

# PHY 316M

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## 1 Capacitors

$$C = \left| \frac{Q}{V} \right|$$

## 2 Current

Is the flow of charge in on direction. Current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{\delta Q}{\delta t}$$

Current Density (current per unit area):  $J = \frac{I}{A}$

$n$  = charge carrier density,  $q$  = charge per carrier,  $v_d$  = drift velocity This can give us,  $J = nqv_d$

### 2.1 Ohm's Law

Usually we see Ohm's law in different forms, i.e. for a particular chunk.

Consider some block with volume  $A \cdot l$ , some source of energy (battery) forces current through by applying an electric field.

For a uniform electric field:  $V = E \cdot l$

$$\begin{aligned} J &= \frac{I}{A} \\ &= \sigma E \\ &= \sigma \frac{V}{l} \end{aligned}$$

$$\frac{V}{I} = \frac{1}{\sigma} \frac{l}{A} = \rho \frac{l}{A} = R = \text{resistance}$$

$\frac{V}{I} = R$  Resistance is not resistivity (opposition of current flow of a particular material  $[\rho]$ ) Ohmic material is a material that has a constant slope on Voltage to Current graph. Most common materials like copper behave like this.

**Example** The resistivity of nichrome wire( heaters, toasters) is  $1.5 \times 10^{-6} \Omega m$ . If a household voltage of  $115V$  is applied across a  $0.2mm$  radius wire,  $1.0m$  long, what current flows?

$$R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2} = \frac{1.5 \times 10^{-6} \Omega m \cdot 1.0m}{(\pi(2 \times 10^{-4}m)^2)} = 11.9 \Omega$$

## 2.2 Model for electric conduction

- electron undergo many rapid collisions when  $E = 0$
- when  $E \neq 0$ , the electrons accelerate between collisions
- $F = ma = qE \Rightarrow a = \frac{qE}{m}$
- $v = v_0 + at = v_0 + \frac{qE}{m}t$

Let  $\tau$  = average collision time =  $R \cdot C$

the  $v_d = v_{avg} = \langle v_0 \rangle + \frac{qE}{m}\tau$

so,  $J = nqv_d = nq = \frac{qE}{m}\tau = \sigma E$

so conductivity  $\sigma = \frac{nq^2\tau}{m}$

Called the Drude model or free electron model

$$\sigma = \frac{nq^2\tau}{m}$$

$$\frac{1}{\sigma} = \rho$$

**Example** Assume for copper that each atom donates one free electron. What is the average time between collision for electrons in copper.?

Given:

- Density =  $8.98 \frac{g}{cm^3}$
- Atomic Weight =  $63.54 \frac{g}{mole}$
- $\rho = 1.7 \times 10^{-8} \Omega \cdot m$

$$\tau = \frac{m}{nq^2\rho} = \frac{9.14 \times 10^{-31} kg}{(8.5 \times 10^{22})(1.6 \times 10^{-19})^2 1.7 \times 10^{-8} \Omega m} = 2.5 \times 10^{-14} s$$

## 2.3 Temperature Dependence of resistivity

- resistivities tabulated for  $20^\circ \text{Celsius}$
- for metals,  $\rho$  is higher and  $T$  is higher
- $\alpha$  = linear temperature coefficient
- over some range,  $\rho = \rho_0(1 + \alpha(T - T_0))$

As  $T$  increases, the scattering time decreases due to collisions with vibrating atoms

At higher temperatures the  $\rho$  to temperature graph is linear. But at the beginning there is residual resistivity due to impurities.

**Semiconductors** The number of carriers decreases as the temperature decreases, this means that all the electrons are sticking to their atoms.

### 3 Resistors in Series and Parallel

Circuit symbol: 

#### 3.1 Resistors in series

For resistors in series the resistivities add

$$R_{tot} = R_1 + R_2 + \dots$$
$$R_{tot} = \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} = R_1 + R_2$$

The current ( $I$ ) is the same everywhere too. **Resistors don't add in parallel. Capacitors do.**

$$V = V_1 + V_2$$

The voltage divider

$$V_1 = I \cdot R_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

#### 3.2 Resistors in parallel

For resistors in parallel halve the resistance if two exact resistors are put in parallel

- In parallel have the same voltage across each element
- In parallel also the current divides among branches

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow R_{tot} = \frac{V}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

**Superconductors** Electrons pair up and when electron jumps to the lattice another electron pulls it right back.

### 3.3 Resistors Disipate Energy

Electrons undergo collisions, and give up energy as heat. A steady release of current ( $I$ ) causes a steady realease of energy.

$$\Delta U = \Delta QV$$

Better to discess the rate, which is really known as:

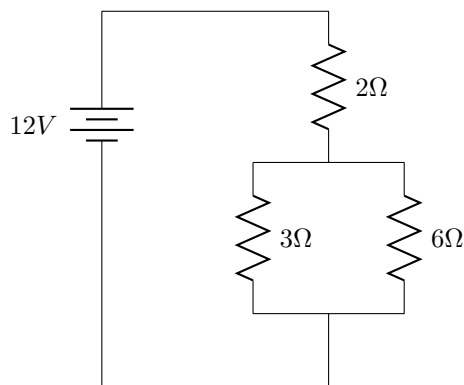
$$\text{Power} = P = \frac{\Delta U}{\Delta t} = \frac{\Delta U}{\Delta t} V = IV$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

This power is also known as Joule heating.

**Putting this into practice:** many circuits can be analyzed with just Ohm's Law and Resistance.

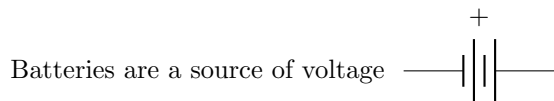
- What is the total power delivered?
- What is the power dissipated in each R?



$$P_{\text{dissipated in } 3\Omega} = \frac{V_3^2}{R} = \frac{(6V)^2}{3\Omega} = 12W$$

### 3.4 Direct Current Circuits

Real battery is an ideal  $\mathcal{E}$ MF plus intended resistance



Source of voltage = "electromotive force" =  $\mathcal{E}$ MF =  $\mathcal{E}$

$$V = \mathcal{E} - Ir$$

"Open-circuit voltage", where  $I = 0$

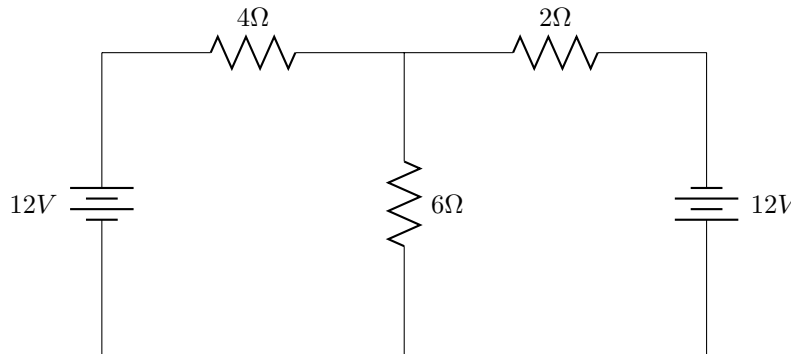
**Analysis of circuits** Any circuit can be analyzed with Kirchhoffs Rules:

- Junction Rule, algebraic sum of currents into a junction = sum of currents  
 $\Sigma i_{in} = \Sigma i_{out}$
- Loop Rule, algebraic sum of voltages around any closed loop is zero.  
 Where voltage rises are positive (- to +) and voltage drops are negative (+ to -)

For resistors the current direction determines voltage drop(negative)

**Example**

- What is the current in the  $6\Omega$  resistor?
- Is it flowing up or down?



(I) Junction at A:  $i_1 = i_2 + i_3$

(II) Loop A:  $12V - i_1(4\Omega) - i_3(6\Omega) = 0$

(III) Loop B:  $I_3(6\Omega) - i_2(2\Omega) + 12V = 0$

$$i_3 = \frac{-12V}{22\Omega} = -\frac{6}{11}A$$

**2 other techniques:**

- same, except use fictitious “loop currents”
- Source suppressing - can look at effects of sources separately

**Next: Circuits with Capacitors** will see time-depended behavior Before “transient phenomena”, look at *Steady-state*: “after a long time”. The capacitor starts acting like the current is 0.

## 4 Magnetism

We saw:

$$F = qv \frac{\mu_0 I}{2\pi r} = 0$$

for  $\vec{v}$  tangent to circle. where  $\mu_0 = 4\pi \times 10^{-7} \frac{Ns^2}{C^2}$

Rewrite this as

$$\boxed{\vec{F} = q\vec{v} \times \vec{B}}$$

where  $\boxed{B = \frac{\mu_0 I}{2\pi r}}$  magnetic field due to a long wire where direction of  $\vec{B}$  is tangent to circle.

If also electric force, combination of electric and magnetic force is called "Lorentz Force"

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

$$\vec{F} = q\vec{v} \times \vec{B}$$

- Like  $\vec{E}$ ,  $\vec{F}$  opposite for  $+q, -q$
- Depends on  $\vec{v}$ :  $\vec{F} = 0$  for  $\vec{v} = 0$
- Depends on angle:  $\boxed{|\vec{F}| = |q|vB\sin\Theta}$   
notice  $\vec{F} = 0$  for  $\vec{v} \parallel \vec{B}$
- Direction  $\vec{F} = q\vec{v} \times \vec{B}$  given by right hand rule

Magnetic field does no work because the magnetic force is perpendicular to displacement.

So  $\vec{B}$  changes direction of  $\vec{v}$  but not its magnitude ( $KE = \frac{1}{2}mv^2$ )

Unit of B =  $\frac{Ns}{Cm}$  = tesla = T also gauss = G =  $10^{-4}T$

### 4.1 Evaluating Cross Products

1. Get general equation by expanding determinant  $\begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} =$   

$$\vec{v} \times \vec{B} = i(v_y B_z - v_z B_y) - j(v_x B_z - v_z B_x) + k(v_x B_y - v_y B_x)$$
2. Just multiply out.  
 know that  $i \times j = k$  and that  $j \times i = -k$
5. Magnetic field does no work

## 4.2 Two ways to Find $\vec{B}$

1. Ampere's Law: for high symmetry
2. Biot Savart Law

In general we will only look at: center of arcs and circles, or due to straight segments, or on axis of loop.

### 4.2.1 $\vec{B}$ at the center of a circle

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s} \times \hat{r}}{r^2}$$

$$B = \int dB = \int \frac{\mu_0 I}{4\pi r^2} ds = \frac{\mu_0 I}{2R}$$

Since  $r = R$ , the integral is constant and is easy to integrate with respect to  $s$ .

**Also:** for a fraction  $f$  of a circle  $\vec{B} = f B_{\text{full circle}}$

### 4.2.2 $\vec{B}$ near a finite straight wire

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{ds \sin\theta}{r^2}$$

1.  $\sin\theta = \sin\theta' = \cos\phi$
2.  $\cos\phi = \frac{R}{r} \rightarrow r = \frac{R}{\cos\phi}$
3.  $\tan\phi = \frac{s}{R} \rightarrow s = R \cdot \tan\phi \rightarrow \frac{R}{\cos^2\phi d\phi}$

$$B = \int dB$$

$$= \int \frac{\mu_0 I \frac{R}{\cos^2\phi} d\phi \cos\phi}{4\pi (\frac{R}{\cos\phi})^2}$$

$$= \frac{\mu_0 I}{4\pi R} \int_{-\phi_1}^{+\phi_2} \cos\phi d\phi$$

$$= \frac{\mu_0 I}{4\pi R} \sin\phi \Big|_{-\phi_1}^{+\phi_2}$$

$$= \frac{\mu_0 I}{4\pi R} [\sin\phi_2 + \sin\phi_1]$$

### 4.2.3 B field due to a square loop of side a

$$B_{loop} = \frac{2\sqrt{2}\mu_0 I}{\pi a}$$

### 4.2.4 B on axis of loop

Off axis compnents cancel around circle.

$\phi$  is the angle at the bottom right of triangle formed by circle and axis

$$r = \sqrt{x^2 + R^2} \quad \text{Using pythagorean theorem}$$

$$\sin\phi = \frac{R}{\sqrt{x^2 + R^2}} \quad \text{So we can define } \sin\phi$$

We only have to integrate along the x compnents

$$dB_x = dB \sin\phi$$

$$= \frac{\mu_0 I ds}{4\pi r^2} \sin\phi$$

We can say that  $\int ds = 2\pi R$  since the radius is constant

$$B = \int \frac{\mu_0 I}{4\pi r^2} \sin\phi ds$$

$$= \frac{\mu_0 I R}{4\pi (x^2 + R^2)^{3/2}} \int ds$$

$$= \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

Also, consider yourself very close to the field,  $x \gg R$   $\frac{1}{x^3}$  dipole field

## 4.3 Motion in a uniform B

$$|\vec{F}| = |q\vec{v} \times \vec{B}| = qvB$$

$\vec{F}$  is  $\perp$  to  $\vec{v}$  centripetal with accelerations  $a_r = \frac{v^2}{r}$  We saw that

$$B_{\text{long wire}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{solenoid}} = \mu_0 n I$$

$$B_{\text{loop center}} = \frac{\mu_0 I}{2R}$$

angular velocity  $\omega = ?$

$$r = \frac{mv}{qB}$$



**Example: Mass Spectrometer** *Note: Electrons travel in a semi-circle in a spectrometer* Electrons are accelerated through potential of  $10^3V$  ("a 1 keV electron"). They enter a region of uniform  $B = 10^{-2}T$ . What is the distance they are displaced?

$$x = 2r = 2\frac{mv}{qB}$$

Know  $m, q, B$  need to find  $v$

$$\begin{aligned}\Delta KE = \Delta PE &\rightarrow \frac{1}{2}mv^2 = eV_0 = \sqrt{\frac{2eV}{m}} \\ x &= 2\frac{m}{eB}\sqrt{\frac{2eV_0}{m}} = \frac{2}{B}\sqrt{\frac{2mV_0}{e}} = \frac{2}{(10^{-2}T)}\sqrt{\frac{2(9.11 \times 10^{-31})(10^3V)}{(1.6 \times 10^{-19}C)}}\end{aligned}$$

In practice however, we are given  $v, B, x$  to get  $(\frac{q}{m})$

**Example: Velocity Selector** region of crossed  $\vec{E} + \vec{B}$

All we have to consider is  $qE$  -vs-  $qvB$

if  $qE = qvB$  then  $\boxed{v = \frac{E}{B}}$

#### 4.4 Force on a current

Consider positive charge travelling along x axis with  $v_0$  with a  $-\vec{B}\hat{y}$

$$\begin{aligned}F_{\text{on wire}} &= q\vec{v} \times \vec{B} \\ &= \frac{1}{n}\vec{j} \times \vec{B} \\ F_{\text{on wire}} &= (\# \text{ charges})F_{\text{on 1 charge}} \\ &= (nAl)\frac{1}{n}\hat{j} \times \hat{B} \\ &= lA\vec{j} \times \vec{B} \\ &= I\vec{l} \times \vec{B}\end{aligned}$$

**Example** Net force on a current loop in a uniform  $\vec{B}$  is **zero**. This is true for any loop.

**Example** A current  $I$  flows from the origin to  $(x, y, z) = (1m, 1m, 0)$  and then straight to  $(2m, 0, 0)$ . In a uniform field of  $\vec{B} = 5T\hat{i}$

$$\begin{aligned}\vec{F}_1 &= (10N)(-\hat{j} + \hat{i}) \\ \vec{F}_2 &= (10N)(-\hat{j} - \hat{i}) \\ \vec{F} &= (20N) - \hat{j}\end{aligned}$$

**Example** Force on a wire segment due to a large  $\parallel$  wire

$$\begin{aligned} B &= \frac{\mu_0 I}{2\pi r} \\ \vec{F} &= I_2 \vec{l} \times \frac{\mu_0 I_1}{2\pi r} \\ &= I_2 \times \frac{l \mu_0 I_1}{2\pi r} \\ \frac{\text{force}}{\text{length}} &= \frac{\mu_0 I_1 I_2}{2\pi r} \end{aligned}$$

**Definition:** Magnetic Moment

Dipole moment ( $\mu = IA$ ) due to a magnetic loop.