Escape Speed

If you fire a projectile upward, usually it will slow, stop momentarily, and return to Earth. There is, however, a certain minimum initial speed that will cause it to move upward forever, theoretically coming to rest only at infinity. For a space rocket, this corresponds roughly to the minimum initial speed required for a space trip to the moon or a neighbouring planet.

We try to find an answer to the following question:
What is a rocket's minimum initial speed to overcome Earth's gravitational attraction?
This specific speed is called the *escape speed* of an object from Earth.

Goals:

- You sketch the changes in different energy forms during a space trip.
- By means of the law of conservation of energy, you find a formula to calculate the escape speed.
- You carefully handle the huge numerical values involved in this problem.

Time: You have 20 minutes to work on this problem.

Problem

A space rocket starts from Earth's surface. We suppose that it reaches its initial speed v_i almost immediately after its take-off and continues its journey without further propulsion. The initial speed is just high enough to allow the rocket to escape to infinity. Calculate the value of v_i (its escape speed).

Instructions

1. In an appropriate diagram (energy vs. distance), sketch the changes of the rocket's kinetic, gravitational and total energy during its journey. Use different colours for the different energy forms.
2. Find algebraic expressions for the rocket's total energy in its initial and final state. Using the law of conservation of energy, deduct a condition for its initial speed v_i (its escape speed).
3. If you're a perfectionist: Express the gravitational constant with the constant of free fall on Earth's surface (cf. last lesson) and use the result in the above condition for v_i.
4. Calculate the numerical value of the escape speed from Earth's surface with the formula of either step 2 or 3.
5. Check the validity of your formula by calculating the escape speed from Moon (2.3 km/s), Mercury (4.3 km/s) and Jupiter (60.0 km/s).
6. To reduce the required initial speed for a mission to Mars, maximum advantage of Earth's own (rotational) motion is made. Discuss this idea briefly.

Additional Problem

The results you found above lead to a qualitative understanding of a problem in astrophysics: A *black hole* is a collapsed star with an enormous gravitation. Nothing, not even light, is fast enough to escape from its surface.

As an example, let's calculate the so-called *Schwarzschild radius* (i.e. the radius of the region around the star from which not even light can escape) of a black hole with a mass of 3.5 solar masses.

7. Take the equation of conservation of energy (step 2) as a starting point. Solve the equation for the radius. Remember to use the speed of light c as the escape speed. Calculate the numerical value of the black hole's Schwarzschild radius.
8. Although neither the sun nor the earth are massive enough to collapse and become a black hole, calculate their respective Schwarzschild radius.
9. Calculate your weight on the black hole's surface.