Linear Waves

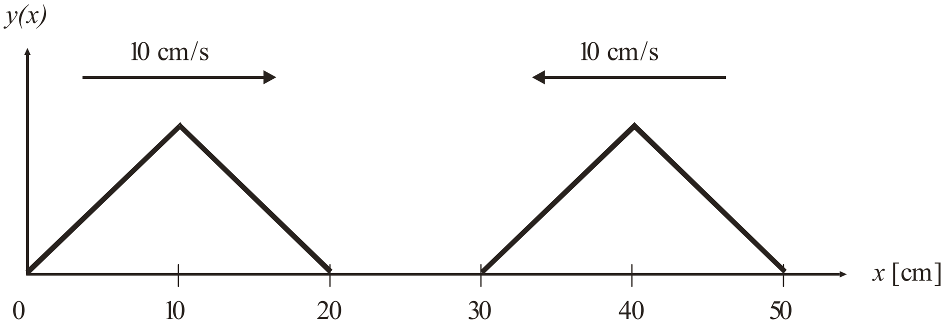
Basic Problems

1. The shape of a wave travelling at 20 cm/s can be described by the function *f(x)* with the following piecewise definition: *f(x)* = *x* for *x* between 0 cm and 10 cm, *f(x)* = 20 cm - *x* for *x* between 10 cm and 20 cm; and *f(x)* = 0 everywhere else.

   Draw the displacement versus position diagram for the fixed time $t = 2$ s and the displacement versus time diagram for the fixed position $x = 30$ cm (*t* between 0 s and 5 s).

2. A single wave crest can be described by a Gaussian function. In its simplest form this is described by $f(x) = e^{-x^2}$. Find a physically correct Gaussian shape function and interpret the parameters. Derive the corresponding wave function and plot this with the calculator or a function plotter for several different times.

3. Two triangular wave crests on a rope move at 10 cm/s in opposite directions (see figure). Draw the shape of the rope after 1 s.



4. A wave on an 11 m long rope is produced by harmonically moving one end up and down. After 3.4 s the first wave crest arrives at the other end. In the same time the first end has moved up and down twelve times. Calculate the wave's frequency, wavelength and speed.

5. A harmonic wave with wavelength 3.5 cm travels at 8.2 cm/s. Calculate its frequency.

6. The radio station DRS 3 broadcasts on 103.4 MHz. Calculate the radio wave's wavelength.

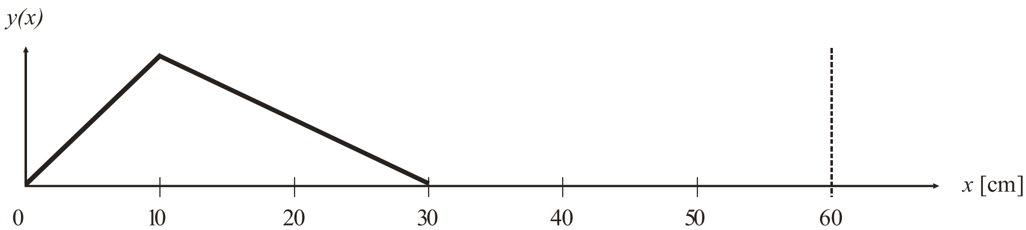
7. A guitar's A string (length 650 mm) is tuned to a fundamental frequency of 440 Hz. Calculate the speed of waves propagating on the string. What can you hear when you touch the string at one third of its length?

8. Calculate the frequencies of the fundamental and the first and second overtone of a pan flute 16.4 cm long.

Additional Problems

9. A wave on a rope can be described by the function $y(x, t) = 3.5\text{ cm} \cdot \sin(3\text{ s}^{-1} \cdot t + 1.5\text{ m}^{-1} \cdot x + 0.5)$. Determine amplitude, frequency, wavelength and speed of the wave. At what time does the point at $x = 1$ m on the rope cross the equilibrium position for the first time?

10. The figure below shows a wave propagating at 20 cm/s to the right at time $t = 0$ s. It is reflected on a loose end at $x = 60$ cm.
    a) Draw the shape of the wave after 2 s.
    b) Draw the displacement vs time diagram for the fixed position $x = 50$ cm (*t* between 0 s and 4 s).



Solutions: 4. 3.5 Hz, 0.92 m, 3.2 m/s; 5. 2.3 Hz; 6. 2.9 m; 7. 572 m/s, 1.32 kHz (2nd overtone); 8. 524 Hz, 1.57 kHz, 2.62 kHz; 9. 3.5 cm, 0.48 Hz, 4.2 m, 2.0 m/s, 0.38 s

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9. $A = 3.5\ cm$, $\omega = 3\ s^{-1}$, $k = 1.5\ m^{-1}$, $\varphi_0 = 0.5$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{3\ s^{-1}}{2\pi} = 0.48\ Hz$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.5\ m^{-1}} = 4{,}2\ m$$

$$v = \lambda \cdot f = 2{,}0\ m/s$$

$$y(x_0, t) = A \cdot \sin(\omega \cdot t + k \cdot x_0 + \varphi_0) = 0$$

$$\Rightarrow \omega \cdot t + k \cdot x_0 + \varphi_0 = m \cdot \pi \quad (m \in \mathbb{Z})$$

$$t = \frac{m \cdot \pi - \varphi_0 - k \cdot x_0}{\omega} = \frac{m \cdot \pi - 0.5 - 1.5\ m^{-1} \cdot 1m}{3 s^{-1}}$$

$$= \frac{m \cdot \pi - 2}{3 s^{-1}} \rightarrow \text{first positive value for } m = 1$$

$$t = \frac{\pi - 2}{3 s^{-1}} = 0{,}38\ s$$

10.a)