

PHY 316M

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1 Capacitors

$$C = \left| \frac{Q}{V} \right|$$

2 Current

Is the flow of charge in on direction. Current:

$$I = \frac{\Delta Q}{\Delta t} = \frac{\delta Q}{\delta t}$$

Current Density (current per unit area): $J = \frac{I}{A}$

n = charge carrier density, q = charge per carrier, v_d = drift velocity This can give us, $J = nqv_d$

2.1 Ohm's Law

Usually we see Ohm's law in different forms, i.e. for a particular chunk.

Consider some block with volume $A \cdot l$, some source of energy (battery) forces current through by applying an electric field.

For a uniform electric field: $V = E \cdot l$

$$\begin{aligned} J &= \frac{I}{A} \\ &= \sigma E \\ &= \sigma \frac{V}{l} \end{aligned}$$

$$\frac{V}{I} = \frac{1}{\sigma} \frac{l}{A} = \rho \frac{l}{A} = R = \text{resistance}$$

$\frac{V}{I} = R$ Resistance is not resistivity (opposition of current flow of a particular material $[\rho]$) Ohmic material is a material that has a constant slope on Voltage to Current graph. Most common materials like copper behave like this.

Example The resistivity of nichrome wire(heaters, toasters) is $1.5 \times 10^{-6} \Omega m$. If a household voltage of $115V$ is applied across a $0.2mm$ radius wire, $1.0m$ long, what current flows?

$$R = \rho \frac{l}{A} = \rho \frac{l}{\pi r^2} = \frac{1.5 \times 10^{-6} \Omega m \cdot 1.0m}{(\pi(2 \times 10^{-4}m)^2)} = 11.9 \Omega$$

2.2 Model for electric conduction

- electron undergo many rapid collisions when $E = 0$
- when $E \neq 0$, the electrons accelerate between collisions
- $F = ma = qE \Rightarrow a = \frac{qE}{m}$
- $v = v_0 + at = v_0 + \frac{qE}{m}t$

Let τ = average collision time

the $v_d = v_{avg} = \langle v_0 \rangle + \frac{qE}{m}\tau$

so, $J = nqv_d = nq = \frac{qE}{m}\tau = \sigma E$

so conductivity $\sigma = \frac{nq^2\tau}{m}$

Called the Drude model or free electron model

$$\sigma = \frac{nq^2\tau}{m}$$

$$\frac{1}{\sigma} = \rho$$

Example Assume for copper that each atom donates one free electron. What is the average time between collision for electrons in copper.?

Given:

- Density = $8.98 \frac{g}{cm^3}$
- Atomic Weight = $63.54 \frac{g}{mole}$
- $\rho = 1.7 \times 10^{-8} \Omega \cdot m$

$$\tau = \frac{m}{nq^2\rho} = \frac{9.14 \times 10^{-31} kg}{(8.5 \times 10^{22})(1.6 \times 10^{-19})^2 1.7 \times 10^{-8} \Omega m} = 2.5 \times 10^{-14} s$$

2.3 Temperature Dependence of resistivity

- resistivities tabulated for 20°Celsius
- for metals, ρ is higher and T is higher
- α = linear temperature coefficient
- over some range, $\rho = \rho_0(1 + \alpha(T - T_0))$

As T increases, the scattering time decreases due to collisions with vibrating atoms

At higher temperatures the ρ to temperature graph is linear. But at the beginning there is residual resistivity due to impurities.

Semiconductors The number of carriers decreases as the temperature decreases, this means that all the electrons are sticking to their atoms.

3 Resistors in Series and Parallel

Circuit symbol: 

3.1 Resistors in series

For resistors in series the resistivities add

$$R_{tot} = R_1 + R_2 + \dots$$
$$R_{tot} = \frac{V}{I} = \frac{V_1}{I_1} + \frac{V_2}{I_2} = R_1 + R_2$$

The current (I) is the same everywhere too. **Resistors don't add in parallel. Capacitors do.**

$$V = V_1 + V_2$$

The voltage divider

$$V_1 = I \cdot R_1 = \frac{V}{R_1 + R_2} \cdot R_1$$

3.2 Resistors in parallel

For resistors in parallel halve the resistance if two exact resistors are put in parallel

- In parallel have the same voltage across each element
- In parallel also the current divides among branches

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} \Rightarrow R_{tot} = \frac{V}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

Superconductors Electrons pair up and when electron jumps to the lattice another electron pulls it right back.

3.3 Resistors Disipate Energy

Electrons undergo collisions, and give up energy as heat. A steady release of current (I) causes a steady realease of energy.

$$\Delta U = \Delta QV$$

Better to discess the rate, which is really known as:

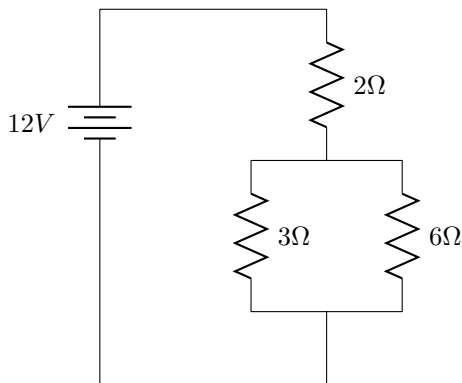
$$\text{Power} = P = \frac{\Delta U}{\Delta t} = \frac{\Delta U}{\Delta t} V = IV$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

This power is also known as Joule heating.

Putting this into practice: many circuits can be analyzed with just Ohm's Law and Resistance.

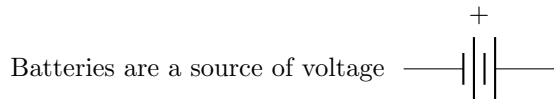
- What is the total power delivered?
- What is the power dissipated in each R?



$$P_{\text{dissipated in } 3\Omega} = \frac{V_3^2}{R} = \frac{(6V)^2}{3\Omega} = 12W$$

3.4 Direct Current Circuits

Real battery is an ideal \mathcal{E} MF plus intended resistance



Source of voltage = "electromotive force" = \mathcal{E} MF = \mathcal{E}

$$V = \mathcal{E} - Ir$$

"Open-circuit voltage", where $I = 0$

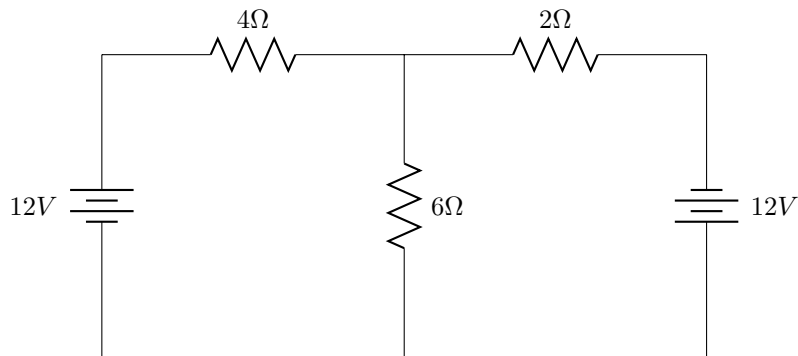
Analysis of circuits Any circuit can be analyzed with Kirchhoffs Rules:

- Junction Rule, algebraic sum of currents into a junction = sum of currents
 $\Sigma i_{in} = \Sigma i_{out}$
- Loop Rule, algebraic sum of voltages around any closed loop is zero.
 Where voltage rises are positive (- to +) and voltage drops are negative (+ to -)

For resistors the current direction determines voltage drop(negative)

Example

- What is the current in the 6Ω resistor?
- Is it flowing up or down?



(I) Junction at A: $i_1 = i_2 + i_3$

(II) Loop A: $12V - i_1(4\Omega) - i_3(6\Omega) = 0$

(III) Loop B: $i_3(6\Omega) - i_2(2\Omega) + 12V = 0$

$$i_3 = \frac{-12V}{22\Omega} = -\frac{6}{11}A$$

2 other techniques:

- same, except use fictitious “loop currents”
- Source suppressing - can look at effects of sources separately

Next: Circuits with Capacitors will see time-depended behavior Before “transient phenomena”, look at *Steady-state*: “after a long time”. The capacitor starts acting like the current is 0.