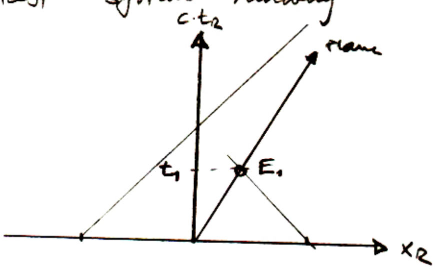
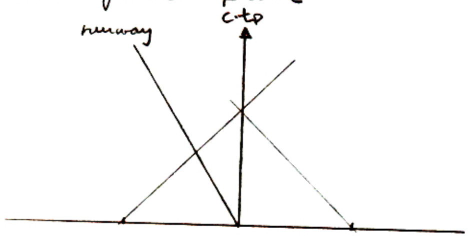
Relativistic Kinematics

1. Rest system runway

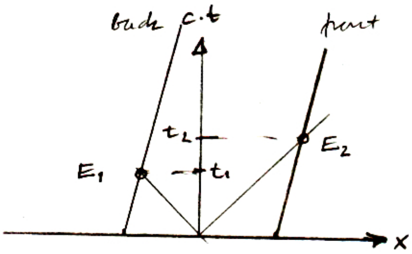


Rest system plane



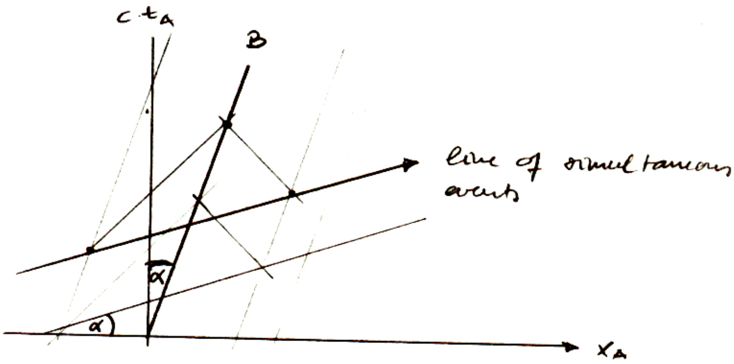
From the point of view of an observer in the reference frame of the runway the front flash is earlier for the plane than the back flash.
For an observer in the plane they are simultaneous.

2.



In the reference frame of the observer next to the train the light reaches the back end of the train **before** it hits the front.
(For someone in the train the two events are simultaneous, though)

3.



All lines of simultaneous events are parallel.
By construction the angles α are the same.

4. $\gamma = \frac{1}{\sqrt{1-\beta^2}} \cong \frac{1}{1-\frac{1}{2}\beta^2} \cong 1 + \frac{1}{2}\beta^2 \quad (\text{for } \beta \ll 1)$

$\Rightarrow \gamma - 1 \cong \frac{1}{2}\beta^2 \quad (\text{for } \beta \ll 1)$

$\rightarrow$ for $\beta = \frac{(2'000/3{,}6)\ m/s}{3 \cdot 10^8\ m/s} = 1{,}85 \cdot 10^{-6}$: $\gamma \cong \underline{1 + 1{,}7 \cdot 10^{-12}}$

$\gamma = 1 + 10^{-6} \Rightarrow \beta \cong \sqrt{2 \cdot 10^{-6}} = 1{,}4 \cdot 10^{-3} \rightarrow v \cong \underline{420\ km/s}$

for $\beta = 0{,}999\ c$: $\gamma = \frac{1}{\sqrt{1-(0{,}999)^2}} = \underline{22}$

5. $\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \left(\frac{T}{t}\right)^2} = \sqrt{1 - \left(\frac{1{,}5}{44}\right)^2} = \underline{0{,}9994}$

6. $\omega = \frac{2\pi}{T}$ in rest system of transmitter

$$\omega' = \frac{2\pi}{T'} = \frac{2\pi}{\gamma \cdot T} \quad \text{in rest system of observer flying by}$$

$$= \frac{\omega}{\gamma} = \omega \cdot \sqrt{1-\beta^2} = 0{,}25 \text{ rad/s} \cdot \sqrt{1-(0{,}8)^2} = \underline{0{,}15 \text{ rad/s}}$$

7. $$l = \frac{\lambda}{\gamma} = \lambda \cdot \sqrt{1-\beta^2} = 9{,}5 \text{ km} \cdot \sqrt{1-\left(\frac{1{,}3}{3{,}0}\right)^2} = \underline{8{,}6 \text{ km}}$$

8. rest system Earth: life time of muons is stretched by Lorentz factor γ

rest system muon: distance is shortened by Lorentz factor γ

9. $$\beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\left(\frac{l}{\lambda}\right)^2} = \sqrt{1-\left(\frac{99}{100}\right)^2} = \underline{0{,}14}$$

Relativistic kinematics

10. see exercise 4: $\gamma \cong 1 + \frac{1}{2}\beta^2$ (for $\beta \ll 1$)

$$\rightarrow \gamma \cong 1 + \frac{1}{2}\left(\frac{5}{3\cdot 10^8}\right)^2 = 1 + 1{,}4 \cdot 10^{-16}$$

$$\rightarrow \Delta t = t - \tau = (\gamma - 1)\cdot\tau = \underline{1.4\ 10^{-16}\cdot\tau}$$

11. $t = \gamma \cdot \tau$

$$t' = \gamma' \cdot \tau = \frac{\gamma'}{\gamma}\cdot t = \sqrt{\frac{1-\beta^2}{1-\beta'^2}}\cdot t = \sqrt{\frac{1-(0{,}75)^2}{1-(0{,}94)^2}}\cdot 37.0\,h$$

$$= \underline{71{,}7\,h}$$

12. From the driver's point of view, the doors close and open at different times:
    1. front of car reaches front door → front door closes (and reopens)
    2. back of car reaches back door → back door closes (and reopens)

13. $\tan\alpha = \frac{\Delta y}{\Delta x}$ in rest system

$\tan\alpha' = \frac{\Delta y}{\Delta x'}$ (no contraction perpendicular to motion)

$$= \frac{\Delta y}{\Delta x/\gamma} = \gamma \cdot \tan\alpha = \frac{1}{\sqrt{1-(0{,}73)^2}}\cdot \tan 30° = 0{,}84$$

$$\Rightarrow \alpha' = \underline{40°}$$

14. a) $u' = c/2, \quad v = c/2$

$$\rightarrow u = \frac{c/2 + c/2}{1 + \frac{c/2 \cdot c/2}{c^2}} = \frac{c}{1+\frac{1}{4}} = \underline{\frac{4}{5}\cdot c}$$

b) $u' = c$

$$\rightarrow u = \frac{c + v}{1 + \frac{c v}{c^2}} = c \cdot \frac{1 + v/c}{1 + v/c} = c \quad \square$$