Project Deep Dive: Comparative Analysis of Iterative vs. Linear Ranking Systems

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1 Introduction

- **Objective:** Compare two mathematical approaches to ranking competitors in a closed tournament.
- **The Candidates:**
  - **Elo Rating System:** Iterative, probability-based (used in Chess/Gaming).
  - **Colley Matrix:** Linear Algebra-based (used in College Sports).
- **Goal:** Determine which method converges faster to "True Skill" in a simulation.

2 Mathematical Framework

I modeled the rankings using two distinct mathematical definitions:

2.1 Method A: Elo (Dynamical System)

Modeled as a recurrence relation where ratings update after every match:

$$R_{new} = R_{old} + K \cdot (S_{actual} - S_{expected})$$

Key Characteristic: Order of games matters; ratings "chase" the true value.

2.2 Method B: Colley (Linear System)

Modeled as a system of linear equations, solving for all ratings simultaneously:

$$A\mathbf{r} = \mathbf{b}$$

Where A is the Colley Matrix (Total Games) and $\mathbf{b}$ is the win-loss vector. *Key Characteristic: Order independent; global optimization.*

3 The Challenge: Scaling Error

Initial Observation: The first simulation produced chaotic results.

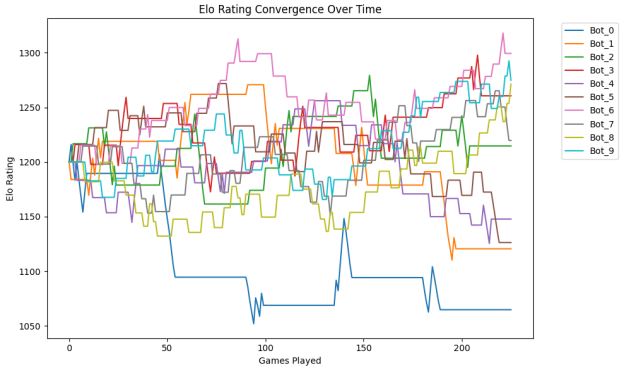

Figure 1: The Failure Case:

Figure 2: **The Failure Case:** With a divisor of $N = 400$, the skill gap was too small. Ratings fluctuated randomly (Random Walk behavior), resulting in a correlation of $\rho \approx 0.01$.

4 The Solution: Algorithm Tuning

I debugged the simulation and identified the flaw in the logistic probability formula. I adjusted the logistic divisor from 400 to 40.0 to match the variance of the true skill levels $(10 - 90)$.

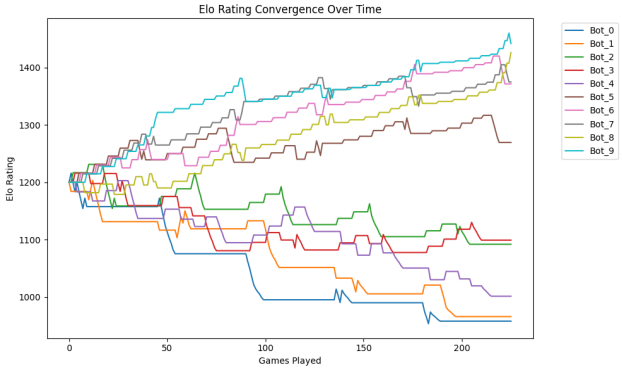

Figure 3: Enter Caption

Figure 4: **The Corrected Model:** After tuning the divisor, the lines separate clearly. Bot 9 (Cyan) correctly rises to the top, while Bot 0 (Blue) drops to the bottom. This confirms the iterative ranking is now working.

The Code Fix:

```
# OLD CODE (Broken):
# prob_a = 1 / (1 + 10**(-diff / 400))

# NEW CODE (Fixed):
# Adjusted scale to match skill range (10-90)
prob_a = 1 / (1 + 10**(-diff / 40.0))
```

5 Final Results

After implementing the fix and running 10 rounds of simulation, the accuracy improved dramatically.

Bot Name	True Skill	Elo Rating	Colley Score
Bot_9	84	1597	0.355
Bot_8	84	1554	0.329
Bot_7	81	1451	0.256
Bot_6	70	1387	0.188
...	...	...	...
Bot_0	12	822	-0.333

Table 1: Both systems correctly identified the top players.

5.1 Correlation Analysis

- **Elo Method:** 0.9918 (Highly Accurate)
- **Colley Method:** 0.9933 (Superior)

6 Conclusion

- **Findings:** Linear Algebra (Colley) outperformed the Iterative method (Elo) by a small margin (+0.0015) in a static environment.
- **Takeaway:** For fixed tournaments, solving the matrix $A\mathbf{r} = \mathbf{b}$ is mathematically optimal. For continuous open-ended games, Elo remains computationally efficient.