Relativity

Basic Problems

1. Calculate the Lorentz factor for an airplane flying at a speed of 2'000 km/h and a particle moving at 99.9 % of the speed of light. What is the maximum speed at which the Lorentz factor is within 1 ppm of 1?
2. Your friend goes on a space trip and moves at 92 % of the speed of light with respect to the earth for five years of her proper time. How much older will you have got when you meet again?
3. During the last billion years the sun has been emitting radiation at a more or less constant rate of $3.8 \cdot 10^{26}$ W. Calculate the percentage decrease of its mass.
4. Calculate the mass defect in the fusion reaction $^2_1\mathrm{H} + ^2_1\mathrm{H} \longrightarrow {}^3_2\mathrm{He} + \mathrm{n}$.
5. Calculate the binding energy of a lead nucleus. Compare the result to the value read from the corresponding diagram in "Formeln und Tafeln".
6. Why is iron considered to be among the most stable elements?
7. Two particles of identical mass *m* and whose kinetic energy is twice the rest energy each collide and form a new particle. What is the new particle‘s maximum mass?
8. An electron is accelerated to 0.999 c. Calculate its kinetic energy.
9. An astronaut moves at 1.5 % of the speed of light. Calculate her rest energy, total energy and kinetic energy.
10. Calculate the speed of an α-particle with a kinetic energy of 5 MeV.
11. A particle is accelerated from 90 % to 91 % of the speed of light. Calculate the percentage change of the particle‘s momentum and energy. What is the result for an acceleration from 98 % to 99 % of the speed of light?
12. Calculate the momentum of 20 keV protons.

Supplementary Problems

13. Calculate the deviation of a clock moving at 5 m/s with respect to an observer from its own proper time. Solve the exercise using an approximation for the Lorentz factor.
14. A Klingon spaceship travels at 75 % of the speed of light with respect to the Earth. The Klingon crew measures a time interval of 37.0 h between two events on Earth. Calculate the time interval they would measure between the same two events if they travelled at 94 % of the speed of light.
15. Look up the formula for the relativistic addition of velocities (e.g. Cutnell & Johnson, section 28.7) and answer the following questions:
    a) What is the speed of an astronaut walking at half the speed of light to the front of a spaceship moving at half the speed of light with respect to an outside observer?
    b) Prove that the speed of light is the same for both the astronaut and the outside observer.
16. Show that the force acting on a particle parallel to its motion can be written as $F = \gamma^3 \cdot m \dfrac{\mathrm{d}v}{\mathrm{d}t}$.
17. The nuclear power station in Leibstadt has an average output power of 1 GW at an efficiency of about 30 %.
    a) Calculate the mass equivalent to the energy produced per day. Where does this mass “disappear”?
    b) The fuel elements become also lighter when they are contained in a gas-tight container. Give a reason for this phenomenon. Can this effect be measured?
18. A deuterium ($^2_1\mathrm{H}$) and a tritium ($^3_1\mathrm{H}$) nucleus combine in a fusion reactor to form a helium nucleus ($^4\mathrm{He}$), leaving one excess neutron. Using the binding energies or the nuclear masses, calculate the energy set free in this reaction.

Numerical Solutions: 1. $1 + 1.7 \cdot 10^{-12}$, 22, 420 km/s; 2. 13 y; 3. $6.7 \cdot 10^{-3}$ %; 4. $- 5.8 \cdot 10^{-30}$ kg; 5. - 1.6 GeV; 7. $6 \cdot m$; 18. 10.9 MeV; 9. 5.4 EJ, 5.4 EJ, 0.61 PJ (for m = 60 kg); 10. 0.052 c; 11. + 5.1 %, + 41 %; 12. 6 MeV/c; 13. $1.4 \cdot 10^{-16}$; 14. 71.7 h; 15. $0.8 \cdot c$; 17. 0.96 g; 18. 18 MeV