Ableitung 2 : Lösung A 1 – 6

1a) $f'(x) = 10x^9$ b) $f'(x) = \frac{2}{3}x^{-1/3}$ c) $f'(x) = -\frac{1}{x^2}$

d) $f'(x) = \frac{2}{3}x^{-1/3}$ e) $f'(x) = -10x^{-11}$ f) $f'(x) = -\frac{3}{2}x^{-5/2}$

g) $f'(x) = -\frac{6}{5}x^{-2/5}$

2a) $\frac{d}{dx}x^{2/5} = \frac{2}{5}x^{-3/5}$ b) $\frac{d}{dx}x^{5/3} = \frac{5}{3}x^{2/3}$ c) $\frac{d}{dx}x^{-0,3} = -0{,}3\,x^{-1,3}$

d) $\frac{d}{dx}\frac{3k}{x^2} = -\frac{6k}{x^3}$ e) $\frac{d}{dt}t^{-1/2} = -\frac{1}{2}t^{-3/2}$ f) $\frac{d}{dt}t^{-1/4} = -\frac{1}{4}t^{-5/4}$

g) $\frac{d}{dt}t^6 = 6t^5$ h) $\frac{d}{dk}mk^{-2} = -2mk^{-3}$

3) a) $y' = 2x - 3$ $y'(1) = -1 = m$

b) $y' = 3x^2 + 2$ $y'(0) = 2 = m$

c) $y' = 6x^2 + 2x$ $y'(-1) = 4 = m$

d) $y' = -\frac{1}{2}x^{-3/2} = -\frac{1}{2}\frac{1}{\sqrt{x^3}}$ $y'\left(-\frac{1}{4\sqrt{2}}\right) \in \emptyset$

weil $\sqrt{}$ von neg. Zahlen nicht existiert.

4) $f(x) = \dfrac{x+1}{x-1}$ in $x_0 = 2$

$$\lim_{h\to 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} = \lim_{h\to 0} \frac{1}{h} \frac{(x+h+1)(x-1) - (x+1)(x+h-1)}{(x+h-1)(x-1)} =$$

$$= \lim_{h\to 0} \frac{1}{h} \frac{x^2+hx+x-x-h-1-x^2-xh+x-x-h+1}{(x+h-1)(x-1)} = \lim_{h\to 0} \frac{1}{h} \frac{-2h}{(x+h-1)(x-1)} =$$

$$= \frac{-2}{(x-1)^2} \qquad f'(2) = \frac{-2}{(2-1)^2} = -2$$

5) $f(x) = \sqrt{x}$

$$\lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} = \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})}{h} \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h\to 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

6) $f(x) = \sqrt[3]{x}$ ⚠ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$\lim_{h\to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h} = \lim_{h\to 0} \frac{(x+h)^{1/3} - x^{1/3}}{h} \cdot \frac{(x+h)^{2/3} + (x+h)^{1/3} \cdot x^{1/3} + x^{2/3}}{(x+h)^{2/3} + (x+h)^{1/3} x^{1/3} + x^{2/3}} =$$

$$= \lim_{h\to 0} \frac{(x+h)^{3/3} - x^{3/3}}{h\left((x+h)^{2/3} + (x+h)^{1/3} x^{1/3} + x^{2/3}\right)} = \lim_{h\to 0} \frac{\cancel{x} + \overset{1}{\cancel{h}} - \cancel{x}}{\cancel{h}\left((x+h)^{2/3} + (x+h)^{1/3} x^{1/3} + x^{2/3}\right)}$$

$$= \frac{1}{x^{2/3} + x^{1/3} x^{1/3} + x^{2/3}} = \frac{1}{3x^{2/3}} = \frac{x^{-2/3}}{3}$$

$f(0) \in \emptyset$ da $\lim_{x\to 0} \dfrac{1}{x^{2/3}} = \infty$