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Boyle-Mariotte

We investigated the relation between pressure and volume of an ideal gas under an isothermal process and measured the universal gas constant *R*.

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Boyle-Mariotte

The law of Robert Boyle and Edmé Mariotte describes the behaviour of an ideal gas undergoing an isothermal change of volume. It provides us with a simple way of determining the universal gas constant.

Goal

Discover the advantages of different graphical representations and regressions.

Practise estimating and calculating errors

Devices

- Glass cylinder with moveable piston
- Thermometer
- Thread and meter stick

Experiment

a Determine the air temperature in the laboratory.

b Measure the circumference of the glass cylinder with the thread. Estimate the thickness of the glass.

c Place the piston at the outmost position, ventilate the cylinder and close the valve.

d Read the air pressure in the cylinder from the pressure gauge for at least ten different positions of the piston.

e Place the piston near the closed end of the cylinder, ventilate the cylinder and close the valve.

f Read the air pressure for at least five positions of the piston.

Analysis:

1. Calculate the cross sectional area of the air column in the cylinder (with errors).
2. Enter the two series of measurements in tables, preferably in a spreadsheet. Add a column for the volume of the air in the cylinder.
3. Draw a diagram (pressure vs. volume) for each of the two series. Fit a power function to the data points. Is the regression in accordance with theory?
4. Draw a diagram (pressure vs. volume^{-1}) for each of the two series. Use this to prove that the product $p \cdot V$ is constant for each series and determine the value of this constant for both series. Discuss the quality of your measurements.
5. Using the results of 4, calculate the value of the universal gas constant R.
6. Discuss the influence of possible error sources in the calculation of R. Calculate the error of R, considering only the most important errors.

Requirements

If you write a report on this experiment, hand it in with the complete interpretation by **Tuesday, 15 March 11**.

Otherwise, work at least on steps 2 and 3 and hand in your interpretation by **Tuesday, 15 March 11**.

Introduction

Contrary to solid bodies and liquids, which are more or less incompressible, the volume of a gas can change considerably under even slight pressure changes. After systematic investigation of this phenomenon, British scientist Robert Boyle (1627 – 1691) found in 1660 that the product of pressure an volume is a constant as long as the temperature is not varied. Independently, French physicist Edmé Mariotte (1620 – 1684) arrived at the same result in 1676. He furthermore made important contributions to the interpretation of the effect [1].

Among other laws describing the behaviour of a gas under temperature changes (Amontons, Gay-Lussac, …), the law of Boyle and Mariotte finally lead to the ideal gas law. Even if it is only a model for a hypothetical, idealised gas, it is adequately precise for most real gases (such as oxygen, hydrogen or air) at room temperature and allows, for example, the prediction of the pressure rise in a heated gas bottle.

The ideal gas law was crucial for the theoretical investigation of heat engines at the beginning of industrialisation. French physicist and engineer Sadi Carnot (1796 – 1832) proved that the maximum efficiency of any cyclic engine transforming heat from a hot reservoir into work and transferring the waste heat into a cold reservoir is limited by the temperatures of the two reservoirs.

Theory

A gas is said to be *ideal* if the following conditions hold:

- The gas particles are pointlike, i.e. their volume is negligibly small.
- There are no forces acting between the particles apart from contact forces.
- Collisions between particles are elastic.
- The particles move in every direction with the same probability.

No real gas is a perfectly ideal gas, but most real gases approximately fulfil the conditions under certain conditions. A good example is air under standard conditions.

An ideal gas is characterised by the *ideal gas law*

$$pV = nRT\,, \tag{1}$$

where p is the gas pressure, V the gas volume, n the number of moles, R the universal gas constant and T the gas temperature (in Kelvin).

To calculate the universal gas constant, we have to solve equation (1) for R:

$$R = \frac{pV}{nT}\,. \tag{2}$$

Putting $pV = k$ and $n = m/M$ (m is the gas mass and M the molar mass of air) in equation (2) we find

$$R = \frac{k}{m/M \cdot T} = \frac{kM}{mT}\,. \tag{3}$$

Replacing the gas mass m by the density ρ_0 and the volume V_0 for air under standard conditions, we finally find:

$$R = \frac{k\,M}{\rho_0\,V_0\,T}\,. \tag{4}$$

All quantities in equation (4) can either be measured in our experiment or are known from the literature.

Experiment

We use a 30 cm long glass cylinder. A pressure gauge and a valve are connected to one end. A gas-tight piston closes the cylinder at the other end. It can be moved by means of a screw thread. By moving the piston only very slowly, the temperature is hold at room temperature and does not change in spite of the compression or expansion of the air. At the outside of the cylinder, there is a scale with millimetre division to measure the length of the air column in the cylinder. If the cross sectional area of the glass cylinder is known, we can calculate the enclosed air volume.

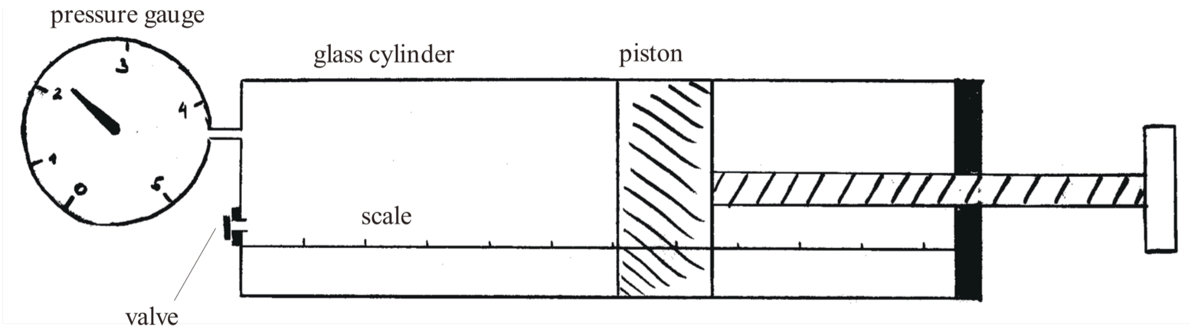

Figure 1: Experimental setup

During the experiment, the volume of the air in the cylinder is changed by moving the piston. For a given volume, we read the corresponding pressure from the pressure gauge. The valve allows for adjusting the "reference" volume at the pressure of the surrounding air.

Boyle - Mariotte

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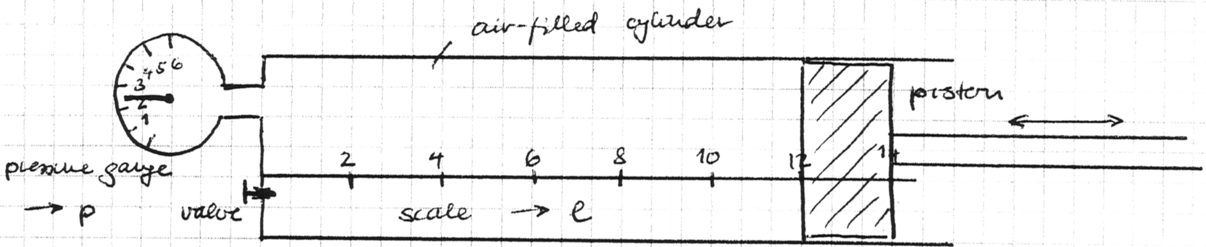
S. Bylaud, 3a

Experimental setup



The piston can be moved in the cylinder very slowly by means of a screw thread.

We read the pressure p for different positions of the piston (→ l → volume V)

Measurements

Air temperature : $\vartheta_A = (19,8 \pm 0,1)\,°C$ (measured with digital thermometer)

circumference of cylinder : $C = (16,4 \pm 0,2)$ cm (measured with thread)

thickness of glass : $t = (0,10 \pm 0,05)$ cm (estimate)

I: initial position of piston at $l = 20,0$ cm

length of air column l [cm]	20,0	19,0	18,0	17,0	16,0
gas pressure p [bar]	1,00	1,05	1,10	1,20	1,25

l [cm]	15,0	14,0	13,0	12,0	11,0
p [bar]	1,35	1,45	1,55	1,70	1,80

D (continued)

ℓ [cm]	10,0	9,0	8,0	7,0
p [bar]	2,00	2,20	2,45	2,80

ℓ [cm]	6,5	6,0	5,5	5,0
p [bar]	3,00	3,20	3,50	3,80

errors :

$\Delta \ell = 0{,}1$ cm

$\Delta p = 0{,}05$ bar

F Initial position of piston at $\ell = 5{,}0$ cm

ℓ [cm]	5,0	5,5	6,0	7,0	8,0
p [bar]	1,00	0,90	0,80	0,70	0,60

ℓ [cm]	10,0	12,0	14,0	20,0
p [bar]	0,50	0,40	0,35	0,25

errors : same as in D

Bom

Interpretation

1. Cross sectional area of air column

$$A = r^2 \cdot \pi = \left(\frac{c'}{2\pi}\right)^2 \cdot \pi = \frac{c'^2}{4\pi}$$

c' is smaller than C because thickness t has not be taken into account, yet.

$$c' = C - 2\pi \cdot t = 16{,}4 \text{ cm} - 2\pi \cdot \cancel{0{,}05}\ 0{,}10 \text{ cm} = 15{,}8 \text{ cm}$$

$$\Delta c' = \Delta C + 2\pi \cdot \Delta t = 0{,}2 \text{ cm} + 2\pi \cdot 0{,}05 \text{ cm} = 0{,}5 \text{ cm}$$

$$\rightarrow \quad c' = (15{,}8 \pm 0{,}5) \text{ cm}$$

$$A = \frac{c'^2}{4\pi} = \frac{(15{,}8 \text{ cm})^2}{4\pi} = 19{,}9 \text{ cm}^2$$

$$\Delta A = A \cdot r_A = A \cdot 2 \cdot r_{c'} = 2 \cdot A \cdot \frac{\Delta c'}{c'} = 2 \cdot 19{,}9 \text{ cm}^2 \cdot \frac{0{,}5}{15{,}8} = 1{,}3 \text{ cm}^2$$

$$\Rightarrow \quad A = \underline{(19{,}9 \pm 1{,}3) \text{ cm}^2}$$

2. Volume of air column

$$V = A \cdot l$$

$$\Delta V = V \cdot r_V = V \cdot (r_A + r_l) = V \cdot \left(\frac{\Delta A}{A} + \frac{\Delta l}{l}\right)$$

$\rightarrow$ table 1

3. power regression

$\rightarrow$ diagrams 1 and 2

$\rightarrow$ $p \propto \left(\frac{V}{V_0}\right)^r$ with $r_1 = -0{,}9625$ (series 1)

$r_2 = -0{,}9963$ (series 2)

$\Rightarrow$ near to theoretical value $r = -1$
$\left(p \propto \frac{1}{V} = V^{-1} \right)$

4. $p \cdot V$ = constant

$\longrightarrow$ diagrams 3 and 4

⚠ proportionality $\rightarrow$ straight line must pass through origin

$p = k \cdot \frac{1}{V}$ with $k_1 = 417{,}75$ bar·cm³ (series 1)
$k_2 = 105{,}12$ bar·cm³ (series 2)

straight line passes through all error bars $\rightarrow$ in accordance with hypothesis.

5. Universal gas constant

ideal gas law : $p \cdot V = n \cdot R \cdot T$

$$\Rightarrow R = \frac{p \cdot V}{n \cdot T} = \frac{k}{m/M \cdot T} = \frac{k \cdot M}{\rho_0 \cdot V_0 \cdot T}$$

$T = T_0 + \vartheta_A = 273{,}15\ K + 19{,}8\ K = (293{,}0 \pm 0{,}1)\ K$
$p_0 = (1{,}00 \pm 0{,}05)$ bar
$V_{01} = (397 \pm 27)\ cm^3$ (series 1)
$V_{02} = (99 \pm 8)\ cm^3$ (series 2)

density of air (at 20°C, 1013,25 mbar) : $\rho_0 = 1{,}205\ kg/m^3$
molar mass of air : $M = 28{,}96\ g/mol$

Compared to the error of the air volume, all other errors can be neglected. $\Rightarrow \Delta R \cong R \cdot r_R = R \cdot r_{V_0} = R \cdot \frac{\Delta V_0}{V_0}$

$$R_1 = \frac{417{,}75 \cdot 10^5\ Pa \cdot 10^{-6}\ m^3 \cdot 0{,}02896\ kg/mol}{1{,}205\ kg/m^3 \cdot 397 \cdot 10^{-6}\ m^3 \cdot 293{,}0\ K} = \underline{(8{,}6 \pm 0{,}6)\ J/(mol \cdot K)}$$

$$R_2 = \frac{105{,}12 \cdot 10^5\ Pa \cdot 10^{-6}\ m^3 \cdot 0{,}02896\ kg/mol}{1{,}205\ kg/m^3 \cdot 99 \cdot 10^{-6}\ m^3 \cdot 293{,}0\ K} = \underline{(8{,}7 \pm 0{,}7)\ J/(mol \cdot K)}$$

Table 1: Calculations for Series 1

Circumference	***C'* [cm]**	15.8	**Δ*C'* [cm]**	0.5
Radius	***r* [cm]**	2.51	**Δ*r* [cm]**	0.080
Cross Sectional Surface	***A* [cm²]**	19.9	**Δ*A* [cm²]**	1.26

Pressure		Length	
***p* [bar]**	**Δ*p* [bar]**	***L* [cm]**	**Δ*L* [cm]**
1.00	0.05	20.0	0.1
1.05	0.05	19.0	0.1
1.10	0.05	18.0	0.1
1.20	0.05	17.0	0.1
1.25	0.05	16.0	0.1
1.35	0.05	15.0	0.1
1.45	0.05	14.0	0.1
1.55	0.05	13.0	0.1
1.70	0.05	12.0	0.1
1.80	0.05	11.0	0.1
2.00	0.05	10.0	0.1
2.20	0.05	9.0	0.1
2.45	0.05	8.0	0.1
2.80	0.05	7.0	0.1
3.00	0.05	6.5	0.1
3.20	0.05	6.0	0.1
3.50	0.05	5.5	0.1
3.80	0.05	5.0	0.1

	Volume		(Volume)$^{-1}$	
	***V* [cm³]**	**Δ*V* [cm³]**	**V^{-1} [cm^{-3}]**	**Δ(V^{-1}) [cm^{-3}]**
V_0 ------------>	397	27.1	2.52E-03	1.7E-04
	377	25.9	2.65E-03	1.8E-04
	358	24.6	2.80E-03	1.9E-04
	338	23.4	2.96E-03	2.0E-04
	318	22.1	3.15E-03	2.2E-04
	298	20.8	3.36E-03	2.3E-04
	278	19.6	3.60E-03	2.5E-04
	258	18.3	3.87E-03	2.7E-04
	238	17.1	4.19E-03	3.0E-04
	219	15.8	4.58E-03	3.3E-04
	199	14.6	5.03E-03	3.7E-04
	179	13.3	5.59E-03	4.2E-04
	159	12.0	6.29E-03	4.8E-04
	139	10.8	7.19E-03	5.6E-04
	129	10.2	7.74E-03	6.1E-04
	119	9.5	8.39E-03	6.7E-04
	109	8.9	9.15E-03	7.5E-04
	99	8.3	1.01E-02	8.4E-04

Table 2: Calculations for Series 2

Circumference	*C'* [cm]	15.8		Δ*C'* [cm]	0.5
Radius	*r* [cm]	2.51		Δ*r* [cm]	0.080
Cross Sectional Area	*A* [cm^2]	19.9		Δ*A* [cm^2]	1.26

Pressure		Length	
p [bar]	Δ*p* [bar]	*L* [cm]	Δ*L* [cm]
1.00	0.05	5.0	0.1
0.90	0.05	5.5	0.1
0.80	0.05	6.0	0.1
0.70	0.05	7.0	0.1
0.60	0.05	8.0	0.1
0.50	0.05	10.0	0.1
0.40	0.05	12.0	0.1
0.35	0.05	14.0	0.1
0.25	0.05	20.0	0.1

	Volume		$(Volume)^{-1}$	
	V [cm^3]	Δ*V* [cm^3]	V^{-1} [cm^{-3}]	$\Delta(V^{-1})$ [cm^{-3}]
V_0 ------------>	99	8.3	1.01E-02	8.4E-04
	109	8.9	9.15E-03	7.5E-04
	119	9.5	8.39E-03	6.7E-04
	139	10.8	7.19E-03	5.6E-04
	159	12.0	6.29E-03	4.8E-04
	199	14.6	5.03E-03	3.7E-04
	238	17.1	4.19E-03	3.0E-04
	278	19.6	3.60E-03	2.5E-04
	397	27.1	2.52E-03	1.7E-04

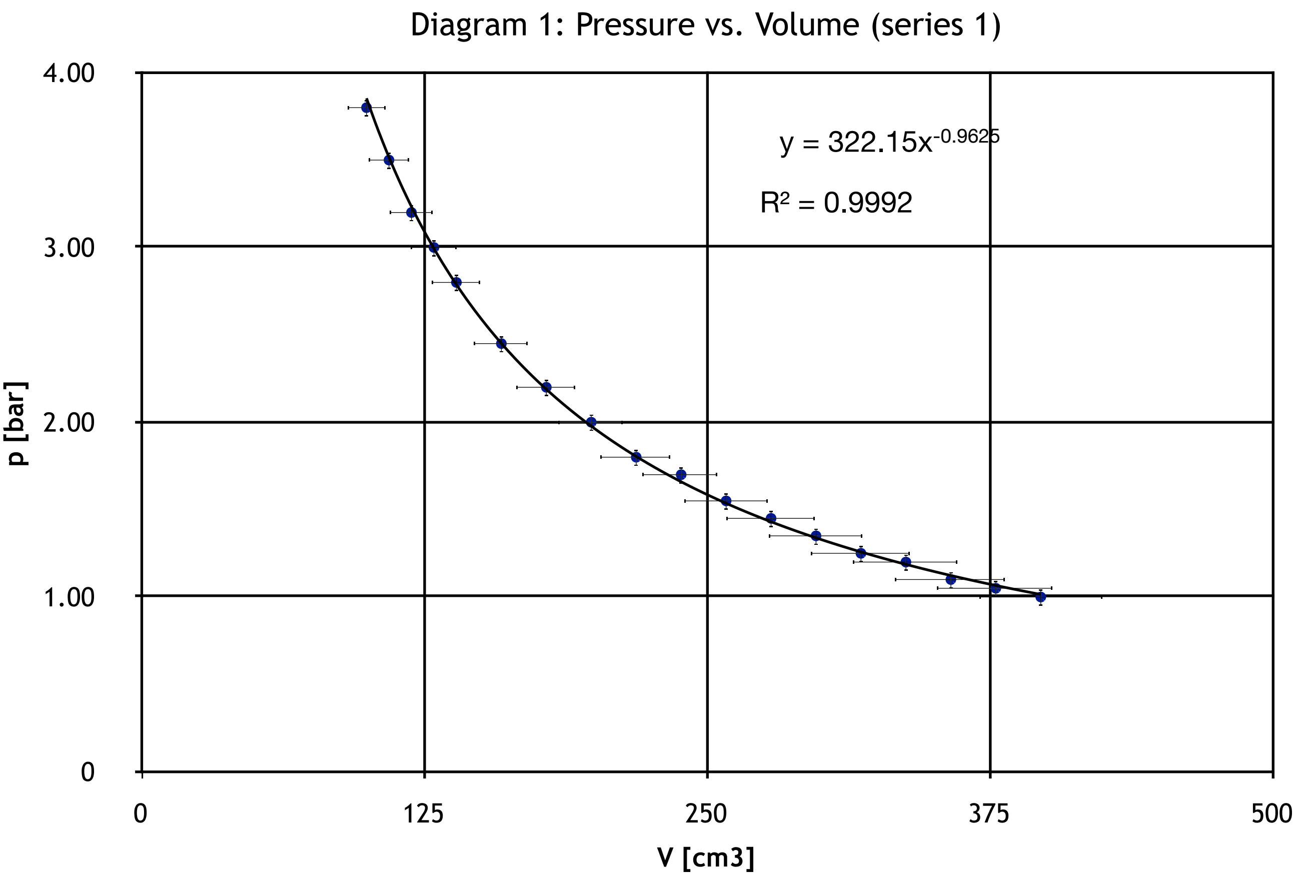


Diagram 1: Pressure vs. Volume (series 1)
y = 322.15x^-0.9625
R² = 0.9992
p [bar]
V [cm3]
4.00
3.00
2.00
1.00
0
0
125
250
375
500

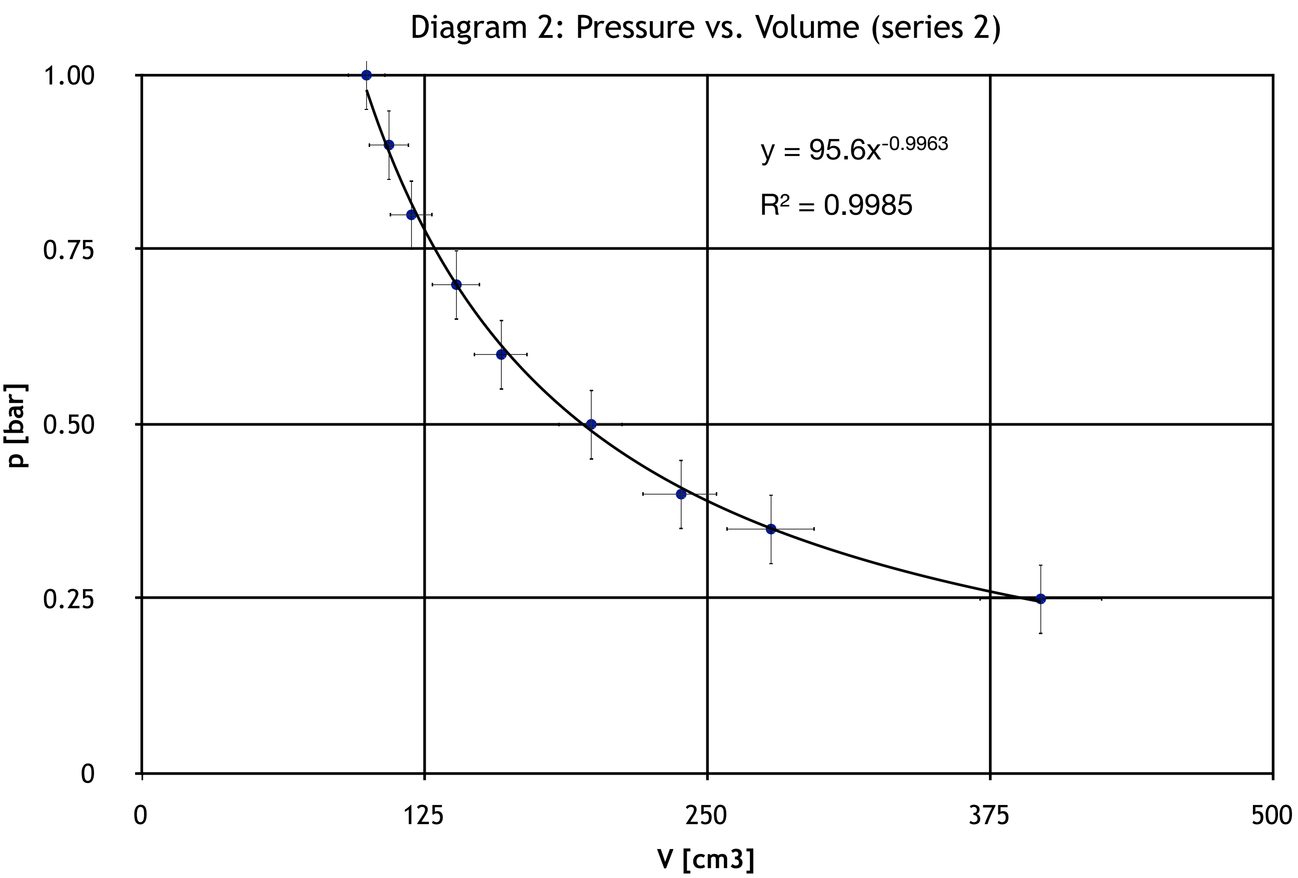


Diagram 2: Pressure vs. Volume (series 2)
y = 95.6x^-0.9963
R² = 0.9985
p [bar]
V [cm3]
1.00
0.75
0.50
0.25
0
0
125
250
375
500

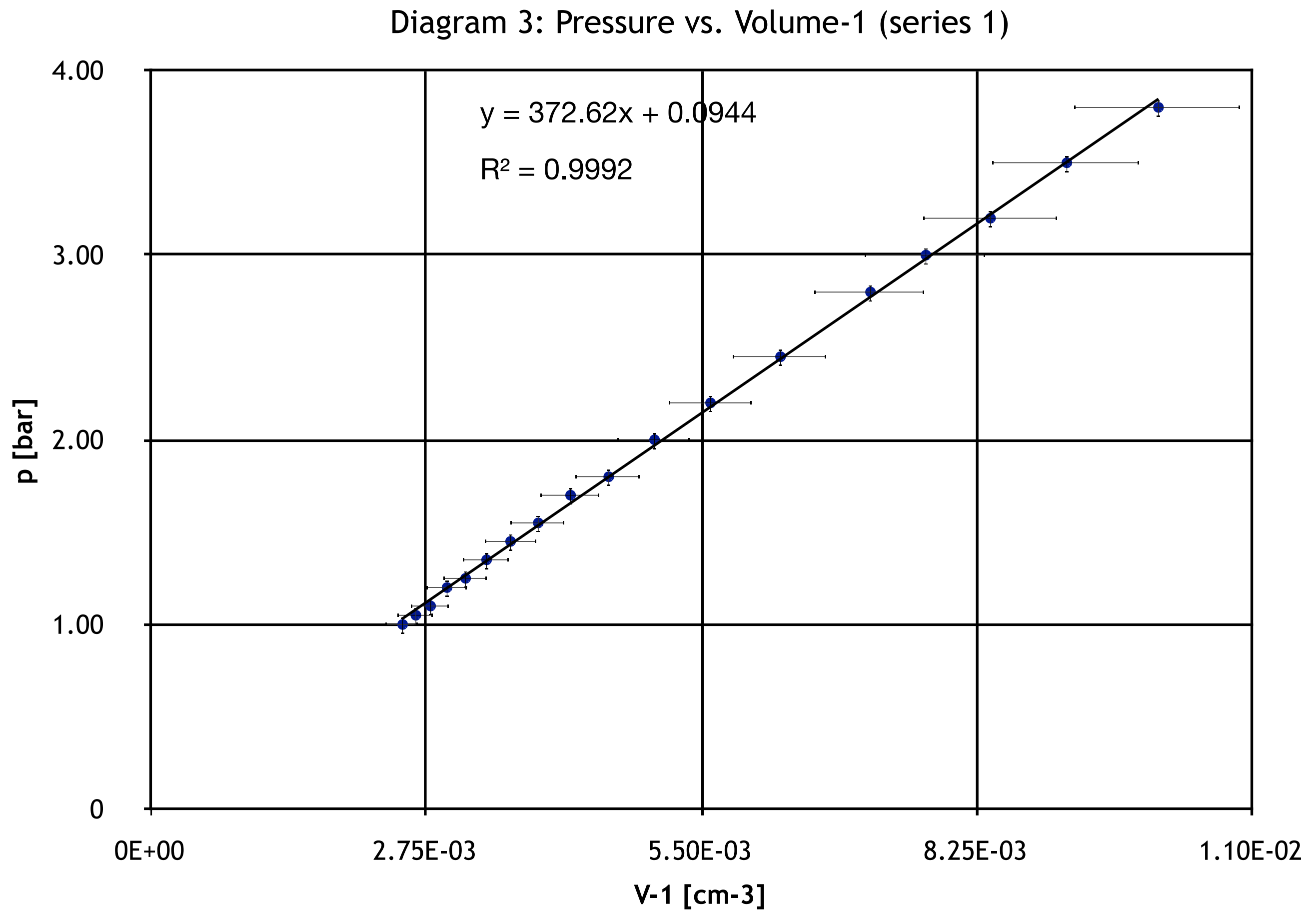


Diagram 3: Pressure vs. Volume-1 (series 1)
y = 372.62x + 0.0944
R² = 0.9992
4.00
3.00
2.00
1.00
0
p [bar]
0E+00
2.75E-03
5.50E-03
8.25E-03
1.10E-02
V-1 [cm-3]

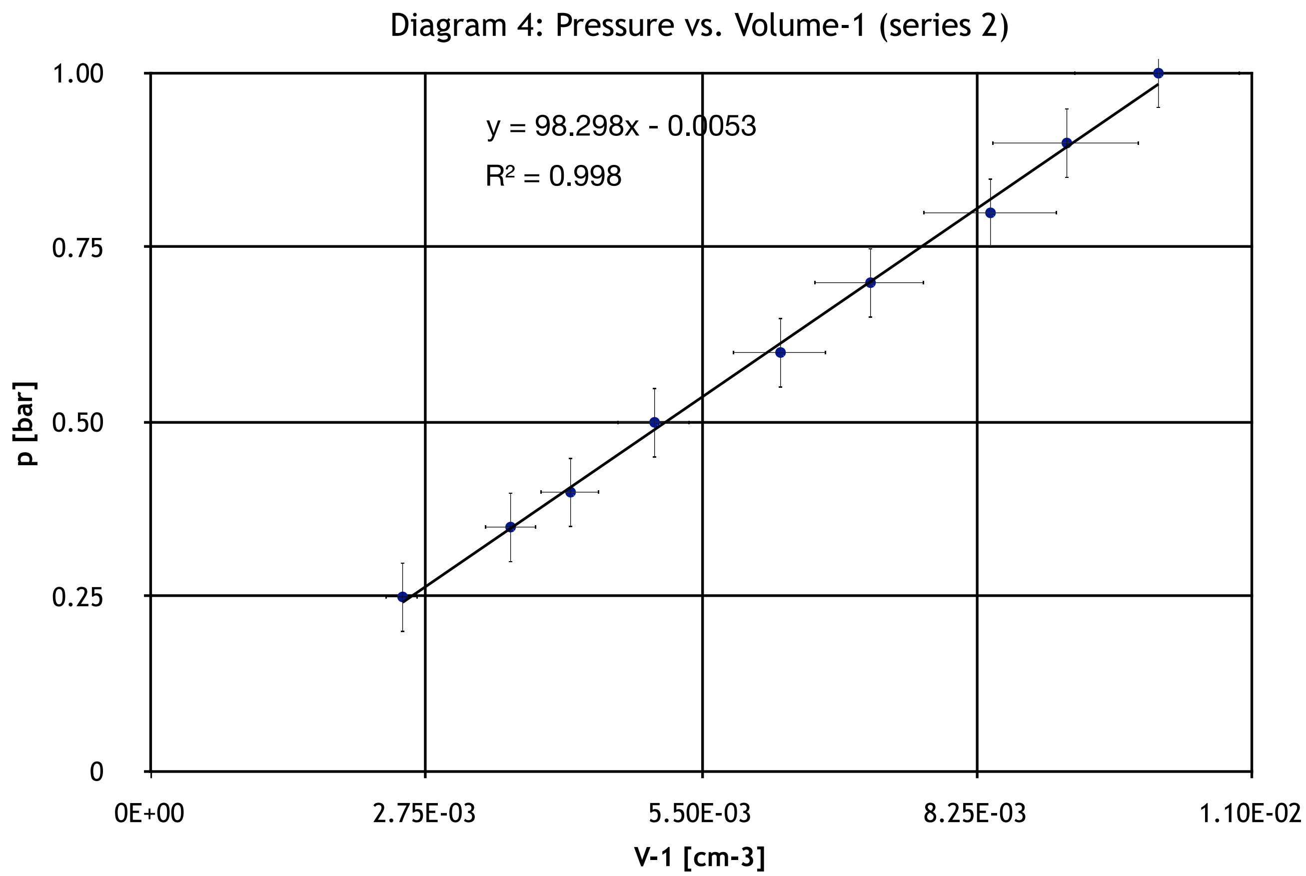


Diagram 4: Pressure vs. Volume-1 (series 2)
y = 98.298x - 0.0053
R² = 0.998
1.00
0.75
0.50
0.25
0
p [bar]
0E+00
2.75E-03
5.50E-03
8.25E-03
1.10E-02
V-1 [cm-3]

Conclusions

Results

The experiment confirms the law of Boyle and Mariotte to a good precision. The power regressions result in powers of $r_1 = -0.9625$ and $r_2 = -0.9963$, which deviate only slightly from the theoretical value $r = -1$, meaning that pressure varies inversely proportional to volume.

The linear regression in the pressure vs. volume^{-1} diagram is also in accordance with theory: The straight line passes through the error bars of all data points. The precision of the experiment even seems to be better than the assumed errors suggest.

We calculated two values for the universal gas constant: $R_1 = (8.6 \pm 0.6)$ J/(mol · K) from the first series of measurements and $R_2 = (8.0 \pm 0.6)$ J/(mol · K) from the second series. The accepted value $R = 8.31$ J/(mol · K) [2] lies within the error boundaries in both cases.

Reflexion

In this lab session we investigated the relation between pressure and volume for air. We learnt how to collect systematic data series and how to take complete lab notes. We used different ways to graphically analyse the data in order to find the universal gas constant.

The experiment could be heavily improved if we measured the air volume more precisely. One possibility would be to determine the volume of the air leaving the cylinder through the valve with a balloon.

Bibliography

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3 Physik 11 (Ausgabe C, Gymnasium Sek II); Dorn, Bader (Herausgeber); Schroedel Verlag, Hannover, 2000