Computational Approaches to *D. rerio* Retinal Organogenesis

by

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Abstract

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Increasing integration of statistical and computational techniques into the analysis of cell and molecular biological data has created opportunities for explaining organogenic phenomena by numerical analysis. This thesis first examines the suitability of the most developed of these explanations for the development of the *D. rerio* eye, the stochastic mitotic mode explanation (SMME) for retinal progenitor cell (RPC) function, and demonstrates that it is neither the best available explanation, nor theoretically sound. It is shown that a deterministic alternative model is an improved explanation for the data, and that the SMME cannot explain postembryonic RPC function. These studies support earlier hypotheses of a linear progression of competency phases over the hypothesized "stochastic" nature of RPC lineage commitment.

A Bayesian model comparison framework, relying centrally on the Galilean Monte Carlo nested sampling algorithm for estimation of model evidence and posterior parameter distributions, is subsequently elaborated as an improved method for testing models in organogenesis. The utility of this approach is proved by comparing models of *D. rerio* RPC populations in the postembryonic Circumferential Marginal Zone (CMZ). The inferential framework is used to show that the activity of CMZ RPCs in the postembryonic period is best characterised by a decaying cell cycle rate, with similar activity across morphological axes, despite asymmetrical population dynamics. Changes to RPC lineage commitment over this time suggest an explanation for the changing photoreceptor mosaic. These studies establish that CMZ RPC behaviours change over time, whose postembryonic function is not adequately described as a recapitulation of an embryonic program.

Finally, the *D. rerio* mutant *rys* is shown to arise from a lesion in the npat locus, and the apparently paradoxically enlarged CMZ RPC population observed in these animals is demonstrated to arise from a failure of these cells to specify and exit the proliferative niche, and not from a proliferative defect. Nested sampling is used to show that unique populations of nucleosome positions in *rys* mutants and sibling arise from separate causal processes. A mechanism for these observations involving simultaneous changes to the subunit composition of the nucleosome pool, and loss of positioning control, is suggested.

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

This work is dedicated to the memory of my brother Gareth Akerman.

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Introduction

Thesis Guide

This document is split into three parts. Part I contains results and discussion pertaining to empirical modelling studies that will be of interest to committee members and developmental biologists. Readers wishing to confine themselves to results chapters should examine Chapter 2, Chapter 4, and Chapter 5. Part II contains technical reports pertaining to novel software that was written in order to perform the analyses presented in Part I. Part III contains supplementary materials arising from Parts I and II. These include detailed descriptions of the methods used in Part I, less-technical explanations of relevant statistical and model theory, the source code of all software and analyses, and the bibliography.

Synopses have been provided in italics at the top of each chapter in Part I. These consist of numbered statements, which summarise the accordingly numbered section in the text.

Readers of the .pdf document will find some text highlighted in plum throughout the thesis. Section indices highlighted in this way link to the specified section. Because empirical results in Part I are separated from relevant methods and code in Part III, links to the methods and code used to generate the figure's data and analysis are given in figure captions. Technical terms have also been highlighted when pertinent material is available in Part III to explain them for the reader who may be unfamiliar.

Readers who wish to replicate analyses, inspect datasets, etc., will find guidance in Chapter 14.

Terminology and Style

Throughout Chapter 1, Chapter 2, and Chapter 3, I have used the apellation "Harris" when referring to the output of William Harris' research group. This is a large body of research spanning several decades, and involves many co-authors. My use of "Harris" here is a convenience, as Harris is the only common author across the period in question, and presumably the agent carrying these ideas forward from project to project. It is not intended to slight or minimize the contributions of any of the other members of Harris' group (many of whom are now senior scientists in their own right). I have used "Raff" in an identical sense in referring to Martin Raff research group's work in Chapter 3.

I have preferred the terms "specification", "determination", and their derivatives, to refer to cells assuming a particular lineage fate. "Differentiation" is well-understood, but ambiguous, and often understood to relate to the mitotic event itself, which I generally do not intend. "A cell has specified" means it has assumed a stable macromolecular identity or "fate". This term is intended to correspond exactly with the appearance of stable markers of cell type.

Unless otherwise noted, formatting of quoted material is preserved, so that italic emphases appear in the original.

Part I

Modelling Studies

Chapter 1

Canonical retinal progenitor cell phenomena and their explanations

Synopsis

(1) The Stochastic Mitotic Mode Explanation (SMME) for retinal progenitor cell (RPC) function is the first computational model which claims to explain RPC behaviours throughout the life of a vertebrate, and so, retinal morphogenesis. (2) This would resolve decades of theoretical deadlock between competing explanations for RPC behaviours. (3) The majority of RPC phenomena are understood in terms of proliferative and fate specification outcomes of RPC lineages. (4) Macromolecular explanations for these outcomes are diverse, and difficult to reconcile into a harmonious explanation. (5) The SMME can be understood as an effort to find order by blurring out some pertinent explanatory details. (6) The SMME promises to clarify the macromolecular underpinnings of morphogenesis in a complex neural tissue by explaining it in terms of a stochastic molecular process.

1.1 The Stochastic Mitotic Mode Explanation (SMME)

The work presented in Chapter 2 examines the best-developed theory of *D. rerio* retinal progenitor cell (RPC) function. The theory in question is the work of preëminent retinal biologist William Harris' research group, referred to hereafter Stochastic Mitotic Mode Explanation (SMME). This theory purports to explain the function of zebrafish RPCs in terms of stochastic effects on mitotic mode, which specifies the mitotic state and fate propensity of offspring.

The SMME for RPC function is of fundamental theoretical and practical interest. It is a concerted effort to explain how a complex tissue like a retina can arise from a field of similar proliferating cells. In 2009, Harris coauthored a detailed review chapter, documenting the bewildering array of macromolecules and cellular processes thought to be involved in RPC function [AH09]. This enumeration contains many caveats and notes that the effects of particular macromolecules routinely differ between developmental stages, cell types, organisms, and so forth. At this time, no clear, detailed, comprehensive models of RPC function had been advanced, and the review is typified by statements like "It is difficult to reconcile

all the studies on the initiation and spread of neurogenesis in a single model."[1] It is therefore remarkable that, over the next nine years, a simple stochastic model of zebrafish RPC function invoking two named macromolecules would dominate the RPC literature.

The explanation aims to advance a simple, comprehensible, "mind-sized" model of RPC function, which is a microcosm of the broader promise of "Systems Biology" to make sense of the contemporary welter of conflicting datasets. By using sophisticated mathematical methods drawn from information and complexity theories, the apparent confusion will be clarified and underlying molecular mechanisms will be revealed. Given Harris' track record and preëminence in the field, we have good reason to take seriously the possibility that the SMME has succeeded. Examining whether this is the case is our first priority. To appreciate the scope of his theoretical maneuver, and possible alternatives to it, we begin by summarising the state of the art at the time of his 2009 review (as well as relevant subsequent additions).

1.2 Explanations for RPC function in 2009 and the drive to unification

Molecular biologists have sought explanations for the same remarkable features of retinal progenitor cells for decades. Animal retinas, having well-understood functions and stereotyped structures, promise tractable targets for typical molecular biological explanations. With well defined cell types present in tightly regulated proportions and topological organisations, both theoretically-inclined molecular biologists and clinically-inclined regenerative medical practitioners have found the retina to be a useful model tissue[2]. The regular, easily detected order in eyes seemed to suggest a similar level of order and regularity in the macromolecular processes which underlay the formation of the tissue. Older explanations commonly suggested that RPCs are more-or-less identical and go through rigidly stereotypical macromolecular processes. It is, however, the persistently observed departures from this conception that have occupied much of our attention.

The simplest explanation for the regularity of retinal development is that any given RPC is executing the same strict "developmental program" as its neighbours. If every progenitor produces a similar number of cells, and the progeny are specified in similar proportions, well-understood principles of cellular adhesion could give rise to the characteristic laminar organisation observed in animal retinas. However, by the 1980s, vertebrate lineage tracing experiments revealed a surprising degree of inter-lineage variability in many neural progenitor systems, not least of which was the retina. In their seminal 1987 paper, David Turner and Connie Cepko, using retroviral lineage labelling techniques, demonstrated that individual RPC lineages in rats had diverse proliferative and fate specification outcomes [TC87]; Harris' group confirmed this result in *Xenopus* the subsequent year [HBEH88], suggesting this variability was a common feature of vertebrate RPC function.

Indeed, at this point, we find fairly clear accounts of what retinal biologists took their theoretical options to be in explaining this variability. As Harris' 1988 report states:

> Changes in cell character associated with cell type diversification may be controlled in an autonomous way, reflecting either a temporal program inside the cell (Temple and Raff, 1986),

[1] This was in no sense a problem with Harris' understanding. A similarly high-level review coauthored by Pam Raymond [AR08] concluded, with regard to models of photoreceptor fate specification: "The data reviewed in the preceding sections indicate that a 'one-size-fits-all' model is not possible..."

[2] Indeed, if central neuroregenerative medicine is to become a clinical possibility, it seems likely that the theoretical and practical issues are most likely to be resolved in eyes before other areas of the CNS.

> the asymmetrical segregation of cytoplasmic determinants (Strome and Wood, 1983; Sulston and Horvitz, 1977), stochastic events inside the cell (Suda et al., 1984), or some combination of these processes. Alternatively, cell type may be controlled in a nonautonomous way, as in cases in which the extracellular environment (Doupe et al., 1985) or cellular interactions (Ready et al., 1976) elicit or limit cell fate. With its multiplicity of cell types, the vertebrate nervous system would seem to require the ultimate sophistication in its means of cellular determination. [HBEH88]

It is striking, then, that Harris' review of the literature two decades later describes the situation similarly:

> Once differentiation is initiated, regulatory mechanisms within the retina ensure that progenitors retain the capacity to undergo more divisions, in parallel with churning out differentiated cells, and that progenitors cease dividing at variable times. There is still debate about the extent of early programming that allows progenitors to step through a series of stereotypical divisions and the extent of regulation from within the whole retina. The production of differentiated cells alters the retinal environment with time...
>
> Moreover, cells from the same clone do not all differentiate at the same time, suggesting three possibilities: a stochastic mechanism for the decision to differentiate, exposure of the two daughters to different environments, or asymmetric inheritance of determinants. [AH09]

In the same paper, he states that the "simple structure and accessibility of the retina make it a useful model to study cell division and differentiation, and as a result most aspects of this have been studied, from lineage tracing of progenitors, to the morphological aspects of division, to the molecular mechanisms involved." Thus, by Harris' own account, some twenty years of additional research into almost every variety of macromolecular explanation for a huge range of RPC-related phenomena had not provided any means to narrow down the possibilities he had already laid out in 1988. We still have Raff's temporal program ("early programming ... step[ping] through a series of steretypical divisions"), asymmetric segregation of cytoplasmic inheritants during mitotic events, "stochastic" events internal to the cells, and possible "environmental" extracellular determinants. While the number of particular macromolecules functionally implicated in proliferative, specificative, and organisational RPC phenomena had greatly expanded, this had not provided any means to differentiate between these theoretical options. The incremental ennumeration of macromolecules involved in neurogenesis has not given rise to a coherent model of RPC function. Harris' SMME therefore represents an example of a "Systems" biological explanation, in which biologists apply the analytical and interpretative methods of the physical and mathematical sciences in an effort to resolve the problems posed by biological complexity, without attempting to assemble all of the relevant observations of previous studies into a single explanation [Mor09].

Before proceeding to the SMME itself, let us briefly summarise the diversity of phenomena implicated in RPC function by 2009, as well as the panoply of mechanisms offered as explanations. In doing so, it will become clear what has been elided in the SMME, and what may need to be restored in any alternative modelling approach.

1.3 Canonical vertebrate RPC phenomena: the RPC "morphogenetic alphabet"

The bulk of our knowledge of RPC behaviour stems from histological observation employing a limited number of techniques. Simple observations of mitotic figures in a variety of animals had, by the 1950s, revealed the surprising diversity of RPC proliferative phenomena across vertebrate clades. However, it was the advent of lineage tracing techniques, particularly those marking single clonal lineages in whole retinae, and the extensive use of these techniques in the 1980s-90s, that formed most of the basic body of observations that any macromolecular explanation is now called upon to account for.

Since the majority of vertebrate retinas of biomedical interest are mammalian, and these retinae are fully formed in an early developmental period, RPC behaviour has been best-studied in an embryonic and early developmental context. Here, vertebrate RPCs are derived from the eye field population of the early neural plate and later neural tube. This population is separated into left and right eye primordia, which in turn pouch outwards toward the ectoderm, and, in conjunction with the lens placode (itself induced from the ectoderm), form the optic vesicle. The primitive eye is formed when this vesicle completes a complex morphological folding process, resulting in the cup-shaped structure of the retina [Cav18]. During this process, the cells of the neural retina are differentiated from the overlying retinal pigmented epithelium (RPE). Sometime after the formation of the retinal cup, RPCs begin to exit the cell cycle and are specified as retinal neurons. Studies of this early period revealed numerous difficult-to-explain features of RPC behaviour. Following Larsen's observation that tissue form is attributable to only six behaviours [Lar92], which she describes as the "morphogenetic alphabet", I have categorised RPC phenomena as relating to proliferation, fate specification, migration, growth, death, and extracellular matrix formation. Since the vast majority of reported phenomena fall under the first two categories, proliferation and specification, these are given separate sections below, while those belonging to the last four are described collectively.

1.3.1 Proliferative phenomena

Clonal lineage tracing experiments reliably find that vertebrate RPCs give rise to highly variable numbers of offspring over the collective "lifetime" of the lineage. The most dramatic of these findings demonstrate that rat RPC lineage sizes vary across two orders of magnitude in vivo, from 1 to over 200 [TSC90]. While at least some of this variability must be related to differential integration of lineage markers into "older" (giving fewer offspring) and "newer" (giving more) RPCs, vertebrate RPC lineages nevertheless differ widely in their fecundity. The physical organisation of these clones is complex; as detailed below, RPC progeny may appear in any of the 3 retinal layers in a wide variety of specified fate combinations, and may engage in short-range migrations to appropriate positions for their specified fates. Most clonal lineages are "extinguished"; that is, after some time, all of its members have become postmitotic. However, it has long been noted that not all cells produced by RPCs are strictly postmitotic neurons; specified Müller glia retain the ability to reenter the cell cycle in response to stimuli (normally, retinal damage) [DC00, FR03], and peripheral circumferential marginal zone (CMZ) RPCs remain proliferative in those vertebrates whose eyes grow beyond early development (notably in frogs and fish, while the chick retina has a CMZ of more limited output [FR00]). Therefore, some clonal lineages may be organised into clumps associated with Müller responses, while others in frogs and fish may continue to be "plated out" in a more-or-less linear manner at the retinal periphery for as long as the lineage "lives" [CHW11]. This

ongoing RPC contribution to peripheral neurogenesis has long been recognised, so that by 1954 we find the following statement introducing a study of unusual mitoses in the retina of a deepsea fish:

> It is conventional[3] to hold that the growth of the vertebrate retina is only possible due to the presence, in this tissue, of a peripheral germinal zone. In this region, young elements actively multiply, and, by subsequent differentiation, give rise to the diverse nervous and sensory constituents of the retina. [VL54]
>
> [author's translation from the French]

Despite this, the proliferating RPCs in this "peripheral germinal zone" (also known as the "ciliary marginal zone", or CMZ, for its proximity to the retinal ciliary body) have not received the same level of attention as those associated with the central retina. As a consequence, these RPCs are generally treated as though they are a type of "frozen" progenitor population, recapitulating spatially, along the peripheral-central axis, the process which RPCs in the central retina undergo in a time-dependent fashion [HP98]. Figure 1.1 displays a coronal cryosection through a zebrafish eye, with the proliferating RPCs of the CMZ highlighted in magenta; these cells contribute to all three of the cellular layers of the neural retina in adulthood.

The length of the RPC cell cycle is of considerable interest, since the evolution of this parameter in time, in conjunction with the RPC population size (the number of cells specified in the eye field), determines the eventual size of differentiated retinal neural population, and therefore the retina. RPC cell cycle length is usually inferred from clonal lineage size, although it has also been assayed directly in cumulative thymidine analogue labelling experiments [AC96]. Vertebrate RPCs undergo a period of relative quiescence, in which the cell cycle lengthens, before the neural retina begins to be specified (in zebrafish, this period is 16-24 hpf). The cell cycle shortens as RPCs begin to exit the cell cycle [HH91, LHO+00]. After the central retina is specified, the RPC cell cycle again lengthens, and is presumed to continue to slow until RPCs have completed specifying [4].

Finally, the orientation of the RPC division plane in mitosis is also implicated in retinal organisation. The orientation of divisions is associated with the both proliferation and fate specification of RPC progeny. For instance, interfering with spindle orientation in the developing rat retina, such that more RPC divisions occur parallel to the neuroepithelial plane (rather than along the apico-basal axis) results in more proliferative and fewer postmitotic, specified progeny [ŽCC+05]. That said, it seems that whatever effects are attributed to mitotic orientation are likely species-specific, as zebrafish RPCs display a different pattern of axis orientation, dividing mainly in the epithelial plane [DPCH03].

1.3.2 Fate specification phenomena

Offspring of vertebrate RPC lineages may enter any of the three cellular layers of the retina. Moreover, single lineages can include any possible combination of cell fates, so that RPCs cannot be readily divided into "types" on the basis of lineage fate outcomes [HBEH88, TSC90, WF88]. While some progenitors have propensities to generate similar cell types, these relations seem species-specific, and may not define separate progenitor pools [AR08]. In general, then, RPCs are taken to be multipotent with respect to the neural retina- all of the cell types[5] of the differentiated retina are derived from similar RPC

[3] Unfortunately, I am unable to locate the source of this convention, likely due to the poor preservation of many of these older reports. That this required no citation in 1954 suggests the original observations of CMZ proliferation may be in the early 20th century.

[4] The work presented in Section 8.4 subtantially confirms this presumption, at least after 3dpf in the zebrafish.

[5] The "cell type" concept is unusually well-defined in the retina, as there are an abundance of distinct morphological and molecular features which differentiate numerous subtypes of the seven general types of retinal neuron.

Figure 1.1: **Section of 3dpf zebrafish eye, displaying CMZ RPCs**
Maximum intensity projection of micro14 [metre] central coronal cryosection of 3dpf *D. rerio* eye, displaying proliferative cells labelled with anti-PCNA (magenta) against a Hoechst 33342 nuclear counterstain (green). CMZ: Circumferential marginal zone. GCL: Ganglion cell layer. INL: Inner nuclear layer. ONL: Outer nuclear layer. RPE: Retinal pigmented epithelium. In animals with a CMZ, the retina is grown by peripheral addition to the GCL, INL, and ONL by the CMZ.

lineages. Little about this picture has changed since its initial development, using a variety of lineage tracers (including retroviruses, thymidine analogues, and injectable dyes) and histochemical markers to supplement morphological identification of specified neurons. In particular, sophisticated modern live imaging experiments in zebrafish (many pioneered by Harris), have broadly confirmed the findings of the 80s and 90s in mammalian fixed specimens, explants, and live animals [BRD+15].

Of particular note are the observations of the Raff group [WR90, CBR03], who demonstrated that dissociated rat RPCs, cultured at clonal density, took on morphological and histochemical features associated with different specified neural types in similar numbers and proportions, and on a similar schedule, to same-aged RPCs cultured in retinal explants. These results dramatically suggested that both the proliferative and fate specification behaviour of RPCs depends less on intercellular contact, and the complex signalling environment of the developing retina, than on factors intrinsic to the RPCs themselves. These studies contain the essential germ of Harris' eventual commitment to SSM explanations, purporting as they do to "test the relative importance of cell-intrinsic mechanisms and extracellular signals in cell fate choice", and providing convincing evidence for the preponderant importance of the purported cell-intrinsic mechanisms.

Figure 1.2: **Histogenetic birth order of retinal neurons**
Adapted from [You85] by [CAY+96], Copyright 1996 National Academy of Sciences. G.C., Ganglion Cells; H.C., Horizontal Cells. Data from embryonic and perinatal mouse eyes.

In spite of the apparent ability of RPCs to produce offspring specified to any of the possible neural cell fates, at any time during their lineage history[6], vertebrate retinal development displays a temporal ordering, such that in any particular location, retinal ganglion cells (RGCs) tend to be produced first, followed by the other cell types in what has been described as an overlapping "histogenetic order", pictured in Figure 1.2. As noted above, RPCs also exit the cell cycle in a spatiotemporally defined order (from central to peripheral over time), and specification follows the same pattern[7]. This naturally gave rise to questions about the origin of the "overlap" observed in the sequential production of various cell types; conceptually, this overlap could be produced by identical RPCs executing identical rigid specification programs if they begin to execute it along the spatiotemporal gradient noted above, as originally suggested by Cepko et al. [CAY+96]. However, it is impossible to reconcile this notion with more recent results from Harris' *in vivo* zebrafish lineage tracing studies [DPCH03, HZA+12, BRD+15], which confirm mammalian cell culture work in demonstrating that vertebrate RPC lineages do not execute similar specificative programs. More recent articulations of Cepko's concept of linear specification

[6]Even in papers arguing for a strict, linear sequence of specificative outcomes in all RPC lineages, the actual data show RPCs occasionally giving rise to "late-born" photoreceptors subsequent to their first division [WR09], giving lie to the notion that the process is particularly strict.

[7]In many studies, cell cycle exit is conflated with specification such that evidence of the former is taken as evidence of the latter. There are, however, reasons to believe cell cycle exit and specification are not the same process, discussed below.

programs incorporate variable sub-programs to account for this [Cep14].

1.3.3 Other morphogenetic phenomena

The outcomes of RPC proliferation and lineage commitment are often taken as sufficient explanation for the formation of the neural retina. Indeed, after the eye cup has been formed (prior to the specification of any neurons), RPCs are, in effect, already "in place". Therefore, there are few documented RPC phenomena outside of the "proliferation" and "specification" categories of the morphogenetic alphabet [Lar92], which ennumerates the cellular processes contributing to tissue form and structure. RPCs do not seem inclined to migrate long distances , they seem not to generate much in the way of extracellular material[8], and, in non-pathological conditions, they are rarely seen to die. Still, there are a number of phenomena which appear to be important for proper retinal organisation that fall into these other "alphabet" categories.

The most notable of these is interkinetic nuclear migration (INM), in which RPC nuclei move back and forth between the apical and basal surfaces of the retina. Common in many neural tissues, INM affects both proliferation and fate specification, but is dissociable from them- both cell cycle and specification proceed (albeit in a precocious manner) when INM is disrupted [MZLF02]. The apical retinal surface provides a microenvironment which appears to be required for RPC mitosis to occur[9], and INM seems to consist of a directed, probably actomyosin-mediated movement to the apical surface, followed by an undirected "random walk" away from the apical surface and hence toward the basal retina [NYLH09]. This "random walk" may arise from displacements due to the directed INM of neighbouring progenitors [AHW+20]. More committed RPCs also appear to actively migrate to positions appropriate for the specified cell type [CAR+15, IKRN16], although it is unclear if this short-range migration is caused by INM undergone by more actively proliferating progenitors. It seems likely that these short-range migrations of more specified cells are especially significant in the early retina, before the neural plexiform layers (consisting of axons and other neural processes) have begun to divide the cellular layers into bounded compartments.

Notable gaps in the study of the RPC "morphogenetic alphabet" include the regulation of cell size and growth, and potential roles for cell death, both of which have received little attention. Cell size and growth are closely linked to proliferative behaviour in yeast [YDH+11], and studies have been conducted in Schwann cells and oligodendrocytes [CDMR01], but related effects in RPCs have not been elucidated. This may be a significant oversight, given the requirement for RPCs to continuously grow during their proliferative lifespan. Cell death does not seem to play the same "pruning" role for RPCs that it often does in other neural tissues, and observed rates of cell death in normal RPC populations are very low, so few studies have been conducted.

[8] The formation of a laminin-rich basement membrane seems to be necessary for optic vesicle formation [ICW13], but it is not clear that RPCs produce this. ECM function in eye formation remains under-studied.

[9] Nonapical divisions do occur, notably in specified, but proliferative cells [GWC+07]. The extent to which RPCs depend on the apical surface may depend on how "RPC" is defined. In general, RPCs are no longer considered as such when they acquire characters associated with differentiated neurons (cell type markers, morphological traits, etc.), although it is clear that the acquisition of these characters does not necessarily imply that the cell is postmitotic. Any complete explanation of how RPCs give rise to the structure of the eye must also consider these nonapical divisions.

1.4 Macromolecular mechanistic explanations for RPC phenomena

Having surveyed the cellular phenomena pertaining to RPC function in retinal development, we proceed to a selection of the many macromolecular mechanistic explanations (MEx) [Fag12] that have been offered to explain them. Unsurprisingly, given the field's focus on the early events of eye development, these MEx are mainly targeted at this period. Thus, the majority explain tissue-level phenomena like the initial "wave" of cell cycle exit and specification, without necessarily seeking a global explanation for RPC behaviour irrespective of context, so it is unknown how many of these might pertain to ongoing peripheral neurogenesis or other, adult neurogenic phenomena like those exhibited by Müller glia. That said, we now turn to examine some of the best-developed of these explanations.

1.4.1 Transcription factor networks

Perhaps the most notable MEx offered to explain RPC specification and development is the eye field transcription factor network, or EFTFN. The roots of this explanation are found in the Pax6 "master gene" explanation popularised in the 1990s [Geh96]. This explanation revolved around the apparently universal involvement of Pax6 gene products in eye formation in model organisms, and the promiscuous inter-species effects of Pax6 (with mouse RNA able to induce ectopic formation of eye structures in *Drosophila* imaginal discs, for instance [HCG95]), so that it appeared to be a highly conserved genetic "switch" for eye development.

Importantly, this explanation purported to resolve what Darwin regarded as a serious problem for his theory, the apparent implausibility of the gradual evolution of eyes (and other "organs of extreme perfection") from some primitive ancestral structure [Dar88, p.143-4]. In particular, Pax6 suggested to some theorists an alternative to the surprising hypothesis of Mayr and Salvini-Plawen, that differences in eye structure and function across clades indicate the independent appearance of eyes in more than 40 clades [vM77]. Pax6 thus provided a molecular pointer to a potential common ancestor for all animal eyes [EHML09].

Subsequent investigations revealed that vertebrate Pax6 is a conserved member of a complex network of cross-activating and inhibiting transcription factors, including Pax6, Rx1, Six3, Six6, Lhx2, ET, and Tll [Zub03]. Members of this network tend to promote proliferation and suppress markers of differentiated neurons, and their loss commonly results in the failure to form the eye field at all [AH09]. The expansion of this explanation to include other TFs in a network revealed significant differences between species [Wag07]- while the role of Pax6 is conserved between *Drosophila* and vertebrates, the roles of other members of the EFTFN are not. Moreover, the universality of Pax6 was only apparent, and not real, as there are bilaterian eyes whose development is Pax6 independent (including in *Platynereis, Branchiostoma*, and planarians) [Koz08]. This highlighted the great difficulty in connecting morphological characters such as those observed by Mayr with a genetic basis- it is simply not clear that Pax6 conservation points to a common ancestor for all eyes, or even all photoreceptive neurons[10]. Moreover, expansion of the monocausal "master gene" explanation, to include a network of TFs with broad gene regulatory effects, highlighted the problems of complexity in offering MEx for RPC function. The components of this network interact in complex, context-dependent ways. While the EFTFN as a whole is

[10]Indeed, the relevant "unit" of homology for evolutionary explanations for eyes remains contested, with some arguing for the cell itself over any particular set of gene sequences [EHML09]

taken to promote RPC proliferation and to delay specification[11], its components have also been held responsible for the specification of particular classes of differentiated neurons, and remain expressed in those postmitotic cells. Notably, Pax6 is implicated in the expression of bHLH TFs required to specify multiple classes of retinal neuron [MAA+01], and is known to directly activate Ath5, necessary for RGC specification [WSP+09]. The EFTFN has thus been offered as an explanation for the maintenance of the multipotent, proliferative RPC state, but how this network is disassembled, and its components repurposed to promote specification, remains obscure.

The EFTFN is not the only transcription factor network offered as a MEx for RPC function. Another well-developed explanation involves Chx10 (aka vsx2), a transcription factor important for normal proliferation of RPCs, its loss causing microopthalmia in the mouse [BNL+96]. Chx10 was subsequently found to repress Mitf, involved in RPE specification, and hence to promote neural retinal fates over pigmented epithelial ones [Hor04]; in the absence of Chx10 the early eye cup does not stratify properly between apical pigmented cells and the neural retina. Much like the multifunctional EFTFN components, Chx10/vsx2 has also been implicated in the specification of particular neural fates, notably bipolar neurons [BNL+96] and the regulation of Vsx1 (a paralogue of Chx10), Foxn4, and Ath5, associated with specification of subpopulations of bipolar cells, horizontal and amacrine cells, and RGCs and PRs, respectively [CYV+08, VJM+09].

Clear hypotheses advocating for particular relationships between different TF-centric explanations are rarely stated. It is tempting simply to arrange them in some kind of "developmental order", perhaps with the Chx10-Mitf network "downstream" of the EFTFN. That this would be facile is evident from the changing roles of these transcription factors, depending on developmental and cellular context. To date, no unifying framework has been applied. Obvious candidates include the "developmental gene regulatory network" concept [LD09], a type of cybernetic explanation which assembles genes into feedback networks. Given the popularity of this type of explanation, it is worth noting that no one has yet had any success in offering one for RPC function.

These transcription factor network MEx frequently incorporate extracellular signals (often as an explanation for the appearance or "set-up" of the TF network), and it is generally recognised that these signals have a profound influence on these networks, and on RPC behaviour generally. We therefore turn to explanations invoking these signalling mechanisms.

1.4.2 Intercellular signalling networks

Virtually every developmentally significant class of signal had been implicated in RPC function by 2009 [AH09]. These include BMP, CNTF, FGF, Glucagon, Hedgehog, IGF, Notch, TGFα, TGFβ, VEGF, wnt, and a host of neurotransmitters. This diversity of signalling pathways has proved to be a formidable problem for integrated explanations, since almost all of these pathways converge on the same two cellular outcomes in RPCs, that is, proliferation and specification. Thus, most signalling MEx for RPC function elide the majority of other signals which are known, or thought, to affect the same processes. That said, let us explore a few of the more detailed signalling explanations.

In developmental terms, the first phenomenon requiring explanation is the appearance of the eye field to begin with- what is it that accounts for the differentiation of RPCs from the rest of the anterior neural

[11] This is sometimes referred to as "promoting RPC fate", since RPCs are taken to be those cells which proliferate but do not yet display markers of specification. Since it is, by now, widely recognised that cells that appear to be well-specified may remain in cell cycle [GWC+07, ESY+17] this terminology should probably be jettisoned.

plate and tube? Wnt signalling MEx have been offered to explain the appearance of the *Xenopus* eye field. Fz3 signalling seems to promote expression of eye field transcription factors (see below) [RDT+01], while an unspecified non-canonical interaction between Wnt11 and Fz5 inhibits canonical β-catenin signalling through Wnt8b/Fz8a, which would otherwise promote prospective anterior forebrain fates [CCY+05]. Inhibition of FGFR2 signalling, and activation of ephrinB1 signalling have also been implicated in early *Xenopus* eye field cell movements [MMDM04]. Subsequent experiments determined that the xenopus ADP signalling through the P2Y1 receptor directly activates the EFTFN [MBE+07], described above. More recent experiments in zebrafish suggest that precocious acquisition of neuroepithelial apicobasal polarity, probably driven by interactions with a Laminin1 basement membrane, distinguishes the early eye field [ICW13].

Subsequent to the appearance of eye field RPCs and their rearrangement into the optic cup, the apparent central-to-peripheral "wave" of RPC exit from cell cycle and specification of early RGCs [HE99], has had detailed MEx advanced to explain it. In both zebrafish and chick retina, FGF3 and FGF8, originating from the optic stalk, initiate this early cell cycle exit and specification [MDBN+05], while inhibiting FGF signalling prevents this from occurring, and ectopic expression of FGF can cause it to occur inappropriately. The progression of this "wave" of cell cycle exit and specification has been separately explained, by Sonic Hedgehog (Shh) signalling from the newly specified RPCs inducing cell cycle exit and specification in adjacent cells [Neu00]. This process is dependent on, and downstream of, the above-mentioned FGF induction [MDBN+05]. The role of Hh signalling has been challenged on the basis that Hh inhibition in subsequent experiments did not display the same effect size [SF03], and that its effects on Ath5 expression, required for RGC specification, are ambiguous [AH09]. More recent MEx have suggested that Hh signals may decrease the length of the cell cycle, resulting in increased proliferation and earlier cell cycle exit and specification [LAA+06, ALHP07].

A well-developed "local" signalling MEx invokes the classic Notch/Delta lateral inhibition model, with small fluctuations in Notch/Delta activity giving rise to a positive feedback response that differentiates neighbouring cells. Cells which have high Delta expression tend to be specified as retinal neurons, while those with high Notch tend to remain proliferative, either as RPCs or Müller glia [DRH95, DCRH97]. More recent results implicate asymmetric inheritance of Sara-positive endosomes in the partitioning of this signal [NRN20].Such a mechanism could regulate the activity of both early, central RPCs, as well as peripheral RPCs, and may contribute to inter-RPC variability. These differences between RPCs located in different parts of the developing retina have been of significant interest, and it is worth briefly examining patterning MEx that may also explain spatial differentiation between RPCs.

1.4.3 Patterning mechanisms

Among the most interesting features of RPCs is that they reliably give rise to specified neurons, in particular RGCs, that seem to "know where they are" in the retina, enabling them to wire their axons in correct retinotopic order in the superior colliculus (SC) or optic tectum (OT). The most developed MEx explaining this refer to gradients of EphA and EphB receptors expressed in RGCs, and their respective ephrin ligands expressed in the SC or OT. In the retinal RGC population, an increasing nasotemporal gradient of EphA is paired with an increasing dorsoventral gradient of EphB. A corresponding increasing rostrocaudal gradient of ephrin-A is paired with an increasing lateromedial gradient of ephrin B in the SC/OT. This allows for a two-axis encoding of an RGCs' position in the retina [TK06]. As the RGCs' axon pathfinding depends on repulsive effects mediated by Eph receptors, this code is sufficient to allow

correct wiring of even single RGCs [GNB08]. The action of Gdf6a seems to establish this code in RPCs themselves, prior to specification [FEF+09].

Indeed, there are numerous similar observations of expression gradients that create spatial differences between RPCs themselves. Most relevant to the proliferation dynamics highlighted in this chapter is the observation that, in *Xenopus* eyes, a decreasing dorsoventral gradient of type III deiodinase renders the cells of the dorsal CMZ refractory to thyroid hormone (as the deiodinase inactivates TH) [MHR+99]. The effect of this is to set up a differential response to TH in post-metamorphic RPCs, so that the ventral population selectively expands in response to TH [BJ79].

These patterning mechanisms are of particular interest here, in large part because they clearly establish that the RPC population is heterogenous, both with respect to proliferative and specificative behaviours, and perhaps others as well. This is of critical importance for any modelling effort, as virtually all mathematical models used by stem cell biologists (and those used to justify Harris' SMME) assume, at least initially, homogenous populations of stem or progenitor cells. Since RPCs do not meet this condition, special care is needed to use these models.

1.4.4 Chromatin dynamics

In recent years, the great importance of chromatin conformation in RPC proliferation and specification has become more clear. Indeed, chromatin dynamics are now widely invoked in explaining stem and progenitor cell behaviour, and suggested as a target for cell reprogramming [Kon06, TR14]. In RPCs, detailed accounts of three-dimensional chromatin dynamics have yet to appear. However, a number of studies point to the importance of chromatin state in informing the overall cellular state. In particular, histone deacetylation seems to be important for RPC specification, as the loss of histone deacetylase 1 (HDAC1) in zebrafish results in overproliferation and decreased specification, and correlated increases in Wnt and Notch activity [Yam05]. In mouse retinal explants, decreased proliferation and specification result from pharmacological inhibition of HDAC [CC07]. Additionally, the chromatin remodelling complex SWI/SNF has repeatedly been implicated in RPC function. Notably, one particular component of this complex seems to be particularly associated with vertebrate RPCs (BAF60c, an accessory subunit) [LHKR08]. A switch to other subunits seems to be necessary for specification [LWR+07]. Details regarding the subunits involved in specification and their downstream effects are complex and context dependent, much like the signalling pathways mentioned above.

1.5 A unified theory of RPC function? "Blurring" to order

From the foregoing discussion, we can see the theoretical conundrum. Macromolecular explanations for RPC behaviours, like those throughout the molecular biological tradition, have generally been built outwards from particular transcription factors, receptors, etc. The result is an archipelago of MEx, at best connected by tenuous speculation, and in most cases, without any known means to form an integrated model. Furthermore, the degree of complexity and context-dependence evident from the literature might seem to preclude such a model. As we have seen, multiple reviews found that the evidence did not allow for clear discrimination between logically distinct types of mechanisms for producing the observed variability in RPC lineage outcomes.

In this situation, there were two theoretical options. The first is simply to "crank the handle"- to generate more and more facts describing the difference particular molecules make to RPC outcomes

in dozens of relevant contexts, piling up exceptions and idiosyncracies, in the hope that doing so will eventually bridge the explanatory "islands" of the MEx archipelago. This has been referred as the enumerationist approach by Kaneko, who ably explains why it is doomed to failure [Kan06, pp.31-32]. The second option, the one actually chosen by Harris, is more theoretically sophisticated. As Nicholas Rescher has noted, regarding in-principle limits to scientific knowledge, the phenomenal universe has infinite descriptive complexity- one can always add more detail to a description of some phenomenon, and no such description is ever complete [Res00, p.22-9]. Moreover, "even as the introduction of greater detail can dissolve order, so the neglect of detail can generate it." [Res00, p.62] As Rescher goes on to comment:

> [W]e realize that in making the shift to greater detail we may well lose information that was, in its own way, adequate enough ... information at the grosser level may well be lost when we shift to the more sophisticated level of greater fine-grained detail. The 'advance' achieved in the wake of 'superior' knowledge can be - and often is - purchased only at a substantial cognitive loss.
>
> ...
>
> It is tempting on first thought to accept the idea that we secure more - and indeed more useful and more reliable - information by examining matters in greater precision and detail. And this is often so. But the reality is that this is not necessarily the case. It is entirely possible that the sort of information we need or want is available at our 'natural' level of operation but comes to be dissolved in the wake of greater sophistication. [Res00, p.65-6]

Rescher's greater point is that "blurring" detail, at levels below the phenomenal one under consideration (for RPCs, generally, the cell or lineage), may be necessary to produce an ordered explanation that is useful for some objective. An example of this practice in macromolecular modelling is coarse-graining, where irrelevant atomic detail is abstracted in favour of more computationally tractable approximations [ILU+14]. "Blurring" in this fashion also enhances the cognitive value of models by keeping them "mind-sized"; at our natural level of operation, as Rescher has it. Given the number of overlapping and contradictory MEx for RPC behaviour, we have the situation Rescher is describing- more sophistication, and more detail, has dissolved order, not revealed it[12]. The inability to assemble an unified explanation for RPC function has left us without fundamental understanding of how highly ordered neural tissues like eyes are generated from composites of units with highly variable, temporally and spatially ordered outcomes like RPC lineages. As this is a common feature of vertebrate neurogenesis more generally, this leaves us without the ability to produce complete models of neurodevelopmental processes in many species. Moreover, in the absence of a clear framework for comparing the explanatory power of the diverse array of MEx so far advanced, practical contributions of clinical relevance have been scanty and tentative, with RPC transplantation, and more recently, gene therapies taking little note of complex MEx for RPC function [CAI+04, GS07, YQW+18].

In a situation of this kind, this type of "blurring" is required, and it seems that by cutting down to the simplest possible explanation, the SMME hopes to bring into view order that was previously obscured by detail. There is, of course, a significant danger here: how does one decide what is "blurred out" and what remains? We can easily understand how a practicioner's biases could lead to a sort of relativism, where

[12] At least part of this problem is likely related to the fact that the majority of biomedical findings cannot be replicated [Ioa05]. The finding, mentioned above, that Shh effect sizes on RPC function were not as large as initially reported when subsequently investigated, is typical and symptomatic of this replication problem. "Blurring" may therefore be necessary not only because of fundamental epistemic limits, but also because it is often difficult to distinguish bona fide results and explanations from spurious ones.

the "blurring" makes apparent a spurious order that conforms to these biases rather than to reality as such. With this in mind, let us survey the general thrust of the SMME, before proceeding to examine it in detail, in Chapter 2.

1.6 Explanatory Strategy and Intent of the SMME

As we have seen in Section 1.2, Harris' long-held understanding of the explanatory options for RPC function divides them into four broad categories:

1. A linear algorithmic "program" of proliferation and specification
2. Asymmetric segregation of specificative determinants
3. "Stochastic processes" internal to the cells
4. Influences of extracellular factors

Harris' sophisticated discussions of RPC MEx rarely treat these categories as exclusive, and concede that good explanations for RPC behaviour may involve phenomena from more than one of them. Indeed, the SMME necessarily contains elements that Harris concedes are "linear" and "deterministic" [HZA+12]. Still, his overall strategy for the SMME is, first, to substantiate the predominant influence of one of these categories of phenomena (that is, category 3, internal stochastic processes or effects), and subsequently to specify an actual macromolecular system that could plausibly be such an "internal stochastic process". These two theoretical maneuvers, while tightly linked, serve different purposes within Harris' overall explanatory framework, which must be examined separately.

The SMME for zebrafish RPC function has been developed across three separate papers [HZA+12, BRD+15, WAR+16]. Each builds on the earlier publications, collectively purporting to explain the behaviour of RPCs wherever, and whenever, they may be found in the zebrafish eye. The underlying model is originally derived from an earlier paper pertaining to rat RPCs [GZC+11]. He et al. [HZA+12] and Wan et al. [WAR+16] use essentially the same model and make up the substance of the first of these two maneuvers. Boije et al. [BRD+15] substantially modifies this model, specifying the activity of two known transcription facts (Ath5 and Ptf1a) as the model's biological referents. This paper constitutes the second theoretical thrust.

The first maneuver intends to support the contention that zebrafish RPCs are equipotent progenitors with variable lineage outcomes that depend on independent "stochastic" processes within each of these cells. This is to be provided by demonstrating that a Simple Stochastic Model (SSM) of an RPC, numerically simulated many times by Monte Carlo methods to represent a population of RPCs, produces similar outcomes to populations of RPCs in vivo. This explanatory strategy is common in the stem cell literature, the original example having been published in 1964 by Till, McCulloch, and Siminovitch [TMS64]. The SMME therefore represents an example of a traditional scientific logic- an explanatory pattern deployed by stem cell biologists in diverse contexts, and widely accepted because of its ongoing use in the literature, although with varying interpretations.

The success of this first maneuver thus depends on two outcomes. Firstly, the output of the SSM should accurately reflect the observed proliferative and specificative outcomes of zebrafish RPC lineages, giving weight to Harris' claim that it "provides a complete quantitative description of the generation of

a CNS structure in a vertebrate in vivo" [HZA+12][13] Secondly, the internal structure of the SSM should provide good reason to believe that one of the stochastic options is a better explanation for RPC lineage outcomes than those identified by the other three categories of theoretical options ennumerated above.

The second theoretical maneuver is the specification of particular biological referents for entities in the model. This offers an opportunity to move beyond a purely conceptual argument about the kind of process that might produce variable RPC lineage outcomes, and to begin the work of explaining how the behaviour of a particular macromolecular system constitutes such a process, so that empirically verifiable hypotheses may be generated. The success of this maneuver depends on the biological plausibility of the identification between model structure and the biological function of transcription factors, Ath5 and Ptf1a, that the model names. A good SSM-based explanation would point the way for further research by identifying *how* so-called "stochastic processes" in RPCs might function, and make some predictions about this. In order to address these issues, we use model comparison, which is introduced and detailed in Section 15.1. With that said, let us turn to the SMME and determine how well these manuevers have succeeded.

[13]This claim is somewhat extravagant; the SSM, by definition, includes no spatial information, so it is unclear how one could be a "complete description" of any spatially organised structure. Still, it can be complete with regard to cell population numbers.

Chapter 2

"Stochastic mitotic mode" models do not explain zebrafish retinal progenitor lineage outcomes

Synopsis

(1) The SMME is composed of Simple Stochastic Models, intended to model zebrafish RPC lineage outcomes, and purports to explain these outcomes with a "stochastic process" mechanism. (2) SMME models diverge from the explanatory rationale of their immediate forebears, introducing an unexplained structure of temporal phases, and become increasingly dominated by the mitotic mode model-element. (3) Performing an AIC-based model comparison demonstrates that an alternative model with a deterministic mitotic mode performs better than the central He et al. model. (4) The suggestion of Wan et al., that the He et al. model also explains postembryonic, CMZ-driven growth, is tested by simulation, and cannot be reconciled with observations. (5) These analyses suggest that the source of variability in RPC lineage outcomes cannot be identified by the construction of SSMs, and that older theories invoking phases of RPC behaviour should be rehabilitated.

2.1 Introduction

Mechanistic explanations (MEx) derive their utility from the resemblance of the conceptual mechanism's output to empirically observed outcomes. As maps to biological territories, biological MEx are habitually identified with the living systems they represent. Most well-developed MEx take the form of a model, whose internal structure is taken to reflect the underlying causal structure of a biological phenomenon. The nature of the causal relationship between a mechanistic model and the phenomenon it purports to explain remains a topic of active discussion in the philosophy of biology [Fag15]. Biologists, nonetheless, usually accept that a model which explains empirical observations well (usually measured by statistical or information theoretic methods), and reliably predicts the results of interventions, bears a meaningful structural resemblance to the actual causal process giving rise to the modelled phenomenon.

Biological phenomena are notable for exhibiting both complex order and unpredictable variability. A

significant challenge for MEx in multicellular systems is to explain how complex, highly ordered tissues, like those produced by neural progenitors, can arise from unpredictably variable cellular outcomes. Stem cell biologists have often used to Simple Stochastic Models (SSMs) in order to explain the observed unpredictable variability in clonal outcomes of putative stem cells [Fag13]. Because SSMs are susceptible to Monte Carlo numerical analysis as Galton-Watson branching processes, they have been convenient explanatory devices, appearing in the literature for more than half a century [TMS64]. By specifying the probability distributions of symmetric proliferative (PP), symmetric postmitotic (DD), and asymmetric proliferative/postmitotic (PD) mitotic modes, SSMs allow cell lineage outcomes to be simulated.

SSMs are "stochastic" insofar as they incorporate parametric random variables. As Jaynes has noted, "[b]elief ... that the property of being 'stochastic' rather than 'deterministic' is a real physical property of a process, that exists independently of human information, is [an] example of the mind projection fallacy: attributing one's own ignorance to Nature instead." [JBE03, pp.506] Despite this, macromolecular processes are often described as "stochastic" in the stem cell literature. Generally speaking, the behaviour of an SSM's random variable is taken to represent sequences of outcomes that are produced by multiple, causally independent events. Recently, the influence of "transcriptional noise" on progenitor specification has been identified as a candidate macromolecular process that may produce unpredictable variability in cellular fate specification. Therefore, one explanatory strategy for stem and progenitor cell function compares SSM model output to observed lineage outcomes, in order to argue that "noisy", causally independent events give rise to the proliferative and specificative outcomes of progenitor lineages.

In this report, we evaluate the best-developed of these explanation, dubbing it the "Stochastic Mitotic Mode Explanation" (SMME) for zebrafish retinal progenitor cell (RPC) function. The SMME is noteworthy because it claims: (1) to "provid[e] a complete quantitative description of the generation of a CNS structure in a vertebrate in vivo" [HZA+12]; (2) to have established the functional equivalency of embryonic RPCs and their descendants in the postembryonic circumferential marginal zone (CMZ) [WAR+16]; and (3) to have established the involvement of causally independent transcription factor signals in the production of unpredictably variable RPC lineage outcomes [BRD+15]. Moreover, the SMME is taken to supply evidence for the predominance of stochastic effects over other explanations for RPC lineage outcomes, such as the classical explanation of a temporal succession of competency states [TR86]. These would be significant achievements with important consequences for both our fundamental understanding of CNS tissue morphogenesis and for retinal regenerative medicine. However, these models were not subjected to model optimization or selection procedures, as advocated by model selection theorists [BA02], and widely adopted in ecology and evolutionary biology [JO04].

In order to evaluate of the two SSMs which form the SMME's MEx for RPC function, we have re-expressed the models as cellular agent simulations conducted using the open source C++-based CHASTE cell simulation framework [MAB+13]. We explored the structure of the SSMs, dubbed the He and Boije SSM respectively, compared to their explanatory forebear, dubbed the Gomes SSM [GZC+11]. This analysis suggested the progression of temporal "phases" in the models was largely responsible for the SMME model fits to data. In order to investigate this possibility, we built an alternative model with a deterministic mitotic mode and compared its output to the He SSM. We found that the deterministic alternative model was a better explanation for the observations, demonstrating that SMME fails when compared to alternatives. We therefore suggest that an explanatory approach based on the use of SSMs is incapable of distinguishing between theoretical alternatives for the causal structure of RPC lineage be-

haviours. Furthermore, by comparing the output of the models with novel postembryonic measurements of proliferative actvity, we find that the SMME explanation cannot account for quantitative majority of retinal growth in the zebrafish, driven by CMZ activity postembryonically. Finally, we discuss the place of SSMs, the concept of "mitotic mode", and the role of "noise" in explaining RPC behaviour, and suggest ways to avoid the modelling pitfalls exemplified by the SMME.

2.2 Metatheoretical analysis

The SMME for zebrafish RPC function has been advanced using two SSMs. One first appears in He et al., and again, unmodified, in Wan et al. [HZA+12, WAR+16]; it explains lineage population statistics and time-dependent rates of the three generically construed mitotic modes (that is, PP, PD, DD). We have called this the He SSM. The second appears in Boije et al. [BRD+15]; its intent is both to introduce the role of specified macromolecules into the mitotic mode process, and to explain neuronal fate specification in terms of the process. We have called this the Boije SSM. The He model is directly descended an SSM advanced to investigate causally independent fate specification in late embryonic rat RPCs, formulated in Gomes et al. [GZC+11]. The Boije SSM differs substantially from the He and Gomes models, but inherits its general structure from the He SSM.

The metascientific analysis of the development of biological explanations remains undertheorized. Perhaps most refined tool for global evaluation of biological theories (Schaffner's "Extended Theories" [Sch93]), treats biological explanations as hierarchically organised logical structures, similar to the method of Imre Lakatos [Lak76], and proposes the use of Bayesian logic to distinguish between them. However, biologists rarely offer explanations in this form; we rather prefer mechanisms, expressed in diagrammatic form or as mathematical model-objects.

We adopt Fagan's view [Fag15], that MEx for the behaviour of stem and progenitor cells consist of assemblages ("mechanisms") of explanatory components which are understood to be causally organised by virtue of their intermeshing properties. While Fagan treats SSMs seperately from macromolecular MEx, and we find that the Gomes SSM was not deployed in this role, the He and Boije SSMs were used as explanations for the behaviour of RPCs. As Feyerabend famously observed, scientists operate as epistemological anarchists; the development of our explanatory logic is not bound by a set of rules, but rather arises organically from our scientific objectives, extrascientific context, and so on [Fey93].

We have therefore chosen to examine the structure of the Gomes, He, and Boije SSMs arranged in chronological order, to highlight how the explanatory logic of zebrafish SMME SSMs differs from their immediate ancestor, and from other uses of SSMs in stem cell biology. We have diagrammatically presented these SSMs as MEx, consisting of components describing the proliferative and fate specification behaviour of cellular agents. The proliferative and specificative components of the MEx are causally organised by their intermeshing property, mitotic mode. We have used abbreviations to denote important classes of model components and inputs, informed by the emphasis of Feyerabend on the persuasive role of metaphysical ingredients and auxiliary scientific material; these are as follows:

- MI - Model ingredient, making reference to some conceptual or metaphysical construct
- AS - Auxiliary scientific content
- RV - Random variable

- PM - Parameter measurement, model parameter set by measurements, independently of model output considerations
- PF - Parameter fit, model parameter set without reference to measurement, in order to produce model output agreement with observations

We draw the reader's attention to the expansion of the "mitotic mode" intermeshing property in later SSMs; this explanatory component comes to dominate the later models, which makes its interpretation critical to the success of the mechanistic explanation in meaningfully representing a real biological process.

Numerous explanations for variability in RPC lineage outcomes have been considered. Harris has argued that these belong to three cell-autonomous categories of process, in addition to extracellular influences: (1) a linear temporal progression of competency states, (2) asymmetric segregation of determinants during mitoses, and (3) intracellular "stochastic events" [HBEH88, AH09]. All of the SSMs are used to argue for the predominant influence of the third type of process in RPC lineage outcomes, essentially by demonstrating that the output of the SSM resembles observations.

2.2.1 Gomes SSM: Ancestral Model of the SMME SSMs

The Gomes SSM is presented in Fig 2.1. The model's structure is straightforward; there are three independent random variables, drawing from an empirically-derived Log-Normal probability distribution for the time each cell takes to divide, and categorical distributions on the mitotic mode of the division and the specified neural fate of any postmitotic progeny. The random mitotic mode variable functions as the "intermeshing property" linking cycle behaviour to fate specification. In this scenario, the processes governing proliferation, leading to cell cycle exit, and governing cell fate commitment are causally independent of each other and of their history. The objective of the Gomes et al. study is to compare the lineage outcomes of dozens of individual E20 rat RPCs, in clonal-density dissociated culture, with the model output, developing earlier work in this system [CBR03]. Although the model incorporates conventional proper time (clock-time), none of the RVs reference it to determine their values. The abstract "cells" represented by this model do not have any timer or any source of information about their relative lineage position. Fate specification is construed in terms of conventional histochemical markers of stable cell fates; only the neural types generated late in the retinal histogenetic order are represented, as these are the only neurons specified by E20 rat RPCs.

This SSM is explicitly built as a null hypothesis, or a model of background noise. It represents an extreme case in which all of the specificative and proliferative behaviours of an RPC are totally independent of all other RPCs and events in its clonal lineage. The stated purpose of this model is "to calibrate the data", serving "as a benchmark" [GZC+11], a purely hypothetical apparatus to produce sequences of causally unrelated lineage outcomes. Substantial deviations of the observed data from the fully independent events of the SSM may then be interpreted as causal dependencies between RPC outcome and their relative lineage relationships, as might be observed in a developmental "program" or algorithmic process. Finding that, with some exceptions, observed proliferative and specificative outcomes generally fall within the plausible range of Gomes SSM output, Gomes et al. conclude that RPC lineage outcomes in late embryonic rat RPCs seem to be dominated by causally independent events, suggesting that causally isolated "noisy" macromolecular processes may give rise to the unpredictable variability in the fate outcomes of these cells.

Figure 2.1: **Structure of Gomes SSM**
Structure of the Gomes SSM [GZC+11]. pRPh, probability of rod photoreceptor specification. pBi, probability of bipolar cell specification. pAm, probability of amacrine cell specification. pMü, probability of Müller glia specification.

2.2.2 He SSM: Explaining variability in zebrafish neural retina lineage size

The He SSM, shown in Fig 2.2, is deployed in a explanatory role rhetorically identical to the Gomes SSM. The extent to which model output "captures.. aspects of the data" is taken to obviate the need for explicit "causative hypothes[e]s". On this account, only residual error between model output and observations may be ascribed to non-stochastic processes like "histogene[tic ordering] of cell types or a signature of early fate specification". That is, if the model can be fit to observations, this is taken to exclude the presence of any cell-autonomous temporal program, asymmetric segregation of fate determinants, extracellular influences, and the like.

There are fewer direct empirical inputs to the He model's parameters than the Gomes SSM, limited to the duration of the cell cycle, which is modelled by a shifted Gamma distribution [1]. The close synchrony of zebrafish RPC divisions is modelled by assigning sister RPCs the same cycle length, shifted by a normal distribution with a variance of one hour, in contrast to Gomes RPCs, which are treated as fully independent. The lineage outcomes the He SSM is called on to explain differ significantly from those referred to by the Gomes SSM. Most notably, the Gomes SSM does not account for the early appearance of RGCs, which are not produced by the late E20 progenitors examined in that study. The zebrafish RPC lineages studied by He et al. produce all of the retinal neural types, including RGCs, which are typically produced by PD-type divisions. The He SSM does not model particular cell fates, supposing that mitotic mode is "decoupled" from fate specification. The mitotic mode RV linking cell cycle to fate specification in the Gomes SSM has thus subsumed the specification outcomes of RPC lineages entirely,

[1] The change of this distribution to Gamma in He et al. from Log-Normal in Gomes et al. is not explained, but presumably was felt to reflect the zebrafish context better.

Figure 2.2: **Structure of He SSM**
Structure of the He SSM [HZA+12]. σ: sqrt(variance) of the location-shifted gamma-distributed cycle time. TiL, Time in Lineage.

and the model is concerned only to explain the sizes of lineages (marked by an inducible genetic marker at various times), and the observed progression of mitotic modes in these early RPCs.

The He model is, therefore, called on to explain the temporal structure of the proliferative behaviour of early zebrafish RPCs that dissociated late rat RPCs do not exhibit. A Gomes-type SSM, in which the RVs determining RPC behaviour are independent of any measure of time, cannot account for this temporal progression. In order to address this, He et al., assume a linear progression of three phases which cells in each lineage pass through, the timing of these phases being determined relative to the first division of the RPC lineage, called here "Time in Lineage" or TiL. The parameters of the mitotic mode RV are determined by these TiL phases. The temporal structure of the phases and their effect on the mitotic mode RV are selected to produce a model fit. While He et al. acknowledge that the model therefore represents a "combination of stochastic and programmatic decisions taken by a population of equipotent RPCs," no test is performed to determine the relative contribution of the model's stochastic vs. linear programmatic elements. Instead, the purportedly stochastic nature of mitotic mode determination is emphasized throughout the report.

2.2.3 Boije SSM: Explaining variability in zebrafish RPC fate outcomes

The second SMME model advanced to explain zebrafish RPC behaviour, the Boije SSM, is diplayed in Fig 2.3. This model is primarily concerned with the lineage fate outcomes that the He SSM does not treat, while abandoning the explicit proper time of the He and Gomes SSMs in favour of abstract generation-counting. The mitotic mode model-ingredient now subsumes all RPC behaviours. Boije et al. make a laudable effort to specify the particular macromolecules ostensibly involved in determining mitotic mode, nominating the transcription factors (TFs) Atoh7, known to be involved in RGC specification, and Ptf1a, known to be involved in the specification of amacrine and horizontal cells, as primary candidates. The contribution of vsx2 is taken to determine the balance between PP and DD divisions late in the lineage, with the latter resulting in the specification of bipolar or photoreceptor cells. The binary presence or absence of these signals is determined by independent RVs structured by the phase structure present in the He SSM, translated into generational time from proper time.

Figure 2.3: **Structure of Boije SSM**
Structure of the Boije SSM [BRD+15]. RPC, retinal progenitor cell. RGC, retinal ganglion cell. AC, amacrine cell. HC, horizontal cell. BC, bipolar cell. PR, photoreceptor. pNG- parameter representing contributions of Vsx1 and Vsx2 to specification of BC & PR fates in absence of Atoh7 or Ptf1a signals.

By this point, the SMME has become a very different type of explanation from its Gomes SSM forebear. We are no longer dealing with RVs that model causally and temporally independent processes for different aspects of RPC behaviour. There is, rather, one temporally dependent process, the determination of mitotic mode, which is explained by unpredictable subsets of each lineage generation

expressing particular TF signals. Where the Gomes SSM takes its parameters directly from empirical measurements of lineage outcomes, asking whether it is sufficient to assume that these are independently determined, the Boije SSM's parameters are derived solely from model fit considerations. In spite of these considerable differences, the explanatory role of the SSM is effectively the same: the model's fit to observations is taken as evidence of the predominant influence of stochastic processes determining mitotic mode on RPC behaviour. While Boije et al. acknowledge that whether some process is called "deterministic" or "stochastic" is "a matter of the level of description" [BRD+15] (i.e. is a property of the model-description and not of the physical process), the explanatory role of stochasticity for RPC lineage outcomes is emphasized throughout.

2.3 Model selection demonstrates the SMME is not the best available explanation for RPC lineage outcomes

It is notable that neither of the reports which use the He SSM [HZA+12, WAR+16], nor that using the Boije SSM [BRD+15] report their fitting procedures in detail, nor do any of the above report any statistical measures of model quality. Additionally, no models representing the alternative "theoretical options" are compared to those advanced as evidence for stochastic processes. Given that the He SSM and Boije SSM depart from the Gomes SSM by the addition of an unexplained temporal structure to the mitotic mode model ingredient, it is striking that the overall argument remains similar to Gomes et al.'s, despite the persuasive force of the latter deriving from the lack of such structures. While He et al. and Boije et al. acknowledge that their models involve both stochastic and linear programmatic elements, their relative influence on model output is not measured, and macromolecular explanation is only applied to the stochastic elements. Moreover, the emphasis on this mitotic mode model construct increases with each successive model, to the extent that in the Boije model there are no other elements that are used to explain RPC behaviours. All cellular behaviours are, in effect, progressively collapsed into the stochastic mitotic mode concept. Finally, the He and Boije SSMs contain more parameters, determined by fewer empirical measurements than the Gomes SSM. It is therefore important to test whether the most important ingredient in these models is the stochastic mitotic mode, against the possibility the unexplained temporal structure is the truly explanatory element.

To examine this, we compare the He SSM to an alternative model. To emulate the ad-hoc fitting procedure reported by He et al. [HZA+12], we fit the models to a training dataset, comprising the induced lineage size and mitotic mode rate data from He et al. We hold out a test dataset, consisting of the Atoh7 morpholino observations from He et al., and the CMZ lineage size data from Wan et al., which played no role in obtaining He et al.'s fitted model parameters. This allows us to test how well the models hold up under novel experimental conditions. Since the Boije SSM draws on the He SSM for its temporal phase-parameterisation and stochastic mitotic mode model ingredient, this analysis of the He SSM also bears directly on the validity of the Boije SSM.

As an alternative to the SMME He SSM, we constructed a model that has a deterministic mitotic mode with variable phase lengths. Rather than variability arising from a stochastic-process mitotic mode changing across phases of fixed length, we simply supposed that mitotic mode is deterministic in each phase (guaranteed PP mitoses in the first phase, PD in the second, and DD in the third), but the phase lengths are variable between lineages and shift slightly between sister cells. That is, we represented this linear progression of deterministic mitotic mode phases using the same type of statistical

construct the He SSM applies to model cell cycle length, to avoid introducing any novel or contentious elements into the model comparison. More specifically, each lineage has a first PP phase length drawn from a shifted gamma distribution, followed by a second phase length drawn from a standard gamma distribution. Upon mitosis, these phase lengths are shifted in sister cells by a normally distributed time period, exactly like cell cycle lengths in the He SSM. The effect of this alternative parameterisation on the mitotic mode RV is summarised in Figure 2.4.

Figure 2.4: **An alternative, deterministic mitotic mode model**
Mitotic mode random variable in He et al.'s stochastic model ("He" panels, top) and a deterministic alternative ("Det. alt." panels, bottom). Blue bands represent variable phase boundaries arising from gamma-distributed and sister-shifted phase boundaries.

This is a representation of the classic suggestion that RPCs step through a linear succession of competency phases. If this temporal program is governed by RPCs passing through a stereotypical series of chromatin configurations which allow for PP, then PD, then DD mitoses in turn, along with the associated competence to produce the particular cell fates associated with PD and DD mitoses, it seems plausible to suggest that RPCs differ in the lengths of time they occupy each state. Moreover, since these chromatin configurations must be broken down and rebuilt with each mitosis, the re-use of the "sister shift" model ingredient from the He SSM's cell cycle RV is congenial, representing the same sort of cell-to-cell variability that results in the small differences between sister cells in cycle timing.

While the code used to implement the SMME models has not been published, the reports provide enough detail to reconstruct these models. We did this using the CHASTE cell-based simulation framework. Because the values selected for the He SSMs' parameters were selected by hand, with only the value for the probability of PD-type mitoses in the second model phase being varied to produce the fit, we suspected that the fit would not be at or near the local minimum for a loss function. That is, a model fit produced in this manner is likely to be located in a region of the parameter space that is highly sensitive to small perturbations, and therefore may depend strongly on implementation-specific

idiosyncracies. He et al. report that they experienced difficulty in obtaining a good fit to their 32 hour induction data, with changes in the phase two PD probability producing large differences in fit quality. When we estimated the He SSM with the original fit parameterisation, we substantially reproduced the original fit, except for the 32 hour data, where the model output diverges substantially from that reported in He et al. (see S1 Fig). Since this is not the best fit available for the He SSM, and we wish to directly compare the best fits for the He SSM and our putative alternative model, we used the simultaneous perturbation stochastic approximation (SPSA) algorithm [Spa98] to optimise both models against observations. SPSA is convenient for complex, multi-phase models like the He SSM, because no knowledge of the relationship between the model's parameters and the loss function is required. We used Akiake's information criterion (AIC) as the loss function to be minimised, in order to provide a comparison metric between the two differently-parameterised models (the deterministic alternative has two fewer parameters than the He SSM).

Figure 2.5: **Model comparison: the SPSA-optimised He SSM and a deterministic alternative** Empirical observations (black crosses and bars) and SPSA-optimised model output (magenta, stochastic mitotic mode model; green, deterministic mitotic mode model). Model output in panels A-G is displayed as mean ± 95% CI. Panels A-F: Training dataset, to which models were fit. Panels G-I: Test dataset, included in AIC calculation. Panels A-C: Probability of observing lineages of a particular size ("count") after inducing single RPC lineage founders at (A) 24, (B) 32, and (C) 48 hours with an indelible genetic marker, tallying their size at 72hpf. Panels D-F: Probability density of mitotic events of modes (D) PP, (E) PD, and (F) DD over the period of retinal development observed by He et al. Panel G: Probability of observing lineages of a particular size ("count") originating from the CMZ; RPCs are taken to be resident in the CMZ for 17 hours before being forced to differentiate, lineage founders are assumed to be evenly distributed in age across this 17 hour time period. Panel H: Average clonal lineage size of wild type and Ath5 morpholino-treated RPCs. Ath5mo treatment is taken to convert 80% of PD divisions, which occur in the second phase of the He model, to PP divisions, resulting in larger lineages. Panel I: Ratio of even to odd sized clones in wild type and Ath5 morpholino-treated RPCs. Methods in Section 11.1.7. Code in Section 17.1.15.

After fitting to the training dataset, we calculated AIC for the He SSM (hereafter SM, for stochastic model) and our deterministic mitotic mode alternative (hereafter DM), for both training and test datasets together. The optimized output for the two models is displayed in Fig 2.5, and presented separately in S2 Fig and S3 Fig. Remarkably, the DM closely recapitulates the output of the SM for both datasets. Moreover, the DM proves to be a better explanation of the overall dataset, as summarised in Table 2.1. Given this model selection scheme, we should choose the DM over the SM as an explanatory model of the data.

Table 2.1: **AIC values for stochastic and deterministic alternative models**

Model	**AIC**
Stochastic (He fit)	1564.4
Stochastic (SPSA fit)	453.6
Deterministic (SPSA fit)	**438.7**

AIC: Akiake's information criterion. Lower values reflect better model explanations of the noted dataset. Lowest value noted in bold. Methods in Section 11.1.7. Code in Section 17.1.15.

We conclude that the SMME is not the best available explanation for variability in zebrafish RPC lineage outcomes. The assumed, but unexplained, linear succession of mitotic mode phases provides the overall structure of the model output. Variability in lineage outcomes may be supplied by different model ingredients without any loss of explanatory power on new datasets. To conclude that these models support a stochastic process governing mitotic mode, ruling out other types of explanation, is unjustified. It is likely that any number of different types of SSMs, representing other sorts of processes (such as asymmetric segregation of fate determinants, or differential spatial exposure to extracellular signals), can produce identical model output. Given this, we suggest SSM models of this type are inadequate for the task of locating the source of variability in RPC lineage outcomes.

2.4 SMME SSMs cannot explain the post-embryonic phase of CMZ-driven zebrafish retinal formation

The zebrafish retina, like other fish retinas, and unlike the mammalian retina, continues to grow long after the early developmental period, indeed, well past the organism's sexual maturity. This may be unsurprising, given that zebrafish increase in length almost ten-fold over the first year of life [PEM+09], necessitating a continuously growing retina during this period. In fact, the quantitative majority of zebrafish retinal growth occurs post-metamorphosis, outside of the early developmental period. This growth occurs due to the persistence of a population of proliferative RPCs present in an annulus at the periphery of the retina, called the ciliary or circumferential marginal zone (CMZ), which plates out the retina in annular cohorts. A typical "tree-ring" analysis from our studies, marking the newly-synthesised DNA of cohorts of cells contributed to the retina at particular times with indelible thymidine analogues, is shown in Fig 2.6. These experiments show the structure of the adult zebrafish retina is dominated by contributions from the CMZ in the period between one and three months of age. Why peripheral RPCs in zebrafish remain proliferative, while those in mammals are quiescent [Tro00], and whether and how their behaviour might differ from embryonic RPCs, remain unresolved. Answers to these questions may have significant fundamental and therapeutic implications, especially given e.g. the possibility of

harnessing endogenous, quiescent, peripheral RPCs in humans for regenerative retinal medicine.

Figure 2.6: **Most zebrafish retinal neurons are contributed by the CMZ between one and three months of age**
Maximum intensity projection derived from confocal micrographs of a zebrafish whole retina dissected at 5 months post fertilisation (mpf). The animal was treated with BrdU at 1, 2, 3, 4, and 5 mpf for 24hr. Anti-BrdU staining of the whole retina reveals the extent to which the CMZ (which is responsible for retinal growth after approximately 72hpf) contributes new neurons in tree-ring fashion, extending out from the center of the retina (CR), which is formed before 72hpf. Monthly cohorts are labelled appropriately. Scale bar, 100 μm. Methods in Section 11.1.2.3.

The He SSM was deployed in Wan et al. [WAR+16], with the claim that the He SSM explains the behaviour of CMZ RPCs. Wan et al. argue that a slowly mitosing population of bona fide stem cells, at the utmost retinal periphery, divides asymmetrically to populate the CMZ with He-SSM-governed RPCs. In other words, the usual suggestion that CMZ RPCs undergo a somewhat different process than embryonic RPCs, perhaps recapitulating across the peripheral-central axis some progression of states or lineage phases that embryonic RPCs pass through in time [HP98], is repudiated in favour of one model which describes the behaviour of all RPCs throughout the life of the organism, with the addition of a small population of stem cells to keep the CMZ stocked with RPCs. If so, this might suggest that the problem of activating quiescent stem cells in the retina is simply that- one need only sort out how to throw the proliferative switch in these cells, since the proliferative and fate specification behaviours of the resultant RPCs will reliably be the same as those observed in development.

While we determined that the SMME is not the best available explanation for RPC lineage outcomes, we still felt that the SSMs associated with this explanation might be used to elucidate this point. In particular, if it is the case that the He SSM provides good estimates of lineage size and proliferative dynamics in the early zebrafish retina, it should be possible, using this model, and the estimates of putative stem cell proliferative behaviour provided by Wan et al., to simulate the population dynamics of the CMZ, at least through the first few weeks of the organism's life.

In pursuing this point, we noted a peculiar feature of the He SSM not documented by any of the SMME reports: the cell cycle model overstates the *per-lineage* rate of mitoses by as much as a factor of 3. That is, the mitotic mode rate data presented in He et al., recapitulated here in Fig 2.5, panels D, E, and F, and used to optimise both the SM and DM, are probability density functions that are not

standardised on a per-lineage basis. These data simply indicate the distribution of mitotic events of a particular type. When we take all of the mitotic events documented by He et al. and calculate the probability of any such event occuring per lineage, per hour, we obtain the values presented in Fig 2.7.

Figure 2.7: **Per-lineage probabilities of mitoses, He et al. observations compared to model output**
Probability of observing a mitotic event over the period studied by He et al., per hour, per lineage. Model output is presented as mean ± 95% CI. Methods in Section 11.1.7. Code in Section 17.1.17.

Similarly, when we performed cumulative thymidine analogue labelling of the 3dpf CMZ, (using the assumptions of Nowakowski et al. [NLM89], which treat the proliferating population as a homogenous, and dividing asymmetrically; these assumptions are flawed but adequate for a rough estimate) we obtain an average cell cycle length of approximately 15 hours, more than twice as long as the He SSM's mean cycle length. These data are displayed in 11.2. Therefore, the He SSM (in both its original and refit parameterisation, as well as the deterministic alternative, since they all rely on the same proliferative model elements) substantially overstates the proliferative potential of both embryonic and early CMZ RPCs. Since we observed a massive build-up of proliferating RPCs between two and four weeks post-fertilisation, we thought this might actually suit this later context better.

To estimate the total annular CMZ population in zebrafish retinas over time, we counted proliferating RPCs present in central coronal sections of zebrafish retinas throughout the first year of life, treating

these as samples of the annulus, and calculated the total number of cells that would be present given the diameter of the spherical lens measured at these times. Our simulated CMZ populations were constructed at 3dpf by drawing an initial population of RPCs governed by the He SSM (using the original fit parameters, which further exaggerate the proliferative potential of these lineages) from the observed distribution. We added immortal stem cells amounting to one tenth of this total. This is likely an overestimate, given that these putative stem cells are thought to be those in the very peripheral ring of cells around the lens, of which typically two to four may be observed in our central sections with an average of over one hundred proliferating RPCs. Moreover, these simulated stem cells were given a mean cycle time of 30 hours, proliferating about twice as quickly as Wan et al. suggest. Finally, to reflect the fact that these stem cells contribute to the retina in linear cohorts [CAH+14], and more of them are therefore required as the retina grows, the stem cells were permitted to divide symmetrically when necessary to maintain the same density of stem cells around the annulus of the lens. Two hundred such CMZ populations were simulated across one year of retinal growth.

The results of these simulations are displayed in Fig 2.8, overlaid over CMZ population estimates derived from observations. Given the generous parameters of the population model, consistently overestimating the proliferative potential of embryonic and early CMZ RPCs, it is surprising that the He SSM proves completely unable to keep up with the growth of the CMZ in the first two months of life. Indeed, the unrealistically active stem cell population the simulated CMZs are provided with is unable to prevent a near-term collapse in RPC numbers, only catching up after the months later as the number of stem cells increases with the growth of the lens. Thus, even given permissive model parameters, the He SSM is not able to recapitulate the quantitatively most important period of retinal growth in the zebrafish.

This analysis suggests that observations of RPCs in embryogenesis and early larval development are unlikely to provide a good quantitative model of the development of the zebrafish retina, even abstracting away spatial and extracellular factors as SSMs necessarily do. Our data point to a second, quantitatively more important phase of retinal development, between approximately one and four months of age, in which RPCs contribute more cells per lineage than in early development. It is likely that models that closely associate proliferative behaviour with fate specification, like those of the SMME, will be unable to explain this period. Recent evidence suggests that mitotic and fate specification behaviours in RPCs may be substantially uncoupled [ESY+17]. This would permit the CMZ population to scale appropriately with the growing retina. Alternatively, it is possible that RPCs have heterogenous proliferative behaviour, and that this heterogeneity is mainly apparent later in development.

2.5 Conclusion

Simple stochastic models are familiar tools for stem cell biologists. Introduced by Till, McCulloch, and Siminovitch in 1964 [TMS64], they proved their utility in describing variability in clonal lineage outcomes of putative stem cells, originating from macromolecular processes beyond the scope of cellular models (and beyond the reach of the molecular techniques of the time). More prosaically, their simplicity afforded computational tractability in an era when processing time was relatively scarce, allowing early access to Monte Carlo simulation techniques. That said, their abstract nature emphasizes an aspatial, lineage-centric view of tissue development, which cannot account for the generation of structurally complex tissues beyond cell numbers, and, perhaps, fate composition. This relatively loose relationship to the

Figure 2.8: **CMZ population of proliferating RPCs: estimates from observations and simulated Wan-type CMZs**
Annular CMZ population was estimated from empirical observations of central coronal cryosections stained with anti-PCNA, a marker of proliferating cells, standardising by the diameter of the spherical lens observed in these animals. "Wan-type" CMZs were initialised with a number of RPCs drawn from from the observed 3dpf population distribution, and given an additional $1/10^{\text{th}}$ of this number of immortal, asymmetrically dividing stem cells, as described in the text, and simulated for the first year of life. All data is presented as mean ± 95% CI. Methods in Section 11.1.2.1, Section 11.1.7. Code in Section 17.1.18.

complex morphogenetic environment provides few constraints on the model configurations that may produce similar outputs. This can result in modellers being led astray by their apparently good fits to observations, which is why the fit of one highly parameterised model cannot be taken, alone, as evidence for some theory; it is very rare that there is not a model representing an alternative theory that cannot be made to produce a reasonable model fit. Indeed, as we demonstrate here, the use of standard model selection techniques may make plain that a completely contradictory theory is a better explanation for the data.

To some extent, this can be ameliorated by careful attention to the specific macromolecular or cellular entities that model constructs represent. The "mitotic mode" construct present in all SSMs is an ambiguous one in this regard. "Mitotic mode" is not, itself, a property of a mitotic event, but is rather

a retrospective classification of the event, after an experimenter observes whether progeny resulting from the event continue to proliferate. Its original appearance in SSMs was simply to allow calculation of clonal population sizes; it was never intended to represent a particular type of process or "decision" made by cells at the time of mitosis to continue proliferating or not. Any number of pre- or post-mitotic signals and processes may result in a particular mitosis being classified as PP, PD, or DD, without anything about the mitotic event itself determining this. The use of such a retrospective classification, rather than the identification of some physical property of the mitotic event (such as the asymmetric inheritance of fate determinants), is a concession that the actual macromolecular determinants of cellular fate are outside the scope of the model.

The SMME represents an attempt to connect this abstract, retrospective model construct to observations of noisy gene transcription [Rv08]. Motivated by the observation that momentary mRNA transcript expression in RPCs is highly variable from cell-to-cell [TSC08], the suggestion is that this transcriptional noise may be responsible for the observed variability in RPC lineage outcomes. Since this noise may be "tuned" to a degree by e.g. promoter sequences [RO04], it could plausibly be under selective pressure. There are two significant problems in identifying the SMME's stochastic mitotic mode with this type of process. The first is demonstrated by constructing an alternative model with deterministic mitotic mode but variable phase lengths: the source of variability may be located elsewhere without compromising the explanatory power of the model. It is impossible to determine what sort of process might give rise to variability in RPC lineage outcomes by using SSMs in this fashion. The second problem is that a causal explanation of the presence of the signal, for which noise is a property, is elided in favour of emphasis on "stochasticity". This is most apparent in the Boije SSM. In this model, Atoh7 and Ptf1a TFs are available to provide their noisy signal in the 4th and 5th lineage generations, and at no other time. We do not dispute that a noisy signal may contribute to variability in that signals' effects; rather, we suggest that it is the temporal structure of such a signal (assuming this structure can be empirically demonstrated, rather than assumed) that calls for causal explanation. The SMME reports explicitly disclaim the necessity for "causative hypotheses" in the case that a stochastic model provides a good fit to observations. As Jaynes remarked in his classic text on probability theory, "[stochasticity] is always presented in verbiage that implies one is describing an objectively true property of a real physical process. To one who believes such a thing literally, there could be no motivation to investigate the causes more deeply ... and so the real processes at work might never be discovered." [JBE03]

In identifying the model construct with the physical processes determining RPC outcomes, the SMME obscures what seems to us to be the primary lesson to be drawn from the Harris groups' beautiful in vivo studies of zebrafish RPCs. That is, compared to late rat RPCs in dissociated clonal culture, RPCs in intact zebrafish retinas produce far more orderly outcomes. To the extent that these outcomes are variable, the source of variability remains unidentified. We suggest that resort to "stochasticity" as an explanatory element should not be made in the absence of model comparisons that rule out alternatives with well-defined causal structures, lest we fall into the trap Jaynes warned us about.

Chapter 3

Toward a computational CMZ model comparison framework

Synopsis

(1) The SMME introduces a phased structure in order to explain early RGC generation by Danio RPCs, which is why it misapplies the explanatory logic of Gomes et al. [GZC+11] (2) A general-purpose statistical approach to testing many different kinds of models is desireable, and available in nested sampling. (3) Models of RPC activity in the postembryonic CMZ should prioritise simplicity, which could be achieved with "slice models". (4) Applying nested sampling to these models allows us to compare their explanatory quality rigorously.

3.1 SMME Postmortem: a wrong turn at Gomes

To return to the question posed in Section 1.6: has the SMME succeeded in finding order by blurring out the chaotic welter of mechanistic explanations for RPC function in retinogenesis? Chapter 2 argues the structure of the SMME models support explanations based on the original idea of a linear, deterministic series of stages through which RPCs progress, originally put forward by Cepko et al. [CAY+96], rather than stochastic processes. The important feature of the SMME models is where they depart from the original Gomes SSM: the assumption of the linear structure of temporal phases. The succession of stages is the model ingredient that produces the structure resembling the data, not the various random variables associated with mitotic mode, which we have proven by demonstrating that a deterministic progression of mitotic mode models the observations better than its stochastic counterpart. By introducing variability into the lengths of the linear temporal program already assumed by the SMME models, we can remove variability from the mitotic mode and produce a superior model with fewer parameters. Only by the process of counterinduction [Fey93], the comparison of models with different propositional structures about reality, does this become clear. The introduction of the linear phase structure is required to model the early contribution of RGCs in zebrafish. If this is left without some biological rationale, retinogenesis remains unexplained. The SMME thus does not achieve its aim of being a complete description of lineage outcomes in retinogenesis, and this is confirmed by our observation that the Wan

model [WAR+16] has little explanatory power outside the first few days of life, which makes up a small portion of the total retinal contribution to the zebrafish retina.

The first theoretical maneuver of the SMME, explaining the data with a model with structure that supports a stochastic mitotic mode process, cannot be achieved. The underlying data used to inform these models speak better to an entirely different selection from the array of theoretical options outlined in Section 1.2, the linear progression of competencies. Because the SSM model form does not usually have spatial dimensions, we did not test models involving variablility in extracellular signals, but it is a good bet that such a model could be made to fit about as well as either of the ones tested in the previous chapter. In effect, if we are to follow the SMME's inferential logic, variability in RPC outcomes can be explained by "stochastic variability" in any model parameter which affects fate outcomes. Because of the significance of this problem for biological inferences, an explanation of the Bayesian epistemological view of probability has been provided in Section 15.1.1. Moreover, a thorough argument that "stochasticity" cannot be a property of real existents is provided in Section 16.1. It suffices here to conclude that the SMME models do not achieve the aim of supporting "stochastic processes" as explanations over the alternatives.

The second theoretical maneuver, to nominate a particular macromolecular system as the physical locus of the stochastic process, suggests Atoh7 and Ptf1a as labels for abstract, Bernoulli distributed processes. It remains obscure how model variables relate to their namesake transcription factors (transcription of the TF itself? activation of other genes?), and no measurements of these factors inform parameter selection. Plainly, these factors are involved in relevant RPC behaviours, but it is unclear why they have been nominated as causally upstream of the "mitotic mode" selection. Since the Boije model inherits the assumption of a linear progression of stages, it is very likely that a model with similar explanatory power could be built using the same strategy outlined above, locating random variability outside mitotic mode, although this is outside the scope of the present work.

We conclude that the sole formally testable model of RPC function in the zebrafish retina is not adequate for our purposes. But how did we get here? As noted in Section 1.3.2, the notion that the most significant behaviours associated with RPCs are produced by mechanisms intrinsic to the cells derives much of its empirical support from the Raff group's work. This story began with the observation that co-culturing E15 rat RPCs in dissociated pellet cultures with P1 cells did not accelerate the appearance of the first rods derived from the E15 progenitors (which occurred at a similar time as *in vivo*), suggesting a partially intrinsic commitment "schedule" for these cells[1] [WR90]. The scope of these observations were dramatically expanded by Raff's subsequent work, intended to address the relative significance of intrinsic versus extrinsic processes in RPC function by comparing clonal RPC lineages in fully dissociated clonal-density cell culture to those in intact explants [CBR03]. The remarkable finding of this study was that dissociated E16-17 rat RPC lineages produce very similar numbers and types of retinal neurons as their tissue-embedded counterparts, albeit without morphological or molecular markers of mature neurons. This observation provided strong evidence for the predominance of RPC-intrinsic processes in determining both mitotic and fate outcomes for late RPC lineages. The complex, spatially organised context of intact explanted tissue seemed to only be required for the maturation of neurons, and was not required to regulate proliferation, or the initial commitment to an appropriate distribution of lineage outcomes. As correct cell numbers and types are the two most important parameters that must be

[1] Co-culturing with P1 cells, did, however, significantly increase the proportion of E15 RPCs specified as rods, resulting in Raff's suggestion here that both intrinsic and extrinsic factors are important.

controlled for RPCs to produce a functional retina of the appropriate size, the suggestion that both are largely determined by intrinsic processes seemed to confirmation of Williams and Goldwitz's much earlier suggestion [WG92], against the prevailing view of the day, that lineage had a greater role to play than cellular microenvironment in RPC contributions. Interestingly, Raff's interpretation of their 2003 data was that RPCs were most likely stepping through a linear, programmed developmental sequence, rather than undergoing shifts in the probabilities of variable outcomes over time. It is notable that this interpretation arises not from the data collected in their study, but from considerations of a single unusual clone reported by [TSC90], and by analogy with drosophila neuroblasts. In retrospect, these arguments for linear sequences of deterministic RPC outcomes do not seem particularly strong, and it is perhaps unsurprising that these studies are remembered mainly for highlighting the importance of RPC-intrinsic processes.

The Gomes study, with its detailed study of particular rat late-embryonic RPC lineages, thus seemed to solidify the notion that unpredictable RPC-intrinsic processes dominate lineage outcomes [GZC+11]. But this study, like its forebears originating in Raff's work, is concerned with a population of RPCs that is too old to produce RGCs. This is the essential point: the SMME does not try to explain the early appearance of RGCs in terms of some macromolecular mechanism, although this is the single most significant difference between the zebrafish RPC lineages studied by Harris and the rat lineages studied by Raff and Cayouette [CBR03, GZC+11]. It is only by assuming the unexplained linear temporal structure of RPC specification that the SMME models achieve their apparently good explanatory quality.

3.2 Implications of the SMME study for modelling CMZ RPCs

Having taken seriously the possibility that an adequate model of zebrafish retinogenesis already exists, and concluded in the negative, we may ask how to improve on this state of affairs. Firstly, careful model estimation is needed, and to have any confidence about our interpretation of the model, we must test alternatives that make different assumptions about the causal structure of the phenomenon being explained by the models. Can we conclude that the approach used in Chapter 2 is adequate to our task? There are two elements to consider: the appropriateness of the modelling approach represented by the SSM in relation to the hypotheses we wish to test, as well as the soundness of the statistical procedures.

Taking up the SSM, its most attractive features are its computational efficiency and the ease of producing a model to fit some particular case- the SMME models include some elements like the correlation of cell cycle lengths in mitotic sisters that greatly improve the temporal modelling of RPC lineage outcomes, for instance, which we will adopt in simulations presented below. The observations arising from the SMME strongly suggest that we would like to test hypotheses about RGC specification in order to find better models of RPC function. Carefully reconsidering the hypothesis of Neumann et al. [Neu00], that Shh from nearby RGCs induces cell cycle exit and RGC specification, would seem to be a high priority, for instance. Can such a scenario be represented in an SSM? One could model the probability of being within Shh induction range of an RGC in the early part of a lineage's life, for instance. Doubtless, a model that incorporated variable RPC specification outcomes determined on this basis could be made to fit data about as well as the variable mitotic mode or variable phase length models tested above. That said, it is not very clear that a highly abstract representation of signalling would constrain the inferred RGC production rate well. Moreover, while we showed that our deterministic mitotic mode model fit test data somewhat better than the stochastic model, the modest size of the calculated AIC differences

suggests that comparing SSMs is not a particularly good way to distinguish even alternative hypotheses about the structure of processes the SSM models explicitly, like cell cycle length and mitotic mode.

It seems that we will be unable to test important models, including those involving documented macromolecular explanations of RGC specification, without the ability to represent some spatial information. Still, the computational benefits of the SSM are too appealing to leave out of the toolkit entirely, and the type of spatial information that is ultimately best used to constrain alternatives on processes like RGC fate commitment could be highly abstract. We would like to leave the door open for models of many forms, prioritising simpler ones. Taken together, this suggests that we need a very general method of testing models of potentially very different structures against one another.

Turning then to our statistical procedures, are these adequate to our goals? We can broadly characterize the approach taken as estimating the local optimum of a loss function for model output, given the dataset; specifically, minimizing Akiake's information criterion by simultaneous perturbation stochastic approximation (SPSA). This procedure has some advantages. AIC is a well-understood measure, well-grounded in information theory, which penalizes superfluous model complexity in a consistent and rigorous way. SPSA is a widely used, well understood algorithm that can be applied to any reasonable euclidean parameter space; cases where parameter spaces are bounded (typical of biological models where negative parameter values are usually nonsensical) are explicitly accounted for, and so on. This procedure is better than many that are available.

Still, after the experience of the SMME model comparison, caution is warranted. The AIC calculation is based on a single estimate for the locally optimal parameterisation of the model. Relative AIC rankings are strongly influenced by the "well depth" of the AIC surface at the local minimum in parameter space[2]. Indeed, blind interpretation of AIC rankings has lead to its use in ecology being described as a "cult" [BB20]. It is easy to imagine practitioners being reduced to "AIC hacking", in the same manner that "p hacking" occurs, in order to achieve some arbitrary value for a hypothesis.

Moreover, while SPSA is a practical algorithm its statistical guarantee is only that it will almost-surely find the local minimum of the loss function. For highly parameterised models with complex loss function surfaces, there will be many local minima for SPSA to get "stuck" in that are far from the global optimum[3]. If we are evaluating hypotheses in order to make decisions about potentially years-long research projects, more certainty about the reliability of the method is necessary. Finally, while SPSA is requires much less computational effort than more global Monte Carlo parameter estimation techniques like simulated annealing or Hamiltonian Monte Carlo, it requires about as much application-specific tuning.

Fortunately, a complete system of Bayesian inference, which addresses all of the problems mentioned above, has been promulgated over the last 15 years: nested sampling. Originally introduced by John Skilling [Ski06], this use of this system has became widespread in cosmology [Tro08, FHB09, HHHL19], where its generality and ability to cope with complex, high dimensional parameter spaces has been well proven. This system is discussed in detail in Section 15.2.7. We have implemented two separate samplers for the exploration of parameter spaces using the overall nested sampling approach: `GMC_NS.jl`, described in Chapter 7, and `BioMotifInference.jl`, described in Chapter 10. `GMC_NS.jl` is a prototype general purpose Galilean Monte Carlo sampler intended to perform nested sampling of arbitrary tissue- or cell-level biological models, while `BioMotifInference.jl` is a highly specialized, well-tested, ad hoc

[2]These problems are discussed in more detail in Section 15.2.1 and Section 15.1.3.

[3]This is part of what is meant by "the curse of dimensionality", when speaking of the difficulty of sampling the loss function in high dimensional parameter spaces.

sampler for evidence calculation and MAP estimation of independent component analysis models of genetic sequence emission. `GMC_NS.jl`, while less mature, is of greater general interest, and having it in hand allows us to broadly consider the features of CMZ models we might like to test, before proceeding to test the function of the package and to determine its general utility in tissue-scale models.

3.3 CMZ models in a putative model comparison framework

The simulations presented in Chapter 2 consisted solely of abstract collections of lineages. The members of these model colonies have no activities beyond proliferation, and no functional attributes beyond their proliferative status[4]. Despite the underlying code consisting of relatively high performance C++, these simple models nonetheless occupied the local component of the cluster used in this work for more than a week. Because the introduction of explicit three-dimensional spatial simulation implies much more computational expense, this could strongly limit the ability to test inferences with local resources. This constraint dominates other considerations in building models of the CMZ. Since funding constraints limit the cloud-export of computational burden from the confines of the research institution, it is reasonable to proceed upwards in model computational expense, seeking the minimum model complexity required to compare hypotheses of interest to us.

3.3.1 Spatial dimension of the models: the ”slice model”

Although decomposing the RPC population of the CMZ into a collection of unordered, independently-proliferating SSMs prevents us from assessing many interesting hypotheses, a computational model of the entire CMZ or retina may not be required. Because observations suggest that RPC lineages contribute to the retina in linear cohorts of neurons, any particular centrally-oriented slice of the CMZ annulus will be responsible for the generation of the neurons central to it. The retina can represented by a series of these ”slice units”, lined up radially, like slices of pie. A complete ”slice unit” would include the central-most larval remnant, contributed by embryonic retinogenesis, surrounded by the CMZ’s more ordered neural contribution from the postembryonic period, and, peripherally, the CMZ itself. Depending on the hypotheses to be tested, the differentiated central retina may be mostly irrelevant, so the modelled slice may consist only of the CMZ and its interface with these central neurons. It is important to note that these slices are conceptually different from the linear cohorts generated from particular lineages, the so-called ‘ArCCoS’, which justify the slices conceptually [CHW11]. It is not necessarily the case that a slice model would consider only one RPC lineage; the slices are better thought of as spatial boxes that sample the CMZ, the conceptual equivalent of the histological section through the eye[5].

Slice models are especially attractive because the proliferative status, position, specified fate, etc. of simulated cells can be directly compared to measurements of fixed sections of retinal tissue. This is especially important for studying the postembryonic CMZ, which rapidly becomes optically inaccessible due to the increasing thickness and pigmentation of overlying tissue, so that live imaging often cannot be used [6].

[4] I.e. the specified identity of post-proliferative cells in these models has no function within the model.

[5] The case of only one simulated lineage may still arise given thin enough sample boxes or old enough animals, but it is a limiting case and probably would not be the norm.

[6] It is worth noting that the spatial parameters of such models would pertain to fixed and not live retinas; it is probable that some scaling relation compensating for fixative shrinkage could allow the mixture of live imaging data in evidence calculations.

If the retina can be thought of as a series of slice models, and the activity of the CMZ is basically homogenous around its circumference, the activity of the CMZ may be usefully represented with a single such model. Moreover, the slice need only include one portion of the retinal periphery, the orientation of which is irrelevant (i.e. the model parameterisation for the dorsal portion of a coronal slice could be identical to the ventral). It may be erroneous to abstract such a slice from its context, because the modelled cells are in contact with adjacent slices. In this thesis, in order to model the interaction between the simulated slice and the rest of the eye, we use a model of lens growth to produce an estimate of the number of additional fractional CMZ slices the modelled slice would have had to produce in order to maintain the CMZ annulus. The appropriate number of RPCs are subtracted from the slice's population, which models the lateral, slice-adjacent interface of the CMZ in addition to niche exit from the CMZ into the neural retina central to the CMZ slice. The various models used in Chapter 4 are summarised in an inferential pipeline schematic in Figure 3.1.

The zebrafish retina is a manifestly asymmetrical structure, with an optic nerve positioned ventro-temporally relative to the center of the optic cup. The CMZ itself is generally understood to be an asymmetric structure, with a larger dorsal than ventral population. This calls into question the assumption of CMZ homogeneity outlined above. It is nevertheless plausible that the appearance and maintenance of these structural features of the zebrafish retina could be explained with a set of slice models, for example, one for each of the dorsal, ventral, nasal, and temporal extrema[7]. Ultimately, these considerations are only relevant if our objective is to compare model explanations for retinogenesis as a whole. If we restrict ourselves to the more modest goal ranking causal influences on the lineage outcomes of RPCs in the dorsal extremity of the CMZ, a slice model could still be valuable even if it fails to explain aspects of tissue-level zebrafish retinogenesis.

[7]Such models are implemented by `CMZNicheSims` as `MultiSlice_Models`.

Figure 3.1: **Whole-eye and slice model abstractions of CMZ RPC populations used in Chapter 4**
. D/V: Dorsoventral axis. N/T: Nasotemporal axis. Red torus: CMZ. Grey circle:Lens. Colored dots denote the data sources used in each model. Magenta text gives the parameters of the model. CT: Cycle time. ϵ: exit rate. See Chapter 8 for more.
Data to estimate models were acquired from immunohistochemical analyses of cryosections through *Danio* eyes. Confocal stacks were segmented and analysed to count PCNA+ve and (if applicable) EdU-labelled CMZ cells in the slice as well as to measure the lens and neural retina. These slice data were used to estimate whole-eye measurements for a fully abstract `CMZ_Model`, representing the entire peripheral RPC population, as well as directly compared to the output of three different types of `Slice_Model`. Quality estimates from these models were sampled by GMC-NS, producing model evidence, posteriors, and MAP output. Model evidence was compared to produce estimates of the evidence ratio and significance.

3.3.2 Temporal resolution of the models

Another important consideration for any CMZ model comparison framework is the time-scale of any phenomena which are to be admitted as possible causal contributors to retinogenesis. While it is reasonable to think that proliferative events may be well-described with a model that operates on a scale of days and fractions thereof, and that such a model would be well suited to describing the full sweep of CMZ activity across the life of the organism, very few of the relevant macromolecular processes are likely to be well-described with a resolution more coarse than seconds or minutes. The implied difference in the number of calculations required to simulate any given time period is thus several orders of magnitude. A simplifying assumption of the temporal homogeneity of CMZ activity would allow us to abstract long developmental time frames; an explanation that pertains to a few hours can perhaps be extended without recalculation.

If an assumption of temporal homogeneity proves inadequate, the functional structure of the simulation will be determined by this consideration. To illustrate this, consider a case where we wish to incorporate measurements of eye pressure, or membrane tension across the retina, as inputs into the proliferative activity of the CMZ, by way of progenitor cortical tension [Win15]. This would permit assessing whether modelling tissue-mechanical inputs to cellular activities allows us to extract additional information from our observations. If such physical model elements are to be included, the functions which relate physical parametric data to the proliferative behaviour of CMZ progenitors must not operate on a time scale too short to cover the period of interest. Suppose we are mainly interested in explaining the assembly of functional units of the mature, specified retina by the CMZ, from the first division of the presumed distal stem cell responsible for the unit to the determination of the last neuron of its functional column. If this process takes days, finite element approximation of cortical tension with a resolution small enough to capture important details of relevant cellular movements (e.g. interkinetic nuclear migration) will probably be too expensive to allow for effective application of Monte Carlo techniques, requiring thousands or millions of simulations per call of the model likelihood function.

The models in this thesis mostly are of very coarse resolution, because they model the CMZ over significant portions of the organism's life; the exception is the `Thymidine_Model`, which is a cell-based intended to model thymidine analogue labelling of cells passing through S-phase, and is not intended to model RPCs leaving the niche at all. The `Thymidine_Model` is also by far the slowest of the CMZNicheSims to estimate.

3.4 Structuring models under uncertainty

The extent to which model simplifications are justified, and the types of phenomena that could be explained with these models, depend heavily on considerations that can only be informed by observations. As has been demonstrated in Chapter 2, the growth of the CMZ population cannot be explained by SMME models fitted to embryonic RPC activity. However, it remains unclear what sort of alternative structure is justified by observations, given our uncertainty about inferred parameters of the populations being measured. Because we have, in `GMC_NS.jl` a general method of comparing differently parameterised models against our observations, we can seek to select models from among alternatives on the basis of their total explanatory power for the observations, independent of the particular model structure or parameter values. Because this process also produces samples from the posterior distribution of the model's parameters, we are also able to account for our uncertainty on these parameters. This allows

us to broadly answer the question of what sorts of hypotheses may be addressed by the conventional immunohistological techniques we have used thus far, and what changes to methodology or analysis may be necessary to produce data that adequately constrains models of interest.

Our general approach is to estimate model evidence over broad prior distributions on model parameters. We assume as little as possible about the processes underlying our results. The values used in these comparisons are the logarithm of the model evidence, logZ, also understood as the logarithm of the marginal probability of the model. Likelihoods and probabilities are often compared in ratios; evidence is the same, and since the logarithm of a ratio is the logarithm of the numerator minus that of the denominator, this log-ratio, logZR, may be calculated by subtracting the evidence of one model from another. This is discussed in more detail in Section 15.2.7. This general model-comparison logic is iterated to address most of the important hypotheses we test in the following chapters.

Chapter 4

Bayesian characterisation of postembryonic CMZ activity

Synopsis

(1) Variability in CMZ RPC population sizes and whole retinal volume estimates is better modelled Log-Normally than Normally. (2) The dynamics of CMZ RPC population measurements over the first year of life suggest that CMZ RPCs may be passing through a succession of phases with different cell cycle and niche exit rates. (3) Whole-eye modelling of CMZ population and retinal volume estimates suggests that only two such phases are necessary to explain these dynamics. (4) Slice modelling of the CMZ population more successfully recapitulates data, and suggest there is no timing difference in proliferative schedule across the CMZ's dorso-ventral axis. (5) Modelling RPC dynamics with a single phase of decaying cycle rate and constant exit rate is a superior description to the 2-phase models; decay models also suggest that there is no dorso-ventral gradient of cell cycle and niche exit parameters. (6) Nowakowski-type modelling of cumulative thymidine analogue labelling gives overstated cycle rates, comparable to the defective whole-eye models, but confirm no dorso-ventral differences in these parameters. More accurate estimates can be obtained using cell-based slice modelling. (7) Cohorts of neurons contributed to the retina by the CMZ are divided among layers in fractions that remain stable over time; however, the neural fates specified vary over time. (8) Retinal neurons contributed by the CMZ are turned over by microglia at a rate too low to be detected in labelled cohorts. (9) Explaining postembryonic CMZ dynamics requires different models from proliferatively-restricted embryonic lineage simulation. Slice models allow abstraction of spatial complexity. (10) Many opportunities for improved slice modelling exist.

4.1 Independent Log-Normal modelling of CMZ parameters

To take up the question of how CMZ RPC activity evolves over time, we measured CMZ and retinal parameters in histological studies of cryosections of zebrafish eyes harvested over the first year of the animal's life. The initial approach is to geometrically impute measurements whole-eye CMZ and retinal volume from sample central cryosections, calculated as described in Section 11.1.4. These derived measurements form the first dataset we consider.

While it remains common to assume that population census data are Normally distributed, it is been known [Hea67] that Log-Normal distributions are usually better models of the outcomes produced by additive processes with small, variable steps (like population sizes or income distributions). Since all of our models require modelling distributions of population data, we start by selecting the most explanatory population model.

As described in more detail in Section 12.0.1, we conclude that population variability in CMZ census and retinal volume estimates are best described by independent Log-Normal distributions. Because Log-Normal distributions are simply transformed Normal Gaussian distributions, we may model our uncertainty about their parameters with Normal-Gamma distributions over the mean and variance of the underlying Normal distribution of the Log-Normal population model [Mur07]. This is explained in more detail in Section 15.2.4. A useful analytic feature of the Normal-Gamma prior is that the marginal posterior distribution of the mean, assuming an uninformative (ignorance) prior, is a location-scaled T distribution. Most of the descriptive statistics in the next section, therefore, calculate the 95% credible interval for the posterior mean of the underlying by T distributions (with the correct change of variables by exponential transformation to produce the features of the correct Log-Normal distribution). Unfortunately, differences of T distributions are not, themselves, necessarily T-distributed, so we have relied on Monte Carlo estimation of rates of change of the these posterior means over time.

4.2 Survey of CMZ population and gross retinal contribution

If it is true that the majority of zebrafish retinogenesis occurs postembryonically, and that models trained on embryonic data do not describe this period well, as seen in Chapter 2, what characterises this CMZ-driven phase of retinogenesis? A schematic overview of the appearance of the CMZ during the first three months of post-embryonic life is presented in Figure 4.1, displaying the progressive expansion of the neural retina and the relative size of the CMZ over this time period.

We first estimate mean CMZ annulus population and retinal volume over the first year of life, in Figure 4.2, panels A and C. The lack of growth observed in the first week of life suggests an initial quiescent period in the population history of the CMZ. Observations in Figure 5.3, showing CMZ RPCs are proliferating too slowly to be labelled by a 12 hour pulse of a thymidine analogue at 10dpf in the siblings of npat mutants, also suggest this quiescence. On the other hand, we have good confidence that retinal volume continues to grow over this time; 99.56% of the marginal posterior mass of the mean 5dpf estimate is above the 3dpf mean, which may indicate any proliferative pause is too short to appear in the retinal volume data.

The first two to three weeks of life (magnified in lens insets in panels A and C) see a slow build in both estimated CMZ annulus population and retinal volume compared to the large increase which follows. While zebrafish are better staged by size than age [PEM+09], the onset of this growth seems to come earlier than the metamorphic transition from larval to juvenile stages at 45dpf [SH14]. This raises the question of how the CMZ contributes to the retina during the critical period of exponential growth of the organism, between about 45 and 90dpf. It is plausible, for instance, that the growth of the CMZ population mainly reflects the increased cell cycle rate, with RPCs specifying at some steady rate, or that the CMZ builds itself up for a wave of increased niche exit and specification somewhat later, similarly to the sequence of events in the embryonic central retina.

To ascertain this timing, we simulated the mean daily rate of change of the CMZ annulus population

Figure 4.1: **Micrographic overview of CMZ development in *D.rerio*, 3-90dpf** Representative confocal micrograph maximum intensity projections (MIPs) of 14 µm coronal eye cryosections, processed via IHC, of zebrafish of noted ages. Magenta – anti-PCNA staining. Green – Hoechst 33342 nuclear staining. Note changing scale bars. Methods in Section 11.1.2.1.

and retinal volume, by performing Monte Carlo difference operations between samples from the marginal posterior means of subsequent timepoints. These simulated mean rates and their associated, empirically determined confidence intervals are plotted in Figure 4.2 panels B and D. These show the time-structure of the phenomenon; the increase in simulated daily CMZ population growth rate occurs before the increase in retinal volume, but the CMZ population estimate does not begin to drop off until 60dpf, by which time the majority of volumetric growth is complete. This seems to substantiate some combination of the scenarios outlined above: there is an early buildup of CMZ population around 18-30dpf, before CMZ RPCs begin to make their primary contribution to the retina between 30-60dpf, which seems to be characterised by much slower population growth and more rapid volumetric growth, implying steady and elevated specification and niche exit of RPCs.

Subsequent to the bulk of the CMZ buildup and contribution to the retina, the population of the CMZ declines at about 39% the rate of its peak ascent, with an estimated -90.1 $\frac{cells}{d}$ by 90dpf. This thins out the CMZ population to below its inital value, spread out over a much larger peripheral annulus, for a much less dense mature CMZ. Interestingly, the 95% CI on the marginal posterior mean of estimated retinal volume at 180 dpf (ranging from 1.84e8 μm^3 to 2.75e8 μm^3) completely encompasses the 95% CI on volume at 90dpf (1.96e8 μm^3-2.48e8 μm^3), while the 360dpf mean (3.85e8 μm^3) is greater than six standard deviations from the 180dpf mean (2.25e8 μm^3). This suggests a possible second period of quiescence from approximately 90-180dpf, followed by steady contribution to the retina without a buildup in the CMZ population subsequently.

Figure 4.2: **Population and activity of the CMZ over the first year of *D.rerio* life**
Panel A: Marginal posterior mean CMZ annulus population. Panel B: Marginal posterior mean retinal volume estimate. Insets in Panels A & B display data from 3-17 dpf. Panel C: Marginal posterior mean of the proliferative index of the CMZ annulus, assayed by specified retinal neurons with incorporated thymidine from an 8hr pulse at the indicated ages. Panel D: Mean daily rate of volumetric increase of the neural retina, calculated as the difference in volumes between two ages over the number of elapsed days. All means are displayed in a band representing the ±95% credible interval for the marginal posterior distribution of the mean. Methods in Section 12.1.8, Section 12.1.9. Code in Section 17.8.9.

Given this description, the ontogeny of the CMZ as a stem cell niche could be well-described by different phases of activity, with different rates of proliferation and specification. It is not immediately clear what sort of periodization is justified by the data. It seems plausible that as few as two phases could explain the data well enough: an initial phase of logarithmic growth, with short cell cycle time

and lower exit rate of RPCs from the CMZ into the specified neural retina, followed by a second phase of decay with longer cycle time and higher exit rate. On the other hand, perhaps some of the data features noted above justify a more complex model that captures, for instance, the initial quiescent period, or the post-180dpf growth of the retina. Because this is a question of how much model structure is justified by our data, we may address it as a model selection problem.

4.3 Two-phase periodization of postembryonic CMZ activity by phased difference equation modelling

To perform our model selection task, we simulate the time-evolution of CMZ population and retinal volume. This requires initial values for the size of the simulated CMZ populations and the volumes of the simulated retinae they are associated with. Given the findings presented in Figure 12.1, we are justified in initializing CMZ population and retinal volume by independent samples from the Log-Normal models of their interindividual variability at 3dpf, the end of embryogenesis and the beginning of the period of CMZ-driven retinal growth. To produce new, simulated values of CMZ population and retinal volume, we apply a system of difference equations as follows, where pop_n is the population at n dpf, CT is the mean cell cycle time of the population in hours, and ϵ is the proportion of the population at time $n-1$ that exits cycle and contributes to the volume of the specified neural retina, and μ_{cv} is the mean volume per cell contributed to the retina in µm^3:

$$p_n = p_{n-1} \cdot 2^{\frac{24}{CT}} - p_{n-1} \cdot \epsilon \tag{4.1}$$

$$v_n = v_{n-1} + p_{n-1} \cdot \epsilon \cdot \mu_{cv} \tag{4.2}$$

A model "phase" can then be defined by the CT and ϵ parameters it applies to update the population and volume, over the appropriate number of days. The full parameterisation of a model with p phases is given by p pairs of CT and ϵ values, and $p-1$ phase transition dates. μ_{cv}, which is taken to apply equally to all phases, is estimated from 3 dpf nuclear measurements, as described in Section 12.1.8. This model is described further in Section 8.2.

Given an initial population and volume sampled from the Log-Normal models of their interindividual distributions, Equation 4.1 and Equation 4.2 are be applied to these values to produce simulated sample measurements. Many such samples obtained by Monte Carlo can be used to estimate Log-Normal distributions for simulated CMZ populations and retinal volume, which can be used to score the model against observations. By defining prior distributions over the model parameters, we may sample from the prior to initialize a model ensemble. The ensemble can then be compressed by nested sampling, moving each model-particle over the parameter space by Galilean Monte Carlo, as described in Section 15.2.6. Using typical procedures applied in nested sampling [Ski06], we estimated the Bayesian evidence, maximum a posteriori model output, and marginal posterior distributions on parameters for 2 and 3-phase models.

The maximum a posteriori model output shows the major problem with this simple model; the model relationship between changes in CMZ population and changes in volume breaks down rapidly. That is, the later volume estimates are too large for the CMZ to produce, given the calculated cellular volume (μ_{cv}) at 3dpf. The models fit the early population and volume data better, but the population peak is dragged upward to produce more-likely volume output at later ages. While it is possible that the later

Figure 4.3: **Maximum a posteriori output of periodization models**
Population and volume estimates from observations (crosses) plotted with mean ± 95% probability density model output, for the 2-phase model (A panels) and 3-phase model (B panels). A1,B1: population estimates. A2,B2: volume estimates. Insets provide magnified views of data from the first 30dpf. Methods in Section 12.1.11. Code in Section 17.8.8.

CMZ contributes more volume per neuron to the cellular retina, it seems more likely that the volume approximation applies better to more-nearly spherical eyes at younger ages, than to the flattened eyes of later ages. When we tried floating μ_{cv} as a variable within the model, the MAP results were similar (data not shown), suggesting that a constant value for μ_{cv} across ages is the problem in achieving good model fits, not the particular value chosen. This reinforces the idea that the problem relates to the breakdown of the retinal volume estimate at later ages.

While this limitation prevents either model from explaining the combined estimate datasets very well, they are in this sense under the same constraint, and so some inference about the number of phases justified by the data is still possible. Evidence estimates for the 2-phase and 3-phase models, given these data, are presented in Table 4.1. There are greater than 3500 orders of magnitude more evidence for the 2-phase model; this result has over 200 standard deviations of significance. This reflects the expanded parameter space in the 3-phase model, which has 8 parameters, compared to the 2-phase models' 5. The additional flexibility afforded by the 3rd phase in fitting the later volume data is unable to overcome the evidentiary penalty associated with the larger parameter space. We conclude that the 2-phase model is a better-justified description of these data.

Table 4.1: **Evidence favours a 2-phase periodization of CMZ activity**

2-phase logZ	**3-phase logZ**	**logZR**	**σ Significance**
-7147.3 ± 9.4	-10743.0 ± 13.0	3596.0 ± 16.0	227.624

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratios in favour of the 2-phase model. Largest evidence value bolded. Methods in Section 12.1.11. Code in Section 17.8.8.

While the parameter estimates associated with these models are clearly unreliable, they are useful to inspect in for comparison to the simulations to follow. We present the parameterization of the MAP model output displayed above in Table 4.2. Unsurprisingly, the selected 2-phase model begins with a first phase of rapid proliferation, with a CT of 14.5 hr, followed by a second, slower phase of 17.5 hr. The imputed exit rate ϵ is greater than 200% of the day's starting population in the first phase, suggesting that new cells exit the CMZ after about 12 hours, around one cycle, while the second phase exit rate is lower, with 160% of the day's starting population exiting the CMZ, again suggesting a residency time of about one cycle. The imputed phase transition age is about 23dpf. Due to the volume estimate problem noted above, it is reasonable to believe that the CT estimates are likely too short, the ϵ estimates too high, all favoured in order to produce higher volume estimates at later ages.

Because nested sampling produces weighted samples from the posterior, we can estimate posterior distributions on parameters. We did so by kernel density estimation, to investigate whether the marginal posterior distributions for the selected model are polymodal. This can reveal how the evidence supports multiple hypotheses about the parameters of the two imputed phases. Kernel density estimates (KDEs) for marginal posterior distributions on the 2-phase models' parameters are presented in Figure 4.4.

These estimates reveal monomodal posterior distributions over the parameters. CT posteriors are well constrained to a small part of the sampling space (left panels, X axes, and top right panel), with the MAP parameter estimates given in Table 4.2 falling within the most dense regions. However, it is evident that the data do not constrain phase exit rates ϵ (left panels, Y axes) very well. That is, the marginal posterior probability distributions for these parameters are very widely spread. The marginal posterior distribution on the age at which the transition between phases occurs is plotted in the bottom

Table 4.2: **Maximum a posteriori parameter estimates for periodization models**

Parameter	**2-phase MAP**	**3-phase MAP**
Phase 1 CT (h)	14.5	14.9
Phase 1 ϵ	2.02	1.95
Phase 2 CT (h)	17.5	75.6
Phase 2 ϵ	1.6	0.29
Phase 3 CT (h)	NA	68.4
Phase 3 ϵ	NA	0.27
Transition 1 age	23.2	47.9
Transition 2 age	NA	179.4

CT: cycle time. ϵ: niche exit rate. Methods in Section 12.1.11. Code in Section 17.8.8.

right of Figure 4.4; the KDE suggests that the age of phase transition is likely around 23 dpf.

The lower exit rate associated with MAP second phase highlights the fact that the relative cycle time or exit rate between phases is irrelevant to the overall behaviour of the system. What matters is whether the cycle time and exit rate within a phase produce a growing or shrinking CMZ. We conclude that, while our global model of CMZ population and volumetric retinal contribution is too flawed to make good parameter estimates, a 2-phase model of this activity is better substantiated by the evidence than a 3-phase model. We proceed on this basis, tentatively accepting the 2-phase model, and taking up the idea of the "slice model" introduced in Section 3.3.1, to investigate modelling the CMZ RPC population more concretely, directly from sectional observations, rather than from the calculated population and retinal volume estimates presented above.

Figure 4.4: **Kernel density estimates of marginal posterior parameter distributions, 2-phase model**
Prior distributions (magenta) apply to parameters of both phases. Maximum a posteriori model parameterisation indicated by vertical lines. Panel A: Marginal posterior distributions on Phase 1 and 2 CT cycle time parameters. Panel B: Marginal posterior distributions on Phase 1 and 2 ϵ niche exit rate parameters. Panel C: Marginal posterior distribution on Phase 1 to 2 transition age parameter. Methods in Section 12.1.12. Code in Section 17.8.8.

4.4 Phased slice-models of CMZ population dynamics suggest subtle shifts across stable-population contour

In the course of the preceding investigations, it became apparent that the CMZ population asymmetry mentioned in Chapter 3 was not a static phenomenon, with the dorsal lobe of the CMZ annulus being consistently more populous than the ventral lobe, as generally implied by the sources covered in Chap-

ter 1. Rather, both the extent and orientation of asymmetry change over time. Sectional population totals for the dorsal and ventral CMZ are presented in Figure 4.5, Panel A, alongside the related intra-individual asymmetry ratio in Panel B. The initially pronounced dorsal population and reduced ventral population both seem to go through the overall boom-bust progression of CMZ population, but their relative proportion within individuals reverses itself over the period from 17-90dpf. We also observed a similar phenomenon occurring across the naso-temporal axis over the same time period (Figure 12.3).

Figure 4.5: **Developmental progression of dorso-ventral population asymmetry in the CMZ.** Marginal posterior distribution of mean dorsal (D) and ventral (V) population size in 14μm coronal cryosections (panel A) or intra-individual D/V count asymmetry ratio (panel B), ±95% credible interval, n=5 animals per age. Data points represent mean counts from three central sections of an experimental animal's eye. Methods in Section 11.1.2.1. Code in Section 17.8.4.

Inspected closely, these data provide a possible rationale for the reversal of asymmetry in the proliferative dynamics of the niche itself: the sectional (or "slice") population of the dorsal CMZ is increasing beyond its postembryonic minimum by 12dpf, while the ventral CMZ takes until 17dpf to exhibit a noticeable increase in size; moreover, the peak dorsal population is achieved by 23dpf, whilst ventrally the peak is only achieved at 30dpf. This suggests that the dorsal and ventral CMZ populations may undergo similar, time-shifted processes of proliferation from different starting populations. If this is so,

an explanation for this time-shifted phenomenon could have fundamental relevance to predicting and controlling the proliferative behaviour of peripheral RPCs and stem cells.

To test this hypothesis, we used a "slice model" of the CMZ, where the thickness of the slice is taken to be the same as the observed cryosection thickness (14 µm). The population of the CMZ is modelled with a difference equation, as above, but with an additional exit term representing lateral, circumferential contributions of the CMZ to the generation of new, adjacent "slices". The value of this term is calculated from the difference in CMZ annulus diameter over the calculated time period, as implied by a power-law model of lens growth fitted to observations, discussed in Section 11.1.4. The resultant difference equation is Equation 4.3. Terms are as defined above, except that p_n is the sectional population at n dpf, and not the total CMZ annulus population; additionally, η is defined as the daily circumferential exit rate implied by the power-law model. The model is detailed in Section 8.3.

$$p_n = p_{n-1} \cdot 2^{\frac{24}{CT}} - p_{n-1} \cdot \epsilon - \eta \tag{4.3}$$

We reasoned that, if the phase transition occurs earlier in the dorsal CMZ than in the ventral CMZ, there should be some informational gain in separating these observations vs. both slices being governed by one model. We therefore estimated the evidence, MAP, and posterior marginals for 2-phase models given the sectional dorsal and ventral populations, both combined and as separately-parameterised slices. To focus on the most informative subset of the data for our hypothesis, we restricted this analysis to the population data within the first three months of life.

The slice model proves to have greater success at explaining sectional counts than the whole-eye model does at explaining the annulus population estimates; maximum a posteriori model output is presented in Figure 4.6. In particular, all of the models adequately represent the early decline in sectional populations, arising from rapid early growth of the eye that exceeds the CMZs' proliferative capacity, without further ado; there is no justification for a separate early quiescent period, as suggested above.

The hypothesis that there is a time-shift in the phase transition across the dorso-ventral axis is not supported by these models. First, the evidence estimates demonstrate that we are not justified in separating the dorsal and ventral populations. The total-population slice model receives greater than 500 orders of magnitude more evidence than the joint evidence for the separate dorsal and ventral models, with greater than 20 standard deviations of significance, as displayed in Table 4.3. It is interesting to note that the evidence for the dorsal model (-579.9 ± 0.9) is substantially less than for the ventral model (-328.9 ± 0.5). This may indicate a causal influence on the dorsal population that is neither in the model nor acting on the ventral population, or it may be an uninformative sampling effect. A second indication that this hypothesis is unsupported are the MAP model parameter values, summarized in Table 4.4. The dorsal MAP transition is actually later than the ventral date, which further suggests the original model-idea of a time-shifted late ventral phase change is unsupported by these data.

Table 4.3: **Evidence favours a combined slice model over separate dorsal and ventral models**

Combined D/V model logZ	**Separate D/V model logZ**	**logZR**	**σ Significance**
-874.3 ± 1.2	-908.8 ± 1.0	34.5 ± 1.6	21.9

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratio in favour of the combined model. Largest evidence value bolded. Methods in Section 12.1.11. Code in Section 17.8.5.

Figure 4.6: **Maximum a posteriori output of total, dorsal, and ventral CMZ slice models** Population and volume estimates from observations (crosses) plotted with mean ± 95% probability density model output, for the total CMZ population model (panels A & B), dorsal CMZ population model (panel C), and ventral CMZ population model (panel D). Methods in Section 12.1.11. Code in Section 17.8.5.

The MAP parameters for the combined slice model suggest that those found for the 2-phase MAP in Figure 4.4 overstate cell cycle speed and exit rate. The combined slice model gives CTs of 34.4 and 23.3 hours for the first and second phases, compared to 14.5 and 17.5 estimated for the whole-eye model. Moreover, the niche exit rates ϵ are estimated at .54 and 1.07 for the combined slice model, compared to 2.02 and 1.6 for the whole-eye model. The values of the slice model seem more realistic in general; it seems unlikely that over one day, fully 200% of the day's starting CMZ population has exited the niche and entered the specified neural retina, for instance, as implied by the first phase exit rate of the whole-eye model. As well, given that CT represents an average cycle time over a heterogenous population of progenitors, many of whom are becoming postmitotic, CT values over 24 hr seem much more plausible than the 15 hr parameter required for the whole-eye model to produce its unrealistically exaggerated population.

The marginal posterior distributions of the total slice model, presented in Figure 4.7, are substantially more polymodal than those in Figure 4.4. This illustrates how data can support multiple hypotheses about model parameters to different degrees. When the phase marginal CT and ϵ are plotted together,

Table 4.4: **Maximum a posteriori parameter estimates for slice models**

Parameter	**Combined MAP**	**Dorsal MAP**	**Ventral MAP**
Phase 1 CT (h)	34.4	33.6	17.8
Phase 1 ϵ	0.54	0.57	1.43
Phase 2 CT (h)	23.3	35.0	11.2
Phase 2 ϵ	1.07	0.67	3.42
Transition age	32.2	41.0	27.8

CT: cycle time. ϵ: niche exit rate. Methods in Section 12.1.11. Code in Section 17.8.5.

they display the characteristic density curves displayed in Panels A and B of Figure 4.7. This supplies the most interesting information available by estimating this slice model. Firstly, the posterior marginal distributions of the first and second phase are very close to one another. Importantly, when the CT-ϵ contour which gives a stable population is plotted (dotted lines), it becomes obvious that the bulk of the first phase posterior lies just under this curve, while the bulk of the second phase lies just above it. This strongly suggests that the population dynamics of the CMZ may be characterised by a relatively subtle shift across the stable population contour, rather than distinct phases.

It should be noted that the MAP estimate does not occupy the most credible KDE kernel. An important property of nested sampling is that the accuracy of evidence calculations is traded off against the accuracy of estimating the posterior distribution [Spe19], as noted in Section 15.2.6. Since we have here prioritized evidence estimation, this is one cause of this discrepancy; the MAP models themselves may have relatively little weight in the KDE estimates. The oversampling of the prior periphery noted in Chapter 7, as well as the very significant polymodality of the model posteriors displayed in Supplementary Figure 12.4 also contribute to this problem[1].

On the basis of the marginal posterior analysis of this model, we suspected that separately parameterised phases is not the best available description of the data. In particular, the relatively small shifts across the balanced population contour implied by the posterior distributions argues strongly for a model with one phase with a slowly decaying cycle rate.

[1] A more conventional approach to reporting posterior estimates is to provide the mean and variance of the distribution, rather than the position of the most likely model in the MAP ensemble. We have eschewed this because of the tendency of `GMC_NS` to oversample the prior volume.

Figure 4.7: **Kernel density estimates of marginal posterior parameter distributions, total slice model**
Panel A: Bivariate KDE of Phase 1 posterior CT vs. ϵ density. Univariate marginals, with priors, are plotted to the left and below of the bivariate panel.
Panel B: Bivariate KDE of Phase 2 posterior CT vs. ϵ density. Univariate marginals, with priors, are plotted to the left and below of the bivariate panel.
Panel C: Univariate KDE of Phase 1-2 transition age.
Line height (univariate panels) or color scale (bivariate panels) indicates the estimated marginal posterior density present at the indicated parameter value. MAP parameters are given with magenta crosses (bivariate panels). Methods in Section 12.1.12. Code in Section 17.8.5.

4.5 Slice models of decaying cell cycle support anatomical homogeneity of RPC behaviour

To model the CMZ as a population of cells with a continuously lengthening cell cycle time, we modify the slice model described above by removing the phases and introducing an exponential decay equation for cycle time. This results in a slice model with three parameters: the initial cycle time (CT_i), a constant exit rate (ϵ), and a decay constant (κ). Population values are updated daily as per Equation 4.3, including the use of the lens model, except that the cycle time t days after initialising is determined by $CT_t = CT_i \cdot e^{\kappa \cdot t}$. More detail is provided in Section 8.4

Figure 4.8: **Maximum a posteriori output of total, dorsal, and ventral CMZ decay models** Population and volume estimates from observations (crosses) plotted with mean ± 95% probability density model output, for the total CMZ population model (panel A & B), dorsal CMZ population model (panel C), and ventral CMZ population model (panel D). Methods in Section 12.1.11. Code in Section 17.8.3.

We performed the same sampling routine on the same 3-90dpf dataset used for the earlier slice models, to enable direct comparison between them. The evidence calculations are presented in Table 4.5. The combined decay model substantially outperforms the combined slice model selected above, as well as separate decay models. Therefore, the model evidence supports the use of the decay model for describing RPC behaviour in general; however, there is insufficient evidence to distinguish between dorsal and

ventral populations using this model.

Table 4.5: **Evidence supports separate dorsal and ventral decay models**

Combined slice logZ	Combined decay logZ	D/V decay logZ	decay logZR	σ Sig.
-874.3 ± 1.2	**-762.6 ± 0.7**	-794.6 ± 0.8	32.0 ± 1.1	29.3

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio between separate D/V decay models and a combined decay model; positive ratio in favour of the combined model. Combined slice model from Table 4.3 included in left column for comparison. Largest evidence value bolded. Methods in Section 12.1.11. Code in Section 17.8.3.

MAP model output for the combined and separate decay models is presented in Figure 4.8, while the MAP parameter estimates are summarised in Table 4.6. Generally speaking, the values estimated for the parameters of the decay model suggest a much gentler proliferative schedule for RPCs, with initial cycle times measured in days, and daily niche exit rates of between 15-30% of the RPC population. Cumulative thymidine analogue labelling results, presented below, suggest these values are unrealistically long. Interestingly, MAP parameterisations for the split Dorsal and Ventral models do not imply that dorsal RPCs are more active than those found ventrally. Probably, if consistent regional biases in CMZ RPC behaviour exist, better data would be required to constrain niche exit rate. Indeed, it seems likely that the lack of effective constraint on the model's niche exit rate ϵ allows for the unrealistically long CT_i suggested by both combined and separate decay models. We conclude that explanations invoking differences in CMZ RPC behaviour across morphological axes cannot be supported with our models and datasets, indicating that morphological homogeneity of these parameters is descriptively adequate at this level.

Table 4.6: **Maximum a posteriori parameter estimates for decay models**

Parameter	Combined MAP	Dorsal MAP	Ventral MAP
CT_i (h)	52.0	69.7	62.1
κ decay	0.0067	0.0124	0.0091
ϵ exit	0.276	0.164	0.189

CT_i: Initial cycle time. κ: Exponential cycle time decay constant. ϵ: niche exit rate. Methods in Section 12.1.11. Code in Section 17.8.3.

Marginal distributions on these parameters are summarised in Figure 4.9. We again note that the data are informative for parameters of cell cycle (CT_i and κ), but give little information regarding niche exit rate. The MAP models found by GMC are well outside the kernel density estimates of the posterior distributions supplied by nested sampling this model. It is not clear why this should consistently be the case. The decay model may be characterised by small, high credibility modes that are outside the bulk of the less credible posterior mass; possibly, these are underweighted if they come late in the nested sampling process. In any case, the posterior marginals seem to confirm that there is little reason to separately distinguish dorsal and ventral subpopulations of CMZ RPCs.

Figure 4.9: **Kernel density estimates of marginal posterior parameter distributions, decay models**
Panel A: Marginal prior and posterior distributions on CT_i. Panel B: Marginal prior and posterior distributions on κ. Panel C: Marginal prior and posterior distributions on ϵ.
MAP model parameters are given by vertical lines. Methods in Section 12.1.12. Code in Section 17.8.3.

4.6 Bayesian inference of cycle parameters from cumulative thymidine labelling experiments supports slice model cycle time estimates

We sought to confirm the plausibility of the CT estimates obtained from modelling CMZ population measurements with an independent line of evidence. We examined 3dpf CMZ RPCs cumulatively labelled with a 10.5 hour pulse of the thymidine analogue EdU [CMD+09]. We used the Empirical Bayes approach

to estimate the evidence for separate Nowakowski-style [NLM89] linear models for the dorsal and ventral CMZ, against a model for both subpopulations combined. While this model is inadequate for reasons outlined in Chapter 2, it can serve to substantiate the first phase cycle time parameter. These results are summarized in Table 4.7, with the relevant linear regressions displayed in Supplementary Figure 12.5.

Table 4.7: **Evidence favours whole-CMZ linear cycle models over separate D/V models**

Model	**Implied T_c (hr)**	**Implied T_s (hr)**	**logZ**
Dorsal	14.7 ± 1.6	1.38 ± 0.76	7.778
Ventral	14.0 ± 1.2	0.8 ± 0.58	15.202
Combined	14.6 ± 1.1	1.25 ± 0.53	**26.165**

T_c: calculated cell cycle time. T_s: calculated s-phase length. logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence value bolded. Methods in Section 12.1.10. Code in Section 17.8.12.

Firstly, there are approximately 3 orders of magnitude of evidence in favour of the combined model vs. the joint evidence for separate models (ie. 26.165 vs. 22.980). In general, this suggests that there is little traction to be gained by separating these observations along the D/V axis. The Nowakowski-calculated cell cycle time T_c for the combined model, 14.6 ± 1.1 hr, includes the whole-eye MAP first phase TC, 14.5 hr, within one standard deviation. This suggests the Nowakowski model has similar problems to the whole-eye models presented in Figure 4.3, substantially overstating cycle rate. Calculated S phase lengths are also unrealistically short. Finally, the data seem to diverge from a linear trend toward the end of the pulse (see Figure 12.5), showing the limitations of this model.

Because the Nowakowski model is clearly inadequate for estimating RPC cycle parameters in the CMZ, we implemented a cell-based cycle simulator, similar to those in Chapter 2, to model discrete cumulative thymidine labelling counts, in lineages with explicitly simulated cycle events. This model is described further in Section 8.6. The MAP model output for this model is presented in Figure 4.10; it does a better job of describing the observations than the Nowakowski method, particularly at the earliest timepoints, due to simulation of cycle time correlation between sibling cells, as well as detector sensitivity.

KDE estimation of marginal distributions over the model parameters are presented in Figure 4.11. These estimates give us a clear picture of what can be learnt from these data; mean cycle time and the length of S phase as a fraction of the cycle time are significantly compressed relative to the prior, but little can be gleaned about the variance of the cycle time distribution, and almost no information is available about the G1 or sister shift fraction. That is, for the parameters which the data offer little information, the posterior distribution correctly collapses back onto the prior, which indicates that this approach adequately copes with unconstrained parameters[2].

The MAP parameters for the LogNormal distribution of cycle times produces a mean cycle time of 24.0 hr. This seems to be a more realistic estimate of RPC cell cycle time, and tends to support the longer values estimated in the slice models, although it does suggest the 50+ hour CT_i estimates for the decay models may be excessively long. Furthermore, this result supports our observation in Figure 2.7 that the He model examined in Chapter 2 substantially overstates the per-lineage proliferative rate of

[2]The model is overparameterised relative to the available dataset. This was done deliberately to allow the model to be used with richer data than were available here; however, it does have the effect of making sampling against these observations less efficient than is achievable.

Figure 4.10: **MAP model output and observations for thymidine slice model of 3dpf cumulative EdU labelling**
Count of EdU-positive cells observed in 14µm coronal cryosections through *Danio* CMZs, at indicated chase times, during a 10 mM EdU pulse (magenta crosses), overlaid with MAP model output. Probability mass distribution of the discrete non-parametric output is shown by color scale; yellow values include counts with >.05 mass. Output mean and 95% mass are indicated by solid and dashed white lines. Methods in Section 13.1.7, Section 12.1.11. Code in Section 17.8.13.

RPCs, at least in the CMZ; the He model's mean cycle time of 6 hours is much too fast to correctly model this population. It is likely that most reports of similar, very short cycle times for embryonic RPCs are underestimates of the actual distribution of cycle times.

Because the thymidine simulator's output resembles observations more closely than the Nowakowski model, produces more realistic parameter estimates than it, and clearly conveys how much information is available in the dataset, we suggest that this explicit simulation approach should be preferred to Nowakowski's method wherever the computational cost can be justified.

4.7 The CMZ contributes stably to each cellular layer with time-variable lineage composition

By labelling CMZ RPCs with the thymidine analogue EdU in a pulse at 3, 23, and 90 dpf, followed by histochemical analysis for known zebrafish retinal neural lineage markers after a 7 day chase, we investigated the possibility that RPC lineage outcomes change over the life of the organism. This

Figure 4.11: **Marginal posterior distributions for thymidine slice model**
Marginal prior (magenta) and posterior (green) distributions on thymidine simulator cycle parameters, noted on the x axis. MAP model parameters are displayed with vertical black lines.
Methods in Section 13.1.7, Section 12.1.12. Code in Section 17.8.13

hypothesis is of particular interest, as differences in the mosaic organisation of embryonically-contributed central retina and CMZ-contributed peripheral retina remain unexplained [ABS+10]. It may, moreover, have clinical significance, were quiescent peripheral stem cells to be entrained for regnerative medical purposes, as their lineage outcomes may be different than embryonic RPCs. We used antibodies raised against Pax6 and Isl2b to mark retinal ganglion cells (RGCs) of the ganglion cell layer and amacrine cells

of the inner nuclear layer. Anti-glutamine synthetase (GS) and anti-PKCβ were used to mark Müller glia (MG) and bipolar cell (BPC) populations of the INL. The unique flattened nuclear morphology of horizontal neurons was used to identify them. Lastly, the antibody Zpr1, directed against an unknown antigen present in photoreceptors with double cone morphology, was used to mark these cells.

Observations were collected in "staining groups", which combined histological markers; representative confocal micrographs from this study in animals pulsed at 23dpf are displayed in Figure 4.12, while data from all ages are plotted in Figure 4.13.

These data take the form of fractions of the thymidine-labelled cohort entering each of the three cellular layers (panel A of Figure 4.13), or the subfraction of the cohort within a given layer expressing a particular cellular marker (Panels B-I). While some variability is apparent in all of the measurements, it is unclear whether it is well-described as time-dependent in most cases. In order to address the question of whether CMZ layer or lineage outcomes differ over time, we needed to assess the joint evidence for separate models of each measurement at each age, against the evidence for a single model of the measurement for all of the assessed ages.

Because it is not obvious that these fractional measurements are better described Log-Normally, as the underlying population counts are, we first assessed the joint evidence for Normal and Log-Normal models of the data, summarized in Supplementary Table 12.4. This supports Log-Normal modelling of all measurements aside from the INL and ONL overall fractions, as well as GCL-resident Pax6- and ONL-resident Zpr1-positive cells. We noted that the evidence estimates differs from the simple likelihood ratio tests in some cases, as summarized in Supplementary Table 12.3. Without a clear fundamental justification for uniformly preferring one model or the other, we selected the best-supported model for each measurement. We estimated the evidence for an age-marginalized model, representing stable contribution to the layer or lineage over time, and compared this to the joint evidence for separate models at each age, representing a time-varying contribution model. These estimates are presented in Table 4.8.

Table 4.8: **Evidence supports stable layer contributions with time-varying fate commitment**

Layer	Marker	Cell type	Stable logZ	Time-vary logZ	logZR	σ sign.
GCL	Cohort	All GCL cells	**-0.84 ± 0.71**	-30.96 ± 1.0	30.1 ± 1.2	24.7
GCL	Isl2b	RGC	-48.68 ± 0.26	**-23.76 ± 0.59**	-24.92 ± 0.64	39.0
GCL	Pax6	Displaced am.	**-23.85 ± 0.44**	-40.08 ± 0.71	16.24 ± 0.84	19.3
GCL	Isl2b/Pax6	RGC subtype	-32.97 ± 0.28	**-16.82 ± 0.83**	-16.14 ± 0.87	18.5
INL	Cohort	All INL cells	**-41.15 ± 0.49**	-52.65 ± 0.66	11.5 ± 0.82	14.0
INL	Pax6	Amacrine cell	-15.37 ± 0.57	**-9.14 ± 0.46**	-6.23 ± 0.73	8.5
INL	PKCβ	Bipolar cell	-5.99 ± 0.3	**2.81 ± 0.54**	-8.79 ± 0.62	14.2
INL	GS	Müller glia	14.34 ± 0.35	**10.18 ± 0.6**	4.17 ± 0.7	6.0
INL	HM	Horizontal cell	**12.0 ± 0.31**	3.02 ± 0.6	8.98 ± 0.68	13.3
ONL	Cohort	All ONL cells	**-47.42 ± 0.45**	-63.19 ± 0.7	15.77 ± 0.83	19.1
ONL	Zpr1	Double cones	**-18.79 ± 0.37**	-31.63 ± 0.6	12.84 ± 0.7	18.3

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of stable model. Methods in Section 12.1.4, Section 12.1.11. Code in Section 17.8.10.

These calculations demonstrate that there is evidence for time-varying contributions to the retina, in

particular, to RGC, amacrine, bipolar, and Müller glial fates. All of these findings have between 6 and 39 standard deviations of significance. In the INL, the trend of the posterior mean for PKCβ-positive bipolar neurons (panel F) and GS-positive Müller glia of these lineages declines from the early postembryonic period (3dpf) to the late juvenile period (90dpf). Most of the time-varying fates show a downward trend in the fraction of the cohort expressing the relevant marker. These time-varying, declining lineages may be functionally related. However, we have no specific evidence that would implicate Isl2b/Pax6 double-positive RGCs in circuits with bipolar or amacrine cells, for instance. This nonetheless provides an explanation for the observation of a more-ordered retinal mosaic in later retinal contributions relative to the embryonic central remnant: since one or more sublineages in each layer are depleted in older fish, a different overall mosaic pattern as the neurons associate and pack together is expected. This may occur by changing the pattern of crystalline defects in this packing order, recently suggested as a mode of mosaic organisation [NNM+19]. Based on the timing of these changes, we tentatively ascribe this differential lineage production to the later phase of CMZ contribution, after the balance between proliferation and niche exit has shifted to the latter.

Lastly, we investigated the possibility that there might be detectable differences in layer or lineage contributions across the dorso-ventral axis; we tested this by measuring the evidence for age-marginalized combined models of fractional contribution against age-marginalized models split along the dorso-ventral axis. We found no evidence to support this hypothesis. These results are presented in Summary Table 12.5.

Figure 4.12: **Representative 23dpf lineage marker confocal micrographs**
Panel A1: RGC/Amacrine staining group. A2: Isl2b channel. A3: Pax6 channel.
Panel B1: MG/BPC staining group. B2: PKCβ channel. B3: GS channel.
Panel C: Double cone staining group. Zpr1 channel.
GCL: Ganglion cell layer. INL: Inner nuclear layer. ONL: Outer nuclear layer. Methods in Section 12.1.4.

Figure 4.13: **Second-phase declines in CMZ-contributed IslPax6+ RGCs, PKCβ+ bipolar neurons, and Zpr1+ double cones**
Overall fraction of CMZ-contributed cohort entering cellular layers (Panel A), or fraction of the layer subcohort expressing the noted immunohistochemical marker of lineage. All mean values are presented as marginal posterior means ± 95%CI.
λ: Fractional lineage contribution is modelled Log-Normally
†: Evidence supports time-varying model of fractional lineage contribution
Magenta: GCL measurements; Blue: INL measurements; Green: ONL measurements
Methods in Section 12.1.4, Section 12.1.11. Code in Section 17.8.10.

4.8 Early retinal cohorts of the *D. rerio* retina are turned over at a low rate by 4C4-positive microglia

Recently, extensive neural death has been reported in older zebrafish retinae [VGV+18]. This was described as a "neurodegenerative pathology" and suggested as a model of age-related neurodegeneration. However, during our thymidine analogue pulse-chase studies, CMZ-contributed cohorts often appeared less numerous only a month or two after their entry into the neural retina, even in juveniles of 30-90 days of age. Moreover, we observed numerous 4C4-positive microglia associating with the CMZ, as displayed in Supplementary Figure 12.7, which suggested that microglia may prune CMZ contributions. If neural retinal turnover occurs throughout the life of the organism, this would belie view that it should be treated as pathological, rather than constitutive. However, the thinning phenomenon could be explained by the changing morphology and geometry of the neural retina during this period. In particular, the neural retina thickens noticeably over this time, as displayed in Supplementary Figure 12.2. Although this increase is due, in large part, to the lengthening of photoreceptor outer segments, the inner nuclear layer is also significantly thickened. It is plausible that this process involves a compaction of the neurons along the coronal axis typically sampled, thus appearing to lose cells over time without this actually occurring.

To investigate, we administered 24 hr pulses of EdU to 1dpf embryos, and followed with 24 hr pulses of BrdU at 23dpf to mark a CMZ-contributed cohort near the height of its activity. By taking both coronal and transverse sections through the eyes of animals at 30, 60, and 90 dpf, we sampled these cohorts from both morphological axes of the retina, and counted labelled sectional totals. Data were totalled across morphological axes to account for the possibility of the cohorts being compacted on the coronal plane and extended on the transverse one. In order to test the hypothesis of early turnover, we estimated the evidence for linearly correlated and uncorrelated models of cohort population over time, using the Empirical Bayes regression method. If the addition of a time-dependent term in the correlated regression model is justified by the evidence at these early ages, this supports the notion that *D. rerio* retinal turnover is a lifelong phenomenon; if not, the apparent contraction of the cohorts is more likely a function of the alternative explanations noted above, or is occurring at too low a rate to be detected by these means. The results are summarized in Table 4.9, with the regressions plotted in Supplementary Figure 12.6.

Table 4.9: **Evidence for linear regression models supports early cohort population stability**

Measurement	Stable logZ	Declining logZ	logZR
1dpf Central Remnant	**-212.953**	-214.167	1.213
30dpf Cohort	**-107.567**	-110.827	3.261

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of stable model. Methods in Section 12.1.3, Section 12.1.10. Code in Section 17.8.15.

There is more evidence for a model of cohort population that is stable over time than one correlated with time, indicating that the additional model complexity implied by allowing turnover is unjustified, for this early period. Despite this, we did find isolated instances where members of these cohorts were visibly being engulfed by 4C4-positive microglia; one such event is depicted in Figure 4.14. Moreover,

we also observed TUNEL-positive nuclei in the central retina of *rys* siblings in the early postembryonic period (Figure 5.8). This indicates that some level of turnover is occuring during this earlier period. The evidence ratio in favour of the uncorrelated models is not overwhelming, which suggests that the cohorts are not turned over at a high enough rate to be detectable in the early period, although the process does occur even at this early age. In order to confirm this finding, we estimated the evidence for age-marginalized Log-Normal models of the population counts against age-differentiated models, representing time-constant and time-varying population models. This investigation resolves any ambiguity: no time-varying model is justified by the available evidence, as summarized in Table 4.10.

Figure 4.14: **4C4+ microglia associate with and engulf EdU-labelled CMZ contributions in the specified neural retina**
Representative maximum intensity projections from confocal micrographs of 14µm coronal cryosections through 30dpf zebrafish eyes labelled at 23dpf with EdU.
Blue: Hoechst 33258 nuclear counterstain. Green: Microglia labelled with 4C4 antibody. Red-orange-yellow: Intensity scale of EdU staining, indicating cellular origin in the 23dpf CMZ.
Chevrons: microglial nuclei. Arrows: EdU-positive nuclei found within 4C4-labelled extent of microglial cell body and appendages.
Methods in Section 12.1.5.

Table 4.10: **GMC-NS evidence estimates confirm Empirical Bayes analysis of early cohort stability**

Cohort	Time-constant logZ	Time-varying logZ	logZR	σ Significance
Embryonic central remnant	**-857.08 ± 0.29**	-868.02 ± 0.6	10.94 ± 0.67	16.4
1 month CMZ	**-402.38 ± 0.31**	-405.76 ± 0.43	3.39 ± 0.53	6.4

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratios in favour of the 2-phase model. Largest evidence value bolded. Methods in Section 12.1.3, Section 12.1.11. Code in Section 17.8.14.

4.9 Summary: Modelling the postembryonic CMZ

CMZ population dynamics, as encapsulated in our datasets, are best explained by a model of slowly decaying cell cycle rate and constant niche exit rate. This parameterisation is superior to phased models of CMZ activity, although both imply an initial period characterised by CMZ population growth, followed by a second phase of during which the CMZ is depleted. These dynamics can be achieved with a a wide range of cycle times and niche exit rates, which we show by establishing posterior distributions on these parameters. These demonstrate that our estimates of cell cycle lengths are better constrained by our data than estimates of exit rates. The population of the CMZ displays an asymmetric structure which reverses over time; we have shown this is not due to time-shifting of the two proliferative phases across anatomical axes, nor can it be explained by different initial cell cycle times and decay rates. The proportional layer composition of CMZ-driven contributions is stable over this time, and does not display variability over the dorso-ventral axis. However, by estimating the evidence for time-stable and time-varying models of lineage contribution, we identified a decline in particular retinal lineage subtypes, particularly those of the INL, but including the GCL. This a potential explanation for the teleost postembryonic change in retinal mosaic pattern. Finally, we have shown that microglia-mediated turnover of retinal neurons is occurring at a rate too low to have a measurable effect on cohort sizes during this time.

Thus, the niche history of the CMZ is not adequately explained by a "frozen" population of RPC progenitors, homeostatically recapitulating a program of development found in embryonic progenitors, as has been suggested previously [HP98, WAR+16], for the proliferative potential implied by these models is too low. Instead, a model that does not limit the size of lineages, but has a slowly decaying cell cycle rate, is sufficient to produce the observed niche population history. This suggests that the postembryonic CMZ is under a separate regulatory regime from the embryonic RPCs, which contribute the central, larval remnant of the neural retina. This period of RPC activity benefits from separate modelling and investigation. Moreover, the decaying cycle rate model suggests an explanation for the difference between animals that have sustained CMZ RPC activity in adulthood, like teleosts, versus those that do not, such as mammals: the rate of decay in mammals may simply be much higher.

In addition to discovering unique features of postembryonic CMZ activity, we have proved out a general framework for Bayesian evaluation and selection of arbitrary biological models, which can inform both future experiments as well as modelling and theoretical choices. For instance, slice models more successfully combine population and morphological information, to answer questions about CMZ population structure, than do abstract whole-population models of the CMZ. From the foregoing analyses, we conclude that treating the CMZ population as a combined unit with shared proliferative parameter evolution is likely well justified, suggesting that much of the apparent complexity of the niche's population history can be usefully abstracted away.

4.10 Future directions

These studies show that the simple datasets we have collected are sufficient to suggest a general model of the postembryonic CMZ, which does not explicitly limit the proliferative potential of RPCs by a measure of time in lineage, like the SMME models. However, they also reveal that these datasets provide little constraint on many important parameters of interest. This is an important insight, as future experiments can be designed with the nested sampling model comparison in mind. For instance,

our models consistently exhibit poorly constrained niche exit rates. The kinetics of niche exit could be examined by thymidine pulse-chase assays, timed to catch cohorts of RPC progeny as they cross the boundary from the CMZ into the neural retina, and this would likely significantly improve our estimates. Cell-based models of cycle parameters can glean some information from cumulative thymidine analogue labelling alone, but these would be greatly improved with costaining for markers of other cycle phases, for instance using phosphorylated histone H3 as a marker of M phase [RTL+18]. Experiments using multiple different thymidine analogue labels [HZP18] are another avenue for refining these estimates.

In general, the model comparison approach worked out here seems promising for testing competing explanations for observed phenomena. It would be worthwhile to improve the accuracy of `GMC_NS.jl`, and to continue developing models that make reference to the widest available array of observations and interventions. Given some adjustments to the experimental approach, and a refined sampler, this approach could replace obsolete models of cell cycle like the one described by Nowakowski et al. [NLM89], while giving a more realistic picture of the information actually provided by particular datasets.

As a final remark, it should be emphasized that the conclusions of this chapter always refer to the quality distribution jointly implied by the model and the data. Despite our failure to distinguish between RPC behaviours in different morphological regions of the CMZ, these rejected hypotheses should be returned to and re-tested with refined models and datasets. The same is true of phased and single-regime models of RPC cell cycle regulation. By proceeding in this manner, as Feyerabend suggested, we can "maximise the empirical content of [our] views" by "try[ing] to improve rather than discard the views that have failed in the competition." [Fey93, p.20]

Chapter 5

Mutant npat results in nucleosome positioning defects in *D. rerio* CMZ progenitors, blocking fate specification but not proliferation

Data and images presented in Figure 5.1, Figure 5.8, Figure 5.10, Figure 5.12, and Supplementary Figure 13.6 were generated by Monica Dixon, who also collected npat ISH and pax6 IHC observations in Figure 5.9 and prepared samples for Figure 5.7. Maria Augusta Sartori prepared Figure 5.11, collected the data presented in Figure 5.13, Figure 5.14, Supplementary Figure 13.8, and performed morpholino injections underlying datasets in Figure 5.15 and Supplementary Figure 13.7.

Synopsis

(1) The Danio mutant rys *has a small eye with an apparently enlarged CMZ. (2) Mutant* rys *RPCs remain mitotic, but cycle slowly and fail to contribute to the neural retina. (3) The* rys *phenotype is primarily characterised by expanded nuclei with disorganised chromatin, expanded progenitor identity, and marginal cell death. (4) The causative mutation of* rys *lies in the npat gene, which produces elevated levels of H2A and H2B histone transcripts. (5)* rys *nucleosomes are displaced relative to siblings, and there is a subpopulation of position present in mutants but not siblings, and another in siblings but not mutants (the "differential sets"). (6) Estimation of Bayesian evidence for ICA models of nucleosome sequence preference suggest that the differential sets are produced by separate causal processes. (7) Maximum a posteriori estimates of these models suggest that the* rys *differential set positions are much more strongly influenced by nucleosome-DNA interaction than the sibling differential set. (8) This suggests a mechanism for npat's effects on specification through the disruption of normal chromatin organisation, possibly by altering the available pool of histones.*

5.1 Introduction

The zebrafish (*D. rerio*) circumferential marginal zone (CMZ), located in the retinal periphery, contains the retinal stem cells and progenitors responsible for the lifelong retinal neurogenesis observed in this cyprinid. Analagous to CMZs in other model organisms, such as *X. laevis* [PKVH98], it has been of particular interest to us since the discovery of quiescent stem cells at the mammalian retinal periphery [Tro00], as an understanding of the molecular regulation of this proliferative zone may shed light on whether these mammalian cells might be harnessed for the purpose of regenerative retinal medicine. While significant progress has been made in this direction [RBBP06], molecular lesions in a plethora of zebrafish mutants displaying defects in CMZ development and activity remain largely uncharacterised. We examine here one such micropthalmic line identified in an ENU screen, *rys* [WSM+05], characterised by Wehman et al. as a Class IIA CMZ mutant, with a small eyes (see Figure 5.1) and apparently paradoxically enlarged CMZ.

Figure 5.1: ***rys* mutants exhibit a small-eye phenotype**
8 dpf *rys* sibling (Sib) and mutant (Rys), head area (top panels), whole body (bottom panels). Red dashes highlight the overall reduced eye size in homozygous *rys* mutant animals.

Mapping revealed the causative *rys* mutation in the zebrafish npat gene, the nuclear protein associated with the ataxia-telangiectasia locus in mammals [IYS+96]. Although npat is heretofore uncharacterised in zebrafish, its mammalian homologues, human NPAT and mouse Npat, have been extensively examined. These studies demonstrated that NPAT plays a critical role in coordinating events associated with the G1/S phase transition in proliferating cells [YWNH03]. S-phase entry requires tight co-ordination between the onset of genomic DNA synthesis and histone production, in order to achieve normal chromatin packaging and assembly. NPAT, found in the nucleus [STN+02] and localised, in a cell-cycle dependent manner, to histone locus bodies [GDL+09], induces S-phase entry [ZDI+98] and activates replication-dependent histone gene transcription by direct interaction with histone gene clus-

ters [ZKL+00] in association with histone nuclear factor P (HiNF-P) [MXM+03]. The protein's effects on S-phase entry and histone transcription are associated with distinct domains at the C-terminus and N-terminus, respectively [WJH03]. NPAT is also known to associate with the histone acetyltransferase CBP/p300 [WIEO04] and directs histone acetylation by this enzyme [HYSL11].

NPAT is involved in E2F transcriptional regulation [GBB+03] and a is substrate of cyclin E/CDK2 [ZDI+98], although E2F-independent activation by cyclin D2/CDK4 in human ES cells has also been described [BGL+10]. The expression of NPAT protein peaks at the G1/S boundary [ZDI+98], as does its phosphorylation, which promotes its transcriptional activation of replication-dependent H2B [Ma00] and H4 [MGv+09] genes, while its effect on low, basal levels of H4 transcription is phosphorylation-independent [YWNH03]. Of particular interest, NPAT has recently been found to be required for CDK9 recruitment to replication-dependent histone genes [PJ10]; CDK9 and monoubiquitinated H2B are essential for proper 3' end processing of stem-loop histone transcripts [PSS+09]. NPAT and HiNF-P have also been found associated with the U7 snRNP complexes that perform this function [GDL+09]. The replication-dependent activities of NPAT are thought to be terminated by WEE1 phosphorylation of H2B, which excludes NPAT from histone clusters [MFKM12].

All of these studies were performed in tissue culture, perhaps due to the challenges associated with studying this critical protein in vivo; mouse embryos with provirally inactivated Npat arrest at the 8-cell stage, for instance [DFWV+97]. The availability of *rys*, a zebrafish npat mutant which develops well into the larval stage, is therefore of considerable interest, as it allows for the study of npat's function within complete tissues. We demonstrate here that npat is critical for fate specification of CMZ neural progenitors. We find substantial evidence for the role of *D. rerio* npat involvement with histone transcription, nucleosome positioning, and scheduling of mitotic activity; we suggest that the *rys* phenotype is brought about by a failure to coordinate the genomic states required to specify, independently of proliferative capacity, in postembryonic zebrafish RPCs of the CMZ.

5.2 Results

5.2.1 The *rys* CMZ phenotype is characterised failure of RPCs to specify, altered nuclear morphology, aberrant proliferation and expanded early progenitor identity

rys reportedly has an enlarged CMZ [WSM+05], but whether this is a consequence of more RPCs, larger RPCs, or other causes, is unknown. The overall appearance of *rys* retinal sections is displayed in Figure 5.2. Two *rys* mutant eyes are displayed below a typical sibling eye. The bottom-most *rys* retina displays the extensive disorganisation which is the typical retinal endpoint of the *rys* pathology.

Figure 5.2: **Typical *rys* sibling and mutant retinal organisation and appearance**
Left panels display maximum intensity projections of confocal micrographs taken of 14 µm central coronal cryosections through a representative 5dpf sibling eye (top panels) and two *rys* eyes, displaying less (middle panels) and greater (bottom panels) retinal disorganisation. Red staining of PCNA+ve nuclei against a blue Hoechst 33342 counterstain. Right panels display schematics of sectional nuclear organisation, with nuclear positions given by blue dots and PCNA+ve RPC nuclei highlighted with yellow. Methods in Section 13.1.5.

We examined niche ontogeny during the early life of *rys* animals using two histochemical markers of cycle activity: Proliferating Cell Nuclear Antigen (PCNA), which in *D. rerio* is expressed throughout the cell cycle; and the genomic incorporation of the thymidine analogue EdU over a 12 hour pulse, marking passage through S-phase during this time. These data are displayed in Figure 5.3. During this period, the PCNA positive population of the sibling CMZ declines (Panel A), with a corresponding decrease in proliferative activity as measured by the incorporation of EdU (Panel B). Whether or not the mutant *rys* CMZ population is enlarged by comparison depends on the age at which it is sampled. There is a 99.6% probability that the posterior mean RPC population in mutants is less than the sib mean at 4dpf, while there is a 99.8% probability of the 10dpf *rys* mean being greater than that of sibs. Therefore, the *rys* CMZ population is better described as achieving its peak periembryonic size later than siblings; its population is only numerically larger than sibs at later ages[1].

Figure 5.3: ***rys* CMZ populations start relatively small and quiescent, end abberantly large and proliferative**
Panel A: Counts and estimated mean ±95% credible intervals of PCNA-positive cells in the peripheral CMZ of *rys* (magenta) and their siblings (green). Counts obtained from one 14 µm central coronal cryosection per individual. Panel B: As above, but for counts of double PCNA-,EdU-positive cells in the CMZ after a 12 hour pulse of EdU. Methods in Section 13.1.5. Code in Section 17.8.25.

Few *rys* RPCs are actively cycling at 4dpf, by comparison to the rapidly proliferating sibling cells; a mean estimate of 96.2±3.4% of sib RPCs are labelled during the 12 hr pulse at 4dpf, while only 24.1±37.2% of *rys* RPCs are. The situation is broadly reversed at 10dpf, with only 24.4±14.9% of sib RPCs labelled, compared to 65.8±16.2% of *rys* RPCs. Confirming this late-stage increase in *rys* thymidine analogue labelling represents bona fide mitotic activity, we found mitotic figures in labelled mutant RPCs at 10dpf, shown in Supplementary Figure 13.1. In *rys* animals which have actively proliferating RPCs at early ages, the CMZ fails to contribute to the central neural retina. Confocal

[1] *rys* animals universally die by approximately 3 weeks of age.

micrographs displaying *rys* CMZ cohorts labelled with BrdU at 3dpf that have failed to enter the neural retina after 7 days of chase time are displayed in Figure 5.4.

Figure 5.4: ***rys* CMZ RPCs fail to contribute to the neural retina**
14µm coronal cryosections through representative sib (left panels) and *rys* (right panels) eyes at 10dpf, 7 days after an 8hr BrdU pulse at 3dpf. Green: anti-BrdU labelling. Blue: Hoechst 33342 counterstain. Methods in Section 13.1.6.

The thymidine analogue pulse-chase studies described above give little direct information about cell cycle characteristics, so we performed a cumulative thymidine labelling experiment in *rys* mutants and siblings at 5dpf, when similar proportions of the PCNA+ve CMZ population retain thymidine label. We inferred cycle parameters by nested sampling, using the thymidine slice model simulator presented in Chapter 4. Kernel density estimates of relevant cycle posterior distributions are presented in Figure 5.5, while maximum a posteriori model output for sib and mutant data are presented in Supplementary Figure 13.2 and Figure 13.3, respectively. These analyses indicate that the mean cycle time in *rys* mutant cells is likely to be longer than that of siblings, and a larger fraction of this time may be spent in S phase.

Examining 5dpf CMZ RPC nuclear number and morphology (Figure 5.6) suggests the enlarged appearance of *rys* CMZs is the failure to contribute to the neural retina, leading to *rys* RPCs contributing

Figure 5.5: **Cycling RPCs in 5dpf *rys* mutant retinas have longer cell cycle times** Shared prior and estimated posterior marginal distributions for parameters of sib and rys cell cycle models, given cumulative EdU labelling data. The position of the maximum a posteriori model in the ensemble is marked with a vertical line. Top panel: Marginal posterior on the mean of the LogNormal model of cycle time. Middle panel: Marginal posterior on the variance of the LogNormal model of cycle time. Bottom panel: Marginal posterior on the fraction of the cell cycle spent in S phase. Methods in Section 11.1.2.2 and Section 12.1.12. Code in Section 17.8.16.

half as many central neurons as their siblings (Panel B), despite having similar overall populations at his age (Panel A). This appearance may be enhanced in the dorsal CMZ population, which tends to be more numerous in *rys* than sibs, although this tendency is reversed in the ventral population (Panels C and D). More significantly, *rys* nuclei are usually larger than their siblings (Panel E), as well as consistently less spherical (Panel F).

To rank these contributions to the *rys* nuclear phenotype, we calculated the Bayesian evidence (marginal probability) for combined Log-Normal (population measurements) and Normal (nuclear mea-

Figure 5.6: **7dpf *rys* CMZ RPCs display enhanced asymmetry, increased volume, decreased sphericity, and relative but not absolute enlargement**
All panels display sib observations as green circles and *rys* as magenta triangles. Plotted behind the underlying observations are the calculated marginal posterior distributions of the mean, given log-Normal models of the population data and Normal models of the volume and sphericity data. The y-axis shows the relative likelihood of underlying mean values on the x-axis, given the data. Panel A: Total PCNA-positive RPC population per central coronal section. Panel B: Number of PCNA negative, specified central retinal neurons per PCNA postive RPC. Panels C and D: Dorsal and Ventral RPC populations per central coronal cryosection. Panel E: Mean volume of nuclei in a given individual's central coronal cryosection. Panel F: Mean sphericity of nuclei in a given individual's central coronal cryosection. Methods in Section 13.1.5. Code in Section 17.8.18.

surements) models of sib and rys data, against the joint evidence for separate models. These estimates

are presented in Table 5.1. Of the effects that differentiate *rys*from sibs (positive evidence ratio, logZR in Table 5.1), the largest is decreased nuclear sphericity, followed by decreased nuclear volume. These are followed by differences in dorsal and ventral CMZ populations, then by the mutant's decreased number of central retinal neurons relative to the RPC population. These models suggest the nuclear morphological changes are the most important contributors to the *rys* enlarged-CMZ phenotype.

Table 5.1: **Evidence-based ranking of phenomenal contributors to *rys* phenotype**

Parameter	**Separate rys-sib logZ**	**Combined rys-sib logZ**	**logZR**	**σ sign.**
Sectional CMZ pop.	-299.9 ± 1.1	**-203.4 ± 1.0**	-96.6 ± 1.5	-65.4
Central pop./RPC	**-56.11 ± 0.27**	-79.52 ± 0.17	23.41 ± 0.32	73.7
Dorsal CMZ pop.	**-175.7 ± 0.92**	-228.4 ± 1.4	52.7 ± 1.6	32.2
Ventral CMZ pop.	**-136.6 ± 0.24**	-167.15 ± 0.13	30.55 ± 0.27	113.8
Nuclear Volume	**-164.57 ± 0.22**	-224.3 ± 1.7	59.7 ± 1.7	34.4
Nuclear Sphericity	**-87.19 ± 0.27**	-265.5 ± 2.3	178.3 ± 2.3	78.4

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratios in favour of separate modelling of the sib and *rys* data. Largest evidence values bolded. Methods in Section 13.1.8. Code in Section 17.8.24.

Enlarged *rys* RPC nuclei have a billowy appearance suggestive of chromosomal disorganization. We used electron microscopy to examine the nuclear ultrastructure of RPC nuclei in *rys* and sibling CMZs. Representative electron micrographs are presented in Figure 5.7. At 36000x magnification, the stretched and pancaked structure of *rys* nuclei is apparent, in contrast with the consistently teardrop-shaped nuclei of sibling RPCs. When the chromatin is imaged at 210000x magnification, it appears less electron-dense in *rys* mutants, with larger tracts of presumptive euchromatin, and less regular spacing of chromosomal material.

Figure 5.7: **RPC nuclei of the *rys* CMZ display disorganized, loosely packed chromatin** Representative electron micrographs of *rys* sibling and mutant CMZ RPC nuclei. Left panels: siblings. Right panels: *rys*. Top panels: 36000x magnification, general overview of the area around the nucleus, dashed black. Area displayed in middle panels dashed white. Middle panels: 110000x magnification nuclear detail. Area displayed in bottom panels dashed white. Bottom panels: 210000x magnification, chromosomal ultrastructure. Methods in Section 13.1.11.

If RPCs in *rys* CMZs fail to enter the specified neural retina, but by 7dpf are becoming mitotically active, an explanation for the proliferative niche's population declining by 10dpf is required. We suspected *rys* RPCs may be apoptosing in situ. Although we did not detect pyknotic nuclear fragments in our EM investigations, it may be that the apoptotic events are too rare in *rys* to detect in this manner. We therefore assayed the presence of the apoptotic marker caspase-3 in *rys* and sib CMZs. As displayed in Figure 5.8, caspase-3-positive nuclei can be detected in the *rys* CMZ at both 4 and 6 dpf (mean 6.5 ± 7.5 and 2.6 ± 1.1 cells, respectively) but are not found in sib CMZs, though both display a similar level of central apoptotic activity (4dpf *rys*: 1.25 ± 1.0; 4dpf sib: 1.4 ± 1.1; 6dpf *rys* 0.6 ± 0.9; 6dpf sib: 1.0 ± 1.4). The lack of debris observable in *rys* CMZs is likely attributable to the activity of 4C4-positive

microglia active in the area; we observed one such cell actively phagocytosing TUNEL-labelled *rys* RPC nuclear fragments in the CMZ, displayed in Supplementary Figure 13.4.

Figure 5.8: ***rys* mutant CMZs have increased caspase-3 positive nuclei**
Number of caspase-3 positive nuclei counted in CMZ, the border between the CMZ and specified central retina, the specified central neural retina, and the overall total, at 4 dpf (left panel) and 6 dpf (right panel). One central 20 µm section per sibling (S) and *rys* (R) larva. Methods in Section 13.1.10. Code in Section 17.8.20.

Suspecting that *rys* CMZ RPCs may maintain an early progenitor identity, we examined pax6a immunostaining of this population, which is normally restricted to putative progenitors in the peripheral and middle CMZ, as well as the retinal ganglion cell layer [RBBP06]. The Pax6-stained region was enlarged in *rys* CMZs relative to their siblings (Figure 5.9, center panels). As vsx2 is also known to be a marker of retinal progenitor cells in the CMZ [RBBP06], we also generated a transgenic vsx2::eGFP *rys* line, which produces eGFP expression in the utmost retinal periphery in siblings. Mutant fish from this line displayed substantially expanded eGFP expression in the CMZ (Figure 5.9, right panels).

Figure 5.9: **Mutant *rys* RPCs display expanded expression of early progenitor markers** Representative transmitted light and confocal micrographs of *rys* sibling and mutant CMZ RPCs. Top panels: siblings. Bottom panels: *rys* mutants. Left panels: in-situ hybridization using npat probe (blue) on 20µm coronal cryosections. Middle panels: anti-Pax6 (red) on Hoechst 33342 nuclear counterstain. Right panels: anti-GFP (green) on Hoechst 3342 nuclear counterstain, in a Tg(vsx2:GFP) line introgressed into *rys*. Methods in Section 13.1.9.

5.2.2 The micropthalmic zebrafish line *rys* is an npat mutant

We performed linkage mapping to identify the causative mutation responsible for the *rys* phenotype, followed by PCR analysis of the transcript products of candidate genes. This study revealed a single G>A transition at position 24862961 on chromosome 15 (NC_007126.5, Zv9), annotated as the first base of intron 9 in the zebrafish npat gene, in a canonical GU splice donor site.

We observed that this mutation reliably results in the retention of npat intron 8, and, less frequently, in the retention of both introns 8 and 9, in 6dpf *rys* mutants, shown in Figure 5.10. The predicted mutant protein is truncated by a stop codon at residue 283, which would preclude the translation of predicted phosphorylation sites and nuclear localisation signals cognate to those identified in human

NPAT [Ma00, STN+02], as displayed in Figure 5.11.

Figure 5.10: **RT-PCR analysis reveals two aberrant intron retention variants in *rys* mutant npat transcripts**
Top panel: Agarose gel electrophoresis of mRNAs prepared from 3dpf homozygous wild type (+/+), heterozygote (+/-), and homozygous mutant (-/-) animals, as identified by genomic PCR. Fragments were amplified from a forward primer sited in exon 8 and a reverse primer sited in exon 11. Bottom panel: exon/intron layout schematics of the putative transcripts detected. Methods in Section 13.1.12.

Figure 5.11: **Functional domains of Human NPAT compared to predicted wild-type and *rys* *Danio* npat**
Proteins presented as linear domain schematics with number of amino acids along the horizontal axis.

Because *D. rerio* is a teleost known to have undergone genome duplication in an ancestral clade, we used the Synteny Database tool [CCP09] identify any possible npat duplicates. A duplicated gene could explain the lessened severity of the mutant *Danio* phenotype when compared to mammalian proviral inactivants. The results of this analysis are presented in Supplementary Figure 13.5. While npat is in the midst of a region which appears to be duplicated on chromosomes 5 and 15, relative to the homologous syntenic run on *H. sapiens* chromosome 11, it is not, itself, duplicated.

We performed in situ hybridisation against npat transcript to confirm that the gene is indeed expressed in wild type CMZs; we found that npat expression is progressively restricted to the CMZ from 4 to 6 dpf in wild-type fish. A representative time-course of 20 µm cryosections through ISH-treated embryos is depicted in Figure 5.12, focusing on the retina at the times when it has formed. Although npat remains transcribed in both the specified GCL and amacrine-rich inner INL to a degree, it is most intensely expressed in the proliferative CMZ, as suggested by its cell cycle functions.

Given unusual chromatin ultrastructure in *rys* CMZ RPCs, and npat's role in histone regulation, we investigated the transcriptional status of npat and its histone regulatory targets by RT-PCR. Homozygous *rys* mutants overtranscribe npat by about 3-fold compared to their wild-type counterparts at both 6dpf and 8dpf. Sibling overabundance compared to wild-type declines from about 2.5-fold to 1.5-fold over this time period, as shown in Supplementary Figure 13.6.

Figure 5.12: **In situ hybridization reveals progressive restriction of npat expression to the CMZ**
20µm sections of wild-type embryo (1dpf) and retinae (4 and 6 dpf), displaying progressive restriction of npat expression (blue) as assayed by in-situ hybridisation. Methods in Section 13.1.9.

Since mammalian NPAT regulates histone transcription and is critical for coordinating expression of replication-dependent histones required to package genomic DNA during S-phase [ZKL+00], we hypothesized that the altered nuclear morphology we observed in *rys* CMZ RPCs may be a consequence of perturbed histone transcription. To test this, we performed qPCR on random-hexamer-primed cDNAs produced from 6 and 8dpf wild type, sibling, and *rys* embryo mRNA extracts. These qPCR assays were performed using degenerate primers[2] directed toward zebrafish core histone gene families H2A, H2B, H3 and H4. As a role for NPAT in 3' end processing of histone transcripts has been identified [PJ10], we also set out to determine whether 3' end processing of histone transcripts is altered in *rys*. By repeating these qPCR assays on oligo-dT-primed cDNAs, we examined the population of polyadenylated histone transcripts in isolation.

These results demonstrated that both mutant *rys* and siblings overexpress histone transcripts to various degrees. In order to determine which histone families are most plausibly involved in the *rys* mutant phenotype, we calculated the statistical significance for mean rys transcript levels exceeding mean sibling transcript level or the WT standard (set to 1.0). To be implicated in the phenotype, mutant transcript levels should be higher than both siblings and wild-type animals. We used a cutoff of 2 SDs of significance for both the sibling and WT tests for a given transcript family. These values are presented in Table 5.2.

Both total and polyadenylated *rys* H2A are overexpressed in mutants relative to both siblings and WT controls at 8 dpf, while total H2B is overexpressed at 6dpf and polyadenylated H2B at 8dpf, using the criterion of 2 SDs of significance on both tests. We note that the overabundances of polyadenylated H2A and H2B are approximately 2-20 fold greater than those observed for any other transcript pool or histone family.

The joint probability that mean *rys* mutant total H2A is greater than the sib mean at both 6dpf and

[2]Zebrafish have notably populous histone clusters with numerous variants and pseudogenes not present in non-genomically-duplicated vertebrates, so degenerate primers are an appropriate way to survey the population of transcripts. A full catalogue has not been undertaken, to my knowledge.

Figure 5.13: ***rys* overexpress total and polyadenylated core histone transcripts** qPCR results for degenerately-primed families of core histone total transcripts (left panels), and polyadenylated transcripts (right panels). From top to bottom, H2A, H2B, H3, and H4 family results are displayed in separate panels. Animal age in dpf on the x axis, fold change vs. wild-type standard on the y axis. Methods in Section 13.1.13. Code in Section 17.8.23.

8dpf to be 99.7% ± 5.0; the same figure for H2B is 98.5% ± 5.1. The joint probability that mean total H2A and H2B are elevated in *rys* mutants compared to sibs at both ages is therefore 98.2% ± 7.1. For the polyadenylated transcripts, the joint probability that the *rys* mutant polyA H2A mean is greater than the sib mean at both ages is 94.1% ± 9.9; the same figure for H2B is 92.6% ± 3.3. The joint probability that mean polyadenylated H2A and H2B are elevated in *rys* relative to sibs at both ages is then 87.1% ± 9.7.

On the basis of these calculations, the most plausible causal contributors to the *rys* phenotype are H2A and H2B family transcripts. While the relative magnitude of total H2A and H2B transcript overexpression is less than that of the polyadenylated pool, we have less uncertainty about these figures. Still, the polyadenylated transcript results suggest mutant npat increases polyadenylation of these transcripts, possibly as a result of a loss of regulation of this system. Unlike the total transcript pool, siblings retain tight, WT-like control over polyadenylated transcripts. *rys* mutants display much more extensive

Table 5.2: **Standard deviations of significance for *rys* mutant transcript > sib or WT**

Transcript Family	Age (dpf)	Total		polyA	
		Sib mean σ	WT std σ	Sib mean σ	WT std σ
H2A	6	1.59	3.72	1.68	1.68
H2A	8	**2.75**	**5.4**	**2.18**	**2.3**
H2B	6	**2.12**	**2.37**	1.47	1.52
H2B	8	1.9	7.59	**2.76**	**2.89**
H3	6	0.77	3.39	0.6	0.97
H3	8	1.21	1.83	0.78	0.54
H4	6	-0.26	-0.58	1.15	1.46
H4	8	0.79	8.83	0.45	0.35

Standard deviations of significance for differences between mean *rys* mutant transcript expression, relative to siblings or WT standard (1.0), at 6 and 8 dpf. Values for histone transcript pools with >2 SDs of significance for both sib and WT tests in bold. Methods in Section 13.1.13. Code in Section 17.8.23.

variability in polyA transcript expression, even where the mean is not elevated above siblings or WT controls.

We further investigated npat's involvement by perturbing its transcription using morpholino injections of wild-type (AB strain) fish. As we do not suppose early morpholino transcriptional blockade will exactly replicate the cellular conditions of a genetic null some days after birth, particularly given the maternal contribution of npat transcript [HSK+13], we did not seek to recapitulate the *rys* phenotype. Rather, we sought to determine if any of *rys*'s constituent phenomena would appear after an early blockage of *rys* transcription, further substantiating the general involvement of npat in *rys*. We tested morpholinos directed both to the npat start codon and to the splice site affected in *rys*, alongside control morpholinos and uninjected animals. We used both a morpholino directed to the transcript ATG start site, as well as to the affected splice site.

The splice morpholino replicated the *rys* overexpression of both total and polyadenylated core histone transcript, while the ATG morpholino more narrowly replicated the effect on polyadenylated transcript (Supplementary Figure 13.8). The ATG morpholino also produced some animals with small eyes by 72dpf, as shown in Figure 5.14, although, in general, the disorganisation observed in *rys* retinae was not recapitulated, as displayed in Supplementary Figure 13.7. We also conducted an analysis of the nuclear morphological parameters measured in Figure 5.6 for *rys* mutants and siblings. For each of the ATG and splice morpholinos, we estimated the joint evidence for separate experimental morpholino and control morpholino models against a combined model. The evidence calculations are presented in Supplementary Table 13.1. We found substantial evidence in favour of separate models for the ATG and control morpholino effects on the number of central retinal neurons per proliferating RPC, suggesting that the ATG morpholino tends to decrease CMZ contribution to the specified retina. This result is displayed in Figure 5.15; the effect is more subtle than that displayed by mutants in Figure 5.6. These results establish that the identification of npat as the causative mutation in *rys* is plausible, as experimental perturbation to npat in WT fish can result in *rys* phenotypic phenomena, notably overexpression of core histone transcripts and small eyes arising from a decrease in the population of the central retina without a concomittant decrease in CMZ population.

Figure 5.14: **48hpf embryos injected with an npat ATG-targeted morpholino display a small-eye phenotype**
48 hpf wild-type embryos injected at the 1-2 cell stage with a control morpholino (CTRL), a morpholino targeting the npat splice site affected by the *rys* mutation (Spl MO), and a morpholino targeting the npat start codon (ATG Mo). Methods in Section 13.1.3.

5.2.3 *rys* siblings and mutants have unique sets of nucleosome positions, best explained by different sequence preferences and increased sequence-dependent positioning in mutants

Perturbed histone expression in *rys* embryos suggests disrupted chromatin organisation as an explanation for the mutant nuclear phenotype, a possible consequence of altering the dynamic composition of the histone pool available for nucleosome formation during cell cycle. We therefore characterised nucleosome positioning in *rys* and wild-type siblings by micrococcal nuclease (MNase) digestion of pooled genomic DNA (gDNA) [CZ12]. Nucleosome-protected fragments from the MNase digests were sequenced and positions called as described previously [CXP+13].

Figure 5.15: **ATG morpholino perturbation of npat recapitulates decline in mean central retinal population relative to CMZ**
Populations of 14µm central coronal cryosections from retinas of animals injected with control (green triangles), npat splice-directed (yellow diamonds), and npat start codon-directed (magenta circles), standardized by number of CMZ RPCs. Plotted behind the underlying observations are the calculated marginal posterior distributions of the mean, given log-Normal models of the data. The y-axis displays the relative likelihood of underlying mean values on the x-axis, given the data. Methods in Section 13.1.3, Section 13.1.5. Code in Section 17.8.19.

We first examined the genomic disposition of nucleosome positions within wild-type siblings and *rys*. Nucleosome positions are distributed relatively evenly over sib genomic material, with only tiny deviations from a neutral assumption of a uniform distribution, as displayed in Fig. 5.16, panel A. Bulk *rys* chromatin displays minor differences from the proportions of positions found in each sib scaffold (panel B), with a tendency toward more even distribution of positions. Sib nucleosome positions are differentially occupied across scaffolds, with Chr 10 and scaffolds not yet mapped to chromosomes (NC) being notably more occupied than expected from scaffold length alone (panel C). Similarly to the distribution of positions, *rys* chromatin is more evenly occupied than sibs; scaffolds with positions that are more heavily occupied in sibs tend to be depleted in *rys* and vice versa (panel D).

By mapping the called nucleosome positions in rys to those in sibs, and calculating the number of bases the *rys* position is displaced from its mapped position, we characterised the translational displacement from the arrangement found in siblings. The probability distribution of this displacement parameter for the population of *rys* positions is displayed in Figure 5.17. The notable bimodality of this distribution arises as a result of the inclusion of unmapped positions, to show the relative size of this population: 23% of positions are not mapped to any sibling position at all; these are located at 141, greater than the full length of a called nucleosome position in our pipeline. The other three-quarters of the positions are mapped to sibling positions, with 11% of overall *rys* positions coinciding entirely with the sibling positions, 19% displaced 10 bases or less from a sib position, and a further 21% displaced between 10 and 30 bases. The last quarter of *rys* positions display more extreme displacements, up to

the entire length of the position. The periodic multimodality observed in the displacement distance of *rys* positions relative to sibs is commonly observed in nucleosome positioning and arises from the 10-base periodicity of nucleosome contacts with DNA [WC17]. This distribution of population displacement suggests that the npat mutation in *rys* results both in a loss of translational control of nucleosomes that are in approximately the expected positions, most commonly by a nucleosome "roll"of one contact, as well as dysregulation of the chromatin remodelling processes which get nucleosomes to the correct positions to begin with, represented by the novel *rys* positions, with no mapped sibling counterpart.

We sought to identify position subpopulations that might represent those involved in the nuclear phenotype of mutant *rys* RPCs. Interestingly, not only are there *rys* nucleosome positions which are not found in siblings, but there is likewise a subset of sib positions which are never found to be occupied in mutant *rys*. Intriguingly, sib positions not observed in *rys* are compensated for quite evenly across the genome by new positions gained in the mutants, with a small excess of new rys positions on most chromosomes, as shown in Supplementary Figure 13.9. Moreover, the novel positions in *rys* are much less typical of the genome, relative to lost sibling positions, than are *rys* positions which overlap ones found in sibs, summarised in Supplementary Table 13.2. This result suggested that a particular subpopulation of wild type nucleosomes are mislocalised in *rys*. This could occur for a variety of reasons, which may not be reflected in the position sequences themselves. For instance, mutant npat could affect chromatin remodelling enzymes without this having a systematic effect on the primary sequence of positions. Mutant npat could also alter the pool of available histones in proliferating progenitors, resulting in altered physico-chemical interactions with DNA, which could manifest as a preponderence of nucleosomes with unusual sequence preferences. If so, this would explain the disappearance of a subset of positions in *rys* and the appearance of a new, similarly sized subset of positions (the 'differential set').

To distinguish between these different possibilities, we calculated the Bayesian evidence ratio for models of separate emission processes for sib and *rys* position sequences, against a combined process for both sets of sequences. If the effect of npat is unrelated to direct interaction between nucleosomes and DNA, the combined model should receive the most support, since any signals appearing in the separate models would be unrelated to the status of npat. On the other hand, if separate models receive more support, DNA sequence preferences are implicated in the differntial positioning of mutant and sib nucleosomes. If the latter proves to be the case, the maximum a posteriori parameters of these models can be compared to reveal the specific changes in the prevalence and identity of detected sequence signals.

The emission model used was the Independent Component Analysis form described by Down and Hubbard [DH05], with a fixed number of independent, variable length position weight matrices related to observations by a Boolean mixing matrix. An observation is scored by the background likelihood of its sequence, given a model of genomic noise, convolved with a sequence-length- and cardinality-penalized score[3] for each source which the mix matrix indicates is present in the observation. Therefore, we first needed to construct background models of *D. rerio*'s genomic "noise" from which repetitive sequence signals characteristic of nucleosome positions could be extracted. Following the suggestion of Down and Hubbard [DH05] that a principled approach to the selection of background models is to train and test a variety of them on relevant sequence, we used the Julia package BioBackgroundModels (presented in Chapter 9) to screen a panel of 1,2,4, and 6-state HMMS against 0th, 1st, and 2nd order encodings of samples from the zebrafish genome, partitioned grossly into exonic, periexonic, and intergenic sequences.

[3]That is, the additional score provided by a source to an observation is penalized both by the expected number of motif occurrences given an observation of that length, as well as by the number of sources which explain the observation in the model.

We found that each of these partitions is best represented by 6-state HMMs trained on a 0th order genome encoding (i.e. the HMMs emit the 4 mononucleotides), as determined by model likelihood given an independent test sample, displayed in Supplementary Figure 13.10.

We used the Julia library BioMotifInference (presented in Chapter 10) to sample from the posterior distributions for these models, given the differential set of *rys* nucleosome positions and the composite background model of genomic noise. We initialized separate ensembles from uninformative priors on the *rys* and sib data alone, as well as the combined *rys* and sib data, allowing for 8 independent PWM sources in all cases. We compressed three model ensembles to within 125 orders of magnitude between the maximum likelihood model sampled and the minimum ensemble likelihood[4]. This process produced the model evidence estimates summarized in Table 5.3, as well as maximum a posteriori samples for each of the ensembles. We estimate that there are greater than 360 orders of magnitude of evidence in favour of separate generative processes for the differential sib and *rys* than for a combined model, with an estimated 525 standard deviations of significance. This large evidentiary weight and signficance give us statistical almost-certainty in selecting separate models for these sequences over a combined model. We present the MAP PWM source samples from the better-evidentiated separate *sib* and *rys* models in Figure 5.18 and Figure 5.19 respectively, while the inferred combined sources are available in Supplementary Figure 13.11.

Table 5.3: **Evidence favours separate emission models for the *rys* mutant and sibling differential position sets**

Sib logZ	*rys* logZ	Joint Sib/*rys* logZ	Combined logZ	logZR
-1.117572e6 ± 0.46	-1.161415e6 ± 0.37	**-2.278988e6 ± 0.59**	-2.279356e6 ± 0.45	368.49 ± 0.74

Methods in Section 13.1.17. Code in Section 17.8.33.

BioMotifInference's source detection was highly conservative. The background models were found to explain a majority of observed nucleosome positions adequately, without any PWM sources, in most models of the maximum a posteriori estimate, for both sibling and rys ensembles. In siblings, the top three sources are the only ones detected in more than 10% of observed sequences, suggesting that the influence of sequence preferences is weakly explanatory for these sites. By contrast, in the mutant *rys* positions, we found twenty-eight separate PWM sources detected in more than 10% of observations, in a variety of posterior modes. This indicates sequence preferences are more explanatory for the *rys* differential position set.

The CWG motifs detected in sibling sequences are commonly reported in nucleosome positions, and have been described as promoting nucleosome formation [Has03]. The majority of the sources detected in the various posterior modes were of this form, with rarer CT- and CA- dinucleotide repeats, as well as a rare ATGG repeat. *rys* positions have a much more diverse set of sources detectable above background genomic noise; notably, AG- dinucleotide repeats that are not found in *sib* sources at all. The CWG motifs also exhibit a preference for flanking A positions that is not evident in the sibling differential position set. CA- dinucleotide repeats are more commonly detected in *rys* positions, and the rare ATGG repeat is not found at all.

[4]That is, the convergence criterion was that the ensemble difference $log(L_{max}) - log(L_{min}$ be <125).

Figure 5.16: ***rys* chromosomes are differentially enriched and depleted of nucleosome position density and occupancy.**
Pie charts of nucleosome position density and occupancy by chromosome. Width of pie slices in panels A and B indicates the fraction of the total number of positions occuring in the numbered chromosomes and nonchromosomal scaffolds (NC). In panel A, depicting the sib genome, slices are colored according to the extent of deviation from an assumption of even distribution of nucleosomes across the genome. In panel B, depicting the *rys* genome, slices are colored according to the extent of deviation from the sib distribution. The width of slices in slices B and D indicate the fraction of the total nucleosome occupancy signal detected. Slice coloration depicts deviations from nucleosome occupancy distributions analogous to position distributions in A and B. Blue and white diagonal bars in the NC slice of panel C denote an out-of-scale positive deviation from the assumption of even distribution, i.e., sib NC scaffolds have 1.67-fold more of the total nucleosome occupancy signal than is expected from their length alone. Methods in Section 13.1.15. Code in Section 17.8.22.

Figure 5.17: **PWM sources detected in sibling differential nucleosome positions.** Probability distribution of *rys* position displacement distance from mapped sibling positions, in base pairs. Methods in Section 13.1.16. Code in Section 17.8.39.

Figure 5.18: **PWM sources detected in sibling differential nucleosome positions.** PWMs presented as sequence logos with letter height proportional to position informational contentemission probability.
PP: Posterior prevalence; proportion of models within the compressed maximum a posteriori ensemble which have this PWM as a signal source
OP: Observation prevalence; mean proportion of observed position sequences this sequence is used to explain, over all posterior samples
Methods in Section 13.1.17. Code in Section 17.8.33.

Figure 5.19: **PWM sources detected in *rys* mutant differential sources.**
PWMs presented as sequence logos with letter height proportional to position informational contentemission probability.
PP: Posterior prevalence; proportion of models within the compressed maximum a posteriori ensemble which have this PWM as a signal source
OP: Observation prevalence; mean proportion of observed position sequences this sequence is used to explain, over all posterior samples
Methods in Section 13.1.17. Code in Section 17.8.33.

5.3 Discussion

We began our investigations by revising the original description of the *rys* mutant CMZ phenotype. The apparently increased size of the CMZ in *rys* mutants, relative to siblings, is produced as an effect of one or more of the following phenomena:

1. Inappropriately retained RPCs in animals older than 7dpf
2. Reduced neural population of the specified central retina, relative to the CMZ
3. Expanded and spread-out RPC nuclei

The first two effects are consistent with RPCs failing to specify as particular retinal neural lineages, demonstrated by retention of early progentior markers in these cells. *rys* RPCs are not, however, arrested in cell cycle. Unlike systems where elevated polyadenylated histone transcript levels resulting in cell cyle arrest [KKT+13], we find that the mitotic activity of these cells actually accelerates under an overabundance of polyadenylated H2A and H2B transcript. We therefore suggest that the *rys* phenotype is best characterised by chromatin disorganisation (phenomenon 3) and a failure of RPCs to correctly specify (which covers phenomena 1 and 2) as retinal neural subtypes. The enlarged appearance of the CMZ is a result of these effects, and not a general increase in the CMZ population, RPC cytoplasmic volume[5], or other variables.

Moreover, in identifying npat as the lesioned gene underlying the *rys* phenotype, we nominate a plausible macromolecular mechanism underlying phenomenon 3 which could produce phenomena 1 and 2. Mutant npat's effects on cell-cycle dependent histone transcription and stability may result in abberrant nucleosome positioning within proliferating cells, by altering the pool of histone proteins available for nucleosome formation. We posit that this is what causes the observed loss of a subpopulation of sibling nucleosome positions in *rys*, together with the gain of novel, abberant positions. When we investigated whether the sequences of these subpopulations were better modelled by separate emissions processes than a shared emissions process, we found substantial evidence in favour of separate emissions processes, suggesting that nucleosome localisation to unique *rys* positions occurs by different mechanisms than those unique to sibs. Interestingly, the PWM signals we detected in this differential nucleosome position set suggest some detail as to how changes in histone expression might result in the observed nuclear disorganisation of *rys*. The observation that the background models explain the positions unique to siblings better than those unique to mutant *rys* strongly suggests that sequence preference is less important in the process generating sibling positions than in the one generating mutant positions. This explanation is summarized in Figure 5.20.

The relative importance of primary sequence in nucleosome positioning has been hotly debated; some have advocated for a nucleosome positioning code intrinsic to primary sequence [KMF+09], while others have disavowed the existence of any such code [ZMR+09]. In general, the nucleosome dynamics community has moved on from these discussions in favour of emphasis on rotational sequence preferences [TCM+07] and translational nucleosome positioning by *trans*-acting factors [KKZ+18]; a common interpretation of the earlier debate is that sequence preferences tend to dominate only at limiting concentrations of histones, when chromatin formation is rare [Poi13]. The fact that sequence preference is less explanatory for the sibling positions than for the mutant ones is suggestive against this background.

[5] That is, alone, without an increase in nucleus size. We did not directly measure this parameter.

Figure 5.20: **Summary of differential nucleosome position analysis in *rys* mutants and siblings.**
Dynamic positioning by trans-acting factors (green ellipse) is a greater influence on the positioning of sibling-exclusive nucleosome positions, overriding physico-chemical nucleosome-DNA sequence preferences (magenta box), and rendering these positions more representative of the genome overall. In *rys* mutant-exclusive positions, sequence preferences are more dominant.

It seems likely that the shifted and novel nucleosome positions observed in mutants reflects all of the following:

1. The appearance of nucleosomes with aberrant subunit composition
2. A loss of translational control over *rys* mutant nucleosomes by *trans*-acting factors
3. The limiting concentration of aberrantly-expressed nucleosomes, resulting in increased influence of "default" sequence-preference positioning

All of the above could plausibly be caused by disrupted histone expression arising from altered npat activity. As regulation of chromatin dynamics is known to be important for both replication timing [GTR+10] and cell identity and fate [SVT13], altered chromatin architecture could explain how mutated npat protein produces the observed shift in proliferative activity and failure of RPCs to specify, via altered histone expression. Moreover, chromatin density has recently been specifically implicated as fundamentally involved in the process by which pluripotent cells restrict gene expression to achieve their stable specified fates [GJH+17]; it may be this overall process which has been disrupted in mutant *rys*.

It is possible that the sequence-level inference is compromised by the ad-hoc nature of the sampler used in our nested sampling procedure. For reasons explained in Chapter 10, the ICA model structure makes euclidean space representations difficult, and ensuring sampling is in detailed balance may not be possible, given changing PWM signal lengths. We suggest that it is appropriate to magnify our error estimate. 100-fold more error than estimated has the effect of reducing the significance of the result from 498 to 4.98 standard deviations, which is still in the vicinity of the 5σ significance typically accepted as a discovery in the physical sciences. We conclude that it is more likely that the differential nucleosome populations found in *rys* mutants and their phenotypically normal siblings are the result of separate generative processes. It is also possible that our identification of the differential subset of nucleosome positions with the disordered chromatin present in *rys* RPCs is inaccurate. We have not conducted a full survey of *rys* proliferative niches, and it is not clear how widely affected other cell types might be. Still, within the retina, only proliferative cells display the chromatin phenotype, and it is on this basis that we make the identification, which is the most parsimonious available.

There are a number of important caveats for this explanation. It remains unclear what form of the npat protein is expressed, if any. The overall effect on the expressed pool of histones in RPCs also remains uncharacterised. It is likely thhat there are macromolecules invovled in the *rys* pathology which intermediate between the npat lesion and the observed chromatin phenotype which we have not examined. We suggest, however, that the overall effect on specification must be due to a failure of RPCs to organize chromatin for specification, and that, interestingly, this failure does not prevent proliferation. *rys* therefore presents a useful model of the dissociability of proliferative and specificative behaviours in neural progenitors, which has recently been documented elsewhere in the developing *D. rerio* retina [ESY+17]. Given the methodological limitations we encountered in probing protein expression in *rys* RPCs, the most interesting and productive avenues of research to pursue in *rys* pertain to this chromatin organisation phenotype. Further experimentation is required to determine how particular changes in chromosome organisation (e.g. alterations in 3-dimensional chromatin organisation of gene expression, number of replication foci, etc.) produce specific features of the *rys* phenotype.

The zebrafish *rys* mutant model provides a unique opportunity to study the role of a human NPAT homologue in a complete, developing tissue. This has previously proven difficult using mouse Npat,

proviral inactivation of which resulted in early embryonic arrest, prior to tissue formation [DFWV+97]. The survival and development of *rys* mutant embryos beyond this stage may reflect the presence of wild-type npat transcript contributed maternally [HSK+13]; *rys* mutants nevertheless do not survive beyond metapmorhosis (~21dpf), so npat seems to be similarly obligatory for normal development in zebrafish, if over a longer timeframe. Still, we observed many differences between the function of npat in zebrafish and documented effects in other vertebrates, which require some explication.

As teleost fish are known to have undergone whole-genome duplication subsequent to their radiation from vertebrates, it is possible that zebrafish may have multiple npat paralogues. We have excluded this possibility by failing to identify any significantly similar CDS sequences in the Zv9 zebrafish genome using BLAST, and the synteny analysis in Figure 13.5. The zebrafish npat gene does have a substantially different genomic context from human NPAT, however: it is not associated with the eponymous ATM locus (which has been duplicated, and is present in this duplicate form elsewhere on chromosome 15). ATM's 5' position is, in zebrafish, occupied by hif1al, with the intergenic region lacking canonical E2F promoter sites. Of the 80 vertebrate genomes currently available from Ensembl, this organisation is shared only with the cave fish (A. mexicanus), with the human-like ATM-NPAT association preserved in all other species. This apparently evolutionarily novel genomic organisation for npat may be responsible for some of the differences in npat function we report here, relative to its homologues.

We identified alterations in histone mRNA transcription and 3' end processing as likely mechanistically involved in the *rys* phenotype. In both 6 and 8 dpf *rys* larvae as well as npat-morpholino treated 1dpf embryos, we observe increased abundances of total and polyadenylated histone transcripts. The mechanism by which these effects are produced in *rys* remains unclear. The increases in histone transcript abundance observed in both mutant and morpholino-treated animals are at odds with observations that NPAT knockouts display decreases in histone transcription [YWNH03], and that the destruction of CDK phosphorylation sites on NPAT protein, which would be the case in the putatively truncated *rys* npat protein, result in similar declines in histone transcript abundance [Ma00, MGv+09]. This implies the role of zebrafish npat in regulating histone transcription differs from human NPAT. We are unable to determine from these data whether the overall increases in histone transcript abundance are a consequence of increased histone transcription, or of the greater stability of improperly polyadenylated histone transcript. The increased abundance of polyadenylated transcript in *rys* mutants and npat morpholino-treated embryos is consistent with the observed role of NPAT in recruiting CDK9 to replication-dependent histone gene clusters, known to be important in generating the normal stem-loop structure at the 3' end of these transcripts [PSS+09], suggesting that this role is conserved in zebrafish. It is also possible that particular histone genes give rise to polyadenylated transcripts in zebrafish, as observed in a variety of cell lines [KKT+13]. Increased transcription of these particular genes might also account for the observed increase in polyadenylated histone transcript abundance. Although increased abundance of polyadenylated histone transcript has been associated with both cell cycle arrest and differentiation [KKT+13], we found that in the *rys* mutant CMZ, this was associated with altered cell cycle parameters, failure to differentiate, and ultimately, cell death by apoptosis. This suggests that the presence of polyadenylated histone transcripts are not directly related to particular cell cycle states or differentiated fates, but rather that a particular regime of coordination and control of the expression of polyadenylated and replication-dependent, stem-looped histone transcripts is required to achieve appropriate transitions between these states. In the case of *rys*, this seems to be related to the effect of altered histone expression on chromatin structure. Future experiments focusing on npat could include

the testing of cell autonomous versus non-autonomous effects by the use of conditional knockouts in mice, for example by using a floxed npat gene in combination with tissue-specific promoter expression of Cre. Another avenue in zebrafish could be the selective interference of e.g. H2A and H2B expression by morpholino, to directly assay the effect of histone expression on chromatin organisation.

Chapter 6

Inferring and modelling specification dynamics in retinal progenitors

6.1 Specification as a function of non-proliferative chromatin dynamics

The npat mutant *rys* investigated in Chapter 5 reinforces a basic theme running through this thesis, that lineage commitment or specification can be, and often is, dissociable from the proliferative behaviour of retinal progenitors, as has been suggested by Engerer et al. [ESY+17]. *rys* illustrates this: RPCs undergo a final burst of proliferation before the death of the animal[1], but never contribute appropriately to the neural retina. Indeed, older *rys* larvae display a proliferative spillage of abberant RPCs into the intraocular space between the lens and GCL; the progeny of these abberant RPCs literally cannot be integrated into the tissue.

The importance of this dissociability for our inferences about RPCs can be gleaned from the basic problems encountered in each of the modelling studies. To begin with, the Stochastic Mitotic Mode Explanation models examined in Chapter 2 failed, in large part, because the underlying Simple Stochastic Models conflate proliferation and specification under the "mitotic mode" concept. This leads immediately to the introduction of model structure to explain specification outside of "mitotic mode", assuming a phased temporal structure to RPC behaviour, without attempting to explain it. Lineage specification and niche exit processes need to be modelled separately from mitotic processes to differentiate their effects on outcomes. Next, proliferative data and morphological measurements in Chapter 4 fail to constrain the posterior distributions on the rate of niche exit in simple phased models, demonstrating that this parameter cannot be inferred with any certainty from overall population-level proliferative dynamics. To accurately model the behaviour of RPCs, data which provides accurate volumetric or temporal constraints on modelled specification or niche exit rate is of paramount importance. Finally, in *rys*, we encounter the problem of explaining RPCs that proliferate but fail to specify; we explain this phenomenon in terms of the effects of mutant npat on chromatin organization by way of effects on histone expression. This directs us to the relevant level of organization for explanation of RPC lineage outcomes: the progression of chromatin states that underlay long-term changes in gene expression, which

[1] Likely, on this view, arising from systemic failure of progenitors to specify appropriately.

are required for the generation of the specified protein complement. Our approach, while providing evidence for the chromatin dynamics we suggest arise from the effects of npat on the available histone pool, does not give us insight into how the appropriate progression of chromatin states might be usefully parameterised.

While a large number of techniques for studying aspects of chromatin structure exist, few suggest a means to describe an overall "nuclear state". That is, if we consider Waddington's idea of a cell's lineage state being a ball rolling down a canalized terrain [Wad57], it remains unclear what sort of parameter space would allow us to infer where the ball is. Older ideas of cellular state space, that rely exclusively on measures of protein expression, are unable to address hypotheses about chromatin structure. It remains unclear what aspects of chromatin organisation are causally important in fate commitment, as little formal model comparison has taken place. One might propose a joint structure-sequence-expression space with thousands of variables drawn from the vast literature on chromatin organisation. Such a project is unlikely to succeed with reasonable compute budgets, even with the sophisticated Bayesian inference tools developed here.

Still, if it is not obvious how one might build a general model of the evolution of "nuclear state", we may suggest some more limited approaches arising from the work presented in this thesis, that may make some headway here.

6.2 Future directions

rys provides a useful platform for investigating the processes required to organize the progression of chromatin through the sequence of states we presume to be required for progression through the series of stages implied by the studies presented in Chapter 2 and Chapter 4, and the associated shifts in RPC lineage contributions. Despite the evidence amassed for the involvement of npat effects on nucleosome positioning in *rys* mutants, many aspects of the suggested mechanism remain obscure. This arises, in part, from the lack of reliable macromolecular tools for investigating npat and histone proteins in *D. rerio*. It would be useful to focus inquiry here on a small number of candidate histones. However, the large number of zebrafish histone genes, with no crystallographic data available, suggests that prediction of the involvement of particular histones would be difficult.

This situation is now rapidly changing, however. The AlphaFold algorithm has recently been recognized as a solution to the Critical Assessment of protein Structure Prediction (CASP) problem set in the CASP13 competition [AlQ19]. This is the first time protein folding has been adequately predicted *in silico* from sequence data alone. If it is possible to use this algorithm to predict *D. rerio* nucleosome structures from histone folding predictions, this may obviate the need for crystallographic data. Such structural predictions could be used with the posterior estimates of signals putatively arising from nucleosome-DNA contact produced in Figure 5.19 to, in turn, predict which nucleosome compositions are most likely to favour the detected sequences. This might be assisted by 3d pattern matching for DNA sequences [HPG07], or roll-and-slide modelling of the contact motifs [TCM+07] Such a study would produce hypotheses that, if interesting enough to pursue *in vivo*, would supply estimates of how likely various histones are to be involved in the *rys* mutant phenotype, and in what regard, which would help prioritise resources for the development of assays for interrogating the specific transcript and protein status of the most likely loci.

A more fundamental way of interrogating *rys* "nuclear state" might be provided by focusing on

transcription itself, which has been proposed as a primary driver of chromatin organisation [CM18]. Such a project would likely require a transgenic line developed to monitor the transcription of one or more loci of interest in live cells, perhaps using bio-orthogonally labelled transcripts, as has recently been demonstrated in zebrafish [WCG+20]. This approach could possibly be used either to investigate the status and trafficking of npat and histone transcripts themselves, or to determine the impact of the npat lesion on the transcriptional shaping of chromatin organisation by tracking the transcription of progenitor markers, for instance.

Ultimately, it is likely that models that allow the prediction and control of RPC behaviour *in situ* in zebrafish retinae will treat the chromatin organisation of RPC nuclei as an integrator of multiple relevant sources of information, including physical forces, cytoskeletal dynamics, and extracellular signals. This type of functional organisation, in which biological "meaning" arises from the integration of salient cellular information by a combination of nonarbitrary physicochemical relations and arbitrary biological coding reltaions, is currently best described by the subdiscipline of biosemiotics [Hof08, Fav15, Hof15]. Of particular interest here is the clear-eyed and unobscurantist work of Marcello Barbieri, which may well provide the metaphysical framework necessary to plumb the depths of this topic [Bar14].

Part II

Software Technical Reports

Chapter 7

GMC_NS.jl: Nested sampling by Galilean Monte Carlo

`GMC_NS.jl` implements a basic Galilean Monte Carlo nested sampler [Ski12, Ski19] in pure Julia. It uses the improved Galilean reflection scheme of Skilling's 2019 GMC publication [Ski19], with increased performance and detailed balance preserved. It uses the natural attrition of model-particles getting stuck in posterior modes to regulate the number of live particles in the model ensemble. This approach partially achieves the same end as algorithms which dynamically adjust the ensemble size based on parameters of the ensemble [FHB09, HHHL19]. A minimum ensemble size may be supplied by the user, which is maintained by initializing a new trajectory isotropically from the position of an existing particle; this ensures that the ensemble may be sampled to convergence.

It should be noted that `GMC_NS.jl` is a prototype; while it finds posteriors in relatively few iterates, it oversamples less important early regions of the posterior. See Section 7.3.5 for more information.

7.1 Implementation notes

The basic sampling scheme followed is, first, to initialize an ensemble of models with parameters sampled evenly from across the prior, then to compress the ensemble to the posterior by applying GMC moves to the least-likely model in the ensemble at a given iterate, constrained within its current likelihood contour. GMC tends to move model-particles across the parameter space evenly and efficiently, particularly in the initial portion of compression across the uninformative bulk of the prior mass. That said, GMC particles may get "stuck" in widely separated minima, so there are instances when the least-likely particle has no valid GMC moves. In this case, the trajectory is not continued. Instead, the trajectory is removed from the ensemble and sampling continues, unless the number of particles in the ensemble would decline below the specified minimum. In this case, a new trajectory is begun by resampling from the remaining prior mass within the ensemble, as in vanilla nested sampling. This is accomplished by random "diffusive" proposals, in contrast to the ordinary, directed Galilean movements. A random particle remaining within the ensemble is selected, and isotropic velocity vectors are sampled from this particle to find a starting location for the new trajectory (this vector is conferred to the new particle).

`GMC_NS.jl` follows the common convention of transforming parameter space to a hypercube. The density of the prior is uniform, that is $\pi(x) = 1$, over the -1:+1 range of the hypercube dimension (this

is referred to as the "unit ball" in the code). Utility functions `to_unit_ball` and `to_prior` transform prior coordinates to the unit ball, and unit ball positions to the prior coordinate, respectively. In future versions, this scheme is likely to be modified to a 0:1 unit hypercube, which does not require operations beyond obtaining the cumulative probability of a parameter value on the prior distribution. The user may also specify a sampling "box" to bound particles from illegal or nonsensical areas of the parameter space (eg. negative values for continuous distributions on positive-valued physical variables). The box reflects particles specularly in the same manner as the likelihood boundary, with the exception that because the orientation of the nth-dimensional box "side" is known, the reflection is performed by stopping the particle at the box side and giving it reversing its velocity vector in dimension n.

`GMC_NS.jl` attempts to maintain efficient GMC sampling by per-particle PID[1] tuning of the GMC timestep [VV12]. GMC treats models as particles defined by their parameter vectors, which give their positions in parameter space, and a velocity vector, initalised isotropically and only changed when the particle "reflects" off the ensemble's likelihood contour. In GMC, when a model-particle is moved through the parameter space (in this case, because it is the least likely particle in the ensemble), a new position is proposed by extending some distance along its velocity vector. This distance is determined by scaling the velocity vector by a "timestep" value, which defines the per-iterate rate at which the particle traverses the parameter space. As the model ensemble is compressed to the posterior, this timestep must decline in order for proposals to be acceptable, as the amount of available parameter space declines rapidly. Additionally, GMC particles may enter convoluted regions of parameter space that form small likelihood-isthmuses to other, more likely regions. In order to deal with this, each trajectory is assigned its own PID tuner, which maintains an independent timestep value for the trajectory. The PID algorithm is tuned to target an unobstructed move rate. In short, PID tuning will reduce the timestep if the particle is frequently reflecting off the likelihood boundary (in which case it is crossing the remaining prior mass without sampling much from within it, or it is in a highly convoluted area of the local likelihood surface), and extend the timestep if it the particle frequently moves without encountering the boundary, so that particles that are closely sampling an open region without encountering the boundary begin to "speed up".

PID constants and default GMC settings have been crudely tuned to produce the best possible result with the stated sampling scheme on the eggbox problem, as described in Section 7.3.5.

7.2 Ensemble, Model, and Model Record interfaces

In operation, `GMC_NS.jl` maintains a `GMC_NS_Ensemble` mutable struct in memory. The directory to which the sampler ought to save the ensemble is specified by its `path` field. `GMC_NS.jl` does not maintain calculated models in memory, instead serializing them to this directory. In order to track the positions and likelihoods of model-particles within the ensemble and as samples from the posterior, a `GMC_NS_Ensemble` maintains two vectors of `GMC_NS_Model_Records` as fields; `models` for live particles, and `posterior_samples` for the previous positions along trajectories. Observations against which models are being scored, prior distributions on model parameters, model constants (if any), and the sampling box are specified by the `GMC_NS_Ensemble`'s `obs`, `priors`, `constants`, and `box` fields, respectively. The

[1] PID stands for Proportional, Integral, Derivative, which nominate the three constants used to tune a process variable by the error of a measurement variable from a supplied setpoint. The P term is directly proportional to this error, the I term is proprotional to the integral of the error over time, and the D term is proportional to the rate of change of the error.

`GMC_NS_Ensemble` also specifies settings for the GMC sampler and the PID tuner. Sensible defaults for these values may be passed (usually, splatted[2]) en masse to an ensemble constructor with the `GMC_DEFAULTS` constant exported by `GMC_NS.jl`.

Therefore, to use `GMC_NS.jl` with their model, the user must write a model-appropriate version of these three structs (the ensemble, model, and model record) with the fields specified by the docstrings in the `/src/ensemble/GMC_NS_Ensemble.jl` file, as well as `/src/GMC_NS_Model.jl`. The user is responsible for an appropriate constructor and likelihood function for the model, any required parameter bounding functions, and so on. The user may optionally write a overloaded `Base.show(io::IO,m::GMC_NS_Model)` function for displaying the model, or a `Base.show(io::IO,m::GMC_NS_Model,e::GMC_NS_Ensemble)` function for displaying the model with observations or other ensemble data. This function can be used with the package's `ProgressMeter` displays for on-line monitoring of the current most likely model.

7.3 Usage notes

7.3.1 Setting up for a run

Given an appropriately coded model, as outlined above, the user must define a number of important values for the sampler before a `GMC_NS_Ensemble` may be constructed. These values are stored in fields of the ensemble. Firstly, the user must prepare the observation data in whatever format is suitable for the model's likelihood function. Next, one must define a vector of `Distributions` on parameters of the model as the `prior` field of the ensemble. These must be univariate, as the CDF of the distribution is used to determine position on that dimension. Some functions to produce univariate marginal distributions from multivariate Normal Gamma and Normal Inverse Gamma priors are available in `NGRefTools.jl`. Next, any constants used in the likelihood function must be available in the `constants` field of the ensemble. A `box` matrix with n-dimensional rows containing minimum and maximum values in the first and second columns defines the maximum extent of parameter space to be explored. The sampler expects the box in the transformed hypercoordinates mentioned above; a one-dimensional box across the entire prior would be [-1 1]. It is convenient to compose the matrix in the original parameter space and perform the transformation with a broadcast call to `to_unit_ball()`, either in the ensemble constructor or in the calculation script.

7.3.2 GMC and PID tuner settings

The GMC sampler is governed by settings specified in `GMC_NS_Ensemble` fields. These include the PID tuner settings, and are summarized below:

`GMC_Nmin` (Integer; Default 50): The minimum number of active model-particles in the ensemble. After the initial sample from the prior, new particles will only be generated when required to maintain this value. Otherwise, when a valid proposal cannot be found for the current least-likely trajectory (i.e. the PID tuner drives the timestep below `GMC_`τ`_death`), the nested sampling step consists of removing that particle to the posterior samples and moving on to the next live particle, without replacement.

[2] "Splatting" in Julia refers to the expansion of a argument vector into a function's call by using the ... operator following the vector's name, i.e. `function(arguments...)`.

`GMC_`τ`_death` (Float; Default $1e-5$): Value of trajectory's τ timestep, below which the particle is considered to be dead. A dead particle's trajectory is not continued. This value is denominated in -1:1 hypercube coordinates. If this value is very small ($\leq 1e-6$), trajectories may carry on for thousands of iterates in relatively unimportant regions.

`GMC_init_`τ (Float; Default 1.): Initial value of the timestep for all trajectories. This applies to the particles assembled on ensemble construction; particles generated from diffusional resampling inherit the τ of their parent, but with the isotropic vector which generated the proposal for their new location.

`GMC_tune_`μ (Integer; Default 4): Length of tuner memory; the proposal acceptance rate α is calculated over the previous μ constructor reports for this trajectory. Tends to produce oversampling if it is much more than 10.

`GMC_tune_`α (Float; Default .25): Target acceptance rate for proposals. Sampler efficiency benefits from this being fairly low (.2-.4).

`GMC_tune_PID` (NTuple3,Float; Default (.3,0.,0.)): kP, kI, and kD constants for the tuner. kP can be somewhat larger. kI and kD are probably detrimental when using short tuner memory, but could be useful if an efficient longer memory regime is available.

`GMC_timestep_`η (Float; Default .25): If this value is >0., a normally distributed error with a standard devation of $\tau\cdot$`GMC_timestep_`η is applied to τ before calculating the GMC proposals, as suggested by [Ski12].

`GMC_reflect_`η (Float; Default .005: If this value is >0., it is used to perturb reflection vectors, as suggested by [Ski12]. It is applied to sampling box reflections as well as likelihood contour reflections. It should be a small value. If it is too large, sampling box reflections can "bend" the wrong way and produce an inappropriate series of samples at the box boundary.

`GMC_exhaust_`σ (Float; Default 10.): Scale factor multiplied into ensemble size to determine the maximum number of unsuccessful searches to perform before terminating sampling. This virtually never happens.

`GMC_chain_`κ (Integer, Default typemax(Int64)). Number of iterates at which to terminate a sampling chain/trajectory. By default, this does not occur. Can prevent oversampling in some cases.

7.3.3 Parallelization

`GMC_NS.jl` does not have an explicit parallelization scheme; the sampler proceeds linearly to the next least-likely particle and performs the appropriate Galilean trajectory sampling procedure. Parallelization may nonetheless be achieved at two levels; the likelihood function may be parallel according to the needs of the user, and individual nested sampling runs may be arbitrarily combined. That is, if one desires to parallelize a `GMC_NS.jl` job, it is best to think in terms of the total number of trajectories to be sampled, dividing this by the number of machines available to perform the sampling work, to obtain the number of particles to be sampled on each machine. The sampling runs can then be combined post hoc to achieve the desired final accuracy. In this scheme, the likelihood function is best parallelized by threading on a single machine.

7.3.4 Displays

Parameters of the ensemble and most-likely model can be monitored on-line, while `GMC_NS.jl` is working. This is done by specifying a vector of function vectors (ie a `Vector{Vector{Function}}`). One such

vector may be supplied to specify the displays to be shown above the `ProgressMeter`, one for those below, supplied as the `upper_displays` or `lower_displays` keyword arguments to `converge_ensemble!()`. `GMC_NS.jl` will rotate the upper and lower displays to the next vector of display functions every `disp_rot_its` sampling iterates, supplied as another `converge_ensemble!()` keyword argument. Default display functions are available in `/src/utilities/progress_displays.jl` and are exported for easy composition of the function vectors.

7.3.5 Eggbox diagnostic

The "eggbox test" is a toy model designed by Feroz et al. [FHB09], and used by others for use in characterising nested sampling algorithms [Buc16]. It uses the likelihood function $logL = (2 + cos(5 \cdot x1) \cdot cos(5 \cdot x2))^5$. Within the unit square, this function has 18 optima and a canonical logZ evidence score of 235.88. The most efficient solution to this problem is afforded by MultiNest; it accurately sums the model evidence in approximately 10000 iterates with 400 particles. `GMC_NS.jl` correctly finds the 18 optima of the eggbox, and converges in fewer iterates, as displayed in Figure 7.1. However, it substantially underestimates the model's evidence; typical `GMC_NS.jl` logZ evidence estimates for the eggbox are consistently 229. This tendency to systematically underestimate model evidence should be kept in mind when examining the evidence estimates in this thesis. The cause of this underestimation is likely an oversampling of the prior volume arising from the minimum allowable timestep not being tied to the volume of prior mass remaining; this allows trajectories to contribute many closely-sampled points from the peripheral prior volume where the trajectory repeatedly encounters the likelihood contour (in the "corners" or less-likely peripheral modes). Suggestions for fixing this issue are given in Section 7.4.

The eggbox may be estimated with the following code:

```
using GMC_NS, Distributions

#compose priors
prior=Uniform(0,1); priors=[prior,prior]

#sampling box
box=[-1. 1.
     -1. 1.]

#monitor convergence
lds=Vector{Vector{Function}}([[ensemble_display]])

#terminate slowly moving particles
gmc=copy(GMC_DEFAULTS)
gmc[2]=1e-3

#assemble 400 particle ensemble
e=Eggbox_Ensemble("eggbox_test", 400, priors, box, gmc...)

```

Figure 7.1: **GMC-NS.jl nested sampling the eggbox toy model**

```
#converge the ensemble
converge_ensemble!(e,backup=(true,100), lower_displays=lds, converge_factor=1e-6)
```

7.3.6 Example use

`GMC_NS.jl` is supplied with two simple, but useful, statistical models by default. These are the `Normal_Model` and `LogNormal_Model` subtypes of `GMC_NS_Model`. Users may familiarize themselves with `GMC_NS.jl`'s operation by the use of these models. A simple demonstration is supplied in the `/test` directory of the package. The program is detailed with additional comments below. The demonstration involves

converging an ensemble of 10 `Normal_Model` chains initialized from an inaccurate `NormalGamma` prior on data generated from a known `Normal` distribution.

```
#Import dependencies
using GMC_NS, Distributions, ConjugatePriors

#Generate some data from a Normal model with \mu 100. and \sigma 5.
sample_dist=Normal(100.,5.)
samples=rand(sample_dist, 10000)

#An inaccurate ConjugatePrior
prior=NormalGamma(300.,2e-3,1.,10.)

#Box for the two parameters of the Normal model.
#First column: parameter minimums.
#Second column: parameter maximums
#First row: \mu
#Second row: precision, ie 1/sqrt(\sigma)
#eps() is machine epsilon, ~1e-16; a small number
box=[0. 1000.
     1/1000. 1/eps()]

#Start with the default sampler settings
#Index 1: Lower the minimum number of ensemble particles to 5
gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=1.e-15

#Assemble the ensemble. Begin with 10 particles
e=Normal_Ensemble("NE_test", 10, samples, prior, box, gmc...)

#Define the upper displays to be rendered above the status line. Here we rotate a single display between three readouts.
uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]])

#Define the lower displays to be rendered below. We want to look at the most likely model in the ensemble plotted against the observations the whole time.
lds=Vector{Vector{Function}}([[model_obs_display]])

#Converge the ensemble, serializing to pwd/NE\_test every 100 iterates and rotating the chosen displays every 1000 iterates.
converge_ensemble!(e,backup=(true,100),upper_displays=uds, lower_displays=lds, disp_rot_its=1000)
```

Typical output after executing this program is presented in Figure 7.2. The maximum a priori parameters may be read off the θ line of the `model_obs_display` below the status line. The algorithm estimates that the model from which the sample was drawn has a μ of ~ 100.29 and a precision of $\sim .045$, corresponding to a σ of ~ 4.7; the global optimum has been found despite the poor starting prior.

Figure 7.2: **Example GMC-NS.jl output**

7.4 Future Directions

`GMC_NS.jl` could be improved in at least four ways. Firstly, it ought to be trajectory- or sampling chain-centric rather than ensemble-centric, so that ensembles can be dynamically assembled in the manner suggested by Speagle [Spe19], which might fix the evidence accuracy problem this sampler exhibits. Briefly, this involves constructing an initial ensemble with a small number of trajectories or sampling chains, then adding to this ensemble by initializing new trajectories in the region of the prior volume that is most of interest to the user, prioritizing either evidence estimation by adding chains in the posterior mass, or posterior estimation by adding chains in the likelihood density. Secondly, an ellipsoidal search routine has been coded for `GMC_NS.jl`, but remains to be implemented; this could be

used both to initialize new chains in a dynamic sampling routine, and to estimate the size of posterior mode particles are currently in, for tuning purposes. Thirdly, the GMC PID tuning system likely needs improvement. The pI and pD constants are largely superfluous, and there may be a more fundamentally justified way to confer the information supplied by boundary contact to the particle by way of timestep tuning. Moreover, the minimum allowable timestep (`GMC_`τ`_death`) should be adjusted as the ensemble is built, such that it disallows small timesteps early in the sampling process. Lastly, the sampler should be refactored to use multiple different random number generation algorithms, not solely the standard Mersenne twister, which may cause problems with resonance under certain unusual circumstances [John Skilling, personal correspondence, 2020]. Beyond improvements to the implementation, `GMC_NS.jl` ought to be benchmarked against Dynesty and MultiNest to ensure that the sampler overhead is comparable or smaller to these implementations.

Chapter 8

CMZNicheSims.jl: *D. rerio* CMZ RPC niche simulators

8.1 Introduction

`CMZNicheSims.jl` (hereafter `CNS.jl`) comprises a set of three simulators of zebrafish retinal progenitor cells (RPCs), representing the postembryonic Circumferential Marginal Zone niche at varying levels of abstraction. These simulators were used in Chapter 4 and Chapter 5. This package extends `GMC_NS.jl`, described in Chapter 7, and requires it. The simulators are organized around the nested sampling procedures implemented in that package. This means that simulations are executed as sample chains ("trajectories") from the simulations' parameter space in an ensemble. The basic operation of this package is accomplished by assembling an ensemble of models from the appropriate simulator submodule of `CNS.jl`, then elongating the trajectorie of this ensemble by Galilean Monte Carlo using `GMC_NS.jl`'s `converge_ensemble` function. Note that `GMC_NS.jl`'s sampler settings are specified by fields of the `CNS.jl` model ensemble struct and not as arguments to `converge_ensemble`; even if default sampler settings are used, sampler operation should be understood before model priors are selected.

The three model ensemble types specified by `CNS.jl` are as follows:

- `CMZ_Ensemble`: Phased difference function models of CMZ population and overall retinal volume, intended to model total retinal CMZ population and its contributions to overall retinal volume over long time periods (months)
- `Slice_Ensemble`: Basic implementation of the "slice model" concept elaborated in Chapter 3 for phased difference function CMZ models, uses a submodel of lens growth to determine "out-of-slice" growth contributions, intended to represent the population of a CMZ slice over long time periods (months)
- `Thymidine_Ensemble`: "Slice model" of proliferative CMZ population exposed to a pulse of thymidine analogue, intended to model cell cycle kinetics in the CMZ over short time periods (hours).

The operation of these simulators is described in more detail below. In general, the workflow is to assemble the ensemble, converge it, and to estimate the model evidence and posterior parameters from the converged ensemble. For more on evidence and posterior estimation by `GMC_NS.jl`, see Chapter 7.

8.2 `CMZ_Ensemble` model and usage

A `CMZ_Ensemble` consists of `CMZ_Models` whose likelihood is assessed by Monte Carlo evaluation of a system of difference equations for simulated retinae, initialized from independent samples from the population and volume distributions specified in the ensemble constants vector. The update functions are specified in general by Equation 4.1 and Equation 4.2. However, the implementation of these functions in the model likelihood function (`CMZ_mc_llh`, found in Section 17.4.3) differs from this presentation; the equations are not evaluated stepwise day-by-day, but are rather updated for the next model event, either an observation timepoint or a phase transition date. The specific equations used in this function are of the computationally efficient factorial form suggested by Langtangen [Lan12, p. 559], so that the equation for the CMZ population n days after some time 0 becomes:

$$p_n = p_0 \cdot (2^{\frac{24}{CT}} - \epsilon)^n$$

while the equation for retinal volume becomes:

$$v_n = v_0 + p_0 \cdot \epsilon \cdot \mu_{cv} \cdot \frac{1 - (2^{\frac{24}{CT}-\epsilon})^{n-1}}{1 - (2^{\frac{24}{CT}-\epsilon})}$$

, where p_n and v_n are the population and volume at day n, p_0 and v_0 are the initial population and volumes, CT is the cycle time in hours, ϵ is the exit rate from the niche (expressed as a fraction of the population), and μ_{cv} is a constant expressing mean cellular volume contributed to the retina by exiting cells.

After Monte Carlo iterative simulation of many retinae, the resultant simulated population and volume distributions at the observed time points are estimated by fitting `LogNormal Distributions`, after which the joint log likelihood of the observations given the simulation is calculated. The likelihood function is threaded so that the total number of Monte Carlo iterates for any given model parameter vector is divided equally between threads. For very large numbers of iterates, this can result in some load balancing inefficiency; in general, it is unneccessary to specify more than 5 million iterates in the ensemble constants vector (item 3 below). 1 million should be adequate for most purposes and allows the efficient sampling of ≥ 3 phase models on one performant machine.

A `CMZ_Ensemble` may be assembled by specifying the path of the folder in which its constituent model files will be serialized, the starting number of chains in the ensemble (a few thousand may reasonably be expected to converge within a day or two on a single high performance machine), the observations to estimate the models against, a vector of prior distributions, a vector of constants, a box matrix, and the sampler settings vectors. The parameter vectors must follow these guidelines:

1. Observations vector: Must be a vector of vector tuples. The first member of the tuple is the population observation vector (total CMZ population at the vector's date index, see 3), the second member is the date's total retinal volume observation vector (in cubicmicrometers). Ie. the observations vector takes the form of a `AbstractVector{<:Tuple{<:AbstractVector{<:Float64},<:AbstractVector{<:Float64}}}`; the tuple of population and volume data found at index `x` corresponds from animals measured on date `constants_vec[1][x]`.

2. Prior vector: For each desired phase of the model, the prior vector must contain a distribution over that phase's mean population cycle time (in hours) and its mean exit rate ($\geq 0.$, may exceed 1.).

Paired cycle time and exit rate for each phase is followed by one less phase transition time in days than the number of phases. Ie. a two-phase prior vector might be, in pseudocode, `[Phase1CT, Phase1ER,Phase2CT,Phase2ER,Phase1End]`.

3. Constants vector: A `Vector{<:Any}` containing, in order:
   (a) T vector: Vector of integers or round floats, age in days of animals for each observations tuple
   (b) Population distribution: `Distribution` defining the initial range of population values. Should normally be a fit from the initial population size observations
   (c) Volume distribution: `Distribution` defining the initial range of retinal volume values. Should normally be a fit from the initial retinal volume observations
   (d) Monte Carlo iterates: Integer specifying the number of retinae to be simulated at each point sampled from the parameter space. 1e6-5e6 is usually fine.
   (e) Phases: Integer defining the number of phases specified by the prior vector
4. Box matrix: Two columns each of the same length as the prior, defining the minimum (column 1) and maximum (column 2) bounds for each parameter dimension to be sampled.

As an example, an ensemble with 3000 chains and using the default GMC sampler settings may be initialized and sampled as follows, where variable names correspond to the vectors described above:

```
e=CMZ_Ensemble(path, 3000, observations, prior, constants, box, GMC_DEFAULTS)
converge_ensemble!(e,backup=(true,50), mc_noise=.3)
```

The `mc_noise` parameter for `CNS.jl` models can be estimated by calculating the standard deviation of repeated evaluations of a parameter vector with a reasonably high likelihood, given the data vectors. The suggested value is roughly appropriate for 1e6 Monte Carlo iterates, given the data vectors used in Section 17.8.8.

8.3 `Slice_Ensemble` model and usage

A `Slice_Ensemble` consists of `Slice_Model`s, whose likelihood is assessed by Monte Carlo evaluation of a hybrid system of a difference equation and a power-law equation, representing a simulated CMZ RPC population in a retinal slice of arbitrary thickness. The `Slice_Model` likelihood function uses the same factorial solution to calculate the CMZ population after time steps of n days as that described for `CMZ_Model`s. However, these population values are penalized by a value determined by using the power-law model of lens growth to determine the change in CMZ circumference over this time period, and the per-slice contribution required to expand the CMZ by this amount, given the present slice population and slice thickness (see Section 17.4.10 for this calculation).

`Slice_Model`s are broadly similar to `CMZ_Model`s. The parameter vectors required to assemble a `Slice_Ensemble` differ from those mentioned for a `CMZ_Ensemble`, however, and are summarised below.

1. Observations vector: Must be a vector of vector tuples. The first member of the tuple is the population observation vector (total CMZ population at the vector's date index, see 3), the second

member is the date's total retinal volume observation vector (in cubicmicrometers). Ie. the observations vector takes the form of a `AbstractVector{<:Tuple{<:AbstractVector{<:Float64},<:AbstractVector{<:Float64}}}`; the tuple of population and volume data found at index `x` corresponds from animals measured on date `constants_vec[1][x]`.

2. Prior vector: For each desired phase of the model, the prior vector must contain a distribution over that phase's mean population cycle time (in hours) and its mean exit rate ($\geq 0.$, may exceed 1.). Paired cycle time and exit rate for each phase is followed by one less phase transition time in days than the number of phases. Ie. a two-phase prior vector might be, in pseudocode, `[Phase1CT, Phase1ER,Phase2CT,Phase2ER,Phase1End]`.

3. Constants vector: A `Vector{<:Any}` containing, in order:

   (a) T vector: Vector of integers or round floats, age in days of animals for each observations tuple

   (b) Population distribution: `Distribution` defining the initial range of population values. Should normally be a fit from the initial population size observations

   (c) Lens model: `Lens_Model` defining the lens circumferential growth power law and the slice's sectional thickness.

   (d) Monte Carlo iterates: Integer specifying the number of retinae to be simulated at each point sampled from the parameter space. 1e6-5e6 is usually fine.

   (e) Phases: Integer defining the number of phases specified by the prior vector

4. Box matrix: Two columns each of the same length as the prior, defining the minimum (column 1) and maximum (column 2) bounds for each parameter dimension to be sampled.

8.4 `Decay_Ensemble` model and usage

`Decay_Model`s are `Slice_Model`s that are parameterised to represent a continuous process of cell cycle rate decay (that is, lengthening cycle times). They use identical `Lens_Model`s to abstract the effects of the stretching 3-dimensional retina on the CMZ slice. Unlike the phased `CMZ_Model`s and `Slice_Model`s, `Decay_Model`s are parameterised solely by an initial cycle time CT_i, an exponential decay constant κ, and a constant exit rate ϵ. Daily cycle time is evaluated as $CT_t = CT_i \cdot e^{\kappa t}$. This is used to update the population value by the `Slice_Model` difference equation, penalized by the exit rate for the amount of CMZ growth implied by the `Lens_Model`.

A `Decay_Ensemble` is identical to a `Slice_Ensemble` except that it has only the three parameters in the order mentioned, and the corresponding prior vector, and omits the Phases constant.

8.5 `MultiSlice_Ensemble` model and usage

A `MultiSlice_Model` consists of more than one slice observations set which are governed by identical parameter vectors. This allows independent observations from different areas of the CMZ to be collectively modelled by one `Slice_Model` or `Decay_Model` parameter vector. This can be used to represent a CMZ with morphological asymmetry but regulatory homogeneity, and is used as an alternative to separate models of morphologically distinguished slices in Chapter 4.

A `MultiSlice_Ensemble` is parameterised identically to the corresponding `Slice_Ensemble` or `Decay_Ensemble`, except that a vector of per-slice observations subvectors, and a vector of per-slice population distribution is provided. The `MultiSlice_Model` uses the paired observations and population distributions in vector order to build a vector of the appropriate slice model type, of the same length.

8.6 `Thymidine_Ensemble` model and usage

Unlike the population-level models described above, the `Thymidine_Model` is a cell-based lineage simulator. It retains the slice model concept of comparing simulated observations directly to counts obtained from cryosections. Because these counts are obtained over a short time period (1 day), no effort is made to account for the growth of the eye. It uses an explicitly modelled cell cycle with G1, S, and G2M phases.

The model assumes a LogNormal distribution of overall cell cycle lengths. It is parameterised by TC_μ and TC_{σ^2} values for this LogNormal distribution (a Normal Inverse Gamma prior should be used), a fraction s which gives the length of S-phase by multiplying into the sampled TC, a fraction $g1$, which gives the fraction of the remaining time which comes before S-phase as G1, and implies the length of the G2M phase afterwards. Finally, the fraction sis is randomly added to or subtracted from the cell cycle length determined for one sibling of a mitosis to give the cell cycle length of the other. This represents the correlated sibling cell cycle times which characterise *Danio* RPC populations, as usefully observed and modelled by He et al. [HZA+12]. The parameter vector is given in the order `[tc`μ`,tc`σ^2`,g1_frac,s_frac,sis_frac]`.

The model assumes linear labelling of nuclear contents over the cell cycle, and implements a minimum value for detection as a thymidine-labelled cell (`DETECTION_THRESHOLD`, set to .15 by default). This represents cells with a dusting of thymidine signal that are not counted as labelled, which normally occurs in confocal stack segmentation routines.

To simulate more than one subpopulation with independent cycle parameters, additional sets of the 5 parameters outlined above can be concatenated, with this vector appended by a parameter denoting the fraction of the population to which each additional parameterisation applies.

The `Thymidine_Ensemble` is constructed similarly to the other models, except that it requires a constants vector as follows:

Constants vector: A `Vector{<:Any}` containing, in order:

1. T vector: Vector of integers or round floats, chase time in hours from beginning of pulse for each observations subvector

2. Pulse: Float giving the length of the thymidine pulse in hours

3. Monte Carlo iterates: Integer specifying the number of retinae to be simulated at each point sampled from the parameter space. 1e5-5e5 usually gives sufficient density over the discrete distribution to avoid noise associated with simulated count values that were not observed at all.

4. End time: Float defining how long to run the simulation- should usually be the time of the last set of observations

Unlike the population-level models, which fit LogNormal distributions to samples of output collected by Monte Carlo execution, the `Thymidine_Model` constructs discrete nonparametric distributions of

simulated counts of labelled cells. This allows the model parameters to produce population distributions that diverge from smooth, continuous distributions. This is particularly notable in the banding pattern of some output, where some counts are very unlikely to be observed. This can allow the model to better reflect the prevalence of even-sized RPC lineages, for instance [HZA+12].

Chapter 9

BioBackgroundModels.jl: Parallel optimization of Hidden Markov Model zoos of genomic background noise

`BioBackgroundModels.jl` (`BBM.jl` hereafter) is a pure Julia package intended to automate the optimization and selection of large numbers ("zoos") of Hidden Markov Models (HMMs), using sequence sampled from specified partitions of a given genome, on arbitrary collections of hardware. It is supplied with extensive utility functions for genomic sampling, high performance implementations of both the Baum-Welch and Churbanov-Winters algorithms for optimization of HMMs by expectation maximization (EM), as well as reporting functions for summarizing the results of the optimized model zoos. These tasks are necessary to select of models of genomic background noise, from which motif signals may be detected using a package such as `BioMotifInference.jl`.

HMMs generated by `BBM.jl` are standard ≥ 1 state HMMs that emit symbols corresponding to stretches of 1 or more base pairs. Following Down et al. [DH05], the number of bases encoded by an HMM symbol is denoted by the "order" of the HMM. 0th order HMM emit 4 symbols, one for each of the genomic nucleotides, while 1st order HMMs emit 16 symbols, one for each 2-base combination, and 2nd order HMMs emit 64 symbols, one for each 3-base combination, and so on. It is expected that users will be interested in comparing the explanatory value of a variety of background HMM state numbers and orders in selecting appropriate background models. These can all be optimized in one batch; because high state and, to a lesser extent, high order numbers impose much more significant memory costs, the use of the Churbanov-Winters linear memory algorithm is strongly recommended[1]. A basic load balancing system has been provided to allow users to prefer particular machines for some optimization jobs over others, which permits the efficient use of clusters of dissimilar hardware in the batch context.

[1]While the theoretical performance of the Churbanov-Winters algorithm is lower than Baum-Welch [CW08], the implementation herein is highly optimized and performant. For many use cases, it is both less memory-intensive and faster than Baum-Welch.

9.1 Genome partitioning and sampling

The optimization of an HMM zoo is performed against a sample of a "genome partition", which defines a portion of a genome in relation to a genomic feature set. The required files to sample from a genome partition are:

- A whole-genome sequence in valid FASTA format (eg. a .fna or other appropriate nucleotide sequence file).
- An index for the genomic sequence file (eg. an .fna.fai file)
- A genomic feature file, annotating the genomic sequence, in GFF3 format

At present, `BBM.jl` offers a default strategy of sampling without replacement from exonic, periexonic, and intergenic partitions of genomes. This partition scheme is intended to differentiate between genomic sequences that are likely to differ in terms of the appropriateness of HMMs of different orders as background models. That is, the selective pressure imposed on genomic sequence by the codon code (exonic partition) and other higher-order functional features of gene organization (periexonic partition) are likely to favour different sorts of background HMM model than the simpler repetitive structure of intergenic material.

Sampling functions should normally be accessed through the API provided to the user. Firstly, sampling jobs may be set up by using the `setup_sample_jobs` function, which uses user-supplied paths to the files listed above, as well as user-specified partitioning variables, to divide the supplied genome into the partitions described and set up channels for these sampling jobs. For example:

```julia
using BioBackgroundModels
#FILE PATHS
danio_gff_path = "Danio_rerio.GRCz11.94.gff3"
danio_genome_path = "GCA_000002035.4_GRCz11_genomic.fna"
danio_gen_index_path = "GCA_000002035.4_GRCz11_genomic.fna.fai"

#CONSTANTS FOR GENOMIC SAMPLING
const sample_set_length = Int64(4e6)
const sample_window_min = 10
const sample_window_max = 3000
const perigenic_pad = 500

channels = setup_sample_jobs(danio_genome_path, danio_gen_index_path, danio_gff_path,
↪  sample_set_length, sample_window_min, sample_window_max, perigenic_pad)
```

Here, `sample_set_length` defines the overall number of bases to be sampled from each partition, while `sample_window_min` and `sample_window_max` define the minimum and maximum length of each individual sample to be taken, without replacement, from the partitions. `sample_set_length` should be chosen to supply both training and test sets for background model selection; ie. it should be twice the desired length of the observation set in training. Lastly, `perigenic_pad` defines the number of bases up-

and down-stream of the first and last codon to consider, along with intronic material, as constituting the "perigenic" partition. This will include promoter elements, splicing signals, and the like.

Secondly, the partitions are sampled (in parallel, by worker, if desired), to produce dataframes of samples and relevant coordinate and strand orientation information:

```
using Distributed,Serialization
sample_output = "samples"
worker_pool = addprocs(3)
sample_record_dfs=execute_sample_jobs(channels, worker_pool)
serialize(sample_output, sample_record_dfs)
```

Here, one worker per partition is used to sample; lower numbers of workers will also work while larger numbers are of no benefit. In this example, the sample dataframes are serialized to the file "samples" for later use; they can also be used directly as described below.

9.2 Optimizing BHMMs by EM algorithms

Background HMM optimization by the expectation-maximization algorithms is performed by a similar two-step setup-and-execute functional pattern as genome sampling. Firstly, `setup_EM_jobs!` is used to transform supplied "job IDs" and the observations supplied by genome sampling into sets of EM chains to be elongated by the EM algorithm. `BBM.jl` "job IDs" take the form of `Chain_ID` structs, whose fields specify a string denoting the partition that forms the observations for the optimization, the number of states in the HMM, the order of the HMM's emitted symbols, and an integer denoting the replicate number of the chain. In general, for any given partition, number of states, and emission order, three or more replicates are advised, to confirm that the converged values of any particular HMM are likely to be near the global optimum [YBW15]. That is, if replicates of some job ID chain all terminate with similar parameter values (see Section 9.3 to examine this using `BBM.jl`), one such chain may reasonably be selected as a representative of the likely global optimum present in this region of the parameter space.

The helper function `split_obs_sets` is used to divide the genomic sample dataframes generated by the sampling API, described above, into training and test sets for optimization. The training set is supplied for the use of the EM algorithms, while the test set is used solely to calculate the final likelihoods of optimized models.

By default, `BBM.jl` initialises new HMM chains by sampling randomly from possible emission vectors, with strong priors on transition matrices with robust autotransition. Generally speaking, this tends to prevent chains from being optimized to trivial local minima with very short state residency times. This reflects our exclusive interest in those HMMs whose states represent stretches of nucleotides with shared biological significance; HMMs which tend to have state residency emission lengths of less than a few bases are of questionable biological relevance. If desired, the user may supply their own initialisation function as the `setup_EM_jobs!` keyword argument `init_function`, which defaults to `autotransition_init`. The default function should serve as the template for such user-defined initialisation functions.

`BBM.jl` operation is designed to be robust to most kinds of interruption, and the `setup_EM_jobs!` function may be used safely on existing sets of EM chains generated by BBM. This allows for resumption

from interruption, reconfiguration of hardware, decreasing the desired chain termination threshold, and so on. The dict[2] containing existing serialized chains, if any, should be supplied to the setup function as the `chains` keyword variable to allow this.

```julia
using BioBackgroundModels, DataFrames, Distributed, Serialization

#FILE PATHS
hmm_output = "chains"
sample_output = "samples"

#JOB CONSTANTS
const replicates = 3 #repeat optimisation from this many seperately initialised samples from the prior
const Ks = [1,2,4,6] #state #s to test
const order_nos = [0,1,2] #DNA kmer order #s to test
const delta_thresh=1e-5 #stopping/convergence criterion (log probability difference btw subsequent EM iterates)
const max_iterates=50000

#PROGRAMATICALLY GENERATE Chain_ID Vector
job_ids=Vector{Chain_ID}()
for (obs_id, obs) in training_sets, K in Ks, order in order_nos, rep in 1:replicates
    push!(job_ids, Chain_ID(obs_id, K, order, rep))
end

#SPLIT GENOME SAMPLES INTO TRAINING AND TEST SETS
sample_dfs = deserialize(sample_output)
training_sets, test_sets = split_obs_sets(sample_dfs)

#INTIIALIZE HMMS
if isfile(hmm_output) #if some results have already been collected, load them
    hmm_results_dict = deserialize(hmm_output)
else #otherwise, pass a new results dict
    hmm_results_dict = Dict{Chain_ID,Vector{EM_step}}()
end

em_jobset = setup_EM_jobs!(job_ids, training_sets; delta_thresh=delta_thresh, chains=hmm_results_dict)
```

Once the EM jobset is set up, it is splatted[3] into the `execute_EM_jobs!` function with any valid worker pool and the path for the Dict of EM chains to be serialized. All workers in the pool should be in

[2] A "Dict" in Julia is a dictionary of key-value pairs.

[3] "Splatting" in Julia refers to the expansion of a argument vector into a function's call by using the ... operator following the vector's name, i.e. `function(arguments...)`.

`:master_worker` topology with the master process. `BBM.jl` workers will obtain new chains to elongate by EM until no more are available; the number of jobs, therefore, defines the useful upper limit for the number of workers in the pool.

Load balancing is achieved by using `BBM.jl`'s LoadConfig struct, whose fields define acceptable ranges of states, orders, and optional vectors of blacklisted and whitelisted `Chain_IDs`. A Dict of LoadConfigs keyed by Integer defines the appropriate LoadConfig to use for any given worker's integer id (ie. the output of `Distributed.myid()`).

The results of EM optimizations are serialized as they are "pulled off the wire", so that they cannot be lost by the algorithm being interrupted, network failure etc. Consequently, `execute_EM_jobs!` does not return anything that needs to be serialized for subsequent analyses; results are always located at the path provided to the function and may be freely inspected by the tools described in Section 9.3 at any time during or after the optimization.

An example of `execute_EM_jobs!` usage, following from the setup example given above:

```
#SETUP LOAD BALANCING
local_config=LoadConfig(1:6,0:2)
remote_config=LoadConfig(1:4,0:1)

load_dict=Dict{Int64,LoadConfig}()

worker_pool=addprocs(no_local_processes, topology=:master_worker)
for worker in worker_pool
    load_dict[worker]=local_config
end

remote_pool=addprocs([(remote_machine,no_remote_processes)], tunnel=true,
↪   topology=:master_worker)

for worker in remote_pool
    load_dict[worker]=remote_config
end

worker_pool=vcat(worker_pool, remote_pool)

#EXECUTE EM JOBS
execute_EM_jobs!(worker_pool, em_jobset..., hmm_output; load_dict=load_dict,
↪   delta_thresh=delta_thresh, max_iterates=max_iterates)
```

9.3 BHMM analysis and display

A report generation and display API is provided in order to analyse the results of optimizing a zoo of background HMMs using `BBM.jl`. One function, `generate_reports`, serves to prepare all of the

available reports on the zoo. Its use is straightforward:

```
chains=deserialize(hmm_output)
sample_dfs = deserialize(sample_output)
training_sets, test_sets = split_obs_sets(sample_dfs)

report_folders=generate_reports(chains, test_sets)
serialize(survey_folders, report_folders) #save reports
```

`generate_reports` creates a Dict of `Report_Folder`s, keyed by partition id string. Each `Report_Folder` contains a `Partition_Report` with information about all HMMs trained on that partition, a `Replicate_Report` which enables the comparison of the replicates of the best model available for the partition, and a Dict of `Chain_Report`s keyed by `Chain_ID`, which give specific information concerning each chain produced for the partition. Examples of report output follow, derived from the model zoo training task whose results are summarised Figure 13.10. The contents of the reports on background models trained on the *D. rerio* "exonic" partition follow.

Figure 9.1 displays partial output from the `Partition_Report` associated with BHMM training on the exonic partition. The report identifies the partition in question, and, for each order of BHMM trained on the partition, displays a plot of the likelihood of the models vs. the number of states in the model. In this case, 0th and 2nd order models are displayed. The likelihood of a naive, 0th-order, 1-state model with equal emission probability for each symbol serves as a benchmark for BHMM likelihood; this value is plotted as a magenta line across the state axis. In this case, all 0th-order models are more likely descriptions of *D. rerio* exonic material than a naive model, while all 2nd-order models fall below this benchmark likelihood. This reflects the greatly expanded parameter space associated with 64 emission symbols. Thus, the additional exonic order capturable by 3mers is insufficient to offset the decreased probability of any given sequence using this many symbols.

Individual models are plotted as scatterplot points. Converged models are rendered in green, unconverged models in red[4]. In this case, no unconverged models are present. The 0th-order models are tightly converged such that their likelihoods are not differentiable, while the 2nd-order models are more spread out across the larger parameter space. The intended use of this report is to perform a quick visual assessment of the state of the zoo trained on the specified partition, before proceeding to the `Replicate_Report` and particular `Chain_Report`s.

Figure 9.2 displays partial output from the exonic `Replicate_Report`. The chains displayed in the report are the most likely replicate set for the partition. The report consists of paired convergence plots for the autotransition probabilities and 2-dimensional symbol emission probabilities for each state and each replicate in the set. Because there is no guarantee that HMMs from th same optima will have the same ordering of states, similar states are identified by minimizing the Euclidean distance between state emission vectors across the trained replicates. In this case, replicate 1's second state (red) is also identified with the second state of replicate 2 (green), but the sixth state of replicate 3 (blue).

The autotransition probability subplot displays the transition matrix diagonal probability for the designated state and replicates (that is, the probability that the HMM will stay in the state). If the

[4]The reports API is agnostic about the state of the model zoo reported on; it can be used to monitor zoos before they are fully converged.

Figure 9.1: **Example `BBM.jl` Partition Report**

HMMs are correctly converged on a global optimum, the autotransition probabilities for all chains should terminate at nearly the same value. Given that this probability will be stationary on converged chains, this can be visually confirmed by looking for substantial horizontal overlap of the chain autotransition probabilities for the state, as seen in Figure 9.2.

The symbol probability subplot displays the evolution of the probability of the first symbol emitted by the state plotted against the probability of the second symbol, providing a simple two-dimensional view into the convergence of the state's emission vector. If the chains are correctly converged on the global optimum, the plot's appearance should show chains from divergent areas of this two-dimensional parameter subset converging on the same values for the two symbols, as seen in Figure 9.2.

`Chain_Report`s for all HMMs trained on the partition are available in a Dict keyed by `Chain_ID`, found at the `chain_reports` field of the report folder. Example output for replicate 1 from the above-described replicate set is presented below. `Chain_Report`s consist of three major sections, described in turn.

The first `Chain_Report` section begins by stating the state number and order of the BHMM, as well

Figure 9.2: **Example `BBM.jl` Replicate Report**

as the genomic partition on which it was trained. This is followed by the likelihood of the model given the test observation set, compared to the naive model. The line stating these likelihoods will be green if the BHMM is better than the naive model, red if not. The general information concludes with the replicate number of the BHMM, as well as the convergence status of the chain and the final EM step size.

The section continues by stating the initial state probability vector a and the state transition matrix A for the converged BHMM. Emission vectors for the BHMMs' state are summarized by their informational content; symbols with $\geq .7$ bits of information are printed in bold capital characters, those with $.7$ information $\geq .25$ are printed in capital characters, those with $.25$ information $\geq .05$ are printed with lowercase characters, and those with less than .05 bits are not printed at all. In this case, state 2 emits adenine and cytosine symbols more frequently than guanine or thymidine, while this is reversed in state 4. The other states have similar enough emission probabilities for the four bases that they do not register on the informative symbols display. This gives a rough idea of what sort of information the BHMM

```
julia> e.chain_reports[Chain_ID("exon",6,0,1)]
Base.TTY(RawFD(0x00000017) open, 0 bytes waiting)BHMM EM Chain Results
6-state, 0th order BHMM
Trained on observation set "exon"
Test logP(O|θ): -2.677474495850883e6, greater than the naive model's -2.7733248445454747e6
Replicate 1
Converged with final step δ 0.0009887791238725185
 ──────────────────────────────────────────────────────────────────────────────
Last θ:
Background BHMM
State Initial and Transition Probabilities
a: [0.31786232049327706, 0.08311610163606636, 0.14481102538589713, 0.1087121512373124, 0.09122496723393655, 0.2542734340135114]
A: 6×6 Array{Float64,2}:
 0.776466     0.00134197   0.000597762   9.3932e-17   1.12908e-29  0.221594
 4.9891e-18   0.993806     0.000114231   0.00035994   0.000666623  0.00505363
 0.00501082   2.53527e-23  0.993775      0.00119238   2.16171e-5   1.21753e-8
 4.75045e-22  1.85006e-24  9.91045e-118  0.979917     0.000395467  0.0196875
 7.04204e-5   0.000262805  0.00112526    0.000411873  0.996586     0.00154403
 0.174564     0.000300059  0.000513614   0.00213432   0.00127718   0.82121

INFORMATIVE SYMBOLS BY STATE
K1 <<< >>>
K2 <<< a c >>>
K3 <<< >>>
K4 <<< g t >>>
K5 <<< >>>
K6 <<< >>>
State Mean Feature Length (bp)
K1: 4.647
K2: 166.237
K3: 148.03
K4: 49.701
K5: 300.917
K6: 5.396
```

Figure 9.3: **Example `BBM.jl` Chain Report - Part 1**

states encode.

The last part of this section presents an estimate of the mean feature length for each state, meaning the mean number of bases emitted from the state before transitioning to a new state (otherwise known as "state residency time" and similar concepts). This is estimated by Monte Carlo sampling of run lengths, given the autotransition probabilities for the BHMM. This is intended to give an idea of the biological relevance of the sequence features emitted by the BHMM's states. Generally speaking, most "background noise" features, like intergenic CpG islands, are held to be on the order of hundreds of bases. Users should expect good BHMMs to have some states that tend to emit features around this size.

The second part of the `Chain_Report`, displayed in Figure 9.4, displays the evolution of the model's likelihood and state autotransition probabilities over the chain iterates. This is primarily intended as a visual aid to the convergence tests presented in the third part of the report. Both the model likelihood and autotransition probabilities should be stationary over the last portion of the chain for it to be considered legitimately converged. This can be numerically confirmed in the third part of the report.

The third part of the `Chain_Report` displays the output of the Heidelberger and Welch convergence diagnostic [HW81], as implemented by `MCMCChains.jl` [Pfi20]. Both the BHMM likelihood and the autotransition probability for each state are tested for stationarity. A chain from a good replicate set (as assessed by the `Replicate_Report`) that passes these diagnostic tests is reasonably considered properly converged on the global optimum for the partition samples used as the training set. If this is so, a green test-passing statement will be printed, if not, a red failure statement. The specific reason for failure

Figure 9.4: **Example `BBM.jl` Chain Report - Part 2**

Convergence Diagnostics

Likelihood and autotransition probabilites converged and passing tests.

Heidelberger and Welch Diagnostic - Chain 1

parameters	burnin	stationarity	pvalue	mean	halfwidth	test
Symbol	Float64	Float64	Float64	Float64	Float64	Float64
logP(O|θ)	0.0000	1.0000	0.9391	-2686293.3768	6528.6612	1.0000
K1	0.0000	1.0000	0.2135	0.7959	0.0262	1.0000
K2	0.0000	1.0000	0.7116	0.9938	0.0002	1.0000
K3	0.0000	1.0000	0.2333	0.9936	0.0006	1.0000
K4	0.0000	1.0000	0.7631	0.9712	0.0161	1.0000
K5	0.0000	1.0000	0.1643	0.9966	0.0000	1.0000
K6	0.0000	1.0000	0.1011	0.8615	0.0501	1.0000

Figure 9.5: **Example `BBM.jl` Chain Report - Part 3**

can be determined by finding the parameters with a test value of 0. Taken together, these reports can be used to verify that the EM optimization is functioning properly, and that appropriately converged models are chosen to represent genomic background noise from the partition of interest.

9.4 Numerical accuracy and tests

Because `BBM.jl` contains two different EM algorithms, these are used internally to check for numerical accuracy and consistency. In general, input to either Baum-Welch or Churbanov algorithms should produce nearly-identical output (i.e. isapprox() will return true). Additionally, some lower-performance implementations are available under `BBM.jl/src/test/ref_fns.jl` for comparison. A fairly comprehensive test suite is provided, and can be executed using `]testBioBackgroundModels`. So long as tests are passing, numerical accuracy of the EM algorithms is established. The tests make use of a synthetic FASTA-format genome to test the genome sampling routines. Passing tests indicates that this system is working as intended.

9.5 Future directions

`BBM.jl` is largely feature complete. However, the genomic sampling regime implemented for this thesis is unnecessarily restrictive. In particular, there is no provision for common tasks like assembling the upstream promoter sequences of some group of genes with a common functional annotation. This could be improved by implementing a partition definition system that uses user-defined feature- and coordinate-based masking of the genome to assemble custom partitions. This would make `BBM.jl` a useful generalist tool for establishing background models when inferring genomic signals. Finally, the Churbanov and Baum-Welch algorithms should be benchmarked against other multi-observation implementations.

Chapter 10

BioMotifInference.jl: Independent component analysis motif inference by nested sampling

10.1 Introduction

While many methods to detect overrepresented motifs in nucleotide sequences, only one is a rigorously validated system of statistical inference that allows us to calculate the evidence for these motif models, given genomic observations: nested sampling [Ski06]. Computational biologists almost immediately benefitted from Skilling's original publication, with the release of the Java-coded nMICA by Down et al. in 2005 [DH05]. Unfortunately, this code base is no longer being maintained. It is, moreover, desireable to take advantage of modern languages and programming techniques to improve the maintainability and productivity of important bioinformatic code. We therefore re-implemented Skilling's original algorithm, for use in inference of DNA motifs in Julia [BEKS15], as `BioMotifInference.jl`, hereafter `BMI.jl`.

In brief, `BMI.jl` is intended to estimate the evidence for Independent Component Analysis models of Position Weight Matrix (PWM) signals on Hidden Markov Models of genomic background noise (summarized in Figure 10.1), and to estimate the maximum a priori parameters of detected signals. This allows the user to perform Bayesian model selection and comparison for generative hypotheses about genomic sequences of interest. The ICA approach is discussed in more detail in the relevant sections of Chapter 15.

10.2 Implementation of the nested sampling algorithm

Skilling's nested sampling algorithm is accompanied with an almost-certain guaranteed to converge an ensemble of models on the global optimum likelihood in the parameter space [Ski06]. An important assumption underpinning this guarantee is that the sampling density (in effect, the size of the ensemble) be high enough that widely separated modes are populated by at least one model each. Although early suggestions for the generation of new models within the ensemble were generally to decorrelate existing models by diffusional Monte Carlo permutation, later work generally acknowledges that this can be

Figure 10.1: **Schematic representation of BioMotifInference likelihood algorithm.**

highly inefficient [Ski12], particularly in large parameter spaces with many modes, of which the sequence parameter spaces explored in Chapter 5 are good examples. A number of ways of addressing this problem have arisen in the cosmology and physics literature. These include improved sampling methods, such as sampling within an ellipsoidal hypersphere encompassing the positions of the ensemble models within the parameter space [FH08, FHB09] or Galiliean Monte Carlo (GMC) [Ski12], as well as methods of increasing computational efficiency, such as dynamic adjustment of ensemble size [HHHL19].

Generally speaking, these methods are not good candidates for `BMI.jl` because the PWM representation of sequence signal in the ICA PWM model (IPM) is makes ordinary parameter space representation much less efficient. This is for at least four reasons:

1. Identical IPMs may be expressed with their vector of PWM sources (or "channels") in different orders, so the parameter space becomes increasingly degenerate with higher numbers of sources.

2. Closely equivalent repetitive signals can be represented by PWMs on opposite ends of the parameter space (e.g. ATAT vs TATA), reducing the efficiency of ellipsoidal sampling methods and introducing a further degeneracy.

3. If model likelihoods are being calculated on the reverse strand of observations, sources on opposite ends of the parameter space (ie. reverse complements) can also represent closely equivalent signals, introducing another source of degeneracy.

4. IPM sources may be of different lengths. Even if we can relate sources in different models to one another by some consistent distance rule, it is unclear how one would decide which dimensions of longer sources to project the parameters of shorter ones into. BioMotifInference does maintain an index for each source against its prior, but this is no guarantee that sources remain in some way "aligned". Rather, some type of alignment would have to be done, probably significantly increasing computational cost[1].

[1]Probably some combination of dimensional analysis and fast alignment algorithms could make some headway here.

While it is possible to perform Markov Chain Monte Carlo in a manner that allows jumps between differently-dimensioned models in detailed balance, this applies only to diffeomorphic transformations [HG12]. PWM models are not merely matrices of unrelated parameters, but are an ordered sequence of 4-parameter discrete Categorical distributions. If the addition or removal of a discrete, indexed set of 4 parameters to an existing vector of such sets can be made reversible in this manner, the proof is beyond the scope of the present work. Therefore, BioMotifInference (BMI) is not likely in strict detailed balance over a well behaved parameter space, given its default permute functions and configuration. Permute functions are designed around pragmatic concerns associated with the flow of model information in the IPM ensemble, and are tuned for computational efficiency in producing permuted models that are more likely than the starting model. The justification for this is simply feasibility; doing this allows an ensemble to be converged on larger samples from genome-scale datasets. Because of the flexible nature of the sampler, the user may easily specify `BMI.jl`-compatible permute functions, should better ones for their application be apparent to them.

It should be noted that these considerations do not entirely preclude the operation of the `BMI.jl` in detailed balance over the prior. This can be achieved by the user specifying a single length for the PWM sources (rather than a range of permissible lengths), and using a permutation routine consisting solely of the functions `random_decorrelate` and `reinit_src`, along with any user-defined functions coded with the detailed balance stricture in mind. Operation in this fashion may improve the accuracy of both evidence estimates and the sources found in the maximum a posteriori (MAP) sample defined by the converged ensemble. That said, unless the supplied prior is unusually good, this mode of operation is only well suited to very small observation sets, on the order of kilobases. With uninformative priors and good background models, the vast majority of sources sampled from the prior will receive significant likelihood penalties relative to the background model alone. Performing nested sampling on such an ensemble by diffusional techniques, in good detailed balance, will usually fail to produce models more likely than the background model within any reasonable compute budget, and will generally never converge. The feasibility constraint thus justifies the use of bespoke sampling routines for genome-scale datasets.

Like its predecessor `nMICA`, `BMI.jl` samples new models by applying one of a collection of permutation functions to an existing model. Additionally, although `BMI.jl` is supplied prior distributions on IPM parameters from which the initial ensemble is sampled, there is no way provided to calculate a posterior from the models generated by the nested sampling process, largely because of the identical source issue, 1. The primary outputs of interest are the estimation of the Bayesian evidence ratios between models (the absolute values of evidence are unlikely to be accurate)[2], as well as the models in the final ensemble (which constitute samples from the MAP mode or modes).

Because the more conventional methods to address the inefficiency of nested sampling by Monte Carlo used in cosmological models described above are unavailable, `BMI.jl` uses an ad hoc method of adjusting the mixture of permute patterns that are applied to models. The basic permute logic (encoded by `Permute_Instruct` arguments to the `permute_IPM` function) is as follows:

1. Select a random model from the ensemble.

2. Apply a randomly selected permutation function from the instruction's function list according to the probability weights given to the functions by the instruction's weight vector.

[2] It is not clear that evidence ratios solve the accuracy problem [Kevin Knuth, personal correspondence, 2021]. The best that can be said about this is that a larger ratio in favour of some model is more likely to reflect an actual preponderence of evidence in its favour. For this reason, the MAP sample may be the most informative part of BMI output.

3. Repeat 2 until a model more likely than the ensemble contour, given observations, is found, or the instruction's `func_limit` is reached.

4. If the `func_limit` is reached without a new model being found, return to 1 and repeat 2 until a model more likely than the ensemble's contour is found, or until the instruction's `model_limit` is reached.

10.3 Usage notes

10.3.1 Preparation of observation set

`BMI.jl` relies on `BioBackgroundModels.jl` both for selecting appropriate background models for motif inference, as well as to code observations and prepare an appropriate matrix of background scores. The selection of background models is covered in Chapter 9. The coding of observations requires the construction of a `DataFrame` containing a column of type `Vector{LongSequence{DNAAlphabet{2}}}`, the unambiguous DNA alphabet of `BioSequences.jl`. This is accomplished as follows:

```
using BioBackgroundModels, DataFrames, Serialization
obspath=/path/to/observations_dataframe
obs_df=deserialize(obspath)
```

Note that `observation_setup` must be supplied an `order` keyword argument of 0; while `BMI.jl` supports background HMMs of any order, signal motifs are 0th order. If the name of the sequence column of the observations `DataFrame` differs from "Seq", a symbol specifying the column name should be passed as the `symbol` keyword argument.

To construct the background matrix, the observations must be masked by partition, and the positional likelihoods of the sequences calculated:

```
BioBackgroundModels.add_partition_masks!(obs_df, genome_gff_path, 500,
↪  (:chr,:seq,:start))

BHMM_dict = Dict{String,BHMM}()
reports_folder=deserialize(/path/to/BBM_Reports_Folder)
for (partition, folder) in reports_folder
    BHMM_dict[partition]=folder.partition_report.best_model[2]
end
```

`add_partition_masks!` adds a mask column to the observations `DataFrame`, which is used by `BGHMM_likelihood_calc` to determine the correct background model to use for any given base in the observations sequence. It requires a path to a valid GFF3 annotation file for the genome. In order for the observations to be masked correctly, a tuple of column names containing the scaffold ID the observation is found on[3], the sequence on the positive strand, and the start position of the sequence on the scaffold.

[3]It must be ensured that this ID corresponds to the one used by the GFF3

The calculation of background scores requires a `Dict` of `BBM.jl` background models keyed by the partition ID. In the example, this is constructed programmatically from a `Report_Folder`; see Chapter 9 for the generation of these reports.

10.3.2 `IPM_Ensemble` assembly

Once the coded observation set and the matrix of background scores are prepared, an `IPM_Ensemble` may be initialised by sampling from a prior. Priors must first be assembled. This process is executed as follows:

```
num_sources=5
source_length_range=3:10
uninformative_mixing_prior=.1
source_prior = BioMotifInference.assemble_source_priors(num_sources,
↪  Vector{Matrix{Float64}}())
mix_prior = (falses(0,0), uninformative_mixing_prior)

e = BioMotifInference.IPM_Ensemble(worker_pool, "/path/to/ensemble/",
↪  ensemble_size, source_prior, (falses(0,0), mixing_prior), background_scores,
↪  coded_obs, source_length_range, posterior_switch=true)
```

Here, an uninformative prior is assembled for 5 sources. If we wanted to supply some PWM samples create informative Dirichlet priors from, these would be supplied in the vector of matrices passed as the second positional argument to `assemble_source_priors`. Informative priors are produced by multiplying the PWM values by a fixed weight, which may be passed as the keyword argument `wt`.

An uninformative prior is supplied for the mixing matrix. The prior tuple consists of an empty `BitMatrix`; if an informative prior is desired, the appropriate `BitMatrix` for the informative sources is passed instead. Sources for which no informative prior is supplied have their mix vectors sampled from a fractional floating point success rate prior. Here, the `uninformative_mixing_prior` is .1, meaning that an average of 10% of observations will be initialized with each source.

Sampling and likelihood calculations may be performed in parallel by submitting a valid integer vector worker pool as the first positional argument ot `IPM_Ensemble`, or conducted by the master process if this is omitted. Passing `true` for the keyword argument `posterior_switch` will ensure that the least likely models, discarded from the ensemble during the sampling procedure, are collected as samples from the posterior distribution. This is `false` by default, as no rigorous method for estimating posterior distributions is available.

10.3.3 Permute routine setup

In order to converge an assembled `IPM_Ensemble` by nested sampling, it is necessary to provide an initial `Permute_Instruct`, which determines sampler behaviour, and is tuned by the `Permute_Tuner` to prioritize permute functions which currently produce the largest positive particle displacements over the likelihood surface in parameter space, per computational time. Some application-specific tweaking is usually necessary. Depending on how the computation is to be parallelized, the weights assigned

ot network-intensive functions by the tuner, in particular, may benefit from being clamped. In order to convey how to set up an initial `Permute_Instruct` successfully, notes regarding the operation of the stock permute functions on `ICA_PWM_Model`s (IPMs) is summarized below. In this discussion, "premature homogenisation" refers to the loss of sampling density in relevant parts of the posterior, so that the ensemble particles are trapped in some local optimum.

- `permute_source`: randomly permutes a single source in the IPM by altering the PWM weights or changing the length of the source. Distributions for the frequency and size of these permutations may be supplied. If the size of the perturbations is small, and the frequency of length changes is set to 0., this function is useful for finding sources that are slightly more probable than the current ones.

- `permute_mix`: flip a random number of mix matrix positions. Retains flips that make observations more probable. If the range of flip moves is kept small, this is a useful function for "polishing" the mix matrix, particularly of models with highly probable sources. Effective throughout the sampling process.

- `perm_src_fit_mix`: as `permute_source`, but the permuted source is supplied with a new mix vector which mixes the source solely into observations that it makes more probable. Efficient and effective throughout the sampling process.

- `fit_mix`: checks each source against each observation, mixes only those sources that make the observation more probable in isolation. By far the most important permute during the early sampling process, since it can quickly move improbable models sampled from uninformative priors into the portion of the parameter space with likelihoods greater than the background model. In effect, this allows the extraction of maximum information about the data from the initial sampling process, which typically produces many widely divergent sources. This prevents premature homogenisation of the ensemble after extensive sampling below the background model likelihood, and subsequent distortions of evidence and maximum a posteriori parameter estimates.

- `random_decorrelate`: perform the operations of `permute_source` and `permute_mix` on the same model. This is an inefficient permute in general, but it is useful for introducing new sources during the early part of a sampling run, in particular, working to prevent premature homogenisation.

- `shuffle_sources`: copy the source PWM from another model in the ensemble, leaving the original mix matrix. This can be an effective permute early in the sampling process. It is network intensive.

- `accumulate_mix`: randomly select a source from the model to be permuted. From another model in the ensemble, select the source which is most similar to the one selected in the original model. Add the two mixvectors together, such that the original source is now mixed in to any observations in the second model that had the similar source. This is an extremely effective permute when the ensemble begins to have many similar sources in different models, and often is the best way to produce models more likely than those produced by `fit_mix`. Network intensive. This is sometimes the last permute that can find model parameterisations more probable than those in a well-converged ensemble. Network intensive.

- `distance_merge`: Randomly select a source from the model to be permuted. From another model in the ensemble, select the most dissimilar source, and overwrite the selected source in the original

model with the PWM and mixvector from this dissimilar source. This can be an effective permute throughout the sampling process. It tends to encourage the propagation of likely sources through the ensemble. Network intensive.

- `similarity_merge`: As `distance_merge`, except the most similar source is selected, and the original mixvector is retained. This tends to be less efficent than `distance_merge`, but can find new models throughout the sampling process. Network intensive.
- `reinit_src`: Resample a random source in the model to be permuted from the prior. This is a very inefficient permute, mostly useful in the early part of the sampling process to reintroduce sources from undersampled parts of the prior.
- `erode_model`: Trim a source's PWM in the model to be permuted to only those positions whose weight vectors express information above a threshold. This has the effect of generating high-likelihood short sources from longer ones with stretches of uninformative postions. It is mostly useful in the early part of the sampling process.
- `info_fill`: For a random source in the permute model, find the least informative PWM position, and set the probability of the most informative base to 1. This function assists in preventing premature homogenisation by rapidly increasing the likelihood of sources that are well-supported by observations but are unlikely to achieve appropriate high PWM weights by random permutation alone.

A `Permute_Instruct` requires a vector of the functions to be used in the permutation routine, a corresponding vector of their initial weights (ie. the frequency with which they will be called), a limit on the number of models to permute before the worker abandons sampling, and a limit on the number of functions to apply to any given model before selecting a new one to attempt permuting. The annotated example provided below also passes, as keyword arguments, vectors of minimum and maximum weights (`min_clmps` and `max_clmps`), as well as a vector of keyword arguments for the vector of functions (`args`).

```
funcvec=full_perm_funcvec #exported by BMI.jl
models_to_permute=10000
func_limit=25

#add two ``polishing functions'' to the function vector
push!(funcvec, BioMotifInference.perm_src_fit_mix)
push!(funcvec, BioMotifInference.random_decorrelate)

#all functions will receive a minimum of 1 in 100 function calls
min_clamps=fill(.01,length(funcvec))

#give some important functions a minimum of 10 in 100 function calls
min_clamps[2:3].=.1 #perm_src_fit_mix & permute_mix
min_clamps[8]=.1 #difference_merge

```

```
    #no function will receive more than 50 in 100 function calls
    max_clamps=fill(.5,length(funcvec))

    #prevent network intensive functions from swamping the cluster
    max_clamps[6:7].=.15 #shuffle_source and accumulate_mix
    max_clamps[9]=.15 #similarity_merge

    #evenly distributed initial weights
    initial_weights= ones(length(funcvec))./length(funcvec)

    #empty arguments vectors for all arguments in the funcvec
    args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcvec)]

    #add arguments for the ``polishing functions''

    ↪  args[end-1]=[(:weight_shift_freq,0.),(:length_change_freq,1.),(:length_perm_range,1:1)]
    args[end]=[(:iterates,50),(:source_permute_freq,.3),(:mix_move_range,1:10)]

    #instantiate the Permute_Instruct
    instruct = Permute_Instruct(funcvec, initial_weights, models_to_permute,
    ↪  func_limit;min_clmps=min_clamps, max_clmps=max_clamps, args=args)
```

10.3.4 Nested sampling of `IPM_Ensembles`

When all of the above tasks are complete, the `IPM_Ensemble` may be converged by nested sampling using the specified permutation routine. This can be performed in parallel; any number of permutation workers may be used. However, because the master process is responsible for disk access, many such workers performing permutation functions which frequently require the deserialization of other models in the ensemble (those noted as "network intensive" above) will rapidly reduce the computational efficiency of the cluster. This can be mitigated by clamping network intensive permute functions to a lower frequency of total function calls.

`BMI.jl` is supplied with a display system similar to, but less customisable than `GMC_NS.jl`. An example of display setup and nested sampling execution is presented below:

```
display_rotation=[true,10,1,[[:tuning_disp,:lh_disp,:src_disp],[:conv_plot,:liwi_disp,:ens_disp]]]

logZ = converge_ensemble!(e, instruct, worker_pool, backup=(true,25),
↪  tuning_disp=true, lh_disp=true, src_disp=true, disp_rotate_inst=display_rotation)
```

10.4 Test of model comparison logic with spiked motifs

Because the proper functioning of `BMI.jl` is difficult to verify, we constructed a test problem to ensure that the package can, in principle, be used to distinguish between synthetic observation sets spiked with two different sets of motifs, representing separate causal processes. This is done by generating two sets of 500 synthetic observations of 100-200 bases in length, and spiking them with different periodic, repetitive signals, as well as aperiodic, randomly occuring transcription factor motifs (a TATA box and a CAAT box). Ensembles of 250 ICA models with two PWM sources between 3 and 12 bases in length are assembled, one for each of the two separate observations sets, and one to model the combined set. Code to estimate the evidence for these ensembles in available in Section 17.8.38. Execution of this code gives a logZR of 110.87 $\pm$ 0.88 in favour of the separate models. While the uncertainty on this value is unreliable, the result suggests that the algorithm is capable of distinguishing properly between observations sets generated by separate causal processes on similar backgrounds.

10.5 Future Directions

The prospects for `BMI.jl` are limited by the difficulty in ensuring the sampling routines are in detailed balance. It is possible that this could be resolved. One possibility might be to implement a form of Galilean Monte Carlo by giving models a three-dimensional coordinate for each base pair in each source channel (fixing the number of bases per source). This coordinate would encode the location of the model within a tetrahedral probability 3-simplex (this is discussed further in Section 15.1.7, about Position Weight Motif models). The boolean mixing matrix could, perhaps, also be subject to GMC, if at each mix matrix position the model had either positive, negative, or zero velocity.

This solution would not directly solve the degeneracy issues mentioned above, but it may allow for more accuracy in evidence estimation. The dimensionality implied by this parameterisation is nonetheless high. The models used in Chapter 5 contain eight PWM sources of 3-10 bases in length. If we fix the PWM length at 5 bases, there are 120 parameters for the PWM sources alone. Practically speaking, this kind of solution is likely to be best employed where only a few, short, repetitive sequences are of interest.

One way to address the error associated with ad hoc sampling routines might be to construct small toy problems with known model evidence; different sampling routines could be tested on these to perform a stretch test, measuring actual sampler performance against the theoretical distribution of compressive steps [Buc16]. If this is possible, the effects of the `BMI.jl` tuning regimen could be characterised and error minimised.

Part III

Supplementary Materials

Chapter 11

Supplementary materials for Chapter 2

11.1 Materials and methods

11.1.1 Zebrafish husbandry

Zebrafish used in this study were of a wild type AB genetic background. Embryos were derived from pairwise crossings. Larvae were collected upon crossing and held in a dark incubator at 28°C. At 1dpf, embryos were treated with 100μl of bleach diluted in 170ml of embryo medium for 3 minutes, rinsed and dechorionated. Embryo medium was changed at 3dpf. After this, all animals were maintained at 28°C on a 14-hour light/10-hour dark cycle (light intesity of 300 lux) in an automated recirculating aquaculture system (Aquaneering). Animals were reared using standard protocols [Wes00]. Fish were sacrificed by tricaine overdose at the appropriate timepoints indicated in the text. All animal experiments were performed with the approval of the University of Toronto Animal Care Committee in accordance with guidelines from the Canadian Council for Animal Care (CCAC).

11.1.2 Proliferative RPC Histochemistry

11.1.2.1 Anti-PCNA histochemistry

In order to assay the number of proliferating cells in zebrafish CMZs, we used anti-PCNA histochemical labelling, as PCNA is, in zebrafish, detectable throughout the cell cycle in proliferating cells. After sacrifice as described above, fish (or their razor-decapitated heads, in the case of fish older than 30dpf) were fixed in 1:9 37% formaldehyde:95% ethanol at room temperature (RT) for 30 minutes, followed by overnight incubation in a fridge at 4°C. Subsequently, the samples were removed from fix and washed 3 times in PBS. They were then cryoprotected by successive 30 minute rinses at RT on a rocker in 5%, 13%, 17.5%, 22%, and 30% sucrose in PBS. Samples were allowed to incubate overnight at 4°C in 30% sucrose. The next day, samples were removed from the sucrose rinse and infiltrated with 2:1 30% sucrose:OCT compound (TissueTek) for 30 minutes at RT. Samples were then embedded for cryosectioning and frozen. All animals younger than 14dpf were embedded side-by-side in groups of 6 under a dissection microscope using a brass mold with milled 3mm channels for maintaining their precise axial orientation.

The cryosectioning block was completed by removing the frozen larval block from the mold and layering more sucrose:OCT mixture between the channels and on top of this filled-in block. Older animals were carefully oriented by hand under a dissection microscope. 14μm coronal cryosections were cut through the fish heads and collected on Superfrost slides (Fisherbrand). These were stored in a freezer at -20°C until staining.

Staining was begun by allowing the slides to dry briefly at RT. Sections were outlined with a PAP pen, then rehydrated in PBS for 30 minutes at RT. Subsequently, sections were blocked in 0.2% Triton X-100 + 2% goat serum in PBS for 30 minutes at RT. This was followed by incubation in mouse monoclonal (PC10) anti-PCNA primary antibody (Sigma), diluted 1:1000 in blocking solution, at 4°C overnight. The next day, primary antibody was removed, and slides were rinsed five times in PDT (PBS + 1% DMSO + 0.1% Tween-20). Cy5-conjugated goat-anti-mouse secondary antibody (Jackson Laboratories), diluted 1:100 in blocking solution, was applied to the sections, which were incubated at 37°C for 2 hours. Secondary antibody was removed, followed by five rinses in PDT as above. Sections were counterstained with Hoechst 33258 diluted to 100 μg/mL in PBS for 15 minutes at RT. Counterstain was then removed, five rinses in PDT were performed as above, followed by a final rinse in PBS. Slides were then mounted in ProLong Gold antifade mounting medium (ThermoFisher Scientific), coverslipped, and sealed with clear nail polish. Slides were kept at 4°C until imaging.

11.1.2.2 Cumulative thymidine analogue labelling for estimation of CMZ RPC cell cycle length

In order to obtain an estimate of cell cycle length in CMZ RPCs at 3dpf, we used cumulative thymidine analogue labelling following the method of Nowakowski et al. [NLM89]. 3dpf zebrafish were divided into ten groups of n5 animals and held in 10 mM EdU for 0.5, 1.5, 2.5, 3.5, 4.5, 5.5, 7.5, 8.5, 9.5, and 10.5 hours, after which they were sacrificed as described above. Samples were processed as described under "Anti-PCNA histochemistry", except that goat-anti-mouse Cy2-conjugated secondary antibody (Jackson Laboratories) was used to label anti-PCNA, after which the Alexa Fluor 647-conjugated azide from a Click-iT kit (ThermoFisher Scientific) was added to the sections for 30 minutes at room temperature to label EdU-bearing nuclei. This was followed by five PDT rinses as described above, and resumption of the protocol with counterstaining, mounting, etc. The raw data is available in `/empirical_data/cumulative_edu.xlsx`

11.1.2.3 Whole retina thymidine analogue labelling of post-embryonic CMZ contributions

We used thymidine analogue labelling as an indelible marker of CMZ contributions to the retina in Fig 2.6. Because thymidine analogues are incorporated into DNA during S-phase, postmitotic progeny of proliferating RPCs which take up the label may be detected at any point after this treatment. Dosing zebrafish with a thymidine analogue for a limited period of time ensures that the label is diluted to undetectable levels in cells which continue to proliferate. Repetitive dosing thus gives rise to labelled cohorts which represent a limited set of generations of cells which become postmitotic after the dose is provided. n=3 fish were held in 10mM BrdU (Sigma) diluted in facility water for 8 hours at 30, 60, 90, 120, and 150 days post fertilisation. 10 days after the last BrdU incubation, the fish were sacrificed as described above. One representative whole retina dissection is presented.

11.1.3 Confocal microscopy and image analysis

All histochemically processed samples (coronal sections of retinas and whole retinas) were imaged on a Leica TCS SP5 II laser confocal imaging microscope, using the Leica LAS AF software for acquisition. For coronal sections, central sections were identified by their position in the ribbon of cryosections. For Fig 2.8, the central section was imaged together with the flanking section on either side of it. Confocal data was analysed using Imaris Bitplane software (v. 7.1.0). Cell counting analyses were performed by segmenting the relevant channels using the Surfaces tool to produce counts of PCNA positive (Figs 2.8, S4 Fig) or PCNA/EdU double-positive (S4 Fig) nuclei. Lens diameters were measured in the DIC channel using the linear measurement tools, across the widest point of the section of spherical lens.

11.1.4 Estimation of CMZ annular population size

In order to estimate the total population of the CMZ in zebrafish eyes sampled throughout the first year of life, we counted PCNA-positive cells in the dorsal and ventral CMZ of central and flanking sections of these eyes as described above. We added the dorsal and ventral populations and took the average of the central and flanking section per-section populations to reduce the possibility of sampling error. We treated this as a sample of a torus mapped to the 14μm sphere segment of lens intersected by the average section. As the surface area of this zone is given by $S = \pi \cdot d_{lens} \cdot h_{section}$, and the total surface area of the lens by $A = \pi \cdot d_{lens}^2$, where d is the diameter of the lens and h the section thickness, we may calculate an estimate of the total population by $pop_{total} = \frac{pop_{section}}{S} \cdot A$. We therefore obtained our annular CMZ population estimates using our measurements with the simplified formula $pop_{total} = \frac{pop_{section} \cdot d_{lens}}{h_{section}}$, calculating pop_{total} on a per-eye basis to account for covariance of the measurements, and subsequently averaging across n=6 eyes per sampled timepoint (3, 5, 8, 12, 17, 23, 30, 60, 90, 120, 180, and 360 dpf). These data are presented in Fig 2.8.

11.1.5 Modelling lens growth

To simulate the annular CMZ population, we wanted to be able to simulate the stem cells at the periphery of the retina occasionally undergoing symmetric divisions, so as to keep up the same density of stem cells in the first ring around the lens. We did this by normalising our lens diameter measurements to the 72hpf value, to produce a measure of relative increase in lens size over the first year of CMZ activity. We performed a linear regression of the logarithm of this relative increase vs the logarithm of time using the Python statsmodels library, using the constants derived from this regression for a power-law model of lens growth. This model was used to supply the `WanStemCellCycleModel` with a target population value as described below.

11.1.6 Estimation of 3dpf CMZ cell cycle length

To produce an estimate of the length of the cell cycle in 3dpf RPCs, we first took the total count of EdU-positive cells in dorsal and ventral CMZ from our cumulative labelling experiment described above, and divided by the total dorsal and ventral CMZ RPC population, as measured by the number of PCNA positive cells. This gave the labelled fraction measurements displayed in S4 Fig. Following Nowakowski et al. [NLM89], we performed an ordinary least squares linear regression on these data using the Python statsmodels library. We then calculated the total cell cycle time $T_c = \frac{1}{slope}$ and $T_s = T_c \times intercept_y$.

While this method is not suited for heterogenous populations of proliferating cells, its use is justified here, as the He SSM to which the estimate is being applied makes similar assumptions as Nowakowski et al., in particular the homogeneity of the population. It should not be considered more than a rough estimate.

11.1.7 CHASTE Simulations

11.1.7.1 Project code

All simulations were performed in an Ubuntu 16.04 environment using the CHASTE C++ simulation package version 2017.1 (git repository at `https://chaste.cs.ox.ac.uk/git/chaste.git`). The CHASTE package is a modular simulation suite for computational biology and has been described previously [MAB+13]. All of the code, simulator output, and empirical data used in this paper is available in the associated CHASTE project git repository at `http://github.com/mmattocks/SMME`. In brief, the approach taken was to encapsulate the SSMs examined here as separate concrete child classes inheriting from the CHASTE `AbstractSimpleCellCycleModel` class. An additional model, representing the simple immortal stem cell proposed in Wan et al. [WAR+16], was produced similarly. Each model is generic and provided with public methods to set their parameters and output relevant simulation outcomes (and per-lineage debug data, if necessary). While these may be used in the ordinary CHASTE unit test framework, permitting their use in any kind of cell-based simulation, we wrote standalone simulator executables (apps, in CHASTE parlance) to permit Python scripting of the simulator scenarios used in this study. This allows for the multi-threaded Monte Carlo simulations performed herein. Therefore, the Python fixtures available in the project's `python_fixtures` directory were used to operate the single-lineage simulators (`GomesSimulator`, `HeSimulator`, and `BoijeSimulator`) and single-annular-CMZ simulator (`WanSimulator`) in `apps/`, including the CellCycleModels and related classes in `src/`; the simulators output their data into `python_fixtures/testoutput/`. These data, along with the empirical observations (both from the Harris groups' papers and our own described herein) available in `empirical_data/`, were processed by the analytical Python scripts in `python_fixtures/figure_plots/` to produce all of the figures used in this study.

We have attempted to make the project fully reproducible and transparent. CHASTE uses the C++ boost implementation of the Mersenne Twister random number generator (RNG) to provide cross-platform reproducibility of simulations. All of our simulators make use of this feature, permitting user-specified ranges of seeds for the RNG. We have also used the numpy libary's RandomState Mersenne Twister container to provide reproducible results for the Python fixtures, notably the SPSA optimisation fixture. The output of the project code will, therefore, be identical every time it is run, on any platform.

It should be noted that none of the code Harris' group used in their reports has been published. We have made every effort to replicate the functional logic of the models as closely as possible. Nonetheless, it is not possible to determine precisely why, for instance, the original He SSM parameterisation failed so notably in our implementation.

11.1.7.2 SPSA optimisation of models

In order to make a fair comparison between the He SSM and our deterministic mitotic mode alternative model, we coded an implementation of the SPSA algorithm described by Spall [Spa98]. This is available in `/python_fixtures/SPSA_fixture.py`. This algorithm, properly tuned, will converge almost-surely

on a Karush-Kuhn-Tucker optimum in the parameter space, given sufficient iterates, even with noisy output (eg. due to RNG noise from low numbers of Monte Carlo samples, permitting conservation of computational resources). It does so by approximating the loss function gradient around a point in parameter space, selecting two sampling locations at each iterate, then moving the next iterate's point in parameter space "downhill" along this gradient.

Our loss function was a modified AIC; we weighted the residual sum of squares from the PD-type mitotic probabilities by 1.5 in order to improve convergence, as the PD data are the most informative part of the dataset regarding the critical phase boundaries (PP mitoses occur in all 3 phases of the He model, DD occur in phases 2 and 3, PD mitoses occur only in phase 2). We also compared model induction count output (that is, panels A-C in Fig 2.5) up to count values of 1000 to penalise parameters producing very large lineage totals.

The values of the SPSA gain sequence constants were selected to be as large as possible without resulting in instability and failure to converge; this ensured that the algorithm stepped over the widest possible range of the parameter space in finding optima. The α and γ coefficients were initially set at the noise-tolerant suggested values of .602 and .101 respectively [Spa98]. 200 iterations were performed. Initially, each iterate involved 250 seeds of each induction timepoint for the count data (panels A-C in Fig 2.5) and 100 seeds of the entire 23-72 hr simulation time for mitotic mode rate data (panels D-F). At iterate 170, these were increased to 1000 and 250 seeds, respectively, decreasing RNG noise to a low level. For the last 10 iterations, RNG noise was reduced to close to nil by increasing the seed numbers to 5000 and 1250, respectively; α and γ were set to the asymptotically optimal 1 and $\frac{1}{6}$ for these iterates, as well. The particular seeds used were kept constant throughout in order to take advantage of the improved convergence rate this affords [KSN99].

Because the simple SPSA can assign any value to parameters, our implementation uses constraints, projecting nonsensical values for parameters (e.g. negative values, mitotic mode probabilities summing to > 1) back into legal parameter space. The use of constraints has no effect on the algorithm's ability to almost-surely converge within the given parameter space [Sad97] As we felt that the deterministic mitotic mode models' "sister shift" should be relatively small in order to be biologically plausible, we also constrained this parameter such that 95% of sister shifts would be less than the mean length of the shortest phase.

The numerical results of the SPSA algorithm are available in `/python_fixtures/testoutput/SPSA/HeSPSAOutput`, alongside png images of output from each iterate, enabling the visualisation of the progress of the algorithm.

Monte Carlo simulations Subsequent to SPSA optimisation, the parameters derived from this procedure were used to perform Monte Carlo simulations of large numbers of individual lineages, using ranges of 10000 non-overlapping seeds for each of panels A, B, C, G, H, and I, and 1000 for each of panels D, E, and F of Fig 2.5 (using the SPSA-optimised parameter sets) and S1 Fig (using the original He et al. parameterisation). This was performed using `/python_fixtures/He_output_fixture.py`.

Similarly, 100 seeds were used in Monte Carlo fashion to simulate individual annular CMZ populations using `/python_fixtures/Wan_output_fixture.py`, which scripts the WanSimulator. Each of these simulations is initialised with an RPC population drawn from a normal distribution given the mean and standard deviation of the CMZ torus estimated from our 3dpf observations, as described above. Each RPC is given a TiL value randomly chosen from 0 (newly born from a stem cell) to 17hr (exiting the

CMZ, "forced to differentiate", in the terms of Wan et al.). A further population of stem cells, sized at $\frac{1}{10}$ of the RPC population, is added using the `WanStemCellCycleModel`. This stem population divides asymmetrically except when it falls under its target population value determined by the model of lens growth described above; as long as this condition holds, the stem cells will divide symmetrically to keep up their numbers.

Simulation data analysis Not all of the numerical data used in He et al. and Wan et al. has been published. As such, we reconstructed the lineage data from He et al.'s Figure 5C, used by He et al. to obtain their mitotic mode probabilities; our reconstructed values are available in `/empirical_data/empirical_lineages\`. We also reconstructed the He et al. WT and Ath5 morpholino data (Fig 2.5 panels H, I) and Wan et al. lineage count probabilities (panel G) from the relevant figures. These values are entered into `/python_fixtures/figure_plots/He_output_plot.py`, where they are used in the AIC calculations described below.

In order to assess the performance of the models used in our simulations, we used Akiake's Information Criterion (AIC). This allows us to compare models with dissimilar numbers of parameters, as AIC trades off goodness-of-fit against parameterisation. The values reported in Table 2.1 were produced by `/python_fixtures/figure_plots/He_output_plot.py`. Residual sums of squares were combined across the training and test datasets.

The 95% CIs for the model output presented in this paper were estimated by repetitive sampling of the output data, using the same number of lineages observed empirically. 5000 such samples were performed. This provides a reasonable estimate of the confidence interval given the limited observational sample size.

11.2 Supporting figures

S1 Fig. He SSM original parameterisation model output. As Fig 2.5; model output uses the parameterisation given in He et al. [HZA+12]. Note significant deviation from observations in panel B, reflecting excess proliferative activity of modelled RPCs.

S2 Fig. SPSA-optimised He SSM model output. As Fig 2.5; stochastic model output displayed alone.

S3 Fig. SPSA-optimised deterministic mitotic mode model output. As Fig 2.5; deterministic model output displayed by itself.

S4 Fig. Cumulative EdU Labelling of 3dpf CMZ RPCs Fraction of PCNA-labelled CMZ RPCs which bear EdU label over time, as determined by histochemical labelling of central coronal cryosections of 3dpf zebrafish retinas. Dots represent results from individual retinas. Line is the ordinary least squares fit ± 95% CI. Cell cycle length (Tc) and S-phase length (Ts) are estimated from this fit following the method of Nowakowski et al. [NLM89]

Chapter 12

Supplementary materials for Chapter 4

12.0.1 Normal and LogNormal models of population data

We first investigated the appropriateness of Normal vs. LogNormal models for measured and calculated parameters of the CMZ and the neural retina, presented in Table 12.1.

Table 12.1: **Likelihood ratio comparison between Normal and Log-Normal models of retinal population parameters**

Parameter	**$\mathcal{N}$ logLH**	**Log-$\mathcal{N}$ logLH**	**logLR**
Sectional PCNA+ve	-285.611	**-283.214**	2.397
Lens diameter	**-203.102**	-203.854	-0.752
CMZ annular pop.(†)	-484.768	**-482.733**	2.035
RPE length	**-368.232**	-368.609	-0.377
CR thickness (†)	**-240.858**	-241.032	-0.174
CR volume (†)	-1091.615	**-1091.146**	0.469

$\mathcal{N}$: Normal distribution. logLH: logarithm of p(D|M), the likelihood of the data given the model. logLR: logarithm of the likelihood ratio; positive ratios in favour of the log-$\mathcal{N}$ model. Largest likelihoods are bolded. †: Calculated quantities. Sectional PCNA+ve: population of PCNA-positive CMZ RPCs per 14μm cryosection. CMZ annular pop: population of annular CMZ. RPE: retinal pigmented epithelium. CR: cellular retina. Methods in Section 11.1.2.1 and Section 12.1.7. Code in Section 17.8.9.

These results suggest that the organism-level population distribution of eye-level CMZ population counts, as assayed by PCNA immunostaining, is better modelled Log-Normally than Normally. This is true whether we test the primary per-section count meaurements, or the estimated whole-annulus population, even though this quantity is calculated using the Normally-distributed lens diameter[1]. The most likely Log-Normal representations of the CMZ populations are about two orders of magnitude more likely than the Normal alternative. Additionally, although Normal models are slightly favoured for describing the population-level distributions of RPE length and cellular retina thickness, the derived cellular retina volume quantity is better modeled by a Log-Normal distribution.

[1]The apparent superiority of the Normal model for lens diameter may be due to the paucity of data at later time points; as described in Section 12.1.7, lenses are difficult to retain in this histological context.

Since the choice of model describing the estimated annular population and retinal volume is critical to the success of later inferences, we used Galilean Monte Carlo-Nested Sampling (GMC-NS) to estimate the Bayesian evidence (the marginal probability of the data over all model parameterisations) for these hypotheses. Evidence estimates and ratios are presented in Table 12.2.

Table 12.2: **Evidence favours Log-Normal models of retinal population parameters**

Parameter	$\mathcal{N}$ **logZ**	**Log-$\mathcal{N}$ logZ**	**logZR**	σ **significance**
CMZ Population	-1457.0 ± 3.7	**-1005.0 ± 3.5**	452.1 ± 5.1	89.45479335402516
Estimated Retinal Volume	-2042.2 ± 3.3	**-1525.2 ± 4.1**	517.0 ± 5.3	97.87822790648408

$\mathcal{N}$: Normal distribution. logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratios in favour of the log-$\mathcal{N}$ model. Largest evidence values bolded. CR: cellular retina. Methods in Section 12.1.8. Code in Section 17.8.7.

Full estimation of the Bayesian evidence for the Normal vs. Log-Normal hypotheses for our calculated parameters produces confirms the rough calculation of likelihood ratio from the single-fit MAP models, although the extent to which LogNormal models are better justified than Normal differs between the methods.

If between-individual variation in retinal CMZ population and retinal volume are both well-modelled Log-Normally, the question of their independence arises. It seems plausible that the size of the CMZ population would be roughly proportional to the overall volume of the retina, after postembryonic central specification, and establishment of the peripheral CMZ remnant. Moreover, since the growth of retinal volume over the life of the organism is driven primarily by the CMZ[2], it seems likely that this volume is well correlated with the size of the CMZ. Since the manner in which we model the CMZ differ depending on our assessment of the dependence of these retinal parameters, we performed Bayesian model selection by the Empirical Bayes method for linear regression, which provides for direct estimation of the evidence for models consisting of linear equations of variables [Bis06].

We find that individual CMZ population and retinal volume estimates are best described by uncorrelated models at in all ages, with the exception of 23.0 dpf, where the evidence weakly favours correlation. It is interesting that the evidence in favour of non-correlation is weakest between 17 and 30 dpf, suggesting the correlation at 23.0 dpf is not spurious, but is related to CMZ activity at this time. These data are displayed in Table 12.3. Of particular importance are the data for 3dpf embryos, as we intend to seed model retinae with CMZ populations and volumes drawn from these distributions. The data for these animals are plotted in Figure 12.1. The general lack of correlation between CMZ population and retinal volume estimates may be due to the loss of information involved in the estimation calculations; on the other hand, it is more plausible that CMZ population should be associated with the rate of retinal volume growth rather than the volume of the retina itself, which suggests that the rate of retinal contribution from the CMZ is probably highest between 17 and 23 days. Because growth rate data are unavailable for single individuals, we cannot make this inference directly.

[2]One observes occasional proliferative clusters in the central retina throughout the life of the fish; these are typically ascribed to Müller glial repair processes. I am unaware of any estimate as to the relative contribution of these clusters vs. the CMZ. As we shall see, there is probably more turnover in the specified retina than previously believed. As a result, the relative contribution of these central clusters should probably be subject to statistical estimation; they may be more significant than mere lesion-repair sites.

Table 12.3: **Evidence favours uncorrelated linear models of CMZ-population and retinal volume over time**

Age (dpf)	Uncorrelated logZ	Correlated logZ	logZR
3.0	**-87.651**	-91.902	4.252
5.0	**-90.0**	-92.362	2.362
8.0	**-90.918**	-96.013	5.095
12.0	**-91.183**	-99.049	7.866
17.0	**-94.818**	-96.668	1.85
23.0	-103.386	**-103.219**	0.167
30.0	**-103.511**	-104.092	0.581
60.0	**-115.025**	-118.169	3.144
90.0	**-113.533**	-122.427	8.894
180.0	**-116.778**	-124.547	7.769
360.0	**-121.016**	-128.637	7.621

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. logZR: evidence ratio; positive ratios in favour of the uncorrelated model. Largest evidence values bolded. Methods in Section 12.1.8, Section 12.1.10. Code in Section 17.8.1.

Figure 12.1: **CMZ population and retinal volume estimates are uncorrelated at 3dpf** Individual CMZ population estimate vs retinal volume estimate for 3dpf animals. Uncorrelated and correlated linear models of these variables are plotted as the mean±95% CI of the predictive distribution of the fitted model. Methods in Section 12.1.8, Section 12.1.10. Code in Section 17.8.1.

12.1 Materials and methods

12.1.1 Zebrafish husbandry

Zebrafish husbandry was performed as described in Section 11.1.1.

12.1.2 PCNA Immunohistochemistry

Anti-PCNA immunohistochemistry was performed as described in Section 11.1.2.1.

12.1.3 Thymidine analogue histochemistry

Both cumulative thymidine analogue histochemistry presented in Section 4.6, as well as the pulse-chase histochemistry used for the study of neural fate in Section 4.8 were performed by the methods described in Section 11.1.2.2 and Section 11.1.2.3, respectively, except that the data from the whole retinal labelling experiment derives from 14micrometre transverse and coronal cryosections (n=3-6 animals per axis, per age).

12.1.4 Lineage tracing immunohistochemistry

Embryos from wild-type AB crosses (staining groups 2 and 3) and *Tg(Isl2b:GFP)* crosses (staining group 1). The *Tg(Isl2b:GFP)* line was the kind gift of the late Dr. Chi-Bin Chien. Embryos were collected and reared as described in Section 11.1.1. Embryos were divided into three groups, to be pulsed with EdU at 3, 23, and 90 dpf and sacrificed after a 7 day chase period. In 3 and 23dpf larvae, animals were pulsed by allowing them to swim freely in a 10mM EdU solution for 2 hours. In 90dpf animals, 10microliters of 10mM EdU was injected intraperitoneally. After 7 days, animals were sacrificed and coronal cryosections were obtained as described in Section 11.1.2.1. These cryosections were divided into staining groups according to their genetic origin, as mentioned above.

Cryosections from all staining groups were allowed to dry briefly at room temperature (RT), before rehydration in PBS for 30 minutes at RT. Subsequently, sections were blocked in 0.2% Triton X-100 + 2% goat serum in PBS for 30 minutes at RT. Primary antibody incubations were as follows for each numbered staining group:

1. Covance rabbit anti-Pax6 primary antibody diluted 1:200 in blocking solution incubated at 4 overnight.
2. Chemicon mouse anti-GS primary antibody diluted 1:500 in blocking solution at 4 overnight.
3. ZIRC mouse anti-Zpr1 primary antibody diluted 1:400 in blocking solution at 4 overnight.

After these overnight incubations, primary antibodies were removed and all sections were rinsed five times in PDT (PBS + 1% DMSO + 0.1% Tween-20). Secondary antibody incubations were as follows for each numbered staining group:

1. Goat anti-rabbit Cy3-conjugated 2º antibody diluted 1:100 in blocking solution at 37°C for 2 hr.
2. Goat anti-mouse Cy2-conjugated 2º antibody diluted 1:100 in blocking solution at 37°C for 2 hr.
3. Goat anti-mouse Cy2-conjugated 2º antibody diluted 1:100 in blocking solution at 37°C for 2 hr.

Staining group 2 was subsequently incubated with Santa Cruz rabbit anti-PKCβ 1º antibody diluted 1:100 in blocking solution at 4°C overnight. The next day, this second primary antibody was removed, and the group sections were rinsed in PDT as above. A further goat anti-mouse Cy2-conjugated 2º antibody diluted 1:100 in blocking sol'n was applied at 37°C for 2 hr.

After each staining group's final secondary antibody was removed, slides were rinsed 3x in PDT, then stained for EdU with an Alexa 647 azide. After another three PDT rinses, 100 ug/mL Hoechst 33258 was applied for 15 minutes at RT. Five PDT rinses, after which all slides were mounted, slipped, and sealed.

12.1.5 4C4 immunohistochemistry

The 4C4 antibody was a generous gift of Dr. Pamela Raymond. It has been empirically determined to mark microglia [BB01]. It was used according to the methods described above with a Cy2 secondary antibody.

12.1.6 Confocal micrograph acquisition and processing

All confocal micrographs presented in Chapter 4 were acquired on a Leica TCS SP5 II microscope, operated using the LAS AF microscopy software (Leica). Data was assessed by using Bitplane Imaris software (v1.5.1). Specifically, the Imaris watershed segmentation algorithm was applied to the nuclear counterstain channel to segment cellular nuclei. Colocalisation of particular markers (ie. PCNA, EdU, or lineage markers) was assessed by thresholding signal from these channels with the segmented nuclear volumes to produce a subpopulation of colabelled nuclei. The segmented Imaris scenes and the associated settings are available in the thesis HDD archive.

12.1.7 Retinal measurements

The developmental morphology study presented in Figure 12.2 was conducted by using the LAS AF software measurement tools on transmitted light data collected in the process of conducting the confocal survey presented in Figure 4.2. Each measurement point is, therefore, likewise the average of the value three central coronal cryosections. Retinal thickness was the linear thickness of the entire cellular retina and its plexiform layers at the center of the section, and the layer thicknesses reflect segments along this line, with the addition of RPE thickness. RPE length was measured in approximating segments from the dorsal to the ventral extrema. Lens diameter was measured in any sections which retained the lens; this was rare at older ages, when the lens sections tend to separate readily as soon as the cryosection is rehydrated. Optic nerve diameter was measured at its thickest point.

12.1.8 Estimation of overall CMZ population and retinal volume

The overall CMZ population estimates were derived by the calculations presented in Section 11.1.4. The volume of the cellular retina was crudely estimated by subtracting the volume of two spheres. The circumference of the outer sphere is given by the RPE length plus the diameter of the lens. The radius of the inner sphere is the half total thickness of the cellular retina subtracted from radius of the outer sphere[3]. The difference in volume between the two spheres is calculated; four-fifths this value is taken as the volume estimate in order to partially compensate for the flattening of the eye at older ages.

[3]Halving serves as crude compensation for the lack of cellular retina on the lens' part of the sphere, as well as the thinning of the cellular retina toward the periphery

12.1.9 Monte Carlo estimation of CMZ population and retinal volume rates of change

In order to estimate the mean daily rate of change of the CMZ population and retinal volume estimates, we performed to Monte Carlo difference operations between samples drawn from the estimated LogNormal distributions at subsequent timepoints. This allows us to empirically estimate the uncertainty on these rates, as the difference between two T-distributions is not, itself, T-distributed. The relevant code is found in Section 17.8.9, and uses functions from NGRefTools.jl.

12.1.10 Evidence estimation by Empirical Bayes linear regression

Evidence comparisons presented without estimated standard deviations (σ) of significance are produced by the Empirical Bayes method of linear regression, as presented in Bishop's standard machine learning textbook [Bis06]. Calculations were performed using the Julia package `BayesianLinearRegression.jl`, which implements Bishop's algorithm. Values are presented in log base e.

12.1.11 Evidence estimation by Galilean Monte Carlo nested sampling

Evidence comparisons presented with estimated standard deviations (σ) of significance are produced by converging ensembles of Galilean Monte Carlo sample chains, which collectively constitute a sample from the posterior in rigorous detailed balance [Ski19]. The Bayesian evidence, or marginal probability of the model over the posterior sample is calculated by the nested sampling algorithm [Ski06], and is presented in log base e. Calculations were performed by the use of the Julia packages `GMC_NS.jl` and `CMZNicheSims.jl`, written for this thesis and presented in Chapter 7 and Chapter 8, respectively. Maximum a posteriori outputs are calculated from the most likely model parameterisation sampled during this process.

12.1.12 Estimation of posterior distributions of phase model parameters

Posterior distributions presented in Figure 4.4 and Figure 4.7 were produced by Kernel Density Estimation (KDE) [Bis06, p. 122]. The KDE algorithm used was the implementation available in the Julia Robotics library `KernelDensityEstimate.jl`, described by Sudderth et al. [SMFW10]. Briefly, this uses multiscale nonmparametric Gibbs sampling of supplied weighted "point clouds" to generate the kernel density estimate. Chains from `GMC_NS.jl` were used in this context by weighting the points in parameter space by their estimated evidence. It is important to note that, for this thesis, `GMC_NS.jl` was configured to prioritise the accuracy of evidence estimates over these posterior estimates; as a consequence, samples from around the maximum a posteriori have little weight in the KDE estimates presented herein [HHHL18]. Alternatives to this scheme are possible; one such algorithm is described by Higson et al. [HHHL19]

12.2 Supplementary Figures

Figure 12.2: **Overview of *D. rerio* retinal parameters during morphogenesis**
All estimates are presented as the marginal posterior distribution of the mean, from uninformative priors, with 95% credibility intervals. Panel A: Thickness of the retina, measured in its center. Panel B: Length of the retinal pigmented epithelium, measured from one side of the lens to the other. Panel C: Diameter of the lens. Panel D: Diameter of the optic nerve, measured in its center. Panel E: total thickness of the retina, divided by the thicknesses of the layers that compose it. GCL: Ganglion cell layer. IPL: Inner plexiform layer. INL: Inner nuclear layer. OPL: Outer plexiform layer. ONL: Outer nuclear layers. RPE: Retinal pigmented epithelium.
Methods in Section 12.1.7. Code in Section 17.8.6.

Figure 12.3: **Developmental progression of naso-temporal population asymmetry in the CMZ.**
Marginal posterior distribution of mean nasal (N) and temporal (T) population size in 14μm transverse cryosections (panel A) or intra-individual N/T count asymmetry ratio (panel B), ±95% credible interval, n=6 animals per age. Data points represent mean counts from three central sections of an experimental animal's eye. Code in Section 17.8.11.

Figure 12.4: **Polymodality of CMZ model MAP estimates**
Four of the six most likely models found in the MAP sample of the combined decay slice model presented in Figure 4.9, located on the marginal distribution of initial cycle time (top panel), and with model output displayed in bottom panels. Note that very different initial CT values produce comparable explanations of the data. Code in Section 17.8.2.

Figure 12.5: **Linear regressions performed on cumulative labelling data from dorsal, ventral, and combined CMZ sectional populations**

Figure 12.6: **Linear regressions of temporally correlated and uncorrelated models of central retinal and 30dpf CMZ-contributed cohorts**

Figure 12.7: **4C4+ microglia are associated with the CMZ**
Representative maximum intensity projection from confocal micrographs of 14µm coronal cryosections through 90dpf zebrafish eyes.
Blue: Hoechst 33258 nuclear counterstain. Red: Microglia labelled with 4C4 antibody. Note extensive presence of 4C4+ microglia around the peripheral CMZ.

12.3 Supplementary Tables

Table 12.4: **Evidence for Normal vs. Log-Normal models of layer and lineage contribution**

Layer	Marker	Cell type	$\mathcal{N}$ logZ	Log-$\mathcal{N}$ logZ	logZR	σ sign.
GCL	Cohort	All GCL cells	-65.52 ± 0.66	**-30.96 ± 1.0**	34.6 ± 1.2	28.9
GCL	Isl2b	RGC	-36.64 ± 0.54	**-23.76 ± 0.59**	12.88 ± 0.79	16.2
GCL	Pax6	Displaced am.	**-40.08 ± 0.71**	-40.28 ± 0.46	-0.19 ± 0.85	0.2
GCL	Isl2b/Pax6	RGC subtype	-22.09 ± 0.39	**-16.82 ± 0.83**	5.27 ± 0.92	5.8
INL	Cohort	All INL cells	**-52.65 ± 0.66**	-120.1 ± 1.4	-67.5 ± 1.5	44.2
INL	Pax6	Amacrine cell	-23.09 ± 0.39	**-9.14 ± 0.46**	13.95 ± 0.61	23.0
INL	PKCβ	Bipolar cell	-21.59 ± 0.34	**2.81 ± 0.54**	24.4 ± 0.64	38.0
INL	GS	Müller glia	-21.45 ± 0.32	**10.18 ± 0.6**	31.63 ± 0.68	46.4
INL	HM	Horizontal cell	-22.52 ± 0.36	**3.02 ± 0.6**	25.55 ± 0.7	36.5
ONL	Cohort	All ONL cells	**-63.19 ± 0.7**	-81.6 ± 1.3	-18.5 ± 1.5	12.3
ONL	Zpr1	Double cones	**-31.63 ± 0.6**	-26.29 ± 0.6	5.34 ± 0.85	6.3

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of Log-$\mathcal{N}$ model.

Table 12.5: **Evidence for combined vs. separate models of layer and lineage contribution across the dorso-ventral axis**

Layer	Marker	Cell type	Combined D-V logZ	Split D-V logZ	logZR	σ sign.
GCL	Cohort	All GCL cells	**-53.473 ± 0.055**	-184.34 ± 0.13	130.86 ± 0.14	911.5
GCL	Isl2b	RGC	**-54.58 ± 0.9**	-67.72 ± 0.94	13.1 ± 1.3	10.1
GCL	Pax6	Displaced am.	**-31.09 ± 0.12**	-46.41 ± 0.3	15.32 ± 0.33	46.9
GCL	Isl2b/Pax6	RGC subtype	**-43.76 ± 0.76**	-109.0 ± 1.3	65.3 ± 1.5	43.3
INL	Cohort	All INL cells	**-131.4 ± 1.4**	-184.4 ± 1.6	52.9 ± 2.1	25.0
INL	Pax6	Amacrine cell	**-15.8 ± 0.024**	-52.42 ± 0.69	36.62 ± 0.69	53.3
INL	PKCβ	Bipolar cell	**-3.86 ± 0.2**	-19.81 ± 0.23	15.96 ± 0.31	51.9
INL	GS	Müller glia	**15.78 ± 0.25**	28.03 ± 0.41	-12.24 ± 0.48	-25.6
INL	HM	Horizontal cell	**10.79 ± 0.41**	-3.65 ± 0.19	14.45 ± 0.45	32.2
ONL	Cohort	All ONL cells	**-79.87 ± 0.97**	-158.2 ± 1.3	78.3 ± 1.6	48.5
ONL	Zpr1	Double cones	**-54.66 ± 0.92**	-64.27 ± 0.87	9.6 ± 1.3	7.6

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of combined model.

Layer	**Marker**	**Cell type**	**$\mathcal{N}$ MLE**	**Log-$\mathcal{N}$ MLE**	**lhR**
GCL	Cohort	All GCL cells	54.194	**55.835**	1.642
GCL	Isl2b	RGC	14.439	**19.106**	4.667
GCL	Pax6	Displaced am.	8.287	**11.158**	2.871
GCL	Isl2b/Pax6	RGC subtype	20.292	**28.139**	7.847
INL	Cohort	All INL cells	44.942	**45.179**	0.237
INL	Pax6	Amacrine cell	**23.808**	23.178	-0.63
INL	PKCβ	Bipolar cell	**27.243**	26.119	-1.124
INL	GS	Müller glia	30.847	**31.818**	0.97
INL	HM	Horizontal cell	27.65	**29.431**	1.78
ONL	Cohort	All ONL cells	41.488	**43.195**	1.706
ONL	Zpr1	Double cones	13.256	**18.483**	5.227

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of stable model.

Chapter 13

Supplementary material for Chapter 5

13.1 Materials and methods

13.1.1 Zebrafish husbandry

Zebrafish husbandry was performed as described in Section 11.1.1.

13.1.2 Generation of transgenic vsx2:eGFP *rys* line

Figure 5.9 makes use of coronal cryosections of *Tg(vsx2:eGFP) rys* animals. Transgenesis was performed using reagents from the Tol2kit [KFG+07], which uses the Invitrogen Gateway plasmid construction system to assemble three-part constructs from donor plasmids. Briefly, the Gateway system uses BP clonase to introduce fragments of interest into sets of 3 entry vectors, which are recombined in predictable order by LR clonase. Tol2kit consists of a numbered, standardized set of preconstructed entry vectors intended for use in the construction of transposable Tol2 transgenesis vectors.

The construct of interest was built from a ∼1.1kb fragment of the 5' promoter sequence of vsx2 in the 5' entry vector, an eGFP expression cassette in the middle entry vector (Tol2kit #383), and a polyadenylation signal in the 3' entry vector (Tol2kit #302), recombined into pdestTol2pA2 (Tol2kit #394).

The vsx2 promoter fragment was isolated by PCR amplification from the BAC (). BP clonase was bypassed in favour of restriction endonuclease sites inserted into the fragment's primers; this was necessitated by the instability of the ccd cassette in the BP vectors. The promoter fragment was amplified with a XhoI site 5' and a AgeI site 3'; the Tol2kit 5' Multiple Cloning Site entry vector (#228) was digested with SalI and XmaI in order to produce compatible sites for ligation. All restriction enzymes and the Quick Ligation kit were from NEB.

Injections into 1-2 cell embryos were conducted by standard techniques [Wes00]. Animals with stable expression of vsx2 in postembryonic peripheral RPCs and in central retinal bipolar cells, as expected, were identified and bred out. These animals were subsequently introgressed into *rys* in order to produce *Tg(vsx2:eGFP) rys* animals.

13.1.3 Morpholino injections

Morpholino injections were performed by Maria Augusta Sartori by methods previously described [WPMT15].

13.1.4 Morphological photography

The photographs presented in Figure 5.1 were obtained with the transmitted light optics of a Zeiss fluorescent stereoscope (SteREO Lumar.V12), using ZEN Imaging software.

13.1.5 PCNA and EdU proliferative histochemistry

PCNA and EdU proliferative histochemistry giving rise to the data presented in Figure 5.3 was performed as described in Section 11.1.2.1 and Section 11.1.2.2, except that the length of the EdU pulse was 24 hours, and was not administered to 5dpf animals, which were set aside for the nuclear morphology study presented in Figure 5.6.

13.1.6 BrdU pulse-chase assay

The BrdU pulse-chase assay presented in Figure 5.4 was conducted by soaking 3dpf *rys* larvae in 10mmol BrdU for 8 hours. After a subsequent 7 day chase period, the 10dpf larvae were sacrificed and processed as described in Section 11.1.2.3.

13.1.7 Cumulative EdU labelling assay

The cumulative EdU labelling experiment which was used to estimate cell cycle parameters in Figure 5.5 was conducted as described in Section 11.1.2.2.

13.1.8 Analysis of *rys* nuclear parameters by Galilean Monte Carlo Nested Sampling

The estimation of evidence for separate and combined mutant and sibling models of nuclear parameters presented in Table 5.1 was produced using the Julia package GMC_NS.jl, presented in Chapter 7.

13.1.9 Progenitor identity marker immunohistochemistry

Pax6 immunohistochemistry was performed as described in Section 12.1.4. In situ hybridization for npat transcript was performed by Monica Dixon by standard methods [Wes00].

13.1.10 Caspase-3 immunohistochemistry

Caspase-3 immunohistochemistry was performed by Monica Dixon, using histochemical methods described in Section 12.1.4, substituting the appropriate anti-caspase-3 primary antibody.

13.1.11 Electron microscopy

Images presented in Figure 5.7 were produced from ultrathin sections for transmission electron microscopy, prepared as described previously [LDT12] by Monica Dixon and Audrey Darabie. These

sections were imaged on a Hitachi H-7000 transmission electron microscope using AMT Image Capture Engine software.

13.1.12 qPCR analysis of *rys* npat expression

The RT-PCR analysis of *rys* npat expression presented in Figure 13.6 was conducted by Monica Dixon by standard methods [SFM89].

13.1.13 qPCR analysis of *rys* histone expression

The qPCR analysis of *rys* histone expression presented in Figure 5.13 was conducted by Maria Augusta Sartori. Briefly, total mRNA was extracted from groups of sib and rys embryos at the appropriate age. Two separate pools of cDNAs were generated from the isolated mRNAs, primed with random hexamers and oligo-dTs. These cDNA pools were used as the templates for the amplification of total histone transcript and polyadenylated transcript, respectively. This was done using primers degenerate for the named class of histone; individual transcripts are not identified. A Bayesian analysis of these data was perfomed in Section 17.8.23.

13.1.14 *rys* MNase digestions and NGS

Two 10-animal pools each of 5dpf *rys* sib and rys animals had fresh nucleosome-protected MNase fragments produced using the Takara Biotech micrococcal nuclease kit. These digests were submitted to the SickKids TCAG facility for Illumina HiSeq 2500 NGS, with two lanes run per pool.

13.1.15 Nucleosome position calling

The nucleosome position calling pipeline used in Chapter 5 was automated using the Java machine learning framework KNIME [DB16] workspace incorporating KNIME4NGS nodes [HJH+17]. Node settings are largely default and are available in the `knime_workspace` folder of the thesis archive.

Briefly, FastQC (v0.11.8) [And18] was used to characterise the quality of the fastq sequence files describing the nucleosome-protected fragment pool from the MNase digestions described above. No sequence from any of the two duplicates of the two pools was rejected due to low quality. Sequences were found to be of uniformly high quality, with low adapter content. These results are available in the `fastqc` thesis git archive. TrimGalore (v0.5.0) [And18] was used to remove trailing adapter bases, using Adapter sequence: 'AGATCGGAAGAGC' (Illumina TruSeq, Sanger iPCR). The trim reports are available in the thesis git archive. Bowtie 2 [LS12] with default settings was used to map reads to the zebrafish genome (GRCz11). Nucleosome positions were called from the resulting sequence alignment map (SAM) files by using DANPOS2 (v2.2.2) [CXP+13]. As DANPOS2 is no longer supported, and the code contained errors preventing execution, a fixed version of this code has been uploaded to `https://github.com/mmattocks/DANPOS2fix` and filed in the thesis HDD archive, along with the called output positions.

The DANPOS nodes of the KNIME workflow are python nodes which execute DANPOS2 itself, as well as the nucleosome position analyses described below.

13.1.16 Analyses of *rys* nucleosome position disposition

Figure 5.16 was produced from the DANPOS2-called positions for the pooled *rys* sibling and mutant MNase-protected fragment datasets. For each numbered *D. rerio* chromosomal scaffold (1-25), as well as for scaffolds not assigned to any chromsome (NC), the number of sibling positions was divided by the total length of the scaffold. These values were compared to the naively expected number of positions per scaffold, determined by dividing the total number of positions per kilobase of genomic material, then multiplying the particular scaffold length. The per-chromosome relative number of positions, as well as fold-differences from the naive expected value, are displayed in pie chart format in Panel A. The Panel C chart displays the result of normalizing these position values by relative occupancy, as determined by DANPOS2.

13.1.16.1 Background Hidden Markov modelling of *D. rerio* genomic sequence emission

The Julia package `BioBackgroundModels.jl` was used to perform background Hidden Markov Model selection on samples of *D. rerio* genomic material, in order to serve as a model of genomic noise for subsequent Independent Component Analysis (ICA) modelling of nucleosome positions, described below. The general strategy pursued was to first train a zoo of 3 replicates each of all HMMs with 1-6 states, emitting 0th-2nd order DNA kmers, on each of three broad partitions of the GRCz11 zebrafish genome, as described in Chapter 9, and initializing the zoo's EM chains using the default `autotransition_init` function, which samples from an uninformative prior on state emission vectors, but a transition matrix prior heavily favouring state autotransition. This zoo was converged to an EM step likelihood delta of $1e{-}3$ on 2e6 bp sampled without replacement from each of these partitions, then evaluated by testing against a further 2e6 bp sampled as a test set. The best model triplicate for each partition, in every case the 6 state, 0th order triplicate, was selected for further refinement by EM training on a new 8e6 bp sampled from the appropriate partitions. After this refinement process was complete, the triplicate chains were inspected using `BioBackgroundModels.jl`'s reports. The convergence of the triplicate on the same region of the parameter space, and the stationarity of the chains were verified before selecting the most likely models of the refined triplicate to represent *D. rerio* genomic background noise in the `BioMotifInference.jl` ICA models.

The code to sample GRCz11 for the initial zoo training task is available in Section 17.8.28, for the refinement task in Section 17.8.27. EM execution code is found in Section 17.8.29 and Section 17.8.31. Analysis code is given in Section 17.8.30 and Section 17.8.26.

13.1.17 Evidence and maximum a posteriori estimation of ICA models of *rys* nucleosome position sequence emission

The Julia package BioMotifInference.jl, presented in Chapter 10 was used to perform evidence and MAP estimation of Independent Component Analysis PWM source models of nucleosome position emission. The ICA models used had 8 PWM sources ranging from 3 to 10 positions in length. Model ensembles were initialized by sampling from uninformative source priors and an mixing prior of .07 (ie. for each source, mean 7% of observations will begin mixed with that source).

Observation sets comprised samples of 7092 nucleosome positions from each of the *rys* mutant and sibling differential position sets, corresponding to approximately 1 Mbp for each of the mutant and sibling sets, and 2 Mbp for the combined set.

Ensembles were converged to within 125 orders of magnitude likelihood compression.

The code used to produce the differential position set samples is available in Section 17.8.34. Ensembles are assembled in Section 17.8.32, and converged in Section 17.8.33.

13.2 Supplementary figures

Figure 13.1: **10dpf mutant *rys* RPCs are mitotic**
3d volume reconstruction of a complete mitotic figure in a *rys* mutant CMZ RPC at 10dpf, from a confocal micrograph of a 14µm coronal cryosection. Blue channel: Hoechst 33258; Green channel: PCNA; Red channel: EdU. Scale bars: 4 µm.

Figure 13.2: **MAP model output and observations for *rys* sibling thymidine slice model of 5dpf cumulative EdU labelling**
Count of EdU-positive cells observed in 14µm coronal cryosections through *rys* mutant CMZs at indicated chase times during a 10 mM EdU pulse (magenta crosses), overlaid with MAP model output. Probability mass distribution of the discrete non-parametric output is shown by color scale; yellow values include counts with >.05 mass. Output mean and 95% mass are indicated by solid and dashed white lines. Methods in Section 13.1.7. Code in Section 17.8.16.

Figure 13.3: **MAP model output and observations for *rys* mutant thymidine slice model of 5dpf cumulative EdU labelling**
Count of EdU-positive cells observed in 14µm coronal cryosections through *rys* mutant CMZs at indicated chase times during a 10 mM EdU pulse (magenta crosses), overlaid with MAP model output. Probability mass distribution of the discrete non-parametric output is shown by color scale; yellow values include counts with >.05 mass. Output mean and 95% mass are indicated by solid and dashed white lines. Methods in Section 13.1.7. Code in Section 17.8.16.

Figure 13.4: **4C4-positive microglia engulf *rys* mutant RPCs**
Maximum intensity projection of *rys* mutant retina at 5dpf, from a confocal micrograph of a 14µm coronal cryosection. Labelled in red, a 4C4-positive microglia engulfs TUNEL-positive apoptotic fragments in the vicinity of the CMZ. Blue channel: Hoechst 33258; Green channel: TUNEL; Red channel: 4C4.

Figure 13.5: **Synteny Database output for the syntenic region containing *D. rerio* npat**

Dre#: *Danio rerio chromosome #* Hsa#: *Homo sapiens chromosome #*

The relative position of the displayed genomic neighbourhoods on their chromosome scaffolds is displayed inset, top right. *D. rerio* npat is highlighted with a red square, visible on the right of the selected Dre15 region, annotated "im:6901964". *H. sapiens* NPAT is highlighted similarly on the left of Hsa11. landmarks of the duplication and rearrangement event that brackets the position of npat on Dre15 are highlighted with red lines.

On the scaffold of *H. sapiens* chromosome 11, NPAT occurs upstream of the highlighted duplication cluster bracketed by ARCN1 and MIZF. Zebrafish npat is located in the midst of this rearranged, duplicated cluster, but does not appear to have been duplicated itself; at least if it was, no paralogue can be found by syntenic or similarity analysis.

Figure 13.6: **npat is overexpressed in *rys***
qPCR data for npat transcript amplified from pools of *rys* mutant (red), sibling (green), and wild-type control (blue), at 6dpf (left) and 8dpf of age (right). Error bars indicate standard deviation.

Figure 13.7: **npat morpholino injectants do not recapitulate *rys* retinal disorganisation phenotype**
Representative 14 µm central coronal cryosections through eyes of zebrafish embryos injected at the 1-2 cell stage with control, ATG- and splice-directed npat morpholinos, alongside uninjected siblings. Maximum intensity projection micrographs, with red staining of PCNA+ve nuclei against a blue Hoechst 33342 counterstain, are presented alongside dot schematics, displaying the positions of nuclei in blue, with PCNA+ve CMZ nuclei highlighted in yellow.

Figure 13.8: **Morpholinos directed to npat result in histone transcript overexpression similar to *rys***
qPCR analysis of fold changes in histone family transcript expression, both polyadenylated (left panel), and total (right panel), after injection at the one-cell stage with morpholino directed to the npat start codon.

Figure 13.9: **Novel nucleosome positions in *rys* occur in similar numbers to those lost from sibs.**
Counts of positions found in sib but not *rys* (red bars, represented as negative numbers, as these are 'lost' in *rys*) and those found in *rys* but not sib (blue bars, 'gained'). The magnitude of the difference between the counts is represented with a yellow bar.

Figure 13.10: **Test likelihoods for background HMMs trained on *D. rerio* genome partitions** Panels are gridded by genomic partition (left-to-right, exonic, intergenic, and periexonic sequences) and emission symbol order (top-to-bottom, 0th order, 1st order, 2nd order). Within each panel, increasing state number is represented on the x axis, while test likelihood is on the y axis. Each cross gives the likelihood of one of the three replicates optimised by EM for each order/state/partition combination. Magenta line indicates the likelihood of a naive model (1-state, equiprobable emission of each base). Only 0th order models prove to be more plausible than this naive control.

Figure 13.11: **PWM sources detected in combined sib and rys differential sources.** PWMs presented as sequence logos with letter height proportional to position informational contentemission probability.
PP: Posterior prevalence; proportion of models within the compressed maximum a posteriori ensemble which have this PWM as a signal source
OP: Observation prevalence; mean proportion of observed position sequences this sequence is used to explain, over all posterior samples

13.3 Supplementary tables

Table 13.1: **Evidence for separate models of retinal parameters in anti-npat morpholino-injected animals**

Morpholino	Measurement	Separate logZ	Combined logZ	logZR	σ sign.
ATG	CMZ pop.	-236.4 ± 2.3	**-198.3 ± 2.2**	-38.1 ± 3.2	12.1
ATG	Total sectional pop.	-412.4 ± 7.8	**-225.3 ± 4.9**	-187.1 ± 9.2	20.3
ATG	Sectional pop. per CMZ cell	**-70.18 ± 0.86**	-78.0 ± 0.86	7.8 ± 1.2	6.4
ATG	Dorsal CMZ pop.	-169.5 ± 1.4	**-151.75 ± 0.8**	-17.8 ± 1.6	11.2
ATG	Ventral CMZ pop.	-223.1 ± 1.5	**-165.3 ± 1.0**	-57.8 ± 1.8	32.5
ATG	Nuclear volume	-290.6 ± 1.8	**-192.7 ± 1.8**	-97.9 ± 2.5	39.6
ATG	Nuclear sphericity	**-186.13 ± 0.79**	-215.66 ± 0.57	29.53 ± 0.97	30.3
Spl	CMZ pop.	-272.6 ± 2.4	**-197.0 ± 2.6**	-75.7 ± 3.6	21.2
Spl	Total sectional pop.	-455.9 ± 8.3	**-205.6 ± 4.7**	-250.3 ± 9.5	26.3
Spl	Sectional pop. per CMZ cell	-87.04 ± 0.17	**-66.22 ± 0.29**	-20.82 ± 0.34	62.0
Spl	Dorsal CMZ pop.	-226.2 ± 1.7	**-171.0 ± 1.1**	-55.2 ± 2.0	27.7
Spl	Ventral CMZ pop.	-234.97 ± 0.81	**-222.3 ± 1.7**	-12.7 ± 1.9	6.8
Spl	Nuclear volume	-277.5 ± 1.6	**-223.2 ± 2.1**	-54.3 ± 2.7	20.4
Spl	Nuclear sphericity	-94.3 ± 1.1	**-67.71 ± 0.39**	-26.6 ± 1.1	23.2

logZ: logarithm of p(D), the marginal likelihood of the data, or model evidence. Largest evidence values bolded. logZR: evidence ratio; positive values in favour of stable model.

Table 13.2: ***rys* mutant differential nucleosome positions are less typical of the genome than those mapped to sibling positions**

Sequence set	*rys* naive LH	sib naive LH	LH ratio
Differential	-6.3402e7	-5.271e7	-1.0692e7
Mapped	-2.2921e8	-2.2617e8	-3.0407e6

naive LH: log-Likelihood of sequences, given genomic background noise models alone. Higher values are more likely a priori, indicating sequences more typical of the genome. LH ratio given with *rys* mutant sequences as the denominator. Note that the ratio of the differential sequences is much greater than that of the mapped sequences, indicating that the *rys* differential position set is much less typical of the genome than mutant sequences which are found overlapping sibling positions. Code in Section 17.8.36.

Chapter 14

Supplementary materials for all chapters

14.1 Thesis git and HDD archives

The resources used to compile and typeset this document are available at https://github.com/mmattocks/Thesis. These include most observational datasets referred to in figures. Most of the code used to generate figures, available in Section 17.8, can be executed without additional resources, beyond the package dependencies included by the script. These are easily installed in Julia, as one may `]add` packages from the interpreter. The scripts may then be `include()`d, with appropriate changes to paths where required (normally declared at the top of the script). Not included in the git archive are raw confocal microscopy stacks from any chapter, nor the bioinformatic data from Chapter 5), due to the large size of these datasets. The fully calculated ensemble posterior samples are also not included, although they can be replicated from the analysis scripts. These larger datasets are available in the HDD data archive, available in the Tropepe lab at the Department of Cell and Systems Biology, kept with Dr. Vince Tropepe. The git archive will host the most up-to-date and accurate version of the thesis.

14.2 Code notes

Most of the code used in this thesis is presented in Chapter 17. Three languages are represented: C++ for the CHASTE simulators presented in Chapter 2, Python for the SPSA optimization algorithm and general scripting in Chapter 2, as well as for the generation of Figure 5.16 in Chapter 5, and Julia for everything else. The Julia code is by far the easiest to execute. We have found it to be a productive, performant, and educational language to code, and the lack of a well-developed cell-based simulation system like CHASTE is the primary reason the thesis is not entirely Julian.

The C++ code requires the CHASTE framework [MAB+13] to compile. If replication of the simulation pipeline in Chapter 2 is required, CHASTE can be downloaded from https://www.cs.ox.ac.uk/chaste/index.html and used to compile the simulators in the `SMME/apps` folder of the SMME repo, available in Section 17.1. The simulators are managed from the python script fixtures available in `SMME/python_fixtures`. Python used for the SMME work is python3.

We also used the Java-based KNIME scripting framework to manage the execution of the bioinformatics pipeline for calling nucleosome positions in Chapter 5. This made use of the KNIME4NGS package [HJH+17]. The KNIME workspace used is available in the HDD archive at `/knime_workspace`. This workspace includes a THiCweed analysis pipeline [ASNS17], which is not presented in this thesis, and can be deleted or ignored. The pipeline requires both python2 and python3 environments, because danpos, the nucleosome position calling algorithm we used, is written in python2. danpos [CXP+13] is no longer available in functioning form from the author's archive. We have uploaded a version that fixes execution errors to `https://github.com/mmattocks/danpos-fixed`.

14.3 Computational cluster description and discussion

The computational cost of the model sampling performed in this thesis varies widely. The ICA models estimated in the *rys* work of Chapter 5 are comparatively enormous problems with expensive solutions, and require more power to solve in a reasonable timeframe than is typically available in a molecular biology lab. On the other hand, the CMZ slice models used in Chapter 4 can be estimated in a day on typical higher-end PCs used for microscopy acquisition and the like. This thesis used two machines as the local cluster for the non-ICA problems. None of the problems made use of GPU processing, although some of them could possibly benefit from the substitution of CUDA arrays.

The first machine was an obsolete watercooled 4-core i5-4670K overclocked to 4.4 GHz, with 16 GB RAM. It has good thermal characteristics and maintains full clock under 100% load at 60°C. This machine is adequate for estimating most of the CMZNicheSims.jl models, on the datasets of this thesis, in less than a day. The CHASTE/SPSA approach presented in Chapter 2 is slow; it takes 7-10 days to execute the required 15000 iterates on this machine. The comparable model in `CMZNicheSims.jl` is the `Thymidine_Model`, which is the slowest of the simulators presented in Chapter 8. Generally, this machine will complete about 1500 iterates a day using this purpose-coded model in Julia. Direct comparison between SPSA and GMC estimation processes is difficult, but this suggests that the overhead from using the object-oriented spatial modelling framework CHASTE does not dominate computational cost. Still, given Julia's C++-equivalent performance and much higher coding productivity, it seems the purpose-built, functionally-coded model will usually be faster and cheaper to develop and execute, overall. In any case, for the types of relatively simple observational datasets collected from microscopy studies, lab machines will suffice; for larger bioinformatic problems like those tackled in Chapter 5, this machine is best used as a combined master and fast calculation node in a cluster of 50+ dedicated calculation nodes. A particular limitation of older machines is the relatively low cap on the size of available RAM DIMMs. Generally, this prevents maximum parallelisation of BioBackgroundModels.jl models across cores. Parallelisation is easier to achieve on more modern machines with more cores/threads and RAM.

The second machine was a more modern 12-threaded i7-9750 in a Tongfang mobile chassis, clocked to 4.0 GHz, with 32 GB RAM. This machine is strongly thermally limited[1], typically running at 3.4GHz and 90°C under full load. Still, the additional threads and RAM make higher-end laptops a useful calculation node. Machines like this can expect to estimate two or more of the `CMZNicheSims.jl` models simultaneously in less than a day, given similarly sized datasets. They can expect to supply about three 4-threaded workers for distributed `BioBackgroundModels.jl` or BioMotifInference.jl jobs,

[1]This machine cooked its own motherboard during the thesis work, requiring replacement with a revised board less vulnerable to overheating. This can be partially overcome in Canadian winters by calculating outside.

with significantly better performance than the AWS nodes described below.

Both BBM.jl and BMI.jl assume that the user will want to distribute the job across a cluster of dissimilar workers. Because Julia has an extremely elegant Distributed package as part of its Base distribution, any type of julia `ClusterManager` with master-worker topology can be used. For this thesis, we chose to purchase compute time on the spot market at Amazon Web Services (AWS). A small module with helper functions for purchasing node time on this market this is supplied in Section 17.7. The AMI image used in this thesis is available in the HDD archive. We used `c5a.24xlarge` nodes. These have 48 cores per socket, clocked at 2.8GHz, and 196 GB of RAM. Most AWS instances are slower than enthusiast hardware, but the sheer number of available cores and RAM makes them an effective way to quickly assemble a large worker pool. We usually used both `BBM.jl` and `BMI.jl` with 4 cores per worker, as the threading overhead decreases efficiency beyond this point. Some `CNS.jl` simulations can usefully use 24-36 threads, but these should be benchmarked. `Thymidine_Ensemble`s do noticeably poorly with large thread numbers, for instance. The model comparison performed in Chapter 5 required five `c5a.24xlarge` instances, added to the two machines described above. These ran for approximately two weeks (half a week for each of the sib and *rys* observations sets and a further week for the combined set). Typical spot prices for these nodes are about USD $1.hr^{-1}$ at the time of writing, giving an approximate cost of USD 1700 for a problem of this size. On-demand pricing is typically 4-5 times this cost; obviously, it is preferable to use the spot market.

This presents the risk that nodes may be stopped at any time, either because the spot price exceeds the submitted maximum bid, or because AWS requires the compute power for the on-demand market [2]. As a result, all of Julia algorithms available in this thesis back up to disk and are robust to dropped workers, power outages, and other kinds of interruption. To resume an interrupted calculation performed from one of the Julia analysis scripts in this thesis, simply execute it again.

Users of `BBM.jl` and `BMI.jl` clusters should be aware that observations data transfer to workers can take a very long time, particularly with the relatively slow connections provided by AWS. This presents an additional complexity to budgeting for the spot market: interruptions require the re-transfer of observations to each worker in turn. `BMI.jl` waits for observations to finish transferring to each worker before starting the next; this prevents the situation where attempting to transfer to all nodes simultaneously on the same connection prevents any of them from starting on the problem. Probably, for larger datasets and larger models, the master node should be one of the cloud devices, to allow the observations to be local to those machines.

[2] High Netflix viewing times seem to be associated with this.

Chapter 15

Theoretical Appendix A: Model theory and statistical methods

15.1 Model Theory

This thesis is concerned with a particular kind of scientific model: those that are subject to mathematical analysis, and tractable subjects for numerical simulation techniques. Most biological models do not fit this description, but increasing computing power has expanded the potential for, in particular, cellular and macromolecular explanations to be formalized and tested in this way. While doing so is sometimes derided as "physics envy", the reality is more complex. Simple heuristics are, sometimes, adequate biological explanations; much of differential diagnosis consists of reliable descriptive relations between qualitatively characterised signs in patients, and the presence of a pathogen, for instance. More often, the complexity of biological systems, and the sophistication of the instruments with which we probe them, give rise to observations which support more than one plausible causal explanation for the features of the data. The question of model comparison then immediately arises. Without numerical analysis, we may only make reference to the use of the model in applied domains where the model failure has commercial consequences. Because biological models are rarely required to succeed as engineering tools, for the prediction and control of outcomes of practical significance, an inability to analyse them numerically means that we are left only with rhetoric to assess their relative explanatory value. This approach to "model comparison" necessarily devolves into the anarchic Feyerabendian discursive chaos discussed in Section 16.2.1. Feyerabend correctly points out that scientific models can only be compared against one another, and that the selection of comparative criteria can never be "objective" in the sense of being fully independent of the contingent situation and goals of the observer. Nothing can be done about this; either we look for relatively-better criteria and relatively-better models as judged by those criteria, or we remain in the realm of pure metaphysical rhetoric, without investigating their implications, what Feyerabend describes as the realm of the "crank":

> [T]he distinction between the crank and the respectable thinker lies in the research that is done once a certain point of view is adopted. The crank usually is content with defending the point of view in its original, undeveloped, metaphysical form, and he is not prepared to test its usefulness in all those cases which seem to favor the opponent, or even admit that there exists a problem. It is this further investigation, the details of it, the knowledge

> of the difficulties, of the general state of knowledge, the recognition of objections, which distinguishes the 'respectable thinker' from the crank. The original content of his theory does not. [Fey81, p.199]

The support of complex biological systems for multiple explanations gives rise to calls for explanatory pluralism [Bri10]. Very differently parameterised models can be adequate explanations for the same system. This is common for explanations at different descriptive depths. For instance, the models used in Chapter 4 do not descend below cellular level, and make no reference to macromolecules. We would still like to have macromolecular explanations of RPC function, particularly because these may supply us with means to intervene onto RPC proliferative and lineage outcomes, which is a reason to have different models for investigating the molecular phenomena of *rys* in Chapter 5.

In other plural explanations, adequate, differently-parameterised models of the same system may be describing different aspects of the same level of organization. Pharmacological kinetic studies of G-protein association with receptors, paired with crystallographic studies of the same phenomenon, supply a good example. These models allow experimenters to pursue different descriptive and interventional objectives. Indeed, as Nicholas Rescher has noted, attempts to synthesize many models into a single overarching explanation often result in descriptive chaos [Res00, p.65-6]. Rescher argues the adequacy of some model in its domain usually requires the blurring out of at least some pertinent details of the system that could have been included; the computational tractability of the model requires the same.

If we all accept these forms of pluralism, we are still left with the cases where models are making contradictory claims about reality[1]. Pluralism cannot coherently extend to abandoning the fundamental logical law of non-contradiction, without compromising the entire endeavour of rationalism. We cannot accept logically contradictory notions about the structure of reality without precluding a broadly consistent view of the way the world works, what Rescher refers to as "Cognitive Harmony" [Res05]. Given the complexity of biological systems, we have no option except to express models formally, and to test them rigorously against one another. This is the task of model selection.

Model selection requires that we be able to score models against observations that represent phenomena, to measure their quality. This requires the selection of a quality function. Loss functions, like AIC, used in Chapter 2, express the relative amount of information in the dataset lost by the model, given a set of parameters. Likelihood functions, used elsewhere, express the credibility of a parameterised model, given the observations. The parameters of the model define an n-dimensional "parameter space"; the quality function expresses a hypersurface within this space. By sampling within this space, we may estimate the shape of the surface. This sampling information can be used in three ways: we may propose new, higher quality parameter space locations to sample from, in search of the highest quality parameterisation of the model (model optimisation); we may derive marginal likelihoods for particular parameter values (parameter estimation), or we may estimate the quality of the model over all parameterisations (evidence estimation). These topics are discussed below.

15.1.1 Bayesian Epistemological View on Model Comparison

The analyses presented in the data chapters express two views on model comparison. The first, in Chapter 2, is drawn from information theory, while the second, taken up in Chapter 4, is a Bayesian view, albeit with more sophisticated tools and more computing power than has been available to Bayesians in

[1] That is, they have incompatible metaphysical content; they are therefore subject to counterinduction as mentioned in Section 16.2.1

the past. In the same way that frequentist analyses may be expressed as a subset of Bayesian analyses (e.g. they seek maximum a priori model parameterisation and likelihood from uninformative priors), informational theoretical approaches to model comparison can be expressed as a subset of Bayesian model comparison theory. In fact, the loss function used in Chapter 2, Akiake Information Criterion, has been adapted to refer to a prior distribution, as the Bayesian Information Criterion [PB04]. The intent of these criteria is to take Occam's Razor to the maximum-likelihood approach, by penalizing the maximum-likelihood value (or maximum a priori score, in the case of BIC) by the number of free parameters.

The general approach of optimizing a model for a quality against a dataset, then penalizing the best model quality by the parameterisation of the model, allows us to overcome an important problem with many maximum-likelihood approaches to model selection: the requirement for models to be parametrically nested. Model nesting precludes counter-induction; we cannot compare models which express fundamentally different views of how the described system is parameterised, only whether adding more parameters improves a particular view. Escaping this limitation allows us to compare stochastic and deterministic mitotic mode models in Chapter 2; these models express different views of how reality is organised. Still, in important ways, this approach shares the basic problem of simply calculating the MLE: the score does not summarize the robustness of the model fits. That is, a model which is a terrible description of a dataset over most of its plausible parameter space, but an excellent one in a tiny region, can appear to be a better explanation than a model which is a broadly good description over the whole parameter space. Simply using the number of parameters to penalize the best model found fails to capture how justified inclusion of those parameters is, by improving the overall likelihood of sampled models.

This leads us to what may be regarded as the completion of the Bayesian view on model selection[2], John Skilling's system of Bayesian inference, nested sampling [Ski06, Ski12, Ski19]. By rearranging the usual presentation of Bayes' rule, Skilling demonstrates how it specifies computational inputs and outputs associated with the activities of model sampling, parameter estimation, and evidence estimation. Bayes' rule is typically written as follows, where x is a logical proposition about the data (e.g. specific values for the cell cycle length and exit rates of the CMZ), and $Pr(a|b)$ denotes the probability of a given b:

$$Pr(x|data) = \frac{Pr(data|x)Pr(x)}{Pr(data)}$$

This can be read aloud as "the posterior probability of the proposition, given the data, $Pr(x|data)$, is equal to the likelihood of the data given the proposition, $Pr(data|x)$, multiplied by the prior probability of the proposition, $Pr(x)$, and divided by the marginal probability of the data over all such propositions, $Pr(data)$." This gives the impression that the principal task in statistical analysis is the calculation of the marginal posterior distributions over model parameters, and gives rise to the treatment of the marginal probability, $Pr(data)$, as a mere normalizing constant. Skilling rearranges this to put computational inputs on the left, and outputs on the right:

$$Pr(x)Pr(data|x) = Pr(data)Pr(x|data)$$

[2]If nested sampling is not *the* completion of the Bayesian system, it is at least *a* complete Bayesian system.

This shows us that the evaluation of a model consists of supplying a prior probability for the proposition, $Pr(x)$, and a likelihood function, to assess the probability of the data given that proposition, $(Pr(data|x))$. As computed output: (over many propositions x), the total evidentiary mass of the data for this model, $(Pr(data))$, as well as the posterior parameter estimates, $Pr(x|data)$. Sample model parameters are drawn from the prior; the likelihood of these propositions about the data are calculated. By accumulating many such samples, the marginal probability of the model over all of these propositions (the evidence for the model) can be estimated. Because these samples may be weighted by their calculated likelihoods and position on the prior, we may also use them to estimate the marginal posterior probabilities of parameters of interest.

This encapsulates both the numerical procedures involved in model analysis, as well as the epistemological view implied by Bayesian statistics. That is, a model analysis is the joint product of the model and the data, which expresses our belief about the overall evidence for the model $(Pr(data))$[3], as well as allowing us to estimate the distribution of credibility we should assign to various values for parameters of the model (propositions), $Pr(x|data)$. These are the two fundamental levels on which quantitative measurements of natural systems allow us to make inferences. We may distinguish between models of the systems in a general sense by their evidence, when applied to the same overall dataset. This allows us to counterinductively test contradictory descriptions of the structure of the phenomenon against one another, inferring which is a better map to the territory. The second level of inference is the ranking of propositions for the parameterisation of models achieved by the sampling procedure. This allows us to determine which particular propositions about the system are supported, given the model and observations. Because the posterior distributions need not be unimodal, we can evaluate these modes as separate hypotheses that are supported to varying degrees by the data. An extensive review of the Bayesian approach to model evidence and model selection has been provided by Knuth et al., covering other methods of estimation not touched on here [KHM+15].

This view dispenses with interpretations of model selection as being about finding the "true model" of reality, or of estimating the actual, objective probabilities inhering in things or processes (a view disputed in Section 16.1). Instead, we are guided to focus on the relative quality of models in explaining all of the relevant data we can gather; we may then evaluate the relative quality of specific propositions about the system within those models (i.e., the posterior distributions on model parameters) as we see fit.

15.1.2 Model sampling and optimization

There are many techniques used to sample the parameter space of a statistical model, given a quality function which scores the model. These techniques have a variety of purposes. Commonly, one wishes to "optimize" a model by finding the parameter vector which produces the best objective function result, given some dataset. This is often referred to as Maximum Likelihood Estimation, the product being a Maximum Likelihood Estimate, with MLE used to refer to these interchangeably. SPSA is used in this thesis to optimize cell-based models of RPC activity in Chapter 2. The likelihood scores are penalized by the number of model parameters to produce an AIC score. MLE methods are broadly useful for many applications, with some tuning. SPSA is used in control systems, to maintain the consistency of modelled processes by estimating the inputs most likely to achieve a setpoint, for instance [ZZLG08].

[3] ie. a better model gives higher marginal probability to observations than a worse one

More sophisticated techniques, like Nested Sampling, involve the estimation of the distribution of quality values over the parameter space, rather than focusing solely on estimating the optimal value.

The process of model sampling requires an algorithmic means to generate proposed parameter vectors to be supplied to the quality function. Most effective sampling methods operate by performing an initial sample from some type of prior information[4], and then iteratively generating new proposals from these initial ones. This iterative proposal generation forms a chain of linked positions within parameter space. Generally, new proposals are made considering information related only to the last accepted proposal[5]. Because the process is "memoryless" in the Markovian sense, and involves the iterative generation of models, it is often called Markov Chain Monte Carlo (MCMC).

Many algorithms for proposal generation exist, and the accuracy and efficiency of sampling procedures depends on their properties. Most offer a physical interpretation of the quality surface in parameter space. Random-walk proposal generation, like the original Metropolis-Hastings implementations of MCMC, is often too inefficient to be usable for high-dimensional parameter spaces, and has broadly been replaced by these physical sampling methods. SPSA, for instance, is a simple gradient method. New proposals are generated by bracketing an existing sample with a pair of samples that define an n-dimensional "slope" at the current location. By proposing a new location down-slope, we receive a new sample which is closer to the local optimum, and we may inexorably approach it in this way. Other, more sophisticated interpretations seek to improve on random-walk proposal generation by mechanical interpretations of proposal movement through parameter space, like Hamiltonian Monte Carlo, or thermodynamic interpretations, like simulated annealing [ADH10]. These methods are better suited to the estimation of the distribution of objective function values than an algorithm like SPSA.

In order to ensure the accuracy of any such estimated distribution across parameter space, the algorithm should, ideally, produce proposals in "detailed balance", which refers to the physical concept of reversibility found in classical mechanics. The purpose of constraining proposal generation in this way is to guarantee that particular areas of the parameter space are not over- or under-sampled, distorting the final estimate, as a consequence of the manner in which the sampling algorithm generates proposals. Galilean Monte Carlo has the property of being in detailed balance, although, as Chapter 7 demonstrates, this is not a guarantee of accurate evidence estimation.

15.1.3 Overfitting

Overfitting is a common problem in, particularly, frequentist statistical modelling [Bis06, p.9-11]. It can arise from an unjustified excess of model structure relative to the information present in the data. This excess structure allows the model to capture measurement noise in the dataset the model is being scored against. This produces models which appear highly explanatory, but which reflects the particular structure of the measurement errors in the training data better than the overall phenomenon. This problem will typically become apparent in cases of sequential inference; an overfit model will fail to explain new observations as well as it explained previous ones.

A second source of overfitting is the failure to properly account for uncertainty and prior information.

[4] In the SPSA optimization performed in Chapter 2, this took the form of the original, poorly optimized fit for the stochastic model and a best guess for a related vector for the deterministic model. In subsequent nested sampling analyses, initialization is performed by sampling randomly from a defined prior distribution.

[5] Proposal rejection can occur for a variety of reasons, depending on how the parameter space is interpreted. In Galilean Monte Carlo, this occurs if proceeding along the specified velocity vector would produce a proposal less likely than the last one.

Division type:	"PP"		"PD"		"DD"	
# progeny in P:	2		1		0	
Probability:	p	+	r	+	q	= 1.0

Figure 15.1: **Simple stochastic stem cell model, representing probabilities of cell division events.**
Black circles denote proliferative cells, while white and grey circles denote different types of postmitotic offspring. "Number of progeny in P" is the number of mitotic offspring produced by each type of division. The probability of each division type must sum to 1, as all possibilities are represented, granting that the division types are defined by the postdivisional mitotic history of the offspring.

For instance, if unidimensional measurements are modelled with a Normal Gaussian distribution, the MLE Normal Gaussian (whose parameters arise from calculating the mean and standard deviation of the measurements, given the assumption of Normality) can be an overfit to the data [Bis06, p.28]. In this simple example, this usually means that the MLE Normal underestimates overly influenced by "outliers". This is one case of the general overfitting problem seen in MLE methods [Gre17]. Accounting for available prior information is the best solution to this problem. As this thesis demonstrates, appropriate accounting for our uncertainty about the parameters of the Normal model, and comparison of the Normal case with appropriate alternatives that might better reflect the underlying causal structure that produces the "problem" data (in this case, Log-Normal models), both help in producing more accurate estimates of model parameters.

15.1.4 Simple Stochastic Models

The Simple Stochastic Model is schematised in Figure 15.1. This is the basic structure of the majority of formal models in the stem cell literature, derived from post-hoc analyses of populations taken to include stem and progenitor cells. The population-level approach is usually explicit, as no differentiation is made between types of proliferating cell; in general, no particular cell is identified with a stem cell, nor can any be identified from the necessarily retrospective population data used to infer the parameters of the model.

The central concept of the model is that mitotic divisions can be categorised by the number of progeny which remain mitotic after the division. This categorisation is necessarily retrospective. This must be kept in mind when considering models of this type, as this categorisation does not necessarily imply that there is some mechanism by which the cell specifies the fate of offspring *at the time of mitosis*, although there is extensive evidence for the coupling of mitotic and specification processes at the molecular level [TFC13, TR14, Dal15, DD17].

In effect, the model compresses the process of fate specification into individual mitotic events. Since the primary distinction between cells in the model is simply whether they are proliferating or not, the

model also elides any heterogeneity within the proliferating population. Beyond not identifying particular cells as "stem cells", this may make models derived from the SSM inappropriate for proliferative populations with a large degree of heterogeneity. The classic idea of a small number of slowly proliferating "true" stem cells and a larger population of rapidly dividing "transit amplifying" progenitors can only be represented by compartmentalised, independent SSMs (as implemented in Chapter 4).

As Fagan notes, in the SSM, "relations among p, r, and q values entail general predictions about cell population size (growth, decrease, or 'steady-state'), and equations that predict mean and standard deviation in population size, probability of [lineage] extinction, and features of steady-state populations are derived."[6] [Fag13, p.60]

Typically, this type of model has been employed to describe population dynamics of proliferating cells in assays generating clonal data, where a "clone" here refers to the population constituted by all offsping descended from some particular (usually "initial" and sometimes therefore taken for "stem") proliferating cell. This population is the *lineage* generated by the cell. The mitotic events which give rise to a lineage are summarised as a Galton-Watson branching process, a statistical model originally intended to represent the lineage extinction of surnames. In the case of branching process models applied to proliferative cells, the random variable determines the mode of division of mitosing cells, defined by the proliferative state (construed in the model as being either mitotic or postmitotic) of progeny. For any given division, a cell may produce two mitotic, one mitotic and one postmitotic, or two postmitotic progeny, and each of these division modes is given a defined (often, but not always, static) probability. Given these values, the history of a cell lineage may be simulated; the output of many of these simulations pooled together, in Monte Carlo fashion, allows the statistical properties of the dynamics of population of simulated cells to be estimated.

15.1.5 Systems of difference equations

Models defined by difference equations are used in Chapter 4. A difference equation is the discrete counterpart of a continuous differential equation. Both types of equations can be used to describe the time-evolution of a system, because they can be recursively applied to calculate the value of model outputs at a time t, given the value of the outputs at $t - p$, p time units in the past [KP01, p.1]. In the models presented in this thesis, the simulation's unit of resolution is the day, which motivates the use of the algorithmically simpler analytic solutions to difference equations. As implied above, the time-dependent value of a model output may depend on a output of another equation. This is the case for the model of estimated CMZ annulus population and retinal volume presented in Figure 4.2, where the volume estimate depends on resolving the value of the day's starting population before evaluating the volume contribution of that population, given the exit rate. The slice models (see `Slice_Model`) used to interrogate phase transition timing in Figure 4.6 are not fully systems of difference equations, since the population difference equation depends on evaluating a continuous power law model of lens growth. Moreover, the slice model of decaying cycle rate (see `Decay_Model`) depends on the daily evaluation of the continuous exponential cycle time function.

[6] While Fagan refers to "stem cell" extinction, the model does not specifically define stem cells, nor does it imply intergenerational continuity, such that a particular intergenerationally identified stem cell should be said to have become extinct. The unit which survives or is made extinct is the lineage derived from some particular proliferative cell.

15.1.6 Independent Component Analysis models of sequence emission

The models of *rys* nucleosome position sequence emission presented in Chapter 5 are products of Independent Component Analysis (ICA), a technique which is used to infer the independent contributions of multiple signal sources on a single multiplex channel; it can be thought of as an application of information theory [LGBS00]. The most common example offered to illustrate this is the separation of the individual streams of speech produced by multiple speakers conversing in a noisy room [JHTS00]. ICA can be used, in this context, to model the process by which a listener separates the independent signals of the speakers from background noises. In this thesis, the *D. rerio* genome is treated in this fashion. In effect, any given sample of genomic sequence may be treated as a multiplex channel produced by a host of independent causal processes. In this sense, each nucleosome position is a "noisy room", with its own local structure of background noise and conversational participants.

The distinction between background noise and signal in an ICA model is made both conceptually, and in the numerical representation of these parts of the model. I have closely followed the early, pioneering example of Down et al. [DH05]. Down et al. use Hidden Markov Models, optimized on promoter sequences, as background models for promoter sequences in general. Against this, a foreground of Position Weight Matrix signals, representing binding motifs within the promoter region, is inferred by nested sampling. Similarly, in Chapter 5, separate HMMs were used to represent genomic sequences from the exonic, "periexonic" (introns and ±500bp intervals around the CDS), and intergenic sequences, against which the foreground of repetitive PWM structural signals is inferred, representing characteristic preferred contact points between the core nucleosome and genomic DNA. The HMM background models and s PWM signal models are linked in their description of a set of o observations by an $o \cdot s$ mixing matrix. While many implementations of source and background mixing are possible, in this case, Down et al.'s simple binary implementation is retained. Each observation o is associated with a "mix vector" of s binary values, indicating whether or not s source is to be scored in that observation.

The likelihood scoring function for this model is numerically equivalent to Down et al., and is available in full in Section 17.6.3. Briefly, for every observed sequence whose mix vector indicates a source is present, a score matrix is produced for that source and observation. This involves "scanning" the PWM down the length of the observation, offering a score for the entire PWM at each base (in the default reverse-complement operation of BioMotifInference.jl, this is repeated with the reverse complement of the motif, for motif detection on the reverse strand). This feature of likelihood function confers the "multiple uncounted" nature of the motif representation in these models. This means that multiple occurrences of a PWM source motif in a sequence can be detected, and that no particular number of occurrences must be specified. These features make this model particularly well suited to the detection of the repetitive structural signals inferred in Chapter 5. The PWM source score matrices are subsequently "woven" by combination with background scores, an expectation value for motif observation, given the length of the sequence, and a penalty for every source included in the observation, which prevents overfitting.

15.1.7 Position Weight Matrices

Position weight matrices (PWMs) were used to model signals arising from nucleosome contacts with their DNA positions in Chapter 5. For a sequence signal of length λ, the PWM which defines it is a $\lambda \cdot 4$ matrix, with each of the four columns representing the categorical probability weights of A, C, G,

and T[7] base emission at the position represented by row λ [SSGE82]. The PWM is typically used in a frequentist context, without prior distributions over its parameters. However, in a Bayesian context, the PWM is easily interpreted as a λ-base length vector of discrete categorical distributions over base emission frequencies[8]. A computationally useful conjugate prior distribution for discrete categoricals is the Dirichlet distribution [Min00]. Vectors of Dirichlets have been used as the priors for signals in `BMI.jl` models. Because of their conjugacy, Dirichlet priors can theoretically be updated to produce full posterior distributions over the PWM's parameters. `BMI.jl` does not provide for this, for reasons related to the ICA PWM model implementation discussed in Chapter 10. The 4-parameter Dirichlet over the 4-category probability simplex can be usefully thought of as a density cloud in a 3-dimensional tetrahedron (i.e., an ordinary one with 4 vertices). The relative density within the tetrahedron expresses the prior (or posterior) probability of a particular categorical distribution at base λ. The points of the tetrahedron represent categorical distributions with unit probabilities for one of the four bases, while points in the center of the tetrahedron represent categoricals with identical .25 probabilites of emitting each base.

15.1.8 Hidden Markov Models

Hidden Markov Models (HMMs) were used to model *D. rerio* genomic background noise in Chapter 5. An HMM is a state machine; it can be in one of a finite number of states at any given time. Each state is associated with a probability distribution on the model's emission of symbols. The HMM generates a sequence of such symbols by transitioning from state to state (or remaining in the same state, called autotransition), emitting after each transition. The states are "hidden" in the sense that their presence is inferred from the sequence of data upon which the model is trained, and not from direct observations of the states themselves. HMMs are, therefore, well-suited to modelling sequences of outcomes with obscure or irrelevant causal structures. A k-state HMM that emits s symbols is defined by an initial state probability vector of length k, a $k{\cdot}k$ state transition probability matrix, and the outcome probability vectors for each state [Rab89]. HMMs can be used in a variety of contexts, each of which is associated with different algorithmic procedures. For instance, the Viterbi algorithm can be used, given an HMM and a sequence of observations, to infer the most likely state the HMM would have resided in at each position in the sequence, had it been generated by that HMM. An example of this use is the inference of exon and intron features from an HMM trained on annotated CDSes [HSF97]. In this thesis, HMMs are optimized by an Expectation Maximization algorithm, and are subsequently used to score the likelihood of *rys* sibling and mutant nucleosome position sequences, given their genomic context.

15.2 Statistical Methods

15.2.1 Akiake Information Criterion scoring

An Akiake Information Criterion (AIC) score for a model depends on the number of free parameters in the model, k. AIC differences between models can be readily calculated from the residual sum of squares (RSS), which is used as the quality function for measuring model output divergence from observations.

[7] This base ordering is computationally significant, since it allows for trivial reverse complementing by reversing the matrix values across both dimensions.

[8] That is, a vector of nonparametric categorical distributions, themselves on a support of ACGT vector of the 4 nucleotide base emission frequencies. `BMI.jl` encodes the support as the integers 1234 for computational efficiency

For n observations and model parameters θ, $RSS = \sum_{i=1}^{n}(y_i - f(x_i, \theta))^2$. For model comparison, AIC is calculated as:

$mAIC = 2k + n \cdot ln(RSS)$ [BA02]

This is not the "true" value of AIC but assumes that a constant associated with the same data cancels out during model comparison (i.e. by subtraction). AIC is a flawed model comparison criterion. It takes no account of uncertainty in parameters, and tends to be biased toward simple models [Bis06, p.33]. AIC does not fare well for the evaluation of models whose parameters are not well-constrained by the data [Bis06, p.217]. AIC calculations are implemented in Section 17.1.9.

15.2.2 Monte Carlo simulations

Monte Carlo refers to the repeated sampling of a statistical model's output to estimate quantities related to it. Many of the simulations presented in this thesis involve two distinct uses of Monte Carlo techniques. The first use is the exploration of parameter spaces by Markov Chain Monte Carlo (MCMC), with sequential sampling of locations in parameter space, such that the probability of a proposed new location depends only the current position in parameter space (ie. it is a Markovian trajectory). These proposals can be generated by various means; in this thesis, Simultaneous Perturbation Stochastic Approximation, Galilean Monte Carlo, and an ad-hoc permutation routine are used. The second use of Monte Carlo is the estimation of model outcomes distributions, which involves repetitively sampling the same position in parameter space to empirically determine a distribution, against which observations are scored. This is how the majority of the models in the thesis are scored.

15.2.3 Simultaneous Perturbation Stochastic Approximation (SPSA)

SPSA is a model optimization technique. It is a reliable gradient-descent method for finding the local optimum in parameter space, given an objective function and a sample starting position. It has been proven useful in a wide variety of engineering applications [KHI97, ZZLG08], not least because it is tolerant to measurement noise in the objective function. This makes it useful for applications where exact determination of the measurement function's value is not possible; these include on-line intervention into systems with real measurement noise [ZZLG08], as well as the calculation of model likelihoods by Monte Carlo.

The implementation of SPSA in Section 17.1.9 follows the example of Spall exactly [Spa98], extended with the modifications of Sadegh for bounded parameter spaces [Sad97], as suggested by Spall. As noted above, SPSA is a gradient-based method. It moves a single model-particle through parameter space. At each iterate, the local quality gradient is estimated by taking two samples on either "side" of the particle, given by adding and subtracting a perturbation vector to/from the current location. The perturbation vector is composed by random selection of positive and negative magnitudes in each model dimension. This is "Simultaneous Perturbation", the alternative being to perturb dimensions one at a time, which the technique Finite Difference Stochastic Approximation (FDSA) does, instead. SPSA is more accurate and efficient, especially for high-dimensional problems [Chi97]. Given the quality estimates at the two bracketing samples, the model-particle's position is updated so as to move it downslope (in the case of AIC), toward the local optimum. In order for ever-smaller, higher-quality regions of the parameter space to be consistently found, the distance sampled from the particle for the gradient estimation, and for moving the particle itself, must both consistently decline. This is achieved by tuning coefficients,

from which gain sequences are derived. Spall provides useful guidelines on how these may be selected, even automatically [Spa98]. Experience with the SMME models suggests that a degree of hand-tuning is probably usually necessary.

15.2.4 Bayesian treatment of Normal models of unknown mean and variance

As noted in the section on overfitting, the usual MLE method for characterising a variable population with a Normal gaussian (calculating the mean μ and variance σ^2) is susceptible to overfitting. In particular, these methods fail to convey the wide uncertainty that usually obtains on the variance parameter of the Normal model [Mur07]. Normal models figure heavily in the description of variable population-level data. While these models are well-justified both by their ubiquitous success in parameter estimation, and by information theoretic considerations [JBE03], and need not reflect the actual distribution of population measures, we use them in this thesis to model these parameters directly. In the main, this is done using LogNormal models, which are Normal gaussians fitted to the logarithm of the data. This change of variable allows more accurate modelling of population data with long right tails, which is common for the outcomes of growth processes. As Heath notes, long right tails are to be expected for values on ranges that are closed at the 0 minimum and open at the high end [Hea67]. Heath suggests that the Log-Normal distribution should be preferred by default for data of this type. We instead measure the evidence for LogNormal against Normal models for the distributions of our measurements in Section 12.0.1.

A valid Bayesian means of modeling the credibility of mean and variance parameters on Normal gaussian distributions is the use of Normal-Gamma (NG) prior distributions. This assigns normally distributed uncertainty on μ and gamma distributed uncertainty on precision, λ, giving rise to a joint normal-gamma (NG) distribution:

$$p(\mu, \lambda) = NG(\mu, \lambda | \mu_0, \kappa_0, \alpha_0, \beta_0) \stackrel{\text{def}}{=} \mathcal{N}(\mu | \mu_0, (\kappa_0 \lambda)^{-1}) Ga(\lambda | \alpha_0, rate = \beta_0) \text{ [Mur07]}$$

In some cases, we have made use of the more convenient Normal-Inverse-Gamma (NIG) distribution, which takes an inverse gamma distribution over the variance σ^2 rather than a gamma distribution over the inverse of variance, λ:

$$p(\mu, \sigma^2) = NIG(\mu, \sigma^2 | \mu_0, V_0, a_0, b_0) \stackrel{\text{def}}{=} \mathcal{N}(\mu | \mu 0, \sigma^2 V_0) IG(\sigma^2 | a_0, b_0)) \text{ [Mur07]}.$$

These prior distributions have useful properties. One of these is conjugacy; posterior distributions in the same form as the NG or NIG prior may be calculated directly, allowing for sequential inference, although this is not performed in this thesis. Another useful property is that the marginal credibility distribution over the normal model's mean is given by a T distribution, being a weighted sum of an infinite series of Normal distributions with different variances. The marginal distribution of the model mean, calculated from an uninformative NG or NIG prior, is identical to the frequentist Standard Error of the mean [Mur07].

We use the Julia package `ConjugatePriors.jl` for the NG and NIG models. However, this package is not provided with functions for fitting these distributions from uninformative priors, nor for calculating marginal distributions. We provide a series of utility functions for these and related tasks in the `NGRefTools.jl` package, included in Section 17.2.

15.2.5 Empirical Bayes linear regression

Linear regression models are those that model the response of some dependent variable to a series of independent variables, which are assigned weights. The Empirical Bayes method of linear regression used in Chapter 4 allows the estimation of posterior distributions on the weights, and on the noise of observations about the mean, by specifying the maximum likelihood prior hyperparameters, given the data. Determination of prior distributions from observations is what gives the Empirical Bayes family of methods its name [BSB00, p.373]. Normally, priors are fixed without reference to the dataset to be examined.

We used the iterative solution to likelihood maximization provided by the analysis of Bishop [Bis06]. Briefly, this consists of a series of steps in which α distributions on weights and β distributions on the response variable's noise are estimated from the data. This process maximizes the probability of the data y given the α and β distributions, over possible values for weights w:

$p(y|\alpha, \beta) = \int p(y|w, \alpha, \beta)dw$

Bishop's method naturally produces an estimate of the model evidence, logZ, similar to the more general nested sampling, albeit without an estimate of the error in the evidence.

15.2.6 Galilean Monte Carlo

Galilean Monte Carlo (GMC) is an algortihm for the sequential Monte Carlo generation of new sample proposals within a parameter space. While GMC was introduced in 2012 [Ski12], the algorithm used in `GMC_NS.jl`, presented in Chapter 7, is Skillings' 2019 revision [Ski19], which prevents model-particles from leaving the likelihood contour in order to find a reflection, and adds an additional reflection direction.

GMC operates by assigning model-particles (ie. a particular parameterisation of that model at some point in parameter space) a velocity vector, chosen isotropically (ie. uniformly, without preference for any direction). New proposals are generated by moving the particle along the velocity vector. A proposed position's likelihood is calculated, and the proposal is accepted if this value is greater than or equal to the contour. Equality is required to allow particles to search flat likelihood "plateaus", which are prominently seen in the models of `CMZNicheSims.jl`, presented in Chapter 8, due to the discretised (timepoint-bound) nature of the observations.

If a proposal is rejected, its current vector crosses the likelihood contour at a distance less than that of the proposal. If this occurs, the particle reflects off the boundary and carries on down the reflected vector with the same velocity. Unlike MCMC methods that rely on differentiating the likelihood surface, in GMC, nothing about the underlying surface is used to propose positions, so it samples evenly from within the likelihood contour, fulfilling detailed balance [Ski12]. This means that particles are equally likely to transition between positions in the forward process as the reverse process; in GMC the proposal process to move up-vector is identical to the one to move down vector, eg. a particle reflecting normal to the boundary.

When a reflection is being proposed, samples are sequentially taken in three "directions", with the first of these to fall within the likelihood contour used as the reflected model-particle. A schematic summary of this scheme is presented in Figure 15.2. The orientation of the likelihood boundary is inferred by the difference in likelihood between the current particle position and the forward position outside the contour, which is used to estimate a normal vector. The "east" reflection is calculated first,

Figure 15.2: **The four directions of GMC particle movement.**
"North" is forward movement along the trajectory; East, South ,and West are reflected vectors. Excerpted from [Ski19] under CC-BY 4.0.

followed by the "west" reflection, which uses the reversed reflection vector, followed by the "south" reflection, which reverses the particle velocity rather than relying on the boundary normal estimate. In `GMC_NS.jl`'s implementation, each time a forward or reflected model is rejected because it is outside the likelihood contour, this is logged in the particle's PID tuner memory, which reduces the timestep for another round of forward, east, west, and south searches. Eventually, if no new model can be found above the ensemble's minimum timestep, the particle's trajectory is terminated.

15.2.7 Nested sampling

Nested sampling is an algorithmic process for estimating the quality of a model, given observations. It arises from fundamental considerations of imposing a measure on statements about a model of a phenomenon [Ski12]. The most basic such statement is an atomic true/false proposition. A measure is a function that assigns a value to a set. A measure of a model's evidence values a statement or a set of statements about the model, given some observations. A set of statements about the model can be represented as a boolean lattice of the statements and their OR combinations [KS12]. The sum and product rules arise in combining measures assigned to these statements into measures of their sets: ie. the probability of A OR B is $p(A)+p(B)$, while the probability of A AND B is $p(A)\times p(B)$. These operators are required in order to preserve consistent ordering between measures and their combinations. This lattice-oriented view thus gives rise to a unique calculus for measuring the divergence of model output from observations over sets of model-propositions[KS12]. This calculus of divergence leads naturally to the concept of information [Ski12], which can be understood in a nested sampling context as related to the compression ratio of the final posterior sample on the prior[9]. Moreover, it allows the measure of the total quality of the model, given data: Bayesian evidence or logZ. The uniqueness, basic symmetry, and non-arbitrary nature of the axioms underlying nested sampling strongly suggest that this approach is fundamentally superior to other, more ad hoc methods of estimating model quality (eg. AIC). While nested sampling can use any quality function, normally a measure of likelihood is used.

[9]Information, H, is the negative logarithm of the compression ratio. [Ski06]

The most basic nested sampling procedure involves initialising an ensemble of model-particles by sampling evenly from prior distributions on their parameters. Given this initial ensemble, "learning" is performed by compression, where the least likely particle is removed, and replaced by one from inside the likelihood contour established by the least likely particle. Over many such iterates, the ensemble of model-particles is compressed into more and more likely areas of the parameter space. These samples allow us to reconstruct the posterior from their likelihood-weighted parameter locations. More importantly, they allow the estimation of the Bayesian model evidence for the dataset.

Integrating under the quality surface of the parameter space is likely to be an intractable problem for high-dimensional models. However, the dimensionality of the problem can be reduced. If the samples are considered solely in terms of the amount of prior mass enclosed within their likelihood contour, and the likelihood itself, the problem becomes a two-dimensional integration. To do this, the samples taken from the ensemble must be put in order from least to most likely (hence the requirement for consistent ordering in the underlying quality measure, mentioned above). Nested sampling produces this ordering by removing the least likely particle from the ensemble with each iterate. A schematic of the evidence integration is displayed in Figure 15.3.

Figure 15.3: **Schematic of evidence integration by nested sampling.**
L, likelihood. X gives the fraction of prior mass remaining. Excerpted from [Ski06], copyright John Skilling.

Both samplers implemented in this thesis use the common crude approximation of the sequence of enclosed prior mass values, distributed along the X axis in Figure 15.3. In this approximation, the fraction of prior mass enclosed by iterate i in an ensemble of N particles is given as $X_i = exp(-i/N)$ [Ski06]. Very accurate samplers use this approximation to good effect, including MultiNest [FHB09]. Having an estimate of the prior mass enclosed at each iterate allows us to determine how much of the prior lies between two iterates by subtracting the inner contour's enclosed mass from the outer[10]. Doing so estimates the width of the prior between the two iterates. Multiplying the iterate's sampled likelihood by its prior width gives an estimate of the mass of the evidence between these two contours. Summing the logarithm of the masses of these nested shells between iterates estimates the model evidence, logZ. As Skilling notes, this value is not dimensionless, but rather has data units [Ski12]. That is, the logZ value depends on the observations supplied to the likelihood function. However, by taking the evidence

[10] `GMC_NS.jl` and `BMI.jl` both use a trapezoidal approximation of the shell width, rather than simple subtraction, although the difference this makes to the evidence estimates is slight.

ratio between models (by subtracting one evidence value from another), we produce a dimensionless quantity, logZR. This expresses the extent to which the data favour one model over the other (a logZR of 0. would indicate the models are equally credible in light of the data). The uncertainty of logZ values, and hence logZR, are available by using the information, H of the ensemble. This uncertainty is estimated as $\sqrt{H/N}$ for an ensemble of N particles. The uncertainty of a logZR value allows us to express the crediblity range of the logZR estimate as a measure of significance. We calculate standard deviations σ of significance by taking the ratio of the uncertainty over the absolute value of logZR. This suggests the probability that the logZR ratio includes 0., indicating equal credibility of the models, assuming normally distributed errors. For instance, a 3 σ significance on a logZR value indicates a 99.7% probability that the favoured model in fact has greater model evidence than the disfavoured model.

15.2.8 Expectation-Maximization optimization of HMMs

To select background models for *D. rerio* genomic noise in Chapter 5, HMMs must be optimised against samples of genomic sequences. HMMs are initialised from prior distributions on their parameters. The prior on symbol emission is usually uninformative. However, because the genomic features we wish to capture with these models consist of runs of bases with particular nucleotide biases, we supply a prior favouring larger values for the transition matrix diagonal, making autotransition (state residency) more probable. Once models are initialised, new model-proposals are generated by iterating expectation-maximization algorithm steps on the HMM. EM algorithms monotonically produce more probable models with every step [Rab89], without any models being rejected. However, they are generally thought to be susceptible to models getting "stuck" at saddle points and in local minima, although EM algorithms will reliably find global optima for some HMMs, under certain conditions [YBW15]. It is also possible to use Markov Chain Monte Carlo to perform this task, although doing so requires a carefully constructed sampler [Ryd08].

Expectation-maximization algorithms work by iteratively cycling through (E)xpectation and (M)aximization steps. In the E step, the most likely hidden states of the HMM, given the observations, are calculated. In the M step, the model parameters are set to those that make the inferred hidden state sequences most likely. More than one way of calculating these quantities is available. `BioBackgroundModels.jl` implements the classic Baum-Welch (BW) algorithm [BP66], closely following Rabiner [Rab89], with the corrections for errata in multiple observations calculation provided by Rahimi [Rah]. It also implements the linear memory algorithm of Churbanov et al., [CW08].

As the name suggests, the Churbanov algorithm's memory use increases linearly with the number of HMM states, while BW's increases geometrically. This arises from the different way in which the calculations are performed. For O observations of T length, a K-state HMM requires an array of $O \cdot T \cdot K_i \cdot K_j$ to calculate ϵs, which are the probabilities of being in states i and j at subsequent times, given the model and observations. Memory use increases over the length of the observations vector, resulting in large spikes of memory use over the course of the E-M iterte. The relevant code is found in Section 17.5.7, line 26. For large (genome-scale) O and T, memory use on $\geq 6 - K$ HMMs is prohibitive, especially if more than one model is being learnt on the same machine. By contrast, the Churbanov algorithm cumulates the relevant probabilities using two $O \cdot K_i \cdot K_j \cdot K_m$ arrays, representing the marginal probability of all state paths emitting the observed sequence, that pass through an i-j transition at least once, and terminate in state m. These two arrays are the memory commit for observations of any length (i.e. there is no T dimension). The relevant code in found in Section 17.5.9, line 12. Usually,

this represents a trivial memory commitment, and longer observations do not increase it. Theoretically, this algorithm can be used to learn HMMs on sequences of any length, unlike standard BW. Although the linear memory algorithm is theoretically less performant than BW [CW08], our implementation benchmarks faster than BW in most realistic observation sets. BW may be more appropriate for small numbers of short observations, but Churbanov should usually be the first choice for learning background models on large samples of genomic sequence.

Chapter 16

Theoretical Appendix B: Metaphysical Arguments

16.1 Chance is not a valid explanation for biological phenomena; Randomness is a measure of sequence compressibility and not an explanation

Part of this document examines Harris' regular invocation of "stochastic" and related adjectives to describe the behaviour of RPCs. This is what lead me to characterise the theory primarily in those terms- the Stochastic Mitotic Mode Explanation. The significance of this may not be obvious; most scientists assume that they know what words like "stochastic" and "random" mean well enough to use them in rigorous technical publications. We may not be aware that there has been a sprawling debate on the meaning of these terms since the earliest statistical formulations begain to appear in the 19th century. However, even the simplest examples (as current today as they were in Laplace's time) reveal how difficult this topic can be.

If we take the example of the coin flip, a process whose outcome we generally regard as being in some way "stochastic" or due to "chance", we immediately face the question of whether these descriptions refer to our inability to know the outcome of the process, or to properties of the process itself. In other words, if we could specify the mechanics of the coin toss with sufficient precision, could we predict the outcome? This reflects two senses in which we may describe a process as "stochastic": an epistemic sense (it is stochastic because we are unable to predict its outcome *a priori* precisely and so must make resort to stochastic modelling), or an ontological one (we describe the process as stochastic because this, in some way, describes how it *really is* independent of our knowledge of it).

Complicating matters is the sheer number of implications that we tend to associate with "stochasticity" and "randomness". We may be saying something about the causal structure of an event with deep metaphysical implications. It is common to distinguish between "deterministic" and "stochastic" processes, as though "stochastic" literally meant "indeterministic"- something like the Copenhagen interpretation of quantum physics. We may mean something about the apparent disorderliness of a series of outcomes of some process, with mathematical and information theoretical implications. What is an apparently simple observation- cellular fate distribution in RPC lineages is "stochastic", now seems to

require at least a little clarification or interpretation. We may say that stochasticity, for Harris, applies to at least the following entities:

1. The population-level outcomes of the RPC fate specification process (the phenomenon itself)
2. The overall behaviour of the macromolecular system whose operations produces these outcomes (the macromolecular mechanism itself)
3. The particular behaviour of some component of the macromolecular system, eg. stochastic expression of transcription factors (the macromolecules themselves)
4. The statistical generalisations used to characterise relevant aspects of the behaviour of the mechanism or the macromolecules (the model itself)

It is hardly fair to expect the SMME to be advancing a coherent theory about the ontological, objective basis of randomness or probability. Still, this leaves us in the awkward position of not knowing quite what the leading explanation for retinal formation is actually saying about its explanandum. The SMME is thus at risk of circularity- the explanandum (unpredictable variability in clonal outcomes) has as explanans a mechanistic explanation containing an abstract mathematical model tuned to produce this unpredictable variability. This may, in other words, turn out to be a be a case of Jaynes' "mind-projection fallacy" [JBE03, p.506], where a property of a statistical model is taken as a property of nature. Before considering this, we need to define our terms more carefully to avoid the pervasive confusion mentioned above.

16.1.0.1 Chance versus Randomness

A commonplace belief is that randomness refers to outcomes produced by chance events. In an extensive and useful discussion, Antony Eagle reviews the evidence for this Commonplace Thesis, or **(CT)** [Eag18], drawing on discussions in the physics literature. Importantly, he notes that chance and randomness are not identical, and that one can conceivably exist without the other. This, in effect, disproves the (CT)- it is very difficult to imagine how the two concepts can be directly related in this productive fashion. I will attempt a brief summary of Eagle's argument:

Chance is used to refer to processes. Exemplars are coin flips and die rolls. We can think of these as "single-case" probabilities that we take to inhere in the process. For instance, we may say that an evenly weighted coin has a .5 probability of returning a value of heads on a flip, even if it is only flipped once. That is, probabilities can be taken to be objective properties of individual instances of processes, and not only descriptions of the frequencies of the process' outcomes over many repetitions. This is closely related to the logical concept of "possibility". If something is possible, it has a chance of occurring. However, possibility is a logical binary; something is either possible or impossible. A "single-case" probability is understood as something like an objective feature of a system as a whole, given its actual configuration and the relevant natural laws.

Randomness, by contrast, mainly refers to process *outcomes*. That is, randomness is a property of a series of outcomes of some process. In general, we may say that a random sequence of outcomes is one that cannot be generated by an description shorter than the sequence itself. That is, there is no set of rules that can generate a genuinely random sequence from a shorter sequence. In algorithmic information theory, the length of the ruleset required to produce some piece of information (like a

sequence of measured outcomes) is called the Kolmogorov complexity of that object; if the Kolmogorov complexity of the object is equal to the object's length, the object definitionally has the property of algorithmic or Kolmogorov randomness[1].

Eagle produces numerous examples of the dissociability of these concepts, from which I have selected two concise illustrations:

> **Chance Without Randomness**
>
> ...
>
> A fair coin, tossed 1000 times, has a positive chance of landing heads more than 700 times. But any outcome sequence of 1000 tosses which contains more than 700 heads will be compressible (long runs of heads are common enough to be exploited by an efficient coding algorithm, and 1000 outcomes is long enough to swamp the constants involved in defining the universal prefix-free Kolmogorov complexity). So any such outcome sequence will not be random, even though it quite easily could come about by chance.
>
> ...
>
> **Randomness Without Chance**
>
> ...
>
> Open or dissipative systems, those which are not confined to a state space region of constant energy, are one much studied class [of the objects of deterministic classical physics- notably, biological systems are dissipative], because such systems are paradigms of chaotic systems ... the behaviour of a chaotic system will be intuitively random ... [t]he sensitive dependence on initial conditions means that, no matter how accurate our finite discrimination of the initial state of a given chaotic system is, there will exist states indiscriminable from the initial state (and so consistent with our knowledge of the initial state), but which would diverge arbitrarily far from the actual evolution of the system. No matter, then, how well we know the initial condition (as long as we do not have infinite powers of discrimination)[2], there is another state the system could be in for all we know that will evolve to a discriminably different future condition. Since this divergence happens relatively quickly, the system is unable to be predicted ... Just as before, the classical physical theory underlying the dynamics of these chaotic systems is one in which probability does not feature. [Eag18]

Therefore, the (CT) is untenable. Processes are "chancy"; collections of process outcomes, "trials", or instantiations are "random". It is tempting to say that Harris is explaining random fate outcomes with descriptions of chancy processes occuring internally to RPCs. Let us examine whether this is plausible.

16.1.0.2 Chance in molecular mechanisms

What might it mean to describe the behaviour of a biological macromolecular system as "chancy"? Let us again distinguish between the ontological and epistemic dimensions of this description. There is a sense in which the operation of a macromolecular mechanism could be said to be objectively chancy, and one in which the "chancy" outcome reflects our ignorance of some source of variability in the process.

Eagle proffers two common lines of argument in favour of chanciness as an objective property of processes. The first is the notion of the "single-case" probability mentioned above. The examples given

[1] The Kolmogorov complexity of a sequence can be estimated, contrary to common belief [LV08]. Minimum Message Length expresses a similar concept [WD99]. More prosaically, one may simply compress a serialized representation of the sequence on one's hard drive with a reasonably good compression algorithm; the degree of compression achieved is a good indicator of the degree of non-random order available to shorten the sequence's description.

[2] Note that this condition defines chaotic randomness as an epistemic, rather than ontological, feature of complex systems- a being with infinite powers of discrimination *could* predict the evolution of a complex classical system with perfect accuracy.

are single coin flips, and the decay of single radioactive atoms, which are commonly taken to have chancy outcomes irrespective of anyone's beliefs about them. As Eagle notes, this is closely related to frequentist ideas about stable processes, or trials:

> It is the stable trial principle that has the closest connection with single-case chance, however. For in requiring that duplicate trials should receive the same chances, it is natural to take the chance to be grounded in the properties of that trial, plus the laws of nature. It is quite conceivable that the same laws could obtain even if that kind of trial has only one instance, and the very same chances should be assigned in that situation. But then there are well-defined chances even though that type of event occurs only once.
>
> ...
>
> The upshot of this discussion is that chance is a *process* notion, rather than being entirely determined by features of the outcome to which the surface grammar of chance ascriptions assigns the chance. For if there can be a single-case chance of $\frac{1}{2}$ for a coin to land heads on a toss even if there is only one actual toss, and it lands tails, then surely the chance cannot be fixed by properties of the outcome 'lands heads', as that outcome does not exist. The chance must rather be grounded in features of the process that can produce the outcome: the coin-tossing trial, including the mass distribution of the coin and the details of how it is tossed, in this case, plus the background conditions and laws that govern the trial. Whether or not an event happens by chance is a feature of the process that produced it, not the event itself. The fact that a coin lands heads does not fix that the coin landed heads by chance, because if it was simply placed heads up, as opposed to tossed in a normal fashion, we have the same outcome not by chance. Sometimes features of the outcome event cannot be readily separated from the features of its causes that characterise the process by means of which it was produced.
>
> [Eag18]

To address this argument, we may pose the question: if we knew enough about the mechanics of the toss, could we predict its outcome? The answer is yes, we can- the Bell Labs statistician Persi Diaconis has built a coin tossing machine that reliably produces heads or tails [Kes04]. We know that tightly controlling the mechanics of a coin toss allows us to treat this system as entirely deterministic, without any significant element of chance in the outcome. A coin toss is only chancy when the human doing it does not have full control over the mechanical parameters of the process. Conceptually, there is no *a priori* reason why a coin-tosser should not be able to regularise the angular momentum of their thumb-flick by training with a strain gauge, place the coin on a stable surface in a wind-free box allowing flicking, and achieve the same effect as the coin-tossing machine. In this case, Eagle's suggestion that "[s]ometimes features of the outcome event cannot be readily separated from the features of its causes that characterise the process" seems obviously wrong- the "chancy" element of coin tossing is fully separable from the rest of the coin tossing process, and replaceable with a non-chancy component.

If the foregoing argument is correct, it seems that the coin toss is an example of the epistemic, rather than ontological, dimension of chance. The process appears to be chancy, or random, because the human tossing the coin is not able to precisely control the mechanical parameters of the process. Indeed, as Diaconis notes, these epistemically-limited tossers do not actually produce unbiased random outcomes- human coin tosses come up as they were started slightly more often than with the obverse face [DHM07].

Eagle's second example of an "objective single-case chance", is the decay of a radioactive atom. There is no known physical process whose parameters are thought to define the lifetime of individual radioactive atoms, in the way that there is a well-specified physical process that produces a particular coin toss outcome. This is an appeal to the Copenhagen theoretical principle that it is *a priori* impossible

to predict the lifetimes of individual atoms. As appeals to quantum theory to ground "objective chance" in biological processes are becoming more common, let us consider whether a quantum theoretical explanation might plausibly underpin the "stochasticity" of RPC fate specification.

16.1.0.3 Can macromolecular chanciness be rooted in quantum indeterminacy?

Eagle suggests that, because the Copenhagen interpretation of quantum physics has wide currency among physicists, the theory's implied indeterminacy of physical phenomena at the quantum level could ground "objective chance". While common, this argument downplays the fact that quantum theory is not a homogenous scientific tradition. The conventional history of mid 20th-century quantum theory holds that John Stewart Bell, in the demonstration of his famous inequality, conclusively proved that deterministic (so-called "hidden variable") theories of quantum mechanics were incorrect. As demonstrated in another section of the publication Eagle's argument appears in, this is an incomplete view. Bohmian theorist Sheldon Goldstein shows that Bell was an advocate of deterministic Bohmian mechanical theory, and thought his famous inequality demonstrated that quantum phenomena could not be *local*, not that they could not be *deterministic* [Gol17].

Indeed, Bohmian quantum mechanics are fully deterministic, describe all of the same phenomena as Copenhagen, and in several cases resolve problems that orthodox quantum theory cannot [Gol17]. We are not, therefore, facing unanimous expert consensus that there is objective chance at the quantum level. We rather have a situation where physical phenomena are described by two different theory-sets, one of which takes its statistical generalisations to be descriptions of ontological indeterminacy (Copenhagen), and the other to reflect epistemic uncertainty about a determinate universe (Bohm). Moreover, there is no obvious reason to prefer the Copenhagen approach, given that the Bohmian theory explains Copenhagen's paradoxical results "without further ado" [Gol17], deals with empirically verified phenomena of physical and biological interest that Copenhagen does not (eg. electron tunneling), and was the preferred approach of Bell himself.

Therefore, Eagle's premise fails. There is no reason to suppose that quantum theoretical models that posit objective chance are good evidence for the *reality* of objective chance. Moreover, there are good reasons to suppose that the converse is true. In sum, then, we must reject Harris' argument to the extent it argues for *objective, ontological* chance, since the arguments for the existence of both single-case objective chance or quantum chance are weak and biologically irrelevant. Clearly, however, the *epistemic* dimension of chance is in play here.

16.1.0.4 Randomness in RPC fate specification

Having dealt with how the concept of chance might apply to Harris' SMME above, let us consider how the term "random" might relate to the process of RPC fate specification and differentiation. As introduced earlier, the technical meaning of "randomness" pertains to sequences of process outcomes. The process outcomes Harris is concerned with are the temporally-arranged fate outcomes of some particular RPC lineage. Therefore, we can ask whether these sequences are likely to be algorithmically random.

The structure of the SMME models precludes algorithmic randomness. That is, the sequence of outcomes has a structure that can be meaningfully compressed by rules which produce typical RPC fate outcomes (Harris' mathematical models are such rule sets). One might object that Harris' meaning is that the particular rules which give rise to cellular fate "choice" in his models involve random number generation. In this case, the claim is trivially about the model and not about the sequence of outcomes

that is actually observed in zebrafish eyes. Indeed, all of Harris' later models assume a tripartite temporal structure to the differentiation process[3]. Therefore, the output of the SMME models themselves demonstrate that RPC fate specification is not an algorithmically random process. One could quantify the Kolmogorov complexity of short lineage outcome sequences in order to ask *how* random the sequence is [SZDG14], but this would not establish randomness as an explanation for variability, but rather would use randomness to describe it.

We should further note that the question of how predictably ordered, i.e. non-random processes like fate specification arise in biological systems is a fundamental question of the biological sciences. In many ways, then, it is the extent to which variable sequences of outcomes like RPC fate specification *depart* from algorithmic randomness which of interest when we are asking questions like "how does the ordered structure of a retina arise from RPC activity?"

16.1.0.5 Summary: "Stochastic" or "variable"?

Above, I argue that the RPC fate specification process is not objectively chancy (since objective chance is an empirically unsupported concept), nor algorithmically random (since the sequence of RPC lineage outcomes is structured and therefore non-random). What, then, should we make of the argument that this process is "stochastic"? Let us consider the forceful argument of the great Bayesian statistician Edwin Thompson Jaynes:

> "Belief in the existence of 'stochastic processes' in the real world; i.e. that the property of being 'stochastic' rather than 'deterministic' is a real physical property of a process, that exists independently of human information, is [an] example of the mind projection fallacy: attributing one's own ignorance to Nature instead. The current literature of probability theory is full of claims to the effect that a 'Gaussian random process' is fully determined by its first and second moments[4]. If it were made clear that this is only the defining property for an abstract mathematical model, there could be no objection to this; but it is always presented in verbiage that implies that one is describing an objectively true property of a real physical process. To one who believes such a thing literally, there could be no motivation to investigate the causes more deeply than noting the first and second moments, and so the real processes at work might never be discovered. This is not only irrational because one is throwing away the very information that is essential to understand the physical process; if carried into practice it can have disastrous consequences. Indeed, there is no such thing as a 'stochastic process' in the sense that the individual events have no specific causes." [JBE03]

It is, thus, important to emphasize that the utility of statistical modelling techniques should not be taken to suggest that the modelled phenomenon is actually, i.e. irreducibly, random and without causal structure. When speaking of biological "randomness" or "stochasticity", biologists rarely precisely define what is meant by these terms. This vagueness sometimes arises from, or results in, a theoretical deficit where properties of statistical models are understood to directly reflect the system being modelled; the scientist has failed to heed Korbzysky's dictum insisting that "a map *is not* the territory it represents, but, if correct, it has a *similar structure* to the territory, which accounts for its usefulness" [Kor05] (italics in original). The "structural similarity" here is between the model's outcomes and the collection of actually-observed population outcomes, *not* the underlying biological process giving rise to measured outcomes. I conclude by suggesting that this particular example applies to all such explanations in

[3]That is, an early bias in RGC production is produced in these models by the a priori commitment to a "rule" which results in early RGC production.

[4]That is, the mean and the variance of the Normal Gaussian model are said to determine the process being modeled.

the biological sciences. There is presently no good reason to accept scientific explanations rooted in chance. As for explanations rooted in randomness, to be credible, these must actually estimate the degree of randomness present in the relevant outcomes, show how it produces the outcomes, and explain departures from pure incompressibility.

16.2 Macromolecular mechanistic explanations in the Systems era

Figure 16.1: **Cellular systems model-construction.**
Excerpted from [Fag15, p.7], under CC-BY-NC-ND 4.0. The system of equations and subsequent steps are based on a 2-element wiring diagram.

The data chapters of this thesis are concerned with two issues in scientific practice: the comparison of models with different structures, and their interpretation. I began my career in stem cell biology with a background in molecular pharmacology, and the explanatory worldview that training inscribed. For me, biological explanation was about elucidating mechanisms, which consisted of descriptions of macromolecules and their accessory small molecule messengers interacting. ”Models,” in the sense of numerical simulations susceptible to formal statistical testing, were only the formal encoding of a body of knowledge that was first proved out at the bench, in interventional experiments. That these results could be encoded by a system of differential equations only confirmed the rigour of the original analysis, which composed the mechanistic explanation in the first place. That engineers should bring their numerical techniques into our laboratories, only to have these analyses confirm the plain-sense interpretations of the benchworkers producing the data, served to reinforce the sense that the pharmacological approach was fundamentally the correct one. It hardly ever entered our minds that model comparison was not occurring at all; it seemed that null hypotheses had already been slain by the flurry of t-tests and ANOVAe applied to the underlying datasets before the eggheads showed up.

That there were problems with this approach was already apparent by the beginning of my graduate education; Faculty of Medicine graduate pharmacology seminars in the early 2000s routinely included dark warnings about the collapse of antidepressant effect sizes over time, and the need for new statistical approaches. Reports that most biomedical research results are not replicable were met with a sort of palpable relief [Ioa05], if not much practical change; at least the growing welter of conflicting results could be explained this way, rather than risking interpretative collapse. Ultimately, the power of the macromolecular mechanism in pharmacology proved overstated; the productivity lull of the 2000-2010 era has been overcome not by rational mechanistic design of traditional small molecules, but by an influx of new classes of drugs, often with unknown or poorly characterised mechanisms of action [Mun19]. The arrival of new statistics has not helped matters much; serious Bayesian analyses tend to doubt whether medical pharmacological

interventions are effective at all in most cases [Ste18]. Still, by the time I had left for greener pastures with the stem cell biologists at the new Department of Cell and Systems Biology, it was clear that the pharmacological view of biological explanation had won significant discursive battles.

A fascinating artefact of this discursive victory is present in the best available study on the use of mechanistic explanations in stem cell biology, Melinda Fagan's "Philosophy of stem cell biology: knowledge in flesh and blood". In concluding chapters outlining the future of stem cell biology, Fagan provides a diagrammatic summary of what she takes to be the field's consensus on its general direction into its future "Systems Biology" incarnation, which I reproduce here in Figure 16.1.

The perspicacious reader will observe that this is nothing other than the pharmacological view I outline above: the practice of molecular biology is the elucidation of mechanisms, the practice of systems biology is the formalization of these mechanisms as systems of differential equations. What was most peculiar about the presence of this view is that it makes no account of the Simple Stochastic Model, which receives significant coverage elsewhere in Fagan's book. This was particularly important to me because I was trying to understand how to use Harris' models to explain my results. What Harris was doing was clearly intended to be both a mechanistic explanation, in the sense that it eventually nominated a pair of particular transcription factors as the causative agents, and also a Systems explanation, in the sense that it relied, for persuasive effect, on complex numerical modelling techniques. It was formulated in SSM terms, however, not as a SODE, and critically was about the metaphysical interpretation of a mathematical construct, not about the joint action of the named macromolecules, as Fagan suggests we should understand mechanistic explanations [Fag13][5].

Moreover, much of the sophisticated literature on mathematical models of stem cell variability are based on interpretations of dynamical systems theory or chaos theory [FK12, HLZQ17] that deal with the macromolecular constituents of cells as bulk expression values; that is, a protein may be represented in one of these models as a coordinate on an dimension representing the amount of the protein expressed. It is extraordinarily rare for these models to be very concerned about crystal structures, phosphorylation states, or any of the other mechanistic details the pharmacological view is replete with. Other approaches rely on control systems theory [SK15, YSK15], others happily use technical information-theoretic terms like "noise" without any effort to define or measure it [CHB+08], still others treat the expression of macromolecules as the endpoints of tissue-mechanical forces [PLM+17], and so on. The actual variety of "systems" explanatory strategies found "in the field" is bewildering; to become proficient in even one is a years-long project. As I began to grasp the sheer number of technical and theoretical subdomains implicated in these explanations, the full force of Nicholas Reschers' argument on this point became apparent:

> The ramifications and implications of philosophical contentions do not respect a discipline's taxonomic boundaries. And we all too easily risk losing sight of this interconnectedness when we pursue the technicalities of a narrow subdomain. In actuality, the stance we take on questions in one domain will generally have substantial implications and ramification for very different issues in other, seeming unrelated domains. And this is exactly why systematization is so important in philosophy - because the way we *do* answer some questions will have limiting repercussions for the way we *can* answer others. We cannot emplace our philosophical convictions into conveniently delineated compartments in the comfortable expectation that what we maintain in one area of the field will have no unwelcome implications for what we

[5] Arguably, the problem here is with the attempt to reify SODE models of macromolecular mechanisms as the standard form for these explanations; however, many relevant explanations, including Harris', and those developed in this thesis, do not descend to the "basement level" of macromolecular interaction at all.

are inclined to maintain in others. [Res05, p.97]

Indeed, the sudden injection of the philosophical contents of the "complexity sciences" into biological discourse felt like an invasion; suddenly, it was not very clear what a mechanism or a biological explanation might consist of after all. Michel Morange has argued persuasively that molecular biology is essentially mature [Mor03] and that "systems" explanations consist mainly in the complementation of ordinary molecular practice with sophisticated mathematical models [Mor08], while acknowledging the increasing space available for the entry of physical explanations in molecular biology [Mor11]. This account is appealing, but it belies the chaotic nature of the recent scientific scene, and cannot make sense of why there are so many contending, often quite incompatible, views on how to explain cellular and molecular biological phenomena, and why so few of those views include any idea on *how those explanations might be tested against one another*.

16.2.1 The Feyerabendian modeller

Chapter 2 cites Paul Feyerabend in establishing that scientific theories exert their cognitive effects on us by making metaphysical claims about reality. This point informs the model-analysis in that chapter, showing how the models of the Stochastic Mitotic Mode Explanation (SMME) become progressively more abstract and restricted to the mitotic mode concept. Mitotic mode is not a physical existent, but rather a conceptual compound of the fate of multiple cells- it is in this sense plainly meta-physical. Feyerabend has a number of essential insights on scientific metaphysics, which, when judiciously understood, allow us to make sense of what is happening within stem cell biology and more broadly, in the "systems biology" era. The central, seminal insight of Feyerabend's *Against Method* is that the observed historical succession of scientific theories occurs by counterpositional advancement of incompatible opposing theories. Only counterinductive comparisons *between* theories are capable of showing up their implicit assumptions and allowing them to be challenged. Feyerabend explains:

> ... it emerges that the evidence that might refute a theory can often be unearthed only with the help of an incompatible alternative: the advice (which goes back to Newton and which is still very popular today) to use alternatives only when refutations have already discredited the orthodox theory puts the cart before the horse. Also, some of the most important formal properties of a theory are found by contrast, and not by analysis. A scientist who wishes to maximize the empirical content of the views he holds and who wants to understand them as clearly as he possibly can must therefore introduce other views; that is, he must adopt a *pluralistic methodology*. He must compare ideas with other ideas rather than with 'experience' and he must try to improve rather than discard the views that have failed in the competition. [Fey93, p.20]

Feyerabend is interested in debunking the notion that science differs from other discursive social practices by showing that no universal method vouchsafes the progressive replacement of earlier, inferior theories with later, improved ones. He famously establishes that Galileo used a form of metaphysical propaganda, particularly in the ad hoc invocation of the now-discredited concept of circular inertia, to establish the credibility of the Copernican model against its orthodox Aristotlean competitor, championed by the Catholic Church [Fey93]. Although we commonly think of the Copernican Revolution as the replacement of an obviously defective theological explanation by a properly formed theory from the empirical sciences, Feyerabend shows that this conceals the actual means by which Galileo makes his persuasive case for the (itself badly defective) Copernican model. Galileo's theory substantially contradicted the available evidence, erroneously asserted the reliability of some telescopic observations and

discredited others[6], made extensive use of ad hoc hypotheses, and was advanced by propagandistic and even dishonest means. Much of this was unavoidable. Ad hoc hypotheses are necessary for new theories, because the auxiliary sciences associated with them have not been developed. Scientific development is intrisically uneven, obligating the use of these makeshift theoretical devices. Without a certain level of rhetorical sleight of hand on Galileo's part, the Copernican program would have succumbed to the greater development and argumentative weight of the Scholastic tradition. As Feyerabend notes, if any of the typically suggested criteria for a universal scientific method were applied, the Church would have won the debate, and we might still have an Aristotlean cosmology.

Because of his concern to attack what he sees as the overweening social influence of scientific and technical experts, Feyerabend puts emphasis on the discursive process of establishing orthodoxy within a field, and the commonalities between scientific and nonscientific domains [Fey75, Fey81, Fey93]. That is, the practice of the natural sciences are susceptible to the same irreducibly subjective and historically contingent forces as other domains of human culture. In this sense there is nothing special about scientific practice that shields its conclusions from error introduced by these means. These extra-scientific forces have significant effects on the forms scientific models take and the interpretations they are given, and it helps to remember this when reading modelling papers mainly published to keep the lights on. Still, I suggest that it is important not to interpret these arguments too pessimistically. Feyerabend genuinely wanted to promote what he called "Open Exchange" between different scientific and metaphysical traditions, centered around a noncoercive exchange of, in effect, models and standards for judging them [Fey93]. He also may not have taken heed of the unique aspects of the natural sciences; statistical descriptions of collections of outcomes and of uncertainty, and a self-reinforcing dynamic of scaffolding methodological complexity, are both unique to the modern natural sciences.

Ultimately, the value in Feyerabend's perspective is seeing that scientists routinely operate as epistemological anarchists, finding what works in their local institutional surroundings, and that we must expect that this will be the case. In other words, Fagan's (extremely carefully argued) conceptualization of the orthodox molecular mechanism, as found in stem cell biology, is useless for interpreting Harris' theories. This is so because Harris does not care about adhering to this standard of orthodoxy. The field has, in typical anarchic fashion, begun adapting models and numerical techniques from a panoply of engineering, physics, statistical mechanics, AI, etc. subdisciplines, more or less willy nilly, and often without any sort of analysis of the adequacy of the model relative to alternatives. Moreover, these models pertain to a variety of levels of biological organisation, from the tissue down to subcellular nuclear compartments, and frequently do not refer in any concrete way to biological macromolecules.

I argue that, from this point of view, the macromolecular mechanistic explanation Fagan reifies is, in fact, already dead. The era of painstaking, effector-by-effector construction of macromolecular pathways, to be virtually enshrined in their ultimate SODE incarnation, perhaps ultimately to be assembled into some Monodian model cell-as-cybernetic-factory, is over, if it ever existed. This is not to say that Morange is wrong- as he asserts, molecular biology is not dead, it has not been replaced by "systems biology". In practice, however, the traditional form of molecular biological explanation is rapidly being replaced by explanatory forms from other fields, and it is not always clear that this has been salutory. For Feyerabend, science, like other human social practices consists of a variety of interacting traditions with differing assumptions, methods, sensory interpretations, and so on. The development of science is

[6]Galileo asserted that comets are optical illusions, for instance, since their non-circular orbits disconfirm the Copernican system, which insists on the circularity of orbital motion.

thus "not the interaction of a practice with something different and external, *but the development of one tradition under the impact of others.*" [Fey93, p.232]. The concept of "systems biology" is fundamentally an artefact of the impact of advanced statistics and statistical mechanics on biology. The resolution to the question of how this impact is mediated will determine the institutional and intellectual landscape that results from it.

To have any agency in this process, molecular biologists must be able to assess the theoretical contents that are being imported into our field for ourselves. If we do not, we cannot assess the impact of the metaphysical commitments of these theories on the rest of our theorising, as Rescher persuasively insists that we must. Indeed, it was Feyerabend's perspective that led me to realise that there were at least three scientific orthodoxies hindering me from understanding the metaphysical contents of Harris' theories. These were the mechanism-to-SODE pipeline instilled by my pharmacological training and recapitulated by Fagan, the unsupported insistence on the actual reality of chanciness by Copenhagen theorists, and the inadequate statistical approaches recommended by frequentist statisticians. By realising that any conceptual element could appear in a mechanistic explanation, by rejecting the objective reality of chance, and embracing the original formulation of statistical orthodoxy suggested by LaPlace (that is, Bayesian statistics), I was able to counter-inductively discover the fundamental problems with the SMME. In continuing to test models with fundamentally different structures, I attempt to improve older theories about linear stages of RPC activity in order to counterinductively weigh these against alternatives.

Moreover, by the conscious adoption of Bayesian methods from cosmology to address local problems in stem cell biology, I attempt to model Feyerabend's Open Exchange between two scientific traditions. My selection of Skilling's nested sampling system of inference is driven by my appreciation for its logical consistency, simplicity, deep roots in fundamental logic and information theory, and broad applicability to a wide range of problems. While Feyerabend is no doubt correct that, ultimately, the selection of criteria to distinguish between scientific theories must itself be subjective, the deep comportment of Skilling's system with the fundamental human drive toward cognitive harmony [Res05] compels me to suggest that it may serve as a near-universal yardstick of the adequacy of scientific models with respect to their underlying datasets in the not-too-distant future.

Chapter 17

Code Appendix

17.1 SMME

Github repository: https://github.com/mmattocks/SMME

17.1.1 setup_project.py

```
"""Copyright (c) 2005-2017, University of Oxford.
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University of Oxford means the Chancellor, Masters and Scholars of the
University of Oxford, having an administrative office at Wellington
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```
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HOWEVER CAUSED AND ON ANY THEORY OF LIABILITY, WHETHER IN CONTRACT, STRICT
LIABILITY, OR TORT (INCLUDING NEGLIGENCE OR OTHERWISE) ARISING IN ANY WAY OUT
OF THE USE OF THIS SOFTWARE, EVEN IF ADVISED OF THE POSSIBILITY OF SUCH DAMAGE.
"""

import os


def find_and_replace(filename, old_string, new_string):
    # Read the file
    f = open(filename, 'r')
    file_contents = f.read()
    f.close()

    # Write to the file
    f = open(filename, 'w')
    f.write(file_contents.replace(old_string, new_string))
    f.close()

    return


def ask_for_response(question):
    # Display the question
    print(question)

    # Define permitted yes/no answers
    yes = {'yes', 'y', 'ye', ''}
    no = {'no', 'n'}
```

```
57
58      # Take the lower case raw input
59      choice = raw_input().lower()
60
61      # Decide on the choice
62      if choice in yes:
63          return True
64      elif choice in no:
65          return False
66      else:
67          ask_for_response("Please respond with yes or no:")
68
69
70  def main():
71      # The absolute path to the project directory
72      path_to_project = os.path.dirname(os.path.realpath(__file__))
73
74      # Identify the name of the project
75      project_name = os.path.basename(path_to_project)
76
77      # Paths to the CMakeLists.txt files
78      base_cmakelists = os.path.join(path_to_project, 'CMakeLists.txt')
79      apps_cmakelists = os.path.join(path_to_project, 'apps',
        ↪  'CMakeLists.txt')
80      test_cmakelists = os.path.join(path_to_project, 'test',
        ↪  'CMakeLists.txt')
81
82      # Perform the find-and-replace tasks to update the template project
83      find_and_replace(base_cmakelists,
        ↪  'chaste_do_project(template_project', 'chaste_do_project(' +
        ↪  project_name)
84      find_and_replace(apps_cmakelists,
        ↪  'chaste_do_apps_project(template_project',
        ↪  'chaste_do_apps_project(' + project_name)
85      find_and_replace(test_cmakelists,
        ↪  'chaste_do_test_project(template_project',
        ↪  'chaste_do_test_project(' + project_name)
86
87      # Amend the components
88      components_list = []
89
90      if ask_for_response("Does this project depend on the cell_based
        ↪  component? [Y/n] "):
```

```
        components_list.append('cell_based')

    if ask_for_response("Does this project depend on the crypt component? [Y/n] "):
        components_list.append('crypt')

    if ask_for_response("Does this project depend on the heart component? [Y/n] "):
        components_list.append('heart')

    if ask_for_response("Does this project depend on the lung component? [Y/n] "):
        components_list.append('lung')

    # If the list is non-empty, replace the default components
    if components_list:
        components_string = ' '.join(components_list)
        default_components = 'continuum_mechanics global io linalg mesh ode pde'

        find_and_replace(base_cmakelists, default_components, components_string)


if __name__ == "__main__":
    main()
```

17.1.2 /apps/src/BoijeSimulator.cpp

```
#include <iostream>
#include <string>

#include <cxxtest/TestSuite.h>
#include "ExecutableSupport.hpp"
#include "Exception.hpp"
#include "PetscTools.hpp"
#include "PetscException.hpp"

#include "BoijeCellCycleModel.hpp"
#include "OffLatticeSimulationPropertyStop.hpp"

#include "AbstractCellBasedTestSuite.hpp"
```

```
14
15 #include "WildTypeCellMutationState.hpp"
16 #include "TransitCellProliferativeType.hpp"
17 #include "DifferentiatedCellProliferativeType.hpp"
18 #include "BoijeRetinalNeuralFates.hpp"
19
20 #include "CellsGenerator.hpp"
21 #include "HoneycombMeshGenerator.hpp"
22 #include "NodesOnlyMesh.hpp"
23 #include "NodeBasedCellPopulation.hpp"
24 #include "VertexBasedCellPopulation.hpp"
25
26 #include "ColumnDataWriter.hpp"
27
28 int main(int argc, char *argv[])
29 {
30     ExecutableSupport::StartupWithoutShowingCopyright(&argc, &argv);
31     //main() returns code indicating sim run success or failure mode
32     int exit_code = ExecutableSupport::EXIT_OK;
33
34     if (argc != 13)
35     {
36         ExecutableSupport::PrintError(
37                 "Wrong arguments for simulator.\nUsage (replace<> with
                 ↪ values, pass bools as 0 or 1):\n BoijeSimulator
                 ↪ <directoryString> <filenameString>
                 ↪ <outputModeUnsigned(0=counts,1=events,2=sequence)>
                 ↪ <debugOutputBool> <startSeedUnsigned>
                 ↪ <endSeedUnsigned> <endGenerationUnsigned>
                 ↪ <phase2GenerationUnsigned> <phase3GenerationUnsigned>
                 ↪ <pAtoh7Double(0-1)> <pPtf1aDouble(0-1)>
                 ↪ <pngDouble(0-1)>",
38                 true);
39         exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
40         return exit_code;
41     }
42
43     /***********************
44      * SIMULATOR PARAMETERS
45      ***********************/
46     std::string directoryString, filenameString;
47     int outputMode; //0 = counts; 1 = mitotic mode events; 2 = mitotic
       ↪ mode sequence sampling
```

```
    bool debugOutput;
    unsigned startSeed, endSeed, endGeneration, phase2Generation, phase3Generation;
    double pAtoh7, pPtf1a, png; //stochastic model parameters

    //PARSE ARGUMENTS
    directoryString = argv[1];
    filenameString = argv[2];
    outputMode = std::stoi(argv[3]);
    debugOutput = std::stoul(argv[4]);
    startSeed = std::stoul(argv[5]);
    endSeed = std::stoul(argv[6]);
    endGeneration = std::stoul(argv[7]);
    phase2Generation = std::stoul(argv[8]);
    phase3Generation = std::stoul(argv[9]);
    pAtoh7 = std::stod(argv[10]);
    pPtf1a = std::stod(argv[11]);
    png = std::stod(argv[12]);

    /************************
     * PARAMETER/ARGUMENT SANITY CHECK
     ************************/
    bool sane = 1;

    if (outputMode != 0 && outputMode != 1 && outputMode != 2)
    {
        ExecutableSupport::PrintError(
                "Bad outputMode (argument 3). Must be 0 (counts) 1 (mitotic events) or 2 (sequence sampling)");
        sane = 0;
    }

    if (endSeed < startSeed)
    {
        ExecutableSupport::PrintError("Bad start & end seeds (arguments, 5, 6). endSeed must not be < startSeed");
        sane = 0;
    }

    if (endGeneration <= 0)
    {
        ExecutableSupport::PrintError("Bad endGeneration (argument 7). endGeneration must be > 0");
```

```
        sane = 0;
    }

    if (phase3Generation < phase2Generation)
    {
        ExecutableSupport::PrintError(
                "Bad phase2Generation or phase3Generation (arguments 8, 9). phase3Generation must be > phase2Generation. Both must be >0");
        sane = 0;
    }

    if (pAtoh7 < 0 || pAtoh7 > 1)
    {
        ExecutableSupport::PrintError("Bad pAtoh7 (argument 10). Must be 0-1");
        sane = 0;
    }

    if (pPtf1a < 0 || pPtf1a > 1)
    {
        ExecutableSupport::PrintError("Bad pPtf1a (argument 11). Must be 0-1");
        sane = 0;
    }

    if (png < 0 || png > 1)
    {
        ExecutableSupport::PrintError("Bad png (argument 12). Must be 0-1");
        sane = 0;
    }

    if (sane == 0)
    {
        ExecutableSupport::PrintError("Exiting with bad arguments. See errors for details");
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /************************
     * SIMULATOR OUTPUT SETUP
```

```
124      ************************/
125
126 //Set up singleton LogFile
127     LogFile* p_log = LogFile::Instance();
128     p_log->Set(0, directoryString, filenameString);
129
130     ExecutableSupport::Print("Simulator writing file " + filenameString +
        ↪  " to directory " + directoryString);
131
132 //Log entry counter
133     unsigned entry_number = 1;
134
135 //Write appropriate headers to log
136     if (outputMode == 0) *p_log << "Entry\tSeed\tCount\n";
137     if (outputMode == 1) *p_log << "Time (hpf)\tSeed\tCellID\tMitotic
        ↪  Mode (0=PP;1=PD;2=DD)\n";
138     if (outputMode == 2) *p_log << "Entry\tSeed\tSequence\n";
139
140 //Instance RNG
141     RandomNumberGenerator* p_RNG = RandomNumberGenerator::Instance();
142
143 //Initialise pointers to relevant singleton ProliferativeTypes and
    ↪  Properties
144     MAKE_PTR(WildTypeCellMutationState, p_state);
145     MAKE_PTR(TransitCellProliferativeType, p_Mitotic);
146     MAKE_PTR(DifferentiatedCellProliferativeType, p_PostMitotic);
147     MAKE_PTR(RetinalGanglion, p_RGC_fate);
148     MAKE_PTR(AmacrineHorizontal, p_AC_HC_fate);
149     MAKE_PTR(ReceptorBipolar, p_PR_BC_fate);
150     MAKE_PTR(CellLabel, p_label);
151
152     /************************
153      * SIMULATOR SETUP & RUN
154      ************************/
155
156 //iterate through supplied seed range, executing one simulation per seed
157     for (unsigned seed = startSeed; seed <= endSeed; seed++)
158     {
159         if (outputMode == 2) *p_log << entry_number << "\t" << seed <<
            ↪  "\t"; //write seed to log - sequence written by
            ↪  cellcyclemodel objects
160
161         //initialise pointer to debugWriter
```

```
        ColumnDataWriter* debugWriter;

        //initialise SimulationTime (permits cellcyclemodel setup)
        SimulationTime::Instance()->SetStartTime(0.0);

        //Reseed the RNG with the required seed
        p_RNG->Reseed(seed);

        //Initialise a HeCellCycleModel and set it up with appropriate TiL values
        BoijeCellCycleModel* p_cycle_model = new BoijeCellCycleModel;

        if (debugOutput)
        {
            //Pass ColumnDataWriter to cell cycle model for debug output
            boost::shared_ptr<ColumnDataWriter> p_debugWriter(
                    new ColumnDataWriter(directoryString, filenameString + "DEBUG_" + std::to_string(seed), false, 10));
            p_cycle_model->EnableModelDebugOutput(p_debugWriter);
            debugWriter = &*p_debugWriter;
        }

        //Setup lineages' cycle model with appropriate parameters
        p_cycle_model->SetDimension(2);
        p_cycle_model->SetPostMitoticType(p_PostMitotic);

        //Setup vector containing lineage founder with the properly set up cell cycle model
        std::vector<CellPtr> cells;
        CellPtr p_cell(new Cell(p_state, p_cycle_model));
        p_cell->SetCellProliferativeType(p_Mitotic);
        p_cycle_model->SetModelParameters(phase2Generation, phase3Generation, pAtoh7, pPtf1a, png);
        p_cycle_model->SetSpecifiedTypes(p_RGC_fate, p_AC_HC_fate, p_PR_BC_fate);
        if (outputMode == 2) p_cycle_model->EnableSequenceSampler(p_label);
        if (outputMode == 2) p_cell->AddCellProperty(p_label);
        p_cell->InitialiseCellCycleModel();
        cells.push_back(p_cell);

        //Generate 1×1 mesh for single-cell colony
        HoneycombMeshGenerator generator(1, 1);
```

```
        MutableMesh<2, 2>* p_generating_mesh = generator.GetMesh();
        NodesOnlyMesh<2> mesh;
        mesh.ConstructNodesWithoutMesh(*p_generating_mesh, 1.5);

        //Setup cell population
        NodeBasedCellPopulation<2>* cell_population(new
        ↪  NodeBasedCellPopulation<2>(mesh, cells));

        //Setup simulator & run simulation
        boost::shared_ptr<OffLatticeSimulationPropertyStop<2>>
        ↪  p_simulator(
                new
                ↪  OffLatticeSimulationPropertyStop<2>(*cell_population));
        p_simulator->SetStopProperty(p_Mitotic); //simulation to stop if
        ↪  no mitotic cells are left
        p_simulator->SetDt(0.25);
        p_simulator->SetEndTime(endGeneration);
        p_simulator->SetOutputDirectory("UnusedSimOutput" +
        ↪  filenameString); //unused output
        p_simulator->Solve();

        //Count lineage size
        unsigned count = cell_population->GetNumRealCells();

        if (outputMode == 0) *p_log << entry_number << "\t" << seed <<
        ↪  "\t" << count << "\n";
        if (outputMode == 2) *p_log << "\n";

        //Reset for next simulation
        SimulationTime::Destroy();
        delete cell_population;
        entry_number++;

        if (debugOutput)
        {
            debugWriter->Close();
        }

    }

    p_RNG->Destroy();
    LogFile::Close();

```

```
    return exit_code;
}
;
```

17.1.3 /apps/src/GomesSimulator.cpp

```
#include <iostream>
#include <string>

#include <cxxtest/TestSuite.h>
#include "ExecutableSupport.hpp"
#include "Exception.hpp"
#include "PetscTools.hpp"
#include "PetscException.hpp"

#include "GomesCellCycleModel.hpp"
#include "OffLatticeSimulationPropertyStop.hpp"

#include "AbstractCellBasedTestSuite.hpp"

#include "WildTypeCellMutationState.hpp"
#include "TransitCellProliferativeType.hpp"
#include "DifferentiatedCellProliferativeType.hpp"
#include "GomesRetinalNeuralFates.hpp"

#include "CellsGenerator.hpp"
#include "HoneycombMeshGenerator.hpp"
#include "NodesOnlyMesh.hpp"
#include "NodeBasedCellPopulation.hpp"
#include "VertexBasedCellPopulation.hpp"

#include "ColumnDataWriter.hpp"

int main(int argc, char *argv[])
{
    ExecutableSupport::StartupWithoutShowingCopyright(&argc, &argv);
    //main() returns code indicating sim run success or failure mode
    int exit_code = ExecutableSupport::EXIT_OK;

    if (argc != 15)
    {
        ExecutableSupport::PrintError(
```

```
                "Wrong arguments for simulator.\nUsage (replace<> with values, pass bools as 0 or 1):\n GomesSimulator <directoryString> <filenameString> <outputModeUnsigned(0=counts,1=events,2=sequence)> <debugOutputBool> <startSeedUnsigned> <endSeedUnsigned> <endTimeDoubleHours> <cellCycleNormalMeanDouble> <cellCycleNormalStdDouble> <pPPDouble(0-1)> <pPDDouble(0-1)> <pBCDouble(0-1)> <pACDouble(0-1)> <pMGDouble(0-1)>",
                true);
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /**********************
     * SIMULATOR PARAMETERS
     **********************/
    std::string directoryString, filenameString;
    int outputMode; //0 = counts; 1 = mitotic mode events; 2 = mitotic mode sequence sampling
    bool debugOutput;
    unsigned startSeed, endSeed;
    double endTime;
    double normalMu, normalSigma, pPP, pPD, pBC, pAC, pMG; //stochastic model parameters

    //PARSE ARGUMENTS
    directoryString = argv[1];
    filenameString = argv[2];
    outputMode = std::stoi(argv[3]);
    debugOutput = std::stoul(argv[4]);
    startSeed = std::stoul(argv[5]);
    endSeed = std::stoul(argv[6]);
    endTime = std::stod(argv[7]);
    normalMu = std::stod(argv[8]);
    normalSigma = std::stod(argv[9]);
    pPP = std::stod(argv[10]);
    pPD = std::stod(argv[11]);
    pBC = std::stod(argv[12]);
    pAC = std::stod(argv[13]);
    pMG = std::stod(argv[14]);

```

```
    /************************
     * PARAMETER/ARGUMENT SANITY CHECK
     ************************/
    bool sane = 1;

    if (outputMode ≠ 0 && outputMode ≠ 1 && outputMode ≠ 2)
    {
        ExecutableSupport::PrintError(
                "Bad outputMode (argument 3). Must be 0 (counts) 1 (mitotic events) or 2 (sequence sampling)");
        sane = 0;
    }

    if (endSeed < startSeed)
    {
        ExecutableSupport::PrintError("Bad start & end seeds (arguments, 5, 6). endSeed must not be < startSeed");
        sane = 0;
    }

    if (endTime ≤ 0)
    {
        ExecutableSupport::PrintError("Bad endTime (argument 7). endTime must be > 0");
        sane = 0;
    }

    if (normalMu ≤ 0 || normalSigma ≤ 0)
    {
        ExecutableSupport::PrintError("Bad cell cycle normal mean or std (arguments 8, 9). Must be  >0");
        sane = 0;
    }

    if (pPP + pPD > 1 || pPP > 1 || pPP < 0 || pPD > 1 || pPD < 0)
    {
        ExecutableSupport::PrintError(
                "Bad mitotic mode probabilities (arguments 10, 11). pPP + pPD should be ≥0, ≤1, sum should not exceed 1");
        sane = 0;
    }

```

```
    if (pBC + pAC + pMG > 1 || pBC > 1 || pBC < 0 || pAC > 1 || pAC < 0 || pMG > 1 || pMG < 0)
    {
        ExecutableSupport::PrintError(
                "Bad specification probabilities (arguments 12, 13, 14). pBC, pAC, pMG should be ⩾0, ⩽1, sum should not exceed 1");
        sane = 0;
    }

    if (sane == 0)
    {
        ExecutableSupport::PrintError("Exiting with bad arguments. See errors for details");
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /************************
     * SIMULATOR OUTPUT SETUP
     ************************/

//Set up singleton LogFile
    LogFile* p_log = LogFile::Instance();
    p_log->Set(0, directoryString, filenameString);

    ExecutableSupport::Print("Simulator writing file " + filenameString + " to directory " + directoryString);

//Log entry counter
    unsigned entry_number = 1;

//Write appropriate headers to log
    if (outputMode == 0) *p_log << "Entry\tSeed\tCount\n";
    if (outputMode == 1) *p_log << "Time (hpf)\tSeed\tCellID\tMitotic Mode (0=PP;1=PD;2=DD)\n";
    if (outputMode == 2) *p_log << "Entry\tSeed\tSequence\n";

//Instance RNG
    RandomNumberGenerator* p_RNG = RandomNumberGenerator::Instance();

//Initialise pointers to relevant singleton ProliferativeTypes and Properties
```

```
    MAKE_PTR(WildTypeCellMutationState, p_state);
    MAKE_PTR(TransitCellProliferativeType, p_Mitotic);
    MAKE_PTR(DifferentiatedCellProliferativeType, p_PostMitotic);
    MAKE_PTR(RodPhotoreceptor, p_RPh_fate);
    MAKE_PTR(AmacrineCell, p_AC_fate);
    MAKE_PTR(BipolarCell, p_BC_fate);
    MAKE_PTR(MullerGlia, p_MG_fate);
    MAKE_PTR(CellLabel, p_label);

    /************************
     * SIMULATOR SETUP & RUN
     ************************/

//iterate through supplied seed range, executing one simulation per seed
    for (unsigned seed = startSeed; seed ≤ endSeed; seed++)
    {
        if (outputMode == 2) *p_log << entry_number << "\t" << seed << "\t"; //write seed to log - sequence written by cellcyclemodel objects

        //initialise pointer to debugWriter
        ColumnDataWriter* debugWriter;

        //initialise SimulationTime (permits cellcyclemodel setup)
        SimulationTime::Instance()->SetStartTime(0.0);

        //Reseed the RNG with the required seed
        p_RNG->Reseed(seed);

        //Initialise a HeCellCycleModel and set it up with appropriate TiL values
        GomesCellCycleModel* p_cycle_model = new GomesCellCycleModel;

        if (debugOutput)
        {
            //Pass ColumnDataWriter to cell cycle model for debug output
            boost::shared_ptr<ColumnDataWriter> p_debugWriter(
                    new ColumnDataWriter(directoryString, filenameString + "DEBUG_" + std::to_string(seed), false, 10));
            p_cycle_model->EnableModelDebugOutput(p_debugWriter);
            debugWriter = &*p_debugWriter;
        }

```

```
        //Setup lineages' cycle model with appropriate parameters
        p_cycle_model->SetDimension(2);
        p_cycle_model->SetPostMitoticType(p_PostMitotic);

        //Setup vector containing lineage founder with the properly set up cell cycle model
        std::vector<CellPtr> cells;
        CellPtr p_cell(new Cell(p_state, p_cycle_model));
        p_cell->SetCellProliferativeType(p_Mitotic);
        p_cycle_model->SetModelParameters(normalMu, normalSigma, pPP, pPD, pBC, pAC, pMG);
        p_cycle_model->SetModelProperties(p_RPh_fate, p_AC_fate, p_BC_fate, p_MG_fate);
        if (outputMode == 2) p_cycle_model->EnableSequenceSampler(p_label);
        if (outputMode == 2) p_cell->AddCellProperty(p_label);
        p_cell->InitialiseCellCycleModel();
        cells.push_back(p_cell);

        //Generate 1×1 mesh for single-cell colony
        HoneycombMeshGenerator generator(1, 1);
        MutableMesh<2, 2>* p_generating_mesh = generator.GetMesh();
        NodesOnlyMesh<2> mesh;
        mesh.ConstructNodesWithoutMesh(*p_generating_mesh, 1.5);

        //Setup cell population
        NodeBasedCellPopulation<2>* cell_population(new NodeBasedCellPopulation<2>(mesh, cells));

        //Setup simulator & run simulation
        boost::shared_ptr<OffLatticeSimulationPropertyStop<2>> p_simulator(
                new OffLatticeSimulationPropertyStop<2>(*cell_population));
        p_simulator->SetStopProperty(p_Mitotic); //simulation to stop if no mitotic cells are left
        p_simulator->SetDt(0.25);
        p_simulator->SetEndTime(endTime);
        p_simulator->SetOutputDirectory("UnusedSimOutput" + filenameString); //unused output
        p_simulator->Solve();

        //Count lineage size
```

```
        unsigned count = cell_population->GetNumRealCells();

        if (outputMode == 0) *p_log << entry_number << "\t" << seed <<
        ↪  "\t" << count << "\n";
        if (outputMode == 2) *p_log << "\n";

        //Reset for next simulation
        SimulationTime::Destroy();
        delete cell_population;
        entry_number++;

        if (debugOutput)
        {
            debugWriter->Close();
        }

    }

    p_RNG->Destroy();
    LogFile::Close();

    return exit_code;
}
;
```

17.1.4 /apps/src/HeSimulator.cpp

```
#include <iostream>
#include <string>

#include <cxxtest/TestSuite.h>
#include "ExecutableSupport.hpp"
#include "Exception.hpp"
#include "PetscTools.hpp"
#include "PetscException.hpp"

#include "HeCellCycleModel.hpp"
#include "OffLatticeSimulationPropertyStop.hpp"

#include "AbstractCellBasedTestSuite.hpp"

#include "WildTypeCellMutationState.hpp"
```

```
16 #include "TransitCellProliferativeType.hpp"
17 #include "DifferentiatedCellProliferativeType.hpp"
18
19 #include "CellsGenerator.hpp"
20 #include "HoneycombMeshGenerator.hpp"
21 #include "NodesOnlyMesh.hpp"
22 #include "NodeBasedCellPopulation.hpp"
23 #include "VertexBasedCellPopulation.hpp"
24
25 #include "ColumnDataWriter.hpp"
26
27 int main(int argc, char *argv[])
28 {
29     ExecutableSupport::StartupWithoutShowingCopyright(&argc, &argv);
30     //main() returns code indicating sim run success or failure mode
31     int exit_code = ExecutableSupport::EXIT_OK;
32
33     if (argc != 22 && argc != 20)
34     {
35         ExecutableSupport::PrintError(
```

```
                "Wrong arguments for simulator.\nUsage (replace<> with values, pass bools as 0 or 1):\nStochastic Mode:\nHeSimulator <directoryString> <filenameString> <outputModeUnsigned(0=counts,1=events,2=sequence)> <deterministicBool=0> <fixtureUnsigned(0=He;1=Wan;2=test)> <founderAth5Mutant?Bool> <debugOutputBool> <startSeedUnsigned> <endSeedUnsigned> <inductionTimeDoubleHours> <earliestLineageStartDoubleHours> <latestLineageStartDoubleHours> <endTimeDoubleHours> <mMitoticModePhase2Double> <mMitoticModePhase3Double> <pPP1Double(0-1)> <pPD1Double(0-1)> <pPP1Double(0-1)> <pPD1Double(0-1)> <pPP1Double(0-1)> <pPD1Double(0-1)>\nDeterministic Mode:\nHeSimulator <directoryString> <filenameString> <outputModeUnsigned(0=counts,1=events,2=sequence)> <deterministicBool=1> <fixtureUnsigned(0=He;1=Wan;2=test)> <founderAth5Mutant?Bool> <debugOutputBool> <startSeedUnsigned> <endSeedUnsigned> <inductionTimeDoubleHours> <earliestLineageStartDoubleHours> <latestLineageStartDoubleHours> <endTimeDoubleHours> <phase1ShapeDouble(>0)> <phase1ScaleDouble(>0)> <phase2ShapeDouble(>0)> <phase2ScaleDouble(>0)> <phaseBoundarySisterShiftWidthDouble>\n",
                true);
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /***********************
     * SIMULATOR PARAMETERS
     ***********************/
    std::string directoryString, filenameString;
    int outputMode; //0 = counts; 1 = mitotic mode events; 2 = mitotic mode sequence sampling
    bool deterministicMode, ath5founder, debugOutput;
    unsigned fixture, startSeed, endSeed; //fixture 0 = He2012; 1 = Wan2016
    double inductionTime, earliestLineageStartTime, latestLineageStartTime, endTime;
```

```
    double mitoticModePhase2, mitoticModePhase3, pPP1, pPD1, pPP2, pPD2,
    ↪  pPP3, pPD3; //stochastic model parameters
    double phase1Shape, phase1Scale, phase2Shape, phase2Scale,
    ↪  phaseSisterShiftWidth, phaseOffset;

    //PARSE ARGUMENTS
    directoryString = argv[1];
    filenameString = argv[2];
    outputMode = std::stoi(argv[3]);
    deterministicMode = std::stoul(argv[4]);
    fixture = std::stoul(argv[5]);
    ath5founder = std::stoul(argv[6]);
    debugOutput = std::stoul(argv[7]);
    startSeed = std::stoul(argv[8]);
    endSeed = std::stoul(argv[9]);
    inductionTime = std::stod(argv[10]);
    earliestLineageStartTime = std::stod(argv[11]);
    latestLineageStartTime = std::stod(argv[12]);
    endTime = std::stod(argv[13]);

    if (deterministicMode == 0)
    {
        mitoticModePhase2 = std::stod(argv[14]);
        mitoticModePhase3 = std::stod(argv[15]);
        pPP1 = std::stod(argv[16]);
        pPD1 = std::stod(argv[17]);
        pPP2 = std::stod(argv[18]);
        pPD2 = std::stod(argv[19]);
        pPP3 = std::stod(argv[20]);
        pPD3 = std::stod(argv[21]);
    }
    else if (deterministicMode == 1)
    {
        phase1Shape = std::stod(argv[14]);
        phase1Scale = std::stod(argv[15]);
        phase2Shape = std::stod(argv[16]);
        phase2Scale = std::stod(argv[17]);
        phaseSisterShiftWidth = std::stod(argv[18]);
        phaseOffset = std::stod(argv[19]);
    }
    else
    {
```

```
        ExecutableSupport::PrintError("Bad deterministicMode (argument 4). Must be 0 or 1");
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /************************
     * PARAMETER/ARGUMENT SANITY CHECK
     ************************/
    bool sane = 1;

    if (outputMode != 0 && outputMode != 1 && outputMode != 2)
    {
        ExecutableSupport::PrintError(
                "Bad outputMode (argument 3). Must be 0 (counts) 1 (mitotic events) or 2 (sequence sampling)");
        sane = 0;
    }

    if (fixture != 0 && fixture != 1 && fixture != 2)
    {
        ExecutableSupport::PrintError("Bad fixture (argument 5). Must be 0 (He), 1 (Wan), or 2 (validation/test)");
        sane = 0;
    }

    if (ath5founder != 0 && ath5founder != 1)
    {
        ExecutableSupport::PrintError("Bad ath5founder (argument 6). Must be 0 (wild type) or 1 (ath5 mutant)");
        sane = 0;
    }

    if (endSeed < startSeed)
    {
        ExecutableSupport::PrintError("Bad start & end seeds (arguments, 8, 9). endSeed must not be < startSeed");
        sane = 0;
    }

    if (inductionTime ≥ endTime)
    {
```

```
        ExecutableSupport::PrintError("Bad latestLineageStartTime (argument 10). Must be <endTime(arg13)");
        sane = 0;
    }
    if (earliestLineageStartTime ≥ endTime || earliestLineageStartTime ≥ latestLineageStartTime)
    {
        ExecutableSupport::PrintError(
                "Bad earliestLineageStartTime (argument 11). Must be <endTime(arg13), <latestLineageStarTime (arg12)");
        sane = 0;
    }
    if (latestLineageStartTime > endTime)
    {
        ExecutableSupport::PrintError("Bad latestLineageStartTime (argument 12). Must be ≤endTime(arg13)");
        sane = 0;
    }

    if (deterministicMode == 0)
    {
        if (mitoticModePhase2 < 0)
        {
            ExecutableSupport::PrintError("Bad mitoticModePhase2 (argument 14). Must be >0");
            sane = 0;
        }
        if (mitoticModePhase3 < 0)
        {
            ExecutableSupport::PrintError("Bad mitoticModePhase3 (argument 15). Must be >0");
            sane = 0;
        }
        if (pPP1 + pPD1 > 1 || pPP1 > 1 || pPP1 < 0 || pPD1 > 1 || pPD1 < 0)
        {
            ExecutableSupport::PrintError(
                    "Bad phase 1 probabilities (arguments 16, 17). pPP1 + pPD1 should be ≥0, ≤1, sum should not exceed 1");
            sane = 0;
        }
```

```
        if (pPP2 + pPD2 > 1 || pPP2 > 1 || pPP2 < 0 || pPD2 > 1 || pPD2 < 0)
        {
            ExecutableSupport::PrintError(
                    "Bad phase 2 probabilities (arguments 18, 19). pPP2 + pPD2 should be ⩾0, ⩽1, sum should not exceed 1");
            sane = 0;
        }
        if (pPP3 + pPD3 > 1 || pPP3 > 1 || pPP3 < 0 || pPD3 > 1 || pPD3 < 0)
        {
            ExecutableSupport::PrintError(
                    "Bad phase 3 probabilities (arguments 20, 21). pPP3 + pPD3 should be ⩾0, ⩽1, sum should not exceed 1");
            sane = 0;
        }
    }

    if (deterministicMode == 1)
    {
        if (phase1Shape ⩽ 0)
        {
            ExecutableSupport::PrintError("Bad phase1Shape (argument 14). Must be >0");
            sane = 0;
        }
        if (phase1Scale ⩽ 0)
        {
            ExecutableSupport::PrintError("Bad phase1Scale (argument 15). Must be >0");
            sane = 0;
        }
        if (phase2Shape ⩽ 0)
        {
            ExecutableSupport::PrintError("Bad phase1Shape (argument 16). Must be >0");
            sane = 0;
        }
        if (phase2Scale ⩽ 0)
        {
```

```
            ExecutableSupport::PrintError("Bad phase1Scale (argument 17). Must be >0");
            sane = 0;
        }
        if (phaseSisterShiftWidth ≤ 0)
        {
            ExecutableSupport::PrintError("Bad phaseSisterShiftWidth (argument 18). Must be >0");
            sane = 0;
        }
    }

    if (sane == 0)
    {
        ExecutableSupport::PrintError("Exiting with bad arguments. See errors for details");
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;

    }

    /************************
     * SIMULATOR OUTPUT SETUP
     ************************/

//Set up singleton LogFile
    LogFile* p_log = LogFile::Instance();
    p_log->Set(0, directoryString, filenameString);

    ExecutableSupport::Print("Simulator writing file " + filenameString + " to directory " + directoryString);

//Log entry counter
    unsigned entry_number = 1;

//Write appropriate headers to log
    if (outputMode == 0) *p_log << "Entry\tInduction Time (h)\tSeed\tCount\n";
    if (outputMode == 1) *p_log << "Time (hpf)\tSeed\tCellID\tMitotic Mode (0=PP;1=PD;2=DD)\n";
    if (outputMode == 2) *p_log << "Entry\tSeed\tSequence\n";

//Instance RNG
```

```
    RandomNumberGenerator* p_RNG = RandomNumberGenerator::Instance();

//Initialise pointers to relevant singleton ProliferativeTypes and Properties
    MAKE_PTR(WildTypeCellMutationState, p_state);
    MAKE_PTR(TransitCellProliferativeType, p_Mitotic);
    MAKE_PTR(DifferentiatedCellProliferativeType, p_PostMitotic);
    MAKE_PTR(Ath5Mo, p_Morpholino);
    MAKE_PTR(CellLabel, p_label);

    /************************
     * SIMULATOR SETUP & RUN
     ************************/

//iterate through supplied seed range, executing one simulation per seed
    for (unsigned seed = startSeed; seed <= endSeed; seed++)
    {
        if (outputMode == 2) *p_log << entry_number << "\t" << seed << "\t"; //write seed to log - sequence written by cellcyclemodel objects

        //initialise pointer to debugWriter
        ColumnDataWriter* debugWriter;

        //initialise SimulationTime (permits cellcyclemodel setup)
        SimulationTime::Instance()->SetStartTime(0.0);

        //Reseed the RNG with the required seed
        p_RNG->Reseed(seed);

        //Initialise a HeCellCycleModel and set it up with appropriate TiL values
        HeCellCycleModel* p_cycle_model = new HeCellCycleModel;

        if (debugOutput)
        {
            //Pass ColumnDataWriter to cell cycle model for debug output
            boost::shared_ptr<ColumnDataWriter> p_debugWriter(
                    new ColumnDataWriter(directoryString, filenameString + "DEBUG_" + std::to_string(seed), false, 10));
            p_cycle_model->EnableModelDebugOutput(p_debugWriter);
            debugWriter = &*p_debugWriter;
        }
```

```
268
269         double currTiL; //Time in Lineage offset for lineages induced
            ↪  after first mitosis
270         double lineageStartTime; //first mitosis time (hpf)
271         double currSimEndTime; //simulation end time (hpf);
272
273
            ↪  /*********************************************************************
274          * Time in Lineage Generation Fixtures & Cell Cycle Model Setup
275
    ↪  *********************************************************************/
276
277         if (fixture == 0) //He 2012-type fixture - even distribution
            ↪  across nasal-temporal axis
278         {
279             //generate lineage start time from even random distro across
                ↪  earliest-latest start time figures
280             lineageStartTime = (p_RNG->ranf() * (latestLineageStartTime -
                ↪  earliestLineageStartTime))
281                     + earliestLineageStartTime;
282             //this reflects induction of cells after the lineages' first
                ↪  mitosis
283             if (lineageStartTime < inductionTime)
284             {
285                 currTiL = inductionTime - lineageStartTime;
286                 currSimEndTime = endTime - inductionTime;
287                 if (outputMode == 1)
                    ↪  p_cycle_model->EnableModeEventOutput(inductionTime,
                    ↪  seed);
288             }
289             //if the lineage starts after the induction time, give it
                ↪  zero TiL & run the appropriate-length simulation
290             //(ie. the endTime is reduced by the amount of time after
                ↪  induction that the first mitosis occurs)
291             if (lineageStartTime ≥ inductionTime)
292             {
293                 currTiL = 0.0;
294                 currSimEndTime = endTime - lineageStartTime;
295                 if (outputMode == 1)
                    ↪  p_cycle_model->EnableModeEventOutput(lineageStartTime,
                    ↪  seed);
296             }
297
```

```
298         }
299         else if (fixture == 1) //Wan 2016-type fixture - each lineage
            ↪  founder selected randomly across residency time, simulator
            ↪  allowed to run until end of residency time
300         //passing residency time (as latestLineageStartTime) and endTime
            ↪  separately allows for creation of "shadow CMZ" population
301         //this allows investigation of different assumptions about how
            ↪  Wan et al.'s model output was generated
302         {
303             //generate random lineage start time from even random distro
                ↪  across CMZ residency time
304             currTiL = p_RNG→ranf() * latestLineageStartTime;
305             currSimEndTime = std::max(.05, endTime - currTiL); //minimum
                ↪  1 timestep, prevents 0 timestep SimulationTime error
306             if (outputMode == 1) p_cycle_model→EnableModeEventOutput(0,
                ↪  seed);
307         }
308         else if (fixture == 2) //validation fixture- all founders have
            ↪  TiL given by induction time
309         {
310             currTiL = inductionTime;
311             currSimEndTime = endTime;
312         }
313
314         //Setup lineages' cycle model with appropriate parameters
315         p_cycle_model→SetDimension(2);
316         //p_cycle_model→SetPostMitoticType(p_PostMitotic);
317
318         if (!deterministicMode)
319         {
320             p_cycle_model→SetModelParameters(currTiL, mitoticModePhase2,
                ↪  mitoticModePhase2 + mitoticModePhase3, pPP1,
321                                                 pPD1, pPP2, pPD2, pPP3,
                                                    ↪  pPD3);
322         }
323         else
324         {
325             //Gamma-distribute phase3 boundary
326             double currPhase2Boundary = phaseOffset +
                ↪  p_RNG→GammaRandomDeviate(phase1Shape, phase1Scale);
327             double currPhase3Boundary = currPhase2Boundary +
                ↪  p_RNG→GammaRandomDeviate(phase2Shape, phase2Scale);
328
```

```
            p_cycle_model->SetDeterministicMode(currTiL, currPhase2Boundary, currPhase3Boundary, phaseSisterShiftWidth);
        }

        if (outputMode == 2) p_cycle_model->EnableSequenceSampler();

        //Setup vector containing lineage founder with the properly set up cell cycle model
        std::vector<CellPtr> cells;
        CellPtr p_cell(new Cell(p_state, p_cycle_model));
        p_cell->SetCellProliferativeType(p_Mitotic);
        if (ath5founder == 1) p_cell->AddCellProperty(p_Morpholino);
        if (outputMode == 2) p_cell->AddCellProperty(p_label);
        p_cell->InitialiseCellCycleModel();
        cells.push_back(p_cell);

        //Generate 1×1 mesh for single-cell colony
        HoneycombMeshGenerator generator(1, 1);
        MutableMesh<2, 2>* p_generating_mesh = generator.GetMesh();
        NodesOnlyMesh<2> mesh;
        mesh.ConstructNodesWithoutMesh(*p_generating_mesh, 1.5);

        //Setup cell population
        NodeBasedCellPopulation<2>* cell_population(new NodeBasedCellPopulation<2>(mesh, cells));

        //Setup simulator & run simulation
        boost::shared_ptr<OffLatticeSimulationPropertyStop<2>> p_simulator(
                new OffLatticeSimulationPropertyStop<2>(*cell_population));
        p_simulator->SetStopProperty(p_Mitotic); //simulation to stop if no mitotic cells are left
        p_simulator->SetDt(0.05);
        p_simulator->SetEndTime(currSimEndTime);
        p_simulator->SetOutputDirectory("UnusedSimOutput" + filenameString); //unused output
        p_simulator->Solve();

        //Count lineage size
        unsigned count = cell_population->GetNumRealCells();

```

```
        if (outputMode == 0) *p_log << entry_number << "\t" << inductionTime << "\t" << seed << "\t" << count << "\n";
        if (outputMode == 2) *p_log << "\n";

        //Reset for next simulation
        SimulationTime::Destroy();
        delete cell_population;
        entry_number++;

        if (debugOutput)
        {
            debugWriter->Close();
        }

    }

    p_RNG->Destroy();
    LogFile::Close();

    return exit_code;
}
;
```

17.1.5 /apps/src/WanSimDebug.cpp

```
#include <iostream>
#include <string>

#include <cxxtest/TestSuite.h>
#include "ExecutableSupport.hpp"
#include "Exception.hpp"
#include "PetscTools.hpp"
#include "PetscException.hpp"

#include "WanStemCellCycleModel.hpp"
#include "HeCellCycleModel.hpp"
#include "OffLatticeSimulation.hpp"

#include "AbstractCellBasedTestSuite.hpp"

#include "WildTypeCellMutationState.hpp"
#include "StemCellProliferativeType.hpp"
```

```
#include "TransitCellProliferativeType.hpp"
#include "DifferentiatedCellProliferativeType.hpp"

#include "CellsGenerator.hpp"
#include "HoneycombMeshGenerator.hpp"
#include "NodesOnlyMesh.hpp"
#include "NodeBasedCellPopulation.hpp"
#include "VertexBasedCellPopulation.hpp"

#include "CellProliferativeTypesCountWriter.hpp"


int main(int argc, char *argv[])
{
    ExecutableSupport::StartupWithoutShowingCopyright(&argc, &argv);
    //main() returns code indicating sim run success or failure mode
    int exit_code = ExecutableSupport::EXIT_OK;

    /************************
     * SIMULATOR PARAMETERS
     ************************/
    std::string directoryString, filenameString;

    //PARSE ARGUMENTS
    directoryString = argv[1];
    filenameString = argv[2];

    /************************
     * SIMULATOR OUTPUT SETUP
     ************************/

//Set up singleton LogFile
    LogFile* p_log = LogFile::Instance();
    p_log->Set(0, directoryString, filenameString);

    ExecutableSupport::Print("Simulator writing file " + filenameString +
     ↪  " to directory " + directoryString);

//Log entry counter
    //unsigned entry_number = 1;

//Write appropriate header to log
```

```
    *p_log << "Entry\tTotalCells\tStemCount\tProgenitorCount\tPostMitoticCount\n";

//Instance RNG
    RandomNumberGenerator* p_RNG = RandomNumberGenerator::Instance();

//Initialise pointers to relevant singleton ProliferativeTypes and Properties
    boost::shared_ptr<AbstractCellProperty> p_state(CellPropertyRegistry::Instance()->Get<WildTypeCellMutationState>());
    boost::shared_ptr<AbstractCellProperty> p_Stem(CellPropertyRegistry::Instance()->Get<StemCellProliferativeType>());
    boost::shared_ptr<AbstractCellProperty> p_Transit(CellPropertyRegistry::Instance()->Get<TransitCellProliferativeType>
    boost::shared_ptr<AbstractCellProperty> p_PostMitotic(CellPropertyRegistry::Instance()->Get<DifferentiatedCellProlife

    /************************
     * SIMULATOR SETUP & RUN
     ************************/

    //initialise SimulationTime (permits cellcyclemodel setup)
    SimulationTime::Instance()->SetStartTime(0.0);

    std::vector<CellPtr> cells;


    WanStemCellCycleModel* p_stem_model = new WanStemCellCycleModel;

    boost::shared_ptr<ColumnDataWriter> p_debugWriter(
            new ColumnDataWriter(directoryString, filenameString + "DEBUG_WAN", false, 10));

    p_stem_model->SetDimension(2);
    p_stem_model->EnableModelDebugOutput(p_debugWriter);
    CellPtr p_cell(new Cell(p_state, p_stem_model));
    p_cell->InitialiseCellCycleModel();
    cells.push_back(p_cell);

    //Generate 1x#cells mesh for abstract colony
    HoneycombMeshGenerator generator(1, 1);
    MutableMesh<2, 2>* p_generating_mesh = generator.GetMesh();
    NodesOnlyMesh<2> mesh;
```

```
    mesh.ConstructNodesWithoutMesh(*p_generating_mesh, 1.5);

    //Setup cell population
    boost::shared_ptr<NodeBasedCellPopulation<2>> cell_population(new
    ↪  NodeBasedCellPopulation<2>(mesh, cells));

    ↪  cell_population->AddCellPopulationCountWriter<CellProliferativeTypesCountWrit

    p_stem_model->EnableExpandingStemPopulation(1, cell_population);

    //Setup simulator & run simulation
    boost::shared_ptr<OffLatticeSimulation<2>> p_simulator(
            new OffLatticeSimulation<2>(*cell_population));
    p_simulator->SetDt(1);
    p_simulator->SetOutputDirectory(directoryString + "/WanDebug");
    p_simulator->SetEndTime(100);
    p_simulator->Solve();

    std::vector<unsigned> prolTypes =
    ↪  cell_population->GetCellProliferativeTypeCount();
    unsigned realCells = cell_population->GetNumRealCells();
    unsigned allCells = cell_population->GetNumAllCells();

    Timer::Print("stem: " + std::to_string(prolTypes[0]) + " transit: " +
    ↪  std::to_string(prolTypes[1]) + " postmitotic: " +
    ↪  std::to_string(prolTypes[2]));
    Timer::Print("realcells: " + std::to_string(realCells) + " allcells:
    ↪  " + std::to_string(allCells));

    //Reset for next simulation
    SimulationTime::Destroy();
    cell_population.reset();

    p_RNG->Destroy();
    LogFile::Close();

    return exit_code;
}
;
```

17.1.6 /apps/src/WanSimulator.cpp

```
#include <iostream>
#include <string>

#include <cxxtest/TestSuite.h>
#include "ExecutableSupport.hpp"
#include "Exception.hpp"
#include "PetscTools.hpp"
#include "PetscException.hpp"

#include "WanStemCellCycleModel.hpp"
#include "HeCellCycleModel.hpp"
#include "OffLatticeSimulationPropertyStop.hpp"

#include "AbstractCellBasedTestSuite.hpp"

#include "WildTypeCellMutationState.hpp"
#include "StemCellProliferativeType.hpp"
#include "TransitCellProliferativeType.hpp"
#include "DifferentiatedCellProliferativeType.hpp"

#include "CellsGenerator.hpp"
#include "HoneycombMeshGenerator.hpp"
#include "NodesOnlyMesh.hpp"
#include "NodeBasedCellPopulation.hpp"
#include "VertexBasedCellPopulation.hpp"

#include "CellProliferativeTypesCountWriter.hpp"

int main(int argc, char *argv[])
{
    ExecutableSupport::StartupWithoutShowingCopyright(&argc, &argv);
    //main() returns code indicating sim run success or failure mode
    int exit_code = ExecutableSupport::EXIT_OK;

    if (argc != 23)
    {
        ExecutableSupport::PrintError(
```

```
                "Wrong arguments for simulator.\nUsage (replace<> with values, pass bools as 0 or 1):\n WanSimulator <directoryString> <startSeedUnsigned> <endSeedUnsigned> <cmzResidencyTimeDoubleHours> <stemDivisorDouble> <meanProgenitorPopualtion@3dpfDouble> <stdProgenitorPopulation@3dpfDouble> <stemGammaShiftDouble> <stemGammaShapeDouble> <stemGammaScaleDouble> <progenitorGammaShiftDouble> <progenitorGammaShapeDouble> <progenitorGammaScaleDouble> <progenitorSisterShiftDouble> <mMitoticModePhase2Double> <mMitoticModePhase3Double> <pPP1Double(0-1)> <pPD1Double(0-1)> <pPP1Double(0-1)> <pPD1Double(0-1)> <pPP1Double(0-1)> <pPD1Double(0-1)>",
                true);
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /***********************
     * SIMULATOR PARAMETERS
     ***********************/
    std::string directoryString;
    unsigned startSeed, endSeed;
    double cmzResidencyTime;
    double stemDivisor, progenitorMean, progenitorStd;
    double stemGammaShift, stemGammaShape, stemGammaScale, progenitorGammaShift, progenitorGammaShape,
            progenitorGammaScale, progenitorGammaSister;
    double mitoticModePhase2, mitoticModePhase3, pPP1, pPD1, pPP2, pPD2, pPP3, pPD3; //stochastic He model parameters

    //PARSE ARGUMENTS
    directoryString = argv[1];
    startSeed = std::stoul(argv[2]);
    endSeed = std::stoul(argv[3]);
    cmzResidencyTime = std::stod(argv[4]);
    //starting cell number distributions
    stemDivisor = std::stod(argv[5]);
    progenitorMean = std::stod(argv[6]);
    progenitorStd = std::stod(argv[7]);
```

```
    //cycle duration params
    stemGammaShift = std::stod(argv[8]);
    stemGammaShape = std::stod(argv[9]);
    stemGammaScale = std::stod(argv[10]);
    progenitorGammaShift = std::stod(argv[11]);
    progenitorGammaShape = std::stod(argv[12]);
    progenitorGammaScale = std::stod(argv[13]);
    progenitorGammaSister = std::stod(argv[14]);
    //He model params
    mitoticModePhase2 = std::stod(argv[15]);
    mitoticModePhase3 = std::stod(argv[16]);
    pPP1 = std::stod(argv[17]);
    pPD1 = std::stod(argv[18]);
    pPP2 = std::stod(argv[19]);
    pPD2 = std::stod(argv[20]);
    pPP3 = std::stod(argv[21]);
    pPD3 = std::stod(argv[22]);

    std::vector<double> stemOffspringParams = { mitoticModePhase2, mitoticModePhase2 + mitoticModePhase3, pPP1, pPD1,
                                                pPP2, pPD2, pPP3, pPD3, progenitorGammaShift, progenitorGammaShape,
                                                progenitorGammaScale, progenitorGammaSister };

    /************************
     * PARAMETER/ARGUMENT SANITY CHECK
     ************************/
    bool sane = 1;

    if (endSeed < startSeed)
    {
        ExecutableSupport::PrintError("Bad start & end seeds (arguments, 3, 4). endSeed must not be < startSeed");
        sane = 0;
    }

    if (cmzResidencyTime ≤ 0)
    {
        ExecutableSupport::PrintError("Bad CMZ residency time (argument 5). cmzResidencyTime must be positive-valued");
```

```
        sane = 0;
    }

    if (stemDivisor ≤ 0)
    {
        ExecutableSupport::PrintError("Bad stemDivisor (argument 6). stemDivisor must be positive-valued");
        sane = 0;
    }

    if (progenitorMean ≤ 0 || progenitorStd ≤ 0)
    {
        ExecutableSupport::PrintError("Bad progenitorMean or progenitorStd (arguments 7,8). Must be positive-valued");
        sane = 0;
    }

    if (stemGammaShift < 0 || stemGammaShape ≤ 0 || stemGammaScale ≤ 0)
    {
        ExecutableSupport::PrintError(
                "Bad stemGammaShift, stemGammaShape, or stemGammaScale (arguments 9, 10, 11). Shifts must be ≥0, cycle shape and scale params must be positive-valued");
        sane = 0;
    }

    if (progenitorGammaShift < 0 || progenitorGammaShape ≤ 0 || progenitorGammaScale ≤ 0 || progenitorGammaSister < 0)
    {
        ExecutableSupport::PrintError(
                "Bad progenitorGammaShift, progenitorGammaShape, progenitorGammaScale, or progenitorGammaSisterShift (arguments 12,13,14,15). Shifts must be ≥0, cycle shape and scale params must be positive-valued");
        sane = 0;
    }

    if (mitoticModePhase2 < 0)
    {
        ExecutableSupport::PrintError("Bad mitoticModePhase2 (argument 14). Must be >0");
        sane = 0;
    }
```

```
    if (mitoticModePhase3 < 0)
    {
        ExecutableSupport::PrintError("Bad mitoticModePhase3 (argument 15). Must be >0");
        sane = 0;
    }
    if (pPP1 + pPD1 > 1 || pPP1 > 1 || pPP1 < 0 || pPD1 > 1 || pPD1 < 0)
    {
        ExecutableSupport::PrintError(
                "Bad phase 1 probabilities (arguments 16, 17). pPP1 + pPD1 should be ⩾0, ⩽1, sum should not exceed 1");
        sane = 0;
    }
    if (pPP2 + pPD2 > 1 || pPP2 > 1 || pPP2 < 0 || pPD2 > 1 || pPD2 < 0)
    {
        ExecutableSupport::PrintError(
                "Bad phase 2 probabilities (arguments 18, 19). pPP2 + pPD2 should be ⩾0, ⩽1, sum should not exceed 1");
        sane = 0;
    }
    if (pPP3 + pPD3 > 1 || pPP3 > 1 || pPP3 < 0 || pPD3 > 1 || pPD3 < 0)
    {
        ExecutableSupport::PrintError(
                "Bad phase 3 probabilities (arguments 20, 21). pPP3 + pPD3 should be ⩾0, ⩽1, sum should not exceed 1");
        sane = 0;
    }

    if (sane == 0)
    {
        ExecutableSupport::PrintError("Exiting with bad arguments. See errors for details");
        exit_code = ExecutableSupport::EXIT_BAD_ARGUMENTS;
        return exit_code;
    }

    /************************
     * SIMULATOR SETUP & RUN
     ************************/

    ExecutableSupport::Print("Simulator writing files to directory " + directoryString);

```

```
//Instance RNG
    RandomNumberGenerator* p_RNG = RandomNumberGenerator::Instance();

//Initialise pointers to relevant singleton ProliferativeTypes and Properties
    boost::shared_ptr<AbstractCellProperty> p_state(CellPropertyRegistry::Instance()->Get<WildTypeCellMutationState>());
    boost::shared_ptr<AbstractCellProperty> p_Stem(CellPropertyRegistry::Instance()->Get<StemCellProliferativeType>());
    boost::shared_ptr<AbstractCellProperty> p_Transit(CellPropertyRegistry::Instance()->Get<TransitCellProliferativeType>()
    boost::shared_ptr<AbstractCellProperty> p_PostMitotic(CellPropertyRegistry::Instance()->Get<DifferentiatedCellProliferative

//iterate through supplied seed range, executing one simulation per seed
    for (unsigned seed = startSeed; seed <= endSeed; seed++)
    {
        //initialise SimulationTime (permits cellcyclemodel setup)
        SimulationTime::Instance()->SetStartTime(0.0);

        //Reseed the RNG with the required seed
        p_RNG->Reseed(seed);

        //unsigned numberStem = int(std::round(p_RNG->NormalRandomDeviate(stemMean, stemStd)));
        unsigned numberProgenitors = int(std::round(p_RNG->NormalRandomDeviate(progenitorMean, progenitorStd)));
        unsigned numberStem = int(std::round(numberProgenitors / stemDivisor));

        std::vector<CellPtr> stems;
        std::vector<CellPtr> cells;

        for (unsigned i = 0; i < numberStem; i++)
        {
            WanStemCellCycleModel* p_stem_model = new WanStemCellCycleModel;
            p_stem_model->SetDimension(2);
```

```
            p_stem_model->SetModelParameters(stemGammaShift, stemGammaShape, stemGammaScale, stemOffspringParams);

            CellPtr p_cell(new Cell(p_state, p_stem_model));
            p_cell->InitialiseCellCycleModel();
            stems.push_back(p_cell);
            cells.push_back(p_cell);
        }

        for (unsigned i = 0; i < numberProgenitors; i++)
        {
            double currTiL = p_RNG->ranf() * cmzResidencyTime;

            HeCellCycleModel* p_prog_model = new HeCellCycleModel;
            p_prog_model->SetDimension(2);
            p_prog_model->SetModelParameters(currTiL, mitoticModePhase2, mitoticModePhase2 + mitoticModePhase3, pPP1,
                                             pPD1, pPP2, pPD2, pPP3, pPD3);
            p_prog_model->EnableKillSpecified();

            CellPtr p_cell(new Cell(p_state, p_prog_model));
            p_cell->InitialiseCellCycleModel();
            cells.push_back(p_cell);
        }

        //Generate 1x#cells mesh for abstract colony
        HoneycombMeshGenerator generator(1, (numberProgenitors + numberStem));
        MutableMesh<2, 2>* p_generating_mesh = generator.GetMesh();
        NodesOnlyMesh<2> mesh;
        mesh.ConstructNodesWithoutMesh(*p_generating_mesh, 1.5);

        //Setup cell population
        boost::shared_ptr<NodeBasedCellPopulation<2>> cell_population(new NodeBasedCellPopulation<2>(mesh, cells));

        cell_population->AddCellPopulationCountWriter<CellProliferativeTypesCount

        //Give Wan stem cells the population & base stem pop size
        for (auto p_cell : stems)
        {
```

```
            WanStemCellCycleModel* p_cycle_model = dynamic_cast<WanStemCellCycleModel*>(p_cell->GetCellCycleModel());
            p_cycle_model->EnableExpandingStemPopulation(numberStem, cell_population);
        }

        //Setup simulator & run simulation
        boost::shared_ptr<OffLatticeSimulationPropertyStop<2>> p_simulator(
                new OffLatticeSimulationPropertyStop<2>(*cell_population));
        p_simulator->SetStopProperty(p_Transit); //simulation to stop if no RPCs are left
        p_simulator->SetDt(1);
        p_simulator->SetOutputDirectory(directoryString + "/Seed" + std::to_string(seed) + "Results");
        p_simulator->SetEndTime(8568); // 360dpf - 3dpf simulation start time
        p_simulator->Solve();

        //Reset for next simulation
        SimulationTime::Destroy();
        cell_population.reset();
    }

    p_RNG->Destroy();

    return exit_code;
}
;
```

17.1.7 /python_fixtures/He_output_fixture.py

```
import multiprocessing
import os
import subprocess
import datetime

import numpy as np
from imageio.plugins._bsdf import BsdfSerializer

executable = '/home/main/chaste_build/projects/ISP/apps/HeSimulator'
```

```
10
11 if not(os.path.isfile(executable)):
12     raise Exception('Could not find executable: ' + executable)
13
14 #########################
15 # SIMULATION PARAMETERS
16 #########################
17
18 #Define start and end RNG seeds; determines:
19 #No. lineages per loss function run
20 #unique sequence of RNG results for each lineage
21 start_seed = 0
22 end_seed_counts = 9999
23 end_seed_events = 999
24 run_modes = [0,1,2] #1=deterministic mode, 0=refit stochastic mode,
   ↪  2=original fit
25 debug_output = 0 #0=off;1=on
26
27 count_directory = "HeCounts"
28 count_filenames = ["induction", "wan", "ath5", "validate"]
29 event_directory = "HeModeEvent"
30 event_filenames = ["inductionMode", "validateMode"]
31 induction_times = [24, 32, 48]
32
33 ##########################
34 #GLOBAL MODEL PARAMETERS
35 ##########################
36
37 #Values defining different marker induction timepoints & relative start
   ↪  time of TiL counter
38 earliest_lineage_start_time = 23.0 #RPCs begin to enter "He model regime"
   ↪  nasally at 23hpf
39 latest_lineage_start_time = 39.0 #last temporal retinal RPC has entered
   ↪  "He model regime" at 39hpf
40 counts_end_time = 72.0
41 events_end_time = 80.0
42 wan_residency_time = 17.0
43
44 ########################################
45 # SPECIFIC MODEL PARAMETERS - THETAHAT
46 ########################################
47
48 #These parameters are the results of the SPSA optimisation fixture
```

```

#STOCHASTIC MITOTIC MODE
#HE ORIGINAL PARAMETERS
#mitotic mode per-phase probabilities
#3-phase mitotic mode time periodisation
o_mitotic_mode_phase_2 = 8 #These are phase lengths, so
o_mitotic_mode_phase_3 = 7 #Phase3 boundary = mmp2 + mmp3
o_phase_1_pPP = 1.0
o_phase_1_pPD = 0.0
o_phase_2_pPP = 0.2
o_phase_2_pPD = 0.4
o_phase_3_pPP = 0.2
o_phase_3_pPD = 0.0

#SPSA REFIT PARAMETERS
#mitotic mode per-phase probabilities
#3-phase mitotic mode time periodisation
mitotic_mode_phase_2 = 4.1483 #These are phase lengths, so
mitotic_mode_phase_3 = 11.6416 #Phase3 boundary = mmp2 + mmp3
phase_1_pPP = 1.0
phase_1_pPD = 0.0
phase_2_pPP = 0.1959
phase_2_pPD = 0.5168
phase_3_pPP = 0.2934
phase_3_pPD = 0.0
he_model_params = 15

#DETERMINISTIC MITOTIC MODE
#Phase boundary shift parameters
phase_1_shape = 3.7371
phase_1_scale = 1.8114
phase_2_shape = 2.5769
phase_2_scale = 1.6814
phase_sister_shift_widths = 1.6326
phase_offset = 1.2333
det_model_params = 12

original_theta_string = str(o_mitotic_mode_phase_2) + " " +
    str(o_mitotic_mode_phase_3) + " " +str(o_phase_1_pPP) + " " +
    str(o_phase_1_pPD) + " " + str(o_phase_2_pPP) + " " +
    str(o_phase_2_pPD) + " " + str(o_phase_3_pPP) + " " +
    str(o_phase_3_pPD)
```

```
stochastic_theta_string = str(mitotic_mode_phase_2) + " " + str(mitotic_mode_phase_3) + " " +str(phase_1_pPP) + " " + str(phase_1_pPD) + " " + str(phase_2_pPP) + " " + str(phase_2_pPD) + " " + str(phase_3_pPP) + " " + str(phase_3_pPD)
deterministic_theta_string = str(phase_1_shape) + " " + str(phase_1_scale) + " " + str(phase_2_shape) + " " + str(phase_2_scale) + " " + str(phase_sister_shift_widths) + " " + str(phase_offset)

def main():

    command_list = []

    for m in range(0,len(run_modes)):
        curr_list = []

        run_mode_string = str(run_modes[m])
        deterministic_mode = run_modes[m]
        if run_modes[m] == 2:
            deterministic_mode = 0
        command_count = 0 #for iterating seed numbers
        base_command = executable

        #induction count commands
        for i in range(0,len(induction_times)):

            output_mode = 0 #0=lineage counts;1=mitotic event logging;2=sequence sampling
            fixture = 0 #0=He 2012;1=Wan 2016
            curr_start_seed = start_seed + command_count * (end_seed_counts+1)
            curr_end_seed = curr_start_seed + end_seed_counts
            ath5founder = 0

            command = base_command+" "\
                        +count_directory+" "\
                        +count_filenames[0]+str(induction_times[i])+"SDMode"+run_
                        "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
```

```
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(induction_times[i])+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(latest_lineage_start_time)+" "\
                        +str(counts_end_time)+" "
            curr_list.append(command)
            command_count += 1

        #wan command
        output_mode = 0
        fixture = 1
        curr_start_seed = start_seed + command_count * (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_counts
        ath5founder = 0
        wan_command = base_command+" "\
                        +count_directory+" "\
                        +count_filenames[1]+"SDMode"+run_mode_string+" "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(0)+" "\
                        +str(0)+" "\
                        +str(wan_residency_time)+" "\
                        +str(counts_end_time)+" "
        curr_list.append(wan_command)
        command_count += 1

        #wan- 17 hr constraint- no "shadow RPCs"
        curr_start_seed = start_seed + command_count * (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_counts
        wan_command = base_command+" "\
                        +count_directory+" "\

                        +count_filenames[1]+"NoShadSDMode"+run_mode_string+" "\
```

```
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(0)+" "\
                        +str(0)+" "\
                        +str(wan_residency_time)+" "\
                        +str(wan_residency_time)+" "
        curr_list.append(wan_command)
        command_count += 1

        #ath5 command
        output_mode = 0
        fixture = 2
        curr_start_seed = start_seed + command_count *
         ↪  (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_counts
        ath5founder = 1

        ath5_command = base_command+" "\
                        +count_directory+" "\
                        +count_filenames[2]+"SDMode"+run_mode_string+" "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(0)+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(latest_lineage_start_time)+" "\
                        +str(counts_end_time)+" "
        curr_list.append(ath5_command)
        command_count += 1

        #validate counts command
        curr_start_seed = start_seed + command_count *
         ↪  (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_counts
```

```
        ath5founder = 0

        validate_command = base_command+" "\
                        +count_directory+" "\
                        +count_filenames[3]+"SDMode"+run_mode_string+" "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(0)+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(latest_lineage_start_time)+" "\
                        +str(counts_end_time)+" "
        curr_list.append(validate_command)
        command_count += 1

        #mitotic mode rate command
        output_mode = 1
        fixture = 0
        curr_start_seed = start_seed + command_count *
         ↪  (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_events
        mode_rate_command = base_command+" "\
                        +event_directory+" "\
                        +event_filenames[0]+"SDMode"+run_mode_string+" "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(latest_lineage_start_time)+" "\
                        +str(events_end_time)+" "

        curr_list.append(mode_rate_command)
        command_count += 1

```

```
        #validate rate command
        fixture = 2
        curr_start_seed = start_seed + command_count *
         ↪  (end_seed_counts+1)
        curr_end_seed = curr_start_seed + end_seed_events
        validate_rate_command = base_command+" "\
                        +event_directory+" "\
                        +event_filenames[1]+"SDMode"+run_mode_string+" "\
                        +str(output_mode)+" "\
                        +str(deterministic_mode)+" "\
                        +str(fixture)+" "\
                        +str(ath5founder)+" "\
                        +str(debug_output)+" "\
                        +str(curr_start_seed)+" "\
                        +str(curr_end_seed)+" "\
                        +str(0)+" "\
                        +str(earliest_lineage_start_time)+" "\
                        +str(latest_lineage_start_time)+" "\
                        +str(events_end_time)+" "

        curr_list.append(validate_rate_command)
        command_count += 1

        for i in range(0,len(curr_list)):
            if run_modes[m] == 2:
                curr_list[i] = curr_list[i] + original_theta_string
            if run_modes[m] == 0:
                curr_list[i] = curr_list[i] + stochastic_theta_string
            if run_modes[m] == 1:
                curr_list[i] = curr_list[i] + deterministic_theta_string

        command_list = command_list + curr_list

    # Use processes equal to the number of cpus available
    cpu_count = multiprocessing.cpu_count()

    print(command_list)

    print("Starting simulations with " + str(cpu_count) + " processes")

    # Generate a pool of workers
    pool = multiprocessing.Pool(processes=cpu_count)

```

```
    # Pass the list of bash commands to the pool, block until pool is complete
    pool.map(execute_command, command_list, 1)


# This is a helper function for run_simulation that runs bash commands in separate processes
def execute_command(cmd):
    print("Executing command: " + cmd)
    return subprocess.call(cmd, shell=True)


if __name__ == "__main__":
    main()
```

17.1.8 /python_fixtures/Kolmogorov_fixture.py

```
import multiprocessing
import os
import subprocess
import datetime
import numpy as np
from imageio.plugins._bsdf import BsdfSerializer

gomes_executable = '/home/main/chaste_build/projects/ISP/apps/GomesSimulator'
he_executable = '/home/main/chaste_build/projects/ISP/apps/HeSimulator'
boije_executable = '/home/main/chaste_build/projects/ISP/apps/BoijeSimulator'

empirical_data = '/home/main/git/chaste/projects/ISP/empirical_data/empirical_lineages.csv'

if not(os.path.isfile(gomes_executable)):
    raise Exception('Could not find executable: ' + gomes_executable)
if not(os.path.isfile(he_executable)):
    raise Exception('Could not find executable: ' + he_executable)
if not(os.path.isfile(boije_executable)):
    raise Exception('Could not find executable: ' + boije_executable)

#########################
# SIMULATION PARAMETERS
```

```
##########################

#Define start and end RNG seeds; determines:
#No. lineages per loss function run
#unique sequence of RNG results for each lineage

directory = "KolmogorovSequences"
empirical_sequences_name = "EOsequences"
output_mode = 2 #sequence sampler
debug_output = 0 #0=off;1=on
start_seed = 0
end_seed = 9999

##########################
#GLOBAL MODEL PARAMETERS
##########################

number_traversal_lineages = 60

############################
# SPECIFIC MODEL PARAMETERS
############################

#GOMES MODEL
g_end_time = 480
g_normal_mean = 3.9716
g_normal_sigma = .32839
g_pPP = .055
g_pPD = .221
g_pBC = .128
g_pAC = .106
g_pMG = .028

g_string = gomes_executable + " " + directory + " Gomes " + str(output_mode) + " " + str(debug_output) + " " + str(start_seed) + " " + str(end_seed) + " " + str(g_end_time) + " " + str(g_normal_mean) + " " + str(g_normal_sigma) + " " + str(g_pPP) + " " + str(g_pPD) + " " + str(g_pBC) + " " + str(g_pAC) + " " + str(g_pMG)

#HE MODEL - GENERAL
h_fixture = 2
h_ath5founder = 0
```

```
h_start_time = 0
h_end_time = 80

#HE MODEL - STOCHASTIC
h_deterministic_mode = 0
h_mitotic_mode_phase_2 = 8 #These are phase lengths, so
h_mitotic_mode_phase_3 = 7 #Phase3 boundary = mmp2 + mmp3
h_phase_1_pPP = 1.0
h_phase_1_pPD = 0.0
h_phase_2_pPP = 0.2
h_phase_2_pPD = 0.4
h_phase_3_pPP = 0.2
h_phase_3_pPD = 0.0

h_string = he_executable + " " + directory + " He " + str(output_mode) +
↪  " " + str(h_deterministic_mode) + " " + str(h_fixture) + " " +
↪  str(h_ath5founder) + " " + str(debug_output) + " " + str(start_seed)
↪  + " " + str(end_seed) + " " + str(h_start_time) + " " +
↪  str(h_start_time) + " " + str(h_start_time+1) + " " + str(h_end_time)
↪  + " " + str(h_mitotic_mode_phase_2) + " " +
↪  str(h_mitotic_mode_phase_3) + " " +str(h_phase_1_pPP) + " " +
↪  str(h_phase_1_pPD) + " " + str(h_phase_2_pPP) + " " +
↪  str(h_phase_2_pPD) + " " + str(h_phase_3_pPP) + " " +
↪  str(h_phase_3_pPD)

#HE MODEL -STOCHASTIC (REFIT)
hr_mitotic_mode_phase_2 = 4.1483 #These are phase lengths, so
hr_mitotic_mode_phase_3 = 11.6416 #Phase3 boundary = mmp2 + mmp3
hr_phase_1_pPP = 1.0
hr_phase_1_pPD = 0.0
hr_phase_2_pPP = 0.1959
hr_phase_2_pPD = 0.5168
hr_phase_3_pPP = 0.2934
hr_phase_3_pPD = 0.0
```

```
hr_string = he_executable + " " + directory + " HeRefit " + str(output_mode) + " " + str(h_deterministic_mode) + " " + str(h_fixture) + " " + str(h_ath5founder) + " " + str(debug_output) + " " + str(start_seed) + " " + str(end_seed) + " " + str(h_start_time) + " " + str(h_start_time) + " " + str(h_start_time+1) + " " + str(h_end_time) + " " + str(hr_mitotic_mode_phase_2) + " " + str(hr_mitotic_mode_phase_3) + " " +str(hr_phase_1_pPP) + " " + str(hr_phase_1_pPD) + " " + str(hr_phase_2_pPP) + " " + str(hr_phase_2_pPD) + " " + str(hr_phase_3_pPP) + " " + str(hr_phase_3_pPD)

#HE MODEL - DETERMINISTIC ALTERNATIVE
d_deterministic_mode = 1
#Phase boundary shift parameters
d_phase_1_shape = 3.7371
d_phase_1_scale = 1.8114
d_phase_2_shape = 2.5769
d_phase_2_scale = 1.6814
d_phase_sister_shift_widths = 1.6326
d_phase_offset = 1.2333

d_string = he_executable + " " + directory + " Deterministic " + str(output_mode) + " " + str(d_deterministic_mode) + " " + str(h_fixture) + " " + str(h_ath5founder) + " " + str(debug_output) + " " + str(start_seed) + " " + str(end_seed) + " " + str(h_start_time) + " " + str(h_start_time) + " " + str(h_start_time+1) + " " + str(h_end_time) + " " + str(d_phase_1_shape) + " " + str(d_phase_1_scale) + " " + str(d_phase_2_shape) + " " + str(d_phase_2_scale) + " " + str(d_phase_sister_shift_widths) + " " + str(d_phase_offset)

#BOIJE MODEL
b_end_generation = 250
b_phase_2_generation = 3
b_phase_3_generation = 5
b_pAtoh7 = .32
b_pPtf1a = .3
b_png = .8
```

```
b_string = boije_executable + " " + directory + " Boije " + str(output_mode) + " " + str(debug_output) + " " + str(start_seed) + " " + str(end_seed) + " " + str(b_end_generation) + " " + str(b_phase_2_generation) + " " + str(b_phase_3_generation) + " " + str(b_pAtoh7) + " " + str(b_pPtf1a) + " " + str(b_png)

#setup the log file, appending to any existing results
e_filename = "/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" +directory +"/" + empirical_sequences_name
os.makedirs(os.path.dirname(e_filename), exist_ok=True)

e_file = open(e_filename,"w")
e_file.write("Entry\tSequence\n")

def main():

    command_list = []
    command_list.append(g_string)
    command_list.append(h_string)
    command_list.append(hr_string)
    command_list.append(d_string)
    command_list.append(b_string)

    # Use processes equal to the number of cpus available
    cpu_count = multiprocessing.cpu_count()

    print(command_list)

    print("Starting simulations with " + str(cpu_count) + " processes")

    # Generate a pool of workers
    pool = multiprocessing.Pool(processes=cpu_count)

    # Pass the list of bash commands to the pool, block until pool is complete
    pool.map(execute_command, command_list, 1)

    traverse_lineages(empirical_data)

def traverse_lineages(data_filename):

    mode_sequences = [None]*(end_seed+1)
```

```python

    #load lineage tracing data
    e_data = np.loadtxt(data_filename, skiprows = 1, usecols = (0,1,2,3))

    #reproducible RNG
    p_RNG = np.random.RandomState(seed = 0)

    k=0

    for curr_seed in range (start_seed,end_seed+1):
        random_lineage_number = p_RNG.randint(1,number_traversal_lineages)
        lineage_events = np.array(e_data[np.where(e_data[:,0]==random_lineage_number)])

        #find mitotic mode of first event and write to log
        current_event = 1
        mitotic_mode = int(lineage_events[np.where(lineage_events[:,1]==current_event)][:,3])
        mode_sequences[k] = f'{mitotic_mode:.0f}'

        while mitotic_mode != 2:
            if mitotic_mode == 1 and p_RNG.random_sample() < .5: break

            child_events = lineage_events[np.where(lineage_events[:,2]==current_event)]
            random_child_row = p_RNG.randint(0,np.ma.size(child_events,0))
            current_event= child_events[random_child_row,1]
            mitotic_mode = child_events[random_child_row,3]
            mode_sequences[k] = mode_sequences[k] + f'{mitotic_mode:.0f}'

        e_file.write(str(k+1)+"\t"+mode_sequences[k]+"\n")
        k += 1

    return mode_sequences

# This is a helper function for run_simulation that runs bash commands in separate processes
def execute_command(cmd):
    print("Executing command: " + cmd)
    return subprocess.call(cmd, shell=True)

```

```
if __name__ == "__main__":
    main()
```

17.1.9 /python_fixtures/SPSA_fixture.py

```
import multiprocessing
import os
import subprocess
import datetime

import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import bernoulli
from fileinput import filename
from statsmodels.sandbox.distributions.gof_new import a_st70_upp

executable = '/home/main/chaste_build/projects/ISP/apps/HeSimulator'

if not(os.path.isfile(executable)):
    raise Exception('Could not find executable: ' + executable)

#####################
# SPSA COEFFICIENTS
#####################

a_s = .025
c_s = .1

a_d = .0019
c_d = .1

A = 20 #10% of expected # iterations
alpha = .602
gamma = .101

###########################
# SPSA & AIC UTILITY VARS
###########################

max_iterations = 199
```

```
number_comparisons = 3030 #total number of comparison points between model output and empirical data (1000 per induction time, 10 per rate mode)
number_comparisons_per_induction = 1000
error_samples = 5000 #number of samples to draw when estimating plausibility interval for simulations

#########################
# SIMULATION PARAMETERS
#########################

#Define start and end RNG seeds; determines:
#No. lineages per loss function run
#unique sequence of RNG results for each lineage
start_seed = 0
end_seed = 249
rate_end_seed = 99
directory_name = "SPSA"
file_name_he = "HeSPSA"
file_name_det = "DetSPSA"
log_name = "HeSPSAOutput"
deterministic_modes = [0, 1] #1=det. mode enabled
count_output_mode = 0 #0=lineage counts;1=mitotic event logging;2=sequence sampling
event_output_mode = 1
fixture = 0 #0=He 2012;1=Wan 2016
debug_output = 0 #0=off;1=on
ath5founder = 0 #0=no morpholino 1=ath5 morpholino

##########################
#GLOBAL MODEL PARAMETERS
##########################

#Values defining different marker induction timepoints & relative start time of TiL counter
earliest_lineage_start_time = 23.0 #RPCs begin to enter "He model regime" nasally at 23hpf
latest_lineage_start_time = 39.0 #last temporal retinal RPC has entered "He model regime" at 39hpf
induction_times = [ 24, 32, 48 ]
end_time = 72.0
rate_end_time = 80

```

```
########################################
# SPECIFIC MODEL PARAMETERS - THETAHAT
########################################

#STOCHASTIC MITOTIC MODE
#mitotic mode per-phase probabilities
#3-phase mitotic mode time periodisation
mitotic_mode_phase_2 = 8 #These are phase lengths, so
mitotic_mode_phase_3 = 7 #Phase3 boundary = mmp2 + mmp3
phase_1_pPP = 1.0
phase_1_pPD = 0.0
phase_2_pPP = 0.2
phase_2_pPD = 0.4
phase_3_pPP = 0.2
phase_3_pPD = 0.0
he_model_params = 15

#DETERMINISTIC MITOTIC MODE
#Phase boundary shift parameters
phase_1_shape = 3
phase_1_scale = 2
phase_2_shape = 2
phase_2_scale = 2
phase_sister_shift_widths = .25
phase_offset = 0
det_model_params = 13

#array "theta_spsa" is manipulated during SPSA calculations, these are starting point references
stochastic_theta_zero = np.array([mitotic_mode_phase_2, mitotic_mode_phase_3, phase_2_pPP, phase_2_pPD, phase_3_pPP ])
determinsitic_theta_zero = np.array([phase_1_shape, phase_1_scale, phase_2_shape, phase_2_scale, phase_sister_shift_widths, phase_offset])

#scales ak gain sequence for probability variables
prob_scale_vector = np.array([ 1, 1, .1, .1, .1])
shift_scale_vector = np.array([1, 1, 1, 1, 1, 3])

##############################
# HE ET AL EMPIRICAL RESULTS
##############################

```

```
count_bin_sequence = np.arange(1,32,1)
count_x_sequence = np.arange(1,31,1)
count_trim_value = 30

rate_bin_sequence = np.arange(30,85,5)
rate_x_sequence = np.arange(30,80,5)
rate_trim_value = 10

#Count probability arrays
raw_counts = np.loadtxt('/home/main/git/chaste/projects/ISP/empirical_data/empirical_counts.cs
skiprows=1, usecols=(3,4,5,6,7,8,9,10)) #collect the per-cell-type counts
raw_counts_24 = raw_counts[0:64,:]
raw_counts_32 = raw_counts[64:233,:]
raw_counts_48 = raw_counts[233:396,:]

prob_24,bin_edges = np.histogram(np.sum(raw_counts_24,axis=1),count_bin_sequence,density=True) #obtain probability density histogram for counts, retabulating by summing across types
prob_32,bin_edges = np.histogram(np.sum(raw_counts_32,axis=1),count_bin_sequence,density=True)
prob_48,bin_edges = np.histogram(np.sum(raw_counts_48,axis=1),count_bin_sequence,density=True)

#extend histogram to 1000 - allows large lineages of some iterates to be included in AIC comparison
prob_empirical_24 = np.concatenate([prob_24, np.zeros(int(number_comparisons_per_induction) - prob_24.size)])
prob_empirical_32 = np.concatenate([prob_32, np.zeros(int(number_comparisons_per_induction) - prob_32.size)])
prob_empirical_48 = np.concatenate([prob_48, np.zeros(int(number_comparisons_per_induction) - prob_48.size)])

count_prob_list = [prob_empirical_24, prob_empirical_32, prob_empirical_48]

#no. of lineages observed per induction timepoint/event group
lineages_sampled_24 = 64
lineages_sampled_32 = 169
lineages_sampled_48 = 163
lineages_sampled_events = 60
```

```
lineages_sampled_list = [lineages_sampled_24,lineages_sampled_32,lineages_sampled_48]

#Mitotic mode rate probability arrays
raw_events = np.loadtxt('/home/main/git/chaste/projects/ISP/empirical_data/empirical_lineages.
skiprows=1, usecols=(3,5,8))
observed_events = raw_events[np.where(raw_events[:,2]==1)] #exclude any mitosis whose time was too early for recording

observed_PP = observed_events[np.where(observed_events[:,0]==0)]
observed_PD = observed_events[np.where(observed_events[:,0]==1)]
observed_DD = observed_events[np.where(observed_events[:,0]==2)]

histo_PP,bin_edges = np.histogram(observed_PP,rate_bin_sequence,density=False)
histo_PD,bin_edges = np.histogram(observed_PD,rate_bin_sequence,density=False)
histo_DD,bin_edges = np.histogram(observed_DD,rate_bin_sequence,density=False)

#hourly per-lineage probabilities- NOT probability density function
prob_empirical_PP = np.array((histo_PP/lineages_sampled_events)/5)
prob_empirical_PD = np.array((histo_PD/lineages_sampled_events)/5)
prob_empirical_DD = np.array((histo_DD/lineages_sampled_events)/5)

event_prob_list = [prob_empirical_PP,prob_empirical_PD,prob_empirical_DD]

#setup the log file, appending to any existing results
log_filename = "/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" +directory_name +"/" + log_name
os.makedirs(os.path.dirname(log_filename), exist_ok=True)

log = open(log_filename,"w")

#plotting utility stuff
#interactive plot mode
plt.ion()
#setup new iterate plot
fig, ((plt0, plt1, plt2),(plt3, plt4, plt5)) = plt.subplots(2,3,figsize=(12,6))
plot_list = [plt0,plt1,plt2,plt3,plt4,plt5]
```

```


def main():
    global theta_spsa,
     ↪  a,c,file_name,scale_vector,end_seed,rate_end_seed,alpha,gamma

    for m in range(0,len(deterministic_modes)):

        #traceable RNG
        p_RNG = np.random.RandomState(seed=786)

        deterministic_mode = deterministic_modes[m]
        if deterministic_mode == 0:
            theta_spsa = stochastic_theta_zero
            a = a_s
            c = c_s
            file_name = file_name_he
            scale_vector = prob_scale_vector
            now = datetime.datetime.now()
            log.write("Began SPSA optimisation of He model @\n" +
             ↪  str(datetime.datetime.now()) + "\n")
            log.write("k\tphase2\tphase3\tPP2\tPD2\tPP3\n")
            number_params = he_model_params
        if deterministic_mode == 1:
            theta_spsa = determinsitic_theta_zero
            a = a_d
            c = c_d
            file_name = file_name_det
            scale_vector = shift_scale_vector
            now = datetime.datetime.now()
            log.write("Began SPSA optimisation of deterministic model
             ↪  @\n" + str(datetime.datetime.now()) + "\n")
            log.write("k\tp1Sh\tp1Sc\tp2Sh\tp2Sc\tsisterShift\toffset\n")
            number_params = det_model_params

        #SPSA algorithm iterator k starts at 0
        k=0

        while k ≤ max_iterations:
            if k == 0:
                end_seed = 249
                rate_end_seed = 99

```

```
            #@defined iterate, increase # of seed to decrease RNG noise to low level
            if k == 169:
                end_seed = 999
                rate_end_seed = 249


            #@defined iterate, increase # of seeds to decrease RNG noise to close to nil, switch to asymptotically optimal alpha and gamma
            if k == 189:
                end_seed = 4999
                rate_end_seed = 1249
                alpha = 1
                gamma = (1/6)

            #write the parameter set to be evaluated to file
            if deterministic_mode == 0: log.write(str(k) + "\t" + str(theta_spsa[0]) + "\t" + str(theta_spsa[1]) + "\t" + str(theta_spsa[2]) + "\t" + str(theta_spsa[3]) + "\t" + str(theta_spsa[4]) + "\n")
            if deterministic_mode == 1: log.write(str(k) + "\t" + str(theta_spsa[0]) + "\t" + str(theta_spsa[1]) + "\t" + str(theta_spsa[2]) + "\t" + str(theta_spsa[3]) + "\t" + str(theta_spsa[4])+ "\t" + str(theta_spsa[5]) + "\n")

            #populate the deltak perturbation vector with samples from a .5p bernoulli +1 -1 distribution
            delta_k = bernoulli.rvs(.5, size=theta_spsa.size, random_state=p_RNG)
            delta_k[delta_k == 0] = -1

            ak = a / ((A + k + 1)**alpha) #calculate ak from gain sequence
            scaled_ak = ak * scale_vector #scale ak appropriately for parameters expressed in hrs & percent

            ck = c / ((k + 1)**gamma) #calculate ck from gain sequence
            scaled_ck = ck * scale_vector

            #Project theta_spsa into space bounded by ck to allow gradient sampling at bounds of probability space
```

```
            projected_theta = project_theta(theta_spsa, scaled_ck, deterministic_mode)

            #Calculate theta+ and theta- vectors for gradient estimate
            theta_plus = projected_theta + scaled_ck * delta_k
            theta_minus = projected_theta - scaled_ck * delta_k

            AIC_gradient = evaluate_AIC_gradient(k, theta_plus, theta_minus, deterministic_mode, number_params, file_name)

            ghat = ((AIC_gradient) / (2 * ck)) * delta_k

            log.write("ghat0: " + str(ghat[0]) + "\n")

            #update new theta_spsa
            theta_spsa = theta_spsa - scaled_ak * ghat

            #constrain updated theta_spsa
            theta_spsa = project_theta(theta_spsa, np.zeros(theta_spsa.size), deterministic_mode)

            k+=1

        #write final result and close log
        if deterministic_mode == 0: log.write(str(k) + "\t" + str(theta_spsa[0]) + "\t" + str(theta_spsa[1]) + "\t" + str(theta_spsa[2]) + "\t" + str(theta_spsa[3]) + "\t" + str(theta_spsa[4]) + "\n")
        if deterministic_mode == 1: log.write(str(k) + "\t" + str(theta_spsa[0]) + "\t" + str(theta_spsa[1]) + "\t" + str(theta_spsa[2]) + "\t" + str(theta_spsa[3]) + "\t" + str(theta_spsa[4])+ "\t" + str(theta_spsa[5]) + "\n")

    log.close

def project_theta(theta, boundary, deterministic_mode):
# Required projection is onto unit triangle modified by boundary (ck value)
# Values outside the lower bounds are reset first
# Then, for phase 2 probabilities, we project (PPa,PDa)→(PPb,PDb) such that if( (PPa+ck) + (PDa+ck) >1), PPb+ck + PDb +ck = 1.
# This takes care of the upper bound
```

```
    #project all negative parameters back into bounded space
    if deterministic_mode == False:
        for i in range(0, theta.size):
            if i == 0:
                if theta[i] < boundary[i] + 4.0: theta[i] = boundary[i] + 4.0 #project mitotic_mode_phase_2 to a minimum of 4- below this param has no effect (due to refractory period after first division)
            else:
                if theta[i] < boundary[i]: theta[i] = boundary[i]

        #if PP3 exceeds 1-boundary, project back to 1-boundary
        if theta[4] > 1 - boundary[4]: theta[4] = 1 - boundary[4]

        if theta[2] + theta[3] > 1 - boundary[2]: #if pPP2 + pPD2 gives a total beyond the current boundary-reduced total of 1.0

            if theta[2] > (1 - 2 * boundary[2]) and theta[2] ≤ theta[3] - (1 - 2 * boundary[2]): # these (PP,PD) points will project into negative PD space
                theta[2] = 1 - 2 * boundary[2]
                theta[3] = boundary[2]

            elif theta[3] > (1 - 2 * boundary[2]) and theta[2] ≤ theta[3] - (1 - 2 * boundary[2]): # these (PP,PD) points will project into negative PP space
                theta[3] = 1 - 2 * boundary[2]
                theta[2] = boundary[2]

            else: # the (PP,PD) points can be projected onto the line PD = -PP + 1 and modified by the boundary;
                v1 = [ 1, -1 ] # vector from (0,1) to (1,0)
                v2 = [ theta[2], theta[3] - 1 ] #vector from (0,1) to (PP,PD)
                dot_product = np.dot(v1,v2)
                lengthv1 = 2
                theta[2] = (dot_product / lengthv1) - boundary[2]
                theta[3] = (1 - dot_product / lengthv1) - boundary[2]

    if deterministic_mode == True:
        for i in range(0, theta.size-1): #exclude offset param
```

```
            if theta[i] < boundary[i]: theta[i] = boundary[i] + 0.1 #prevents non→0 arguments for deterministic mode

        #projects sister shift value such that 95% of sister shift values will be less than smallest mean phase time
        minPhase = min(theta[0]*theta[1],theta[2]*theta[3])
        if theta[4] > minPhase/2 - boundary[4]: theta[4] = minPhase/2 - boundary[4]

    return theta

def evaluate_AIC_gradient(k, theta_plus, theta_minus, deterministic_mode, number_params, file_name):
    #Form the simulator commands for current thetas and deterministic modes
    command_list = []
    base_command = executable

    rate_settings = str(event_output_mode)+" "\
                    +str(deterministic_mode)+" "\
                    +str(fixture)+" "\
                    +str(ath5founder)+" "\
                    +str(debug_output)+" "\
                    +str(start_seed)+" "\
                    +str(rate_end_seed)+" "\
                    +str(earliest_lineage_start_time)+" "\
                    +str(earliest_lineage_start_time)+" "\
                    +str(latest_lineage_start_time)+" "\
                    +str(rate_end_time)+" "

    count_settings_1 = str(count_output_mode)+" "\
                       +str(deterministic_mode)+" "\
                       +str(fixture)+" "\
                       +str(ath5founder)+" "\
                       +str(debug_output)+" "\
                       +str(start_seed)+" "\
                       +str(end_seed)+" "

    count_settings_2 = str(earliest_lineage_start_time)+" "\
                       +str(latest_lineage_start_time)+" "\
                       +str(end_time)+" "

    if deterministic_mode == 0:
```

```
343
344         stochastic_params_plus = str(theta_plus[0])+" "\
345                     +str(theta_plus[1])+" "\
346                     +str(phase_1_pPP)+" "\
347                     +str(phase_1_pPD)+" "\
348                     +str(theta_plus[2])+" "\
349                     +str(theta_plus[3])+" "\
350                     +str(theta_plus[4])+" "\
351                     +str(phase_3_pPD)
352
353         stochastic_params_minus = str(theta_minus[0])+" "\
354                     +str(theta_minus[1])+" "\
355                     +str(phase_1_pPP)+" "\
356                     +str(phase_1_pPD)+" "\
357                     +str(theta_minus[2])+" "\
358                     +str(theta_minus[3])+" "\
359                     +str(theta_minus[4])+" "\
360                     +str(phase_3_pPD)
361
362         command_rate_plus = base_command\
363                     +" "+directory_name+" "+ file_name+"RatePlus "\
364                     +rate_settings\
365                     +stochastic_params_plus
366
367         command_rate_minus = base_command\
368                     +" "+directory_name+" "+ file_name+"RateMinus "\
369                     +rate_settings\
370                     +stochastic_params_minus
371
372         command_list.append(command_rate_plus)
373         command_list.append(command_rate_minus)
374
375         for i in range(0,len(induction_times)):
376             command_plus = base_command\
377                         +" "+directory_name+"
                             ↪  "+file_name+str(induction_times[i])+"Plus "\
378                         +count_settings_1\
379                         +str(induction_times[i])+" "\
380                         +count_settings_2\
381                         +stochastic_params_plus
382
383             command_minus = base_command\
```

```
                        +" "+directory_name+"
                         ↪  "+file_name+str(induction_times[i])+"Minus "\
                        +count_settings_1\
                        +str(induction_times[i])+" "\
                        +count_settings_2\
                        +stochastic_params_minus

            command_list.append(command_plus)
            command_list.append(command_minus)

    if deterministic_mode == 1:

        deterministic_params_plus = str(theta_plus[0])+" "\
                    +str(theta_plus[1])+" "\
                    +str(theta_plus[2])+" "\
                    +str(theta_plus[3])+" "\
                    +str(theta_plus[4])+" "\
                    +str(theta_plus[5])

        deterministic_params_minus = str(theta_minus[0])+" "\
                    +str(theta_minus[1])+" "\
                    +str(theta_minus[2])+" "\
                    +str(theta_minus[3])+" "\
                    +str(theta_minus[4])+" "\
                    +str(theta_minus[5])

        command_rate_plus = base_command\
                    +" "+directory_name+" "+ file_name+"RatePlus "\
                    +rate_settings\
                    +deterministic_params_plus

        command_rate_minus = base_command\
                    +" "+directory_name+" "+ file_name+"RateMinus "\
                    +rate_settings\
                    +deterministic_params_minus

        command_list.append(command_rate_plus)
        command_list.append(command_rate_minus)

        for i in range(0,len(induction_times)):
            command_plus = base_command\
                        +" "+directory_name+"
                         ↪  "+file_name+str(induction_times[i])+"Plus "\
```

```
                        +count_settings_1\
                        +str(induction_times[i])+" "\
                        +count_settings_2\
                        +deterministic_params_plus

            command_minus = base_command\
                        +" "+directory_name+"
                         ↪  "+file_name+str(induction_times[i])+"Minus "\
                        +count_settings_1\
                        +str(induction_times[i])+" "\
                        +count_settings_2\
                        +deterministic_params_minus

            command_list.append(command_plus)
            command_list.append(command_minus)


    # Use processes equal to the number of cpus available
    cpu_count = multiprocessing.cpu_count()

    print("Starting simulations for iterate " + str(k) + " with " +
     ↪  str(cpu_count) + " processes, " + str(end_seed+1) + " lineages
     ↪  simulated for counts, " + str(rate_end_seed+1) + " lineages for
     ↪  events")

    log.flush() #required to prevent pool from jamming up log for some
     ↪  reason

    # Generate a pool of workers
    pool = multiprocessing.Pool(processes=cpu_count)

    # Pass the list of bash commands to the pool, block until pool is
     ↪  complete
    pool.map(execute_command, command_list, 1)

    #these numpy arrays hold the individual RSS vals for timepoints +
     ↪  rates
    rss_plus = np.zeros(len(induction_times)+3)
    rss_minus = np.zeros(len(induction_times)+3)

    #clear the plots on the interactive figure
    for i in range(0, len(plot_list)):
        plt.sca(plot_list[i])
```

```
        plt.cla()

    #calculate RSS for each induction timepoint
    for i in range(0,len(induction_times)):
        counts_plus = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" + directory_name + "/" + file_name + str(induction_times[i]) + "Plus", skiprows=1, usecols=3)
        counts_minus = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" + directory_name + "/" + file_name + str(induction_times[i]) + "Minus", skiprows=1, usecols=3)
        prob_histo_plus, bin_edges = np.histogram(counts_plus, bins=1000, range=(1,1001),density=True)
        prob_histo_minus, bin_edges = np.histogram(counts_minus, bins=1000, range=(1,1001),density=True)

        residual_plus = prob_histo_plus - count_prob_list[i]
        residual_minus = prob_histo_minus - count_prob_list[i]

        rss_plus[i] = np.sum(np.square(residual_plus))
        rss_minus[i] = np.sum(np.square(residual_minus))

        plotter(plot_list[i], counts_plus, counts_minus, count_prob_list[i],lineages_sampled_list[i], 0)


    rates_plus = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" + directory_name + "/" + file_name + "RatePlus", skiprows=1, usecols=(0,3))
    rates_minus = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/" + directory_name + "/" + file_name + "RateMinus", skiprows=1, usecols=(0,3))

    for i in range (0,3):
        mode_rate_plus = np.array(rates_plus[np.where(rates_plus[:,1]==i)])
        mode_rate_minus = np.array(rates_minus[np.where(rates_minus[:,1]==i)])

```

```
        histo_rate_plus, bin_edges = np.histogram(mode_rate_plus[:,0], rate_bin_sequence, density=False)
        histo_rate_minus, bin_edges = np.histogram(mode_rate_minus[:,0], rate_bin_sequence, density=False)

        #hourly per-lineage probabilities
        prob_histo_rate_plus = np.array(histo_rate_plus / ((rate_end_seed + 1)*5))
        prob_histo_rate_minus = np.array(histo_rate_plus / ((rate_end_seed + 1)*5))

        residual_plus = prob_histo_rate_plus - event_prob_list[i]
        residual_minus = prob_histo_rate_minus - event_prob_list[i]

        #PD RESIUDAL WEIGHTING
        if i == 1:
            rss_plus[i+3] = np.sum(1.5*np.square(residual_plus))
            rss_minus[i+3] = np.sum(1.5*np.square(residual_minus))

        else:
            rss_plus[i+3] = np.sum(np.square(residual_plus))
            rss_minus[i+3] = np.sum(np.square(residual_minus))

        plotter(plot_list[i+3], mode_rate_plus[:,0], mode_rate_minus[:,0], event_prob_list[i],lineages_sampled_events, 1)

    AIC_plus = 2 * number_params + number_comparisons * np.log(np.sum(rss_plus))
    AIC_minus = 2 * number_params + number_comparisons * np.log(np.sum(rss_minus))

    plt.show()

    plt.savefig("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/"+directory_name +"/" + "SDmode" + str(deterministic_mode) + "iterate" + str(k) + ".png")

    log.write("theta_plus:\n")
    if deterministic_mode == 0: log.write(str(k) + "\t" + str(theta_plus[0]) + "\t" + str(theta_plus[1]) + "\t" + str(theta_plus[2]) + "\t" + str(theta_plus[3]) + "\t" + str(theta_plus[4]) + "\n")
```

```
    if deterministic_mode == 1: log.write(str(k) + "\t" + str(theta_plus[0]) + "\t" + str(theta_plus[1]) + "\t" + str(theta_plus[2]) + "\t" + str(theta_plus[3]) + "\t" + str(theta_plus[4]) + "\t" + str(theta_plus[5]) +"\n")
    log.write("theta_minus:\n")
    if deterministic_mode == 0: log.write(str(k) + "\t" + str(theta_minus[0]) + "\t" + str(theta_minus[1]) + "\t" + str(theta_minus[2]) + "\t" + str(theta_minus[3]) + "\t" + str(theta_minus[4]) + "\n")
    if deterministic_mode == 1: log.write(str(k) + "\t" + str(theta_minus[0]) + "\t" + str(theta_minus[1]) + "\t" + str(theta_minus[2]) + "\t" + str(theta_minus[3]) + "\t" + str(theta_minus[4]) + "\t" + str(theta_minus[5]) + "\n")

    log.write("PositiveAIC: " + str(AIC_plus) + " NegativeAIC: " + str(AIC_minus) + "\n")

    AIC_gradient_sample = AIC_plus - AIC_minus;

    return AIC_gradient_sample

#data plotter function for monitoring SPSA results
def plotter(subplot,plus,minus,empirical_prob,samples,mode):
    global plt0,plt1,plt2,plt3,plt4,plt5, prob_histo_plus, prob_histo_minus
    bin_sequence = []
    x_sequence = []
    trim_value = 0

    if mode == 0:
        bin_sequence = count_bin_sequence
        x_sequence = count_x_sequence
        trim_value = count_trim_value
        prob_histo_plus, bin_edges = np.histogram(plus, bin_sequence, density=True)
        prob_histo_minus, bin_edges = np.histogram(minus, bin_sequence, density=True)

    if mode == 1:
        bin_sequence = rate_bin_sequence
        x_sequence = rate_x_sequence
        trim_value = rate_trim_value
```

```
        histo_plus, bin_edges = np.histogram(plus, bin_sequence, density=False)
        histo_minus, bin_edges = np.histogram(minus, bin_sequence, density=False)
        prob_histo_plus = np.array(histo_plus / ((rate_end_seed + 1)*5))
        prob_histo_minus = np.array(histo_plus / ((rate_end_seed + 1)*5))

    trimmed_prob = empirical_prob[0:trim_value]

    subplot.plot(x_sequence,trimmed_prob, 'k+')
    plt.pause(0.0001)

    interval_plus = sampler(plus,samples,bin_sequence)

    subplot.plot(x_sequence,prob_histo_plus, 'g-')
    plt.pause(0.0001)
    subplot.fill_between(x_sequence, (prob_histo_plus - interval_plus), (prob_histo_plus + interval_plus), alpha=0.2, edgecolor='#008000', facecolor='#00FF00')
    plt.pause(0.0001)

    interval_minus = sampler(minus,samples,bin_sequence)

    subplot.plot(x_sequence,prob_histo_minus, 'm-')
    plt.pause(0.0001)
    subplot.fill_between(x_sequence, (prob_histo_minus - interval_minus), (prob_histo_minus + interval_minus), alpha=0.2, edgecolor='#800080', facecolor='#FF00FF')
    plt.pause(0.0001)

    if subplot==plt0: plt0.set_ylim((0,.20))
    if subplot==plt1: plt1.set_ylim(0,.20)
    if subplot==plt2: plt2.set_ylim(0,.7)
    if subplot==plt3: plt3.set_ylim(0, .30)
    if subplot==plt4: plt4.set_ylim(0, .15)
    if subplot==plt5: plt5.set_ylim(0, .35)

def sampler(data,samples,bin_sequence):

    if len(data) == 0: #catches edge case w/ no entries for a mitotic mode
        data=[0]

```

```
    base_sample=np.zeros((error_samples,len(bin_sequence)-1))

    for i in range(0,error_samples):
        new_data_sample = np.random.choice(data,samples)
        new_histo_prob, bin_edges = np.histogram(new_data_sample,
        ↪  bin_sequence, density=True)
        base_sample[i,:] = new_histo_prob

    sample_95CI = np.array(2 * (np.std(base_sample,0)))

    return sample_95CI

# This is a helper function for run_simulation that runs bash commands in
↪  separate processes
def execute_command(cmd):
    return subprocess.call(cmd, shell=True)

if __name__ == "__main__":
    main()
```

17.1.10 /python_fixtures/Wan_output_fixture.py

```
import multiprocessing
import os
import subprocess
import datetime

import numpy as np
from openpyxl.styles.builtins import output
from sklearn.datasets.tests.test_svmlight_format import currdir

executable = '/home/main/chaste_build/projects/ISP/apps/WanSimulator'

if not(os.path.isfile(executable)):
    raise Exception('Could not find executable: ' + executable)

#########################
# SIMULATION PARAMETERS
#########################

#Define start and end RNG seeds; determines:
#No. simulated CMZs per run
```

```
#unique sequence of RNG results for each lineage
start_seed = 0
end_seed = 99 #total seeds should be divisible by # cores

output_directory = "WanOutput"

##########################
#MODEL PARAMETERS
##########################

#CMZ residency time
wan_residency_time = 17.0
#factor to divide 3dpf progenitor population by to obtain estimate of stem cell population
stem_divisor = 10
#3dpf CMZ progenitor population mean and standard deviation
progenitor_mean = 792
progenitor_std = 160

#stem cell cycle parameters- cell cycle RV is a shifted gamma distribution
stem_gamma_shift = 4
stem_gamma_shape = 6.5 #mean 30 hr stem cell time - in reality is probably more like 60+
stem_gamma_scale = 4

progenitor_gamma_shift = 4  #default He et al. values, mean 6 hr cycle time
progenitor_gamma_shape = 2
progenitor_gamma_scale = 1
progenitor_gamma_sister = 1

cmz_theta_string = str(wan_residency_time) + " " + str(stem_divisor) + " " + str(progenitor_mean) + " " + str(progenitor_std) + " " + str(stem_gamma_shift) + " " + str(stem_gamma_shape) + " " + str(stem_gamma_scale) + " " + str(progenitor_gamma_shift) + " " + str(progenitor_gamma_shape) + " " + str(progenitor_gamma_scale) + " " + str(progenitor_gamma_sister)

#STOCHASTIC MITOTIC MODE
#HE ORIGINAL PARAMETERS
#mitotic mode per-phase probabilities
#3-phase mitotic mode time periodisation
```

```
mitotic_mode_phase_2 = 8 #These are phase boundaries rather than lengths as in eg. He_output_fixture.py
mitotic_mode_phase_3 = 15
phase_1_pPP = 1.0
phase_1_pPD = 0.0
phase_2_pPP = 0.2
phase_2_pPD = 0.4
phase_3_pPP = 0.2
phase_3_pPD = 0.0

stochastic_theta_string = str(mitotic_mode_phase_2) + " " + str(mitotic_mode_phase_3) + " " +str(phase_1_pPP) + " " + str(phase_1_pPD) + " " + str(phase_2_pPP) + " " + str(phase_2_pPD) + " " + str(phase_3_pPP) + " " + str(phase_3_pPD)

def main():

    command_list = []

    # Use processes equal to the number of cpus available
    cpu_count = multiprocessing.cpu_count()
    seeds_per_command = (end_seed + 1) / cpu_count
    base_command = executable + " " + output_directory

    for i in range(0,cpu_count):
        curr_start_seed = i * seeds_per_command
        curr_end_seed = curr_start_seed + (seeds_per_command - 1)

        command = base_command + " "\
            +str(curr_start_seed) + " "\
            +str(curr_end_seed) + " "\
            +cmz_theta_string + " "\
            +stochastic_theta_string

        #command_list.append(command)


        command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator WanOutput 18.0 24.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0 0.2 0.4 0.2 0.0")
```

```
88
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 42.0 49.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
89
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 51.0 57.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
90
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 94.0 99.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
91
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 58.0 64.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
92
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 65.0 70.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
93
      ↪ command_list.append("/home/main/chaste_build/projects/ISP/apps/WanSimulator
      ↪ WanOutput 71.0 74.0 17.0 10 792 160 4 6.5 4 4 2 1 1 8 15 1.0 0.0
      ↪ 0.2 0.4 0.2 0.0")
94
95      print("Starting simulations with " + str(cpu_count) + " processes")
96
97      # Generate a pool of workers
98      pool = multiprocessing.Pool(processes=cpu_count)
99
100     # Pass the list of bash commands to the pool, block until pool is
        ↪ complete
101     pool.map(execute_command, command_list, 1)
102
103
104 # This is a helper function for run_simulation that runs bash commands in
    ↪ separate processes
105 def execute_command(cmd):
106     print("Executing command: " + cmd)
107     return subprocess.call(cmd, shell=True)
108
109
110 if __name__ == "__main__":
```

```
    main()
```

17.1.11 /python_fixtures/diagram_utility_scripts/Gomes_He_cycle_plots.py

```
import os
import numpy as np
import matplotlib.pyplot as plt
from fileinput import filename
from scipy.stats import lognorm
from scipy.stats import gamma
from statsmodels.sandbox.distributions.gof_new import a_st70_upp

#Model parameters
gomes_normal_sigma = .32839
gomes_normal_mu = 3.9716

he_gamma_shape = 2
he_gamma_scale = 1
he_gamma_offset = 4

def main():
    gomes_x_range = np.arange(1,151,1)
    gomes_y = lognorm.pdf(gomes_x_range, gomes_normal_sigma, 0,
    ↪  np.exp(gomes_normal_mu))

    he_x_range = np.arange(1,16,.1)
    he_y = gamma.pdf(he_x_range, he_gamma_shape, he_gamma_offset,
    ↪  he_gamma_scale)

    fig, (ax_g, ax_h) = plt.subplots(1,2,figsize=(6,2))

    ax_g.plot(gomes_x_range,gomes_y,'k-')
    ax_h.plot(he_x_range, he_y, 'k-')

    plt.sca(ax_g)
    plt.xlabel("Cell cycle duration (h)")
    plt.ylabel("Probability")

    plt.sca(ax_h)
    plt.xlabel("Cell cycle duration (h)")
    plt.ylabel("Probability")
```

```python

    plt.tight_layout()
    plt.savefig('/home/main/Desktop/utility.png', transparent=True)
    plt.show()

if __name__ == "__main__":
    main()
```

17.1.12 /python_fixtures/diagram_utility_scripts/He_Boije_signal_plots.py

```python
import os
import numpy as np
import matplotlib.pyplot as plt
from fileinput import filename
from scipy.stats import lognorm
from scipy.stats import gamma
from statsmodels.sandbox.distributions.gof_new import a_st70_upp

#Model parameters
he_x = [0,8,15,25]
he_pp_y = [1,.2,.2,.2]
he_pd_y = [0,.4,0,0]
he_dd_y = [0,.4,.8,.8]

boije_x = [0,3,5,9]
boije_pAtoh7 = [0,.32,0,0]
boije_pPtf1a = [0,.3,0,0]
boije_pNg = [0,0,.8,.8]

def main():
    fig, ((pp,pd,dd))= plt.subplots(1,3,figsize=(6,2),sharey=True)

    fig2, ((pAtoh7),(pPtf1a),(pNg)) = plt.subplots(3,1,figsize=(2,4),sharex=True)

    pp.set_xlim((0,24))
    pd.set_xlim((0,24))
    dd.set_xlim((0,24))

    pp.set_xticks([8,15])
    pd.set_xticks([8,15])
```

```
    dd.set_xticks([8,15])

    pp.set_ylim((-.1,1.1))
    pp.set_yticks(np.arange(0,1.2,.2))
    pp.set_ylabel("Probability")
    pp.set_xlabel("TiL (h)")
    pp.axvline(8,linestyle='--',color='.1', alpha=.2)
    pp.axvline(15,linestyle='--',color='.1', alpha=.2)
    pp.step(he_x,he_pp_y, 'k-', where='post', linewidth=2)

    pd.set_ylim((-.1,1.1))
    pd.set_yticks(np.arange(0,1.2,.2))
    pd.set_xlabel("TiL (h)")
    pd.axvline(8,linestyle='--',color='.1', alpha=.2)
    pd.axvline(15,linestyle='--',color='.1', alpha=.2)
    pd.step(he_x,he_pd_y, 'k-', where='post', linewidth=2)

    dd.set_ylim((-.1,1.1))
    dd.set_yticks(np.arange(0,1.2,.2))
    dd.set_xlabel("TiL (h)")
    dd.axvline(8,linestyle='--',color='.1', alpha=.2)
    dd.axvline(15,linestyle='--',color='.1', alpha=.2)
    dd.step(he_x,he_dd_y, 'k-', where='post', linewidth=2)

    pAtoh7.set_xlim((0,8))
    pNg.set_xlim((0,8))
    pPtf1a.set_xlim((0,8))

    pAtoh7.set_xticks([3,5])
    pNg.set_xticks([3,5])
    pPtf1a.set_xticks([3,5])


    pNg.set_xlabel("Generation")


    pAtoh7.set_ylim((-.1,1.1))
    pAtoh7.set_yticks(np.arange(0,1.25,.25))
    pAtoh7.set_ylabel("Probability")
    pAtoh7.axvline(3,linestyle='--',color='.1',alpha=.2)
    pAtoh7.axvline(5,linestyle='--',color='.1',alpha=.2)
    pAtoh7.step(boije_x,boije_pAtoh7, 'k-', where='post', linewidth=2)

```

```
    pPtf1a.set_ylim((-.1,1.1))
    pPtf1a.set_yticks(np.arange(0,1.25,.25))
    pPtf1a.set_ylabel("Probability")
    pPtf1a.axvline(3,linestyle='--',color='.1',alpha=.2)
    pPtf1a.axvline(5,linestyle='--',color='.1',alpha=.2)
    pPtf1a.step(boije_x,boije_pPtf1a, 'k-', where='post', linewidth=2)

    pNg.set_ylim((-.1,1.1))
    pNg.set_yticks(np.arange(0,1.25,.25))
    pNg.set_ylabel("Probability")
    pNg.axvline(3,linestyle='--',color='.1',alpha=.2)
    pNg.axvline(5,linestyle='--',color='.1',alpha=.2)
    pNg.step(boije_x,boije_pNg, 'k-', where='post', linewidth=2)


    plt.tight_layout()
    fig.savefig('Hemode.png', transparent=True)
    fig2.savefig('Boijesignals.png', transparent=True)
    plt.show()

if __name__ == "__main__":
    main()
```

17.1.13 /python_fixtures/diagram_utility_scripts/He_Det_signal_plots.py

```
import os
import numpy as np
import matplotlib.pyplot as plt
from fileinput import filename
from scipy.stats import lognorm
from scipy.stats import gamma
from statsmodels.sandbox.distributions.gof_new import a_st70_upp

#Model parameters
he_x = [0,8,15,25]
he_pp_y = [1,.2,.2,.2]
he_pd_y = [0,.4,0,0]
he_dd_y = [0,.4,.8,.8]

d_pp_y = [1,0,0,0]
d_pd_y = [0,1,0,0]
```

```
d_dd_y = [0,0,1,1]

fig, ((spp,spd,sdd))= plt.subplots(1,3,figsize=(6,2),sharey=True)
fig2, ((dpp,dpd,ddd))= plt.subplots(1,3,figsize=(6,2),sharey=True)

for [pp,pd,dd] in [[spp,spd,sdd],[dpp,dpd,ddd]]:
    for p in [pp,pd,dd]:
        p.set_xlim((0,24))
        p.set_xticks([8,15])
        p.set_ylim((-.1,1.1))
        p.set_yticks(np.arange(0,1.2,.2))
        p.set_xlabel("TiL (h)")
        if p==spp or p==spd or p==sdd:
            p.axvline(8,linestyle='--',color='.1', alpha=.2)
            p.axvline(15,linestyle='--',color='.1', alpha=.2)
        else:
            p.set_xticklabels(["⟷", "⟷"])
            p.tick_params(axis="x",colors="red")
            p.axvline(8,linestyle='-',color='blue', alpha=.15,
             ↪  linewidth=15)
            p.axvline(15,linestyle='-',color='blue', alpha=.15,
             ↪  linewidth=15)
    pp.set_ylabel("Probability")
    if pp==spp:
        pp.step(he_x,he_pp_y, 'k-', where='post', linewidth=2)
    else:
        pp.step(he_x,d_pp_y, 'r-', where='post', linewidth=2)
    if pd==spd:
        pd.step(he_x,he_pd_y, 'k-', where='post', linewidth=2)
    else:
        pd.step(he_x,d_pd_y, 'r-', where='post', linewidth=2)
    if dd==sdd:
        dd.step(he_x,he_dd_y, 'k-', where='post', linewidth=2)
    else:
        dd.step(he_x,d_dd_y, 'r-', where='post', linewidth=2)

fig.tight_layout()
fig2.tight_layout()
fig.savefig('Hemode.png')
fig2.savefig('Detmode.png')
plt.show()
```

17.1.14 /python_fixtures/figure_plots/Cumulative_EdU.py

```
import os
import numpy as np
import statsmodels.api as sm
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

from PIL import Image
from io import BytesIO


#PLoS formatting stuff
plt.rcParams['font.size'] = 12
plt.rcParams['font.family'] = 'sans-serif'
plt.rcParams['font.sans-serif'] = ['Arial']

def main():
    #Read & parse raw data file
    raw_data = pd.read_excel('/home/main/git/chaste/projects/ISP/empirical_data/cumulative_e
    #group data by dorsal/ventral
    dv_grouped = raw_data.groupby('D/V')
    #make new dataframe for totals and labelled fraction
    totals_frame = dv_grouped.get_group('D')
    #reset indices to ventral frame for addition operatinos
    totals_frame = totals_frame.set_index(dv_grouped.get_group('V').index)
    #sum D + V PCNA & EdU counts
    totals_frame.PCNA = totals_frame.PCNA + dv_grouped.get_group('V').PCNA
    totals_frame.EdU = totals_frame.EdU + dv_grouped.get_group('V').EdU
    #create new column for labelled fraction
    totals_frame['labelled_fraction'] = totals_frame.EdU / totals_frame.PCNA

    #setup x for linear regression with y-intercept constant
    X = totals_frame.Time
    X = sm.add_constant(X)

    model = sm.OLS(totals_frame.labelled_fraction, X).fit()
    print(model.summary())
```

```
    fit_results = model.params

    tc = 1/fit_results[1]
    ts = tc * fit_results[0]

    sns.regplot(totals_frame.Time,totals_frame.labelled_fraction)

    plt.text(7,.2,"Tc: " + f'{tc:.2f}')
    plt.text(7,.1,"Ts: " + f'{ts:.2f}')

    plt.xlabel("Time (h)")
    plt.ylabel("Fraction of CMZ RPCs labelled by EdU")

    plt.savefig("cumulative_edu.png")
    png_memory = BytesIO()
    plt.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('cumulative_edu.tiff')
    png_memory.close()

    plt.show()

if __name__ == "__main__":
    main()
```

17.1.15 /python_fixtures/figure_plots/He_output_plot.py

```
import os
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.patches as patches
from fileinput import filename
from statsmodels.sandbox.distributions.gof_new import a_st70_upp

from PIL import Image
from io import BytesIO

#PLoS formatting stuff
plt.rcParams['font.size'] = 10
plt.rcParams['font.family'] = 'sans-serif'
plt.rcParams['font.sans-serif'] = ['Arial']

```

```
#AIC & Plotting utility params
count_seeds = 10000
event_seeds = 1000
error_samples = 5000 #number of samples to draw when estimating plausibility interval for simulations

he_model_params = 15
det_model_params = 13

#stats & line_plotter utility arrays
bin_sequence_24_32 = np.arange(1,26,1)
x_sequence_24_32 = np.arange(1,25,1)

bin_sequence_48 = np.arange(1,11,1)
x_sequence_48 = np.arange(1,10,1)

bin_sequence_wan = np.arange(2,17,1)
x_sequence_wan = np.arange(2,16,1)

bin_sequence_events = np.arange(30,85,5)
x_sequence_events = np.arange(30,80,5)

##############################
# HE ET AL EMPIRICAL RESULTS
##############################

#Count probability arrays
raw_counts = np.loadtxt('/home/main/git/chaste/projects/ISP/empirical_data/empirical_counts.cs
skiprows=1, usecols=(3,4,5,6,7,8,9,10)) #collect the per-cell-type counts
raw_counts_24 = raw_counts[0:64,:]
raw_counts_32 = raw_counts[64:233,:]
raw_counts_48 = raw_counts[233:396,:]

prob_empirical_24,bin_edges = np.histogram(np.sum(raw_counts_24,axis=1),bin_sequence_24_32,density=True) #obtain probability density histogram for counts, retabulating by summing across types
prob_empirical_32,bin_edges = np.histogram(np.sum(raw_counts_32,axis=1),bin_sequence_24_32,density=True)
prob_empirical_48,bin_edges = np.histogram(np.sum(raw_counts_48,axis=1),bin_sequence_48,density=True)
```

```
50
51 #no. of lineages EO per induction timepoint/event group
52 lineages_sampled_24 = 64
53 lineages_sampled_32 = 169
54 lineages_sampled_48 = 163
55 lineages_sampled_events = 60
56
57 #Mitotic mode rate probability arrays
58 raw_events =
   ↪  np.loadtxt('/home/main/git/chaste/projects/ISP/empirical_data/empirical_lineages.
   ↪  skiprows=1, usecols=(3,5,8))
59 EO_events = raw_events[np.where(raw_events[:,2]==1)] #exclude any mitosis
   ↪  whose time was too early for recording
60
61 EO_PP = EO_events[np.where(EO_events[:,0]==0)]
62 EO_PD = EO_events[np.where(EO_events[:,0]==1)]
63 EO_DD = EO_events[np.where(EO_events[:,0]==2)]
64
65 prob_empirical_PP,bin_edges =
   ↪  np.histogram(EO_PP,bin_sequence_events,density=True)
66 prob_empirical_PD,bin_edges =
   ↪  np.histogram(EO_PD,bin_sequence_events,density=True)
67 prob_empirical_DD,bin_edges =
   ↪  np.histogram(EO_DD,bin_sequence_events,density=True)
68
69 empirical_fit_list =
   ↪  [prob_empirical_24,prob_empirical_32,prob_empirical_48,prob_empirical_PP,prob_emp
70
71 #mutant results
72 wt_mean_clone_size = 6
73 ath5_mean_clone_size = 8
74 wt_even_odd_ratio = 1.1
75 ath5_even_odd_ratio = 2.2
76
77 ##############################
78 # WAN ET AL EMPIRICAL RESULTS
79 ##############################
80 #(derived from figure)
81 prob_wan =
   ↪  [.3143,.1857,.1757,.0871,.0871,.0786,.0271,.0179,.0071,0,0,.0071];
82 lineages_sampled_wan = 40 #not reported- approximation based on 95% CI
83
```

```
empirical_test_list = [prob_wan, wt_mean_clone_size, ath5_mean_clone_size, wt_even_odd_ratio, ath5_even_odd_ratio]

def main():
    #LOAD & PARSE HE MODEL OUTPUT FILES
    #original fit stochastic mitotic mode files
    o_counts_24 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    o_counts_32 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    o_counts_48 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    o_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeM skiprows=1, usecols=(0,3))
    o_events_PP = np.array(o_events[np.where(o_events[:,1]==0)])
    o_events_PD = np.array(o_events[np.where(o_events[:,1]==1)])
    o_events_DD = np.array(o_events[np.where(o_events[:,1]==2)])
    o_counts_wan_no_shadow = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    o_counts_ath5 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    o_counts_validate = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)

    #refit stochastic mitotic mode files
    s_counts_24 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    s_counts_32 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
    s_counts_48 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeC skiprows=1, usecols=3)
```

```
    s_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeMo skiprows=1, usecols=(0,3))
    s_events_PP = np.array(s_events[np.where(s_events[:,1]==0)])
    s_events_PD = np.array(s_events[np.where(s_events[:,1]==1)])
    s_events_DD = np.array(s_events[np.where(s_events[:,1]==2)])
    s_counts_wan_no_shadow = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    s_counts_ath5 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    s_counts_validate = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)

    #deterministic mitotic mode files
    d_counts_24 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    d_counts_32 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    d_counts_48 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    d_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeMo skiprows=1, usecols=(0,3))
    d_events_PP = np.array(d_events[np.where(d_events[:,1]==0)])
    d_events_PD = np.array(d_events[np.where(d_events[:,1]==1)])
    d_events_DD = np.array(d_events[np.where(d_events[:,1]==2)])
    d_counts_wan_no_shadow = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    d_counts_ath5 = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)
    d_counts_validate = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeCo skiprows=1, usecols=3)

    #calculate Ath5 clone size and even-odd ratios
```

```
    o_wt_mean_clone_size = np.mean(o_counts_validate)
    o_ath5_mean_clone_size = np.mean(o_counts_ath5)

    s_wt_mean_clone_size = np.mean(s_counts_validate)
    s_ath5_mean_clone_size = np.mean(s_counts_ath5)

    d_wt_mean_clone_size = np.mean(d_counts_validate)
    d_ath5_mean_clone_size = np.mean(d_counts_ath5)

    o_wt_even_odd_ratio = len(np.where(o_counts_validate % 2 == 0)[0])/len(np.where(o_counts_validate % 2 == 1)[0])
    o_ath5_even_odd_ratio = len(np.where(o_counts_ath5 % 2 == 0)[0])/len(np.where(o_counts_ath5 % 2 == 1)[0])

    s_wt_even_odd_ratio = len(np.where(s_counts_validate % 2 == 0)[0])/len(np.where(s_counts_validate % 2 == 1)[0])
    s_ath5_even_odd_ratio = len(np.where(s_counts_ath5 % 2 == 0)[0])/len(np.where(s_counts_ath5 % 2 == 1)[0])

    d_wt_even_odd_ratio = len(np.where(d_counts_validate % 2 == 0)[0])/len(np.where(d_counts_validate % 2 == 1)[0])
    d_ath5_even_odd_ratio = len(np.where(d_counts_ath5 % 2 == 0)[0])/len(np.where(d_counts_ath5 % 2 == 1)[0])

    o_fit_output_list = [o_counts_24, o_counts_32, o_counts_48, o_events_PP, o_events_PD, o_events_DD]
    o_test_output_list = [o_counts_wan_no_shadow, o_wt_mean_clone_size, o_ath5_mean_clone_size, o_wt_even_odd_ratio, o_ath5_even_odd_ratio]

    s_fit_output_list = [s_counts_24, s_counts_32, s_counts_48, s_events_PP, s_events_PD, s_events_DD]
    s_test_output_list = [s_counts_wan_no_shadow, s_wt_mean_clone_size, s_ath5_mean_clone_size, s_wt_even_odd_ratio, s_ath5_even_odd_ratio]

    d_fit_output_list = [d_counts_24, d_counts_32, d_counts_48, d_events_PP, d_events_PD, d_events_DD]
    d_test_output_list = [d_counts_wan_no_shadow, d_wt_mean_clone_size, d_ath5_mean_clone_size, d_wt_even_odd_ratio, d_ath5_even_odd_ratio]

    #SETUP FIGURES
```

```
#4 figures to be generated; 3 show output of individual model modes, 4th overlays stochastic refit on deterministic fit
o_fig, ((o_24,o_32),(o_48,o_PP),(o_PD,o_DD),(o_wan_noshad, o_ath5size),(o_ratio,o_unused)) = plt.subplots(5,2,figsize=(7.5,8.75))
s_fig, ((s_24,s_32),(s_48,s_PP),(s_PD,s_DD),(s_wan_noshad, s_ath5size),(s_ratio,s_unused)) = plt.subplots(5,2,figsize=(7.5,8.75))
d_fig, ((d_24,d_32),(d_48,d_PP),(d_PD,d_DD),(d_wan_noshad, d_ath5size),(d_ratio,d_unused)) = plt.subplots(5,2,figsize=(7.5,8.75))
sd_fig, ((sd_24,sd_32),(sd_48,sd_PP),(sd_PD,sd_DD),(sd_wan_noshad, sd_ath5size),(sd_ratio,sd_unused)) = plt.subplots(5,2,figsize=(7.5,8.75))

#turn off unused axes

o_unused.axis('off');s_unused.axis('off');d_unused.axis('off');sd_unused.axis

#set xlabels

o_24.set_xlabel('Count');o_32.set_xlabel('Count');o_48.set_xlabel('Count');o_
o_PP.set_xlabel('Age (hpf)');o_PD.set_xlabel('Age (hpf)');o_DD.set_xlabel('Age (hpf)')


s_24.set_xlabel('Count');s_32.set_xlabel('Count');s_48.set_xlabel('Count');s_
s_PP.set_xlabel('Age (hpf)');s_PD.set_xlabel('Age (hpf)');s_DD.set_xlabel('Age (hpf)')


d_24.set_xlabel('Count');d_32.set_xlabel('Count');d_48.set_xlabel('Count');d_
d_PP.set_xlabel('Age (hpf)');d_PD.set_xlabel('Age (hpf)');d_DD.set_xlabel('Age (hpf)')


sd_24.set_xlabel('Count');sd_32.set_xlabel('Count');sd_48.set_xlabel('Count')
sd_PP.set_xlabel('Age (hpf)');sd_PD.set_xlabel('Age (hpf)');sd_DD.set_xlabel('Age (hpf)')

#set ylabels

o_24.set_ylabel('Probability');o_32.set_ylabel('Probability');o_48.set_ylabel
```

```
o_PP.set_ylabel('PP Probability');o_PD.set_ylabel('PD Probability');o_DD.set_ylabel('DD Probability');
o_ath5size.set_ylabel('Clone size');o_ratio.set_ylabel('Clone even/odd ratio')


s_24.set_ylabel('Probability');s_32.set_ylabel('Probability');s_48.set_ylabel
s_PP.set_ylabel('PP Probability');s_PD.set_ylabel('PD Probability');s_DD.set_ylabel('DD Probability');
s_ath5size.set_ylabel('Clone size');s_ratio.set_ylabel('Clone even/odd ratio')


d_24.set_ylabel('Probability');d_32.set_ylabel('Probability');d_48.set_ylabel
d_PP.set_ylabel('PP Probability');d_PD.set_ylabel('PD Probability');d_DD.set_ylabel('DD Probability');
d_ath5size.set_ylabel('Clone size');d_ratio.set_ylabel('Clone even/odd ratio')


sd_24.set_ylabel('Probability');sd_32.set_ylabel('Probability');sd_48.set_yla
sd_PP.set_ylabel('PP Probability');sd_PD.set_ylabel('PD Probability');sd_DD.set_ylabel('DD Probability');
sd_ath5size.set_ylabel('Clone size');sd_ratio.set_ylabel('Clone even/odd ratio')

#PLOT DATA
#plot lines, CIs, and bars
o_objects = line_plotter(o_24, o_counts_24, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_24, prob_empirical_24, 'b-', '#000080', '#0000FF', 'y+', 0)
line_plotter(o_32, o_counts_32, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_32, prob_empirical_32, 'b-', '#000080', '#0000FF', 'y+', 0)
line_plotter(o_48, o_counts_48, bin_sequence_48, x_sequence_48, count_seeds, lineages_sampled_48, prob_empirical_48, 'b-', '#000080', '#0000FF', 'y+', 0)
line_plotter(o_PP, o_events_PP[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PP,'b-', '#000080', '#0000FF', 'y+', 1)
```

```
line_plotter(o_PD, o_events_PD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PD,'b-', '#000080', '#0000FF', 'y+', 1)
line_plotter(o_DD, o_events_DD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_DD,'b-', '#000080', '#0000FF', 'y+', 1)
line_plotter(o_wan_noshad, o_counts_wan_no_shadow, bin_sequence_wan, x_sequence_wan, count_seeds, lineages_sampled_wan, prob_wan, 'b-', '#000080', '#0000FF', 'y+', 2)
o_ath5size.bar([0,1,2,3], [(wt_mean_clone_size/wt_mean_clone_size), (o_wt_mean_clone_size/wt_mean_clone_size), (ath5_mean_clone_size/wt_mean_clone_size), (o_ath5_mean_clone_size/wt_mean_clone_size)],color=['#000000','#000080','#000
WT','OSM WT', 'EO Ath5mo', 'OSM Ath5mo'])
o_ratio.bar([0,1,2,3], [wt_even_odd_ratio, o_wt_even_odd_ratio, ath5_even_odd_ratio, o_ath5_even_odd_ratio],color=['#000000','#000080','#000000','#000080'],edgeco
WT','OFit WT', 'EO Ath5mo', 'OFit Ath5mo'])
plt.setp(o_ath5size.get_xticklabels(), rotation=25, ha='right')
plt.setp(o_ratio.get_xticklabels(), rotation=25, ha='right')

s_objects = line_plotter(s_24, s_counts_24, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_24, prob_empirical_24, 'm-', '#800080', '#FF00FF', 'k+', 0)
line_plotter(s_32, s_counts_32, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_32, prob_empirical_32, 'm-', '#800080', '#FF00FF', 'k+', 0)
line_plotter(s_48, s_counts_48, bin_sequence_48, x_sequence_48, count_seeds, lineages_sampled_48, prob_empirical_48, 'm-', '#800080', '#FF00FF', 'k+', 0)
line_plotter(s_PP, s_events_PP[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PP,'m-', '#800080', '#FF00FF', 'k+', 1)
line_plotter(s_PD, s_events_PD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PD,'m-', '#800080', '#FF00FF', 'k+', 1)
```

```
line_plotter(s_DD, s_events_DD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_DD,'m-', '#800080', '#FF00FF', 'k+', 1)
line_plotter(s_wan_noshad, s_counts_wan_no_shadow, bin_sequence_wan, x_sequence_wan, count_seeds, lineages_sampled_wan, prob_wan, 'm-', '#800080', '#FF00FF', 'k+', 2)
s_ath5size.bar([0,1,2,3], [(wt_mean_clone_size/wt_mean_clone_size), (s_wt_mean_clone_size/wt_mean_clone_size),(ath5_mean_clone_size/wt_mean_clone
'#000000', '#800080'],edgecolor='#000000',tick_label=['EO WT','SM WT', 'EO Ath5mo', 'SM Ath5mo'])
s_ratio.bar([0,1,2,3], [wt_even_odd_ratio, s_wt_even_odd_ratio, ath5_even_odd_ratio, s_ath5_even_odd_ratio],color=['#000000','#000080','#000000','#000080'],edgeco
WT','SM WT', 'EO Ath5mo', 'SM Ath5mo'])
plt.setp(s_ath5size.get_xticklabels(), rotation=25, ha='right')
plt.setp(s_ratio.get_xticklabels(), rotation=25, ha='right')


d_objects = line_plotter(d_24, d_counts_24, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_24, prob_empirical_24, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(d_32, d_counts_32, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_32, prob_empirical_32, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(d_48, d_counts_48, bin_sequence_48, x_sequence_48, count_seeds, lineages_sampled_48, prob_empirical_48, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(d_PP, d_events_PP[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PP,'g-', '#008000', '#00FF00', 'k+', 1)
line_plotter(d_PD, d_events_PD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PD,'g-', '#008000', '#00FF00', 'k+', 1)
line_plotter(d_DD, d_events_DD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_DD,'g-', '#008000', '#00FF00', 'k+', 1)
line_plotter(d_wan_noshad, d_counts_wan_no_shadow, bin_sequence_wan, x_sequence_wan, count_seeds, lineages_sampled_wan, prob_wan, 'g-', '#008000', '#00FF00', 'k+', 2)
```

```
d_ath5size.bar([0,1,2,3], [(wt_mean_clone_size/wt_mean_clone_size), (d_wt_mean_clone_size/wt_mean_clone_size),(ath5_mean_clone_size/wt_mean_clone
WT','DM WT', 'EO Ath5mo', 'DM Ath5mo'])
d_ratio.bar([0,1,2,3], [wt_even_odd_ratio, d_wt_even_odd_ratio, ath5_even_odd_ratio, d_ath5_even_odd_ratio],color=['#000000','#008000','#000000','#008000'],edgeco
WT','DM WT', 'EO Ath5mo', 'DM Ath5mo'])
plt.setp(d_ath5size.get_xticklabels(), rotation=25, ha='right')
plt.setp(d_ratio.get_xticklabels(), rotation=25, ha='right')


sd_objects_s = line_plotter(sd_24, s_counts_24, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_24, prob_empirical_24, 'm-', '#800080', '#FF00FF', 'k+', 0)
sd_objects_d = line_plotter(sd_24, d_counts_24, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_24, prob_empirical_24, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(sd_32, s_counts_32, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_32, prob_empirical_32, 'm-', '#800080', '#FF00FF', 'k+', 0)
line_plotter(sd_32, d_counts_32, bin_sequence_24_32, x_sequence_24_32, count_seeds, lineages_sampled_32, prob_empirical_32, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(sd_48, s_counts_48, bin_sequence_48, x_sequence_48, count_seeds, lineages_sampled_48, prob_empirical_48, 'm-', '#800080', '#FF00FF', 'k+', 0)
line_plotter(sd_48, d_counts_48, bin_sequence_48, x_sequence_48, count_seeds, lineages_sampled_48, prob_empirical_48, 'g-', '#008000', '#00FF00', 'k+', 0)
line_plotter(sd_PP, s_events_PP[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PP,'m-', '#800080', '#FF00FF', 'k+', 1)
line_plotter(sd_PP, d_events_PP[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PP,'g-', '#008000', '#00FF00', 'k+', 1)
line_plotter(sd_PD, s_events_PD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PD,'m-', '#800080', '#FF00FF', 'k+', 1)
```

```
    line_plotter(sd_PD, d_events_PD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_PD,'g-', '#008000', '#00FF00', 'k+', 1)
    line_plotter(sd_DD, s_events_DD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_DD,'m-', '#800080', '#FF00FF', 'k+', 1)
    line_plotter(sd_DD, d_events_DD[:,0], bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events,prob_empirical_DD,'g-', '#008000', '#00FF00', 'k+', 1)
    line_plotter(sd_wan_noshad, s_counts_wan_no_shadow, bin_sequence_wan, x_sequence_wan, count_seeds, lineages_sampled_wan, prob_wan, 'm-', '#800080', '#FF00FF', 'k+', 2)
    line_plotter(sd_wan_noshad, d_counts_wan_no_shadow, bin_sequence_wan, x_sequence_wan, count_seeds, lineages_sampled_wan, prob_wan, 'g-', '#008000', '#00FF00', 'k+', 2)
    sd_ath5size.bar([0,1,2,3,4,5], [(wt_mean_clone_size), (s_wt_mean_clone_size),(d_wt_mean_clone_size),(ath5_mean_clone_size),(s_ath5_
    WT','SM WT','DM WT', 'EO Ath5mo', 'SM Ath5Mo', 'DM Ath5mo'])
    sd_ratio.bar([0,1,2,3,4,5], [wt_even_odd_ratio, s_wt_even_odd_ratio, d_wt_even_odd_ratio, ath5_even_odd_ratio, s_ath5_even_odd_ratio, d_ath5_even_odd_ratio],color=['#000000','#800080','#008000','#000000','#80008
    WT','SM WT','DM WT', 'EO Ath5mo', 'SM Ath5Mo', 'DM Ath5mo'])
    plt.setp(sd_ath5size.get_xticklabels(), rotation=25, ha='right')
    plt.setp(sd_ratio.get_xticklabels(), rotation=25, ha='right')

    #calculate AICs, write table to file
    o_fit_AIC = calculate_fit_AIC(o_fit_output_list, empirical_fit_list, he_model_params)
    o_test_AIC = calculate_test_AIC(o_test_output_list, empirical_fit_list, he_model_params)

    s_fit_AIC = calculate_fit_AIC(s_fit_output_list, empirical_fit_list, he_model_params)
    s_test_AIC = calculate_test_AIC(s_test_output_list, empirical_test_list, he_model_params)

    d_fit_AIC = calculate_fit_AIC(d_fit_output_list, empirical_fit_list, det_model_params)
    d_test_AIC = calculate_test_AIC(d_test_output_list, empirical_test_list, det_model_params)
```

```
    AIC_table('AIC_table.tex', o_fit_AIC, o_test_AIC, s_fit_AIC, s_test_AIC, d_fit_AIC, d_test_AIC)

    #plot annotiations
    plot_annotater(s_fig,s_objects,'Stochastic Model')
    plot_annotater(o_fig,o_objects,'Stochastic Model (original fit)')
    plot_annotater(d_fig,d_objects, 'Deterministic Model')
    plot_annotater(sd_fig,sd_objects_s, 'Stochastic Model', sd_objects_d, 'Deterministic Model')

    #export as .tiff, .png
    o_fig.savefig('o_fig.png',dpi=600)
    png_memory = BytesIO()
    o_fig.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('o_fig.tiff')
    png_memory.close()

    s_fig.savefig('s_fig.png',dpi=600)
    png_memory = BytesIO()
    s_fig.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('s_fig.tiff')
    png_memory.close()

    d_fig.savefig('d_fig.png',dpi=600)
    png_memory = BytesIO()
    d_fig.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('d_fig.tiff')
    png_memory.close()

    sd_fig.savefig('sd_fig.png',dpi=600)
    png_memory = BytesIO()
    sd_fig.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('sd_fig.tiff')
    png_memory.close()

    plt.show()
```

```
def calculate_fit_AIC(output_list, empirical_list, number_params):
    #function calculates AIC for He et al counts & event probability density (NOT per-lineage event probability)
    #this numpy array holds the individual RSS vals for timepoints + events
    rss = np.zeros(6)
    number_comparisons = 0

    for i in range(0,3):
        output_counts = output_list[i]
        empirical_prob = empirical_list[i]

        output_histo,bin_edges = np.histogram(output_counts, np.arange(1,len(empirical_prob)+2), density = False)
        output_prob = np.array(output_histo/count_seeds)

        residual = np.array(output_prob- empirical_prob)
        number_comparisons += len(residual)
        rss[i] = np.sum(np.square(residual))

    for i in range (0,3):
        output_events = output_list[i+3]
        empirical_prob = empirical_list[i+3]

        histo_events,bin_edges = np.histogram(output_events, bin_sequence_events, density=True)
        output_prob = np.array(histo_events/event_seeds)

        residual = np.array(output_prob - empirical_prob)
        number_comparisons += len(residual)
        rss[i+3] = np.sum(np.square(residual))

    AIC = 2 * number_params + number_comparisons * np.log(np.sum(rss))

    return AIC

def calculate_test_AIC(output_list, empirical_list, number_params):
    #function calculates AIC for He mutant data & Wan counts
    rss=np.zeros(3)
    number_comparisons = 0

    #Wan data
    for i in range (0,1):
```

```
        output_counts = output_list[i]
        empirical_prob = empirical_list[i]

        output_histo,bin_edges = np.histogram(output_counts, np.arange(2,len(empirical_prob)+3), density = False)
        output_prob = np.array(output_histo/count_seeds)

        residual = np.array(output_prob- empirical_prob)
        number_comparisons += len(residual)
        rss[i] = np.sum(np.square(residual))

    #He data
    for i in range(1,3):
        output_measure = output_list[i]
        empirical_measure = empirical_list[i]

        residual = np.array(output_measure - empirical_measure)
        number_comparisons += 1
        rss[i] = np.sum(np.square(residual))

    AIC = 2 * number_params + number_comparisons * np.log(np.sum(rss))

    return AIC

#line plotter plots average counts or event event +- 2 standard deviations (He et al's "95% probability interval")
#"mode" 1 gives correct hourly output for 5-hour event bins. mode 2 begins Wan plots at clone size 2, reflecting unavailable data
def line_plotter(subplot, data, bin_sequence, x_sequence, seeds, samples, empirical_prob, line_colour_string, fill_edge_colour_string, fill_face_colour_string, empirical_colour_string, mode):

    legend_objects = []

    prob_histo, bin_edges = np.histogram(data, bin_sequence, density=True)

    #estimate of 95% CI for empirical observations of model output from repeated sampling of the data w/ the appropriate number of empirically observed lineages
    interval = sampler(data,samples,bin_sequence)
    legend_objects.append(subplot.plot(x_sequence,prob_histo, line_colour_string))
```

```
    subplot.fill_between(x_sequence, (prob_histo - interval), (prob_histo + interval), alpha=0.1, edgecolor=fill_edge_colour_string, facecolor=fill_face_colour_string)

    if mode == 0:

        legend_objects.append(subplot.plot(np.arange(1,len(empirical_prob)+1,1),em empirical_colour_string))

    if mode == 1:

        legend_objects.append(subplot.plot(np.arange(30,80,5),empirical_prob, empirical_colour_string))

    if mode == 2:

        legend_objects.append(subplot.plot(np.arange(2,len(empirical_prob)+2,1), empirical_prob, empirical_colour_string))

    return legend_objects

def sampler(data,samples,bin_sequence):

    if len(data) == 0: #catches edge case w/ no entries for a mitotic mode
        data=[0]

    base_sample=np.zeros((error_samples,len(bin_sequence)-1))

    for i in range(0,error_samples):
        new_data_sample = np.random.choice(data,samples)
        new_histo_prob, bin_edges = np.histogram(new_data_sample, bin_sequence, density=True)
        base_sample[i,:] = new_histo_prob

    sample_95CI = np.array(2 * (np.std(base_sample,0)))

    return sample_95CI

def plot_annotater(fig_to_ann, legend_objects_1, legend_objects_1_name, legend_objects_2=None, legend_objects_2_name=None):
    #subplot labels
    labels = ['A','B','C','D','E','F','G','H','I','']
```

```
    for i, ax in enumerate(fig_to_ann.axes):
        ax.text(.01,.95, labels[i], transform=ax.transAxes, fontweight='bold', va='top')

        time_rectangle_x = .70
        time_rectangle_y = .65
        time_rectangle_width = .2
        time_rectangle_height = .03
        time_caret_half_width = .02
        time_caret_height = np.sqrt(3)*(time_caret_half_width*2)

        if i < 3:
            #add the time bar and label 72 hr caret on appropriate plots

            ax.add_patch(plt.Rectangle((time_rectangle_x,time_rectangle_y),time_r color='k', transform=ax.transAxes))

            ax.add_patch(plt.Polygon(((time_rectangle_x+time_rectangle_width,time fill=False, transform=ax.transAxes))

            ax.text(time_rectangle_x+time_rectangle_width,time_rectangle_y+time_r transform=ax.transAxes, fontsize='8',va='bottom',ha='center')

        if i == 0:
            #add 24hr caret

            ax.add_patch(plt.Polygon(((time_rectangle_x,time_rectangle_y+time_rec fill=False, transform=ax.transAxes))

            ax.text(time_rectangle_x,time_rectangle_y+time_rectangle_height+time_ transform=ax.transAxes, fontsize='8',va='bottom',ha='center')

        if i == 1:
            #add 32hr caret

            ax.add_patch(plt.Polygon(((time_rectangle_x+(8/48)*time_rectangle_wid fill=False, transform=ax.transAxes))
```

```
431
            ↪ ax.text(time_rectangle_x+(8/48)*time_rectangle_width,time_rectangle_y
            ↪ transform=ax.transAxes,
            ↪ fontsize='8',va='bottom',ha='center')
432
433         if i == 2:
434
            ↪ ax.add_patch(plt.Polygon(((time_rectangle_x+(24/48)*time_rectangle_wi
            ↪ fill=False, transform=ax.transAxes))
435
            ↪ ax.text(time_rectangle_x+(24/48)*time_rectangle_width,time_rectangle_
            ↪ transform=ax.transAxes,
            ↪ fontsize='8',va='bottom',ha='center')
436
437         parent_circle_coord = (.85,.8)
438         child_1_coord = (.8,.6)
439         child_2_coord = (.9,.6)
440         ellipse_width = .03
441         ellipse_height = 2.5*ellipse_width
442
443         if i == 3:
444
            ↪ ax.add_patch(plt.Polygon(((parent_circle_coord),child_1_coord),color=
445
            ↪ ax.add_patch(plt.Polygon(((parent_circle_coord),child_2_coord),color=
446
            ↪ ax.add_patch(patches.Ellipse(parent_circle_coord,ellipse_width,ellips
            ↪ color='k', transform=ax.transAxes))
447
            ↪ ax.add_patch(patches.Ellipse(child_1_coord,ellipse_width,ellipse_heig
            ↪ color='k', transform=ax.transAxes))
448
            ↪ ax.add_patch(patches.Ellipse(child_2_coord,ellipse_width,ellipse_heig
            ↪ color='k', transform=ax.transAxes))
449           ax.text(.91, .7, 'PP', transform=ax.transAxes, va='center',
            ↪ ha='left')
450
451         if i == 4:
452
            ↪ ax.add_patch(plt.Polygon(((parent_circle_coord),child_1_coord),color=
453
            ↪ ax.add_patch(plt.Polygon(((parent_circle_coord),child_2_coord),color=
```

```
                ax.add_patch(patches.Ellipse(parent_circle_coord,ellipse_width,ellipse
                color='k', transform=ax.transAxes))

                ax.add_patch(patches.Ellipse(child_1_coord,ellipse_width,ellipse_heig
                color='k', transform=ax.transAxes))

                ax.add_patch(patches.Ellipse(child_2_coord,ellipse_width,ellipse_heig
                edgecolor='k', facecolor='w', transform=ax.transAxes))
            ax.text(.91, .7, 'PD', transform=ax.transAxes, va='center',
                ha='left')


        if i == 5:

                ax.add_patch(plt.Polygon(((parent_circle_coord),child_1_coord),color=

                ax.add_patch(plt.Polygon(((parent_circle_coord),child_2_coord),color=

                ax.add_patch(patches.Ellipse(parent_circle_coord,ellipse_width,ellipse
                color='k', transform=ax.transAxes))

                ax.add_patch(patches.Ellipse(child_1_coord,ellipse_width,ellipse_heig
                edgecolor='k', facecolor='w', transform=ax.transAxes))

                ax.add_patch(patches.Ellipse(child_2_coord,ellipse_width,ellipse_heig
                edgecolor='k', facecolor='w', transform=ax.transAxes))
            ax.text(.91, .7, 'DD', transform=ax.transAxes, va='center',
                ha='left')


        if i == 6:
            ax.text(.80,.85,'CMZ', transform=ax.transAxes,
                fontweight='bold', va='top')

    if legend_objects_2 == None:

        fig_to_ann.legend((legend_objects_1[1][0],legend_objects_1[0][0]),('Obser
        bbox_to_anchor=(.59,.10,.1,.1), loc='upper left')
    else:
```

```
        fig_to_ann.legend((legend_objects_1[1][0],legend_objects_1[0][0],legend_o
        legend_objects_2_name), bbox_to_anchor=(.59,.1,.1,.1),
        loc='upper left')

    #data group box annotations
    fig_to_ann.patches.extend([plt.Rectangle((0.005,0.41),0.99,0.58,
                                fill=False, color='k', linestyle='-',
                                transform=fig_to_ann.transFigure,
                                figure=fig_to_ann)])
    fig_to_ann.patches.extend([plt.Polygon(((0.005, .4),(.995,.4),
    (.995,.21), (.5,.21), (.5, .01), (.005, .01)),
                                fill=False, color='k', linestyle=':',
                                transform=fig_to_ann.transFigure,
                                figure=fig_to_ann)])

    fig_to_ann.text(.605, .075, 'Training Data', fontsize=12, transform =
    fig_to_ann.transFigure, figure=fig_to_ann)
    fig_to_ann.patches.extend([plt.Rectangle((0.6,0.07),0.143,0.029,
                                fill=False, color='k', linestyle='-',
                                transform=fig_to_ann.transFigure,
                                figure=fig_to_ann)])


    fig_to_ann.text(.605, .025, 'Test Data', fontsize=12, transform =
    fig_to_ann.transFigure, figure=fig_to_ann)
    fig_to_ann.patches.extend([plt.Rectangle((0.6,0.02),0.111,0.029,
                                fill=False, color='k', linestyle=':',
                                transform=fig_to_ann.transFigure,
                                figure=fig_to_ann)])

    fig_to_ann.subplots_adjust(wspace=0.4, hspace=0.3)
    fig_to_ann.tight_layout()

def AIC_table (filename, o_training_AIC, o_test_AIC, s_training_AIC,
s_test_AIC, d_training_AIC, d_test_AIC):
    #writes LaTeX tabular snippet with AIC values
    table_file = open(filename, 'w')
    table_file.write(r'\begin{tabular}{|l|l|l|}'+'\n')
    table_file.write(r'\hline'+'\n')
    table_file.write(r'{\bf Model} & {\bf Training AIC} & {\bf Test AIC}
    \\ \thickhline'+'\n')
```

```
    table_file.write(r'Stochastic (He fit) & $'+f'{o_training_AIC:.2f}'+r'$ & $' + f'{o_test_AIC:.2f}' + r'$\\ \hline'+'\n')
    table_file.write(r'Stochastic (SPSA fit) & $'+f'{s_training_AIC:.2f}'+r'$ & $' + f'{s_test_AIC:.2f}' + r'$\\ \hline'+'\n')
    table_file.write(r'Deterministic (SPSA fit) & $'+f'{d_training_AIC:.2f}'+r'$ & $' + f'{d_test_AIC:.2f}' + r'$\\ \hline'+'\n')
    table_file.write(r'\end{tabular}'+'\n')

if __name__ == "__main__":
    main()
```

17.1.16 /python_fixtures/figure_plots/Kolmogorov_plot.py

```
import os
import numpy as np
import matplotlib.pyplot as plt

#AIC & Plotting utility params
event_seeds = 1000
error_samples = 5000 #number of samples to draw when estimating plausibility interval for simulations

def main():
    #LOAD & PARSE KOLMOGOROV COMPLEXITY FILES
    gomes_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol skiprows=1, usecols=1)
    he_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol skiprows=1, usecols=1)
    he_refit_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol skiprows=1, usecols=1)
    deterministic_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol skiprows=1, usecols=1)
    boije_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol skiprows=1, usecols=1)
```

```
    eo_kol = np.loadtxt('/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/Kol
    skiprows=1, usecols=1)

    data_sequence = [gomes_kol,he_kol,he_refit_kol,deterministic_kol,boije_kol,eo_kol]

    plt.violinplot(data_sequence, showextrema=True)

    plt.ylabel ('Estimated Kolmogorov Complexity')

    plt.show()

if __name__ == "__main__":
    main()
```

17.1.17 /python_fixtures/figure_plots/Mitotic_rate_plot.py

```
import os
import numpy as np
import matplotlib.pyplot as plt

from PIL import Image
from io import BytesIO

#PLoS formatting stuff
plt.rcParams['font.size'] = 12
plt.rcParams['font.family'] = 'sans-serif'
plt.rcParams['font.sans-serif'] = ['Arial']

#AIC & Plotting utility params
event_seeds = 1000
error_samples = 5000 #number of samples to draw when estimating plausibility interval for simulations

bin_sequence_events = np.arange(30,85,5)
x_sequence_events = np.arange(30,80,5)

##############################
# HE ET AL EMPIRICAL RESULTS
##############################

```

```
rate_bin_sequence = np.arange(30,85,5)
rate_x_sequence = np.arange(30,80,5)
rate_trim_value = 10

#no. of lineages EO per induction timepoint/event group
lineages_sampled_events = 60

#Mitotic mode rate probability arrays
raw_events = np.loadtxt('/home/main/git/chaste/projects/ISP/empirical_data/empirical_lineages.c skiprows=1, usecols=(3,5,8))
EO_events = raw_events[np.where(raw_events[:,2]==1)] #exclude any mitosis whose time was too early for recording

event_counts, bin_edges = np.histogram(EO_events,bin_sequence_events,density=False)
#hourly values
empirical_events_per_lineage = np.array(event_counts/(lineages_sampled_events*5))

def main():
    #LOAD & PARSE HE MODEL OUTPUT FILES
    #original fit stochastic mitotic mode files
    o_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeM skiprows=1, usecols=(0))

    #refit stochastic mitotic mode files
    s_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeM skiprows=1, usecols=(0))

    #deterministic mitotic mode files
    d_events = np.loadtxt("/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/HeM skiprows=1, usecols=(0))

    original = line_plotter(o_events,bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events, 'b--', '#000080', '#0000FF')
    refit = line_plotter(s_events,bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events, 'm-', '#800080', '#FF00FF')
```

```
    deterministic = line_plotter(d_events,bin_sequence_events, x_sequence_events, event_seeds, lineages_sampled_events, 'g-', '#008000', '#00FF00')

    empirical = plt.plot(np.arange(30,80,5),empirical_events_per_lineage, 'k+')

    plt.legend((original[0], refit[0], deterministic[0], empirical[0]), ('SM (He fit)', 'SM (SPSA fit)', 'DM', 'Observed'))

    plt.xlabel ('Age (hpf)')
    plt.ylabel ('Probability of mitotic event per lineage per hour')


    plt.savefig("per_lineage_mitotic_rate.png")
    png_memory = BytesIO()
    plt.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('per_lineage_mitotic_rate.tiff')
    png_memory.close()

    plt.show()

#line plotter plots average counts or event event +- 2 standard deviations (He et al's "95% probability interval")
#"mode" 1 gives correct hourly output for 5-hour event bins. mode 2 begins Wan plots at clone size 2, reflecting unavailable data
def line_plotter(data, bin_sequence, x_sequence, seeds, samples, line_colour_string, fill_edge_colour_string, fill_face_colour_string):

    histo, bin_edges = np.histogram(data, bin_sequence, density=False)
    prob_histo = np.array(histo / (seeds * 5))

    #estimate of 95% CI for empirical observations of model output from repeated sampling of the data w/ the appropriate number of empirically observed lineages
    interval = sampler(data,samples,bin_sequence)
    line = plt.plot(x_sequence,prob_histo, line_colour_string)
    plt.fill_between(x_sequence, (prob_histo - interval), (prob_histo + interval), alpha=0.1, edgecolor=fill_edge_colour_string, facecolor=fill_face_colour_string)

```

```
    return line

def sampler(data,samples,bin_sequence):

    if len(data) == 0: #catches edge case w/ no entries for a mitotic mode
        data=[0]

    base_sample=np.zeros((error_samples,len(bin_sequence)-1))

    for i in range(0,error_samples):
        new_data_sample = np.random.choice(data,samples)
        new_histo, bin_edges = np.histogram(new_data_sample, bin_sequence, density=False)
        new_histo_prob = np.array(new_histo/samples)
        base_sample[i,:] = new_histo_prob

    sample_95CI = np.array(2 * (np.std(base_sample,0)))

    return sample_95CI

if __name__ == "__main__":
    main()
```

17.1.18 /python_fixtures/figure_plots/Wan_output_plot.py

```
import os
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import matplotlib.ticker as ticker

from PIL import Image
from io import BytesIO

#PLoS formatting stuff
plt.rcParams['font.size'] = 12
plt.rcParams['font.family'] = 'sans-serif'
plt.rcParams['font.sans-serif'] = ['Arial']

def main():
```

```
    wan_output_dir = '/home/main/git/chaste/projects/ISP/python_fixtures/testoutput/WanOutput'
    wan_results_list = []
    for root,dirs,files in os.walk(wan_output_dir):
        for name in files:
            if name == 'celltypes.dat':
                results = np.loadtxt(os.path.join(root,name), usecols=2)
                if len(results) == 8569:
                    wan_results_list.append(results)

    wan_sim_results = np.array(wan_results_list)
    wan_sim_mean = np.mean(wan_sim_results,axis=0)
    wan_sim_95CI = 2*np.std(wan_sim_results,axis=0)
    wan_sim_x_seq = np.arange(72,8641,1)/24

    empirical_pop_mean = np.array([792.0, 768.4, 906.1, 1159.7, 1630.0, 3157.9, 3480.2, 4105.1, 1003.0, 477.2, 438.8088611111])
    empirical_pop_95CI = 2*np.array([160.1, 200.1, 244.5, 477.6, 444.3, 1414.3, 472.1, 1169.7, 422.8, 367.5, 294.8])
    empirical_x_seq = np.array([72,120,192,288,408,552,720,1440,2160,4320,8640])/24

    fig, ax = plt.subplots(1,1)

    empirical = plt.plot(empirical_x_seq, empirical_pop_mean, 'g-')
    plt.fill_between(empirical_x_seq, (empirical_pop_mean - empirical_pop_95CI), (empirical_pop_mean + empirical_pop_95CI), alpha=0.1, edgecolor='#008000', facecolor='#00FF00')

    wan_simulator = plt.plot(wan_sim_x_seq, wan_sim_mean, 'm-')
    plt.fill_between(wan_sim_x_seq, (wan_sim_mean - wan_sim_95CI), (wan_sim_mean + wan_sim_95CI), alpha=0.1, edgecolor='#800080', facecolor='#FF00FF')

    plt.ylim(bottom=0)
    plt.xlim(left=0)

    plt.ylabel("CMZ population size")
    plt.xlabel("Retina age (dpf)")
    ax.xaxis.set_major_locator(ticker.MultipleLocator(40))
    ax.legend((empirical[0], wan_simulator[0]), ('Observed CMZ population', '"Wan-type" simulated CMZ population'))

```

```
    png_memory = BytesIO()
    fig.savefig(png_memory, format='png', dpi=600)
    PILpng = Image.open(png_memory)
    PILpng.save('wan_fig.tiff')
    png_memory.close()

    plt.show()

if __name__ == "__main__":
    main()
```

17.1.19 /src/BoijeCellCycleModel.cpp

```
#include "BoijeCellCycleModel.hpp"

BoijeCellCycleModel::BoijeCellCycleModel() :
        AbstractSimpleCellCycleModel(), mOutput(false), mEventStartTime(), mSequenceSampler(false), mSeqSamplerLabelSister(
                false), mDebug(false), mTimeID(), mVarIDs(), mDebugWriter(), mGeneration(0), mPhase2gen(3), mPhase3gen(
                5), mprobAtoh7(0.32), mprobPtf1a(0.30), mprobng(0.80), mAtoh7Signal(false), mPtf1aSignal(false), mNgSignal(
                false), mMitoticMode(0), mSeed(0), mp_PostMitoticType(), mp_RGC_Type(), mp_AC_HC_Type(), mp_PR_BC_Type(), mp_label_Type()
{
}

BoijeCellCycleModel::BoijeCellCycleModel(const BoijeCellCycleModel& rModel) :
        AbstractSimpleCellCycleModel(rModel), mOutput(rModel.mOutput), mEventStartTime(rModel.mEventStartTime), mSequenceSampler(
                rModel.mSequenceSampler), mSeqSamplerLabelSister(rModel.mSeqSamplerLabelSister), mDebug(rModel.mDebug), mTimeID(
                rModel.mTimeID), mVarIDs(rModel.mVarIDs), mDebugWriter(rModel.mDebugWriter), mGeneration(
                rModel.mGeneration), mPhase2gen(rModel.mPhase2gen), mPhase3gen(rModel.mPhase3gen), mprobAtoh7(
```

```
                rModel.mprobAtoh7), mprobPtf1a(rModel.mprobPtf1a),
                ↪ mprobng(rModel.mprobng), mAtoh7Signal(
                rModel.mAtoh7Signal), mPtf1aSignal(rModel.mPtf1aSignal),
                ↪ mNgSignal(rModel.mNgSignal), mMitoticMode(
                rModel.mMitoticMode), mSeed(rModel.mSeed),
                ↪ mp_PostMitoticType(rModel.mp_PostMitoticType),
                ↪ mp_RGC_Type(
                rModel.mp_RGC_Type), mp_AC_HC_Type(rModel.mp_AC_HC_Type),
                ↪ mp_PR_BC_Type(rModel.mp_PR_BC_Type), mp_label_Type(
                rModel.mp_label_Type)
{
}

AbstractCellCycleModel* BoijeCellCycleModel::CreateCellCycleModel()
{
    return new BoijeCellCycleModel(*this);
}

void BoijeCellCycleModel::SetCellCycleDuration()
{

    if
    ↪ (mpCell->GetCellProliferativeType()->IsType<DifferentiatedCellProliferativeTy
    {
        mCellCycleDuration = DBL_MAX;
    }
    else
    {
        mCellCycleDuration = 1.0;
    }

}

void BoijeCellCycleModel::ResetForDivision()
{
    mGeneration++; //increment generation counter
    //the first division is ascribed to generation "1"

    RandomNumberGenerator* p_random_number_generator =
    ↪ RandomNumberGenerator::Instance();

    mMitoticMode = 0; //0=PP;1=PD;2=DD

```

```
    /***********************************************
     * TRANSCRIPTION FACTOR RANDOM VARIABLES & RULES
     * 1st phase: only PP divisions (symmetric proliferative) permitted
     * 2nd phase: all division modes permitted, all TF signals avail
     * 3rd phase: only PP/DD modes permitted, only ng has nonzero signal
     ***********************************************/

    //reset RVs and signals
    double atoh7RV = 1, ptf1aRV = 1, ngRV = 1;
    mAtoh7Signal = mPtf1aSignal = mNgSignal = false;


    //PHASE & TF SIGNAL RULES
    if (mGeneration > mPhase2gen && mGeneration ≤ mPhase3gen) //if the
     ↪  cell is in the 2nd model phase, all signals have nonzero
     ↪  probabilities at each division
    {
        //RVs take values evenly distributed across 0-1
        atoh7RV = p_random_number_generator→ranf();
        ptf1aRV = p_random_number_generator→ranf();
        ngRV = p_random_number_generator→ranf();

        if (atoh7RV < mprobAtoh7)
        {
            mAtoh7Signal = true;
        }
        if (ptf1aRV < mprobPtf1a)
        {
            mPtf1aSignal = true;
        }
        if (ngRV < mprobng)
        {
            mNgSignal = true;
        }
    }

    if (mGeneration > mPhase3gen) //if the cell is in the 3rd model
     ↪  phase, only ng signal has a nonzero probability
    {
        ngRV = p_random_number_generator→ranf();
        //roll a probability die for the ng signal
        if (ngRV < mprobng)
        {
```

```
            mNgSignal = true;
        }
    }

    /****************
     * MITOTIC MODE & SPECIFICATION RULES
     * -(additional asymmetric postmitotic rules specified in InitialiseDaughterCell();)
     * *************/

    if(mAtoh7Signal == true)
    {
        mMitoticMode = 1;
    }

    if (mPtf1aSignal == true && mAtoh7Signal == false) //Ptf1A alone gives a symmetrical postmitotic AC/HC division
    {
        mMitoticMode = 2;
        mpCell->SetCellProliferativeType(mp_PostMitoticType);
        mpCell->AddCellProperty(mp_AC_HC_Type);
    }

    if (mPtf1aSignal == false && mAtoh7Signal == false && mNgSignal == true) //ng alone gives a symmetrical postmitotic PR/BC division
    {
        mMitoticMode = 2;
        mpCell->SetCellProliferativeType(mp_PostMitoticType);
        mpCell->AddCellProperty(mp_PR_BC_Type);
    }

    /****************
     * Write mitotic event to file if appropriate
     * mDebug: many files: detailed per-lineage info switch; intended for TestHeInductionCountFixture
     * mOutput: 1 file: time, seed, cellID, mitotic mode, intended for TestHeMitoticModeRateFixture
     * *************/

    if (mDebug)
    {
        WriteDebugData(atoh7RV, ptf1aRV, ngRV);
    }
```

```
130
131     if (mOutput)
132     {
133         WriteModeEventOutput();
134     }
135
136     AbstractSimpleCellCycleModel::ResetForDivision();
137
138     /******************
139      * SEQUENCE SAMPLER
140      ******************/
141     //if the sequence sampler has been turned on, check for the label &
         ↪  write mitotic mode to log
142     //50% chance of each daughter cell from a mitosis inheriting the
         ↪  label
143     if (mSequenceSampler)
144     {
145         if (mpCell->HasCellProperty<CellLabel>())
146         {
147             (*LogFile::Instance()) << mMitoticMode;
148             double labelRV = p_random_number_generator->ranf();
149             if (labelRV <= .5)
150             {
151                 mSeqSamplerLabelSister = true;
152                 mpCell->RemoveCellProperty<CellLabel>();
153             }
154             else
155             {
156                 mSeqSamplerLabelSister = false;
157             }
158         }
159         else
160         {
161             //prevents lost-label cells from labelling their progeny
162             mSeqSamplerLabelSister = false;
163         }
164     }
165 }
166
167 void BoijeCellCycleModel::InitialiseDaughterCell()
168 {
169     //Asymmetric specification rules
170
```

```
    if (mAtoh7Signal == true)
    {
        if (mPtf1aSignal == true)
        {
            mpCell->SetCellProliferativeType(mp_PostMitoticType);
            mpCell->AddCellProperty(mp_AC_HC_Type);
        }
        else
        {
            mpCell->SetCellProliferativeType(mp_PostMitoticType);
            mpCell->AddCellProperty(mp_RGC_Type);
        }
    }

    /******************
     * SEQUENCE SAMPLER
     ******************/
    if (mSequenceSampler)
    {
        if (mSeqSamplerLabelSister)
        {
            mpCell->AddCellProperty(mp_label_Type);
            mSeqSamplerLabelSister = false;
        }
        else
        {
            mpCell->RemoveCellProperty<CellLabel>();
        }
    }
}

void BoijeCellCycleModel::SetGeneration(unsigned generation)
{
    mGeneration = generation;
}

unsigned BoijeCellCycleModel::GetGeneration() const
{
    return mGeneration;
}

```

```
void BoijeCellCycleModel::SetPostMitoticType(boost::shared_ptr<AbstractCellProperty> p_PostMitoticType)
{
    mp_PostMitoticType = p_PostMitoticType;
}
void BoijeCellCycleModel::SetSpecifiedTypes(boost::shared_ptr<AbstractCellProperty> p_RGC_Type,
                                            boost::shared_ptr<AbstractCellPropert p_AC_HC_Type,
                                            boost::shared_ptr<AbstractCellPropert p_PR_BC_Type)
{
    mp_RGC_Type = p_RGC_Type;
    mp_AC_HC_Type = p_AC_HC_Type;
    mp_PR_BC_Type = p_PR_BC_Type;
}

void BoijeCellCycleModel::SetModelParameters(unsigned phase2gen, unsigned phase3gen, double probAtoh7, double probPtf1a,
                                             double probng)
{
    mPhase2gen = phase2gen;
    mPhase3gen = phase3gen;
    mprobAtoh7 = probAtoh7;
    mprobPtf1a = probPtf1a;
    mprobng = probng;
}

void BoijeCellCycleModel::EnableModeEventOutput(double eventStart, unsigned seed)
{
    mOutput = true;
    mEventStartTime = eventStart;
    mSeed = seed;

}

void BoijeCellCycleModel::WriteModeEventOutput()
{
```

```
    double currentTime = SimulationTime::Instance()->GetTime() + mEventStartTime;
    CellPtr currentCell = GetCell();
    double currentCellID = (double) currentCell->GetCellId();
    (*LogFile::Instance()) << currentTime << "\t" << mSeed << "\t" << currentCellID << "\t" << mMitoticMode << "\n";
}

void BoijeCellCycleModel::EnableSequenceSampler(boost::shared_ptr<AbstractCellProperty label)
{
    mSequenceSampler = true;
    mp_label_Type = label;
}

void BoijeCellCycleModel::EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter)
{
    mDebug = true;
    mDebugWriter = debugWriter;

    mTimeID = mDebugWriter->DefineUnlimitedDimension("Time", "h");
    mVarIDs.push_back(mDebugWriter->DefineVariable("CellID", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Generation", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticMode", "Mode"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("atoh7Set", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("atoh7RV", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("ptf1aSet", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("ptf1aRV", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("ngSet", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("ngRV", "Percentile"));

    mDebugWriter->EndDefineMode();
}
```

```
275
276 void BoijeCellCycleModel::WriteDebugData(double atoh7RV, double ptf1aRV,
    ↪ double ngRV)
277 {
278     double currentTime = SimulationTime::Instance()->GetTime();
279     CellPtr currentCell = GetCell();
280     double currentCellID = (double) currentCell->GetCellId();
281
282     mDebugWriter->PutVariable(mTimeID, currentTime);
283     mDebugWriter->PutVariable(mVarIDs[0], currentCellID);
284     mDebugWriter->PutVariable(mVarIDs[1], mGeneration);
285     mDebugWriter->PutVariable(mVarIDs[2], mMitoticMode);
286     mDebugWriter->PutVariable(mVarIDs[3], mprobAtoh7);
287     mDebugWriter->PutVariable(mVarIDs[4], atoh7RV);
288     mDebugWriter->PutVariable(mVarIDs[5], mprobPtf1a);
289     mDebugWriter->PutVariable(mVarIDs[6], ptf1aRV);
290     mDebugWriter->PutVariable(mVarIDs[7], mprobng);
291     mDebugWriter->PutVariable(mVarIDs[8], ngRV);
292     mDebugWriter->AdvanceAlongUnlimitedDimension();
293 }
294
295 /******************
296  * UNUSED FUNCTIONS (required for class nonvirtuality, do not remove)
297  ******************/
298
299 double BoijeCellCycleModel::GetAverageTransitCellCycleTime()
300 {
301     return mCellCycleDuration;
302 }
303
304 double BoijeCellCycleModel::GetAverageStemCellCycleTime()
305 {
306     return mCellCycleDuration;
307 }
308
309 void BoijeCellCycleModel::OutputCellCycleModelParameters(out_stream&
    ↪ rParamsFile)
310 {
311     *rParamsFile << "\t\t\t<CellCycleDuration>" << mCellCycleDuration <<
        ↪ "</CellCycleDuration>\n";
312
313     // Call method on direct parent class
```

```
    ↪ AbstractSimpleCellCycleModel::OutputCellCycleModelParameters(rParamsFile);
}

// Serialization for Boost ≥ 1.36
#include "SerializationExportWrapperForCpp.hpp"
CHASTE_CLASS_EXPORT(BoijeCellCycleModel)
```

17.1.20 /src/BoijeCellCycleModel.hpp

```
#ifndef BOIJECELLCYCLEMODEL_HPP_
#define BOIJECELLCYCLEMODEL_HPP_

#include "AbstractSimpleCellCycleModel.hpp"
#include "RandomNumberGenerator.hpp"
#include "Cell.hpp"
#include "DifferentiatedCellProliferativeType.hpp"
#include "SmartPointers.hpp"
#include "ColumnDataWriter.hpp"
#include "LogFile.hpp"
#include "CellLabel.hpp"

#include "BoijeRetinalNeuralFates.hpp"


/*******************************
* BOIJE CELL CYCLE MODEL
* As described in Boije et al. 2015 [Boije2015] doi:
  ↪ 10.1016/j.devcel.2015.08.011
*
* USE: By default, BoijeCellCycleModels are constructed with the
  ↪ parameters reported in [Boije2015].
* In normal use, the model steps through three phases of probabilistic
  ↪ transcription factor signalling.
* These signals determine the mitotic mode and the fate of offspring.
*
* NB: the Boije model is purely generational and assumes a generation
  ↪ time of 1; time in Boije simulations
* thus represents generation number rather than proper time. Refactor for
  ↪ use with simulations that refer
* to clocktime!!
 *
```

```
 * 2 per-model-event output modes:
 * EnableModeEventOutput() enables mitotic mode event logging-all cells will write to the singleton log file
 * EnableModelDebugOutput() enables more detailed debug output, each seed will have its own file written to
 * by a ColumnDataWriter passed to it from the test
 * (eg. by the SetupDebugOutput helper function in the project simulator)
 *
 * 1 mitotic-event-sequence sampler (only samples one "path" through the lineage):
 * EnableSequenceSampler() - one "sequence" of progenitors writes mitotic event type to a string in the singleton log file
 *
 ********************************/

class BoijeCellCycleModel : public AbstractSimpleCellCycleModel
{
    friend class TestSimpleCellCycleModels;

private:

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the cell-cycle model and random number generator, never used directly - boost uses this.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractSimpleCellCycleModel>(*this);

        SerializableSingleton<RandomNumberGenerator>* p_wrapper = RandomNumberGenerator::Instance()->GetSerializationWrapper();
        archive & p_wrapper;
        archive & mCellCycleDuration;
        archive & mGeneration;
    }

```

```
    //Private write functions for models
    void WriteModeEventOutput();
    void WriteDebugData(double atoh7RV, double ptf1aRV, double ngRV);

protected:
    //mode/output variables
    bool mOutput;
    double mEventStartTime;
    bool mSequenceSampler;
    bool mSeqSamplerLabelSister;
    //debug writer stuff
    bool mDebug;
    int mTimeID;
    std::vector<int> mVarIDs;
    boost::shared_ptr<ColumnDataWriter> mDebugWriter;
    //model parameters and state memory vars
    unsigned mGeneration;
    unsigned mPhase2gen;
    unsigned mPhase3gen;
    double mprobAtoh7;
        double mprobPtf1a;
        double mprobng;
        bool mAtoh7Signal;
        bool mPtf1aSignal;
        bool mNgSignal;
    unsigned mMitoticMode;
    unsigned mSeed;
    boost::shared_ptr<AbstractCellProperty> mp_PostMitoticType;
    boost::shared_ptr<AbstractCellProperty> mp_RGC_Type;
    boost::shared_ptr<AbstractCellProperty> mp_AC_HC_Type;
    boost::shared_ptr<AbstractCellProperty> mp_PR_BC_Type;
    boost::shared_ptr<AbstractCellProperty> mp_label_Type;

    /**
     * Protected copy-constructor for use by CreateCellCycleModel().
     *
     * The only way for external code to create a copy of a cell cycle model
     * is by calling that method, to ensure that a model of the correct subclass is created.
     * This copy-constructor helps subclasses to ensure that all member variables are correctly copied when this happens.
     *
```

```
     * This method is called by child classes to set member variables for a daughter cell upon cell division.
     * Note that the parent cell cycle model will have had ResetForDivision() called just before CreateCellCycleModel() is called,
     * so performing an exact copy of the parent is suitable behaviour. Any daughter-cell-specific initialisation
     * can be done in InitialiseDaughterCell().
     *
     * @param rModel the cell cycle model to copy.
     */
    BoijeCellCycleModel(const BoijeCellCycleModel& rModel);

public:

    /**
     * Constructor - just a default, mBirthTime is set in the AbstractCellCycleModel class.
     */
    BoijeCellCycleModel();

    /**
     * SetCellCycleDuration() method to set length of cell cycle (default 1.0 as Boije's model is purely generational)
     */
    void SetCellCycleDuration();

    /**
     * Overridden builder method to create new copies of
     * this cell-cycle model.
     *
     * @return new cell-cycle model
     */
    AbstractCellCycleModel* CreateCellCycleModel();

    /**
     * Overridden ResetForDivision() method.
     * Contains general mitotic mode logic
     **/
    void ResetForDivision();

    /**
     * Overridden InitialiseDaughterCell() method.
```

```
     * Used to implement asymmetric mitotic mode
     * */
    void InitialiseDaughterCell();

    /**
     * Sets the cell's generation.
     *
     * @param generation the cell's generation
     **/
    void SetGeneration(unsigned generation);

    /**
     * @return the cell's generation.
     */
    unsigned GetGeneration() const;

    /****************************
     * Model setup functions:
     * Set TF signal probabilities with SetModelParameters
     * Set Postmitotic and specification AbstractCellProperties w/ the relevant functions
     * By default these are available in BoijeRetinalNeuralFates.hpp
     *
     * NB: The Boije model lumps together amacrine & horizontal cells, photoreceptors & bipolar neurons
     ****************************/

    void SetModelParameters(unsigned phase2gen = 3, unsigned phase3gen = 5, double probAtoh7 = 0.32, double probPtf1a = 0.3, double probng = 0.8);
    void SetPostMitoticType(boost::shared_ptr<AbstractCellProperty> p_PostMitoticType);
    void SetSpecifiedTypes(boost::shared_ptr<AbstractCellProperty> p_RGC_Type, boost::shared_ptr<AbstractCellProperty> p_AC_HC_Type, boost::shared_ptr<AbstractCellProperty> p_PR_BC_Type);

    //Functions to enable per-cell mitotic mode logging for mode rate & sequence sampling fixtures
    //Uses singleton logfile
    void EnableModeEventOutput(double eventStart, unsigned seed);
    void EnableSequenceSampler(boost::shared_ptr<AbstractCellProperty> label);

```

```
    //More detailed debug output. Needs a ColumnDataWriter passed to it
    //Only declare ColumnDataWriter directory, filename, etc; do not set up otherwise
    void EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter);

    /**
     * Overridden GetAverageTransitCellCycleTime() method.
     *
     * @return the average of mMinCellCycleDuration and mMaxCellCycleDuration
     */
    double GetAverageTransitCellCycleTime();

    /**
     * Overridden GetAverageStemCellCycleTime() method.
     *
     * @return the average of mMinCellCycleDuration and mMaxCellCycleDuration
     */
    double GetAverageStemCellCycleTime();

    /**
     * Overridden OutputCellCycleModelParameters() method.
     *
     * @param rParamsFile the file stream to which the parameters are output
     */
    virtual void OutputCellCycleModelParameters(out_stream& rParamsFile);
};

#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(BoijeCellCycleModel)

#endif /*BOIJECELLCYCLEMODEL_HPP_*/
```

17.1.21 /src/BoijeRetinalNeuralFates.cpp

```
#include "../../../../projects/ISP/src/BoijeRetinalNeuralFates.hpp"

RetinalGanglion::RetinalGanglion(unsigned colour)
```

```
    : AbstractCellProperty(),
      mColour(colour)
{
}

RetinalGanglion::~RetinalGanglion()
{
}

unsigned RetinalGanglion::GetColour() const
{
    return mColour;
}

AmacrineHorizontal::AmacrineHorizontal(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
{
}

AmacrineHorizontal::~AmacrineHorizontal()
{
}

unsigned AmacrineHorizontal::GetColour() const
{
    return mColour;
}

ReceptorBipolar::ReceptorBipolar(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
{
}

ReceptorBipolar::~ReceptorBipolar()
{
}

unsigned ReceptorBipolar::GetColour() const
{
    return mColour;
}
```

```

#include "SerializationExportWrapperForCpp.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(RetinalGanglion)
CHASTE_CLASS_EXPORT(AmacrineHorizontal)
CHASTE_CLASS_EXPORT(ReceptorBipolar)
```

17.1.22 /src/BoijeRetinalNeuralFates.hpp

```
#ifndef BOIJERETINALNEURALFATES_HPP_
#define BOIJERETINALNEURALFATES_HPP_

#include <boost/shared_ptr.hpp>
#include "AbstractCellProperty.hpp"
#include "ChasteSerialization.hpp"
#include <boost/serialization/base_object.hpp>

class RetinalGanglion : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

```

```
public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in the visualizer (defaults to 6)
     */
    RetinalGanglion(unsigned colour=3);

    /**
     * Destructor.
     */
    virtual ~RetinalGanglion();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};

class AmacrineHorizontal : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
```

```
        archive & mColour;
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in
 ↪  the visualizer (defaults to 6)
     */
    AmacrineHorizontal(unsigned colour=4);

    /**
     * Destructor.
     */
    virtual ~AmacrineHorizontal();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};

class ReceptorBipolar : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
```

```
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in the visualizer (defaults to 6)
     */
    ReceptorBipolar(unsigned colour=5);

    /**
     * Destructor.
     */
    virtual ~ReceptorBipolar();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};

#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(RetinalGanglion)
CHASTE_CLASS_EXPORT(AmacrineHorizontal)
CHASTE_CLASS_EXPORT(ReceptorBipolar)

#endif /* BOIJERETINALNEURALFATES_HPP_ */
```

17.1.23 /src/GomesCellCycleModel.cpp

```
#include "GomesCellCycleModel.hpp"
#include "GomesRetinalNeuralFates.hpp"

GomesCellCycleModel::GomesCellCycleModel() :
```

```
        AbstractSimpleCellCycleModel(), mOutput(false), mEventStartTime(), mSequenceSampler(false), mSeqSamplerLabelSister(
                false), mDebug(false), mTimeID(), mVarIDs(), mDebugWriter(), mNormalMu(3.9716), mNormalSigma(0.32839), mPP(
                .055), mPD(0.221), mpBC(.128), mpAC(.106), mpMG(.028), mMitoticMode(), mSeed(), mp_PostMitoticType(), mp_RPh_Type(), mp_BC_Type(), mp_AC_Type(), mp_MG_Type(), mp_label_Type()
{
}

GomesCellCycleModel::GomesCellCycleModel(const GomesCellCycleModel& rModel) :
        AbstractSimpleCellCycleModel(rModel), mOutput(rModel.mOutput), mEventStartTime(rModel.mEventStartTime), mSequenceSampler(
                rModel.mSequenceSampler), mSeqSamplerLabelSister(rModel.mSeqSamplerLabelSister), mDebug(rModel.mDebug), mTimeID(
                rModel.mTimeID), mVarIDs(rModel.mVarIDs), mDebugWriter(rModel.mDebugWriter), mNormalMu(
                rModel.mNormalMu), mNormalSigma(rModel.mNormalSigma), mPP(rModel.mPP), mPD(rModel.mPD), mpBC(
                rModel.mpBC), mpAC(rModel.mpAC), mpMG(rModel.mpMG), mMitoticMode(rModel.mMitoticMode), mSeed(
                rModel.mSeed), mp_PostMitoticType(rModel.mp_PostMitoticType), mp_RPh_Type(rModel.mp_RPh_Type), mp_BC_Type(
                rModel.mp_BC_Type), mp_AC_Type(rModel.mp_AC_Type), mp_MG_Type(rModel.mp_MG_Type), mp_label_Type(
                rModel.mp_label_Type)
{
}

AbstractCellCycleModel* GomesCellCycleModel::CreateCellCycleModel()
{
    return new GomesCellCycleModel(*this);
}

void GomesCellCycleModel::SetCellCycleDuration()
{
    /*************************************
```

```
31      * CELL CYCLE DURATION RANDOM VARIABLE
32      ************************************/
33
34     RandomNumberGenerator* p_random_number_generator =
       ↪  RandomNumberGenerator::Instance();
35
36     //Gomes cell cycle length determined by lognormal distribution with
       ↪  default mean 56 hr, std 18.9 hrs.
37     mCellCycleDuration =
       ↪  exp(p_random_number_generator->NormalRandomDeviate(mNormalMu,
       ↪  mNormalSigma));
38 }
39
40 void GomesCellCycleModel::ResetForDivision()
41 {
42     /****************
43      * Mitotic mode rules
44      * *************/
45     RandomNumberGenerator* p_random_number_generator =
       ↪  RandomNumberGenerator::Instance();
46
47     //Check time in lineage and determine current mitotic mode phase
48     mMitoticMode = 0; //0=PP;1=PD;2=DD
49
50     /******************************
51      * MITOTIC MODE RANDOM VARIABLE
52      ******************************/
53     //initialise mitoticmode random variable, set mitotic mode
       ↪  appropriately after comparing to mode probability array
54     double mitoticModeRV = p_random_number_generator->ranf();
55
56     if (mitoticModeRV > mPP && mitoticModeRV <= mPP + mPD)
57     {
58         mMitoticMode = 1;
59     }
60     if (mitoticModeRV > mPP + mPD)
61     {
62         mMitoticMode = 2;
63     }
64
65     /****************
66      * Write mitotic event to file if appropriate
```

```
     * mDebug: many files: detailed per-lineage info switch; intended for TestHeInductionCountFixture
     * mOutput: 1 file: time, seed, cellID, mitotic mode, intended for TestHeMitoticModeRateFixture
     * *************/

    if (mDebug)
    {
        WriteDebugData(mitoticModeRV);
    }

    if (mOutput)
    {
        WriteModeEventOutput();
    }

    //set new cell cycle length (will be overwritten with DBL_MAX for DD divisions)
    AbstractSimpleCellCycleModel::ResetForDivision();

    /****************
     * Symmetric postmitotic specification rule
     * -(asymmetric postmitotic rule specified in InitialiseDaughterCell();)
     * *************/

    if (mMitoticMode == 2)
    {
        mpCell->SetCellProliferativeType(mp_PostMitoticType);
        mCellCycleDuration = DBL_MAX;
        /****************************
         * SPECIFICATION RANDOM VARIABLE
         ****************************/
        double specificationRV = p_random_number_generator->ranf();
        if (specificationRV <= mpMG)
        {
            mpCell->AddCellProperty(mp_MG_Type);
        }
        if (specificationRV > mpMG && specificationRV <= mpMG + mpAC)
        {
            mpCell->AddCellProperty(mp_AC_Type);
        }
```

```
        if (specificationRV > mpMG + mpAC && specificationRV ≤ mpMG + mpAC + mpBC)
        {
            mpCell→AddCellProperty(mp_BC_Type);
        }
        if (specificationRV > mpMG + mpAC + mpBC)
        {
            mpCell→AddCellProperty(mp_RPh_Type);
        }
    }

    /******************
     * SEQUENCE SAMPLER
     ******************/
    //if the sequence sampler has been turned on, check for the label & write mitotic mode to log
    //50% chance of each daughter cell from a mitosis inheriting the label
    if (mSequenceSampler)
    {
        if (mpCell→HasCellProperty<CellLabel>())
        {
            (*LogFile::Instance()) << mMitoticMode;
            double labelRV = p_random_number_generator→ranf();
            if (labelRV ≤ .5)
            {
                mSeqSamplerLabelSister = true;
                mpCell→RemoveCellProperty<CellLabel>();
            }
            else
            {
                mSeqSamplerLabelSister = false;
            }
        }
        else
        {
            //prevents lost-label cells from labelling their progeny
            mSeqSamplerLabelSister = false;
        }
    }
}

void GomesCellCycleModel::InitialiseDaughterCell()
```

```
{
    if (mMitoticMode == 0)
    {
        //daughter cell's mCellCycleDuration is copied from parent; reset to new value from gamma PDF here
        SetCellCycleDuration();
    }

    /************
     * PD-type division
     **********/

    if (mMitoticMode == 1)
    {
        RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();
        mpCell->SetCellProliferativeType(mp_PostMitoticType);
        mCellCycleDuration = DBL_MAX;
        /********************
         * SPECIFICATION RULES
         ********************/
        double specificationRV = p_random_number_generator->ranf();
        if (specificationRV <= mpMG)
        {
            mpCell->AddCellProperty(mp_MG_Type);
        }
        if (specificationRV > mpMG && specificationRV <= mpMG + mpAC)
        {
            mpCell->AddCellProperty(mp_AC_Type);
        }
        if (specificationRV > mpMG + mpAC && specificationRV <= mpMG + mpAC + mpBC)
        {
            mpCell->AddCellProperty(mp_BC_Type);
        }
        if (specificationRV > mpMG + mpAC + mpBC)
        {
            mpCell->AddCellProperty(mp_RPh_Type);
        }
    }

    if (mMitoticMode == 2)
    {
```

```
        RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();
        //remove the fate assigned to the parent cell in ResetForDivision, then assign the sister fate as usual
        mpCell->RemoveCellProperty<AbstractCellProperty>();
        mpCell->SetCellProliferativeType(mp_PostMitoticType);

        /*********************
         * SPECIFICATION RULES
         ********************/
        double specificationRV = p_random_number_generator->ranf();
        if (specificationRV <= mpMG)
        {
            mpCell->AddCellProperty(mp_MG_Type);
        }
        if (specificationRV > mpMG && specificationRV <= mpMG + mpAC)
        {
            mpCell->AddCellProperty(mp_AC_Type);
        }
        if (specificationRV > mpMG + mpAC && specificationRV <= mpMG + mpAC + mpBC)
        {
            mpCell->AddCellProperty(mp_BC_Type);
        }
        if (specificationRV > mpMG + mpAC + mpBC)
        {
            mpCell->AddCellProperty(mp_RPh_Type);
        }
    }

    /******************
     * SEQUENCE SAMPLER
     ******************/
    if (mSequenceSampler)
    {
        if (mSeqSamplerLabelSister)
        {
            mpCell->AddCellProperty(mp_label_Type);
            mSeqSamplerLabelSister = false;
        }
        else
        {
            mpCell->RemoveCellProperty<CellLabel>();
```

```
        }
    }
}

void GomesCellCycleModel::SetModelParameters(const double normalMu, const double normalSigma, const double PP,
                                             const double PD, const double pBC, const double pAC, const double pMG)
{
    mNormalMu = normalMu;
    mNormalSigma = normalSigma;
    mPP = PP;
    mPD = PD;
    mpBC = pBC;
    mpAC = pAC;
    mpMG = pMG;

}

void GomesCellCycleModel::SetModelProperties(boost::shared_ptr<AbstractCellProperty> p_RPh_Type,
                                             boost::shared_ptr<AbstractCellProper p_AC_Type,
                                             boost::shared_ptr<AbstractCellProper p_BC_Type,
                                             boost::shared_ptr<AbstractCellProper p_MG_Type)
{
    mp_RPh_Type = p_RPh_Type;
    mp_AC_Type = p_AC_Type;
    mp_BC_Type = p_BC_Type;
    mp_MG_Type = p_MG_Type;
}

void GomesCellCycleModel::SetPostMitoticType(boost::shared_ptr<AbstractCellProperty> p_PostMitoticType)
{
```

```
    mp_PostMitoticType = p_PostMitoticType;
}

void GomesCellCycleModel::EnableModeEventOutput(double eventStart, unsigned seed)
{
    mOutput = true;
    mEventStartTime = eventStart;
    mSeed = seed;

}

void GomesCellCycleModel::WriteModeEventOutput()
{
    double currentTime = SimulationTime::Instance()->GetTime() + mEventStartTime;
    CellPtr currentCell = GetCell();
    double currentCellID = (double) currentCell->GetCellId();
    (*LogFile::Instance()) << currentTime << "\t" << mSeed << "\t" << currentCellID << "\t" << mMitoticMode << "\n";
}

void GomesCellCycleModel::EnableSequenceSampler(boost::shared_ptr<AbstractCellProperty label)
{
    mSequenceSampler = true;
    mp_label_Type = label;
}

void GomesCellCycleModel::EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter)
{
    mDebug = true;
    mDebugWriter = debugWriter;

    mTimeID = mDebugWriter->DefineUnlimitedDimension("Time", "h");
    mVarIDs.push_back(mDebugWriter->DefineVariable("CellID", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("CycleDuration", "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("PP", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("PD", "Percentile"));
```

```
    mVarIDs.push_back(mDebugWriter->DefineVariable("Dieroll", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticMode", "Mode"));

    mDebugWriter->EndDefineMode();
}

void GomesCellCycleModel::WriteDebugData(double percentileRoll)
{
    double currentTime = SimulationTime::Instance()->GetTime();
    CellPtr currentCell = GetCell();
    double currentCellID = (double) currentCell->GetCellId();

    mDebugWriter->PutVariable(mTimeID, currentTime);
    mDebugWriter->PutVariable(mVarIDs[0], currentCellID);
    mDebugWriter->PutVariable(mVarIDs[1], mCellCycleDuration);
    mDebugWriter->PutVariable(mVarIDs[2], mPP);
    mDebugWriter->PutVariable(mVarIDs[3], mPD);
    mDebugWriter->PutVariable(mVarIDs[4], percentileRoll);
    mDebugWriter->PutVariable(mVarIDs[5], mMitoticMode);
    mDebugWriter->AdvanceAlongUnlimitedDimension();
}

/******************
 * UNUSED FUNCTIONS (required for class nonvirtuality, do not remove)
 ******************/

double GomesCellCycleModel::GetAverageTransitCellCycleTime()
{
    return (0.0);
}

double GomesCellCycleModel::GetAverageStemCellCycleTime()
{
    return (0.0);
}

void GomesCellCycleModel::OutputCellCycleModelParameters(out_stream& rParamsFile)
{
    *rParamsFile << "\t\t\t<CellCycleDuration>" << mCellCycleDuration << "</CellCycleDuration>\n";
```

```

    // Call method on direct parent class

    AbstractSimpleCellCycleModel::OutputCellCycleModelParameters(rParamsFile);
}

// Serialization for Boost >= 1.36
#include "SerializationExportWrapperForCpp.hpp"
CHASTE_CLASS_EXPORT(GomesCellCycleModel)
```

17.1.24 /src/GomesCellCycleModel.hpp

```
#ifndef GOMESCELLCYCLEMODEL_HPP_
#define GOMESCELLCYCLEMODEL_HPP_

#include "AbstractSimpleCellCycleModel.hpp"
#include "RandomNumberGenerator.hpp"
#include "Cell.hpp"
#include "DifferentiatedCellProliferativeType.hpp"
#include "GomesRetinalNeuralFates.hpp"
#include "SmartPointers.hpp"
#include "ColumnDataWriter.hpp"
#include "LogFile.hpp"
#include "CellLabel.hpp"

/*****************************************
 * GOMES CELL CYCLE MODEL
 * As described in Gomes et al. 2011 [Gomes2011] doi: 10.1242/dev.059683
 *
 * USE: By default, GomesCellCycleModels are constructed with the parameter fit reported in [Gomes2011].
 * Cell cycle length, mitotic mode, and postmitotic fate of cells are determined by independent random variables
 * PP = symmetric proliferative mitotic mode, both progeny remain mitotic
 * PD = asymmetric proliferative mitotic mode, one progeny exits the cell cycle and differentiates
 * DD = symmetric differentiative mitotic mode, both progeny exit the cell cycle and differentiate
 *
 * Change default model parameters with SetModelParameters(<params>);
 * Set AbstractCellProperties for differentiated neural types with SetModelProperties();
```

```
 *
 * 2 per-model-event output modes:
 * EnableModeEventOutput() enables mitotic mode event logging-all cells will write to the singleton log file
 * EnableModelDebugOutput() enables more detailed debug output, each seed will have its own file written to
 * by a ColumnDataWriter passed to it from the test
 * (eg. by the SetupDebugOutput helper function in the project simulator)
 *
 * 1 mitotic-event-sequence sampler (only samples one "path" through the lineage):
 * EnableSequenceSampler() - one "sequence" of progenitors writes mitotic event type to a string in the singleton log file
 *
 ***********************************************/

class GomesCellCycleModel : public AbstractSimpleCellCycleModel
{
    friend class TestSimpleCellCycleModels;

private:

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the cell-cycle model and random number generator, never used directly - boost uses this.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractSimpleCellCycleModel>(*this);

        SerializableSingleton<RandomNumberGenerator>* p_wrapper = RandomNumberGenerator::Instance()->GetSerializationWrapper();
        archive & p_wrapper;
        archive & mCellCycleDuration;
    }
```

```
62
63      //Private write functions for models
64      void WriteModeEventOutput();
65      void WriteDebugData(double percentile);
66
67  protected:
68      //mode/output variables
69      bool mOutput;
70      double mEventStartTime;
71      bool mSequenceSampler;
72      bool mSeqSamplerLabelSister;
73      //debug writer stuff
74      bool mDebug;
75      int mTimeID;
76      std::vector<int> mVarIDs;
77      boost::shared_ptr<ColumnDataWriter> mDebugWriter;
78      //model parameters and state memory vars
79      double mNormalMu;
80      double mNormalSigma;
81      double mPP;
82      double mPD;
83      double mpBC;
84      double mpAC;
85      double mpMG;
86      unsigned mMitoticMode;
87      unsigned mSeed;
88      boost::shared_ptr<AbstractCellProperty> mp_PostMitoticType;
89      boost::shared_ptr<AbstractCellProperty> mp_RPh_Type;
90      boost::shared_ptr<AbstractCellProperty> mp_BC_Type;
91      boost::shared_ptr<AbstractCellProperty> mp_AC_Type;
92      boost::shared_ptr<AbstractCellProperty> mp_MG_Type;
93      boost::shared_ptr<AbstractCellProperty> mp_label_Type;
94
95      /**
96       * Protected copy-constructor for use by CreateCellCycleModel().
97       *
98       * The only way for external code to create a copy of a cell cycle
    ↪   model
99       * is by calling that method, to ensure that a model of the correct
    ↪   subclass is created.
100      * This copy-constructor helps subclasses to ensure that all member
    ↪   variables are correctly copied when this happens.
101      *
```

```
     * This method is called by child classes to set member variables for a daughter cell upon cell division.
     * Note that the parent cell cycle model will have had ResetForDivision() called just before CreateCellCycleModel() is called,
     * so performing an exact copy of the parent is suitable behaviour. Any daughter-cell-specific initialisation
     * can be done in InitialiseDaughterCell().
     *
     * @param rModel the cell cycle model to copy.
     */
    GomesCellCycleModel(const GomesCellCycleModel& rModel);

public:

    /**
     * Constructor - just a default, mBirthTime is set in the AbstractCellCycleModel class.
     */
    GomesCellCycleModel();

    /**
     * SetCellCycleDuration() method to set length of cell cycle (lognormal distribution as specified in [Gomes2011])
     */
    void SetCellCycleDuration();

    /**
     * Overridden builder method to create new copies of
     * this cell-cycle model.
     *
     * @return new cell-cycle model
     */
    AbstractCellCycleModel* CreateCellCycleModel();

    /**
     * Overridden ResetForDivision() method.
     * Contains general mitotic mode logic
     **/
    void ResetForDivision();

    /** Overridden InitialiseDaughterCell() method. Used to implement asymmetric mitotic mode*/
```

```
    void InitialiseDaughterCell();

    /* Model setup functions
     * Set lognormal cell cycle curve properties with *mean and std of corresponding NORMAL curve*
     * Model requires valid AbstractCellProperties to assign postmitotic fate;
     * Defaults are found in GomesRetinalNeuralFates.hpp
     * (RodPohotreceptor, AmacrineCell, BipolarCell, MullerGlia)
     */

    void SetModelParameters(const double normalMu = 3.9716, const double normalSigma = .32839, const double PP = .055,
                            const double PD = .221, const double pBC = .128, const double pAC = .106, const double pMG =
                                    .028);
    void SetModelProperties(boost::shared_ptr<AbstractCellProperty> p_RPh_Type,
                            boost::shared_ptr<AbstractCellProperty> p_AC_Type,
                            boost::shared_ptr<AbstractCellProperty> p_BC_Type,
                            boost::shared_ptr<AbstractCellProperty> p_MG_Type);

    //This should normally be a DifferentiatedCellProliferativeType
    void SetPostMitoticType(boost::shared_ptr<AbstractCellProperty> p_PostMitoticType);

    //Functions to enable per-cell mitotic mode logging for mode rate & sequence sampling fixtures
    //Uses singleton logfile
    void EnableModeEventOutput(double eventStart, unsigned seed);
    void EnableSequenceSampler(boost::shared_ptr<AbstractCellProperty> label);

    //More detailed debug output. Needs a ColumnDataWriter passed to it
    //Only declare ColumnDataWriter directory, filename, etc; do not set up otherwise
    void EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter);

```

```
    //Not used, but must be overwritten lest GomesCellCycleModels be abstract
    double GetAverageTransitCellCycleTime();
    double GetAverageStemCellCycleTime();

    /**
     * Overridden OutputCellCycleModelParameters() method.
     *
     * @param rParamsFile the file stream to which the parameters are output
     */
    virtual void OutputCellCycleModelParameters(out_stream& rParamsFile);
};

#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(GomesCellCycleModel)

#endif /*GOMESCELLCYCLEMODEL_HPP_*/
```

17.1.25 /src/GomesRetinalNeuralFates.cpp

```
#include "GomesRetinalNeuralFates.hpp"

RodPhotoreceptor::RodPhotoreceptor(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
{
}

RodPhotoreceptor::~RodPhotoreceptor()
{
}

unsigned RodPhotoreceptor::GetColour() const
{
    return mColour;
}

AmacrineCell::AmacrineCell(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
```

```
{
}

AmacrineCell::~AmacrineCell()
{
}

unsigned AmacrineCell::GetColour() const
{
    return mColour;
}

BipolarCell::BipolarCell(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
{
}

BipolarCell::~BipolarCell()
{
}

unsigned BipolarCell::GetColour() const
{
    return mColour;
}

MullerGlia::MullerGlia(unsigned colour)
    : AbstractCellProperty(),
      mColour(colour)
{
}

MullerGlia::~MullerGlia()
{
}

unsigned MullerGlia::GetColour() const
{
    return mColour;
}

#include "SerializationExportWrapperForCpp.hpp"
```

```
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(RodPhotoreceptor)
CHASTE_CLASS_EXPORT(AmacrineCell)
CHASTE_CLASS_EXPORT(BipolarCell)
CHASTE_CLASS_EXPORT(MullerGlia)
```

17.1.26 /src/GomesRetinalNeuralFates.hpp

```
#ifndef GOMESRETINALNEURALFATES_HPP_
#define GOMESRETINALNEURALFATES_HPP_

#include <boost/shared_ptr.hpp>
#include "AbstractCellProperty.hpp"
#include "ChasteSerialization.hpp"
#include <boost/serialization/base_object.hpp>

class RodPhotoreceptor : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

public:
```

```
34
35      /**
36       * Constructor.
37       *
38       * @param colour  what colour cells with this property should be in
   ↪   the visualizer (defaults to 6)
39       */
40      RodPhotoreceptor(unsigned colour=3);
41
42      /**
43       * Destructor.
44       */
45      virtual ~RodPhotoreceptor();
46
47      /**
48       * @return #mColour.
49       */
50      unsigned GetColour() const;
51  };
52
53  class AmacrineCell : public AbstractCellProperty
54  {
55  private:
56
57      /**
58       * Colour for use by visualizer.
59       */
60      unsigned mColour;
61
62      /** Needed for serialization. */
63      friend class boost::serialization::access;
64      /**
65       * Archive the member variables.
66       *
67       * @param archive the archive
68       * @param version the current version of this class
69       */
70      template<class Archive>
71      void serialize(Archive & archive, const unsigned int version)
72      {
73          archive &
            ↪  boost::serialization::base_object<AbstractCellProperty>(*this);
74          archive & mColour;
```

```
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in
↪   the visualizer (defaults to 6)
     */
    AmacrineCell(unsigned colour=4);

    /**
     * Destructor.
     */
    virtual ~AmacrineCell();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};

class BipolarCell : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
```

```
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in the visualizer (defaults to 6)
     */
    BipolarCell(unsigned colour=5);

    /**
     * Destructor.
     */
    virtual ~BipolarCell();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};

class MullerGlia : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
```

```
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in the visualizer (defaults to 6)
     */
    MullerGlia(unsigned colour=6);

    /**
     * Destructor.
     */
    virtual ~MullerGlia();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};
#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(RodPhotoreceptor)
CHASTE_CLASS_EXPORT(AmacrineCell)
CHASTE_CLASS_EXPORT(BipolarCell)
CHASTE_CLASS_EXPORT(MullerGlia)

#endif /* GOMESRETINALNEURALFATES_HPP_ */
```

17.1.27 /src/HeAth5Mo.cpp

```
#include "../../../../projects/ISP/src/HeAth5Mo.hpp"

Ath5Mo::Ath5Mo(unsigned colour)
```

```
    : AbstractCellProperty(),
      mColour(colour)
{
}

Ath5Mo::~Ath5Mo()
{
}

unsigned Ath5Mo::GetColour() const
{
    return mColour;
}

#include "SerializationExportWrapperForCpp.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(Ath5Mo)
```

17.1.28 /src/HeAth5Mo.hpp

```
#ifndef HEATH5MO_HPP_
#define HEATH5MO_HPP_

#include <boost/shared_ptr.hpp>
#include "AbstractCellProperty.hpp"
#include "ChasteSerialization.hpp"
#include <boost/serialization/base_object.hpp>

class Ath5Mo : public AbstractCellProperty
{
private:

    /**
     * Colour for use by visualizer.
     */
    unsigned mColour;

    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the member variables.
     *
```

```
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractCellProperty>(*this);
        archive & mColour;
    }

public:

    /**
     * Constructor.
     *
     * @param colour  what colour cells with this property should be in the visualizer (defaults to 6)
     */
    Ath5Mo(unsigned colour=3);

    /**
     * Destructor.
     */
    virtual ~Ath5Mo();

    /**
     * @return #mColour.
     */
    unsigned GetColour() const;
};


#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(Ath5Mo)
#endif /* HEATH5MO_HPP */
```

17.1.29 /src/HeCellCycleModel.cpp

```
#include "HeCellCycleModel.hpp"

```

```
3 HeCellCycleModel::HeCellCycleModel() :
4         AbstractSimpleCellCycleModel(), mKillSpecified(false),
          ↪  mDeterministic(false), mOutput(false), mEventStartTime(
5                 24.0), mSequenceSampler(false),
                  ↪  mSeqSamplerLabelSister(false), mDebug(false),
                  ↪  mTimeID(), mVarIDs(), mDebugWriter(), mTiLOffset(
6                 0.0), mGammaShift(4.0), mGammaShape(2.0),
                  ↪  mGammaScale(1.0), mSisterShiftWidth(1),
                  ↪  mMitoticModePhase2(
7                 8.0), mMitoticModePhase3(15.0), mPhaseShiftWidth(2.0),
                  ↪  mPhase1PP(1.0), mPhase1PD(0.0), mPhase2PP(0.2),
                  ↪  mPhase2PD(
8                 0.4), mPhase3PP(0.2), mPhase3PD(0.0), mMitoticMode(0),
                  ↪  mSeed(0), mTimeDependentCycleDuration(false),
                  ↪  mPeakRateTime(), mIncreasingRateSlope(),
                  ↪  mDecreasingRateSlope(), mBaseGammaScale()
9 {
10     mReadyToDivide = true; //He model begins with a first division
11 }
12
13 HeCellCycleModel::HeCellCycleModel(const HeCellCycleModel& rModel) :
14         AbstractSimpleCellCycleModel(rModel),
          ↪  mKillSpecified(rModel.mKillSpecified), mDeterministic(
15                rModel.mDeterministic), mOutput(rModel.mOutput),
                  ↪  mEventStartTime(rModel.mEventStartTime),
                  ↪  mSequenceSampler(
16                rModel.mSequenceSampler),
                  ↪  mSeqSamplerLabelSister(rModel.mSeqSamplerLabelSister),
                  ↪  mDebug(rModel.mDebug), mTimeID(
17                rModel.mTimeID), mVarIDs(rModel.mVarIDs),
                  ↪  mDebugWriter(rModel.mDebugWriter), mTiLOffset(
18                rModel.mTiLOffset), mGammaShift(rModel.mGammaShift),
                  ↪  mGammaShape(rModel.mGammaShape), mGammaScale(
19                rModel.mGammaScale),
                  ↪  mSisterShiftWidth(rModel.mSisterShiftWidth),
                  ↪  mMitoticModePhase2(
20                rModel.mMitoticModePhase2),
                  ↪  mMitoticModePhase3(rModel.mMitoticModePhase3),
                  ↪  mPhaseShiftWidth(
21                rModel.mPhaseShiftWidth), mPhase1PP(rModel.mPhase1PP),
                  ↪  mPhase1PD(rModel.mPhase1PD), mPhase2PP(
22                rModel.mPhase2PP), mPhase2PD(rModel.mPhase2PD),
                  ↪  mPhase3PP(rModel.mPhase3PP), mPhase3PD(
```

```
                rModel.mPhase3PD), mMitoticMode(rModel.mMitoticMode), mSeed(rModel.mSeed), mTimeDependentCycleDuration(
                rModel.mTimeDependentCycleDuration), mPeakRateTime(rModel.mPeakRateTime), mIncreasingRateSlope(
                rModel.mIncreasingRateSlope), mDecreasingRateSlope(rModel.mDecreasingRateSlope), mBaseGammaScale(
                rModel.mBaseGammaScale)
{
}

AbstractCellCycleModel* HeCellCycleModel::CreateCellCycleModel()
{
    return new HeCellCycleModel(*this);
}

void HeCellCycleModel::SetCellCycleDuration()
{
    RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();

    /*************************************
     * CELL CYCLE DURATION RANDOM VARIABLE
     *************************************/

    if (!mTimeDependentCycleDuration) //Normal operation, cell cycle length stays constant
    {
        //He cell cycle length determined by shifted gamma distribution reflecting 4 hr refractory period followed by gamma pdf
        mCellCycleDuration = mGammaShift + p_random_number_generator->GammaRandomDeviate(mGammaShape, mGammaScale);
    }

    /****
     * Variable cycle length
     * Give -ve mIncreasingRateSlope and +ve mDecreasingRateSlope,
     * cell cycle length linearly declines (increasing rate), then increases, switching at mPeakRateTime
     ****/
    else
```

```
    {
        double currTime = SimulationTime::Instance()->GetTime();
        if (currTime <= mPeakRateTime)
        {
            mGammaScale = std::max((mBaseGammaScale - currTime *
             ↪  mIncreasingRateSlope), .0000000000001);
        }
        if (currTime > mPeakRateTime)
        {
            mGammaScale = std::max(
                    ((mBaseGammaScale - mPeakRateTime *
                     ↪  mIncreasingRateSlope)
                            + (mBaseGammaScale + (currTime -
                             ↪  mPeakRateTime) * mDecreasingRateSlope)),
                    .0000000000001);
        }
        mCellCycleDuration = mGammaShift +
         ↪  p_random_number_generator->GammaRandomDeviate(mGammaShape,
         ↪  mGammaScale);
    }

}

void HeCellCycleModel::ResetForDivision()
{
    /*****************************************************
     * TIME IN LINEAGE DEPENDENT MITOTIC MODE PHASE RULES
     * *****************************************************/
    RandomNumberGenerator* p_random_number_generator =
     ↪  RandomNumberGenerator::Instance();

    double currentTiL = SimulationTime::Instance()->GetTime() +
     ↪  mTiLOffset;

    /*Rule logic defaults to phase 1 behaviour, checks for currentTiL >
     ↪  phaseBoundaries and changes
     currentPhase and subsequently mMitoticMode as appropriate*/
    unsigned currentPhase = 1;
    mMitoticMode = 0;

    //Check time in lineage and determine current mitotic mode phase
    /**************
     * Phase boundary & deterministic mitotic mode rules
```

```
90      **************/
91     if (currentTiL > mMitoticModePhase2 && currentTiL <
        ↪  mMitoticModePhase3)
92     {
93         //if current TiL is > phase 2 boundary time, set the currentPhase
            ↪  appropriately
94         currentPhase = 2;
95
96         //if deterministic mode is enabled, PD divisions are guaranteed
            ↪  unless this is an Ath5 morphant
97         if (mDeterministic)
98         {
99             mMitoticMode = 1; //0=PP;1=PD;2=DD
100            if (mpCell→HasCellProperty<Ath5Mo>()) //Ath5 morphants
                ↪  undergo PP rather than PD divisions in 80% of cases
101            {
102                double ath5RV = p_random_number_generator→ranf();
103                if (ath5RV ≤ .8)
104                {
105                    mMitoticMode = 0;
106                }
107            }
108        }
109    }
110
111    if (currentTiL > mMitoticModePhase3)
112    {
113        //if current TiL is > phase 3 boundary time, set the currentPhase
            ↪  appropriately
114        currentPhase = 3;
115        if (mDeterministic)
116        {
117            //if deterministic mode is enabled, DD divisions are
                ↪  guaranteed
118            mMitoticMode = 2;
119        }
120    }
121
122    /******************************
123     * MITOTIC MODE RANDOM VARIABLE
124     ******************************/
125    //initialise mitoticmode random variable, set mitotic mode
        ↪  appropriately after comparing to mode probability matrix
```

```
    double mitoticModeRV = p_random_number_generator->ranf(); //0-1 evenly distributed RV

    if (!mDeterministic)
    {
        //construct 3×2 matrix of mode probabilities arranged by phase
        double modeProbabilityMatrix[3][2] = { { mPhase1PP, mPhase1PD }, { mPhase2PP, mPhase2PD }, { mPhase3PP,




        //if the RV is > currentPhasePP && ≤ currentPhasePD, change mMitoticMode from PP to PD
        if (mitoticModeRV > modeProbabilityMatrix[currentPhase - 1][0]
                && mitoticModeRV
                        ≤ modeProbabilityMatrix[currentPhase - 1][0] + modeProbabilityMatrix[currentPhase - 1][1])
        {
            mMitoticMode = 1;
            if (mpCell->HasCellProperty<Ath5Mo>()) //Ath5 morphants undergo PP rather than PD divisions in 80% of cases
            {
                double ath5RV = p_random_number_generator->ranf();
                if (ath5RV ≤ .8)
                {
                    mMitoticMode = 0;
                }
            }
        }
        //if the RV is > currentPhasePP + currentPhasePD, change mMitoticMode from PP to DD
        if (mitoticModeRV > modeProbabilityMatrix[currentPhase - 1][0] + modeProbabilityMatrix[currentPhase - 1][1])
        {
            mMitoticMode = 2;
        }
    }

    /****************
     * Write mitotic event to relevant files
     * *************/
```

```
    if (mDebug)
    {
        WriteDebugData(currentTiL, currentPhase, mitoticModeRV);
    }

    if (mOutput)
    {
        WriteModeEventOutput();
    }

    //set new cell cycle length (will be overwritten with DBL_MAX for DD divisions)
    AbstractSimpleCellCycleModel::ResetForDivision();

    /****************
     * Symmetric postmitotic specification rule
     * -(asymmetric postmitotic rule specified in InitialiseDaughterCell();)
     * *************/
    if (mMitoticMode == 2)
    {
        boost::shared_ptr<AbstractCellProperty> p_PostMitoticType = mpCell->rGetCellPropertyCollection().GetCellPropertyRegistry()->G
        mpCell->SetCellProliferativeType(p_PostMitoticType);
        mCellCycleDuration = DBL_MAX;

        if(mKillSpecified)
        {
            mpCell->Kill();
        }
    }

    /******************
     * SEQUENCE SAMPLER
     ******************/
    //if the sequence sampler has been turned on, check for the label & write mitotic mode to log
    //50% chance of each daughter cell from a mitosis inheriting the label
    if (mSequenceSampler)
    {
        if (mpCell->HasCellProperty<CellLabel>())
```

```
        {
            (*LogFile::Instance()) << mMitoticMode;
            double labelRV = p_random_number_generator->ranf();
            if (labelRV <= .5)
            {
                mSeqSamplerLabelSister = true;
                mpCell->RemoveCellProperty<CellLabel>();
            }
            else
            {
                mSeqSamplerLabelSister = false;
            }
        }
        else
        {
            //prevents lost-label cells from labelling their progeny
            mSeqSamplerLabelSister = false;
        }
    }
}

void HeCellCycleModel::Initialise()
{
    boost::shared_ptr<AbstractCellProperty> p_Transit =
                ↪ mpCell->rGetCellPropertyCollection().GetCellPropertyRegistry()->Get<T
    mpCell->SetCellProliferativeType(p_Transit);

    if (mTiLOffset == 0) //the "regular" case, set cycle duration
    ↪ normally
    {
        SetCellCycleDuration();
    }

    if (mTiLOffset < 0) //these cells are offspring of Wan stem cells
    {
        mReadyToDivide = false;
        SetCellCycleDuration();
    }

    if (mTiLOffset > 0.0) //if the TiL is > 0, the first division has
    ↪ already occurred
    {
```

```
        mReadyToDivide = false;

        RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();

        /**
         * This calculation "runs time forward" by subtracting appropriately generated cell lengths from TiLOffset
         * Ultimately c is subtracted from a final cell length calculation to give the appropriate reduced cycle length
         **/

        double c = mTiLOffset;
        while (c > 0)
        {
            c = c - (mGammaShift + p_random_number_generator->GammaRandomDeviate(mGammaShape, mGammaScale));
        }

        mCellCycleDuration = (mGammaShift + p_random_number_generator->GammaRandomDeviate(mGammaShape, mGammaScale))
                + c;
    }

}

void HeCellCycleModel::InitialiseDaughterCell()
{
    RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();

    /************
     * PD-type division & shifted sister cycle length & boundary adjustments
     **********/

    if (mMitoticMode == 1) //RPC becomes specified retinal neuron in asymmetric PD mitosis
    {
        boost::shared_ptr<AbstractCellProperty> p_PostMitoticType =
```

```
269                 ↪ mpCell→rGetCellPropertyCollection().GetCellPropertyRegistry()→G
270         mpCell→SetCellProliferativeType(p_PostMitoticType);
271         mCellCycleDuration = DBL_MAX;
272
273         if(mKillSpecified)
274         {
275             mpCell→Kill();
276         }
277     }
278
279     //daughter cell's mCellCycleDuration is copied from parent; modified
        ↪ by a normally distributed shift if it remains proliferative
280     if (mMitoticMode == 0)
281     {
282         double sisterShift =
            ↪ p_random_number_generator→NormalRandomDeviate(0,
            ↪ mSisterShiftWidth); //random variable mean 0 SD 1 by default
283         mCellCycleDuration = std::max(mGammaShift, mCellCycleDuration +
            ↪ sisterShift); // sister shift respects 4 hour refractory
            ↪ period
284     }
285
286     //deterministic model phase boundary division shift for daughter
        ↪ cells
287     if (mDeterministic)
288     {
289         //shift phase boundaries to reflect error in "timer" after
            ↪ division
290         double phaseShift =
            ↪ p_random_number_generator→NormalRandomDeviate(0,
            ↪ mPhaseShiftWidth);
291         mMitoticModePhase2 = mMitoticModePhase2 + phaseShift;
292         mMitoticModePhase3 = mMitoticModePhase3 + phaseShift;
293     }
294
295     /******************
296      * SEQUENCE SAMPLER
297      ******************/
298     if (mSequenceSampler)
299     {
300         if (mSeqSamplerLabelSister)
301         {
```

```
            boost::shared_ptr<AbstractCellProperty> p_label_type =
                    ↪ mpCell→rGetCellPropertyCollection().GetCellPropertyRegistry(
            mpCell→AddCellProperty(p_label_type);
            mSeqSamplerLabelSister = false;
        }
        else
        {
            mpCell→RemoveCellProperty<CellLabel>();
        }
    }

    if (mMitoticMode == 2 && mKillSpecified) mpCell→Kill();
}

void HeCellCycleModel::SetModelParameters(double tiLOffset, double
 ↪ mitoticModePhase2, double mitoticModePhase3,
                                          double phase1PP, double
                                           ↪ phase1PD, double phase2PP,
                                           ↪ double phase2PD,
                                          double phase3PP, double
                                           ↪ phase3PD, double
                                           ↪ gammaShift, double
                                           ↪ gammaShape,
                                          double gammaScale, double
                                           ↪ sisterShift)
{
    mTiLOffset = tiLOffset;
    mMitoticModePhase2 = mitoticModePhase2;
    mMitoticModePhase3 = mitoticModePhase3;
    mPhase1PP = phase1PP;
    mPhase1PD = phase1PD;
    mPhase2PP = phase2PP;
    mPhase2PD = phase2PD;
    mPhase3PP = phase3PP;
    mPhase3PD = phase3PD;
    mGammaShift = gammaShift;
    mGammaShape = gammaShape;
    mGammaScale = gammaScale;
    mSisterShiftWidth = sisterShift;
}
```

```
void HeCellCycleModel::SetDeterministicMode(double tiLOffset, double mitoticModePhase2, double mitoticModePhase3,
                                            double phaseShiftWidth, double gammaShift, double gammaShape,
                                            double gammaScale, double sisterShift)
{
    mDeterministic = true;
    mTiLOffset = tiLOffset;
    mMitoticModePhase2 = mitoticModePhase2;
    mMitoticModePhase3 = mitoticModePhase3;
    mPhaseShiftWidth = phaseShiftWidth;
    mGammaShift = gammaShift;
    mGammaShape = gammaShape;
    mGammaScale = gammaScale;
    mSisterShiftWidth = sisterShift;
}

void HeCellCycleModel::SetTimeDependentCycleDuration(double peakRateTime, double increasingSlope,
                                                     double decreasingSlope)
{
    mTimeDependentCycleDuration = true;
    mPeakRateTime = peakRateTime;
    mIncreasingRateSlope = increasingSlope;
    mDecreasingRateSlope = decreasingSlope;
    mBaseGammaScale = mGammaScale;
}

void HeCellCycleModel::EnableKillSpecified()
{
    mKillSpecified = true;
}

void HeCellCycleModel::EnableModeEventOutput(double eventStart, unsigned seed)
{
    mOutput = true;
    mEventStartTime = eventStart;
    mSeed = seed;
}
```

```
372
373 void HeCellCycleModel::WriteModeEventOutput()
374 {
375     double currentTime = SimulationTime::Instance()->GetTime() +
        ↪  mEventStartTime;
376     CellPtr currentCell = GetCell();
377     double currentCellID = (double) currentCell->GetCellId();
378     (*LogFile::Instance()) << currentTime << "\t" << mSeed << "\t" <<
        ↪  currentCellID << "\t" << mMitoticMode << "\n";
379 }
380
381 void HeCellCycleModel::EnableSequenceSampler()
382 {
383     mSequenceSampler = true;
384     boost::shared_ptr<AbstractCellProperty> p_label_type =
385
                ↪  mpCell->rGetCellPropertyCollection().GetCellPropertyRegistry()->Get<C
386     mpCell->AddCellProperty(p_label_type);
387 }
388
389 void
    ↪  HeCellCycleModel::PassDebugWriter(boost::shared_ptr<ColumnDataWriter>
    ↪  debugWriter, int timeID,
390                                          std::vector<int> varIDs)
391 {
392     mDebug = true;
393     mDebugWriter = debugWriter;
394     mTimeID = timeID;
395     mVarIDs = varIDs;
396 }
397
398 void
    ↪  HeCellCycleModel::EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter>
    ↪  debugWriter)
399 {
400     mDebug = true;
401     mDebugWriter = debugWriter;
402
403     mTimeID = mDebugWriter->DefineUnlimitedDimension("Time", "h");
404
405     mVarIDs.push_back(mDebugWriter->DefineVariable("CellID", "No"));
406     mVarIDs.push_back(mDebugWriter->DefineVariable("TiL", "h"));
```

```
    mVarIDs.push_back(mDebugWriter->DefineVariable("CycleDuration", "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase2Boundary", "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase3Boundary", "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticModeRV", "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticMode", "Mode"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Label", "binary"));

    mDebugWriter->EndDefineMode();
}

void HeCellCycleModel::WriteDebugData(double currentTiL, unsigned phase, double mitoticModeRV)
{
    double currentTime = SimulationTime::Instance()->GetTime();
    double currentCellID = mpCell->GetCellId();
    unsigned label = 0;
    if (mpCell->HasCellProperty<CellLabel>()) label = 1;

    mDebugWriter->PutVariable(mTimeID, currentTime);
    mDebugWriter->PutVariable(mVarIDs[0], currentCellID);
    mDebugWriter->PutVariable(mVarIDs[1], currentTiL);
    mDebugWriter->PutVariable(mVarIDs[2], mCellCycleDuration);
    mDebugWriter->PutVariable(mVarIDs[3], mMitoticModePhase2);
    mDebugWriter->PutVariable(mVarIDs[4], mMitoticModePhase3);
    mDebugWriter->PutVariable(mVarIDs[5], phase);
    if (!mDeterministic)
    {
        mDebugWriter->PutVariable(mVarIDs[6], mitoticModeRV);
    }
    mDebugWriter->PutVariable(mVarIDs[7], mMitoticMode);
    if (mSequenceSampler)
    {
        mDebugWriter->PutVariable(mVarIDs[8], label);
    }
    mDebugWriter->AdvanceAlongUnlimitedDimension();
}
```

```
/******************
 * UNUSED FUNCTIONS (required for class nonvirtuality, do not remove)
 ******************/

double HeCellCycleModel::GetAverageTransitCellCycleTime()
{
    return (0.0);
}

double HeCellCycleModel::GetAverageStemCellCycleTime()
{
    return (0.0);
}

void HeCellCycleModel::OutputCellCycleModelParameters(out_stream&
↪   rParamsFile)
{
    *rParamsFile << "\t\t\t<CellCycleDuration>" << mCellCycleDuration <<
     ↪  "</CellCycleDuration>\n";

    // Call method on direct parent class

     ↪  AbstractSimpleCellCycleModel::OutputCellCycleModelParameters(rParamsFile);
}

// Serialization for Boost ≥ 1.36
#include "SerializationExportWrapperForCpp.hpp"
CHASTE_CLASS_EXPORT(HeCellCycleModel)
```

17.1.30 /src/HeCellCycleModel.hpp

```
#ifndef HECELLCYCLEMODEL_HPP_
#define HECELLCYCLEMODEL_HPP_

#include "AbstractSimpleCellCycleModel.hpp"
#include "RandomNumberGenerator.hpp"
#include "Cell.hpp"
#include "TransitCellProliferativeType.hpp"
#include "DifferentiatedCellProliferativeType.hpp"
#include "SmartPointers.hpp"
#include "ColumnDataWriter.hpp"
#include "LogFile.hpp"
```

```
#include "CellLabel.hpp"
#include "HeAth5Mo.hpp"

/***********************************
 * HE CELL CYCLE MODEL
 * As described in He et al. 2012 [He2012] doi: 10.1016/j.neuron.2012.06.033
 *
 * USE: By default, HeCellCycleModels are constructed with the parameter fit reported in [He2012].
 * In normal use, the model steps through three phases of mitotic mode probability parameterisation.
 * PP = symmetric proliferative mitotic mode, both progeny remain mitotic
 * PD = asymmetric proliferative mitotic mode, one progeny exits the cell cycle and differentiates
 * DD = symmetric differentiative mitotic mode, both progeny exit the cell cycle and differentiate
 *
 * Change default model parameters with SetModelParameters(<params>);
 * Enable deterministic model alternative with EnableDeterministicMode(<params>);
 *
 * 2 per-model-event output modes:
 * EnableModeEventOutput() enables mitotic mode event logging-all cells will write to the singleton log file
 * EnableModelDebugOutput() enables more detailed debug output, each seed will have its own file written to
 * by a ColumnDataWriter passed to it from the test
 * (eg. by the SetupDebugOutput helper function in the project simulator)
 *
 * 1 mitotic-event-sequence sampler (only samples one "path" through the lineage):
 * EnableSequenceSampler() - one "sequence" of progenitors writes mitotic event type to a string in the singleton log file
 *
 ***********************************/

class HeCellCycleModel : public AbstractSimpleCellCycleModel
{
    friend class TestSimpleCellCycleModels;

private:

```

```
    /** Needed for serialization. */
    friend class boost::serialization::access;
    /**
     * Archive the cell-cycle model and random number generator, never used directly - boost uses this.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
        archive & boost::serialization::base_object<AbstractSimpleCellCycleModel>(*this);

        SerializableSingleton<RandomNumberGenerator>* p_wrapper = RandomNumberGenerator::Instance()->GetSerializationWrapper();
        archive & p_wrapper;
        archive & mCellCycleDuration;
    }

    //Private write functions for models
    void WriteModeEventOutput();
    void WriteDebugData(double currTiL, unsigned phase, double percentile);

protected:
    //mode/output variables
    bool mKillSpecified;
    bool mDeterministic;
    bool mOutput;
    double mEventStartTime;
    bool mSequenceSampler;
    bool mSeqSamplerLabelSister;
    //debug writer stuff
    bool mDebug;
    int mTimeID;
    std::vector<int> mVarIDs;
    boost::shared_ptr<ColumnDataWriter> mDebugWriter;
    //model parameters and state memory vars
    double mTiLOffset;
    double mGammaShift;
```

```
    double mGammaShape;
    double mGammaScale;
    double mSisterShiftWidth;
    double mMitoticModePhase2;
    double mMitoticModePhase3;
    double mPhaseShiftWidth;
    double mPhase1PP;
    double mPhase1PD;
    double mPhase2PP;
    double mPhase2PD;
    double mPhase3PP;
    double mPhase3PD;
    unsigned mMitoticMode;
    unsigned mSeed;
    bool mTimeDependentCycleDuration;
    double mPeakRateTime;
    double mIncreasingRateSlope;
    double mDecreasingRateSlope;
    double mBaseGammaScale;

    /**
     * Protected copy-constructor for use by CreateCellCycleModel().
     *
     * The only way for external code to create a copy of a cell cycle model
     * is by calling that method, to ensure that a model of the correct subclass is created.
     * This copy-constructor helps subclasses to ensure that all member variables are correctly copied when this happens.
     *
     * This method is called by child classes to set member variables for a daughter cell upon cell division.
     * Note that the parent cell cycle model will have had ResetForDivision() called just before CreateCellCycleModel() is called,
     * so performing an exact copy of the parent is suitable behaviour. Any daughter-cell-specific initialisation
     * can be done in InitialiseDaughterCell().
     *
     * @param rModel the cell cycle model to copy.
     */
    HeCellCycleModel(const HeCellCycleModel& rModel);

```

```
public:

    /**
     * Constructor - just a default, mBirthTime is set in the AbstractCellCycleModel class.
     */
    HeCellCycleModel();

    /**
     * SetCellCycleDuration() method to set length of cell cycle
     */
    void SetCellCycleDuration();

    /**
     * Overridden builder method to create new copies of
     * this cell-cycle model.
     *
     * @return new cell-cycle model
     */
    AbstractCellCycleModel* CreateCellCycleModel();

    /**
     * Overridden ResetForDivision() method.
     * Contains general mitotic mode logic
     **/
    void ResetForDivision();

    /**
     * Overridden Initialise() method
     * Used to give an appropriate mCellCycleDuration to cells w/ TiL offsets
     * sets mReadytoDivide to false as appropriate
     * Initialises as transit proliferative type
     **/
    void Initialise();

    /**
     * Overridden InitialiseDaughterCell() method.
     * Used to apply sister-cell time shifting (cell cycle duration, deterministic phase boundaries)
     * Used to implement asymmetric mitotic mode
     * */
    void InitialiseDaughterCell();
```

```
    /*Model setup functions for standard He (SetModelParameters) and deterministic alternative (SetDeterministicMode) models
     * Default parameters are from refits of He et al + deterministic alternatives
     * He 2012 params: mitoticModePhase2 = 8, mitoticModePhase3 = 15, p1PP = 1, p1PD = 0, p2PP = .2, p2PD = .4, p3PP = .2, p3PD = 0
     * gammaShift = 4, gammaShape = 2, gammaScale = 1, sisterShift = 1
     */
    void SetModelParameters(double tiLOffset = 0, double mitoticModePhase2 = 8, double mitoticModePhase3 = 15,
                            double phase1PP = 1, double phase1PD = 0, double phase2PP = .2, double phase2PD = .4,
                            double phase3PP = .2, double phase3PD = 0, double gammaShift = 4, double gammaShape = 2,
                            double gammaScale = 1, double sisterShift = 1);
    void SetDeterministicMode(double tiLOffset = 0, double mitoticModePhase2 = 8, double mitoticModePhase3 = 15,
                              double phaseShiftWidth = 1, double gammaShift = 4, double gammaShape = 2,
                              double gammaScale = 1, double sisterShift = 1);
    void SetTimeDependentCycleDuration(double peakRateTime, double increasingSlope, double decreasingSlope);

    //Function to set mKillSpecified = true; marks specified neurons for death and removal from population
    //Intended to help w/ resource consumption for WanSimulator
    void EnableKillSpecified();

    //Functions to enable per-cell mitotic mode logging for mode rate & sequence sampling fixtures
    //Uses singleton logfile
    void EnableModeEventOutput(double eventStart, unsigned seed);
    void EnableSequenceSampler();

    //More detailed debug output. Needs a ColumnDataWriter passed to it
    //Only declare ColumnDataWriter directory, filename, etc; do not set up otherwise
```

```
    //Use PassDebugWriter if the writer is already enabled elsewhere (ie. in a Wan stem cell cycle model)
    void EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter);
    void PassDebugWriter(boost::shared_ptr<ColumnDataWriter> debugWriter, int timeID, std::vector<int> varIDs);

    //Not used, but must be overwritten lest HeCellCycleModels be abstract
    double GetAverageTransitCellCycleTime();
    double GetAverageStemCellCycleTime();

    /**
     * Overridden OutputCellCycleModelParameters() method.
     *
     * @param rParamsFile the file stream to which the parameters are output
     */
    virtual void OutputCellCycleModelParameters(out_stream& rParamsFile);
};

#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(HeCellCycleModel)

#endif /*HECELLCYCLEMODEL_HPP_*/
```

17.1.31 /src/OffLatticeSimulationPropertyStop.cpp

```
#include "OffLatticeSimulationPropertyStop.hpp"

#include <boost/make_shared.hpp>

#include "CellBasedEventHandler.hpp"
#include "ForwardEulerNumericalMethod.hpp"
#include "StepSizeException.hpp"

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::OffLatticeSimulationPropertyS rCellPopulation,
                                                        bool deleteCellPopulationInDestructor,
```

```
                                                    bool initialiseCells
                                                    )
    : AbstractCellBasedSimulation<ELEMENT_DIM,SPACE_DIM>(rCellPopulation,
    ↪  deleteCellPopulationInDestructor, initialiseCells),
    p_property()
{
    if
    ↪  (!dynamic_cast<AbstractOffLatticeCellPopulation<ELEMENT_DIM,SPACE_DIM>*>(&rCe
    {
        EXCEPTION("OffLatticeSimulations require a subclass of
        ↪  AbstractOffLatticeCellPopulation.");
    }
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
bool
↪  OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::StoppingEventHasOccurred
{
    if(p_property->GetCellCount()<1){
                return true;
        }
        else{
                return false;
        }
}

//Public access to Stopping Event bool
template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
bool
↪  OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::HasStoppingEventOccurred
{
    return StoppingEventHasOccurred();
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪  OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::SetStopProperty(boost::s
↪  stopPropertySetting)
{
         p_property = stopPropertySetting;
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
```

```
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::AddForce(boost::shared_p
↪ > pForce)
{
    mForceCollection.push_back(pForce);
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::RemoveAllForces()
{
    mForceCollection.clear();
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::AddCellPopulationBoundar
↪ > pBoundaryCondition)
{
    mBoundaryConditions.push_back(pBoundaryCondition);
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::RemoveAllCellPopulationB
{
    mBoundaryConditions.clear();
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::SetNumericalMethod(boost
↪ SPACE_DIM> > pNumericalMethod)
{
    mpNumericalMethod = pNumericalMethod;
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
const boost::shared_ptr<AbstractNumericalMethod<ELEMENT_DIM, SPACE_DIM> >
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::GetNumericalMethod()
↪ const
{
    return mpNumericalMethod;
```

```
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
const std::vector<boost::shared_ptr<AbstractForce<ELEMENT_DIM, SPACE_DIM>
↪ > >&
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::rGetForceCollection()
↪ const
{
    return mForceCollection;
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::UpdateCellLocationsAndTo
{
    CellBasedEventHandler::BeginEvent(CellBasedEventHandler::POSITION);

    double time_advanced_so_far = 0;
    double target_time_step  = this->mDt;
    double present_time_step = this->mDt;

    while (time_advanced_so_far < target_time_step)
    {
        // Store the initial node positions (these may be needed when
        ↪ applying boundary conditions)
        std::map<Node<SPACE_DIM>*, c_vector<double, SPACE_DIM> >
        ↪ old_node_locations;

        for (typename AbstractMesh<ELEMENT_DIM, SPACE_DIM>::NodeIterator
        ↪ node_iter =
        ↪ this->mrCellPopulation.rGetMesh().GetNodeIteratorBegin();
             node_iter !=
             ↪ this->mrCellPopulation.rGetMesh().GetNodeIteratorEnd();
             ++node_iter)
        {
            old_node_locations[&(*node_iter)] =
            ↪ (node_iter)->rGetLocation();
        }

        // Try to update node positions according to the numerical method
        try
        {
            mpNumericalMethod->UpdateAllNodePositions(present_time_step);
```

```
            ApplyBoundaries(old_node_locations);

            // Successful time step! Update time_advanced_so_far
            time_advanced_so_far += present_time_step;

            // If using adaptive timestep, then increase the present_time_step (by 1% for now)
            if (mpNumericalMethod->HasAdaptiveTimestep())
            {
                ///\todo #2087 Make this a settable member variable
                double timestep_increase = 0.01;
                present_time_step = std::min((1+timestep_increase)*present_time_step, target_time_step - time_advanced_so_far);
            }

        }
        catch (StepSizeException& e)
        {
            // Detects if a node has travelled too far in a single time step
            if (mpNumericalMethod->HasAdaptiveTimestep())
            {
                // If adaptivity is switched on, revert node locations and choose a suitably smaller time step
                RevertToOldLocations(old_node_locations);
                present_time_step = std::min(e.GetSuggestedNewStep(), target_time_step - time_advanced_so_far);
            }
            else
            {
                // If adaptivity is switched off, terminate with an error
                EXCEPTION(e.what());
            }
        }
    }

    CellBasedEventHandler::EndEvent(CellBasedEventHandler::POSITION);
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
```

```
void OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::RevertToOldLocations(std::vector<double, SPACE_DIM> > oldNodeLoctions)
{
    for (typename AbstractMesh<ELEMENT_DIM, SPACE_DIM>::NodeIterator node_iter = this->mrCellPopulation.rGetMesh().GetNodeIteratorBegin();
        node_iter != this->mrCellPopulation.rGetMesh().GetNodeIteratorEnd();
        ++node_iter)
    {
        (node_iter)->rGetModifiableLocation() = oldNodeLoctions[&(*node_iter)];
    }
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::ApplyBoundaries(std::map<unsigned, std::vector<boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT_DIM,
SPACE_DIM> > oldNodeLoctions)
{
    // Apply any boundary conditions
    for (typename std::vector<boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT_DIM,SPACE_DIM> >::iterator bcs_iter = mBoundaryConditions.begin();
         bcs_iter != mBoundaryConditions.end();
         ++bcs_iter)
    {
        (*bcs_iter)->ImposeBoundaryCondition(oldNodeLoctions);
    }

    // Verify that each boundary condition is now satisfied
    for (typename std::vector<boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT_DIM,SPACE_DIM> >::iterator bcs_iter = mBoundaryConditions.begin();
         bcs_iter != mBoundaryConditions.end();
         ++bcs_iter)
    {
        if (!((*bcs_iter)->VerifyBoundaryCondition()))
        {
            EXCEPTION("The cell population boundary conditions are incompatible.");
        }
```

```
    }
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::WriteVisualizerSetupFile
{
    if (PetscTools::AmMaster())
    {
        for (unsigned i=0; i<this->mForceCollection.size(); i++)
        {
            ↪ this->mForceCollection[i]->WriteDataToVisualizerSetupFile(this->mViz
        }


        ↪ this->mrCellPopulation.WriteDataToVisualizerSetupFile(this->mpVizSetupFil
    }
}

template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::SetupSolve()
{
    // Clear all forces
    for (typename AbstractMesh<ELEMENT_DIM, SPACE_DIM>::NodeIterator
    ↪ node_iter =
    ↪ this->mrCellPopulation.rGetMesh().GetNodeIteratorBegin();
        node_iter !=
        ↪ this->mrCellPopulation.rGetMesh().GetNodeIteratorEnd();
        ++node_iter)
    {
        node_iter->ClearAppliedForce();
    }

    // Use a forward Euler method by default, unless a numerical method
    ↪ has been specified already
    if (mpNumericalMethod == nullptr)
    {
        mpNumericalMethod =
        ↪ boost::make_shared<ForwardEulerNumericalMethod<ELEMENT_DIM,
        ↪ SPACE_DIM> >();
    }
```

```
212
        ↪  mpNumericalMethod→SetCellPopulation(dynamic_cast<AbstractOffLatticeCellPopul
213     mpNumericalMethod→SetForceCollection(&mForceCollection);
214 }
215
216 template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
217 void
    ↪  OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::OutputAdditionalSimulati
    ↪  rParamsFile)
218 {
219     // Loop over forces
220     *rParamsFile << "\n\t<Forces>\n";
221     for (typename
        ↪  std::vector<boost::shared_ptr<AbstractForce<ELEMENT_DIM,SPACE_DIM>
        ↪  > >::iterator iter = mForceCollection.begin();
222          iter ≠ mForceCollection.end();
223          ++iter)
224     {
225         // Output force details
226         (*iter)→OutputForceInfo(rParamsFile);
227     }
228     *rParamsFile << "\t</Forces>\n";
229
230     // Loop over cell population boundary conditions
231     *rParamsFile << "\n\t<CellPopulationBoundaryConditions>\n";
232     for (typename
        ↪  std::vector<boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT_
        ↪  > >::iterator iter = mBoundaryConditions.begin();
233          iter ≠ mBoundaryConditions.end();
234          ++iter)
235     {
236         // Output cell boundary condition details
237         (*iter)→OutputCellPopulationBoundaryConditionInfo(rParamsFile);
238     }
239     *rParamsFile << "\t</CellPopulationBoundaryConditions>\n";
240
241     // Output numerical method details
242     *rParamsFile << "\n\t<NumericalMethod>\n";
243     mpNumericalMethod→OutputNumericalMethodInfo(rParamsFile);
244     *rParamsFile << "\t</NumericalMethod>\n";
245 }
246
247 template<unsigned ELEMENT_DIM, unsigned SPACE_DIM>
```

```
void
↪ OffLatticeSimulationPropertyStop<ELEMENT_DIM,SPACE_DIM>::OutputSimulationParamete
↪ rParamsFile)
{
    // No new parameters to output, so just call method on direct parent
    ↪ class

    ↪ AbstractCellBasedSimulation<ELEMENT_DIM,SPACE_DIM>::OutputSimulationParameter
}

// Explicit instantiation
template class OffLatticeSimulationPropertyStop<1,1>;
template class OffLatticeSimulationPropertyStop<1,2>;
template class OffLatticeSimulationPropertyStop<2,2>;
template class OffLatticeSimulationPropertyStop<1,3>;
template class OffLatticeSimulationPropertyStop<2,3>;
template class OffLatticeSimulationPropertyStop<3,3>;

// Serialization for Boost ≥ 1.36
#include "SerializationExportWrapperForCpp.hpp"
EXPORT_TEMPLATE_CLASS_ALL_DIMS(OffLatticeSimulationPropertyStop)
```

17.1.32 /src/OffLatticeSimulationPropertyStop.hpp

```
#ifndef OFFLATTICESIMULATIONPROPERTYSTOP_HPP_
#define OFFLATTICESIMULATIONPROPERTYSTOP_HPP_

#include "AbstractCellBasedSimulation.hpp"
#include "AbstractForce.hpp"
#include "AbstractCellPopulationBoundaryCondition.hpp"
#include "AbstractNumericalMethod.hpp"

#include "ChasteSerialization.hpp"
#include <boost/serialization/base_object.hpp>
#include <boost/serialization/set.hpp>
#include <boost/serialization/vector.hpp>

/**
 * Run an off-lattice 2D or 3D cell-based simulation using an off-lattice
 * cell population.
 *
 * In cell-centre-based cell populations, each cell is represented by a
```

```
 * single node (corresponding to its centre), and connectivity is defined
 * either by a Delaunay triangulation or a radius of influence. In vertex-
 * based cell populations, each cell is represented by a polytope
 * (corresponding to its membrane) with a variable number of vertices.
 * Alternative cell populations may be defined by the user.
 *
 * The OffLatticeSimulation is constructed with a CellPopulation, which
 * updates the correspondence between each Cell and its spatial representation
 * and handles cell division (governed by the CellCycleModel associated
 * with each cell). Once constructed, one or more Force laws may be passed
 * to the OffLatticeSimulation object, to define the mechanical properties
 * of the CellPopulation. Similarly, one or more CellKillers may be passed
 * to the OffLatticeSimulation object to specify conditions in which Cells
 * may die, and one or more CellPopulationBoundaryConditions to specify
 * regions in space beyond which Cells may not move.
 */
template<unsigned ELEMENT_DIM, unsigned SPACE_DIM = ELEMENT_DIM>
class OffLatticeSimulationPropertyStop : public AbstractCellBasedSimulation<ELEMENT_DIM, SPACE_DIM>
{
private:

    /** Needed for serialization. */
    friend class boost::serialization::access;
    friend class TestOffLatticeSimulation;
    friend class TestOffLatticeSimulationWithNodeBasedCellPopulation;

    /**
     * Save or restore the simulation.
     *
     * @param archive the archive
     * @param version the current version of this class
     */
    template<class Archive>
    void serialize(Archive & archive, const unsigned int version)
    {
```

```
        archive & boost::serialization::base_object<AbstractCellBasedSimulation<ELEMENT_DIM, SPACE_DIM> >(*this);
        archive & mForceCollection;
        archive & mBoundaryConditions;
        archive & mpNumericalMethod;
    }

protected:
    boost::shared_ptr<AbstractCellProperty> p_property;
    /** The mechanics used to determine the new location of the cells, a list of the forces. */
    std::vector<boost::shared_ptr<AbstractForce<ELEMENT_DIM, SPACE_DIM> > > mForceCollection;

    /** List of boundary conditions. */
    std::vector<boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT, SPACE_DIM> > > mBoundaryConditions;

    /** The numerical method to use in this simulation. Defaults to the explicit forward Euler method. */
    boost::shared_ptr<AbstractNumericalMethod<ELEMENT_DIM, SPACE_DIM> > mpNumericalMethod;

    /**
     * Overridden UpdateCellLocationsAndTopology() method.
     *
     * Calculate forces and update node positions.
     */
    virtual void UpdateCellLocationsAndTopology();

    /**
     * Sends nodes back to the positions given in the input map. Used after a failed step
     * when adaptivity is turned on.
     *
     * @param oldNodeLoctions A map linking nodes to their old positions.
     */
    void RevertToOldLocations(std::map<Node<SPACE_DIM>*, c_vector<double, SPACE_DIM> > oldNodeLoctions);

    /**
```

```
     * Applies any boundary conditions.
     *
     * @param oldNodeLoctions Mapping between node indices and old node locations
     */
    void ApplyBoundaries(std::map<Node<SPACE_DIM>*, c_vector<double, SPACE_DIM> > oldNodeLoctions);

    /**
     * Overridden SetupSolve() method to clear the forces applied to the nodes.
     */
    virtual void SetupSolve();

    /**
     * Overridden WriteVisualizerSetupFile() method.
     */
    virtual void WriteVisualizerSetupFile();

    bool StoppingEventHasOccurred();

public:

    /**
     * Constructor.
     *
     * @param rCellPopulation Reference to a cell population object
     * @param deleteCellPopulationInDestructor Whether to delete the cell population on destruction to
     *     free up memory (defaults to false)
     * @param initialiseCells Whether to initialise cells (defaults to true, set to false when loading
     *     from an archive)
     */
    OffLatticeSimulationPropertyStop(AbstractCellPopulation<ELEMENT_DIM, SPACE_DIM>& rCellPopulation,
                                     bool deleteCellPopulationInDestructor = false, bool initialiseCells = true);

    void SetStopProperty(boost::shared_ptr<AbstractCellProperty> stopPropertySetting);
```

```
    bool HasStoppingEventOccurred();

    /**
     * Add a force to be used in this simulation (use this to set the mechanics system).
     *
     * @param pForce pointer to a force law
     */
    void AddForce(boost::shared_ptr<AbstractForce<ELEMENT_DIM, SPACE_DIM> > pForce);

    /**
     * Remove all the forces.
     */
    void RemoveAllForces();

    /**
     * Add a cell population boundary condition to be used in this simulation.
     *
     * @param pBoundaryCondition pointer to a boundary condition
     */
    void AddCellPopulationBoundaryCondition(boost::shared_ptr<AbstractCellPopulationBoundaryCondition<ELEMENT_DIM, SPACE_DIM> > pBoundaryCondition);

    /**
     * Method to remove all the cell population boundary conditions
     */
    void RemoveAllCellPopulationBoundaryConditions();

    /**
     * Set the numerical method to be used in this simulation (use this to solve the mechanics system).
     *
     * @param pNumericalMethod pointer to a numerical method object
     */
    void SetNumericalMethod(boost::shared_ptr<AbstractNumericalMethod<ELEMENT_DIM, SPACE_DIM> > pNumericalMethod);

```

```
    /**
     * @return the current numerical method.
     */
    const boost::shared_ptr<AbstractNumericalMethod<ELEMENT_DIM,
     ↪  SPACE_DIM> > GetNumericalMethod() const;

    /**
     * Overridden OutputAdditionalSimulationSetup() method.
     *
     * Output any force, boundary condition or numerical method
↪   information.
     *
     * @param rParamsFile the file stream to which the parameters are
↪   output
     */
    void OutputAdditionalSimulationSetup(out_stream& rParamsFile);

    /**
     * Overridden OutputSimulationParameters() method.
     *
     * @param rParamsFile the file stream to which the parameters are
↪   output
     */
    virtual void OutputSimulationParameters(out_stream& rParamsFile);

    /**
     * Directly access the forces attached to this simulation, to allow
↪   their manipulation after archiving.
     *
     * @return mForceCollection the vector of pointers to forces attached
↪   to this simulation
     */
    const std::vector<boost::shared_ptr<AbstractForce<ELEMENT_DIM,
     ↪  SPACE_DIM> > >& rGetForceCollection() const;
};

// Serialization for Boost ≥ 1.36
#include "SerializationExportWrapper.hpp"
EXPORT_TEMPLATE_CLASS_ALL_DIMS(OffLatticeSimulationPropertyStop)

namespace boost
{
namespace serialization
```

```
{
/**
 * Serialize information required to construct an OffLatticeSimulation.
 */
template<class Archive, unsigned ELEMENT_DIM, unsigned SPACE_DIM>
inline void save_construct_data(Archive & ar, const OffLatticeSimulationPropertyStop<ELEMENT_DIM, SPACE_DIM> * t,
                                const unsigned int file_version)
{
    // Save data required to construct instance
    const AbstractCellPopulation<ELEMENT_DIM, SPACE_DIM>* p_cell_population = &(t->rGetCellPopulation());
    ar & p_cell_population;
}

/**
 * De-serialize constructor parameters and initialise an OffLatticeSimulation.
 */
template<class Archive, unsigned ELEMENT_DIM, unsigned SPACE_DIM>
inline void load_construct_data(Archive & ar, OffLatticeSimulationPropertyStop<ELEMENT_DIM, SPACE_DIM> * t,
                                const unsigned int file_version)
{
    // Retrieve data from archive required to construct new instance
    AbstractCellPopulation<ELEMENT_DIM, SPACE_DIM>* p_cell_population;
    ar >> p_cell_population;

    // Invoke inplace constructor to initialise instance, middle two variables set extra
    // member variables to be deleted as they are loaded from archive and to not initialise cells.
    ::new (t) OffLatticeSimulationPropertyStop<ELEMENT_DIM, SPACE_DIM>(*p_cell_population, true, false);
}
}
} // namespace

#endif /*OFFLATTICESIMULATIONPROPERTYSTOP_HPP_*/
```

17.1.33 /src/WanStemCellCycleModel.cpp

```
#include "WanStemCellCycleModel.hpp"

WanStemCellCycleModel::WanStemCellCycleModel() :
        AbstractSimpleCellCycleModel(), mExpandingStemPopulation(false), mPopulation(), mOutput(false), mEventStartTime(
                72.0), mDebug(false), mTimeID(), mVarIDs(), mDebugWriter(), mBasePopulation(), mGammaShift(4.0), mGammaShape(
                2.0), mGammaScale(1.0), mMitoticMode(0), mSeed(0), mTimeDependentCycleDuration(false), mPeakRateTime(), mIncreasingRateSlope(), mDecreasingRateSlope(), mBaseGammaScale(), mHeParamVector(
                { 8, 15, 1, 0, .2, .4, .2, 0, 4, 2, 1, 1 })
{
}

WanStemCellCycleModel::WanStemCellCycleModel(const WanStemCellCycleModel& rModel) :
        AbstractSimpleCellCycleModel(rModel), mExpandingStemPopulation(rModel.mExpandingStemPopulation), mPopulation(
                rModel.mPopulation), mOutput(rModel.mOutput), mEventStartTime(rModel.mEventStartTime), mDebug(
                rModel.mDebug), mTimeID(rModel.mTimeID), mVarIDs(rModel.mVarIDs), mDebugWriter(rModel.mDebugWriter), mBasePopulation(
                rModel.mBasePopulation), mGammaShift(rModel.mGammaShift), mGammaShape(rModel.mGammaShape), mGammaScale(
                rModel.mGammaScale), mMitoticMode(rModel.mMitoticMode), mSeed(rModel.mSeed), mTimeDependentCycleDuration(
                rModel.mTimeDependentCycleDuration), mPeakRateTime(rModel.mPeakRateTime), mIncreasingRateSlope(
                rModel.mIncreasingRateSlope), mDecreasingRateSlope(rModel.mDecreasingRateSlope), mBaseGammaScale(
                rModel.mBaseGammaScale), mHeParamVector(rModel.mHeParamVector)
{
}

```

```
AbstractCellCycleModel* WanStemCellCycleModel::CreateCellCycleModel()
{
    return new WanStemCellCycleModel(*this);
}

void WanStemCellCycleModel::SetCellCycleDuration()
{
    RandomNumberGenerator* p_random_number_generator = RandomNumberGenerator::Instance();

    mCellCycleDuration = mGammaShift + p_random_number_generator->GammaRandomDeviate(mGammaShape, mGammaScale);

    /*************************************
     * CELL CYCLE DURATION RANDOM VARIABLE
     *************************************/
    //Wan stem cell cycle length determined by the same formula as He RPCs
    /*
     if (!mTimeDependentCycleDuration) //Normal operation, cell cycle length stays constant
     {
     //He cell cycle length determined by shifted gamma distribution reflecting 4 hr refractory period followed by gamma pdf
     mCellCycleDuration = mGammaShift + p_random_number_generator->GammaRandomDeviate(mGammaShape, mGammaScale);
     }*/

    /****
     * Variable cycle length
     * Give -ve mIncreasingRateSlope and +ve mDecreasingRateSlope,
     * cell cycle length linearly declines (increasing rate), then increases, switching at mPeakRateTime
     ****/
    /*
     else
     {
     double currTime = SimulationTime::Instance()->GetTime();
     if (currTime <= mPeakRateTime)
     {
```

```
56      mGammaScale = std::max((mBaseGammaScale - currTime *
  ↪   mIncreasingRateSlope), .0000000000001);
57      }
58      if (currTime > mPeakRateTime)
59      {
60      mGammaScale = std::max(((mBaseGammaScale - mPeakRateTime *
  ↪   mIncreasingRateSlope)
61      + (mBaseGammaScale + (currTime - mPeakRateTime) *
  ↪   mDecreasingRateSlope)),.0000000000001);
62      }
63      Timer::Print("mGammaScale: " + std::to_string(mGammaScale));
64      mCellCycleDuration = mGammaShift +
  ↪   p_random_number_generator->GammaRandomDeviate(mGammaShape,
  ↪   mGammaScale);
65      }
66      */
67 }
68
69 void WanStemCellCycleModel::ResetForDivision()
70 {
71
72     mMitoticMode = 1; //by default, asymmetric division giving rise to He
        ↪   cell (mode 1)
73
74     if (mExpandingStemPopulation)
75     {
76         double currRetinaAge = SimulationTime::Instance()->GetTime() +
            ↪   mEventStartTime;
77         double lensGrowthFactor = .09256 * pow(currRetinaAge, .52728); //
            ↪   power law model fit for lens growth
78         unsigned currentPopulationTarget = int(std::round(mBasePopulation
            ↪   * lensGrowthFactor));
79
80         unsigned currentStemPopulation =
            ↪   (mPopulation->GetCellProliferativeTypeCount())[0];
81         if (currentStemPopulation < currentPopulationTarget)
82         {
83             mMitoticMode = 0; //if the current population is < target,
                ↪   symmetrical stem-stem division occurs (mode 0)
84         }
85     }
86
87     /****************
```

```
88      * Write mitotic event to relevant files
89      * *************/
90     if (mDebug)
91     {
92         WriteDebugData();
93     }
94
95     if (mOutput)
96     {
97         WriteModeEventOutput();
98     }
99
100    AbstractSimpleCellCycleModel::ResetForDivision();
101
102 }
103
104 void WanStemCellCycleModel::Initialise()
105 {
106
107    boost::shared_ptr<AbstractCellProperty> p_Stem =
108
               ↪  mpCell→rGetCellPropertyCollection().GetCellPropertyRegistry()→Get<S
109    mpCell→SetCellProliferativeType(p_Stem);
110
111    SetCellCycleDuration();
112 }
113
114 void WanStemCellCycleModel::InitialiseDaughterCell()
115 {
116
117    if (mMitoticMode == 1)
118    {
119        /*******************************************
120         * RPC-fated cells are given HeCellCycleModel
121         *******************************************/
122
123        double tiLOffset = -(SimulationTime::Instance()→GetTime());
124        //Initialise a HeCellCycleModel and set it up with appropriate
           ↪  TiL value & parameters
125        HeCellCycleModel* p_cycle_model = new HeCellCycleModel;
126        p_cycle_model→SetModelParameters(tiLOffset, mHeParamVector[0],
           ↪  mHeParamVector[1], mHeParamVector[2],
```

```
                                                mHeParamVector[3], mHeParamVector[4], mHeParamVector[5], mHeParamVector[6],
                                                mHeParamVector[7], mHeParamVector[8], mHeParamVector[9], mHeParamVector[10],
                                                mHeParamVector[11]);
        p_cycle_model->EnableKillSpecified();

        //if debug output is enabled for the stem cell, enable it for its progenitor offspring
        if (mDebug)
        {
            p_cycle_model->PassDebugWriter(mDebugWriter, mTimeID, mVarIDs);
        }

        mpCell->SetCellCycleModel(p_cycle_model);
        p_cycle_model->Initialise();
    }

    else
    {
        SetCellCycleDuration();
    }
}

void WanStemCellCycleModel::SetModelParameters(double gammaShift, double gammaShape, double gammaScale,
                                                std::vector<double> heParamVector)
{
    mGammaShift = gammaShift;
    mGammaShape = gammaShape;
    mGammaScale = gammaScale;
    mHeParamVector = heParamVector;
}

void WanStemCellCycleModel::EnableExpandingStemPopulation(int basePopulation,
```

```
158
                                                                    ↪ boost::shared_ptr<Abstr
                                                                    ↪ p_population)
159 {
160     mExpandingStemPopulation = true;
161     mBasePopulation = basePopulation;
162     mPopulation = p_population;
163 }
164
165 void WanStemCellCycleModel::SetTimeDependentCycleDuration(double
    ↪ peakRateTime, double increasingSlope,
166                                                             double
                                                                    ↪ decreasingSlope)
167 {
168     mTimeDependentCycleDuration = true;
169     mPeakRateTime = peakRateTime;
170     mIncreasingRateSlope = increasingSlope;
171     mDecreasingRateSlope = decreasingSlope;
172     mBaseGammaScale = mGammaScale;
173 }
174
175 void WanStemCellCycleModel::EnableModeEventOutput(double eventStart,
    ↪ unsigned seed)
176 {
177     mOutput = true;
178     mEventStartTime = eventStart;
179     mSeed = seed;
180 }
181
182 void WanStemCellCycleModel::WriteModeEventOutput()
183 {
184     double currentTime = SimulationTime::Instance()->GetTime() +
        ↪ mEventStartTime;
185     CellPtr currentCell = GetCell();
186     double currentCellID = (double) currentCell->GetCellId();
187     (*LogFile::Instance()) << currentTime << "\t" << mSeed << "\t" <<
        ↪ currentCellID << "\t" << mMitoticMode << "\n"; //
188 }
189
190 void
    ↪ WanStemCellCycleModel::EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter>
    ↪ debugWriter)
191 {
```

```
    mDebug = true;
    mDebugWriter = debugWriter;

    mTimeID = mDebugWriter->DefineUnlimitedDimension("Time", "h");
    mVarIDs.push_back(mDebugWriter->DefineVariable("CellID", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("TiL", "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("CycleDuration",
     ↪  "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase2Boundary",
     ↪  "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase3Boundary",
     ↪  "h"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Phase", "No"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticModeRV",
     ↪  "Percentile"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("MitoticMode",
     ↪  "Mode"));
    mVarIDs.push_back(mDebugWriter->DefineVariable("Label", "binary"));

    mDebugWriter->EndDefineMode();
}

void WanStemCellCycleModel::WriteDebugData()
{
    double currentTime = SimulationTime::Instance()->GetTime();
    double currentCellID = mpCell->GetCellId();

    mDebugWriter->PutVariable(mTimeID, currentTime);
    mDebugWriter->PutVariable(mVarIDs[0], currentCellID);
    mDebugWriter->PutVariable(mVarIDs[1], 0);
    mDebugWriter->PutVariable(mVarIDs[2], mCellCycleDuration);
    mDebugWriter->PutVariable(mVarIDs[3], 0);
    mDebugWriter->PutVariable(mVarIDs[4], 0);
    mDebugWriter->PutVariable(mVarIDs[5], 0);
    mDebugWriter->PutVariable(mVarIDs[7], mMitoticMode);
    mDebugWriter->AdvanceAlongUnlimitedDimension();
}

/******************
 * UNUSED FUNCTIONS (required for class nonvirtuality, do not remove)
 ******************/

double WanStemCellCycleModel::GetAverageTransitCellCycleTime()
```

```
{
    return (0.0);
}

double WanStemCellCycleModel::GetAverageStemCellCycleTime()
{
    return (0.0);
}

void WanStemCellCycleModel::OutputCellCycleModelParameters(out_stream&
 ↪  rParamsFile)
{
    *rParamsFile << "\t\t\t<CellCycleDuration>" << mCellCycleDuration <<
     ↪  "</CellCycleDuration>\n";

// Call method on direct parent class

     ↪  AbstractSimpleCellCycleModel::OutputCellCycleModelParameters(rParamsFile);
}

// Serialization for Boost ≥ 1.36
#include "SerializationExportWrapperForCpp.hpp"
CHASTE_CLASS_EXPORT(WanStemCellCycleModel)
```

17.1.34 /src/WanStemCellCycleModel.hpp

```
#ifndef WANSTEMCELLCYCLEMODEL_HPP_
#define WANSTEMCELLCYCLEMODEL_HPP_

#include "AbstractCellPopulation.hpp"
#include "AbstractSimpleCellCycleModel.hpp"
#include "RandomNumberGenerator.hpp"
#include "Cell.hpp"
#include "StemCellProliferativeType.hpp"
#include "SmartPointers.hpp"
#include "ColumnDataWriter.hpp"
#include "LogFile.hpp"

#include "HeCellCycleModel.hpp"

/***********************************
 * WAN STEM CELL CYCLE MODEL
```

```
17  * A simple model standing in for the CMZ "stem cells" described in'
18  * Wan et al. 2016 [Wan2016]
19  *
20  * USE: By default, WanStemCellCycleModels consistently divide
    ↪  asymmetrically.
21  * InitialiseDaughterCell() marks offspring for RPC fate
22  * So-marked cells are given HeCellCycleModels for their next division.
23  *
24  * Change default model parameters with SetModelParameters(<params>);
25  *
26  * 2 per-model-event output modes:
27  * EnableModeEventOutput() enables mitotic mode event logging-all cells
    ↪  will write to the singleton log file
28  * EnableModelDebugOutput() enables more detailed debug output, each seed
    ↪  will have its own file written to
29  * by a ColumnDataWriter passed to it from the test
30  * (eg. by the SetupDebugOutput helper function in the project simulator)
31  *
32  * 1 mitotic-event-sequence sampler (only samples one "path" through the
    ↪  lineage):
33  * EnableSequenceSampler() - one "sequence" of progenitors writes mitotic
    ↪  event type to a string in the singleton log file
34  *
35  ************************************/
36
37 class WanStemCellCycleModel : public AbstractSimpleCellCycleModel
38 {
39     friend class TestSimpleCellCycleModels;
40
41 private:
42
43     /** Needed for serialization. */
44     friend class boost::serialization::access;
45     /**
46      * Archive the cell-cycle model and random number generator, never
    ↪  used directly - boost uses this.
47      *
48      * @param archive the archive
49      * @param version the current version of this class
50      */
51     template<class Archive>
52     void serialize(Archive & archive, const unsigned int version)
53     {
```

```
        archive & boost::serialization::base_object<AbstractSimpleCellCycleModel>(*this);

        SerializableSingleton<RandomNumberGenerator>* p_wrapper = RandomNumberGenerator::Instance()->GetSerializationWrapper();
        archive & p_wrapper;
        archive & mCellCycleDuration;
    }

    //Private write functions for models
    void WriteModeEventOutput();
    void WriteDebugData();

protected:
    //mode/output variables
    bool mExpandingStemPopulation;
    boost::shared_ptr<AbstractCellPopulation<2>> mPopulation;
    bool mOutput;
    double mEventStartTime;
    //debug writer stuff
    bool mDebug;
    int mTimeID;
    std::vector<int> mVarIDs;
    boost::shared_ptr<ColumnDataWriter> mDebugWriter;
    //model parameters and state memory vars
    int mBasePopulation;
    double mGammaShift;
    double mGammaShape;
    double mGammaScale;
    unsigned mMitoticMode;
    unsigned mSeed;
    bool mTimeDependentCycleDuration;
    double mPeakRateTime;
    double mIncreasingRateSlope;
    double mDecreasingRateSlope;
    double mBaseGammaScale;
    std::vector<double> mHeParamVector;

    /**
     * Protected copy-constructor for use by CreateCellCycleModel().
     *
```

```
     * The only way for external code to create a copy of a cell cycle model
     * is by calling that method, to ensure that a model of the correct subclass is created.
     * This copy-constructor helps subclasses to ensure that all member variables are correctly copied when this happens.
     *
     * This method is called by child classes to set member variables for a daughter cell upon cell division.
     * Note that the parent cell cycle model will have had ResetForDivision() called just before CreateCellCycleModel() is called,
     * so performing an exact copy of the parent is suitable behaviour. Any daughter-cell-specific initialisation
     * can be done in InitialiseDaughterCell().
     *
     * @param rModel the cell cycle model to copy.
     */
    WanStemCellCycleModel(const WanStemCellCycleModel& rModel);

public:

    /**
     * Constructor - just a default, mBirthTime is set in the AbstractCellCycleModel class.
     */
    WanStemCellCycleModel();

    /**
     * SetCellCycleDuration() method to set length of cell cycle
     */
    void SetCellCycleDuration();

    /**pro
     * Overridden builder method to create new copies of
     * this cell-cycle model.
     *
     * @return new cell-cycle model
     */
    AbstractCellCycleModel* CreateCellCycleModel();

    /**
     * Overridden ResetForDivision() method.
```

```
     **/
    void ResetForDivision();

    /**
     * Overridden Initialise() method.
     * Sets proliferative type as stem cell
     **/

    void Initialise();

    /**
     * Overridden InitialiseDaughterCell() method.
     * Used to implement asymmetric mitotic mode
     * Daughter cells are initialised w/ He cell cycle model
     * */
    void InitialiseDaughterCell();

    /*Model setup functions for standard He (SetModelParameters) and deterministic alternative (SetDeterministicMode) models
     * Default parameters are from refits of He et al + deterministic alternatives
     * He 2012 params: mitoticModePhase2 = 8, mitoticModePhase3 = 15, p1PP = 1, p1PD = 0, p2PP = .2, p2PD = .4, p3PP = .2, p3PD = 0
     * gammaShift = 4, gammaShape = 2, gammaScale = 1, sisterShift = 1
     */
    void SetModelParameters(double gammaShift = 4, double gammaShape = 2, double gammaScale = 1, std::vector<double> heParamVector = { 8, 15, 1, 0, .2, .4, .2, 0, 4, 2, 1, 1 });
    void EnableExpandingStemPopulation(int basePopulation, boost::shared_ptr<AbstractCellPopulation<2>> p_population);
    void SetTimeDependentCycleDuration(double peakRateTime, double increasingSlope, double decreasingSlope);

    //Functions to enable per-cell mitotic mode logging for mode rate & sequence sampling fixtures
    //Uses singleton logfile
    void EnableModeEventOutput(double eventStart, unsigned seed);

    //More detailed debug output. Needs a ColumnDataWriter passed to it
    //Only declare ColumnDataWriter directory, filename, etc; do not set up otherwise
    void EnableModelDebugOutput(boost::shared_ptr<ColumnDataWriter> debugWriter);
```

```

    //Not used, but must be overwritten lest WanStemCellCycleModels be abstract
    double GetAverageTransitCellCycleTime();
    double GetAverageStemCellCycleTime();

    /**
     * Overridden OutputCellCycleModelParameters() method.
     *
     * @param rParamsFile the file stream to which the parameters are output
     */
    virtual void OutputCellCycleModelParameters(out_stream& rParamsFile);
};

#include "SerializationExportWrapper.hpp"
// Declare identifier for the serializer
CHASTE_CLASS_EXPORT(WanStemCellCycleModel)

#endif /*WANSTEMCELLCYCLEMODEL_HPP_*/
```

17.2 NGRefTools

Github repository: https://github.com/mmattocks/NGRefTools.jl

17.2.1 /src/LogNormalUtils.jl

```
function get_lognormal_params(desired_μ, desired_σ)
    μ²=desired_μ^2
    σ²=desired_σ^2
    ln_μ=log(μ²/sqrt(μ²+σ²))
    ln_σ=sqrt(log(1+(σ²/μ²)))
    return ln_μ, ln_σ
end

function get_lognormal_desc(d::LogNormal)
    return mean(d), std(d)
end
```

17.2.2 /src/MarginalTDist.jl

```
"""
    MarginalTDist(ν,μ,σ)

Return a shifted, scaled T Distribution with degrees of freedom `ν`, mean `μ`, and standard deviation `σ`.

This is the marginal distribution of the posterior mean for an uninformative (reference) Normal Gamma prior distribution. It is also the distribution of the posterior predictive for m=1 new observations.

    MarginalTDist(ν)        #Standard TDist with ν dof
    MarginalTDist(ν,μ,σ)    #TDist with ν dof shifted by μ, scaled by σ

    params(d)               #Return d.ν, d.μ, d.σ
    mean(d)                 #Return d.μ (median, mode are identical)
    std(d)                  #Return d.σ
    quantile(d,p)           #Return quantile at probability p

Reference: Kevin P. Murphy, Conjugate Bayesian Analysis of the Gaussian Distribution. 2007. https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf
"""
MarginalTDist=LocationScale{Float64,TDist{Float64}}

Distributions.dof(d::MarginalTDist) = d.ρ.ν

Distributions.std(d::MarginalTDist) = d.σ

"""
    fit(MarginalTDist, x; PP=false)

Assuming an uninformative (reference) Normal Gamma prior, return the marginal posterior distribution of the mean of a Normal model of observations `x`.

Equivalent to the frequentist sampling distribution of the MLE mean. If `PP=true`, return the posterior predictive distribution for m=1 additional observations sampled from the marginal posterior mean instead.

```

```
Example:

    julia> PMM_MarginalTDist(rand(10))
    MarginalTDist{Float64}(
    t: TDist{Float64}(ν=9.0)
    μ: 0.4789484996068401
    σ: 0.08491142725619677
    )

Reference: Kevin P. Murphy, Conjugate Bayesian Analysis of the Gaussian Distribution. 2007. https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf
"""
function Distributions.fit(::Type{MarginalTDist}, x::AbstractVector{<:Real}; PP=false)
    PP ? PPM_MTDist(x) : PMM_MTDist(x)
end

#Posterior marginal mean distribution
function PMM_MTDist(x)
    n=length(x)
    μ=mean(x)
    ssr=sum((x.-μ).^2)
    σ=sqrt(ssr/(n*(n-1)))
    return MarginalTDist(μ,σ,TDist(n-1))
end

#Posterior predictive mean distribution
function PPM_MTDist(x)
    n=length(x)
    μ=mean(x)
    ssr=sum((x.-μ).^2)
    αn=(n-1)/2
    βn=.5*ssr
    return MarginalTDist(μ,sqrt((βn*(n+1))/(αn*n)),TDist(2*αn))
end

"""
    MTDist_MC_func(func, xs... ; lower=.025, upper=.975, mc_its=1e6, summary=false)

```

```
Monte Carlo execute `func` with a random sample from a MarginalTDist fitted to each x in xs, over `mc_its` iterates, returning the calculated results (if `summary` is `true`) or the mean and quantiles specified by `lower` and `upper`.

Example:

    julia> NGRefTools.MTDist_MC_func(*,[rand(10),rand(10)],summary=true)
    (0.09491370397229557, 0.20712818357976473, 0.3385677491994645)

See also: [`fit(MarginalTDist,x)`](@ref)
"""
function MTDist_MC_func(func::Function, xs; lower=.025, upper=.975, mc_its=1e7, summary=false)
    dists=[fit(MarginalTDist,x) for x in xs]
    results=Vector{Float64}()
    for it in 1:mc_its
        push!(results, func(rand.(dists)...))
    end
    summary ? (return quantile(results, lower), mean(results), quantile(results,upper)) : (return results)
end

function plot_logn_MTDist(xs, colors, markers, labels, xlabel, ylabel; args...)
    plt=plot(;args...)
    for (x,color) in zip(xs,colors)
        min=floor(minimum(x)-.15*mean(x))
        max=ceil(maximum(x)+.20*mean(x))
        X=collect(min:max)
        pmdist=fit(MarginalTDist, log.(x))
        y=pdf(pmdist,log.(X))
        plot!(plt,X,y,ribbon=(y,zeros(length(y))),label=:none,color=color,xlabel=
    end
    for (x,color,marker,label) in zip(xs,colors, markers,labels)
        pmdist=fit(MarginalTDist, log.(x))
        min=floor(minimum(x)-.15*mean(x))
        max=ceil(maximum(x)+.20*mean(x))
        X=collect(min:max)
        y=pdf(pmdist,log.(X))
        scaty=[mean(y) for x in 1:length(x)]
        plt=scatter!(plt,x,scaty,color=color, marker=marker, label=label)
```

```
    end

    return plt
end

function plot_n_MTDist(xs, colors, markers, labels, xlabel, ylabel; args ... )
    plt=plot(;args ... )
    for (x,color) in zip(xs,colors)
        pmdist=fit(MarginalTDist, x)
        min=quantile(pmdist, .01)
        max=quantile(pmdist, .99)
        X=collect(min:(max-min)/1000:max)
        y=pdf(pmdist,X)

        plot!(plt,X,y,ribbon=(y,zeros(length(y))),color=color,label=:none,xlabel=
    end
    for (x,color,marker,label) in zip(xs,colors, markers,labels)
        pmdist=fit(MarginalTDist, x)
        scaty=[pdf(pmdist,quantile(pmdist,.25)) for x in 1:length(x)]
        plt=scatter!(plt,x,scaty,color=color, marker=marker,label=label)
    end

    return plt
end

function plot_mt_timeseries(X, obs, colors, markers, labels; log=false, plotargs ... )
    plt=plot(;plotargs ... )
    for (ovec, color, marker, label) in zip(obs, colors, markers, labels)
        log ? (mts=[fit(MarginalTDist,log.(ovec[t])) for t in 1:length(X)]) :
            (mts=[fit(MarginalTDist,ovec[t]) for t in 1:length(X)])
        if log
            means=[exp(mean(mt)) for mt in mts]
            lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in mts]
            upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in mts]
        else
            means=[mean(mt) for mt in mts]
            lower=[mean(mt)-quantile(mt,.025) for mt in mts]
            upper=[quantile(mt,.975)-mean(mt) for mt in mts]
        end

```

```
        scatter!(plt, vcat([[X[t] for i in 1:length(ovec[t])] for t in 1:length(X)] ... ),vcat(ovec ... ), marker=marker, color=color, markersize=2, label=nothing)
        plot!(plt, X, means, ribbon=(lower, upper), color=color, label=label)
    end
    return plt
end

function mean_mass_comparator(x,y;labels=["x","y"])

    xdist=fit(MarginalTDist,x)
    ydist=fit(MarginalTDist,y)
    if mean(xdist)>mean(ydist)
        frac=ccdf(xdist,mean(ydist))
        println("$(round(frac*100,digits=1))% of the marginal posterior mean density of $(labels[1]) is above the mean of $(labels[2]).")
    else
        frac=cdf(xdist,mean(ydist))
        println("$(round(frac*100,digits=1))% of the marginal posterior mean density of $(labels[1]) is below the mean of $(labels[2]).")
    end
end

function log_mean_mass_comparator(x,y;labels=["x","y"])
    xdist=fit(MarginalTDist,log.(filter(n→!iszero(n),x)))
    ydist=fit(MarginalTDist,log.(filter(n→!iszero(n),y)))
    if mean(xdist)>mean(ydist)
        frac=ccdf(xdist,mean(ydist))
        println("$(round(frac*100,digits=1))% of the marginal posterior mean density of $(labels[1]) is above the mean of $(labels[2]).")
    else
        frac=cdf(xdist,mean(ydist))
        println("$(round(frac*100,digits=1))% of the marginal posterior mean density of $(labels[1]) is below the mean of $(labels[2]).")
    end
end
```

17.2.3 /src/NGRef.jl

```julia
"""
    fit(NormalGamma, x)

Return the posterior `NormalGamma` distribution on the unknown mean and precision of a Normal model of data vector `x`, assuming an uninformative (reference) prior.

Example:

    julia> fit(NormalGamma,rand(10))
    NormalGamma{Float64}(mu=0.5052725306750604, nu=10.0, shape=4.5, rate=0.5057485400844524)

Reference: Kevin P. Murphy, Conjugate Bayesian Analysis of the Gaussian Distribution. 2007. https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf
"""
    function Distributions.fit(::Type{NormalGamma},x::AbstractVector)
        n=length(x)
        μ=mean(x)
        ssr=sum((x.-μ).^2)
        α=(n-1)/2
        β=ssr/2
        return NormalGamma(μ,n,α,β)
    end

    function Distributions.params(d::NormalGamma)
        return d.mu, d.nu, d.shape, d.rate
    end

    function marginals(d::NormalGamma)

        m_T=MarginalTDist(d.mu,sqrt(d.rate/(d.shape*d.nu)),TDist(2*d.shape))
        m_Ga=Gamma(d.shape,1/d.rate)
        return m_T,m_Ga
    end
```

17.2.4 /src/NGRefTools.jl

```
module NGRefTools
    using ConjugatePriors, Distributions, Plots
    #import Makie:surface

    include("NGRef.jl")
    export marginals
    include("NIGRef.jl")
    #export NGplot
    include("MarginalTDist.jl")
    export MarginalTDist, MTDist_MC_func, mean_mass_comparator, log_mean_mass_comparator, plot_n_MTDist, plot_logn_MTDist
    include("LogNormalUtils.jl")
    export get_lognormal_params, get_lognormal_desc
end # module
```

17.2.5 /src/NIGRef.jl

```
"""
    fit(NormalInverseGamma, x)

Return the posterior `NormalInverseGamma` distribution on the unknown mean and variance of a Normal model of data vector `x`, assuming an uninformative (reference) prior.

Example:

    julia> fit(NormalInverseGamma,rand(10))
    NormalInverseGamma{Float64}(mu=0.5052725306750604, nu=10.0, shape=4.5, rate=0.5057485400844524)

Reference: Kevin P. Murphy, Conjugate Bayesian Analysis of the Gaussian Distribution. 2007. https://www.cs.ubc.ca/~murphyk/Papers/bayesGauss.pdf
"""
    function Distributions.fit(::Type{NormalInverseGamma},x::AbstractVector)
        n=length(x)
        Vn = 1/n
        μ=mean(x)
        an = -.5 + (n/2)
```

```
        bn = .5 * (sum(x.^2)-(μ^2/Vn))
        return NormalInverseGamma(μ,Vn,an,bn)
    end

    function Distributions.params(d::NormalInverseGamma)
        return d.mu, d.v0, d.shape, d.scale
    end

    function marginals(d::NormalInverseGamma)

        ↪  m_T=MarginalTDist(d.mu,sqrt(d.scale*d.v0/d.shape),TDist(2*d.shape))
        m_Ga=InverseGamma(d.shape,d.scale)
        return m_T,m_Ga
    end

    # function NGplot(d::NormalInverseGamma; grid=1000, upper=.95,
    ↪  lower=.05)
    #     marg_mean, marg_var = marginals(d)
    #     xmax=quantile(marg_mean,upper)
    #     xmin=quantile(marg_mean,lower)
    #     ymax=quantile(marg_var,upper)
    #     ymin=quantile(marg_var,lower)
    #     xs=LinRange(xmin, xmax, grid)
    #     ys=LinRange(ymin, ymax, grid)
    #     zs=[pdf(d,x,y) for x in xs, y in ys]

    #     return surface(xs,ys,zs)
    # end
```

17.3 GMC_NS

Github repository: https://github.com/mmattocks/GMC_NS.jl

17.3.1 /src/GMC_NS.jl

```
module GMC_NS
    using  Distributions, Serialization, UnicodePlots
    import ProgressMeter: AbstractProgress, Progress, @showprogress,
    ↪  next!, move_cursor_up_while_clearing_lines, printover,
    ↪  durationstring
    import Printf: @sprintf
    import Random: rand
```

```
import Statistics: unscaled_covzm, det, eigen
import Serialization: serialize, deserialize
import LinearAlgebra: dot, normalize
import BioBackgroundModels: lps
import StatsFuns: logaddexp, logsumexp, logsubexp
import Base: show
import Measurements: measurement
import ConjugatePriors: NormalGamma, NormalInverseGamma
import NGRefTools: MarginalTDist
import KernelDensity: kde, AbstractKDE

#GMC settings vector fields: [GMC_Nmin::Int64, GMC_τ_death::Float64, GMC_init_τ::Float64, GMC_tune_μ::Int64, GMC_tune_α::Float64, GMC_tune_PID::NTuple{3,Float64}, GMC_timestep_η::Float64, GMC_reflect_η::Float64, GMC_exhaust_σ::Float64, GMC_chain_κ::Int64]

#should GMC_reflect_η be smaller?
GMC_DEFAULTS=Vector{Any}([50,1e-5,1., 4,.25,(.3,0.,.0), .25, .005, 10., typemax(Int64)])
export GMC_DEFAULTS

include("ensemble/GMC_NS_Ensemble.jl")
export GMC_NS_Ensemble
include("model/GMC_NS_Model.jl")
export GMC_NS_Model, GMC_NS_Model_Record
include("model/model_utilities.jl")
include("GMC/galilean_trajectory.jl")
include("utilities/t_Tuner.jl")
include("utilities/ellipsoid.jl")
include("nested_sampler/search_patterns.jl")
include("nested_sampler/nested_step.jl")
include("nested_sampler/converge_ensemble.jl")
export converge_ensemble!
include("utilities/coordinate_utils.jl")
export box_bound!, to_prior, to_unit_ball, box_reflect!
include("ensemble/ensemble_utilities.jl")
export ensemble_history, clean_ensemble_dir, measure_evidence, reset_ensemble!, move_ensemble!, copy_ensemble, rewind_ensemble, show_models, show_models_e, get_model, rectify_ensemble!, posterior_kde
include("utilities/ns_progressmeter.jl")
include("utilities/progress_displays.jl")
```

```
    export tuning_display,evidence_display,convergence_display,info_display,lh_display,l
    include("utilities/stats.jl")
    include("model/normal/Normal_Model.jl")
    include("model/normal/Normal_Ensemble.jl")
    export Normal_Ensemble
    include("model/normal/LogNormal_Model.jl")
    include("model/normal/LogNormal_Ensemble.jl")
    export LogNormal_Ensemble
    include("model/eggbox/Eggbox_Model.jl")
    include("model/eggbox/Eggbox_Ensemble.jl")
    export Eggbox_Ensemble
end # module
```

17.3.2 /src/GMC/galilean_trajectory.jl

```
"""
    galilean_trajectory_sample!(m, e, tuner, cache)

Given 'm <: GMC_NS_Model', 'e <: GMC_NS_Ensemble', 'tuner <: GMC_Tuner', and 'cache <: Union{Nothing,GMC_NS_Model}', return the next model-sample from a Gailean trajectory through parameter space, as well as any cached reflection model, obeying 'new_m.log_Li ≥ e.contour'.

Implements Skilling's 2019 update to the GMC algorithm. Briefly: proceed "forward" along the trajectory's velocity vector unless the likelihood contour is hit; in this case, attempt to reflect "east", then "west", then "south". If a reflection is found, return the model in the original position with the new velocity vector (maintaining detailed balance), as well as the cached reflection model "forward" along this new vector. If no model with likelihood greater than contour can be found, return the original model (as long as the trajectory is alive it will be retried at lower timestep τ).

Skilling, John. “Galilean and Hamiltonian Monte Carlo.” In Proceedings, 33:8. Garching, Germany: MDPI, 2019.
"""
function galilean_trajectory_sample!(m, e, tuner, cache)
    t=m.trajectory; next_i=m.i+1; cache_i=m.i+2

```

```
    e.GMC_timestep_η > 0. ? (τ=max(tuner.τ+rand(Normal(0,tuner.τ*e.GMC_timestep_η)),eps())) : (τ=tuner.τ) #perturb τ if specified
    d=τ*m.v #distance vector for given timestep
    fwd_pos,adj_d,box_rflct=box_move(m.pos,d,e.box)

    if !box_rflct #if we're not stopped by the sampling box
        fwd_m=e.model_initλ(t, next_i, to_prior.(fwd_pos,e.priors), fwd_pos, m.v, e.obs, e.constants... )  #try to proceed along distance vector
        if m.θ==fwd_m.θ #if forward motion is no longer making a difference to the parameter vector (ie timestep is too small), kill the trajectory and bail out
            tuner.τ=e.GMC_τ_death
            return m, nothing
        end
    else
        fwd_m=m #placeholder, no effect on box reflection
    end

    if box_rflct || (fwd_m.log_Li < m.log_Li) #if no model is available from the position along the distance vector, search for reflections and store in cache
        process_report!(tuner, false) #report a reflection off a lh contour

        lhδ=lps(m.log_Li,-fwd_m.log_Li) #get the likelihood difference between the two points
        n=boundary_norm(adj_d,lhδ) #get the gradient normal for the distance and lhδ
        box_rflct ? (v⁇=box_reflect(m.pos,e.box,m.v,e.GMC_reflect_η)) : v⁇=reflect(m.v,n,e.GMC_reflect_η) #get the reflected velocity vector off the boundary "east", or off the sampling box
        rd=τ*v⁇ #reflected distance given the timestep
        east_pos,_=box_move(m.pos,rd,e.box)
        cache=e.model_initλ(t, cache_i, to_prior.(east_pos,e.priors), east_pos, v⁇, e.obs, e.constants... ) #try going>

        if cache.log_Li < m.log_Li && !box_rflct #no west reflection off the sampling box
            process_report!(tuner, false)

            west_pos,_=box_move(m.pos,-rd,e.box)
```

```
            cache=e.model_initλ(t, cache_i, to_prior.(west_pos,e.priors), west_pos, -v?, e.obs, e.constants... )  #if east reflection fails, try west
        end

        if cache.log_Li < m.log_Li
            process_report!(tuner, false)

            e.GMC_reflect_η > 0. ? (v?=r_perturb(-m.v, e.GMC_reflect_η)) : (v?=-m.v)
            sd=τ*v? #south distance along perturbed reflexn vec (inverted m.v)

            south_pos,_=box_move(m.pos,sd,e.box)
            cache=e.model_initλ(t, cache_i, to_prior.(south_pos,e.priors),south_pos,v?, e.obs, e.constants... )  #if west or box reflection fails, try south (reversed along the particle's vector)
        end
        cache.log_Li < m.log_Li && (process_report!(tuner, false)) #if that fails wait for smaller τ
    end

    if cache!==nothing && cache.log_Li ≥ m.log_Li #a reflection that results in a model above contour exists, so return a model in place with the appropriate velocity vector and the cached model down the reflection vector
        r_m=construct_reflection(m, next_i, cache.v)
        return (r_m, cache)
    else #if no reflections occurred or no reflected positions above the contour were found - either forward search succeeded or the whole search failed
        cache!==nothing && (cache=nothing) #reset cache if it's been set with a model below contour
        fwd_m.i == next_i && fwd_m.log_Li ≥ m.log_Li ? (process_report!(tuner, true); return fwd_m, cache) : (return m, cache) #if the forward model is a new iterate and at or above the contour, return that, otherwise return the original model
    end
end

                function box_move(pos,d,box)
```

```
        if !(all(box[:,1].<pos+d.<box[:,2])) #if not all coordinates after move would be in the box
            boundary_d=box.-pos #get distances to box boundaries
            b=isapprox.(pos,box) #find any boundaries that have been exceeded due to numerical creep
            boundary_d[b].=0 #correct these distances
            boundary_t=boundary_d./d #get time to hit boundaries, given particle displacment d

            for idx in findall(x->x≥0.,boundary_t) #for any boundaries the particle is riding (distance 0), give a small positive time if the particle is headed into the boundary to allow that dimension to be selected for bounding
                (dim,bound)=(idx[1],idx[2])
                boundary_t[idx]==0. && ((bound==1 && d[dim] < 0.) || (bound==2 && d[dim] > 0.)) && (boundary_t[idx]=nextfloat(0.))
            end


            first_b_idx=findfirst(isequal(minimum(boundary_t[findall( #find the index of the first boundary that would be hit
            first_b_d=boundary_d[first_b_idx]
            d=d.*(first_b_d/d[first_b_idx[1]])
        end

        all(d.==0.) ? box_reflect=true : box_reflect=false

        new_pos=pos+d
        !(all(box[:,1].<new_pos.<box[:,2])) && box_bound!(new_pos,box)

        return new_pos, d, box_reflect
    end

    function box_reflect(pos,box,v,η)
        b=isapprox.(pos,box)
        low_idxs=[v[i]<0&&b[i,1] for i in 1:length(v)]
        hi_idxs=[v[i]>0&&b[i,2] for i in 1:length(v)]

```

```
                    v?=copy(v)
                    v?[low_idxs]=-v[low_idxs]
                    v?[hi_idxs]=-v[hi_idxs]

                    η > 0. ? (return r_perturb(v?,η)) : (return v?)
                end

                function boundary_norm(v, lhδ)
                    b=lhδ./v
                    b[b.==Inf]•=prevfloat(Inf) #rectify overflow
                    b[b.==0.]•=rand([nextfloat(0.),prevfloat(0.)]) #rectify exact 0.
                    b[b.==-Inf]•=nextfloat(-Inf) #rectify underflow
                    return normalize(b)
                end

                function reflect(v, n, η)
                    v?=v-(2*n*dot(n,v))
                    η > 0. ? (return r_perturb(v?,η)) : (return v?)
                end

                function r_perturb(v?, η)
                    return v?*cos(η) + rand(MvNormal(length(v?),1.))*sin(η)
                end
```

17.3.3 /src/ensemble/GMC_NS_Ensemble.jl

```
"""
    GMC_NS_Ensemble

Abstract supertype for model ensembles intended to be subjected to Galilean Monte Carlo Nested Sampling ('GMC_NS'). Subtyped ensemble structs must implement the following fields as their interface with 'GMC_NS':

    mutable struct Minimal_Ensemble <: GMC_NS_Ensemble
        path::String #ensemble records serialized here

        models::Vector{<:GMC_NS_Model_Record} #records of model paths & lhs
        model_initλ::Function #GMC_NS_Model constructor
```

```

    contour<:AbstractFloat #current log_Li[end], init minimum([record.log_Li for record in models])
    log_Li::Vector{<:AbstractFloat} #likelihood of lowest-ranked model at iterate i, init [-Inf]
    log_Xi::Vector{<:AbstractFloat} #amt of prior mass included in ensemble contour at Li, init [0]
    log_wi::Vector{<:AbstractFloat} #width of prior mass covered in this iterate, init [-Inf]
    log_Liwi::Vector{<:AbstractFloat} #evidentiary weight of the iterate, init [-Inf]
    log_Zi::Vector{<:AbstractFloat} #ensemble evidence, init [typemin(FloatType)]
    Hi::Vector{<:AbstractFloat} #ensemble information, init [0]

    obs::Any #observations formatted for GMC_NS_Model likelihood Function
    priors::Vector{<:Distribution} #prior distributions on GMC_NS_Model parameters, formatted for model_initλ
    constants::Vector #fixed parameters to be passed to GMC_NS_Model constructor

    sample_posterior::Bool #switch for posterior_samples collection
    posterior_samples::Vector{<:GMC_NS_Model_Record}

    #GMC tuning settings
    GMC_tune_ι<:Integer #maximum number of initial τ tuning steps
    GMC_tune_ρ<:AbstractFloat #use ensemble size*ρ GMC samples per initial tuning step
    GMC_tune_μ<:Integer #tune τ on the basis of this number of the most recent steps
    GMC_tune_α<:AbstractFloat #tune τ such that the acceptance rate is between this value and 1.
    GMC_tune_β<:AbstractFloat #multiply τ by 1+β (sampling acceptance rate=1.) or 1-β (rate <α)

    #GMC Sampler settings
    GMC_timestep_η<:AbstractFloat #>0. to apply an perturbation term randomly drawn from Normal(0, η*τ) to τ at each step (can assist equilibration)
    GMC_reflect_η<:AbstractFloat #>0. to apply perturbation to reflection vectors, keep value small (can assist equilibration)
```

```
        GMC_exhaust_σ<:AbstractFloat #apply this scale parameter to the ensemble size to obtain the maximum number of GMC iterates to perform before terminating nested sampling
        GMC_chain_κ<:Integer #maximum chain length

        t_counter<:Integer #counter for next instantiated trajectory number
    end

"""
abstract type GMC_NS_Ensemble end


"""
    show(e<:GMC_NS_Ensemble)
    show(io,e<:GMC_NS_Ensemble)

Display the ensemble path, likelihood histogram, contour, maximum log likelihood, and log evidence (marginal likelihood).
"""
function Base.show(io::IO, e::GMC_NS_Ensemble; progress=false)
        livec=[model.log_Li for model in e.models]
        maxLH=maximum(livec)
        printstyled(io, "$(typeof(e)) @ $(e.path)\n", bold=true)
        msg = @sprintf "Contour: %3.6e MaxLH:%3.3e log Evidence:%3.6e" e.contour maxLH e.log_Zi[end]
    println(io, msg)
    if length(unique(livec)) > 1
        try
            hist=UnicodePlots.histogram(livec, title="Ensemble Likelihood Distribution")
            show(io, hist)
            println()
            progress && return(nrows(hist.graphics)+6)
        catch
            printstyled("HISTOGRAM UNAVAILABLE", color=:green)
            println()
            progress && return 3
        end
    else
        printstyled("HISTOGRAM UNAVAILABLE", color=:green)
        println()
        progress && return 3
```

```
    end
end
```

17.3.4 /src/ensemble/ensemble_utilities.jl

```
function ensemble_history(e::GMC_NS_Ensemble, bins=25)
    !e.sample_posterior && throw(ArgumentError("This ensemble has no posterior samples to show a history for!"))
    livec=vcat([model.log_Li for model in e.models],[model.log_Li for model in e.posterior_samples])
    show(histogram(livec, nbins=bins))
end

function e_backup(e::GMC_NS_Ensemble, tuner::Dict{Int64,<:GMC_Tuner})
    cp(string(e.path,'/',"ens"),string(e.path,'/',"ens.bak"), force=true)
    serialize(string(e.path,'/',"ens"), e)
    serialize(string(e.path,'/',"tuner"), tuner)
end

function clean_ensemble_dir!(e::GMC_NS_Ensemble)
    if e.sample_posterior
        ens_paths=vcat(
            [basename(m.path) for m in e.models],
            [basename(m.path) for m in e.posterior_samples],
            "ens", "tuner"
        )
    else
        ens_paths=vcat(
            [basename(m.path) for m in e.models],
            "ens", "tuner"
        )
    end

    for file in readdir(e.path)
        !(file in ens_paths) && rm(e.path*'/'*file)
    end
end

function complete_evidence(e::GMC_NS_Ensemble)
    log_Z=e.log_Zi[end]
    H=e.Hi[end]
    live_weight=lps(e.log_Xi[length(e.log_Li)], -log(length(e.models)))
```

```
    lis=sort([model.log_Li for model in e.models])
    liwis=sort(lps.(lis, live_weight))

    for (li,liwi) in zip(lis,liwis)
        last_Z=log_Z
        log_Z=logaddexp(log_Z,liwi)
        H=lps(
            (exp(lps(liwi,-log_Z)) * li),
            (exp(lps(last_Z,-log_Z)) * lps(H,last_Z)),
            -log_Z)
    end

    return log_Z, H
end

function measure_evidence(e::GMC_NS_Ensemble)
    log_Z, H = complete_evidence(e)
    return measurement(log_Z,sqrt(abs(H)/length(e.models)))
end

function reset_ensemble!(e::GMC_NS_Ensemble)

    Ni=Int64.(round.(collect(-1:-1:-length(e.log_Li)+1)./e.log_Xi[2:end]))
    N=Ni[1]

    new_models=Vector{typeof(e.models[1])}()

    for traj in 1:N
        if string(traj)*".1" in [basename(record.path) for record in e.models]

            push!(new_models,e.models[findfirst(isequal(string(traj)*".1"), [basename(record.path) for record in e.models])])
        else

            push!(new_models,e.posterior_samples[findfirst(isequal(string(traj)*"
            [basename(record.path) for record in e.posterior_samples])])
        end
    end

    e.models=new_models
    e.posterior_samples=Vector{typeof(e.models[1])}()
```

```

    e.contour=minimum([record.log_Li for record in e.models])

    e.log_Li=[e.log_Li[1]]
    e.log_Xi=[e.log_Xi[1]]
    e.log_wi=[e.log_wi[1]]
    e.log_Liwi=[e.log_Liwi[1]]
    e.log_Zi=[e.log_Zi[1]]
    e.Hi=[e.Hi[1]]

    e.t_counter=length(e.models)+1

    clean_ensemble_dir!(e)
    isfile(e.path*"/tuner") && rm(e.path*"/tuner")
    serialize(e.path*"/ens", e)

    return e
end

function move_ensemble!(e::GMC_NS_Ensemble,path::String)
    !isdir(path) && mkpath(path)
    rectype=typeof(e.models[1])

    for file in readdir(e.path)
        mv(e.path*'/'*file,path*'/'*file)
    end

    for (n,model) in enumerate(e.models)
        e.models[n]=rectype(model.trajectory, model.i, model.pos, path*'/'*basename(model.path), model.log_Li)
    end
    if e.sample_posterior
        for (n,model) in enumerate(e.posterior_samples)
            e.posterior_samples[n]=rectype(model.trajectory, model.i, model.pos, path*'/'*basename(model.path), model.log_Li)
        end
    end

    rm(e.path)
    e.path=path
    serialize(e.path*"/ens",e)
    return e
end
```

```julia

function copy_ensemble(e::GMC_NS_Ensemble,path::String)
    new_e=deepcopy(e)
    !isdir(path) && mkdir(path)
    rectype=typeof(e.models[1])
    for file in readdir(e.path)
        cp(e.path*'/'*file,path*'/'*file)
    end

    for (n,model) in enumerate(e.models)
        new_e.models[n]=rectype(model.trajectory, model.i, model.pos, path*'/'*basename(model.path), model.log_Li)
    end
    if e.sample_posterior
        for (n,model) in enumerate(e.posterior_samples)
            new_e.posterior_samples[n]=rectype(model.trajectory, model.i, model.pos, path*'/'*basename(model.path), model.log_Li)
        end
    end

    new_e.path=path
    serialize(new_e.path*"/ens",new_e)
    return new_e
end

function rewind_ensemble!(e::GMC_NS_Ensemble,rewind_idx)
    !e.sample_posterior && throw(ArgumentError("An ensemble not retaining posterior samples cannot be rewound!"))
    rewind_idx ≥ length(e.log_Li) && throw(ArgumentError("rewind_idx must be less than the current iterate!"))


    Ni=Int64.(round.(collect(-1:-1:-length(e.log_Li)+1)./e.log_Xi[2:end]))
    insert!(Ni,1,Ni[1])

    for it in length(e.log_Li)-1:-1:rewind_idx
        last_post=pop!(e.posterior_samples)

        if Ni[it]==Ni[it-1]
            models_idx=findfirst(isequal(last_post.trajectory), [rec.trajectory for rec in e.models])
            insert!(e.models, models_idx, last_post)
            deleteat!(e.models, models_idx+1)
```

```
        else
            push!(e.models, last_post)
        end

        !(length(e.models)==Ni[it-1]) && throw(DomainError("it: $it Ni[it]:$(Ni[it]) Ni[it-1]:$(Ni[it-1])"))
    end

    e.contour=e.log_Li[rewind_idx]
    e.log_Li=e.log_Li[1:rewind_idx]
    e.log_Xi=e.log_Xi[1:rewind_idx]
    e.log_wi=e.log_wi[1:rewind_idx]
    e.log_Liwi=e.log_Liwi[1:rewind_idx]
    e.log_Zi=e.log_Zi[1:rewind_idx]
    e.Hi=e.Hi[1:rewind_idx]

    traj_nos=vcat([rec.trajectory for rec in e.models],[rec.trajectory for rec in e.posterior_samples])

    e.t_counter=maximum(traj_nos)

    return e
end

function show_models(e::GMC_NS_Ensemble,idxs)
    liperm=sortperm([model.log_Li for model in e.models],rev=true)
    for idx in idxs
        m=deserialize(e.models[liperm[idx]].path)
        show(m)
    end
end

function show_models_e(e::GMC_NS_Ensemble,idxs)
    liperm=sortperm([model.log_Li for model in e.models],rev=true)
    for idx in idxs
        m=deserialize(e.models[liperm[idx]].path)
        show(stdout,m,e)
    end
end

function get_model(e::GMC_NS_Ensemble,no)
    return deserialize(e.path*'/'*string(no))
end
```

```julia

function reestimate_ensemble!(e::GMC_NS_Ensemble, wi_mode="trapezoidal")

    Ni=Int64.(round.(collect(-1:-1:-length(e.log_Li)+1)./e.log_Xi[2:end]))
    push!(Ni, length(e.models))

    #fix any borked starting values
    e.log_Li[1]=-Inf #L0 = 0
        e.log_Xi[1]=0. #X0 = 1
        e.log_wi[1]=-Inf #w0 = 0
        e.log_Liwi[1]=-Inf #Liwi0 = 0
        e.log_Zi[1]=-Inf #Z0 = 0
        e.Hi[1]=0. #H0 = 0,

    for i in 1:length(e.log_Li)-1
        j=i+1

        e.log_Li[j]=e.posterior_samples[i].log_Li

        e.log_Xi[j]=-i/Ni[i]

        if wi_mode=="trapezoidal"
            e.log_wi[j]= logsubexp(e.log_Xi[i], -j/Ni[j]) - log(2) #log width of prior mass spanned by the last step-trapezoidal approx
        elseif wi_mode=="simple"
            e.log_wi[j]= logsubexp(e.log_Xi[i], e.log_Xi[j]) #log width of prior mass spanned by the last step-simple approx
        else
            throw(ArgumentError("Unsupported wi_mode!"))
        end
        e.log_Liwi[j]=lps(e.log_Li[j],e.log_wi[j]) #log likelihood + log width = increment of evidence spanned by iterate
        e.log_Zi[j]=logaddexp(e.log_Zi[i],e.log_Liwi[j])    #log evidence
        #information- dimensionless quantity
        Hj=lps(
            (exp(lps(e.log_Liwi[j],-e.log_Zi[j])) * e.log_Li[j]),
            (exp(lps(e.log_Zi[i],-e.log_Zi[j])) * lps(e.Hi[i],e.log_Zi[i])),
            -e.log_Zi[j])
        Hj === -Inf ? (e.Hi[j]=0.) : (e.Hi[j]=Hj)
    end
end
```

```julia

#get a posterior kernel density estimate from the ensemble
function posterior_kde(e::GMC_NS_Ensemble; bivar=Vector{Pair{<:Integer}}())
    !e.sample_posterior && throw(ArgumentError("This ensemble is not set to collect posterior samples!"))


    θ_mat=zeros(length(e.models[1].pos),length(e.models)+length(e.posterior_sampl
    weight_vec=zeros(length(e.models)+length(e.posterior_samples))

    logz, _=complete_evidence(e)

    for (n,mrec) in enumerate(e.posterior_samples)
        m=deserialize(mrec.path)
        θ_mat[:,n]=m.θ
        weight_vec[n]=lps([m.log_Li,e.log_Xi[n+1],-logz])
    end

    for (n,mrec) in enumerate(e.models)
        m=deserialize(mrec.path)
        p=length(e.posterior_samples)
        θ_mat[:,n+p]=m.θ
        weight_vec[n+p]=lps([m.log_Li, e.log_Xi[end], -log(length(e.models)),-logz])
    end

    kdes=Vector{AbstractKDE}()
    for t in 1:size(θ_mat,1)
        push!(kdes,kde(θ_mat[t,:],weights=exp.(weight_vec)))
    end

    for bv in bivar

        push!(kdes,kde((θ_mat[bv[1],:],θ_mat[bv[2],:]),weights=exp.(weight_vec)))
    end

    return kdes
end
```

17.3.5 /src/model/GMC_NS_Model.jl

```
"""
    GMC_NS_Model_Record

Abstract supertype for records of serialised 'GMC_NS_Model's. Subtyped record structs must implement the following fields as their interface with 'GMC_NS':

    struct Minimal_Record
        trajectory::Integer #id assigned at construction, identifies trajectory
        i::Integer #id integer assigned at construction, identifies position on trajectory
        pos::Vector{<:AbstractFloat} #hypersphere position of model-particle
        path::String #model is serialized to ensembledir/trajectory.i
        log_Li::AbstractFloat #log likelihood calculated by 'GMC_NS_Model' constructor
    end
"""
abstract type GMC_NS_Model_Record end

"""
    GMC_NS_Model_Record

Abstract supertype for models in 'GMC_NS_Ensemble's. Subtyped model structs must NOT be mutable, and must not have inner constructors that make field-by-field construction inaccessible. Subtyped model structs must implement the following fields as their interface with 'GMC_NS', in order, and all additional fields must follow:

    mutable struct Minimal_Model
        trajectory::Integer #id assigned at construction, identifies trajectory
        i::Integer #id integer assigned at construction, identifies position on trajectory

        θ::Vector{<:AbstractFloat} #model's parameter Vector
        log_Li<:AbstractFloat #model's likelihood, calculated by constructor

        pos::Vector{<:AbstractFloat}
```

```
        v::Vector{<:AbstractFloat} #model's velocity in hypersphere coords
    end
"""
abstract type GMC_NS_Model end
```

17.3.6 /src/model/model_utilities.jl

```
function construct_reflection(m, i, r_v)
    params=[getfield(m,pn) for pn in propertynames(m)]
    params[2]=i
    params[6]=r_v
    return typeof(m)(params... )
end
```

17.3.7 /src/model/eggbox/Eggbox_Ensemble.jl

```
mutable struct Eggbox_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{Eggbox_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::Nothing
    priors::Vector{<:Distribution}
    constants::Vector{<:Real}
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Eggbox_Record}

    GMC_Nmin::Int64

```

```
    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

Eggbox_Ensemble(path::String, no_models::Integer, prior, box, GMC_settings...; sample_posterior::Bool=true) =
Eggbox_Ensemble(
    path,
    construct_eggbox_model,
    assemble_EMs(path, no_models, prior, box, Vector{Real}())...,
    [-Inf], #L0 = 0
        [0.], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0.], #H0 = 0,
    nothing,
    length(prior)==1 ? ([GMC_NS.marginals(prior)...]) : (prior),
    Vector{Real}(),
    length(prior)==1 ? (to_unit_ball.(box,[GMC_NS.marginals(prior)...])) : (to_unit_ball.(box,prior)),
    sample_posterior,
    Vector{Eggbox_Record}(),
    GMC_settings...,
        no_models+1)

function Base.show(io::IO, e::Eggbox_Ensemble; progress=true)
    x1o=[0.0, 0.4, 0.8, 0.2, 0.6, 1.0, 0.0, 0.4, 0.8, 0.2, 0.6, 1.0, 0.0,0.4,0.8,0.2,0.6,1.0]
    x2o=[0.0, 0.0, 0.0, 0.2, 0.2, 0.2, 0.4, 0.4, 0.4, 0.6, 0.6, 0.6, 0.8, 0.8, 0.8,1.0, 1.0, 1.0]

    x1m=[m.pos[1] for m in e.models]
```

```
    x2m=[m.pos[2] for m in e.models]
    x1m.+=1.; x2m.+=1.
    x1m./=2.; x2m./=2

    plt=scatterplot(x1o,x2o,title="Eggbox Ensemble: Contour $(round(e.contour,digits=2))",color=:green,name="Optima")
    scatterplot!(plt, x1m, x2m, color=:magenta, name="Model Particles")
    show(io, plt)
    println()

    (progress && return 19)
end

function assemble_EMs(path::String, no_trajectories::Integer, prior, box, constants)
        ensemble_records = Vector{Eggbox_Record}()
    !isdir(path) && mkpath(path)
    length(prior)==1 ? (marginals=[GMC_NS.marginals(prior)...]) : (marginals=prior)
    box=to_unit_ball.(box,marginals)

    @showprogress 1 "Assembling Normal Model ensemble..." for trajectory_no in 1:no_trajectories
                model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=[rand.(prior)...]
            pos=to_unit_ball.(proposal,marginals)
            box_bound!(pos,box)
            θvec=to_prior.(pos,marginals)

            model = construct_eggbox_model(trajectory_no, 1, θvec, pos, [0.], nothing, constants...; v_init=true)

                        serialize(model_path, model) #save the model to the ensemble directory
                        push!(ensemble_records, Eggbox_Record(trajectory_no,1,pos,model_path,model.log_Li
                else #interrupted assembly pick up from where we left off
                        model = deserialize(model_path)
                        push!(ensemble_records, Eggbox_Record(trajectory_no,1,model.pos,model_path,model.
                end
        end
```

```

        return ensemble_records, minimum([record.log_Li for record in
        ↪  ensemble_records])
end
```

17.3.8 /src/model/eggbox/Eggbox_Model.jl

```
mutable struct Eggbox_Record <: GMC_NS_Model_Record
    trajectory::Int64
    i::Int64
    pos::Vector{Float64}
    path::String
    log_Li::Float64
end

struct Eggbox_Model <: GMC_NS_Model
    trajectory::Integer #id integer assigned at construction
    i::Integer #id of parent model, if any

    θ::Vector{Float64} #model's parameter Vector
    log_Li::Float64 #model's likelihood, calculated by constructor

    pos::Vector{Float64}
    v::Vector{Float64} #model's velocity in parameter space
end

function construct_eggbox_model(trajectory::Integer, i::Integer,
↪   θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64},
↪   obs::Nothing; v_init=false)
    x1,x2=θ
    log_lh=(2 + cos(5π * x1) * cos(5π * x2))^5
    v_init && (v=rand(MvNormal(length(θ),1.)))

    Eggbox_Model(trajectory, i, θ, log_lh, pos, v)
end

function Base.show(io::IO, m::Eggbox_Model; xsteps=100)
    println("Eggbox Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)")
    println("θ: $(m.θ)")
    println("v: $(m.v)")
end
```

17.3.9 /src/model/normal/LogNormal_Ensemble.jl

```
mutable struct LogNormal_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{Normal_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::Vector{Float64}
    priors::Vector{<:Distribution}
    constants::Vector{<:Real}
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Normal_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

LogNormal_Ensemble(path::String, no_models::Integer,
↪   obs::AbstractVector{<:AbstractFloat}, prior, box, GMC_settings...;
↪   sample_posterior::Bool=true) =
```

```
LogNormal_Ensemble(
    path,
    construct_lognormal_model,
    assemble_lNMs(path, no_models, obs, prior, box, Vector{Real}()) ... ,
    [-Inf], #L0 = 0
        [0.], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0.], #H0 = 0,
    obs,
    length(prior)==1 ? ([GMC_NS.marginals(prior) ... ]) : (prior),
    Vector{Real}(),
    length(prior)==1 ? (to_unit_ball.(box,[GMC_NS.marginals(prior) ... ]))
    ↪ : (to_unit_ball.(box,prior)),
    sample_posterior,
    Vector{Normal_Record}(),
    GMC_settings ... ,
        no_models+1)

function Base.show(io::IO, m::LogNormal_Model, e::LogNormal_Ensemble;
↪ xsteps=100, progress=true)
    μ,λ=m.θ
    lμ=log(μ^2/sqrt(μ^2 + inv(λ)))
    lσ=sqrt(log(1+(inv(λ)/μ^2)))
    n=LogNormal(lμ,lσ)

    ↪ X=[quantile(n,.025):(quantile(n,.975)-quantile(n,.025))/xsteps:quantile(n,.97
    y=pdf.(n,X)

    scattermin=maximum(y)*.25
    scattermax=maximum(y)*.75

    ↪ scatterrange=[scattermin:(scattermax-scattermin)/length(e.obs):scattermax ... ]
    scattery=[rand(scatterrange) for i in 1:length(e.obs)]

    xlims=(min(minimum(e.obs),minimum(X)),max(maximum(e.obs),maximum(X)))

    plt=lineplot(X,y,xlabel="X",ylabel="p",title="LogNormal Model
    ↪ $(m.trajectory).$(m.i), log_Li $(m.log_Li)", name="Model",
    ↪ xlim=xlims)
    scatterplot!(plt, e.obs, scattery, name="Obs")

```

```
    show(io, plt)
    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")

    progress && return nrows(plt.graphics)+8
end

function assemble_lNMs(path::String, no_trajectories::Integer, obs, prior, box, constants)
        ensemble_records = Vector{Normal_Record}()
    !isdir(path) && mkpath(path)
    length(prior)==1 ? (marginals=[GMC_NS.marginals(prior)...]) : (marginals=prior)
    box=to_unit_ball.(box,marginals)

    @showprogress 1 "Assembling log-Normal Model ensemble..." for trajectory_no in 1:no_trajectories
                model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=[rand.(prior)...]
            pos=to_unit_ball.(proposal,marginals)
            box_bound!(pos,box)
            θvec=to_prior.(pos,marginals)

            model = construct_lognormal_model(trajectory_no, 1, θvec, pos, [0.], obs, constants...; v_init=true)

                        serialize(model_path, model) #save the model to the ensemble directory
                        push!(ensemble_records, Normal_Record(trajectory_no,1,pos,model_path,model.log_Li
                else #interrupted assembly pick up from where we left off
                        model = deserialize(model_path)
                        push!(ensemble_records, Normal_Record(trajectory_no,1,model.pos,model_path,model.
                end
        end

        return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end
```

17.3.10 /src/model/normal/LogNormal_Model.jl

```julia
#NOTE: LogNormal_Models reuse Normal_Records

struct LogNormal_Model <: GMC_NS_Model
    trajectory::Integer #id integer assigned at construction
    i::Integer #id of parent model, if any

    θ::Vector{Float64} #model's parameter Vector
    log_Li::Float64 #model's likelihood, calculated by constructor

    pos::Vector{Float64}
    v::Vector{Float64} #model's velocity in parameter space
end

function construct_lognormal_model(trajectory::Integer, i::Integer, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Float64}; v_init=false)
    μ,λ=θ
    lμ=log(μ^2/sqrt(μ^2 + inv(λ)))
    lσ=sqrt(log(1+(inv(λ)/μ^2)))
    mod_lnormal=LogNormal(lμ,lσ)
    log_lh=lps(logpdf.(mod_lnormal, obs))
    v_init && (v=rand(MvNormal(length(θ),1.)))

    LogNormal_Model(trajectory, i, θ, log_lh, pos, v)
end

function Base.show(io::IO, m::LogNormal_Model; xsteps=100)
    μ,λ=m.θ
    lμ=log(μ^2/sqrt(μ^2 + inv(λ)))
    lσ=sqrt(log(1+(inv(λ)/μ^2)))
    n=LogNormal(lμ,lσ)
    X=[quantile(n,.025):(quantile(n,.975)-quantile(n,.025))/xsteps:quantile(n,.97
    y=pdf.(n,X)

    show(io, lineplot(X,y,xlabel="X",ylabel="p",title=title="LogNormal Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)"))
    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")
end
```

17.3.11 /src/model/normal/Normal_Ensemble.jl

```
mutable struct Normal_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{Normal_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::Vector{Float64}
    priors::Vector{<:Distribution}
    constants::Vector{<:Real}
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Normal_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

```

```
Normal_Ensemble(path::String, no_models::Integer, obs::AbstractVector{<:AbstractFloat}, prior, box, GMC_settings...; sample_posterior::Bool=true) =
Normal_Ensemble(
    path,
    construct_normal_model,
    assemble_NMs(path, no_models, obs, prior, box, Vector{Real}())...,
    [-Inf], #L0 = 0
        [0.], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0.], #H0 = 0,
    obs,
    length(prior)==1 ? ([GMC_NS.marginals(prior)...]) : (prior),
    Vector{Real}(),
    length(prior)==1 ? (to_unit_ball.(box,[GMC_NS.marginals(prior)...])) : (to_unit_ball.(box,prior)),
    sample_posterior,
    Vector{Normal_Record}(),
    GMC_settings...,
        no_models+1)

function Base.show(io::IO, m::Normal_Model, e::Normal_Ensemble; xsteps=100, progress=true)
    μ,λ=m.θ
    σ=sqrt(1/λ)
    n=Normal(μ,σ)
    X=[μ-4σ:((μ+4σ)-(μ-4σ))/xsteps:μ+4σ...]
    y=pdf.(n,X)

    scattermin=y[Int(floor(xsteps/2)-floor(xsteps/3))]
    scattermax=y[Int(floor(xsteps/2)-floor(xsteps/6))]

    scatterrange=[scattermin:(scattermax-scattermin)/length(e.obs):scattermax...]
    scattery=[rand(scatterrange) for i in 1:length(e.obs)]

    xlims=(min(minimum(e.obs),minimum(X)),max(maximum(e.obs),maximum(X)))

    plt=lineplot(X,y,xlabel="X",ylabel="p",title="Normal Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)", name="Model", xlim=xlims)
    scatterplot!(plt, e.obs, scattery, name="Obs")
```

```julia

    show(io, plt)
    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")

    progress && return nrows(plt.graphics)+8
end

function assemble_NMs(path::String, no_trajectories::Integer, obs, prior, box, constants)
        ensemble_records = Vector{Normal_Record}()
    !isdir(path) && mkpath(path)
    length(prior)==1 ? (marginals=[GMC_NS.marginals(prior)...]) : (marginals=prior)
    box=to_unit_ball.(box,marginals)

    @showprogress 1 "Assembling Normal Model ensemble..." for trajectory_no in 1:no_trajectories
                model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=[rand.(prior)...]
            pos=to_unit_ball.(proposal,marginals)
            box_bound!(pos,box)
            θvec=to_prior.(pos,marginals)

            model = construct_normal_model(trajectory_no, 1, θvec, pos, [0.], obs, constants...; v_init=true)

                        serialize(model_path, model) #save the model to the ensemble directory
                        push!(ensemble_records, Normal_Record(trajectory_no,1,pos,model_path,model.log_Li
                else #interrupted assembly pick up from where we left off
                        model = deserialize(model_path)
                        push!(ensemble_records, Normal_Record(trajectory_no,1,model.pos,model_path,model.
                end
        end

        return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end
```

17.3.12 /src/model/normal/Normal_Model.jl

```
mutable struct Normal_Record <: GMC_NS_Model_Record
    trajectory::Int64
    i::Int64
    pos::Vector{Float64}
    path::String
    log_Li::Float64
end

struct Normal_Model <: GMC_NS_Model
    trajectory::Integer #id integer assigned at construction
    i::Integer #id of parent model, if any

    θ::Vector{Float64} #model's parameter Vector
    log_Li::Float64 #model's likelihood, calculated by constructor

    pos::Vector{Float64}
    v::Vector{Float64} #model's velocity in parameter space
end

function construct_normal_model(trajectory::Integer, i::Integer,
↪   θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64},
↪   obs::Vector{Float64}; v_init=false)
    μ,λ=θ
    mod_normal=Normal(μ,sqrt(1/λ))
    log_lh=lps(logpdf.(mod_normal, obs))
    v_init && (v=rand(MvNormal(length(θ),1.)))

    Normal_Model(trajectory, i, θ, log_lh, pos, v)
end

function Base.show(io::IO, m::Normal_Model; xsteps=100)
    μ,λ=m.θ
    σ=sqrt(1/λ)
    n=Normal(μ,σ)
    X=[μ-4σ:((μ+4σ)-(μ-4σ))/xsteps:μ+4σ ... ]
    y=pdf.(n,X)

    show(io, lineplot(X,y,xlabel="X",ylabel="p",title="Normal Model
     ↪  $(m.trajectory).$(m.i), log_Li $(m.log_Li)"))
```

```
    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")
end
```

17.3.13 /src/nested_sampler/converge_ensemble.jl

```
"""
    converge_ensemble!(e; max_iterates, backup, clean, converge_criterion, converge_factor, mc_noise, progargs)

Given 'e <: GMC_NS_Ensemble', recursively execute nested sampling steps ('nested_step!()') until ensemble convergence criterion is obtained, or 'max_iterates' is reached.

Convergence criteria specied by 'converge_criterion <: String': "standard" for evidence convergence, "compression" for ensemble likelihood range compression (can be used to improve MAP parameter estimation after evidence has converged). Factor to use specified by 'converge_factor <: AbstractFloat'. "Standard" defined in Skilling 2006.

For likelihood estimates generated by Monte Carlo model evaluation, 'mc_noise <: AbstractFloat' defines the expected standard deviation of likelihood in the MAP region of parameter space; this will be used as an alternative convergence criterion if the ensemble is converged to within this value.

Keyword arguments to GMC_NS_Progress are passed from progargs.

Skilling, John. “Nested Sampling for Bayesian Computations.” In Proc. Valencia. Benidorm (Alicante, Spain): inference.org.uk, 2006. https://www.inference.org.uk/bayesys/Valencia.pdf.
"""
function converge_ensemble!(e::GMC_NS_Ensemble; max_iterates=typemax(Int64), backup::Tuple{Bool,Integer}=(false,0), clean::Tuple{Bool,Integer,Integer}=(false,0,0), converge_criterion::String="standard", converge_factor::AbstractFloat=1e-3, mc_noise::AbstractFloat=0., progargs... )
    N = length(e.models); curr_it=length(e.log_Li)

```

```
    if curr_it==1 || !isfile(e.path*"/tuner")
        serialize(e.path*"/ens",e)
        tuner_dict=get_tuner_dict(e)
    else
        tuner_dict=deserialize(e.path*"/tuner") #restore tuner from saved if any
    end

    meter = GMC_NS_Progress(e, .1; start_it=curr_it, progargs... )

    converge_check = get_convfunc(converge_criterion)

    reflect_cache = nothing #init cache for models precalculated in galilean reflection

    cln_switch = clean[1] && !e.sample_posterior #ignore clean arguments if posterior samples are to be collected

    while !converge_check(e, converge_factor, mc_noise) && (curr_it ≤ max_iterates)
        warn, reflect_cache = nested_step!(e, tuner_dict, reflect_cache)
        warn == 1 && (@error "Failed to find new models, aborting at current iterate."; return e)
        curr_it += 1

        backup[1] && curr_it%backup[2] == 0 && e_backup(e,tuner_dict) #every backup interval, serialise the ensemble and tuner
        cln_switch && curr_it%clean[2] == 0 && clean_ensemble_dir(e,clean[3]) #every clean interval, remove old discarded models

        update!(meter, converge_check(e,converge_factor, vals=true)... )
    end

    if converge_check(e,converge_factor, mc_noise)
        final_logZ=measure_evidence(e)
        @info "Job done, sampled to convergence. Final logZ $(final_logZ.val) ± $(final_logZ.err)"

        e_backup(e,tuner_dict) #final backup
        cln_switch && clean_ensemble_dir(e,0) #final clean
        return final_logZ
    elseif curr_it==max_iterates
```

```
        @info "Job done, sampled to maximum iterate $max_iterates. Convergence criterion not obtained."

        e_backup(e,tuner_dict) #final backup
        cln_switch && clean_ensemble_dir(e,0) #final clean
        return e.log_Zi[end]
    end
end
                function evidence_converge(e, evidence_fraction, mc_noise=0.; vals=false)
                    mc_noise > 0. && noise_check(e,mc_noise) && return true

                    val=lps(findmax([model.log_Li for model in e.models])[1],  e.log_Xi[end])
                    thresh=lps(log(evidence_fraction),e.log_Zi[end])

                    vals ? (return val, thresh) : (return val<thresh)
                end

                function compress_converge(e, compression_ratio, mc_noise=0.; vals=false)
                    mc_noise > 0. && noise_check(e,mc_noise) && return true

                    val=findmax([model.log_Li for model in e.models])[1]-e.contour
                    thresh=compression_ratio

                    val===-Inf && (val=nextfloat(val))
                    vals ? (return val, thresh) : (return val<thresh)
                end

                function get_convfunc(criterion)
                    if criterion == "standard"
                        return evidence_converge
                    elseif criterion == "compression"
                        return compress_converge
                    else
                        throw(ArgumentError("Convergence criterion $criterion not supported! Try \"standard\" or \"compression\"."))
                    end
```

```
                end

                #if the entire ensemble is within 99.7% of the noise distribution of its mean likelihood, it is converged to the extent possible given Monte Carlo noise
                function noise_check(e, mc_noise)
                    mod_lhs=[m.log_Li for m in e.models]
                    std(mod_lhs) ≤ mc_noise ? (return true) : (return false)
                end
```

17.3.14 /src/nested_sampler/nested_step.jl

```
"""
    nested_step!(e<:GMC_NS_Ensemble)

Step and update ensemble e by sampling the e.contour-bounded prior mass via Galilean Monte Carlo.

"""

function nested_step!(e::GMC_NS_Ensemble, tuners::Dict{Int64,τ_PID}, cache::Union{GMC_NS_Model,Nothing})
    N = length(e.models); N?=N #number of sample models/particles on the posterior surface
    i = length(e.log_Li) #iterate number, index for last values
    j = i+1 #index for newly pushed values

    e.contour, least_likely_idx = findmin([model.log_Li for model in e.models])
    Li_model = e.models[least_likely_idx]
    m = deserialize(Li_model.path)
    t=tuners[Li_model.trajectory]

    #SELECT NEW MODEL, SAVE TO ENSEMBLE DIRECTORY, CREATE RECORD AND PUSH TO ENSEMBLE

    cache===nothing ? (model_selected=false; cache_insert=false) : (model_selected=true; cache_insert=true)
    samples=1; sample_limit=Int64(floor(length(e.models)*e.GMC_exhaust_σ))

```

```
    while !model_selected && samples≤sample_limit
        if t.τ > e.GMC_τ_death && m.i ≤ e.GMC_chain_κ #while the particle's timestep is greater than the minimum to keep a chain alive, and the chain is below the maximum allowed length, sample by advancing it along a galilean trajectory
            model_selected,cache = galilean_search!(m,e,t,least_likely_idx,samples,sample_limit,cache) #ensemble is modified by galilean_search! directly
        elseif (N-1)<e.GMC_Nmin #if removing the particle would put the sample under the minimum size, resample by diffusion from an existing particle

            model_selected=diffusion_search!(e,tuners,least_likely_idx,N,e.GMC_τ_ #ensemble modified by diffusion_search!
            samples+=1
        else #if not, push to the posterior samples and let the trajectory chain die
            model_selected=true
            e.sample_posterior && push!(e.posterior_samples, e.models[least_likely_idx])#if sampling posterior, push the model record to the ensemble's posterior samples vector
            deleteat!(e.models, least_likely_idx) #remove least likely model record from the active particles in the ensemble
            N?-=1 #update for trapezoidal log_wi calc
        end
    end

    if cache_insert #in this case a cached precalculated reflection model can be used
        insert_reflection!(e, cache) #insert the model into the ensemble
        process_report!(t, true)
        cache=nothing #reset the cache
    end

    samples==sample_limit && (return 1, cache) #failed to find a new sample after sampling up to the limit, return an error code

    #UPDATE ENSEMBLE QUANTITIES
    push!(e.log_Li, e.contour) #log likelihood of the least likely model - the current ensemble ll contour at Xi
    push!(e.log_Xi, -i/N) #log Xi - crude estimate of the iterate's enclosed prior mass
```

```
    push!(e.log_wi, logsubexp(e.log_Xi[i], -j/N?) - log(2)) #log width of prior mass spanned by the last step-trapezoidal approx
    push!(e.log_Liwi, lps(e.log_Li[j],e.log_wi[j])) #log likelihood + log width = increment of evidence spanned by iterate
    push!(e.log_Zi, logaddexp(e.log_Zi[i],e.log_Liwi[j]))    #log evidence
    #information- dimensionless quantity. cf. MultiNest @ https://github.com/farhanferoz/MultiNest/blob/master/MultiNest_v3.12/nested.F MultiNest now gives info only at the final step
    Hj=lps(
        (exp(lps(e.log_Liwi[j],-e.log_Zi[j])) * e.log_Li[j]), #information contribution of this step
        (exp(lps(e.log_Zi[i],-e.log_Zi[j])) * lps(e.Hi[i],e.log_Zi[i])), #rescale last information by evidence
        -e.log_Zi[j])
    Hj === -Inf ? push!(e.Hi,0.) : push!(e.Hi, Hj) #prevent problems from early strings of models with -Inf log likelihoods

    return 0, cache
end
```

17.3.15 /src/nested_sampler/search_patterns.jl

```
function galilean_search!(m,e,t,llidx,samples,s_limit,cache)
    new_m,cache=galilean_trajectory_sample!(m,e,t,cache)
    m.i==1 && ((m,new_m,cache)=start_trim!(m,new_m,cache))


    while new_m.i==m.i && samples≤s_limit && t.τ > e.GMC_τ_death#failure to find any new step, retune tau and try again until sample limit reached or trajectory's tuner reaches taudeath
        new_m,cache=galilean_trajectory_sample!(m,e,t,cache)
        samples+=1
        m.i==1 && ((m,new_m,cache)=start_trim!(m,new_m,cache))
    end

    if new_m.i==m.i+1
        e.sample_posterior && push!(e.posterior_samples, e.models[llidx])#if sampling posterior, push the model record to the ensemble's posterior samples vector

        rectype=typeof(e.models[1])
```

```
        deleteat!(e.models, llidx) #remove least likely model record

        new_model_record = rectype(new_m.trajectory,new_m.i, new_m.pos, string(e.path,'/',new_m.trajectory,'.',new_m.i), new_m.log_Li)
        push!(e.models, new_model_record) #insert new model record
        serialize(new_model_record.path, new_m)

        return true, cache
    else
        return false, cache
    end
end

function start_trim!(m,new_m,cache)
    if new_m.i!=m.i && isapprox(new_m.pos,m.pos)
        params=[getfield(m,pn) for pn in propertynames(m)]
        params[6]=new_m.v
        m=typeof(m)(params ... )
        params=[getfield(cache,pn) for pn in propertynames(m)]
        params[2]=2;
        new_m=typeof(m)(params ... )
        cache=nothing
    end
    return m,new_m,cache
end

function insert_reflection!(e, cache)
    t_idx=findfirst(t→t==cache.trajectory,[m.trajectory for m in e.models])

    e.sample_posterior && push!(e.posterior_samples, e.models[t_idx])#if sampling posterior, push the model record to the ensemble's posterior samples vector
    rectype=typeof(e.models[1])
    deleteat!(e.models, t_idx) #remove least likely model record

    new_model_record = rectype(cache.trajectory,cache.i, cache.pos, string(e.path,'/',cache.trajectory,'.',cache.i), cache.log_Li)
    push!(e.models, new_model_record) #insert new model record
    serialize(new_model_record.path, cache)
end

```

```
function diffusion_search!(e, tuners, llidx, sample_limit, τ_limit)
    sample=0; model_selected=false; new_m=0; src_traj=0
    while !(sample > sample_limit) && !model_selected
        sample +=1
        mrec=rand(e.models)
        model=deserialize(mrec.path)
        randdir=rand(MvNormal(length(model.θ),1.))
        τ=tuners[model.trajectory].τ
        new_pos=model.pos+randdir*τ
        box_bound!(new_pos,e.box)
        new_m=e.model_initλ(e.t_counter, 1, to_prior.(new_pos,e.priors), new_pos, randdir, e.obs, e.constants ... )

        while τ*.7>τ_limit && new_m.log_Li<e.contour
            τ*=.7
            randdir=rand(MvNormal(length(model.θ),1.))
            new_pos=model.pos+randdir*τ
            box_bound!(new_pos,e.box)
            new_m=e.model_initλ(e.t_counter, 1, to_prior.(new_pos,e.priors), new_pos, randdir, e.obs, e.constants ... )
        end

        new_m.log_Li ≥ e.contour && (model_selected=true; src_traj=model.trajectory)
    end

    if new_m.log_Li ≥ e.contour
        e.sample_posterior && push!(e.posterior_samples, e.models[llidx])#if sampling posterior, push the model record to the ensemble's posterior samples vector
        rectype=typeof(e.models[1])
        deleteat!(e.models, llidx) #remove least likely model record

        new_model_record = rectype(new_m.trajectory,new_m.i, new_m.pos, string(e.path,'/',new_m.trajectory,'.',new_m.i), new_m.log_Li)
        push!(e.models, new_model_record) #insert new model record
        serialize(new_model_record.path, new_m)
        tuners[e.t_counter]=τ_PID(e)
        tuners[e.t_counter].τ=tuners[src_traj].τ
        e.t_counter +=1
        return true
```

```
    else
        return false
    end
end

function ellipsoid_resample!(e,tdict,llidx,samples,s_limit)
    posmat=zeros(length(e.models[1].pos),length(e.models))
    for (n,rec) in enumerate(e.models)
        posmat[:,n]=rec.pos
    end
    e_ellip=bounding_ellipsoid(posmat)

    new_pos=sample_ellipsoid(e_ellip)
    box_bound!(new_pos,e.box)
    new_m=e.model_initλ(e.t_counter, 1, to_prior.(new_pos,e.priors), new_pos, [0.], e.obs, e.box, e.constants... , v_init=true)
    samples+=1

    while new_m.log_Li < e.contour && samples < s_limit
        new_pos=sample_ellipsoid(e_ellip)
        box_bound!(new_pos,e.box)
        new_m=e.model_initλ(e.t_counter, 1, to_prior.(new_pos,e.priors), new_pos, [0.], e.obs, e.box, e.constants... , v_init=true)
        samples+=1
    end

    if new_m.log_Li > e.contour
        e.sample_posterior && push!(e.posterior_samples, e.models[llidx])#if sampling posterior, push the model record to the ensemble's posterior samples vector
        rectype=typeof(e.models[1])
        deleteat!(e.models, llidx) #remove least likely model record

        new_model_record = rectype(new_m.trajectory,new_m.i, new_m.pos, string(e.path,'/',new_m.trajectory,'.',new_m.i), new_m.log_Li)
        push!(e.models, new_model_record) #insert new model record
        serialize(new_model_record.path, new_m)
        tdict[e.t_counter]=τ_PID(e)
        e.t_counter +=1
        return true
    else
        return false
```

```
    end
end
```

17.3.16 /src/utilities/coordinate_utils.jl

```
function to_prior(pos,prior)
    return quantile(prior, .5+.5*pos)
end

function to_unit_ball(pos,prior)
    return (cdf(prior,pos)-.5)/.5
end


function box_bound!(pos,box)
    if !(all(box[:,1].<pos.<box[:,2]))
        pos[pos.<box[:,1]]•=box[:,1][pos.<box[:,1]]
        pos[pos.>box[:,2]]•=box[:,2][pos.>box[:,2]]
    end
end
```

17.3.17 /src/utilities/ellipsoid.jl

```
# Represents an N-dimensional ellipsoid
struct Ellipsoid
    ctr::Vector{Float64}  # center coordinates
    cov::Array{Float64, 2}
    icov::Array{Float64, 2}  # inverse of cov
    vol::Float64
end

# Draw a random point from within a unit N-ball
function randnball(ndim)
    z = randn(ndim)
    r2 = 0.
    for i=1:ndim
        r2 += z[i]*z[i]
    end
    factor = rand()^(1. /ndim) / sqrt(r2)
    for i=1:ndim
        z[i] *= factor
```

```
    end
    return z
end

# proportionality constant depending on dimension
# for n even:      (2pi)^(n    /2) / (2 * 4 * ... * n)
# for n odd :  2 * (2pi)^((n-1)/2) / (1 * 3 * ... * n)
function nball_vol_factor(ndim::Int)
    if ndim % 2 == 0
        c = 1.
        for i=2:2:ndim
            c *= 2pi / i
        end
        return c
    else
        c = 2.
        for i = 3:2:ndim
            c *= 2pi / i
        end
        return c
    end
end

function ellipsoid_volume(scaled_cov::Matrix{Float64})
    ndim = size(scaled_cov, 1)
    return nball_vol_factor(ndim) * sqrt(det(scaled_cov))
end

# find the bounding ellipsoid of points x where
function bounding_ellipsoid(x::Matrix{Float64}, enlarge=1.0)

    ndim, npoints = size(x)

    ctr = mean(x, dims=2)[:, 1]
    delta = x .- ctr
    cov = unscaled_covzm(delta, 2)
    icov = inv(cov)

    # Calculate expansion factor necessary to bound each point.
    # This finds the maximum of (delta_i' * icov * delta_i)
    fmax = -Inf
    for k in 1:npoints
        f = 0.0
```

```
        for j=1:ndim
            for i=1:ndim
                f += icov[i, j] * delta[i, k] * delta[j, k]
            end
        end
        fmax = max(fmax, f)
    end

    fmax *= enlarge
    cov .*= fmax
    icov .*= 1. /fmax
    vol = ellipsoid_volume(cov)

    return Ellipsoid(ctr, cov, icov, vol)
end

function sample_ellipsoid(ell::Ellipsoid)
    ndim = length(ell.ctr)

    # Get scaled eigenvectors (in columns): vs[:,i] is the i-th eigenvector.
    f = eigen(ell.cov)
    v, w = f.vectors, f.values
    for j=1:ndim
        tmp = sqrt(abs(w[j]))
        for i=1:ndim
            v[i, j] *= tmp
        end
    end

    return v*randnball(ndim) + ell.ctr
end
```

17.3.18 /src/utilities/ns_progressmeter.jl

```
const CONVERGENCE_MEMORY=500

mutable struct GMC_NS_Progress{T<:Real} <: AbstractProgress
    interval::T
    dt::AbstractFloat
    start_it::Integer
    counter::Integer
```

```
    triggered::Bool
    tfirst::AbstractFloat
    tlast::AbstractFloat
    tstp::AbstractFloat
    printed::Bool        # true if we have issued at least one status update
    desc::AbstractString # prefix to the percentage, e.g.  "Computing..."
    color::Symbol        # default to green
    output::IO           # output stream into which the progress is written
    numprintedvalues::Integer   # num values printed below progress in last iteration
    offset::Integer             # position offset of progress bar (default is 0)

    e::GMC_NS_Ensemble
    #tuner::GMC_Tuner
    top_m::GMC_NS_Model

    mean_stp_time::AbstractFloat

    upper_displays::Vector{Vector{Function}}
    lower_displays::Vector{Vector{Function}}
    display_rotate_iterates::Int64
    upperidx::Integer
    loweridx::Integer

    convergence_history::Vector{Float64}
    time_history::Vector{Float64}

    function GMC_NS_Progress{T}(e::GMC_NS_Ensemble,
                                top_m::GMC_NS_Model,
                                #tuner::GMC_Tuner,
                                interval::T;
                                dt::Real=0.1,
                                desc::AbstractString="Nested Sampling::",
                                color::Symbol=:green,
                                output::IO=stdout,
                                offset::Int=0,
                                start_it::Int=1,
                                upper_displays=[[evidence_display]],
                                lower_displays=[[ensemble_display]],
                                disp_rot_its=0,
```

```
                                ) where T
        tfirst = tlast = time()
        printed = false
        new{T}(interval,
         dt,
         start_it,
         start_it,
         false,
         tfirst,
         tlast,
         0.,
         printed,
         desc,
         color,
         output,
         0,
         offset,
         e,
         #tuner,
         top_m,
         0.,
         upper_displays,
         lower_displays,
         disp_rot_its,
         1,
         1,
         zeros(CONVERGENCE_MEMORY),
         zeros(CONVERGENCE_MEMORY))
    end
end

"""
    GMC_NS_Progress(e, interval; kwargs ... )

Return a GMC_NS.jl ProgressMeter for the given ensemble, to be updated at 'interval'. Important keyword arguments:

'upper_displays <: AbstractVector{<:AbstractVector{Function}}': define a vector of above-status-line display functions to rotate through
'lower_displays <: AbstractVector{<:AbstractVector{Function}}': define a vector of below-status-line display functions to rotate through
'disp_rot_its <: Integer': define the number of iterates to rotate to the next upper and lower display function vectors
```

```julia

"""
function GMC_NS_Progress(e::GMC_NS_Ensemble,
    #tuner::GMC_Tuner,
     interval::Real; dt::Real=0.1, desc::AbstractString="GMC-NS::",
      ↪ color::Symbol=:green, output::IO=stderr, offset::Integer=0,
      ↪ start_it::Integer=1,
      ↪ upper_displays::AbstractVector{<:AbstractVector{Function}}=Vector{Vector{Fun
      ↪ lower_displays::AbstractVector{<:AbstractVector{Function}}=Vector{Vector{Fun
      ↪ disp_rot_its::Integer=0)
    top_m = deserialize(e.models[findmax([model.log_Li for model in
     ↪ e.models])[2]].path)

    return GMC_NS_Progress{typeof(interval)}(e, top_m,
    # tuner,
     interval, dt=dt, desc=desc, color=color, output=output,
      ↪ offset=offset, start_it=start_it, upper_displays=upper_displays,
      ↪ lower_displays=lower_displays, disp_rot_its=disp_rot_its)
end


function update!(p::GMC_NS_Progress, val, thresh; options...)
    p.counter += 1

    p.tstp=time()-p.tlast
    popfirst!(p.time_history)
    push!(p.time_history, p.tstp)

    p.interval = val - thresh
    isinf(p.interval) && (p.interval = 0.)
    popfirst!(p.convergence_history)
    push!(p.convergence_history, p.interval)

    string(p.top_m.trajectory,'.',p.top_m.i) ≠
     ↪ basename(p.e.models[findmax([model.log_Li for model in
     ↪ p.e.models])[2]].path) &&
     ↪ (p.top_m=deserialize(p.e.models[findmax([model.log_Li for model
     ↪ in p.e.models])[2]].path))

    updateProgress!(p; options...)
end

```

```
function updateProgress!(p::GMC_NS_Progress; offset::Integer = p.offset, keep = (offset == 0))
    p.offset = offset
    t = time()
    p.display_rotate_iterates > 0 && p.counter%p.display_rotate_iterates == 0 && rotate_displays(p)

    if p.interval < 0 && !p.triggered
        p.triggered = true
        if p.printed
            p.triggered = true
            dur = durationstring(t-p.tfirst)
            msg = @sprintf "%s Converged. Time: %s (%d iterations). logZ: %s\n" p.desc dur p.counter p.e.log_Zi[end]

            print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
            move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
            length(p.upper_displays)>0 ? (upper_lines=display_dash(p, p.upper_displays[p.upperidx])) : (upper_lines=0)
            printover(p.output, msg, :magenta)
            length(p.lower_displays)>0 ? (lower_lines=display_dash(p, p.lower_displays[p.loweridx])) : (lower_lines=0)

            p.numprintedvalues=upper_lines + lower_lines + 1

            if keep
                println(p.output)
            else
                print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))
            end
        end
        return
    end

    if t > p.tlast+p.dt && !p.triggered
        p.counter < CONVERGENCE_MEMORY ? mean_step_time=mean(p.time_history[end-(p.counter-1):end]) : mean_step_time=mean(p.time_history)
```

```
        msg = @sprintf "%s Iterate: %s Recent step time μ: %s Convergence Interval: %g\n" p.desc p.counter hmss(mean_step_time) p.interval

        print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
        move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
        length(p.upper_displays)>0 ? (upper_lines=display_dash(p, p.upper_displays[p.upperidx])) : (upper_lines=0)
        printover(p.output, msg, p.color)
        length(p.lower_displays)>0 ? (lower_lines=display_dash(p, p.lower_displays[p.loweridx])) : (lower_lines=0)

        p.numprintedvalues=upper_lines + lower_lines + 1
        print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))

        # Compensate for any overhead of printing. This can be
        # especially important if you're running over a slow network
        # connection.
        p.tlast = t + 2*(time()-t)
        p.printed = true
    end
end
                function rotate_displays(p)
                    p.upperidx < length(p.upper_displays) ? p.upperidx+=1 : p.upperidx=1
                    p.loweridx < length(p.lower_displays) ? p.loweridx+=1 : p.loweridx=1
                end

                function hmss(dt)
                    dt<0 ? (dt=-dt; prfx="-") : (prfx="")
                    isnan(dt) && return "NaN"
                    (h,r) = divrem(dt,60*60)
                    (m,r) = divrem(r, 60)
                    (isnan(h)||isnan(m)||isnan(r)) && return "NaN"
                    string(prfx,Int(h),":",Int(m),":",Int(ceil(r)))
                end

                function display_dash(p::GMC_NS_Progress, funcvec::Vector{Function})
                    lines=0
                    for func in funcvec
                        lines+=func(p)
```

```
                    end
                    return lines
                end
```

17.3.19 /src/utilities/progress_displays.jl

```
function tuning_display(p)
    lines=show(p.output, p.tuner, progress=true)
    println();
    return lines
end

function convergence_display(p)
    try
        ciplot=lineplot([p.counter-(length(p.convergence_history)-1):p.counter ...
        p.convergence_history, title="Convergence Interval Recent History", xlabel="Iterate",ylabel="CI", color=:yellow)
        lines=nrows(ciplot.graphics)+5
        show(p.output, ciplot); println()
        return lines
    catch
        printstyled(p.output, "CONVERGENCE PLOT UNAVAILABLE. STANDBY\n", bold=true, color=:yellow); println()
        return 2
    end
end

function evidence_display(p)
    try
        log_Zis=p.e.log_Zi[2:end]
        posidx=findfirst(i→i > -Inf, log_Zis)
        log_Zis[1:posidx-1]•=log_Zis[posidx]
        evplot=lineplot(log_Zis, title="Evidence History", xlabel="Iterate", color=:red, name="Ensemble logZ")
        lines=nrows(evplot.graphics)+5
        show(p.output, evplot); println()
        return lines
    catch
        printstyled(p.output, "EVIDENCE PLOT UNAVAILABLE. STANDBY\n", bold=true, color=:red); println()
        return 2
```

```
    end
end

function info_display(p)
    try
        infoplot=lineplot(p.e.Hi[2:end], title="Information History", xlabel="Iterate", color=:green, name="Ensemble H")
        lines=nrows(infoplot.graphics)+5
        show(p.output, infoplot); println()
        return lines
    catch
        printstyled(p.output, "INFORMATION PLOT UNAVAILABLE. STANDBY\n", bold=true, color=:green); println()
        return 2
    end
end

function lh_display(p)
    try
        lhplot=lineplot(p.e.log_Li[2:end], title="Contour History", xlabel="Iterate", color=:magenta, name="Ensemble logLH")
        lines=nrows(lhplot.graphics)+5
        show(p.output, lhplot); println()
        return lines
    catch
        printstyled(p.output, "CONTOUR HISTORY UNAVAILABLE. STANDBY\n", bold=true, color=:magenta); println()
        return 2
    end
end

function liwi_display(p)
    try

        liwiplot=lineplot([max(2,p.counter-(CONVERGENCE_MEMORY-1)):p.counter ... ],
        title="Recent iterate evidentiary weight", xlabel="Iterate", name="Ensemble log Liwi", color=:cyan)
        lines=nrows(liwiplot.graphics)+5
        show(p.output, liwiplot); println()
        return lines
    catch
        printstyled(p.output, "EVIDENTIARY HISTORY UNAVAILABLE. STANDBY\n", bold=true, color=:cyan); println()
```

```
        return 2
    end
end

function ensemble_display(p)
    return lines=show(p.output, p.e, progress=true)
end

function model_display(p)
    try
        println("Current MAP model:")
        return lines=show(p.output, p.top_m, progress=true)
    catch
        printstyled(p.output, "MODEL DISPLAY UNAVAILABLE\n", bold=true, color=:blue); println()
        return 3
    end
end

function model_obs_display(p)
    try
        println("Current MAP model")
        return lines=show(p.output, p.top_m, p.e, progress=true)
    catch
        printstyled(p.output, "MODEL DISPLAY UNAVAILABLE\n", bold=true, color=:blue); println()
        return 3
    end
end
```

17.3.20 /src/utilities/stats.jl

```
function get_lognormal_params(desired_μ, desired_σ)
    μ²=desired_μ^2
    σ²=desired_σ^2
    ln_μ=log(μ²/sqrt(μ²+σ²))
    ln_σ=sqrt(log(1+(σ²/μ²)))
    return ln_μ, ln_σ
end

function marginals(d::NormalInverseGamma)
```

```
    m_T=MarginalTDist(d.mu,sqrt((d.shape*inv(d.v0))/d.scale),TDist(2*d.shape))
    m_Ga=InverseGamma(d.shape,d.scale)
    return m_T,m_Ga
end

function marginals(d::NormalGamma)
    m_T=MarginalTDist(d.mu,sqrt(d.rate/(d.shape*d.nu)),TDist(2*d.shape))
    m_Ga=Gamma(d.shape,1/d.rate)
    return m_T,m_Ga
end
```

17.3.21 /src/utilities/t_Tuner.jl

```
abstract type GMC_Tuner end

mutable struct τ_Tuner <: GMC_Tuner
    τ::Float64
    τ_history::Vector{Float64}
    α::Float64
    β::Float64
    a::BitVector
    memory::Integer

    function τ_Tuner(τ1, e::GMC_NS_Ensemble)
        new(τ1,
        zeros(0),
        e.GMC_tune_α,
        e.GMC_tune_β,
        falses(0),
        e.GMC_tune_μ)
    end
end

mutable struct τ_PID <: GMC_Tuner
    τ::Float64
    τ_history::Vector{Float64}
    α::Float64
    β::Float64

    a::BitVector
    a_history::BitVector
```

```
    ? :: Vector{Float64}
    memory :: Integer
    Kp :: Float64
    Ki :: Float64
    Kd :: Float64

    ϵ :: Float64
    pterm :: Float64
    iterm :: Float64
    dterm :: Float64

    function τ_PID(e :: GMC_NS_Ensemble)
        new(e.GMC_init_τ,
        zeros(0),
        e.GMC_tune_α,
        1.,
        falses(0),
        falses(0),
        Vector{Float64}(),
        e.GMC_tune_μ,
        e.GMC_tune_PID ... ,
        0.,0.,0.,0.)
    end
end

function process_report!(t :: τ_PID, report :: Bool)
    push!(t.a_history, report)
    length(t.a_history)>t.memory ? (t.a=t.a_history[end-t.memory:end]) :
     ↪  (t.a=t.a_history)
    push!(t.?,mean(t.a))
    τ_update!(t)
end

function τ_update!(t :: τ_PID)
    push!(t.τ_history,t.τ)

    t.ϵ=t.?[end] - t.α
    t.pterm=t.Kp * t.ϵ
    !(t.τ==eps()) && (t.iterm+=(t.Ki * t.ϵ))

```

```
    length(t.⍰) > Int64(round(.1*t.memory)) ? (d = t.⍰[end-Int64(round(.1*t.memory))] - t.⍰[end]) : #d is positive if acceptance is declining- in this case want smaller beta and tau
        d=0.
    t.dterm=t.Kd * d

    t.β=max(eps(), 1. + t.pterm + t.iterm - t.dterm)
    t.τ=max(eps(), t.τ * t.β)
end


function init_tune(e::GMC_NS_Ensemble)
    @info "Performing initial GMC timestep tuning on ensemble..."
    τ=1.; tuned=false; τd=1.1; accept=1.; it=1
    while !tuned && it<e.GMC_tune_ι
        test_report=falses(0)
        println(stdout, "τ: $τ, It: $it, Accept: $accept")

        while length(test_report)<Int64(floor(length(e.models)*e.GMC_tune_ρ))
            m=deserialize(rand([m.path for m in e.models]))
            candidate=galilean_trajectory_sample!(m,e,τ)

            candidate.id==m.id ? push!(test_report,0) : push!(test_report,1)
            candidate=0
            GC.gc()
        end

        last_accept=accept
        accept=sum(test_report)/length(test_report)

        if accept == 1.
            τ*=τd
            last_accept == 1. && (τd*=1+e.GMC_tune_β)
            last_accept < e.GMC_tune_α && (τd=min(1.000001,1-e.GMC_tune_β*τd))
        elseif accept < e.GMC_tune_α
            τ/=τd
            last_accept == 1. && (τd=min(1.000001,1-e.GMC_tune_β*τd))
            last_accept < e.GMC_tune_α && (τd*=1+e.GMC_tune_β)
        else
```

```
            return τ
        end

        it+=1
        move_cursor_up_while_clearing_lines(stdout, 1)
    end
    println("Done. τ=$τ")
    return τ
end

function tune_τ!(t::τ_Tuner, step_report::BitVector)
    push!(t.τ_history,t.τ)
    t.a=vcat(t.a,step_report)
    ls=length(t.a)
    ls>t.memory && (t.a=t.a[ls-t.memory:end])

    accept=sum(t.a)/ls
    if accept==1.
        t.τ=t.τ*(1+t.β)
    elseif accept<t.α
        t.τ=t.τ*(1-t.β)
    end
end

function Base.show(io::IO, t::τ_PID; progress=false)
    try
        plot=lineplot(t.?, title="Recent Unobstructed Moves",
        ↪  xlabel="Proposals", ylabel="Rate", color=:white, name="Free
        ↪  moves")

        lbls=plot.labels_left
        (arrow="")
        all(t.α .> [v for v in parse.(Float64,values(lbls))]) &&
        ↪  (arrow="↑")
        all(t.α .< [v for v in parse.(Float64,values(lbls))]) &&
        ↪  (arrow="↓")

        lineplot!(plot, [t.α for i in 1:length(t.?)], name="α setpoint
        ↪  "*arrow, color=:red)
        show(io, plot)
        println()
        printstyled("τ:$(t.τ), α:$(t.α), β:$(t.β)",bold=true)
        println()
```

```
        printstyled("Kp:$(t.Kp), Ki:$(t.Ki), Kd:$(t.Kd)",color=:green)
        println()
        printstyled("ptrm:$(t.pterm), itrm:$(t.iterm),
         ↪  dtrm:$(t.dterm)",color=:magenta)
        println()


        progress && return nrows(plot.graphics)+9
    catch
        printstyled("τ_PID NOT AVAILABLE. STANDBY", bold=true)
        progress && return 1
    end
end

function get_tuner_dict(e)
    d=Dict{Int64,τ_PID}()
    for rec in e.models
        d[rec.trajectory]=τ_PID(e)
    end
    return d
end
```

17.3.22 /test/eggbox_test.jl

```
using GMC_NS, Distributions

prior=Uniform(0,1)
priors=[prior,prior]

box=[0. 1.
     0. 1.]

gmc=GMC_DEFAULTS
gmc[1]=180
gmc[2]=eps()

e=Eggbox_Ensemble("Eggbox_test", 1000, priors, box, gmc ... )

uds=Vector{Vector{Function}}([[evidence_display],[liwi_display],[convergence_display]
lds=Vector{Vector{Function}}([[ensemble_display]])

```

```
converge_ensemble!(e,backup=(true,100),upper_displays=uds,lower_displays=lds,
↪ disp_rot_its=1000, converge_factor=1e-6)
```

17.3.23 /test/ensemble_tests.jl

```
@testset "GMC_NS_Ensemble tests ... " begin
    sample_dist=Normal(100.,5.)
    samples=rand(sample_dist, 10000)

    n_chains=10

    max_μ=1000
    min_μ=eps()
    max_λ=100
    min_λ=1e-4

    ln_max_μ=log(max_μ^2/sqrt(max_μ^2 + inv(max_λ)))
    ln_min_μ=log(min_μ^2/sqrt(min_μ^2 + inv(min_λ)))
    ln_max_λ=1/log(1+(inv(max_λ)/max_μ^2))
    ln_min_λ=1/log(1+(inv(min_λ)/min_μ^2))

    n_priors=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
    n_box=[min_μ max_μ;min_λ max_λ]
    ln_priors=[Uniform(ln_min_μ,ln_max_μ),Uniform(ln_min_λ,ln_max_λ)]
    ln_box=[ln_min_μ ln_max_μ;ln_min_λ ln_max_λ]

    gmc=GMC_DEFAULTS
    gmc[1]=3
    gmc[2]=1e-5
    gmc[end]=1e4

    ne=Normal_Ensemble("NE_test", n_chains, samples, n_priors, n_box,
    ↪ gmc ... )
    lne=LogNormal_Ensemble("LNE_test", n_chains, samples, ln_priors,
    ↪ ln_box, gmc ... )

    uds=Vector{Vector{Function}}()

    lds=Vector{Vector{Function}}()

    ne_ev = converge_ensemble!(ne,backup=(true,100),upper_displays=uds,
    ↪ lower_displays=lds)
```

```
    lne_ev = converge_ensemble!(lne,backup=(true,100),upper_displays=uds,
    ↪ lower_displays=lds)

    @test ne_ev > lne_ev

    for e in [ne, lne]
        @info "Testing detailed balance at $(e.path)..."

        @test issorted(e.log_Li)
        @test all(e.log_Li.<0.)
        @test issorted(e.log_Xi,rev=true)
        @test all(e.log_Xi.≤0.)
        @test all(e.log_wi.<0.)
        @test all(e.log_Liwi.<0.)
        @test all(e.log_Zi.<0.)

        Nvec=Vector{Int64}()
        for i in 2:length(e.log_Li)
            Xi=e.log_Xi[i]
            N=round(-(i-1)/Xi)
            push!(Nvec, N)
        end

        @test all(e.GMC_Nmin.≤Nvec.≤n_chains)

        for chain in 1:e.t_counter-1
            ch_pfx=e.path*'/'*string(chain)*'.'
            total_spls=0
            while isfile(ch_pfx*string(total_spls+1))
                total_spls+=1
            end

            lastm=deserialize(ch_pfx*"1")

            for spl in 2:total_spls
                splm=deserialize(ch_pfx*string(spl))

                @test !(splm.θ!=lastm.θ && splm.v!=lastm.v)
                @test !(splm.θ==lastm.θ && splm.v==lastm.v)
                if splm.θ==lastm.θ
                    @test splm.v!=lastm.v
                    @test splm.log_Li==lastm.log_Li
                elseif splm.v==lastm.v
```

```
                    @test splm.θ!=lastm.θ
                    @test splm.log_Li ≥ lastm.log_Li
                else
                    @error "Galilean trajectory generation is broken!"
                end
                lastm=splm
            end
        end
    end

    rm("NE_test", recursive=true)
    rm("LNE_test", recursive=true)
end

```

17.3.24 /test/galilean_unit_tests.jl

```
@testset "Galilean trajectory unit tests ..." begin
    #test box movement
    pos=[.5,.5]
    box=[0 1.
         0 1.]
    allow_d=[.1,.1]
    box_d=[1.,1.]

    allowed_pos, allowed_d=box_move(pos,allow_d,box)
    @test allowed_pos==pos+allow_d
    @test allowed_d == allow_d

    boxed_pos, boxed_d=box_move(pos, box_d, box)
    @test boxed_pos==[1.,1.]
    @test boxed_d==[.5,.5]

    box_reflect_v=box_reflect([.5,.99999999],box,[.1,.1],0.)
    @test box_reflect_v==[.1,-.1]

    #test boundary norm finding
    norm=1000
    v=[.5,.5]
    bn=boundary_norm(v, norm)
    @test bn == normalize(norm./v)

```

```
    #test reflection given norm
    rv=reflect(v,bn,0.)
    @test isapprox(rv, -v)

    #test reflection perturbaton
    pv=r_perturb(v,.01)
    @test v!=pv
end
```

17.3.25 /test/normal_demo.jl

```
using GMC_NS, Distributions, ConjugatePriors

sample_dist=Normal(100.,5.)
samples=rand(sample_dist, 10000)

prior=NormalGamma(300.,2e-3,1.,10.) #a lousy prior

box=[0. 1000.
     1/1000. 1/eps()]

gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=eps()

e=Normal_Ensemble("NE_test", 10, samples, prior, box, gmc ... )

uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]

lds=Vector{Vector{Function}}([[model_obs_display]])

converge_ensemble!(e,backup=(true,100),upper_displays=uds,
↪  lower_displays=lds, disp_rot_its=1000)
```

17.3.26 /test/runtests.jl

```
@info "Loading test packages ... "
using GMC_NS, Test, Distributions, Serialization
import GMC_NS:box_move, box_reflect, boundary_norm, reflect, r_perturb
import LinearAlgebra:normalize

```

```
@info "Beginning tests ... "
using Random
Random.seed!()

include("galilean_unit_tests.jl")
include("ensemble_tests.jl")

@info "Tests complete!"
```

17.4 CMZNicheSims

Github repository: https://github.com/mmattocks/CMZNicheSims.jl

17.4.1 /src/CMZNicheSims.jl

```
module CMZNicheSims
    using Distributions, GMC_NS, UnicodePlots, StatsPlots
    import ProgressMeter:
     ↪  @showprogress,move_cursor_up_while_clearing_lines
    import BioBackgroundModels:lps
    import StatsFuns:logsumexp
    import Serialization:serialize,deserialize
    import KernelDensity:kde,AbstractKDE

    const MAXVAL=prevfloat(Inf) #don't permit infinite CMZ populations or
     ↪  retinal volumes

    include("thymidine_sim/thymidine_cell.jl")
    include("thymidine_sim/thymidine_model.jl")
    include("thymidine_sim/thymidine_ensemble.jl")
    export Thymidine_Ensemble
    include("thymidine_sim/thymidine_sim.jl")
    include("thymidine_sim/thymidine_lh.jl")
    include("CMZ_sim/CMZ_lh.jl")
    include("CMZ_sim/CMZ_model.jl")
    include("CMZ_sim/CMZ_ensemble.jl")
    export CMZ_Ensemble
    include("slice_sim/lens_model.jl")
    export Lens_Model
    include("slice_sim/slice_lh.jl")
    include("slice_sim/slice_model.jl")
    include("slice_sim/slice_ensemble.jl")
```

```
    export Slice_Ensemble
    include("cycle_decay_sim/decay_lh.jl")
    include("cycle_decay_sim/decay_constructor.jl")
    include("cycle_decay_sim/decay_ensemble.jl")
    export Decay_Ensemble
    include("multislice_sim/multislice_model.jl")
    include("multislice_sim/multislice_ensemble.jl")
    export MultiSlice_Ensemble, MultiSlice_Decay_Ensemble
end # module
```

17.4.2 /src/CMZ_sim/CMZ_ensemble.jl

```
mutable struct CMZ_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{CMZ_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}


    ↪  obs::AbstractVector{<:Tuple{<:AbstractVector{<:Float64},<:AbstractVector{<:Fl
    priors::Vector{<:Distribution}
    constants::Vector{<:Any}
    #T, popdist, voldist, mc_its, phs
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{CMZ_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
```

```
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

CMZ_Ensemble(path::String, no_models::Integer, obs::AbstractVector{<:Tuple{<:AbstractVector{<:Float64},<:AbstractVector{<:Float6
priors::AbstractVector{<:Distribution}, constants, box, GMC_settings; sample_posterior::Bool=true) =
CMZ_Ensemble(
    path,
    construct_CMZ,
    assemble_CMs(path, no_models, obs, priors, constants, box)...,
    [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
    obs,
    priors,
    constants, #T, popdist, voldist, mc_its, phs
    box,
    sample_posterior,
    Vector{CMZ_Record}(),
    GMC_settings...,
    no_models+1)

function assemble_CMs(path::String, no_trajectories::Integer, obs, priors, constants, box)
    ensemble_records = Vector{CMZ_Record}()
    !isdir(path) && mkpath(path)
    phs=constants[6]
    @showprogress 1 "Assembling CMZ_Model ensemble..." for trajectory_no in 1:no_trajectories
        model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
```

```
                proposal[2*phs+1:(2*phs+1)+(phs-2)]=sort(proposal[2*phs+1:(2*phs+1)+(

            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)

            model = construct_CMZ(trajectory_no, 1, θvec, pos, [0.], obs, constants ... ; v_init=true)

            serialize(model_path, model) #save the model to the ensemble directory
            push!(ensemble_records, CMZ_Record(trajectory_no, 1, pos,model_path,model.log_Li))
        else #interrupted assembly pick up from where we left off
            model = deserialize(model_path)
            push!(ensemble_records, CMZ_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
        end
    end

    return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end


function Base.show(io::IO, m::CMZ_Model, e::CMZ_Ensemble; progress=false)
    T=e.constants[1]

    catpobs=vcat([e.obs[t][1] for t in 1:length(T)] ... )
    ymax=max(maximum(m.disp_mat[:,:,1]),maximum(catpobs))
    ymin=min(minimum(m.disp_mat[:,:,1]),minimum(catpobs))

    plt=lineplot(T,m.disp_mat[:,2,1],title="CMZ_Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)",color=:green,name="μ pop", ylim=[ymin,ymax])
    lineplot!(plt,T,m.disp_mat[:,1,1],color=:magenta,name="95% CI")
    lineplot!(plt,T,m.disp_mat[:,3,1],color=:magenta)

    Ts=vcat([[T[n] for i in 1:length(e.obs[n][1])] for n in 1:length(T)] ... )
    scatterplot!(plt,Ts, catpobs, color=:yellow, name="Obs")

```

```
    show(io, plt)
    println()

    catvobs=vcat([e.obs[t][2] for t in 1:length(T)]...)
    ymax=max(maximum(m.disp_mat[:,:,1]),maximum(catvobs))
    ymin=min(minimum(m.disp_mat[:,:,1]),minimum(catvobs))

    plt=lineplot(T,m.disp_mat[:,2,2],color=:blue,name="μ vol",
    ↪  ylim=[ymin,ymax])
    lineplot!(plt,T,m.disp_mat[:,1,2],color=:yellow,name="95% CI")
    lineplot!(plt,T,m.disp_mat[:,3,2],color=:yellow)

    Ts=vcat([[T[n] for i in 1:length(e.obs[n][2])] for n in
    ↪  1:length(T)]...)
    scatterplot!(plt,Ts, catvobs, color=:yellow, name="Obs")

    show(io, plt)

    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")

    (progress && return nrows(plt.graphics)+26);
end
```

17.4.3 /src/CMZ_sim/CMZ_lh.jl

```
function CMZ_mc_llh(popdist::Distribution, voldist::Distribution,
↪  volconst::Float64, phase_ends::Vector{Float64},
↪  pparams::Vector{Float64}, mc_its::Int64, T::Vector{<:AbstractFloat},
↪  obs::Vector{<:Tuple{<:AbstractVector{<:Float64},<:AbstractVector{<:Float64}}})
    times=length(T)
    pends=floor.(phase_ends)
    events=Int64.(sort(unique(vcat(T,pends))))
    results=zeros(mc_its,length(events),2)

    nt=Threads.nthreads()
    Threads.@threads for t in 1:nt
        t==nt ? (idxs=1+(t-1)*Int(floor(mc_its/nt)):mc_its) :
        ↪  (idxs=(1+(t-1)*Int(floor(mc_its/nt))):t*Int(floor(mc_its/nt)))

        phase=1
```

```
        cycle_time,exit_rate=pparams[1+((phase-1)*2):2+((phase-1)*2)]

        results[idxs,1,1]•=rand.(popdist)
        results[idxs,1,2]•=rand.(voldist)

        last_ph_ch=1;
        next_event=2;
        while next_event ≤ length(events)
            n=events[next_event]-events[last_ph_ch]

            pop_factor=max(eps(),(2^(24/cycle_time)-exit_rate))
            pop_factor==1. && (pop_factor=pop_factor+eps())

            vol_factor=volconst*exit_rate*(1-pop_factor^(n-1))/(1-pop_factor)

            lastpops=view(results,idxs,last_ph_ch,1)
            lastvols=view(results,idxs,last_ph_ch,2)


            results[idxs,next_event,1]•=max.(min.(lastpops.*pop_factor^n,MAXVAL),

            results[idxs,next_event,2]•=min.(lastvols.+(lastpops.*vol_factor),MAX

            if events[next_event] in pends #after updating pop and vol to t, update phase parameters for next event, make t₀ vol₀ and pop₀ the phase change vals
                phase+=1; last_ph_ch=next_event
                phase ≤ length(pparams)/2 && ((cycle_time,exit_rate)=pparams[1+((phase-1)*2):2+((phase-1)*2)])
            end #advance phase if necessary

            next_event+=1
        end
    end

    tidxs=[findfirst(t→t==time,events) for time in T]

    pop_lns=Vector{LogNormal}(undef,times)
    vol_lns=Vector{LogNormal}(undef,times)
    pop_lns[1]=popdist
    vol_lns[1]=voldist

    Threads.@threads for t in 2:times
```

```
            pop_lns[t]=fit(LogNormal,results[:,tidxs[t],1])
            vol_lns[t]=fit(LogNormal,results[:,tidxs[t],2])
    end

    pop_lhs=Vector{Float64}(undef,times-1)
    vol_lhs=Vector{Float64}(undef,times-1)

    Threads.@threads for t in 1:times-1
        pop_lhs[t]=lps(logpdf(pop_lns[t+1],obs[t+1][1]))
        vol_lhs[t]=lps(logpdf(vol_lns[t+1],obs[t+1][2]))
    end

    log_lh=lps(lps(pop_lhs),lps(vol_lhs))

    disp_mat=zeros(times,3,2)
    Threads.@threads for t in 1:times

        ↪ disp_mat[t,:,1]=[quantile(pop_lns[t],.025),mean(pop_lns[t]),quantile(pop_

        ↪ disp_mat[t,:,2]=[quantile(vol_lns[t],.025),mean(vol_lns[t]),quantile(vol_
    end

    return log_lh, disp_mat
end
```

17.4.4 /src/CMZ_sim/CMZ_model.jl

```
mutable struct CMZ_Record <: GMC_NS_Model_Record
    trajectory::Int64
    i::Int64
    pos::Vector{Float64}
    path::String
    log_Li::Float64
end

struct CMZ_Model <: GMC_NS_Model
    trajectory::Int64
    i::Int64

    θ::Vector{Float64}
    log_Li::Float64

```

```
    pos::Vector{Float64}
    v::Vector{Float64}

    disp_mat::Array{Float64} #matrix for mean & 95%CI plot of model output
end

function construct_CMZ(trajectory::Int64, i::Int64, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Tuple{Vector{Float64},Vector{Float64}}}, T::Vector{Float64}, popdist::Distribution, voldist::Distribution, volconst::Float64, mc_its::Int64, phs::Int64; v_init=false)
    pparams=θ[1:2*phs];phase_ends=θ[2*phs+1:(2*phs+1)+(phs-2)]

    log_lh,disp_mat=CMZ_mc_llh(popdist, voldist, volconst, phase_ends, pparams, mc_its, T, obs)

    v_init && (v=rand(MvNormal(length(θ),1.)))

    CMZ_Model(trajectory, i, θ, log_lh, pos, v, disp_mat)
end
```

17.4.5 /src/cycle_decay_sim/decay_constructor.jl

```
function construct_decay_slice(trajectory::Int64, i::Int64, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Vector{Float64}}, T::Vector{Float64}, popdist::Distribution, lens_model::Lens_Model, mc_its::Int64; v_init=false)
    log_lh,disp_mat=decay_slice_mc_llh(popdist, lens_model, θ, mc_its, T, obs)

    v_init && (v=rand(MvNormal(length(θ),1.)))

    return Slice_Model(trajectory, i, θ, log_lh, pos, v, disp_mat)
end
```

17.4.6 /src/cycle_decay_sim/decay_ensemble.jl

```
Decay_Ensemble(path::String, no_models::Integer, obs::AbstractVector{<:AbstractVector{<:Float64}}, priors::AbstractVector{<:Distribution}, constants, box, GMC_settings; sample_posterior::Bool=true) =
Slice_Ensemble(
    path,
    construct_decay_slice,
    assemble_DMs(path, no_models, obs, priors, constants, box)...,
    [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
    obs,
    priors,
    constants, #T, popdist, lens_model, mc_its
    box,
    sample_posterior,
    Vector{Slice_Record}(),
    GMC_settings...,
    no_models+1)

function assemble_DMs(path::String, no_trajectories::Integer, obs, priors, constants, box)
    ensemble_records = Vector{Slice_Record}()
    !isdir(path) && mkpath(path)
    @showprogress 1 "Assembling Slice_Model ensemble..." for trajectory_no in 1:no_trajectories
        model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)

            model = construct_decay_slice(trajectory_no, 1, θvec, pos, [0.], obs, constants...; v_init=true)

            serialize(model_path, model) #save the model to the ensemble directory
```

```
            push!(ensemble_records, Slice_Record(trajectory_no, 1, pos,model_path,model.log_Li))
        else #interrupted assembly pick up from where we left off
            model = deserialize(model_path)
            push!(ensemble_records, Slice_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
        end
    end

    return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end
```

17.4.7 /src/cycle_decay_sim/decay_lh.jl

```
function decay_slice_mc_llh(popdist::Distribution, lm::Lens_Model, dparams::Vector{Float64}, mc_its::Int64, T::Vector{Float64}, obs::Vector{Vector{Float64}})
    times=length(T)
    events=Int64.(sort(T))
    results=zeros(mc_its,length(events))

    nt=Threads.nthreads()

    init_day=events[1]
    init_ct, decay_const, exit_rate=dparams
    cts=[init_ct*exp(decay_const*elapsed) for elapsed in collect(events[1]:1:events[end])•-init_day]

    Threads.@threads for t in 1:nt
        t==nt ? (idxs=1+(t-1)*Int(floor(mc_its/nt)):mc_its) : (idxs=(1+(t-1)*Int(floor(mc_its/nt))):t*Int(floor(mc_its/nt)))

        results[idxs,1]•=rand.(popdist)

        day=init_day; next_event=2
        curr_pops=copy(results[idxs,1,1])
        while day ≤ events[end]
            days_elapsed = day - init_day
            cycle_time=cts[day-init_day+1]
            pop_factor=max(eps(),(2^(24/cycle_time)-exit_rate))
            circexit=circumferential_exit(lm,day-1,day,curr_pops)
```

```
24
25                 ↪  curr_pops=max.(min.(curr_pops.*pop_factor,MAXVAL)•-circexit,eps())
26
27             day in events[2:end] && (
28                 results[idxs,next_event]•=curr_pops;
29                 next_event+=1)
30             day+=1
31         end
32     end
33
34     tidxs=[findfirst(t→t==time,events) for time in T]
35
36     pop_lns=Vector{LogNormal}(undef,times)
37     pop_lns[1]=popdist
38     Threads.@threads for t in 2:times
39             pop_lns[t]=fit(LogNormal,results[:,tidxs[t]])
40     end
41
42     pop_lhs=Vector{Float64}(undef,times-1)
43
44     Threads.@threads for t in 1:times-1
45         pop_lhs[t]=lps(logpdf(pop_lns[t+1],obs[t+1]))
46     end
47
48     log_lh=lps(pop_lhs)
49
50     disp_mat=zeros(times,3)
51     Threads.@threads for t in 1:times
52
           ↪  disp_mat[t,:]=[quantile(pop_lns[t],.025),mean(pop_lns[t]),quantile(pop_ln
53     end
54
55     return log_lh, disp_mat
56 end
```

17.4.8 /src/multislice_sim/multislice_ensemble.jl

```
1 mutable struct MultiSlice_Ensemble <: GMC_NS_Ensemble
2     path::String
3
4     model_initλ::Function
```

```
    models::Vector{Slice_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::AbstractVector{<:AbstractVector{<:AbstractVector{<:Float64}}}
    priors::Vector{<:Distribution}
    constants::Vector{<:Any}
    #T, popdist, lens_model, mc_its, phs
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Slice_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

MultiSlice_Ensemble(path::String, no_models::Integer,
    obs::AbstractVector{<:AbstractVector{<:AbstractVector{<:Float64}}},
    priors::AbstractVector{<:Distribution}, constants, box, GMC_settings;
    sample_posterior::Bool=true) =
MultiSlice_Ensemble(
    path,
    construct_multislice,
    assemble_MSMs(path, no_models, obs, priors, constants, box)...,
```

```
    [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
    obs,
    priors,
    constants, #T, popdist, lens_model, mc_its, phs
    box,
    sample_posterior,
    Vector{Slice_Record}(),
    GMC_settings ... ,
    no_models+1)

MultiSlice_Decay_Ensemble(path::String, no_models::Integer,
↪   obs::AbstractVector{<:AbstractVector{<:AbstractVector{<:Float64}}},
↪   priors::AbstractVector{<:Distribution}, constants, box, GMC_settings;
↪   sample_posterior::Bool=true) =
MultiSlice_Ensemble(
        path,
        construct_decay_multislice,
        assemble_MSDMs(path, no_models, obs, priors, constants, box) ... ,
        [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
        obs,
        priors,
        constants, #T, popdist, lens_model, mc_its
        box,
        sample_posterior,
        Vector{Slice_Record}(),
        GMC_settings ... ,
        no_models+1)

function assemble_MSMs(path::String, no_trajectories::Integer, obs,
↪   priors, constants, box)
    ensemble_records = Vector{Slice_Record}()
    !isdir(path) && mkpath(path)
    phs=constants[5]
```

```
    @showprogress 1 "Assembling Slice_Model ensemble ..." for trajectory_no in 1:no_trajectories
        model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
            proposal[2*phs+1:(2*phs+1)+(phs-2)]=sort(proposal[2*phs+1:(2*phs+1)+(
            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)
            model = construct_multislice(trajectory_no, 1, θvec, pos, [0.], obs, constants ... ; v_init=true)
            serialize(model_path, model) #save the model to the ensemble directory
            push!(ensemble_records, Slice_Record(trajectory_no, 1, pos,model_path,model.log_Li))
        else #interrupted assembly pick up from where we left off
            model = deserialize(model_path)
            push!(ensemble_records, Slice_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
        end
    end

    return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end

function assemble_MSDMs(path::String, no_trajectories::Integer, obs, priors, constants, box)
    ensemble_records = Vector{Slice_Record}()
    !isdir(path) && mkpath(path)
    @showprogress 1 "Assembling Slice_Model ensemble ..." for trajectory_no in 1:no_trajectories
        model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)

```

```
            model = construct_decay_multislice(trajectory_no, 1, θvec, pos, [0.], obs, constants... ; v_init=true)

            serialize(model_path, model) #save the model to the ensemble directory
            push!(ensemble_records, Slice_Record(trajectory_no, 1, pos,model_path,model.log_Li))
        else #interrupted assembly pick up from where we left off
            model = deserialize(model_path)
            push!(ensemble_records, Slice_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
        end
    end

    return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end

function Base.show(io::IO, m::MultiSlice_Model, e::MultiSlice_Ensemble; progress=false)
    T=e.constants[1]

    println("MultiSlice_Model $(m.trajectory).$(m.i)")

    rows=0
    for (n,(obs, slice)) in enumerate(zip(e.obs,m.slices))
        catpobs=vcat([obs[t] for t in 1:length(T)]... )
        ymax=max(maximum(slice.disp_mat[:,:]),maximum(catpobs))
        ymin=min(minimum(slice.disp_mat[:,:]),minimum(catpobs))

        plt=lineplot(T,slice.disp_mat[:,2],title="Slice $n",color=:green,name="μ pop", ylim=[ymin,ymax])
        lineplot!(plt,T,slice.disp_mat[:,1],color=:magenta,name="95% CI")
        lineplot!(plt,T,slice.disp_mat[:,3],color=:magenta)

        Ts=vcat([[T[n] for i in 1:length(obs[n])] for n in 1:length(T)]... )
        scatterplot!(plt,Ts, catpobs, color=:yellow, name="Obs")

        rows+=nrows(plt.graphics)
        show(io, plt)
        println()
    end
```

```julia

    println()
    println("log_Li: $(m.log_Li)")
    println("θ: $(m.θ)")
    println("v: $(m.v)")

    progress ? (return rows+14) : (return)
end
```

17.4.9 /src/multislice_sim/multislice_model.jl

```julia
struct MultiSlice_Model <: GMC_NS_Model
    trajectory::Int64
    i::Int64

    θ::Vector{Float64}
    log_Li::Float64

    pos::Vector{Float64}
    v::Vector{Float64}

    slices::Vector{Slice_Model}
end

function construct_multislice(trajectory::Int64, i::Int64, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Vector{Vector{Float64}}}, T::Vector{Float64}, popdists::Vector{<:Distribution}, lens_model::Lens_Model, mc_its::Int64, phs::Int64; v_init=false)
    pparams=θ[1:2*phs];phase_ends=θ[2*phs+1:(2*phs+1)+(phs-2)]

    log_lh=0.

    slices=Vector{Slice_Model}()
    for (popdist,pobs) in zip(popdists, obs)
        llh,disp_mat=slice_mc_llh(popdist, lens_model, phase_ends, pparams, mc_its, T, pobs)
        log_lh+=llh

        push!(slices,Slice_Model(trajectory, i, θ, log_lh, pos, v, disp_mat))
    end
```

```julia

    v_init && (v=rand(MvNormal(length(θ),1.)))

    return MultiSlice_Model(trajectory, i, θ, log_lh, pos, v, slices)
end

function construct_decay_multislice(trajectory::Int64, i::Int64, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Vector{Vector{Float64}}}, T::Vector{Float64}, popdists::Vector{<:Distribution}, lens_model::Lens_Model, mc_its::Int64; v_init=false)
    log_lh=0.

    slices=Vector{Slice_Model}()
    for (popdist,pobs) in zip(popdists, obs)
        llh,disp_mat=decay_slice_mc_llh(popdist, lens_model, θ, mc_its, T, pobs)
        log_lh+=llh

        push!(slices,Slice_Model(trajectory, i, θ, log_lh, pos, v, disp_mat))
    end

    v_init && (v=rand(MvNormal(length(θ),1.)))

    return MultiSlice_Model(trajectory, i, θ, log_lh, pos, v, slices)
end
```

17.4.10 /src/slice_sim/lens_model.jl

```julia
struct Lens_Model
    factor::Float64
    power::Float64
    section::Float64
end

function circumferential_exit(lm,t1,t2,pops)
    t1c=lm.factor*(t1^lm.power) #lens circumference at time 1, according to lens model
    t2c=lm.factor*(t2^lm.power) #lens circumference at time 2
    nc=t2c-t1c #new circumference grown btw t1 and t2

```

```julia

    sliceno=t1c/lm.section #number of slices implied by lm's section size for t1 lens diameter
    psg=nc/sliceno #amount of circumferential growth each CMZ slice is responsible for (per slice growth)
    return (psg/lm.section).*pops #per slice growth divided by slice section thickness supplies the implied number of RPCs leaving CMZ to supply growing retina
end

function Base.length(lm::Lens_Model)
    return 1
end
```

17.4.11 /src/slice_sim/slice_ensemble.jl

```julia
mutable struct Slice_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{Slice_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::AbstractVector{<:AbstractVector{<:Float64}}
    priors::Vector{<:Distribution}
    constants::Vector{<:Any}
    #T, popdist, lens_model, mc_its, phs
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Slice_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
```

```
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

Slice_Ensemble(path::String, no_models::Integer, obs::AbstractVector{<:AbstractVector{<:Float64}}, priors::AbstractVector{<:Distribution}, constants, box, GMC_settings; sample_posterior::Bool=true) =
Slice_Ensemble(
    path,
    construct_slice,
    assemble_SMs(path, no_models, obs, priors, constants, box)...,
    [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
    obs,
    priors,
    constants, #T, popdist, lens_model, mc_its, phs
    box,
    sample_posterior,
    Vector{Slice_Record}(),
    GMC_settings...,
    no_models+1)

function assemble_SMs(path::String, no_trajectories::Integer, obs, priors, constants, box)
    ensemble_records = Vector{Slice_Record}()
    !isdir(path) && mkpath(path)
    phs=constants[5]
    @showprogress 1 "Assembling Slice_Model ensemble..." for trajectory_no in 1:no_trajectories
```

```
        model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
            proposal[2*phs+1:(2*phs+1)+(phs-2)]=sort(proposal[2*phs+1:(2*phs+1)+(

            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)

            model = construct_slice(trajectory_no, 1, θvec, pos, [0.], obs, constants... ; v_init=true)

            serialize(model_path, model) #save the model to the ensemble directory
            push!(ensemble_records, Slice_Record(trajectory_no, 1, pos,model_path,model.log_Li))
        else #interrupted assembly pick up from where we left off
            model = deserialize(model_path)
            push!(ensemble_records, Slice_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
        end
    end

    return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end


function Base.show(io::IO, m::Slice_Model, e::Slice_Ensemble; progress=false)
    T=e.constants[1]

    catpobs=vcat([e.obs[t] for t in 1:length(T)]...)
    ymax=max(maximum(m.disp_mat[:,:]),maximum(catpobs))
    ymin=min(minimum(m.disp_mat[:,:]),minimum(catpobs))

    plt=lineplot(T,m.disp_mat[:,2],title="Slice_Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)",color=:green,name="μ pop", ylim=[ymin,ymax])
    lineplot!(plt,T,m.disp_mat[:,1],color=:magenta,name="95% CI")
    lineplot!(plt,T,m.disp_mat[:,3],color=:magenta)

```

```
    Ts=vcat([[T[n] for i in 1:length(e.obs[n])] for n in 1:length(T)]...)
    scatterplot!(plt,Ts, catpobs, color=:yellow, name="Obs")

    show(io, plt)
    println()
    println("θ: $(m.θ)")
    println("v: $(m.v)")

    (progress && return nrows(plt.graphics)+16);
end
```

17.4.12 /src/slice_sim/slice_lh.jl

```
function slice_mc_llh(popdist::Distribution, lm::Lens_Model,
↪   phase_ends::Vector{Float64}, pparams::Vector{Float64}, mc_its::Int64,
↪   T::Vector{Float64}, obs::Vector{Vector{Float64}})
    times=length(T)
    pends=floor.(phase_ends)
    events=Int64.(sort(unique(vcat(T,pends))))
    results=zeros(mc_its,length(events))

    nt=Threads.nthreads()
    Threads.@threads for t in 1:nt
        t==nt ? (idxs=1+(t-1)*Int(floor(mc_its/nt)):mc_its) :
        ↪  (idxs=(1+(t-1)*Int(floor(mc_its/nt))):t*Int(floor(mc_its/nt)))

        phase=1
        cycle_time,exit_rate=pparams[1+((phase-1)*2):2+((phase-1)*2)]

        results[idxs,1]⋅=rand.(popdist)

        next_event=2;
        while next_event≤length(events)
            n=events[next_event]-events[next_event-1]

            pop_factor=max(eps(),(2^(24/cycle_time)-exit_rate))

            lastpops=view(results,idxs,next_event-1)

            results[idxs,next_event]⋅=min.(lastpops.*pop_factor^n,MAXVAL)

```

```
            results[idxs,next_event]•=max.(view(results,idxs,next_event)•-circumf

            if events[next_event] in pends #after updating pop and vol to t, update phase parameters for next event, make t₀ vol₀ and pop₀ the phase change vals
                phase+=1;
                phase ≤ length(pparams)/2 && ((cycle_time,exit_rate)=pparams[1+((phase-1)*2):2+((phase-1)*2)])
            end #advance phase if necessary

            next_event+=1
        end
    end

    tidxs=[findfirst(t→t==time,events) for time in T]

    pop_lns=Vector{LogNormal}(undef,times)
    pop_lns[1]=popdist
    Threads.@threads for t in 2:times
            pop_lns[t]=fit(LogNormal,results[:,tidxs[t]])
    end

    pop_lhs=Vector{Float64}(undef,times-1)

    Threads.@threads for t in 1:times-1
        pop_lhs[t]=lps(logpdf(pop_lns[t+1],obs[t+1]))
    end

    log_lh=lps(pop_lhs)

    disp_mat=zeros(times,3)
    Threads.@threads for t in 1:times

        disp_mat[t,:]=[quantile(pop_lns[t],.025),mean(pop_lns[t]),quantile(pop_ln
    end

    return log_lh, disp_mat
end
```

17.4.13 /src/slice_sim/slice_model.jl

```
struct Slice_Record <: GMC_NS_Model_Record
    trajectory::Int64
    i::Int64
    pos::Vector{Float64}
    path::String
    log_Li::Float64
end

struct Slice_Model <: GMC_NS_Model
    trajectory::Int64
    i::Int64

    θ::Vector{Float64}
    log_Li::Float64

    pos::Vector{Float64}
    v::Vector{Float64}

    disp_mat::Array{Float64} #matrix for mean & 95%CI plot of model output
end

function construct_slice(trajectory::Int64, i::Int64, θ::Vector{Float64}, pos::Vector{Float64}, v::Vector{Float64}, obs::Vector{Vector{Float64}}, T::Vector{Float64}, popdist::Distribution, lens_model::Lens_Model, mc_its::Int64, phs::Int64; v_init=false)
    pparams=θ[1:2*phs];phase_ends=θ[2*phs+1:(2*phs+1)+(phs-2)]

    log_lh,disp_mat=slice_mc_llh(popdist, lens_model, phase_ends, pparams, mc_its, T, obs)

    v_init && (v=rand(MvNormal(length(θ),1.)))

    return Slice_Model(trajectory, i, θ, log_lh, pos, v, disp_mat)
end
```

17.4.14 /src/thymidine_sim/thymidine_cell.jl

```julia
abstract type AbstractCell end

mutable struct LabelCell <: AbstractCell
    time::Float64
    tδ?::Float64
    g1?::Float64
    s?::Float64
    label::Float64
end
```

17.4.15 /src/thymidine_sim/thymidine_ensemble.jl

```julia
mutable struct Thymidine_Ensemble <: GMC_NS_Ensemble
    path::String

    model_initλ::Function
    models::Vector{Thymidine_Record}

    contour::Float64
    log_Li::Vector{Float64}
    log_Xi::Vector{Float64}
    log_wi::Vector{Float64}
    log_Liwi::Vector{Float64}
    log_Zi::Vector{Float64}
    Hi::Vector{Float64}

    obs::Vector{Vector{<:Integer}}
    priors::Vector{<:Distribution}
    constants::Vector{<:Any}
    #T, pulse, mc_its, end_time
    box::Matrix{Float64}

    sample_posterior::Bool
    posterior_samples::Vector{Thymidine_Record}

    GMC_Nmin::Int64

    GMC_τ_death::Float64
    GMC_init_τ::Float64
    GMC_tune_μ::Int64
```

```
    GMC_tune_α::Float64
    GMC_tune_PID::NTuple{3,Float64}

    GMC_timestep_η::Float64
    GMC_reflect_η::Float64
    GMC_exhaust_σ::Float64
    GMC_chain_κ::Int64

    t_counter::Int64
end

Thymidine_Ensemble(path::String, no_models::Integer, obs::AbstractVector{<:AbstractVector{<:Integer}}, priors::AbstractVector{<:Distribution}, constants, box, GMC_settings; sample_posterior::Bool=true) =
Thymidine_Ensemble(
    path,
    thymidine_constructor,
    assemble_TMs(path, no_models, obs, priors, constants, box)...,
    [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
    obs,
    priors,
    constants, #popdist, T, pulse, mc_its, end_time
    box,
    sample_posterior,
    Vector{Thymidine_Record}(),
    GMC_settings...,
        no_models+1)

function assemble_TMs(path::String, no_trajectories::Integer, obs, priors, constants, box)
        ensemble_records = Vector{Thymidine_Record}()
        !isdir(path) && mkpath(path)
    @showprogress 1 "Assembling Thymidine_Model ensemble..." for trajectory_no in 1:no_trajectories
                model_path = string(path,'/',trajectory_no,'.',1)
        if !isfile(model_path)
            proposal=rand.(priors)
```

```julia

            pos=to_unit_ball.(proposal,priors)
            box_bound!(pos,box)
            θvec=to_prior.(pos,priors)

            model = thymidine_constructor(trajectory_no, 1, θvec, pos, [0.], obs, constants... ; v_init=true)

                        serialize(model_path, model) #save the model to the ensemble directory
                        push!(ensemble_records, Thymidine_Record(trajectory_no, 1, pos,model_path,model.log_Li))
                else #interrupted assembly pick up from where we left off
                        model = deserialize(model_path)
                        push!(ensemble_records, Thymidine_Record(trajectory_no, 1, model.pos,model_path,model.log_Li))
                end
        end

        return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end

function Base.show(io::IO, m::Thymidine_Model, e::Thymidine_Ensemble; progress=false)
    T=e.constants[2]

    catobs=vcat(e.obs... )
    ymax=max(maximum(m.disp_mat),maximum(catobs))
    ymin=min(minimum(m.disp_mat),minimum(catobs))

    plt=lineplot(T,m.disp_mat[:,2],title="Thymidine_Model $(m.trajectory).$(m.i), log_Li $(m.log_Li)",color=:green,name="μ count", ylim=[ymin,ymax])
    lineplot!(plt,T,m.disp_mat[:,1],color=:magenta,name="95% CI")
    lineplot!(plt,T,m.disp_mat[:,3],color=:magenta)

    Ts=vcat([[T[n] for i in 1:length(e.obs[n])] for n in 1:length(T)]... )
    scatterplot!(plt,Ts, catobs, color=:yellow, name="Obs")

    show(io, plt)
```

```
    println()
    println("θ: $(m.θ)")

    (progress && return nrows(plt.graphics)+8);
end

function print_MAP_output(e::Thymidine_Ensemble, path::String=e.path;
↪   its::Int64=Int64(1e6), size=(900,600), clims=(0.,.05))
    MLEmod=deserialize(e.models[findmax([m.log_Li for m in
     ↪  e.models])[2]].path)
    θ=MLEmod.θ
    lt=length(e.priors)

    n_pops=1
    lt>5 && (n_pops+=(lt-5)/6)

    pparams=Vector{Tuple{LogNormal, Float64, Float64, Float64}}()
    for pop in 1:n_pops
        tcμ, tcσ², g1_frac, s_frac, sis_frac=
         ↪  θ[Int64(1+((pop-1)*5)):Int64(5*pop)]
        tcσ=sqrt(tcσ²);
        push!(pparams,(LogNormal(tcμ,tcσ),g1_frac,s_frac,sis_frac))
    end

    n_pops > 1 ? (pop_fracs=θ[Int64(5*n_pops)+1:end]) : (pop_fracs=[])

    popdist, T, pulse, mc_its, end_time = e.constants

    log_lh, disp_mat, joint_DNPs=thymidine_mc_llh(popdist, pop_fracs,
     ↪  pparams, its, end_time, pulse, T, e.obs, true)

    max_ct=0
    for DNP in joint_DNPs
        max_ct<maximum(DNP.support) && (max_ct=maximum(DNP.support))
    end
    cts=LinRange(0,max_ct,max_ct+1)
    probs=zeros(length(T),length(cts))
    for (t,DNP) in enumerate(joint_DNPs)
        for (ct, p) in zip(DNP.support,DNP.p)
            ct_idx=findfirst(isequal(ct),cts)
            ct_idx ≢ nothing && (probs[t,ct_idx]=p)
        end
    end
```

```
    catobs=vcat(e.obs ... )
    ymax=Int64(round(maximum(catobs)*1.1))
    ys=[0,ymax]
    Ts=vcat([[T[n] for i in 1:length(e.obs[n])] for n in 1:length(T)] ... )


    MAPout=Plots.heatmap(T,cts,transpose(probs),ylims=ys,color=:viridis,size=size, xlabel="Post-pulse chase time (hr)", ylabel="Labelled cells (#)", colorbar_title="Probability mass", legend=:topleft, foreground_color_legend=nothing, background_color_legend=nothing, legendfontcolor=:white, xticks=T)
    Plots.scatter!(MAPout,Ts,catobs,marker=:cross,markercolor=:magenta, label="Observations")
    Plots.plot!(MAPout, T, disp_mat[:,2], color=:white, width=2, label="Simulated count mean")
    Plots.plot!(MAPout, T, disp_mat[:,1], color=:white, style=:dash, label="Simulated count 95% probability mass")
    Plots.plot!(MAPout, T, disp_mat[:,3], color=:white, style=:dash, label=:none)

    savefig(MAPout, path)
end

function print_marginals(e::Thymidine_Ensemble, path::String; param_names=["LogNormal Tc μ", "LogNormal Tc σ²", "G1 Fraction", "S Fraction", "Sister Shift Fraction"])
    wtvec=zeros(0)
    lt=length(e.priors)
    θvecs=[zeros(0) for param in 1:lt]
    for (n,rec) in enumerate(e.posterior_samples)
        m=deserialize(rec.path)
        for i in 1:length(θvecs)
            push!(θvecs[i],m.θ[i])
        end
        push!(wtvec,e.log_Liwi[n+1])
    end

    for rec in e.models
        m=deserialize(rec.path)
        for i in 1:length(θvecs)
            push!(θvecs[i],m.θ[i])
        end
```

```
        push!(wtvec,(lps(m.log_Li,lps(e.log_Xi[end],-log(length(e.models))))))
    end

    n_pops=1
    lt>5 && (n_pops+=Int64((lt-5)/6))
    n_pops>1 ? (plotrows=6; og_pn=copy(param_names)) : (plotrows=5)

    for pop in 2:n_pops
        param_names=vcat(param_names,vcat(og_pn,"Population $pop Fraction"))
    end

    mapm=deserialize(e.models[findmax([m.log_Li for m in e.models])[2]].path)

    plots=Vector{Plots.Plot}()
    for (n,θvec) in enumerate(θvecs)
        p_label=param_names[n]
        occursin("Fraction",p_label) ? (xls=[0,1]) : (xls=[quantile(e.priors[n],.01),quantile(e.priors[n],.99)])


        θkde=kde(θvec,weights=exp.(wtvec))
        n==1 ? (lblpos=:right) : (lblpos=:none)
        plt=StatsPlots.plot(e.priors[n], color=:darkmagenta, fill=true, fillalpha=.5, label="Prior", xlabel=p_label, ylabel="Density", xlims=xls)
        plot!(θkde.x,θkde.density, color=:green, fill=true, fillalpha=.75, label="Posterior")

        plot!([mapm.θ[n],mapm.θ[n]],[0,maximum(θkde.density)],color=:black, label="MAP", legend=lblpos)
        push!(plots,plt)
        n_pops>1 && n==5 && push!(plots,plot())
    end

    combined=plot(plots...,layout=grid(plotrows,n_pops),size=(600*n_pops,800))

    savefig(combined, path)
end
```

17.4.16 /src/thymidine_sim/thymidine_lh.jl

```
function thymidine_mc_llh(pop_dist, pop_fracs, pparams, mc_its, end_time, pulse, T::Vector{<:AbstractFloat}, obs::Vector{<:AbstractVector{<:Integer}}, display=false)
    n_pops=length(pparams); times=length(T)
    popset_labelled=zeros(Int64,mc_its,n_pops,times)
    max_labelled=maximum.(obs)

    Threads.@threads for it in 1:mc_its
        pop_lins=zeros(Int64,n_pops)
        pop_lins[1]=Int64(max(1,round(rand(pop_dist))))
        for p in 2:n_pops
            pop_lins[p]=Int64(round(pop_fracs[p-1]*pop_lins[p-1]))
            pop_lins[p-1]-=pop_lins[p]
        end

        for p in 1:n_pops
            tc_dist,g1_frac,s_frac,sis_frac = pparams[p]
            plv=view(popset_labelled,it,p,:)

            for l in 1:Int64(floor(pop_lins[p]/2))
                !all(max_labelled .< sum(popset_labelled[it,:,:],dims=1)[1,:]) && sim_lin_pair!(plv, T, end_time, pulse, tc_dist, g1_frac, s_frac, sis_frac)
            end

            for l in 1:pop_lins[p]%2
                !all(max_labelled .< sum(popset_labelled[it,:,:],dims=1)[1,:]) && sim_lineage!(plv, T, end_time, pulse, tc_dist, g1_frac, s_frac, sis_frac)
            end
        end
    end

    pop_DNPs=Matrix{DiscreteNonParametric}(undef,n_pops,times)
    for p in 1:n_pops
        Threads.@threads for t in 1:times

            pop_DNPs[p,t]=fit(DiscreteNonParametric,popset_labelled[:,p,t])
        end
```

```
    end

    if n_pops > 1
        joints=Vector{DiscreteNonParametric}(undef,times)
        Threads.@threads for t in 1:times
            joints[t]=joint_DNP_sum(pop_DNPs[:,t])
        end
    else
        joints=pop_DNPs[1,:]
    end

    log_lhs=Vector{Float64}(undef,times)
    Threads.@threads for t in 1:times
        log_lhs[t]=lps(logpdf(joints[t],obs[t]))
    end

    log_lh=lps(log_lhs)

    disp_mat=zeros(times,3)
    Threads.@threads for t in 1:times

        ↪  disp_mat[t,:]=[quantile(joints[t],.025),mean(joints[t]),quantile(joints[t
    end
    display ? (return log_lh, disp_mat, joints) : (return log_lh,
    ↪  disp_mat)
end

function joint_DNP_sum(cs)
    npops=length(cs)
    ct_supports=Vector{Vector{Int64}}()
    for c in cs
        push!(ct_supports,c.support[[c.p ... ].>0.])
    end

    possible_sums=collect(Iterators.product(ct_supports ... ))
    possible_totals=unique(sum.(possible_sums))
    total_mat=sum.(possible_sums)

    probs=zeros(length(possible_totals))

    Threads.@threads for (n,pt) in collect(enumerate(possible_totals))
        psums=possible_sums[findall(t→t==pt,total_mat)]
```

```
        probs[n]=logsumexp([lps([logpdf(cs[p],ps[p]) for p in 1:npops])
        ↪  for ps in psums])
    end

    return DiscreteNonParametric(possible_totals, exp.(probs))
end
```

17.4.17 /src/thymidine_sim/thymidine_model.jl

```
mutable struct Thymidine_Record <: GMC_NS_Model_Record
    trajectory::Int64
    i::Int64
    pos::Vector{Float64}
    path::String
    log_Li::Float64
end

struct Thymidine_Model <: GMC_NS_Model
    trajectory::Int64
    i::Int64

    θ::Vector{Float64}
    log_Li::Float64

    pos::Vector{Float64}
    v::Vector{Float64}

    disp_mat::Matrix{Float64} #matrix for mean & 95%CI plot of model
    ↪  output
end

function thymidine_constructor(trajectory, i, θ, pos, v, obs, popdist, T,
↪  pulse, mc_its, end_time; v_init=false)
    lt=length(θ)

    lt!=5 && mod(lt-5,6)≠0 && throw(ArgumentError("θ must contain tcμ,
    ↪  tcσ², g1_frac, s_frac, sis_frac values per lineage population,
    ↪  and one popfrac per population beyond the first!"))
    pulse<0 && throw(ArgumentError("pulse length must be ≥0!"))
```

```

    n_pops=1
    lt>5 && (n_pops+=(lt-5)/6)

    pparams=Vector{Tuple{LogNormal, Float64, Float64, Float64}}()
    for pop in 1:n_pops
        tcμ, tcσ², g1_frac, s_frac, sis_frac=
        ↪  θ[Int64(1+((pop-1)*5)):Int64(5*pop)]
        tcσ=sqrt(tcσ²);
        push!(pparams,(LogNormal(tcμ,tcσ),g1_frac,s_frac,sis_frac))
    end

    n_pops > 1 ? (pop_fracs=θ[Int64(5*n_pops)+1:end]) : (pop_fracs=[])

    log_lh,disp_mat=thymidine_mc_llh(popdist, pop_fracs, pparams, mc_its,
    ↪  end_time, pulse, T, obs)

    v_init && (v=rand(MvNormal(length(θ),1.)))

    Thymidine_Model(trajectory, i, θ, log_lh, pos, v, disp_mat)
end
```

17.4.18 /src/thymidine_sim/thymidine_sim.jl

```
DETECTION_THRESHOLD=.15
MIN_CT=4.

function sim_lin_pair!(plv, T, end_time, pulse, Tc, g1_frac, s_frac,
↪  sis_frac)
    tδ1, g1_1, s1 = cycle_model(Tc, g1_frac, s_frac)
    tδ2, g1_2, s2 = (1+rand([-1,1])*sis_frac).*(tδ1, g1_1, s1)

    mint=-min(tδ1,tδ2)
    mint==0. ? (birth_time=mint) : (birth_time=rand(Uniform(mint,0.)))

    cells1=[LabelCell(birth_time,tδ1,g1_1,s1,0.)]
    cells2=[LabelCell(birth_time,tδ2,g1_2,s2,0.)]

    for cellvec in [cells1,cells2]
        ci=1
        while ci ≤ length(cellvec)
```

```
            cycle_sim!(plv, ci, cellvec, T, end_time, pulse, Tc, g1_frac,s_frac, sis_frac)
            ci+=1
        end
    end
end


function sim_lineage!(plv, T, end_time, pulse, Tc, g1_frac, s_frac, sis_frac)
    tδ1, g1, s1 = cycle_model(Tc, g1_frac, s_frac)
    birth_time=rand(Uniform(-tδ1,0.))
    cells=[LabelCell(birth_time,tδ1,g1,s1,0.)]

    ci=1
    while ci ≤ length(cells)
        cycle_sim!(plv, ci, cells, T, end_time, pulse, Tc, g1_frac, s_frac, sis_frac)
        ci+=1
    end
end

function cycle_sim!(plv, ci::Int64, cells::Vector{LabelCell}, T::Vector{Float64}, end_time::Float64, pulse::Float64, Tc::LogNormal, g1_frac::Float64, s_frac::Float64, sis_frac::Float64)
    n_times=length(T);

    cell=cells[ci]
    t=cell.time; tδ?=cell.tδ?; g1?=cell.g1?; s?=cell.s?; l=cell.label

    first_cycle=true

    while t<end_time
        tδ,s=0.,0.
        first_cycle ? (tδ=tδ?;g1=g1?;s=s?;first_cycle=false) : ((tδ, g1, s)=cycle_model(Tc, g1_frac, s_frac))

        s_start=t+g1
        s_end=s_start+s
        cycle_mitosis=t+tδ
        cycle_events=[s_start,s_end,cycle_mitosis]

        oldl=l; cycle_label=false
```

```
        if (s_start ≤ pulse && s_end ≥ 0) #some s-phase overlaps with the pulse in this case
            cycle_label=true
            l=update_label(l, s, s_start, s_end, pulse)
        end

        tidxs=t.<T.<cycle_mitosis
        n_idxs=sum(tidxs)
        (length(tidxs)>0 && (oldl > DETECTION_THRESHOLD || l > DETECTION_THRESHOLD))

        if n_idxs>0 && (oldl > DETECTION_THRESHOLD || l > DETECTION_THRESHOLD)
            cycle_label ? (label_outcomes=[oldl,-Inf,l]) : (label_outcomes=[oldl,oldl,oldl])
            cubuf=falses(n_idxs)
            for (n,timept) in enumerate(T[tidxs])
                count_labelled_at_T!(n,cubuf,timept, cycle_events, label_outcomes, oldl, s_start, s, pulse)
            end
            plv[tidxs]+=cubuf
        end

        l⁄=2 #mitosis halves label for the daughters

        tδ2, g1_2, s2 = (1+rand([-1,1])*sis_frac).*(tδ, g1, s)
        tδ2 < MIN_CT && (tδ2=tδ; g1_2=g1; s2 = s;) #prevent sister shift from resulting in unrealistically fast proliferation

        cycle_mitosis < end_time && push!(cells,LabelCell(cycle_mitosis,tδ2,g1_2,s2,l))
        t+=tδ
    end
end

function cycle_model(Tc, g1_frac, s_frac)
    tδ=max(MIN_CT,rand(Tc)) #floor of 1 hr for any distribution of cycle times
    s=tδ * s_frac
    g1=g1_frac * (tδ - s)
    return tδ, g1, s
end

```

```
function update_label(label, s, s_start, s_end, pulse)
    s_overlap=min(pulse,s_start+s_end)-max(0,s_start)
    nuc_label_frac=s_overlap/s
    return min(nuc_label_frac+label,1.) #label before mitosis time will be what's been accumulated from past + this cycle's s phase
end

function count_labelled_at_T!(n, cubuf, timept, cycle_events, label_outcomes, oldl, s_start, s, pulse)
    outcome_idx=timept.<cycle_events
    timept_label=label_outcomes[outcome_idx][1]
    timept_label==-Inf && (timept_label=partial_label(timept,oldl,s_start,s,pulse))
    timept_label ≥ DETECTION_THRESHOLD && (cubuf[n]=true)
end

function partial_label(timept, label, s_start, s, pulse)
    s_overlap=min(pulse,timept)-max(0.,s_start)
    nuc_label_frac=s_overlap/s
    return min(nuc_label_frac+label,1.)
end
```

17.5 BioBackgroundModels

Github repository: https://github.com/mmattocks/BioBackgroundModels.jl

17.5.1 /src/BioBackgroundModels.jl

```
module BioBackgroundModels

import Base:copy,size
import Distances: euclidean
import Distributions:Univariate,Dirichlet,Categorical,logpdf,isprobvec
import Distributed: RemoteChannel, myid, remote_do, rmprocs
import HMMBase: AbstractHMM, assert_hmm, istransmat
import MCMCChains: Chains, ChainDataFrame, heideldiag
import Printf: @sprintf
import Random: rand, shuffle
import Serialization: serialize
import StatsFuns: logsumexp, logaddexp
import Statistics: mean
import UnicodePlots: lineplot,lineplot!, scatterplot,scatterplot!
```

```
using BioSequences, DataFrames, FASTX, GFF3, ProgressMeter

include("BHMM/BHMM.jl")
export BHMM
include("EM/chain.jl")
export Chain_ID, EM_step
include("API/genome_sampling.jl")
export setup_sample_jobs, execute_sample_jobs
include("API/EM_master.jl")
export setup_EM_jobs!, execute_EM_jobs!

include("reports/chain_report.jl")
include("reports/partition_report.jl")
include("reports/replicate_convergence.jl")
include("API/reports.jl")
export generate_reports

include("EM/baum-welch.jl")
include("EM/churbanov.jl")
include("utilities/load_balancer.jl")
export LoadConfig
include("EM/EM_converge.jl")
include("genome_sampling/partition_masker.jl")
include("genome_sampling/sequence_sampler.jl")
include("likelihood_funcs/bg_lh_matrix.jl")
export BGHMM_likelihood_calc
include("likelihood_funcs/hmm.jl")
export obs_lh_given_hmm
include("utilities/observation_coding.jl")
include("utilities/BBG_analysis.jl")
include("utilities/BBG_progressmeter.jl")
include("utilities/HMM_init.jl")
include("utilities/model_display.jl")
include("utilities/utilities.jl")
export split_obs_sets
include("utilities/log_prob_sum.jl")
export lps
end # module
```

17.5.2 /src/API/EM_master.jl

```
"""
    setup_EM_jobs!(job_ids, obs_sets; delta_thresh, chains, init_function)

Given a vector of 'Chain_IDs', and the appropriate obs_sets dictionary, return the appropriate tuple of channels and chains for the 'execute_EM_jobs!' function. Given an existing chains dict, resumes any existing non-converged chains (as assessed by the provided 'delta_thresh' value), otherwise initialises hmms for new chains with optionally user-specified 'init_function' (default 'autotransition_init').

"""
function setup_EM_jobs!(job_ids::Vector{Chain_ID}, obs_sets::Dict{String,Vector{LongSequence{DNAAlphabet{2}}}}; delta_thresh::Float64=1e-3, chains::Dict{Chain_ID,Vector{EM_step}}=Dict{Chain_ID,Vector{EM_step}}(), init_function::Function=autotransition_init)
    #argument checking
    length(job_ids) < 1 && throw(ArgumentError("Empty job id vector!"))
    length(obs_sets) < 1 && throw(ArgumentError("Empty observation sets!"))

    no_input_hmms = length(job_ids)
    input_channel= RemoteChannel(()->Channel{Tuple}(no_input_hmms*3)) #channel to hold BHMM learning jobs
    output_channel= RemoteChannel(()->Channel{Tuple}(Inf)) #channel to take EM iterates off of

    code_dict = code_job_obs(job_ids, obs_sets)

    @showprogress 1 "Setting up HMMs..." for id in job_ids #for each jobid, add an initial BHMM to input_channel for EM_workers
        if haskey(chains, id) && length(chains[id]) > 0 #true if resuming from incomplete chain
            chain_end=chains[id][end]
            if !chain_end.converged || (chain_end.converged && chain_end.delta > delta_thresh)#push the last hmm iterate for nonconverged chains to the input channel with coded observations and values for chain resumption
```

```
                put!(input_channel, (id, chain_end.iterate, chain_end.hmm, chain_end.log_p, code_dict[(id.obs_id, id.order)]))
            else #skip any jobs that have converged from previous runs
                no_input_hmms -= 1
            end

        else
            hmm = init_function(id.K, id.order, id.obs_id) #initialise first BHMM in chain
            chains[id] = Vector{EM_step}() #initialise the relevant chain
            obs=code_dict[(id.obs_id,id.order)]
            lh=obs_lh_given_hmm(obs,hmm,linear=false)
            put!(input_channel, (id, 1, hmm, lh, obs))
        end
    end
    return no_input_hmms, chains, input_channel, output_channel
end

"""
    execute_EM_jobs!(worker_pool, no_input_hmms, chains, input_channel, output_channel, chains_path; load_dict, EM_func, delta_thresh, max_iterates, verbose)

Given: a vector of worker IDs ([1] to use the master), the splatted output of 'setup_EM_jobs' (no_input_hmms, chains, input_channel, output_channel), and a valid path to save chains ('chains_path'); assign chains to workers using 'load_dict' to iteratively execute 'EM_func' (default linear_step) until 'delta_thresh' convergence criterion is obtained or 'max_iterates' have been calculated. Specifying 'verbose=true' gives some additional debug output from 'EM_converge!'.
"""
function execute_EM_jobs!(worker_pool::Vector{Int64}, no_input_hmms::Integer, chains::Dict{Chain_ID,Vector{EM_step}}, input_channel::RemoteChannel, output_channel::RemoteChannel, chains_path::String; load_dict=Dict{Int64,LoadConfig}(), EM_func::Function=linear_step, delta_thresh=1e-3, max_iterates=5000, verbose=false)
    #argument checking
    length(worker_pool) < 1 && throw(ArgumentError("Worker pool must contain one or more worker IDs!"))
```

```
    no_input_hmms < 1 && throw(ArgumentError("Zero input HMMs reported, likely continuing from chains already converged beyond default delta_thresh for setup_EM_jobs"))
    length(chains) < 1 && throw(ArgumentError("No chains supplied, likely job set from setup_EM_jobs passed incorrectly"))
    !isready(input_channel) && throw(ArgumentError("BHMM input channel has no contents, likely job set from setup_EM_jobs already executed"))

    #SEND BHMM FIT JOBS TO WORKERS
    if isready(input_channel) > 0
        @info "Fitting HMMs.."
        #WORKERS FIT HMMS
        for worker in worker_pool
            if worker in keys(load_dict)
                remote_do(EM_converge!, worker, input_channel, output_channel, no_input_hmms, load_config=load_dict[worker], EM_func=EM_func, delta_thresh=delta_thresh, max_iterates=max_iterates, verbose=verbose)
            else
                remote_do(EM_converge!, worker, input_channel, output_channel, no_input_hmms, EM_func=EM_func, delta_thresh=delta_thresh, max_iterates=max_iterates, verbose=verbose)
            end
        end
    else
        @warn "No input HMMs (all already converged?), skipping fitting.."
    end

    #GET LEARNT HMMS OFF REMOTECHANNEL, SERIALISE AT EVERY ITERATION, UPDATE PROGRESS METERS
    job_counter=no_input_hmms
    learning_meters=Dict{Chain_ID, ProgressHMM}()
    overall_meter=Progress(no_input_hmms,"Overall batch progress:")

    while job_counter > 0
        wait(output_channel)
        workerid, jobid, iterate, hmm, log_p, delta, converged, steptime = take!(output_channel)
```

```
        #either update an existing ProgressHMM meter or create a new one for the job
        if haskey(learning_meters, jobid) && iterate > 2
                update!(learning_meters[jobid], delta, steptime)
        else
            offset = workerid - 1
            if iterate ≤2
                learning_meters[jobid] = ProgressHMM(delta_thresh, "$jobid on Wk $workerid:", offset, 2)
                update!(learning_meters[jobid], delta, steptime)
            else
                learning_meters[jobid] = ProgressHMM(delta_thresh, "$jobid on Wk $workerid:", offset, iterate)
                update!(learning_meters[jobid], delta, steptime)
            end
        end
        #push the hmm and related params to the results_dict
        push!(chains[jobid], EM_step(iterate, hmm, log_p, delta, converged))
        #try to serialize the results; catch interrupts and other errors to prevent corruption
        try
            serialize(chains_path, chains)
        catch e
            @warn "Serializing failed!"
            println(e)
        end

        #decrement the job counter, update overall progress meter, and save the current results dict on convergence or max iterate
        if converged || iterate == max_iterates
            job_counter -= 1
            ProgressMeter.update!(overall_meter, (no_input_hmms-job_counter))
            if !isready(input_channel) #if there are no more jobs to be learnt, retire the worker
                workerid!=1 && rmprocs(workerid)
            end
        end
    end

    #count converged & unconverged jobs, report results
    converged_counter = 0
```

```
    unconverged_counter = 0
    for (id, chain) in chains
        chain[end].converged == true ? (converged_counter += 1) : (unconverged_counter += 1)
    end

    @info "Background HMM batch learning task complete, $converged_counter converged jobs, $unconverged_counter jobs failed to converge in $max_iterates iterates since job start."
end
```

17.5.3 /src/API/genome_sampling.jl

```
"""
    setup_sample_jobs(genome_path, genome_index_path, gff3_path, sample_set_length, sample_window_min, sample_window_max, perigenic_pad; deterministic)

Return sample job channels for 'execute_sample_jobs', given 'genome_path' to a properly formatted FASTA genome, 'genome_index_path' to the associated .FAI, 'gff3_path' to the genome's GFF3 feature database, a total number of bases to sample, 'sample_set_length', as well as 'sample_window_min' and 'sample_window_max' lengths for individual samples. 'perigenic_pad' specifies the number of bases up- and down-stream of exonic sequences to be considered 'perigenic'; if 0, the perigenic partition will be entirely intronic. Optional boolean 'deterministic' may be set to 'true' to always return '+' strand sequences from unstranded samples; otherwise unstranded samples may be returned with either orientation.
"""

#function to partition genome and set up Distributed RemoteChannels so partitions can be sampled simultaneously
function setup_sample_jobs(genome_path::String, genome_index_path::String, gff3_path::String, sample_set_length::Integer, sample_window_min::Integer, sample_window_max::Integer, perigenic_pad::Integer; deterministic::Bool=false)
    #argument checking
    !ispath(genome_path) && throw(ArgumentError("Bad genome path!"))
    !ispath(genome_index_path) && throw(ArgumentError("Bad genome index path!"))
```

```
    !ispath(gff3_path) && throw(ArgumentError("Bad gff3 path!"))
    sample_set_length < 1 && throw(ArgumentError("Sample set length must be a positive integer!"))
    sample_window_min < 1 || sample_window_max < 1 && throw(ArgumentError("Sample window minimum and maximum bounds must be positive integers!"))
    sample_window_min ≥ sample_window_max && throw(ArgumentError("Sample window minimum size must be smaller than maximum size"))
    perigenic_pad < 0 && throw(ArgumentError("Perigenic pad must be 0 or positive!"))

    coordinate_partitions = partition_genome_coordinates(gff3_path, perigenic_pad)
    sample_sets = DataFrame[]
    input_sample_jobs = RemoteChannel(()→Channel{Tuple}(length(coordinate_partitions))) #channel to hold sampling jobs
    completed_sample_jobs = RemoteChannel(()→Channel{Tuple}(length(coordinate_partitions))) #channel to hold completed sample dfs
    for (partitionid, partition) in coordinate_partitions
        add_metacoordinates!(partition)
        put!(input_sample_jobs, (genome_path, genome_index_path, partition, partitionid, sample_set_length, sample_window_min, sample_window_max, deterministic))
    end
    progress_channel = RemoteChannel(()→Channel{Tuple}(20))
    return (input_sample_jobs, completed_sample_jobs, progress_channel, sample_set_length)
end

"""
    execute_sample_jobs!(channels, worker_pool)

Return sample dataframes for each genomic partition, given appropriately set up job channels (from setup_sample_jobs!) and a vector of worker ids ('worker_pool', [1] for master).
"""

function execute_sample_jobs(channels::Tuple{RemoteChannel,RemoteChannel,RemoteChannel,Int
worker_pool::Vector{Int64}; partitions::Integer=3)
```

```
    input_sample_channel, completed_sample_channel, progress_channel, sample_set_length = channels

    #argument checking
    length(worker_pool) < 1 && throw(ArgumentError("Worker pool must contain one or more worker IDs!"))
    !isready(input_sample_channel) && throw(ArgumentError("Input sample channel not ready, likely channel set from setup_sample_jobs passed incorrectly"))
    sample_set_length < 1 && throw(ArgumentError("Sample set length must be a positive integer, likely channel set from setup_sample_jobs passed incorrectly"))

    #send sampling jobs to workers
    if isready(input_sample_channel) > 0
        @info "Sampling.."
        #WORKERS SAMPLE
        for (n, worker) in enumerate(worker_pool)
            if n ≤ partitions #no more workers than partitions to be used
                remote_do(get_sample_set, worker, input_sample_channel, completed_sample_channel, progress_channel)
            end
        end
    else
        @error "No sampling jobs!"
    end

    #progress meters for sampling
    sampling_meters=Dict{String, Progress}()
    overall_sampling_meter=Progress(partitions,"Overall sampling progress:")
    completed_counter = 0
    ProgressMeter.update!(overall_sampling_meter, completed_counter)
    sampling_offset = ones(Bool, partitions)

    #collect progress updates while waiting on completion of sampling jobs
    while completed_counter < partitions
        wait(progress_channel)
        partition_id, progress = take!(progress_channel)
        if haskey(sampling_meters, partition_id)
```

```
            ProgressMeter.update!(sampling_meters[partition_id], progress)
        else
            offset = findfirst(sampling_offset)[1]
            sampling_meters[partition_id] = Progress(sample_set_length, "Sampling partition $partition_id:", offset)
            ProgressMeter.update!(sampling_meters[partition_id], progress)
            sampling_offset[offset] = false
        end
        if progress == sample_set_length
            completed_counter += 1
            ProgressMeter.update!(overall_sampling_meter, completed_counter)
        end
    end

    #collect sample dfs by partition id when ready
    sample_record_dfs = Dict{String,DataFrame}()
    collected_counter = 0
    while collected_counter < partitions
        wait(completed_sample_channel)
        partition_id, sample_df = take!(completed_sample_channel)
        sample_record_dfs[partition_id] = sample_df
        collected_counter += 1
        @info "Partition $partition_id completed sampling..."
    end

    return sample_record_dfs
end
```

17.5.4 /src/API/reports.jl

```
struct Report_Folder
    partition_id::String
    partition_report::Partition_Report
    replicate_report::Replicate_Report
    chain_reports::Dict{Chain_ID,Chain_Report}
end

function generate_reports(chains::Dict{Chain_ID,Vector{EM_step}}, test_sets::Dict{String,Vector{LongSequence{DNAAlphabet{2}}}})
```

```
    #check chains dict for problems

    length(chains)==0 && throw(ArgumentError("Empty chains dict!"))
    any(chain->length(chain)<2, values(chains)) && throw(ArgumentError("Some chains are too short (<2 EM steps)! Probably not all chains have been operated on by EM workers yet. Try EM_converge first!"))
    unconv=sum(!chain[end].converged for chain in values(chains))
    unconv > 0 && @warn "Not all chains are converged to the selected step delta."
    length(test_sets)==0 && throw(ArgumentError("Empty test_sets dict!"))

    chain_reports = report_chains(chains, test_sets)
    partition_reports = report_partitions(chain_reports)

    report_folders=Dict{String, Report_Folder}()

    for part_report in partition_reports
        rep_report=report_replicates(part_report.best_repset, chains)
        chain_subset=Dict{Chain_ID,Chain_Report}()
        for id in keys(chain_reports)
            id.obs_id==part_report.partition_id && (chain_subset[id]=chain_reports[id])
        end

        report_folders[part_report.partition_id]=Report_Folder(part_report.partit
        part_report, rep_report, chain_subset)
    end

    return report_folders
end
```

17.5.5 /src/BHMM/BHMM.jl

```
"""
    BHMM([a::AbstractVector{T}, ]A::AbstractMatrix{T}, B::AbstractVector{<:Categorical}) where F where T
Build an BHMM with transition matrix `A` and observations distributions `B`.
If the initial state distribution `a` is not specified, a uniform distribution is assumed.
```

```
Observations distributions can be of different types (for example `Normal` and `Exponential`).
However they must be of the same dimension (all scalars or all multivariates).
# Example
```julia
hmm = BHMM([0.9 0.1; 0.1 0.9], [Normal(0,1), Normal(10,1)])
```
"""
struct BHMM{T} <: AbstractHMM{Univariate}
    a::AbstractVector{T}
    A::AbstractMatrix{T}
    B::AbstractVector{<:Categorical}
    partition::String
    BHMM{T}(a, A, B, partition="") where {T} = assert_BHMM(a, A, B) && new(a, A, B, partition)
end

BHMM(a::AbstractVector{T}, A::AbstractMatrix{T}, B::AbstractVector{<:Categorical}, partition ...) where {T} = BHMM{T}(a, A, B, partition ...)
BHMM(A::AbstractMatrix{T}, B::AbstractVector{<:Categorical}, partition ...) where {T} = BHMM{T}(ones(size(A)[1])/size(A)[1], A, B, partition ...)

copy(hmm::BHMM) = BHMM(copy(hmm.a), copy(hmm.A), copy(hmm.B), hmm.partition)
size(hmm::BHMM) = (length(hmm.B), length(hmm.B[1].p))

"""
    assert_BHMM(a, A, B)
Throw an `ArgumentError` if the initial state distribution and the transition matrix rows does not sum to 1,
and if the observations distributions does not have the same dimensions.
"""
function assert_BHMM(a::AbstractVector,
                     A::AbstractMatrix,
                     B::AbstractVector{<:Categorical})
    !isprobvec(a) && throw(ArgumentError("Initial state vector a is not a valid probability vector!"))
    !istransmat(A) && throw(ArgumentError("Transition matrix A is not valid!"))
```

```julia

    !all([length(d.p) for d in B] .== length(B[1].p)) && throw(ArgumentError("All distributions must have the same dimensions"))
    !(length(a) == size(A,1) == length(B)) && throw(ArgumentError("Length of initial state vector a, dimension of transition matrix A, and number of distributions B are not the same!"))
    return true
end

function Base.show(io::IO, hmm::BHMM)
    println(io, "Background BHMM")
    println(io, "State Initial and Transition Probabilities")
    print(io, "a: ")
    show(io, hmm.a)
    println(io)
    print(io, "A: ")
    display(hmm.A)
    println(io)
    println(io, "INFORMATIVE SYMBOLS BY STATE")
    for (n,d) in enumerate(hmm.B)
        print(io, "K$n ")
        print_emitters(d)
    end
end
```

17.5.6 /src/EM/EM_converge.jl

```julia
function EM_converge!(hmm_jobs::RemoteChannel, output_hmms::RemoteChannel, no_models::Integer; load_config::LoadConfig=LoadConfig(1:typemax(Int64)-1, 0:typemax(Int64)-1),  EM_func::Function=linear_step, delta_thresh=1e-3, max_iterates=5000, verbose=false)
    while isready(hmm_jobs)
        workerid = myid()
        jobid, start_iterate, hmm, job_norm, observations = load_balancer(no_models, hmm_jobs, load_config)
        jobid == 0 && break #no valid job for this worker according to load_table entry

```

```
        start_iterate > max_iterates - 1 && throw(ArgumentError("BHMM chain $jobid is already longer ($start_iterate iterates) than specified max_iterates!"))
        curr_iterate = start_iterate

        #mask calculations here rather than ms_mle_step to prevent recalculation every iterate
        #build array of observation lengths

        obs_lengths = [findfirst(iszero,observations[o,:])-1 for o in 1:size(observations,1)] #mask calculations here rather than mle_step to prevent recalculation every iterate
        EM_func==bw_step && (observations=transpose(observations))

        start_iterate == 1 && put!(output_hmms, (workerid, jobid, curr_iterate, hmm, job_norm, 0.0, false, 0.0)); #on the first iterate return the initial BHMM immediately
        verbose && @info "Fitting BHMM on Wk $workerid, start iterate $start_iterate, $jobid with $(size(hmm)[1]) states and $(size(hmm)[2]) symbols ... "

        curr_iterate += 1

        start=time()
        new_hmm, last_norm = EM_func(hmm, observations, obs_lengths)
        delta = abs(lps(job_norm, -last_norm))
        put!(output_hmms, (workerid, jobid, curr_iterate, new_hmm, last_norm, delta, false, time()-start))

        for i in curr_iterate:max_iterates
            start=time()
            new_hmm, norm = EM_func(new_hmm, observations, obs_lengths)
            curr_iterate += 1
            delta = abs(lps(norm, -last_norm))
            if delta < delta_thresh
                put!(output_hmms, (workerid, jobid, curr_iterate, new_hmm, norm, delta, true, time()-start))
                verbose && @info "$jobid converged after $(curr_iterate-1) EM steps"
                break
            else
                put!(output_hmms, (workerid, jobid, curr_iterate, new_hmm, norm, delta, false, time()-start))
```

```
                verbose && @info "$jobid EM step $(curr_iterate-1) delta
                ↪  $delta"
                last_norm = norm
            end
        end
    end
end
```

17.5.7 /src/EM/baum-welch.jl

```
"""
    ms_mle_step(hmm::AbstractHMM{F}, observations) where F

Perform one step of the EM (Baum-Welch) algorithm.

# Example
```julia
hmm, log_likelihood = ms_mle_step(hmm, observations)
```
"""

function bw_step(hmm::BHMM, observations, obs_lengths)
    lls = bw_llhs(hmm, observations)
    log_α = messages_forwards_log(hmm.a, hmm.A, lls, obs_lengths)
    log_β = messages_backwards_log(hmm.A, lls, obs_lengths)
    log_A = log.(hmm.A)

    K,Tmaxplus1,O = size(lls) #the last T value is the 0 end marker of
    ↪ the longest T

    #transforms to cut down log_ξ, log_γ assignment times
    lls = permutedims(lls, [2,3,1]) # from (K,T,O) to (T,O,K)
    log_α = permutedims(log_α, [2,3,1])
    log_β = permutedims(log_β, [2,3,1])

    # E-step
    log_ξ = fill(-Inf, Tmaxplus1,O,K,K)
    log_γ = fill(-Inf, Tmaxplus1,O,K)
    log_pobs = zeros(O)

   @inbounds for o = 1:O
        log_pobs[o] = logsumexp(lps.(log_α[1,o,:], log_β[1,o,:]))
```

```
    end

    Threads.@threads for idx in [(i,j,o) for i=1:K, j=1:K, o=1:O]
        i,j,o = idx[1],idx[2],idx[3]
        obsl = obs_lengths[o]
        @inbounds for t = 1:obsl-1 #log_ξ & log_γ calculated to T-1 for each o
           log_ξ[t,o,i,j] = lps(log_α[t,o,i], log_A[i,j], log_β[t+1,o,j], lls[t+1,o,j], -log_pobs[o])
           log_γ[t,o,i] = lps(log_α[t,o,i], log_β[t,o,i], -log_pobs[o])
        end
        t=obsl #log_ξ @ T = 0
        log_ξ[t,o,i,j] = 0
        log_γ[t,o,i] = lps(log_α[t,o,i], log_β[t,o,i], -log_pobs[o])
    end

    ξ = similar(log_ξ)
    ξ .= exp.(log_ξ)
    Σk_ξ = sum(ξ, dims=[3,4])
    nan_mask = Σk_ξ .== 0
    Σk_ξ[nan_mask] .= Inf #prevent NaNs in dummy renorm
    ξ  ./=  Σk_ξ #dummy renorm across K to keep numerical creep from causing isprobvec to fail on new new_A during hmm creation

    γ = similar(log_γ)
    γ .= exp.(log_γ)
    Σk_γ = sum(γ, dims=3)
    Σk_γ[nan_mask[:,:,:]] .= Inf #prevent NaNs in dummy renorm
    γ ./= Σk_γ #dummy renorm

    # M-step
    new_A = zeros(K,K)
    for i=1:K, j=1:K
        Σotξ_vec = zeros(O)
        Σotγ_vec = zeros(O)
        Threads.@threads for o in 1:O
            Σotξ_vec[o] = sum(ξ[1:obs_lengths[o]-1,o,i,j])
            Σotγ_vec[o] = sum(γ[1:obs_lengths[o]-1,o,i])
        end
        new_A[i,j] = sum(Σotξ_vec) / sum(Σotγ_vec)
    end
    new_A ./= sum(new_A, dims=2) #dummy renorm
    new_a = (sum(γ[1,:,:], dims=1)./sum(sum(γ[1,:,:], dims=1)))[1,:]
```

```
    new_a ./= sum(new_a) #dummy renorm

    obs_mask = .!nan_mask
    obs_collection = observations[obs_mask[:,:]]

    B = Categorical[]
    @inbounds for (i, d) in enumerate(hmm.B)
        γ_d = γ[:,:,i]
        push!(B, t_categorical_mle(Categorical, d.support[end], obs_collection, γ_d[obs_mask[:,:]]))
    end

    return typeof(hmm)(new_a, new_A, B), lps(log_pobs)
end
                function bw_llhs(hmm, observations)
                    lls = zeros(length(hmm.B), size(observations)... )
                    Threads.@threads for d in 1:length(hmm.B)
                        lls[d,:,:] = logpdf.(hmm.B[d], observations)
                    end
                    return lls
                end
                #Multisequence competent log implementations of forward and backwards algos
                function messages_forwards_log(init_distn, trans_matrix, log_likelihoods, obs_lengths)
                    log_alphas = zeros(size(log_likelihoods))
                    log_trans_matrix = log.(trans_matrix)
                    log_alphas[:,1,:] = log.(init_distn) .+ log_likelihoods[:,1,:]
                    Threads.@threads for o in 1:size(log_likelihoods)[3]
                        @inbounds for t in 2:obs_lengths[o], j in 1:size(log_likelihoods)[1]

                            log_alphas[j,t,o]=logsumexp(log_alphas[:,t-1,o] .+ log_trans_matrix[:,j]) + log_likelihoods[j,t,o]
                        end
                    end
                    return log_alphas
                end

                function messages_backwards_log(trans_matrix, log_likelihoods, obs_lengths)
```

```
    log_betas = zeros(size(log_likelihoods))
    log_trans_matrix = log.(trans_matrix)
    Threads.@threads for o in 1:size(log_likelihoods)[3]
        @inbounds for t in obs_lengths[o]-1:-1:1
            tmp = view(log_betas, :, t+1, o) .+ view(log_likelihoods, :, t+1, o)
            @inbounds for i in 1:size(log_likelihoods)[1]
                log_betas[i,t,o] = logsumexp(view(log_trans_matrix, i,:) .+ tmp)
            end
        end
    end
    return log_betas
end
#subfunc derived from Distributions.jl categorical.jl fit_mle, threaded
function t_categorical_mle(::Type{<:Categorical}, k::Integer, x::AbstractArray{T}, w::AbstractArray{F}) where T<:Integer where F<:AbstractFloat

    Categorical(t_pnormalize!(t_add_categorical_counts!(zeros(k), x, w)))
end

t_pnormalize!(v::Vector) = (v ./= sum(v); v)

function t_add_categorical_counts!(h::Vector{F}, x::AbstractArray{T}, w::AbstractArray{F}) where T<:Integer where F<:AbstractFloat
    n = length(x)
    if n ≠ length(w)
        throw(DimensionMismatch("Inconsistent array lengths."))
    end
    hlock = ReentrantLock()
    Threads.@threads for i=1:n
        @inbounds xi = x[i]
        @inbounds wi = w[i]
        lock(hlock)
        h[xi] += wi   # cannot use @inbounds, as no guarantee that x[i] is in bound
        unlock(hlock)
```

```
                    end
                    return h
                end
```

17.5.8 /src/EM/chain.jl

```
struct Chain_ID
    obs_id::AbstractString
    K::Integer
    order::Integer
    replicate::Integer
    Chain_ID(obs_id,K,order,replicate) = assert_chain_id(K,order,replicate) && new(obs_id,K,order,replicate)
end

function assert_chain_id(K, order, replicate)
    K < 1 && throw(ArgumentError("Chain_ID K (# of hmm states) must be a positive integer!"))
    order < 0 && throw(ArgumentError("Chain_ID symbol order must be zero or a positive integer!"))
    replicate < 1 && throw(ArgumentError("Chain_ID replicate must be a positive integer!"))
    return true
end

struct EM_step
    iterate::Integer
    hmm::BHMM
    log_p::AbstractFloat
    delta::AbstractFloat
    converged::Bool

    EM_step(iterate,hmm,log_p,delta,converged)=assert_step(iterate,hmm,log_p) && new(iterate, hmm, log_p, delta, converged)
end

function assert_step(iterate, hmm, log_p)
    iterate < 1 && throw(ArgumentError("EM_step iterate number must be a positive integer!"))
    assert_hmm(hmm.a, hmm.A, hmm.B)
```

```
    log_p > 0.0 && throw(ArgumentError("EM_step log probability value must be 0 or negative!"))
    return true
end
```

17.5.9 /src/EM/churbanov.jl

```
function linear_step(hmm, observations, obs_lengths)
    O,T = size(observations);
    a = transpose(log.(hmm.a)); A = log.(hmm.A) #less expensive to transpose transmatrix in the backwards_sweep than to transpose here and take a ranged view ie view(A,m:m,:) is more expensive than transpose(view(A,m,:))
    N = length(hmm.B); Γ = length(hmm.B[1].support);
    mask=observations.≠0
    #INITIALIZATION
    βoi_T = zeros(O,N); βoi_t = zeros(O,N) #log betas at T initialised as zeros
    Eoγim_T = fill(-Inf,O,Γ,N,N); Eoγim_t = fill(-Inf,O,Γ,N,N)
    for m in 1:N, i in 1:N, γ in 1:Γ, o in 1:O
        observations[o, obs_lengths[o]] == γ && m == i && (Eoγim_T[o, γ, i, m] = 0)
    end
    Tijm_T = fill(-Inf,O,N,N,N); Tijm_t = fill(-Inf,O,N,N,N) #Ti,j(T,m) = 0 for all m; in logspace

    #RECURRENCE

    βoi_T,Tijm_T,Eoγim_T=backwards_sweep!(hmm,A,N,Γ,βoi_T,βoi_t,Tijm_T,Tijm_t,Eoγ

    #TERMINATION
    lls = c_llhs(hmm,observations[:,1])
    α1om = lps.(lls,a) #first position forward msgs
    log_pobs = [c_lse(lps.(α1om[o,:], βoi_T[o,:])) for o in 1:O]

    Toij = [c_lse([lps(Tijm_T[o,i,j,m], α1om[o,m], -log_pobs[o]) for m in 1:N]) for o in 1:O, i in 1:N, j in 1:N] #terminate Tijs with forward messages
    Eoiγ = [c_lse([lps(Eoγim_T[o,γ,i,m], α1om[o,m], -log_pobs[o]) for m in 1:N]) for o in 1:O, i in 1:N, γ in 1:Γ] #terminate Eids with forward messages

```

```
    #INTEGRATE ACROSS OBSERVATIONS AND SOLVE FOR NEW BHMM PARAMS
    #INITIAL STATE DIST
    a_o=α1om.+βoi_T•-c_lse.(eachrow(α1om.+βoi_T)) #estimate a for each o
    obs_penalty=log(O) #broadcast subtraction to normalise log prob vals by obs number
    new_a=c_lse.(eachcol(a_o))•-obs_penalty #sum over obs and normalise by number of obs
    #TRANSITION MATRIX
    new_A = [c_lse(view(Toij,:,i,j)) for i in 1:N, j in 1:N]•-c_lse.(eachcol(c_lse.([view(Toij,o,i,:) for o in 1:O, i in 1:N])))
    #EMISSION MATRIX
    new_b=[c_lse(view(Eoiγ,:,i,γ)) for i in 1:N, γ in 1:Γ]•-c_lse.(eachcol(c_lse.([view(Eoiγ,o,i,:) for o in 1:O, i in 1:N])))
    new_B=[Categorical(exp.(new_b[i,:])) for i in 1:N]

    return BHMM(exp.(new_a), exp.(new_A), new_B, hmm.partition), lps(log_pobs)
end
                #LINEAR_STEP SUBFUNCS
                function backwards_sweep!(hmm, A, N, Γ, βoi_T, βoi_t, Tijm_T, Tijm_t, Eoγim_T, Eoγim_t, observations, mask, obs_lengths)
                    for t in maximum(obs_lengths)-1:-1:1
                        last_β=copy(βoi_T)
                        lls = c_llhs(hmm,observations[:,t+1])
                        omask = findall(mask[:,t+1])
                        βoi_T[omask,:] .+= view(lls,omask,:)
                        Threads.@threads for m in 1:N
                            βoi_t[omask,m] = c_lse.(eachrow(view(βoi_T,omask,:).+transpose(view(A,
                            for j in 1:N, i in 1:N
                                Tijm_t[omask, i, j, m] •= c_lse.(eachrow(lps.(view(Tijm_T,omask,i,j,:), view(lls,omask,:), transpose(view(A,m,:)))))
                                i==m && (Tijm_t[omask, i, j, m] •= logaddexp.(Tijm_t[omask, i, j, m], lps.(last_β[omask,j], A[m,j],lls[omask,j])))
                            end
                            for i in 1:N, γ in 1:Γ
```

```
                                Eoγim_t[omask, γ, i, m] •=
                                ↪  c_lse.(eachrow(lps.(view(Eoγim_T,omask,γ,i,:),vie
                                if i==m
                                    symmask =
                                    ↪  findall(observations[:,t].==γ)
                                    Eoγim_t[symmask, γ, i, m] •=
                                    ↪  logaddexp.(Eoγim_t[symmask, γ, i,
                                    ↪  m], βoi_t[symmask,m])
                                end
                            end
                        end
                        βoi_T=copy(βoi_t); Tijm_T=copy(Tijm_t); Eoγim_T =
                        ↪  copy(Eoγim_t);
                    end
                    return βoi_T, Tijm_T, Eoγim_T
                end

                #logsumexp with single dispatch for speed
                function c_lse(X::AbstractArray{T}; dims=:) where
                ↪  {T<:Real}
                    u=maximum(X)
                    u isa AbstractArray || isfinite(u) || return float(u)
                    return u + log(sum(x -> exp(x-u), X))
                end

                #log likelihoods
                function c_llhs(hmm, observation)
                    lls = zeros(length(observation),length(hmm.B))
                    Threads.@threads for d in 1:length(hmm.B)
                        lls[:,d] = logpdf.(hmm.B[d], observation)
                    end
                    return lls
                end
```

17.5.10 /src/genome_sampling/partition_masker.jl

```
function get_partition_code_dict(dict_forward::Bool=true)
    if dict_forward == true
        return partition_code_dict =
        ↪  Dict("intergenic"=>1,"periexonic"=>2,"exon"=>3)
    else
```

```
        return code_partition_dict = Dict(1⇒"intergenic", 2⇒"periexonic", 3⇒"exon")
    end
end

function make_padded_df(position_fasta::String, gff3_path::String, genome_path::String, genome_index_path::String, pad::Integer)
    position_reader = FASTA.Reader(open((position_fasta),"r"))
    genome_reader = open(FASTA.Reader, genome_path, index=genome_index_path)
    scaffold_df = build_scaffold_df(gff3_path)
    position_df = DataFrame(SeqID = String[], Start=Int[], End=Int[], PadSeq = LongSequence[], PadStart=Int[], RelStart=Int[], SeqOffset=Int[])
    scaffold_seq_dict = build_scaffold_seq_dict(genome_path, genome_index_path)

    for entry in position_reader
        scaffold = FASTA.identifier(entry)

        if scaffold ≠ "MT"
            desc_array = split(FASTA.description(entry))
            pos_start = parse(Int, desc_array[2])
            pos_end = parse(Int, desc_array[4])
            scaffold_end = scaffold_df.End[findfirst(isequal(scaffold), scaffold_df.SeqID)]

            pad_start=max(1,pos_start-pad)
            pad_length= pos_start - pad_start
            seq_offset = pad - pad_length
            padded_seq = fetch_sequence(scaffold, scaffold_seq_dict, pad_start, pos_end, '+')

            if !hasambiguity(padded_seq)
                push!(position_df, [scaffold, pos_start, pos_end, padded_seq, pad_start, pad_length, seq_offset])
            end
        end
    end

    close(position_reader)
    close(genome_reader)
    return position_df
```

```
end

function add_partition_masks!(position_df::DataFrame, gff3_path::String, perigenic_pad::Integer=500, columns::Tuple{Symbol,Symbol,Symbol}=(:SeqID, :PadSeq, :PadStart))
    partitions=["exon", "periexonic", "intergenic"]
    partition_coords_dict = partition_genome_coordinates(gff3_path, perigenic_pad)
    partitioned_scaffolds = divide_partitions_by_scaffold(partition_coords_dict)
    maskcol = [zeros(Int64,0,0) for i in 1:size(position_df,1)]
    position_df.MaskMatrix=maskcol

    @Threads.threads for entry in eachrow(position_df)
        scaffold = entry[columns[1]]
        maskLength = length(entry[columns[2]])
        seqStart = entry[columns[3]]

        scaffold_coords_dict = Dict{String,DataFrame}()

        for ((partition, part_scaffold), df) in partitioned_scaffolds
            if scaffold == part_scaffold
                scaffold_coords_dict[partition] = df
            end
        end

        entry.MaskMatrix=mask_sequence_by_partition(maskLength, seqStart, scaffold_coords_dict)
    end

end
                #add_partition_masks!() SUBFUNCTIONS
                function divide_partitions_by_scaffold(partition_coords_dict::Dict{String, DataFrame})
                    scaffold_coords_dict = Dict{Tuple{String, String}, DataFrame}()
                    for (partition_id, partition_df) in partition_coords_dict
                        for scaffold_subframe in groupby(partition_df, :SeqID)
                            scaffold_id = scaffold_subframe.SeqID[1]
                            scaffold_df = copy(scaffold_subframe)
```

```
                        scaffold_coords_dict[(partition_id, scaffold_id)] = scaffold_df
                    end
                end
                return scaffold_coords_dict
            end

            function mask_sequence_by_partition(maskLength::Integer, seqStart::Integer, scaffold_coords_dict::Dict{String, DataFrame})
                partition_code_dict = get_partition_code_dict()
                seqMask = zeros(Integer, (maskLength, 2))
                position = seqStart
                while position ≤ seqStart+maskLength
                    position_partition, partition_extent, position_strand = find_position_partition(position, scaffold_coords_dict)

                    partition_code = partition_code_dict[position_partition]
                    mask_position = position - seqStart + 1

                    seqMask[mask_position:min(maskLength,mask_position + partition_extent),1] ⋅= partition_code
                    if position_strand == '+'

                        seqMask[mask_position:min(maskLength,mask_position + partition_extent),2] ⋅= 1
                    elseif position_strand == '-'

                        seqMask[mask_position:min(maskLength,mask_position + partition_extent),2] ⋅= -1
                    else

                        seqMask[mask_position:min(maskLength,mask_position + partition_extent),2] ⋅= 0
                    end

                    position += partition_extent + 1
                end

                return seqMask
```

```
                end

                function find_position_partition(position::Integer, partition_dict::Dict{String, DataFrame})
                    foundPos = false
                    position_partition_id = ""
                    three_prime_extent = 0
                    sample_strand = 0
                    for (partition_id, partition) in partition_dict
                        hitindex = findfirst(x->x≥position, partition.End)
                        if hitindex ≠ nothing
                            if position ≥ partition.Start[hitindex]
                                foundPos=true
                                position_partition_id = partition_id
                                three_prime_extent = partition.End[hitindex] - position
                                if partition_id == "exon" || partition_id == "periexonic"
                                    sample_strand = partition.Strand[hitindex]
                                end
                            end
                        end
                    end

                    if foundPos == false
                        throw(DomainError("Position $position not found among partition coordinates!"))
                    else
                        return position_partition_id, three_prime_extent, sample_strand
                    end
                end

#function to partition a genome into coordinate sets of:
#merged exons
#"periexonic" sequences (genes with 5' and 3' boundaries projected -/+perigenic_pad bp, minus exons) - includes promoter elements, introns, 3' elements
#intergenic sequences (everything else)
#given a valid gff3
```

```
function partition_genome_coordinates(gff3_path::String, perigenic_pad::Integer=500)
    # construct dataframes of scaffolds and metacoordinates
    scaffold_df = build_scaffold_df(gff3_path)

    #partition genome into intragenic, periexonic, and exonic coordinate sets
    #assemble exonic featureset
    exon_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[], Strand=Char[])
    build_feature_df!(gff3_path, "CDS", "MT", exon_df)

    #project exon coordinates onto the scaffold bitwise, merging overlapping features and returning the merged dataframe
    merged_exon_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[], Strand=Char[])
    @showprogress 1 "Partitioning exons..." for scaffold_subframe in DataFrames.groupby(exon_df, :SeqID) # for each scaffold subframe that has exon features
        scaffold_id  = scaffold_subframe.SeqID[1] #get the scaffold id
        scaffold_bitarray = init_scaffold_bitarray(scaffold_id, scaffold_df, false, stranded=true) #init a stranded bitarray of scaffold length

        project_features_to_bitarray!(scaffold_subframe,scaffold_bitarray) #project features as Trues on bitarray of falses
        merged_subframe = get_feature_df_from_bitarray(scaffold_id, scaffold_bitarray) #get a feature df from the projected bitarray
        append!(merged_exon_df,merged_subframe) #append the merged scaffold df to the overall merged df
    end

    #assemble gene featureset
    gene_df =  DataFrame(SeqID = String[], Start = Integer[], End = Integer[], Strand = Char[])
    build_feature_df!(gff3_path, "gene", "MT", gene_df)

    perigenic_pad > 0 && add_pad_to_coordinates!(gene_df, scaffold_df, perigenic_pad) #if a perigenic pad is specified (to capture promoter/downstream elements etc in the periexonic set), apply it to the gene coords

```

```
    #build intergenic coordinate set by subtracting gene features from scaffold bitarrays
    intergenic_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[])
    @showprogress 1 "Partitioning intergenic regions ..." for scaffold_subframe in DataFrames.groupby(scaffold_df, :SeqID) # for each scaffold subframe that has exon features
        scaffold_id  = scaffold_subframe.SeqID[1] #get the scaffold id
        scaffold_bitarray = init_scaffold_bitarray(scaffold_id, scaffold_df, true) #init a bitarray of scaffold length
        if any(isequal(scaffold_id),gene_df.SeqID) #if any genes on the scafold
            scaffold_genes=gene_df[findall(isequal(scaffold_id), gene_df.SeqID), :] #get the gene rows by finding the scaffold_id

            subtract_features_from_bitarray!(scaffold_genes,scaffold_bitarray) #subtract the gene positions from the scaffold bitarray
        end
        intragenic_subframe = get_feature_df_from_bitarray(scaffold_id, scaffold_bitarray) #get a feature df from the projected bitarray
        append!(intergenic_df,intragenic_subframe) #append the merged scaffold df to the overall merged df
    end

    #build periexonic set by projecting gene coordinates onto the scaffold bitwise, then subtracting the exons
    periexonic_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[], Strand=Char[])
    @showprogress 1 "Partitioning periexonic regions ..." for gene_subframe in DataFrames.groupby(gene_df, :SeqID) # for each scaffold subframe that has exon features
        scaffold_id  = gene_subframe.SeqID[1] #get the scaffold id
        scaffold_bitarray = init_scaffold_bitarray(scaffold_id, scaffold_df, false, stranded=true) #init a bitarray of scaffold length
        project_features_to_bitarray!(gene_subframe,scaffold_bitarray) #project features as Trues on bitarray of falses
        if any(isequal(scaffold_id),merged_exon_df.SeqID)
            scaffold_exons=merged_exon_df[findall(isequal(scaffold_id), merged_exon_df.SeqID), :]
```

```
                subtract_features_from_bitarray!(scaffold_exons,scaffold_bitarray)
        end
        periexonic_subframe = get_feature_df_from_bitarray(scaffold_id, scaffold_bitarray) #get a feature df from the projected bitarray
        append!(periexonic_df,periexonic_subframe) #append the merged scaffold df to the overall merged df
    end

    return Dict("exon"⇒merged_exon_df, "periexonic"⇒periexonic_df, "intergenic"⇒intergenic_df)
end

            #partition_genome_coordinates() SUBFUNCTIONS
            #BITARRAY SCAFFOLD REPRESENTATION SUBFUNCTIONS
            function init_scaffold_bitarray(scaffold_id::String, scaffold_df, value::Bool; stranded::Bool=false)
                scaffold_length = scaffold_df.End[findfirst(isequal(scaffold_id), scaffold_df.SeqID)]
                if stranded
                    value ? (return rscaffold_bitarray = trues(scaffold_length,2)) : (return rscaffold_bitarray = falses(scaffold_length,2))
                else
                    value ? (return rscaffold_bitarray = trues(scaffold_length,1)) : (return rscaffold_bitarray = falses(scaffold_length,1))
                end
            end

            function project_features_to_bitarray!(scaffold_feature_sf::SubDataFrame, scaffold_bitarray::BitArray)
                @inbounds for item in eachrow(scaffold_feature_sf)
                    scaffold_bitarray[item.Start:item.End,1] = [true for base in item.Start:item.End]
                    if size(scaffold_bitarray)[2] == 2 #if the bitarray is stranded
                        if item.Strand == '+'
                            scaffold_bitarray[item.Start:item.End,2] = [true for base in item.Start:item.End]
```

```
                end
            end
        end
    end

    function subtract_features_from_bitarray!(scaffold_feature_sf::DataFrame, scaffold_bitarray::BitArray)
        @inbounds for item in eachrow(scaffold_feature_sf)
            scaffold_bitarray[item.Start:item.End,1] = [false for base in item.Start:item.End]
        end
    end

    function get_feature_df_from_bitarray(scaffold_id::String, scaffold_bitarray::BitArray)
        size(scaffold_bitarray)[2] == 2 ? scaffold_feature_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[], Strand=Char[]) : scaffold_feature_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[])

        new_feature_start = findnext(view(scaffold_bitarray,:,1), 1)
        while new_feature_start ≠ nothing # while new features are still found on the bitarray
            if size(scaffold_bitarray)[2] == 2 #if stranded, get strand info
                scaffold_bitarray[new_feature_start,2] == 1 ? new_feature_strand = '+' : new_feature_strand = '-'
            end
            if findnext(!eval,view(scaffold_bitarray,:,1),new_feature_st ≠ nothing
                new_feature_end = findnext(!eval,view(scaffold_bitarray,:,1),new_featur #find next false after feature start and subtract 1 for feature end
            else
                new_feature_end = size(scaffold_bitarray)[1] #if none is found, the end of the feature is the end of hte scaffold
```

```
                        end
                        size(scaffold_bitarray)[2] == 2 ? push!(scaffold_feature_df,[scaffold_id, new_feature_start, new_feature_end, new_feature_strand]) : push!(scaffold_feature_df,[scaffold_id, new_feature_start, new_feature_end]) #push stranded feature info as appropriate
                        new_feature_start = findnext(view(scaffold_bitarray,:,1), new_feature_end+1)
                    end
                    return scaffold_feature_df
                end
```

17.5.11 /src/genome_sampling/sequence_sampler.jl

```
####SAMPLING FUNCTIONS####
#function for a Distributed worker to produce a set of samples of given parameters from genomic sequences
function get_sample_set(input_sample_jobs::RemoteChannel, completed_sample_jobs::RemoteChannel, progress_updates::RemoteChannel)
    while isready(input_sample_jobs)
        genome_path, genome_index_path, partition_df, partitionid, sample_set_length, sample_window_min, sample_window_max, deterministic = take!(input_sample_jobs)

        stranded::Bool = get_strand_dict()[partitionid]
        scaffold_sequence_record_dict::Dict{String,LongSequence} = build_scaffold_seq_dict(genome_path, genome_index_path)

        sample_df = DataFrame(SampleScaffold=String[],SampleStart=Integer[],SampleEnd=Integer
        metacoordinate_bitarray = trues(partition_df.MetaEnd[end])
        sample_set_counter = 0

        while sample_set_counter < sample_set_length #while we don't yet have enough sample sequence
```

```
            sample_scaffold::String, sample_Start::Integer, sample_End::Integer, sample_metaStart::Integer, sample_metaEnd::Integer, sample_sequence::LongSequence, strand::Char = get_sample(metacoordinate_bitarray, sample_window_min, sample_window_max,  partition_df, scaffold_sequence_record_dict; stranded=stranded, deterministic=deterministic)
            push!(sample_df,[sample_scaffold, sample_Start, sample_End, sample_sequence, strand]) #push the sample to the df
            sample_length = sample_End - sample_Start + 1
            sample_set_counter += sample_length #increase the counter by the length of the sampled sequence
            metacoordinate_bitarray[sample_metaStart:sample_metaEnd] = [false for base in 1:sample_length] #mark these residues as sampled
            put!(progress_updates, (partitionid, min(sample_set_counter,sample_set_length)))
        end
        put!(completed_sample_jobs,(partitionid, sample_df))
    end
end
                #get_sample_set() SUBFUNCTIONS
                #function defining whether partitions respect stranding upon fetching sequence (ie is the sequence fetched in the feature strand orientation, or are we agnostic about the strand we sample?)
                function get_strand_dict()
                    return Dict("exon"⇒true,"periexonic"⇒true,"intergenic"⇒false)
                end
                #function to obtain a dict of scaffold sequences from a FASTA reader
                function build_scaffold_seq_dict(genome_fa, genome_index)
                    genome_reader = open(FASTA.Reader, genome_fa, index=genome_index)
                    seq_dict::Dict{String, LongSequence} = Dict{String,FASTA.Record}()
                    @inbounds for record in genome_reader
                        id = FASTA.identifier(record)
                        seq_dict[rectify_identifier(id)]=FASTA.sequence(record)
                    end
                    close(genome_reader)
```

```
        return seq_dict
    end

    # function to convert scaffold ID from that observed by the masked .fna to the more legible one observed by the GRCz11 GFF3
    function rectify_identifier(scaffold_id::String)
        if length(scaffold_id) ≥ 4 && scaffold_id[1:4] == "CM00" #marks chromosome scaffold
            chr_code = scaffold_id[5:10]
            chr_no = "$(Int((parse(Float64,chr_code)) - 2884.2))"
            return chr_no
        else
            return scaffold_id
        end
    end

    #function to produce a single sample from a metacoordinate set and the feature df
    function get_sample(metacoordinate_bitarray::BitArray, sample_window_min::Integer, sample_window_max::Integer, partition_df::DataFrame, scaffold_seq_dict::Dict{String,LongSequence}; stranded::Bool=false, deterministic::Bool=false)
        proposal_acceptance = false
        sample_metaStart = 0
        sample_metaEnd = 0
        sample_Start = 0
        sample_End = 0
        sample_sequence = LongSequence{DNAAlphabet{2}}("")
        sample_scaffold = ""
        strand = nothing

        while proposal_acceptance == false
            available_indices = findall(metacoordinate_bitarray) #find all unsampled indices
            window = "FAIL"
            start_index,feature_metaStart,feature_metaEnd, feature_length = 0,0,0,0
            strand = '0'
            while window == "FAIL"
```

```
                    start_index = rand(available_indices) #randomly choose from the unsampled indices
                    feature_metaStart, feature_metaEnd, strand = get_feature_params_from_metacoord(start_index, partition_df, stranded)
                    feature_length = length(feature_metaStart:feature_metaEnd) #find the metaboundaries of the feature the index occurs in
                    if feature_length ≥ sample_window_min #don't bother finding windows on features smaller than min
                        window = determine_sample_window(feature_metaStart, feature_metaEnd, start_index, metacoordinate_bitarray, sample_window_min, sample_window_max) #get an appropriate sampling window around the selected index, given the feature boundaries and params
                    end
                end

                sample_scaffoldid, sample_scaffold_start, sample_scaffold_end = meta_to_feature_coord(window[1],window[2],partition_df)

                proposal_sequence = fetch_sequence(sample_scaffoldid, scaffold_seq_dict, sample_scaffold_start, sample_scaffold_end, strand; deterministic=deterministic) #get the sequence associated with the sample window

                if mask_check(proposal_sequence)
                    proposal_acceptance = true #if the sequence passes the mask check, accept the proposed sample
                    sample_scaffold = sample_scaffoldid
                    sample_Start = sample_scaffold_start
                    sample_End = sample_scaffold_end
                    sample_metaStart = window[1]
```

```
                        sample_metaEnd = window[2]
                        sample_sequence=proposal_sequence
                    end
                end
                return sample_scaffold, sample_Start, sample_End, sample_metaStart, sample_metaEnd, sample_sequence, strand
            end

            #function to find a valid sampling window
            function determine_sample_window(feature_metaStart::Integer, feature_metaEnd::Integer, metacoord::Integer, metacoord_bitarray::BitArray, sample_window_min::Integer, sample_window_max::Integer)
                window_start = 0
                window_end = 0
                feature_size = feature_metaEnd - feature_metaStart + 1
                feature_sampled = any(!eval, metacoord_bitarray[feature_metaStart:feature_metaEnd])
                #attempt to construct the sample window by taking the whole feature
                if !feature_sampled && feature_size < sample_window_max && feature_size > sample_window_min
                    window_start = feature_metaStart
                    window_end = feature_metaEnd
                    return window_start, window_end
                else #if this fails, build the biggest window we can from the sampling point, within the feature
                    featurepos = metacoord - feature_metaStart + 1
                    next_sampled_index = findnext(!eval, metacoord_bitarray[feature_metaStart:feature_metaEnd], featurepos)
                    prev_sampled_index = findprev(!eval, metacoord_bitarray[feature_metaStart:feature_metaEnd], featurepos)
                    next_sampled_index === nothing ? (window_end = feature_metaEnd) : (window_end = next_sampled_index + feature_metaStart - 1)
```

```
111                         prev_sampled_index ≡ nothing ? (window_start =
                             ↪  feature_metaStart) : (window_start =
                             ↪  prev_sampled_index + feature_metaStart - 1)
112                         windowsize = window_end - window_start + 1
113                         #check to see if this window is bigger than the
                             ↪  min, if not, return a failure code
114                         windowsize < sample_window_min && return "FAIL"
115                         #check to see if this window is bigger than the
                             ↪  max, if so, trim it before returning,
                             ↪  removing as evenly as possible around the
                             ↪  metacoordinate
116                         if windowsize > sample_window_max
117                             bases_to_trim = windowsize -
                                 ↪  sample_window_max
118                             clearance_5P = metacoord - window_start + 1
119                             clearance_3P = window_end - metacoord + 1
120                             trimmed=0
121                             while trimmed < bases_to_trim
122                                 if clearance_5P ≥ clearance_3P &&
                                     ↪  clearance_5P > 0
123                                     clearance_5P -= 1
124                                 elseif clearance_3P > clearance_5P &&
                                     ↪  clearance_3P > 0
125                                     clearance_3P -= 1
126                                 end
127                                 trimmed +=1
128                             end
129                             window_start = metacoord - clearance_5P + 1
130                             window_end = metacoord + clearance_3P - 1
131                         end
132                         return window_start, window_end
133                     end
134                 end
135
136
137                 # function to check for repetitive stretches or
                     ↪  degenerate bases in proposal sequence
138                 function mask_check(proposal_sequence::LongSequence)
139                     proposal_acceptance = true
140                     if hasambiguity(proposal_sequence) ||
                         ↪  isrepetitive(proposal_sequence,
                         ↪  (length(proposal_sequence) ÷ 10))
141                         proposal_acceptance = false
```

```
                    end
                    return proposal_acceptance
                end

####SHARED SEQUENCE FETCHER####
# function to get proposal sequence from dict of scaffold sequences, given coords and scaffold id
function fetch_sequence(scaffold_id::String, scaffold_seq_dict::Dict{String, LongSequence}, proposal_start::Integer, proposal_end::Integer, strand::Char; deterministic=false)
    if strand == '0' #unstranded samples may be returned with no preference in either orientation
        deterministic ? strand = '+' : (rand(1)[1] ≤ .5 ? strand = '+' : strand = '-')
    end
    if strand == '+'
        proposal_sequence = scaffold_seq_dict[scaffold_id][proposal_start:proposal_end]
    elseif strand == '-'
        proposal_sequence = reverse_complement(scaffold_seq_dict[scaffold_id][proposal_start:proposal_
    else
        throw(ArgumentError("Invalid sample code! Must be '+', '-', or '0' (random strand)"))
    end

    return proposal_sequence
end

####SHARED BASIC COORDINATE SUBFUNCTIONS####
# function to push scaffold ID, start, and end points of given featuretype to supplied dataframe
function build_feature_df!(GFF3_path::String, feature_type::String, scaffold_exclusion::String, feature_df::DataFrame)
    reader = open(GFF3.Reader, GFF3_path) # access the GFF3
    @inbounds for record in reader # iterate over Gff3 records
        if GFF3.isfeature(record) # if the record is a feature
            if GFF3.featuretype(record) == feature_type #if the features is of the requested type, get the following info
                seqID = GFF3.seqid(record)
                if seqID ≠ scaffold_exclusion
                    seq_start = GFF3.seqstart(record)
```

```
                    seq_end = GFF3.seqend(record)
                    if feature_type == "CDS" || feature_type == "gene"
                        seq_strand = convert(Char, GFF3.strand(record))
                        push!(feature_df, [seqID, seq_start, seq_end, seq_strand])
                    else
                        push!(feature_df, [seqID, seq_start, seq_end]) # push relevant info to the df
                    end
                end
            end
        end
    end
    close(reader)
end

#function to assemble dataframe of scaffold coords + metacoords given gff3
function build_scaffold_df(gff3_path)
    scaffold_df = DataFrame(SeqID = String[], Start = Integer[], End = Integer[])
    build_feature_df!(gff3_path, "supercontig", "MT", scaffold_df)
    build_feature_df!(gff3_path, "chromosome", "MT", scaffold_df)
    add_metacoordinates!(scaffold_df)
    return scaffold_df
end

#function to add pad to either side of some featureset
function add_pad_to_coordinates!(feature_df::DataFrame, scaffold_df::DataFrame, pad_size::Integer; col_symbols::Array{Symbol}=[:Start, :End])
    pad_start_array = zeros(Integer,size(feature_df,1))
    pad_end_array = zeros(Integer,size(feature_df,1))
    feature_df[!, col_symbols[1]] = [max(feature_df.Start[i]-pad_size,1) for i in 1:size(feature_df,1)] #truncate pads at beginning and end of scaffolds
    feature_df[!, col_symbols[2]] = [min(feature_df.End[i]+pad_size,scaffold_df.End[findfirst(isequal(feature_df.S for i in 1:size(feature_df,1)]
end

####SHARED METACOORDINATE FUNCTIONS####
```

```
# function to add a metacoordinate column to a dataframe of scaffold positions, allowing sampling across all scaffolds
function add_metacoordinates!(feature_df::DataFrame)
    meta_start_array = Integer[]
    meta_end_array = Integer[]
    metaposition = 1
    @inbounds for feature in eachrow(feature_df)
        push!(meta_start_array, (metaposition))
        metaposition += feature.End - feature.Start
        push!(meta_end_array, (metaposition))
        metaposition += 1
    end
    feature_df.MetaStart = meta_start_array # metacoordinate contains 'start' metaposition across all genomic material
    feature_df.MetaEnd = meta_end_array # metacoordinate contains 'end' metaposition across all genomic material
end

# function to convert metacoordinate set to scaffold-relative coordinates
function meta_to_feature_coord(meta_start::Integer, meta_end::Integer, feature_df::DataFrame)
    feature_row = get_feature_row_index(feature_df, meta_start)
    seqid = feature_df.SeqID[feature_row]
    scaffold_start = feature_df.Start[feature_row] + (meta_start - feature_df.MetaStart[feature_row])
    scaffold_end = feature_df.End[feature_row] - (feature_df.MetaEnd[feature_row] - meta_end)
    return seqid, scaffold_start, scaffold_end
end

#function to obtain the feature boundaries and strand of the feature that a metacoordinate falls within
function get_feature_params_from_metacoord(metacoordinate::Integer, feature_df::DataFrame, stranded::Bool)
    feature_row = get_feature_row_index(feature_df, metacoordinate)
    feature_metaStart = feature_df.MetaStart[feature_row]
    feature_metaEnd = feature_df.MetaEnd[feature_row]
    stranded ? feature_strand = feature_df.Strand[feature_row] : feature_strand = '0'
    return feature_metaStart, feature_metaEnd, feature_strand
end

#function obtain the index of a feature given its metacoordinate
```

```
function get_feature_row_index(feature_df::DataFrame, metacoordinate::Integer)
    total_feature_bases = feature_df.MetaEnd[end]
    if metacoordinate == total_feature_bases #edge case of metacoord at end of range
        return size(feature_df,1) #last index in df
    else
        index = findfirst(end_coord→end_coord>metacoordinate,feature_df.MetaEnd) #find the index of the feature whose end metacoord is > the query metacoord
        if index > 1 && metacoordinate == feature_df.MetaEnd[index-1] #if the metacoordinate is the last base of the previous feature
            if metacoordinate ≥ feature_df.MetaStart[index-1] && metacoordinate ≤ feature_df.MetaEnd[index-1] #confirm that the metacoord is in the previous feature
                return index-1 #return the previous feature index
            end
        elseif metacoordinate ≥ feature_df.MetaStart[index] && metacoordinate ≤ feature_df.MetaEnd[index] #else confirm that the metacoordinate is in the found feature
            return index #return the found feature index
        else
            throw(DomainError("Unexpected metacoordinate $metacoordinate in partition of $total_feature_bases bases, with feature start $(feature_df.MetaStart[index]), end $(feature_df.MetaEnd[index])"))
        end
    end
end
```

17.5.12 /src/likelihood_funcs/bg_lh_matrix.jl

```
#function to obtain positional likelihoods for a sequence under a given BHMM.
function get_BGHMM_symbol_lh(seq::AbstractMatrix, hmm::BHMM)
    @assert size(seq)[1] == 1
    (seq=Array(transpose(seq))) # one sequence at a time only
    symbol_lhs = zeros(length(seq))
    length_mask = [length(seq)-1]

```

```
    lls = bw_llhs(hmm, seq) #obtain log likelihoods for sequences and states
    log_α = messages_forwards_log(hmm.a, hmm.A, lls, length_mask) #get forward messages
    log_β = messages_backwards_log(hmm.A, lls, length_mask) # get backwards messages

    #calculate observation probability and γ weights
    K,Tmaxplus1,Strand = size(lls) #the last T value is the 0 end marker of the longest T

    #transforms to cut down log_ξ, log_γ assignment times
    lls = permutedims(lls, [2,1,3]) # from (K,T) to (T,K)
    log_α = permutedims(log_α, [2,1,3])
    log_β = permutedims(log_β, [2,1,3])

    log_γ = fill(-Inf, Tmaxplus1,K)
    log_pobs = logsumexp(lps.(log_α[1,:], log_β[1,:]))

    @inbounds for i = 1:K, t = 1:length_mask[1]
            log_γ[t,i] = lps(log_α[t,i],log_β[t,i],-log_pobs)
    end

    for t in 1:length_mask[1]
        symbol_lh::AbstractFloat = -Inf #ie log(p=0)
        for k = 1:K #iterate over states
                state_symbol_lh::AbstractFloat = lps(log_γ[t,k], log(hmm.B[k].p[seq[t]])) #state symbol likelihood is the γ weight * the state symbol probability (log implementation)
                symbol_lh = logaddexp(symbol_lh, state_symbol_lh) #sum the probabilities over states
        end
        symbol_lhs[t] = symbol_lh
    end

    return symbol_lhs[1:end-1] #remove trailing index position
end

#function to calculate BGHMM from an observation set and a dict of BGHMMs
function BGHMM_likelihood_calc(observations::DataFrame, BGHMM_dict::Dict, code_partition_dict = get_partition_code_dict(false); symbol=:PadSeq)
```

```
    lh_matrix_size = ((findmax(length.(collect(values(observations[!, symbol]))))[1]), length(observations[!, symbol]))
    BGHMM_lh_matrix = zeros(lh_matrix_size) #T, Strand, O

    BGHMM_fragments = fragment_observations_by_BGHMM(observations[!, symbol], observations.MaskMatrix)


    #@showprogress 1 "Writing frags to matrix.."
    Threads.@threads for (jobid, frag) in BGHMM_fragments
        (frag_start, o, partition, strand) = jobid

        partition_BGHMM::BHMM = BGHMM_dict[code_partition_dict[partition]]
        no_symbols = length(partition_BGHMM.B[1].p)
        order = Int(log(4,no_symbols) - 1)

        order_seq = get_order_n_seqs([frag], order)
        coded_seq = code_seqs(order_seq)

        subseq_symbol_lh = get_BGHMM_symbol_lh(coded_seq, partition_BGHMM)

        if strand == -1
            subseq_symbol_lh = reverse(subseq_symbol_lh)
        end #positive, unstranded frags  are inserted as-is
        BGHMM_lh_matrix[frag_start:frag_start+length(frag)-1,o] = subseq_symbol_lh
    end

    return BGHMM_lh_matrix
end

function fragment_observations_by_BGHMM(seqs::AbstractVector, masks::AbstractVector)
    likelihood_jobs = Vector{Tuple{Tuple,LongSequence{DNAAlphabet{2}}}}()
    @showprogress 1 "Fragmenting observations by partition..." for (o, obs_seq) in enumerate(seqs)
        mask = masks[o]
        frags = Vector{LongSequence{DNAAlphabet{2}}}() #container for all subsequences in observation
        frag = LongSequence{DNAAlphabet{2}}()

```

```
        frag_end=0
        frag_start = 1

        while frag_start < length(obs_seq) # while we're not at the sequence end
            curr_partition = mask[frag_start,1] #get the partition code of the frag start
            curr_strand = mask[frag_start,2] #get the strand of the frag start

            #JOBID COMPOSED HERE
            jobid = (frag_start, o, curr_partition, curr_strand) #compose an identifying index for this frag

            findnext(!isequal(curr_partition),mask[:,1],frag_start) ≠ nothing ? frag_end = findnext(!isequal(curr_partition),mask[:,1],frag_start) -1 : frag_end = length(obs_seq) #find the next position in the frag that has a different partition mask value from hte current one and set that position-1 to frag end, alternately frag end is end of the overall  sequence
            frag = obs_seq[frag_start:frag_end] #get the frag bases
            if curr_strand == -1 #if the fragment is reverse stranded
                reverse_complement!(frag) #use the reverse complement sequence
            end

            push!(likelihood_jobs,(jobid, frag)) #put the frag in the jobs vec
            frag_start = frag_end + 1 #move on
        end
    end
    return likelihood_jobs
end
```

17.5.13 /src/likelihood_funcs/hmm.jl

```
function obs_lh_given_hmm(observations, hmm; linear=true)
    linear ? (return linear_likelihood(observations, hmm)) : (return bw_likelihood(Matrix(transpose(observations)), hmm))
end

```

```
function bw_likelihood(observations, hmm)
    obs_lengths = [findfirst(iszero,observations[:,o])-1 for o in 1:size(observations,2)]

    lls = bw_llhs(hmm, observations)
    log_α = messages_forwards_log(hmm.a, hmm.A, lls, obs_lengths)
    log_β = messages_backwards_log(hmm.A, lls, obs_lengths)

    O = size(lls)[3] #the last T value is the 0 end marker of the longest T
    log_pobs = zeros(O)
    Threads.@threads for o in 1:O
        log_pobs[o] = logsumexp(lps.(log_α[:,1,o], log_β[:,1,o]))
    end

    return sum(log_pobs)
end

function linear_likelihood(observations,hmm)
    O= size(observations,1);
    obs_lengths = [findfirst(iszero,observations[o,:])-1 for o in 1:size(observations,1)] #mask calculations here rather than mle_step to prevent recalculation every iterate

    a = transpose(log.(hmm.a)); A = log.(hmm.A)
    N = length(hmm.B)
    mask=observations.≠0
    #INITIALIZATION
    βoi_T = zeros(O,N); βoi_t = zeros(O,N) #log betas at T initialised as zeros

    #RECURRENCE
    βoi_T = backwards_lh_sweep!(hmm, A, N, βoi_T, βoi_t, observations, mask, obs_lengths)

    #TERMINATION
    lls = c_llhs(hmm,observations[:,1])
    α1om = lls .+ a #first position forward msgs

    return lps([logsumexp(lps.(α1om[o,:], βoi_T[o,:])) for o in 1:O])
end
                #LINEAR_STEP SUBFUNCS
```

```
function backwards_lh_sweep!(hmm, A, N, βoi_T, βoi_t, observations, mask, obs_lengths)
    for t in maximum(obs_lengths)-1:-1:1
        last_β=copy(βoi_T)
        lls = c_llhs(hmm,observations[:,t+1])
        omask = findall(mask[:,t+1])
        βoi_T[omask,:] .+= view(lls,omask,:)
        Threads.@threads for m in 1:N
            βoi_t[omask,m] = logsumexp.(eachrow(view(βoi_T,omask,:).+transpose(vie
        end
        βoi_T=copy(βoi_t)
    end
    return βoi_T
end
```

17.5.14 /src/reports/chain_report.jl

```
struct Chain_Report
    id::Chain_ID
    final_hmm::BHMM
    test_lh::AbstractFloat
    naive_lh::AbstractFloat
    final_delta::AbstractFloat
    state_run_lengths::AbstractVector{AbstractFloat}
    convergence_values::Chains
    convergence_diagnostic::ChainDataFrame
    converged::Bool
end

function Base.show(io::IO, report::Chain_Report)
    nominal_dict=Dict(0=>"th",1=>"st",2=>"nd",3=>"rd",4=>"th",5=>"th")
    haskey(nominal_dict,report.id.order) ? (nom_str=nominal_dict[report.id.order]) : (nom_str="th")
    printstyled(io, stdout, "BHMM EM Chain Results\n"; bold=true)
    println(io, "$(report.id.K)-state, $(report.id.order)$nom_str order BHMM")
    println(io, "Trained on observation set \"$(report.id.obs_id)\"")
    report.test_lh > report.naive_lh ? (lh_str="greater"; lh_color=:green) : (lh_str="less"; lh_color=:red)
    printstyled(io, "Test logP(O|θ): $(report.test_lh), $lh_str than the naive model's $(report.naive_lh)\n"; color=lh_color)
```

```
    println(io, "Replicate $(report.id.replicate)")
    report.converged ? println(io, "Converged with final step δ $(report.final_delta)") : println(io, "Failed to converge!")
    println(io, " ── ─ ─ ─ ── ─ ─ ─ ── ─ ─ ─
```

```
    if report.convergence_diagnostic.name == "short"
        printstyled(io,"Convergence diagnostics unavailable for chains <10 steps!\n", color=:yellow)
    elseif report.convergence_diagnostic.name == "error"
        printstyled(io,"Convergence diagnostics errored! Zeros in autotransition matrix?\n", color=:red)
    else
        all(Bool.(report.convergence_diagnostic.nt.stationarity)) && all(Bool.(report.convergence_diagnostic.nt.test)) ? printstyled(io, "Likelihood and autotransition probabilites converged and passing tests.\n", color=:green) : printstyled(io, "Not all parameters converged or passing tests!\n",color=:red)
        display(report.convergence_diagnostic)
    end
end

function latex_report(report::Chain_Report)

end

function report_chains(chains::Dict{Chain_ID,Vector{EM_step}}, test_sets::Dict{String,Vector{LongSequence{DNAAlphabet{2}}}}; naive_hmm=BHMM(ones(1,1),[Categorical(4)]))
    job_ids=[id for (id,chain) in chains] # make list of chain_ids
    partitions=unique([id.obs_id for id in job_ids]) #get list of unique partitions represented among chain_ids
    naive_order = Integer(log(4,length(naive_hmm.B[1].support))-1)
    for partition in partitions #check that all partitions are available for testing and make sure codes for naive hmm are generated by adding job_ids
        !(partition in keys(test_sets)) && throw(ArgumentError("Observation set $partition required by chains for testing not present in test sets!"))
        push!(job_ids, Chain_ID(partition,length(naive_hmm.a),naive_order, 1))
    end

    code_dict=code_job_obs(job_ids, test_sets) #code all the obs sets that are necessary ot perform our tests
    naive_lhs=Dict{String,Float64}() #construct dict of naive likelihoods for all partitions to be tested prior to individual tests
```

```
    @showprogress 1 "Testing naive hmm ..." for ((partition,order),obs_set) in code_dict
        order==naive_order && (naive_lhs[partition]=obs_lh_given_hmm(code_dict[(partition,naive_order)]
    end

    reports=Dict{Chain_ID,Chain_Report}()
    @showprogress 1 "Testing chains ..." for (id, chain) in chains
        chains_array=zeros(length(chain),1+id.K)
        for (n,step) in enumerate(chain)
            chains_array[n,1]=step.log_p
            chains_array[n,2:end]=get_diagonal_array(step.hmm)
        end
        convergence_values=Chains(chains_array,["logP(O|θ)", ["K"*string(i) for i in 1:id.K] ... ])
        if length(chain) ≥ 10
            try
                convergence_diagnostic=heideldiag(convergence_values)[1]
            catch
                convergence_diagnostic=ChainDataFrame("error",(;:a⇒[1.]))
            end
        else
            convergence_diagnostic=ChainDataFrame("short",(;:a⇒[1.]))
        end

        id.K > 1 ? (mean_run_lengths=sim_run_lengths(get_diagonal_array(chain[end].hmm),1000
        : (mean_run_lengths=[Inf])
        test_lh=obs_lh_given_hmm(code_dict[(id.obs_id,id.order)], chain[end].hmm)
        reports[id]=Chain_Report(id, chain[end].hmm, test_lh, naive_lhs[id.obs_id], chain[end].delta, mean_run_lengths, convergence_values, convergence_diagnostic, chain[end].converged)
    end

    return reports
end

function get_diagonal_array(hmm::BHMM)
    k = length(hmm.a)
    diagonal = zeros(k)
```

```
    @inbounds for i in 1:k
        diagonal[i] = hmm.A[i,i]
    end
    return diagonal
end

#function to simulate run lengths for vector of diagonal values
function sim_run_lengths(diagonal_value::AbstractArray, samples::Integer)
    mean_run_lengths = zeros(length(diagonal_value))
    for (i, value) in enumerate(diagonal_value)
        if value == 0.
            mean_run_lengths[i]=0.
        elseif value ==1.
            mean_run_lengths[i]=Inf
        else
            runlengths = zeros(Integer, samples)
            for s in 1:samples
                run = true
                runlength = 0
                while run
                    runlength += 1
                    if rand(1)[1] > value
                        run = false
                    end
                end
                runlengths[s] = runlength
            end
            mean_run_lengths[i] = mean(runlengths)
        end
    end
    return mean_run_lengths
end
```

17.5.15 /src/reports/partition_report.jl

```
struct Order_Report
    converged_K::Vector{Float64}
    converged_lh::Vector{Float64}
    failed_K::Vector{Float64}
    failed_lh::Vector{Float64}
end

```

```
struct Partition_Report
    partition_id::String
    naive_lh::Float64
    orddict::Dict{Integer,Order_Report}
    best_model::Tuple{Chain_ID,BHMM}
    best_repset::Vector{Chain_ID}
end

function Base.show(io::IO, report::Partition_Report)
    printstyled(io, "BACKGROUND HMM TRAINING REPORT\n", bold=true)
    printstyled(io, "Genome partition id: $(report.partition_id)\n", bold=true, color=:magenta)

    for (order, ordreport) in report.orddict
        if length(ordreport.failed_lh)>0
            ordplot=scatterplot(ordreport.converged_K, ordreport.converged_lh; name="Converged", title="Order $order HMMs", xlabel="K# States", ylabel="P(O|θ)", color=:green, xlim=(1,maximum(ordreport.converged_K)), ylim=(floor(min(minimum(ordreport.converged_lh),minimum(ordreport.fai
            length(ordreport.failed_K)≥1 && scatterplot!(ordplot, ordreport.failed_K, ordreport.failed_lh; name="Unconverged",color=:red)
            lineplot!(ordplot, [report.naive_lh for i in 1:maximum(ordreport.converged_K)], name="Naive",color=:magenta)
            show(ordplot)
            println()
        else
            ordplot=scatterplot(ordreport.converged_K, ordreport.converged_lh; name="Converged", title="Order $order HMMs", xlabel="K# States", ylabel="P(O|θ)", color=:green, xlim=(1,maximum(ordreport.converged_K)), ylim=(floor(min(minimum(ordreport.converged_lh),report.naive_lh),sigd
            lineplot!(ordplot, [report.naive_lh for i in 1:maximum(ordreport.converged_K)], name="Naive",color=:magenta)
            show(ordplot)
            println()
        end
    end
end

```

```
function report_partitions(chain_reports::Dict{Chain_ID,Chain_Report})
    partitions=Vector{String}()
    orders=Vector{Integer}()
    Ks=Vector{Integer}()
    reps=Vector{Integer}()
    for id in keys(chain_reports)
        !in(id.obs_id, partitions) && push!(partitions, id.obs_id)
        !in(id.order, orders) && push!(orders, id.order)
        !in(id.K, Ks) && push!(Ks, id.K)
        !in(id.replicate, reps) && push!(reps, id.replicate)
    end

    reports=Vector{Partition_Report}()
    for partition in partitions
        best_lh,best_rep=-Inf,Chain_ID("empty",1,0,1)
        naive_lh=1.
        orddict=Dict{Integer,Order_Report}()
        for order in orders
            conv_statevec=Vector{Float64}()
            fail_statevec=Vector{Float64}()
            conv_lh_vec=Vector{Float64}()
            fail_lh_vec=Vector{Float64}()
            for (id,report) in chain_reports
                if id.obs_id==partition && id.order==order
                    report.test_lh > best_lh && (best_lh=report.test_lh;best_rep=id)
                    naive_lh==1. && (naive_lh=chain_reports[id].naive_lh)
                    chain_reports[id].converged ? (push!(conv_statevec,id.K);
                    push!(conv_lh_vec,chain_reports[id].test_lh)) : (push!(fail_statevec,id.K);
                    push!(fail_lh_vec,chain_reports[id].test_lh))
                end
            end

            orddict[order]=Order_Report(conv_statevec,conv_lh_vec,fail_statevec,f
        end
        best_model=(best_rep,chain_reports[best_rep].final_hmm)
        best_repset=[Chain_ID(partition, best_rep.K, best_rep.order, rep) for rep in reps]

        push!(reports,Partition_Report(partition,naive_lh,orddict,best_model,best_
    end
```

```
    return reports
end
```

17.5.16 /src/reports/replicate_convergence.jl

```
struct Replicate_Report
    ids::Vector{Chain_ID} #replicates
    state_vecs::Dict{Chain_ID,Matrix{Float64}} #vectors of autotransition probabilities evolving by iterate, concatanated to matrices, indexed by Chain_ID
    emission_arrays::Dict{Chain_ID,Array{Float64}} #iterate*symbol*state_vecs
    sorted_id1_states::Vector{Integer}
    sort_dicts::Dict{Chain_ID,Dict{Integer,Integer}}#emission channels sorted on the basis of euclidean closeness to ids[1]
    sorted_symbols::Dict{Integer,Vector{Integer}}


    Replicate_Report(ids,state_vecs,emission_arrays, sorted_id1_states,sort_dicts,sorted_symbols) = assert_repreport(ids) && new(ids,state_vecs,emission_arrays,sorted_id1_states,sort_dicts,sorted_symbol
end

function assert_repreport(ids::Vector{Chain_ID})
    Ks=Vector{Int64}()
    orders=Vector{Int64}()
    replicates=Vector{Int64}()
    obsids=Vector{String}()

    for id in ids
        push!(Ks,id.K)
        push!(orders,id.order)
        push!(obsids,id.obs_id)
        push!(replicates, id.replicate)
    end

    length(unique(Ks)) > 1 && throw(ArgumentError("Replicate set includes chains with different K state numbers! All chains must have HMMs with the same number of states."))
```

```
    length(unique(orders)) > 1 && throw(ArgumentError("Replicate set includes chains with different order coding numbers! All chains must have HMMs with the same order coding."))
    !allunique(replicates) && throw(ArgumentError("Replicate set includes chains with the same replicate #. Replicates should be unique!"))

    return true
end

function report_replicates(repset::Vector{Chain_ID},chains::Dict{Chain_ID,Vector{EM_step}}
    assert_repreport(repset) #check that the repset will pass its constructor before doing the work
    emission_arrays=Dict{Chain_ID,AbstractArray{AbstractFloat}}()
    state_arrays=Dict{Chain_ID,AbstractArray{AbstractFloat}}()

    for id in repset
        emission_array=zeros(length(chains[id]),4^(id.order+1),id.K)
        state_array=zeros(length(chains[id]),id.K)
        for (it, step) in enumerate(chains[id])
            for k in 1:id.K
                emission_array[it,:,k]=step.hmm.B[k].p
                state_array[it,k]=step.hmm.A[k,k]
            end
        end
        emission_arrays[id]=emission_array
        state_arrays[id]=state_array
    end

    println(repset)
    println(emission_arrays)
    println(state_arrays)

    sort_dicts,sorted_id1_states,sorted_symbols=sort_emitters_by_distance!(repset

    return Replicate_Report(repset,state_arrays,emission_arrays,sorted_id1_states, sort_dicts,sorted_symbols)
end

function sort_emitters_by_distance!(repset,emission_arrays)
    id1=repset[1]
```

```
    length(repset)==1 && (return Dict{Chain_ID,Dict{Integer,Integer}}(),[1:id1.K ... ],[1:4^id1.order+1])

    final_emissions = zeros(length(repset),size(emission_arrays[id1])[2:3] ... )
    for (n,id) in enumerate(repset)
        final_emissions[n,:,:] = emission_arrays[id][end,:,:]
    end

    distances=zeros(length(repset)-1,id1.K,id1.K)
    for rep in 1:length(repset)-1
        for k in 1:id1.K, j in 1:id1.K
            distances[rep,k,j]=euclidean(final_emissions[1,:,k],final_emissions[1
        end
    end

    sortdicts=Dict{Chain_ID,Dict{Integer,Integer}}()
    sorted_id1_states=Vector{Integer}()
    sorted_symbols=Dict{Integer,Vector{Integer}}()
    for rep in 1:length(repset)-1
        sortdicts[repset[rep+1]]=Dict{Integer,Integer}()
    end

    for i in 1:id1.K
        id1_mindist_K=findmin([sum([minimum(distances[rep,k,:]) for rep in 1:length(repset)-1]) for k in 1:id1.K])[2]#find the state from the first replicate that has minimum cumulative distance to the closest state in the other replicates, excluding already-chosen states by spiking their values
        push!(sorted_id1_states,id1_mindist_K)
        for rep in 1:length(repset)-1
            rep_k=findmin(distances[rep,id1_mindist_K,:])[2]
            sortdicts[repset[rep+1]][id1_mindist_K]=rep_k
            distances[rep,:,rep_k]•=1.0 #mask
            distances[1,id1_mindist_K,:]•=1.0 #mask
        end


        symbol_distances=sum(hcat([euclidean.(final_emissions[1,:,id1_mindist_K], for (n,id) in enumerate(repset[2:end])] ... ),dims=2)
        sorted_symbols[id1_mindist_K]=sortperm(symbol_distances[:,1])
    end
```

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    return sortdicts,sorted_id1_states,sorted_symbols
end


function Base.show(io::IO, report::Replicate_Report; top_states::Integer=report.ids[1].K, top_symbols::Integer=2)
    printstyled(io, "REPLICATE CONVERGENCE REPORT\n", color=:green)
    println("────────────────────────────────────────────────────────────────────────────────────────────────────")
    for i in 1:top_states
        id=report.ids[1]
        id1_state=report.sorted_id1_states[i]
        itmax=maximum([size(report.state_vecs[id],1) for id in report.ids])

        autoplt=lineplot(report.state_vecs[id][:,id1_state],
                        title="Autotransition prob. convergence",
                        name="$(report.ids[1]) K$id1_state",
                        xlim=(0,ceil(itmax, sigdigits=2)),
                        ylim=(0,1),
                        xlabel="iterate",
                        ylabel="p")


        symplot=lineplot(report.emission_arrays[id][:,report.sorted_symbols[id1_s

                        report.emission_arrays[id][:,report.sorted_symbols[id1_st
                        title="Symbol prob. convergence",
                        name="$(report.ids[1]) K$id1_state",
                        xlim=(0,1),
                        ylim=(0,1),
                        xlabel="Symbol 1 p",
                        ylabel="S2 p")

        for (n,id) in enumerate(report.ids)
            if n > 1
                lineplot!(autoplt, report.state_vecs[id][:,report.sort_dicts[id][id1_state]],
                            name="$(report.ids[n]) K$(report.sort_dicts[id][id1_state])")
                lineplot!(symplot,
```

```
                                ↪  report.emission_arrays[id][:,report.sorted_symbols[id
                                ↪  report.emission_arrays[id][:,report.sorted_symbols[id
                               name="$(report.ids[n])
                                ↪  K$(report.sort_dicts[id][id1_state])")
            end
        end

        show(autoplt)
        println("\n")
        show(symplot)
        println("\n")
    end
end
```

17.5.17 /src/utilities/BBG_analysis.jl

```
#function to produce matrix of hmm chain parameter coordinate evolution-
 ↪  selects 3 matched emission parameters from 3+ state HMMs, one per
 ↪  state for 3 states. Matching is performed by minimizing euclidean
 ↪  distance between the state emission probability vectors B for the
 ↪  states to be matched (as EM-optimised replicates will have similar
 ↪  states organized in different orders)

function chain_3devo_coords(chains::Vector{Vector{Any}})
    length(chains)≤0 && throw(ArgumentError, "Argument must be vector of
     ↪  more than one hmm chain")
    coords=[Vector{Tuple{AbstractFloat,AbstractFloat,AbstractFloat}}()
     ↪  for i in 1:length(chains)]

    for step in chains[1]
        hmm=step[2]
        length(hmm.B)<3 && throw(ArgumentError, "3- or greater state hmms
         ↪  required")
        push!(coords[1], (hmm.B[1].p[1],hmm.B[2].p[1],hmm.B[3].p[1]))
    end

    for (c, chain) in enumerate(chains[2:end])
        ref_idxs=Vector{Integer}()

         ↪  ref_vecs=[chains[1][end][2].B[1].p,chains[1][end][2].B[2].p,chains[1][end
```

```
        end_hmm=chain[end][2]
        length(end_hmm.B)<3 && throw(ArgumentError, "3- or greater state hmms required")

        for vec in ref_vecs
            euclideans=Vector{AbstractFloat}()
            for d in 1:length(end_hmm.B)
                push!(euclideans, euclidean(vec, end_hmm.B[d].p))
            end
            min_euc_idx = findmin(euclideans)[2]
            while min_euc_idx in ref_idxs
                euclideans[min_euc_idx]=1.
                min_euc_idx = findmin(euclideans)[2]
            end
            push!(ref_idxs,findmin(euclideans)[2])
        end

        for step in chain
            hmm=step[2]
            push!(coords[c+1], (hmm.B[ref_idxs[1]].p[1],hmm.B[ref_idxs[2]].p[1],hmm.B[ref_idxs[3]].p
        end
    end

    return coords
end
```

17.5.18 /src/utilities/BBG_progressmeter.jl

```
#UTILITY PROGRESSMETER, REPORTS WORKER NUMBER AND CURRENT ITERATE
mutable struct ProgressHMM{T<:Real} <: ProgressMeter.AbstractProgress
    thresh::T
    dt::AbstractFloat
    val::T
    counter::Integer
    triggered::Bool
    tfirst::AbstractFloat
    tlast::AbstractFloat
    printed::Bool        # true if we have issued at least one status update
    desc::AbstractString # prefix to the percentage, e.g.  "Computing..."
    color::Symbol        # default to green
```

```
    output::IO            # output stream into which the progress is written
    numprintedvalues::Integer   # num values printed below progress in last iteration
    offset::Integer             # position offset of progress bar (default is 0)
    steptime::AbstractFloat
    function ProgressHMM{T}(thresh;
                                dt::Real=0.1,
                                desc::AbstractString="Progress: ",
                                color::Symbol=:green,
                                output::IO=stderr,
                                offset::Integer=0,
                                start_it::Integer=1) where T
        tfirst = tlast = time()
        printed = false
        new{T}(thresh, dt, typemax(T), start_it, false, tfirst, tlast, printed, desc, color, output, 0, offset, 0.0)
    end
end

ProgressHMM(thresh::Real, dt::Real=0.1, desc::AbstractString="Progress: ",
         color::Symbol=:green, output::IO=stderr;
         offset::Integer=0, start_it::Integer=1) = ProgressHMM{typeof(thresh)}(thresh, dt=dt, desc=desc, color=color, output=output, offset=offset, start_it=start_it)

ProgressHMM(thresh::Real, desc::AbstractString, offset::Integer=0, start_it::Integer=1) = ProgressHMM{typeof(thresh)}(thresh, desc=desc, offset=offset, start_it=start_it)

function update!(p::ProgressHMM, val, steptime; options...)
    p.val = val
    p.counter += 1
    p.steptime = steptime
    updateProgress!(p; options...)
end

function updateProgress!(p::ProgressHMM; showvalues = Any[], valuecolor = :blue, offset::Integer = p.offset, keep = (offset == 0))
    p.offset = offset
```

```
    t = time()
    if p.val ≤ p.thresh && !p.triggered
        p.triggered = true
        if p.printed
            p.triggered = true
            dur = ProgressMeter.durationstring(t-p.tfirst)
            msg = @sprintf "%s Time: %s (%d iterations)" p.desc dur p.counter
            print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
            ProgressMeter.move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
            ProgressMeter.printover(p.output, msg, p.color)
            ProgressMeter.printvalues!(p, showvalues; color = valuecolor)
            if keep
                println(p.output)
            else
                print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))
            end
        end
        return
    end

    if t > p.tlast+p.dt && !p.triggered
        elapsed_time = t - p.tfirst
        msg = @sprintf "%s (convergence: %g ⇒ %g, iterate: %g, step time: %s)" p.desc p.val p.thresh p.counter hmss(p.steptime)
        print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
        ProgressMeter.move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
        ProgressMeter.printover(p.output, msg, p.color)
        ProgressMeter.printvalues!(p, showvalues; color = valuecolor)
        print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))
        # Compensate for any overhead of printing. This can be
        # especially important if you're running over a slow network
        # connection.
        p.tlast = t + 2*(time()-t)
        p.printed = true
    end
end

            function hmss(dt)
                isnan(dt) && return "NaN"
```

```
                (h,r) = divrem(dt,60*60)
                (m,r) = divrem(r, 60)
                (isnan(h)||isnan(m)||isnan(r)) && return "NaN"
                string(Int(h),":",Int(m),":",Int(ceil(r)))
            end
```

17.5.19 /src/utilities/HMM_init.jl

```
function autotransition_init(K::Integer, order::Integer, partition::String="")
        a = rand(Dirichlet(ones(K)/K)) #uninformative prior on initial state probabilities
        A = strong_autotrans_matrix(K)
        no_emission_symbols = Int(4^(order+1)) #alphabet size for the order
        emission_dists = [generate_emission_dist(no_emission_symbols) for i in 1:K]
        #generate the BHMM with the appropriate transition matrix and emissions distributions
        hmm = BHMM(a, A, emission_dists, partition)
end
            #function to construct BHMM transition matrix with strong priors on auto-transition
            function strong_autotrans_matrix(states::Integer, prior_dope::AbstractFloat=(states*250.0), prior_background::AbstractFloat=.1)
                transition_matrix=zeros(states,states)
                for k in 1:states
                    dirichlet_params = fill(prior_background, states)
                    dirichlet_params[k] = prior_dope
                    transition_matrix[k,:] = rand(Dirichlet(dirichlet_params))
                end
                return transition_matrix
            end

            #function to construct BHMM state emission distribution from uninformative dirichlet over the alphabet size
            function generate_emission_dist(no_emission_symbols, prior=Dirichlet(ones(no_emission_symbols)/no_emission_symbols))
                return Categorical(rand(prior))
            end
```

17.5.20 /src/utilities/load_balancer.jl

```
struct LoadConfig
    k_range::UnitRange
    o_range::UnitRange
    blacklist::Vector{Chain_ID}
    whitelist::Vector{Chain_ID}
    function LoadConfig(k_range, o_range; blacklist=Vector{Chain_ID}(), whitelist=Vector{Chain_ID}())
        assert_loadconfig(k_range, o_range) && new(k_range, o_range, blacklist, whitelist)
    end
end

function assert_loadconfig(k_range,o_range)
    k_range[1] < 1 && throw(ArgumentError("Minimum value of LoadConfig k_range is 1!"))
    o_range[1] < 0 && throw(ArgumentError("Minimum value of LoadConfig o_range is 0!"))
    return true
end

#subfunc to handle balancing memory load on dissimilar machines in cluster
function load_balancer(no_models::Integer, hmm_jobs::RemoteChannel, config::LoadConfig, timeout::Integer=300)
    jobid::Chain_ID, start_iterate::Integer, hmm::BHMM, job_norm::AbstractFloat, observations::Matrix = take!(hmm_jobs)

    starttime=time()
    while (jobid.K < config.k_range[1] || jobid.K > config.k_range[end] || jobid.order < config.o_range[1] || jobid.order > config.o_range[end] || jobid in config.blacklist || (length(config.whitelist)>0 && !(jobid in config.whitelist))) && (time()-starttime ≤ timeout) #while a job prohibited by load table, keep putting the job back and drawing a new one
        put!(hmm_jobs, (jobid, start_iterate, hmm, job_norm, observations))
        jobid, start_iterate, hmm, job_norm, observations = take!(hmm_jobs)
    end
```

```
    (time()-starttime > timeout) ? (return 0,0,0,0,0) : (return jobid, start_iterate, hmm, job_norm, observations)
end
```

17.5.21 /src/utilities/log_prob_sum.jl

```
                #subfuncs to handle sums of log probabilities that may include -Inf (ie p=0), returning -Inf in this case rather than NaNs
                function lps(adjuvants)
                    prob = sum(adjuvants) ; isnan(prob) ? - Inf : prob
                end

                function lps(base, adjuvants... )
                    prob = base+sum(adjuvants) ; isnan(prob) ? -Inf : prob
                end
```

17.5.22 /src/utilities/model_display.jl

```
cs_dict=Dict('A'⇒:green,'C'⇒:blue, 'G'⇒:yellow, 'T'⇒:red)

function print_emitters(state::Categorical) #print informative symbols from the state's emission distribution, if any
    order=Integer(log(4,length(state.p))-1)
    alphabet=CompoundAlphabet(ACGT,order)
    bits=log(2,length(state.p)) #higher order hmms express more information per symbol
    dummy_p=state.p.+10^-99 #prevent -Inf or NaN from zero probability symbols
    infoscore=(bits+sum(x*log(2,x) for x in dummy_p))
    infovec=[x*infoscore for x in dummy_p]
    infovec./=bits
    print("<<< ")
    for (symbol,symbol_prob) in enumerate(infovec)
        if symbol_prob ≥ .05
            str=string(alphabet.integers[symbol])
            if symbol_prob ≥ .7
                for char in str
                    printstyled(stdout, char; bold=true, color=cs_dict[char])
```

```
                end
            elseif .7 > symbol_prob ≥ .25
                for char in str
                    printstyled(stdout, char; color=cs_dict[char])
                end
            elseif .25 > symbol_prob
                for char in str
                    printstyled(stdout, lowercase(char);
                    ↪  color=cs_dict[char])
                end
            end
            print(" ")
        end
    end
    println(">>>")
end
```

17.5.23 /src/utilities/observation_coding.jl

```
#KMER ORDER/SEQUENCE INTEGER CODING UTILITIES
#higher order DNA alphabet
struct CompoundAlphabet
  symbols::Dict{Mer{DNAAlphabet{2}},Integer}
  integers::Dict{Integer,Mer{DNAAlphabet{2}}}
  #build a CompoundAlphabet for DNA of some order_no
    function CompoundAlphabet(alphabet::Tuple, order_no::Integer)
        symbols = Dict{Mer{DNAAlphabet{2}},Integer}()
        integers = Dict{Integer,Mer{DNAAlphabet{2}}}()

        tuples = Array{Tuple}
        if order_no > 0
            tuples = collect(Iterators.product([alphabet for order in
            ↪  0:order_no]...))
        else #0th order compound product of an alphabet is that alphabet
            tuples = collect(Iterators.product(alphabet))
        end

        @inbounds for index in eachindex(tuples)
            tuple_seq = Mer{DNAAlphabet{2}}(collect(tuples[index]))
            integers[index] = tuple_seq
            symbols[tuple_seq] = index
        end
```

```julia

        new(symbols,integers)
    end
end


struct N_Order_ntSequence
    alphabet::CompoundAlphabet
    seq_lengths::Vector{Integer}
    order_kmers::Vector{Vector{Mer{DNAAlphabet{2}}}}
  end


function code_job_obs(job_ids::Vector{Chain_ID}, obs_sets::Dict{String,Vector{LongSequence{DNAAlphabet{2}}}})
    code_jobs=Vector{Tuple{String,Integer}}()
    for id in job_ids #assemble a vector of observation encoding jobs
        code_job=(id.obs_id, id.order)
        !(code_job in code_jobs) && push!(code_jobs, code_job)
    end

    code_dict = Dict{Tuple{String,Integer}, AbstractArray}()
    @showprogress 1 "Encoding observations..." for (obs_id, order) in code_jobs #build the appropriate sample sets once
        order_seqs = get_order_n_seqs(obs_sets[obs_id],order) #get the kmer sequences at the appropriate order
        coded_seqs = code_seqs(order_seqs, sorted=true) #numerically code the sequences in trainable format
        code_dict[(obs_id, order)] = coded_seqs
    end
    return code_dict
end

#from a vector of LongSequences, get
function get_order_n_seqs(seqs::Vector{LongSequence{DNAAlphabet{2}}}, order_no::Integer, base_tuple::Tuple=ACGT)
    kmer_vecs = Vector{Vector{Mer{DNAAlphabet{2}}}}()
    length_vec = Vector{Integer}()
    window = order_no + 1

    for seq in seqs
        kmer_vec = Vector{Mer{DNAAlphabet{2}}}()
```

```
        @inbounds for (i, kmer) in collect(each(Mer{DNAAlphabet{2},window},seq))
            push!(kmer_vec, kmer)
        end

        push!(kmer_vecs, kmer_vec)
        push!(length_vec, length(seq))
    end

    return nordseqs = N_Order_ntSequence(CompoundAlphabet(base_tuple, order_no), length_vec, kmer_vecs)
end

#convert tuple kmers to symbol codes
function code_seqs(input::N_Order_ntSequence, offsets::Vector=[0 for i in 1:length(input.order_kmers)]; sorted::Bool=false)
    symbol_no=length(input.alphabet.symbols)
    if symbol_no ≤ typemax(UInt8)
        integer_type = UInt8
    elseif typemax(UInt8) < symbol_no < typemax(UInt16)
        integer_type = UInt16
    else
        integer_type = UInt32
    end

    alphabet = input.alphabet
    output = zeros(integer_type,  length(input.order_kmers), (maximum([length(seq) for seq in input.order_kmers])+1)) #leave 1 missing value after the longest sequence forindexing sequence length in CLHMM messages
    sorted && (sort_idxs = sortperm(input.seq_lengths,rev=true))
    sorted ? input_seqs = input.order_kmers[sort_idxs] : input_seqs = input.order_kmers

    for (i, seq) in enumerate(input_seqs)
        for t in 1:length(seq)
            curr_kmer = seq[t]
            curr_code = alphabet.symbols[curr_kmer]
            output[i,t+offsets[i]]=curr_code
        end
    end
    return output
end
```

17.5.24 /src/utilities/utilities.jl

```
"""
Utility functions for learning and using background genomic hidden markov models
"""
#function to split random sample dataframe into training and test sets (divide total sequence length by half)
function split_obs_sets(sample_dfs::Dict{String,DataFrame})
    training_sets = Dict{String,Vector{LongSequence{DNAAlphabet{2}}}}()
    test_sets = Dict{String,Vector{LongSequence{DNAAlphabet{2}}}}()

    for (partition_id, partition) in sample_dfs
        partition = partition[shuffle(1:size(partition, 1)),:] #shuffle the samples to avoid any effect from sampling without replacement
        partition.sampleLength = (partition.SampleEnd - partition.SampleStart) .+ 1
        midway = sum(partition.sampleLength)÷2
        split_index = 0
        counter = 0
        while split_index == 0
            counter += 1
            length_sum = sum(partition.sampleLength[1:counter])
            if length_sum > midway
                split_index = counter
            end
        end

        training_sets[partition_id]  = partition.SampleSequence[1:split_index-1]
        test_sets[partition_id] = partition.SampleSequence[split_index:end]
    end
    return training_sets, test_sets
end
```

17.5.25 /test/ref_fns.jl

```
#slow algo written in mimicry of Churbanov & Winters
function old_linear(hmm, observations, obs_lengths)
    O = size(observations)[2]
    a = log.(hmm.A); a = log.(hmm.a)
    N = length(hmm.B); B = length(hmm.B[1].support); b = [log(hmm.B[m].p[γ]) for m in 1:N, γ in 1:B]
    α1oi = zeros(O,N); β1oi = zeros(O,N); Eoi = zeros(O,N,B); Toij = zeros(O,N,N); Aoi = zeros(O,N); log_pobs=zeros(O); γt=0

    for o in 1:O
        #INITIALIZATION
        T = obs_lengths[o]; βoi_T = zeros(N) #log betas at T initialised as zeros
        EiT = fill(-Inf,B,N,N); TijT = fill(-Inf,N,N,N) #Ti,j(T,m) = 0 for all m; in logspace

        @inbounds for m in 1:N, i in 1:N, γ in 1:B
            observations[T, o] == γ && m == i ? (EiT[γ, i, m] = 0) :
                (EiT[γ, i, m] = -Inf) #log Ei initialisation
        end

        #RECURRENCE
        @inbounds for t in T-1:-1:1
            βoi_t = similar(βoi_T); Tijt = similar(TijT); Eit = similar(EiT)
            Γ = observations[t+1,o]; γt = observations[t,o]
            Γ==0 && println("hooo!! $o $t")
            for m in 1:N
                βoi_t[m] = logsumexp([lps(a[m,j], b[j,Γ], βoi_T[j]) for j in 1:N])
                for i in 1:N
                    for j in 1:N
                        Tijt[i, j, m] = logsumexp([lps(a[m,n], TijT[i, j, n], b[n,Γ]) for n in 1:N])
                        i==m && (Tijt[i, j, m] = logaddexp(Tijt[i, j, m], lps(βoi_T[j], a[m,j], b[j, Γ])))
                    end
                    for γ in 1:B
                        Eit[γ, i, m] = logsumexp([lps(b[n,Γ], a[m,n], EiT[γ, i, n]) for n in 1:N])
```

```
                        i==m && γ==γt && (Eit[γ, i, m] = logaddexp(Eit[γ, i, m], βoi_t[m]))
                    end
                end
            end
            βoi_T = βoi_t; TijT = Tijt; EiT = Eit
        end

        #TERMINATION
        Γ = observations[1,o]
        α1oi[o,:] = [lps(a[i], b[i, Γ]) for i in 1:N]
        β1oi[o,:] = βoi_T
        log_pobs[o] = logsumexp(lps.(α1oi[o,:], βoi_T[:]))
        Eoi[o,:,:] = [logsumexp([lps(EiT[γ,i,m], a[m], b[m,γt]) for m in 1:N]) for i in 1:N, γ in 1:B]
        Toij[o,:,:] = [logsumexp([lps(TijT[i,j,m], a[m], b[m,γt]) for m in 1:N]) for i in 1:N, j in 1:N]
    end

    #INTEGRATE ACROSS OBSERVATIONS AND SOLVE FOR NEW BHMM PARAMS
    obs_penalty=log(O)
    a_o=α1oi.+β1oi•-logsumexp.(eachrow(α1oi.+β1oi))
    new_a=logsumexp.(eachcol(a_o))•-obs_penalty

    a_int = Toij•-logsumexp.([Toij[o,i,:] for o in 1:O, i in 1:N])
    new_a = logsumexp.([a_int[:,i,j] for i in 1:N, j in 1:N])•-obs_penalty

    e_int=Eoi•-logsumexp.([Eoi[o,j,:] for o in 1:O, j in 1:N])
    new_b=logsumexp.([e_int[:,j,d] for d in 1:B, j in 1:N])•-obs_penalty
    new_D=[Categorical(exp.(new_b[:,i])) for i in 1:N]

    return typeof(hmm)(exp.(new_a), exp.(new_a), new_D), lps(log_pobs)
end


#HMMBase
function mouchet_mle_step(hmm::BHMM, observations) where F
    # NOTE: This function works but there is room for improvement.

    log_likelihoods = mouchet_log_likelihoods(hmm, observations)

    log_α = mouchet_messages_forwards_log(hmm.a, hmm.A, log_likelihoods)
```

```
    log_β = mouchet_messages_backwards_log(hmm.A, log_likelihoods)
    log_A = log.(hmm.A)

    normalizer = logsumexp(log_α[1,:] + log_β[1,:])

    # E-step

    T, K = size(log_likelihoods)
    log_ξ = zeros(T, K, K)

    @inbounds for t = 1:T-1, i = 1:K, j = 1:K
        log_ξ[t,i,j] = log_α[t,i] + log_A[i,j] + log_β[t+1,j] + log_likelihoods[t+1,j] - normalizer
    end

    ξ = exp.(log_ξ)
    ξ ./= sum(ξ, dims=[2,3])

    # M-step
    new_A = sum(ξ[1:end-1,:,:], dims=1)[1,:,:] #index fix-MM
    new_A ./= sum(new_A, dims=2)

    new_a = exp.((log_α[1,:] + log_β[1,:]) .- normalizer)
    new_a ./= sum(new_a)

    # TODO: Cleanup/optimize this part
    γ = exp.((log_α .+ log_β) .- normalizer)

    B = Categorical[]
    for (i, d) in enumerate(hmm.B)
        # Super hacky ...
        # https://github.com/JuliaStats/Distributions.jl/issues/809
        push!(B, fit_mle(Categorical, permutedims(observations), γ[:,i]))
    end

    typeof(hmm)(new_a, new_A, B), normalizer
end


function mouchet_log_likelihoods(hmm, observations)
    hcat(map(d -> logpdf.(d, observations), hmm.B)...)
end

```

```julia


function mouchet_messages_forwards_log(init_distn, trans_matrix, log_likelihoods)
    # OPTIMIZE
    log_alphas = zeros(size(log_likelihoods))
    log_trans_matrix = log.(trans_matrix)
    log_alphas[1,:] = log.(init_distn) .+ log_likelihoods[1,:]
    @inbounds for t = 2:size(log_alphas)[1]
        for i in 1:size(log_alphas)[2]
            log_alphas[t,i] = logsumexp(log_alphas[t-1,:] .+ log_trans_matrix[:,i]) + log_likelihoods[t,i]
        end
    end
    log_alphas
end

function mouchet_messages_backwards_log(trans_matrix, log_likelihoods)
    # OPTIMIZE
    log_betas = zeros(size(log_likelihoods))
    log_trans_matrix = log.(trans_matrix)
    @inbounds for t = size(log_betas)[1]-1:-1:1
        tmp = view(log_betas, t+1, :) .+ view(log_likelihoods, t+1, :)
        @inbounds for i in 1:size(log_betas)[2]
            log_betas[t,i] = logsumexp(view(log_trans_matrix, i, :) .+ tmp)
        end
    end
    log_betas
end
```

17.5.26 /test/runtests.jl

```
using BioBackgroundModels,BioSequences,DataFrames,Distributed,Distributions,FASTX,Progr
import BioBackgroundModels: autotransition_init, load_balancer, CompoundAlphabet, get_order_n_seqs, code_seqs, code_job_obs, make_padded_df, add_partition_masks!, partition_genome_coordinates, divide_partitions_by_scaffold, mask_sequence_by_partition, find_position_partition, setup_sample_jobs, rectify_identifier, add_metacoordinates!, meta_to_feature_coord, get_feature_row_index, get_strand_dict, build_scaffold_seq_dict, get_feature_params_from_metacoord, determine_sample_window, fetch_sequence, get_sample_set, get_sample, assert_hmm, linear_step, get_BGHMM_symbol_lh,make_padded_df,add_partition_masks!,BGHMM_likelihood_calc,lin
t_add_categorical_counts!
import Serialization: deserialize
include("synthetic_sequence_gen.jl")
include("ref_fns.jl")

#JOB FILEPATHS
#GFF3 feature database, FASTA genome and index paths
Sys.islinux() ? genome = (@__DIR__) * "/synthetic.fna" : genome = (@__DIR__) * "\\synthetic.fna"
Sys.islinux() ? index = (@__DIR__) * "/synthetic.fna.fai" : index = (@__DIR__) * "\\synthetic.fna.fai"
Sys.islinux() ? gff = (@__DIR__) * "/synthetic.gff3" : gff = (@__DIR__) * "\\synthetic.gff3"
Sys.islinux() ? posfasta = (@__DIR__) * "/syntheticpos.fa" : posfasta = (@__DIR__) * "\\syntheticpos.fa"

!isfile(genome) && print_synthetic_fasta(genome)
!isfile(index) && print_synthetic_index(index)
!isfile(gff) && print_synthetic_gff(gff)
!isfile(posfasta) && print_synthetic_position(posfasta)

Random.seed!(1)

@testset "BHMM/BHMM.jl constructors" begin
    good_a = [.20,.30,.20,.30]
    bad_probvec_a = [.25,.25,.25,.27]
    good_A = fill(.25,4,4)
    bad_probvec_A = deepcopy(good_A)
    bad_probvec_A[:,3]•=.27
```

```
    bad_size_A = fill(.25,5,4)
    good_D = Categorical([.25,.25,.25,.25])
    bad_size_D = Categorical([.2,.2,.2,.2,.2])
    good_Dvec = [good_D for i in 1:4]
    bad_size_Dvec =[good_D, good_D, good_D, bad_size_D]
    bad_length_Dvec=[good_D for i in 1:5]

    @test_throws ArgumentError BHMM(bad_probvec_a, good_A, good_Dvec)
    @test_throws ArgumentError BHMM(good_a, bad_probvec_A, good_Dvec)
    @test_throws ArgumentError BHMM(good_a, bad_size_A, good_Dvec)
    @test_throws ArgumentError BHMM(good_a, good_A, bad_size_Dvec)
    @test_throws ArgumentError BHMM(good_a, good_A, bad_length_Dvec)

    hmm=BHMM(good_a, good_A, good_Dvec)
    @test typeof(hmm)==BHMM{Float64}
    @test hmm.a==good_a
    @test hmm.A==good_A
    @test hmm.B==good_Dvec
    @test size(hmm) == (4,4)

    hmm_copy=copy(hmm)
    @test hmm.a==hmm_copy.a
    @test hmm.A==hmm_copy.A
    @test hmm.B==hmm_copy.B

    hmm2=BHMM(good_A, good_Dvec)
    @test hmm2.a==[.25,.25,.25,.25]
    @test hmm2.A==good_A
    @test hmm2.B==good_Dvec
end

@testset "BHMM/chain.jl Chain_ID and EM_step constructors" begin
    obs_id="test"
    K,zero_k,neg_K=4,0,-1
    order,badorder=0,-1
    replicate,zero_rep,neg_rep=1,0,-1

    @test_throws ArgumentError Chain_ID(obs_id, zero_k, order, replicate)
    @test_throws ArgumentError Chain_ID(obs_id, neg_K, order, replicate)
    @test_throws ArgumentError Chain_ID(obs_id, K, badorder, replicate)
    @test_throws ArgumentError Chain_ID(obs_id, K, order, zero_rep)
    @test_throws ArgumentError Chain_ID(obs_id, K, order, neg_rep)

```

```
    id=Chain_ID(obs_id, K, order, replicate)
    @test typeof(id)==Chain_ID
    @test id.obs_id==obs_id
    @test id.K==K
    @test id.order==order
    @test id.replicate==replicate

    good_A = fill(.25,4,4)
    good_D = Categorical([.25,.25,.25,.25])
    good_Dvec = [good_D for i in 1:4]
    hmm=BHMM(good_A,good_Dvec)
    log_p=-.001

    @test_throws ArgumentError EM_step(0,hmm,0.0,0.0,true)
    @test_throws ArgumentError EM_step(-1,hmm,0.0,0.0,true)
    @test_throws ArgumentError EM_step(1,hmm,1.0,0.0,true)

    steppe=EM_step(1,hmm,log_p,0.0,false)
    @test typeof(steppe)==EM_step
    @test steppe.iterate==1
    @test steppe.hmm==hmm
    @test steppe.log_p==log_p
    @test steppe.delta==0.0
    @test steppe.converged==false
end

@testset "utilities/log_prob_sum.jl Log probability summation functions" begin
    @test isapprox(lps([.1, .1, .1]), .3)
    @test lps([-Inf, .1, .1]) == -Inf
    @test isapprox(lps(.1, .1, .1), .3)
    @test lps(-Inf, .1, .1) == -Inf
    @test lps(.1, -Inf, .1) == -Inf
end

@testset "utilities/HMM_init.jl initialization functions" begin
    K=4
    order=2
    hmm=autotransition_init(K,order)
    @test typeof(hmm)==BHMM{Float64}
    @test size(hmm)==(4,64)
end

```

```
@testset "utilities/load_balancer.jl EM_converge load balancing structs and functions" begin
    @test_throws ArgumentError LoadConfig(0:5,1:5)
    @test_throws ArgumentError LoadConfig(1:5,-1:5)


    job_ids=[Chain_ID("test",6,2,1),Chain_ID("test",4,2,1),Chain_ID("test",2,0,1)
    obs_sets=Dict("test"⇒[LongSequence{DNAAlphabet{2}}("AAAAAAAAAAAAAA")])

    K_exclusive_config=LoadConfig(1:1,0:0)
    O_exclusive_config=LoadConfig(1:6,3:4)
    blacklist_config=LoadConfig(1:6,0:2,blacklist=job_ids)

    whitelist_config=LoadConfig(1:6,0:2,whitelist=[Chain_ID("test",6,2,2)])
    high_config=LoadConfig(4:6,0:2,whitelist=job_ids)

    no_input_hmms, chains, input_channel, output_channel = setup_EM_jobs!(job_ids,obs_sets)
    @test load_balancer(no_input_hmms, input_channel, K_exclusive_config, 3) == (0,0,0,0,0)

    no_input_hmms, chains, input_channel, output_channel = setup_EM_jobs!(job_ids,obs_sets)
    @test load_balancer(no_input_hmms, input_channel, O_exclusive_config, 3) == (0,0,0,0,0)

    no_input_hmms, chains, input_channel, output_channel = setup_EM_jobs!(job_ids,obs_sets)
    @test load_balancer(no_input_hmms, input_channel, blacklist_config, 3) == (0,0,0,0,0)

    no_input_hmms, chains, input_channel, output_channel = setup_EM_jobs!(job_ids,obs_sets)
    @test load_balancer(no_input_hmms, input_channel, whitelist_config, 3) == (0,0,0,0,0)

    no_input_hmms, chains, input_channel, output_channel = setup_EM_jobs!(job_ids,obs_sets)
    high_set=load_balancer(no_input_hmms, input_channel, high_config)

    @test high_set[1]==Chain_ID("test",6,2,1)
    @test high_set[2]==1
```

```
    @test typeof(high_set[3])==BHMM{Float64}
    @test high_set[4]==obs_lh_given_hmm(high_set[5],high_set[3],linear=false)
    @test typeof(high_set[5])==Matrix{UInt8}
end

@testset "utilities/observation_coding.jl Order coding structs and functions" begin
    compound_alphabet=CompoundAlphabet(ACGT, 2)
    @test typeof(compound_alphabet.symbols)==Dict{Mer{DNAAlphabet{2}},Integer}
    @test length(compound_alphabet.symbols)==64
    @test compound_alphabet.symbols[Mer{DNAAlphabet{2}}("AAA")]==1
    @test compound_alphabet.symbols[Mer{DNAAlphabet{2}}("TTT")]==64

    test_seqs = [BioSequences.LongSequence{DNAAlphabet{2}}("ACGTACGTACGTACGT"),BioSequences.L
    target0= [1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 0; 4 4 4 4 4 4 4 0 0 0 0 0 0 0 0 0 0]
    target2= [37 58 15 20 37 58 15 20 37 58 15 20 37 58 0; 64 64 64 64 64 0 0 0 0 0 0 0 0 0 0]

    order0_seqs = get_order_n_seqs(test_seqs,0)
    @test typeof(order0_seqs)==BioBackgroundModels.N_Order_ntSequence
    for (mer, code) in CompoundAlphabet(ACGT,0).symbols
        @test order0_seqs.alphabet.symbols[mer]==code
    end
    @test order0_seqs.seq_lengths==[16,7]
    @test order0_seqs.order_kmers[1][1]==Mer{DNAAlphabet{2}}("A")
    code0_seqs = code_seqs(order0_seqs)
    @test target0 == code0_seqs

    order2_seqs = get_order_n_seqs(test_seqs,2)
    @test typeof(order2_seqs)==BioBackgroundModels.N_Order_ntSequence
    for (mer, code) in CompoundAlphabet(ACGT,2).symbols
        @test order2_seqs.alphabet.symbols[mer]==code
    end
    @test order2_seqs.seq_lengths==[16,7]
    @test order2_seqs.order_kmers[1][1]==Mer{DNAAlphabet{2}}("ACG")
    code2_seqs = code_seqs(order2_seqs)
    @test target2 == code2_seqs

    bw_o0_seqs = get_order_n_seqs(test_seqs[end:-1:1], 0)
```

```
    sorted_code_seqs = code_seqs(bw_o0_seqs, sorted=true)
    @test target0 == sorted_code_seqs

    obs_sets=Dict("test1"⇒test_seqs,
                  "test2"⇒test_seqs)

    job_ids=[Chain_ID("test1",4,0,1),Chain_ID("test2",4,0,1),Chain_ID("test1",2,2

    code_dict=code_job_obs(job_ids,obs_sets)
    @test ("test1",0) in keys(code_dict)
    @test ("test1",2) in keys(code_dict)
    @test ("test2",0) in keys(code_dict)
    @test !(("test2",2) in keys(code_dict))
    @test code_dict[("test1",0)]==target0
    @test code_dict[("test1",2)]==target2
    @test code_dict[("test2",0)]==target0

    #test type selection for high order numbers
    o5_nos=get_order_n_seqs(test_seqs,5)
    o5_codes=code_seqs(o5_nos)
    @test typeof(o5_codes)==Matrix{UInt16}

    o7_nos=get_order_n_seqs(test_seqs,7)
    o7_codes=code_seqs(o7_nos)
    @test typeof(o7_codes)==Matrix{UInt32}

end

@testset "genome_sampling/partition_masker.jl functions" begin
    synthetic_seq = BioSequences.LongSequence{DNAAlphabet{2}}(generate_synthetic_seq())
    position_start = 501
    position_length=141
    position_pad=350
    perigenic_pad = 250

    position_df = make_padded_df(posfasta, gff, genome, index, position_pad)
    add_partition_masks!(position_df, gff, perigenic_pad)

    @test position_df.SeqID[1]=="1"
```

```
    @test length(position_df.PadSeq[1]) == position_pad+position_length ==
    length(position_df.PadStart[1]:position_df.PadStart[1]+position_length+positi
    == position_df.End[1]-position_df.Start[1]+1+position_pad ==
    size(position_df.MaskMatrix[1])[1]
    @test synthetic_seq[position_df.PadStart[1]:position_df.End[1]]==position_df.PadSeq

    mm = position_df.MaskMatrix[1]
    idx=1#pos 151
    @test sum(mm[idx:idx+99,1] .== 1)==length(mm[idx:idx+99,1])
    idx+=100#pos 251
    @test sum(mm[idx:idx+259,1] .== 2)==length(mm[idx:idx+259,1])
    idx+=260 #pos 511
    @test sum(mm[idx:idx+59,1] .== 3)==length(mm[idx:idx+59,1])
    idx+=60#pos 571
    @test sum(mm[idx:idx+29,1] .== 2)==length(mm[idx:idx+29,1])
    idx+=30#pos601
    @test sum(mm[idx:idx+40,1] .== 3)==length(mm[idx:idx+40,1])

    #test exception
    position_df = make_padded_df(posfasta, gff, genome, index, position_pad)

    partition_coords_dict=partition_genome_coordinates(gff, perigenic_pad)

    partitioned_scaffolds=divide_partitions_by_scaffold(partition_coords_dict)
    scaffold_coords_dict = Dict{String,DataFrame}()
    scaffold="1"
    for ((partition, part_scaffold), df) in partitioned_scaffolds
        if scaffold == part_scaffold
            scaffold_coords_dict[partition] = df
        end
    end

    @test_throws DomainError mask_sequence_by_partition(1001,151,scaffold_coords_dict)
end

@testset "genome_sampling/sequence_sampler.jl functions & API/genome_sampling.jl interface" begin
    #rectifier tests
```

```
    @test rectify_identifier("1")=="1"

    partitions = 3 #exonic, periexonic, intragenic
    sample_set_length=100
    min_sample_window=5
    max_sample_window=25
    perigenic_pad=250
    syn_intergenic_starts = [1]
    syn_intergenic_ends = [250]
    syn_periexonic_starts = [251,571,661,761,861,961]
    syn_periexonic_ends = [510,600,700,800,900,1000]
    syn_periexonic_strands = ['-','-','-','-','-','-']
    syn_exonic_starts = [511,601,701,801,901]
    syn_exonic_ends = [570,660,760,860,960]
    syn_exonic_strands = ['+','-','-','-','-']
    partition_lengths = Dict("exon"=>5*60,"intergenic"=>250,"periexonic"=>450)

    # "Testing sequence sampler fns..."
    synthetic_seq = BioSequences.LongSequence{DNAAlphabet{2}}(generate_synthetic_seq())

    # "Partitioning synthetic genome coordinates..."
    coordinate_partitions = partition_genome_coordinates(gff, perigenic_pad)

    @test coordinate_partitions["intergenic"].Start == syn_intergenic_starts
    @test coordinate_partitions["intergenic"].End == syn_intergenic_ends

    @test coordinate_partitions["periexonic"].Start == syn_periexonic_starts
    @test coordinate_partitions["periexonic"].End == syn_periexonic_ends
    @test coordinate_partitions["periexonic"].Strand == syn_periexonic_strands

    @test coordinate_partitions["exon"].Start == syn_exonic_starts
    @test coordinate_partitions["exon"].End == syn_exonic_ends
    @test coordinate_partitions["exon"].Strand == syn_exonic_strands

    # "Checking sampling functions at all synthetic indices..."

```

```
    input_test_channel, completed_test_channel, progress_channel, setlength = setup_sample_jobs(genome, index, gff, sample_set_length, min_sample_window, max_sample_window, perigenic_pad; deterministic=true)
    while isready(input_test_channel)
        genome_path, genome_index_path, partition_df, partitionid, sample_set_length, sample_window_min, sample_window_max, deterministic = take!(input_test_channel)

        for feature in eachrow(partition_df)
            seqid, scaffold_start, scaffold_end = meta_to_feature_coord(feature.MetaStart, feature.MetaEnd, partition_df)
            @test scaffold_start == feature.Start && scaffold_end == feature.End
        end

        stranded = get_strand_dict()[partitionid]
        @test typeof(stranded) == Bool

        scaffold_sequence_record_dict = build_scaffold_seq_dict(genome, index)
        @test scaffold_sequence_record_dict["1"] == synthetic_seq

        partition_length = partition_df.MetaEnd[end]
        @test partition_length == partition_lengths[partitionid]

        metacoordinate_bitarray = trues(partition_df.MetaEnd[end])

        for bitindex in findall(metacoordinate_bitarray)
            feature_metaStart, feature_metaEnd, strand = get_feature_params_from_metacoord(bitindex, partition_df, stranded)
            @test 1 ≤ feature_metaStart < feature_metaEnd ≤ length(metacoordinate_bitarray)
            @test feature_metaStart in partition_df.MetaStart
            @test feature_metaEnd in partition_df.MetaEnd
            if partitionid == "periexonic"
                @test strand == '-'
            elseif partitionid == "exon"
                @test strand in ['-','+']
            end
            feature_length = length(feature_metaStart:feature_metaEnd)
```

```
            window = determine_sample_window(feature_metaStart, feature_metaEnd, bitindex, metacoordinate_bitarray, sample_window_min, sample_window_max) #get an appropriate sampling window around the selected index, given the feature boundaries and params
            @test 1 ≤ feature_metaStart ≤ window[1] < window[1]+sample_window_min-1 < window[1]+sample_window_max-1≤ window[2] ≤ feature_metaEnd ≤ length(metacoordinate_bitarray)
            sample_scaffoldid, sample_scaffold_start, sample_scaffold_end = meta_to_feature_coord(window[1],window[2],partition_df)
            @test sample_scaffoldid == "1"
            @test 1 ≤ sample_scaffold_start ≤ sample_scaffold_start+sample_window_min ≤ sample_scaffold_end ≤ min(sample_scaffold_start+sample_window_max,1000) ≤ 1000

            strand == '-' ? target_seq=reverse_complement(synthetic_seq[sample_scaffold_start:sam
            :
                target_seq = synthetic_seq[sample_scaffold_start:sample_scaffold_end]

            proposal_sequence = fetch_sequence(sample_scaffoldid, scaffold_sequence_record_dict, sample_scaffold_start, sample_scaffold_end, strand; deterministic=deterministic) #get the sequence associated with the sample window

            @test proposal_sequence == target_seq
        end
    end

    # "Verifying sampling channels..."

    input_sample_channel, completed_sample_channel, progress_channel, setlength = setup_sample_jobs(genome, index, gff, sample_set_length, min_sample_window, max_sample_window, perigenic_pad; deterministic=true)
    get_sample_set(input_sample_channel, completed_sample_channel, progress_channel)

    #collect sample dfs by partition id when ready
    collected_counter = 0
```

```
    sample_record_dfs = Dict{String,DataFrame}()
    while collected_counter < partitions
        wait(completed_sample_channel)
        partition_id, sample_df = take!(completed_sample_channel)
        sample_record_dfs[partition_id] = sample_df
        collected_counter += 1
    end

    #get_sample entire feature selection
    coordinate_partitions = partition_genome_coordinates(gff, perigenic_pad)
    partition_id="exon"
    partition=coordinate_partitions[partition_id]
    add_metacoordinates!(partition)
    metacoordinate_bitarray=trues(partition.MetaEnd[end])
    scaffold_sequence_record_dict = build_scaffold_seq_dict(genome, index)

    #test fetch_sequence strand char ArgumentError
    @test_throws ArgumentError fetch_sequence("1", Dict{String,LongSequence}("1"⇒LongSequence{DNAAlphabet{2}}("AAAAAAAAAAAAA")) 1, 5, 'L')

    #test get_feature_row_index ArgumentError
    @test_throws DomainError get_feature_row_index(partition,0)

    @test length(get_sample(metacoordinate_bitarray, 1, 250, partition, scaffold_sequence_record_dict)[6])==60

    synthetic_seq = BioSequences.LongSequence{DNAAlphabet{2}}(generate_synthetic_seq())
    for (partid, df) in sample_record_dfs
        for sample in eachrow(df)
            target_seq = synthetic_seq[sample.SampleStart:sample.SampleEnd]
            strand = sample.Strand
            if sample.Strand == '-'
                target_seq = reverse_complement(target_seq)
            end
            @test sample.SampleSequence == target_seq
        end
    end

```

```
    #execute_sample_jobs API
    wkpool=addprocs(1)
    @everywhere using BioBackgroundModels

    channels = setup_sample_jobs(genome, index, gff, sample_set_length,
    ↪  min_sample_window, max_sample_window, perigenic_pad;
    ↪  deterministic=true)
    records=execute_sample_jobs(channels, wkpool)

    rmprocs(wkpool)
end

@testset "likelihood_funcs/hmm.jl & bg_lh_matrix.jl functions" begin
    pvec = [.4,.3,.2,.1]
    trans = ones(1,1)
    B = [Categorical(pvec)]
    hmm = BHMM(trans, B)

    testseq=zeros(Int64,1,5)
    testseq[1:4] = [1,2,3,4]
    @test isapprox(get_BGHMM_symbol_lh(testseq, hmm)[1], log.(pvec[1]))
    @test
    ↪  isapprox(obs_lh_given_hmm(testseq,hmm),obs_lh_given_hmm(testseq,hmm,linear=fa
    @test isapprox(sum(get_BGHMM_symbol_lh(testseq,hmm)),
    ↪  obs_lh_given_hmm(testseq,hmm))
    @test isapprox(sum(get_BGHMM_symbol_lh(testseq,hmm)),
    ↪  obs_lh_given_hmm(testseq,hmm,linear=false))

    pvec=[.25,.25,.25,.25]
    trans = [.9 .1
         .1 .9]
    B = [Categorical(pvec), Categorical(pvec)]
    hmm = BHMM(trans, B)

    @test
    ↪  isapprox(obs_lh_given_hmm(testseq,hmm),obs_lh_given_hmm(testseq,hmm,linear=fa
    @test isapprox(sum(get_BGHMM_symbol_lh(testseq,hmm)),
    ↪  obs_lh_given_hmm(testseq,hmm))
    @test isapprox(sum(get_BGHMM_symbol_lh(testseq,hmm)),
    ↪  obs_lh_given_hmm(testseq,hmm,linear=false))

    testseq=zeros(Int64,1,1001)
    testseq[1,1:1000]=rand(1:4,1000)
```

```

    @test isapprox(sum(get_BGHMM_symbol_lh(testseq, hmm)),obs_lh_given_hmm(testseq,hmm))

    Dex = [Categorical([.3, .1, .3, .3]),Categorical([.15, .35, .35, .15])]
    Dper = [Categorical([.15, .35, .35, .15]),Categorical([.4, .1, .1, .4])]
    Dint = [Categorical([.4, .1, .1, .4]),Categorical([.45, .05, .05, .45])]
    BGHMM_dict = Dict{String,BHMM}()
    BGHMM_dict["exon"] = BHMM(trans, Dex)
    BGHMM_dict["periexonic"] = BHMM(trans, Dper)
    BGHMM_dict["intergenic"] = BHMM(trans, Dint)

    position_length=141;perigenic_pad=250;
    position_df = make_padded_df(posfasta, gff, genome, index, position_length)

    reader=open(FASTA.Reader, genome, index=index)
    seq=FASTA.sequence(reader["CM002885.2.1"])
    @test position_df.Start[1]==501
    @test position_df.End[1]==641
    @test position_df.PadSeq[1]==seq[360:641]
    @test position_df.PadStart[1]==360
    @test position_df.RelStart[1]==141
    @test position_df.SeqOffset[1]==0

    offset_position_df = make_padded_df(posfasta, gff, genome, index,600)
    @test offset_position_df.Start[1]==501
    @test offset_position_df.End[1]==641
    @test offset_position_df.PadSeq[1]==seq[1:641]
    @test offset_position_df.PadStart[1]==1
    @test offset_position_df.RelStart[1]==500
    @test offset_position_df.SeqOffset[1]==100

    add_partition_masks!(position_df, gff, perigenic_pad)
    @test all(position_df.MaskMatrix[1][1:151,1].==2)
    @test all(position_df.MaskMatrix[1][152:211,1].==3)
    @test all(position_df.MaskMatrix[1][212:241,1].==2)
    @test all(position_df.MaskMatrix[1][242:end,1].==3)
    @test all(position_df.MaskMatrix[1][1:151,2].==-1)
    @test all(position_df.MaskMatrix[1][152:211,2].==1)
```

```
    @test all(position_df.MaskMatrix[1][212:end,2].==-1)

    lh_matrix=BGHMM_likelihood_calc(position_df,BGHMM_dict)
    @test size(lh_matrix)==(position_length*2,1)


    periexonic_frag=reverse_complement!(LongSequence{DNAAlphabet{2}}(seq[360:510]
    pno=get_order_n_seqs([periexonic_frag],0)
    pcode=code_seqs(pno)
    plh=get_BGHMM_symbol_lh(pcode, BGHMM_dict["periexonic"])
    @test isapprox(lh_matrix[1:151,1], reverse(plh))

    exonic_frag=LongSequence{DNAAlphabet{2}}(seq[511:570])
    eno=get_order_n_seqs([exonic_frag],0)
    ecode=code_seqs(eno)
    elh=get_BGHMM_symbol_lh(ecode, BGHMM_dict["exon"])
    @test isapprox(lh_matrix[152:211,1],elh)
end

@testset "EM/baum-welch.jl MS_HMMBase equivalency and functions" begin
    A = [.5 .5
         .5 .5]
    B = [Categorical(ones(4)/4), Categorical([.7,.1,.1,.1])]
    hmm = BHMM(A, B)
    log_A = log.(hmm.A)

    obs = zeros(UInt8, 22,1)
    obs[1:21] = [4,3,3,2,3,2,1,1,2,3,3,3,4,4,2,3,2,3,4,3,2]
    obs_lengths=[21]

    lls = BioBackgroundModels.bw_llhs(hmm,obs)
    m_lls = mouchet_log_likelihoods(hmm,obs[1:21])

    K,Tmaxplus1,O = size(lls)
    T=Tmaxplus1-1

    #TEST LOG LIKELIHOOD FN
    @test transpose(lls[:,1:end-1,1]) == m_lls

    log_α = BioBackgroundModels.messages_forwards_log(hmm.a, hmm.A, lls, obs_lengths)
    m_log_α = mouchet_messages_forwards_log(hmm.a, hmm.A, m_lls)

```

```
    #TEST FORWARD MESSAGES FN
    @test transpose(log_α[:,1:end-1,1]) == m_log_α

    log_β = BioBackgroundModels.messages_backwards_log(hmm.A, lls, obs_lengths)
    m_log_β = mouchet_messages_backwards_log(hmm.A, m_lls)

    #TEST BACKWARDS  MESSAGES FN
    @test isapprox(transpose(log_β[:,1:end-1,1]),  m_log_β)

    lls = permutedims(lls, [2,3,1]) # from (K,T,O) to (T,O,K)
    log_α = permutedims(log_α, [2,3,1])
    log_β = permutedims(log_β, [2,3,1])

    # "Testing obs probability calcs ..."
    #TEST OBSERVATION PROBABILITY CALC
    normalizer = logsumexp(m_log_α[1,:] + m_log_β[1,:])
    log_pobs = logsumexp(lps.(log_α[1,1,:], log_β[1,1,:]))

    @test isapprox(normalizer, log_pobs)

    # "Testing ξ, γ calcs ..."

    log_ξ = fill(-Inf, Tmaxplus1,O,K,K)
    log_γ = fill(-Inf, Tmaxplus1,O,K)

    m_log_ξ = zeros(T, K, K)

    for t = 1:T-1, i = 1:K, j = 1:K
        m_log_ξ[t,i,j] = m_log_α[t,i] + log_A[i,j] + m_log_β[t+1,j] + m_lls[t+1,j] - normalizer
    end

    for i = 1:K, j = 1:K, o = 1:O
        obsl = obs_lengths[o]
        for t = 1:obsl-1 #log_ξ & log_γ calculated to T-1 for each o
           log_ξ[t,o,i,j] = lps(log_α[t,o,i], log_A[i,j], log_β[t+1,o,j], lls[t+1,o,j], -log_pobs[o])
           log_γ[t,o,i] = lps(log_α[t,o,i], log_β[t,o,i], -log_pobs[o])
        end
        t=obsl #log_ξ @ T = 0
        log_ξ[t,o,i,j] = 0
        log_γ[t,o,i] = lps(log_α[t,o,i], log_β[t,o,i], -log_pobs)
```

```
    end

    ξ = exp.(log_ξ)
    k_ξ = sum(ξ, dims=[3,4])
    nan_mask = k_ξ .== 0; k_ξ[nan_mask] ⋅= Inf #prevent NaNs in dummy renorm arising from multiple sequences indexing
    ξ  ./=  k_ξ #dummy renorm across K to keep numerical creep from causing isprobvec to fail on new new_A during hmm creation

    m_ξ = exp.(m_log_ξ)
    m_ξ ./= sum(m_ξ, dims=[2,3])

    #check equivalency of xi calc methods
    @test isapprox(reshape(ξ,Tmaxplus1,K,K)[1:21,:,:],m_ξ)

    γ = exp.(log_γ)
    k_γ = sum(γ, dims=3); k_γ[nan_mask[:,:,:]] ⋅= Inf #prevent NaNs in dummy renorm
    γ ./= k_γ

    m_γ = exp.((m_log_α .+ m_log_β) ⋅- normalizer)

    #check equivalency of gamma calculations
    @test isapprox(reshape(γ,Tmaxplus1,K)[1:21,:,:],m_γ)

    # "Testing initial and transition matrix calcs…"

    m_new_A = sum(m_ξ[1:end-1,:,:], dims=1)[1,:,:]
    m_new_A ./= sum(m_new_A, dims=2)

    new_A = zeros(K,K)
    for i=1:K, j=1:K
        Σotξ_vec = zeros(O)
        Σotγ_vec = zeros(O)
        for o in 1:O
            Σotξ_vec[o] = sum(ξ[1:obs_lengths[o]-1,o,i,j])
            Σotγ_vec[o] = sum(γ[1:obs_lengths[o]-1,o,i])
        end
        new_A[i,j] = sum(Σotξ_vec) / sum(Σotγ_vec)
    end
    new_A ./= sum(new_A, dims=[2]) #dummy renorm

    #check equivalency of transition matrix calculations
```

```
    @test isapprox(m_new_A,new_A)

    m_new_a = exp.((m_log_α[1,:] + m_log_β[1,:]) .- normalizer)
    m_new_a ./= sum(m_new_a)

    new_a = (sum(γ[1,:,:], dims=1)./sum(sum(γ[1,:,:], dims=1)))[1,:]
    new_a ./= sum(new_a) #dummy renorm

    @test isapprox(m_new_a,new_a)

    # "Testing distribution calcs ..."

    F=Univariate
    m_D = Distribution{F}[]
    for (i, d) in enumerate(hmm.B)
        # Super hacky ...
        # https://github.com/JuliaStats/Distributions.jl/issues/809
        push!(m_D, fit_mle(eval(typeof(d).name.name),
         ↪  permutedims(obs[1:21]), m_γ[:,i]))
    end

    obs_mask = .!nan_mask
    obs_collection = obs[obs_mask[:,:]]

    B = Distribution{F}[]
    @inbounds for (i, d) in enumerate(hmm.B)
        # Super hacky ...
        # https://github.com/JuliaStats/Distributions.jl/issues/809
        γ_d = γ[:,:,i]
        push!(B, fit_mle(eval(typeof(d).name.name), obs_collection,
         ↪  γ_d[obs_mask[:,:]]))
        #slowest call by orders of magnitude
    end

    #test maximization equivalency
    for (d, dist) in enumerate(B)
        @test isapprox(dist.support, m_D[d].support)
        @test isapprox(dist.p, m_D[d].p)
    end

    # "Testing mle_step ..."

```

```
    #verify that above methods independently produce equivalent output, and that this is true of multiple identical obs, but not true of different obs sets
    mouchet_hmm = mouchet_mle_step(hmm, obs[1:21])

    new_hmm = BioBackgroundModels.bw_step(hmm, obs, obs_lengths)

    dblobs = zeros(UInt8, 22,2)
    dblobs[1:21,1] = [4,3,3,2,3,2,1,1,2,3,3,3,4,4,2,3,2,3,4,3,2]
    dblobs[1:21,2] = [4,3,3,2,3,2,1,1,2,3,3,3,4,4,2,3,2,3,4,3,2]
    dblobs_lengths=[21,21]
    dbl_hmm = BioBackgroundModels.bw_step(hmm, dblobs, dblobs_lengths)

    otherobs = dblobs
    otherobs[1:21,2] = [1,2,3,4,1,2,3,4,1,2,3,4,1,2,3,4,1,2,3,4,1]
    other_hmm = BioBackgroundModels.bw_step(hmm, otherobs, dblobs_lengths)

    for n in fieldnames(typeof(new_hmm[1]))
        if n == :B
            for (d, dist) in enumerate(new_hmm[1].B)
                @test dist.support == mouchet_hmm[1].B[d].support
                @test isapprox(dist.p,mouchet_hmm[1].B[d].p)
                @test dist.support == dbl_hmm[1].B[d].support
                @test isapprox(dist.p,dbl_hmm[1].B[d].p)
                @test dist.support == other_hmm[1].B[d].support
                @test !isapprox(dist.p, other_hmm[1].B[d].p)
            end
        elseif n == :partition
            @test getfield(new_hmm[1],n) == getfield(mouchet_hmm[1],n)
            @test getfield(new_hmm[1],n) == getfield(dbl_hmm[1],n)
            @test getfield(new_hmm[1],n) == getfield(other_hmm[1],n)
        else
            @test isapprox(getfield(new_hmm[1],n), getfield(mouchet_hmm[1],n))
            @test isapprox(getfield(new_hmm[1],n), getfield(dbl_hmm[1],n))
            @test !isapprox(getfield(new_hmm[1],n), getfield(other_hmm[1],n))
        end
    end

    @test new_hmm[2] == mouchet_hmm[2] ≠ dbl_hmm[2] ≠ other_hmm[2]
```

```

    # "Testing fit_mle! ..."

    # "test fit_mle! function"
    input_hmms= RemoteChannel(()→Channel{Tuple}(1))
    output_hmms = RemoteChannel(()→Channel{Tuple}(30))
    chainid=Chain_ID("Test",2,0,1)
    put!(input_hmms, (chainid, 2, hmm, 0.0, transpose(obs)))
    BioBackgroundModels.EM_converge!(input_hmms, output_hmms, 1;
    ↪  EM_func=BioBackgroundModels.bw_step, max_iterates=4,
    ↪  verbose=true)
    wait(output_hmms)
    workerid, jobid, iterate, hmm3, log_p, epsilon, converged =
    ↪  take!(output_hmms)
    @test jobid == chainid
    @test iterate == 3
    @test assert_hmm(hmm3.a, hmm3.A, hmm3.B)
    @test size(hmm3) == size(hmm) == (2,4)
    @test log_p < 1
    @test log_p == obs_lh_given_hmm(transpose(obs),hmm, linear=false)
    @test converged == false
    wait(output_hmms)
    workerid, jobid, iterate, hmm4, log_p, epsilon, converged =
    ↪  take!(output_hmms)
    @test jobid == chainid
    @test iterate == 4
    @test assert_hmm(hmm4.a, hmm4.A, hmm4.B)
    @test size(hmm4) == size(hmm) == (2,4)
    @test log_p < 1
    @test log_p == obs_lh_given_hmm(transpose(obs),hmm3,linear=false)
    @test converged == false

    # "Test convergence.."
    obs=zeros(UInt8, 101, 2)
    for i in 1:size(obs)[2]
        obs[1:100,i]=rand(1:4,100)
    end
    input_hmms= RemoteChannel(()→Channel{Tuple}(1))
    output_hmms = RemoteChannel(()→Channel{Tuple}(30))
    put!(input_hmms, (chainid, 2, hmm, 0.0, transpose(obs)))
    BioBackgroundModels.EM_converge!(input_hmms, output_hmms, 1;
    ↪  EM_func=BioBackgroundModels.bw_step, delta_thresh=.05,
    ↪  max_iterates=100, verbose=true)
```

```
    wait(output_hmms)
    workerid, jobid, iterate, hmm4, log_p, epsilon, converged = take!(output_hmms)
    while isready(output_hmms)
        workerid, jobid, iterate, hmm4, log_p, epsilon, converged = take!(output_hmms)
    end
    @test converged==1

    #test t_add_categorical_counts DimensionMismatch
    @test_throws DimensionMismatch t_add_categorical_counts!(zeros(4), zeros(UInt8,2,3), zeros(2,2))

end

@testset "EM/churbanov.jl Baum-Welch equivalency and functions" begin
    # "Setting up for MLE function tests.."
    A = fill((1/6),6,6)
    B = [Categorical(ones(4)/4), Categorical([.7,.05,.15,.1]),Categorical([.15,.35,.4,.1]), Categorical([.6,.15,.15,.1]),Categorical([.1,.4,.4,.1]), Categorical([.2,.2,.3,.3])]
    hmm = BHMM(A, B)
    log_A = log.(hmm.A)

    obs = zeros(Int64,1,250)
    obs[1:249] = rand(1:4,249)
    obs_lengths=[249]
    # "Testing mle_step..."

    #verify that above methods independently produce equivalent output, and that this is true of multiple identical obs, but not true of different obs sets
    mouchet_hmm = mouchet_mle_step(hmm, obs[1:249])

    new_hmm = linear_step(hmm, obs, obs_lengths)

    ms_sng = BioBackgroundModels.bw_step(hmm, Array(transpose(obs)), obs_lengths)

    dblobs = zeros(Int64, 2,250)
    dblobs[1,1:249] = obs[1:249]
    dblobs[2,1:249] = obs[1:249]
```

```
    dblobs_lengths=[249,249]
    dbl_hmm = linear_step(hmm, dblobs, dblobs_lengths)


    ms_dbl =BioBackgroundModels.bw_step(hmm,
     ↪  Array(transpose(dblobs)),dblobs_lengths)

    otherobs = deepcopy(dblobs)
    otherobs[2,1:249] = rand(1:4,249)
    if otherobs[1,1]==otherobs[2,1]
        otherobs[1,1]==1&&(otherobs[2,1]=2)
        otherobs[1,1]==2&&(otherobs[2,1]=3)
        otherobs[1,1]==3&&(otherobs[2,1]=4)
        otherobs[1,1]==4&&(otherobs[2,1]=1)
    end

    other_hmm = linear_step(hmm, otherobs, dblobs_lengths)

    ms_hmm = BioBackgroundModels.bw_step(hmm,
     ↪  Array(transpose(otherobs)),dblobs_lengths)

    for n in fieldnames(typeof(new_hmm[1]))
        if n == :B
            for (d, dist) in enumerate(new_hmm[1].B)
            @test dist.support==mouchet_hmm[1].B[d].support
            @test isapprox(dist.p,mouchet_hmm[1].B[d].p)
            @test dist.support==ms_sng[1].B[d].support
            @test isapprox(dist.p,ms_sng[1].B[d].p)
            @test dist.support==dbl_hmm[1].B[d].support
            @test isapprox(dist.p,dbl_hmm[1].B[d].p)
            @test dist.support==ms_dbl[1].B[d].support
            @test isapprox(dist.p,ms_dbl[1].B[d].p)
            @test dist.support==other_hmm[1].B[d].support
            @test ms_hmm[1].B[d].support==other_hmm[1].B[d].support
            @test !isapprox(dist.p, other_hmm[1].B[d].p)
            @test isapprox(ms_hmm[1].B[d].p, other_hmm[1].B[d].p)

            end
        elseif n == :partition
            @test getfield(new_hmm[1],n) == getfield(mouchet_hmm[1],n)
            @test getfield(new_hmm[1],n) == getfield(dbl_hmm[1],n)
            @test getfield(new_hmm[1],n) == getfield(other_hmm[1],n)
        else
```

```
            @test isapprox(getfield(new_hmm[1],n), getfield(mouchet_hmm[1],n))
            @test isapprox(getfield(new_hmm[1],n), getfield(ms_sng[1],n))

            @test isapprox(getfield(new_hmm[1],n), getfield(dbl_hmm[1],n))
            @test isapprox(getfield(new_hmm[1],n), getfield(ms_dbl[1],n))

            @test !isapprox(getfield(new_hmm[1],n), getfield(other_hmm[1],n))
            @test isapprox(getfield(ms_hmm[1],n), getfield(other_hmm[1],n))

        end
    end

    @test ms_sng[2] == new_hmm[2] == mouchet_hmm[2] ≠ dbl_hmm[2] == ms_dbl[2] ≠ other_hmm[2] == ms_hmm[2]

    # "Testing fit_mle! ..."

    #test fit_mle! function
    input_hmms= RemoteChannel(()→Channel{Tuple}(1))
    output_hmms = RemoteChannel(()→Channel{Tuple}(30))
    chainid=Chain_ID("Test",6,0,1)
    put!(input_hmms, (chainid, 2, hmm, 0.0, obs))
    BioBackgroundModels.EM_converge!(input_hmms, output_hmms, 1; max_iterates=4, verbose=true)
    wait(output_hmms)
    workerid, jobid, iterate, hmm3, log_p, delta, converged = take!(output_hmms)
    @test jobid == chainid
    @test iterate == 3
    @test BioBackgroundModels.assert_hmm(hmm3.a, hmm3.A, hmm3.B)
    @test size(hmm3) == size(hmm) == (6,4)
    @test log_p < 1
    @test log_p == linear_likelihood(obs, hmm)
    @test converged == false
    wait(output_hmms)
    workerid, jobid, iterate, hmm4, log_p, delta, converged = take!(output_hmms)
    @test jobid == chainid
    @test iterate == 4
```

```
    @test BioBackgroundModels.assert_hmm(hmm4.a, hmm4.A, hmm4.B)
    @test size(hmm4) == size(hmm) == (6,4)
    @test log_p < 1
    @test log_p == linear_likelihood(obs, hmm3)
    @test converged == false

    # "Test convergence.."
    obs=zeros(Int64, 4, 1001)
    for o in 1:size(obs)[1]
        obsl=rand(100:1000)
        obs[o,1:obsl]=rand(1:4,obsl)
    end
    input_hmms= RemoteChannel(()→Channel{Tuple}(1))
    output_hmms = RemoteChannel(()→Channel{Tuple}(Inf))
    put!(input_hmms, (chainid, 2, hmm, 0.0, obs))
    BioBackgroundModels.EM_converge!(input_hmms, output_hmms, 1; delta_thresh=.05, max_iterates=100, verbose=true)
    wait(output_hmms)
    workerid, jobid, iterate, hmm4, log_p, delta, converged = take!(output_hmms)
    while isready(output_hmms)
        workerid, jobid, iterate, hmm4, log_p, delta, converged = take!(output_hmms)
    end
    @test converged==1
end

@testset "API/EM_master.jl and reports.jl API tests" begin
    #CONSTANTS FOR BHMM LEARNING
    replicates = 4 #repeat optimisation from this many seperately initialised samples from the prior
    Ks = [1,2,4,6] #mosaic class #s to test
    order_nos = [0,1,2] #DNA kmer order #s to test
    sample_set_length=100
    min_sample_window=5
    max_sample_window=25
    perigenic_pad=250

    wkpool=addprocs(2)
    @everywhere using BioBackgroundModels

```

```
    channels = setup_sample_jobs(genome, index, gff, sample_set_length, min_sample_window, max_sample_window, perigenic_pad; deterministic=true)
    sample_record_dfs=execute_sample_jobs(channels, wkpool)
    training_sets, test_sets = split_obs_sets(sample_record_dfs)

    rmprocs(wkpool)


    job_ids=Vector{Chain_ID}()
    for (obs_id, obs) in training_sets, K in Ks, order in order_nos, rep in 1:replicates
        push!(job_ids, Chain_ID(obs_id, K, order, rep))
    end

    no_input_hmms, chains, input_hmms, output_hmms = setup_EM_jobs!(job_ids, training_sets)
    @test no_input_hmms == replicates*length(Ks)*length(order_nos)*length(training_sets)

    while isready(input_hmms)
        jobid, start_iterate, hmm, last_norm, observations = take!(input_hmms)
        @test last_norm == linear_likelihood(observations, hmm)
        obs_lengths = [findfirst(iszero,observations[o,:])-1 for o in 1:size(observations)[1]]
        #make sure input HMMs are valid and try to mle_step them and ensure their 1-step children are valid
        @test assert_hmm(hmm.a, hmm.A, hmm.B)
        new_hmm, prob = linear_step(hmm,observations,obs_lengths)
        @test assert_hmm(hmm.a, hmm.A, hmm.B)
        @test prob < 0
    end

    genome_reader = open(FASTA.Reader, genome, index=index)

    seq=LongSequence{DNAAlphabet{2}}(FASTA.sequence(genome_reader["CM002885.2.1"]
    seqdict = Dict("test"⇒[seq for i in 1:3])
    seqdict["blacklist"]=seqdict["test"]

    job_ids=Vector{Chain_ID}()
    push!(job_ids, Chain_ID("blacklist", 1, 0, 1))
    push!(job_ids, Chain_ID("blacklist", 1, 0, 2))
```

```
    Ks=[1,2]; order_nos=[0,1]
    for K in Ks, order in order_nos, replicate in 1:2
        push!(job_ids, Chain_ID("test", K, order, replicate))
    end

    wkpool=addprocs(2, topology=:master_worker)
    @everywhere using BioBackgroundModels

    load_dict=Dict{Int64,LoadConfig}()
    for (n,wk) in enumerate(wkpool)
        n==1 &&
         ↪  (load_dict[wk]=LoadConfig(1:2,0:1,blacklist=[Chain_ID("blacklist",1,0,1)]
        n==2 && (load_dict[wk]=LoadConfig(1:2,0:1))
    end
    #test work resumption, load configs

    em_jobset=setup_EM_jobs!(job_ids, seqdict, delta_thresh=10000.)
    execute_EM_jobs!(wkpool, em_jobset..., "testchains",
     ↪  delta_thresh=10000., verbose=true, load_dict=load_dict)

    for (chain_id,chain) in em_jobset[2]
        @test chain[end].converged == true
        @test chain[end].delta ≤ 10000
        @test typeof(chain[end].hmm)==BHMM{Float64}
        @test 1 ≤ chain[end].iterate ≤ 5000
    end

    wkpool=addprocs(2, topology=:master_worker)
    @everywhere using BioBackgroundModels

    em_jobset=setup_EM_jobs!(job_ids, seqdict,
     ↪  chains=deserialize("testchains"), delta_thresh=.1)
    execute_EM_jobs!(wkpool, em_jobset..., "testchains", delta_thresh=.1,
     ↪  verbose=true)

    for (chain_id,chain) in em_jobset[2]
        @test chain[end].converged == true
        @test chain[end].delta ≤ .1
        @test typeof(chain[end].hmm)==BHMM{Float64}
        @test 1 ≤ chain[end].iterate ≤ 5000
    end

    #test for already converged warning
```

```
    wkpool=addprocs(2, topology=:master_worker)
    @everywhere using BioBackgroundModels
    @test_throws ArgumentError execute_EM_jobs!(wkpool, em_jobset ... ,
     ↪  "testchains", delta_thresh=.01, verbose=true)

    rm("testchains")

    @test_throws ArgumentError
     ↪  generate_reports(Dict{Chain_ID,Vector{EM_step}}(),seqdict)
    @test_throws ArgumentError
     ↪  generate_reports(em_jobset[2],Dict{String,Vector{LongSequence{DNAAlphabet{2}}}

    @test_throws ArgumentError
     ↪  BioBackgroundModels.report_chains(em_jobset[2],Dict{String,Vector{LongSequenc

    report_folders=generate_reports(em_jobset[2],seqdict)
    folder=report_folders["test"]
    @test "test"==folder.partition_id
    show(folder.partition_report)
    show(folder.replicate_report)
    for id in folder.partition_report.best_repset
        show(folder.chain_reports[id])
    end
end

rm(genome)
rm(index)
rm(gff)
rm(posfasta)
```

17.5.27 /test/synthetic_sequence_gen.jl

```
function print_synthetic_fasta(path::String,line_length::Integer=80)
        header=">CM002885.2.1 BioBackgroundModels Synthetic chromosome
         ↪  for tests\n"
        write(path, header,
         ↪  format_lines(generate_synthetic_seq(),line_length))
end

function print_synthetic_index(path::String)
        write(path,
         ↪  "CM002885.2.1        1000        5        80        81\n")
```

```
end

function print_synthetic_position(path::String)
        write(path, ">1 start 501 end 641 smt_pos 111 smt_value 205.11821 fuzziness_score 43.53698\n",generate_synthetic_seq()[501:641],"\n")
end

function print_synthetic_gff(path::String)
gff_txt = "##gff-version 3
##sequence-region   1 1 1000
#!genome-build BGHMM Synthetic
#!genome-version Synth1.0
#!genome-date 2019-08
#!genome-build-accession NCBI:FAKE
#!genebuild-last-updated 2019-08
1	Ensembl	chromosome	1	1000	.	.	.	gene	[Source:FAKESRC];gene_id=ENSDARG00000000001;logic_name=ensembl_havana_gene;version
###
1	ensembl_havana	gene	501	1000	.	-	.
1	ensembl_havana	mRNA	501	1000	.	-	.
1	ensembl_havana	three_prime_UTR	501	510	.
1	ensembl_havana	exon	501	570	.	+	.
1	ensembl_havana	CDS	511	570	.	+	0
1	ensembl_havana	exon	601	660	.	-	.
1	ensembl_havana	CDS	601	660	.	-	1
1	ensembl_havana	exon	701	760	.	-	.
1	ensembl_havana	CDS	701	760	.	-	0
1	ensembl_havana	exon	801	860	.	-	.
1	ensembl_havana	CDS	801	860	.	-	0
1	ensembl_havana	exon	901	975	.	-	.
1	ensembl_havana	CDS	901	960	.	-	1
1	ensembl_havana	five_prime_UTR	961	975	.	-
###\n"
        write(path,gff_txt)
end
```

```
function generate_synthetic_seq(gene_start::Integer=501, UTR3L::Integer=10, exon_length::Integer=60, intron_length::Integer=40, no_exons::Integer=5, UTR5L::Integer=25, OAL::Integer=1000, (iseq,pseq,eseq)::Tuple{String,String,String}=("AT","CG","CAT"); verbose::Bool=false)
        @assert length(iseq) == 2
        @assert length(pseq) == 2
        @assert length(eseq) == 3
        @assert mod(gene_start-1,2) == 0
        @assert mod(exon_length,3) == 0
        @assert mod(intron_length,2) == 0
        gene_length=gene_start+UTR3L+(no_exons*exon_length)+((no_exons-1)*intron_leng
        @assert gene_length ≤ OAL
        seq=""
        for i in 1:((gene_start-1) / 2)
                seq *= iseq
        end
        verbose && @info "Intergenic length $(length(seq))"
        for i in 1:UTR3L / 2
                seq *= pseq
        end
        verbose && @info "Added 3'UTR ... $(length(seq))"
        for ex in 1:no_exons-1
                for i in 1:(exon_length/3)
                        seq*=eseq
                end
                ex == 1 ? ilength = 30 : ilength = 40
                for i in 1:(ilength/2)
                        seq*=pseq
                end
        end
        for i in 1:(exon_length/3)
                seq*=eseq
        end
        verbose && @info "Added exons and introns ... $(length(seq))"
        for i in 1:UTR5L / 2
                seq *= pseq
        end
        verbose && @info "Added 5'UTR ... $(length(seq))"

        for i in 1:((OAL-length(seq))/2)
                seq*=iseq
```

```
        end

        verbose && @info "Generated synthetic genome sequence of length $(length(seq))"

        return seq
end

  function format_lines(seq::String, line_length::Integer)
        a=join((SubString(seq,i,min(i+line_length-1,length(seq))) for i=1:line_length:length(seq)),'\n')
        return(a)
  end
```

17.6 BioMotifInference

Github repository: https://github.com/mmattocks/BioMotifInference.jl

17.6.1 /src/BioMotifInference.jl

```
module BioMotifInference
    using BioBackgroundModels, BioSequences, Distributed, Distributions, Serialization, UnicodePlots
    import DataFrames:DataFrame
    import ProgressMeter: AbstractProgress, Progress, @showprogress, next!, move_cursor_up_while_clearing_lines, printover, durationstring
    import Printf: @sprintf
    import StatsFuns: logaddexp, logsumexp, logsubexp
    import Random: rand, seed!, shuffle!
    import Distances: euclidean
    import Measurements: measurement

    #CONSTANTS AND PERMUTE FUNCTION ARGUMENT DEFAULTS GIVING RISE TO IMPLEMENTATION-SPECIFIC SAMPLING EFFECTS
    global MOTIF_EXPECT=1. #motif expectation- this value to be divided by obs lengths to obtain penalty factor for scoring

    global REVCOMP=true #calculate scores on both strands?

```

```
global TUNING_MEMORY=20 #coefficient multiplied by Permute_Instruct function call limit to give total number of calls remembered by tuner per function
global CONVERGENCE_MEMORY=500 #number of iterates to display for convergence interval history

global SRC_PERM_FREQ=.5 #frequency with which random_decorrelate will permute a source

global PWM_SHIFT_DIST=Weibull(.5,.1) #distribution of weight matrix permutation magnitudes
global PWM_SHIFT_FREQ=.2 #proportion of positions in source to permute weight matrix
global PWM_LENGTHPERM_FREQ=.2 #proportion of sources to permute length
global LENGTHPERM_RANGE=1:3

global PRIOR_WT=3. #estimate prior dirichlets from product of this constant and sample "mle" wm
global PRIOR_LENGTH_MASS=.8

global EROSION_INFO_THRESH=1.

global CONSOLIDATE_THRESH=.035

include("IPM/ICA_PWM_Model.jl")
export Model_Record
export ICA_PWM_Model
include("IPM/IPM_likelihood.jl")
include("IPM/IPM_prior_utilities.jl")
assemble_source_priors
include("ensemble/IPM_Ensemble.jl")
export IPM_Ensemble, assemble_IPMs
include("permutation/permute_utilities.jl")
include("permutation/orthogonality_helper.jl")
include("permutation/permute_functions.jl")
export full_perm_funcvec
include("permutation/permute_control.jl")
export Permute_Instruct
include("permutation/Permute_Tuner.jl")
include("ensemble/ensemble_utilities.jl")
```

```
    export ensemble_history, reset_ensemble!, move_ensemble!, copy_ensemble!, rewind_ensemble, complete_evidence, get_model, show_models, reestimate_ensemble!
    include("utilities/model_display.jl")
    include("utilities/worker_diagnostics.jl")
    include("utilities/ns_progressmeter.jl")
    include("utilities/synthetic_genome.jl")
    include("utilities/worker_sequencer.jl")
    export synthetic_sample
    include("nested_sampler/nested_step.jl")
    include("nested_sampler/converge_ensemble.jl")
    export converge_ensemble!

end # module
```

17.6.2 /src/IPM/ICA_PWM_Model.jl

```
struct Model_Record #record struct to associate a log_Li with a saved, calculated model
    path::String
    log_Li::AbstractFloat
end

struct ICA_PWM_Model #Independent component analysis position weight matrix model
    name::String #designator for saving model to posterior
    origin::String #functions instantiating IPMs should give an informative desc of the means by which the model was generated
    sources::Vector{Tuple{<:AbstractMatrix{<:AbstractFloat},<:Integer}} #vector of PWM signal sources (LOG PROBABILITY!!!) tupled with an index denoting the position of the first PWM base on the prior matrix- allows us to permute length and redraw from the appropriate prior position
    source_length_limits::UnitRange{<:Integer} #min/max source lengths for init and permutation
    mix_matrix::BitMatrix # obs x sources bool matrix
    log_Li::AbstractFloat
    permute_blacklist::Vector{Function} #blacklist of functions that ought not be used to permute this model (eg. because to do so would not generate a different model for IPMs produced from fitting the mix matrix)
```

```
    function ICA_PWM_Model(name, origin, sources, source_length_limits, mix_matrix, log_Li, permute_blacklist=Vector{Function}())
        verify_srcs(name, origin, sources)
        new(name, origin, sources, source_length_limits, mix_matrix, log_Li, permute_blacklist)
    end
end

#ICA_PWM_Model FUNCTIONS
ICA_PWM_Model(name::String, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
mix_prior::Tuple{BitMatrix,<:AbstractFloat}, bg_scores::AbstractArray{<:AbstractFloat}, observations::AbstractArray{<:Integer}, source_length_limits::UnitRange{<:Integer}) = init_IPM(name, source_priors,mix_prior,bg_scores,observations,source_length_limits)

function verify_srcs(name, ori, sources::Vector{<:Tuple{<:AbstractMatrix{<:AbstractFloat},<:Integer}})
    for (n,(pwm,pi)) in enumerate(sources)
        !all(isprobvec(exp.(pwm[pos,:])) for pos in 1:size(pwm,1)) && throw(DomainError("Bad probvec in model $name, source $n, origin $(ori):$(exp.(pwm))"))
    end
end

#MODEL INIT
function init_IPM(name::String, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
mix_prior::Tuple{BitMatrix,<:AbstractFloat}, bg_scores::AbstractArray{<:AbstractFloat}, observations::AbstractArray{<:Integer}, source_length_limits::UnitRange{<:Integer})
    assert_obs_bg_compatibility(observations,bg_scores)
    T,O = size(observations)
    S=length(source_priors)
    obs_lengths=[findfirst(iszero,observations[:,o])-1 for o in 1:size(observations)[2]]
    sources=init_logPWM_sources(source_priors, source_length_limits)
    mix=init_mix_matrix(mix_prior,O,S)
    log_lh = IPM_likelihood(sources, observations, obs_lengths, bg_scores, mix)

```

```
    return ICA_PWM_Model(name, "init_IPM", sources, source_length_limits, mix, log_lh)
end
                #init_IPM SUBFUNCS
                function assert_obs_bg_compatibility(obs, bg_scores)
                    T,O=size(obs)
                    t,o=size(bg_scores)
                    O!=o && throw(DomainError("Background scores and observations must have same number of observation columns!"))
                    T!=t+1 && throw(DomainError("Background score array must have the same observation lengths as observations!"))
                end

                function init_logPWM_sources(prior_vector::AbstractVector{<:Union{<:Abstra
                source_length_limits::UnitRange{<:Integer})
                    srcvec = Vector{Tuple{Matrix{Float64},Int64}}()
                        for prior in prior_vector
                            if typeof(prior)<:AbstractVector{<:Dirichlet{<:AbstractF
                                if length(prior) < source_length_limits[1]
                                    length_dist=DiscreteNonParametric(
                                        [source_length_limits ... ],
                                        [PRIOR_LENGTH_MASS,ones(length(source_len
                                    PWM_length=rand(length_dist)
                                elseif source_length_limits[1] < length(prior) < source_length_limits[end]
                                    length_dist=DiscreteNonParametric(
                                        [source_length_limits ... ],
                                        [
                                            (ones(length(prior)-source_length_lim
                                            (ones(source_length_limits[end]-lengt
                                        ]
                                    )
                                    PWM_length=rand(length_dist)
                                else
```

```
                                    PWM_length=rand(source_length_limits)
                                end

                                if PWM_length>length(prior)

                                    prior_coord=rand(-(PWM_length-length(prior)):
                                else

                                    prior_coord=rand(1:length(prior)-PWM_length+1
                                end

                                PWM = zeros(PWM_length,4)

                                curr_pos=prior_coord
                                for pos in 1:PWM_length
                                    curr_pos < 1 || curr_pos >
                                    length(prior) ?
                                    dirichlet=Dirichlet(ones(4)/4) :
                                        dirichlet=prior[curr_pos]
                                    PWM[pos, :] = rand(dirichlet)
                                end
                                push!(srcvec, (log.(PWM), prior_coord))
                                #push the source PWM to the source
                                vector with the prior coord idx to
                                allow drawing from the appropriate
                                prior dirichlets on permuting source
                                length
                            elseif prior==false
                                PWM_length=rand(source_length_limits)
                                PWM=zeros(PWM_length,4)
                                dirichlet=Dirichlet(ones(4)/4)
                                for pos in 1:PWM_length
                                    PWM[pos,:] = rand(dirichlet)
                                end
                                push!(srcvec, (log.(PWM), 1))
                            else
                                throw(ArgumentError("Bad prior supplied
                                for ICA_PWM_Model!"))
                            end
                        end
                    return srcvec
                end

```

```
                function init_mix_matrix(mix_prior::Tuple{BitMatrix,<:AbstractFloat}, no_observations::Integer, no_sources::Integer)
                    inform,uninform=mix_prior
                    if size(inform,2) > 0
                        @assert size(inform,1)==no_observations && size(inform,2)≤no_sources "Bad informative mix prior dimensions!"
                    end
                    @assert 0.0 ≤ uninform ≤1.0 "Uninformative mix prior not between 0.0 and 1.0!"
                    mix_matrix = falses(no_observations, no_sources)
                    if size(inform,2)>0
                        mix_matrix[:,1:size(inform,2)]=inform
                    end
                    for index in CartesianIndices((1:no_observations,size(inform,2)+1:no_sourc
                        rand() ≤ uninform && (mix_matrix[index] = true)
                    end
                    return mix_matrix
                end

function Base.show(io::IO, m::ICA_PWM_Model; nsrc::Integer=length(m.sources), progress=false)
    nsrc == 0 && (nsrc=length(m.sources))
    nsrc>length(m.sources) && (nsrc=length(m.sources))
    nsrc==length(m.sources) ? (srcstr="All") : (srcstr="Top $nsrc")

    printidxs,printsrcs,printfreqs=sort_sources(m,nsrc)

    printstyled(io, "ICA PWM Model $(m.name) w/ logLi $(m.log_Li)\n", bold=true)
    println(io, srcstr*" sources:")
    for src in 1:nsrc
        print(io, "S$(printidxs[src]), $(printfreqs[src]*100)%: ")
        pwmstr_to_io(io, printsrcs[src])
        println(io)
    end
    println(m.origin)

    progress && return(nsrc+4)
end
```

```
function sort_sources(m, nsrc)
    printidxs=Vector{Integer}()
    printsrcs=Vector{Matrix{Float64}}()
    printfreqs=Vector{Float64}()

    freqs=vec(sum(m.mix_matrix,dims=1)); total=size(m.mix_matrix,1)
    sortfreqs=sort(freqs,rev=true); sortidxs=sortperm(freqs,rev=true)
    for srcidx in 1:nsrc
        push!(printidxs, sortidxs[srcidx])
        push!(printsrcs, m.sources[sortidxs[srcidx]][1])
        push!(printfreqs, sortfreqs[srcidx]/total)
    end
    return printidxs,printsrcs,printfreqs
end
```

17.6.3 /src/IPM/IPM_likelihood.jl

```
#LIKELIHOOD SCORING FUNCS
function IPM_likelihood(sources::AbstractVector{<:Tuple{<:AbstractMatrix{<:AbstractFloat},
observations::AbstractMatrix{<:Integer},
obs_lengths::AbstractVector{<:Integer},
bg_scores::AbstractArray{<:AbstractFloat}, mix::BitMatrix,
revcomp::Bool=REVCOMP, returncache::Bool=false,
cache::AbstractVector{<:AbstractFloat}=zeros(0),
clean::AbstractVector{<:Bool}=Vector(falses(size(observations)[2])))
    source_wmls=[size(source[1])[1] for source in sources]
    O = size(bg_scores)[2]
    source_stops=[obsl-wml+1 for wml in source_wmls, obsl in obs_lengths] #stop scannng th source across the observation as the source reaches the end
    L=maximum(obs_lengths)+1

    obs_src_idxs=mix_pull_idxs(mix) #get vectors of sources emitting in each obs

```

```
    revcomp ? (srcs=[cat(source[1],revcomp_pwm(source[1]),dims=3) for source in sources]; motif_expectations = [((MOTIF_EXPECT/2)/obsl) for obsl in obs_lengths]; mat_dim=2) : (srcs=[source[1] for source in sources]; ; motif_expectations = [(MOTIF_EXPECT/obsl) for obsl in obs_lengths]; mat_dim=1) #setup appropriate reverse complemented sources if necessary and set log_motif_expectation-nMica has 0.5 per obs for including the reverse complement, 1 otherwise

    lme_vec=zeros(length(sources))

    obs_lhs=Vector{Vector{Float64}}() #setup likelihood vecs for threaded operation-reassembled on return
    nt=Threads.nthreads()
    for t in 1:nt-1
        push!(obs_lhs,zeros(Int(floor(O/nt))))
    end
    push!(obs_lhs, zeros(Int(floor(O/nt)+(O%nt))))

    Threads.@threads for t in 1:nt
        revcomp && (weavevec=zeros(3))
        revcomp ? (score_mat=zeros(maximum(source_stops),2)) : (score_mat=zeros(maximum(source_stops)))
        opt = floor(O/nt) #obs per thread
        score_matrices=Vector{typeof(score_mat)}(undef, length(sources)) #preallocate
        osi_emitting=Vector{Int64}() #preallocate
        lh_vec = zeros(L) #preallocated likelihood vector is one position (0 initialiser) longer than the longest obs

        for i in 1:Int(opt+(t==nt)*(O%nt))
            o=Int(i+(t-1)*opt)
            if clean[o]
                obs_lhs[t][i]=cache[o]
            else
                obsl = obs_lengths[o]
                oidxs=obs_src_idxs[o]
                mixwmls=source_wmls[oidxs]

                obs_cardinality = length(oidxs) #the more sources, the greater the cardinality_penalty
                if obs_cardinality > 0
```

```
                    revcomp ? score_sources_ds!(score_mat, score_matrices, view(srcs,oidxs), view(observations,:,o), view(source_stops,oidxs,o)) : score_sources_ss!(score_mat, score_matrices, view(srcs,oidxs), view(observations,:,o), view(source_stops,oidxs,o)) #get scores for this observation


                    lme_vec•=motif_expectations[o]
                    penalty_sum = sum(lme_vec[1:obs_cardinality])
                    penalty_sum > 1. && (penalty_sum=1.)
                    cardinality_penalty=log(1.0-penalty_sum)
                else
                    cardinality_penalty=0.0
                end

                revcomp ? (obs_lhs[t][i]=weave_scores_ds!(weavevec, lh_vec, obsl, view(bg_scores,:,o), score_matrices, oidxs, mixwmls, log(motif_expectations[o]), cardinality_penalty, osi_emitting)) : (obs_lhs[t][i]=weave_scores_ss!(lh_vec, obsl, view(bg_scores,:,o), score_matrices, oidxs, mixwmls, log(motif_expectations[o]), cardinality_penalty, osi_emitting))

                empty!(osi_emitting)

            end
        end
    end

    returncache ? (return lps([lps(obs_lhs[t]) for t in 1:nt]), vcat(obs_lhs ... )) : (return lps([lps(obs_lhs[t]) for t in 1:nt]))
end
                @inline function mix_pull_idxs(A::AbstractArray{Bool})
                    n=count(A)
                    S=[Vector{Int64}() for o in 1:size(A,1)]
                    cnt=1
                    for (i,a) in pairs(A)
                        if a
                            push!(S[i[1]],i[2])
```

```
                        cnt+=1
                    end
                end
                return S
            end

            @inline function revcomp_pwm(pwm::AbstractMatrix{<:AbstractFloat}) #in order to find a motif on the reverse strand, we scan the forward strand with the reverse complement of the pwm, reordered 3' to 5', so that eg. an PWM for an ATG motif would become one for a CAT motif
                return pwm[end:-1:1,end:-1:1]
            end

            @inline function score_sources_ds!(score_mat, score_matrices::AbstractVector{<:AbstractMatrix{<:AbstractFloat}}, sources, observation::AbstractVector{<:Integer}, source_stops)
                for (s,source) in enumerate(sources)
                    for t in 1:source_stops[s]
                        for position in 1:size(source,1)
                            score_loc = t+position-1 #score_loc is the position of the obs to be scored by PWM
                            score_mat[t,1] += source[position,observation[score_loc],1] #add the appropriate log PWM value from the source to the score
                            score_mat[t,2] += source[position,observation[score_loc],2] #add the appropriate log PWM value from the source to the score

                        end
                    end
                    score_matrices[s]=score_mat[1:source_stops[s],:] #copy score matrix to vector
                    score_mat•=0. #reset score matrix
                end
            end

```

```
95          @inline function score_sources_ss!(score_mat,
            ↪ score_matrices::AbstractVector{<:AbstractArray{<:AbstractFloat}},
            ↪ sources, observation::AbstractVector{<:Integer},
            ↪ source_stops)
96              for (s,source) in enumerate(sources)
97                  for t in 1:source_stops[s]
98                      for position in 1:size(source,1)
99                          score_loc = t+position-1 #score_loc is
                            ↪ the position of the obs to be scored
                            ↪ by PWM
100                         score_mat[t] +=
                            ↪ source[position,observation[score_loc]]
                            ↪ #add the appropriate log PWM value
                            ↪ from the source to the score
101
102                     end
103                 end
104                 score_matrices[s]=score_mat[1:source_stops[s]]
                    ↪ #copy score matrix to vector
105                 score_mat•=0. #reset score matrix
106             end
107         end
108
109         @inline function weave_scores_ds!(weavevec, lh_vec,
            ↪ obsl::Integer, bg_scores::SubArray,
            ↪ score_mat::AbstractVector{<:AbstractMatrix{<:AbstractFloat}},
            ↪ obs_source_indices::AbstractVector{<:Integer},
            ↪ source_wmls::AbstractVector{<:Integer},
            ↪ log_motif_expectation::AbstractFloat,
            ↪ cardinality_penalty::AbstractFloat, osi_emitting)
110             for i in 2:obsl+1 #i=1 is the lh_vec initializing 0,
                ↪ i=2 is the score of the first background position
                ↪ (ie t=1)
111                 t=i-1
112                 score = lps(lh_vec[i-1], bg_scores[t],
                    ↪ cardinality_penalty)
113
114                 #logic: all observations are scored from t=wml to
                    ↪ the end of the obs-therefore check at each
                    ↪ position for new sources to add (indexed by
                    ↪ vector position to retrieve source wml and
                    ↪ score matrix)
```

```
                if length(osi_emitting)<length(obs_source_indices)
                    for n in 1:length(obs_source_indices)
                        if !(n in osi_emitting)
                            t ≥ source_wmls[n] && (push!(osi_emitting,n))
                        end
                    end
                end

                for n in osi_emitting
                    wml = source_wmls[n]
                    from_score = lh_vec[i-wml+1] #score at the first position of the PWM
                    score_array = score_mat[n] #get the source score matrix
                    score_idx = t - wml + 1 #translate t to score_array idx for emission score
                    f_emit_score = score_array[score_idx,1] #emission score at the last position of the PWM
                    r_emit_score = score_array[score_idx,2]

                    weavevec .= score, lps(from_score, f_emit_score, log_motif_expectation), lps(from_score, r_emit_score, log_motif_expectation)

                    score=logsumexp(weavevec)
                end
                lh_vec[i] = score
            end
            return lh_vec[obsl+1]
        end

        @inline function weave_scores_ss!(lh_vec, obsl::Integer, bg_scores::SubArray, score_mat::AbstractVector{<:AbstractVector{<:AbstractFloat}}, obs_source_indices::AbstractVector{<:Integer}, source_wmls::AbstractVector{<:Integer}, log_motif_expectation::AbstractFloat, cardinality_penalty::AbstractFloat, osi_emitting)
```

```
141                     for i in 2:obsl+1 #i=1 is ithe lh_vec initializing 0,
                         ↪  i=2 is the score of the first background position
                         ↪  (ie t=1)
142                         t=i-1
143                         score = lps(lh_vec[i-1], bg_scores[t],
                             ↪  cardinality_penalty)
144                         #logic: all observations are scored from t=wml to
                             ↪  the end of the obs-therefore check at each
                             ↪  position for new sources to add (indexed by
                             ↪  vector position to retrieve source wml and
                             ↪  score matrix)
145                         if
                             ↪  length(osi_emitting)<length(obs_source_indices)
146                             for n in 1:length(obs_source_indices)
147                                 if !(n in osi_emitting)
148                                     t ⩾ source_wmls[n] &&
                                         ↪  (push!(osi_emitting,n))
149                                 end
150                             end
151                         end
152
153                         for n in osi_emitting
154                             wml = source_wmls[n]
155                             from_score = lh_vec[i-wml+1] #score at the
                                 ↪  first position of the PWM
156                             score_array = score_mat[n] #get the source
                                 ↪  score matrix
157                             score_idx = t - wml + 1 #translate t to
                                 ↪  score_array idx for emission score
158                             f_emit_score = score_array[score_idx]
                                 ↪  #emission score at the last position of
                                 ↪  the PWM
159
160                             score=logaddexp(score, lps(from_score,
                                 ↪  f_emit_score, log_motif_expectation))
161                         end
162                         lh_vec[i] = score
163                     end
164                     return lh_vec[obsl+1]
165                 end
```

17.6.4 /src/IPM/IPM_prior_utilities.jl

```
function read_fa_wms_tr(path::String)
    wms=Vector{Matrix{Float64}}()
    wm=zeros(1,4)
    f=open(path)
    for line in eachline(f)
        prefix=line[1:2]
        prefix == "01" && (wm=transpose([parse(Float64,i) for i in split(line)[2:end]]))
        prefix ≠ "01" && prefix ≠ "NA" && prefix ≠ "PO" && prefix ≠ "//" && (wm=vcat(wm, transpose([parse(Float64,i) for i in split(line)[2:end]])))
        prefix == "//" && push!(wms, wm)
    end
    return wms
end

#wm_samples are in decimal probability space, not log space
function assemble_source_priors(no_sources::Integer, wm_samples::AbstractVector{<:AbstractMatrix{<:AbstractFloat}}, prior_wt::AbstractFloat=PRIOR_WT) #estimate a dirichlet prior on wm_samples inputs; if the number of samples is lower than the number of sources, return a false bool for init and permutation functions
    source_priors = Vector{Union{Vector{Dirichlet{Float64}},Bool}}()
    for source in 1:no_sources
        if source ≤ length(wm_samples)
            push!(source_priors, estimate_dirichlet_prior_on_wm(wm_samples[source], prior_wt))
        else
            push!(source_priors, false)
        end
    end
    return source_priors
end

            function estimate_dirichlet_prior_on_wm(wm::AbstractMatrix{<:AbstractFloat}, wt::AbstractFloat=PRIOR_WT)
                for i in 1:size(wm)[1]
```

```
                    !(isprobvec(wm[i,:])) && throw(DomainError("Bad weight vec supplied to estimate_dirichlet_prior_on_wm! $(wm[i,:])"))
                end
                prior = Vector{Dirichlet{Float64}}()
                for position in 1:size(wm)[1]
                    normvec=wm[position,:]
                    zero_idxs=findall(isequal(0.),wm[position,:])
                    normvec[zero_idxs].+=10^-99
                    push!(prior, Dirichlet(normvec.*wt))
                end
                return prior
            end

function cluster_mix_prior!(df::DataFrame, wms::AbstractVector{<:AbstractMatrix{<:AbstractFloat}})
    mix=falses(size(df,1),length(wms))
    for (o, row) in enumerate(eachrow(df))
        row.cluster ≠ 0 && (mix[o,row.cluster]=true)
    end

    represented_sources=unique(df.cluster)
    return mix[:,represented_sources]
end

function infocenter_wms_trim(wm::AbstractMatrix{<:AbstractFloat}, trimsize::Integer)
    !(size(wm,2)==4) && throw(DomainError("Bad wm! 2nd dimension should be size 4"))
    infovec=get_pwm_info(wm, logsw=false)
    maxval, maxidx=findmax(infovec)
    upstream_extension=Int(floor((trimsize-1)/2))
    downstream_extension=Int(ceil((trimsize-1)/2))
    1+upstream_extension+downstream_extension > size(wm,1) && throw(DomainError("Src too short for trim! $upstream_extension $downstream_extension"))
    return wm[max(1,maxidx-upstream_extension):min(maxidx+downstream_extension,size(wm,1
end

function filter_priors(target_src_no::Integer, target_src_size::Integer, prior_wms::AbstractVector{<:AbstractMatrix{<:AbstractFloat}}, prior_mix::BitMatrix)
```

```
    wms=Vector{Matrix{Float64}}(undef, target_src_no)
    freqsort_idxs=sortperm([sum(prior_mix[:,s]) for s in 1:length(prior_wms)])
    for i in 1:target_src_no
        target_src_idx=freqsort_idxs[i]
        wms[i]=infocenter_wms_trim(prior_wms[target_src_idx], target_src_size)
    end
    return wms
end

function combine_filter_priors(target_src_no::Integer, target_src_size::Integer, prior_wms::Tuple{<:AbstractVector{<:AbstractMatrix{<:AbstractFloat}},AbstractVect
prior_mix::Tuple{BitMatrix,BitMatrix})
    wms=Vector{Matrix{Float64}}(undef, target_src_no)
    cat_wms=vcat(prior_wms[1],prior_wms[2])
    first_freq=[sum(prior_mix[1][:,s]) for s in 1:length(prior_wms[1])]
    second_freq=[sum(prior_mix[2][:,s]) for s in 1:length(prior_wms[2])]
    freqsort_idxs=sortperm(vcat(first_freq,second_freq))
    for i in 1:target_src_no
        target_src_idx=freqsort_idxs[i]
        wms[i]=infocenter_wms_trim(cat_wms[target_src_idx], target_src_size)
    end
    return wms
end
```

17.6.5 /src/ensemble/IPM_Ensemble.jl

```
mutable struct IPM_Ensemble
        path::String #ensemble models and popped-out posterior samples serialised here
        models::Vector{Model_Record} #ensemble keeps paths to serialised models and their likelihood tuples rather than keeping the models in memory

        contour::AbstractFloat#current log_Li[end]
        log_Li::Vector{AbstractFloat} #likelihood of lowest-ranked model at iterate i
        log_Xi::Vector{AbstractFloat} #amt of prior mass included in ensemble contour at Li
```

```
        log_wi::Vector{AbstractFloat} #width of prior mass covered in this iterate
        log_Liwi::Vector{AbstractFloat} #evidentiary weight of the iterate
        log_Zi::Vector{AbstractFloat} #ensemble evidence
        Hi::Vector{AbstractFloat} #ensemble information

        obs_array::Matrix{Integer} #observations Txo
        obs_lengths::Vector{Integer}

        source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:Abstract #source pwm priors
        mix_prior::Tuple{BitMatrix,AbstractFloat} #prior on %age of observations that any given source contributes to

        bg_scores::Matrix{AbstractFloat} #precalculated background HMM scores, same dims as obs

        sample_posterior::Bool
        posterior_samples::Vector{Model_Record} #list of posterior sample records

        model_counter::Integer

        naive_lh::AbstractFloat #the likelihood of the background model without any sources
end

####IPM_Ensemble FUNCTIONS
IPM_Ensemble(path::String, no_models::Integer, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa mix_prior::Tuple{BitMatrix,<:AbstractFloat}, bg_scores::AbstractMatrix{<:AbstractFloat}, obs::AbstractArray{<:Integer}, source_length_limits; posterior_switch::Bool=false) =
IPM_Ensemble(
        path,
        assemble_IPMs(path, no_models, source_priors, mix_prior, bg_scores, obs, source_length_limits)...,
        [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
```

```
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
        obs,
        [findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]],
        source_priors,
        mix_prior,
        bg_scores, #precalculated background score
        posterior_switch,
        Vector{String}(),
        no_models+1,
        IPM_likelihood(init_logPWM_sources(source_priors,
         ↪  source_length_limits), obs, [findfirst(iszero,obs[:,o])-1 for
         ↪  o in 1:size(obs)[2]], bg_scores,
         ↪  falses(size(obs)[2],length(source_priors))))

IPM_Ensemble(worker_pool::AbstractVector{<:Integer}, path::String,
 ↪  no_models::Integer,
 ↪  source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
 ↪  mix_prior::Tuple{BitMatrix,<:AbstractFloat},
 ↪  bg_scores::AbstractMatrix{<:AbstractFloat}, obs::Array{<:Integer},
 ↪  source_length_limits; posterior_switch::Bool=false) =
IPM_Ensemble(
        path,
        distributed_IPM_assembly(worker_pool, path, no_models,
         ↪  source_priors, mix_prior, bg_scores, obs,
         ↪  source_length_limits)...,
        [-Inf], #L0 = 0
        [0], #X0 = 1
        [-Inf], #w0 = 0
        [-Inf], #Liwi0 = 0
        [-Inf], #Z0 = 0
        [0], #H0 = 0,
        obs,
        [findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]],
        source_priors,
        mix_prior,
        bg_scores, #precalculated background score
        posterior_switch,
        Vector{String}(),
        no_models+1,
```

```
        IPM_likelihood(init_logPWM_sources(source_priors, source_length_limits), obs, [findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]], bg_scores, falses(size(obs)[2],length(source_priors))))

function assemble_IPMs(path::String, no_models::Integer, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
mix_prior::Tuple{BitMatrix,<:AbstractFloat}, bg_scores::AbstractArray{<:AbstractFloat}, obs::AbstractArray{<:Integer}, source_length_limits::UnitRange{<:Integer})
        ensemble_records = Vector{Model_Record}()
        !isdir(path) && mkpath(path)

        @assert size(obs)[2]==size(bg_scores)[2]

        @showprogress 1 "Assembling IPM ensemble..." for model_no in 1:no_models
                model_path = string(path,'/',model_no)
                if !isfile(model_path)
                        model = ICA_PWM_Model(string(model_no), source_priors, mix_prior, bg_scores, obs, source_length_limits)
                        serialize(model_path, model) #save the model to the ensemble directory
                        push!(ensemble_records, Model_Record(model_path,model.log_Li))
                else #interrupted assembly pick up from where we left off
                        model = deserialize(model_path)
                        push!(ensemble_records, Model_Record(model_path,model.log_Li))
                end
        end

        return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end

```

```
function distributed_IPM_assembly(worker_pool::Vector{Int64}, path::String, no_models::Integer, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
mix_prior::Tuple{BitMatrix,<:AbstractFloat}, bg_scores::AbstractArray{<:AbstractFloat}, obs::AbstractArray{<:Integer}, source_length_limits::UnitRange{<:Integer})
        ensemble_records = Vector{Model_Record}()
        !isdir(path) && mkpath(path)

        @assert size(obs)[2]==size(bg_scores)[2]

    model_chan=RemoteChannel(()->Channel{ICA_PWM_Model}(length(worker_pool)))
    job_chan = RemoteChannel(()->Channel{Union{Tuple,Nothing}}(1))
        put!(job_chan,(source_priors, mix_prior, bg_scores, obs, source_length_limits))

        sequence_workers(worker_pool, worker_assemble, job_chan, model_chan)

        assembly_progress=Progress(no_models, desc="Assembling IPM ensemble ... ")

        model_counter=check_assembly!(ensemble_records, path, no_models, assembly_progress)

        while model_counter ≤ no_models
                wait(model_chan)
                candidate=take!(model_chan)
                model = ICA_PWM_Model(string(model_counter), candidate.origin, candidate.sources, candidate.source_length_limits, candidate.mix_matrix, candidate.log_Li)
                model_path=string(path,'/',model_counter)
                serialize(model_path,model)
                push!(ensemble_records, Model_Record(model_path,model.log_Li))
                model_counter+=1
                next!(assembly_progress)
        end

        take!(job_chan),put!(job_chan,nothing)
```

```

        return ensemble_records, minimum([record.log_Li for record in ensemble_records])
end
                                function check_assembly!(ensemble_records::AbstractVector{path::String, no_models::Integer, assembly_progress::Progress)
                                        counter=1
                                        while counter ≤ no_models
                                                model_path=string(path,'/',counter)
                                                if isfile(model_path)
                                                        model=deserialize(model_path)
                                                        push!(ensemble_records, Model_Record(model_path,m
                                                        counter+=1
                                                        next!(assembly_progress)
                                                else
                                                        return counter
                                                end
                                        end
                                        return counter
                                end

                                function worker_assemble(job_chan::RemoteChannel, models_chan::RemoteChannel, comms_chan::RemoteChannel)
                                        put!(comms_chan,myid())
                                        wait(job_chan)
                                        params=fetch(job_chan)
                                        while !(fetch(job_chan) === nothing)
                                                model=ICA_PWM_Model(string(myid()),pa
                                                put!(models_chan,model)
                                        end
                                end

function Base.show(io::IO, e::IPM_Ensemble; nsrc=0, progress=false)
        livec=[model.log_Li for model in e.models]
        maxLH=maximum(livec)
        printstyled(io, "ICA PWM Model Ensemble @ $(e.path)\n", bold=true)
```

```
        msg = @sprintf "Contour: %3.6e MaxLH:%3.3e Max/Naive:%3.3e log Evidence:%3.6e" e.contour maxLH (maxLH-e.naive_lh) e.log_Zi[end]
        println(io, msg)
        hist=UnicodePlots.histogram(livec, title="Ensemble Likelihood Distribution")
        show(io, hist)
        println()
        progress && return(nrows(hist.graphics)+6)
end
```

17.6.6 /src/ensemble/ensemble_utilities.jl

```
function ensemble_history(e::IPM_Ensemble, bins=25)
    !e.sample_posterior && throw(ArgumentError("This ensemble has no posterior samples to show a history for!"))
    livec=vcat([model.log_Li for model in e.models],[model.log_Li for model in e.posterior_samples])
    show(histogram(livec, nbins=bins))
end

function e_backup(e::IPM_Ensemble, tuner::Permute_Tuner)
    cp(string(e.path,'/',"ens"),string(e.path,'/',"ens.bak"), force=true)
    serialize(string(e.path,'/',"ens"), e)
    serialize(string(e.path,'/',"tuner"), tuner)
end

function clean_ensemble_dir(e::IPM_Ensemble, model_pad::Integer; ignore_warn=false)
    !ignore_warn && e.sample_posterior && throw(ArgumentError("Ensemble is set to retain posterior samples and its directory should not be cleaned!"))
    for file in readdir(e.path)
        !(file in vcat([basename(model.path) for model in e.models],"ens",[string(number) for number in e.model_counter-length(e.models)-model_pad:e.model_counter-1])) && rm(e.path*'/'*file)
    end
end

function complete_evidence(e::IPM_Ensemble)
    log_Z=e.log_Zi[end]
```

```
    H=e.Hi[end]
    live_weight=lps(e.log_Xi[length(e.log_Li)], -log(length(e.models)))
    lis=sort([model.log_Li for model in e.models])
    liwis=sort(lps.(lis, live_weight))

    for (li,liwi) in zip(lis,liwis)
        last_Z=log_Z
        log_Z=logaddexp(log_Z,liwi)
        H=lps(
            (exp(lps(liwi,-log_Z)) * li),
            (exp(lps(last_Z,-log_Z)) * lps(H,last_Z)),
            -log_Z)
    end

    return log_Z, H
end

function measure_evidence(e::IPM_Ensemble)
    log_Z, H = complete_evidence(e)
    return measurement(log_Z,sqrt(abs(H)/length(e.models)))
end

function reset_ensemble!(e::IPM_Ensemble)
    new_e=deepcopy(e)
    for i in 1:length(e.models)
        if string(i) in [basename(record.path) for record in e.models]
            new_e.models[i]=e.models[findfirst(isequal(string(i)),
            ↪ [basename(record.path) for record in e.models])]
        else

            ↪ new_e.models[i]=e.posterior_samples[findfirst(isequal(string(i)),
            ↪ [basename(record.path) for record in
            ↪ e.posterior_samples])]
        end
    end

    new_e.contour=minimum([record.log_Li for record in new_e.models])

    new_e.log_Li=[new_e.log_Li[1]]
    new_e.log_Xi=[new_e.log_Xi[1]]
    new_e.log_wi=[new_e.log_wi[1]]
    new_e.log_Liwi=[new_e.log_Liwi[1]]
    new_e.log_Zi=[new_e.log_Zi[1]]
```

```
    new_e.Hi=[new_e.Hi[1]]

    new_e.posterior_samples=Vector{Model_Record}()

    new_e.model_counter=length(new_e.models)+1

    clean_ensemble_dir(new_e, 0; ignore_warn=true)
    isfile(e.path*"/tuner") && rm(e.path*"/tuner")
    serialize(e.path*"/ens", new_e)

    return new_e
end

function move_ensemble!(e::IPM_Ensemble,path::String)
    !isdir(path) && mkdir(path)
    for file in readdir(e.path)
        mv(e.path*'/'*file,path*'/'*file)
    end

    for (n,model) in enumerate(e.models)
        e.models[n]=Model_Record(path*'/'*basename(model.path),
        ↪  model.log_Li)
    end
    if e.sample_posterior
        for (n,model) in enumerate(e.posterior_samples)

            ↪  e.posterior_samples[n]=Model_Record(path*'/'*basename(model.path),
            ↪  model.log_Li)
        end
    end

    rm(e.path)
    e.path=path
    serialize(e.path*"/ens",e)
    return e
end

function copy_ensemble!(e::IPM_Ensemble,path::String)
    new_e=deepcopy(e)
    !isdir(path) && mkdir(path)
    for file in readdir(e.path)
        cp(e.path*'/'*file,path*'/'*file, force=true)
    end
```

```

    for (n,model) in enumerate(e.models)
        new_e.models[n]=Model_Record(path*'/'*basename(model.path), model.log_Li)
    end
    if e.sample_posterior
        for (n,model) in enumerate(e.posterior_samples)
            new_e.posterior_samples[n]=Model_Record(path*'/'*basename(model.path), model.log_Li)
        end
    end

    new_e.path=path
    serialize(new_e.path*"/ens",new_e)
    return new_e
end

function rewind_ensemble(e::IPM_Ensemble,rewind_idx)
    !e.sample_posterior && throw(ArgumentError("An ensemble not retaining posterior samples cannot be rewound!"))
    rewind_idx ≥ length(e.log_Li) && throw(ArgumentError("rewind_idx must be less than the current iterate!"))

    n=length(e.models)
    max_model_no=length(e.log_Li)+length(e.models)-1
    rewind_model_no=rewind_idx+length(e.models)-1
    new_e = deepcopy(e)

    rm_models=[string(name) for name in rewind_model_no+1:max_model_no]

    filter!(model→!(basename(model.path) in rm_models),new_e.models)
    filter!(model→!(basename(model.path) in rm_models),new_e.posterior_samples)

    while length(new_e.models) < n
        push!(new_e.models,pop!(new_e.posterior_samples))
    end
    new_e.contour=new_e.log_Li[rewind_idx]
    new_e.log_Li=new_e.log_Li[1:rewind_idx]
    new_e.log_Xi=new_e.log_Xi[1:rewind_idx]
    new_e.log_wi=new_e.log_wi[1:rewind_idx]
    new_e.log_Liwi=new_e.log_Liwi[1:rewind_idx]
```

```
    new_e.log_Zi=new_e.log_Zi[1:rewind_idx]
    new_e.Hi=new_e.Hi[1:rewind_idx]

    new_e.model_counter=length(new_e.models)+rewind_idx

    return new_e
end

function show_models(e::IPM_Ensemble,idxs)
    liperm=sortperm([model.log_Li for model in e.models],rev=true)
    for idx in idxs
        m=deserialize(e.models[liperm[idx]].path)
        show(m)
    end
end

function get_model(e::IPM_Ensemble,no)
    return deserialize(e.path*'/'*string(no))
end

function reestimate_ensemble!(e::IPM_Ensemble, wi_mode="trapezoidal")
    Ni=Int64.(round.(collect(-1:-1:-length(e.log_Li)+1)./e.log_Xi[2:end]))
    push!(Ni, length(e.models))

    #fix any borked starting values
    e.log_Li[1]=-Inf #L0 = 0
        e.log_Xi[1]=0. #X0 = 1
        e.log_wi[1]=-Inf #w0 = 0
        e.log_Liwi[1]=-Inf #Liwi0 = 0
        e.log_Zi[1]=-Inf #Z0 = 0
        e.Hi[1]=0. #H0 = 0,

    for i in 1:length(e.log_Li)-1
        j=i+1

        e.log_Li[j]=e.posterior_samples[i].log_Li

        e.log_Xi[j]=-i/Ni[i]

        if wi_mode=="trapezoidal"
```

```
            e.log_wi[j]= logsubexp(e.log_Xi[i], -j/Ni[j]) - log(2) #log width of prior mass spanned by the last step-trapezoidal approx
        elseif wi_mode=="simple"
            e.log_wi[j]= logsubexp(e.log_Xi[i], e.log_Xi[j]) #log width of prior mass spanned by the last step-simple approx
        else
            throw(ArgumentError("Unsupported wi_mode!"))
        end
        e.log_Liwi[j]=lps(e.log_Li[j],e.log_wi[j]) #log likelihood + log width = increment of evidence spanned by iterate
        e.log_Zi[j]=logaddexp(e.log_Zi[i],e.log_Liwi[j])    #log evidence
        #information- dimensionless quantity
        Hj=lps(
            (exp(lps(e.log_Liwi[j],-e.log_Zi[j])) * e.log_Li[j]),
            (exp(lps(e.log_Zi[i],-e.log_Zi[j])) * lps(e.Hi[i],e.log_Zi[i])),
            -e.log_Zi[j])
        Hj === -Inf ? (e.Hi[j]=0.) : (e.Hi[j]=Hj)
    end
end
```

17.6.7 /src/nested_sampler/converge_ensemble.jl

```
function converge_ensemble!(e::IPM_Ensemble, instruction::Permute_Instruct; max_iterates=typemax(Int64), backup::Tuple{Bool,Integer}=(false,0), clean::Tuple{Bool,Integer,Integer}=(false,0,0), verbose::Bool=false, converge_criterion::String="standard", converge_factor::AbstractFloat=.001, progargs... )
    N = length(e.models); curr_it=length(e.log_Li)

    curr_it>1 && isfile(e.path*"/tuner") ? (tuner=deserialize(e.path*"/tuner")) : (tuner = Permute_Tuner(instruction)) #restore tuner from saved if any
    wk_mon = Worker_Monitor([1])
    meter = ProgressNS(e, wk_mon, tuner, 0.; start_it=curr_it, progargs... )

    converge_check = get_convfunc(converge_criterion)
    while !converge_check(e, converge_factor) && (curr_it ≤ max_iterates)
```

```
        warn,step_report = nested_step!(e, instruction) #step the ensemble
        warn == 1 && #"1" passed for warn code means no workers persist; all have hit the permute limit
            (@error "Failed to find new models, aborting at current iterate."; return e) #if there is a warning, iust return the ensemble and print info
        curr_it += 1

        tune_weights!(tuner, step_report)
        instruction = tuner.inst

        backup[1] && curr_it%backup[2] == 0 && e_backup(e,tuner) #every backup interval, serialise the ensemble and instruction
        clean[1] &&  !e.sample_posterior && curr_it%clean[2] == 0 && clean_ensemble_dir(e,clean[3]) #every clean interval, remove old discarded models

        update!(meter, converge_check(e,converge_factor,vals=true)... )
    end

    if converge_check(e,converge_factor)
        final_logZ = measure_evidence(e)
        @info "Job done, sampled to convergence. Final logZ $(final_logZ.val) ± $(final_logZ.err)"

        e_backup(e,tuner)
        clean[1] && !e.sample_posterior && clean_ensemble_dir(e,0) #final clean
        return final_logZ
    elseif curr_it==max_iterates
        @info "Job done, sampled to maximum iterate $max_iterates. Convergence criterion not obtained."

        e_backup(e,tuner)
        clean[1] && !e.sample_posterior && clean_ensemble_dir(e,0) #final clean
        return e.log_Zi[end]
    end
end

```

```
function converge_ensemble!(e::IPM_Ensemble, instruction::Permute_Instruct, wk_pool::Vector{Int64}; max_iterates=typemax(Int64), backup::Tuple{Bool,Integer}=(false,0), clean::Tuple{Bool,Integer,Integer}=(false,0,0), verbose::Bool=false, converge_criterion::String="standard", converge_factor::AbstractFloat=.001, progargs... )
    N = length(e.models)

    converge_check = get_convfunc(converge_criterion)
    model_chan= RemoteChannel(()->Channel{Tuple{Union{ICA_PWM_Model,String},Integer, AbstractVector{<:Tuple}}}(10*length(wk_pool))) #channel to take EM iterates off of
    job_chan = RemoteChannel(()->Channel{Tuple{<:AbstractVector{<:Model_Record}, Float64, Union{Permute_Instruct,String}}}(1))
    put!(job_chan,(e.models, e.contour, instruction))

    if !converge_check(e,converge_factor) #sequence workers only if not already converged
        @async sequence_workers(wk_pool, permute_IPM, e, job_chan, model_chan)
    end

    curr_it=length(e.log_Li)
    wk_mon=Worker_Monitor(wk_pool)
    curr_it>1 && isfile(e.path*"/tuner") ? (tuner=deserialize(e.path*"/tuner")) : (tuner = Permute_Tuner(instruction)) #restore tuner from saved if any
    meter = ProgressNS(e, wk_mon, tuner, 0.; start_it=curr_it, progargs... )

    while !converge_check(e, converge_factor) && (curr_it ≤ max_iterates)
        warn, step_report = nested_step!(e, model_chan, wk_mon) #step the ensemble
        warn == 1 && #"1" passed for warn code means no workers persist; all have hit the permute limit
                (@error "All workers failed to find new models, aborting at current iterate."; return e) #if there is a warning, iust return the ensemble and print info
        curr_it += 1
        tune_weights!(tuner, step_report)
```

```
        instruction = tuner.inst
        take!(job_chan); put!(job_chan,(e.models,e.contour,instruction))
        backup[1] && curr_it%backup[2] == 0 && e_backup(e,tuner) #every backup interval, serialise the ensemble and instruction
        clean[1] && !e.sample_posterior && curr_it%clean[2] == 0 && clean_ensemble_dir(e,clean[3]) #every clean interval, remove old discarded models

        update!(meter, converge_check(e,converge_factor,vals=true)... )
    end

    take!(job_chan); put!(job_chan, (e.models, e.contour, "stop")) #stop instruction terminates worker functions

    if converge_check(e,converge_factor)
        final_logZ = measure_evidence(e)
        @info "Job done, sampled to convergence. Final logZ $(final_logZ.val) ± $(final_logZ.err)"

        e_backup(e,tuner)
        clean[1] && !e.sample_posterior && clean_ensemble_dir(e,0) #final clean
        return final_logZ
    elseif curr_it==max_iterates
        @info "Job done, sampled to maximum iterate $max_iterates. Convergence criterion not obtained."

        e_backup(e,tuner)
        clean[1] && !e.sample_posterior && clean_ensemble_dir(e,0) #final clean
        return e.log_Zi[end]
    end
end

                function evidence_converge(e, evidence_fraction; vals=false)
                    val=lps(findmax([model.log_Li for model in e.models])[1],  e.log_Xi[end])
                    thresh=lps(log(evidence_fraction),e.log_Zi[end])
                    vals ? (return val, thresh) : (return val<thresh)
                end

```

```
                function compress_converge(e, compression_ratio; vals=false)
                    val=findmax([model.log_Li for model in e.models])[1]-e.contour
                    thresh=compression_ratio
                    vals ? (return val, thresh) : (return val<thresh)
                end

                function get_convfunc(criterion)
                    if criterion == "standard"
                        return evidence_converge
                    elseif criterion == "compression"
                        return compress_converge
                    else
                        throw(ArgumentError("Convergence criterion $criterion not supported! Try \"standard\" or \"compression\"."))
                    end
                end
```

17.6.8 /src/nested_sampler/nested_step.jl

```
#### IMPLEMENTATION OF JOHN SKILLINGS' NESTED SAMPLING ALGORITHM ####
function nested_step!(e::IPM_Ensemble, instruction::Permute_Instruct)
    N = length(e.models) #number of sample models/particles on the posterior surface
    i = length(e.log_Li) #iterate number, index for last values
    j = i+1 #index for newly pushed values

    e.contour, least_likely_idx = findmin([model.log_Li for model in e.models])
    Li_model = e.models[least_likely_idx]

    #SELECT NEW MODEL, SAVE TO ENSEMBLE DIRECTORY, CREATE RECORD AND PUSH TO ENSEMBLE
    model_selected=false; step_report=0
    while !model_selected
        candidate,step_report=permute_IPM(e, instruction)
        if !(candidate===nothing)
            if !(candidate.log_Li in [m.log_Li for m in e.models])
                model_selected=true

```

```
                new_model_record = Model_Record(string(e.path,'/',e.model_counter), candidate.log_Li);
                e.sample_posterior && push!(e.posterior_samples, Li_model)#if sampling posterior, push the model record to the ensemble's posterior samples vector
                deleteat!(e.models, least_likely_idx)
                push!(e.models, new_model_record);

                final_model=ICA_PWM_Model(string(e.model_counter), candidate.origin, candidate.sources, candidate.source_length_limits, candidate.mix_matrix, candidate.log_Li, candidate.permute_blacklist)
                serialize(new_model_record.path, final_model)

                e.model_counter +=1
            end
        else
            push!(e.models, Li_model)
            return 1, step_report
        end
    end

    #UPDATE ENSEMBLE QUANTITIES
    push!(e.log_Li, e.contour) #log likelihood of the least likely model - the current ensemble ll contour at Xi
    push!(e.log_Xi, -i/N) #log Xi - crude estimate of the iterate's enclosed prior mass
    push!(e.log_wi, logsubexp(e.log_Xi[i], -j/N?) - log(2)) #log width of prior mass spanned by the last step-trapezoidal approx
    push!(e.log_Liwi, lps(e.log_Li[j],e.log_wi[j])) #log likelihood + log width = increment of evidence spanned by iterate
    push!(e.log_Zi, logaddexp(e.log_Zi[i],e.log_Liwi[j]))    #log evidence
    #information- dimensionless quantity. cf. MultiNest @ https://github.com/farhanferoz/MultiNest/blob/master/MultiNest_v3.12/nested.F MultiNest now gives info only at the final step
    Hj=lps(
        (exp(lps(e.log_Liwi[j],-e.log_Zi[j])) * e.log_Li[j]), #information contribution of this step
        (exp(lps(e.log_Zi[i],-e.log_Zi[j])) * lps(e.Hi[i],e.log_Zi[i])), #rescale last information by evidence
        -e.log_Zi[j])
```

```
    Hj === -Inf ? push!(e.Hi,0.) : push!(e.Hi, Hj) #prevent problems from early strings of models with -Inf log likelihoods

    return 0, step_report
end

function nested_step!(e::IPM_Ensemble, model_chan::RemoteChannel, wk_mon::Worker_Monitor)
    N = length(e.models)+1 #number of sample models/particles on the posterior surface- +1 as one has been removed in the distributed dispatch for converge_ensemble
    i = length(e.log_Li) #iterate number, index for last values
    j = i+1 #index for newly pushed values

    e.contour, least_likely_idx = findmin([model.log_Li for model in e.models])
    Li_model = e.models[least_likely_idx]

    #SELECT NEW MODEL, SAVE TO ENSEMBLE DIRECTORY, CREATE RECORD AND REPLACE LEAST LIKELY MODEL
    model_selected=false;wk=0;step_report=0
    while !model_selected
        @async wait(model_chan)
        candidate,wk,step_report = take!(model_chan)
        if !(candidate=="quit")
            if (candidate.log_Li > e.contour) && !(candidate.log_Li in [m.log_Li for m in e.models])
                model_selected=true

                new_model_record = Model_Record(string(e.path,'/',e.model_counter), candidate.log_Li);
                e.sample_posterior && push!(e.posterior_samples, Li_model)#if sampling posterior, push the model record to the ensemble's posterior samples vector
                deleteat!(e.models, least_likely_idx)
                push!(e.models, new_model_record);

                final_model=ICA_PWM_Model(string(e.model_counter), candidate.origin, candidate.sources, candidate.source_length_limits, candidate.mix_matrix, candidate.log_Li, candidate.permute_blacklist)
                serialize(new_model_record.path, final_model)
```

```julia

                e.model_counter +=1
            end
            update_worker_monitor!(wk_mon,wk,true)
        else
            update_worker_monitor!(wk_mon,wk,false)
            !any(wk_mon.persist) && (return 1,step_report)
        end
    end

    #UPDATE ENSEMBLE QUANTITIES
    push!(e.log_Li, minimum([model.log_Li for model in e.models])) #log likelihood of the least likely model - the current ensemble ll contour at Xi
    push!(e.log_Xi, -i/N) #log Xi - crude estimate of the iterate's enclosed prior mass
    push!(e.log_wi, lps(e.log_Xi[i], -((j+1)/N)-log(2))) #log width of prior mass spanned by the last step-trapezoidal approx
    push!(e.log_Liwi, lps(e.log_Li[j],e.log_wi[j])) #log likelihood + log width = increment of evidence spanned by iterate
    push!(e.log_Zi, logaddexp(e.log_Zi[i],e.log_Liwi[j]))    #log evidence

    #information- dimensionless quantity
    push!(e.Hi, lps(
            (exp(lps(e.log_Liwi[j],-e.log_Zi[j])) * e.log_Li[j]), #term1
            (exp(lps(e.log_Zi[i],-e.log_Zi[j])) * lps(e.Hi[i],e.log_Zi[i])), #term2
            -e.log_Zi[j])) #term3

    return 0, step_report
end
```

17.6.9 /src/permutation/Permute_Tuner.jl

```julia
mutable struct Permute_Tuner
    inst::Permute_Instruct
    velocities::Matrix{Float64}
    successes::BitMatrix #(memoryxfunc)
    tabular_display::DataFrame
    time_history::Vector{Float64}
    override::Bool
```

```
end

"""
    Permute_Tuner(instruction)

Generate a Permute_Tuner for the given instruction.
"""
function Permute_Tuner(instruction::Permute_Instruct)
    nfuncs=length(instruction.funcs)
    funcnames=Vector{String}()
    vels=ones(TUNING_MEMORY*instruction.func_limit,nfuncs)
    succs=trues(TUNING_MEMORY*instruction.func_limit,nfuncs)
    for (idx,func) in enumerate(instruction.funcs)
        length(instruction.args[idx])>0 ? (kwstr="(+$(length(instruction.args[idx]))kwa)") : (kwstr="")
        push!(funcnames, string(nameof(func),kwstr))
    end

    tabular_display=DataFrame("Function"=>funcnames, "Succeed"=>zeros(Int64,nfuncs), "Fail"=>zeros(Int64,nfuncs),"Velocity"=>ones(nfuncs), "Weights"=>instruction.weights)

    instruction.override_time>0. ? (override=true) : (override=false)
    return Permute_Tuner(instruction,vels,succs,tabular_display,zeros(CONVERGENCE_MEMORY
end

"""
    tune_weights!(tuner, call_report)

Given a call_report from permute_IPM(), adjust tuner's Permute_Instruct weights for function success rate and likelihood surface velocity.
"""
function tune_weights!(tuner::Permute_Tuner, call_report::Vector{Tuple{Int64,Float64,Float64}})
    for call in call_report
        funcidx,time,distance=call
        distance!==-Inf && (tuner.velocities[:,funcidx]=update_velocity!(tuner.velocities[:,funcidx] #do not push velocity to array if it has -Inf probability (usu no new model found)
        if call===call_report[end]
```

```
            tuner.successes[:,funcidx]=update_sucvec!(tuner.successes[:,funcidx],
            tuner.tabular_display[funcidx,"Succeed"]+=1
        else
            tuner.successes[:,funcidx]=update_sucvec!(tuner.successes[:,funcidx],
            tuner.tabular_display[funcidx,"Fail"]+=1
        end
    end
    tuner.override && mean(tuner.time_history)>tuner.inst.override_time ? (tuner.inst.weights=tuner.inst.override_weights; tuner.tabular_display[!,"Weights"]=tuner.inst.override_weights) :
        update_weights!(tuner)
end

function update_velocity!(velvec,time,distance)
    popfirst!(velvec) #remove first value
    vel = distance - log(time)
    push!(velvec,distance-log(time))
end

function update_sucvec!(sucvec, bool)
    popfirst!(sucvec)
    push!(sucvec, bool)
end

function update_weights!(t::Permute_Tuner)
    mvels=[mean(t.velocities[:,n]) for n in 1:length(t.inst.funcs)]
    t.tabular_display[!,"Velocity"]=copy(mvels)

    any(i->i<0,mvels) && (mvels.+=-minimum(mvels)+1.) #to calculate weights, scale negative values into >1.
    pvec=[mvels[n]*(sum(t.successes[:,n])/length(t.successes[:,n])) for n in 1:length(t.inst.funcs)]
    pvec.//=sum(pvec)
    (any(pvec.<t.inst.min_clmps) || any(pvec.>t.inst.max_clmps)) && clamp_pvec!(pvec,t.inst.min_clmps,t.inst.max_clmps)

    @assert isprobvec(pvec)
    t.inst.weights=pvec; t.tabular_display[!,"Weights"]=pvec
end

"""
```

```
    clamp_pvec(pvec, tuner)

Clamp the values of a probability vector between the minimums and maximums provided by a Permute_Tuner.
"""
            function clamp_pvec!(pvec, min_clmps, max_clmps)
                #logic- first accumulate on low values, then distribute excess from high values
                any(pvec.<min_clmps) ? (low_clamped=false) : (low_clamped=true)
                while !low_clamped
                    vals_to_accumulate=pvec.<min_clmps
                    vals_to_deplete=pvec.>min_clmps


                    depletion=sum(min_clmps[vals_to_accumulate]•-pvec[vals_to_acc

                    pvec[vals_to_accumulate]•=min_clmps[vals_to_accumulate]

                    pvec[vals_to_deplete]•-=depletion/sum(vals_to_deplete)

                    !any(pvec.<min_clmps)&&(low_clamped=true)
                end

                any(pvec.>max_clmps) ? (high_clamped=false) : (high_clamped=true)
                while !high_clamped
                    vals_to_deplete=pvec.>max_clmps
                    vals_to_accumulate=pvec.<max_clmps


                    depletion=sum(pvec[vals_to_deplete]•-max_clmps[vals_to_deplet
                    pvec[vals_to_deplete]•=max_clmps[vals_to_deplete]

                    pvec[vals_to_accumulate].+=depletion/sum(vals_to_accumulate)

                    !any(pvec.>max_clmps)&&(high_clamped=true)
                end
            end

function Base.show(io::IO, tuner::Permute_Tuner; progress=false)
    show(io, tuner.tabular_display, rowlabel=:I, summary=false)
    progress && return(size(tuner.tabular_display,1)+4)
```

```
end
```

17.6.10 /src/permutation/orthogonality_helper.jl

```
##ORTHOGONALITY HELPER
function consolidate_srcs(con_idxs::Dict{Integer,Vector{Integer}}, m::ICA_PWM_Model, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat, models::AbstractVector{<:Model_Record}; iterates::Integer=length(m.sources)*2, remote=false)
    new_log_Li=-Inf;  iterate = 1
    T,O = size(obs_array); T=T-1; S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
    a, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true)

    while new_log_Li ≤ contour && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))

        for host_src in filter(!in(vcat(values(con_idxs)...)), keys(con_idxs)) #copy mix information to the source to be consolidated on as host
            for cons_src in con_idxs[host_src]
                new_mix[:,host_src]⋅=[new_mix[o,host_src] || new_mix[o,cons_src] for o in 1:size(new_mix,1)]
            end
        end

        remote ? (merger_m = deserialize(rand(models).path)) : (merger_m = remotecall_fetch(deserialize, 1, rand(models).path)) #randomly select a model to merge
        used_m_srcs=Vector{Int64}()

        for src in unique(vcat(values(con_idxs)...)) #replace all non-host sources with sources from a merger model unlike the one being removed
            distvec=[pwm_distance(m.sources[src][1],m_src[1]) for m_src in merger_m.sources]
            m_src=findmax(distvec)[2]
```

```
            while m_src in used_m_srcs
                distvec[m_src]=0.; m_src=findmax(distvec)[2]
                length(used_m_srcs)==length(merger_m.sources) && break;break
            end

            clean[new_mix[:,src]]•=false #mark dirty any obs that start with the source
            new_sources[src]=merger_m.sources[m_src]
            new_mix[:,src]=merger_m.mix_matrix[:,m_src]
            clean[new_mix[:,src]]•=false #mark dirty any obs that end with the source
            push!(used_m_srcs, m_src)
        end

        if consolidate_check(new_sources)[1] #if the new sources pass the consolidate check
            new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true, cache, clean) #assess likelihood
        end

        iterate += 1
    end

    return ICA_PWM_Model("candidate","consolidated $(m.origin)", new_sources, m.source_length_limits, new_mix, new_log_Li)
end

function consolidate_check(sources::AbstractVector{<:Tuple{<:AbstractMatrix{<:AbstractFloa
thresh=CONSOLIDATE_THRESH, revcomp=REVCOMP)
    pass=true
    lengthδmat=[size(src1[1],1) - size(src2[1],1) for src1 in sources, src2 in sources]
    cons_idxs=Dict{Integer,Vector{Integer}}()
    for src1 in 1:length(sources), src2 in src1+1:length(sources)
        if lengthδmat[src1,src2]==0
```

```
            revcomp ? (info_condition = (pwm_distance(sources[src1][1],sources[src2][1]) < thresh || pwm_distance(sources[src1][1],revcomp_pwm(sources[src2][1])) < thresh)) : (info_condition = (pwm_distance(sources[src1][1],sources[src2][1]) < thresh))
            if info_condition
                if !in(src1,keys(cons_idxs))
                    cons_idxs[src1]=[src2]; pass=false
                else
                    push!(cons_idxs[src1], src2)
                end
            end
        end
    end

    return pass, cons_idxs
end

                function pwm_distance(pwm1,pwm2)
                    minwml=min(size(pwm1,1),size(pwm2,1))
                    return sum([euclidean(exp.(pwm1[pos,:]), exp.(pwm2[pos,:])) for pos in 1:minwml])/minwml
                end
```

17.6.11 /src/permutation/permute_control.jl

```
mutable struct Permute_Instruct
    funcs::AbstractVector{<:Function}
    weights::AbstractVector{<:AbstractFloat}
    args::AbstractVector{<:AbstractVector{<:Tuple{<:Symbol,<:Any}}}
    model_limit::Integer
    func_limit::Integer
    min_clmps::AbstractVector{<:AbstractFloat}
    max_clmps::AbstractVector{<:AbstractFloat}
    override_time::AbstractFloat
    override_weights::AbstractVector{<:AbstractFloat}
    Permute_Instruct(funcs,
                    weights,
                    model_limit,
                    func_limit;
```

```
                    min_clmps=fill(.01,length(funcs)),
                    max_clmps=fill(1.,length(funcs)),
                    override_time=0.,
                    override_weights=zeros(length(funcs)),
                    args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcs)])=assert_permute_instruct(funcs,weights,args,m && new(funcs,weights,args,model_limit,func_limit,min_clmps,max_cl
end

function assert_permute_instruct(funcs,weights,args,model_limit,func_limit,min_clmps, max_clmps, override_time, override_weights)

    !(length(funcs)==length(args)==length(weights)==length(min_clmps)==length(max_ && throw(ArgumentError("A valid Permute_Instruct must have as many tuning weights and argument vectors as functions!"))
    model_limit<1 && throw(ArgumentError("Permute_Instruct limit on models to permute must be positive Integer!"))
    func_limit<1 && throw(ArgumentError("Permute_Instruct limit on fuction calls per model permtued must be positive Integer!"))
    any(min_clmps.≥max_clmps) && throw(ArgumentError("Permute_Instruct max_clmps must all be greater than corresponding min_clmps!"))
    sum(min_clmps)>1. && throw(ArgumentError("Sum of minimum clamps must be <1.0 to maintain a valid probability vector!"))
    any(max_clmps.>1.) && throw(ArgumentError("Permute_Tuner maximum clamps cannot be >1.0!"))
    override_time<0. && throw(ArgumentError("Permute_Instruct override_time must be 0 (disabled) or positive float!"))
    override_time>0. && !isprobvec(override_weights) && throw(ArgumentError("Permute_Instruct override_weights must be valid probvec!"))
    return true
end

function permute_IPM(e::IPM_Ensemble, instruction::Permute_Instruct)
    call_report=Vector{Tuple{Int64,Float64,Float64}}()
        for model = 1:instruction.model_limit
                m_record = rand(e.models)
        m = deserialize(m_record.path)

        model_blacklist=copy(m.permute_blacklist)
```

```
        filteridxs, filtered_funcs, filtered_weights, filtered_args = filter_permutes(instruction, model_blacklist)

        for call in 1:instruction.func_limit
            start=time()
            funcidx=rand(filtered_weights)
            permute_func=filtered_funcs[funcidx]

            pos_args,kw_args=get_permfunc_args(permute_func,e,m,filtered_args[fun
            new_m=permute_func(pos_args ... ;kw_args ... )

            push!(call_report,(filteridxs[funcidx],time()-start,new_m.log_Li - e.contour))

            if length(new_m.permute_blacklist) > 0 && new_m.log_Li == -Inf #in this case the blacklisted function will not work on this model at all; it should be removed from the functions to use
                vcat(model_blacklist,new_m.permute_blacklist)
                filteridxs, filtered_funcs, filtered_weights, filtered_args = filter_permutes(instruction, model_blacklist)
            end

                        dupecheck(new_m,m) && new_m.log_Li > e.contour && return new_m, call_report
                end
        end
        return nothing, call_report
end

function permute_IPM(e::IPM_Ensemble, job_chan::RemoteChannel, models_chan::RemoteChannel, comms_chan::RemoteChannel) #ensemble.models is partially updated on the worker to populate arguments for permute funcs
        persist=true
    id=myid()
    put!(comms_chan,id)
    while persist
        wait(job_chan)
        start=time()
        e.models, e.contour, instruction = fetch(job_chan)
        instruction == "stop" && (persist=false; break)
```

```
71
72          call_report=Vector{Tuple{Int64,Float64,Float64}}()
73          for model=1:instruction.model_limit
74                          found::Bool=false
75              m_record = rand(e.models)
76
77              remotecall_fetch(isfile,1,m_record.path) ? m =
                ↪  (remotecall_fetch(deserialize,1,m_record.path)) : (break)
78
79              model_blacklist=copy(m.permute_blacklist)
80              filteridxs, filtered_funcs, filtered_weights, filtered_args =
                ↪  filter_permutes(instruction, model_blacklist)
81
82              for call in 1:instruction.func_limit
83                  start=time()
84                  funcidx=rand(filtered_weights)
85                  permute_func=filtered_funcs[funcidx]
86
                    ↪  pos_args,kw_args=get_permfunc_args(permute_func,e,m,filtered_args
87                  new_m=permute_func(pos_args ... ;kw_args ... )
88
                    ↪  push!(call_report,(filteridxs[funcidx],time()-start,new_m.log_Li
                    ↪  - e.contour))
89
90                  if length(new_m.permute_blacklist) > 0 && new_m.log_Li ==
                    ↪  -Inf #in this case the blacklisted function will not
                    ↪  work on this model at all; it should be removed from
                    ↪  the functions to use
91                      vcat(model_blacklist,new_m.permute_blacklist)
92                      filteridxs, filtered_funcs, filtered_weights,
                        ↪  filtered_args = filter_permutes(instruction,
                        ↪  model_blacklist)
93                  end
94
95                  dupecheck(new_m,m) && new_m.log_Li > e.contour &&
                    ↪  ((put!(models_chan, (new_m ,id, call_report)));
                    ↪  found=true; break)
96
97                          end
98              found==true && break;
99              wait(job_chan)
100             fetch(job_chan)≠e.models && (break) #if the ensemble has
                ↪  changed during the search, update it
```

```
                        model==instruction.model_limit && (put!(models_chan, ("quit", id, call_report));persist=false)#worker to put "quit" on channel if it fails to find a model more likely than contour
                    end
        end
end
                    function filter_permutes(instruction, model_blacklist)

                        filteridxs=findall(p->!in(p,model_blacklist),instruction.func
                        filtered_funcs=instruction.funcs[filteridxs]
                        filtered_weights=filter_weights(instruction.weights, filteridxs)
                        filtered_args=instruction.args[filteridxs]
                        return filteridxs, filtered_funcs, filtered_weights, filtered_args
                    end

                    function filter_weights(weights, idxs)
                        deplete=0.
                        for (i, weight) in enumerate(weights)
                            !in(i, idxs) && (deplete+=weight)
                        end
                        weights[idxs].+=deplete/length(idxs)
                        new_weights=weights[idxs]./sum(weights[idxs])
                        return Categorical(new_weights)
                    end

                    function get_permfunc_args(func::Function,e::IPM_Ensemble, m::ICA_PWM_Model, args::Vector{Tuple{Symbol,Any}})
                        pos_args=[]
                        argparts=Base.arg_decl_parts(methods(func).ms[1])
                        argnames=[Symbol(argparts[2][n][1]) for n in 2:length(argparts[2])]
                        for argname in argnames #assemble basic positional arguments from ensemble and model fields
                            if argname == Symbol('m')
                                push!(pos_args,m)
                            elseif argname in fieldnames(IPM_Ensemble)
                                push!(pos_args,getfield(e,argname))
                            elseif argname in fieldnames(ICA_PWM_Model)
```

```
                            push!(pos_args,getfield(m,argname))
                        else
                            throw(ArgumentError("Positional argument $argname of $func not available in the ensemble or model!"))
                        end
                    end

                    kw_args = NamedTuple()
                    if length(args) > 0 #if there are any keyword arguments to pass
                        sym=Vector{Symbol}()
                        val=Vector{Any}()
                        for arg in args
                            push!(sym, arg[1]); push!(val, arg[2])
                        end
                        kw_args = (;zip(sym,val)... )
                    end

                    return pos_args, kw_args
                end

                function dupecheck(new_model, model)
                    (new_model.sources==model.sources && new_model.mix_matrix==model.mix_matrix) ? (return false) : (return true)
                end
```

17.6.12 /src/permutation/permute_functions.jl

```
#DECORRELATION SEARCH PATTERNS
function permute_source(m::ICA_PWM_Model, models::Vector{Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
iterates::Integer=length(m.sources)*2, weight_shift_freq::AbstractFloat=PWM_SHIFT_FREQ, weight_shift_dist::Distributions.ContinuousUnivariateDistribution=PWM_SHIFT_DIST, length_change_freq::AbstractFloat=PWM_LENGTHPERM_FREQ, length_perm_range::UnitRange{<:Integer}=LENGTHPERM_RANGE, remote=false)
```

```
#weight_shift_dist is given in decimal probability values- converted to log space in permute_source_lengths!
    new_log_Li=-Inf;  iterate = 1
    O = size(obs_array,2);S = length(m.sources)
    new_sources=deepcopy(m.sources);

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    while new_log_Li ≤ contour && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        new_sources=deepcopy(m.sources);
        s = rand(1:S)
        clean=Vector{Bool}(trues(O))
        clean[m.mix_matrix[:,s]]•=false #all obs with source are dirty

        weight_shift_freq > 0 && (new_sources[s]=permute_source_weights(new_sources[s], weight_shift_freq, weight_shift_dist))
        rand() < length_change_freq && (new_sources[s]=permute_source_length(new_sources[s], source_priors[s], m.source_length_limits, length_perm_range))

        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean)
        iterate += 1
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate", "PS from $(m.name)", new_sources, m.source_length_limits, m.mix_matrix, new_log_Li)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate", "PS from $(m.name)", new_sources, m.source_length_limits, m.mix_matrix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

```

```
function permute_mix(m::ICA_PWM_Model, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; iterates::Integer=10, mix_move_range::UnitRange=1:length(m.mix_matrix), remote=false)
    new_log_Li=-Inf;  iterate = 1; O = size(obs_array,2);
    new_mix=falses(size(m.mix_matrix))

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)
    dirty=false

    while (new_log_Li ≤ contour || !dirty) && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        mix_moves=rand(mix_move_range)
        mix_moves > length(m.mix_matrix) && (mix_moves = length(m.mix_matrix))

        new_mix, clean = mix_matrix_decorrelate(m.mix_matrix, mix_moves) #generate a decorrelated candidate mix
        c_log_li, c_cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, new_mix, true, true, cache, clean) #calculate the model with the candidate mix
        positive_indices=c_cache.>(cache) #obtain any obs indices that have greater probability than we started with
        clean=Vector{Bool}(trues(O)-positive_indices)
        if any(positive_indices) #if there are any such indices
            new_log_Li, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, new_mix, true, true, cache, clean) #calculate the new model
            dirty=true
        end
        iterate += 1
    end

    return ICA_PWM_Model("candidate","PM from $(m.name)", m.sources, m.source_length_limits, new_mix, new_log_Li, Vector{Function}()) #no consolidate check is necessary as sources havent changed
end

```

```
function perm_src_fit_mix(m::ICA_PWM_Model, models::Vector{Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat},contour::AbstractFloat, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa iterates::Integer=length(m.sources)*2, weight_shift_freq::AbstractFloat=PWM_SHIFT_FREQ, length_change_freq::AbstractFloat=PWM_LENGTHPERM_FREQ, length_perm_range::UnitRange{<:Integer}=LENGTHPERM_RANGE,weight_shift_dist::Distr remote=false)
    new_log_Li=-Inf;  iterate = 1
    O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
    fit_mix in m.permute_blacklist ? (new_bl=m.permute_blacklist) : (new_bl=Vector{Function}())

    while new_log_Li ≤ contour && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix); tm_one=deepcopy(m.mix_matrix);
        clean=Vector{Bool}(trues(O))
        s = rand(1:S);
        clean[new_mix[:,s]]•=false #all obs starting with source are dirty

        new_mix[:,s]•=false;tm_one[:,s]•=true

        weight_shift_freq > 0 && (new_sources[s]=permute_source_weights(new_sources[s], weight_shift_freq, weight_shift_dist))
        rand() < length_change_freq && (new_sources[s]=permute_source_length(new_sources[s], source_priors[s], m.source_length_limits, length_perm_range))

        l,zero_cache=IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true)
        l,one_cache=IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, tm_one, true, true)
        fit_mix=one_cache.≥zero_cache

        if REVCOMP #if we're looking at reverse strands, we want to see if sources fit better on the reverse strand, and if so switch over to the revcomp source and use its fitted mix
```

```
            revsrc=(revcomp_pwm(new_sources[s][1]),new_sources[s][2])
            revsrcs=deepcopy(new_sources)
            revsrcs[s]=revsrc
            l, revsrc_cache=IPM_likelihood(revsrcs, obs_array, obs_lengths, bg_scores, tm_one, true, true)
            rfit_mix=revsrc_cache.≥zero_cache
            lps(revsrc_cache[rfit_mix])>lps(one_cache[fit_mix]) && (new_sources=revsrcs;fit_mix=rfit_mix) #if the total likelihood contribution for the revsource fit is greater than the source fit, take the revsource
        end

        new_mix[:,s]=fit_mix

        clean[fit_mix]•=false #all obs ending with source are dirty

        new_log_Li = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, false, zero_cache, clean)
        iterate += 1
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","PSFM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li, new_bl)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","PSFM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li, new_bl), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function fit_mix(m::ICA_PWM_Model, models::Vector{Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}; remote=false)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix); test_mix=falses(size(m.mix_matrix))
    new_bl=[fit_mix]

    l,zero_cache=IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, test_mix, true, true)
    for (s,source) in enumerate(m.sources)
        test_mix=falses(size(m.mix_matrix))
        test_mix[:,s]•=true
```

```
        l,src_cache=IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, test_mix, true, true)
        fit_mix=src_cache.≥zero_cache

        if REVCOMP #if we're looking at reverse strands, we want to see if sources fit better on the reverse strand, and if so switch over to the revcomp source and use its fitted mix
            revsrc=(revcomp_pwm(source[1]),source[2])
            revsrcs=deepcopy(new_sources)
            revsrcs[s]=revsrc
            l, revsrc_cache=IPM_likelihood(revsrcs, obs_array, obs_lengths, bg_scores, test_mix, true, true)
            rfit_mix=revsrc_cache.≥zero_cache
            lps(revsrc_cache[rfit_mix])>lps(src_cache[fit_mix]) && (new_sources=revsrcs;fit_mix=rfit_mix) #if the total likelihood contribution for the revsource fit is greater than the source fit, take the revsource
        end

        new_mix[:,s]=fit_mix
    end
    new_mix==m.mix_matrix ? (new_log_Li=-Inf) : (new_log_Li = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, new_mix, true)) #bail if the mix matrix doesnt change
    new_log_Li in [model.log_Li for model in models] && (new_log_Li=-Inf) #an existing identical log_Li is almost certainly a fitted model from another origin with the same sources, in this case abort to prevent dupes

    return ICA_PWM_Model("candidate","FM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li, new_bl) #no consolidate check necessary as no change to sources
end

```

```
function random_decorrelate(m::ICA_PWM_Model, models::Vector{Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa iterates::Integer=length(m.sources)*2, source_permute_freq::AbstractFloat=SRC_PERM_FREQ, weight_shift_freq::AbstractFloat=PWM_SHIFT_FREQ, length_change_freq::AbstractFloat=PWM_LENGTHPERM_FREQ, weight_shift_dist::Distributions.ContinuousUnivariateDistribution=PWM_SHIFT_DIST, mix_move_range::UnitRange=1:size(m.mix_matrix,1), remote=false)
    new_log_Li=-Inf;  iterate = 1
    O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)

    a, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true)

    while new_log_Li ≤ contour && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))
        s = rand(1:S)
        clean[new_mix[:,s]]•=false #all obs starting with source are dirty
        rand() < source_permute_freq && (new_sources[s]=permute_source_weights(new_sources[s], weight_shift_freq, weight_shift_dist))
        rand() < length_change_freq && (new_sources[s]=permute_source_length(new_sources[s], source_priors[s], m.source_length_limits))

        new_mix[:,s]=mixvec_decorrelate(new_mix[:,s],rand(mix_move_range))
        clean[new_mix[:,s]]•=false #all obs ending with source are dirty
        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true, cache, clean)
        iterate += 1
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
```

```
    cons_check ? (return ICA_PWM_Model("candidate", "RD from $(m.name)", new_sources, m.source_length_limits, new_mix, new_log_Li)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","RD from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function shuffle_sources(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; remote=false)
    new_log_Li=-Inf; O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    remote ? (merger_m = deserialize(rand(models).path)) : (merger_m = remotecall_fetch(deserialize, 1, rand(models).path))#randomly select a model to merge

    svec=[1:S ... ]

    while new_log_Li ≤ contour && length(svec)>0 #until we produce a model more likely than the lh contour or no more sources to attempt merger
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))

        s = popat!(svec,rand(1:length(svec))) #randomly select a source to merge

        new_sources[s]=merger_m.sources[s]; new_mix[:,s]•=merger_m.mix_matrix[:,s] #shuffle in the merger model source from this index

        clean[m.mix_matrix[:,s]]•=false #mark dirty any obs that have the source
```

```
        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","SS from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li,Vector{Function}())) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","SS from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function accumulate_mix(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; remote=false)
    new_log_Li=-Inf; O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
    new_bl=Vector{Function}()

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    remote ? (merger_m = deserialize(rand(models).path)) : (merger_m = remotecall_fetch(deserialize, 1, rand(models).path))#randomly select a model to merge

    svec=[1:S ... ]

    while new_log_Li ≤ contour && length(svec)>0 #until we produce a model more likely than the lh contour or no more sources to attempt merger
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))

        s = popat!(svec,rand(1:length(svec))) #randomly select a source to merge
```

```
        distvec=[pwm_distance(src[1],m_src[1]) for src in m.sources, m_src in merger_m.sources]
        S > 1 ? merge_s=findmin(distvec)[2][2] : merge_s=findmin(distvec)[2]

        new_mix[:,s]•=[new_mix[o,s] || merger_m.mix_matrix[o,merge_s] for o in 1:size(new_mix,1)] #accumulate the mix vector for this source

        clean[m.mix_matrix[:,s]]•=false #mark dirty any obs that have the source
        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood

        if new_log_Li ≤ contour #if accumulating on the host source doesnt work, try copying over the merger source
            new_sources[s]=merger_m.sources[merge_s]
            new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood
        end
    end

    (new_log_Li == m.log_Li || new_log_Li in [model.log_Li for model in models]) && (new_log_Li=-Inf) #accumulate can sometimes duplicate models if source mix is identical

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","AM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","AM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function distance_merge(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; remote=false)
```

```
    new_log_Li=-Inf; O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)

    a, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true)

    remote ? (merger_m = deserialize(rand(models).path)) : (merger_m = remotecall_fetch(deserialize, 1, rand(models).path)) #randomly select a model to merge

    svec=[1:S ... ]

    while new_log_Li ≤ contour && length(svec)>0 #until we produce a model more likely than the lh contour or no more sources to attempt merger
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))

        s = popat!(svec,rand(1:length(svec))) #randomly select a source to merge
        merge_s=most_dissimilar(new_mix[:,s],merger_m.mix_matrix) #find the source in the merger model whose mixvec is most dissimilar to the one selected

        clean[new_mix[:,s]]•=false #mark dirty any obs that start with the source
        new_sources[s] = merger_m.sources[merge_s] #copy the source
        new_mix[:,s] = merger_m.mix_matrix[:,merge_s] #copy the mixvector (without which the source will likely be highly improbable)
        clean[new_mix[:,s]]•=false #mark dirty any obs that end with the source

        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true, cache, clean) #assess likelihood
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
```

```
    cons_check ? (return ICA_PWM_Model("candidate","DM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li,Vector{Function}())) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","DM from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function similarity_merge(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; remote=false)
    new_log_Li=-Inf; O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources)

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    remote ? (merger_m = deserialize(rand(models).path)) : (merger_m = remotecall_fetch(deserialize, 1, rand(models).path))#randomly select a model to merge

    svec=[1:S...]

    while new_log_Li ≤ contour && length(svec)>0 #until we produce a model more likely than the lh contour or no more sources to attempt merger
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)
        clean=Vector{Bool}(trues(O))

        s = popat!(svec,rand(1:length(svec))) #randomly select a source to merge
        merge_s=most_similar(m.mix_matrix[:,s],merger_m.mix_matrix) #obtain the source in the merger model whose mixvec is most similar to the one in the original

        clean[m.mix_matrix[:,s]]•=false #mark dirty any obs that have the source
        new_sources[s] = merger_m.sources[merge_s] #copy the source, but dont copy the mixvector
```

```
        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","SM from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li,Vector{Function}())) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","SM from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function reinit_src(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat},contour::AbstractFloat, source_priors::AbstractVector{<:Union{<:AbstractVector{<:Dirichlet{<:AbstractFloa
iterates::Integer=length(m.sources)*2, remote=false)
    new_log_Li=-Inf;  iterate = 1
    O = size(obs_array,2); S = length(m.sources)
    new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix)

    while new_log_Li ≤ contour && iterate ≤ iterates #until we produce a model more likely than the lh contour or exceed iterates
        new_sources=deepcopy(m.sources); new_mix=deepcopy(m.mix_matrix); tm_one=deepcopy(m.mix_matrix);
        clean=Vector{Bool}(trues(O))
        s = rand(1:S);
        clean[new_mix[:,s]]•=false #all obs starting with source are dirty

        new_mix[:,s]•=false;tm_one[:,s]•=true

        new_sources[s] = init_logPWM_sources([source_priors[s]], m.source_length_limits)[1] #reinitialize the source

        l,zero_cache=IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, true)
```

```
        l,one_cache=IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, tm_one, true, true)

        fit_mix=one_cache.≥zero_cache

        new_mix[:,s]=fit_mix

        clean[fit_mix]•=false #all obs ending with source are dirty

        new_log_Li = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, new_mix, true, false, zero_cache, clean)
        iterate += 1
    end

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","RS from $(m.name)", new_sources, m.source_length_limits, new_mix, new_log_Li,Vector{Function}())) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","RS from $(m.name)",new_sources, m.source_length_limits, new_mix, new_log_Li), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

function erode_model(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; info_thresh::AbstractFloat=EROSION_INFO_THRESH, remote=false)
    new_log_Li=-Inf; O = size(obs_array,2);
    new_sources=deepcopy(m.sources)

    erosion_sources=Vector{Integer}()
    for (s,src) in enumerate(m.sources)
        pwm,pi=src
        if size(pwm,1)>m.source_length_limits[1]  #do not consider eroding srcs at min length limit
            infovec=get_pwm_info(pwm)
            any(info→<(info, info_thresh),infovec) && push!(erosion_sources,s)
        end
    end
```

```
    length(erosion_sources)==0 && return ICA_PWM_Model("candidate","EM from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li,[erode_model])#if we got a model we cant erode bail out with a model marked -Inf lh and with EM blacklisted

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    while new_log_Li ≤ contour && length(erosion_sources) > 0 #until we produce a model more likely than the lh contour or there are no more sources to erode
        clean=Vector{Bool}(trues(O))
        s=popat!(erosion_sources,rand(1:length(erosion_sources)))

        new_sources[s]=erode_source(new_sources[s], m.source_length_limits, info_thresh)
        clean[m.mix_matrix[:,s]]•=false

        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood
    end

    new_log_Li ≤ contour ? (blacklist=[erode_model]) : (blacklist=Vector{Function}())

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","EM from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li, blacklist)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","EM from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li, blacklist), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end
```

```
function info_fill(m::ICA_PWM_Model, models::AbstractVector{<:Model_Record}, obs_array::AbstractMatrix{<:Integer}, obs_lengths::AbstractVector{<:Integer}, bg_scores::AbstractMatrix{<:AbstractFloat}, contour::AbstractFloat; remote=false)
    new_log_Li=-Inf; O = size(obs_array,2); S = length(m.sources); iterate=1
    new_sources=deepcopy(m.sources)

    a, cache = IPM_likelihood(m.sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true)

    svec=[1:S ... ]

    while new_log_Li ≤ contour && length(svec)>0 #until we produce a model more likely than the lh contour or no more sources to attempt infofill
        new_sources=deepcopy(m.sources)
        clean=Vector{Bool}(trues(O))
        s = popat!(svec,rand(1:length(svec)))
        pwm=new_sources[s][1]
        fill_idx=findmin(get_pwm_info(pwm))[2]
        fill_bases=[1,2,3,4]
        fill_candidate=findmax(pwm[fill_idx,:])[2]
        deleteat!(fill_bases, findfirst(b→b==fill_candidate,fill_bases))
        new_sources[s][1][fill_idx,:]⋅=-Inf; new_sources[s][1][fill_idx,fill_candidate]=0.
        clean[m.mix_matrix[:,s]]⋅=false
        new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood

        while new_log_Li ≤contour && length(fill_bases)>0
            fill_candidate=popat!(fill_bases,rand(1:length(fill_bases)))
            new_sources[s][1][fill_idx,:]⋅=-Inf; new_sources[s][1][fill_idx,fill_candidate]=0.
            new_log_Li, cache = IPM_likelihood(new_sources, obs_array, obs_lengths, bg_scores, m.mix_matrix, true, true, cache, clean) #assess likelihood
        end
    end

```

```
    new_log_Li ≤ contour ? (blacklist=[info_fill]) : (blacklist=Vector{Function}())

    cons_check, cons_idxs = consolidate_check(new_sources)
    cons_check ? (return ICA_PWM_Model("candidate","IF from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li, blacklist)) : (return consolidate_srcs(cons_idxs, ICA_PWM_Model("candidate","IF from $(m.name)",new_sources, m.source_length_limits, m.mix_matrix, new_log_Li, blacklist), obs_array, obs_lengths, bg_scores, contour, models; remote=remote))
end

full_perm_funcvec=[permute_source, permute_mix, perm_src_fit_mix, fit_mix, random_decorrelate, shuffle_sources, accumulate_mix, distance_merge, similarity_merge, reinit_src, erode_model, info_fill]
```

17.6.13 /src/permutation/permute_utilities.jl

```
##BASIC UTILITY FUNCTIONS
#SOURCE PERMUTATION
function permute_source_weights(source::Tuple{<:AbstractMatrix{<:AbstractFloat},<:Integer}, shift_freq::AbstractFloat, PWM_shift_dist::Distribution{Univariate,Continuous})
    dirty=false; source_length=size(source[1],1)
    new_source=deepcopy(source)

    for source_pos in 1:source_length
        if rand() ≤ shift_freq
            pos_WM = exp.(source[1][source_pos,:]) #leaving logspace, get the wm at that position
            new_source[1][source_pos,:] = log.(wm_shift(pos_WM, PWM_shift_dist)) #accumulate probabilty at a randomly selected base, reassign in logspace and carry on
            !dirty && (dirty=true)
        end
    end

    if !dirty #if no positions were shifted, pick one and shift
        rand_pos=rand(1:source_length)
        pos_WM = exp.(source[1][rand_pos,:])
```

```
        new_source[1][rand_pos,:]=log.(wm_shift(pos_WM, PWM_shift_dist))
    end

    return new_source
end

                function wm_shift(pos_WM::AbstractVector{<:AbstractFloat}, PWM_shift_dist::Distribution{Univariate,Continuous})
                    base_to_shift = rand(1:4) #pick a base to accumulate probability
                    permute_sign = rand(-1:2:1)
                    shift_size = rand(PWM_shift_dist)
                    new_wm=zeros(4)

                    for base in 1:4 #ACGT
                        if base == base_to_shift
                            new_wm[base] =
                            clamp(0, #no lower than 0 prob
                            (pos_WM[base]          #selected PWM posn
                            + permute_sign * shift_size), #randomly permuted by size param
                            1) #no higher than prob 1
                        else
                            size_frac = shift_size / 3 #other bases shifted in the opposite direction by 1/3 the shift accumulated at the base to permute
                            new_wm[base] =
                            clamp(0,
                            (pos_WM[base]
                            - permute_sign * size_frac),
                            1)
                        end
                    end
                    new_wm = new_wm ./ sum(new_wm) #renormalise to sum 1 - necessary in case of clamping at 0 or 1
                    !isprobvec(new_wm) && throw(DomainError(new_wm, "Bad weight vector generated in wm_shift!")) #throw assertion exception if the position WM is invalid
                    return new_wm
                end

```

```
function permute_source_length(source::Tuple{<:AbstractMatrix{<:AbstractFloat},<:Integer},
    prior::Union{<:AbstractVector{<:Dirichlet{<:AbstractFloat}},<:Bool},
    length_limits::UnitRange{<:Integer},
    permute_range::UnitRange{<:Integer}=LENGTHPERM_RANGE,
    uninformative::Dirichlet=Dirichlet([.25,.25,.25,.25]))
    source_PWM, prior_idx = source
    source_length = size(source_PWM,1)

    permute_sign, permute_length = get_length_params(source_length, length_limits, permute_range)

    permute_sign==1 ? permute_pos = rand(1:source_length+1) :
        permute_pos=rand(1:source_length-permute_length)

    if permute_sign == 1 #if we're to add positions to the PWM
        ins_WM=zeros(permute_length,4)
        if prior==false
            for pos in 1:permute_length
                ins_WM[pos,:] = log.(transpose(rand(uninformative)))
            end
        else
            for pos in 1:permute_length
                prior_position=permute_pos+prior_idx-1
                prior_position<1 || prior_position>length(prior) ?
                    ins_WM[pos,:] = log.(transpose(rand(uninformative))) :
                    ins_WM[pos,:] = log.(transpose(rand(prior[prior_position])))
            end
        end
        upstream_source=source_PWM[1:permute_pos-1,:]
        downstream_source=source_PWM[permute_pos:end,:]
        source_PWM=vcat(upstream_source,ins_WM,downstream_source)
        permute_pos==1 && (prior_idx-=permute_length)
    else #if we're to remove positions
        upstream_source=source_PWM[1:permute_pos-1,:]
        downstream_source=source_PWM[permute_pos+permute_length:end,:]
        source_PWM=vcat(upstream_source,downstream_source)
        permute_pos==1 && (prior_idx+=permute_length)
    end
```

```

    return (source_PWM, prior_idx) #return a new source
end

                function get_length_params(source_length::Integer, length_limits::UnitRange{<:Integer}, permute_range::UnitRange{<:Integer})
                    extendable = length_limits[end]-source_length
                    contractable =  source_length-length_limits[1]

                    if extendable == 0 && contractable > 0
                        permute_sign=-1
                    elseif contractable == 0 && extendable > 0
                        permute_sign=1
                    else
                        permute_sign = rand(-1:2:1)
                    end

                    permute_sign==1 && extendable<permute_range[end] && (permute_range=permute_range[1]:extendable)
                    permute_sign==-1 && contractable<permute_range[end] && (permute_range=permute_range[1]:contractable)
                    permute_length = rand(permute_range)

                    return permute_sign, permute_length
                end

function erode_source(source::Tuple{<:AbstractMatrix{<:AbstractFloat},<:Integer},length_li
    pwm,prior_idx=source
    infovec=get_pwm_info(pwm)
    start_idx,end_idx=get_erosion_idxs(infovec, info_thresh, length_limits)

    return new_source=(pwm[start_idx:end_idx,:], prior_idx+start_idx-1)
end

    function get_pwm_info(pwm::AbstractMatrix{<:AbstractFloat}; logsw::Bool=true)
        wml=size(pwm,1)
        infovec=zeros(wml)
        for pos in 1:wml
```

```
            logsw ? wvec=deepcopy(exp.(pwm[pos,:])) : wvec=deepcopy(pwm[pos,:])
            !isprobvec(wvec) && throw(DomainError(wvec, "Bad wvec in get_pwm_info -Original sources must be in logspace!!"))
            wvec.+=10^-99
            infscore = (2.0 + sum([x*log(2,x) for x in wvec]))
            infovec[pos]=infscore
        end
        return infovec
    end

    function get_erosion_idxs(infovec::AbstractVector{<:AbstractFloat}, info_thresh::AbstractFloat, length_limits::UnitRange{<:Integer})
        srcl=length(infovec)
        contractable =  srcl-length_limits[1]
        contractable ≤ 0 && throw(DomainError(contractable, "erode_source passed a source at its lower length limit!"))
        centeridx=findmax(infovec)[2]

        start_idx=findprev(info→<(info,info_thresh),infovec,centeridx)
        start_idx===nothing ? (start_idx=1) : (start_idx+=1)
        end_idx=findnext(info→<(info, info_thresh),infovec,centeridx)
        end_idx===nothing ? (end_idx=srcl) : (end_idx-=1)

        pos_to_erode=srcl-(end_idx-start_idx)
        if pos_to_erode > contractable
            pos_to_restore = pos_to_erode-contractable
            while pos_to_restore>0
                end_die=rand()
                if end_die ≤ .5
                    start_idx>1 && (pos_to_restore-=1; start_idx-=1)
                else
                    end_idx<srcl && (pos_to_restore-=1; end_idx+=1)
                end
            end
        end

        return start_idx, end_idx
    end

#MIX MATRIX FUNCTIONS
function mixvec_decorrelate(mix::BitVector, moves::Integer)
    new_mix=deepcopy(mix)
```

```
    idxs_to_flip=rand(1:length(mix), moves)
    new_mix[idxs_to_flip] ⊻= .!mix[idxs_to_flip]
    return new_mix
end

function mix_matrix_decorrelate(mix::BitMatrix, moves::Integer)
    clean=Vector{Bool}(trues(size(mix,1)))
    new_mix=deepcopy(mix)
    indices_to_flip = rand(CartesianIndices(mix), moves)
    new_mix[indices_to_flip] ⊻= .!mix[indices_to_flip]
    clean[unique([idx[1] for idx in indices_to_flip])] ⊻= false #mark all obs that had flipped indices dirty
    return new_mix, clean
end


# function most_dissimilar(mix1, mix2)
#     S1=size(mix1,2);S2=size(mix2,2)
#     dist_mat=zeros(S1,S2)
#     for s1 in 1:S1, s2 in 1:S2
#         dist_mat[s1,s2]=sum(mix1[:,s1].==mix2[:,s2])
#     end
#     scores=vec(sum(dist_mat,dims=1))
#     return findmin(scores)[2]
# end


function most_dissimilar(src_mixvec, target_mixmat)
    src_sim = [sum(src_mixvec.==target_mixmat[:,s]) for s in 1:size(target_mixmat,2)] #compose array of elementwise equality comparisons between mixvectors and sum to score
    merge_s=findmin(src_sim)[2] #source from merger model will be the one with the highest equality comparison score
end

function most_similar(src_mixvec, target_mixmat)
    src_sim = [sum(src_mixvec.==target_mixmat[:,s]) for s in 1:size(target_mixmat,2)] #compose array of elementwise equality comparisons between mixvectors and sum to score
    merge_s=findmax(src_sim)[2] #source from merger model will be the one with the highest equality comparison score
end
```

17.6.14 /src/utilities/model_display.jl

```
#THIS CODE BASED ON N. REDDY'S THICWEED.JL DISPLAY SCRIPT

#CONSTANTS
# each tuple specifies the x, y, fontsize, xscale to print the character
# in a 100×100 box at position 100,100.
charvals_dict = Dict{Char,Tuple}('A'⇒ (88.5,199.,135.,1.09),
                 'C'⇒ (79.,196.,129.,1.19),
                 'G'⇒ (83.,196.,129.,1.14),
                 'T'⇒ (80.,199.,135.,1.24))

colour_dict = Dict{Char,String}('A'⇒"(0,128,0)", #green
                 'C'⇒"(0,0,128)", #blue
                 'G'⇒"(255,69,0)", #yellow-brown
                 'T'⇒"(150,0,0)")  #red

#char output params for string display of PWMs in nested sampler instrumentation
lwcs_vec=['a','c','g','t']
upcs_vec=['A','C','G','T']
cs_vec=[:green,:blue,:yellow,:red]
thresh_dict=Dict([(0.,'_'),(.25,'.'),(.5,"lwcs"),(1,"upcs")])

source_left_x = 10
xlen = 20
yscale = 250
scale_factor = 0.9
ypixels_per_source = 250
ypad = 10
xpad = 20
xpixels_per_position = 40
fontsize1=60
fontsize2=40

#function to convert ICA_PWM_Model sources lists to PWM sequence logo diagrams
function logo_from_model(model::ICA_PWM_Model,svg_output::String;freq_sort::Bool=false)
    source_tups = Vector{Tuple{AbstractFloat, Integer, Matrix{AbstractFloat}}}() #(%of sequences w/ source, prior index, weight matrix)
    mix = model.mix_matrix #o x s
```

```
    for (prior, source) in enumerate(model.sources)
        push!(source_tups, (sum(model.mix_matrix[:,prior])/size(model.mix_matrix)[1], prior, exp.(source[1])))
    end

    freq_sort && sort(source_tups)

    file = open(svg_output, "w")
    write(file, svg_header(xpad+xpixels_per_position*maximum([length(source[1]) for source in model.sources]),ypixels_per_source*length(model.sources)+ypad))

    curry = 0
    for (frequency, index, source) in source_tups
        curry += yscale
        font1y = curry-190
        ndig = ndigits(index+1)
        write(file, pwm_to_logo(source,source_left_x,curry,xlen,yscale*scale_factor))
        write(file, "<text x=\"10\" y=\"$font1y\" font-family=\"Helvetica\" font-size=\"$fontsize1\" font-weight=\"bold\" >$(index)</text>")
        write(file, "<text x=\"$(20+fontsize1*0.6*ndig)\" y=\"$(font1y-fontsize1+1.5*fontsize2)\" font-family=\"Helvetica\" font-size=\"$fontsize2\" font-weight=\"bold\" >$(frequency*100) % of sequences</text>\n")
    end

    write(file, svg_footer())
    @info "Logo written."
    close(file)
end

function svg_header(canvas_size_x, canvas_size_y)
    return """
    <?xml version="1.0" standalone="no"?>
    <!DOCTYPE svg PUBLIC "-//W3C//DTD SVG 1.1//EN"
    "http://www.w3.org/Graphics/SVG/1.1/DTD/svg11.dtd">
    <svg width="$canvas_size_x" height="$canvas_size_y"
     version="1.1" xmlns="http://www.w3.org/2000/svg">
```

```
    """
end

function pwm_to_logo(source,xpos,ypos,xlen,yscale)
    outstr = ""
    for n in 1:size(source,1)
        outstr *= print_weightvec_at_x_y(source[n,:],xpos+xlen*n,ypos,xlen,yscale)
    end
    outstr *= "\n"
    return outstr
end

function pwmstr_to_io(io,source;log=true)
    log && (source=exp.(source))
    for pos in 1:size(source,1)
        char,color=uni_wvec_params(source[pos,:])
        printstyled(io, char; color=color)
    end
end

function uni_wvec_params(wvec) #assign a single unicode char and color symbol for the most informational position in a position weight vector
    wvec.+=10^-99
    infoscore=(2.0 + sum([x*log(2,x) for x in wvec]))
    infovec = [x*infoscore for x in wvec]
    val,idx=findmax(infovec)
    return char,color=get_char_by_thresh(idx,val)
end
                function get_char_by_thresh(idx,val)
                    char = '?'; seen_thresh=0.
                    for (thresh,threshchar) in thresh_dict
                        val ≥ thresh && thresh ≥ seen_thresh && (char=threshchar; seen_thresh=thresh)
                    end
                    char=='?' && println(val)
                    char=="lwcs" && (char=lwcs_vec[idx])
                    char=="upcs" && (char=upcs_vec[idx])
                    color=cs_vec[idx]

                    return char,color
                end
```

```
function print_weightvec_at_x_y(wvec, xpos, ypos, xlen, yscale)
    # xlen is the length occupied by that column
    # yscale is the total height for 2 information bits i.e. the
    # maximum height available for a "perfect" nucleotide
    outstr = ""
    basestr = "ACGT"
    wvec.+=10^-99  #prevent log(0) = -Inf
    wvec = [x/sum(wvec) for x in wvec] #renorm
    infscore = (2.0 + sum([x*log(2,x) for x in wvec]))
    if infscore==0.0
        return ""
    end
    wvec = [x*infscore*yscale/2.0 for x in wvec]
    # at this point, the sum of all wvec is a maximum of yscale
    wveclist = [(wvec[n],basestr[n]) for n in 1:4]
    wveclist = sort(wveclist)
    curr_ypos = ypos
    for n in 1:4
        curr_ypos -= wveclist[n][1]
        outstr *= print_char_in_rect(wveclist[n][2],xpos,curr_ypos,xlen,wveclist[n][1])
    end
    return outstr
end

blo = 1

function print_char_in_rect(c,x,y,width,height)
    raw_x, raw_y, raw_fontsize, raw_xscale = charvals_dict[c]
    raw_x = (raw_x*raw_xscale-100)/raw_xscale
    raw_y = raw_y-100

    xscale = width/100.0 * raw_xscale
    yscale = height/100.0

    scaled_x = x/xscale + raw_x
    scaled_y = y/yscale + raw_y

    return "<text x=\"$scaled_x\" y=\"$scaled_y\" font-size=\"$raw_fontsize\" font-weight=\"bold\" font-family=\"Helvetica\" fill=\"rgb"*colour_dict[c]*"\" transform=\"scale($xscale,$yscale)\">$c</text>\n"
```

```
end

function svg_footer()
    return "</svg>"
end
```

17.6.15 /src/utilities/ns_progressmeter.jl

```
#UTILITY  REPORTS WORKER NUMBER AND CURRENT ITERATE
mutable struct ProgressNS{T<:Real} <: AbstractProgress
    interval::T
    dt::AbstractFloat
    start_it::Integer
    counter::Integer
    triggered::Bool
    tfirst::AbstractFloat
    tlast::AbstractFloat
    tstp::AbstractFloat
    printed::Bool        # true if we have issued at least one status update
    desc::AbstractString # prefix to the percentage, e.g.  "Computing..."
    color::Symbol        # default to green
    output::IO           # output stream into which the progress is written
    numprintedvalues::Integer   # num values printed below progress in last iteration
    offset::Integer             # position offset of progress bar (default is 0)

    e::IPM_Ensemble
    wm::Worker_Monitor
    tuner::Permute_Tuner
    top_m::ICA_PWM_Model

    mean_stp_time::AbstractFloat

    wk_disp::Bool
    tuning_disp::Bool
    conv_plot::Bool
    ens_disp::Bool
    lh_disp::Bool
    liwi_disp::Bool
```

```
    src_disp::Bool
    no_displayed_srcs::Integer

    disp_rotate_inst::Vector{Any}

    convergence_history::Vector{Float64}

    function ProgressNS{T}(     e::IPM_Ensemble,
                                top_m::ICA_PWM_Model,
                                wm::Worker_Monitor,
                                tuner::Permute_Tuner,
                                interval::T;
                                dt::Real=0.1,
                                desc::AbstractString="Nested Sampling::",
                                color::Symbol=:green,
                                output::IO=stdout,
                                offset::Int=0,
                                start_it::Int=1,
                                wk_disp::Bool=false,
                                tuning_disp::Bool=false,
                                conv_plot::Bool=true,
                                ens_disp::Bool=false,
                                lh_disp::Bool=false,
                                liwi_disp::Bool=false,
                                src_disp::Bool=true,
                                nsrcs::Integer=0,

                                ↪  disp_rotate_inst=[false,0,0,Vector{Vector{Symbol}}
                                ↪  where T
        tfirst = tlast = time()
        printed = false
        new{T}(interval,
         dt,
         start_it,
         start_it,
         false,
         tfirst,
         tlast,
         0.,
         printed,
         desc,
         color,
         output,
```

```
        0,
        offset,
        e,
        wm,
        tuner,
        top_m,
        0.,
        wk_disp,
        tuning_disp,
        conv_plot,
        ens_disp,
        lh_disp,
        liwi_disp,
        src_disp,
        nsrcs,
        disp_rotate_inst,
        zeros(CONVERGENCE_MEMORY))
    end
end

function ProgressNS(e::IPM_Ensemble, wm::Worker_Monitor, tuner::Permute_Tuner, interval::Real; dt::Real=0.1, desc::AbstractString="Nested Sampling::", color::Symbol=:green, output::IO=stderr, offset::Integer=0, start_it::Integer=1, wk_disp::Bool=false, tuning_disp::Bool=false, conv_plot::Bool=false, lh_disp::Bool=false, liwi_disp::Bool=false, ens_disp::Bool=false, src_disp::Bool=false, nsrcs=0, disp_rotate_inst::Vector{Any}=[false,0,0,Vector{Vector{Symbol}}()])
    top_m = deserialize(e.models[findmax([model.log_Li for model in e.models])[2]].path)

    return ProgressNS{typeof(interval)}(e, top_m, wm, tuner, interval, dt=dt, desc=desc, color=color, output=output, offset=offset, start_it=start_it, wk_disp=wk_disp, tuning_disp=tuning_disp, conv_plot=conv_plot, lh_disp=lh_disp, liwi_disp=liwi_disp, ens_disp=ens_disp, src_disp=src_disp, nsrcs=nsrcs, disp_rotate_inst=disp_rotate_inst)
end


function update!(p::ProgressNS, val, thresh; options...)
    p.counter += 1

```

```
    p.tstp=time()-p.tlast
    popfirst!(p.tuner.time_history)
    push!(p.tuner.time_history, p.tstp)

    p.interval = val - thresh
    popfirst!(p.convergence_history)
    push!(p.convergence_history, p.interval)

    p.top_m.name ≠ basename(p.e.models[findmax([model.log_Li for model in p.e.models])[2]].path) && (p.top_m=deserialize(p.e.models[findmax([model.log_Li for model in p.e.models])[2]].path))

    updateProgress!(p; options... )
end

function updateProgress!(p::ProgressNS; offset::Integer = p.offset, keep = (offset == 0))
    p.offset = offset
    t = time()
    p.disp_rotate_inst[1] && p.counter%p.disp_rotate_inst[2] == 0 && rotate_displays(p)

    if p.interval ≤ 0 && !p.triggered
        p.triggered = true
        if p.printed
            p.triggered = true
            dur = durationstring(t-p.tfirst)
            msg = @sprintf "%s Converged. Time: %s (%d iterations). logZ: %s\n" p.desc dur p.counter p.e.log_Zi[end]

            print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
            move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
            upper_lines=display_upper_dash(p)
            printover(p.output, msg, :magenta)
            lower_lines=display_lower_dash(p)

            p.numprintedvalues=upper_lines + lower_lines + 1

            if keep
                println(p.output)
            else
```

```
                print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))
            end
        end
        return
    end

    if t > p.tlast+p.dt && !p.triggered
        p.counter < CONVERGENCE_MEMORY ? mean_step_time=mean(p.tuner.time_history[end-(p.counter-1):end]) :

                                                mean_step_time=mean(p.tuner.time_hist
        msg = @sprintf "%s Iterate: %s Recent step time μ: %s Convergence Interval: %g\n" p.desc p.counter hmss(mean_step_time) p.interval

        print(p.output, "\n" ^ (p.offset + p.numprintedvalues))
        move_cursor_up_while_clearing_lines(p.output, p.numprintedvalues)
        upper_lines=display_upper_dash(p)
        printover(p.output, msg, p.color)
        lower_lines=display_lower_dash(p)

        p.numprintedvalues=upper_lines + lower_lines + 1
        print(p.output, "\r\u1b[A" ^ (p.offset + p.numprintedvalues))

        # Compensate for any overhead of printing. This can be
        # especially important if you're running over a slow network
        # connection.
        p.tlast = t + 2*(time()-t)
        p.printed = true
    end
end
                function rotate_displays(p)
                    curr_inst=p.disp_rotate_inst[3]
                    curr_inst+1>length(p.disp_rotate_inst[4]) ? next_inst=1 : next_inst=curr_inst+1
                    next_disps=p.disp_rotate_inst[4][next_inst]
                    for disp in [:wk_disp,:tuning_disp,:conv_plot,:ens_disp,:lh_disp,:liwi_di
                        disp in next_disps ? setproperty!(p, disp, true) : setproperty!(p, disp, false)
                    end
```

```
                    p.disp_rotate_inst[3]=next_inst
                end

                function hmss(dt)
                    dt<0 ? (dt=-dt; prfx="-") : (prfx="")
                    isnan(dt) && return "NaN"
                    (h,r) = divrem(dt,60*60)
                    (m,r) = divrem(r, 60)
                    (isnan(h)||isnan(m)||isnan(r)) && return "NaN"
                    string(prfx,Int(h),":",Int(m),":",Int(ceil(r)))
                end

                function display_upper_dash(p::ProgressNS)
                    wklines = tunelines = cilines = 0
                    p.wk_disp && (wklines=show(p.output, p.wm,
                    ↪  progress=true);println())
                    p.tuning_disp && (tunelines=show(p.output, p.tuner,
                    ↪  progress=true);println())
                    if p.conv_plot

                        ↪  ciplot=lineplot([p.counter-(length(p.convergence_history)
                        ↪  p.convergence_history, title="Convergence
                        ↪  Interval Recent History",
                        ↪  xlabel="Iterate",ylabel="CI", color=:yellow)
                        cilines=nrows(ciplot.graphics)+5
                        show(p.output, ciplot); println()
                    end
                    return wklines + tunelines + cilines
                end

                function display_lower_dash(p::ProgressNS)
                    lhlines = liwilines = ensemblelines = srclines = 0
                    if p.lh_disp
                        lhplot=lineplot(p.e.log_Li[2:end], title="Contour
                        ↪  History", xlabel="Iterate", color=:magenta,
                        ↪  name="Ensemble logLH")
                        lineplot!(lhplot, [p.e.naive_lh for it in
                        ↪  1:length(p.e.log_Li[2:end])], name="Naive
                        ↪  logLH")
                        lhlines=nrows(lhplot.graphics)+5
                        show(p.output, lhplot); println()
                    end
                    if p.liwi_disp && p.counter>2
```

```
                        liwiplot=lineplot([max(2,p.counter-(CONVERGENCE_MEMORY-1)
                        title="Recent iterate evidentiary weight",
                        xlabel="Iterate", name="Ensemble log Liwi",
                        color=:cyan)
                        liwilines=nrows(liwiplot.graphics)+5
                        show(p.output, liwiplot); println()
                    end
                    p.ens_disp && (ensemblelines=show(p.output, p.e, progress=true))
                    p.src_disp && (println("MLE Model Sources:");srclines=show(p.output, p.top_m, nsrc=p.no_displayed_srcs, progress=true))
                    return lhlines + liwilines + ensemblelines + srclines
                end
```

17.6.16 /src/utilities/synthetic_genome.jl

```
function synthetic_sample(no_obs::Integer, obsl, bhmm_vec::AbstractVector{<:BHMM}, bhmm_dist::Categorical, spikes::AbstractVector{<:AbstractMatrix{<:AbstractFloat}}, spike_instruct::AbstractVector{<:Tuple{<:Bool, <:Tuple}})
    !(typeof(obsl) <:UnitRange || typeof(obsl) <:Integer) && throw(ArgumentError("obsl must be Integer or UnitRange"))
    length(spike_instruct)≠length(spikes) && throw(ArgumentError("spike_instruct must be as long as spikes"))

    obs, hmm_truth=obs_array_from_bhmms(no_obs,obsl,bhmm_vec,bhmm_dist)

    spike_truth = spike_obs!(obs, spikes, spike_instruct)

    bg_scores = score_synthetic(obs, bhmm_vec, hmm_truth)

    return obs, bg_scores, hmm_truth, spike_truth
end

function obs_array_from_bhmms(no_obs, obsl, bhmm_vec, bhmm_dist)
    obs=zeros(UInt8,max(obsl...)+1, no_obs)
    truthvec=zeros(UInt8,no_obs)
    for o in 1:no_obs
        hmm_idx=rand(bhmm_dist)
        truthvec[o]=hmm_idx
```

```
        bhmm=bhmm_vec[hmm_idx]
        typeof(obsl)<:UnitRange ? (l=rand(obsl)) : (l=obsl)
        obs[1:l,o]=rand(bhmm, l)
    end
    return obs, truthvec
end

function spike_obs!(obs, spikes, spike_instruct)
    truthmat=falses(size(obs,2),length(spikes))
    for (s,spike) in enumerate(spikes)
        structural,ins=spike_instruct[s]
        structural ? (truthmat[:,s] = spike_struc!(obs, spike, ins ... )) : (truthmat[:,s] = spike_irreg!(obs, spike, ins ... ))
    end
    return truthmat
end

function spike_struc!(obs, spike, frac_obs, periodicity)
    truth=falses(size(obs,2))
    for o in 1:size(obs,2)
        if rand() < frac_obs
            truth[o]=true
            rand()<.5 ? (source=revcomp_pwm(spike)) : (source=spike)
            pos=rand(1:periodicity)
            oidx=findfirst(iszero,obs[:,o])
            while pos<oidx
                pos_ctr=pos
                pwm_ctr=1
                while pos_ctr<oidx&&pwm_ctr≤size(source,1)
                    obs[pos_ctr,o]=rand(Categorical(source[pwm_ctr,:]))
                    pos_ctr+=1
                    pwm_ctr+=1
                end
                pos+=periodicity+pwm_ctr
            end
        end
    end
    return truth
end

function spike_irreg!(obs, spike, frac_obs, recur)
    truth=falses(size(obs,2))
    for o in 1:size(obs,2)
```

```
        oidx=findfirst(iszero,obs[:,o])
        if rand()<frac_obs
            truth[o]=true
            for r in 1:rand(recur)
                rand()<.5 ? (source=revcomp_pwm(spike)) : source=spike
                pos=rand(1:oidx-1)
                pwm_ctr=1
                while pos<oidx && pwm_ctr ≤ size(source,1)
                    obs[pos,o]=rand(Categorical(source[pwm_ctr,:]))
                    pos+=1
                    pwm_ctr+=1
                end
            end
        end
    end
    return truth
end

function score_synthetic(obs, bhmm_vec, hmm_truth)
    lh_mat=zeros(size(obs,1)-1, size(obs,2))
    for o in 1:size(obs,2)
        oidx=findfirst(iszero,obs[:,o])

        ↪ lh_mat[1:oidx-1,o]=BioBackgroundModels.get_BGHMM_symbol_lh(transpose(obs[
    end
    return lh_mat
end
```

17.6.17 /src/utilities/worker_diagnostics.jl

```
struct Worker_Monitor
    idx::Dict{Integer,Integer}
    persist::BitMatrix
    last_seen::Matrix{Float64}
end

function Worker_Monitor(wk_pool::Vector{<:Integer})
    idx=Dict{Integer,Integer}()
    for (n,wk) in enumerate(wk_pool)
        idx[wk]=n
    end
    persist=trues(1,length(wk_pool))
```

```
    last_seen=[time() for x in 1:1, y in 1:length(wk_pool)]
    return Worker_Monitor(idx, persist, last_seen)
end

function update_worker_monitor!(mon,wk,persist)
    mon.persist[1,mon.idx[wk]]=persist
    mon.last_seen[1,mon.idx[wk]]=time()
end

function Base.show(io::IO, mon::Worker_Monitor; progress=false)
    printstyled("Worker Diagnostics\n", bold=true)
    pers=heatmap(float.(mon.persist), colormap=persistcolor,
     ↪  title="Peristence",labels=false)
    ls=heatmap([time()-ls for ls in mon.last_seen], title="Last
     ↪  Seen",labels=false)
    show(pers)
    println()
    show(ls)
    progress && return nrows(pers.graphics)+nrows(ls.graphics)+7
end

                function persistcolor(z, zmin, zmax)
                    z==1. && return 154
                    z==0. && return 160
                end
```

17.6.18 /src/utilities/worker_sequencer.jl

```
#waits for one worker to begin before calling the next so that network is
 ↪  not slammed trying to send huge arrays to many workers simultaneously

function sequence_workers(wk_pool, func, args...)
    comms_chan = RemoteChannel(()->Channel{Integer}(length(wk_pool)))

    for worker in wk_pool
        remote_do(func, worker, args..., comms_chan)
        wait(comms_chan); report=take!(comms_chan)
        @assert worker==report
    end
end
```

17.6.19 /test/consolidate_unit_tests.jl

```
@testset "Orthogonality helper" begin
    bg_scores = log.(fill(.25, (17,3)))
    obs=[BioSequences.LongSequence{DNAAlphabet{2}}("ATGATTACGATGATGCA")
    BioSequences.LongSequence{DNAAlphabet{2}}("TCAGTTACGATGATCAG")
    BioSequences.LongSequence{DNAAlphabet{2}}("TTACGCACAGATGTTAC")]
    order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
    obs=Array(transpose(coded_seqs))
    obsl=[findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]]

    src_ATG = [.7 .1 .1 .1
    .1 .1 .1 .7
    .1 .1 .7 .1]

    src_CAG = [.1 .7 .1 .1
    .7 .1 .1 .1
    .1 .1 .7 .1]

    src_TTAC = [.1 .1 .1 .7
    .1 .1 .1 .7
    .7 .1 .1 .1
    .1 .7 .1 .1]

    src_GCA = [.1 .1 .7 .1
    .1 .7 .1 .1
    .7 .1 .1 .1]

    consolidate_one =
    ↪  [(log.(src_ATG),0),(log.(src_TTAC),0),(log.(src_ATG),0)]
    cons_one_mix = BitMatrix([true true false
                    true false true
                    false true true])

    consolidate_two =
    ↪  [(log.(src_ATG),0),(log.(src_ATG),0),(log.(src_ATG),0)]
    cons_two_mix = BitMatrix([true false false
                    false true false
                    false false true])

    distance_model =
    ↪  [(log.(src_TTAC),0),(log.(src_CAG),0),(log.(src_GCA),0)]
```

```julia

    c1model = ICA_PWM_Model("c1", "c1", consolidate_one, 3:4, cons_one_mix, IPM_likelihood(consolidate_one, obs, obsl, bg_scores, cons_one_mix))
    c2model = ICA_PWM_Model("c2", "c2", consolidate_two, 3:4, cons_two_mix, IPM_likelihood(consolidate_two, obs, obsl, bg_scores, cons_two_mix))
    dmodel = ICA_PWM_Model("d", "d", distance_model, 3:4, trues(3,3), IPM_likelihood(distance_model, obs, obsl, bg_scores, trues(3,3)))

    dpath=randstring()
    serialize(dpath, dmodel)
    drec=Model_Record(dpath,dmodel.log_Li)

    #check distance calculation
    pwmtest_1=zeros(1,4);pwmtest_1[1]=1.
    pwmtest_2=zeros(1,4);pwmtest_2[2]=1.
    @test pwm_distance(log.(pwmtest_1),log.(pwmtest_2)) == euclidean(pwmtest_1,pwmtest_2) == 1.4142135623730951

    #test consolidate check
    @test consolidate_check(distance_model) == (true,Dict{Integer,Vector{Integer}}())
    @test consolidate_check(consolidate_one) == (false, Dict{Integer,Vector{Integer}}(1⇒[3]))
    @test consolidate_check(consolidate_two) == (false, Dict{Integer,Vector{Integer}}(1⇒[2,3], 2⇒[3]))

    #test overall consolidate function
    _,con_idxs=consolidate_check(consolidate_one)
    c1consmod=consolidate_srcs(con_idxs, c1model, obs, obsl, bg_scores, drec.log_Li, [drec])

    @test consolidate_check(c1consmod.sources)[1]
    @test c1consmod.log_Li > dmodel.log_Li
    @test all(c1consmod.mix_matrix[:,1])
    @test c1consmod.sources[1][1]==log.(src_ATG)
    @test c1consmod.sources[3][1]≠log.(src_ATG)
    @test c1consmod.origin=="consolidated c1"

    _,con_idxs=consolidate_check(consolidate_two)
    c2consmod=consolidate_srcs(con_idxs, c2model, obs, obsl, bg_scores, drec.log_Li, [drec])
```

```

    @test consolidate_check(c2consmod.sources)[1]
    @test c2consmod.log_Li > dmodel.log_Li
    @test all(c2consmod.mix_matrix[:,1])
    @test c2consmod.sources[1][1]==log.(src_ATG)
    @test c2consmod.sources[2][1]≠log.(src_ATG)
    @test c2consmod.sources[3][1]≠log.(src_ATG)
    @test c2consmod.origin=="consolidated c2"

    rm(dpath)
end
```

17.6.20 /test/ensemble_tests.jl

```
@testset "Ensemble assembly and nested sampling functions" begin
    ensembledir = randstring()
    spensembledir = randstring()
    distdir = randstring()

    source_pwm = [.7 .1 .1 .1
    .1 .1 .1 .7
    .1 .1 .7 .1]

    source_pwm_2 = [.6 .1 .1 .2
    .2 .1 .1 .6
    .1 .2 .6 .1]

    src_length_limits=2:12
    no_sources=3

    source_priors = assemble_source_priors(no_sources, [source_pwm,
    ↪ source_pwm_2])
    mix_prior=.5

    bg_scores = log.(fill(.1, (30,4)))

    ↪ obs=[BioSequences.LongSequence{DNAAlphabet{2}}("CCGTTGACGATGTGATGAATAATGAAAGA

    ↪ BioSequences.LongSequence{DNAAlphabet{2}}("CCCCGATGATGACCGTTGACCAGATGGATG")

    ↪ BioSequences.LongSequence{DNAAlphabet{2}}("CCCCGATGATGACCCCGATTTTGAAAAAAA")
```

```
    BioSequences.LongSequence{DNAAlphabet{2}}("TCATCATGCTGATGATGAATCAGATGAAAG")
]

order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
obs=Array(transpose(coded_seqs))

ensemble = IPM_Ensemble(ensembledir, 150, source_priors, (falses(0,0),mix_prior), bg_scores, obs, src_length_limits)
ensemble = IPM_Ensemble(ensembledir, 200, source_priors, (falses(0,0),mix_prior), bg_scores, obs, src_length_limits) #test resumption

sp_ensemble = IPM_Ensemble(spensembledir, 200, source_priors, (falses(0,0),mix_prior), bg_scores, obs, src_length_limits, posterior_switch=true)

@test length(ensemble.models) == 200
for model in ensemble.models
    @test -350 < model.log_Li < -150
end

@test length(sp_ensemble.models) == 200
for model in sp_ensemble.models
    @test -350 < model.log_Li < -150
end

assembler=addprocs(1)

@everywhere using BioMotifInference

dist_ensemble=IPM_Ensemble(assembler, distdir, 150, source_priors, (falses(0,0),mix_prior), bg_scores, obs, src_length_limits)
dist_ensemble=IPM_Ensemble(assembler, distdir, 200, source_priors, (falses(0,0),mix_prior), bg_scores, obs, src_length_limits) #test resumption

@test length(dist_ensemble.models) == 200
for model in ensemble.models
    @test -350 < model.log_Li < -150
end

```

```
    rmprocs(assembler)
    rm(distdir, recursive=true)

    models_to_permute = 600
    funclimit=200
    funcvec=full_perm_funcvec

    instruct = Permute_Instruct(funcvec, ones(length(funcvec))./length(funcvec),models_to_permute,200, min_clmps=fill(.02,length(funcvec)))

    @info "Testing convergence displays..."
    sp_logZ = converge_ensemble!(sp_ensemble, instruct, converge_factor=500.,  wk_disp=true, tuning_disp=true, ens_disp=true, conv_plot=true, src_disp=true, lh_disp=true, liwi_disp=true, max_iterates=50)

    sp_ensemble=reset_ensemble!(sp_ensemble)

    @info "Testing threaded convergence..."
    sp_logZ = converge_ensemble!(sp_ensemble, instruct, converge_factor=500.,wk_disp=false, tuning_disp=false, ens_disp=false, conv_plot=false, src_disp=false, lh_disp=false, liwi_disp=false, backup=(true, 150), max_iterates=300)

    @info "Testing resumption..."
    sp_logZ = converge_ensemble!(sp_ensemble, instruct, converge_factor=500.,wk_disp=false, tuning_disp=false, ens_disp=false, conv_plot=false, src_disp=false, lh_disp=false, liwi_disp=false, backup=(true, 150))
    @test length(sp_ensemble.models) == 200
    @test length(sp_ensemble.log_Li) == length(sp_ensemble.log_Xi) == length(sp_ensemble.log_wi) == length(sp_ensemble.log_Liwi) == length(sp_ensemble.log_Zi) == length(sp_ensemble.Hi) == sp_ensemble.model_counter-200
    for i in 1:length(sp_ensemble.log_Li)-1
        @test sp_ensemble.log_Li[i] ≤ sp_ensemble.log_Li[i+1]
    end
    for i in 1:length(sp_ensemble.log_Zi)-1
        @test sp_ensemble.log_Zi[i] ≤ sp_ensemble.log_Zi[i+1]
    end
    @test sp_logZ > -1500.0

```

```
    @info "Testing multiprocess convergence..."
    @info "Spawning worker pool..."
    worker_pool=addprocs(2, topology=:master_worker)
    @everywhere using BioMotifInference

    ####CONVERGE############
    final_logZ = converge_ensemble!(ensemble, instruct, worker_pool, converge_factor=500., wk_disp=false, tuning_disp=false, ens_disp=false, conv_plot=false, src_disp=false, lh_disp=false, liwi_disp=false, backup=(true,500), clean=(true, 500, 1000))

    convits=length(ensemble.log_Li)

    #test converging already converged, wih different converge criterion
    final_logZ = converge_ensemble!(ensemble, instruct, worker_pool, converge_factor=150., converge_criterion="compression", wk_disp=false, tuning_disp=false, ens_disp=false, conv_plot=false, src_disp=false, lh_disp=false, liwi_disp=false, backup=(true,500), clean=(true, 500, 1000))

    length(ensemble.log_Li)==convits

    rmprocs(worker_pool)

    @test length(ensemble.models) == 200
    @test length(ensemble.log_Li) == length(ensemble.log_Xi) == length(ensemble.log_wi) == length(ensemble.log_Liwi) == length(ensemble.log_Zi) == length(ensemble.Hi) == ensemble.model_counter-200
    for i in 1:length(ensemble.log_Li)-1
        @test ensemble.log_Li[i] ≤ ensemble.log_Li[i+1]
    end
    for i in 1:length(ensemble.log_Zi)-1
        @test ensemble.log_Zi[i] ≤ ensemble.log_Zi[i+1]
    end
    @test typeof(final_logZ) == Float64
    @test final_logZ > -1500.0

    rm(ensembledir, recursive=true)
    rm(spensembledir, recursive=true)
end
```

17.6.21 /test/lh_perf.jl

```
using BenchmarkTools, BioMotifInference, BioBackgroundModels, BioSequences
println(Threads.nthreads())
source_pwm = [.7 .1 .1 .1
.1 .1 .1 .7
.1 .1 .7 .1]
source_pwm_2 = [.6 .1 .1 .2
.2 .1 .1 .6
.1 .2 .6 .1]
sources = [(log.(source_pwm), 1),(log.(source_pwm_2), 1)]

bg_scores = log.(fill(.1, (150,10000)))

obs=zeros(UInt8,151,10000)
obs[1:150,1:end]•=1
obsl=[findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]]
mix=trues(10000,2)

println(median(@benchmark (BioMotifInference.IPM_likelihood($sources, $obs, $obsl, $bg_scores, $mix)) samples=5 evals=20))

println(median(@benchmark (BioMotifInference.dev_likelihood($sources, $obs, $obsl, $bg_scores, $BitMatrix(transpose(mix)))) samples=5 evals=20))
```

17.6.22 /test/likelihood_unit_tests.jl

```
@testset "Model scoring and likelihood functions" begin
    #test a trivial scoring example
    obs=[BioSequences.LongSequence{DNAAlphabet{2}}("AAAAA")]
    order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
    obs=Array(transpose(coded_seqs))

    source_pwm = zeros(1,4)
    source_pwm[1,1] = 1
    log_pwm = log.(source_pwm)

    source_stop=5

```

```
    #score prellocates
    ss_srcs = [revcomp_pwm(log_pwm)]
    ds_srcs=[cat(log_pwm,revcomp_pwm(log_pwm),dims=3)]
    score_mat_ds=zeros(source_stop,2)
    score_mat_ss=zeros(source_stop,1)
    score_matrices_ds=Vector{Matrix{Float64}}(undef, 2)
    score_matrices_ss=Vector{Vector{Float64}}(undef, 2)

    score_sources_ds!(score_mat_ds, score_matrices_ds, ds_srcs, obs[:,1],
    ↪  source_stop)
    @test score_matrices_ds[1] == [0.0 -Inf; 0.0 -Inf; 0.0 -Inf; 0.0
    ↪  -Inf; 0.0 -Inf]

    score_sources_ss!(score_mat_ss, score_matrices_ss, ss_srcs, obs[:,1],
    ↪  source_stop)

    @test score_matrices_ss[1] == [-Inf for i in 1:5]

    #test a more complicated scoring example with two sources
    obs=[BioSequences.LongSequence{DNAAlphabet{2}}("ATGATGATGATG")]
    order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
    obs=Array(transpose(coded_seqs))

    source_pwm = [.7 .1 .1 .1
                  .1 .1 .1 .7
                  .1 .1 .7 .1]

    source_pwm_2 = [.6 .1 .1 .2
    .2 .1 .1 .6
    .1 .2 .6 .1]

    target_s1 = [.7^3 .1^3; .1^3 .1^3; .1^3 (.1*.7^2); .7^3 .1^3; .1^3
    ↪  .1^3; .1^3 (.1*.7^2); .7^3 .1^3; .1^3 .1^3; .1^3 (.1*.7^2); .7^3
    ↪  .1^3]
    target_s2 = [.6^3 (.2*.1^2); (.2*.1^2) (.2*.1^2); (.2*.1^2)
    ↪  (.2*.6^2); .6^3 (.2*.1^2); (.2*.1^2) (.2*.1^2); (.2*.1^2)
    ↪  (.2*.6^2); .6^3 (.2*.1^2); (.2*.1^2) (.2*.1^2); (.2*.1^2)
    ↪  (.2*.6^2); .6^3 (.2*.1^2)]
    log_pwms = [log.(source_pwm), log.(source_pwm_2)]

    source_start = 1
    source_stop = 10
```

```

    #score prellocates
    ss_srcs = log_pwms
    ds_srcs=[cat(pwm,revcomp_pwm(pwm),dims=3) for pwm in log_pwms]
    score_mat_ds=zeros(source_stop,2)
    score_mat_ss=zeros(source_stop,1)

    score_sources_ds!(score_mat_ds, score_matrices_ds, ds_srcs, obs[:,1],
     ↪  [source_stop, source_stop])
    @test isapprox(score_matrices_ds[1], log.(target_s1))
    @test isapprox(score_matrices_ds[2], log.(target_s2))

    score_sources_ss!(score_mat_ss, score_matrices_ss, ss_srcs, obs[:,1],
     ↪  [source_stop, source_stop])
    @test isapprox(score_matrices_ss[1], log.(target_s1[:,1]))
    @test isapprox(score_matrices_ss[2], log.(target_s2[:,1]))

    #test score weaving and IPM likelihood calculations
    #trivial example using fake cardinality_penalty
    target=logaddexp(log(.25), 0.)
    lh_vec=zeros(2)
    obsl=1
    bg_scores=view([log(.25)],:,:)
    score_mat=[[0.]]
    osi=[1]
    source_wmls=[1]
    lme=0.
    cardinality_penalty=0. #not correct but for test purposes
    osi_emit=Vector{Int64}()

    @test weave_scores_ss!(lh_vec, obsl, bg_scores, score_mat, osi,
     ↪  source_wmls, lme, cardinality_penalty, osi_emit) == target

    target2=logsumexp([log(.25), log(.5), log(.5)])
    lh_vec=zeros(2)
    weavevec=zeros(3)
    score_mat=[zeros(1,2)]
    empty!(osi_emit)
    lme=log(.5)

    @test weave_scores_ds!(weavevec, lh_vec, obsl, bg_scores, score_mat,
     ↪  osi, source_wmls, lme, cardinality_penalty, osi_emit) == target2

```

```
    #test more complicated weaves
    obs=[BioSequences.LongSequence{DNAAlphabet{2}}("ATGATGATGATG")
    BioSequences.LongSequence{DNAAlphabet{2}}("TGATGATGATGA")]
    order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
    obs=Array(transpose(coded_seqs))

    sources = [(log.(source_pwm), 1),(log.(source_pwm_2), 1)]
    dbl1_sources=[(log.(source_pwm), 1),(log.(source_pwm), 1)]

    ds_srcs=[cat(pwm[1],revcomp_pwm(pwm[1]),dims=3) for pwm in sources]
    dbl1_srcs=[cat(pwm[1],revcomp_pwm(pwm[1]),dims=3) for pwm in
    ↪  dbl1_sources]

    source_stops=[10,10]

    target_o2_s1 = [.1^3 .1^3; .1^3 (.1*.7^2); .7^3 .1^3; .1^3 .1^3; .1^3
    ↪  (.1*.7^2); .7^3 .1^3; .1^3 .1^3; .1^3 (.1*.7^2); .7^3 .1^3; .1^3
    ↪  .1^3]
    target_o2_s2 = [(.2*.1^2) (.2*.1^2); (.2*.1^2) (.2*.6^2); .6^3
    ↪  (.2*.1^2); (.2*.1^2) (.2*.1^2); (.2*.1^2) (.2*.6^2); .6^3
    ↪  (.2*.1^2); (.2*.1^2) (.2*.1^2); (.2*.1^2) (.2*.6^2); .6^3
    ↪  (.2*.1^2); (.2*.1^2) (.2*.1^2)]


    score_mat_ds=zeros(source_stop,2)
    score_matrices_ds=Vector{Matrix{Float64}}(undef, 2)

    score_sources_ds!(score_mat_ds, score_matrices_ds, ds_srcs, obs[:,1],
    ↪  source_stops)

    o1_mats = copy(score_matrices_ds)

    @test isapprox(o1_mats, [log.(target_s1), log.(target_s2)])

    score_sources_ds!(score_mat_ds, score_matrices_ds, ds_srcs, obs[:,2],
    ↪  source_stops)

    o2_mats = copy(score_matrices_ds)

    @test isapprox(o2_mats, [log.(target_o2_s1), log.(target_o2_s2)])

    bg_scores = log.(fill(.5, (12,2)))
```

```
    log_motif_expectation = log(0.5 / size(bg_scores,1))
    osi = [1,2]
    obs_cardinality = length(osi)
    penalty_sum = sum(exp.(fill(log_motif_expectation,obs_cardinality)))
    cardinality_penalty=log(1.0-penalty_sum)
    empty!(osi_emit)
    source_wmls=[3,3]

    lh_target = -8.87035766177774
    lh_vec=zeros(13)

    o1_lh = weave_scores_ds!(weavevec, lh_vec, 12, view(bg_scores,:,1),
     ↪  o1_mats, osi, source_wmls, log_motif_expectation,
     ↪  cardinality_penalty, osi_emit)
    @test isapprox(lh_target,o1_lh)

    empty!(osi_emit)

    o2_lh = weave_scores_ds!(weavevec, lh_vec, 12, view(bg_scores,:,2),
     ↪  o2_mats, osi, source_wmls, log_motif_expectation,
     ↪  cardinality_penalty, osi_emit)

    lh,cache = IPM_likelihood(sources, obs, [12,12], bg_scores,
     ↪  trues(2,2), true,true)
    @test isapprox(o1_lh,cache[1])
    @test isapprox(o2_lh,cache[2])
    @test isapprox(lps(o1_lh,o2_lh),lh)

    #test source penalization
    score_sources_ds!(score_mat_ds, score_matrices_ds, dbl1_srcs,
     ↪  obs[:,1], source_stops)

    dbl_score_mat=copy(score_matrices_ds)

    empty!(osi_emit)

    naive=weave_scores_ds!(weavevec, lh_vec, 12,
     ↪  view(bg_scores,:,1),Vector{Matrix{Float64}}(), Vector{Int64}(),
     ↪  Vector{Int64}(), log_motif_expectation, cardinality_penalty,
     ↪  osi_emit)

    empty!(osi_emit)

```

```
single=weave_scores_ds!(weavevec, lh_vec, 12, view(bg_scores,:,1), [dbl_score_mat[1]], [1], [3], log_motif_expectation, cardinality_penalty, osi_emit)

empty!(osi_emit)

double=weave_scores_ds!(weavevec, lh_vec, 12, view(bg_scores,:,1), dbl_score_mat, [1,2], [3,3], log_motif_expectation, cardinality_penalty, osi_emit)

empty!(osi_emit)

triple=weave_scores_ds!(weavevec, lh_vec, 12, view(bg_scores,:,1), [dbl_score_mat[1] for i in 1:3], [1,2,3], [3,3,3], log_motif_expectation, cardinality_penalty, osi_emit)

@test (single-naive) > (double-single) > (triple-double)

naive_target=-16.635532333438686
naive = IPM_likelihood(sources, obs, [findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]], bg_scores,falses(size(obs)[2],length(sources)))
@test naive==naive_target

#test IPM_likelihood clean vector and cache calculations
mix_matrix=trues(2,2)
baselh,basecache = IPM_likelihood(sources, obs, [12,12], bg_scores, mix_matrix, true,true)

clean=[true,false]

unchangedlh,unchangedcache=IPM_likelihood(sources, obs, [12,12], bg_scores, mix_matrix, true, true, basecache, clean)

changed_lh,changedcache=IPM_likelihood(sources, obs, [12,12], bg_scores, BitMatrix([true true; false false]), true, true, basecache, clean)

indep_lh, indepcache=IPM_likelihood(sources, obs,[12,12], bg_scores, BitMatrix([true true; false false]), true, true)

@test baselh==unchangedlh!=changed_lh==indep_lh
@test basecache==unchangedcache!=changedcache==indepcache
```

```

    #check that IPM_likelihood works in single stranded operation
    IPM_likelihood(sources, obs,[12,12], bg_scores, BitMatrix([true true; false false]), false)
end
```

17.6.23 /test/mix_matrix_unit_tests.jl

```
@testset "Mix matrix initialisation and manipulation functions" begin
    #test mix matrix init
    prior_mix_test=init_mix_matrix((trues(2,10),0.0),2, 20)
    @test all(prior_mix_test[:,1:10])
    @test !any(prior_mix_test[:,11:20])

    @test sum(init_mix_matrix((falses(0,0),1.0), O, S)) == O*S
    @test sum(init_mix_matrix((falses(0,0),0.0), O, S)) == 0
    @test 0 < sum(init_mix_matrix((falses(0,0),0.5), O, S)) < O*S

    #test mix matrix decorrelation
    empty_mixvec=falses(O)
    one_mix=mixvec_decorrelate(empty_mixvec,1)
    @test sum(one_mix)==1

    empty_mix = falses(O,S)
    new_mix,clean=mix_matrix_decorrelate(empty_mix, 500)
    @test 0 < sum(new_mix) ≤ 500
    @test !all(clean)

    full_mix = trues(O,S)
    less_full_mix,clean=mix_matrix_decorrelate(full_mix, 500)

    @test O*S-sum(less_full_mix) ≤ 500
    @test !all(clean)

    #test matrix similarity and dissimilarity functions
    test_mix=falses(O,S)
    test_mix[1:Int(floor(O/2)),:]•=true
    test_idx=3
    compare_mix=deepcopy(test_mix)
    compare_mix[:,test_idx] •= .!compare_mix[:,test_idx]
    @test most_dissimilar(test_mix,compare_mix)==test_idx

```

```
    src_mixvec=falses(O)
    src_mixvec[Int(ceil(O/2)):end]•=true
    @test most_similar(src_mixvec,compare_mix)==test_idx
end
```

17.6.24 /test/permute_func_tests.jl

```
@testset "Model permutation functions" begin
    source_pwm = [.7 .1 .1 .1
    .1 .1 .1 .7
    .1 .1 .7 .1]

    source_pwm_2 = [.6 .1 .1 .2
    .2 .1 .1 .6
    .1 .2 .6 .1]

    pwm_to_erode = [.25 .25 .25 .25
                    .97 .01 .01 .01
                    .01 .01 .01 .97
                    .01 .01 .97 .01
                    .25 .25 .25 .25]

    src_length_limits=2:5

    source_priors = assemble_source_priors(3, [source_pwm, source_pwm_2])
    mix_prior=.5

    bg_scores = log.(fill(.25, (12,2)))
    obs=[BioSequences.LongSequence{DNAAlphabet{2}}("ATGATGATGATG")
    BioSequences.LongSequence{DNAAlphabet{2}}("TGATGATGATGA")]
    order_seqs = BioBackgroundModels.get_order_n_seqs(obs, 0)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)
    obs=Array(transpose(coded_seqs))
    obsl=[findfirst(iszero,obs[:,o])-1 for o in 1:size(obs)[2]]

    test_model = ICA_PWM_Model("test", source_priors,
    ↪  (falses(0,0),mix_prior), bg_scores, obs, src_length_limits)

    ps_model= permute_source(test_model, Vector{Model_Record}(), obs,
    ↪  obsl, bg_scores, test_model.log_Li, source_priors,
    ↪  iterates=1000,weight_shift_freq=.2,length_change_freq=.2)
    @test ps_model.log_Li > test_model.log_Li
```

```
    @test ps_model.sources ≠ test_model.sources
    @test ps_model.mix_matrix = test_model.mix_matrix
    @test ps_model.origin = "PS from test"

    pm_model= permute_mix(test_model, obs, obsl, bg_scores, test_model.log_Li, iterates=1000)
    @test pm_model.log_Li > test_model.log_Li
    @test pm_model.sources = test_model.sources
    @test pm_model.mix_matrix ≠ test_model.mix_matrix
    @test "PM from test" = pm_model.origin

    psfm_model=perm_src_fit_mix(test_model, Vector{Model_Record}(),obs, obsl, bg_scores, test_model.log_Li, source_priors, iterates=1000)
    @test psfm_model.log_Li > test_model.log_Li
    @test psfm_model.sources ≠ test_model.sources
    @test psfm_model.mix_matrix ≠ test_model.mix_matrix
    @test "PSFM from test" = psfm_model.origin

    fm_model=fit_mix(test_model, Vector{Model_Record}(), obs,obsl,bg_scores)
    @test fm_model.log_Li > test_model.log_Li
    @test fm_model.sources = test_model.sources
    @test fm_model.mix_matrix ≠ test_model.mix_matrix
    @test "FM from test" = fm_model.origin
    @test fit_mix in fm_model.permute_blacklist

    post_fm_psfm=perm_src_fit_mix(fm_model, Vector{Model_Record}(),obs, obsl, bg_scores, test_model.log_Li, source_priors, iterates=1000)
    @test "PSFM from candidate" = post_fm_psfm.origin
    @test fit_mix in post_fm_psfm.permute_blacklist

    rd_model=random_decorrelate(test_model, Vector{Model_Record}(),obs, obsl, bg_scores, test_model.log_Li, source_priors, iterates=1000)
    @test rd_model.log_Li > test_model.log_Li
    @test rd_model.sources ≠ test_model.sources
    @test rd_model.mix_matrix ≠ test_model.mix_matrix
    @test "RD from test" = rd_model.origin

    rs_model=reinit_src(test_model, Vector{Model_Record}(),obs, obsl, bg_scores, test_model.log_Li, source_priors, iterates=1000)
    @test rs_model.log_Li > test_model.log_Li
    @test rs_model.sources ≠ test_model.sources
```

```
    @test rs_model.mix_matrix ≠ test_model.mix_matrix
    @test "RS from test" == rs_model.origin


    erosion_sources=[(log.(source_pwm),1),(log.(source_pwm_2),1),(log.(pwm_to_ero

    eroded_mix=trues(2,3)

    erosion_lh=IPM_likelihood(erosion_sources,obs,obsl, bg_scores, eroded_mix)

    erosion_model=ICA_PWM_Model("erode", "", erosion_sources, test_model.source_length_limits,eroded_mix, erosion_lh)

    eroded_model=erode_model(erosion_model, Vector{Model_Record}(), obs, obsl, bg_scores, erosion_model.log_Li)
    @test eroded_model.log_Li > erosion_model.log_Li
    @test eroded_model.sources ≠ erosion_model.sources
    @test eroded_model.mix_matrix == erosion_model.mix_matrix
    @test "EM from erode" == eroded_model.origin
    @test eroded_model.sources[1]==erosion_model.sources[1]
    @test eroded_model.sources[2]==erosion_model.sources[2]
    @test eroded_model.sources[3]≠erosion_model.sources[3]
    @test eroded_model.sources[3][1]==erosion_model.sources[3][1][2:4,:]

    dbl_eroded=erode_model(eroded_model, Vector{Model_Record}(), obs, obsl, bg_scores, eroded_model.log_Li)
    @test dbl_eroded.permute_blacklist == [erode_model]


    merger_srcs=   [(log.([.1 .7 .1 .1
    .1 .7 .1 .1
    .15 .35 .35 .15]),
    1
    ),
    (log.([.1 .7 .1 .1
        .1 .65 .15 .1
        .1 .7 .1 .1]),
        1
    ),
    (log.([.1 .7 .1 .1
        .1 .7 .1 .1
        .1 .7 .1 .1
```

```
        .7 .1 .1 .1]),
        1
    )]

    accurate_srcs=[ (log.([0.90 0.06 0.02 0.02; 0.02 0.02 0.02 0.94; 0.02 0.02 0.94 0.02]), 1),
    (log.([0.94 0.02 0.02 0.02; 0.02 0.07 0.07 0.84; 0.02 0.02 0.94 0.02]), 1),
    (log.([0.95 0.01 0.02 0.02; 0.02 0.02 0.02 0.94; 0.02 0.02 0.94 0.02]), 1)]

    merger_mix = BitMatrix([true false false
    true true false])

    accurate_mix=BitMatrix([true false false
    true true false])

    merger_base=ICA_PWM_Model("merge", "", merger_srcs,src_length_limits, merger_mix, IPM_likelihood(merger_srcs,obs,obsl, bg_scores, merger_mix))

    merger_target=ICA_PWM_Model("target", "", accurate_srcs, src_length_limits, accurate_mix, IPM_likelihood(accurate_srcs,obs,obsl,bg_scores,accurate_mix))

    path=randstring()
    test_record = Model_Record(path, merger_target.log_Li)
    serialize(path, merger_target)

    dm_model=distance_merge(merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li)
    @test dm_model.log_Li > merger_base.log_Li
    @test dm_model.sources ≠ merger_base.sources
    distance_dict=Dict(1⇒3,2⇒1,3⇒1)
    @test all([src==merger_base.sources[n] || src==merger_target.sources[distance_dict[n]] for (n,src) in enumerate(dm_model.sources)])
    @test "DM from merge" == dm_model.origin

    sm_model=similarity_merge(merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li)
    @test sm_model.log_Li > merger_base.log_Li
    @test sm_model.sources ≠ merger_base.sources
```

```
    @test all([src==merger_base.sources[n] || src==merger_target.sources[n] for (n,src) in enumerate(sm_model.sources)])
    @test "SM from merge" == sm_model.origin

    shuffled_model=shuffle_sources(merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li)
    @test sum(shuffled_model.sources.==merger_base.sources)==length(shuffled_model.sour
    for (s,src) in enumerate(shuffled_model.sources)
        @test src==merger_base.sources[s] || src==merger_target.sources[s]
        if src == merger_base.sources[s]
            @test shuffled_model.mix_matrix[:,s]==merger_base.mix_matrix[:,s]
        else
            @test shuffled_model.mix_matrix[:,s]==merger_target.mix_matrix[:,s]
        end
    end
    @test "SS from merge" == shuffled_model.origin

    acc_base_mix=falses(2,1);acc_base_mix[1,1]=true
    acc_merge_mix=falses(2,1);acc_merge_mix[2,1]=true

    acc_base=ICA_PWM_Model("accbase", "", [merger_srcs[1]],src_length_limits, acc_base_mix, IPM_likelihood([merger_srcs[1]],obs,obsl, bg_scores, acc_base_mix))
    acc_merge=ICA_PWM_Model("accmerge", "", [accurate_srcs[1]],src_length_limits, acc_merge_mix, IPM_likelihood([accurate_srcs[1]],obs,obsl, bg_scores, acc_merge_mix))

    accpath=randstring()
    acc_record = Model_Record(accpath, acc_merge.log_Li)
    serialize(accpath, acc_merge)

    acc_model=accumulate_mix(acc_base, [acc_record], obs, obsl, bg_scores, acc_merge.log_Li)
    @test acc_model.mix_matrix==trues(2,1)
    @test acc_model.sources == acc_merge.sources
    @test "AM from accbase" == acc_model.origin
```

```
    testwk=addprocs(1)[1]
    @everywhere import BioMotifInference

    ddm_model=remotecall_fetch(distance_merge, testwk, merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li, remote=true)
    @test ddm_model.log_Li > merger_base.log_Li
    @test ddm_model.sources ≠ merger_base.sources
    @test all([src==merger_base.sources[n] || src==merger_target.sources[distance_dict[n]] for (n,src) in enumerate(ddm_model.sources)])
    @test "DM from merge" == ddm_model.origin

    dsm_model=remotecall_fetch(similarity_merge, testwk, merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li, remote=true)
    @test dsm_model.log_Li > merger_base.log_Li
    @test dsm_model.sources ≠ merger_base.sources
    @test all([src==merger_base.sources[n] || src==merger_target.sources[n] for (n,src) in enumerate(dsm_model.sources)])
    @test "SM from merge" == dsm_model.origin

    dshuffled_model=remotecall_fetch(shuffle_sources, testwk, merger_base, [test_record], obs, obsl, bg_scores, merger_base.log_Li, remote=true)
    @test sum(dshuffled_model.sources.==merger_base.sources)==length(dshuffled_model.so
    for (s,src) in enumerate(dshuffled_model.sources)
        @test src==merger_base.sources[s] || src==merger_target.sources[s]
        if src == merger_base.sources[s]
            @test dshuffled_model.mix_matrix[:,s]==merger_base.mix_matrix[:,s]
        else
            @test dshuffled_model.mix_matrix[:,s]==merger_target.mix_matrix[:,s]
        end
    end
    @test "SS from merge" == dshuffled_model.origin
```

```
    dacc_model=remotecall_fetch(accumulate_mix, testwk, acc_base, [acc_record], obs, obsl, bg_scores, acc_merge.log_Li)
    @test dacc_model.mix_matrix==trues(2,1)
    @test dacc_model.sources == acc_merge.sources
    @test "AM from accbase" == dacc_model.origin

    info_base=ICA_PWM_Model("info_base", "", [accurate_srcs[1]], src_length_limits, accurate_mix[:,1:1], IPM_likelihood([accurate_srcs[1]],obs,obsl,bg_scores,accurate_mix[:,1:1]))

    info_model=info_fill(info_base,Vector{Model_Record}(),obs,obsl,bg_scores,info
    @test info_model.sources[1][1][1,:] == [0., -Inf, -Inf, -Inf]
    @test info_model.log_Li > info_base.log_Li
    @test "IF from info_base" == info_model.origin

    rmprocs(testwk)
    rm(path)
    rm(accpath)
end
```

17.6.25 /test/permute_tuner_tests.jl

```
@testset "Permute Tuner" begin
    funcvec=[random_decorrelate,fit_mix]
    instruct=Permute_Instruct(funcvec, [.5,.5],100,100)
    tuner=Permute_Tuner(instruct)
    #need to test update_weights functionality
    #want to supply some fake data to induce a .8 .2 categorical
    tuner.successes[:,1]=falses(TUNING_MEMORY*instruct.func_limit)
    tuner.successes[:,2]=falses(TUNING_MEMORY*instruct.func_limit)

    tuner.successes[1:Int(floor(TUNING_MEMORY*instruct.func_limit*.8)),1]•=true

    tuner.successes[1:Int(floor(TUNING_MEMORY*instruct.func_limit*.2)),2]•=true
    update_weights!(tuner)
    @test tuner.inst.weights==[.8,.2]    #check clamping
    tuner.successes[:,1]=falses(TUNING_MEMORY*instruct.func_limit)
    @test tuner.inst.min_clmps==[.01,.01]
    @test tuner.inst.max_clmps==[1.,1.]
    update_weights!(tuner)
    @test tuner.inst.weights==[.01,1-.01]

```

```
    #test override
    instruct=Permute_Instruct(funcvec, [.5,.5],100,100,override_time=2.,override_weights=[.75,.25])
    tuner=Permute_Tuner(instruct)
    tune_weights!(tuner, [(1,1.,1.)])
    @test tuner.inst.weights==[.5,.5] #no override
    tuner.time_history=fill(3.,CONVERGENCE_MEMORY)
    tune_weights!(tuner, [(1,1.,1.)])
    @test tuner.inst.weights==[.75,.25]

    #more clamping tests
    testvec=[0.02693244970132842, 0.031512823560878485, 0.050111382705776294, 0.6893314065796903, 0.04206537140157707, 0.02013161919125878, 0.026535815205008566, 0.051758654139053166, 0.03497967993105292, 0.013999262917669668, 0.01264153466670629]
    target=[0.02527900179828276, 0.029859375657832827, 0.04845793480273063, 0.6876779586766446, 0.040411923498531406, 0.02, 0.02488236730196291, 0.050105206236007505, 0.03332623202800726, 0.02, 0.02]


    clamp_pvec!(testvec,fill(.02,length(testvec)),fill(1.,length(testvec)))
    @test isprobvec(testvec)
    @test testvec==target

    #high clamp
    testvec=[.9,.08,.02]
    clamp_pvec!(testvec, fill(.05,length(testvec)), [.45,.45,.45])
    @test testvec==[.45, .2825, .2675]
end
```

17.6.26 /test/pwm_unit_tests.jl

```
@testset "PWM source prior setup, PWM source initialisation and manipulation functions" begin
    #test dirichlet prior estimation from wm inputs
    wm_input = [.0 .2 .3 .5; .0 .2 .3 .5]
    est_dirichlet_vec = estimate_dirichlet_prior_on_wm(wm_input)
    @test typeof(est_dirichlet_vec) == Vector{Dirichlet{Float64}}
    for pos in 1:length(est_dirichlet_vec)
        @test isapprox(est_dirichlet_vec[pos].alpha, wm_input[pos,:].*PRIOR_WT)
```

```
    end

    bad_input = wm_input .* 2
    @test_throws DomainError estimate_dirichlet_prior_on_wm(bad_input)

    wm_input = [.1 .2 .3 .4; .1 .2 .3 .4]
    est_dirichlet_vec = estimate_dirichlet_prior_on_wm(wm_input)
    @test typeof(est_dirichlet_vec) == Vector{Dirichlet{Float64}}
    for pos in 1:length(est_dirichlet_vec)
        @test est_dirichlet_vec[pos].alpha == wm_input[pos,:].*PRIOR_WT
    end

    length_range = 2:2

    #test informative/uninformative source prior vector assembly
    test_priors = assemble_source_priors(2, [wm_input])
    @test length(test_priors)  == 2
    for pos in 1:length(test_priors[1])
        @test test_priors[1][pos].alpha == wm_input[pos,:].*PRIOR_WT
    end
    @test test_priors[2] == false

    #test source wm initialisation from priors
    test_sources = init_logPWM_sources(test_priors, length_range)
    for source in test_sources
        for pos in 1:size(source[1])[1]
            @test isprobvec(exp.(source[1][pos,:]))
        end
    end

    #test that wm_shift is returning good shifted probvecs
    rando_dist=Dirichlet([.25,.25,.25,.25])
    for i in 1:1000
        wm=rand(rando_dist)
        new_wm=wm_shift(wm,Weibull(1.5,.1))
        @test isprobvec(new_wm)
        @test wm!=new_wm
    end

    #test that legal new sources are generated by permute_source_weights
    permuted_weight_sources=deepcopy(test_sources)

    ↪ permuted_weight_sources[1]=permute_source_weights(permuted_weight_sources[1],
```

```
    permuted_weight_sources[2]=permute_source_weights(permuted_weight_sources[2],
    @test permuted_weight_sources ≠ test_sources
    for (s,source) in enumerate(permuted_weight_sources)
        for pos in 1:size(source[1],1)
            @test isprobvec(exp.(source[1][pos,:]))
            @test source[1][pos,:] ≠ test_sources[s][1][pos,:]
        end
    end

    #test that get_length_params returns legal length shifts
    lls=1:10
    pr=1:5
    for i in 1:1000
        srcl=rand(1:10)
        sign, permute_length=get_length_params(srcl, lls, pr)
        @test pr[1]≤permute_length≤pr[end]
        @test sign==-1 || sign==1
        @test lls[1]≤(srcl+(sign*permute_length))≤lls[end]
    end

    permuted_length_sources=deepcopy(test_sources)

    permuted_length_sources[1]=permute_source_length(permuted_length_sources[1],t

    permuted_length_sources[2]=permute_source_length(permuted_length_sources[2],t
    for (s, source) in enumerate(permuted_length_sources)
        @test size(source[1],1) ≠ size(test_sources[1][1],1)
        @test 1≤size(source[1],1)≤3
    end

    info_test_wm=[1. 0. 0. 0.
    .94 .02 .02 .02
    .82 .06 .06 .06
    .7 .1 .1 .1
    .67 .11 .11 .11
    .52 .16 .16 .16
    .4 .2 .2 .2
    .25 .25 .25 .25]

    #test eroding sources by finding most informational position and
    cutting off when information drops below threshold
    infovec=get_pwm_info(log.(info_test_wm))
```

```
    @test infovec==[2.0, 1.5774573308022544, 1.0346297041419121, 0.6432203505529603, 0.5620360019822908, 0.2403724636586433, 0.07807190511263773, 0.0]

    erosion_test_source=(log.([.25 .25 .25 .25
                        .2 .4 .2 .2
                        .7 .1 .1 .1
                        .06 .06 .06 .82
                        .7 .1 .1 .1
                        .25 .25 .25 .25]),1)

    infovec=get_pwm_info(erosion_test_source[1])
    start_idx, end_idx = get_erosion_idxs(infovec, .25, 2:8)
    @test start_idx==3
    @test end_idx==5

    eroded_pwm,eroded_prior_idx=erode_source(erosion_test_source,2:8,.25)
    for pos in 1:size(eroded_pwm,1)
        @test isprobvec(exp.(eroded_pwm[pos,:]))
    end
    @test eroded_prior_idx==3
    @test isapprox(exp.(eroded_pwm),[.7 .1 .1 .1
    .06 .06 .06 .82
    .7 .1 .1 .1])


    #make sure revcomp_pwm is reversing pwms across both dimensions
    revcomp_test_pwm = zeros(2,4)
    revcomp_test_pwm[1,1] = 1
    revcomp_test_pwm[2,3] = 1
    log_revcomp_test_pwm = log.(revcomp_test_pwm)
    @test revcomp_pwm(log_revcomp_test_pwm) == [-Inf 0. -Inf -Inf
                                                -Inf -Inf -Inf 0.]
end
```

17.6.27 /test/runtests.jl

```
@info "Loading test packages ..."

using BioMotifInference, BioBackgroundModels, BioSequences, Distributions, Distributed, Random, Serialization, Test
```

```
import StatsFuns: logsumexp, logaddexp
import BioMotifInference:estimate_dirichlet_prior_on_wm,
↪ assemble_source_priors, init_logPWM_sources, wm_shift,
↪ permute_source_weights, get_length_params, permute_source_length,
↪ get_pwm_info, get_erosion_idxs, erode_source, init_mix_matrix,
↪ mixvec_decorrelate, mix_matrix_decorrelate, most_dissimilar,
↪ most_similar, revcomp_pwm, score_sources_ds!, score_sources_ss!,
↪ weave_scores_ss!, weave_scores_ds!, IPM_likelihood,
↪ consolidate_check, consolidate_srcs, pwm_distance, permute_source,
↪ permute_mix, perm_src_fit_mix, fit_mix, random_decorrelate,
↪ reinit_src, erode_model, reinit_src, shuffle_sources, accumulate_mix,
↪ distance_merge, similarity_merge, info_fill, converge_ensemble!,
↪ reset_ensemble!, Permute_Tuner, PRIOR_WT, TUNING_MEMORY,
↪ CONVERGENCE_MEMORY, tune_weights!, update_weights!, clamp_pvec!
import Distances: euclidean

@info "Beginning tests ..."
using Random
Random.seed!(786)
O=1000;S=50

include("pwm_unit_tests.jl")
include("mix_matrix_unit_tests.jl")
include("likelihood_unit_tests.jl")
include("consolidate_unit_tests.jl")
include("permute_func_tests.jl")
include("permute_tuner_tests.jl")
include("ensemble_tests.jl")
@info "Tests complete!"
```

17.6.28 /test/spike_recovery.jl

```
#Testbed for synthetic spike recovery from example background
using BioBackgroundModels, BioMotifInference, Random, Distributed,
↪ Distributions, Serialization
Random.seed!(786)
#CONSTANTS
no_obs=500
obsl=100:200

const folder_path="/bench/PhD/NGS_binaries/BBM/refined_folders"

```

```
report_folders=deserialize(folder_path)

bhmm_vec=[BHMM([0.40027008799648645, 0.1997894691941005,
↪ 0.006556173444055311, 0.3381222117369811, 0.053302496489949405,
↪ 0.0019595611384158307], [0.9910385961237116 0.001055084535731191
↪ 0.0030117366072612726 0.0025443692116936516 1.875366316590218e-83
↪ 0.002350213521605879; 0.003592562092379169 0.6212087435679351
↪ 1.4862557155404555e-91 0.3738436265294334 0.001355067810251799
↪ 5.674610926019336e-174; 0.03314627345971247 0.00026886477654034587
↪ 0.9628685354878737 3.677787936178394e-5 0.00367954839651186 5.0e-324;
↪ 0.0002903769304330447 0.3737391779744011 1.584645416437449e-35
↪ 0.6225645652084915 0.002839228767872827 0.0005666511188010834;
↪ 0.0005590565306708014 0.006419582454763721 5 0.004061509889975232
↪ 0.01962586251822172 0.9687912312979926 0.0005427573083774576;
↪ 0.0174697691356862 2.0648278441425858e-20 8.008498087945918e-218
↪ 0.004151185943888868 0.0013251598095032858 0.9770538851109234],
↪ [Categorical([0.2565257692018922, 0.24506145230281004,
↪ 0.2649485969649606, 0.23346418153033835]),
↪ Categorical([0.052491168994042534, 0.0987036876775239,
↪ 0.2383241923003866, 0.610480951028046]),
↪ Categorical([0.08371833024203595, 0.07204591736665274,
↪ 0.3854013598343639, 0.4588343925569475]),
↪ Categorical([0.5954709485303519, 0.2497223085607694,
↪ 0.09528006039357192, 0.0595266825153071]),
↪ Categorical([0.4744103622376032, 0.04038264172760258,
↪ 0.04588736323302444, 0.4393196328017719]),
↪ Categorical([0.41173384083242826, 0.44270375576246973,
↪ 0.030612125665857307, 0.11495027773924743])])]

bhmm_dist=Categorical([1.])

struc_sig_1=[.1 .7 .1 .1
           .1 .1 .1 .7
           .1 .7 .1 .1]
periodicity=8
struc_frac_obs=.75


tata_box=[.05 .05 .05 .85
          .85 .05 .05 .05
          .05 .05 .05 .85
          .85 .05 .05 .05
          .425 .075 .075 .425
```

```
          .85 .05 .05 .05
          .425 .075 .075 .425]
motif_frac_obs=.7
motif_recur_range=1:4

@info "Constructing synthetic sample set..."
obs1, bg_scores1, hmm_truth1, spike_truth1 = synthetic_sample(no_obs,obsl,bhmm_vec,bhmm_dist,[struc_sig_1,tata_box],[(true,(st

e1 = "BMISpikeTest"

#JOB CONSTANTS
const ensemble_size = 750
const no_sources = 4
const source_min_bases = 3
const source_max_bases = 12
const source_length_range= source_min_bases:source_max_bases
const mixing_prior = .5
const models_to_permute = ensemble_size * 3
funcvec=full_perm_funcvec
push!(funcvec, BioMotifInference.permute_source)
push!(funcvec, BioMotifInference.permute_source)
args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcvec)]
args[end-1]=[(:weight_shift_freq,0.),(:length_change_freq,1.),(:length_perm_range,1:1
args[end]=[(:weight_shift_freq,.1),(:length_change_freq,0.),(:weight_shift_dist,Unifo


instruct = Permute_Instruct(funcvec, ones(length(funcvec))./length(funcvec),models_to_permute,100; args=args)

@info "Assembling source priors..."
prior_array= Vector{Matrix{Float64}}()
source_priors = BioMotifInference.assemble_source_priors(no_sources, prior_array)

@info "Assembling ensemble..."
isfile(e1*"/ens") ? (ens1 = deserialize(e1*"/ens")) :
    (ens1 = IPM_Ensemble(e1, ensemble_size, source_priors, (falses(0,0), mixing_prior), bg_scores1, obs1, source_length_range))

@info "Converging ensemble..."
```

```
logZ1 = BioMotifInference.converge_ensemble!(ens1, instruct, backup=(true,250), wk_disp=false, tuning_disp=false, ens_disp=true, conv_plot=false, src_disp=true, lh_disp=false, liwi_disp=false)
```

17.7 AWSWrangler

Github repository: https://github.com/mmattocks/AWSWrangler.jl

17.7.1 /src/AWSWrangler.jl

```
module AWSWrangler

using AWSCore, AWSSDK, Dates, Sockets

function spot_wrangle(no_instances::Integer, spot_price::AbstractFloat, sgrp_name::String, sgrp_desc::String, skeys::String, zone::String, ami::String, instance_type::String)
    #ssh permissions
    WANip=chomp(read(pipeline(`dig +short myip.opendns.com @resolver1.opendns.com`), String))
    permissions = [
        [
            "FromPort" => 22,
            "IpProtocol" => "tcp",
            "IpRanges" => [
                [
                    "CidrIp" => WANip*"/32",
                    "Description" => "SSH access"
                ]
            ],
            "ToPort" => 22
        ]
    ]

    @info "Configuring AWS credentials..."
    aws=aws_config()

    @info "Getting security group info..."
    sgrp=Dict()

    try
```

```
29
        ↪ sgrp=AWSSDK.EC2.create_security_group(aws,GroupDescription=sgrp_desc,Grou
30          inret=AWSSDK.EC2.authorize_security_group_ingress(aws,
        ↪ GroupId=sgrp["groupId"], IpPermissions=permissions)
31          outret=AWSSDK.EC2.authorize_security_group_egress(aws,
        ↪ GroupId=sgrp["groupId"], IpPermissions=permissions)
32      catch error
33          if error.code == "InvalidGroup.Duplicate"
34
            ↪ sgrp=AWSSDK.EC2.describe_security_groups(aws,GroupName=sgrp_name)["se
35          else
36              throw(error)
37          end
38      end
39
40      #Launch specs
41      LaunchSpec = [
42          "ImageId" => ami,
43          "InstanceType" => instance_type,
44          "SecurityGroupId" => [
45              sgrp["groupId"]
46          ],
47          "Placement" => [
48              "AvailabilityZone" => zone
49          ],
50          "KeyName" => skeys
51      ]
52
53      @info "Requesting spot instances..."
54
55      spot_reqs=AWSSDK.EC2.request_spot_instances(aws,
    ↪ LaunchSpecification=LaunchSpec, InstanceCount=no_instances,
    ↪ SpotPrice=spot_price, Type="one-time")
56
57      instance_public_ips=Vector()
58
59      @info "Waiting for instances to become active..."
60
61      if no_instances==1
62
        ↪ spot_req_Id=spot_reqs["spotInstanceRequestSet"]["item"]["spotInstanceRequ
63          active=false
64          while !active
```

```
            AWSSDK.EC2.describe_spot_instance_requests(aws, SpotInstanceRequestId=spot_req_Id)["spotInstanceRequestSet"]["item"][ == "active" ? active=true : sleep(10)
        end
        instance_Id=AWSSDK.EC2.describe_spot_instance_requests(aws, SpotInstanceRequestId=spot_req_Id)["spotInstanceRequestSet"]["item"]["ins
        address=AWSSDK.EC2.describe_instances(aws, InstanceId=instance_Id)["reservationSet"]["item"]["instancesSet"]["item"]
        push!(instance_public_ips,address)
    else
        for spot_req in spot_reqs["spotInstanceRequestSet"]["item"]
            spot_req_Id = spot_req["spotInstanceRequestId"]
            active=false
            while !active
                AWSSDK.EC2.describe_spot_instance_requests(aws, SpotInstanceRequestId=spot_req_Id)["spotInstanceRequestSet"]["ite == "active" ? active=true : sleep(10)
            end
            instance_Id = AWSSDK.EC2.describe_spot_instance_requests(aws, SpotInstanceRequestId=spot_req_Id)["spotInstanceRequestSet"]["item"][
            address=AWSSDK.EC2.describe_instances(aws, InstanceId=instance_Id)["reservationSet"]["item"]["instancesSet"]["it
            push!(instance_public_ips,address)
        end
    end

    return instance_public_ips
end

function get_cheapest_zone(instance_type::String)
    arguments=[
        "ProductDescription"=>["Linux/UNIX"]
        "InstanceType"=>[instance_type]
        "StartTime"=>Dates.now(Dates.UTC)
    ]
    history=AWSSDK.EC2.describe_spot_price_history(aws_config(), arguments)

    price_dict=Dict{String,Float64}()
    for record in history["spotPriceHistorySet"]["item"]
        price_dict[record["availabilityZone"]]=parse(Float64, record["spotPrice"])
    end
```

```

    return sort(collect(price_dict), by=x→x[2])[1]
end

export spot_wrangle,get_cheapest_zone

end # module
```

17.8 Analyses

Github repository: https://github.com/mmattocks/Thesis/Analyses

17.8.1 /cmz/a10correlation.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions,NGRefTools,StatsBase,Plots,Measurements
import Measurements:value,uncertainty

default(legendfont = (8), guidefont = (10,), tickfont = (8), guide = "x")

a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["PopEst"]=Vector{Vector{Float64}}()
measure_dict["VolEst"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    pevec,tvec,dvec,vvec,vevec, rtvec, rplvec, rptvec, ldvec, ondvec, onlvec, oplvec, inlvec, iplvec, gclvec = [zeros(0) for i in 1:15]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Lens diameter")...])>0 && push!(ldvec,mean(skipmissing(i_df."Lens diameter")))
```

```
    end

    for i_df in groupby(t_df,"BSR")
        if length([skipmissing(i_df."CMZ Sum") ... ])>0
            push!(tvec,mean(skipmissing(i_df."CMZ Sum")))
            if length([skipmissing(i_df."Lens diameter") ... ])>0
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(skipmissing(i_df."Lens diameter"))/14.)
            else
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(ldvec)/14.)
            end
        end
        if length([skipmissing(i_df."Retina thickness") ... ])>0 &&
            length([skipmissing(i_df."IPL") ... ])>0 &&
            length([skipmissing(i_df."OPL") ... ])>0 &&
            length([skipmissing(i_df."RPE length") ... ])>0

                rthi=mean(skipmissing(i_df."Retina thickness"))-mean(skipmissing(i_df."OPL"))-mean(skipmissing(i_df."
                rpel=mean(skipmissing(i_df."RPE length"))
                if length([skipmissing(i_df."Lens diameter") ... ])>0
                    rcirc=rpel+mean(skipmissing(i_df."Lens diameter"))
                else
                    rcirc=rpel+mean(ldvec)
                end

                or=rcirc/2π
                ir=or-.5*rthi

                ov=(4/3)*π*(or^3)
                iv=(4/3)*π*(ir^3)

                push!(vevec,(ov-iv)*(4/5))
            end
    end

    push!(measure_dict["PopEst"],filter(p→p>0,pevec))
    push!(measure_dict["VolEst"],vevec)
end

x=measure_dict["PopEst"][1]
uncorrX3d=ones(length(x),1)
```

```
corrX3d=hcat(ones(length(x)),x)
y=measure_dict["VolEst"][1]
uncorrm=BayesianLinearRegression.fit!(BayesianLinReg(uncorrX3d,y))
corrm=BayesianLinearRegression.fit!(BayesianLinReg(corrX3d,y))

function blr_plt(x,y,Xs,blrs,colors,labels,q=.975)

    plt=scatter(x,y,marker=:diamond,markersize=8,markerstrokestyle=:bold,markerco
    retinal volume",xlabel="Estimated CMZ Annulus
    Population",legend=:bottomleft)
    for (X,blr,color,label) in zip(Xs, blrs,colors,labels)
        line=zeros(size(X,1))
        ribbon=zeros(size(X,1))
        predictions=BayesianLinearRegression.predict(blr,X)
        for (n,p) in enumerate(predictions)
            normal=Normal(value(p),uncertainty(p))
            ribbon[n]=quantile(normal,q)-mean(normal)
            line[n]=mean((value(p)))
        end
        plot!(plt,x,line,ribbon=ribbon,color=color,label=label)
    end
    return plt
end

p=blr_plt(x,y,[corrX3d,uncorrX3d],[corrm,uncorrm],[:darkmagenta,:green],["Correlated
  Model", "Uncorrelated Model"])

savefig(p, "/bench/PhD/Thesis/images/cmz/3dcorr.png")
@info "Saved fig."

println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf Age (dpf)} & {\\bf Uncorrelated logZ} & {\\bf Correlated
  logZ} & {\\bf logZR}\\\\ \\hline")

for (x,xs,ys) in zip(X,measure_dict["PopEst"],measure_dict["VolEst"])
    uncorrX=ones(length(xs),1)
    corrX=hcat(ones(length(xs)),xs)
    uncorr=BayesianLinearRegression.fit!(BayesianLinReg(uncorrX,ys))
    corr=BayesianLinearRegression.fit!(BayesianLinReg(corrX,ys))
```

```
    println("$x & {\\bf $(round(logEvidence(uncorr),digits=3))} & $(round(logEvidence(corr),digits=3)) & $(round(logEvidence(uncorr)-logEvidence(corr),digits=3))\\\\ \\hline")
end

println("\\end{tabular}")
```

17.8.2 /cmz/a10decayslicediagnostic.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions,CMZNicheSims,GMC_NS, Serialization, Plots

emultipath="/bench/PhD/NGS_binaries/CNS/A10/emulti_decay"

default(legendfont = (10), guidefont = (12), tickfont = (10))

base_scale=[144.,5.]
base_min=[10.,0.]
time_scale=90
time_min=4.

prior=[LogNormal(log(20),log(2)),Uniform(eps(),.03),LogNormal(log(.9),log(1.6))]


em=deserialize(emultipath*"/ens")

models=["132.93","674.18","575.46","745.16"]
MLE=deserialize(em.path*"/132.93").log_Li
X=em.constants[1]
plotticks=[10*i for i in 1:9]

mapplots=Vector{Plots.Plot}()
mapxs=Vector{Float64}()

for model in models
    mapm=deserialize(em.path*"/$model")

```

```
    catdobs=vcat([em.obs[1][t] for t in 1:length(X)]...)
    catvobs=vcat([em.obs[2][t] for t in 1:length(X)]...)

    dXs=vcat([[X[n] for i in 1:length(em.obs[1][n])] for n in 1:length(X)]...)
    vXs=vcat([[X[n] for i in 1:length(em.obs[2][n])] for n in 1:length(X)]...)

    md_mean=mapm.slices[1].disp_mat[:,2]
    md_upper=mapm.slices[1].disp_mat[:,3]•-mapm.slices[1].disp_mat[:,2]
    md_lower=mapm.slices[1].disp_mat[:,2]•-mapm.slices[1].disp_mat[:,1]

    mv_mean=mapm.slices[2].disp_mat[:,2]
    mv_upper=mapm.slices[2].disp_mat[:,3]•-mapm.slices[2].disp_mat[:,2]
    mv_lower=mapm.slices[2].disp_mat[:,2]•-mapm.slices[2].disp_mat[:,1]

    mapmd_plt=scatter(dXs,catdobs, marker=:cross, color=:black, markersize=3, showaxis=:y, xformatter=_→"", xticks=X, legend=nothing, title=model, ylabel="Population")
    plot!(mapmd_plt, X, md_mean, ribbon=(md_lower,md_upper), color=:green)

    mapmv_plt=scatter(vXs,catvobs, marker=:cross, color=:black, markersize=3, ylabel="Population", xticks=X, xlabel="Age (dpf)", legend=nothing)
    plot!(mapmv_plt, X, mv_mean, ribbon=(mv_lower,mv_upper), color=:darkmagenta)

    combined_map=Plots.plot(mapmd_plt,mapmv_plt,layout=grid(2,1), size=(300,600),  link=:xy)

    push!(mapplots, combined_map)
    push!(mapxs,mapm.θ[1])
end

mkdes=posterior_kde(em, bivar=[1⇒2,1⇒3,2⇒3])

ctkmarg=contour(mkdes[4].x,mkdes[4].y,transpose(mkdes[4].density), c=:viridis, xlabel="iCT (hr)", ylabel="κ", colorbar=:false, xlims=[0,144], ylims=[0,.03], levels=200, xticks=24.:24.:144.)
```

```
ctemarg=contour(mkdes[5].x,mkdes[5].y,transpose(mkdes[5].density), c=:viridis, xlabel="iCT (hr)", ylabel="ϵ", colorbar=:false, xlims=[0,144], ylims=[0,5.], levels=200, xticks=24.:24.:144.)

kemarg=contour(mkdes[6].x,mkdes[6].y,transpose(mkdes[6].density), c=:viridis, xlabel="κ", ylabel="ϵ", colorbar=:false, xlims=[0,.03], ylims=[0,5.], levels=200, xticks=24.:24.:144.)

# clim=(0,.5), legend=(.8,.3)

ymarg=map(x→pdf(prior[1][1],x),mkdes[1].x)

ctmarg=plot(mkdes[1].x, ymarg, color=:darkmagenta, fill=true, fillalpha=.5,xlims=[0,144], xlabel="Initial Cycle Time (hr)", ylabel="Probability density", label="Prior")
plot!(mkdes[1].x,mkdes[1].density,color=:green, fill=true, fillalpha=.5, label="Combined  Posterior")

colors=[:darkgreen,:darkred,:darkblue,:darkmagenta]
for (n, model) in enumerate(models)

    plot!([mapxs[n],mapxs[n]],[0,maximum(mkdes[1].density)],color=colors[n], label=model)
end

combined_layout=@layout[ctmarg{.3h}
                        grid(1,4)
                        ]

pmplot=Plots.plot(ctmarg,mapplots ... ,layout=combined_layout,size=(1200,900))

savefig(pmplot, "/bench/PhD/Defense/images/polymodality.png")
```

17.8.3 /cmz/a10dvdecayslice.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions,CMZNicheSims,GMC_NS, Serialization, Plots
a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

edpth="/bench/PhD/NGS_binaries/CNS/A10/ed_decay"
evpth="/bench/PhD/NGS_binaries/CNS/A10/ev_decay"
```

```
empth="/bench/PhD/NGS_binaries/CNS/A10/emulti_decay"
paths=[edpth,evpth,empth]

default(legendfont = (10), guidefont = (12), tickfont = (10))

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["Dorsal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ventral CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Lens circumference"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    if !(Float64(unique(t_df."Time point (d)")[1]) in [180., 360.])

        push!(X,Float64(unique(t_df."Time point (d)")[1]))
        dvec,vvec,lcirc = [zeros(0) for i in 1:4]

        for i_df in groupby(t_df,"BSR")
            length([skipmissing(i_df."Lens diameter")...])>0 && push!(lcirc,mean(skipmissing(i_df."Lens diameter"))*π)
        end

        for i_df in groupby(t_df,"BSR")

            length([skipmissing(i_df."Dorsal CMZ (#)")...])>0 && mean(skipmissing(i_df."Dorsal CMZ (#)")) >0. && push!(dvec,mean(skipmissing(i_df."Dorsal CMZ (#)")))
            length([skipmissing(i_df."Ventral CMZ (#)")...])>0 && mean(skipmissing(i_df."Ventral CMZ (#)")) >0. && push!(vvec,mean(skipmissing(i_df."Ventral CMZ (#)")))
        end

        push!(measure_dict["Dorsal CMZ (#)"],dvec)
        push!(measure_dict["Ventral CMZ (#)"],vvec)
        push!(measure_dict["Lens circumference"],lcirc)
    end
end
```

```

dobs=measure_dict["Dorsal CMZ (#)"]
vobs=measure_dict["Ventral CMZ (#)"]
mobs=[measure_dict["Dorsal CMZ (#)"],measure_dict["Ventral CMZ (#)"]]

obsset=[dobs,vobs,mobs]

dpopdist=fit(LogNormal,measure_dict["Dorsal CMZ (#)"][1])
vpopdist=fit(LogNormal,measure_dict["Ventral CMZ (#)"][1])

logLC=vcat([log.(measure_dict["Lens circumference"][n]) for n in 1:length(X)]...)
logX=vcat([[log.(X[n]) for ms in measure_dict["Lens circumference"][n]] for n in 1:length(X)]...)
logX=hcat(ones(length(logX)),logX)

lm=BayesianLinearRegression.fit!(BayesianLinReg(logX,logLC))
w1,w2=posteriorWeights(lm)
factor=10^w1.val
power=w2.val

lm=Lens_Model(factor,power,14.)

mc_its=Int64(1e6)

ed_constants=[X,dpopdist,lm,mc_its]
ev_constants=[X,vpopdist,lm,mc_its]
em_constants=[X,[dpopdist,vpopdist],lm,mc_its]
constants=[ed_constants,ev_constants,em_constants]

priors=[LogNormal(log(20),log(2)),Uniform(eps(),.03),LogNormal(log(.9),log(1.6))]

box=[10. 144.
     eps() .03
     0. 5.]
box=GMC_NS.to_unit_ball.(box,priors)

uds=Vector{Vector{Function}}([[],[liwi_display],[convergence_display]])
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display],[ensemble_displa

gmc_settings=GMC_DEFAULTS
gmc_settings[end]=100

```

```
for (pth,constants,obs) in zip(paths,constants,obsset)
    if isfile(pth*"/ens")
        e=deserialize(pth*"/ens")
    else
        if !occursin("emulti",pth)
            e=Decay_Ensemble(pth,1000,obs, priors, constants, box, gmc_settings)
        else
            e=MultiSlice__Decay_Ensemble(pth,1000,obs, priors, constants, box, gmc_settings)
        end
    end

    converge_ensemble!(e,backup=(true,20),upper_displays=uds, lower_displays=lds, disp_rot_its=50, mc_noise=.3, converge_factor=1e-6)
end

ed=deserialize(edpth*"/ens")
ev=deserialize(evpth*"/ens")
em=deserialize(empth*"/ens")

edev=measure_evidence(ed)
evev=measure_evidence(ev)
emev=measure_evidence(em)

evidence_ratio=emev-(edev+evev)

println("\\begin{tabular}{|l|l|l|l|l|}")
println("\\hline")
println("{\\bf Combined D/V model logZ} & {\\bf Separate D/V model logZ} & {\\bf logZR} & {\\bf \$\\sigma\$ Significance}\\\\ \\hline")
println("\\textbf{$(round(emev,digits=3))} & $(round(edev+evev,digits=3)) & $(round(evidence_ratio,digits=3)) & $(round(evidence_ratio.val/evidence_ratio.err,digits=3))\\\\ \\hline")
println("\\end{tabular}")

mapd=deserialize(ed.models[findmax([m.log_Li for m in ed.models])[2]].path)
mapv=deserialize(ev.models[findmax([m.log_Li for m in ev.models])[2]].path)
```

```
mapm=deserialize(em.models[findmax([m.log_Li for m in em.models])[2]].path)

println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf Parameter} & {\\bf Combined MAP} & {\\bf Dorsal MAP} & {\\bf Ventral MAP}\\ \\hline")
println("\$CT_{i}\$ (h) & $(round(mapm.θ[1], digits=1)) & $(round(mapd.θ[1], digits=1)) & $(round(mapv.θ[1], digits=1))\\\\ \\hline")
println("\$\\kappa\$ decay& $(round(mapm.θ[2], digits=4)) & $(round(mapd.θ[2], digits=4)) & $(round(mapv.θ[2], digits=4))\\\\ \\hline")
println("\$\\epsilon\$ exit& $(round(mapm.θ[3], digits=3)) & $(round(mapd.θ[3], digits=3)) & $(round(mapv.θ[3], digits=3))\\\\ \\hline")

X=ed.constants[1]

plotticks=[10*i for i in 1:9]

catdobs=vcat([ed.obs[t] for t in 1:length(X)]...)
catvobs=vcat([ev.obs[t] for t in 1:length(X)]...)

dXs=vcat([[X[n] for i in 1:length(ed.obs[n])] for n in 1:length(X)]...)
vXs=vcat([[X[n] for i in 1:length(ev.obs[n])] for n in 1:length(X)]...)
dvXs=vcat([[X[n] for i in 1:length(em.obs[1][n])] for n in 1:length(X)]...)

d_mean=mapd.disp_mat[:,2]
d_upper=mapd.disp_mat[:,3].-mapd.disp_mat[:,2]
d_lower=mapd.disp_mat[:,2].-mapd.disp_mat[:,1]

v_mean=mapv.disp_mat[:,2]
v_upper=mapv.disp_mat[:,3].-mapv.disp_mat[:,2]
v_lower=mapv.disp_mat[:,2].-mapv.disp_mat[:,1]

md_mean=mapm.slices[1].disp_mat[:,2]
md_upper=mapm.slices[1].disp_mat[:,3].-mapm.slices[1].disp_mat[:,2]
md_lower=mapm.slices[1].disp_mat[:,2].-mapm.slices[1].disp_mat[:,1]

mv_mean=mapm.slices[2].disp_mat[:,2]
mv_upper=mapm.slices[2].disp_mat[:,3].-mapm.slices[2].disp_mat[:,2]
```

```
mv_lower=mapm.slices[2].disp_mat[:,2]•-mapm.slices[2].disp_mat[:,1]

mapmd_plt=scatter(dXs,catdobs, marker=:cross, color=:black, markersize=3, label="Dorsal CMZ population data", showaxis=:y, xformatter=_→"", xticks=X, ylabel="Population")
plot!(mapmd_plt, X, md_mean, ribbon=(md_lower,md_upper), color=:green, label="Combined model D population")
annotate!([(8,140,Plots.text("A",18))])

mapmv_plt=scatter(vXs,catvobs, marker=:cross, color=:black, markersize=3, label="Ventral CMZ population data", showaxis=:y, ylabel="Population", xformatter=_→"",xticks=X)
plot!(mapmv_plt, X, mv_mean, ribbon=(mv_lower,mv_upper), color=:darkmagenta, label="Combined model V population")
annotate!([(8,120,Plots.text("B",18))])

mapd_plt=scatter(dXs,catdobs, marker=:cross, color=:black, markersize=3, label="Dorsal CMZ population data", xlabel="Age (dpf)", xticks=X, ylabel="Population")
plot!(mapd_plt, X, d_mean, ribbon=(d_lower,d_upper), color=:green, label="Separate model D population")
annotate!([(8,125,Plots.text("C",18))])

mapv_plt=scatter(vXs,catvobs, marker=:cross, color=:black, markersize=3, label="Ventral CMZ population data", xlabel="Age (dpf)", ylabel="Population", xticks=X)
plot!(mapv_plt, X, v_mean, ribbon=(v_lower,v_upper), color=:darkmagenta, label="Separate model V population")
annotate!([(8,140,Plots.text("D",18))])

combined_map=Plots.plot(mapmd_plt,mapmv_plt,mapd_plt,mapv_plt,layout=grid(2,2), size=(1200,900),  link=:x)

savefig(combined_map,"/bench/PhD/Thesis/images/cmz/a10dvdecayMAP.png")

dkdes=posterior_kde(ed)
vkdes=posterior_kde(ev)
mkdes=posterior_kde(em)

ymarg=map(x→pdf(priors[1],x),mkdes[1].x)

```

```
ctmarg=plot(mkdes[1].x, ymarg, color=:darkmagenta, fill=true, fillalpha=.5,xlims=[0,144], xlabel="Initial Cycle Time (hr)", ylabel="Probability density", label="Prior")
plot!(mkdes[1].x,mkdes[1].density,color=:green, fill=true, fillalpha=.5, label="Combined  Posterior")
plot!(dkdes[1].x,dkdes[1].density,color=:orange, fill=true, fillalpha=.25, label="Dorsal Posterior")
plot!(vkdes[1].x,vkdes[1].density,color=:blue, fill=true, fillalpha=.25, label="Ventral Posterior")
plot!([mapm.θ[1],mapm.θ[1]],[0,maximum(mkdes[1].density)],color=:darkgreen, label="Combined MAP")
plot!([mapd.θ[1],mapd.θ[1]],[0,maximum(dkdes[1].density)],color=:darkred, label="Dorsal MAP")
plot!([mapv.θ[1],mapv.θ[1]],[0,maximum(vkdes[1].density)],color=:darkblue, label="Ventral MAP")
annotate!([(0,.06,Plots.text("A",18))])

ymarg=map(x→pdf(priors[2],x),mkdes[2].x)
kmarg=plot(mkdes[2].x, ymarg, color=:darkmagenta, fill=true, fillalpha=.5, xlims=[0,.03], xlabel="Decay constant κ", ylabel="Probability density", label="Prior")
plot!(mkdes[2].x,mkdes[2].density,color=:green, fill=true, fillalpha=.5, label="Combined  Posterior")
plot!(dkdes[2].x,dkdes[2].density,color=:orange, fill=true, fillalpha=.25, label="Dorsal Posterior")
plot!(vkdes[2].x,vkdes[2].density,color=:blue, fill=true, fillalpha=.25, label="Ventral Posterior")
plot!([mapm.θ[2],mapm.θ[2]],[0,maximum(mkdes[2].density)],color=:darkgreen, label="Combined MAP")
plot!([mapd.θ[2],mapd.θ[2]],[0,maximum(dkdes[2].density)],color=:darkred, label="Dorsal MAP")
plot!([mapv.θ[2],mapv.θ[2]],[0,maximum(vkdes[2].density)],color=:darkblue, label="Ventral MAP")
annotate!([(0,225,Plots.text("B",18))])

ymarg=map(x→pdf(priors[3],x),mkdes[3].x)
ermarg=plot(mkdes[3].x, ymarg, color=:darkmagenta, fill=true, fillalpha=.5, xlims=[0,4], xlabel="Niche exit rate ϵ", ylabel="Probability density", label="Prior")
plot!(mkdes[3].x,mkdes[3].density,color=:green, fill=true, fillalpha=.5, label="Combined  Posterior")
plot!(dkdes[3].x,dkdes[3].density,color=:orange, fill=true, fillalpha=.25, label="Dorsal Posterior")
```

```
plot!(vkdes[3].x,vkdes[3].density,color=:blue, fill=true, fillalpha=.25, label="Ventral Posterior")
plot!([mapm.θ[3],mapm.θ[3]],[0,maximum(mkdes[3].density)],color=:darkgreen, label="Combined MAP")
plot!([mapd.θ[3],mapd.θ[3]],[0,maximum(dkdes[3].density)],color=:darkred, label="Dorsal MAP")
plot!([mapv.θ[3],mapv.θ[3]],[0,maximum(vkdes[3].density)],color=:darkblue, label="Ventral MAP")
annotate!([(0,1.2,Plots.text("C",18))])


ph1c=plot(ctmarg,kmarg,ermarg, layout=grid(3,1),size=(1000,1200))
savefig(ph1c,"/bench/PhD/Thesis/images/cmz/a10decaymarg.png")
```

17.8.4 /cmz/a10dvratio.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions, GMC_NS, Serialization, Plots, NGRefTools
a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["Dorsal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ventral CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ratio"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    rvec,dvec,vvec = [zeros(0) for i in 1:3]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Dorsal CMZ (#)")...])>0 && mean(skipmissing(i_df."Dorsal CMZ (#)")) >0. && push!(dvec,mean(skipmissing(i_df."Dorsal CMZ (#)")))
```

```
        length([skipmissing(i_df."Ventral CMZ (#)")...])>0 && mean(skipmissing(i_df."Ventral CMZ (#)")) >0. && push!(vvec,mean(skipmissing(i_df."Ventral CMZ (#)")))
        length([skipmissing(i_df."Dorsal CMZ (#)")...])>0 && mean(skipmissing(i_df."Dorsal CMZ (#)")) >0. && length([skipmissing(i_df."Ventral CMZ (#)")...])>0 && mean(skipmissing(i_df."Ventral CMZ (#)")) >0. && push!(rvec,mean(skipmissing(i_df."Dorsal CMZ (#)"))/mean(skipmissing(i_df."Ventral CMZ (#)")))
    end

    push!(measure_dict["Ratio"],rvec)
    push!(measure_dict["Dorsal CMZ (#)"],dvec)
    push!(measure_dict["Ventral CMZ (#)"],vvec)
end

d_logn_mts=[fit(MarginalTDist,log.(measure_dict["Dorsal CMZ (#)"][n])) for n in 1:length(X)]
d_mean=[exp(mean(mt)) for mt in d_logn_mts]
d_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in d_logn_mts]
d_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in d_logn_mts]

v_logn_mts=[fit(MarginalTDist,log.(measure_dict["Ventral CMZ (#)"][n])) for n in 1:length(X)]
v_mean=[exp(mean(mt)) for mt in v_logn_mts]
v_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in v_logn_mts]
v_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in v_logn_mts]

r_n_mts=[fit(MarginalTDist,measure_dict["Ratio"][n]) for n in 1:length(X)]
r_mean=[mean(mt) for mt in r_n_mts]
r_lower=[mean(mt)-quantile(mt,.025) for mt in r_n_mts]
r_upper=[quantile(mt,.975)-mean(mt) for mt in r_n_mts]

plotticks=[30*i for i in 1:12]

dv_chart=Plots.plot(X, d_mean, ribbon=(d_lower,d_upper), color=:green, label="Dorsal mean", ylabel="CMZ sectional population size", showaxis=:y, xticks=plotticks, xformatter=_->"")
plot!(X, v_mean, ribbon=(v_lower,v_upper), color=:darkmagenta, label="Ventral mean")
```

```
scatter!(vcat([[X[n] for i in 1:length(measure_dict["Dorsal CMZ (#)"][n])] for n in 1:length(X)] ... ),vcat(measure_dict["Dorsal CMZ (#)"] ... ), marker=:utriangle, color=:green, markersize=3, markerstrokecolor=:green, label="Dorsal data")
scatter!(vcat([[X[n] for i in 1:length(measure_dict["Ventral CMZ (#)"][n])] for n in 1:length(X)] ... ),vcat(measure_dict["Ventral CMZ (#)"] ... ), marker=:dtriangle, color=:darkmagenta, markersize=3, markerstrokecolor=:darkmagenta, label="Ventral data")
annotate!([(1.5,125,Plots.text("A",18))])
lens!([2, 31], [10, 150], inset = (1, bbox(0.3, 0.1, 0.45, 0.6)))

ratio_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["Ratio"][n])] for n in 1:length(X)] ... ),vcat(measure_dict["Ratio"] ... ), marker=:cross, color=:black, markersize=3, markerstrokecolor=:black, label="Ratio data", ylabel="Individual asymmetry ratio", xlabel="Age (dpf)", xticks=plotticks)
plot!(X, r_mean, ribbon=(r_lower,r_upper), color=:green, label="Ratio mean")
plot!(X, [1. for i in 1:length(X)], style=:dot, color=:black, label="Even ratio")
annotate!([(1.5,.5,Plots.text("B",18))])


l=@layout [a{.6h}
           b]

combined=Plots.plot(dv_chart,ratio_chart,layout=l, link=:x, size=(900,600))

savefig(combined, "/bench/PhD/Thesis/images/cmz/DVontology.png")
```

17.8.5 /cmz/a10dvslice.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions,CMZNicheSims,GMC_NS, Serialization, Plots
a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

edpath="/bench/PhD/NGS_binaries/CNS/A10/ed"
evpath="/bench/PhD/NGS_binaries/CNS/A10/ev"
emultipath="/bench/PhD/NGS_binaries/CNS/A10/emulti"
```

```
paths=[edpath,evpath,emultipath]

default(legendfont = (10), guidefont = (12), tickfont = (10))

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
 ↪  eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["Dorsal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ventral CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Lens circumference"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    if !(Float64(unique(t_df."Time point (d)")[1]) in [180., 360.])

        push!(X,Float64(unique(t_df."Time point (d)")[1]))
        dvec,vvec,lcirc = [zeros(0) for i in 1:4]

        for i_df in groupby(t_df,"BSR")
            length([skipmissing(i_df."Lens diameter")...])>0 &&
             ↪  push!(lcirc,mean(skipmissing(i_df."Lens diameter"))*π)
        end

        for i_df in groupby(t_df,"BSR")

            length([skipmissing(i_df."Dorsal CMZ (#)")...])>0 &&
             ↪  mean(skipmissing(i_df."Dorsal CMZ (#)")) >0. &&
             ↪  push!(dvec,mean(skipmissing(i_df."Dorsal CMZ (#)")))
            length([skipmissing(i_df."Ventral CMZ (#)")...])>0 &&
             ↪  mean(skipmissing(i_df."Ventral CMZ (#)")) >0. &&
             ↪  push!(vvec,mean(skipmissing(i_df."Ventral CMZ (#)")))
        end

        push!(measure_dict["Dorsal CMZ (#)"],dvec)
        push!(measure_dict["Ventral CMZ (#)"],vvec)
        push!(measure_dict["Lens circumference"],lcirc)
    end
end

```

```
obsset=[measure_dict["Dorsal CMZ (#)"],measure_dict["Ventral CMZ (#)"],[measure_dict["Dorsal CMZ (#)"],measure_dict["Ventral CMZ (#)"]]]

dpopdist=fit(LogNormal,measure_dict["Dorsal CMZ (#)"][1])
vpopdist=fit(LogNormal,measure_dict["Ventral CMZ (#)"][1])

logLC=vcat([log.(measure_dict["Lens circumference"][n]) for n in 1:length(X)]...)
logX=vcat([[log.(X[n]) for ms in measure_dict["Lens circumference"][n]] for n in 1:length(X)]...)
logX=hcat(ones(length(logX)),logX)

lm=BayesianLinearRegression.fit!(BayesianLinReg(logX,logLC))
w1,w2=posteriorWeights(lm)
factor=10^w1.val
power=w2.val

lm=Lens_Model(factor,power,14.)

mc_its=Int64(1e6)
base_scale=[144.,5.]
base_min=[10.,0.]
time_scale=90
time_min=4.

cycle_prior=[LogNormal(log(20),log(2)),LogNormal(log(.9),log(1.6))]
end_prior=[Uniform(4,90)]

function compose_priors(phases)
    prs=Vector{Vector{Distribution}}()
    bxes=Vector{Matrix{Float64}}()
    for p in phases
        pr=[repeat(cycle_prior,p)...,repeat(end_prior,p-1)...]
        push!(prs,pr)
        bx=hcat([repeat(base_min,p)...,fill(time_min,p-1)...],
        [repeat(base_scale,p)...,fill(time_scale,p-1)...])
        push!(bxes,GMC_NS.to_unit_ball.(bx,pr))
    end
    return prs,bxes
end

prior,box=compose_priors(2)
```

```

d_constants=[X,dpopdist,lm,mc_its,2]
v_constants=[X,vpopdist,lm,mc_its,2]
multi_constants=[X,[dpopdist,vpopdist],lm,mc_its,2]
constants=[d_constants,v_constants,multi_constants]

uds=Vector{Vector{Function}}([[],[liwi_display],[convergence_display]])
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display],[ensemble_displa

gmc_settings=GMC_DEFAULTS
gmc_settings[end]=100

for (pth,constants,obs) in zip(paths,constants,obsset)
    if isfile(pth*"/ens")
        e=deserialize(pth*"/ens")
    else
        if !occursin("emulti",pth)
            e=Slice_Ensemble(pth,1000,obs, prior ... , constants, box ... ,
            ↪ gmc_settings)
        else
            e=MultiSlice_Ensemble(pth,1000,obs, prior ... , constants,
            ↪ box ... , gmc_settings)
        end
    end

    converge_ensemble!(e,backup=(true,20),upper_displays=uds,
    ↪ lower_displays=lds, disp_rot_its=50, mc_noise=.3,
    ↪ converge_factor=1e-6)
end

ed=deserialize(edpath*"/ens")
ev=deserialize(evpath*"/ens")
em=deserialize(emultipath*"/ens")

edev=measure_evidence(ed)
evev=measure_evidence(ev)
emev=measure_evidence(em)

evidence_ratio=emev-(edev+evev)

println("\\begin{tabular}{|l|l|l|l|l|}")
println("\\hline")
```

```
121 println("{\\bf Combined D/V model logZ} & {\\bf Separate D/V model logZ}
    ↪ & {\\bf logZR} & {\\bf \$\\sigma\$ Significance}\\\\ \\hline")
122 println("\\textbf{$(round(emev,digits=3))} & $(round(edev+evev,digits=3))
    ↪ & $(round(evidence_ratio,digits=3)) &
    ↪ $(round(evidence_ratio.val/evidence_ratio.err,digits=3))\\\\
    ↪ \\hline")
123 println("\\end{tabular}")
124
125 mapd=deserialize(ed.models[findmax([m.log_Li for m in
    ↪ ed.models])[2]].path)
126 mapv=deserialize(ev.models[findmax([m.log_Li for m in
    ↪ ev.models])[2]].path)
127 mapm=deserialize(em.models[findmax([m.log_Li for m in
    ↪ em.models])[2]].path)
128
129 println("\\begin{tabular}{|l|l|l|l|}")
130 println("\\hline")
131 println("{\\bf Parameter} & {\\bf Total MAP} & {\\bf Dorsal MAP} & {\\bf
    ↪ Ventral MAP}\\ \\hline")
132 println("Phase 1 \$CT\$ (h) & $(round(mapm.θ[1], digits=1)) &
    ↪ $(round(mapd.θ[1], digits=1)) & $(round(mapv.θ[1], digits=1))\\\\
    ↪ \\hline")
133 println("Phase 1 \$\\epsilon\$ & $(round(mapm.θ[2], digits=2)) &
    ↪ $(round(mapd.θ[2], digits=2)) & $(round(mapv.θ[2], digits=2))\\\\
    ↪ \\hline")
134 println("Phase 2 \$CT\$ (h) & $(round(mapm.θ[3], digits=1)) &
    ↪ $(round(mapd.θ[3], digits=1)) & $(round(mapv.θ[3], digits=1))\\\\
    ↪ \\hline")
135 println("Phase 2 \$\\epsilon\$ & $(round(mapm.θ[4], digits=2)) &
    ↪ $(round(mapd.θ[4], digits=2)) & $(round(mapv.θ[4], digits=2))\\\\
    ↪ \\hline")
136 println("Transition age & $(round(mapm.θ[5], digits=1)) &
    ↪ $(round(mapd.θ[5], digits=1)) & $(round(mapv.θ[5], digits=1))\\\\
    ↪ \\hline")
137 println("\\end{tabular}")
138
139 X=ed.constants[1]
140
141 plotticks=[10*i for i in 1:9]
142
143 catdobs=vcat([ed.obs[t] for t in 1:length(X)]...)
144 catvobs=vcat([ev.obs[t] for t in 1:length(X)]...)
145
```

```
dXs=vcat([[X[n] for i in 1:length(ed.obs[n])] for n in 1:length(X)]...)
vXs=vcat([[X[n] for i in 1:length(ev.obs[n])] for n in 1:length(X)]...)
dvXs=vcat([[X[n] for i in 1:length(em.obs[1][n])] for n in 1:length(X)]...)

d_mean=mapd.disp_mat[:,2]
d_upper=mapd.disp_mat[:,3]•-mapd.disp_mat[:,2]
d_lower=mapd.disp_mat[:,2]•-mapd.disp_mat[:,1]

v_mean=mapv.disp_mat[:,2]
v_upper=mapv.disp_mat[:,3]•-mapv.disp_mat[:,2]
v_lower=mapv.disp_mat[:,2]•-mapv.disp_mat[:,1]

md_mean=mapm.slices[1].disp_mat[:,2]
md_upper=mapm.slices[1].disp_mat[:,3]•-mapm.slices[1].disp_mat[:,2]
md_lower=mapm.slices[1].disp_mat[:,2]•-mapm.slices[1].disp_mat[:,1]

mv_mean=mapm.slices[2].disp_mat[:,2]
mv_upper=mapm.slices[2].disp_mat[:,3]•-mapm.slices[2].disp_mat[:,2]
mv_lower=mapm.slices[2].disp_mat[:,2]•-mapm.slices[2].disp_mat[:,1]

mapmd_plt=scatter(dXs,catdobs, marker=:cross, color=:black, markersize=3, label="Dorsal CMZ population data", showaxis=:y, xformatter=_→"", xticks=X, ylabel="Population")
plot!(mapmd_plt, X, md_mean, ribbon=(md_lower,md_upper), color=:green, label="Combined model D population")
annotate!([(8,200,Plots.text("A",18))])

mapmv_plt=scatter(vXs,catvobs, marker=:cross, color=:black, markersize=3, label="Ventral CMZ population data", showaxis=:y, ylabel="Population", xformatter=_→"",xticks=X)
plot!(mapmv_plt, X, mv_mean, ribbon=(mv_lower,mv_upper), color=:darkmagenta, label="Combined model V population")
annotate!([(8,145,Plots.text("B",18))])

mapd_plt=scatter(dXs,catdobs, marker=:cross, color=:black, markersize=3, label="Dorsal CMZ population data", xlabel="Age (dpf)", xticks=X, ylabel="Population")
plot!(mapd_plt, X, d_mean, ribbon=(d_lower,d_upper), color=:green, label="Separate model D population")
annotate!([(8,150,Plots.text("C",18))])

```

```
mapv_plt=scatter(vXs,catvobs, marker=:cross, color=:black, markersize=3, label="Ventral CMZ population data", xlabel="Age (dpf)", ylabel="Population", xticks=X)
plot!(mapv_plt, X, v_mean, ribbon=(v_lower,v_upper), color=:darkmagenta, label="Separate model V population")
annotate!([(8,250,Plots.text("D",18))])

combined_map=Plots.plot(mapmd_plt,mapmv_plt,mapd_plt,mapv_plt,layout=grid(2,2), size=(1200,900),  link=:x)

savefig(combined_map,"/bench/PhD/Thesis/images/cmz/a10dvMAP.png")

###KDE FIG
kdes=posterior_kde(em,bivar=[1⇒2,3⇒4])

balx=10.:1.:144.
baly=[2^(24/ct)-1 for ct in balx]

ph1marg=contour(kdes[6].x,kdes[6].y,transpose(kdes[6].density), c=:viridis, xlabel="Phase 1 CT (hr)", ylabel="Phase 1 ϵ rate", colorbar=:false, xlims=[0,144], ylims=[0,5], levels=200, clim=(0,.5), legend=(.8,.3), xticks=24.:24.:144.)
plot!(balx,baly,color=:black,linestyle=:dash,label="Balanced pop.")
scatter!([mapm.θ[1]],[mapm.θ[2]], marker=:+, markersize=15, markerstrokestyle=:bold, color=:magenta, label="MAP")
for i in 1:5
    scatter!([mapm.θ[1]],[mapm.θ[2]], marker=:+, markersize=15, markerstrokestyle=:bold, color=:magenta, label=:none, framestyle=:origin)
end
lens!([15, 45], [.35, 1.6], inset = (1, bbox(0.2, 0.05, 0.75, 0.55)), colorbar=:false, framestyle=:box, subplot=2, xformatter=_→"", yformatter=_→"")

annotate!([(4,4.5,Plots.text("A",18))])

ymarg=map(x→pdf(cycle_prior[1],x),kdes[1].x)
ph1ctmarg=plot(kdes[1].x, ymarg, color=:darkmagenta, fill=true,
fillalpha=.5,xlims=[0,144], ylabel="p",xaxis=false, xformatter=_→"", legend=:none)
plot!(kdes[1].x,kdes[1].density,color=:green, fill=true, fillalpha=.5)
plot!([mapm.θ[1],mapm.θ[1]],[0,maximum(kdes[1].density)],color=:black,linewidth=1)
yflip!(ph1ctmarg)
```

```
ymarg=map(x→pdf(cycle_prior[2],x),kdes[2].x)
ph1ermarg=plot(ymarg,kdes[2].x, color=:darkmagenta, fill=true, fillalpha=.5, ylims=[0,5], legend=:none, yaxis=false, yformatter=_→"", xlabel="p")
plot!(kdes[2].density,kdes[2].x,color=:green, fill=true, fillalpha=.5)
plot!([0,maximum(kdes[2].density)],[mapm.θ[2],mapm.θ[2]],color=:black,linewidth=1)
xflip!(ph1ermarg)

marglayout=@layout [ymarg{.1w} bivar{.8h}; empty xmarg]

cbar=heatmap(transpose(ones(101,1).*(0:0.005:.5)), c=:viridis, legend=:none, yticks=:none, xticks=(1:20:101, string.(0:0.1:.5)),xlabel="Probability density")

empty=plot([],fill=true,xaxis=false,yaxis=false,grid=false, color=:darkmagenta, xformatter=_→"", yformatter=_→"", label="Prior")
plot!([],color=:green,fill=true,label="Posterior")
plot!([], color=:black, linewidth=2, label="MAP")
ph1c=plot(ph1ermarg,ph1marg,empty,ph1ctmarg,layout=marglayout, link=:both)

ph2marg=contour(kdes[7].x,kdes[7].y,transpose(kdes[7].density), c=:viridis, xlabel="Phase 2 CT (hr)", ylabel="Phase 2 ϵ rate", colorbar=:false, xlims=[0,144], ylims=[0,5], levels=200, clim=(0,.5), legend=(.8,.3), xticks=24.:24.:144.)
plot!(balx,baly,color=:black,linestyle=:dash,label="Balanced pop.")
scatter!([mapm.θ[3]],[mapm.θ[4]], marker=:+, markersize=15, color=:magenta, label="MAP")
for i in 1:5
    scatter!([mapm.θ[3]],[mapm.θ[4]], marker=:+, markersize=15, color=:magenta, label=:none, framestyle=:origin)
end
lens!([15, 45], [.35, 1.6], inset = (1, bbox(0.2, 0.05, 0.75, 0.55)), colorbar=:false, framestyle=:box, subplot=2, xformatter=_→"", yformatter=_→"")

annotate!(ph2marg,[(4,4.5,Plots.text("B",18))])

ymarg=map(x→pdf(cycle_prior[1],x),kdes[1].x)
ph2ctmarg=plot(kdes[1].x, ymarg, color=:darkmagenta, fill=true,
```

```
fillalpha=.5, ylabel="p", xlims=[0,144], xaxis=false, xformatter=_->"", legend=:none)
plot!(kdes[3].x,kdes[3].density,color=:green, fill=true, fillalpha=.5)
plot!([mapm.θ[3],mapm.θ[3]],[0,maximum(kdes[3].density)],color=:black,linewidth=1)
yflip!(ph2ctmarg)

ymarg=map(x->pdf(cycle_prior[2],x),kdes[4].x)
ph2ermarg=plot(ymarg,kdes[4].x, color=:darkmagenta, fill=true, fillalpha=.5, ylims=[0,5], legend=:none, yaxis=false, yformatter=_->"", xlabel="p")
plot!(kdes[4].density,kdes[4].x,color=:green, fill=true, fillalpha=.5)
plot!([0,maximum(kdes[4].density)],[mapm.θ[4],mapm.θ[4]],color=:black,linewidth=1)
xflip!(ph2ermarg)

marglayout2=@layout [ymarg{.1w} bivar{.8h}; empty xmarg]

ph2c=plot(ph2ermarg,ph2marg,deepcopy(empty),ph2ctmarg,layout=marglayout2, link=:both)

pend=plot(end_prior[1], color=:darkmagenta, fill=true, fillalpha=.5, xlims=[0,90], ylabel="Density", xlabel="Phase transition age", xticks=X,  label="Prior", legend=:none)
plot!(kdes[5].x, kdes[5].density, color=:green, fill=true, fillalpha=.5, label="Posterior")
plot!([mapm.θ[5],mapm.θ[5]],[0,maximum(kdes[5].density)],color=:black, label="MAP")
annotate!([(1,.015,Plots.text("C",18))])

combined_layout=@layout[ph1{.45h}
                        cbar{.01h}
                        ph2{.45h}
                        pend{.09h}]

combined_marg=plot(ph1c,cbar,ph2c,pend,layout=combined_layout,size=(1000,1200))

savefig(combined_marg,"/bench/PhD/Thesis/images/cmz/a10dvmarginals.png")
```

17.8.6 /cmz/a10morphology.jl

```
using CSV,DataFrames,Distributions,NGRefTools,StatsBase,Plots
gr()
default(legendfont = (10), guidefont = (11), tickfont = (10))
```

```
a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["Retina thickness"]=Vector{Vector{Float64}}()
measure_dict["RPE length"]=Vector{Vector{Float64}}()
measure_dict["RPE thickness"]=Vector{Vector{Float64}}()
measure_dict["Lens diameter"]=Vector{Vector{Float64}}()
measure_dict["ON diameter"]=Vector{Vector{Float64}}()
measure_dict["ONL"]=Vector{Vector{Float64}}()
measure_dict["OPL"]=Vector{Vector{Float64}}()
measure_dict["INL"]=Vector{Vector{Float64}}()
measure_dict["IPL"]=Vector{Vector{Float64}}()
measure_dict["GCL"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    rtvec, rplvec, rptvec, ldvec, ondvec, onlvec, oplvec, inlvec, iplvec, gclvec = [zeros(0) for i in 1:10]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Lens diameter") ... ])>0 && push!(ldvec,mean(skipmissing(i_df."Lens diameter")))

        length([skipmissing(i_df."Retina thickness") ... ])>0 && push!(rtvec,mean(skipmissing(i_df."Retina thickness")))
        length([skipmissing(i_df."RPE length") ... ])>0 && push!(rplvec,mean(skipmissing(i_df."RPE length")))
        length([skipmissing(i_df."RPE thickness") ... ])>0 && push!(rptvec,mean(skipmissing(i_df."RPE thickness")))
        length([skipmissing(i_df."ON diameter") ... ])>0 && push!(ondvec,mean(skipmissing(i_df."ON diameter")))
        length([skipmissing(i_df.ONL) ... ])>0 && push!(onlvec,mean(skipmissing(i_df.ONL)))
        length([skipmissing(i_df.OPL) ... ])>0 && push!(oplvec,mean(skipmissing(i_df.OPL)))
```

```
        length([skipmissing(i_df.INL)...])>0 && push!(inlvec,mean(skipmissing(i_df.INL)))
        length([skipmissing(i_df.IPL)...])>0 && push!(iplvec,mean(skipmissing(i_df.IPL)))
        length([skipmissing(i_df.GCL)...])>0 && push!(gclvec,mean(skipmissing(i_df.GCL)))
    end

    push!(measure_dict["Retina thickness"],rtvec)
    push!(measure_dict["RPE length"],rplvec)
    push!(measure_dict["RPE thickness"],rptvec)
    push!(measure_dict["Lens diameter"],ldvec)
    push!(measure_dict["ON diameter"],ondvec)
    push!(measure_dict["ONL"],onlvec)
    push!(measure_dict["OPL"],oplvec)
    push!(measure_dict["INL"],inlvec)
    push!(measure_dict["IPL"],iplvec)
    push!(measure_dict["GCL"],gclvec)
end

#TEST IF DATA IS BETTER MODELLED NORMALLY OR LOGNORMALLY
for ms in ["Retina thickness", "RPE length", "RPE thickness", "Lens diameter", "ON diameter", "ONL", "OPL", "INL", "IPL", "GCL"]
    ms=="ON diameter" ? (idxs=length(X)-1) : (idxs=length(X))

    normal_mts=[fit(Normal,measure_dict[ms][n]) for n in 1:idxs]
    logn_mts=[fit(LogNormal,filter!(i->!iszero(i),measure_dict[ms][n])) for n in 1:idxs]

    normal_lh=sum([sum(logpdf(normal_mts[n],measure_dict[ms][n])) for n in 1:idxs])
    logn_lh=sum([sum(logpdf(logn_mts[n],measure_dict[ms][n])) for n in 1:idxs])

    println("$ms & $(round(normal_lh,digits=3)) & $(round(logn_lh,digits=3)) & $(round(logn_lh-normal_lh,digits=3))\\\\ \\hline")
end

plotticks=[30*i for i in 1:12]

rt_n_mts=[fit(MarginalTDist,measure_dict["Retina thickness"][n]) for n in 1:length(X)]
```

```
rtn_mean=[mean(mt) for mt in rt_n_mts]
rtn_lower=[mean(mt)-quantile(mt,.025) for mt in rt_n_mts]
rtn_upper=[quantile(mt,.975)-mean(mt) for mt in rt_n_mts]

rtn_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["Retina thickness"][n])] for n in 1:length(X)]...),vcat(measure_dict["Retina thickness"]...), marker=:cross, color=:black, markersize=3, label="Retinal thic. data", ylabel="Retinal thickness (μm)", showaxis=:y, xticks=plotticks, xformatter=_→"", legend=:bottomright)
plot!(X, rtn_mean, ribbon=(rtn_lower,rtn_upper), color=:darkmagenta, label="Mean thickness")
annotate!([(8,200,Plots.text("A",18))])

rpl_n_mts=[fit(MarginalTDist,measure_dict["RPE length"][n]) for n in 1:length(X)]
rpln_mean=[mean(mt) for mt in rpl_n_mts]
rpln_lower=[mean(mt)-quantile(mt,.025) for mt in rpl_n_mts]
rpln_upper=[quantile(mt,.975)-mean(mt) for mt in rpl_n_mts]

rpln_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["RPE length"][n])] for n in 1:length(X)]...),vcat(measure_dict["RPE length"]...), marker=:cross, color=:black, markersize=3, label="RPE length data", ylabel="RPE length (μm)", showaxis=:y, xticks=plotticks, xformatter=_→"", legend=:bottomright)
plot!(X, rpln_mean, ribbon=(rpln_lower,rpln_upper), color=:grey, label="Mean thickness")
annotate!([(8,3000,Plots.text("B",18))])

ld_n_mts=[fit(MarginalTDist,measure_dict["Lens diameter"][n]) for n in 1:length(X)]
ldn_mean=[mean(mt) for mt in ld_n_mts]
ldn_mean[end]=ld_n_mts[end].μ
ldn_mean[end-2]=ld_n_mts[end-2].μ
ldn_lower=[mean(mt)-quantile(mt,.025) for mt in ld_n_mts]
ldn_lower[end]=ld_n_mts[end].μ - (ld_n_mts[end].μ-3*ld_n_mts[end].σ)
ldn_lower[end-2]=ld_n_mts[end-2].μ - (ld_n_mts[end-2].μ-3*ld_n_mts[end-2].σ)
ldn_upper=[quantile(mt,.975)-mean(mt) for mt in ld_n_mts]
ldn_upper[end]=(ld_n_mts[end].μ+3*ld_n_mts[end].σ) - ld_n_mts[end].μ
ldn_upper[end-2]=(ld_n_mts[end-2].μ+3*ld_n_mts[end-2].σ) - ld_n_mts[end-2].μ

```

```
ldn_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["Lens diameter"][n])] for n in 1:length(X)]...),vcat(measure_dict["Lens diameter"]...), marker=:cross, color=:black, markersize=3, label="Lens dia. data", ylabel="Lens diameter (μm)", showaxis=:y, xticks=plotticks, xformatter=_→"", legend=:topleft)
plot!(X, ldn_mean, ribbon=(ldn_lower,ldn_upper), color=:darkgreen, label="Diameter mean")
annotate!([(8,600,Plots.text("C",18))])

on_n_mts=[fit(MarginalTDist,measure_dict["ON diameter"][n]) for n in 1:length(X)-1]
onn_mean=[mean(mt) for mt in on_n_mts]
onn_mean[end]=on_n_mts[end].μ
onn_lower=[mean(mt)-quantile(mt,.025) for mt in on_n_mts]
onn_lower[end]=on_n_mts[end].μ - (on_n_mts[end].μ-3*on_n_mts[end].σ)
onn_upper=[quantile(mt,.975)-mean(mt) for mt in on_n_mts]
onn_upper[end]=(on_n_mts[end].μ+3*on_n_mts[end].σ) - on_n_mts[end].μ

onn_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["ON diameter"][n])] for n in 1:length(X)-1]...),vcat(measure_dict["ON diameter"]...), marker=:cross, color=:black, markersize=3, label="ON dia. data", ylabel="Optic nerve diameter (μm)", showaxis=:y, xticks=plotticks, xformatter=_→"", legend=:topleft, ylims=[0,200],xlims=[0,360])
plot!(X[1:end-1], onn_mean, ribbon=(onn_lower,onn_upper), color=:darkblue, label="Diameter mean")
annotate!([(8,105,Plots.text("D",18))])
lens!([2, 31], [0, 30], inset = (1, bbox(0.55, 0.1, .45, .8)))

stackdata=hcat(mean.(measure_dict["RPE thickness"]),mean.(measure_dict["ONL"]),mean.(measure_dict["OPL"]),mean.(measure_

layers=areaplot(X, stackdata, xticks=plotticks, legend=:none, xlabel="Age (dpf)", ylabel="Layer thickness (μm)")
plot!([100,100],[220,240], color=:black)
annotate!([(100,250,Plots.text("GCL",12))])
plot!([130,130],[195,240], color=:black)
annotate!([(130,250,Plots.text("IPL",12))])
plot!([160,160],[130,240], color=:black)
annotate!([(160,250,Plots.text("INL",12))])
plot!([190,190],[100,240], color=:black)
annotate!([(190,250,Plots.text("OPL",12))])
plot!([220,220],[75,240], color=:black)
```

```
annotate!([(220,250,Plots.text("ONL",12))])
plot!([250,250],[25,240], color=:black)
annotate!([(250,250,Plots.text("RPE",12))])
annotate!([(8.,175,Plots.text("E",18))])

combined=plot(rtn_chart, rpln_chart, ldn_chart, onn_chart, layers,
↪ layout=grid(5,1), size=(1000,1200), link=:x)
savefig(combined, "/bench/PhD/Thesis/images/cmz/morphology.png")
```

17.8.7 /cmz/a10nvln.jl

```
using
↪ CSV,DataFrames,Distributions,NGRefTools,StatsBase,Plots,GMC_NS,Serialization,
↪ Measurements

a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"
pne_pth="/bench/PhD/NGS_binaries/GMC_NS/A10/pop_norm"
plne_pth="/bench/PhD/NGS_binaries/GMC_NS/A10/pop_lognorm"
vne_pth="/bench/PhD/NGS_binaries/GMC_NS/A10/vol_norm"
vlne_pth="/bench/PhD/NGS_binaries/GMC_NS/A10/vol_lognorm"

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
↪ eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["PopEst"]=Vector{Vector{Float64}}()
measure_dict["VolEst"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    pevec,ldvec,spvec = [zeros(0) for i in 1:3]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Lens diameter")...])>0 &&
        ↪ push!(ldvec,mean(skipmissing(i_df."Lens diameter")))
    end

    for i_df in groupby(t_df,"BSR")
```

```
        if length([skipmissing(i_df."CMZ Sum")...])>0 && mean(skipmissing(i_df."CMZ Sum")) > 0
            if length([skipmissing(i_df."Lens diameter")...])>0
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(skipmissing(i_df."Lens diameter"))/14.)
            else
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(ldvec)/14.)
            end
        end
        if length([skipmissing(i_df."Retina thickness")...])>0 && length([skipmissing(i_df."IPL")...])>0 && length([skipmissing(i_df."OPL")...])>0 && length([skipmissing(i_df."RPE length")...])>0

            rthi=mean(skipmissing(i_df."Retina thickness"))-mean(skipmissing(i_df."OPL"))-mean(skipmissing(i_df."IPL
            rpel=mean(skipmissing(i_df."RPE length"))
            if length([skipmissing(i_df."Lens diameter")...])>0
                rcirc=rpel+mean(skipmissing(i_df."Lens diameter"))
            else
                rcirc=rpel+mean(ldvec)
            end

            or=rcirc/2π
            ir=or-.5*rthi

            ov=(4/3)*π*(or^3)
            iv=(4/3)*π*(ir^3)

            push!(spvec,(ov-iv)*(4/5))
        end
    end

    push!(measure_dict["PopEst"],pevec)
    push!(measure_dict["VolEst"],spvec)
end

gmc=GMC_DEFAULTS
gmc[1]=5

uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]

```

```
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

evdict=Dict{String,Measurement}()

pmax_μ=2e5
pmin_μ=1
pmax_λ=1e-3
pmin_λ=1e-10
vmax_μ=1e10
vmin_μ=100
vmax_λ=1e-10
vmin_λ=1e-20

pn_priors=[Uniform(pmin_μ,pmax_μ),Uniform(pmin_λ,pmax_λ)]
pn_box=[pmin_μ pmax_μ;pmin_λ pmax_λ]
vn_priors=[Uniform(vmin_μ,vmax_μ),Uniform(vmin_λ,vmax_λ)]
vn_box=[vmin_μ vmax_μ;vmin_λ vmax_λ]

n_models=100

for (pth,prior,box,eststring) in zip([pne_pth, vne_pth],[pn_priors, vn_priors],[pn_box,vn_box],["PopEst","VolEst"])
    for (nx,x) in enumerate(X)
        enspth=pth*"/$x"
        if isfile(enspth*"/ens")
            e=deserialize(enspth*"/ens")
        else
            e=Normal_Ensemble(enspth,n_models,filter(i->!iszero(i), measure_dict[eststring][nx]), prior, box, gmc ... )
        end

        pth in keys(evdict) ? (evdict[pth] += converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)) : (evdict[pth] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6))
    end
end

ln_pmax_μ=log(pmax_μ^2/sqrt(pmax_μ^2 + inv(pmax_λ)))
ln_pmin_μ=log(pmin_μ^2/sqrt(pmin_μ^2 + inv(pmin_λ)))
```

```
ln_pmax_λ=1/log(1+(inv(pmax_λ)/pmax_μ^2))
ln_pmin_λ=1/log(1+(inv(pmin_λ)/pmin_μ^2))

ln_vmax_μ=log(vmax_μ^2/sqrt(vmax_μ^2 + inv(vmax_λ)))
ln_vmin_μ=log(vmin_μ^2/sqrt(vmin_μ^2 + inv(vmin_λ)))
ln_vmax_λ=1/log(1+(inv(vmax_λ)/vmax_μ^2))
ln_vmin_λ=1/log(1+(inv(vmin_λ)/vmin_μ^2))

pln_priors=[Uniform(ln_pmin_μ,ln_pmax_μ),Uniform(ln_pmin_λ,ln_pmax_λ)]
pln_box=[ln_pmin_μ ln_pmax_μ;ln_pmin_λ ln_pmax_λ]
vln_priors=[Uniform(ln_vmin_μ,ln_vmax_μ),Uniform(ln_vmin_λ,ln_vmax_λ)]
vln_box=[ln_vmin_μ ln_vmax_μ;ln_vmin_λ ln_vmax_λ]

for (pth,prior,box,eststring) in zip([plne_pth, vlne_pth],[pn_priors,
↪  vn_priors],[pn_box,vn_box],["PopEst","VolEst"])
    for (nx,x) in enumerate(X)
        enspth=pth*"/$x"
        if isfile(enspth*"/ens")
            e=deserialize(enspth*"/ens")
        else

            ↪  e=LogNormal_Ensemble(enspth,n_models,filter(i→i>0,measure_dict[estst
            ↪  prior, box, gmc ... )
        end

        pth in keys(evdict) ? (evdict[pth] +=
         ↪  converge_ensemble!(e,backup=(true,10000),upper_displays=uds,
         ↪  lower_displays=lds, disp_rot_its=10000,
         ↪  converge_factor=1e-6)) :  (evdict[pth] =
         ↪  converge_ensemble!(e,backup=(true,10000),upper_displays=uds,
         ↪  lower_displays=lds, disp_rot_its=10000,
         ↪  converge_factor=1e-6))
    end
end

pop_evidence_ratio=evdict[plne_pth]-evdict[pne_pth]
vol_evidence_ratio=evdict[vlne_pth]-evdict[vne_pth]

println("CMZ Population & $(round(evdict[pne_pth],digits=3)) &
↪  $(round(evdict[plne_pth],digits=3)) &
↪  $(round(pop_evidence_ratio,digits=3)) &
↪  $(pop_evidence_ratio.val/pop_evidence_ratio.err)\\\\ \\hline")
```

```
println("Estimated Retinal Volume & $(round(evdict[vne_pth],digits=3)) &
↪ $(round(evdict[vlne_pth],digits=3)) &
↪ $(round(vol_evidence_ratio,digits=3)) &
↪ $(vol_evidence_ratio.val/vol_evidence_ratio.err)\\\\ \\hline")
```

17.8.8 /cmz/a10periodisation.jl

```
using CSV,DataFrames,Distributions,StatsBase,GMC_NS,Serialization,
↪ CMZNicheSims, Random, Plots
gr()
default(legendfont = (10), guidefont = (12), tickfont = (10))

Random.seed!(786)

a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"
e2ph="/bench/PhD/NGS_binaries/CNS/A10/e2ph"
e3ph="/bench/PhD/NGS_binaries/CNS/A10/e3ph"

paths=[e2ph, e3ph]

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
↪ eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["PopEst"]=Vector{Vector{Float64}}()
measure_dict["VolEst"]=Vector{Vector{Float64}}()
measure_dict["SphEst"]=Vector{Vector{Float64}}()
measure_dict["CircEst"]=Vector{Vector{Float64}}()
measure_dict["ThiEst"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    pevec,vevec,tvec,ldvec,spvec,cvec,thivec = [zeros(0) for i in 1:7]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Lens diameter")...])>0 &&
        ↪ push!(ldvec,mean(skipmissing(i_df."Lens diameter")))
    end
```

```

    for i_df in groupby(t_df,"BSR")
        if length([skipmissing(i_df."CMZ Sum")...])>0 && mean(skipmissing(i_df."CMZ Sum")) > 0
            push!(tvec,mean(skipmissing(i_df."CMZ Sum")))
            if length([skipmissing(i_df."Lens diameter")...])>0
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(skipmissing(i_df."Lens diameter"))/14.)
            else
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(ldvec)/14.)
            end
        end
        if length([skipmissing(i_df."Retina thickness")...])>0 &&
         length([skipmissing(i_df."IPL")...])>0 &&
         length([skipmissing(i_df."OPL")...])>0 &&
         length([skipmissing(i_df."RPE length")...])>0

            rthi=mean(skipmissing(i_df."Retina thickness"))-mean(skipmissing(i_df."OPL"))-mean(skipmissing(i_df."IPL
            push!(thivec,rthi)
            rpel=mean(skipmissing(i_df."RPE length"))
            if length([skipmissing(i_df."Lens diameter")...])>0
                rcirc=rpel+mean(skipmissing(i_df."Lens diameter"))
            else
                rcirc=rpel+mean(ldvec)
            end

            push!(cvec,rcirc)

            or=rcirc/2π
            ir=or-rthi

            ov=(4/3)*π*(or^3)
            iv=(4/3)*π*(ir^3)

            push!(spvec,(ov-iv)*(4/5))
        end
    end

    push!(measure_dict["PopEst"],pevec)
    push!(measure_dict["SphEst"],spvec)
    push!(measure_dict["CircEst"],cvec)
```

```
    push!(measure_dict["ThiEst"],thivec)
end

obs=[(measure_dict["PopEst"][t],measure_dict["SphEst"][t]) for t in 1:length(X)]

nucpop3d=2075.6 #from a38 sib_measure_dict NucPop mean
or=mean(measure_dict["CircEst"][1])/2π
ir=or-.5*mean(measure_dict["ThiEst"][1])
sxvol_3d=(((π*or^2)-(π*ir^2))*14)*(4/5)
vol_const=sxvol_3d/nucpop3d
mc_its=Int64(1.e6)

popdist=fit(LogNormal,measure_dict["PopEst"][1])
voldist=fit(LogNormal,measure_dict["SphEst"][1])

ph2_constants=[X, popdist, voldist, vol_const, mc_its, 2]
ph3_constants=[X, popdist, voldist, vol_const, mc_its, 3]
constants=[ph2_constants, ph3_constants]

cycle_prior=[Uniform(10.,144.), LogNormal(log(.9),log(2))]
end_prior=[Uniform(4.,359.)]

phase_max=[144.,5.]
phase_min=[10.,0.]
time_max=359
time_min=4.

function compose_priors(phases)
    prs=Vector{Vector{Distribution}}()
    bxes=Vector{Matrix{Float64}}()
    for p in phases
        pr=[repeat(cycle_prior,p)...,repeat(end_prior,p-1)...]
        push!(prs,pr)
        bx=hcat([repeat(phase_min,p)...,fill(time_min,p-1)...],
        [repeat(phase_max,p)...,fill(time_max,p-1)...])
        push!(bxes,GMC_NS.to_unit_ball.(bx,pr))
    end
    return prs,bxes
end

phases=[2,3]
priors,boxes=compose_priors(phases)
```

```

uds=Vector{Vector{Function}}([[],[liwi_display],[convergence_display]])
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display],[ensemble_displa

gmc_settings=GMC_DEFAULTS
gmc_settings[end]=100

for (pth,prior,constants,box) in zip(paths,priors,constants,boxes)
    if isfile(pth*"/ens")
        e=deserialize(pth*"/ens")
    else
        e=CMZ_Ensemble(pth,1000,obs, prior, constants, box, gmc_settings)
    end

    converge_ensemble!(e,backup=(true,50),upper_displays=uds,
    ↪ lower_displays=lds, disp_rot_its=100, mc_noise=.3,
    ↪ converge_factor=1e-6)
end

e2=deserialize(e2ph*"/ens")
e3=deserialize(e3ph*"/ens")

e2ev=measure_evidence(e2)
e3ev=measure_evidence(e3)

evidence_ratio=e2ev-e3ev

println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf 2-phase logZ} & {\\bf 3-phase logZ} & {\\bf logZR} & {\\bf
↪ \$\\sigma\$ Significance}\\ \\hline")
println("\\textbf{$(round(e2ev,digits=3))} & $(round(e3ev,digits=3)) &
↪ $(round(evidence_ratio,digits=3)) &
↪ $(round(evidence_ratio.val/evidence_ratio.err,digits=3))\\\\
↪ \\hline")
println("\\end{tabular}")

map2=deserialize(e2.models[findmax([m.log_Li for m in
↪ e2.models])[2]].path)
map3=deserialize(e3.models[findmax([m.log_Li for m in
↪ e3.models])[2]].path)

println("\\begin{tabular}{|l|l|l|}")
```

```
println("\\hline")
println("{\\bf Parameter} & {\\bf 2-phase MAP} & {\\bf 3-phase MAP}\\ \\hline")
println("Phase 1 \$CT\$ (h) & $(round(map2.θ[1], digits=1)) & $(round(map3.θ[1], digits=1))\\\\ \\hline")
println("Phase 1 \$\\epsilon\$ & $(round(map2.θ[2], digits=2)) & $(round(map3.θ[2], digits=2))\\\\ \\hline")
println("Phase 2 \$CT\$ (h) & $(round(map2.θ[3], digits=1)) & $(round(map3.θ[3], digits=1))\\\\ \\hline")
println("Phase 2 \$\\epsilon\$ & $(round(map2.θ[4], digits=2)) & $(round(map3.θ[4], digits=2))\\\\ \\hline")
println("Phase 3 \$CT\$ (h) & NA & $(round(map3.θ[5], digits=1))\\\\ \\hline")
println("Phase 3 \$\\epsilon\$ & NA & $(round(map3.θ[6], digits=2))\\\\ \\hline")
println("Transition 1 age & $(round(map2.θ[5], digits=1)) & $(round(map3.θ[7], digits=1))\\\\ \\hline")
println("Transition 2 age & NA & $(round(map3.θ[8], digits=1))\\\\ \\hline")
println("\\end{tabular}")

X=e2.constants[1]
plotticks=[30*i for i in 1:12]

catpobs=vcat([e2.obs[t][1] for t in 1:length(X)]...)
catvobs=vcat([e2.obs[t][2] for t in 1:length(X)]...)
Xs=vcat([[X[n] for i in 1:length(e2.obs[n][1])] for n in 1:length(X)]...)

p2_mean=map2.disp_mat[:,2,1]
p2_upper=map2.disp_mat[:,1,1]-map2.disp_mat[:,2,1]
p2_lower=map2.disp_mat[:,2,1]-map2.disp_mat[:,3,1]

map2_popplt=scatter(Xs,catpobs, marker=:cross, color=:black, markersize=3, label="CMZ population estimate", ylabel="Population", showaxis=:y, xticks=plotticks, xformatter=_->"", legend=:topright, background_color_legend=nothing)
plot!(map2_popplt, X, p2_mean, ribbon=(p2_lower,p2_upper), color=:darkmagenta, label="2-phase model population")
lens!([2, 32], [300, 5700], inset = (1, bbox(0.6, 0.0, 0.38, 0.7)))
annotate!([(30,1.25e4,Plots.text("A1",18))])

v2_mean=map2.disp_mat[:,2,2]
v2_upper=map2.disp_mat[:,1,2]-map2.disp_mat[:,2,2]
```

```
v2_lower=map2.disp_mat[:,2,2]-map2.disp_mat[:,3,2]

map2_volplt=scatter(Xs,catvobs, marker=:cross, color=:black, markersize=3, label="Retinal volume estimate", ylabel="Volume (μm^3)", xlabel="Age (dpf)", legend=:topleft, showaxis=:y, xticks=plotticks, xformatter=_->"", background_color_legend=nothing)
plot!(map2_volplt, X, v2_mean, ribbon=(v2_lower,v2_upper), color=:green, label="2-phase model volume")
lens!([2, 32], [2e6, 3e7], inset = (1, bbox(0.6, 0.0, 0.34, 0.5)))
annotate!([(30,3.5e8,Plots.text("A2",18))])


p3_mean=map3.disp_mat[:,2,1]
p3_upper=map3.disp_mat[:,1,1]-map3.disp_mat[:,2,1]
p3_lower=map3.disp_mat[:,2,1]-map3.disp_mat[:,3,1]

map3_popplt=scatter(Xs,catpobs, marker=:cross, color=:black, markersize=3, label="CMZ population estimate", ylabel="Population", showaxis=:y, xticks=plotticks, xformatter=_->"", legend=:top, background_color_legend=nothing)
plot!(map3_popplt, X, p3_mean, ribbon=(p3_lower,p3_upper), color=:darkmagenta, label="3-phase model population")
lens!([2, 32], [300, 5700], inset = (1, bbox(0.6, 0.0, 0.38, 0.7)))
annotate!([(30,1.5e4,Plots.text("B1",18))])

v3_mean=map3.disp_mat[:,2,2]
v3_upper=map3.disp_mat[:,1,2]-map3.disp_mat[:,2,2]
v3_lower=map3.disp_mat[:,2,2]-map3.disp_mat[:,3,2]

map3_volplt=scatter(Xs,catvobs, marker=:cross, color=:black, markersize=3, label="Retinal volume estimate", ylabel="Volume (μm^3)", xlabel="Age (dpf)", legend=:topleft, xticks=plotticks, background_color_legend=nothing)
plot!(map3_volplt, X, v3_mean, ribbon=(v3_lower,v3_upper), color=:green, label="3-phase model volume")
lens!([2, 32], [2e6, 3e7], inset = (1, bbox(0.6, 0.0, 0.34, 0.5)))
annotate!([(30,3.5e8,Plots.text("B2",18))])

combined_map=Plots.plot(map2_popplt,map2_volplt,map3_popplt,map3_volplt,layout=grid(4 size=(1200,1200), link=:x)

savefig(combined_map,"/bench/PhD/Thesis/images/cmz/a10pMAP.png")

```

```
kdes=posterior_kde(e2)

ctmarg=plot(Uniform(10,144), color=:darkmagenta, fill=true, fillalpha=.5,xlims=[0,144], xlabel="Cycle Time (hr)", ylabel="Probability density", label="Prior")
plot!(kdes[1].x,kdes[1].density,color=:green, fill=true, fillalpha=.5, label="Ph1 Posterior")
plot!(kdes[3].x,kdes[3].density,color=:orange, fill=true, fillalpha=.5, label="Ph2 Posterior")
plot!([map2.θ[1],map2.θ[1]],[0,maximum(kdes[1].density)],color=:darkgreen, label="Ph1 MAP")
plot!([map2.θ[3],map2.θ[3]],[0,maximum(kdes[3].density)],color=:darkred, label="Ph2 MAP")
annotate!([(0,.11,Plots.text("A",18))])

ymarg=map(x→pdf(LogNormal(log(.9),log(2)),x),kdes[2].x)

ermarg=plot(kdes[2].x, ymarg, color=:darkmagenta, fill=true, fillalpha=.5, xlims=[0,4], xlabel="Niche exit rate ϵ", ylabel="Probability density", label="Prior")
plot!(kdes[2].x,kdes[2].density,color=:green, fill=true, fillalpha=.5, label="Ph1 Posterior")
plot!(kdes[4].x,kdes[4].density,color=:orange, fill=true, fillalpha=.5, label="Ph2 Posterior")
plot!([map2.θ[2],map2.θ[2]],[0,maximum(kdes[2].density)],color=:darkgreen, label="Ph1 MAP")
plot!([map2.θ[4],map2.θ[4]],[0,maximum(kdes[4].density)],color=:darkred, label="Ph2 MAP")
annotate!([(0,7.,Plots.text("B",18))])


transmarg=plot(Uniform(4.,359.), color=:darkmagenta, fill=true, fillalpha=.5,xlims=[0,360], xlabel="Phase transition age (dpf)", ylabel="Probability density", label="Prior")
plot!(kdes[5].x,kdes[5].density,color=:green, fill=true, fillalpha=.5, label="Ph1-2 posterior")
plot!([map2.θ[5],map2.θ[5]],[0,maximum(kdes[5].density)],color=:darkgreen, label="Ph1-2 MAP")
annotate!([(0,.04,Plots.text("C",18))])


ph1c=plot(ctmarg,ermarg,transmarg, layout=grid(3,1),size=(1000,1200))
savefig(ph1c,"/bench/PhD/Thesis/images/cmz/a10pmarginals.png")
```

17.8.9 /cmz/a10popsurvey.jl

```
using CSV,DataFrames,Distributions,NGRefTools,StatsBase,Plots
gr()
default(legendfont = (10), guidefont = (11), tickfont = (10))

a10pth="/bench/PhD/datasets/A10 measurements 2018update.csv"

a10df=DataFrame(CSV.read(a10pth))
a10df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
 ↪  eachrow(a10df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()

measure_dict["PopEst"]=Vector{Vector{Float64}}()
measure_dict["CMZ Sum"]=Vector{Vector{Float64}}()
measure_dict["Dorsal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ventral CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["VolEst"]=Vector{Vector{Float64}}()
measure_dict["Retina thickness"]=Vector{Vector{Float64}}()
measure_dict["RPE length"]=Vector{Vector{Float64}}()
measure_dict["Lens diameter"]=Vector{Vector{Float64}}()
measure_dict["ON diameter"]=Vector{Vector{Float64}}()
measure_dict["ONL"]=Vector{Vector{Float64}}()
measure_dict["OPL"]=Vector{Vector{Float64}}()
measure_dict["INL"]=Vector{Vector{Float64}}()
measure_dict["IPL"]=Vector{Vector{Float64}}()
measure_dict["GCL"]=Vector{Vector{Float64}}()
measure_dict["Pop/VolEst"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a10df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    pevec,tvec,dvec,vvec,vevec, rtvec, rplvec, rptvec, ldvec, ondvec,
     ↪  onlvec, oplvec, inlvec, iplvec, gclvec, pvest = [zeros(0) for i
     ↪  in 1:16]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Lens diameter")...])>0 &&
         ↪  push!(ldvec,mean(skipmissing(i_df."Lens diameter")))
```

```
    end

    for i_df in groupby(t_df,"BSR")
        ipop=false
        ivol=false
        if length([skipmissing(i_df."CMZ Sum") ... ])>0
            push!(tvec,mean(skipmissing(i_df."CMZ Sum")))
            if length([skipmissing(i_df."Lens diameter") ... ])>0
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(skipmissing(i_df."Lens diameter"))/14.)
                ipop=true
            else
                push!(pevec,mean(skipmissing(i_df."CMZ Sum"))*mean(ldvec)/14.)
                ipop=true
            end
        end

        length([skipmissing(i_df."Dorsal CMZ (#)") ... ])>0 && push!(dvec,mean(skipmissing(i_df."Dorsal CMZ (#)")))
        length([skipmissing(i_df."Ventral CMZ (#)") ... ])>0 && push!(vvec,mean(skipmissing(i_df."Ventral CMZ (#)")))
        if length([skipmissing(i_df."Retina thickness") ... ])>0 &&
         length([skipmissing(i_df."IPL") ... ])>0 &&
         length([skipmissing(i_df."OPL") ... ])>0 &&
         length([skipmissing(i_df."RPE length") ... ])>0

            rthi=mean(skipmissing(i_df."Retina thickness"))-mean(skipmissing(i_df."OPL"))-mean(skipmissing(i_df."IPL
            rpel=mean(skipmissing(i_df."RPE length"))
            if length([skipmissing(i_df."Lens diameter") ... ])>0
                rcirc=rpel+mean(skipmissing(i_df."Lens diameter"))
            else
                rcirc=rpel+mean(ldvec)
            end

            or=rcirc/2π
            ir=or-.5*rthi

            ov=(4/3)*π*(or^3)
            iv=(4/3)*π*(ir^3)

            push!(vevec,(ov-iv)*(4/5))
```

```
        end
        length([skipmissing(i_df."Retina thickness") ... ])>0 && push!(rtvec,mean(skipmissing(i_df."Retina thickness")))
        length([skipmissing(i_df."RPE length") ... ])>0 && push!(rptvec,mean(skipmissing(i_df."RPE length")))
        length([skipmissing(i_df."ON diameter") ... ])>0 && push!(ondvec,mean(skipmissing(i_df."ON diameter")))
        length([skipmissing(i_df.ONL) ... ])>0 && push!(onlvec,mean(skipmissing(i_df.ONL)))
        length([skipmissing(i_df.OPL) ... ])>0 && push!(oplvec,mean(skipmissing(i_df.OPL)))
        length([skipmissing(i_df.INL) ... ])>0 && push!(inlvec,mean(skipmissing(i_df.INL)))
        length([skipmissing(i_df.IPL) ... ])>0 && push!(iplvec,mean(skipmissing(i_df.IPL)))
        length([skipmissing(i_df.GCL) ... ])>0 && push!(gclvec,mean(skipmissing(i_df.GCL)))
        ivol && ipop &&
        push!(pvest,
        mean(skipmissing(i_df."CMZ Sum"))/
            ((mean(skipmissing(i_df."Retina thickness"))-mean(skipmissing(i_df."OPL"))-mean(skipmissing(i_df."IPL
                *mean(skipmissing(i_df."RPE length"))*(π/4)))
    end

    push!(measure_dict["PopEst"],pevec)
    push!(measure_dict["CMZ Sum"],tvec)
    push!(measure_dict["Dorsal CMZ (#)"],dvec)
    push!(measure_dict["Ventral CMZ (#)"],vvec)
    push!(measure_dict["VolEst"],vevec)
    push!(measure_dict["Retina thickness"],rtvec)
    push!(measure_dict["RPE length"],rptvec)
    push!(measure_dict["Lens diameter"],ldvec)
    push!(measure_dict["ON diameter"],ondvec)
    push!(measure_dict["ONL"],onlvec)
    push!(measure_dict["OPL"],oplvec)
    push!(measure_dict["INL"],inlvec)
    push!(measure_dict["IPL"],iplvec)
    push!(measure_dict["GCL"],gclvec)
    push!(measure_dict["Pop/VolEst"],pvest)
end

#TEST IF DATA IS BETTER MODELLED NORMALLY OR LOGNORMALLY
```

```
for ms in ["CMZ Sum", "Lens diameter", "PopEst", "RPE length", "Retina thickness", "VolEst"]
    normal_mts=[fit(Normal,measure_dict[ms][n]) for n in 1:length(X)]
    logn_mts=[fit(LogNormal,filter!(i->!iszero(i),measure_dict[ms][n])) for n in 1:length(X)]

    normal_lh=sum([sum(logpdf(normal_mts[n],measure_dict[ms][n])) for n in 1:length(X)])
    logn_lh=sum([sum(logpdf(logn_mts[n],measure_dict[ms][n])) for n in 1:length(X)])

    println("$ms & $(round(normal_lh,digits=3)) & $(round(logn_lh,digits=3)) & $(round(logn_lh-normal_lh,digits=3))\\\\ \\hline")

end

pe_n_mts=[fit(MarginalTDist,measure_dict["PopEst"][n]) for n in 1:length(X)]
pen_lower=[mean(mt)-quantile(mt,.025) for mt in pe_n_mts]
pen_upper=[quantile(mt,.975)-mean(mt) for mt in pe_n_mts]

pe_logn_mts=[fit(MarginalTDist,log.(filter!(i->!iszero(i),measure_dict["PopEst"][n])) for n in 1:length(X)]
pe_mean=[exp(mean(mt)) for mt in pe_logn_mts]
pe_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in pe_logn_mts]
pe_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in pe_logn_mts]

plotticks=[30*i for i in 1:12]

ann_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["PopEst"][n])] for n in 1:length(X)]...),vcat(measure_dict["PopEst"]...), marker=:cross, color=:black, markersize=3, label="Pop. data", ylabel="Est. CMZ annulus population", showaxis=:y, xticks=plotticks, xformatter=_->"")
plot!(ann_chart, X, pe_mean, ribbon=(pe_lower,pe_upper), color=:darkmagenta, label="Pop. mean")
lens!([2, 20], [500, 2500], inset = (1, bbox(0.3, 0.1, 0.45, 0.45)))
annotate!([(8,4500,Plots.text("A",18))])

poprate_mean,poprate_lower,poprate_upper=[Vector{Float64}() for i in 1:3]
for (nx, x) in enumerate(X)
    if nx > 1
```

```
        d=X[nx]-X[nx-1]
        l,m,u=MTDist_MC_func((a,b)→((exp(a)-exp(b))/d), [log.(filter!(i→!iszero(i),measure_dict["PopEst"][nx])), log.(filter!(i→!iszero(i),measure_dict["PopEst"][nx-1]))], summary=true)

        push!(poprate_mean,m);push!(poprate_lower,m-l);push!(poprate_upper,u-m)
    end
end

poprate_chart=plot(X[2:end],poprate_mean,ribbon=(poprate_lower, poprate_upper), color=:grey, ylabel="Est. CMZ pop. rate of change (cells/d)", label="Simulated mean rate", legend=:topright, showaxis=:y, xticks=plotticks, xformatter=_→"")
annotate!([(8,400,Plots.text("B",18))])


l3,m3,u3=MTDist_MC_func((a,b)→(exp(a)-exp(b))/5., [filter!(!isinf,log.(measure_dict["PopEst"][2])), filter!(!isinf,log.(measure_dict["PopEst"][1]))], summary=true)

ratedist23=MTDist_MC_func((a,b)→(exp(a)-exp(b))/6., [filter!(!isinf,log.(measure_dict["PopEst"][6])), filter!(!isinf,log.(measure_dict["PopEst"][5]))], summary=false)

println("3dpf poprate mean: $(m3) 23dpf poprate mean $(mean(ratedist23))poprate 23dpf quantile : $(quantile(ratedist23,.025))")

ve_logn_mts=[fit(MarginalTDist,log.(filter!(i→!iszero(i),measure_dict["VolEst"][n])) for n in 1:length(X)]
ve_mean=[exp(mean(mt)) for mt in ve_logn_mts]
ve_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in ve_logn_mts]
ve_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in ve_logn_mts]

vol_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["VolEst"][n])] for n in 1:length(X)] ... ),vcat(measure_dict["VolEst"] ... ), marker=:cross, color=:black, markersize=3, label="Vol. data", ylabel="Est. retinal volume (μm³)",  showaxis=:y, xticks=plotticks, xformatter=_→"", legend=:bottomright)
plot!(vol_chart, X, ve_mean, ribbon=(ve_lower,ve_upper), color=:blue, label="Vol. mean")
```

```
lens!([2, 20], [2.5e6, 1e7], inset = (1, bbox(0.12, 0., 0.39, 0.40)))
annotate!([(8,4.5e8,Plots.text("C",18))])

println("$(ccdf(ve_logn_mts[2],log(ve_mean[1]))) % of marginal posterior mass of 5dpf volume estimate is greater than the 3dpf estimated mean")

volrate_mean,volrate_lower,volrate_upper=[Vector{Float64}() for i in 1:3]
for (nx, x) in enumerate(X)
    if nx > 1
        d=X[nx]-X[nx-1]
        l,m,u=MTDist_MC_func((a,b)→(exp(a)-exp(b))/d, [filter!(!isinf,log.(measure_dict["VolEst"][nx])), filter!(!isinf,log.(measure_dict["VolEst"][nx-1]))], summary=true)

        push!(volrate_mean,m);push!(volrate_lower,l);push!(volrate_upper,u)
    end
end
volrate_lower=volrate_mean.-volrate_lower
volrate_upper=volrate_upper.-volrate_mean

volrate_chart=plot(X[2:end],volrate_mean,ribbon=(volrate_lower, volrate_upper), color=:grey, label="Simulated mean rate", legend=:topright, xlabel="Age(dpf)", ylabel="Est. volume rate (μm³d⁻¹)", xticks=plotticks)
annotate!([(8,4.5e6,Plots.text("D",18))])

l=@layout [a{0.3h}
            b{0.2h}
            c{0.3h}
            d]
combined=plot(ann_chart,poprate_chart,vol_chart,volrate_chart,layout=(4,1),size=(1200
savefig(combined, "/bench/PhD/Thesis/images/cmz/CMZoverall.png")

l30,m30,u30=MTDist_MC_func((a,b)→(exp(a)-exp(b))/7., [filter!(!isinf,log.(measure_dict["VolEst"][7])), filter!(!isinf,log.(measure_dict["VolEst"][6]))], summary=true)

ratedist60=MTDist_MC_func((a,b)→(exp(a)-exp(b))/30., [filter!(!isinf,log.(measure_dict["VolEst"][8])), filter!(!isinf,log.(measure_dict["VolEst"][7]))], summary=false)

```

```
println("6*30dpf volrate mean: $(6*m30) volrate 60dpf quantile : $(quantile(ratedist60,.01))")
```

17.8.10 /cmz/a19lineagetrace.jl

```
using CSV,DataFrames,Distributions,NGRefTools,StatsBase,Plots,Measurements,GMC_NS,Serialization
gr()

a19_1pth="/bench/PhD/datasets/A19GR1.csv"
a19_2pth="/bench/PhD/datasets/A19GR2.csv"
a19_3pth="/bench/PhD/datasets/A19GR3.csv"

gr1df=DataFrame(CSV.read(a19_1pth))
gr2df=DataFrame(CSV.read(a19_2pth))
gr3df=DataFrame(CSV.read(a19_3pth))

X=sort(unique(gr1df."Pulse age (dpf)"))
t_measure_dict=Dict{String,Vector{Vector{Float64}}}()
d_measure_dict=Dict{String,Vector{Vector{Float64}}}()
v_measure_dict=Dict{String,Vector{Vector{Float64}}}()

mds=[t_measure_dict,d_measure_dict,v_measure_dict]

for md in mds
    md["ONL"]=[zeros(0) for i in 1:length(X)]
    md["INL"]=[zeros(0) for i in 1:length(X)]
    md["GCL"]=[zeros(0) for i in 1:length(X)]
    md["Isl"]=[zeros(0) for i in 1:length(X)]
    md["Pax6"]=[zeros(0) for i in 1:length(X)]
    md["Isl_Pax6"]=[zeros(0) for i in 1:length(X)]
    md["INL_Pax6"]=[zeros(0) for i in 1:length(X)]
    md["PKCB"]=[zeros(0) for i in 1:length(X)]
    md["GS"]=[zeros(0) for i in 1:length(X)]
    md["HM"]=[zeros(0) for i in 1:length(X)]
    md["Zpr1"]=[zeros(0) for i in 1:length(X)]
end

function check_layerfrac(row)
    !ismissing(row."#GCL") && !ismissing(row."#INL") && !ismissing(row."#PRL") && !ismissing(row."Total") ? (return true) : (return false)
```

```
end

function gr1_dfcheck(subdf)
    pass=falses(size(subdf,1))
    for (n,row) in enumerate(eachrow(subdf))
        !ismissing(row."GCL - Isl") && !ismissing(row."GCL - Pax6") && !ismissing(row."GCL - Isl/Pax6") && !ismissing(row."INL - Pax6") ? (pass[n]=true) : (pass[n]=false)
    end
    all(pass) ? (return true) : (return false)
end

for df in [gr1df,gr2df,gr3df]
    for agedf in groupby(df, "Pulse age (dpf)")
        xidx=findfirst(i→i==unique(agedf."Pulse age (dpf)")[1],X)
        for inddf in groupby(agedf, "Individual")
            if df === gr1df
                if gr1_dfcheck(inddf)
                    clean=falses(2)
                    for row in eachrow(inddf)
                        if row."D/V" == "D"
                            push!(d_measure_dict["Isl"][xidx],row."GCL - Isl"/row."#GCL")
                            push!(d_measure_dict["Pax6"][xidx],row."GCL - Pax6"/row."#GCL")
                            push!(d_measure_dict["Isl_Pax6"][xidx],row."GCL - Isl/Pax6"/row."#GCL")
                            push!(d_measure_dict["INL_Pax6"][xidx],row."INL - Pax6"/row."#INL")
                            if check_layerfrac(row)
                                push!(d_measure_dict["GCL"][xidx],row."#GCL"/row.
                                push!(d_measure_dict["INL"][xidx],row."#INL"/row.
                                push!(d_measure_dict["ONL"][xidx],row."#PRL"/row.
                                clean[1]=true
                            end
                        else
                            push!(v_measure_dict["Isl"][xidx],row."GCL - Isl"/row."#GCL")
```

```
                            push!(v_measure_dict["Pax6"][xidx],row."GCL - Pax6"/row."#GCL")
                            push!(v_measure_dict["Isl_Pax6"][xidx],row."GCL - Isl/Pax6"/row."#GCL")
                            push!(v_measure_dict["INL_Pax6"][xidx],row."INL - Pax6"/row."#INL")
                            if check_layerfrac(row)
                                push!(v_measure_dict["GCL"][xidx],row."#GCL"/row.
                                push!(v_measure_dict["INL"][xidx],row."#INL"/row.
                                push!(v_measure_dict["ONL"][xidx],row."#PRL"/row.
                                clean[2]=true
                            end
                        end
                    end
                    push!(t_measure_dict["Isl"][xidx],sum(inddf."GCL - Isl")/sum(inddf."#GCL"))
                    push!(t_measure_dict["Pax6"][xidx],sum(inddf."GCL - Pax6")/sum(inddf."#GCL"))
                    push!(t_measure_dict["Isl_Pax6"][xidx],sum(inddf."GCL - Isl/Pax6")/sum(inddf."#GCL"))
                    push!(t_measure_dict["INL_Pax6"][xidx],sum(inddf."INL - Pax6")/sum(inddf."#INL"))
                    if all(clean)
                        push!(t_measure_dict["GCL"][xidx],sum(inddf."#GCL")/sum(i
                        push!(t_measure_dict["INL"][xidx],sum(inddf."#INL")/sum(i
                        push!(t_measure_dict["ONL"][xidx],sum(inddf."#PRL")/sum(i
                    end
                end
            elseif df == gr2df
                for row in eachrow(inddf)
                    if row."D/V" == "D"
                        push!(d_measure_dict["PKCB"][xidx],row."INL-PKCB"/row."#I
                        push!(d_measure_dict["GS"][xidx],row."INL-GS"/row."#INL")
```

```
                    push!(d_measure_dict["HM"][xidx],row."INL-HM"/row."#INL")
                    push!(d_measure_dict["GCL"][xidx],row."#GCL"/row."Total")
                    push!(d_measure_dict["INL"][xidx],row."#INL"/row."Total")
                    push!(d_measure_dict["ONL"][xidx],row."#PRL"/row."Total")
                else
                    push!(v_measure_dict["PKCB"][xidx],row."INL-PKCB"/row."#I
                    push!(v_measure_dict["GS"][xidx],row."INL-GS"/row."#INL")
                    push!(v_measure_dict["HM"][xidx],row."INL-HM"/row."#INL")
                    push!(v_measure_dict["GCL"][xidx],row."#GCL"/row."Total")
                    push!(v_measure_dict["INL"][xidx],row."#INL"/row."Total")
                    push!(v_measure_dict["ONL"][xidx],row."#PRL"/row."Total")
                end
            end
            push!(t_measure_dict["PKCB"][xidx],sum(inddf."INL-PKCB")/sum(indd
            push!(t_measure_dict["GS"][xidx],sum(inddf."INL-GS")/sum(inddf."#
            push!(t_measure_dict["HM"][xidx],sum(inddf."INL-HM")/sum(inddf."#
            push!(t_measure_dict["GCL"][xidx],sum(inddf."#GCL")/sum(inddf."To
            push!(t_measure_dict["INL"][xidx],sum(inddf."#INL")/sum(inddf."To
            push!(t_measure_dict["ONL"][xidx],sum(inddf."#PRL")/sum(inddf."To
        elseif df == gr3df
            for row in eachrow(inddf)
                if row."D/V" == "D"
                    push!(d_measure_dict["Zpr1"][xidx],row."PRL - Zpr1"/row."#PRL")
                    push!(d_measure_dict["GCL"][xidx],row."#GCL"/row."Total")
```

```
                        push!(d_measure_dict["INL"][xidx],row."#INL"/row."Total")
                        push!(d_measure_dict["ONL"][xidx],row."#PRL"/row."Total")
                    else
                        push!(v_measure_dict["Zpr1"][xidx],row."PRL - Zpr1"/row."#PRL")
                        push!(v_measure_dict["GCL"][xidx],row."#GCL"/row."Total")
                        push!(v_measure_dict["INL"][xidx],row."#INL"/row."Total")
                        push!(v_measure_dict["ONL"][xidx],row."#PRL"/row."Total")
                    end
                end
                push!(t_measure_dict["Zpr1"][xidx],sum(inddf."PRL - Zpr1")/sum(inddf."#PRL"))
                push!(t_measure_dict["GCL"][xidx],sum(inddf."#GCL")/sum(inddf."To
                push!(t_measure_dict["INL"][xidx],sum(inddf."#INL")/sum(inddf."To
                push!(t_measure_dict["ONL"][xidx],sum(inddf."#PRL")/sum(inddf."To
            end
        end
    end
end

for d in mds
    for (k,v) in d
        for ovec in v
            if any(i->i==0.,ovec)
                ovec[findall(i->i==0.,ovec)]•=1e-3
            end
        end
        d[k]=v
    end
end

onl_n_mts=[fit(MarginalTDist,t_measure_dict["ONL"][n]) for n in 1:length(X)]
onl_mean=[mean(mt) for mt in onl_n_mts]
onl_lower=[mean(mt)-quantile(mt,.025) for mt in onl_n_mts]
```

```
onl_upper=[quantile(mt,.975)-mean(mt) for mt in onl_n_mts]

inl_n_mts=[fit(MarginalTDist,t_measure_dict["INL"][n]) for n in 1:length(X)]
inl_mean=[mean(mt) for mt in inl_n_mts]
inl_lower=[mean(mt)-quantile(mt,.025) for mt in inl_n_mts]
inl_upper=[quantile(mt,.975)-mean(mt) for mt in inl_n_mts]

gcl_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["GCL"][n])) for n in 1:length(X)]
gcl_mean=[exp(mean(mt)) for mt in gcl_ln_mts]
gcl_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in gcl_ln_mts]
gcl_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in gcl_ln_mts]

layers_chart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["GCL"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["GCL"]...), marker=:utriangle, color=:darkmagenta, markersize=3, markerstrokecolor=:darkmagenta, label="GCL data", ylabel="Fraction of cohort in layer", xlabel="Age (dpf)", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:inside)
plot!(X, gcl_mean, ribbon=(gcl_lower,gcl_upper), color=:darkmagenta, label="GCL mean")
scatter!(vcat([[X[n] for i in 1:length(t_measure_dict["INL"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["INL"]...), marker=:cross, color=:blue, markersize=3, label="INL data")
plot!(X, inl_mean,ribbon=(inl_lower,inl_upper), color=:blue, label="INL mean")
scatter!(vcat([[X[n] for i in 1:length(t_measure_dict["ONL"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["ONL"]...), marker=:dtriangle, color=:green, markersize=3, markerstrokecolor=:green, label="ONL data")
plot!(X, onl_mean,ribbon=(onl_lower,onl_upper), color=:green, label="ONL mean")
annotate!([(15,.7,Plots.text("A",18))])

isl_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["Isl"][n])) for n in 1:length(X)]
isl_mean=[exp(mean(mt)) for mt in isl_ln_mts]
isl_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in isl_ln_mts]
isl_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in isl_ln_mts]

```

```
islchart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["Isl"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["Isl"]...), marker=:square, color=:darkmagenta, markerstrokecolor=:darkmagenta, markersize=3, label="Isl+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:bottom,  xformatter=_->"", showaxis=:y)
plot!(X, isl_mean,ribbon=(isl_lower,isl_upper), color=:darkmagenta, label="Isl+ mean", ylims=[0,.5])
annotate!([(15,.05,Plots.text("B(†)",18))])

pax6_n_mts=[fit(MarginalTDist,t_measure_dict["Pax6"][n]) for n in 1:length(X)]
pax6_mean=[mean(mt) for mt in pax6_n_mts]
pax6_lower=[mean(mt)-quantile(mt,.025) for mt in pax6_n_mts]
pax6_upper=[quantile(mt,.975)-mean(mt) for mt in pax6_n_mts]

pax6chart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["Pax6"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["Pax6"]...), marker=:circle, color=:darkmagenta, markersize=3, markerstrokecolor=:darkmagenta, label="Pax6+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:top,  xformatter=_->"", showaxis=:y, ylabel="Fraction of GCL cohort labelled")
plot!(X, pax6_mean,ribbon=(pax6_lower,pax6_upper), color=:darkmagenta, label="Pax6+ mean", ylims=[0,.6])
annotate!([(15,.45,Plots.text("C(N)",18))])

islp_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["Isl_Pax6"][n])) for n in 1:length(X)]
islp_mean=[exp(mean(mt)) for mt in islp_ln_mts]
islp_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in islp_ln_mts]
islp_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in islp_ln_mts]

islpchart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["Isl_Pax6"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["Isl_Pax6"]...), marker=:triangle, color=:darkmagenta, markersize=3, markerstrokecolor=:darkmagenta, label="Isl/Pax6+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:bottom, xlabel="Age (dpf)")
plot!(X, islp_mean,ribbon=(islp_lower,islp_upper), color=:darkmagenta, label="Isl/Pax6+ mean")
annotate!([(15,.05,Plots.text("D(†)",18))])
```

```
gcl_subplot=plot(islchart, pax6chart, islpchart, layout=grid(3,1), link=:x)

pax6i_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["INL_Pax6"][n])) for n in 1:length(X)]
pax6i_mean=[exp(mean(mt)) for mt in pax6i_ln_mts]
pax6i_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in pax6i_ln_mts]
pax6i_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in pax6i_ln_mts]

inlpax6chart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["INL_Pax6"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["INL_Pax6"]...), marker=:square, color=:blue, markersize=3, markerstrokecolor=:blue, label="Pax6+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:bottom, xformatter=_->"", showaxis=:y)
plot!(X, pax6i_mean,ribbon=(pax6i_lower,pax6i_upper), color=:blue, label="Pax6+ mean")
annotate!([(15,.075,Plots.text("E(†)",18))])

pkc_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["PKCB"][n])) for n in 1:length(X)]
pkc_mean=[exp(mean(mt)) for mt in pkc_ln_mts]
pkc_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in pkc_ln_mts]
pkc_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in pkc_ln_mts]

pkcchart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["PKCB"][n])] for n in 1:length(X)]...),vcat(t_measure_dict["PKCB"]...), marker=:circle, color=:blue, markersize=3, markerstrokecolor=:blue, label="PKCβ+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:top, xformatter=_->"", showaxis=:y, ylabel="Fraction of INL cohort labelled")
plot!(X, pkc_mean,ribbon=(pkc_lower,pkc_upper), color=:blue, label="PKCβ+ mean", ylims=[0,.2])
annotate!([(15,.18,Plots.text("F(†)",18))])

gs_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["GS"][n])) for n in 1:length(X)]
gs_mean=[exp(mean(mt)) for mt in gs_ln_mts]
gs_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in gs_ln_mts]
gs_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in gs_ln_mts]
```

```
gschart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["GS"][n])] for n in 1:length(X)] ... ),vcat(t_measure_dict["GS"] ... ), marker=:diamond, color=:blue, markersize=3, markerstrokecolor=:blue, label="GS+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:topright, xformatter=_→"", showaxis=:y)
plot!(X, gs_mean,ribbon=(gs_lower,gs_upper), color=:blue, label="GS+ mean")
annotate!([(15,.125,Plots.text("G(†)",18))])

hm_ln_mts=[fit(MarginalTDist,log.(t_measure_dict["HM"][n])) for n in 1:length(X)]
hm_mean=[exp(mean(mt)) for mt in hm_ln_mts]
hm_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in hm_ln_mts]
hm_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in hm_ln_mts]

hmchart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["HM"][n])] for n in 1:length(X)] ... ),vcat(t_measure_dict["HM"] ... ), marker=:cross, color=:blue, markersize=3, markerstrokecolor=:blue, label="HM+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:topright, xlabel="Age (dpf)")
plot!(X, hm_mean,ribbon=(hm_lower,hm_upper), color=:blue, label="HM+ mean")
annotate!([(15,.13,Plots.text("H",18))])

inl_subplot=plot(inlpax6chart, pkcchart, gschart, hmchart, layout=grid(4,1), link=:x)

z_n_mts=[fit(MarginalTDist,t_measure_dict["Zpr1"][n]) for n in 1:length(X)]
z_mean=[mean(mt) for mt in z_n_mts]
z_lower=[mean(mt)-quantile(mt,.025) for mt in z_n_mts]
z_upper=[quantile(mt,.975)-mean(mt) for mt in z_n_mts]

zchart=scatter(vcat([[X[n] for i in 1:length(t_measure_dict["Zpr1"][n])] for n in 1:length(X)] ... ),vcat(t_measure_dict["Zpr1"] ... ), marker=:square, color=:green, markerstrokecolor=:green, markersize=3, label="Zpr1+ data", xticks=X, foreground_color_legend=nothing, background_color_legend=nothing, legend=:bottom, xlabel="Age (dpf)", ylabel="Frac. OPL labelled")
```

```
plot!(X, z_mean,ribbon=(z_lower,z_upper), color=:green, label="HM+ mean", ylims=[0,.55])
annotate!([(15,.15,Plots.text("I(N)",18))])

l=@layout [[layers{0.5h};gcl] [inl{0.8h};opl]]
combined=plot(layers_chart,gcl_subplot, inl_subplot,zchart, size=(1200,1200),layout=l)
savefig(combined, "/bench/PhD/Thesis/images/cmz/layercontributions.png")

for ms in keys(t_measure_dict)
    normal_mts=[fit(Normal,filter!(i→!isnan(i),t_measure_dict[ms][n])) for n in 1:length(X)]
    logn_mts=[fit(LogNormal,filter!(i→!isnan(i),t_measure_dict[ms][n])) for n in 1:length(X)]

    normal_lh=sum([sum(logpdf(normal_mts[n],t_measure_dict[ms][n])) for n in 1:length(X)])
    logn_lh=sum([sum(logpdf(logn_mts[n],t_measure_dict[ms][n])) for n in 1:length(X)])

    println("$ms & $(round(normal_lh,digits=3)) & $(round(logn_lh,digits=3)) & $(round(logn_lh-normal_lh,digits=3))\\\\ \\hline")
end

gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=1.e-15

uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

n_models=50

evdict=Dict{String,Measurement}()

max_μ=1.
min_μ=0.
max_λ=4000.
min_λ=1.

prior=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
box=[min_μ max_μ;min_λ max_λ]
```

```

for ms in keys(t_measure_dict)
    for (n,ovec) in enumerate(t_measure_dict[ms])
        e_basepth="/bench/PhD/NGS_binaries/GMC_NS/A19/n_"*ms
        x=X[n]
        enspth=e_basepth*"/$x"
        if isfile(enspth*"/ens")
            e=deserialize(enspth*"/ens")
        else
            e=Normal_Ensemble(enspth,n_models,filter(i→i>0,ovec), prior, box, gmc ... )
        end

        "n_"*ms in keys(evdict) ? (evdict["n_"*ms] += converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)) : (evdict["n_"*ms] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6))
    end

    for (n,ovec) in enumerate(t_measure_dict[ms])
        e_basepth="/bench/PhD/NGS_binaries/GMC_NS/A19/ln_"*ms
        x=X[n]
        enspth=e_basepth*"/$x"
        if isfile(enspth*"/ens")
            e=deserialize(enspth*"/ens")
        else
            e=LogNormal_Ensemble(enspth,n_models,filter(i→i>0,ovec), prior, box, gmc ... )
        end

        "ln_"*ms in keys(evdict) ? (evdict["ln_"*ms] += converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)) : (evdict["ln_"*ms] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6))
    end
end

```

```
for ms in keys(t_measure_dict)
    nev=evdict["n_"*ms]
    lnev=evdict["ln_"*ms]
    ratio=lnev-nev
    println("$ms & $(round(nev,digits=3)) & $(round(lnev,digits=3)) & $(round(ratio,digits=3)) & $(round(ratio.val/ratio.err, digits=1))\\\\ \\hline")
end

for ms in keys(t_measure_dict)
    if ms in ["Pax6","INL","ONL","Zpr1"]
        efunc=Normal_Ensemble
        pf="cn_"
    else
        efunc=LogNormal_Ensemble
        pf="cln_"
    end

    ovec=vcat(t_measure_dict[ms]...)
    e_basepth="/bench/PhD/NGS_binaries/GMC_NS/A19/"*pf*ms

    if isfile(e_basepth*"/ens")
        e=deserialize(e_basepth*"/ens")
    else
        e=efunc(e_basepth,n_models,filter(i->i>0,ovec), prior, box, gmc...)
    end

    evdict[pf*ms] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000,  converge_factor=1e-6)
end

for ms in keys(t_measure_dict)
    if ms in ["Pax6","INL","ONL","Zpr1"]
        pf="n_"
    else
        pf="ln_"
    end

    cev=evdict["c"*pf*ms]
    sev=evdict[pf*ms]
    ratio=cev-sev
```

```
    println("$ms & $(round(cev,digits=3)) & $(round(sev,digits=3)) & $(round(ratio,digits=3)) & $(round(ratio.val/ratio.err,digits=1))\\\\ \\hline")
end

dv_evdict=Dict{String,Measurement}()

for ms in keys(d_measure_dict)
    if ms in ["INL_Pax6","INL","Isl","Isl_Pax6","ONL","Zpr1"]
        efunc=Normal_Ensemble
        pf="d_cn_"
    else
        efunc=LogNormal_Ensemble
        pf="d_cln_"
    end

    ovec=vcat(d_measure_dict[ms]...)
    e_basepth="/bench/PhD/NGS_binaries/GMC_NS/A19/"*pf*ms

    if isfile(e_basepth*"/ens")
        e=deserialize(e_basepth*"/ens")
    else
        e=efunc(e_basepth,n_models,filter(i->!isnan(i),ovec), prior, box, gmc...)
    end

    dv_evdict[pf*ms] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000,  converge_factor=1e-6)
end

for ms in keys(v_measure_dict)
    if ms in ["INL_Pax6","INL","Isl","Isl_Pax6","ONL","Zpr1"]
        efunc=Normal_Ensemble
        pf="v_cn_"
    else
        efunc=LogNormal_Ensemble
        pf="v_cln_"
    end

    ovec=vcat(d_measure_dict[ms]...)
    e_basepth="/bench/PhD/NGS_binaries/GMC_NS/A19/"*pf*ms

```

```
    if isfile(e_basepth*"/ens")
        e=deserialize(e_basepth*"/ens")
    else
        e=efunc(e_basepth,n_models,filter(i->!isnan(i),ovec), prior, box, gmc ... )
    end

    dv_evdict[pf*ms] = converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000,  converge_factor=1e-6)
end


for ms in keys(t_measure_dict)
    if ms in ["INL_Pax6","INL","Isl","Isl_Pax6","ONL","Zpr1"]
        pf="n_"
    else
        pf="ln_"
    end

    cev=evdict["c"*pf*ms]
    dev=dv_evdict["d_c"*pf*ms]
    vev=dv_evdict["v_c"*pf*ms]
    sev=dev+vev
    ratio=cev-sev
    println("$ms & $(round(cev,digits=3)) & $(round(sev,digits=3)) & $(round(ratio,digits=3)) & $(round(ratio.val/ratio.err,digits=1))\\\\ \\hline")
end
```

17.8.11 /cmz/a20dvratio.jl

```
using BayesianLinearRegression,CSV,DataFrames,Distributions, GMC_NS, Serialization, Plots, NGRefTools
a20pth="/bench/PhD/Thesis/datasets/a20.csv"

a20df=DataFrame(CSV.read(a20pth))
a20df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a20df)]

X=Vector{Float64}()
measure_dict=Dict{String,Vector{Vector{Float64}}}()
```

```

measure_dict["Nasal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Temporal CMZ (#)"]=Vector{Vector{Float64}}()
measure_dict["Ratio"]=Vector{Vector{Float64}}()

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for t_df in groupby(a20df, "Time point (d)")
    push!(X,Float64(unique(t_df."Time point (d)")[1]))
    rvec,nvec,tvec = [zeros(0) for i in 1:3]

    for i_df in groupby(t_df,"BSR")
        length([skipmissing(i_df."Nasal CMZ (#)")...])>0 &&
        ↪ mean(skipmissing(i_df."Nasal CMZ (#)")) >0. &&
        ↪ push!(nvec,mean(skipmissing(i_df."Nasal CMZ (#)")))
        length([skipmissing(i_df."Temporal CMZ (#)")...])>0 &&
        ↪ mean(skipmissing(i_df."Temporal CMZ (#)")) >0. &&
        ↪ push!(tvec,mean(skipmissing(i_df."Temporal CMZ (#)")))
        length([skipmissing(i_df."Nasal CMZ (#)")...])>0 &&
        ↪ mean(skipmissing(i_df."Nasal CMZ (#)")) >0. &&
        ↪ length([skipmissing(i_df."Temporal CMZ (#)")...])>0 &&
        ↪ mean(skipmissing(i_df."Temporal CMZ (#)")) >0. &&
        ↪ push!(rvec,mean(skipmissing(i_df."Nasal CMZ
        ↪ (#)"))/mean(skipmissing(i_df."Temporal CMZ (#)")))
    end

    push!(measure_dict["Ratio"],rvec)
    push!(measure_dict["Nasal CMZ (#)"],nvec)
    push!(measure_dict["Temporal CMZ (#)"],tvec)
end

n_logn_mts=[fit(MarginalTDist,log.(measure_dict["Nasal CMZ (#)"][n])) for
↪ n in 1:length(X)]
n_mean=[exp(mean(mt)) for mt in n_logn_mts]
n_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in n_logn_mts]
n_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in n_logn_mts]

t_logn_mts=[fit(MarginalTDist,log.(measure_dict["Temporal CMZ (#)"][n]))
↪ for n in 1:length(X)]
t_mean=[exp(mean(mt)) for mt in t_logn_mts]
t_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in t_logn_mts]
t_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in t_logn_mts]

```

```
r_n_mts=[fit(MarginalTDist,measure_dict["Ratio"][n]) for n in 1:length(X)]
r_mean=[mean(mt) for mt in r_n_mts]
r_lower=[mean(mt)-quantile(mt,.025) for mt in r_n_mts]
r_upper=[quantile(mt,.975)-mean(mt) for mt in r_n_mts]

plotticks=[30*i for i in 1:12]

nt_chart=Plots.plot(X, n_mean, ribbon=(n_lower,n_upper), color=:green, label="Nasal mean", ylabel="CMZ sectional population size", showaxis=:y, xticks=plotticks, xformatter=_->"")
plot!(X, t_mean, ribbon=(t_lower,t_upper), color=:darkmagenta, label="Temporal mean")
scatter!(vcat([[X[n] for i in 1:length(measure_dict["Nasal CMZ (#)"][n])] for n in 1:length(X)]...),vcat(measure_dict["Nasal CMZ (#)"]...), marker=:utriangle, color=:green, markersize=3, markerstrokecolor=:green, label="Nasal data")
scatter!(vcat([[X[n] for i in 1:length(measure_dict["Temporal CMZ (#)"][n])] for n in 1:length(X)]...),vcat(measure_dict["Temporal CMZ (#)"]...), marker=:dtriangle, color=:darkmagenta, markersize=3, markerstrokecolor=:darkmagenta, label="Temporal data")
annotate!([(1.5,125,Plots.text("A",18))])

ratio_chart=scatter(vcat([[X[n] for i in 1:length(measure_dict["Ratio"][n])] for n in 1:length(X)]...),vcat(measure_dict["Ratio"]...), marker=:cross, color=:black, markersize=3, markerstrokecolor=:black, label="Ratio data", ylabel="Individual asymmetry ratio", xlabel="Age (dpf)", xticks=plotticks)
plot!(X, r_mean, ribbon=(r_lower,r_upper), color=:green, label="Ratio mean")
plot!(X, [1. for i in 1:length(X)], style=:dot, color=:black, label="Even ratio")
annotate!([(10,.5,Plots.text("B",18))])


l=@layout [a{.6h}
            b]

combined=Plots.plot(nt_chart,ratio_chart,layout=l, link=:x, size=(900,600))

savefig(combined, "/bench/PhD/Thesis/images/cmz/NTontology.png")
```

17.8.12 /cmz/a25dvlinreg.jl

```
using Distributions, Serialization, CSV, BayesianLinearRegression, Plots, DataFrames, Measurements
import Measurements:value,uncertainty


a25path="/bench/PhD/datasets/a25.csv"
a25df=DataFrame(CSV.File(a25path))

X=Vector{Float64}()
yD=Vector{Vector{Float64}}()
yV=Vector{Vector{Float64}}()

for timedf in groupby(a25df,"Time")
    push!(X,timedf[1,"Time"])
    dvec=Vector{Float64}()
    vvec=Vector{Float64}()
    for row in eachrow(timedf)
        dv=row."D/V"
        if dv == "D"
            push!(dvec, row.EdU/row.PCNA)
        else
            push!(vvec,row.EdU/row.PCNA)
        end
    end

    push!(yD,dvec)
    push!(yV,vvec)
end

sp=sortperm(X)
yD=yD[sp]; yV=yV[sp]
sort!(X)

xYD=zeros(0); xYV=zeros(0)
YD=zeros(0); YV=zeros(0)

for (n,vec) in enumerate(yD)
    global xYD=vcat(xYD,[X[n] for i in 1:length(vec)])
    global YD=vcat(YD,vec)
```

```
end

for (n,vec) in enumerate(yV)
    global xYV=vcat(xYV,[X[n] for i in 1:length(vec)])
    global YV=vcat(YV,vec)
end

xYD=hcat(ones(length(xYD)),xYD)
xYV=hcat(ones(length(xYV)),xYV)

xY=vcat(xYD,xYV);Y=vcat(YD,YV)

dm=BayesianLinearRegression.fit!(BayesianLinReg(xYD,YD))
vm=BayesianLinearRegression.fit!(BayesianLinReg(xYV,YV))
tm=BayesianLinearRegression.fit!(BayesianLinReg(xY,Y))

println("Dorsal model logZ: $(logEvidence(dm))")
println("Ventral model logZ: $(logEvidence(vm))")
println("Joint D/V logZ: $(logEvidence(dm) + logEvidence(vm)) ")
println("Combined model logZ: $(logEvidence(tm))")
println("Combined/Joint ratio : $(logEvidence(tm)-(logEvidence(dm) +
↪  logEvidence(vm)))")

function blr_plt(x,y,Xs,blrs,colors,labels,q=.975)

      ↪  plt=scatter(x,y,marker=:cross,markersize=3,markercolor=:black,label="Data",yl
      ↪  labelled fraction",xlabel="Pulse time (hr)",legend=:bottomright,
      ↪  ylims=(-0.1,1.))
    for (X,blr,color,label) in zip(Xs, blrs,colors,labels)
        line=zeros(size(X,1))
        ribbon=zeros(size(X,1))
        predictions=BayesianLinearRegression.predict(blr,X)
        for (n,p) in enumerate(predictions)
            normal=Normal(value(p),uncertainty(p))
            ribbon[n]=quantile(normal,q)-mean(normal)
            line[n]=mean((value(p)))
        end
        plot!(plt,x,line,ribbon=ribbon,color=color,label=label)
    end
    return plt
end

dplt=blr_plt(xYD[:,2],YD,[xYD],[dm],[:green],["Dorsal model"])
```

```
annotate!([(1,.9,Plots.text("A",12))])
vplt=blr_plt(xYV[:,2],YV,[xYV],[vm],[:magenta],["Ventral model"])
annotate!([(1,.9,Plots.text("B",12))])
tplt=blr_plt(xY[:,2],Y,[xY],[tm],[:black],["Combined model"])
annotate!([(1,.9,Plots.text("C",12))])

combined_plt=plot(dplt,vplt,tplt,layout=(3,1), size=(600,900))

savefig(combined_plt, "/bench/PhD/Thesis/images/cmz/3ddvlinreg.png")
@info "Saved fig."


println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf Model} & {\\bf Implied \$T_c\$} & {\\bf Implied \$T_s\$} & {\\bf logZ}\\\\ \\hline")

for (name,model) in zip(["Dorsal","Ventral","Combined"],[dm,vm,tm])
    local intercept,slope=posteriorWeights(model)
    println("$name & $(1/slope) & $(intercept*1/slope) & $(round(logEvidence(model),digits=3))\\\\ \\hline")
end

println("\\end{tabular}")
```

17.8.13 /cmz/a25thymidinesim.jl

```
using GMC_NS, CMZNicheSims, ConjugatePriors, CSV, DataFrames, Distributions, Serialization, NGRefTools

a10path="/bench/PhD/datasets/A10 measurements 2018update.csv"
a10df=DataFrame(CSV.File(a10path))

df3=a10df[a10df."Time point (d)" .== 3, :]

inddict=Dict{Tuple,Vector}()
for row in eachrow(df3)
    ind=(row.Block, row.Slide, row.Row)
    if !(ind in keys(inddict))
        inddict[ind]=[(row["Dorsal CMZ (#)"]+row["Ventral CMZ (#)"])]
    else
```

```
        push!(inddict[ind],((row["Dorsal CMZ (#)"]+row["Ventral CMZ (#)"])))
    end
end

popvec=Vector{Float64}()
for (ind, pops) in inddict
    push!(popvec,mean(pops))
end

a25path="/bench/PhD/datasets/a25.csv"
a25df=DataFrame(CSV.File(a25path))

X=Vector{Float64}()
y=Vector{Vector{Int64}}()

for timedf in groupby(a25df,"Time")
    push!(X,timedf[1,"Time"])
    dict=Dict{Tuple,Vector{Int64}}()
    for row in eachrow(timedf)
        ind=(row.Block,row.Slide,row.Row)
        if !(ind in keys(dict))
            dict[ind]=[row.PCNA,row.EdU]
        else
            dict[ind][1]+=row.PCNA
            dict[ind][2]+=row.EdU
        end
    end

    tcounts=Vector{Int64}()
    for (ind,counts) in dict
        global popvec=vcat(popvec, counts[1])
        push!(tcounts, counts[2])
    end
    push!(y,tcounts)
end

y=y[sortperm(X)]
sort!(X)

pop_dist=fit(LogNormal,popvec)

tc_prior=NormalInverseGamma(3.5,1.,5.,2.)
```

```
g1_prior=Beta(2.,2.)
s_prior=Beta(2.,5.)
sister_prior=Beta(8.,75.)

tc_prior2=NormalInverseGamma(3.75,1.,5.,2.)
pop2_prior=Beta(1.1,10.)

priors_1_pop=[marginals(tc_prior)..., g1_prior, s_prior, sister_prior]
priors_2_pop=[marginals(tc_prior2)..., g1_prior, s_prior,
↪  sister_prior,pop2_prior]

priors_2_pop=vcat(priors_1_pop,priors_2_pop)

p1_box=[2. 8.
        .1 2
        eps() 1.
        eps() 1.
        eps() .5]

p2_box=vcat(p1_box,vcat(p1_box,[eps() 1.]))

p1_box=GMC_NS.to_unit_ball.(p1_box,priors_1_pop)
p2_box=GMC_NS.to_unit_ball.(p2_box, priors_2_pop)

ep1="/bench/PhD/NGS_binaries/CNS/A25/1pop"
ep2="/bench/PhD/NGS_binaries/CNS/A25/2pop"
const pulse_time=10.5
const mc_its=Int64(1e5)
const end_time=10.5
constants=[pop_dist, X, pulse_time, mc_its, end_time]

for (ep,priors,box) in
↪  zip([ep1,ep2],[priors_1_pop,priors_2_pop],[p1_box,p2_box])
    if isfile(ep*"/ens")
        e=deserialize(ep*"/ens")
    else
        @info "Assembling ensemble at $ep"
        gmcd=GMC_DEFAULTS
        gmcd[1]=5
        gmcd[end]=100
        e=Thymidine_Ensemble(ep, 100, y, priors, constants, box, gmcd)
    end

```

```
    uds=Vector{Vector{Function}}([[liwi_display],[convergence_display],[ensemble_
    lds=Vector{Vector{Function}}([[model_obs_display],[model_obs_display],[model_
    converge_ensemble!(e,backup=(true,1),upper_displays=uds, lower_displays=lds, disp_rot_its=5, mc_noise=.14, converge_factor=1e-3)
end

e1=deserialize(ep1*"/ens")
e2=deserialize(ep2*"/ens")

ev1=measure_evidence(e1)
ev2=measure_evidence(e2)

CMZNicheSims.print_MAP_output(e1, "/bench/PhD/Thesis/images/cmz/a25MAP.png", its=Int64(1e7))
CMZNicheSims.print_marginals(e1,"/bench/PhD/Thesis/images/cmz/a25marginals.png")
```

17.8.14 /cmz/a27GMC_NS.jl

```
using Distributions, Serialization, CSV, BayesianLinearRegression, Plots, DataFrames, Measurements, GMC_NS
import Measurements:value,uncertainty

a27path="/bench/PhD/datasets/a27 r.csv"
a27df=DataFrame(CSV.File(a27path))

X=unique(a27df["Age"]).*30.
CR=Dict{Float64,Vector{Float64}}()
mo1=Dict{Float64,Vector{Float64}}()

for x in X
    CR[x]=Vector{Float64}()
    x<90. && (mo1[x]=Vector{Float64}())
end

for row in eachrow(a27df)
    x=row."Age"*30.
    !ismissing(row."LR") && push!(CR[x],row."LR")
```

```
    !ismissing(row."D/N1") && !ismissing(row."V/T1") &&
    ↪ push!(mo1[x],sum([row."D/N1",row."V/T1"]))
end


gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=1.e-15

uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

n_models=50

evdict=Dict{String,Measurement}()

max_μ=2000.
min_μ=0.
max_λ=500.
min_λ=1e-6

prior=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
box=[min_μ max_μ;min_λ max_λ]

cobs=vcat([v for v in values(CR)]...)
mobs=vcat([v for v in values(mo1)]...)

for (md, pstring) in zip([CR,mo1],["CR","mo1"])
    for (k,v) in md
        pfx=pstring*string(k)
        enspth="/bench/PhD/NGS_binaries/GMC_NS/A27/"*pfx
        if isfile(enspth*"/ens")
            e=deserialize(enspth*"/ens")
        else
            e=LogNormal_Ensemble(enspth,n_models,v, prior, box, gmc...)
        end

```

```
        pstring in keys(evdict) ? (evdict[pstring]+=converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)) : (evdict[pstring]=converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6))
    end
end

for (obs,pstring) in zip([cobs,mobs],["cCR","cmo1"])
    enspth="/bench/PhD/NGS_binaries/GMC_NS/A27/"*pstring
    if isfile(enspth*"/ens")
        e=deserialize(enspth*"/ens")
    else
        e=LogNormal_Ensemble(enspth,n_models,obs, prior, box, gmc ... )
    end


    evdict[pstring]=converge_ensemble!(e,backup=(true,10000),upper_displays=uds, lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)
end

for ms in ["CR","mo1"]
    cev=evdict["c"*ms]
    sev=evdict[ms]
    ratio=cev-sev
    println("$ms & $(round(cev,digits=3)) & $(round(sev,digits=3)) & $(round(ratio,digits=3)) & $(round(ratio.val/ratio.err,digits=1))\\\\ \\hline")
end
```

17.8.15 /cmz/a27linreg.jl

```
using Distributions, Serialization, CSV, BayesianLinearRegression, Plots, DataFrames, Measurements
import Measurements:value,uncertainty

a27path="/bench/PhD/datasets/a27 r.csv"
a27df=DataFrame(CSV.File(a27path))

X=unique(a27df["Age"]).*30.
```

```
CR=Dict{Float64,Vector{Float64}}()
mo1=Dict{Float64,Vector{Float64}}()

for x in X
    CR[x]=Vector{Float64}()
    x<90. && (mo1[x]=Vector{Float64}())
end

for row in eachrow(a27df)
    x=row."Age"*30.
    !ismissing(row."LR") && push!(CR[x],row."LR")
    !ismissing(row."D/N1") && !ismissing(row."V/T1") &&
     ↪  push!(mo1[x],sum([row."D/N1",row."V/T1"]))
end

xCR=zeros(0); xmo1=zeros(0)
yCR=zeros(0); ymo1=zeros(0)

for (age,vec) in CR
    println([age for i in 1:length(vec)])
    global xCR=vcat(xCR,[age for i in 1:length(vec)])
    global yCR=vcat(yCR,vec)
end

for (time,vec) in mo1
    global xmo1=vcat(xmo1,[time for i in 1:length(vec)])
    global ymo1=vcat(ymo1,vec)
end

xCR=hcat(ones(length(xCR)),xCR)
xmo1=hcat(ones(length(xmo1)),xmo1)

uncorrCR2=BayesianLinearRegression.fit!(BayesianLinReg(ones(length(yCR),1),yCR))
corrCR2=BayesianLinearRegression.fit!(BayesianLinReg(xCR,yCR))
uncorrCRX=ones(length(X),1)
corrCRX=hcat(ones(length(X)),X)

uncorrmo1=BayesianLinearRegression.fit!(BayesianLinReg(ones(length(ymo1),1),ymo1))
corrmo1=BayesianLinearRegression.fit!(BayesianLinReg(xmo1,ymo1))
uncorrmo1X=ones(length(X[1:2]),1)
corrmo1X=hcat(ones(length(X[1:2])),X[1:2])

function blr_plt(i,x,y,Xs,blrs,colors,labels,q=.975)
```

```
    plt=scatter(x,y,marker=:diamond,markersize=8,markerstrokestyle=:bold,markercolor=:black,
    size",xlabel="Age
    (dpf)",legend=:topright,ylims=[0,maximum(y)+25])
    for (X,blr,color,label) in zip(Xs, blrs,colors,labels)
        line=zeros(size(X,1))
        ribbon=zeros(size(X,1))
        predictions=BayesianLinearRegression.predict(blr,X)
        for (n,p) in enumerate(predictions)
            normal=Normal(value(p),uncertainty(p))
            ribbon[n]=quantile(normal,q)-mean(normal)
            line[n]=mean((value(p)))
        end
        plot!(plt,i,line,ribbon=ribbon,color=color,label=label)
    end
    return plt
end

pCR=blr_plt(X,xCR[:,2],yCR,[corrCRX,uncorrCRX],[corrCR2,uncorrCR2],[:magenta,:green],
    Model - CR", "Uncorrelated Model - CR"])
annotate!([(35,250,Plots.text("A",18))])

pmo1=blr_plt(X[1:2],xmo1[:,2],ymo1,[corrmo1X,uncorrmo1X],[corrmo1,uncorrmo1],[:magenta,:green],
    Model - 1mo", "Uncorrelated Model - 1mo CMZ"])
annotate!([(32,350,Plots.text("B",18))])

combined=plot(pCR,pmo1,layout=grid(2,1),size=(600,800))
savefig(combined,"/bench/PhD/Thesis/images/cmz/a27linreg.png")

println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf Measurement} & {\\bf Uncorrelated logZ} & {\\bf Correlated
    logZ} & {\\bf logZR}\\\\ \\hline")
println("1dpf Central Remnant & {\\bf
    $(round(logEvidence(uncorrCR2),digits=3))} &
    $(round(logEvidence(corrCR2),digits=3)) &
    $(round(logEvidence(uncorrCR2)-logEvidence(corrCR2),digits=3))\\\\
    \\hline")
println("30dpf Cohort & {\\bf $(round(logEvidence(uncorrmo1),digits=3))}
    & $(round(logEvidence(corrmo1),digits=3)) &
    $(round(logEvidence(uncorrmo1)-logEvidence(corrmo1),digits=3))\\\\
    \\hline")
println("\\end{tabular}")
```

17.8.16 /rys/a35thymidinesim.jl

```
using GMC_NS, CMZNicheSims, ConjugatePriors, CSV, DataFrames, Distributions, Serialization, NGRefTools, KernelDensity, Plots, StatsPlots
import BioBackgroundModels: lps
a35path="/bench/PhD/Thesis/datasets/A35.csv"
a35df=DataFrame(CSV.File(a35path))

X=Vector{Float64}()
s_popvec=Vector{Float64}()
r_popvec=Vector{Float64}()
sib_y=Vector{Vector{Int64}}()
rys_y=Vector{Vector{Int64}}()

for timedf in groupby(a35df,"Time")
    push!(X,timedf[1,"Time"])
    r_dict=Dict{Tuple,Vector{Int64}}()
    s_dict=Dict{Tuple,Vector{Int64}}()
    for row in eachrow(timedf)
        if !ismissing(row.PCNA) && !ismissing(row.EdU)
            row.Treatment=="Sib" ? (dict=s_dict; y=sib_y) : (dict=r_dict; y=rys_y)
            ind=(row.Block,row.Slide,row.Row)
            if !(ind in keys(dict))
                dict[ind]=[row.PCNA,row.EdU]
            else
                dict[ind][1]+=row.PCNA
                dict[ind][2]+=row.EdU
            end
        end
    end

    rcounts=Vector{Int64}()
    scounts=Vector{Int64}()
    for (dict,ctvec,pvec) in zip([r_dict,s_dict],[rcounts,scounts],[r_popvec,s_popvec])
        for (ind,counts) in dict
            push!(pvec, counts[1])
            push!(ctvec, counts[2])
        end
```

```
    end

    push!(sib_y,scounts)
    push!(rys_y,rcounts)
end

sib_y=sib_y[sortperm(X)]
rys_y=rys_y[sortperm(X)]
sort!(X)

s_pop_prior=fit(NormalInverseGamma,log.(s_popvec))
r_pop_prior=fit(NormalInverseGamma,log.(r_popvec))

g1_prior=Normal(4.,2.)
tc_prior=NormalInverseGamma(3.5,1.,5.,2.)
s_prior=NormalInverseGamma(4.,2.,1.,1.)
sister_prior=Beta(8.,75.)

s_priors=[marginals(s_pop_prior)..., g1_prior, marginals(tc_prior)..., marginals(s_prior)..., sister_prior]
r_priors=[marginals(r_pop_prior)..., g1_prior, marginals(tc_prior)..., marginals(s_prior)..., sister_prior]

box=[3.3 5.8
        .07 .26
        1. 10.
        .1 10.
        eps() 2.7
        eps() 20.
        .1 200.
        eps() .5]

s_box=GMC_NS.to_unit_ball.(box,s_priors)
r_box=GMC_NS.to_unit_ball.(box,s_priors)

sp="/bench/PhD/NGS_binaries/CNS/A35/sib"
rp="/bench/PhD/NGS_binaries/CNS/A35/rys"
const pulse_time=10.5
const mc_its=Int64(1.5e5)
const end_time=10.5
constants=[X, pulse_time, mc_its, end_time]

```

```
for (ep, priors, bx, y) in zip([sp,rp], [s_priors, r_priors], [s_box,r_box], [sib_y, rys_y])
    if isfile(ep*"/ens")
        e=deserialize(ep*"/ens")
    else
        @info "Assembling ensemble at $ep"
        gmcd=GMC_DEFAULTS
        gmcd[1]=5
        e=Thymidine_Ensemble(ep, 100, y, priors, constants, bx, gmcd)
    end


    uds=Vector{Vector{Function}}([[liwi_display],[convergence_display],[evidence_

    lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display],[ensembl

    converge_ensemble!(e,backup=(true,1),upper_displays=uds, lower_displays=lds, disp_rot_its=5, mc_noise=.14, converge_factor=1e-3)
end

se=deserialize(sp*"/ens")
re=deserialize(rp*"/ens")

measure_evidence(se)
measure_evidence(re)

CMZNicheSims.print_MAP_output(se,"/bench/PhD/Thesis/images/rys/a35sibMAP.png", its=Int64(1e7))
CMZNicheSims.print_MAP_output(re,"/bench/PhD/Thesis/images/rys/a35rysMAP.png", its=Int64(1e7))

lt=length(se.priors)
swt=zeros(0); rwt=zeros(0)
sθ=[zeros(0) for param in 1:lt]
rθ=[zeros(0) for param in 1:lt]

for (e,wtvec,θvecs) in zip([se,re],[swt,rwt],[sθ,rθ])
    for (n,rec) in enumerate(e.posterior_samples)
        m=deserialize(rec.path)
        for i in 1:length(θvecs)
            push!(θvecs[i],m.θ[i])
        end
```

```
        push!(wtvec,e.log_Liwi[n+1])
    end

    for rec in e.models
        m=deserialize(rec.path)
        for i in 1:length(θvecs)
            push!(θvecs[i],m.θ[i])
        end

        push!(wtvec,(lps(m.log_Li,lps(e.log_Xi[end],-log(length(e.models))))))
    end
end

deleteat!(sθ,3); deleteat!(sθ,4)
deleteat!(rθ,3); deleteat!(rθ,4)

n_pops=1; plotrows=5; param_names=["LogNormal Tc μ", "LogNormal Tc σ²", "G1 Fraction", "S Fraction", "Sister Shift Fraction"]

maps=deserialize(se.models[findmax([m.log_Li for m in se.models])[2]].path)
mapr=deserialize(re.models[findmax([m.log_Li for m in re.models])[2]].path)

plots=Vector{Plots.Plot}()
for (n,(sθvec,rθvec)) in enumerate(zip(sθ,rθ))
    n==3 && (n=4)
    p_label=param_names[n]
    occursin("Fraction",p_label) ? (xls=[0,1]) : (xls=[quantile(se.priors[n],.01),quantile(se.priors[n],.99)])


    sθkde=kde(sθvec,weights=exp.(swt))
    rθkde=kde(rθvec,weights=exp.(rwt))
    n==1 ? (lblpos=:right) : (lblpos=:none)
    plt=StatsPlots.plot(se.priors[n], color=:darkmagenta, fill=true, fillalpha=.5, label="Prior", xlabel=p_label, ylabel="Density", xlims=xls)
    plot!(sθkde.x,sθkde.density, color=:green, fill=true, fillalpha=.5, label="Sib Posterior", legend=lblpos)

    plot!([maps.θ[n],maps.θ[n]],[0,maximum(sθkde.density)],color=:darkgreen, label="sib MAP", legend=lblpos)
```

```
    plot!(rθkde.x,rθkde.density, color=:orange, fill=true, fillalpha=.5, label="Rys Posterior", legend=lblpos)

    plot!([mapr.θ[n],mapr.θ[n]],[0,maximum(rθkde.density)],color=:darkred, label="Rys MAP", legend=lblpos)

    push!(plots,plt)
    n_pops>1 && n==5 && push!(plots,plot())
end

combined=plot(plots ... , layout=grid(3,n_pops),size=(600*n_pops,800))

savefig(combined, "/bench/PhD/Thesis/images/rys/a35marginals.png")
```

17.8.17 /rys/a37.jl

```
using CSV,DataFrames,Distributions,Measurements,GMC_NS, NGRefTools, Plots,Measures, Serialization
gr()
default(legendfont = 10, guidefont = (12,:bold), tickfont = 10, guide = "x")


pth="/bench/PhD/Thesis/datasets/a37.csv"

a37df=DataFrame(CSV.read(pth))

atg_measure_dict=Dict{String,Vector{Float64}}()
spl_measure_dict=Dict{String,Vector{Float64}}()
ctl_measure_dict=Dict{String,Vector{Float64}}()
uninj_measure_dict=Dict{String,Vector{Float64}}()

msd=[atg_measure_dict,spl_measure_dict,ctl_measure_dict,uninj_measure_dict]

for dict in msd
    dict["DCMZ"]=Vector{Float64}()
    dict["VCMZ"]=Vector{Float64}()
    dict["TCMZ"]=Vector{Float64}()
    dict["NucPop"]=Vector{Float64}()
    dict["Sphericity"]=Vector{Float64}()
    dict["Volume"]=Vector{Float64}()
end
```

```

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for (gdf,dict) in zip(groupby(a37df, "Group"),msd)
    for row in eachrow(gdf)
        if row."D/V"=="D"
            push!(dict["DCMZ"],row."PCNA +ve")
            !ismissing(row."Total PCNA") && push!(dict["TCMZ"],row."Total PCNA")
            !ismissing(row."Total nuclei") && push!(dict["NucPop"],row."Total nuclei")
            push!(dict["Sphericity"],row."Sphericity")
            push!(dict["Volume"],row."Volume")
        else
            push!(dict["VCMZ"],row."PCNA +ve")
            dict["Sphericity"][end]+=row."Sphericity";dict["Sphericity"][end]/=2
            dict["Volume"][end]+=row."Volume";dict["Volume"][end]/=2
        end
    end
end


for md in msd
    md["NucPer"]=md["NucPop"]./md["TCMZ"]
end

gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=1.e-15
gmcdir="/bench/PhD/NGS_binaries/BSS/A37"


uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

n_models=50

max_μ=300.;min_μ=0.
max_λ=1.;min_λ=1e-5
prior=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
box=[min_μ max_μ;min_λ max_λ]

np_prior=[Uniform(min_μ,5000),Uniform(min_λ,max_λ)]
```

```
np_box=[min_μ 5000;min_λ max_λ]

s_max_μ=1.;s_min_μ=0.
s_max_λ=5e5;s_min_λ=1e3
s_prior=[Uniform(s_min_μ,s_max_μ),Uniform(s_min_λ,s_max_λ)]
s_box=[s_min_μ s_max_μ;s_min_λ s_max_λ]

totalmsvec=["TCMZ","NucPop","NucPer","DCMZ","VCMZ","Volume","Sphericity"]
popmsvec=["TCMZ","NucPop","NucPer","DCMZ","VCMZ"]
normmsvec=["Volume","Sphericity"]

for (md,morph) in zip([atg_measure_dict,spl_measure_dict],["ATG","Spl"])
    evdict=Dict{String,Measurement}()

    for ms in totalmsvec
        evdict[ms*"_Combined"]=measurement(0,0)
        evdict[ms*"_Separate"]=measurement(0,0)
    end


    for ms in popmsvec
        ms=="NucPop" ? (pr=np_prior; bx=np_box) : (pr=prior; bx=box)


        c_ens=gmcdir*"/" * morph * "_c_"*ms
        c_obs=vcat(md[ms],ctl_measure_dict[ms])
        if isfile(c_ens*"/ens")
            e=deserialize(c_ens*"/ens")
        else
            e=LogNormal_Ensemble(c_ens,n_models, c_obs, pr, bx, gmc ... )
        end


        evdict[ms*"_Combined"]+=converge_ensemble!(e,backup=(true,10000),upper_di
        lower_displays=lds, disp_rot_its=10000,
        converge_criterion="compression", converge_factor=1e-6)

        for (msd,p) in zip([md,ctl_measure_dict],["/"*morph*"_","/ctl_"])
            ens=gmcdir*p*ms
            obs=msd[ms]
            if isfile(ens*"/ens")
                e=deserialize(ens*"/ens")
            else
```

```
                e=LogNormal_Ensemble(ens,n_models, obs, pr, bx, gmc ... )
            end

            evdict[ms*"_Separate"]+=converge_ensemble!(e,backup=(true,10000),upper_displays=lds, disp_rot_its=10000, converge_criterion="compression", converge_factor=1e-6)
        end
    end

    for ms in normmsvec
        c_ens=gmcdir*"/" * morph * "_c_"*ms
        c_obs=vcat(md[ms],ctl_measure_dict[ms])
        if isfile(c_ens*"/ens")
            e=deserialize(c_ens*"/ens")
        else
            if ms=="Sphericity"
                e=Normal_Ensemble(c_ens,n_models, c_obs, s_prior, s_box, gmc ... )
            else
                e=Normal_Ensemble(c_ens,n_models, c_obs, prior, box, gmc ... )
            end
        end


        evdict[ms*"_Combined"]+=converge_ensemble!(e,backup=(true,10000),upper_displays=lds, disp_rot_its=10000, converge_criterion="compression", converge_factor=1e-6)

        for (msd,p) in zip([md,ctl_measure_dict],["/"*morph*"_","/ctl_"])
            ens=gmcdir*p*ms
            obs=msd[ms]
            if isfile(ens*"/ens")
                e=deserialize(ens*"/ens")
            else
                if ms=="Sphericity"
                    e=Normal_Ensemble(ens,n_models, obs, s_prior, s_box, gmc ... )
                else
                    e=Normal_Ensemble(ens,n_models, obs, prior, box, gmc ... )
                end
            end
```

```
138
139
                ↪ evdict[ms*"_Separate"]+=converge_ensemble!(e,backup=(true,10000),uppe
                ↪ lower_displays=lds, disp_rot_its=10000,
                ↪ converge_criterion="compression", converge_factor=1e-6)
140         end
141     end
142
143     for ms in totalmsvec
144         sev=evdict[ms*"_Separate"]
145         cev=evdict[ms*"_Combined"]
146         ratio=sev-cev
147         sig=ratio.val/ratio.err
148
149         println("$morph $ms & $(round(sev,digits=3)) &
            ↪ $(round(cev,digits=3)) & $(round(ratio,digits=3)) &
            ↪ $(round(sig,digits=1)) \\\\ \\hline")
150     end
151 end
152
153 colvec=[:darkmagenta,:darkorange,:green]
154
155 np_plt=plot_logn_MTDist([atg_measure_dict["NucPer"],spl_measure_dict["NucPer"],ctl_me
    ↪ Mo.","Spl Mo.","Ctrl Mo."],"Mean Sectional Population per CMZ
    ↪ Cell","Posterior mean lh.")
156
157 savefig(np_plt, "/bench/PhD/Thesis/images/rys/morphnuclei.png")
```

17.8.18 /rys/a38.jl

```
1 using CSV,DataFrames,Distributions,NGRefTools,StatsBase,StatsPlots
2 gr()
3 default(legendfont = 10, guidefont = (12,:bold), tickfont = 10, guide =
  ↪ "x")
4
5
6 pth="/bench/PhD/datasets/A38.csv"
7
8 a38df=DataFrame(CSV.read(pth))
9 a38df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
  ↪ eachrow(a38df)]
10 a38df.Group[1:10]•="sib"
```

```
a38df.Group[11:22]•="rys"

sib_measure_dict=Dict{String,Vector{Float64}}()
rys_measure_dict=Dict{String,Vector{Float64}}()

for dict in [sib_measure_dict,rys_measure_dict]
    dict["DCMZ"]=Vector{Float64}()
    dict["VCMZ"]=Vector{Float64}()
    dict["TCMZ"]=Vector{Float64}()
    dict["NucPop"]=Vector{Float64}()
    dict["Sphericity"]=Vector{Float64}()
    dict["Volume"]=Vector{Float64}()
end

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for (gdf,dict) in zip(groupby(a38df,
↪   "Group"),[sib_measure_dict,rys_measure_dict])
    for row in eachrow(gdf)
        if row."D/V"=="D"
            push!(dict["DCMZ"],row."PCNA +ve")
            push!(dict["TCMZ"],row."Total PCNA")
            push!(dict["NucPop"],row."Total nuclei")
            push!(dict["Sphericity"],row."Sphericity")
            push!(dict["Volume"],row."Volume")
        else
            push!(dict["VCMZ"],row."PCNA +ve")

                ↪  dict["Sphericity"][end]+=row."Sphericity";dict["Sphericity"][end]/=2
            dict["Volume"][end]+=row."Volume";dict["Volume"][end]/=2
        end
    end
end

#TEST IF DATA IS BETTER MODELLED NORMALLY OR LOGNORMALLY
for ms in ["DCMZ", "VCMZ", "NucPop", "Sphericity", "Volume"]
    n_ms=[fit(Normal,dict[ms]) for dict in [sib_measure_dict,
    ↪   rys_measure_dict]]
    logn_ms=[fit(LogNormal,dict[ms]) for dict in [sib_measure_dict,
    ↪   rys_measure_dict]]


    ↪   normal_lh=sum(logpdf(n_ms[1],sib_measure_dict[ms]))+sum(logpdf(n_ms[2],rys_me
```

```
    logn_lh=sum(logpdf(logn_ms[1],sib_measure_dict[ms]))+sum(logpdf(logn_ms[2],ry

    println("$ms & $(round(normal_lh,digits=3)) & $(round(logn_lh,digits=3)) & $(round(logn_lh-normal_lh,digits=3))\\\\ \\hline")
end


TCMZ_plt=plot_logn_MTDist([sib_measure_dict["TCMZ"],rys_measure_dict["TCMZ"]],[:green CMZ Population","Posterior mean lh.")
annotate!([(55,3.8,Plots.text("A",18))])

PopRatio_plt=plot_logn_MTDist([sib_measure_dict["NucPop"]./sib_measure_dict["TCMZ"],r Sectional Population per CMZ Cell","Posterior mean lh.")
annotate!([(7.5,3.,Plots.text("B",18))])

DCMZ_plt=plot_logn_MTDist([sib_measure_dict["DCMZ"],rys_measure_dict["DCMZ"]],[:green CMZ Population","Posterior mean lh.")
annotate!([(35,4.5,Plots.text("C",18))])

log_mean_mass_comparator(rys_measure_dict["DCMZ"],sib_measure_dict["DCMZ"],labels=["r 5dpf sectional DCMZ population","sib 5dpf sectional DCMZ population"])
log_mean_mass_comparator(rys_measure_dict["VCMZ"],sib_measure_dict["VCMZ"],labels=["r 5dpf sectional VCMZ population","sib 5dpf sectional VCMZ population"])

VCMZ_plt=plot_logn_MTDist([sib_measure_dict["VCMZ"],rys_measure_dict["VCMZ"]],[:green CMZ Population","Posterior mean lh.")
annotate!([(15,2.25,Plots.text("D",18))])


Vol_plt=plot_n_MTDist([sib_measure_dict["Volume"],rys_measure_dict["Volume"]],[:green nuclear volume","Posterior mean lh.")
annotate!([(.2,105,Plots.text("E",18))])

Sph_plt=plot_n_MTDist([sib_measure_dict["Sphericity"],rys_measure_dict["Sphericity"]] nuclear sphericity","Posterior mean lh.")
annotate!([(.65,325,Plots.text("F",18))])
plot!(legend=:top)

```

```
combined=plot(TCMZ_plt,PopRatio_plt,DCMZ_plt,VCMZ_plt,Vol_plt,Sph_plt,layout=grid(3,2
↪  size=(900,900))

savefig(combined,"/bench/PhD/Thesis/images/rys/nuclearstudy.png")
```

17.8.19 /rys/a38GMC_NS.jl

```
using CSV,DataFrames,Distributions,GMC_NS,Measurements,Serialization

pth="/bench/PhD/datasets/A38.csv"

a38df=DataFrame(CSV.read(pth))
a38df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in
↪  eachrow(a38df)]
a38df.Group[1:10]•="sib"
a38df.Group[11:22]•="rys"

sib_measure_dict=Dict{String,Vector{Float64}}()
rys_measure_dict=Dict{String,Vector{Float64}}()

for dict in [sib_measure_dict,rys_measure_dict]
    dict["DCMZ"]=Vector{Float64}()
    dict["VCMZ"]=Vector{Float64}()
    dict["TCMZ"]=Vector{Float64}()
    dict["NucPop"]=Vector{Float64}()
    dict["Sphericity"]=Vector{Float64}()
    dict["Volume"]=Vector{Float64}()
end

#EXTRACT ALL MEASUREMENTS FROM DF TO VECTORS
for (gdf,dict) in zip(groupby(a38df,
↪  "Group"),[sib_measure_dict,rys_measure_dict])
    for row in eachrow(gdf)
        if row."D/V"=="D"
            push!(dict["DCMZ"],row."PCNA +ve")
            push!(dict["TCMZ"],row."Total PCNA")
            push!(dict["NucPop"],row."Total nuclei")
            push!(dict["Sphericity"],row."Sphericity")
            push!(dict["Volume"],row."Volume")
        else
            push!(dict["VCMZ"],row."PCNA +ve")
```

```
                ↪  dict["Sphericity"][end]+=row."Sphericity";dict["Sphericity"][end]/=2
            dict["Volume"][end]+=row."Volume";dict["Volume"][end]/=2
        end
    end
end

for md in [sib_measure_dict,rys_measure_dict]
    md["NucPer"]=md["NucPop"]./md["TCMZ"]
end


gmc=GMC_DEFAULTS
gmc[1]=15
gmc[2]=1.e-15
gmcdir="/bench/PhD/NGS_binaries/GMC_NS/A38"


uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

n_models=150

evdict=Dict{String,Measurement}()

totalmsvec=["TCMZ","NucPer","DCMZ","VCMZ","Volume","Sphericity"]
popmsvec=["TCMZ","NucPer","DCMZ","VCMZ"]
normmsvec=["Volume","Sphericity"]

for ms in totalmsvec
    evdict[ms*"_Combined"]=measurement(0,0)
    evdict[ms*"_Separate"]=measurement(0,0)
end

max_μ=300.;min_μ=0.
max_λ=.3;min_λ=0.
prior=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
box=[min_μ max_μ;min_λ max_λ]

s_max_μ=1.;s_min_μ=0.
s_max_λ=5e5;s_min_λ=0.
s_prior=[Uniform(s_min_μ,s_max_μ),Uniform(s_min_λ,s_max_λ)]
s_box=[s_min_μ s_max_μ;s_min_λ s_max_λ]
```

```
for ms in popmsvec
    c_ens=gmcdir*"/c_"*ms
    c_obs=vcat(sib_measure_dict[ms],rys_measure_dict[ms])
    if isfile(c_ens*"/ens")
        e=deserialize(c_ens*"/ens")
    else
        e=LogNormal_Ensemble(c_ens,n_models, c_obs, prior, box, gmc ... )
    end


    evdict[ms*"_Combined"]+=converge_ensemble!(e,backup=(true,10000),upper_displa
    lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)

    for (msd,p) in zip([sib_measure_dict,rys_measure_dict],["/s_","/r_"])
        ens=gmcdir*p*ms
        obs=msd[ms]
        if isfile(ens*"/ens")
            e=deserialize(ens*"/ens")
        else
            e=LogNormal_Ensemble(ens,n_models, obs, prior, box, gmc ... )
        end

        evdict[ms*"_Separate"]+=converge_ensemble!(e,backup=(true,10000),upper_di
        lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)
    end
end

for ms in normmsvec
    c_ens=gmcdir*"/c_"*ms
    c_obs=vcat(sib_measure_dict[ms],rys_measure_dict[ms])
    if isfile(c_ens*"/ens")
        e=deserialize(c_ens*"/ens")
    else
        if ms=="Sphericity"
            e=Normal_Ensemble(c_ens,n_models, c_obs, s_prior, s_box,
            gmc ... )
        else
            e=Normal_Ensemble(c_ens,n_models, c_obs, prior, box, gmc ... )
        end
    end

```

```
    evdict[ms*"_Combined"]+=converge_ensemble!(e,backup=(true,10000),upper_displa
    lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)

    for (msd,p) in zip([sib_measure_dict,rys_measure_dict],["/s_","/r_"])
        ens=gmcdir*p*ms
        obs=msd[ms]
        if isfile(ens*"/ens")
            e=deserialize(ens*"/ens")
        else
            if ms=="Sphericity"
                e=Normal_Ensemble(ens,n_models, obs, s_prior, s_box,
                 gmc ... )
            else
                e=Normal_Ensemble(ens,n_models, obs, prior, box, gmc ... )
            end
        end


        evdict[ms*"_Separate"]+=converge_ensemble!(e,backup=(true,10000),upper_di
        lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)
    end
end

for ms in totalmsvec
    sev=evdict[ms*"_Separate"]
    cev=evdict[ms*"_Combined"]
    ratio=sev-cev
    sig=ratio.val/ratio.err

    println("$ms & $(round(sev,digits=3)) & $(round(cev,digits=3)) &
    $(round(ratio,digits=3)) & $(round(sig,digits=1)) \\\\ \\hline")
end
```

17.8.20 /rys/caspase.jl

```
using CSV, DataFrames, Plots, Distributions, Measures
import Plots:Plot

gr()

caspase_pth="/bench/PhD/Thesis/datasets/rys caspase.csv"
```

```
caspasedf=DataFrame(CSV.read(caspase_pth))

sib_ms_dict=Dict{String,Vector{Vector{Float64}}}()
rys_ms_dict=Dict{String,Vector{Vector{Float64}}}()

X=[4.,6.]

for md in [sib_ms_dict,rys_ms_dict]
    md["CMZ"]=[zeros(0) for i in 1:2]
    md["Border"]=[zeros(0) for i in 1:2]
    md["Central"]=[zeros(0) for i in 1:2]
    md["Total"]=[zeros(0) for i in 1:2]
end

for row in eachrow(caspasedf)
    row.dpf==4 ? (xidx=1) : (xidx=2)
    row.genotype=="sib" ? (md=sib_ms_dict) : (md=rys_ms_dict)
    push!(md["CMZ"][xidx],row.CMZ)
    push!(md["Border"][xidx],row."CMZ/central border region")
    push!(md["Central"][xidx],row."central retina")
    push!(md["Total"][xidx],row.CMZ + row."CMZ/central border region" +
     ↪  row."central retina")
end

plot_halves=Vector{Plot}()
for (n, age) in enumerate(X)
    subplots=Vector{Plot}()
    for (m,ms) in enumerate(["CMZ","Border","Central","Total"])
        s_data=sib_ms_dict[ms][n]
        sl=length(s_data)
        r_data=rys_ms_dict[ms][n]
        rl=length(r_data)
        if m == 1

             ↪  subplt=bar(1:1:sl,s_data,barwidths=1,color=:green,xticks=(1:1:sl+rl,v
             ↪  for i in 1:sl],["R" for i in 1:rl])),legend=:none,
             ↪  xlabel=ms, margin=0mm,ylabel="Caspase-3+ cells")
        else

             ↪  subplt=bar(1:1:sl,s_data,barwidths=1,color=:green,xticks=(1:1:sl+rl,v
             ↪  for i in 1:sl],["R" for i in 1:rl])),legend=:none,
             ↪  xlabel=ms, leftmargin=-7.5mm, yformatter=_→"")
        end
```

```
        bar!(sl+1:1:sl+rl,r_data,color=:darkmagenta, barwidths=1.)
        push!(subplots,subplt)
    end
    title = plot(title = "$(Int64(age)) dpf", grid = false, showaxis = nothing, bottom_margin = -35Plots.px)
    l=@layout [t{.02h};[a{.25w} b{.25w} c{.25w} d{.25w}]]
    half=plot(title, subplots ... ,layout=l,link=:y)
    push!(plot_halves,half)
end

combined=plot(plot_halves ... ,layout=grid(1,2), size=(900,450))

savefig(combined,"/bench/PhD/Thesis/images/rys/caspase.png")

println("Rys CMZ caspase 4dpf: $(mean(rys_ms_dict["CMZ"][1])) ± $(std(rys_ms_dict["CMZ"][1]))")
println("Rys CMZ caspase 6dpf: $(mean(rys_ms_dict["CMZ"][2])) ± $(std(rys_ms_dict["CMZ"][2]))")

println("Rys central caspase 4dpf: $(mean(rys_ms_dict["Central"][1])) ± $(std(rys_ms_dict["Central"][1]))")
println("Rys central caspase 6dpf:  $(mean(rys_ms_dict["Central"][2])) ± $(std(rys_ms_dict["Central"][2]))")
println("Sib central caspase 4dpf: $(mean(sib_ms_dict["Central"][1])) ± $(std(sib_ms_dict["Central"][1]))")
println("Sib central caspase 6dpf: $(mean(sib_ms_dict["Central"][2])) ± $(std(sib_ms_dict["Central"][2]))")
```

17.8.21 /rys/em.jl

```
using Plots, Images, FileIO, Plots.PlotMeasures

sib36=load("/bench/PhD/Thesis/images/rys/C2 cell 1/Rys sib C2 36k cell 1.jpg")
sib36=sib36[1:2465,:]
sib110=load("/bench/PhD/Thesis/images/rys/C2 cell 1/Rys sib C2 110k cell 1.jpg")
sib110=sib110[1:2465,:]
sib210=load("/bench/PhD/Thesis/images/rys/C2 cell 1/Rys sib C2 210k cell 1.jpg")
sib210=sib210[1:2465,:]

```

```
rys36=load("/bench/PhD/Thesis/images/rys/D1 cell 1/Rys mut D1 CMZ 36k.jpg")
rys36=rys36[1:2465,:]
rys110=load("/bench/PhD/Thesis/images/rys/D1 cell 1/Rys mut D1 CMZ 110k.jpg")
rys110=rys110[1:2465,:]
rys210=load("/bench/PhD/Thesis/images/rys/D1 cell 1/Rys mut D1 CMZ 210k.jpg")
rys210=rys210[1:2465,:]


s36p=plot(sib36, axis=nothing, border=:none, margin=0mm)
annotate!([(200,105,Plots.text("SIB",25,:white,:bold))])
s110p=plot(sib110, axis=nothing,  border=:none, margin=0mm)
annotate!([(200,105,Plots.text("SIB",25,:white,:bold))])
s210p=plot(sib210, axis=nothing, border=:none, margin=0mm)
annotate!([(200,105,Plots.text("SIB",25,:white,:bold))])
r36p=plot(rys36, axis=nothing, border=:none, margin=0mm)
annotate!([(200,105,Plots.text("RYS",25,:white,:bold))])
r110p=plot(rys110, axis=nothing,  border=:none, margin=0mm)
annotate!([(200,105,Plots.text("RYS",25,:white,:bold))])
r210p=plot(rys210, axis=nothing, border=:none, margin=0mm)
annotate!([(200,105,Plots.text("RYS",25,:white,:bold))])

combined=plot(s36p, r36p, s110p, r110p, s210p, r210p, layout=grid(3,2),size=(1200,2400))

savefig(combined, "/bench/PhD/Thesis/images/emtogimp.png")
```

17.8.22 /rys/occupancy.py

```
import os
import numpy as np
import pandas as pd
from Bio import SeqIO
import matplotlib.pyplot as plt
from matplotlib.gridspec import GridSpec
from matplotlib import cm
from matplotlib import colors

from PIL import Image
from io import BytesIO
```

```
def get_chr_props(genome):
    genome_dict = SeqIO.index(genome, &quot;fasta&quot;)
    length_dict = {}
    prop_dict = {}

    total_length = 0
    nonchr_length = 0
    for entry in genome_dict:
        length=len(genome_dict[entry].seq)
        if entry.isdigit():
            length_dict[entry] = length
            total_length += length
        else:
            nonchr_length += length
            total_length += length
    length_dict['nonchr']=nonchr_length

    for entry in length_dict:
        prop_dict[entry] = length_dict[entry]/total_length

    return prop_dict

def build_position_dict(df):
    observed_dict={}
    observed_nonchr_positions = 0

    for chr, chrgrp_data in df.groupby('chr'):
        length=len(chrgrp_data)
        if chr.isdigit():
            observed_dict[chr]=length
        else:
            observed_nonchr_positions+=length
    observed_dict['nonchr']=observed_nonchr_positions

    return observed_dict

def build_signal_dict(df):
    observed_dict={}
    observed_nonchr_signal = 0

    for chr, chrgrp_data in df.groupby('chr'):
        signal=chrgrp_data['smt_value'].sum()
```

```
        if chr.isdigit():
            observed_dict[chr]=signal
        else:
            observed_nonchr_signal+=signal
    observed_dict['nonchr']=observed_nonchr_signal

    return observed_dict

#relevant file locations
reference_genome = flow_variables['reference_genome_input']
sib_positions = os.path.join(input_table_1['PathToDanposResults'][0],'pooled',input_table_2['Danp
rys_positions = os.path.join(input_table_1['PathToDanposResults'][0],'pooled',input_table_2['Danp

#read in dataframes
sib_pos_df = pd.read_table(sib_positions, dtype={'chr':'category', 'start':'uint32', 'end':'uint32', 'smt_pos':'uint32', 'smt_value':'float32', 'fuzziness_score':'float32'})
rys_pos_df = pd.read_table(rys_positions, dtype={'chr':'category', 'start':'uint32', 'end':'uint32', 'smt_pos':'uint32', 'smt_value':'float32', 'fuzziness_score':'float32'})

#get proportional lengths of extrachromosomal material from reference genome
prop_dict=get_chr_props(reference_genome)

#get total positions and signals from sib and rys dfs
sib_total_positions=len(sib_pos_df)
sib_total_signal=sib_pos_df['smt_value'].sum()
rys_total_positions=len(rys_pos_df)
rys_total_signal=rys_pos_df['smt_value'].sum()

#build dict of expected position numbers and signal sums for sibs based on percentage of genome comprised by relevant chr
sib_expected_pos_d={}
sib_expected_sig_d={}
for entry in prop_dict:
    sib_expected_pos_d[entry]=sib_total_positions * prop_dict[entry]
    sib_expected_sig_d[entry]=sib_total_signal * prop_dict[entry]

#build dicts of observed position numbers and signal summit sums by chromosome
```

```
sib_posn_d=build_position_dict(sib_pos_df)
sib_sig_d=build_signal_dict(sib_pos_df)
rys_posn_d=build_position_dict(rys_pos_df)
rys_sig_d=build_signal_dict(rys_pos_df)

#build arrays of observed positions, signals, and proportional
↪ differences
sib_observed_positions=[]
sib_observed_signal=[]
rys_observed_positions=[]
rys_observed_signal=[]

rys_expected_posn_from_sib=[]
rys_expected_sig_from_sib=[]

sib_posn_dif_from_expected=[]
sib_sig_dif_from_expected=[]
rys_posn_dif_from_sib=[]
rys_sig_dif_from_sib=[]

#for chr1-25
for i in range (1,26):
    #build observed position &amp; signal arrays
    sib_observed_positions.append(sib_posn_d[str(i)])
    sib_observed_signal.append(sib_sig_d[str(i)])
    rys_observed_positions.append(rys_posn_d[str(i)])
    rys_observed_signal.append(rys_sig_d[str(i)])

    #build rys expected arrays

    ↪ rys_expected_posn_from_sib.append(rys_total_positions*(sib_posn_d[str(i)]/sib

    ↪ rys_expected_sig_from_sib.append(rys_total_signal*(sib_sig_d[str(i)]/sib_tota

    #build proportional difference arrays

    ↪ sib_posn_dif_from_expected.append((sib_posn_d[str(i)]-sib_expected_pos_d[str(

    ↪ sib_sig_dif_from_expected.append((sib_sig_d[str(i)]-sib_expected_sig_d[str(i)

    ↪ rys_posn_dif_from_sib.append((rys_posn_d[str(i)]-rys_expected_posn_from_sib[i

    ↪ rys_sig_dif_from_sib.append((rys_sig_d[str(i)]-rys_expected_sig_from_sib[i-1]
```

```
#lastly append nonchrosomal material to all arrays
sib_observed_positions.append(sib_posn_d['nonchr'])
sib_observed_signal.append(sib_sig_d['nonchr'])
rys_observed_positions.append(rys_posn_d['nonchr'])
rys_observed_signal.append(rys_sig_d['nonchr'])

rys_expected_posn_from_sib.append(rys_total_positions*(sib_posn_d['nonchr']/sib_total
rys_expected_sig_from_sib.append(rys_total_signal*(sib_sig_d['nonchr']/sib_total_sign

sib_posn_dif_from_expected.append((sib_posn_d['nonchr']-sib_expected_pos_d['nonchr'])
sib_sig_dif_from_expected.append((sib_sig_d['nonchr']-sib_expected_sig_d['nonchr'])/s
rys_posn_dif_from_sib.append((rys_posn_d['nonchr']-rys_expected_posn_from_sib[25])/ry
rys_sig_dif_from_sib.append((rys_sig_d['nonchr']-rys_expected_sig_from_sib[25])/rys_e

#!! sib nonchromosomal signal is blown out relative to other scaffolds.
 ↪ store value and annotate, color separately
NC_sig_dif = sib_sig_dif_from_expected[25]
sib_sig_dif_from_expected[25] = 0

#build proportional difference color arrays
sib_norm =
 ↪ colors.Normalize(vmin=-abs(max(sib_posn_dif_from_expected+sib_sig_dif_from_expect
 ↪ key=abs)),
 ↪ vmax=abs(max(sib_posn_dif_from_expected+sib_sig_dif_from_expected,
 ↪ key=abs)))
rys_norm =
 ↪ colors.Normalize(vmin=-abs(max(rys_posn_dif_from_sib+rys_sig_dif_from_sib,
 ↪ key=abs)), vmax=abs(max(rys_posn_dif_from_sib+rys_sig_dif_from_sib,
 ↪ key=abs)))

sib_pos_colors = cm.PiYG(sib_norm(sib_posn_dif_from_expected))
sib_sig_colors = cm.PiYG(sib_norm(sib_sig_dif_from_expected))
sib_sig_colors[25] = [0, .1, 1, 1]
rys_pos_colors = cm.PRGn(rys_norm(rys_posn_dif_from_sib))
rys_sig_colors = cm.PRGn(rys_norm(rys_sig_dif_from_sib))

#build figure
fig = plt.figure(figsize=(6,6.5))

gs1 = GridSpec (2,2, wspace=.1, hspace=.05, left=.05, right=.95, top=.95,
 ↪ bottom=.15)
pos_sib = plt.subplot(gs1[0, 0])
```

```
pos_rys = plt.subplot(gs1[0, 1])
sig_sib = plt.subplot(gs1[1, 0])
sig_rys = plt.subplot(gs1[1, 1])

pos_sib.set_ylabel('Positions', fontsize=12, weight='bold')
sig_sib.set_ylabel('Occupancy', fontsize=12, weight='bold')

pos_sib.text(.01,.95, 'A', transform=pos_sib.transAxes, fontweight='bold', va='top')
pos_rys.text(.01,.95, 'B', transform=pos_rys.transAxes, fontweight='bold', va='top')
sig_sib.text(.01,.95, 'C', transform=sig_sib.transAxes, fontweight='bold', va='top')
sig_rys.text(.01,.95, 'D', transform=sig_rys.transAxes, fontweight='bold', va='top')

gs2 = GridSpec(4, 2, wspace=.1, hspace=.05, left=.05, right=.95, top=.15, bottom=.05)
sib_cbax = plt.subplot(gs2[3, 0])
rys_cbax = plt.subplot(gs2[3, 1])

#colorbar stuff
ssm = cm.ScalarMappable(cmap=cm.PiYG, norm=sib_norm)
ssm._A =[]
sib_cb=plt.colorbar(ssm, cax=sib_cbax, orientation='horizontal', extend='max', ticks=[-abs(max(sib_posn_dif_from_expected+sib_sig_dif_from_expected, key=abs)), -.3, -.2, -.1, 0, .1, .2, .3])
sib_cb.ax.set_xticklabels(['{0:.2f}'.format(-abs(max(sib_posn_dif_from_expected+sib_s key=abs))),'-0.3','-0.2','-0.1','0.0','0.1','0.2','0.3'])
sib_cbax.set_xlabel('Fold difference from per-kb uniformity')
sib_cbax.xaxis.set_label_position('top')

rsm = cm.ScalarMappable(cmap=cm.PRGn, norm=rys_norm)
rsm._A =[]
rsm_cb=plt.colorbar(rsm, cax=rys_cbax, orientation='horizontal', ticks=[-abs(max(rys_posn_dif_from_sib+rys_sig_dif_from_sib, key=abs)), -.05, -.025, 0, .025, .05, abs(max(rys_posn_dif_from_sib+rys_sig_dif_from_sib, key=abs))])
rsm_cb.ax.set_xticklabels(['{0:.2f}'.format(-abs(max(rys_posn_dif_from_sib+rys_sig_di key=abs))),'-0.05','-0.025','0.0','0.025','0.05','{0:.2f}'.format(abs(max(rys_pos key=abs)))])
rys_cbax.set_xlabel('Fold difference from sib proportions')
```

```
rys_cbax.xaxis.set_label_position('top')

#labels for chr wedges
labels=list(range(1,26))
labels.append('NC')

#make the donut plots
pos_sib.pie(sib_observed_positions, colors=sib_pos_colors, radius=1, startangle=90, counterclock=False, wedgeprops=dict(width=.7,edgecolor='w'), labels=labels, textprops=dict(fontsize=6))
pos_sib.text(0.5, 0.5, 'sib', horizontalalignment='center', verticalalignment='center', transform=pos_sib.transAxes, weight='bold', fontsize='16')
pos_rys.pie(rys_observed_positions, colors=rys_pos_colors, radius=1, startangle=90, counterclock=False, wedgeprops=dict(width=.7,edgecolor='w'), labels=labels, textprops=dict(fontsize=6))
pos_rys.text(0.5, 0.5, 'rys', horizontalalignment='center', verticalalignment='center', transform=pos_rys.transAxes, weight='bold', fontsize='16')

wedges, texts = sig_sib.pie(sib_observed_signal, colors=sib_sig_colors, radius=1, startangle=90, counterclock=False, wedgeprops=dict(width=.7,edgecolor='w'), labels=labels, textprops=dict(fontsize=6))
sig_sib.text(0.5, 0.5, 'sib', horizontalalignment='center', verticalalignment='center', transform=sig_sib.transAxes, weight='bold', fontsize='16')
sig_rys.pie(rys_observed_signal, colors=rys_sig_colors, radius=1, startangle=90, counterclock=False, wedgeprops=dict(width=.7,edgecolor='w'), labels=labels, textprops=dict(fontsize=6))
sig_rys.text(0.5, 0.5, 'rys', horizontalalignment='center', verticalalignment='center', transform=sig_rys.transAxes, weight='bold', fontsize='16')

#annotate NC wedge
wedges[25].set_hatch('//')
ang = (wedges[25].theta2 - wedges[25].theta1)/2. + wedges[25].theta1
y = np.sin(np.deg2rad(ang))
x = np.cos(np.deg2rad(ang))
```

```
horizontalalignment = {-1: &quot;right&quot;, 1: &quot;left&quot;}[int(np.sign(x))]
connectionstyle = &quot;angle,angleA=0,angleB={}&quot;.format(ang)
kw = dict(xycoords='data', textcoords='data', arrowprops=dict(arrowstyle=&quot;-&quot;),
            zorder=0, va=&quot;center&quot;, fontsize=9)
kw[&quot;arrowprops&quot;].update({&quot;connectionstyle&quot;: connectionstyle})
sig_sib.annotate('{0:.2f}'.format(NC_sig_dif)+'-fold', xy=(x, y), xytext=(.8*np.sign(x), 1.30*y),
    horizontalalignment=horizontalalignment, **kw)

fig.savefig(os.path.join(input_table_1['PathToDanposResults'][0],'proportional_chromo
png_memory = BytesIO()
fig.savefig(png_memory, format='png', dpi=600)
PILpng = Image.open(png_memory)
PILpng.save(os.path.join(input_table_1['PathToDanposResults'][0],'proportional_chromo
png_memory.close()

# Copy input to output
output_table_1 = input_table_1.copy()
output_table_2 = input_table_2.copy()
```

17.8.23 /rys/qPCR.jl

```
using CSV, DataFrames, Measurements, Plots, Distributions
import Plots:Plot

rand6pth="/bench/PhD/Thesis/datasets/6dpfRys-Histones random primers qPCR.csv"
rand8pth="/bench/PhD/Thesis/datasets/8dpfRys-Histones random primers qPCR.csv"
dt6pth="/bench/PhD/Thesis/datasets/6dpfRys-Histones oligo dT qPCR.csv"
dt8pth="/bench/PhD/Thesis/datasets/8dpfRys-Histones oligo dT qPCR.csv"

rand6=DataFrame(CSV.read(rand6pth))
rand8=DataFrame(CSV.read(rand8pth))
dt6=DataFrame(CSV.read(dt6pth))
dt8=DataFrame(CSV.read(dt8pth))

randdfs=[rand6,rand8]
dtdfs=[dt6,dt8]
```

```
samples=Dict{Tuple{String,Int64},Vector{String}}()

measure_dict=Dict{Tuple{String,Int64,Int64,String,String},Vector{Float64}}()

for (df,d) in zip(randdfs,[6,8])
    samples[("rand",d)]=unique(df.Sample)
    for row in eachrow(df)
        str=("rand",d,row.Plate,row.Sample,row.Detector)
        if !ismissing(row.Ct)
            str in keys(measure_dict) ? push!(measure_dict[str],row.Ct) : measure_dict[str]=[row.Ct]
        end
    end
end

for (df,d) in zip(dtdfs,[6,8])
    samples[("dt",d)]=unique(df."Sample Name")
    for row in eachrow(df)
        str=("dt",d,row.Plate,row."Sample Name",row."Target Name")
        if !ismissing(row.CT)
            str in keys(measure_dict) ? push!(measure_dict[str],row.CT) : measure_dict[str]=[row.CT]
        end
    end
end

X=[6.,8.]
rand_sib=Dict{String,Vector{Vector{Measurement}}}()
rand_rys=Dict{String,Vector{Vector{Measurement}}}()
dt_sib=Dict{String,Vector{Vector{Measurement}}}()
dt_rys=Dict{String,Vector{Vector{Measurement}}}()


for ms in ["H2A","H2B","H3","H4"]
    for d in [6,8]
        xidx=findfirst(isequal(d),X)
        for plate in [1,2,3]
            wt_hk=measure_dict[("rand",d,plate,"WT cDNA 10x","ActinB")]
            length(wt_hk)>1 ? wt_hk_m=measurement(mean(wt_hk),std(wt_hk)) : wt_hk_m=measurement(mean(wt_hk),0.)
            wt_trg=measure_dict[("rand",d,plate,"WT cDNA 10x",ms)]
```

```
            length(wt_hk)>1 ? wt_trg_m=measurement(mean(wt_trg),std(wt_trg)) : wt_trg_m=measurement(mean(wt_trg),0.)
            wt_ΔCt=wt_trg_m-wt_hk_m

            for (smpl, msdict) in zip(["Sib cDNA 10x","Rys cDNA 10x"],[rand_sib,rand_rys])
                smpl_hk=measure_dict[("rand",d,plate,smpl,"ActinB")]
                length(smpl_hk)>1 ? smpl_hk_m=measurement(mean(smpl_hk),std(smpl_hk)) : smpl_hk_m=measurement(mean(smpl_hk),0.)
                smpl_trg=measure_dict[("rand",d,plate,smpl,ms)]
                length(wt_hk)>1 ? smpl_trg_m=measurement(mean(smpl_trg),std(smpl_trg)) : smpl_trg_m=measurement(mean(smpl_trg),0.)
                smpl_ΔCt=smpl_trg_m-smpl_hk_m
                ΔΔCt=smpl_ΔCt-wt_ΔCt
                fold_change=2^-ΔΔCt
                ms in keys(msdict) ? push!(msdict[ms][xidx],fold_change) : msdict[ms]=[[fold_change],Vector{Float64}()]
            end
        end
    end
end

msvec=["H2A","H2B","H3","H4"]

for ms in msvec
    for d in [6,8]
        xidx=findfirst(isequal(d),X)
        for plate in [1,2,3]
            wt_hk=measure_dict[("dt",d,plate,"WT cDNA","ActinB")]
            length(wt_hk)>1 ? wt_hk_m=measurement(mean(wt_hk),std(wt_hk)) : wt_hk_m=measurement(mean(wt_hk),0.)
            wt_trg=measure_dict[("dt",d,plate,"WT cDNA",ms)]
            length(wt_trg)>1 ? wt_trg_m=measurement(mean(wt_trg),std(wt_trg)) : wt_trg_m=measurement(mean(wt_trg),0.)
            wt_ΔCt=wt_trg_m-wt_hk_m

            for (smpl, msdict) in zip(["Sib cDNA","Rys cDNA"],[dt_sib,dt_rys])
                smpl_hk=measure_dict[("dt",d,plate,smpl,"ActinB")]
```

```
                length(smpl_hk)>1 ? smpl_hk_m=measurement(mean(smpl_hk),std(smpl_hk)) : smpl_hk_m=measurement(mean(smpl_hk),0.)
                smpl_trg=measure_dict[("dt",d,plate,smpl,ms)]
                length(smpl_trg)>1 ? smpl_trg_m=measurement(mean(smpl_trg),std(smpl_trg)) : smpl_trg_m=measurement(mean(smpl_trg),0.)
                smpl_ΔCt=smpl_trg_m-smpl_hk_m
                ΔΔCt=smpl_ΔCt-wt_ΔCt
                fold_change=2^-ΔΔCt
                ms in keys(msdict) ? push!(msdict[ms][xidx],fold_change) : msdict[ms]=[[fold_change],Vector{Float64}()]
            end
        end
    end
end

plot_halves=Vector{Plot}()
for ((sib_dict, rys_dict),assay_name) in zip([(rand_sib,rand_rys),(dt_sib,dt_rys)],["Total transcript", "Polyadenylated Transcript"])
    subplots=Vector{Plot}()
    for (n,ms) in enumerate(msvec)
        plt=0.
        mx=0.
        mnn=Inf
        for (dict,color,symbol,leg) in zip([sib_dict, rys_dict],[:green,:darkmagenta],[:circle,:diamond],["Sib","rys"])
            means=[mean(valvec) for valvec in dict[ms]]
            stdevs=[std(valvec) for valvec in dict[ms]]
            upper=Vector{Float64}()
            mns=[mn.val for mn in means]
            stds=[sd.val for sd in stdevs]
            lower=Vector{Float64}()

            for (mn,sd) in zip(mns,stds)
                dist=Normal(mn,sd)
                push!(upper,quantile(dist,.975)-mn)
                push!(lower,mn-quantile(dist,.025))
            end

            xs=vcat([[X[n] for i in 1:length(dict[ms][n])] for n in 1:length(X)] ... )
```

```
            ys=vcat([[m.val for m in dict[ms][n]] for n in 1:length(X)] ... )

            mx=max(mx,maximum(vcat(upper,ys)))
            mnn=min(mnn,minimum(vcat(lower,ys)))

            if dict===sib_dict
                if n!=4 && n!=1
                    plt=scatter(xs,ys, marker=symbol, color=color, markerstrokecolor=color, markersize=3, label=leg*" data", showaxis=:y, xticks=X, xformatter=_->"", ylabel="Fold change",xlims=[5.5,8.5],legend=:none)
                elseif n==1
                    plt=scatter(xs,ys, marker=symbol, color=color, markerstrokecolor=color, markersize=3, label=leg*" data", showaxis=:y, xticks=X, xformatter=_->"", ylabel="Fold change",xlims=[5.5,8.5],legend=:topleft, title=assay_name)
                else
                    plt=scatter(xs,ys, marker=symbol, color=color, markerstrokecolor=color, markersize=3, label=leg*" data", xticks=X, ylabel="Fold change", xlabel="Age (dpf)", xlims=[5.5,8.5],legend=:none)
                end
            else
                scatter!(plt, xs,ys, marker=symbol, color=color, markerstrokecolor=color, markersize=3, label=leg)
            end

            plot!(plt, X, mns, ribbon=(lower,upper), color=color, label=leg*" mean")
        end
        plot!(X, [1. for i in 1:length(X)], linewidth=4., style=:dot, color=:black, label="WT std.")
        annotate!([(5.75,mnn+.25*(mx-mnn),Plots.text(ms,16))])
        println(ms)
        println("max: $mx min: $mnn")
        println(mnn+.25*(mx-mnn))
        push!(subplots,plt)
    end
```

```
    assay_combined=plot(subplots ... ,layout=grid(4,1),link=:x)
    push!(plot_halves,assay_combined)
end

combined=plot(plot_halves ... ,layout=grid(1,2),size=(900,900))
savefig(combined,"/bench/PhD/Thesis/images/rys/qPCR.png")

stdmat=zeros(4,2,2,2)

for ((sib_dict, rys_dict),assay_idx) in zip([(rand_sib,rand_rys),(dt_sib,dt_rys)],[1,2])
    for (n,ms) in enumerate(msvec)
        sib_means=0.
        for dict in [sib_dict, rys_dict]
            means=[measurement(mean(valvec).val,std(valvec).val) for valvec in dict[ms]]
            if dict===sib_dict
                sib_means=means
            else
                sdifs=[measurement(mean(valvec).val,std(valvec).val) for valvec in dict[ms]].-sib_means
                stds=[v.val/v.err for v in sdifs]
                stdmat[n,assay_idx,1,:]=stds

                wtdifs=[measurement(mean(valvec).val,std(valvec).val) for valvec in dict[ms]].-1.
                stds=[v.val/v.err for v in wtdifs]
                stdmat[n,assay_idx,2,:]=stds
            end
        end
    end
end

for (a,assay) in enumerate(["Total","polyA"])
    for (m,ms) in enumerate(msvec)
        for (d,age) in enumerate([6,8])
            println("$assay & $ms & $age & $(round(stdmat[m,a,1,d],digits=2)) & $(round(stdmat[m,a,2,d],digits=2))\\\\ \\hline")
        end
    end
end
```

```

rr_h2a=[Normal(m,s) for (m,s) in zip(mean.(rand_rys["H2A"]),std.(rand_rys["H2A"]))]
rs_h2a=[Normal(m,s) for (m,s) in zip(mean.(rand_sib["H2A"]),std.(rand_sib["H2A"]))]
rand_h2a_joint=exp(sum(logccdf.(rr_h2a,[d.μ for d in rs_h2a])))

rr_h2b=[Normal(m,s) for (m,s) in zip(mean.(rand_rys["H2B"]),std.(rand_rys["H2B"]))]
rs_h2b=[Normal(m,s) for (m,s) in zip(mean.(rand_sib["H2B"]),std.(rand_sib["H2B"]))]
rand_h2b_joint=exp(sum(logccdf.(rr_h2b,[d.μ for d in rs_h2b])))

dtr_h2a=[Normal(m,s) for (m,s) in zip(mean.(dt_rys["H2A"]),std.(dt_rys["H2A"]))]
dts_h2a=[Normal(m,s) for (m,s) in zip(mean.(dt_sib["H2A"]),std.(dt_sib["H2A"]))]
dt_h2a_joint=exp(sum(logccdf.(dtr_h2a,[d.μ for d in dts_h2a])))

dtr_h2b=[Normal(m,s) for (m,s) in zip(mean.(dt_rys["H2B"]),std.(dt_rys["H2B"]))]
dts_h2b=[Normal(m,s) for (m,s) in zip(mean.(dt_sib["H2B"]),std.(dt_sib["H2B"]))]
dt_h2b_joint=exp(sum(logccdf.(dtr_h2b,[d.μ for d in dts_h2b])))

all_joint=rand_h2b_joint*dt_h2a_joint*dt_h2b_joint
```

17.8.24 /rys/rysp_GMC_NS.jl

```
using CSV,DataFrames,Distributions,GMC_NS,Measurements,Serialization

a38pth="/bench/PhD/datasets/A38.csv"
a49pth="/bench/PhD/datasets/A49.csv"

a38df=DataFrame(CSV.read(a38pth))
a38df.Group[1:10]•="sib"
a38df.Group[11:22]•="rys"
a49df=DataFrame(CSV.read(a49pth))
a49df=a49df[2:end,:]

a38df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a38df)]
```

```

sib_measure_dict=Dict{String,Vector{Vector{Float64}}}()
rys_measure_dict=Dict{String,Vector{Vector{Float64}}}()

XCMZ=Vector{Float64}()
XEdU=Vector{Float64}()

for dict in [sib_measure_dict,rys_measure_dict]
    dict["DCMZ"]=Vector{Float64}()
    dict["VCMZ"]=Vector{Float64}()
    dict["TCMZ"]=Vector{Float64}()
    dict["EdU"]=Vector{Float64}()
end

for tdf in groupby(a49df,"Age")
    age=unique(tdf."Age")[1]
    push!(XCMZ,age);push!(XEdU,age)
    xidx=length(XCMZ)
    for sdf in groupby(tdf, "Rys/Sib")
        sr=unique(sdf."Rys/Sib")[1]
        for mdf in groupby(sdf,"D/V/P/WE")
            if unique(mdf."D/V/P/WE")[1] == "D"
                if sr=="sib"
                    push!(sib_measure_dict["DCMZ"],mdf."PCNA +ve")
                    push!(sib_measure_dict["TCMZ"],mdf."PCNA +ve")
                    push!(sib_measure_dict["EdU"],mdf."EdU +ve, PCNA +ve")
                else
                    push!(rys_measure_dict["DCMZ"],mdf."PCNA +ve")
                    push!(rys_measure_dict["TCMZ"],mdf."PCNA +ve")
                    push!(rys_measure_dict["EdU"],mdf."EdU +ve, PCNA +ve")
                end
            elseif unique(mdf."D/V/P/WE")[1] == "V"
                if sr=="sib"
                    push!(sib_measure_dict["VCMZ"],mdf."PCNA +ve")
                    sib_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
                    sib_measure_dict["EdU"][xidx].+=mdf."EdU +ve, PCNA +ve"
                else
                    push!(rys_measure_dict["VCMZ"],mdf."PCNA +ve")
                    rys_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
```

```
                    rys_measure_dict["EdU"][xidx].+=mdf."EdU +ve, PCNA +ve"
                end
            end
        end
    end
end

push!(XCMZ,5.)
for sdf in groupby(a38df,"Group")
    xidx=length(XCMZ)
    if unique(sdf."Group")[1]=="sib"
        for mdf in groupby(sdf,"D/V")
            if unique(mdf."D/V")[1] == "D"
                push!(sib_measure_dict["DCMZ"],mdf."PCNA +ve")
                push!(sib_measure_dict["TCMZ"],mdf."PCNA +ve")
            else
                push!(sib_measure_dict["VCMZ"],mdf."PCNA +ve")
                sib_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
            end
        end
    else
        for mdf in groupby(sdf,"D/V")
            if unique(mdf."D/V")[1] == "D"
                push!(rys_measure_dict["DCMZ"],mdf."PCNA +ve")
                push!(rys_measure_dict["TCMZ"],mdf."PCNA +ve")
            else
                push!(rys_measure_dict["VCMZ"],mdf."PCNA +ve")
                rys_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
            end
        end
    end
end

perm=sortperm(XCMZ)
for ms in ["DCMZ","VCMZ","TCMZ"]
    sib_measure_dict[ms]=sib_measure_dict[ms][perm]
    rys_measure_dict[ms]=rys_measure_dict[ms][perm]
end
XCMZ=[4.,10.]

for msd in [sib_measure_dict,rys_measure_dict]
    for (k,v) in msd
```

```
        for vec in v
            if any(i→i==0,vec)
                vec[findall(i→i==0,vec)]•=1e-3
            end
        end
    end
end

gmc=GMC_DEFAULTS
gmc[1]=5
gmc[2]=1.e-15
gmc[end]=2e4

uds=Vector{Vector{Function}}([[convergence_display],[evidence_display],[info_display]
lds=Vector{Vector{Function}}([[model_obs_display],[ensemble_display]])

n_models=50

evdict=Dict{String,Measurement}()

for ms in ["TCMZ","EdU"]
    evdict[ms*"_Combined"]=measurement(0,0)
    evdict[ms*"_Separate"]=measurement(0,0)
end

max_μ=2000.
min_μ=0.
max_λ=500.
min_λ=1e-6

prior=[Uniform(min_μ,max_μ),Uniform(min_λ,max_λ)]
box=[min_μ max_μ;min_λ max_λ]

gmcdir="/bench/PhD/NGS_binaries/GMC_NS/rysp/"

for ms in ["TCMZ","EdU"]
    for (n,x) in enumerate(XCMZ)
        c_ens=gmcdir*"/c_"*ms*string(Int64(x))
        c_obs=vcat(sib_measure_dict[ms][n],rys_measure_dict[ms][n])
        if isfile(c_ens*"/ens")
            e=deserialize(c_ens*"/ens")
        else
```

```
            e=LogNormal_Ensemble(c_ens,n_models, c_obs, prior, box, gmc ... )
        end


        evdict[ms*"_Combined"]+=converge_ensemble!(e,backup=(true,10000),upper_di
        lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)


        for (msd,p) in zip([sib_measure_dict,rys_measure_dict],["/s_","/r_"])
            ens=gmcdir*p*ms*string(Int64(x))
            obs=msd[ms][n]
            if isfile(ens*"/ens")
                e=deserialize(ens*"/ens")
            else
                e=LogNormal_Ensemble(ens,n_models, obs, prior, box, gmc ... )
            end

            evdict[ms*"_Separate"]+=converge_ensemble!(e,backup=(true,10000),uppe
            lower_displays=lds, disp_rot_its=10000, converge_factor=1e-6)
        end
    end
end
```

17.8.25 /rys/ryspont.jl

```
using CSV,DataFrames,Distributions,NGRefTools,StatsBase,StatsPlots
gr()
default(legendfont = (8), guidefont = (12,:bold), tickfont = (8), guide = "x")


a38pth="/bench/PhD/datasets/A38.csv"
a49pth="/bench/PhD/datasets/A49.csv"

a38df=DataFrame(CSV.read(a38pth))
a38df.Group[1:10]•="sib"
a38df.Group[11:22]•="rys"
a49df=DataFrame(CSV.read(a49pth))
```

```
a49df=a49df[2:end,:]

a38df.BSR=[string(row.Block,',',row.Slide,',',row.Row) for row in eachrow(a38df)]

sib_measure_dict=Dict{String,Vector{Vector{Float64}}}()
rys_measure_dict=Dict{String,Vector{Vector{Float64}}}()

XCMZ=Vector{Float64}()
XEdU=Vector{Float64}()

for dict in [sib_measure_dict,rys_measure_dict]
    dict["DCMZ"]=Vector{Float64}()
    dict["VCMZ"]=Vector{Float64}()
    dict["TCMZ"]=Vector{Float64}()
    dict["EdU"]=Vector{Float64}()
end

for tdf in groupby(a49df,"Age")
    age=unique(tdf."Age")[1]
    push!(XCMZ,age);push!(XEdU,age)
    xidx=length(XCMZ)
    for sdf in groupby(tdf, "Rys/Sib")
        sr=unique(sdf."Rys/Sib")[1]
        for mdf in groupby(sdf,"D/V/P/WE")
            if unique(mdf."D/V/P/WE")[1] == "D"
                if sr=="sib"
                    push!(sib_measure_dict["DCMZ"],mdf."PCNA +ve")
                    push!(sib_measure_dict["TCMZ"],mdf."PCNA +ve")
                    push!(sib_measure_dict["EdU"],mdf."EdU +ve, PCNA +ve")
                else
                    push!(rys_measure_dict["DCMZ"],mdf."PCNA +ve")
                    push!(rys_measure_dict["TCMZ"],mdf."PCNA +ve")
                    push!(rys_measure_dict["EdU"],mdf."EdU +ve, PCNA +ve")
                end
            elseif unique(mdf."D/V/P/WE")[1] == "V"
                if sr=="sib"
                    push!(sib_measure_dict["VCMZ"],mdf."PCNA +ve")
                    sib_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
                    sib_measure_dict["EdU"][xidx].+=mdf."EdU +ve, PCNA +ve"
```

```
                else
                    push!(rys_measure_dict["VCMZ"],mdf."PCNA +ve")
                    rys_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
                    rys_measure_dict["EdU"][xidx].+=mdf."EdU +ve, PCNA +ve"
                end
            end
        end
    end
end

push!(XCMZ,5.)
for sdf in groupby(a38df,"Group")
    xidx=length(XCMZ)
    if unique(sdf."Group")[1]=="sib"
        for mdf in groupby(sdf,"D/V")
            if unique(mdf."D/V")[1] == "D"
                push!(sib_measure_dict["DCMZ"],mdf."PCNA +ve")
                push!(sib_measure_dict["TCMZ"],mdf."PCNA +ve")
            else
                push!(sib_measure_dict["VCMZ"],mdf."PCNA +ve")
                sib_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
            end
        end
    else
        for mdf in groupby(sdf,"D/V")
            if unique(mdf."D/V")[1] == "D"
                push!(rys_measure_dict["DCMZ"],mdf."PCNA +ve")
                push!(rys_measure_dict["TCMZ"],mdf."PCNA +ve")
            else
                push!(rys_measure_dict["VCMZ"],mdf."PCNA +ve")
                rys_measure_dict["TCMZ"][xidx].+=mdf."PCNA +ve"
            end
        end
    end
end

perm=sortperm(XCMZ)
for ms in ["DCMZ","VCMZ","TCMZ"]
    sib_measure_dict[ms]=sib_measure_dict[ms][perm]
    rys_measure_dict[ms]=rys_measure_dict[ms][perm]
end
XCMZ=XCMZ[perm]
```

```
sp_logn_mts=[fit(MarginalTDist,log.(sib_measure_dict["TCMZ"][n])) for n in 1:length(XCMZ)]
sp_mean=[exp(mean(mt)) for mt in sp_logn_mts]
sp_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in sp_logn_mts]
sp_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in sp_logn_mts]

rp_logn_mts=[fit(MarginalTDist,log.(rys_measure_dict["TCMZ"][n])) for n in 1:length(XCMZ)]
rp_mean=[exp(mean(mt)) for mt in rp_logn_mts]
rp_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in rp_logn_mts]
rp_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in rp_logn_mts]

log_mean_mass_comparator(rys_measure_dict["TCMZ"][1],sib_measure_dict["TCMZ"][1],labe
$(XCMZ[1]) dpf sectional CMZ population","sib $(XCMZ[1]) dpf sectional CMZ population"])
log_mean_mass_comparator(rys_measure_dict["TCMZ"][end],sib_measure_dict["TCMZ"][end],
$(XCMZ[end]) dpf sectional CMZ population","sib $(XCMZ[end]) dpf sectional CMZ population"])

println("At 4dpf, $(mean(sib_measure_dict["EdU"][1]./sib_measure_dict["TCMZ"][1])) ± $(std(sib_measure_dict["EdU"][1]./sib_measure_dict["TCMZ"][1])) of sib PCNA+ve are EdU positive")

println("At 4dpf, $(mean(rys_measure_dict["EdU"][1]./rys_measure_dict["TCMZ"][1])) ± $(std(rys_measure_dict["EdU"][1]./rys_measure_dict["TCMZ"][1])) of rys PCNA+ve are EdU positive")

println("At 10dpf, $(mean(sib_measure_dict["EdU"][end]./sib_measure_dict["TCMZ"][end])) ± $(std(sib_measure_dict["EdU"][end]./sib_measure_dict["TCMZ"][end])) of sib PCNA+ve are EdU positive")


println("At 10dpf, $(mean(rys_measure_dict["EdU"][end]./rys_measure_dict["TCMZ"][end])) ± $(std(rys_measure_dict["EdU"][end]./rys_measure_dict["TCMZ"][end])) of rys PCNA+ve are EdU positive")

se_logn_mts=[fit(MarginalTDist,log.(sib_measure_dict["EdU"][n])) for n in 1:length(XEdU)]
```

```
se_mean=[exp(mean(mt)) for mt in se_logn_mts]
se_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in se_logn_mts]
se_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in se_logn_mts]

re_logn_mts=[fit(MarginalTDist,log.(filter(val→!iszero(val),rys_measure_dict["EdU"][
    for n in 1:length(XEdU)]
re_mean=[exp(mean(mt)) for mt in re_logn_mts]
re_lower=[exp(mean(mt))-exp(quantile(mt,.025)) for mt in re_logn_mts]
re_upper=[exp(quantile(mt,.975))-exp(mean(mt)) for mt in re_logn_mts]


pcna_plt=plot(XCMZ,sp_mean,ribbon=(sp_lower,sp_upper),color=:green,
    label="sib mean",xlabel="Age (dpf)",ylabel="PCNA+ve CMZ Cells")
plot!(pcna_plt,XCMZ,rp_mean,ribbon=(rp_lower,rp_upper),color=:darkmagenta,
    label="rys mean",legend=:none)
scatter!(pcna_plt,vcat([[XCMZ[n] for i in
    1:length(sib_measure_dict["TCMZ"][n])] for n in
    1:length(XCMZ)]...),vcat(sib_measure_dict["TCMZ"]...),marker=:circle,markersize=:
    data")
scatter!(pcna_plt,vcat([[XCMZ[n] for i in
    1:length(rys_measure_dict["TCMZ"][n])] for n in
    1:length(XCMZ)]...),vcat(rys_measure_dict["TCMZ"]...),marker=:dtriangle,markersiz
    data")
annotate!([(4.5,175,Plots.text("A",18))])

edu_plt=plot(XEdU,se_mean,ribbon=(se_lower,se_upper),color=:green,
    label="sib mean",xlabel="Age (dpf)",ylabel="EdU+ve CMZ Cells")
plot!(edu_plt,XEdU,re_mean,ribbon=(re_lower,re_upper),color=:darkmagenta,
    label="rys EdU")
scatter!(edu_plt,vcat([[XEdU[n] for i in
    1:length(sib_measure_dict["EdU"][n])] for n in
    1:length(XEdU)]...),vcat(sib_measure_dict["EdU"]...),marker=:circle,markersize=:3
    data")
scatter!(edu_plt,vcat([[XEdU[n] for i in
    1:length(rys_measure_dict["EdU"][n])] for n in
    1:length(XEdU)]...),vcat(rys_measure_dict["EdU"]...),marker=:dtriangle,markersize
    data")
annotate!([(4.5,150,Plots.text("B",18))])


combined_plt=plot(pcna_plt,edu_plt,layout=grid(1,2),size=(900,500))
savefig(combined_plt,"/bench/PhD/Thesis/images/rys/CMZontogeny.png")
```

17.8.26 /rys/bg_models/BBM_refinement_analysis.jl

```
using BioBackgroundModels, Serialization

hmm_output = "/bench/PhD/NGS_binaries/BBM/refined_chains"
sample_output = "/bench/PhD/NGS_binaries/BBM/refinement_samples"

survey_folders = "/bench/PhD/NGS_binaries/BBM/refined_folders"

chains=deserialize(hmm_output)
sample_dfs = deserialize(sample_output)
training_sets, test_sets = split_obs_sets(sample_dfs)

report_folders=generate_reports(chains, test_sets)
serialize(survey_folders, report_folders)
```

17.8.27 /rys/bg_models/BBM_refinement_prep.jl

```
#JOB FILEPATHS
#GFF3 feature database, FASTA genome and index paths
danio_gff_path = "/bench/PhD/seq/GRCz11/Danio_rerio.GRCz11.94.gff3"
danio_genome_path =
↪  "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna"
danio_gen_index_path =
↪  "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna.fai"
#sample record and hmm serialisation output path
sample_output = "/bench/PhD/NGS_binaries/BBM/refinement_samples"
#non registered libs

#GENERAL SETUP
@info "Loading libraries ..."
using Distributed,Serialization

#JOB CONSTANTS
#CONSTANTS FOR GENOMIC SAMPLING
const sample_set_length = Int64(16e6)
const sample_window_min = 10
const sample_window_max = 3000
const perigenic_pad = 500
const partitions = 3 #exonic, periexonic, intragenic

#Setup sampling workers
```

```
@info "Spawning workers ..."
pool_size = partitions                          #number of workers to use
worker_pool = addprocs(pool_size) # add processes up to the worker pool size + 1 control process
@everywhere using BioBackgroundModels, Random
@everywhere Random.seed!(1)

#GET SEQUENCE OBSERVATIONS TO TRAIN AND TEST MODELS - PARTITIONING GENOME INTO EXONIC, PERIEXONIC, INTRAGENIC SEQUENCE
@info "Building observation db ..."
#generate the genomic sample dataframe
if isfile(sample_output) #if the sample DB has already been built for the current project, terminate
    @error "Existing sample dataframe at $sample_output"
else #otherwise, build it from scratch
    @info "Setting up sampling jobs ..."
    #setup worker channels, input gets the genome partitions
    channels = setup_sample_jobs(danio_genome_path, danio_gen_index_path, danio_gff_path, sample_set_length, sample_window_min, sample_window_max, perigenic_pad)

    @info "Sampling genome ..."
    sample_record_dfs=execute_sample_jobs(channels, worker_pool)

    @info "Serializing sample dataframes ..."
    serialize(sample_output, sample_record_dfs) #write the dataframes to file
end

rmprocs(worker_pool)

@info "Done sampling jobs!"
```

17.8.28 /rys/bg_models/BBM_sample_prep.jl

```
#JOB FILEPATHS
#GFF3 feature database, FASTA genome and index paths
danio_gff_path = "/bench/PhD/seq/GRCz11/Danio_rerio.GRCz11.94.gff3"
danio_genome_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna"
danio_gen_index_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna.fai"
```

```
#sample record and hmm serialisation output path
sample_output = "/bench/PhD/NGS_binaries/BBM/survey_samples"
#non registered libs

#GENERAL SETUP
@info "Loading libraries..."
using Distributed,Serialization

#JOB CONSTANTS
#CONSTANTS FOR GENOMIC SAMPLING
const sample_set_length = Int64(4e6)
const sample_window_min = 10
const sample_window_max = 3000
const perigenic_pad = 500
const partitions = 3 #exonic, periexonic, intragenic

#Setup sampling workers
@info "Spawning workers..."
pool_size = partitions                          #number of workers to use
worker_pool = addprocs(pool_size) # add processes up to the worker pool size
@everywhere using BioBackgroundModels, Random
@everywhere Random.seed!(1)

#GET SEQUENCE OBSERVATIONS TO TRAIN AND TEST MODELS - PARTITIONING GENOME INTO EXONIC, PERIEXONIC, INTRAGENIC SEQUENCE
@info "Building observation db..."
#generate the genomic sample dataframe
if isfile(sample_output) #if the sample DB has already been built for the current project, terminate
    @error "Existing sample dataframe at $sample_output"
else #otherwise, build it from scratch
    @info "Setting up sampling jobs..."
    #setup worker channels, input gets the genome partitions
    channels = setup_sample_jobs(danio_genome_path, danio_gen_index_path, danio_gff_path, sample_set_length, sample_window_min, sample_window_max, perigenic_pad)

    @info "Sampling genome..."
    sample_record_dfs=execute_sample_jobs(channels, worker_pool)

    @info "Serializing sample dataframes..."
```

```
    serialize(sample_output, sample_record_dfs) #write the dataframes to file
end

rmprocs(worker_pool)

@info "Done sampling jobs!"
```

17.8.29 /rys/bg_models/BBM_survey.jl

```
#JOB FILEPATHS
#sample record and hmm serialisation output path
sample_output = "/bench/PhD/NGS_binaries/BBM/survey_samples"
hmm_output = "/bench/PhD/NGS_binaries/BBM/survey_chains"

#GENERAL SETUP
@info "Loading libraries..."
using BioBackgroundModels, DataFrames, Distributed, Serialization
include("/srv/git/rys_nucleosomes/aws/aws_wrangler.jl")

#JOB CONSTANTS
const replicates = 3 #repeat optimisation from this many seperately initialised samples from the prior
const Ks = [1,2,4,6] #mosaic class #s to test
const order_nos = [0,1,2] #DNA kmer order #s to test
const delta_thresh=1e-3 #stopping/convergence criterion (log probability difference btw subsequent EM iterates)
const max_iterates=15000

#LOAD SAMPLES
@info "Loading samples from $sample_output..."
sample_dfs = deserialize(sample_output)

#BUILD TRAINING AND TEST SETS FROM SAMPLES
training_sets, test_sets = split_obs_sets(sample_dfs)

#PROGRAMATICALLY GENERATE Chain_ID Vector
job_ids=Vector{Chain_ID}()
for (obs_id, obs) in training_sets, K in Ks, order in order_nos, rep in 1:replicates
    push!(job_ids, Chain_ID(obs_id, K, order, rep))
end
```

```
#DISTRIBUTED CLUSTER CONSTANTS
load_dict=Dict{Int64,LoadConfig}()
local_config=LoadConfig(1:6,2:2)
remote_config=LoadConfig(1:6,0:1)
aws_instance_config=LoadConfig(1:6,0:2)
remote_machine = "10.0.0.3"
no_local_processes = 1
no_remote_processes = 1
#SETUP DISTRIBUTED BAUM WELCH LEARNERS
@info "Spawning local cluster workers ..."
worker_pool=addprocs(no_local_processes, topology=:master_worker)
for worker in worker_pool
    load_dict[worker]=local_config
end

remote_pool=addprocs([(remote_machine,no_remote_processes)], tunnel=true,
↪   topology=:master_worker)

for worker in remote_pool
    load_dict[worker]=remote_config
end

worker_pool=vcat(worker_pool, remote_pool)

#AWS PARAMS
@info "Setting up AWS wrangling ..."

security_group_name="calc1"
security_group_desc="calculation group"
ami="ami-02df7e1881a08f163"
skeys="AWS"
instance_type="c5.4xlarge"
zone,spot_price=get_cheapest_zone(instance_type)
no_instances=1
instance_workers=8
bid=spot_price+.01

@assert bid ≥ spot_price

@info "Wrangling AWS instances ..."
aws_ips = spot_wrangle(no_instances, bid, security_group_name,
↪   security_group_desc, skeys, zone, ami, instance_type)
```

```
@info "Giving instances 90s to boot ..."
sleep(90)

@info "Spawning AWS cluster workers ..."
for ip in aws_ips
    instance_pool=addprocs([(ip, instance_workers)], tunnel=true,
    ↪ topology=:master_worker, sshflags="-o StrictHostKeyChecking=no")
    for worker in instance_pool
        load_dict[worker]=aws_instance_config
    end
    global worker_pool=vcat(worker_pool, instance_pool)
end

@info "Loading worker libraries everywhere ..."
@everywhere using BioBackgroundModels
#INTIIALIZE HMMS
@info "Setting up HMMs ..."
if isfile(hmm_output) #if some results have already been collected, load
↪ them
    @info "Loading incomplete results ..."
    hmm_results_dict = deserialize(hmm_output)
else #otherwise, pass a new results dict
    @info "Initialising new HMM results file at $hmm_output"
    hmm_results_dict = Dict{Chain_ID,Vector{EM_step}}()
end

em_jobset = setup_EM_jobs!(job_ids, training_sets;
↪ chains=hmm_results_dict)
execute_EM_jobs!(worker_pool, em_jobset ... , hmm_output;
↪ load_dict=load_dict, delta_thresh=delta_thresh,
↪ max_iterates=max_iterates)
```

17.8.30 /rys/bg_models/BBM_survey_analysis.jl

```
using BioBackgroundModels, Serialization, Plots, Measures
import Measures:mm
import Plots:Plot

hmm_output = "/bench/PhD/NGS_binaries/BBM/survey_chains"
sample_output = "/bench/PhD/NGS_binaries/BBM/survey_samples"

survey_folders = "/bench/PhD/NGS_binaries/BBM/survey_folders"
```

```julia

chains=deserialize(hmm_output)
sample_dfs = deserialize(sample_output)
training_sets, test_sets = split_obs_sets(sample_dfs)

report_folders=generate_reports(chains, test_sets)
serialize(survey_folders, report_folders)

partplts=Vector{Plot}()

for o in [0,1,2]
    for p in ["exon","intergenic","periexonic"]
        rf=report_folders[p]
        pr=rf.partition_report
        od=pr.orddict
        or=od[o]

        p=="exon" ? (ylbl="Order $o lh"; yfmt=y->y; lm=0mm) : (ylbl=""; yfmt=_->""; lm=-7mm)
        o==2 ? (xlbl=p; xfmt=x->Int64(x);bm=0mm) : (xlbl=""; xfmt=_->""; bm=-7mm)

        plt=scatter(or.converged_K,or.converged_lh, marker=:cross, markersize=15, markercolor=:black, label="Ord. $o model lhs", xlabel=xlbl, ylabel=ylbl, legend=nothing, xformatter=xfmt, yformatter=yfmt, xticks=[1,2,4,6], leftmargin=lm,bottommargin=bm)
        plot!(unique(or.converged_K),[pr.naive_lh for l in 1:length(unique(or.converged_K))], label="Naive lh", color=:darkmagenta,linewidth=2.5)
        push!(partplts,plt)
    end
end
combined=plot(partplts...,layout=grid(3,3),link=:xy, size=(600,600))
savefig(combined,"/bench/PhD/Thesis/images/rys/bghmm.png")
```

17.8.31 /rys/bg_models/BBM_survey_refinement.jl

```julia
#JOB FILEPATHS
hmm_output = "/bench/PhD/NGS_binaries/BBM/survey_chains"
refined_path = "/bench/PhD/NGS_binaries/BBM/refined_chains"
sample_output = "/bench/PhD/NGS_binaries/BBM/refinement_samples"
```

```
5
6 survey_folders = "/bench/PhD/NGS_binaries/BBM/survey_folders"
7
8 #GENERAL SETUP
9 @info "Loading libraries ... "
10 using BioBackgroundModels, DataFrames, Distributed, Serialization
11
12 const delta_thresh=1e-4 #stopping/convergence criterion (log probability
   ↪ difference btw subsequent EM iterates)
13 const max_iterates=15000
14
15 #LOAD SAMPLES
16 @info "Loading samples from $sample_output ... "
17 sample_dfs = deserialize(sample_output)
18
19 #BUILD TRAINING AND TEST SETS FROM SAMPLES
20 training_sets, test_sets = split_obs_sets(sample_dfs)
21
22 #GENERATE Chain_ID Vector AND CHAINS SUBSET
23 @info "Loading reports from $survey_folders"
24 report_folders=deserialize(survey_folders)
25 job_ids=Vector{Chain_ID}()
26 for (partition,folder) in report_folders
27     report=folder.partition_report
28     for id in report.best_repset
29         push!(job_ids,id)
30     end
31 end
32
33 if isfile(refined_path) #if some results have already been collected,
   ↪ load them
34     @info "Loading incomplete results ... "
35     refined_chains=deserialize(refined_path)
36 else
37     @info "Initialising new HMM results file at $refined_path"
38     chains = deserialize(hmm_output)
39     refined_chains=Dict{Chain_ID,Vector{EM_step}}()
40     for job_id in job_ids
41         refined_chains[job_id]=[EM_step(1, chains[job_id][end].hmm, 0, 0,
           ↪ false)]
42     end
43 end
44
```

```
#DISTRIBUTED CLUSTERs CONSTANTS
remote_machine = "10.0.0.3"
no_local_processes = 1
no_remote_processes = 1
load_dict=Dict{Int64,LoadConfig}()
local_config=LoadConfig(1:6,0:2)
remote_config=LoadConfig(1:6,0:2)

#SETUP DISTRIBUTED BAUM WELCH LEARNERS
@info "Spawning local cluster workers ... "
worker_pool=addprocs(no_local_processes, topology=:master_worker)
for worker in worker_pool
    load_dict[worker]=local_config
end

remote_pool=addprocs([(remote_machine,no_remote_processes)], tunnel=true,
↪   topology=:master_worker)
for worker in remote_pool
    load_dict[worker]=remote_config
end

worker_pool=vcat(worker_pool, remote_pool)

@info "Loading worker libraries everywhere ... "
@everywhere using BioBackgroundModels

em_jobset = setup_EM_jobs!(job_ids, training_sets;
↪   delta_thresh=delta_thresh, chains=refined_chains)
execute_EM_jobs!(worker_pool, em_jobset ... , refined_path;
↪   delta_thresh=delta_thresh, max_iterates=max_iterates,
↪   load_dict=load_dict)
```

17.8.32 /rys/nested_sampling/dif_pos_assembly.jl

```
@info "Setting up for job ... "
#JOB FILEPATHS
sib_wms_path =
↪   "/bench/PhD/NGS_binaries/BMI/sib_nuc_position_sequences.fa_wms.tr"
rys_wms_path =
↪   "/bench/PhD/NGS_binaries/BMI/rys_nuc_position_sequences.fa_wms.tr"

sib_df_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_positions"
```

```
rys_df_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_positions"
combined_df_binary =
↪  "/bench/PhD/NGS_binaries/BMI/combined_diff_positions"

sib_code_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_codes"
rys_code_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_codes"
combined_code_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_codes"

sib_diff_bg = "/bench/PhD/NGS_binaries/BMI/sib_diff_bg"
rys_diff_bg = "/bench/PhD/NGS_binaries/BMI/rys_diff_bg"
combined_diff_bg = "/bench/PhD/NGS_binaries/BMI/combined_diff_bg"

sib_e_pth = "/bench/PhD/NGS_binaries/BMI/sib_e"
rys_e_pth = "/bench/PhD/NGS_binaries/BMI/rys_e"
combined_e_pth = "/bench/PhD/NGS_binaries/BMI/combined_e"

#JOB CONSTANTS
const ensemble_size = 500
const no_sources = 8
const source_min_bases = 3
const source_max_bases = 10
@assert source_min_bases < source_max_bases
const source_length_range= source_min_bases:source_max_bases
const mixing_prior = .07
@assert mixing_prior ≥ 0 && mixing_prior ≤ 1

@info "Loading master libraries ... "
using Distributed, Distributions, Serialization

@info "Adding workers ... "
no_local_procs=2
worker_pool=addprocs(no_local_procs, topology=:master_worker)

@info "Loading libraries everywhere ... "
@everywhere using BioMotifInference, Random
Random.seed!(myid()*10000)

@info "Assembling uninformative source priors ... "
sib_ui_sp = BioMotifInference.assemble_source_priors(no_sources,
↪  Vector{Matrix{Float64}}())
rys_ui_sp = BioMotifInference.assemble_source_priors(no_sources,
↪  Vector{Matrix{Float64}}())
```

```
combined_ui_sp = BioMotifInference.assemble_source_priors(no_sources, Vector{Matrix{Float64}}())

@info "Loading BGHMM likelihood matrix binaries ..."
sib_matrix=deserialize(sib_diff_bg)
rys_matrix=deserialize(rys_diff_bg)
combined_matrix=deserialize(combined_diff_bg)

@info "Loading coded observation sets ..."
sib_obs = deserialize(sib_code_binary)
rys_obs = deserialize(rys_code_binary)
combined_obs = deserialize(combined_code_binary)

@info "Assembling sib IPM ensemble on uninformative priors ..."
isfile(string(sib_e_pth,'/',"ens")) ? (sib_e = deserialize(string(sib_e_pth,'/',"ens"))) :
    (sib_e = BioMotifInference.IPM_Ensemble(worker_pool, sib_e_pth, ensemble_size, sib_ui_sp, (falses(0,0), mixing_prior), sib_matrix, sib_obs, source_length_range, posterior_switch=true); serialize(string(sib_e_pth,'/',"ens"),sib_e))
sib_e=[]; Base.GC.gc();

@info "Assembling rys IPM ensemble on uninformative priors ..."
isfile(string(rys_e_pth,'/',"ens")) ? (rys_e = deserialize(string(rys_e_pth,'/',"ens"))) :
    (rys_e = BioMotifInference.IPM_Ensemble(worker_pool, rys_e_pth, ensemble_size, rys_ui_sp, (falses(0,0), mixing_prior), rys_matrix, rys_obs, source_length_range, posterior_switch=true); serialize(string(rys_e_pth,'/',"ens"),rys_e))
rys_e=[]; Base.GC.gc();


@info "Assembling combined IPM ensemble on uninformative priors ..."
isfile(string(combined_e_pth,'/',"ens")) ? (combined_e = deserialize(string(combined_e_pth,'/',"ens"))) :
    (combined_e = BioMotifInference.IPM_Ensemble(worker_pool, combined_e_pth, ensemble_size, combined_ui_sp, (falses(0,0), mixing_prior), combined_matrix, combined_obs, source_length_range, posterior_switch=true); serialize(string(combined_e_pth,'/',"ens"),combined_e))
combined_e=[]; Base.GC.gc();

rmprocs(worker_pool)
```

```

@info "Job done!"
```

17.8.33 /rys/nested_sampling/dif_pos_learner.jl

```
using Distributed, Distributions, Serialization
include("/srv/git/rys_nucleosomes/aws/aws_wrangler.jl")

sib_e_pth = "/bench/PhD/NGS_binaries/BMI/sib_e"
rys_e_pth = "/bench/PhD/NGS_binaries/BMI/rys_e"
combined_e_pth = "/bench/PhD/NGS_binaries/BMI/combined_e"

@info "Assembling worker pool..."

#DISTRIBUTED CLUSTERs CONSTANTS
remote_machine = "10.0.0.3"
no_local_processes = 2
no_remote_processes = 6

@info "Spawning local cluster workers..."
worker_pool=addprocs(no_local_processes, topology=:master_worker)
remote_pool=addprocs([(remote_machine,no_remote_processes)], tunnel=true,
↪  topology=:master_worker)

worker_pool=vcat(worker_pool, remote_pool)

#AWS PARAMS
@info "Setting up AWS wrangling..."

security_group_name="calc1"
security_group_desc="calculation group"
ami="ami-0bb0f5609b995d39e"
skeys="AWS"
instance_type="c5a.24xlarge"
zone,spot_price=get_cheapest_zone(instance_type)
no_instances=5
instance_workers=48
bid=spot_price+.1

@assert bid ≥ spot_price

@info "Wrangling AWS instances..."
```

```
aws_ips = spot_wrangle(no_instances, bid, security_group_name, security_group_desc, skeys, zone, ami, instance_type)
@info "Giving instances 150s to boot ..."
sleep(150)

# aws_ips=["18.223.214.57","18.224.4.81", "3.17.13.150","3.21.113.229","18.222.211.44"]


@info "Spawning AWS cluster workers ..."
for ip in aws_ips
    instance_pool=addprocs([(ip, instance_workers)], tunnel=true, topology=:master_worker, sshflags="-o StrictHostKeyChecking=no")
    global worker_pool=vcat(worker_pool, instance_pool)
end

@info "Loading worker libraries everywhere ..."
@everywhere using BioMotifInference,Random
@everywhere Random.seed!(myid()*100000)

#JOB CONSTANTS
funcvec=full_perm_funcvec
models_to_permute=10000
func_limit=25
push!(funcvec, BioMotifInference.perm_src_fit_mix)
push!(funcvec, BioMotifInference.random_decorrelate)

min_clamps=fill(.01,length(funcvec))
min_clamps[2:3]•=.1 #perm_src_fit_mix & permute_mix
min_clamps[8]=.1 #difference_merge
max_clamps=fill(.5,length(funcvec))
max_clamps[6:7]•=.15 #ss and am
max_clamps[9]=.15 #sim merge

initial_weights= ones(length(funcvec))./length(funcvec)
# override_weights=fill(.034375,length(funcvec))
# override_weights[6:9]•=.1;override_weights[13:14]•=.1625
# override_time=20.

args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcvec)]
args[end-1]=[(:weight_shift_freq,0.),(:length_change_freq,1.),(:length_perm_range,1:1
args[end]=[(:iterates,50),(:source_permute_freq,.3),(:mix_move_range,1:10)]

```

```
instruct = Permute_Instruct(funcvec, initial_weights, models_to_permute, func_limit;min_clmps=min_clamps, max_clmps=max_clamps, args=args)

display_rotation=[true,10,1,[[:tuning_disp,:lh_disp,:src_disp],[:conv_plot,:liwi_disp

@info "Beginning nested sampling for sib ensemble..."
sib_e=deserialize(sib_e_pth*"/ens")
sib_logZ = converge_ensemble!(sib_e, instruct, worker_pool, converge_criterion="compression", converge_factor=150., backup=(true,25), tuning_disp=true,lh_disp=true,src_disp=true, disp_rotate_inst=display_rotation)
sib_e=[]; Base.GC.gc();

@info "Beginning nested sampling for rys ensemble..."
rys_e=deserialize(rys_e_pth*"/ens")
rys_logZ = converge_ensemble!(rys_e, instruct, worker_pool, converge_criterion="compression", converge_factor=150., backup=(true,25), tuning_disp=true,lh_disp=true,src_disp=true, disp_rotate_inst=display_rotation)
rys_e=[]; Base.GC.gc();

@info "Beginning nested sampling for combined ensemble..."
combined_e=deserialize(combined_e_pth*"/ens")
combined_logZ = converge_ensemble!(combined_e, instruct, worker_pool, converge_criterion="compression", converge_factor=150., backup=(true,25), tuning_disp=true,lh_disp=true,src_disp=true,disp_rotate_inst=display_rotation)

joint_logZ=rys_logZ+sib_logZ
evratio=joint_logZ-combined_logZ
sig=evratio.val/evratio.err

println("$(round(sib_logZ,digits=3)) & $(round(rys_logZ,digits=3)) & {\\bf $(round(joint_logZ,digits=3))} & $(round(combined_logZ,digits=3)) &  $(round(evratio,digits=3)) & $(round(sig,digits=1)) \\\\ \\hline")

rmprocs(worker_pool)

@info "Job done!"
```

17.8.34 /rys/nested_sampling/dif_pos_sample_prep.jl

```
using BioSequences, Distributed, DataFrames, BioBackgroundModels, CSV, Serialization
import StatsBase:sample

include("/bench/PhD/Thesis/Analyses/rys/position_analysis/position_overlap.jl")

#JOB PATHS AND CONSTANTS

sample_rows = 7092 #2 million bases

sib_pos = "/bench/PhD/danpos_results/pooled/sib.Fnor.smooth.positions.xls"
rys_pos = "/bench/PhD/danpos_results/pooled/rys.Fnor.smooth.positions.xls"

danio_genome_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna"
danio_gen_index_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna.fai"
danio_gff_path = "/bench/PhD/seq/GRCz11/Danio_rerio.GRCz11.94.gff3"

refined_folders_path = "/bench/PhD/NGS_binaries/BBM/refined_folders"
selected_hmms = "/bench/PhD/NGS_binaries/BBM/selected_hmms"

sib_df_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_positions"
rys_df_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_positions"
combined_df_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_positions"

sib_code_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_codes"
rys_code_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_codes"
combined_code_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_codes"

sib_diff_bg = "/bench/PhD/NGS_binaries/BMI/sib_diff_bg"
rys_diff_bg = "/bench/PhD/NGS_binaries/BMI/rys_diff_bg"
combined_diff_bg = "/bench/PhD/NGS_binaries/BMI/combined_diff_bg"

sib_fa = "/bench/PhD/thicweed_results/sib_nuc_position_sequences.fa"
sib_arch = "/bench/PhD/thicweed_results/sib_nuc_position_sequences.fa_archs.txt"

```

```
rys_fa = "/bench/PhD/thicweed_results/rys_nuc_position_sequences.fa"
rys_arch = "/bench/PhD/thicweed_results/rys_nuc_position_sequences.fa_archs.txt"

@info "Reading from position.xls ..."
sib_df=CSV.read(sib_pos)
rys_df=CSV.read(rys_pos)

@info "Mapping ..."
map_positions!(sib_df, rys_df)
map_positions!(rys_df, sib_df)

@info "Making differential position dataframes ..."
sib_diff_df = deepcopy(sib_df[findall(iszero,sib_df.mapped_pos),:])
sib_diff_df = sib_diff_df[findall(!isequal("MT"), sib_diff_df.chr),:]
rys_diff_df = deepcopy(rys_df[findall(iszero,rys_df.mapped_pos),:])
rys_diff_df = rys_diff_df[findall(!isequal("MT"), rys_diff_df.chr),:]

add_position_sequences!(sib_diff_df, danio_genome_path, danio_gen_index_path)
add_position_sequences!(rys_diff_df, danio_genome_path, danio_gen_index_path)

@info "Filtering ambiguous sequences ..."
deleterows!(sib_diff_df, [hasambiguity(seq) for seq in sib_diff_df.seq])
deleterows!(rys_diff_df, [hasambiguity(seq) for seq in rys_diff_df.seq])

@info "Sampling subsets of sequences ..."

sib_diff_df=sib_diff_df[sample(axes(sib_diff_df, 1), sample_rows; replace=false, ordered=true), :]
rys_diff_df=rys_diff_df[sample(axes(rys_diff_df, 1), sample_rows; replace=false, ordered=true), :]

@info "Masking positions by genome partition and strand ..."
BioBackgroundModels.add_partition_masks!(sib_diff_df, danio_gff_path, 500, (:chr,:seq,:start))
BioBackgroundModels.add_partition_masks!(rys_diff_df, danio_gff_path, 500, (:chr,:seq,:start))

@info "Obtaining cluster data ..."
get_cluster!(sib_diff_df, sib_fa, sib_arch)
get_cluster!(rys_diff_df, rys_fa, rys_arch)
```

```

@info "Serializing dataframes ..."
combined_diff_df = vcat(sib_diff_df, rys_diff_df)

serialize(sib_df_binary, sib_diff_df)
serialize(rys_df_binary, rys_diff_df)
serialize(combined_df_binary, combined_diff_df)

@info "Creating coded observation sets ..."
sib_codes = observation_setup(sib_diff_df, order=0, symbol=:seq)
rys_codes = observation_setup(rys_diff_df, order=0, symbol=:seq)
combined_codes = observation_setup(combined_diff_df, order=0,
↪  symbol=:seq)

@info "Serializing coded observation sets ..."
serialize(sib_code_binary, Matrix(transpose(sib_codes)))
serialize(rys_code_binary, Matrix(transpose(rys_codes)))
serialize(combined_code_binary, Matrix(transpose(combined_codes)))

@info "Setting up for BBM likelihood calculations ..."
BHMM_dict = Dict{String,BHMM}()
refined_folders=deserialize(refined_folders_path)
for (part, folder) in refined_folders
    BHMM_dict[part]=folder.partition_report.best_model[2]
end

serialize(selected_hmms,BHMM_dict)

@info "Performing calculations ..."
sib_lh_matrix = BGHMM_likelihood_calc(sib_diff_df, BHMM_dict,
↪  symbol=:seq)
rys_lh_matrix = BGHMM_likelihood_calc(rys_diff_df, BHMM_dict,
↪  symbol=:seq)
combined_lh_matrix = hcat(sib_lh_matrix,rys_lh_matrix)

@info "Serializing background matrices ..."
serialize(sib_diff_bg, sib_lh_matrix)
serialize(rys_diff_bg, rys_lh_matrix)
serialize(combined_diff_bg, combined_lh_matrix)

@info "Job done!"
```

17.8.35 /rys/nested_sampling/local_test.jl

```
using Distributed, Distributions, Serialization

test_e_pth = "/bench/PhD/NGS_binaries/BMI/test_e"

@info "Assembling worker pool ..."

#DISTRIBUTED CLUSTERs CONSTANTS
remote_machine = "10.0.0.3"
no_local_processes = 1

@info "Spawning local cluster workers ..."
worker_pool=addprocs(no_local_processes, topology=:master_worker)

@info "Loading worker libraries everywhere ..."
@everywhere using BioMotifInference,Random
@everywhere Random.seed!(myid()*100000)

#JOB CONSTANTS
models_to_permute=5000
func_limit=30
clamp=.02
funcvec=full_perm_funcvec
push!(funcvec, BioMotifInference.permute_source)
push!(funcvec, BioMotifInference.permute_source)
args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcvec)]
args[end-1]=[(:weight_shift_freq,0.),(:length_change_freq,1.),(:length_perm_range,1:1
args[end]=[(:weight_shift_freq,.1),(:length_change_freq,0.),(:weight_shift_dist,Unifo

instruct = Permute_Instruct(funcvec,
↪  ones(length(funcvec))./length(funcvec), models_to_permute,
↪  func_limit, clamp; args=args)

display_rotation=[true,10,1,[[:tuning_disp,:lh_disp,:src_disp],[:conv_plot,:liwi_disp

@info "Beginning nested sampling for test ensemble ..."
test_e=deserialize(test_e_pth*"/ens")
test_logZ = converge_ensemble!(test_e, instruct, worker_pool, .001,
↪  backup=(true,1),tuning_disp=true,lh_disp=true,src_disp=true,
↪  disp_rotate_inst=display_rotation)

rmprocs(worker_pool)
```

```

@info "Job done!"
```

17.8.36 /rys/nested_sampling/overall_naive.jl

```
using BioMotifInference, BioSequences, DataFrames, BioBackgroundModels, CSV, Serialization
import BioMotifInference: IPM_likelihood

include("/bench/PhD/Thesis/Analyses/rys/position_analysis/position_overlap.jl")

sib_pos = "/bench/PhD/danpos_results/pooled/sib.Fnor.smooth.positions.xls"
rys_pos = "/bench/PhD/danpos_results/pooled/rys.Fnor.smooth.positions.xls"

danio_genome_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna"
danio_gen_index_path = "/bench/PhD/seq/GRCz11/GCA_000002035.4_GRCz11_genomic.fna.fai"
danio_gff_path = "/bench/PhD/seq/GRCz11/Danio_rerio.GRCz11.94.gff3"

refined_folders_path = "/bench/PhD/NGS_binaries/BBM/refined_folders"
selected_hmms = "/bench/PhD/NGS_binaries/BBM/selected_hmms"

sib_df_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_positions"
rys_df_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_positions"
combined_df_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_positions"

@info "Reading from position.xls ... "
sib_df=CSV.read(sib_pos)
rys_df=CSV.read(rys_pos)

@info "Mapping ... "
map_positions!(sib_df, rys_df)
map_positions!(rys_df, sib_df)

@info "Making differential position dataframes ... "
sib_diff_df = deepcopy(sib_df[findall(iszero,sib_df.mapped_pos),:])
sib_diff_df = sib_diff_df[findall(!isequal("MT"), sib_diff_df.chr),:]
rys_diff_df = deepcopy(rys_df[findall(iszero,rys_df.mapped_pos),:])
```

```
rys_diff_df = rys_diff_df[findall(!isequal("MT"), rys_diff_df.chr),:]
sib_mapped_df = deepcopy(sib_df[findall(!iszero,sib_df.mapped_pos),:])
sib_mapped_df = sib_mapped_df[findall(x->x>0, sib_mapped_df.start),:]
sib_mapped_df = sib_mapped_df[findall(!isequal("MT"), sib_mapped_df.chr),:]
rys_mapped_df = deepcopy(rys_df[findall(!iszero,rys_df.mapped_pos),:])
rys_mapped_df = rys_mapped_df[findall(x->x>0, rys_mapped_df.start),:]
rys_mapped_df = rys_mapped_df[findall(!isequal("MT"), rys_mapped_df.chr),:]

add_position_sequences!(sib_diff_df, danio_genome_path, danio_gen_index_path)
add_position_sequences!(rys_diff_df, danio_genome_path, danio_gen_index_path)
add_position_sequences!(sib_mapped_df, danio_genome_path, danio_gen_index_path)
add_position_sequences!(rys_mapped_df, danio_genome_path, danio_gen_index_path)

@info "Filtering ambiguous sequences..."
deleterows!(sib_diff_df, [hasambiguity(seq) for seq in sib_diff_df.seq])
deleterows!(rys_diff_df, [hasambiguity(seq) for seq in rys_diff_df.seq])
deleterows!(sib_mapped_df, [hasambiguity(seq) for seq in sib_mapped_df.seq])
deleterows!(rys_mapped_df, [hasambiguity(seq) for seq in rys_mapped_df.seq])

@info "Masking positions by genome partition and strand..."
BioBackgroundModels.add_partition_masks!(sib_diff_df, danio_gff_path, 500, (:chr,:seq,:start))
BioBackgroundModels.add_partition_masks!(rys_diff_df, danio_gff_path, 500, (:chr,:seq,:start))
BioBackgroundModels.add_partition_masks!(sib_mapped_df, danio_gff_path, 500, (:chr,:seq,:start))
BioBackgroundModels.add_partition_masks!(rys_mapped_df, danio_gff_path, 500, (:chr,:seq,:start))

@info "Creating coded observation sets..."
sib_codes = Matrix(transpose(observation_setup(sib_diff_df, order=0, symbol=:seq)))
rys_codes = Matrix(transpose(observation_setup(rys_diff_df, order=0, symbol=:seq)))
```

```
sib_mapped_codes = Matrix(transpose(observation_setup(sib_mapped_df, order=0, symbol=:seq)))
rys_mapped_codes = Matrix(transpose(observation_setup(rys_mapped_df, order=0, symbol=:seq)))

@info "Setting up for BBM likelihood calculations..."
BHMM_dict = Dict{String,BHMM}()
refined_folders=deserialize(refined_folders_path)
for (part, folder) in refined_folders
    BHMM_dict[part]=folder.partition_report.best_model[2]
end

sib_lh_matrix = BGHMM_likelihood_calc(sib_diff_df, BHMM_dict, symbol=:seq)
rys_lh_matrix = BGHMM_likelihood_calc(rys_diff_df, BHMM_dict, symbol=:seq)
sibm_lh_matrix= BGHMM_likelihood_calc(sib_mapped_df, BHMM_dict, symbol=:seq)
rysm_lh_matrix = BGHMM_likelihood_calc(rys_mapped_df, BHMM_dict, symbol=:seq)

sib_e_pth = "/bench/PhD/NGS_binaries/BMI/sib_e"
rys_e_pth = "/bench/PhD/NGS_binaries/BMI/rys_e"

sibe=deserialize(sib_e_pth*"/ens")
ryse=deserialize(rys_e_pth*"/ens")

sib_MAP=deserialize(sibe.models[findmax([rec.log_Li for rec in sibe.models])[2]].path)
rys_MAP=deserialize(ryse.models[findmax([rec.log_Li for rec in ryse.models])[2]].path)

sib_bg_lh = IPM_likelihood(sib_MAP.sources, sib_codes, [findfirst(iszero,sib_codes[:,o])-1 for o in 1:size(sib_codes,2)], sib_lh_matrix, falses(size(sib_codes,2),length(sib_MAP.sources)))

sibm_bg_lh = IPM_likelihood(sib_MAP.sources, sib_mapped_codes, [findfirst(iszero,sib_mapped_codes[:,o])-1 for o in 1:size(sib_mapped_codes,2)], sibm_lh_matrix, falses(size(sib_mapped_codes,2),length(sib_MAP.sources)))

```

```
rys_bg_lh = IPM_likelihood(rys_MAP.sources, rys_codes, [findfirst(iszero,rys_codes[:,o])-1 for o in 1:size(rys_codes,2)], rys_lh_matrix, falses(size(rys_codes,2),length(rys_MAP.sources)))

rysm_bg_lh = IPM_likelihood(rys_MAP.sources, rys_mapped_codes, [findfirst(iszero,rys_mapped_codes[:,o])-1 for o in 1:size(rys_mapped_codes,2)], rysm_lh_matrix, falses(size(rys_mapped_codes,2),length(rys_MAP.sources)))

println("\\begin{tabular}{|l|l|l|l|}")
println("\\hline")
println("{\\bf Sequence set} & {\\bf \\textit{rys} naive LH} & {\\bf sib naive LH} & {\\bf LH ratio}\\\\ \\hline")
println("Differential & $(round(rys_bg_lh, sigdigits=5)) & $(round(sib_bg_lh, sigdigits=5)) & $(round(rys_bg_lh-sib_bg_lh,sigdigits=5))\\\\ \\hline")
println("Mapped & $(round(rysm_bg_lh, sigdigits=5)) & $(round(sibm_bg_lh, sigdigits=5)) & $(round(rysm_bg_lh-sibm_bg_lh,sigdigits=5))\\\\ \\hline")
println("\\end{tabular}")
```

17.8.37 /rys/nested_sampling/prior_test.jl

```
@info "Setting up for job ..."
#JOB FILEPATHS
sib_wms_path = "/bench/PhD/NGS_binaries/BMI/sib_nuc_position_sequences.fa_wms.tr"
rys_wms_path = "/bench/PhD/NGS_binaries/BMI/rys_nuc_position_sequences.fa_wms.tr"

sib_df_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_positions"
rys_df_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_positions"
combined_df_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_positions"

sib_code_binary = "/bench/PhD/NGS_binaries/BMI/sib_diff_codes"
rys_code_binary = "/bench/PhD/NGS_binaries/BMI/rys_diff_codes"
combined_code_binary = "/bench/PhD/NGS_binaries/BMI/combined_diff_codes"

sib_diff_bg = "/bench/PhD/NGS_binaries/BMI/sib_diff_bg"
rys_diff_bg = "/bench/PhD/NGS_binaries/BMI/rys_diff_bg"
combined_diff_bg = "/bench/PhD/NGS_binaries/BMI/combined_diff_bg"
```

```
sib_e_ui_pth = "/bench/PhD/NGS_binaries/BMI/sib_e_ui"
sib_e_inf_pth = "/bench/PhD/NGS_binaries/BMI/sib_e_inf"
rys_e_ui_pth = "/bench/PhD/NGS_binaries/BMI/rys_e_ui"
rys_e_inf_pth = "/bench/PhD/NGS_binaries/BMI/rys_e_inf"
combined_e_ui_pth = "/bench/PhD/NGS_binaries/BMI/combined_e_ui"
combined_e_inf_pth = "/bench/PhD/NGS_binaries/BMI/combined_e_inf"


#JOB CONSTANTS
const ensemble_size = 1000
const no_sources = 8
const source_min_bases = 3
const source_max_bases = 10
@assert source_min_bases < source_max_bases
const source_length_range= source_min_bases:source_max_bases
const mixing_prior = .07
@assert mixing_prior ≥ 0 && mixing_prior ≤ 1
const prior_wt=1.2

@info "Loading master libraries ... "
using Distributed, Distributions, Serialization

@info "Adding librarians and workers ... "
no_local_procs=2
no_remote_procs=6
remote_machine = "10.0.0.119"
remote_pool=addprocs([(remote_machine, no_remote_procs)], tunnel=true,
↪ topology=:master_worker)
local_pool=addprocs(no_local_procs, topology=:master_worker)
worker_pool=vcat(remote_pool,local_pool)

@info "Loading libraries everywhere ... "
@everywhere using BioMotifInference, Random
Random.seed!(myid()*10000)

@info "Loading informative source priors ... "
sib_wms = BioMotifInference.read_fa_wms_tr(sib_wms_path)
rys_wms = BioMotifInference.read_fa_wms_tr(rys_wms_path)
sib_mix_prior =
↪ BioMotifInference.cluster_mix_prior!(deserialize(sib_df_binary),
↪ sib_wms)
```

```
rys_mix_prior = BioMotifInference.cluster_mix_prior!(deserialize(rys_df_binary), rys_wms)

@info "Filtering informative priors ..."
sib_prior_wms = BioMotifInference.filter_priors(Int(floor(no_sources/2)), source_max_bases, sib_wms, sib_mix_prior)
rys_prior_wms = BioMotifInference.filter_priors(Int(floor(no_sources/2)), source_max_bases, rys_wms, rys_mix_prior)
combined_prior_wms = BioMotifInference.combine_filter_priors(Int(floor(no_sources/2)), source_max_bases, (sib_wms, rys_wms), (sib_mix_prior, rys_mix_prior))

@info "Assembling informative source priors ..."
sib_inf_sp = BioMotifInference.assemble_source_priors(no_sources, sib_prior_wms, prior_wt)
rys_inf_sp = BioMotifInference.assemble_source_priors(no_sources, rys_prior_wms, prior_wt)
combined_inf_sp = BioMotifInference.assemble_source_priors(no_sources, combined_prior_wms, prior_wt)

@info "Assembling uninformative source priors ..."
sib_ui_sp = BioMotifInference.assemble_source_priors(no_sources, Vector{Matrix{Float64}}())
rys_ui_sp = BioMotifInference.assemble_source_priors(no_sources, Vector{Matrix{Float64}}())
combined_ui_sp = BioMotifInference.assemble_source_priors(no_sources, Vector{Matrix{Float64}}())

@info "Loading BGHMM likelihood matrix binaries ..."
sib_matrix=deserialize(sib_diff_bg)
rys_matrix=deserialize(rys_diff_bg)
combined_matrix=deserialize(combined_diff_bg)

@info "Loading coded observation sets ..."
sib_obs = deserialize(sib_code_binary)
rys_obs = deserialize(rys_code_binary)
combined_obs = deserialize(combined_code_binary)

@info "Assembling sib IPM ensemble on uninformative priors ..."
isfile(string(sib_e_ui_pth,'/',"ens")) ? (sib_e_ui = deserialize(string(sib_e_ui_pth,'/',"ens"))) :
```

```
    (sib_e_ui = BioMotifInference.IPM_Ensemble(worker_pool, sib_e_ui_pth, ensemble_size, sib_ui_sp, (falses(0,0), mixing_prior), sib_matrix, sib_obs, source_length_range); serialize(string(sib_e_ui_pth,'/',"ens"),sib_e_ui))

@info "Assembling sib IPM ensemble on informative priors ..."
isfile(string(sib_e_inf_pth,'/',"ens")) ? (sib_e_inf = deserialize(string(sib_e_inf_pth,'/',"ens"))) :
        (sib_e_inf = BioMotifInference.IPM_Ensemble(worker_pool, sib_e_inf_pth, ensemble_size, sib_inf_sp, (falses(0,0), mixing_prior), sib_matrix, sib_obs, source_length_range); serialize(string(sib_e_inf_pth,'/',"ens"),sib_e_inf))

@info "Assembling rys IPM ensemble on uninformative priors ..."
isfile(string(rys_e_ui_pth,'/',"ens")) ? (rys_e_ui = deserialize(string(rys_e_ui_pth,'/',"ens"))) :
    (rys_e_ui = BioMotifInference.IPM_Ensemble(worker_pool, rys_e_ui_pth, ensemble_size, rys_ui_sp, (falses(0,0), mixing_prior), rys_matrix, rys_obs, source_length_range); serialize(string(rys_e_ui_pth,'/',"ens"),rys_e_ui))

@info "Assembling rys IPM ensemble on informative priors ..."
isfile(string(rys_e_inf_pth,'/',"ens")) ? (rys_e_inf = deserialize(string(rys_e_inf_pth,'/',"ens"))) :
        (rys_e_inf = BioMotifInference.IPM_Ensemble(worker_pool, rys_e_inf_pth, ensemble_size, rys_inf_sp, (falses(0,0), mixing_prior), rys_matrix, rys_obs, source_length_range); serialize(string(rys_e_inf_pth,'/',"ens"),rys_e_inf))

@info "Assembling combined IPM ensemble on uninformative priors ..."
isfile(string(combined_e_ui_pth,'/',"ens")) ? (combined_e_ui = deserialize(string(combined_e_ui_pth,'/',"ens"))) :
    (combined_e_ui = BioMotifInference.IPM_Ensemble(worker_pool, combined_e_ui_pth, ensemble_size, combined_ui_sp, (falses(0,0), mixing_prior), combined_matrix, combined_obs, source_length_range); serialize(string(combined_e_ui_pth,'/',"ens"),combined_e_ui))

@info "Assembling combined IPM ensemble on informative priors ..."
isfile(string(combined_e_inf_pth,'/',"ens")) ? (combined_e_inf = deserialize(string(combined_e_inf_pth,'/',"ens"))) :
```

```
        (combined_e_inf = BioMotifInference.IPM_Ensemble(worker_pool, combined_e_inf_pth, ensemble_size, combined_inf_sp, (falses(0,0), mixing_prior), combined_matrix, combined_obs, source_length_range); serialize(string(combined_e_inf_pth,'/',"ens"),combined_e_inf))

rmprocs(worker_pool)

@info "Fitting distributions..."

sib_ui_dist=fit_mle(Normal,[model.log_Li for model in sib_e_ui.models])
sib_inf_dist=fit_mle(Normal,[model.log_Li for model in sib_e_inf.models])
@info "Is informative a better prior for sibs?"
println(sib_inf_dist.μ > sib_ui_dist.μ && quantile(sib_inf_dist,.5) > sib_ui_dist.μ)

rys_ui_dist=fit_mle(Normal,[model.log_Li for model in rys_e_ui.models])
rys_inf_dist=fit_mle(Normal,[model.log_Li for model in rys_e_inf.models])
@info "Is informative a better prior for rys?"
println(rys_inf_dist.μ > rys_ui_dist.μ && quantile(rys_inf_dist,.5) > rys_ui_dist.μ)

combined_ui_dist=fit_mle(Normal,[model.log_Li for model in combined_e_ui.models])
combined_inf_dist=fit_mle(Normal,[model.log_Li for model in combined_e_inf.models])
@info "Is informative a better prior for combined?"
println(combined_inf_dist.μ > combined_ui_dist.μ && quantile(combined_inf_dist,.5) > combined_ui_dist.μ)

@info "Job done!"
```

17.8.38 /rys/nested_sampling/spike_recovery.jl

```
#Testbed for synthetic spike recovery from example background
using BioBackgroundModels, BioMotifInference, Random, Distributed, Distributions, Serialization
Random.seed!(786)
#CONSTANTS
no_obs=500
obsl=100:200

```

```
const folder_path="/bench/PhD/NGS_binaries/BBM/refined_folders"

report_folders=deserialize(folder_path)

bhmm_vec=Vector{BHMM}()
push!(bhmm_vec,report_folders["intergenic"].partition_report.best_model[2])
push!(bhmm_vec,report_folders["periexonic"].partition_report.best_model[2])
push!(bhmm_vec,report_folders["exon"].partition_report.best_model[2])

bhmm_dist=Categorical([.6,.2,.2])

struc_sig_1=[.1 .7 .1 .1
           .1 .1 .1 .7
           .1 .7 .1 .1]
struc_sig_2=[.1 .1 .7 .1
           .1 .1 .7 .1
           .7 .1 .1 .1]
periodicity=8
struc_frac_obs=.75


tata_box=[.05 .05 .05 .85
          .85 .05 .05 .05
          .05 .05 .05 .85
          .85 .05 .05 .05
          .425 .075 .075 .425
          .85 .05 .05 .05
          .425 .075 .075 .425]
caat_box=[.15 .15 .55 .15
          .15 .15 .55 .15
          .05 .85 .05 .05
          .05 .85 .05 .05
          .85 .05 .05 .05
          .85 .05 .05 .05
          .05 .05 .05 .85
          .15 .55 .15 .15
          .15 .15 .15 .55]
motif_frac_obs=.7
motif_recur_range=1:4

@info "Constructing synthetic sample set 1..."
obs1, bg_scores1, hmm_truth1, spike_truth1 =
↪  synthetic_sample(no_obs,obsl,bhmm_vec,bhmm_dist,[struc_sig_1,tata_box],[(true,(st
```

```
50
51 @info "Constructing synthetic sample set 2 ... "
52 obs2, bg_scores2, hmm_truth2, spike_truth2 =
   ↪  synthetic_sample(no_obs,obsl,bhmm_vec,bhmm_dist,[struc_sig_2,caat_box],[(true,(st
53
54 @info "Assembling combined sample set ... "
55 obs3=hcat(obs1,obs2)
56 bg_scores3=hcat(bg_scores1, bg_scores2)
57
58 @info "Assembling worker pool ... "
59
60 #DISTRIBUTED CLUSTERs CONSTANTS
61 remote_machine = "10.0.0.3"
62 no_local_processes = 2
63 no_remote_processes = 6
64
65 @info "Spawning local cluster workers ... "
66 worker_pool=addprocs(no_local_processes, topology=:master_worker)
67 # remote_pool=addprocs([(remote_machine,no_remote_processes)],
   ↪  tunnel=true, topology=:master_worker)
68
69 # worker_pool=vcat(worker_pool, remote_pool)
70
71 @info "Loading worker libraries everywhere ... "
72 @everywhere using BioMotifInference, Random
73 @everywhere Random.seed!(myid())
74
75 e1 = "/bench/PhD/NGS_binaries/BMI/e1"
76 e2 = "/bench/PhD/NGS_binaries/BMI/e2"
77 e3 = "/bench/PhD/NGS_binaries/BMI/e3"
78
79 #JOB CONSTANTS
80 const ensemble_size = 250
81 const no_sources = 2
82 const source_min_bases = 3
83 const source_max_bases = 12
84 const source_length_range= source_min_bases:source_max_bases
85 const mixing_prior = .5
86 const models_to_permute = ensemble_size * 3
87 funcvec=full_perm_funcvec
88 push!(funcvec, BioMotifInference.permute_source)
89 push!(funcvec, BioMotifInference.permute_source)
90 args=[Vector{Tuple{Symbol,Any}}() for i in 1:length(funcvec)]
```

```
args[end-1]=[(:weight_shift_freq,0.),(:length_change_freq,1.),(:length_perm_range,1:1
args[end]=[(:weight_shift_freq,.1),(:length_change_freq,0.),(:weight_shift_dist,Unifo


instruct = Permute_Instruct(funcvec,
    ones(length(funcvec))./length(funcvec),models_to_permute,100,
    min_clmps=.02; args=args)

@info "Assembling source priors ... "
prior_array= Vector{Matrix{Float64}}()
source_priors = BioMotifInference.assemble_source_priors(no_sources,
    prior_array)

@info "Assembling ensemble 1 ... "
isfile(e1*"/ens") ? (ens1 = deserialize(e1*"/ens")) :
    (ens1 = IPM_Ensemble(worker_pool, e1, ensemble_size, source_priors,
        (falses(0,0), mixing_prior), bg_scores1, obs1,
        source_length_range))

@info "Converging ensemble 1 ... "
logZ1 = BioMotifInference.converge_ensemble!(ens1, instruct, worker_pool,
    backup=(true,250), wk_disp=false, tuning_disp=true, ens_disp=false,
    conv_plot=true, src_disp=true, lh_disp=false, liwi_disp=false)

@info "Assembling ensemble 2 ... "
isfile(e2*"/ens") ? (ens2 = deserialize(e2*"/ens")) :
    (ens2 = IPM_Ensemble(worker_pool, e2, ensemble_size, source_priors,
        (falses(0,0), mixing_prior), bg_scores2, obs2,
        source_length_range))

@info "Converging ensemble 2 ... "
logZ2 = BioMotifInference.converge_ensemble!(ens2, instruct, worker_pool,
    backup=(true,250), wk_disp=false, tuning_disp=true, ens_disp=false,
    conv_plot=true, src_disp=true, lh_disp=false, liwi_disp=false)

@info "Assembling ensemble 3 ... "
isfile(e3*"/ens") ? (ens3 = deserialize(e3*"/ens")) :
    (ens3 = IPM_Ensemble(worker_pool, e3, ensemble_size, source_priors,
        (falses(0,0), mixing_prior), bg_scores3, obs3,
        source_length_range))

@info "Converging ensemble 3 ... "

```

```
logZ3 = BioMotifInference.converge_ensemble!(ens3, instruct, worker_pool, backup=(true,250), wk_disp=false, tuning_disp=true, ens_disp=false, conv_plot=true, src_disp=true, lh_disp=false, liwi_disp=false)

@info "logZR of joint 1&2 / 3:"
ratio=logZ1+logZ2-logZ3
println(ratio)

rmprocs(worker_pool)
```

17.8.39 /rys/position_analysis/position_overlap.jl

```
using DataFrames, BioSequences, BioBackgroundModels, CSV, FASTX, Statistics

function add_position_sequences!(df::DataFrame, genome_path::String, genome_idx_path::String)
    scaffold_seq_record_dict::Dict{String,BioSequences.LongSequence} = BioBackgroundModels.build_scaffold_seq_dict(genome_path, genome_idx_path)

    seqs=[BioSequences.LongSequence{DNAAlphabet{4}}() for i in 1:size(df,1)]
    df[!, :seq] .= seqs

    Threads.@threads for entry in eachrow(df)
        entry.start > 0 ? (estart=entry.start) : (estart=1)
        entry.seq=BioBackgroundModels.fetch_sequence(entry.chr, scaffold_seq_record_dict ,estart, entry.end, '+')
    end

end

function get_cluster!(df::DataFrame, fasta::String, arch::String)
    df[!, :cluster] .= zeros(Int64, size(df,1))
    art=CSV.read(arch, header=0)

    position_vec=Vector{Vector{Int64}}()
    reader=FASTA.Reader(open(fasta))
    for (n, entry) in enumerate(reader)
        scaffold = FASTA.identifier(entry)
        desc_array = split(FASTA.description(entry))
```

```
        pos_start = parse(Int64, desc_array[2])
        pos_end = parse(Int64, desc_array[4])
        seq = FASTA.sequence(entry)

        idx = filter(in(findall(chr→chr==scaffold, df.chr)),
         ↪  findall(start→start==pos_start, df.start))

        if length(idx)==1
            match=df[idx[1],:]
            @assert match.end == pos_end
            @assert match.seq==seq
            match.cluster=art.Column1[n]
        end
    end
end

function make_position_df(position_fasta::String)
    position_reader = FASTA.Reader(open((position_fasta),"r"))
    position_df = DataFrame(SeqID = String[], Start=Int64[], End=Int64[],
     ↪  Seq = LongSequence[])

    for entry in position_reader
        scaffold = FASTA.identifier(entry)

        if scaffold ≠ "MT"
            desc_array = split(FASTA.description(entry))
            pos_start = parse(Int64, desc_array[2])
            pos_end = parse(Int64, desc_array[4])
            seq = FASTA.sequence(entry)

            if !hasambiguity(seq)
                push!(position_df, [scaffold, pos_start, pos_end, seq])
            end
        end
    end

    close(position_reader)
    return position_df
end

function observation_setup(position_df::DataFrame; order::Int64=0,
 ↪  symbol::Symbol=:Seq)
```

```
    order_seqs = BioBackgroundModels.get_order_n_seqs(Vector{LongSequence{DNAAlphabet{2}}}(pos symbol]),order)
    coded_seqs = BioBackgroundModels.code_seqs(order_seqs)

    return coded_seqs
end

function map_positions!(base_df, map_df)
    df_rows = size(base_df,1)
    base_df[!, :mapped_pos] = zeros(Int64, df_rows)
    base_df[!, :overlap_bp] = zeros(Int64, df_rows)
    base_df[!, :rel_overlap] = zeros(df_rows)

    chrgroups=groupby(base_df, :chr)

    for chr_df in chrgroups #split the df by scaffold so base numbers are local to scaffold
        base_chr = chr_df.chr[1]
        matched=false
        for map_chr_df in groupby(map_df, :chr)
            base_chr == map_chr_df.chr[1] && subDFmap!(chr_df, map_chr_df)
            matched=true
        end
        matched=false && (chr_df.mapped_pos=zeros(Int64,length(chr_df.start)); chr_df.overlap_bp=zeros(Int64,length(chr_df.start)); chr_df.rel_overlap=zeros(length(chr_df.start)))
    end

    return base_df
end

                function subDFmap!(chr_df::SubDataFrame, map_chr_df::SubDataFrame)
                    Threads.@threads for position in eachrow(chr_df)
                        search_start = position.start; search_end=position.end

                        start_idxs=findall(in(findall(ende→search_start ≤ ende,ma
                        findall(start→search_start ≥ start,map_chr_df.start))
```

```
end_idxs=findall(in(findall(ende→search_end≤ende,map_ch findall(start→search_end≥start,map_chr_df.start))
mapped_idxs = unique(vcat(start_idxs, end_idxs))

overlaps=Vector{Int64}()
for idx in mapped_idxs
    #println(idx)
    mapped_start=map_chr_df.start[idx]; mapped_end=map_chr_df.end[idx]
    if (mapped_start==search_start && mapped_end==search_end) #complete position overlap (all positions are the same size)
        push!(overlaps,(mapped_end-mapped_start+1))
        #println("same")
    elseif (mapped_start < search_start ≤ mapped_end) #5' end of the searched position overlaps with the mapped position
        push!(overlaps,(mapped_end-search_start+1))
        #println("5' overlap")
    elseif (mapped_start ≤ search_end < mapped_end) #3' end of searched position overlaps with the mapped position
        push!(overlaps,(search_end-mapped_start+1))
        #println("3' overlap")
    end
end

if length(mapped_idxs)>0 #1 or more mapped candidates: take the one with the most overlap
    overlap, oidx = findmax(overlaps)
    map_idx = mapped_idxs[oidx]
    position.mapped_pos=map_idx
    position.overlap_bp=overlap
    position.rel_overlap=overlap / (search_end-search_start+1)
end
```

```
                    end
                end
```

17.8.40 /test/eggbox_plots.jl

```
using GMC_NS, Distributions, Plots

gr()
default(legendfont = (10), guidefont = (12), tickfont = (10))

prior=Uniform(0,1)
priors=[prior,prior]

box=[-1. 1.
     -1. 1.]

gmc=copy(GMC_DEFAULTS)
gmc[2]=1e-3

e=Eggbox_Ensemble("Eggbox_test", 400, priors, box, gmc ... )

x1=0.:.01:1.
x2=0.:.01:1.

f(x1, x2) = begin
        (2 + cos(5π * x1) * cos(5π * x2))^5
    end

function harvest_xs(e)
    x1m=[m.pos[1] for m in e.models]
    x2m=[m.pos[2] for m in e.models]
    x1m.+=1.; x2m.+=1.
    x1m./=2.; x2m./=2
    return x1m, x2m
end

plt=contour(x1,x2,f, xlabel="x1", ylabel="x2", colorbar=:none)

init=deepcopy(plt)
plot!(init, title="Initialisation")
scatter!(init, harvest_xs(e), marker=:cross, markercolor=:black,
↪ legend=:none)
```

```

converge_ensemble!(e,backup=(true,100), max_iterates=1000, converge_factor=1e-6)

thousand=deepcopy(plt)
plot!(thousand, title="1000 iterates")
scatter!(thousand, harvest_xs(e), marker=:cross, markercolor=:black, legend=:none)

converge_ensemble!(e,backup=(true,100), max_iterates=5000, converge_factor=1e-6)

tenthousand=deepcopy(plt)
plot!(tenthousand, title="5000 iterates")
scatter!(tenthousand, harvest_xs(e), marker=:cross, markercolor=:black, legend=:none)

converge_ensemble!(e,backup=(true,100), converge_factor=1e-6)

converged=deepcopy(plt)
plot!(converged, title="Converged ($(length(e.log_Li)) iterates)")
scatter!(converged, harvest_xs(e), marker=:cross, markercolor=:black, legend=:none)

cmb=plot(init, thousand, tenthousand, converged, layout=grid(2,2), size=(900,900))
savefig(cmb, "/bench/PhD/Thesis/images/GMC/eggbox.png")
```

Bibliography

[ABS+10] W. Ted Allison, Linda K. Barthel, Kristina M. Skebo, Masaki Takechi, Shoji Kawamura, and Pamela A. Raymond. Ontogeny of cone photoreceptor mosaics in zebrafish. *The Journal of Comparative Neurology*, 518(20):4182–4195, October 2010.

[AC96] Macrene R. Alexiades and Constance Cepko. Quantitative analysis of proliferation and cell cycle length during development of the rat retina. *Developmental Dynamics*, 205(3):293–307, March 1996.

[ADH10] Christophe Andrieu, Arnaud Doucet, and Roman Holenstein. Particle Markov chain Monte Carlo methods: Particle Markov Chain Monte Carlo Methods. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 72(3):269–342, June 2010.

[AH09] Michalis Agathocleous and William A. Harris. From progenitors to differentiated cells in the vertebrate retina. *Annual Review of Cell and Developmental Biology*, 25:45–69, 2009.

[AHW+20] Afnan Azizi, Anne Herrmann, Yinan Wan, Salvador JRP Buse, Philipp J Keller, Raymond E Goldstein, and William A Harris. Nuclear crowding and nonlinear diffusion during interkinetic nuclear migration in the zebrafish retina. 9(58635):31, 2020.

[ALHP07] Michalis Agathocleous, Morgane Locker, William A. Harris, and Muriel Perron. A General Role of Hedgehog in the Regulation of Proliferation. *Cell Cycle*, 6(2):156–159, January 2007.

[AlQ19] Mohammed AlQuraishi. AlphaFold at CASP13. *Bioinformatics*, 35(22):4862–4865, November 2019.

[And18] Simon Andrews. FastQC A Quality Control tool for High Throughput Sequence Data. https://www.bioinformatics.babraham.ac.uk/projects/fastqc/, 04-10-18.

[And18] Simon Andrews. Trim Galore! https://www.bioinformatics.babraham.ac.uk/projects/trim_galore/, 28-06-18.

[AR08] Ruben Adler and Pamela A. Raymond. Have we achieved a unified model of photoreceptor cell fate specification in vertebrates? *Brain Research*, 1192:134–150, February 2008.

[ASNS17] Ankit Agrawal, Snehal V Sambare, Leelavati Narlikar, and Rahul Siddharthan. THiCweed: Fast, sensitive detection of sequence features by clustering big data sets. *bioRxiv*, November 2017.

[BA02] Kenneth P. Burnham and David Raymond Anderson. *Model Selection and Multimodel Inference: A Practical Information-Theoretic Approach*. Springer, New York, 2nd ed edition, 2002.

[Bar14] Marcello Barbieri. *Code Biology: A New Science of Life*. Springer Berlin Heidelberg, New York, NY, 2014.

[BB01] Thomas Becker and Catherina G. Becker. Regenerating descending axons preferentially reroute to the gray matter in the presence of a general macrophage/microglial reaction caudal to a spinal transection in adult zebrafish. *The Journal of Comparative Neurology*, 433(1):131–147, April 2001.

[BB20] M.J. Brewer and A. Butler. Model selection and the cult of AIC. In *Departmental Seminar*, University of Wollongong, Australia, February 2020.

[BEKS15] Jeff Bezanson, Alan Edelman, Stefan Karpinski, and Viral B. Shah. Julia: A Fresh Approach to Numerical Computing. *arXiv:1411.1607 [cs]*, July 2015.

[BGL+10] Klaus A. Becker, Prachi N. Ghule, Jane B. Lian, Janet L. Stein, Andre J. van Wijnen, and Gary S. Stein. Cyclin D2 and the CDK substrate p220 ^{NPAT} are required for self-renewal of human embryonic stem cells. *Journal of Cellular Physiology*, 222(2):456–464, February 2010.

[Bis06] Christopher M. Bishop. *Pattern Recognition and Machine Learning*. Information Science and Statistics. Springer, New York, 2006.

[BJ79] D. H. Beach and Marcus Jacobson. Influences of thyroxine on cell proliferation in the retina of the clawed frog at different ages. *The Journal of Comparative Neurology*, 183(3):615–623, February 1979.

[BNL+96] M. Burmeister, J. Novak, M. Y. Liang, S. Basu, L. Ploder, N. L. Hawes, D. Vidgen, F. Hoover, D. Goldman, V. I. Kalnins, T. H. Roderick, B. A. Taylor, M. H. Hankin, and R. R. McInnes. Ocular retardation mouse caused by Chx10 homeobox null allele: Impaired retinal progenitor proliferation and bipolar cell differentiation. *Nature Genetics*, 12(4):376–384, April 1996.

[BP66] Leonard E. Baum and Ted Petrie. Statistical Inference for Probabilistic Functions of Finite State Markov Chains. *The Annals of Mathematical Statistics*, 37(6):1554–1563, December 1966.

[BRD+15] Henrik Boije, Steffen Rulands, Stefanie Dudczig, Benjamin D. Simons, and William A. Harris. The Independent Probabilistic Firing of Transcription Factors: A Paradigm for Clonal Variability in the Zebrafish Retina. *Developmental Cell*, 34(5):532–543, September 2015.

[Bri10] Ingo Brigandt. Beyond reduction and pluralism: Toward an epistemology of explanatory integration in biology. *Erkenntnis*, 73(3):295–311, 2010.

[BSB00] José M. Bernardo, Adrian F. M. Smith, and Thomas Bayes. *Bayesian Theory*. Wiley Series in Probability and Mathematical Statistics. Wiley, Chichester, 2000.

[Buc16] Johannes Buchner. A statistical test for Nested Sampling algorithms. *Statistics and Computing*, 26(1-2):383–392, January 2016.

[CAH+14] L. Centanin, J.-J. Ander, B. Hoeckendorf, K. Lust, T. Kellner, I. Kraemer, C. Urbany, E. Hasel, W. A. Harris, B. D. Simons, and J. Wittbrodt. Exclusive multipotency and

preferential asymmetric divisions in post-embryonic neural stem cells of the fish retina. *Development*, 141(18):3472–3482, September 2014.

[CAI+04] Brenda LK Coles, Brigitte Angénieux, Tomoyuki Inoue, Katia Del Rio-Tsonis, Jason R. Spence, Roderick R. McInnes, Yvan Arsenijevic, and Derek van der Kooy. Facile isolation and the characterization of human retinal stem cells. *Proceedings of the National Academy of Sciences of the United States of America*, 101(44):15772–15777, 2004.

[CAR+15] Renee W. Chow, Alexandra D. Almeida, Owen Randlett, Caren Norden, and William A. Harris. Inhibitory neuron migration and IPL formation in the developing zebrafish retina. *Development*, 142(15):2665–2677, August 2015.

[Cav18] Florencia Cavodeassi. Dynamic Tissue Rearrangements during Vertebrate Eye Morphogenesis: Insights from Fish Models. *Journal of Developmental Biology*, 6(1):4, February 2018.

[CAY+96] Constance L. Cepko, Christopher P. Austin, Xianjie Yang, Macrene Alexiades, and Diala Ezzeddine. Cell fate determination in the vertebrate retina. *Proceedings of the National Academy of Sciences*, 93(2):589–595, 1996.

[CBR03] Michel Cayouette, Ben A. Barres, and Martin Raff. Importance of intrinsic mechanisms in cell fate decisions in the developing rat retina. *Neuron*, 40(5):897–904, December 2003.

[CC07] Bo Chen and Constance L Cepko. Requirement of histone deacetylase activity for the expression of critical photoreceptor genes. *BMC Developmental Biology*, 7(1):78, 2007.

[CCP09] J. M. Catchen, J. S. Conery, and J. H. Postlethwait. Automated identification of conserved synteny after whole-genome duplication. *Genome Research*, 19(8):1497–1505, August 2009.

[CCY+05] Florencia Cavodeassi, Filipa Carreira-Barbosa, Rodrigo M. Young, Miguel L. Concha, Miguel L. Allende, Corinne Houart, Masazumi Tada, and Stephen W. Wilson. Early Stages of Zebrafish Eye Formation Require the Coordinated Activity of Wnt11, Fz5, and the Wnt/β-Catenin Pathway. *Neuron*, 47(1):43–56, July 2005.

[CDMR01] Ian J. Conlon, Graham A. Dunn, Anne W. Mudge, and Martin C. Raff. Extracellular control of cell size. *Nature Cell Biology*, 3(10):918–921, October 2001.

[Cep14] Connie Cepko. Intrinsically different retinal progenitor cells produce specific types of progeny. *Nature Reviews Neuroscience*, 15(9):615–627, September 2014.

[CHB+08] Hannah H. Chang, Martin Hemberg, Mauricio Barahona, Donald E. Ingber, and Sui Huang. Transcriptome-wide noise controls lineage choice in mammalian progenitor cells. *Nature*, 453(7194):544–547, May 2008.

[Chi97] D. C. Chin. Comparative study of stochastic algorithms for system optimization based on gradient approximations. *IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics)*, 27(2):244–249, April 1997.

[CHW11] Lázaro Centanin, Burkhard Hoeckendorf, and Joachim Wittbrodt. Fate Restriction and Multipotency in Retinal Stem Cells. *Cell Stem Cell*, 9(6):553–562, December 2011.

[CM18] Peter R Cook and Davide Marenduzzo. Transcription-driven genome organization: A model for chromosome structure and the regulation of gene expression tested through simulations. *Nucleic Acids Research*, 1:12, 2018.

[CMD+09] Fatemah Chehrehasa, Adrian C.B. Meedeniya, Patrick Dwyer, Greger Abrahamsen, and Alan Mackay-Sim. EdU, a new thymidine analogue for labelling proliferating cells in the nervous system. *Journal of Neuroscience Methods*, 177(1):122–130, February 2009.

[CW08] Alexander Churbanov and Stephen Winters-Hilt. Implementing EM and Viterbi algorithms for Hidden Markov Model in linear memory. *BMC Bioinformatics*, 9(1):224, December 2008.

[CXP+13] K. Chen, Y. Xi, X. Pan, Z. Li, K. Kaestner, J. Tyler, S. Dent, X. He, and W. Li. DANPOS: Dynamic analysis of nucleosome position and occupancy by sequencing. *Genome Research*, 23(2):341–351, February 2013.

[CYV+08] Anna M. Clark, Sanghee Yun, Eric S. Veien, Yuan Y. Wu, Robert L. Chow, Richard I. Dorsky, and Edward M. Levine. Negative regulation of Vsx1 by its paralog Chx10/Vsx2 is conserved in the vertebrate retina. *Brain Research*, 1192:99–113, February 2008.

[CZ12] Kairong Cui and Keji Zhao. Genome-Wide Approaches to Determining Nucleosome Occupancy in Metazoans Using MNase-Seq. In Randall H. Morse, editor, *Chromatin Remodeling*, volume 833, pages 413–419. Humana Press, Totowa, NJ, 2012.

[Dal15] Stephen Dalton. Linking the Cell Cycle to Cell Fate Decisions. *Trends in Cell Biology*, 25(10):592–600, October 2015.

[Dar88] Charles Darwin. *The Origin of Species by Means of Natural Selection, or, The Preservation of Favoured Races in the Struggle for Life*. J. Murray,, London :, 1888.

[DB16] Christian Dietz and Michael R. Berthold. KNIME for Open-Source Bioimage Analysis: A Tutorial. In Winnok H. De Vos, Sebastian Munck, and Jean-Pierre Timmermans, editors, *Focus on Bio-Image Informatics*, volume 219, pages 179–197. Springer International Publishing, Cham, 2016.

[DC00] Michael A. Dyer and Constance L. Cepko. Control of Müller glial cell proliferation and activation followingretinal injury. *Nature Neuroscience*, 3(9):873–880, September 2000.

[DCRH97] R. I. Dorsky, W. S. Chang, D. H. Rapaport, and W. A. Harris. Regulation of neuronal diversity in the Xenopus retina by Delta signalling. *Nature*, 385(6611):67–70, January 1997.

[DD17] Jacqueline Deschamps and Denis Duboule. Embryonic timing, axial stem cells, chromatin dynamics, and the Hox clock. *Genes & Development*, 31(14):1406–1416, July 2017.

[DFWV+97] MARCO Di Fruscio, Hans Weiher, Barbara C. Vanderhyden, Takashi Imai, Tadahiro Shiomi, T. A. Hori, Rudolf Jaenisch, and Douglas A. Gray. Proviral inactivation of the Npat gene of Mpv 20 mice results in early embryonic arrest. *Molecular and cellular biology*, 17(7):4080–4086, 1997.

[DH05] T. A. Down and Tim J.P. Hubbard. NestedMICA: Sensitive inference of over-represented motifs in nucleic acid sequence. *Nucleic Acids Research*, 33(5):1445–1453, March 2005.

[DHM07] Persi Diaconis, Susan Holmes, and Richard Montgomery. Dynamical Bias in the Coin Toss. *SIAM Review*, 49(2):211–235, January 2007.

[DPCH03] Tilak Das, Bernhard Payer, Michel Cayouette, and William A. Harris. In vivo time-lapse imaging of cell divisions during neurogenesis in the developing zebrafish retina. *Neuron*, 37(4):597–609, 2003.

[DRH95] Richard I Dorsky, David H Rapaport, and William A Harris. Xotch inhibits cell differentiation in the xenopus retina. *Neuron*, 14(3):487–496, March 1995.

[Eag18] Antony Eagle. Chance versus Randomness. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, spring 2018 edition, 2018.

[EHML09] Ted Erclik, Volker Hartenstein, Roderick R. McInnes, and Howard D. Lipshitz. Eye evolution at high resolution: The neuron as a unit of homology. *Developmental Biology*, 332(1):70–79, August 2009.

[ESY+17] Peter Engerer, Sachihiro C Suzuki, Takeshi Yoshimatsu, Prisca Chapouton, Nancy Obeng, Benjamin Odermatt, Philip R Williams, Thomas Misgeld, and Leanne Godinho. Uncoupling of neurogenesis and differentiation during retinal development. *The EMBO Journal*, 36(9):1134–1146, May 2017.

[Fag12] Melinda Bonnie Fagan. The Joint Account of Mechanistic Explanation. *Philosophy of Science*, 79(4):448–472, October 2012.

[Fag13] Melinda Bonnie Fagan. *Philosophy of Stem Cell Biology: Knowledge in Flesh and Blood*. New Directions in the Philosophy of Science. Palgrave Macmillan, Houndmills, Basingstoke, Hampshire, 2013.

[Fag15] Melinda Bonnie Fagan. Collaborative explanation and biological mechanisms. *Studies in History and Philosophy of Science Part A*, 52:67–78, August 2015.

[Fav15] Donald Favareau. Why this now? The conceptual and historical rationale behind the development of biosemiotics. *Green Letters*, 19(3):227–242, September 2015.

[FEF+09] Curtis R. French, Timothy Erickson, Danielle V. French, David B. Pilgrim, and Andrew J. Waskiewicz. Gdf6a is required for the initiation of dorsal–ventral retinal patterning and lens development. *Developmental Biology*, 333(1):37–47, September 2009.

[Fey75] Paul Feyerabend. How to Defend Society Against Science. *Radical Philosophy*, (11):3–8, 1975.

[Fey81] Paul Feyerabend. *Philosophical Papers*. Cambridge University Press, Cambridge ; New York, 1981.

[Fey93] Paul Feyerabend. *Against Method*. Verso, London ; New York, 3rd ed edition, 1993.

[FH08] Farhan Feroz and M. P. Hobson. Multimodal nested sampling: An efficient and robust alternative to MCMC methods for astronomical data analysis. *Monthly Notices of the Royal Astronomical Society*, 384(2):449–463, January 2008.

[FHB09] F. Feroz, M. P. Hobson, and M. Bridges. MultiNest: An efficient and robust Bayesian inference tool for cosmology and particle physics. *Monthly Notices of the Royal Astronomical Society*, 398(4):1601–1614, October 2009.

[FK12] Chikara Furusawa and Kunihiko Kaneko. A Dynamical-Systems View of Stem Cell Biology. *Science*, 338(6104):215–217, October 2012.

[FR00] Andy J. Fischer and Thomas A. Reh. Identification of a Proliferating Marginal Zone of Retinal Progenitors in Postnatal Chickens. *Developmental Biology*, 220(2):197–210, April 2000.

[FR03] Andy J. Fischer and Thomas A. Reh. Potential of Müller glia to become neurogenic retinal progenitor cells: Retinal Müller Glia as a Source of Stem Cells. *Glia*, 43(1):70–76, July 2003.

[GBB+03] G. Gao, A. P. Bracken, K. Burkard, D. Pasini, M. Classon, C. Attwooll, M. Sagara, T. Imai, K. Helin, and J. Zhao. NPAT Expression Is Regulated by E2F and Is Essential for Cell Cycle Progression. *Molecular and Cellular Biology*, 23(8):2821–2833, April 2003.

[GDL+09] Prachi N. Ghule, Zbigniew Dominski, Jane B. Lian, Janet L. Stein, Andre J. van Wijnen, and Gary S. Stein. The subnuclear organization of histone gene regulatory proteins and $3'$ end processing factors of normal somatic and embryonic stem cells is compromised in selected human cancer cell types. *Journal of Cellular Physiology*, 220(1):129–135, July 2009.

[Geh96] Walter J. Gehring. The master control gene for morphogenesis and evolution of the eye. *Genes to Cells*, 1(1):11–15, January 1996.

[GJH+17] Mahdi Golkaram, Jiwon Jang, Stefan Hellander, Kenneth S. Kosik, and Linda R. Petzold. The Role of Chromatin Density in Cell Population Heterogeneity during Stem Cell Differentiation. *Scientific Reports*, 7(1):13307, October 2017.

[GNB08] Nathan J. Gosse, Linda M. Nevin, and Herwig Baier. Retinotopic order in the absence of axon competition. *Nature*, 452(7189):892–895, April 2008.

[Gol17] Sheldon Goldstein. Bohmian Mechanics. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, summer 2017 edition, 2017.

[Gre17] Michael Green. The truth about Bayesian priors and overfitting. https://towardsdatascience.com/the-truth-about-bayesian-priors-and-overfitting-84e24d3a1153, August 2017.

[GS07] Frédéric Gaillard and Yves Sauvé. Cell-based therapy for retina degeneration: The promise of a cure. *Vision Research*, 47(22):2815–2824, October 2007.

[GTR+10] D. M. Gilbert, S.-I. Takebayashi, T. Ryba, J. Lu, B. D. Pope, K. A. Wilson, and I. Hiratani. Space and Time in the Nucleus: Developmental Control of Replication Timing and Chromosome Architecture. *Cold Spring Harbor Symposia on Quantitative Biology*, 75(0):143–153, January 2010.

[GWC+07] Leanne Godinho, Philip R. Williams, Yvonne Claassen, Elayne Provost, Steven D. Leach, Maarten Kamermans, and Rachel O.L. Wong. Nonapical Symmetric Divisions Underlie Horizontal Cell Layer Formation in the Developing Retina In Vivo. *Neuron*, 56(4):597–603, November 2007.

[GZC+11] Francisco L. A. F. Gomes, Gen Zhang, Felix Carbonell, José A. Correa, William A. Harris, Benjamin D. Simons, and Michel Cayouette. Reconstruction of rat retinal progenitor cell lineages in vitro reveals a surprising degree of stochasticity in cell fate decisions. *Development (Cambridge, England)*, 138(2):227–235, January 2011.

[Has03] Samiul Hasan. *Abstract.Pdf*. PhD thesis, University of Cambridge, Cambridge, 2003.

[HBEH88] Christine E. Holt, Thomas W. Bertsch, Hillary M. Ellis, and William A. Harris. Cellular Determination in the Xenopus Retina is Independent of Lineage and Birth Date. *Neuron*, 1(1):15–26, March 1988.

[HCG95] G Halder, P Callaerts, and W. Gehring. Induction of ectopic eyes by targeted expression of the eyeless gene in Drosophila. *Science*, 267(5205):1788–1792, March 1995.

[HE99] Minjie Hu and Stephen S. Easter. Retinal Neurogenesis: The Formation of the Initial Central Patch of Postmitotic Cells. *Developmental Biology*, 207(2):309–321, March 1999.

[Hea67] D.F. Heath. Normal or Log-normal: Appropriate Distributions. *Nature*, 213:1159–1160, March 1967.

[HG12] David I. Hastie and Peter J. Green. Model choice using reversible jump Markov chain Monte Carlo: *Model choice using reversible jump MCMC*. *Statistica Neerlandica*, 66(3):309–338, August 2012.

[HH91] William A. Harris and Volker Hartenstein. Neuronal determination without cell division in xenopus embryos. *Neuron*, 6:499–515, 1991.

[HHHL18] Edward Higson, Will Handley, Mike Hobson, and Anthony Lasenby. Sampling Errors in Nested Sampling Parameter Estimation. *Bayesian Analysis*, 13(3):873–896, September 2018.

[HHHL19] Edward Higson, Will Handley, Michael Hobson, and Anthony Lasenby. Dynamic nested sampling: An improved algorithm for parameter estimation and evidence calculation. *Statistics and Computing*, 29(5):891–913, September 2019.

[HJH+17] Maximilian Hastreiter, Tim Jeske, Jonathan Hoser, Michael Kluge, Kaarin Ahomaa, Marie-Sophie Friedl, Sebastian J. Kopetzky, Jan-Dominik Quell, H. Werner Mewes, and Robert Küffner. KNIME4NGS: A comprehensive toolbox for Next Generation Sequencing analysis. *Bioinformatics*, page btx003, January 2017.

[HLZQ17] Sui Huang, Fangting Li, Joseph X. Zhou, and Hong Qian. Processes on the emergent landscapes of biochemical reaction networks and heterogeneous cell population dynamics: Differentiation in living matters. *Journal of the Royal Society Interface*, 14(130), May 2017.

[Hof08] Jesper Hoffmeyer, editor. *A Legacy for Living Systems: Gregory Bateson as Precursor to Biosemiotics*. Number volume 2 in Biosemiotics. Springer, New York, 2008.

[Hof15] Jesper Hoffmeyer. Semiotic Scaffolding of Multicellularity. *Biosemiotics*, 8(2):159–171, August 2015.

[Hor04] D. J. Horsford. Chx10 repression of Mitf is required for the maintenance of mammalian neuroretinal identity. *Development*, 132(1):177–187, December 2004.

[HP98] William A. Harris and Muriel Perron. Molecular recapitulation: The growth of the vertebrate retina. *International Journal of Developmental Biology*, 42:299–304, 1998.

[HPG07] J. Herisson, G. Payen, and R. Gherbi. A 3D pattern matching algorithm for DNA sequences. *Bioinformatics*, 23(6):680–686, March 2007.

[HSF97] John Henderson, Steven Salzberg, and Kenneth H. Fasman. Finding Genes in DNA with a Hidden Markov Model. *Journal of Computational Biology*, 4(2):127–141, January 1997.

[HSK+13] Steven A. Harvey, Ian Sealy, Ross Kettleborough, Fruzsina Fenyes, Richard White, Derek Stemple, and James C. Smith. Identification of the zebrafish maternal and paternal transcriptomes. *Development*, 140(13):2703–2710, July 2013.

[HW81] Philip Heidelberger and Peter D. Welch. A spectral method for confidence interval generation and run length control in simulations. *Communications of the ACM*, 24(4):233–245, April 1981.

[HYSL11] Hongpeng He, Fa-Xing Yu, Chi Sun, and Yan Luo. CBP/p300 and SIRT1 Are Involved in Transcriptional Regulation of S-Phase Specific Histone Genes. *PLoS ONE*, 6(7):e22088, July 2011.

[HZA+12] Jie He, Gen Zhang, Alexandra D. Almeida, Michel Cayouette, Benjamin D. Simons, and William A. Harris. How Variable Clones Build an Invariant Retina. *Neuron*, 75(5):786–798, September 2012.

[HZP18] Lachlan Harris, Oressia Zalucki, and Michael Piper. BrdU/EdU dual labeling to determine the cell-cycle dynamics of defined cellular subpopulations. *Journal of Molecular Histology*, 49(3):229–234, June 2018.

[ICW13] Kenzo Ivanovitch, Florencia Cavodeassi, and Stephen W. Wilson. Precocious Acquisition of Neuroepithelial Character in the Eye Field Underlies the Onset of Eye Morphogenesis. *Developmental Cell*, 27(3):293–305, November 2013.

[IKRN16] Jaroslav Icha, Christiane Kunath, Mauricio Rocha-Martins, and Caren Norden. Independent modes of ganglion cell translocation ensure correct lamination of the zebrafish retina. *The Journal of Cell Biology*, 215(2):259–275, October 2016.

[ILU+14] Helgi I. Ingólfsson, Cesar A. Lopez, Jaakko J. Uusitalo, Djurre H. de Jong, Srinivasa M. Gopal, Xavier Periole, and Siewert J. Marrink. The power of coarse graining in biomolecular simulations: The power of coarse graining in biomolecular simulations. *Wiley Interdisciplinary Reviews: Computational Molecular Science*, 4(3):225–248, May 2014.

[Ioa05] John P. A. Ioannidis. Why Most Published Research Findings Are False. *PLoS Medicine*, 2(8):e124, August 2005.

[IYS+96] T Imai, M Yamauchi, N Seki, T Sugawara, T Saito, Y Matsuda, H Ito, T Nagase, N Nomura, and T Hori. Identification and characterization of a new gene physically linked to the ATM gene. *Genome Research*, 6(5):439–447, May 1996.

[JBE03] Edwin T Jaynes, G. Larry Bretthorst, and EBSCO Publishing. *Probability Theory: The Logic of Science*. Cambridge University Press, Cambridge, 2003.

[JHTS00] Jong-Hwan Lee, Ho-Young Jung, Te-Won Lee, and Soo-Young Lee. Speech feature extraction using independent component analysis. In *2000 IEEE International Conference on Acoustics, Speech, and Signal Processing. Proceedings (Cat. No.00CH37100)*, volume 3, pages 1631–1634, Istanbul, Turkey, 2000. IEEE.

[JO04] Jerald B. Johnson and Kristian S. Omland. Model selection in ecology and evolution. *Trends in Ecology & Evolution*, 19(2):101–108, February 2004.

[Kan06] Kunihiko Kaneko. *Life: An Introduction to Complex Systems Biology*. Understanding Complex Systems. Springer, Berlin ; New York, 2006.

[Kes04] David Kestenbaum. The Not So Random Coin Toss. https://www.npr.org/templates/story/story.php?storyId=1697475, 2004.

[KFG+07] Kristen M. Kwan, Esther Fujimoto, Clemens Grabher, Benjamin D. Mangum, Melissa E. Hardy, Douglas S. Campbell, John M. Parant, H. Joseph Yost, John P. Kanki, and Chi-Bin Chien. The Tol2kit: A multisite gateway-based construction kit forTol2 transposon transgenesis constructs. *Developmental Dynamics*, 236(11):3088–3099, November 2007.

[KHI97] N.L. Kleinman, S.D. Hill, and V.A. Ilenda. SPSA/SIMMOD optimization of air traffic delay cost. In *Proceedings of the 1997 American Control Conference (Cat. No.97CH36041)*, pages 1121–1125 vol.2, Albuquerque, NM, USA, 1997. IEEE.

[KHM+15] Kevin H. Knuth, Michael Habeck, Nabin K. Malakar, Asim M. Mubeen, and Ben Placek. Bayesian evidence and model selection. *Digital Signal Processing*, 47:50–67, December 2015.

[KKT+13] Vijayalakshmi Kari, Oleksandra Karpiuk, Bettina Tieg, Malte Kriegs, Ekkehard Dikomey, Heike Krebber, Yvonne Begus-Nahrmann, and Steven A. Johnsen. A Subset of Histone H2B Genes Produces Polyadenylated mRNAs under a Variety of Cellular Conditions. *PLoS ONE*, 8(5):e63745, May 2013.

[KKZ+18] Naeh L. Klages-Mundt, Ashok Kumar, Yuexuan Zhang, Prabodh Kapoor, and Xuetong Shen. The Nature of Actin-Family Proteins in Chromatin-Modifying Complexes. *Frontiers in Genetics*, 9, September 2018.

[KMF+09] Noam Kaplan, Irene K. Moore, Yvonne Fondufe-Mittendorf, Andrea J. Gossett, Desiree Tillo, Yair Field, Emily M. LeProust, Timothy R. Hughes, Jason D. Lieb, Jonathan Widom, and Eran Segal. The DNA-encoded nucleosome organization of a eukaryotic genome. *Nature*, 458(7236):362–366, March 2009.

[Kon06] Toru Kondo. Epigenetic alchemy for cell fate conversion. *Current Opinion in Genetics & Development*, 16(5):502–507, October 2006.

[Kor05] Alfred Korzybski. *Science and Sanity: An Introduction to Non-Aristotelian Systems and General Semantics*. Inst. of General Semantics, Brooklyn, N.Y, 5. ed., 3. print edition, 2005.

[Koz08] Zbynek Kozmik. The role of Pax genes in eye evolution. *Brain Research Bulletin*, 75(2-4):335–339, March 2008.

[KP01] Walter G. Kelley and Allan C. Peterson. *Difference Equations: An Introduction with Applications*. Harcourt Academic Press, San Diego, CA, USA, second edition, 2001.

[KS12] Kevin H. Knuth and John Skilling. Foundations of Inference. *Axioms*, 1(1):38–73, June 2012.

[KSN99] Nathan L. Kleinman, James C. Spall, and Daniel Q. Naiman. Simulation-Based Optimization with Stochastic Approximation Using Common Random Numbers. *Management Science*, 45(11):1570–1578, 1999.

[LAA+06] M. Locker, M. Agathocleous, M. A. Amato, K. Parain, W. A. Harris, and M. Perron. Hedgehog signaling and the retina: Insights into the mechanisms controlling the proliferative properties of neural precursors. *Genes & Development*, 20(21):3036–3048, November 2006.

[Lak76] Imre Lakatos. Falsification and the Methodology of Scientific Research Programmes. In Sandra G. Harding, editor, *Can Theories Be Refuted? Essays on the Duhem-Quine Thesis*, Synthese Library, pages 205–259. Springer Netherlands, Dordrecht, 1976.

[Lan12] Hans Petter Langtangen. *A Primer on Scientific Programming with Python*. Number 6 in Texts in Computational Science and Engineering. Springer Berlin Heidelberg, Berlin, Heidelberg, third edition, 2012.

[Lar92] Ellen W. Larsen. Tissue strategies as developmental constraints: Implications for animal evolution. *Trends in Ecology & Evolution*, 7(12):414–417, December 1992.

[LD09] Enhu Li and Eric H. Davidson. Building developmental gene regulatory networks. *Birth Defects Research Part C: Embryo Today: Reviews*, 87(2):123–130, June 2009.

[LDT12] Benjamin W. Lindsey, Audrey Darabie, and Vincent Tropepe. The cellular composition of neurogenic periventricular zones in the adult zebrafish forebrain. *The Journal of Comparative Neurology*, 520(10):2275–2316, July 2012.

[LGBS00] Te-Won Lee, M. Girolami, A.J. Bell, and T.J. Sejnowski. A unifying information-theoretic framework for independent component analysis. *Computers & Mathematics with Applications*, 39(11):1–21, June 2000.

[LHKR08] Deepak A. Lamba, Susan Hayes, Mike O. Karl, and Thomas Reh. Baf60c is a component of the neural progenitor-specific BAF complex in developing retina. *Developmental Dynamics*, 237(10):3016–3023, September 2008.

[LHO+00] Zheng Li, Minjie Hu, Malgorzata J. Ochocinska, Nancy M. Joseph, and Stephen S. Easter. Modulation of cell proliferation in the embryonic retina of zebrafish (Danio rerio). *Developmental Dynamics*, 219(3):391–401, 2000.

[LS12] Ben Langmead and Steven L Salzberg. Fast gapped-read alignment with Bowtie 2. *Nature Methods*, 9(4):357–359, April 2012.

[LV08] Ming Li and P. M. B. Vitányi. *An Introduction to Kolmogorov Complexity and Its Applications*. Texts in Computer Science. Springer, New York, 3rd ed edition, 2008.

[LWR+07] Julie Lessard, Jiang I. Wu, Jeffrey A. Ranish, Mimi Wan, Monte M. Winslow, Brett T. Staahl, Hai Wu, Ruedi Aebersold, Isabella A. Graef, and Gerald R. Crabtree. An Essential Switch in Subunit Composition of a Chromatin Remodeling Complex during Neural Development. *Neuron*, 55(2):201–215, July 2007.

[Ma00] T. Ma. Cell cycle-regulated phosphorylation of p220NPAT by cyclin E/Cdk2 in Cajal bodies promotes histone gene transcription. *Genes & Development*, 14(18):2298–2313, September 2000.

[MAA+01] Till Marquardt, Ruth Ashery-Padan, Nicole Andrejewski, Raffaella Scardigli, Francois Guillemot, and Peter Gruss. Pax6 Is Required for the Multipotent State of Retinal Progenitor Cells. *Trends in Biochemical Sciences*, 30(3):i, 2001.

[MAB+13] Gary R. Mirams, Christopher J. Arthurs, Miguel O. Bernabeu, Rafel Bordas, Jonathan Cooper, Alberto Corrias, Yohan Davit, Sara-Jane Dunn, Alexander G. Fletcher, Daniel G. Harvey, Megan E. Marsh, James M. Osborne, Pras Pathmanathan, Joe Pitt-Francis, James Southern, Nejib Zemzemi, and David J. Gavaghan. Chaste: An Open Source C++ Library for Computational Physiology and Biology. *PLoS Computational Biology*, 9(3):e1002970, March 2013.

[MBE+07] Karine Massé, Surinder Bhamra, Robert Eason, Nicholas Dale, and Elizabeth A. Jones. Purine-mediated signalling triggers eye development. *Nature*, 449(7165):1058–1062, October 2007.

[MDBN+05] Juan-Ramon Martinez-Morales, Filippo Del Bene, Gabriela Nica, Matthias Hammerschmidt, Paola Bovolenta, and Joachim Wittbrodt. Differentiation of the Vertebrate Retina Is Coordinated by an FGF Signaling Center. *Developmental Cell*, 8(4):565–574, April 2005.

[MFKM12] Kiran Mahajan, Bin Fang, John M Koomen, and Nupam P Mahajan. H2B Tyr37 phosphorylation suppresses expression of replication-dependent core histone genes. *Nature Structural & Molecular Biology*, 19(9):930–937, August 2012.

[MGv+09] Partha Mitra, Prachi N. Ghule, Margaretha van der Deen, Ricardo Medina, Rong-lin Xie, William F. Holmes, Xin Ye, Keiichi I. Nakayama, J. Wade Harper, Janet L. Stein, Gary S. Stein, and Andre J. van Wijnen. CDK inhibitors selectively diminish cell cycle controlled

activation of the histone H4 gene promoter by p220 ^{NPAT} and HiNF-P. *Journal of Cellular Physiology*, 219(2):438–448, May 2009.

[MHR+99] Nicholas Marsh-Armstrong, Haochu Huang, Benjamin F Remo, Tong Tong Liu, and Donald D Brown. Asymmetric Growth and Development of the Xenopus laevis Retina during Metamorphosis Is Controlled by Type III Deiodinase. *Neuron*, 24(4):871–878, December 1999.

[Min00] T.P. Minka. Estimating a Dirichlet Distribution. Technical Report, MIT, 2000.

[MMDM04] Kathryn B. Moore, Kathleen Mood, Ira O. Daar, and Sally A. Moody. Morphogenetic Movements Underlying Eye Field Formation Require Interactions between the FGF and ephrinB1 Signaling Pathways. *Developmental Cell*, 6(1):55–67, January 2004.

[Mor03] Michel Morange. *La Vie Expliquée: 50 Ans Après La Double Hélice*. Sciences. O. Jacob, Paris, 2003.

[Mor08] Michel Morange. What history tells us XIII. Fifty years of the Central Dogma. *Journal of biosciences*, 33(2):171–175, 2008.

[Mor09] Michel Morange. A new revolution? The place of systems biology and synthetic biology in the history of biology. *EMBO Reports*, 10(Suppl 1):S50–S53, August 2009.

[Mor11] Michel Morange. Recent opportunities for an increasing role for physical explanations in biology. *Studies in History and Philosophy of Biol & Biomed Sci*, 42(2):139–144, 2011.

[Mun19] Bernard Munos. 2018 New Drugs Approvals: An All-Time Record, And A Watershed. *Forbes*, January 2019.

[Mur07] Kevin P Murphy. Conjugate Bayesian analysis of the Gaussian distribution. Technical Report, University of British Columbia, October 2007.

[MXM+03] Partha Mitra, Rong-Lin Xie, Ricardo Medina, Hayk Hovhannisyan, S. Kaleem Zaidi, Yue Wei, J. Wade Harper, Janet L. Stein, André J. van Wijnen, and Gary S. Stein. Identification of HiNF-P, a Key Activator of Cell Cycle-Controlled Histone H4 Genes at the Onset of S Phase. *Molecular and Cellular Biology*, 23(22):8110–8123, November 2003.

[MZLF02] A Murciano, J Zamora, J Lopezsanchez, and J Frade. Interkinetic Nuclear Movement May Provide Spatial Clues to the Regulation of Neurogenesis. *Molecular and Cellular Neuroscience*, 21(2):285–300, October 2002.

[Neu00] C. J. Neumann. Patterning of the Zebrafish Retina by a Wave of Sonic Hedgehog Activity. *Science*, 289(5487):2137–2139, September 2000.

[NLM89] R. S. Nowakowski, S. B. Lewin, and M. W. Miller. Bromodeoxyuridine immunohistochemical determination of the lengths of the cell cycle and the DNA-synthetic phase for an anatomically defined population. *Journal of Neurocytology*, 18(3):311–318, June 1989.

[NNM+19] Hayden Nunley, Mikiko Nagashima, Kamirah Martin, Alcides Lorenzo Gonzalez, Sachihiro C. Suzuki, Declan Norton, Rachel O. L. Wong, Pamela A. Raymond, and David K. Lubensky. Defect patterns on the curved surface of fish retinae suggest mechanism of cone mosaic formation. Preprint, Biophysics, October 2019.

[NRN20] Elisa Nerli, Mauricio Rocha-Martins, and Caren Norden. Asymmetric neurogenic commitment of retinal progenitors involves Notch through the endocytic pathway. *eLife*, 9:e60462, November 2020.

[NYLH09] Caren Norden, Stephen Young, Brian A. Link, and William A. Harris. Actomyosin Is the Main Driver of Interkinetic Nuclear Migration in the Retina. *Cell*, 138(6):1195–1208, September 2009.

[PB04] David Posada and Thomas R. Buckley. Model Selection and Model Averaging in Phylogenetics: Advantages of Akaike Information Criterion and Bayesian Approaches Over Likelihood Ratio Tests. *Systematic Biology*, 53(5):793–808, October 2004.

[PEM+09] David M. Parichy, Michael R. Elizondo, Margaret G. Mills, Tiffany N. Gordon, and Raymond E. Engeszer. Normal table of postembryonic zebrafish development: Staging by externally visible anatomy of the living fish. *Developmental Dynamics*, 238(12):2975–3015, December 2009.

[Pfi20] Cameron Pfiffer. TuringLang/MCMCChains.jl. The Turing Language, December 2020.

[PJ10] J. Pirngruber and S. A. Johnsen. Induced G1 cell-cycle arrest controls replication-dependent histone mRNA 3′ end processing through p21, NPAT and CDK9. *Oncogene*, 29(19):2853–2863, 2010.

[PKVH98] Muriel Perron, Shami Kanekar, Monica L. Vetter, and William A Harris. The genetic sequence of retinal development in the ciliary margin of the xenopus eye. *Developmental Biology*, (199):185–200, 1998.

[PLM+17] Tao Peng, Linan Liu, Adam L MacLean, Chi Wut Wong, Weian Zhao, and Qing Nie. A mathematical model of mechanotransduction reveals how mechanical memory regulates mesenchymal stem cell fate decisions. *BMC Systems Biology*, 11, May 2017.

[Poi13] Julia Pointner. *Genome-wide identification of nucleosome positioning determinants in Schizosaccharomyces pombe*. PhD thesis, Ludwig-Maximilians-Universität, München, July 2013.

[PSS+09] Judith Pirngruber, Andrei Shchebet, Lisa Schreiber, Efrat Shema, Neri Minsky, Rob D Chapman, Dirk Eick, Yael Aylon, Moshe Oren, and Steven A Johnsen. CDK9 directs H2B monoubiquitination and controls replication-dependent histone mRNA 3′-end processing. *EMBO reports*, 10(8):894–900, August 2009.

[Rab89] Lawrence R Rabiner. A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition. *PROCEEDINGS OF THE IEEE*, 77(2):30, 1989.

[Rah] Ali Rahimi. An Erratum for "A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition". https://alumni.media.mit.edu/~rahimi/rabiner/rabiner-errata/rabiner-errata.html#complex_xi.

[RBBP06] Pamela A. Raymond, Linda K. Barthel, Rebecca L. Bernardos, and John J. Perkowski. Molecular characterization of retinal stem cells and their niches in adult zebrafish. *BMC developmental biology*, 6(1):36, 2006.

[RDT+01] J. T. Rasmussen, M. A. Deardorff, C. Tan, M. S. Rao, P. S. Klein, and M. L. Vetter. Regulation of eye development by frizzled signaling in Xenopus. *Proceedings of the National Academy of Sciences*, 98(7):3861–3866, March 2001.

[Res00] Nicholas Rescher. *Nature and Understanding.* Oxford University Press, New York, 2000.

[Res05] Nicholas Rescher. *Cognitive Harmony.* University of Pittsburgh Press, Pittsburgh, PA, 2005.

[RO04] Jonathan M. Raser and Erin K. O'Shea. Control of Stochasticity in Eukaryotic Gene Expression. *Science; Washington*, 304(5678):1811–4, June 2004.

[RTL+18] Jiong Ren, Cai-zhi Tang, Xu-Dong Li, Zhi-Bin Niu, Bo-Yang Zhang, Tao Zhang, Mei-Jiao Gao, Xin-Ze Ran, Yong-Ping Su, and Feng-Chao Wang. Identification of G2/M phase transition by sequential nuclear and cytoplasmic changes and molecular markers in mice intestinal epithelial cells. *Cell Cycle*, 17(6):780–791, March 2018.

[Rv08] Arjun Raj and Alexander van Oudenaarden. Stochastic gene expression and its consequences. *Cell*, 135(2):216–226, October 2008.

[Ryd08] Tobias Rydén. EM versus Markov chain Monte Carlo for estimation of hidden Markov models: A computational perspective. *Bayesian Analysis*, 3(4):659–688, December 2008.

[Sad97] Payman Sadegh. Constrained Optimization via Stochastic Approximation with a Simultaneous Perturbation Gradient Approximation. *Automatica*, 33(5):889–892, May 1997.

[Sch93] Kenneth F. Schaffner. *Discovery and Explanation in Biology and Medicine.* Science and Its Conceptual Foundations. University of Chicago Press, Chicago, 1993.

[SF03] Deborah L Stenkamp and Ruth A Frey. Extraretinal and retinal hedgehog signaling sequentially regulate retinal differentiation in zebrafish. *Developmental Biology*, 258(2):349–363, June 2003.

[SFM89] J. Sambrook, E. F. Fritsch, and T. Maniatis. Molecular cloning: A laboratory manual. *Molecular cloning: a laboratory manual.*, (Ed. 2), 1989.

[SH14] Corinna Singleman and Nathalia G. Holtzman. Growth and Maturation in the Zebrafish, *Danio Rerio* : A Staging Tool for Teaching and Research. *Zebrafish*, 11(4):396–406, August 2014.

[SK15] Zheng Sun and Natalia L. Komarova. Stochastic control of proliferation and differentiation in stem cell dynamics. *Journal of Mathematical Biology; Heidelberg*, 71(4):883–901, October 2015.

[Ski06] John Skilling. Nested Sampling for Bayesian Computations. In *Proc. Valencia*, Benidorm (Alicante, Spain), June 2006. inference.org.uk.

[Ski12] John Skilling. Bayesian computation in big spaces-nested sampling and Galilean Monte Carlo. In *BAYESIAN INFERENCE AND MAXIMUM ENTROPY METHODS IN SCIENCE AND ENGINEERING: 31st International Workshop on Bayesian Inference and Maximum Entropy Methods in Science and Engineering*, pages 145–156, Waterloo, Ontario, Canada, 2012.

[Ski19] John Skilling. Galilean and Hamiltonian Monte Carlo. In *Proceedings*, volume 33, page 8, Garching, Germany, 2019. MDPI.

[SMFW10] Erik B Sudderth, Michael I Mandel, William T Freeman, and Alan S Willsky. Distributed Occlusion Reasoning for Tracking with Nonparametric Belief Propagation. *Communications of the ACM*, 53(10):95–103, 2010.

[Spa98] J. C. Spall. Implementation of the simultaneous perturbation algorithm for stochastic optimization. *IEEE Transactions on Aerospace and Electronic Systems*, 34(3):817–823, July 1998.

[Spe19] Joshua S. Speagle. Dynesty: A Dynamic Nested Sampling Package for Estimating Bayesian Posteriors and Evidences. *arXiv:1904.02180 [astro-ph, stat]*, April 2019.

[SSGE82] G. D. Stormo, T. D. Schneider, L. Gold, and A. Ehrenfeucht. Use of the 'Perceptron' algorithm to distinguish translational initiation sites in E. coli. *Nucleic Acids Research*, 10(9):2997–3011, May 1982.

[Ste18] Jacob Stegenga. *Medical Nihilism*. Oxford University Press, Oxford, United Kingdom, first edition edition, 2018.

[STN+02] Masashi Sagara, Eri Takeda, Akiyo Nishiyama, Syunsaku Utsumi, Yoshiroh Toyama, Shigeki Yuasa, Yasuharu Ninomiya, and Takashi Imai. Characterization of functional regions for nuclear localization of NPAT. *The Journal of Biochemistry*, 132(6):875–879, 2002.

[SVT13] Lourdes Serrano, Berta N Vazquez, and Jay Tischfield. Chromatin structure, pluripotency and differentiation. *Experimental Biology and Medicine*, 238(3):259–270, March 2013.

[SZDG14] Fernando Soler-Toscano, Hector Zenil, Jean-Paul Delahaye, and Nicolas Gauvrit. Calculating Kolmogorov Complexity from the Output Frequency Distributions of Small Turing Machines. *PLoS ONE*, 9(5):e96223, May 2014.

[TC87] David L. Turner and Constance L. Cepko. A common progenitor for neurons and glia persists in rat retina late in development. *Nature*, 328(9), July 1987.

[TCM+07] Michael Y. Tolstorukov, Andrew V. Colasanti, David M. McCandlish, Wilma K. Olson, and Victor B. Zhurkin. A Novel Roll-and-Slide Mechanism of DNA Folding in Chromatin: Implications for Nucleosome Positioning. *Journal of Molecular Biology*, 371(3):725–738, August 2007.

[TFC13] Vuong Tran, Lijuan Feng, and Xin Chen. Asymmetric distribution of histones during Drosophila male germline stem cell asymmetric divisions. *Chromosome Research*, 21(3):255–269, May 2013.

[TK06] Dmitry N. Tsigankov and Alexei A. Koulakov. A unifying model for activity-dependent and activity-independent mechanisms predicts complete structure of topographic maps in ephrin-A deficient mice. *Journal of Computational Neuroscience*, 21(1):101–114, August 2006.

[TMS64] J. E. Till, E. A. Mcculloch, and L. Siminovitch. A stochastic model of stem cell proliferation, based on the growth of spleen colony-forming cells. *Proceedings of the National Academy of Sciences of the United States of America*, 51:29–36, January 1964.

[TR86] Sally Temple and Martin C. Raff. Clonal analysis of oligodendrocyte development in culture: Evidence for a developmental clock that counts cell divisions. *Cell*, 44(5):773–779, March 1986.

[TR14] W.-W. Tee and D. Reinberg. Chromatin features and the epigenetic regulation of pluripotency states in ESCs. *Development*, 141(12):2376–2390, June 2014.

[Tro00] V. Tropepe. Retinal Stem Cells in the Adult Mammalian Eye. *Science*, 287(5460):2032–2036, March 2000.

[Tro08] Roberto Trotta. Bayes in the sky: Bayesian inference and model selection in cosmology. *Contemporary Physics*, 49(2):71–104, March 2008.

[TSC90] D. L. Turner, E. Y. Snyder, and C. L. Cepko. Lineage-independent determination of cell type in the embryonic mouse retina. *Neuron*, 4(6):833–845, June 1990.

[TSC08] Jeffrey M. Trimarchi, Michael B. Stadler, and Constance L. Cepko. Individual Retinal Progenitor Cells Display Extensive Heterogeneity of Gene Expression. *PLOS ONE*, 3(2):e1588, February 2008.

[VGV+18] Jessie Van houcke, Emiel Geeraerts, Sophie Vanhunsel, An Beckers, Lut Noterdaeme, Marijke Christiaens, Ilse Bollaerts, Lies De Groef, and Lieve Moons. Extensive growth is followed by neurodegenerative pathology in the continuously expanding adult zebrafish retina. *Biogerontology*, October 2018.

[VJM+09] Marta Vitorino, Patricia R Jusuf, Daniel Maurus, Yukiko Kimura, Shin-ichi Higashijima, and William A Harris. Vsx2 in the zebrafish retina: Restricted lineages through derepression. *Neural Development*, 4(1):14, 2009.

[VL54] V. Vilter and L. Lewis. [Existence and distribution of mitoses in the retina of the deepsea fish Bathylagus benedicti]. - PubMed - NCBI. *C R Seances Soc Biol Fil.*, 148(21-22):1771–5, 1954.

[vM77] L. v. Salvini-Plawen and Ernst Mayr. On the Evolution of Photoreceptors and Eyes. In Max K. Hecht, William C. Steere, and Bruce Wallace, editors, *Evolutionary Biology*, pages 207–263. Springer US, Boston, MA, 1977.

[VV12] Antonio Visioli and Ramon Vilanova, editors. *PID Control in the Third Millennium: Lessons Learned and New Approaches*. Advances in Industrial Control. Springer, London New York, 2012.

[Wad57] C H Waddington. *The Strategy of the Genes : A Discussion of Some Aspects of Theoretical Biology*. Routledge Taylor & Francis Group, London New York, 1957.

[Wag07] Günter P. Wagner. The developmental genetics of homology. *Nature Reviews Genetics*, 8(6):473–479, 2007.

[WAR+16] Y. Wan, A. D. Almeida, S. Rulands, N. Chalour, L. Muresan, Y. Wu, B. D. Simons, J. He, and W. A. Harris. The ciliary marginal zone of the zebrafish retina: Clonal and time-lapse analysis of a continuously growing tissue. *Development*, 143(7):1099–1107, April 2016.

[WC17] Gregory M. Wright and Feng Cui. Nucleosome rotational positioning is influenced by cis- and trans-acting factors. *bioRxiv*, page 35, 2017.

[WCG+20] Kim. J Westerich, K.S. Chandrasekaran, T. Gross-Thebing, N. Kuck, E. Raz, and A. Rentmeister. Bioorthogonal mRNA labeling at the poly(A) tail for imaging localization and dynamics in live zebrafish embryos. *Chemical Science*, 2020.

[WD99] C. S. Wallace and D. L. Dowe. Minimum Message Length and Kolmogorov Complexity. *The Computer Journal*, 42(4):270–283, January 1999.

[Wes00] M. Westerfield. The Zebrafish Book : A Guide for the Laboratory Use of Zebrafish. *http://zfin.org/zf_info/zfbook/zfbk.html*, 2000.

[WF88] R. Wetts and S. E. Fraser. Multipotent precursors can give rise to all major cell types of the frog retina. *Science*, 239(4844):1142–1145, March 1988.

[WG92] Robert W Williams and Dan Goldowitz. Lineage versus environment in embryonic retina: A revisionist perspective. *TINS*, 15(10):6, 1992.

[WIEO04] Aiyan Wang, Tsuyoshi Ikura, Kazuhiro Eto, and Masato S. Ota. Dynamic interaction of p220NPAT and CBP/p300 promotes S-phase entry. *Biochemical and Biophysical Research Communications*, 325(4):1509–1516, December 2004.

[Win15] R. Winklbauer. Cell adhesion strength from cortical tension - an integration of concepts. *Journal of Cell Science*, 128(20):3687–3693, October 2015.

[WJH03] Y. Wei, J. Jin, and J. W. Harper. The Cyclin E/Cdk2 Substrate and Cajal Body Component p220NPAT Activates Histone Transcription through a Novel LisH-Like Domain. *Molecular and Cellular Biology*, 23(10):3669–3680, May 2003.

[WPMT15] Loksum Wong, Namita Power, Amanda Miles, and Vincent Tropepe. Mutual antagonism of the paired-type homeobox genes, vsx2 and dmbx1, regulates retinal progenitor cell cycle exit upstream of ccnd1 expression. *Developmental Biology*, 402(2):216–228, June 2015.

[WR90] Takashi Watanabe and Martin C. Raff. Rod photoreceptor development in vitro: Intrinsic properties of proliferating neuroepithelial cells change as development proceeds in the rat retina. *Neuron*, 4(3):461–467, March 1990.

[WR09] L. L. Wong and D. H. Rapaport. Defining retinal progenitor cell competence in Xenopus laevis by clonal analysis. *Development*, 136(10):1707–1715, May 2009.

[WSM+05] Ann M. Wehman, Wendy Staub, Jason R. Meyers, Pamela A. Raymond, and Herwig Baier. Genetic dissection of the zebrafish retinal stem-cell compartment. *Developmental Biology*, 281(1):53–65, May 2005.

[WSP+09] Minde I. Willardsen, Arminda Suli, Yi Pan, Nicholas Marsh-Armstrong, Chi-Bin Chien, Heithem El-Hodiri, Nadean L. Brown, Kathryn B. Moore, and Monica L. Vetter. Temporal regulation of Ath5 gene expression during eye development. *Developmental Biology*, 326(2):471–481, February 2009.

[Yam05] M. Yamaguchi. Histone deacetylase 1 regulates retinal neurogenesis in zebrafish by suppressing Wnt and Notch signaling pathways. *Development*, 132(13):3027–3043, July 2005.

[YBW15] Fanny Yang, Sivaraman Balakrishnan, and Martin J. Wainwright. Statistical and computational guarantees for the Baum-Welch algorithm. In *2015 53rd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, pages 658–665, Monticello, IL, September 2015. IEEE.

[YDH+11] Jingye Yang, Huzefa Dungrawala, Hui Hua, Arkadi Manukyan, Lesley Abraham, Wesley Lane, Holly Mead, Jill Wright, and Brandt L. Schneider. Cell size and growth rate are major determinants of replicative lifespan. *Cell Cycle*, 10(1):144–155, January 2011.

[You85] Richard W. Young. Cell differentiation in the retina of the mouse. *The Anatomical Record*, 212(2):199–205, June 1985.

[YQW+18] Kai Yao, Suo Qiu, Yanbin V. Wang, Silvia J. H. Park, Ethan J. Mohns, Bhupesh Mehta, Xinran Liu, Bo Chang, David Zenisek, Michael C. Crair, Jonathan B. Demb, and Bo Chen. Restoration of vision after de novo genesis of rod photoreceptors in mammalian retinas. *Nature*, August 2018.

[YSK15] Jienian Yang, Zheng Sun, and Natalia L. Komarova. Analysis of stochastic stem cell models with control. *Mathematical Biosciences*, 266(Supplement C):93–107, August 2015.

[YWNH03] X. Ye, Y. Wei, G. Nalepa, and J. W. Harper. The Cyclin E/Cdk2 Substrate p220NPAT Is Required for S-Phase Entry, Histone Gene Expression, and Cajal Body Maintenance in Human Somatic Cells. *Molecular and Cellular Biology*, 23(23):8586–8600, December 2003.

[ŽCC+05] Mihaela Žigman, Michel Cayouette, Christoforos Charalambous, Alexander Schleiffer, Oliver Hoeller, Dara Dunican, Christopher R. McCudden, Nicole Firnberg, Ben A. Barres, David P. Siderovski, and Juergen A. Knoblich. Mammalian Inscuteable Regulates Spindle Orientation and Cell Fate in the Developing Retina. *Neuron*, 48(4):539–545, November 2005.

[ZDI+98] Jiyong Zhao, Brian Dynlacht, Takashi Imai, Tada-aki Hori, and Ed Harlow. Expression of NPAT, a novel substrate of cyclin E–CDK2, promotes S-phase entry. *Genes & development*, 12(4):456–461, 1998.

[ZKL+00] Jiyong Zhao, Brian K. Kennedy, Brandon D. Lawrence, David A. Barbie, A. Gregory Matera, Jonathan A. Fletcher, and Ed Harlow. NPAT links cyclin E–Cdk2 to the regulation of replication-dependent histone gene transcription. *Genes & development*, 14(18):2283–2297, 2000.

[ZMR+09] Yong Zhang, Zarmik Moqtaderi, Barbara P Rattner, Ghia Euskirchen, Michael Snyder, James T Kadonaga, X Shirley Liu, and Kevin Struhl. Intrinsic histone-DNA interactions are not the major determinant of nucleosome positions in vivo. *Nature Structural & Molecular Biology*, 16(8):847–852, August 2009.

[Zub03] M. E. Zuber. Specification of the vertebrate eye by a network of eye field transcription factors. *Development*, 130(21):5155–5167, August 2003.

[ZZLG08] Ya-Li Zhou, Qi-Zhi Zhang, Xiao-Dong Li, and Woon-Seng Gan. On the use of an SPSA-based model-free feedback controller in active noise control for periodic disturbances in a duct. *Journal of Sound and Vibration*, 317(3-5):456–472, November 2008.