ETH zürich

# Informatik I

## Exercise session 4

Autumn 2020

# Homework

- Questions?

# Introduction Activity

Write the pseudo-code for the following problems.

# Introduction Activity

You are filling up a water container. You want to keep adding water until the reservoir is full.

# Introduction Activity

You have a list of `n` students, where `n` is a strictly positive integer. You want to print the list of all the students names.

# Introduction Activity

You want to compute the sum of all the integers from 1 to `n`, where `n` is a strictly positive integer.

# Introduction Activity

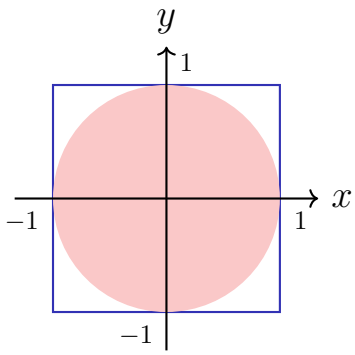
You have a checkerboard of size `n*n`, where `n` is a strictly positive integer. Each cell of the board has a name. You want to print the names of all the cells.

# Monte Carlo Simulation

**Monte Carlo Simulation:** Use randomness to solve problems. Wide area of applications in applied mathematics, all natural sciences and engineering

# Estimate $\pi$ using Monte Carlo Simulation

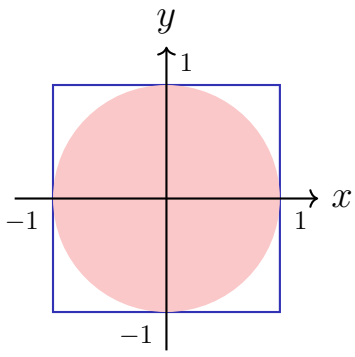
$$\frac{\text{circle area}}{\text{square area}} = \frac{\pi}{4}$$



# Estimate $\pi$ using Monte Carlo Simulation

$$\frac{\text{circle area}}{\text{square area}} = \frac{\pi}{4}$$

Idea: simulate random variable with uniform distribution on the unit square $[0, 1] \times [0, 1]$.

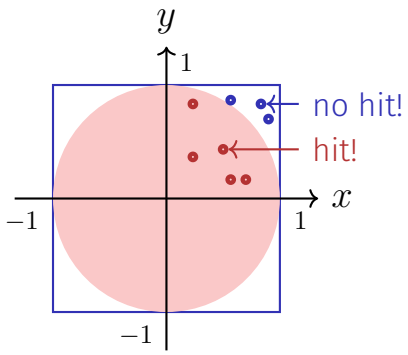


# Estimate $\pi$ using Monte Carlo Simulation

$$\frac{\text{circle area}}{\text{square area}} = \frac{\pi}{4}$$

Idea: simulate random variable with uniform distribution on the unit square $[0, 1] \times [0, 1]$.

$$\frac{\text{number hits}}{\text{number trials}} \cdot 4 \approx \pi.$$



# Task

- Conduct an experiment: estimate $\pi$ using the Monte Carlo method above.
- Execute the experiment for number of trials $1, 2, 4, 8, \ldots, n$, $n$ should be asked from the user.
- Output the number of trials and the estimation of $\pi$ each time.

Hint: a pseudo-random number uniformly disrtibuted on $[0, 1)$ can be obtained via `Math.random()`.

# Extension

Deterministic formulas to approximate $\pi$:

$$Sum_1 : \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$
$$Sum_2 : \frac{\pi}{2} = 1 + \frac{1}{3} + \frac{1 \cdot 2}{3 \cdot 5} + \frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7} + \cdots$$

Compare the approximation accuracy of the different methods.

# Solution on next slide

spoiler ahead – do not open (yet)

# Estimate $\pi$ using Monte Carlo Simulation

```
public class Main {
  public static void main(String[] args){
    int n = In.readInt();
    for (int trials = 1; trials <= n; trials*=2){
      int hits = 0;
      for (int i = 0; i<trials; ++i){
        double x = Math.random();
        double y = Math.random();
        if (x * x + y * y <= 1){
          hits++; }
      }
      double pi = (double)hits / trials * 4;
      Out.println("trials=" + trials + ", pi=" + pi);
    }
  }
```

# Monte Carlo Simulation

Code Examples → Monte Carlo Simulation (Use Case)
`https://expert.ethz.ch/solve/oSd2kgsH3cxcec9Hs`