Hall Effect

Although the definition of the magnetic field already provides a way of measuring this important quantity, it can be very complex and time-consuming. The *Hall effect* allows to build small devices (*Hall probes*) producing a voltage proportional to the magnitude of the magnetic field, which thus can easily be measured.

Another application of the Hall effect is to determine the concentration of free charge carriers in a conductor or a semiconductor.

Goal: You know how the Hall voltage arises and you can calculate it for a given probe. You can describe a measurement used to determine the concentration of free electrons in a metal.

Time: You work on this problem for 20 minutes.

Problem

An electric current of 30 mA flows through a 2 mm thick strip of germanium with a rectangular cross section. The strip is placed in a homogeneous magnetic field with a magnitude of 43 mT.

Calculate the voltage measured between the sides of the strip (see figure).

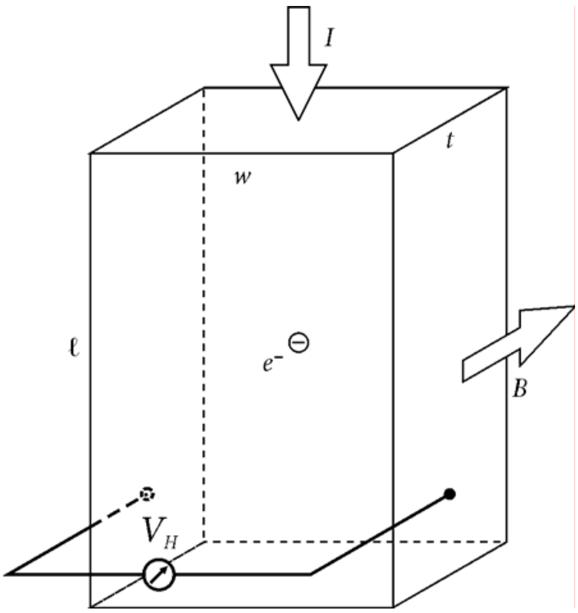

Figure: Section of a Hall probe

Instructions

1. The free electron in the above figure moves with the electric current *I*. Draw its velocity vector and the magnetic force acting on the electron.

   Hint: The conventional direction of current is defined for positive particles!

2. Because of the deflection caused by the magnetic force, there is an excess of electrons on one side of the strip and a lack of electrons on the other side. What is the direction of the arising electric field? Why do not all of the free electrons move to one side?

3. Find a formal expression for the magnitude of the electric field when the electron is in equilibrium. Show that the voltage between the two sides of the strip is proportional to the magnitude of the magnetic field.

4. Explain in your own words the origin of the Hall voltage you just found.

It can be shown (see additional questions) that the electric current flowing through a strip with cross section $w \cdot t$ can be written as

$$I = n_e \cdot w \cdot t \cdot e \cdot v \,,$$

where n_e is the *concentration of free electrons* (number of free electrons per unit volume) and v is their average velocity.

5. Using the expression above, write the Hall voltage as

$$V_H = \frac{I \cdot B}{n_e \cdot t \cdot e} \,.$$

6. Calculate the magnitude of the Hall voltage for the values given at the beginning of the problem.

   HINT: A table displaying some concentrations of free electrons can be found in "Formeln und Tafeln" (page 178).

Additional Questions

7. In order to express the Hall voltage using only easily measurable quantities and an intrinsic property of the metal, we look for a relation between the drift velocity v and the current I.
   - Express the number of electrons in a section of the germanium strip with the *concentration of free electrons* n_e and the dimensions of the cuboid shaped section.
   - Prove the relation for the current used in the derivation of the Hall voltage.
8. Why are Hall probes usually made of semiconducting materials?
9. What do you have to consider if you have to plan a Hall probe appropriate for weak magnetic fields? Giving realistic values, explain how you would design a Hall probe with a sensitivity of 1 mV/µT.
10. Describe how to set up an experiment to measure the concentration of free electrons in a metal.