

Comparative Analysis of Iterative vs. Linear Ranking Systems

MD Masudur Rahman (201961240)

January 14, 2026

Abstract

This project investigates the mathematical structure of competitive ranking systems, specifically comparing iterative algorithms against linear algebraic methods. By simulating a closed tournament, I will analyze the stability and accuracy of the *Elo rating system* (a recurrence relation) versus the *Colley Matrix method* (a system of linear equations). The goal is to determine which approach converges faster to a player's "true" skill level and how each system reacts to unexpected outcomes (upsets).

Keywords: ranking systems, linear algebra, Elo rating, Colley matrix, simulation

1 Introduction

In competitive gaming and sports, accurately ranking participants is a fundamental mathematical challenge. While many different systems exist, they generally fall into two categories: those that update ratings step-by-step after every game, and those that solve for all ratings simultaneously.

In this project, I will compare two specific examples of these categories:

- **The Elo System:** An iterative method used in chess and many video games.
- **The Colley Matrix:** A method based on linear algebra often used in collegiate sports.

Research Question: How does the iterative Elo rating system compare to the linear algebra-based Colley Matrix in a simulated tournament? Specifically, which one converges faster and which one handles "upsets" better?

2 Methods

1. Mathematical Formulation:

- **Elo (Iterative):** I will model this as a discrete dynamical system governed by the recurrence relation:

$$R_{new} = R_{old} + K(S_{actual} - S_{expected})$$

I will investigate how the K -factor influences the stability of the ratings.

- **Colley (Linear System):** I will formulate the ranking problem as a system of linear equations $A\mathbf{r} = \mathbf{b}$. I plan to prove that the matrix A is diagonally dominant and therefore always invertible, guaranteeing a unique solution.
2. **Computational Simulation:** Using Python (specifically the NumPy library), I will:
 - Create a synthetic tournament with 10 “bots,” each assigned a hidden skill level.
 - Simulate a round-robin format where every bot plays every other bot.
 - Implement both algorithms to calculate rankings based on the match results.
 3. **Comparative Analysis:** I will calculate the error between the rankings generated by the algorithms and the bots’ hidden “true” skill levels. I will also analyze specific test cases where a low-ranked bot defeats a high-ranked bot to observe how the systems react.

3 Tools and Version Control

The project will be implemented in **Python**, utilizing standard scientific computing libraries. All progress, including code development and report drafting, will be documented via version control.

GitHub Repository: [<https://github.com/mmasudurr/math3030-ranking-analysis>]

4 Conclusion

This project will demonstrate the trade-offs between recursive numerical methods and exact linear algebraic solutions in the context of competitive rankings. The findings will provide insight into the mathematical fairness of matchmaking systems used in modern gaming.