Relativity

1. $\gamma_1 - 1 = \frac{1}{\sqrt{1-\left(\frac{2000}{3{,}6\cdot 3\cdot 10^8}\right)^2}} - 1 = 1{,}7\cdot 10^{-12} \Rightarrow \gamma_1 = 1 + 1{,}7\cdot 10^{-12}$

(approximation: $\gamma_1 = \frac{1}{\sqrt{1-\beta^2}} \cong \frac{1}{1-\frac{1}{2}\beta^2} \cong 1 + \frac{1}{2}\beta^2 = 1 + \frac{1}{2}\left(\frac{2000}{3{,}6\cdot 3\cdot 10^8}\right)^2 = 1 + 1{,}7\cdot 10^{-12}$)

$\gamma_2 = \frac{1}{\sqrt{1-(0{,}999)^2}} = 22$

$\beta_3 = \sqrt{1-\frac{1}{\gamma_3^2}} = \sqrt{1-\frac{1}{(1{,}000'001)^2}} = 0{,}0014 \rightarrow v_3 = \beta_3 \cdot c = 420\ \text{km/s}$

2. $\omega_B = \frac{2\pi}{T_B} = \frac{2\pi}{\gamma\cdot T_A} = \frac{\omega_A}{\gamma} = \omega_A \cdot \sqrt{1-\beta^2} = 0{,}25\ \text{rad/s}\ \sqrt{1-0{,}8^2} = 0{,}15\ \text{rad/s}$

3. $\ell = \frac{\lambda}{\gamma} = \lambda\cdot\sqrt{1-(v/c)^2} = 9{,}5\ \text{km}\cdot\sqrt{1-\left(\frac{1{,}3}{3}\right)^2} = 8{,}6\ \text{km}$

4. $\ell = \frac{\lambda}{\gamma} \Rightarrow \gamma = \frac{\lambda}{\ell} \Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\left(\frac{\ell}{\lambda}\right)^2} = \sqrt{1-0{,}99^2} = 0{,}14$

5. $\Delta m = \frac{\Delta E_0}{c^2} = \frac{P\cdot\Delta t}{c^2} = \frac{3{,}8\cdot 10^{26}\,W\cdot 10^9\cdot 365\cdot 86400\,s}{(3\cdot 10^8\,m/s)^2} = 1{,}33\cdot 10^{26}\ \text{kg}$

$\frac{\Delta m}{m} = \frac{1{,}33\cdot 10^{26}}{1{,}99\cdot 10^{30}} = 6{,}7\cdot 10^{-3}\,\% = 67\ \text{ppm}$

6. $\Delta m = 2\cdot m_{H-2} - (m_{He3} + m_n) = (2\cdot 2{,}0141018 - 3{,}0160293 - 1{,}0086649157)\,u$

$= 0{,}0035\,u = 5{,}8\cdot 10^{-30}\ \text{kg}$

7. $E_B = (m_{Pb-208} - (82\cdot m_p + 126\cdot m_n) - 82\cdot m_e)c^2 = (207{,}976636 - 82\cdot 1{,}007276467 - 126\cdot 1{,}0086649157 - 82\cdot 0{,}000548580)\,u\cdot c^2$

$= -1{,}76\cdot 1{,}66\cdot 10^{-27}\ \text{kg}\cdot(3\cdot 10^8\,m/s)^2 = -2{,}62\cdot 10^{-10}\,J = -1{,}6\ \text{GeV}$

8. near the maximum of binding energy per nucleon
heavier and lighter nuclei have less binding energy

9. $E = 2\cdot E_0 + 2\cdot E_{kin} = 2\cdot E_0 + 2\cdot 2\cdot E_0 = 6\cdot E_0 = 6\cdot m\cdot c^2 = m'\cdot c^2 + E'_{kin}$

$\rightarrow m' \leq 6\cdot m$

10. $E_{kin} = (\gamma - 1)\cdot E_0 = \left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)\cdot E_0 = \left(\sqrt{\frac{1}{1-0{,}999^2}} - 1\right)\cdot 511\ \text{keV}$

11. $E_0 = m\cdot c^2 = 60\,\text{kg}\cdot(3\cdot 10^8\,m/s)^2 = 5{,}4\cdot 10^{18}\,J = 10{,}9\ \text{MeV}$

$E = \gamma\cdot E_0 = \frac{1}{\sqrt{1-\beta^2}}\cdot E_0 = \frac{1}{\sqrt{1-(0{,}015)^2}}\cdot 5{,}4\cdot 10^{18}\,J = 5{,}4\cdot 10^{18}\,J$

$E_{kin} = E - E_0 = 6{,}1\cdot 10^{14}\,J$

12. $\gamma = 1 + \frac{E_{kin}}{E_0} = 1 + \frac{5}{3727} = 1{,}00134 \Rightarrow \beta = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1{,}00134)^2}} = 0{,}052$

13. $\frac{E'}{E} = \frac{p'}{p} = \frac{\gamma'}{\gamma} = \sqrt{\frac{1-\beta^2}{1-\beta'^2}} = \sqrt{\frac{1-0{,}9^2}{1-0{,}91^2}} = 1{,}051 \rightarrow +5{,}1\,\%$ ($98\,\% \rightarrow 99\,\%$: $+41\,\%$)

14. $p = \frac{1}{c}\cdot\sqrt{E^2 - E_0^2} = \frac{1}{c}\sqrt{(E_0 + E_{kin})^2 - E_0^2} = \sqrt{(938 + 0{,}02)^2 - 938^2}\ \text{MeV}/c$

$= 6{,}1\ \text{MeV}/c$

Relativity

15. $\gamma = \frac{1}{\sqrt{1-\beta^2}} \cong \frac{1}{1-\frac{1}{2}\beta^2} \cong 1+\frac{1}{2}\beta^2$

$\rightarrow \quad \frac{\Delta t}{\tau} = \frac{t}{\tau} - 1 = \gamma - 1 \cong \frac{1}{2}\beta^2 = \frac{1}{2}\left(\frac{5}{3\cdot 10^8}\right)^2 = \underline{1{,}4\cdot 10^{-16}}$

16. $t = \gamma\cdot\tau \;,\quad t' = \gamma'\tau = \gamma'\cdot\frac{t}{\gamma} = t\sqrt{\frac{1-\beta^2}{1-\beta'^2}}$

$= 37{,}0\,h\cdot\sqrt{\frac{1-0{,}75^2}{1-0{,}94^2}} = \underline{71{,}7\,h}$

17. $v_{AB} = \frac{v_{AC}+v_{CB}}{1+\frac{v_{AC}\,v_{CB}}{c^2}}$

a) $v = \frac{c/2+c/2}{1+\frac{c/2\cdot c/2}{c^2}} = \frac{c}{1+1/4} = \frac{4}{5}\cdot c$

b) $c' = \frac{c+c/2}{1+\frac{c\cdot c/2}{c^2}} = \frac{3/2\cdot c}{3/2} = c$ ✓

18. $\vec{F} = \frac{d\vec{p}}{dt} = m\cdot\frac{d\gamma}{dt}\cdot\vec{v} + m\cdot\gamma\cdot\frac{d\vec{v}}{dt}$

$= m\left(-\frac{1}{2}\,\frac{-2\cdot v/c}{(1-(v/c)^2)^{3/2}}\cdot\frac{1}{c}\cdot\frac{dv}{dt}\cdot\vec{v} + \gamma\cdot\frac{d\vec{v}}{dt}\right)$

$= m\cdot\gamma\cdot\left(\gamma^2\cdot\frac{v}{c}\cdot\frac{dv}{dt}\cdot\frac{\vec{v}}{c} + \frac{d\vec{v}}{dt}\right)$

für $\vec{v}\parallel\frac{d\vec{v}}{dt}$: $\quad F = \gamma\cdot m\cdot(\gamma^2\cdot\beta^2+1)\cdot\frac{dv}{dt} = \gamma\cdot m\cdot\frac{\beta^2+1-\beta^2}{1-\beta^2}\cdot\frac{dv}{dt} = \underline{\gamma^3\cdot m\cdot\frac{dv}{dt}}$

19. a) $\Delta m = \frac{\Delta E}{c^2} = \frac{P_{th}\,\Delta t}{c^2} = \frac{P_{el}\cdot\Delta t}{\eta\cdot c^2} = \frac{10^9\,W\cdot 86'400\,s}{0{,}3\cdot(3\cdot 10^8\,m/s)^2} = \underline{3{,}2\,g}$

$\rightarrow$ kinetic energy of decay products, radiation (γ)

b) heat transfer from container to water
in equilibrium: same energy as in a) $\rightarrow$ can be measured

20. ${}^2_1H + {}^3_1H \longrightarrow {}^4_2He + n$

a) $\Delta E = E_B - E_B' = 2\cdot(-1{,}14\,MeV) + 3\cdot(-2{,}86\,MeV) - 4\cdot(-7{,}14\,MeV)$

$= \underline{17{,}7\,MeV}$ (accepted value 17,6 MeV)

b) $\Delta E = \Delta m\cdot c^2 = \left[(m_{H-2}+m_{H-3}) - (m_{He-4}+m_n)\right]\cdot c^2$

$= \left[(2{,}0141018 + 3{,}016049) - (4{,}0026033 + 1{,}0086649)\right]u\cdot c^2$

$= 0{,}01888\cdot 1{,}66\cdot 10^{-27}\,kg\cdot(3{,}00\cdot 10^8\,m/s)^2 = 2{,}8\cdot 10^{-12}\,J = \underline{17{,}6\,MeV}$