POWER IN AN AC CIRCUIT

For a dc circuit the power is related to voltage and current as follows:

$$P = V \cdot I$$

In an ac circuit voltage and current depend on time:

$$V \to v(t) = v_o \cdot \sin(\omega \cdot t)$$
$$I \to i(t) = i_o \cdot \sin(\omega \cdot t - \Delta\varphi)$$

The instantaneous value of the power at time t is therefore:

$$p(t) = v(t) \cdot i(t) = v_o \cdot \sin(\omega \cdot t) \cdot i_o \cdot \sin(\omega \cdot t - \Delta\varphi) \; .$$
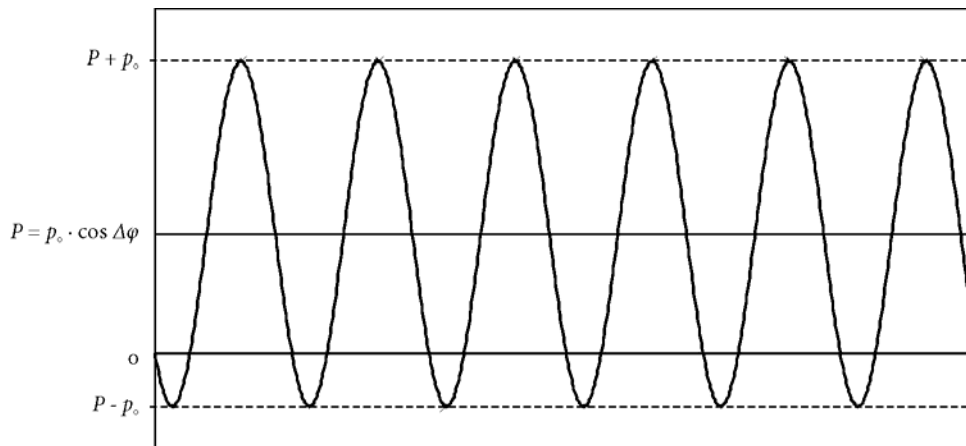
Using the trigonometric relation

$$\sin\alpha \cdot \sin\beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta)) \; ,$$

and the substitutions $\alpha = \omega \cdot t$ and $\beta = \omega \cdot t - \Delta\varphi$ we get

$$p(t) = v_o \cdot i_o \cdot \frac{1}{2} \cdot (\cos(\Delta\varphi) - \cos(2 \cdot \omega \cdot t - \Delta\varphi)) = p_o \cdot (\cos\Delta\varphi - \cos(2 \cdot \omega \cdot t - \Delta\varphi))$$

where $p_o = \frac{1}{2} \cdot v_o \cdot i_o$.

We thus find that the power oscillates with amplitude p_0 and twice the frequency of the applied voltage around the average value $P = \hat{p} \cdot \cos\Delta\varphi$ (see figure).

In most practical applications only the average value of the power is of any importance. From the figure it is obvious that the average power is

$$P = \overline{p(t)} = p_o \cdot \cos\Delta\varphi = \frac{1}{2} \cdot v_o \cdot i_o \cdot \cos\Delta\varphi$$

Using the *root mean squares (rms-values)* or *effective values* $V_{rms} = \dfrac{v_o}{\sqrt{2}}$ and $I_{rms} = \dfrac{i_o}{\sqrt{2}}$ for the voltage and the current we end up with an expression very similar to the one describing the power in a dc circuit:

In an ac circuit with effective voltage V_{rms} and effective current I_{rms}, the *effective power* is

$$P = V_{rms} \cdot I_{rms} \cdot \cos\Delta\varphi \; ,$$

where the product $S = V_{rms} \cdot I_{rms}$ is the *apparent power* and $\cos\Delta\varphi$ the *power factor*.

In the data sheet of ac appliances we always find the rms values for voltage and current and the effective power or the power factor.