Relativistic Kinematics

Basic Problems

1. At the ends of a runway two light flashes are emitted at the same time (in the runway's rest system). Which event takes place earlier in the rest system of a plane just taking off? Draw a diagram for the situation.
2. Calculate the Lorentz factor for an airplane flying at a speed of 2'000 km/h and a particle moving at 99.9 % of the speed of light. What is the maximum speed at which $\gamma - 1 < 10^{-6}$?
3. The half life of muons in their rest system is 1.5 µs. When they are accelerated in a particle accelerator, a half life of 44 µs is measured. Calculate the muons' speed.
4. A radar transmitter rotates at a frequency of 0.25 rad/s (measured on the earth). Calculate the frequency as measured by an observer flying by the earth at 80 % of the speed of light.
5. A pedestrian walks at 1.3 m/s on a 9.5 km long path along a picturesque river. How long would the path be from the pedestrian's point of view if the speed of light was only 3.0 m/s?
6. Muons created by the cosmic radiation some 12 km above the Earth's surface are detected on our planet although they can only travel a distance of some 600 m in their rest system. Explain this phenomenon both from the point of view of the muons and the observers on the Earth.
7. What is the speed at which a meter stick flying by an observer is only 99 cm long?

Additional Problems

8. Calculate the deviation of a clock moving at 5 m/s from its own proper time. Solve the exercise by using an approximation for the Lorentz factor.
9. A Klingon spaceship travels at 75 % of the speed of light with respect to the earth. The Klingon crew measures a time interval of 37.0 h between two events on earth. Calculate the time interval they would measure between the same two events if they travelled at 94 % of the speed of light.
10. A 6 m long car drives through a garage which has two doors at a distance of 5 m. Due to the car's length contraction it is possible that both doors are closed for a short moment when the car is entirely inside the garage. From the driver's point of view, however, it is the distance between the doors which is shorter and is impossible that the car fits between them. How can this apparent contradiction be resolved?
11. A spaceship flies at 73 % of the speed of light by a ramp which is inclined by 30° as measured in its rest system. Calculate the ramp's inclination as measured by the spaceship's crew.
12. An object moves with speed *u'* in a reference frame B which is moving with speed *v* relative to a second reference frame A. The speed *u* at which the object moves in System A can be calculated using the following formula:

$$u = \frac{u' + v}{1 + \frac{u' \cdot v}{c^2}}$$

    a) An astronaut walks with half the speed of light in a space ship flying by at half the speed of light. Calculate the speed at which the astronaut moves from the point of view of an outside observer.

    b) Prove that light has the same speed in both reference frames.

Solutions to Basic Problems: 2. $1 + 1.7 \cdot 10^{-12}$, 22, 420 km/s; 3. $0.9994 \cdot c$; 4. 0.15 rad/s; 5. 8.6 km; 7. $0.14 \cdot c$