Problem Solving Strategy for Simple Harmonic Motions

The following three steps can be applied to a) prove that a physical quantity is a simple harmonic motion; and b) find an expression for the angular frequency of the oscillation.

1. Using the appropriate physical laws (dynamics, heat transfer, electrodynamics, …), derive an expression for the second derivative of the quantity under investigation.
2. The quantity is a simple harmonic motion (i.e. it can be described by the well-known equations of motion) if and only if its second derivative is proportional to the quantity itself and if the constant of proportionality is negative.
3. After writing the characteristic differential equation in the standard form, the angular frequency can easily be identified (pattern matching), thereby fixing most kinematic parameters (period, frequency).
4. The amplitude and the maximum velocity and acceleration are fixed by the initial conditions.

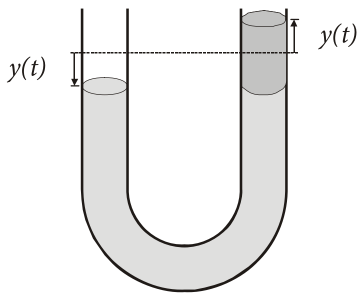
Oscillating Liquid Column

We find innumerable oscillation phenomena in nature and in our everyday life. An important part of them are simple harmonic motions. Investigating a simple example, you learn how to derive a formal expression for the period of the oscillation.

Goals: You derive the characteristic differential equation of a simple harmonic motion and the oscillation period from the dynamics of a mechanical system. You apply basic relations between quantities of motion.

Time: You work on the exercise for 15 minutes.

Problem:
A horseshoe tube with constant cross section is filled with a liquid. In the equilibrium position the liquid columns in both legs have the same height (dashed horizontal line). After disturbing the equilibrium, the liquid starts oscillating in the tube.



Show that the oscillation is a simple harmonic motion and derive an expression for its period.

Instruction

1. Express the restoring force in the situation above by the cross section of the tube, the density of the liquid and the displacement *y(t)* (positive *y*-direction is upwards).

   Express the accelerated mass by the dimensions of the liquid column and the density of the liquid.

2. Combining the results from steps 1 and 2 with Newton's second law, show that the motion of the liquid column fulfils the characteristic equation for a simple harmonic motion.
3. Derive a formal expression for the angular frequency and the period of the oscillation.

   Calculate the period for the 95 cm long mercury column in the demonstration experiment. Compare the result to the measured value.

4. Calculate the maximum velocity and acceleration when the liquid column is started with an initial displacement of 5 cm.