PHY 316M

Marc Matvienko

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1 Capacitors

$C = |\frac{Q}{V}|$

2 Current

Is the flow of charge in on direction. Current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{\delta Q}{\delta t}$$

Current Density (current per unit area): $J = \frac{I}{A}$
$n =$ charge carriere density, $q =$ charge per carrier, $v_d =$ drift velocity This can give us, $\boxed{J = nqv_d}$

2.1 Ohm's Law

Usually we see Ohm's law in different forms, i.e. for a particular chunk.
Consinder some block with volume $A \cdot l$, some source of energy(battery) forces current thorugh by applying an electric field.
For a uniform electric field: $\boxed{V = E \cdot l}$

$$\begin{aligned} J &= \frac{I}{A} \\ &= \sigma E \\ &= \sigma \frac{V}{l} \end{aligned}$$

$$\boxed{\frac{V}{I} = \frac{1}{\sigma}\frac{l}{A} = \rho\frac{l}{A} = R = \text{resistance}}$$

$\boxed{\frac{V}{I} = R}$ Resistance is not resistivity (opisiton of current flow of a particular material[ρ]) Ohmic material is a material that has a constant slope on Voltage to Current graph. Most common materials like copper behave like this.

Example The resistivity of nichrome wire(heaters, toasters) is $1.5 \times 10^{-6}\Omega m$. If a household voltage of $115V$ is applied acros a $0.2mm$ radius write, $1.0m$ long, what current flows?

$R = \rho\frac{l}{A} = \rho\frac{l}{\pi r^2} = \frac{1.5\times10^{-6}\Omega m \cdot 1.0m}{(\pi(2\times10^{-4}m)^2))} = 11.9\Omega$

2.2 Model for electric conduction

- electron unergo many rapid ocllision when $E = 0$
- when $E \neq 0$, the electrons accelrate between collisions
- $F = ma = qE => a = \frac{qE}{m}$
- $v = v_0 + at = v_0 + \frac{qE}{m}t$

Let τ = average collision time $= R \cdot C$
the $v_d = v_{avg} =< v_0 > + \frac{qE}{m}\tau$
so, $J = nqv_d = nq = \frac{qE}{m}\tau = \sigma E$
so conducitvity $\sigma = \frac{nq^2\tau}{m}$
Called the Drude model or free electron mode

$$\boxed{\sigma = \frac{nq^2\tau}{m}}$$

$$\boxed{\frac{1}{\sigma} = \rho}$$

Example Assume for copper that each atom donates one free electron. What is the average time between collision for electrons in copper.?
Given:

- Density$= 8.98\frac{g}{cm^3}$
- Atomic Weight $= 63.54\frac{g}{mole}$
- $\rho = 1.7 \times 10 -^{-8}\ \Omega \cdot m$

$\tau = \frac{m}{nq^2\rho} = \frac{9.14\times10^{-31}kg}{(8.5\times10^22)(1.6\times10^{-19})^2 1.7\times10^{-8}\Omega m} = 2.5 \times 10^{-14}s$

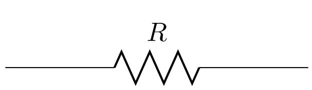
2.3 Temperature Dependence of resistivity

- resistivities tabulated for 20 °celsius
- for metals, ρ is higher and T is higher
- α = linear temprature coefficient
- over some range, $\boxed{\rho = \rho_0(1 + \alpha(T - T_0))}$

As T increases, the scattering time decreases due to collisions with vibrating atoms
At higher temperatures the ρ to temprature graph is linear. But at the beginning there is residual resistvity due to impurities.

Semiconductors The number of carriers decreases as the temperature decreases, this means that all the electrons are sticking to their atoms.

3 Resistors

Circuit symbol: 

3.1 Resistors in series

For resistors in series the resistivities add

$$R_{tot} = R_1 + R_2 + ...$$

$$R_{tot} = \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} = R_1 + R_2$$

The current (I) is the same everywhere too. **Resistors don't add in parallel. Capacitors do.**

$$V = V_1 + V_2$$

The voltage divider

$$V_1 = I \cdot R_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

3.2 Resistors in parallel

For resisitors in parallel halve the resistance if two exact resistors are put in parallel

- In parallel have the same voltage across each element
- In parallel also the current divides among branches

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} => R_{tot} = \frac{V}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Superconductors Electrons pair up and when electron jumps to the lattice another electrons pulls it right back.

3.3 Resistors Disipate Energy

Electrons undergo collisions, and give up energy as heat. A steady release of current (I) causes a steady realease of energy.

$$\Delta U = \Delta Q V$$

Better to discess the rate, which is really known as:

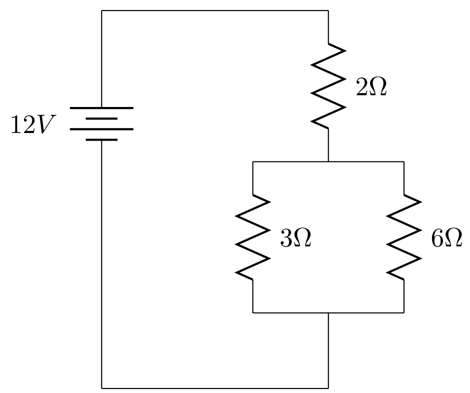
$$\text{Power} = P = \frac{\Delta U}{\Delta t} = \frac{\Delta U}{\Delta t} V = IV$$

$$\boxed{P = IV = I^2 R = \frac{V^2}{R}}$$

This power is also known as Joule heating.

Putting this into practice: many circuits can be analyzed with just Ohm's Law and Resistance.

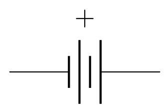
- What is the total power delivered?
- What is the power dissipated in each R?



$$P_{\text{dissapated in }3\Omega} = \frac{V_3^2}{R} = \frac{(6V)^2}{3\Omega} = 12W$$

3.4 Direct Current Circuits

Real battery is an ideal $\mathscr{E}$MF plus intended resistance

Batteries are a source of voltage 

Source of voltage = "electromotive force" = $\mathscr{E}$MF= $\mathscr{E}$

$$\boxed{V = \mathscr{E} - Ir}$$

"Open-circuit voltage", where $I = 0$

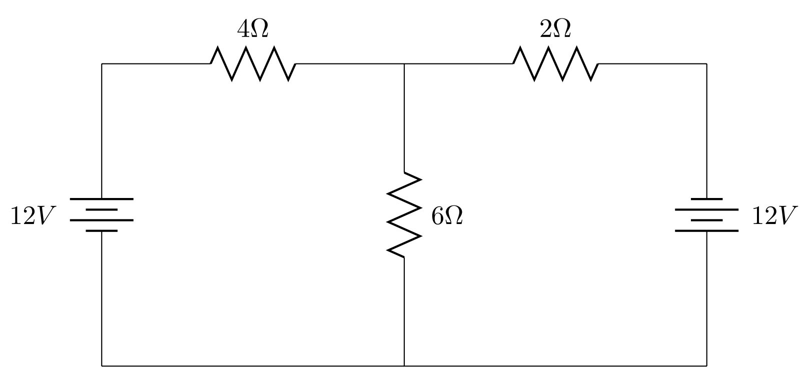
Analysis of circuits Any circuit can be analyzed with Kirchhoffs Rules:

- Junction Rule, algebraic sum of currents into a junction = sum of currents $\Sigma i_{in} = \Sigma i_{out}$
- Loop Rule, algebraic sum of voltages around any closed loop is zero. Where voltage rises are positive (- to +) and votlage drops are negative (+ to -)

For resistors the current directino determines voltage drop(negative)

Example

- What is the current in the 6Ω resisitor?
- Is it flowing up or down?



(I) Junction at A: $i_1 = i_2 + i_3$

(II) Loop A: $12V - i_1(4\Omega) - i_3(6\Omega) = 0$

(III) Loop B: $I_3(6\Omega) - i_2(2\Omega) + 12V = 0$

$i_3 = \frac{-12V}{22\Omega} = -\frac{6}{11}A$

2 other techniques:

i) same, except use ficticious "loop currents"

ii) Source suppressing - can look at effects of sources seperately

Next: Circuits with Capacitors will see time-depedent behavior Before "transient phenomena", look at *Steady-state*: "after a long time". The capacitor starts acting like the current is 0.

4 Magnetism

We saw:

$$F = qv\frac{\mu_0 I}{2\pi r} = 0$$

for $\vec{v}$ tangent to circle. where $\mu_0 = 4\pi \times 10^{-7} \frac{Ns^2}{C^2}$
Rewrite this as

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

where $\boxed{B = \frac{\mu_0 I}{2\pi r}}$ magentic field due to a long wire where direciton of $\vec{B}$ is tangent to circle.
If also electric force, combination of electric and magnetic force is called "Lorentz Force"

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$\vec{F} = q\vec{v} \times \vec{B}$

- Like $\vec{E}, \vec{F}$ opposite for $+q, -q$
- Depends on $\vec{v} : \vec{F} = 0$ for $\vec{v} = 0$
- Depends on angle : $\boxed{|\vec{F}| = |q|vBsin\Theta}$
  notice $\vec{F} = 0$ for $\vec{v} \parallel \vec{B}$
- Direction $\vec{F} = q\vec{v} \times \vec{B}$ given by right hand rule

Magnetic field does no work because the magnetic force is perpendcular to displacement.
So $\vec{B}$ changes direction of $\vec{v}$ but not its magnitude ($KE = \frac{1}{2}mv^2$)
Unit of B $= \frac{Ns}{Cm} =$ tesla $= T$ also gauss $= G = 10^{-4}T$

4.1 Evaluating Cross Products

1. Get general equation by expaning determinant $\begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} =$
   $\vec{v} \times \vec{B} = i(v_y B_z - v_z B_y) - j(v_x B_z - v_z B_x) + k(v_x B_y - v_y B_x)$
2. Just multiply out.
   know that $i \times j = k$ and that $j \times i = -k$
5. Magnetic field does no work

4.2 Ampere's and Biot Savart Law

1. Ampere's Law: for high symmetry
2. Biot Savart Law

In general we will only look at: center of arcs and circles, or due to straight segments, or on axis of loop.

4.2.1 $\vec{B}$ at the center of a circle

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$
$$B = \int dB = \int \frac{\mu_0 I}{4\pi r^2}ds = \frac{\mu_0 I}{2R}$$

Since $r = R$, the integral is constant and is easy to integrate with respect to s.

Also: for a fraction f of a circle $\boxed{\vec{B} = fB_{\text{full circle}}}$

4.2.2 $\vec{B}$ near a finite straight wire

$$\begin{aligned} d\vec{B} &= \frac{\mu_0 I}{4\pi}\frac{d\vec{s}\times\hat{r}}{r^2} \\ &= \frac{\mu_0 I}{4\pi}\frac{dssin\theta}{r^2} \end{aligned}$$

1. $sin\theta = sin\theta' = cos\phi$
2. $cos\phi = \frac{R}{r} \to r\frac{R}{cos\phi}$
3. $tan\phi = \frac{s}{R} \to s = R\cdot tan\phi \to \frac{R}{cos^2\phi d\phi}$

$$\begin{aligned} B &= \int dB \\ &= \int \frac{\mu_0 I\frac{R}{cos^2\phi}d\phi cos\phi}{4\pi(\frac{R}{cos\phi})^2} \\ &= \frac{\mu_0 I}{4\pi R}\int_{-\phi_1}^{+\phi^2} cos\phi d\phi \\ &= \frac{\mu_0 I}{4\pi R}sin\phi|_{-\phi_1}^{+\phi^2} \\ &= \frac{\mu_0 I}{4\pi R}[sin\phi_2 + sin\phi_1] \end{aligned}$$

4.2.3 B field due to a square loop of side a

$$B_{loop}$$
$$\frac{2\sqrt{2}\mu_0 I}{\pi a}$$

4.2.4 B on axis of loop

Off axis compnents cancel around circle.
ϕ is the angle at the bottom right of triangle formed by circle and axis

$$r = \sqrt{x^2+R^2} \qquad \text{Using pythagorean theorem}$$
$$sin\phi = \frac{R}{\sqrt{x^2+R^2}} \qquad \text{So we can define } sin\phi$$

We only have to integrate along the x compnents

$$dB_x = dBsin\phi$$
$$= \frac{\mu_0 I ds}{4\pi r^2} sin\phi$$

We can say that $\int ds = 2\pi R$ since the radius is constant

$$B = \int \frac{\mu_0 I}{4\pi r^2} sin\phi \, ds$$
$$= \frac{\mu_0 IR}{4\pi(x^2 + R^2)^{3/2}} \int ds$$
$$= \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$$

Also, consider yourself very close to the field, $x >> R$ $\frac{1}{x^3}$ dipole field

4.3 Motion in a uniform $\vec{B}$

$$|\vec{F}| = |q\vec{v} \times \vec{B}| = qvB$$

$\vec{F}$ is $\perp$ to $\vec{v}$ centripetal with accelarations $a_r = \frac{v^2}{r}$ We saw that

$$B_{\text{long wire}} = \frac{\mu_0 I}{2\pi r}$$
$$B_{\text{solonoid}} = \mu_0 nI$$
$$B_{\text{loop center}} = \frac{\mu_0 I}{2R}$$

angular velocity $\omega =?$

$$r = \frac{mv}{qB}$$

Example: Mass Spectrometer *Note: Electrons travel in a semi-circle in a spectrometer* Electrons are accelarated through potential of $10^3 V$ ("a 1 keV electron"). They enter a region of uniform $B = 10^{-2}T$. What is the distance they are displaced?

$$x = 2r = 2\frac{mv}{qB}$$

Know m, q, B need to find v

$$\Delta KE = \Delta PE \rightarrow \frac{1}{2}mv^2 = eV_0 = \sqrt{\frac{2eV}{m}}$$
$$x = 2\frac{m}{eB}\sqrt{\frac{2eV_0}{m}} = \frac{2}{B}\sqrt{\frac{2mV_0}{e}} = \frac{2}{(10^{-2}T)}\sqrt{\frac{2(9.11 \times 10^{-31})(10^3 V)}{(1.6 \times 10^{-19}C)}}$$

In practice however, we are given v,B,x to get $(\frac{q}{m})$

Example: Velocity Selector region of crossed $\vec{E} + \vec{B}$
All we have to consider is qE *-vs-* qvB
if $qE = qvB$ then $\boxed{v = \frac{E}{B}}$

4.4 Force on a current

Consider positive charge travelling along x axis with v_0 with a $-\vec{B}\hat{y}$

$$\begin{aligned}
F_{\text{on wire}} &= q\vec{v} \times \vec{B} \\
&= \frac{1}{n}\vec{j} \times \vec{B} \\
F_{\text{on wire}} &= (\#\text{ charges})F_{\text{on 1 charge}} \\
&= (nAl)\frac{1}{n}\hat{\vec{j}} \times \hat{\vec{B}} \\
&= lA\vec{j} \times \vec{B} \\
&= I\vec{l} \times \vec{B}
\end{aligned}$$

Example Net force on a current loop in a uniform $\vec{B}$ is **zero**. This is true for any loop.

Example A current I flows from the origin to $(x, y, z) = (1m, 1m, 0)$ and then straight to $(2m, 0, 0)$. In a uniform field of $\vec{B} = 5T\hat{\imath}$

$$\begin{aligned}
\vec{F_1} &= (10N)(-\hat{\jmath} + \hat{\imath}) \\
\vec{F_2} &= (10N)(-\hat{\jmath} - \hat{\imath}) \\
\vec{F} &= (20N) - \hat{\jmath}
\end{aligned}$$

Example Force on a wire segment due to a large $\|$ wire

$$\begin{aligned}
B &= \frac{\mu_0 I}{2\pi r} \\
\vec{F} &= I_2\vec{l} \times \frac{\mu_0 I_1}{2\pi r} \\
&= I_2 \times \frac{l\mu_0 I_1}{2\pi r} \\
\frac{\text{force}}{\text{length}} &= \frac{\mu_0 I_1 I_2}{2\pi r}
\end{aligned}$$

Definition: Magnetic Moment
Dipole moment ($\mu = IA$) due to a magnetic loop.

4.5 Torque on a current

We saw: $\vec{F} = I\vec{l} \times \vec{B}$

Magnetic Moment $\vec{\mu} = I\vec{A}$
and for N loops we have $\vec{\mu} = NI\vec{A}$

we also saw that for a loop in uniform $B \to \vec{F} = 0$. There is no force, but there is a **net torque**.

Torque on a current
Loop in a uniform B

$$\begin{aligned}\vec{\tau} &= \Sigma\vec{r}\times\vec{F}\\ |\vec{\tau}| &= \Sigma rFsin\theta\\ &= (z)(\frac{l}{z})IlBsin\theta\\ &= Il^2Bsin\theta\\ &= \mu Bsin\theta\\ \vec{\tau} &= \vec{\mu}\times\vec{B}\end{aligned}$$

Where μ is the magnetic moment and $\vec{B}$ is the magnetic field.

5 Magnetism and Matter

Comes from some sort of current loop in matter. Perhaps an electron traveling around a proton, that is a current loop.

Example Consider the oribting elctron as current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{e}{T} = \frac{e}{(2\pi r)/v} = \frac{e\cdot v}{2\pi r}$$

$$\mu = IA = \frac{e\cdot v}{2\pi r}\cdot\pi r^2 = \frac{1}{2}evr$$

Recall that angular momentum is $\vec{l} = \vec{r}\times\vec{p}$ and therefore $\boxed{l = mvr}$

$$\mu = \frac{e}{2m}mvr = \frac{e}{2m}l$$

An atom can have many electrons $\vec{L} = \Sigma\vec{l_i}$ and that $\mu = -\frac{e}{2m}\vec{L}$

5.1 Quantum Mechanics

$\vec{l}$ is quantized: $l = 0, \hbar, 2\hbar, ...$ When $\hbar = \frac{h}{2\pi} = 1.06\times10^{-34}J$ where h = Planck's constant = $6.63\times10^{-34}J$. Ultimately, the spin of atom does not really exist, but it behaves like it is. Its "spin" would be $s = \frac{\hbar}{2}$

Adding total angular momentum $\vec{J} = \vec{L} + \vec{S}$

When $l = \hbar$ we have the smallest possible moment. $\mu = \frac{e\hbar}{2m}$ = Bohr magneton = $9.27\times10^{-24}J/T$

5.2 Magnetization

Definition Magnetization = $M = \frac{\mu}{(volume)}$ = magnetic moment per unit volume.

The total field In an external lfield, some moments align and they produce their own field, $\vec{B}_{matter}$. With our definitions, $B_{\text{matter}} = \mu_0 M$, so the total field $\vec{B} = \vec{B}_{ext} + \mu_0\vec{M}$. For many field $\vec{B}\propto\vec{B}_{ext}$. Really we can say that $\vec{B} = K_m\vec{B}_{ext}$ where K_m = magnetic permeability or just the permeability constant.

Vaccum $K_m = 1$

Diamagnet in most materials $K_m - 1 \approx 10^{-6} - 10^{-4} \approx -10^{-5}$ because they have closed shells

Ferromagnet $K_m >> 1$ have locked moments due to Pauli exclusion principle.

Superconductors Type I superconductors have a $K_m = 0$, which means they shield the cternal field.

Magnetic Susceptibility $\chi_m = K_m - 1$ $\boxed{\frac{\mu_0 M}{B_{ext}}}$

5.3 Hall Effects

5.3.1 Hall effect

is used for measuring the charge carrier dnesity n (or, for known n, can measure B or map B)

Consider a current I and a $\perp \vec{B}$

Carrier experience $\vec{F} = q\vec{v} \times \vec{B}$, deflext and build up on surface with their transverse electric field balances magnetic force until $qE_t = qv_d B$

We measure the (transverse) "Hall Voltage" $V_H = E_t d$

Use current density $j = \frac{I}{A} = \frac{I}{dl} = nqv_l \to v_d = \frac{I}{nqdl}$ (1) and $F_e = F_m \to E_t = v_d B \to v_d = \frac{E_t}{B}$ (2)

Due to (1) and (2) we can say $\frac{E_t}{B} = \frac{I}{nqdl}$ and $V_H = E_t d \to \frac{V_H}{Bd} = \frac{I}{nqdl} \to \boxed{n = \frac{IB}{V_H ql}}$

5.3.2 Motional EMF

This gives us Magnetic Flux and Faraday's Law
Instead of a current procing a Hall Voltage, we can just $\vec{v} \perp \vec{B}$. Ions are stuck, but electrons can move, until $qE_t = qvB$ and get a transverse voltage. Called a **motional EMF**. $\boxed{\mathscr{E} = E_t l = Blv}$. We will see many ways to get EMF by "sweeping past B field"

6 Faraday's Law

Holds for any closed path, even the object producing the flux itself.

6.1 Preperation

$\mathscr{E} = Blv$
Before looking at Faraday's Law define Magnetic Flux $\Phi_m = \int \vec{B} \cdot d\vec{A} = \vec{B}\vec{A} = BAcos\Theta$. We will be interested in "flux through a current loop".

Example A long solenoid with current I and n turns/meter, radius R_2 contains a loop of radius $R_1 \perp$ to solenoid axis. Find flux through loop. Remember: $\boxed{B_{\text{solenoid}} = \mu_0 nI}$.

$$\Phi_m = \int \vec{B} \cdot d\vec{A} = \vec{B} \cdot \vec{A} = BA = (\mu_0 nI)(\pi R_1^2)$$

Will only get an induced voltage if something is moving though.

Example A rectangular loop is ?? in plane of a long wire carrying current I. From wire to one end of rectangle is a, and to other end is b.
Remember that B is not uniform.

$$\Phi_m = \int \vec{B} \cdot d\vec{A} \underset{\vec{B} \perp \vec{A}}{=} \int BdA = \int_a^B \frac{\mu_0 I}{2\pi r} ldr = \frac{\mu_0 Il}{2\pi} \int_a^b \frac{1}{r} dr = \frac{\mu_0 Il}{2\pi} ln(b/a)$$

6.2 Content

A changing magnetic flux induces a current/voltage aroud a loop. If Φ_m is the flux though surface then the voltage around that surface is

$$\mathscr{E} = -\frac{\partial \Phi_m}{\partial t}$$

If N loops on a coil then

$$\mathscr{E} = -N\frac{\partial \Phi_m}{\partial t}$$

Example A loop of wire with resistance R is partly in a region of uniform B and is pushed in at a constant v. What happens?

$$\Phi = BA = Blx$$

Faraday: $\mathscr{E} = \frac{d\Phi}{dt} = -\frac{d}{dt}Blx = Bl\frac{dx}{dt} = -lv$. If loop is closed, current flows. $I = \frac{\mathscr{E}}{R} = -\frac{Blv}{R}$.

Energy designated: $P = I^2R = \frac{B^2l^2v^2}{R}$. Another view: $\vec{F} = I\vec{l}\times\vec{B} = I; B = \frac{Blv}{R}lB$

Hence we can say $P = Fv = \frac{B^2l^2v^2}{R}$

The sign in Faraday's Law The "-" in the equation for $\mathscr{E}$ is called **Lenz' Law**, which states that induced current opposes the change in flux.

We saw the EMF of a moving rod $\mathscr{E} = Blv$

Faraday's Law $\mathscr{E} = -\frac{d\Phi}{dt}$, and Lenz' law tells us that the negative is there.

$\Phi = \int \vec{B}d\vec{A} = BAcos\theta$

$\mathscr{E} = -\frac{d}{dt}(BAcos\theta) = Acos\theta\frac{dB}{dt}$

Example A rectabular loop with horizontal side of length 1m, loop resistance 0.1Ω, and a mass of $0.1kg$, is dropping out of a region of uniform $B = 2T$. What is its terminal velocity?

$F_{net} = 0 = F_g + F_m$

We know induced voltage $\mathscr{E} = Blv \to I = \frac{Blv}{R}$

So $|F_m| = |I\vec{L}\times\vec{B}| = IlB = \frac{B^2l^2v}{R} \to \frac{B^2l^2v}{R} = mg$

$v = \frac{mgR}{B^2l^2} = \frac{(0.1kg)(9.8m/s^2)(0.1\Omega)}{(2T)^2(1m)^2}$

Example Two concentric loops with $R_1 = 1m$ and $R_2 = 1cm$. Current in #1 is increasing is increasing at $2A/s$. What is the Emf induced in loop #2

We know that $B_{\text{center of loop}} = \frac{\mu_0 I_1}{2R_1}$

Then EMF $\mathscr{E} = -\frac{d\Phi}{dt} = -\frac{d}{dt}BA = -\pi R_2^2\frac{dB}{dt}$

or $\mathscr{E}_2 = -\pi R_2^2\frac{d}{dt}\frac{\mu_0 I_1}{2R_1} = -\frac{\pi R_2^2\mu_0 dI_1}{2R_1 dt})$

So we can get $\mathscr{E}_2 = \frac{-\pi(0.01m)^2(4\pi\times10^{-7})}{2(1m)}(2A/s) = -4\times10^{-10}V = -400pV$

Common situation: $\mathscr{E} = -(\text{constant})\frac{dI_1}{dt}$

We define $\mathscr{E}_2 = -M_{12}\frac{dI_1}{dt}$

Big / small loops $= M_{12} = \frac{\pi R_2^2\mu_0}{2R_1}$

6.3 DC generator

Mechanical input creates electric output. Where $\theta = \omega t =$ rotations of the generator

$$\begin{aligned}\mathscr{E} &= -\frac{d}{dt}BAcos\theta \\ &= -BA\frac{d}{dt}cos\omega t \\ &= \omega BAsin\omega t\end{aligned}$$

What is "induced voltage around loop"? Even if no loop (no wire), to have voltage we must have an $\vec{E}$ field in space.

Note Different $\vec{E}$ arrangement than in electrostatics.

Electrostatics: $V = \int \vec{E} \cdot d\vec{s}$, and also $\oint \vec{E} \cdot d\vec{s} = 0$

But $\vec{E}$ fields due to changing $\vec{B}$, hence, $\mathscr{E} = \oint \vec{E} \cdot d\vec{s} \neq 0$

So we can have "non-conservative" fields

Note Similarity beteween Ampere's Law and Faraday's Law

Ampere's

$$\begin{aligned}\oint \vec{B} \cdot d\vec{s} &= \mu_0 I \\ B(2\pi r) &= \mu_0 I \\ B &= \frac{\mu_0 I}{2\pi r}\end{aligned}$$

Faraday's

$$\begin{aligned}\mathscr{E} = \oint \vec{E} \cdot d\vec{s} &= -\frac{d\Phi_m}{dt} \\ E(2\pi r) &= \frac{d\Phi}{dt} \\ E &= -\frac{-\frac{d\Phi}{dt}}{2\pi r}\end{aligned}$$

- $r < R : \Phi = \pi r^2 B \rightarrow E = \frac{rdB}{2dt}$
- $r > R : \Phi = \pi R^2 B \rightarrow E = \frac{E^2 dB}{2rdt}$

Note Electromagnetic safety suggests that one change B slowly to avoid a larg $\vec{E}$

Example Rotating rod in magnetic field. Held at one end and spins in magnetic field. Length l and spin of ω. To get induced voltage do:

$$\begin{aligned}
\mathscr{E} &= -\frac{d}{dt}BA \\
&= -B\frac{dA}{dt} \\
&= -B\frac{\Delta A}{\Delta t} \\
&= -B\frac{(\pi l^2)}{2\pi/\omega} \\
&= -\frac{B\omega l^2}{2}
\end{aligned}$$

7 Inductance

For this recall, $\mathscr{E} = -N\frac{d\Phi_M}{dt}$. This holds true for any closed path around Φ. In particular, the changing current induces a voltage across the source itself.

N is the number of turns

$n = \frac{N}{l}$

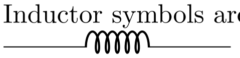
$B = \mu_0 \frac{N}{l} I$

So due to Faraday's law we can say that

$$\begin{aligned}
\mathscr{E} &= -N\frac{d\Phi}{dt} \\
&= -N\frac{d}{dt}(BA) \\
&= -N\frac{d}{dt}(\mu_0\frac{N}{l}I)(A) \\
&= -\frac{\mu_0 N^2 A}{l}\frac{dI}{dt}
\end{aligned}$$

Main feature voltage indeed is proportianl to rate of change of current.
Constant of proportionality = L = "self-inductance" or "inductance". $\boxed{\mathscr{E} = -L\frac{dI}{dt}}$
Similar, but opposite, to capacitance: $q = Cv$ or $\frac{dq}{dt} = C\frac{dV}{dt} = i$. For magnetic fields $N\Phi \propto B \propto I_{\text{changing flux}}$.
We can hence write, $\boxed{N\Phi = LI}$ $\frac{d}{dt} \rightarrow -N\frac{d\Phi}{dt} = -L\frac{dI}{dt} \rightarrow \boxed{v_L = -L\frac{dI}{dt}}$ this is also known as "back emf".
Above for solenoid, $\boxed{L = \frac{\mu_0 N^2}{l}A}$. The unit of L is a henry, is in units of H = $\frac{V-s}{A}$.

Note If you fill coil with magnetic momtvial with permiability K_m then $L = K_m L_0$.
Inductor symbols are:

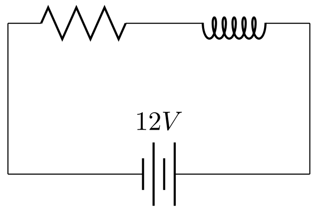


cute inductor

american inductor

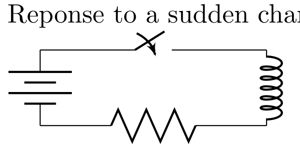
7.1 RL circuits

1. Steady state i.e. "after a long time". i is constant, $\frac{di}{dt}$



$v_L = -L\frac{di}{dt} = 0$

2. Reponse to a sudden change in DC



$$
\begin{aligned}
V_0 - L\frac{dI}{dt} - IR = 0 \\
V_0 - IR = L\frac{dI}{dt} \\
I - \frac{V_0}{R} = -\frac{L}{R}\frac{dI}{dt} \\
\frac{dI}{I - V_0/R} = -\frac{R}{L}dt \\
\int_0^i \frac{dI}{I - V_0/R} = -\frac{R}{L}\int_0^t dt \\
ln[\frac{i - V_0/R}{-V_0/R}] = -\frac{R}{L}t \\
\frac{i - V_0/R}{-V_0/R} = e^{-\frac{R}{L}t}
\end{aligned}
$$

So,

$$
\begin{aligned}
v_L = -L\frac{dI}{dt} &= -L[\frac{d}{dt}(\frac{V_0}{R}(1 - e^{-\frac{R}{L}t}))] \\
&= -L\frac{V_0}{R}(e^{-\frac{R}{L}t})(-\frac{R}{L}) \\
&= V_0 e^{-\frac{R}{L}t}
\end{aligned}
$$

If we have switch in position 1 for a logn time $\frac{R}{L}t \to I = \frac{V_0}{R}$
Switch to position 2 of $t = 0$

7.1.1 Loop Theorem

$-L\frac{dI}{dt} - IR = 0$

$$\int_I^{I_0} \frac{dI}{I} = -\int_0^t \frac{R}{L}dt \to ln(\frac{I}{I_0}) = -\frac{R}{L}t$$

7.2 Energy Stored Inductors

Power is $P = IV = \frac{dV}{dt}$, inductor $\frac{dV}{dt} = I(L\frac{dI}{dt})$, or dI gives dV: $dV = LIdI$. $V = \int dV = L\int_0^I IdI = LI^2\frac{1}{2}$
Energy in inductor $\boxed{V_L = \frac{1}{2}LI^2}$

Potential energy One can think of this U as stored in B field

Definition magnetic energy per unit volume

$$u_b = \frac{U_L}{\text{volume}} = \frac{1}{2\mu_0}B^2$$

8 Oscillators

Remember the formula for induction and L-R circuits Today we will discuss:

1. Free oscillations **LC, LRC**
2. Forced oscillations **R, C , Power**

8.1 Natural/Free

Combination of L and C is an oscillators

- First charge C to $Q_0 = CV_0$
- Connect to L at $t = 0$

Since we know

$$I = -\frac{dQ}{dt}$$

Loop:

$$\begin{aligned} V_c - V_l &= 0 \\ \frac{Q}{C} - L\frac{dI}{dt} &= 0 \\ \frac{Q}{C} + L\frac{d}{dt}(\frac{dQ}{dt}) &= 0 \end{aligned}$$

Ultimately, we get:

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

1. simplest 2nd order diff. eq.
2. like simple harmonic oscillator, similar to solution of $Q = Acos(\omega t) + Bsin(\omega t)$

Only possible solutions $A + B$ are constant to be detemined by initial conditions and (like $\omega = \sqrt{\frac{k}{m}}$ for spring) here, $\boxed{\omega = \frac{1}{\sqrt{LC}}}$ is the angular frequency of oscillator. We can check this by substituting:

$$\begin{aligned} \frac{d^Q}{dt^2} &= \frac{d^2}{dt^2}(Acos(\omega t) + Bsin(\omega t)) \\ &= -\omega^2[Acos(\omega t) + Bsin(\omega t)] \\ &= -\omega Q \\ &= -\frac{1}{LC}Q \\ \omega^2 &= \frac{1}{LC} \\ \omega &= \frac{1}{\sqrt{LC}} \end{aligned}$$

1. at $t = 0, Q_0 = 0$. Using $Q = Acos(\omega t) + Bsin(\omega t)$
   by plugging in time we can get $Q_0 = A$

2. at $t = 0^+, I = 0$(current in an inductor charges continuously)

$$0 = \frac{dQ}{dt}|_{t=0} = \frac{d}{dt}(Acos(\omega t) + Bsin(\omega t))|_{t=0} = -\omega Acos(\omega t) + \omega Bsin(\omega t)|_{t=0} = -\omega A(0) + \omega B(1) = \omega B$$

$$I = \frac{dQ}{dt} = -\frac{d}{dt}(Q_0 cos\omega t) = \omega Q_0 sin\omega t = I_{\max} sin\omega t$$

Note Current Amplitude $I_{\max} = \omega Q_0$

8.2 Energy in LC oscillator ??? back and forth

Between electric: $U_c = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q_0^2}{C}cos^2\omega t$ and magnetic $U_m = \frac{1}{2}LI^2 = \frac{1}{2}LI_{\max}^2 sin^2\omega t$ So total energy $U_e + U_m = \frac{1}{2}\frac{Q_0^2}{C}$. Capacitor with inductor called "Tank circuit"

8.3 RLC Circuit (freely decaying oscillators)

New energt is lost each cucle due to I^2R. Loop theorem tells us $\frac{Q}{C} - IR - L\frac{dI}{dt} = 0$

Equation of a damped harmonic oscillator $\frac{Q}{C} + R\frac{dQ}{dt} + L\frac{d^2Q}{dt^2} = 0$. Solution is complex, but for low damping $Q = Q_0 e^{-t\frac{R}{2L}} cos(\omega_d t)$ since our charge decays. The frequency w_d is given approximately given by $\boxed{\approxeq \sqrt{\frac{1}{LC} - \frac{R}{2L}^2}}$ We say that if $R < \sqrt{\frac{4L}{C}}$ then we have a circuit that is "under damped" if $R = \sqrt{\frac{4L}{C}}$ "critically damped" and if $R > \sqrt{\frac{4L}{C}}$ "over damped"

9 AC Circuits

Forced oscillations: Apply a $v = V_0 cos(\omega t)$. We will study the steady state AC response. We will ignore transience (the part where the circuit takes some time to become steady). Period is $T = \frac{2\pi}{\omega}$

1. Pure Resistive "insert drawing" $i = \frac{v}{R}$, $I_0 = \frac{V_0}{R}$

9.1 Pure C

Consider an alternating circuit with a capacitor that has charge q. $v = V_{max}cos(\omega t)$. Since current is $i = \frac{dq}{dt}(cv) = c\frac{d}{dt}(V_{max}cos\omega t) = -\omega CV_{max}sin(\omega t) = cos(\omega t + \pi/2)$ Hence, $i = \frac{V_{max}}{1/\omega c}cos(\omega t + \pi/2)$

1. Amplitudes are related by $I_{max} = \frac{V_{max}}{(1/\omega c)}$

   Defintion χ_c = "capactive reactance" $= 1/\omega c \to$ opposition to AC current flow du to capacitor. we say in capactive citcuits current leads voltage by $\pi/2 \to IC\mathscr{E}$ So we ultimately get:

$$I_{max} = \frac{V_{max}}{\chi_c}$$

2. and $v = V_{max}cos(\omega t)$ or $i = I_{max}cos(\omega t + \pi/2)$

3. χ_c behavior:
   $\omega \to 0$ gives $\chi_C = \frac{1}{\omega c} \to \infty$
   $\omega \to \infty$ gives $\chi_C = \frac{1}{\omega c} \to 0$
   Blocks low frequency or just DC circuits.

9.2 Pure L

Knowing the loop theorem, we can say $di = \frac{1}{L}vdt$ and so $i = \int di = \frac{1}{L}\int vdt = \frac{1}{\omega L}V_{max}sin(\omega t)$

Defintion $\chi_L =$ "inductive reactance" $= \omega L$ and hence we get $\boxed{I_{max} = \frac{V_{max}}{\chi_L}}$ for amplitude.
In an inductive circuit: $\mathscr{E}LI$ voltage leads current by $\pi/2$
Opposite behavior:
$\omega \to 0$ gives $\chi_L = \frac{1}{\omega c} \to 0$
$\omega \to \infty$ gives $\chi_L = \frac{1}{\omega c} \to \infty$
Blocks high frequency.

9.3 Pure R

Power is $P = IV$ but really his is just $P = i^2R = (I_{max}cos\omega t)^2R$.
Interested in the average so $P_{avg} = I_{max}^2\langle cos^2\omega t\rangle R = 0.5$
So, $P_{avg} = \frac{1}{2}I_{max}^2R$ We often use "rms" calues instead of amplitudes.
rms is the root of the mean of the square. rms really just gives $\frac{1}{\sqrt{2}}$. $I_{rms} = \frac{1}{\sqrt{2}}I_{max}$ the same is true for voltages too. $P_{avg} = I_{rms}^2R$ rms values are like an effective DC (average) value.
To add voltages with idfferent phases, will want to use a **Phasor representation**. A phasor is really just a rotating vector.
Any sinudoid can be represented as the x component of a rotating vector. Which is always $V_{max}cos(\omega t)$

For out AC circuits it is quite simple.

9.4 Examples

These are all going to be series circuits. So lets consider an RC circuit. Voltage and current are in phase across/through resistor. But voltage across capacitor largs current by $\pi/2$. $\mathscr{E} = \mathscr{E}_{max}cos(\omega t + \phi)$ So: $\mathscr{E}_{max} = \sqrt{V_R^2 + V_C^2}$ can be described that votlages add in quadratune. and source hase phase with respect to current $\phi = tan^{-1}(\frac{-V_C}{V_R})$. and divide by I_{max}: $\frac{\mathscr{E}_{max}}{I_{max}} = \sqrt{\frac{V_R}{I_{max}}^2 + \frac{V_C}{I_{max}}^2} = \sqrt{R^2 + \chi_c^2} \cong Z =$ impedeance $=$ total opposition to current flow.

Example $\mathscr{E}$ is houseold,$V_{max} = 120V, 60Hz$. Dimmer circuit: what rang eof C is needed to vary power from 10W to 100W? $P = I_{rms}^2R$, want this to vary from 10 to 100. First we have to figure out the current, and for this we use $I_{max} = \frac{\mathscr{E}_{max}}{Z}$ Solve for C $= \frac{1}{\omega\sqrt{\frac{\mathscr{E}_{max}^2}{P_{avg}} - R^2}}$ For $P_avg = 10W \to C = 7.2\mu F$, $P_avg = 100W \to C = 40\mu F$