Musterlösung A-2K, K = 1, .., 4 (Integralrechnung 5)

K=1, a) $g(x)$ Stammfunktion von $f(x) \iff g'(x) = f(x)$

$$\int_0^x f(t)\,dt = g(x)$$

Ableiten ist einfacher!

$$\frac{d}{dx}\,\frac{ax-b}{e^x} = \frac{a e^x - (ax-b)e^x}{e^{2x}} = \frac{a+b-ax}{e^x} \overset{!}{=} \frac{x-1}{e^x} \Rightarrow \left.\begin{array}{l} -ax = +x \\ a+b = -1 \end{array}\right\}$$

das gilt wenn $a = -1$ und $b = 0 \Rightarrow g(x) = \frac{-x}{e^x}$

b) Skizze ist immer hilfreich!

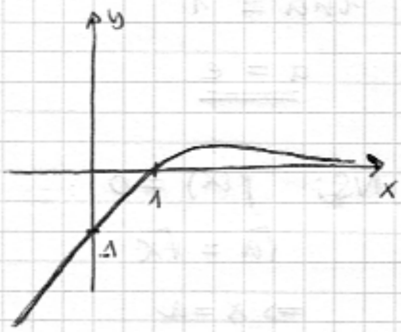
NS: $\frac{x-1}{e^x} = 0 \Rightarrow x = 1$

$$\lim_{x\to\infty} \frac{x-1}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$

Ableiten (Nenner, Zähler)

$f(0) = \frac{-1}{e^0} = -1$

$$\lim_{x\to-\infty} \frac{x-1}{e^x} = -\infty$$



Im ersten Quadranten $\Rightarrow x \in [1, \infty[$

$$\int_1^K f(x)\,dx = g(x)\Big|_1^K = \frac{-x}{e^x}\Big|_1^K = \frac{-K}{e^K} + \frac{1}{e}$$

$$\lim_{K\to\infty} \underbrace{\frac{-K}{e^K}}_{\to 0} + \frac{1}{e} = \frac{1}{e} \quad ✓$$

K=2) $y = \left(1 - \frac{x}{k}\right)\sqrt{x} \quad x \in [0; k],\ k > 0$

$$V = \int_0^K \pi \left[\left(1-\frac{x}{k}\right)\sqrt{x}\right]^2 dx = \pi \int_0^K x\left(1 + \left(\frac{x}{k}\right)^2 - \frac{2x}{k}\right) dx = \pi \int_0^K \left(x + \frac{x^3}{k^2} - \frac{2x^2}{k}\right) dx$$

$$= \pi \left[\frac{x^2}{2} + \frac{x^4}{4k^2} - \frac{2x^3}{3k}\right]_0^K = \pi\left(\frac{k^2}{2} + \frac{k^2}{4} - \frac{2k^2}{3}\right) = \pi k^2 \left(\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right) =$$

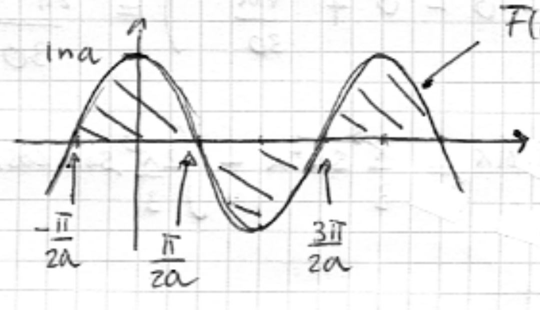
$$= \pi k^2 \cdot \frac{1}{12} \overset{!}{=} 4\frac{\pi}{3} \Rightarrow \quad k^2 = \frac{4}{3} \cdot 12 = 16 \Rightarrow \underline{\underline{k = 4}}$$

K=3) Was heisst Flächenstücke?

$\Rightarrow$ Skizze!

$\ln a$ ist ein Wert.

Flächenstück!



Nullstellen von $\ln a \cos(ax)$ mit $a > 1$ d.h $\ln a > 0$

$$\ln a \cos(ax) = 0 \Rightarrow \cos ax = 0 . \quad \cos\alpha = 0 \Leftrightarrow \alpha = \frac{\pi}{2} \pm k\cdot\pi \quad k \in \mathbb{Z}$$

$$\Rightarrow ax = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{2a} \qquad ax = -\frac{\pi}{2} \Rightarrow x = -\frac{\pi}{2a}$$

Für welche $a > 1$ ist das Flächen-stück für $x \in \left[-\frac{\pi}{2a} ; \frac{\pi}{2a}\right]$ maximal?

(auch $x \in [0, \frac{\pi}{2a}]$ wäre ok)

$$\left[\int_{-\frac{\pi}{2a}}^{\pi/2a} \ln a \cos(ax)\, dx\right]' = \left[\ln a \frac{\sin ax}{a} \Big|_{-\frac{\pi}{2a}}^{\pi/2a}\right]' =$$

$$= \left[\frac{\ln a}{a}\left(\underset{1}{\sin\frac{\pi}{2}} - \underset{(-1)}{\sin\left(-\frac{\pi}{2}\right)}\right)\right]' = \left[\frac{\ln a}{a}\cdot 2\right]' = \frac{d}{da}\left(2\frac{\ln a}{a}\right) =$$

$$= 2\cdot\left(\frac{1}{a^2} - \ln a \cdot \frac{1}{a^2}\right) = \frac{2}{a^2}(1 - \ln a) \overset{!}{=} 0 \Rightarrow \ln a = 1$$

$$\underline{\underline{a = e}}$$

k=4) $y = f(x) = (\sqrt{a} - \sqrt{x})^2 \qquad f(x)^2 = (\sqrt{a} - \sqrt{x})^4$

NS: $f(x) = 0$

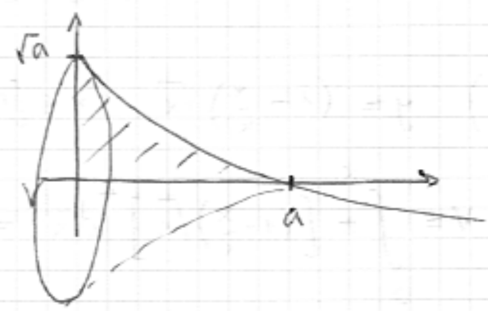
$\sqrt{a} = \sqrt{x}$

$\Rightarrow x = a$

und $f(0)$

$$V = \pi \int_0^a (\sqrt{a} - \sqrt{x})^4\, dx = (*)$$

substitution: $\sqrt{x} = u \qquad du = \frac{dx}{2\sqrt{x}} = \frac{dx}{2u}$



$$(*) = \pi \int_0^a \underbrace{(\sqrt{a} - u)^4}_{g'} \cdot \underset{f}{2u}\, du = \qquad g = -\frac{(\sqrt{a}-u)^5}{5}$$

$$= 2\pi \int_0^a (\sqrt{a} - u)^4 \cdot u\, du =$$

$$= 2\pi \left[-\frac{(\sqrt{a}-u)^5}{5} u \Big|_0^a - \int_0^a -\frac{(\sqrt{a}-u)^5}{5} \cdot 1\, du\right] = 2\pi\left[-\frac{(\sqrt{a}-u)u}{5}\right]_0^a + \left[-\frac{(\sqrt{a}-u)^6}{30}\right]_0^a$$

$$= 2\pi\left(0 - 0 - 0 + \frac{\sqrt{a}^6}{30}\right) = \frac{2\pi\, a^{6/2}}{30} = \underline{\underline{\frac{\pi}{15} \cdot a^3}}$$

A3) $$\int \underset{g'}{x^2} \underset{f}{\sin^2 x}\, dx = \frac{x^3}{3}\sin^2 x - \int \frac{2x^3}{3} \sin x \cos x\, dx = \frac{x^3}{3}\sin^2 x - \underbrace{\int \frac{x^3}{3} \sin 2x\, dx}_{\text{dann 3x partielle} \int \text{mit Pol. als } f.}$$