_a}(t) \pm 2\text{Error} \cdot \mu_{x_a}(t) \tag{39}$$

Note that in Eq. (39), the mean travel time forecasts and forecasting error are $U(\mu_{x_a}(t), \frac{1}{12}(4\text{Error} \cdot \mu_{x_a}(t))^2)$ and $U(0, \frac{1}{12}(4\text{Error} \cdot \mu_{x_a}(t))^2)$, respectively.

7.2.3. Probes for model validation

A total of 17,824 AVI vehicles from 26 selected days between May and August 1997, were used to test the first and second model approximations. Due to the low number of AVI vehicles travelling from link 1 to link 5 and from link 2 to link 5, this study tested the route comprising of links 3, 4, and 5 which is 11.5 km of length. The mean travel times forecasts at each time of day was estimated using the three-point polynomial approximation from mean link travel time forecast for discrete time intervals. The individual route travel time forecasting model was iteratively applied until the destination node was reached.

7.2.4. Performance measurements

In order to measure the accuracy of the proposed approach, three different measures of effectiveness are used: the root mean square error (RMSE), the mean absolute percent error (MAPE), and the correlation coefficients, which are shown in Eqs. (40)–(42), respectively.

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\frac{|\hat{y}_5^i - y_5^i|}{y_5^i} \times 100 \tag{40}$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\hat{y}_5^i - y_5^i)^2} \tag{41}$$

$$\rho = \frac{\sum_{i=1}^{n}(\hat{y}_5^i - E(\hat{y}_5^i))(y_5^i - E(y_5^i))}{n\hat{\sigma}\sigma} \tag{42}$$

where y_5^i is the observed individual route travel time up to the downstream node of link 5; $\hat{y}_5^i$ is the forecast individual route travel time up to the downstream node of link 5; n is the number of test probes (i.e. 17,824).

Note that correlation coefficients (ρ) are used to identify the relationship between the observed and forecast values. That is, the closer the value of ρ to 1, the better the performance of the forecasting model.

In contrast to the estimated mean route travel time forecasts which can be compared with the observed route travel times, the variance of the individual route travel time forecasts do not have a corresponding observed value. In this sense, this study evaluates the accuracy of the proposed approximation models for the variance of the individual route travel time forecasts by examining the probability that the observed individual route travel time belongs to the prediction interval (PI) of the individual route travel time forecasts. Similarly, the accuracy of Fu and Rilett's (1998) models for approximating the variance of the individual route travel time is determined by examining the probability that the observed individual route travel time belongs to the confidence interval (CI) of the individual route travel time estimates.

The confidence interval (CI) and prediction interval (PI), shown in Eqs. (43) and (44), respectively, were used to evaluate the accuracy of the approximation models for the variance of the individual route travel time estimates and forecasts. All tests were conducted at a 95% level of significance.

$$\mathrm{CI} = E(y_5) \pm 1.96\sqrt{\mathrm{Var}(y_5)} \tag{43}$$

$$\mathrm{PI} = \widehat{E}(y_5) \pm 1.96\sqrt{\widehat{\mathrm{Var}}(y_5)} \tag{44}$$

7.3. Analysis of results

7.3.1. Mean of individual route travel time estimates and forecasts

Table 7 shows the accuracy of the first and second order approximation models for the mean of the individual travel time estimation (i.e. previous time of day) and forecasting problems. It was found that the MAPE, RMSE, and ρ of the first order models for estimation problem were 3.928%, 22.810 s, and 0.99, respectively, while those for forecasting problem were 7.619%, 77.460 s, and 0.96, respectively. It may be seen that, as expected, the accuracy of the travel time forecasting models was lower than the accuracy of the travel time estimation models. Figs. 6 and 7 show the relationship between the observed individual route travel times and the (i) estimated

Table 7
Accuracy of the mean individual route travel time estimates and forecasts

Case	Approximation model	MAPE (%)	RMSE (s)	ρ
Individual route travel time estimates (previous time period)	First	3.928	22.810	0.993
	Second	3.927	22.804	0.993
Individual route travel time forecasts (future time period)	First	7.619	77.460	0.959
	Second	7.618	77.404	0.959

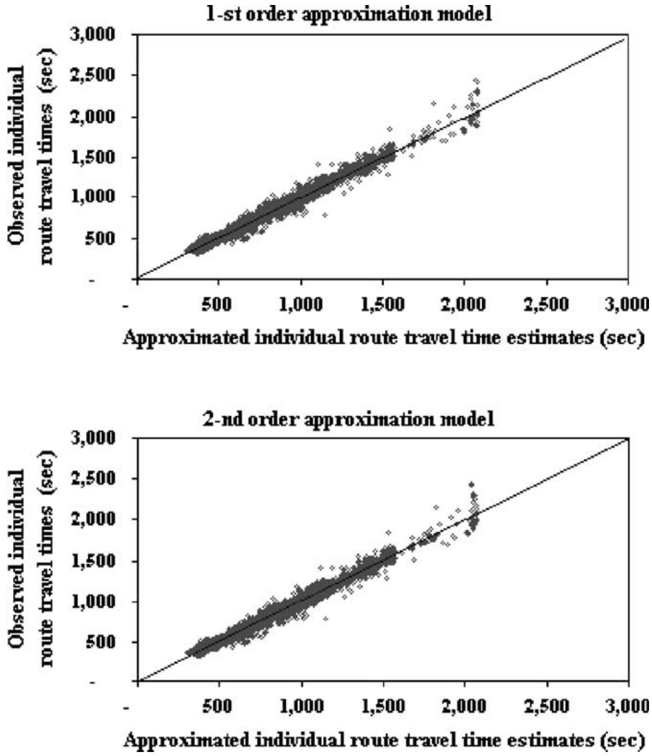

Fig. 6. Observed vs. estimated individual route travel times.

individual route travel times, and (ii) forecast individual route travel times, respectively, for both the first and second order models.

It is hypothesized that the lower accuracy of the forecasting models is mainly due to the mean travel time forecasting errors for each discrete future time periods. Consider link 4 on the test bed freeway corridor. If the travel speed on link 4 at a certain time of day is 60 km/h, it takes 4.6 min to travel the link. If mean travel time forecasts for the next 10–15 min ahead is required, the average travel time forecasting error would be 12% which corresponds to 0.552 min (about 33 s) of forecasting error. However, note that as shown in Table 5 the individual variance assumed for link 4 under V/C ratio of 0.00–1.00 is only about 34 s^2 (e.g. the standard deviation is approximately 6 s). This implies that the overall uncertainty for travel time forecasts results mainly from the mean travel time forecasting error rather than the individual variance.

It was also found that, as was hypothesized by Fu and Rilett (1998), the difference in accuracy between the first and second order models for both route travel time estimation and forecasting problems is fairly small. It is hypothesized that the rather small difference between the two models results from the fact that the second derivative of the forecast mean link travel time around the arrival time (i.e. the third term of Eq. (28)) is close to zero. Note that ideally there should not be an error in the mean estimation problem of the individual route travel times because the mean link travel time estimation errors were assumed to be zero. However, the result shows this is not the case for this test bed. It is hypothesized that the error in the mean estimation problem of the

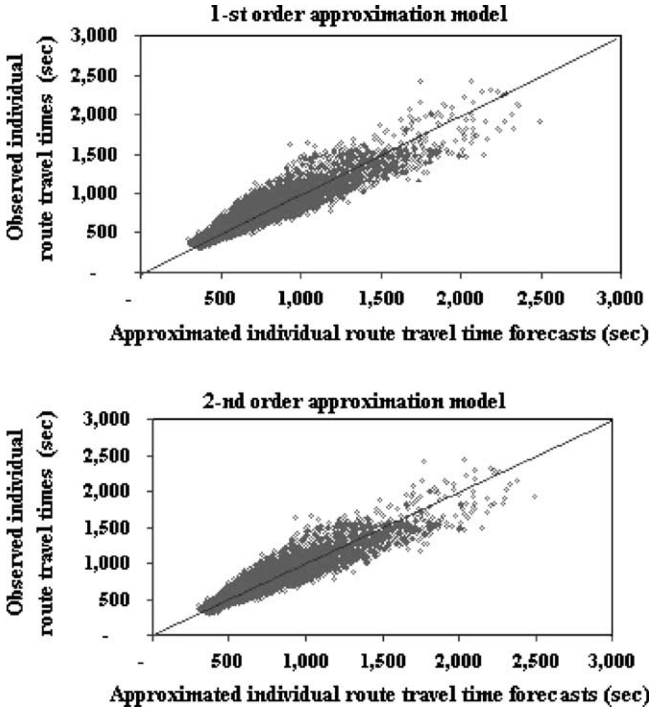

Fig. 7. Observed vs. forecast individual route travel times.

individual route travel times results from two sources. The first are errors associated with using the three-point polynomial approximation procedure to obtain mean link travel time for a given time of day. The second are errors associated with using the approximation models proposed in this study.

7.3.2. Variance of individual route travel time estimates and forecasts

Table 8 shows the accuracy of the estimated variance of individual route travel time estimates (i.e. Fu and Rilett's, 1998 approach) and that of the individual route travel time forecasts (i.e. the approach proposed in this paper). It may be seen that there is a 33% probability that the observed individual route travel times does not belongs to the CI while there is a 16% probability that they

Table 8
Individual route travel time estimates/forecasts and CI/PI

Case	Approximation model	Vehicles out of interval	
		Number	(%)
CI (previous time of day)	First order	5917	33.20
	Second order	5909	33.15
PI (future time of day)	First order	2859	16.04
	Second order	2855	16.02

are not covered in the PI. Given that 95% level of significance was used, these results appears to be unacceptable.

It is hypothesized that the error of the variance approximation model of the individual route travel time estimates is mainly attributed to (i) the errors of the mean approximation model of the individual route travel time estimates, (ii) the errors associated the assumption for the individual variance and its approximation, and (iii) three-point approximation procedure. Note that given these reasons, even if an acceptable error (e.g. $\leqslant 5\%$) was obtained on all links along a given route, the error for the route may be considerably larger. A similar hypothesis may be applied to the case of the variance approximation model of the individual route travel time forecasts.

Note that the error of the variance approximation model of the individual route travel time forecasts was smaller as compared with that of the individual route travel time estimates. This finding may be attributed to the fact that, as discussed earlier, the error of the individual route travel time forecasts due to the IV is overshadowed by the mean travel time forecasting error. Note that the mean travel time forecasting errors were assumed to be uniformly distributed in this study. If an other type of distribution is assumed, both the PI and its associated error will be different.

Not surprisingly, there is very little difference between the first and second order approximation models for both the travel time estimation and the forecasting cases. In this sense, it is considered that from a practical aspective, the first order approximation model is appropriate.

Figs. 8 and 9 show the CI for the individual route travel time estimates and PI for the individual route travel time forecasts of a randomly selected day. Fig. 8 illustrates the CI of individual travel time estimates with respect to the time of day while Fig. 9 shows the boundary of the PI of individual of travel time forecasts. Note that in figure, due to the random forecasting errors assumed for each time of day, the lower and upper bounds of PI range with respect to time of day could not be connected. It may be seen that the CI in Fig. 8 belongs to the PI in Fig. 9.

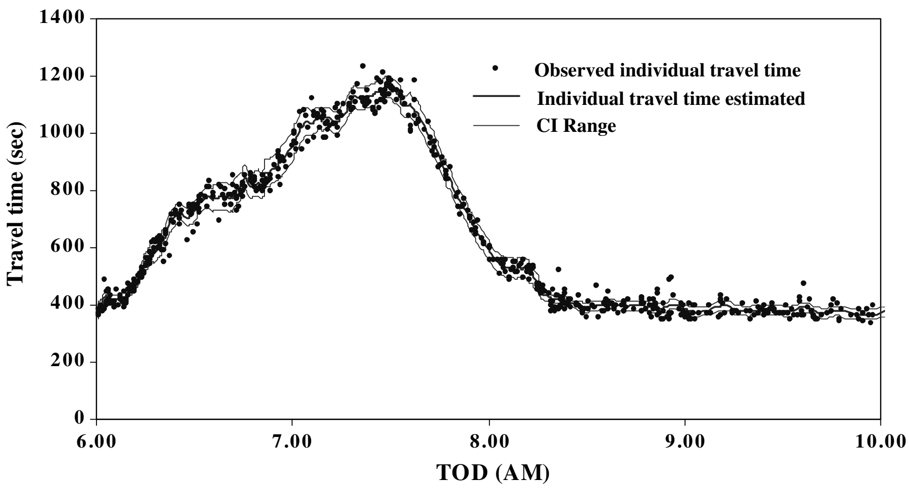

Fig. 8. Sample confidence interval vs. time of day (95% level of significance).

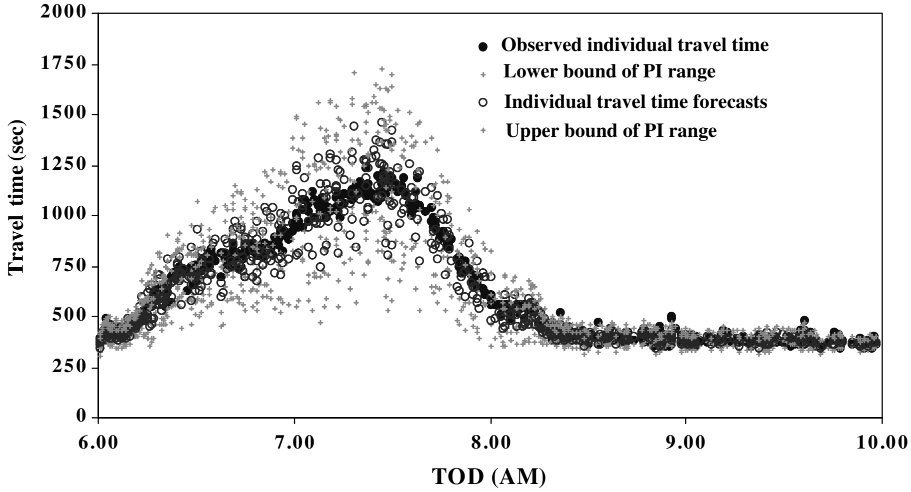

Fig. 9. Sample prediction interval vs. time of day (95% level of significance).

8. Concluding remarks

Given the limitation of the existing algorithms to the DSSPP, this paper firstly decomposed the variance (or uncertainty) of the individual travel time forecasts and proposed mathematical models for estimating the mean and variance of the individual route travel time forecasts for future time period in the context of ATIS. The proposed models were then implemented and the results were evaluated using the travel time data from Houston, Texas.

From the decomposition of travel time uncertainty, it was observed that the uncertainty of individual travel time for each previous time of day comes solely from individual variance. However, uncertainty of the individual travel time forecast consists of two components: the mean travel time forecasting error and the individual vehicles' variance. The results obtained from the first or second order approximation for the variance and mean of the individual travel time forecast showed that the difference was fairly small. Accordingly, the first order approximation model is considered to be acceptable for the US-290 test bed. Another important finding of this study is that the proposed approaches can be readily implemented because the two components of the travel time uncertainty are approximately independent and the individual variance in the future time periods can be approximated by the observations in previous time periods. Lastly, the proposed models can be considered a generalization of Fu and Rilett's (1998) model because they can be applied not only to the previous time periods (i.e. with one component of travel time uncertainty) but also to the future time periods (i.e. with two components of travel time uncertainty).

It should be noted here that, to test the proposed model, the paper assumed uniform distribution for the mean travel time forecasting errors. Additional work should be done on the exact nature and characteristics of mean travel time forecasting errors. The model performances therefore should be evaluated and concluded in the actual forecasting environment.

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