Harmonics of Thin Pipes

In wind instruments the air is oscillating in a tube. The sound waves are reflected on both ends, thereby producing a superposition of standing waves.

The harmonics can easily be found for thin pipes. At the mouth the air's oscillation has a maximum, i.e. there is an anti-node. The other end is either a node (closed pipe) or an anti-node (open pipe).

Draw the standing waves in the figure below, express their wavelength and frequency as a function of the pipe's length and the speed of sound and find a general expression for the i^{th} harmonic.