As you can probably guess, this dataset is based on several equations in astrophysics. So, let's review these equations together.

# Stefan–Boltzmann law

black-body radiant emittance    Stefan-Boltzmann constant    Boltzmann constant

$$j^\star = \varepsilon \sigma T^4 \qquad \sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.670374\ldots \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$$

emissivity    thermodynamic temperature    Planck's constant    Speed of light

This law examines the relationship between the luminosity of the star and the temperature of the objects. This relationship has two constant numbers. The first number that is represented by the small sigma is Stefan Boltzmann's constant, and the other is the emissivity coefficient which is represented by epsilon and depends on the characteristic of the radiating object. The emissivity coefficient is always between zero and one. Now it is obvious that in the data set they use this rule to find the luminosity of a star. This law only applies to black bodies, which in theory absorb all incident thermal radiation. Also, this law played an important role in the investigations that led to Max Planck's quantum theory.

$$L = 4\pi R^2 \sigma T^4 \longrightarrow T = \sqrt[4]{\frac{L}{4\pi R^2 \sigma}}$$

luminosity    the stellar radius    the effective temperature

# Wien's displacement law

constant of proportionality

wavelength peak

$$\lambda_{\text{peak}} = \frac{b}{T}$$

absolute temperature

As you can see in this equation, the wavelength of the heat radiation that is most emitted by a black body is inversely proportional to the absolute temperature of the body. Now it is clear that in the data set they use this rule to find the surface temperature of a star using wavelength. In other words, Wien's displacement law interprets the maximum spectral intensity of this radiation. We know that Planck's law also interprets the radiation emitted by black bodies. Let's look at Planck's law.

Planck's constant

Speed of light

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda k_B T)} - 1}$$

describes the spectral emissive power per unit area, per unit solid angle, and per unit frequency for particular radiation frequencies.

wavelength

the Boltzmann constant

# Absolute magnitude

The apparent magnitude of celestial bodies, which is indicated by the symbol m, is a measure to express their apparent brightness and the light that reaches the earth. The intrinsic energy they emit depends on their distance from the earth, and since the stars are at a different distance from us, this criterion does not indicate their true brightness. It is possible that the brightness of a star is very bright Due to the great distance, it will appear fainter than a faint star located at a short distance from the earth. The absolute magnitude indicated by the letter M is also the apparent magnitude with the condition that the star is hypothetically brought to a distance of ten parsecs or 32.6 light years.

Distance (in parsecs, with 1 pc = 3.2616 light-years)

$$M = m - 5\log_{10}(d_{\mathrm{pc}}) + 5 = m - 5\left(\log_{10} d_{\mathrm{pc}} - 1\right)$$

absolute magnitude

apparent magnitude

Before we begin, we need to know one more thing about the difference between the types of stars in the dataset.

## red dwarf

Red dwarf stars are the smallest and most interesting type of star in the main sequence. The red dwarf star is low-mass and relatively cold. Because red dwarfs are low-mass, they burn hydrogen very slowly and retain their luminosity for trillions of years. These objects remain in the main sequence until they run out of fuel.

## brown dwarf

Brown dwarfs, with a mass ranging from 12 times the mass of Jupiter to half the mass of the Sun, can emit light like stars, but the amount of this light is usually not large. Unlike stars, the glow of brown dwarfs does not come from the heat of nuclear explosions in their cores. Instead, their light and heat remain from their initial formation. Like main sequence stars, these objects form from the collapse of clouds of gas and dust, just like stars, but to a lesser extent, where gravity holds the material tightly together, creating a young protostar at the center.

## white dwarf

Like an ancient tomb, white dwarfs hold a lot of information about the life and death of stars. These highly dense objects are the last stage of the evolution of low- to intermediate-mass stars. The stars in the sky seem unlimited and fixed but eventually, most stars become white dwarfs. White dwarfs are usually very dense. Their mass is comparable to the mass of the sun while they may be the same size as the earth. White dwarfs are very hot at the beginning of life and emit a lot of X-rays and ultraviolet rays into the space around them. Some of these rays are trapped among the gas outflows. The gases react to this phenomenon with fluorescent light and a rainbow of colors called a planetary nebula. White dwarfs have a long and long journey ahead of them. These objects cool slowly until they become a stagnant mass of carbon and oxygen that floats in space without any glow. This mass is called a black dwarf, but the universe is not old enough to witness the formation of black dwarfs [1].

## main sequence

In main sequence stars, nuclear fusion occurs for hydrogen atoms in their cores and produces helium. About ninety percent of the stars in the universe, including our Sun, are main sequence stars. These stars can have a mass from one-tenth to two hundred times that of our Sun. The lifetime of these types of stars depends on their mass. A more massive star has more matter at its disposal, but due to the higher temperature of its core caused by stronger gravity, it burns its hydrogen fuel faster.

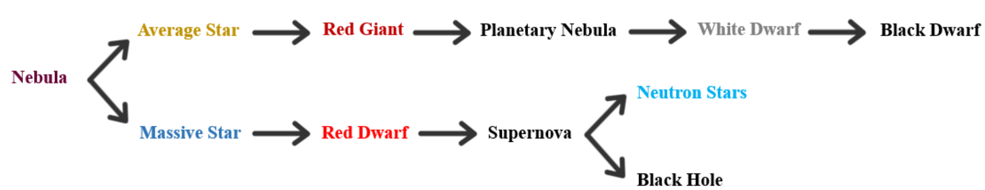
## supergiant

During the main sequence period, stars are in hydrostatic equilibrium, which, as we saw on the previous slide, means that nuclear fusion in their cores, where they combine hydrogen to produce helium, has enough energy and pressure to maintain the weight of the outer layers keeps it from falling inside. When these nuclear reactions stop, the helium core contracts and heat the surrounding shell of hydrogen gas. The radiation resulting from this reaction puts pressure on the outer layers of the star and causes them to expand and cool. The star then evolves into a red giant or a supergiant, depending on its initial mass.

# hypergiant

A hypergiant star is a very massive and radiant star whose radius is usually more than 1000 times that of the Sun. The mass of some supergiants is 150 times that of the Sun, and some of them are millions of times brighter than the Sun. Another important characteristic of hypergiants is their instability, that is, the conditions governing them do not last long, and their temperature and luminosity change continuously.

The process of stars running out of fuel is complex but fascinating. Massive stars, like most stars, start by burning hydrogen and producing helium. Nuclear energy provides the power of the stars (nuclear fusion, four hydrogen nuclei fuse at very high temperatures and form the helium nucleus, this process produces heat). When stars run out of hydrogen, their center collapses due to gravitational pull, raising the temperature enough for nuclear fusion of helium to carbon. In stars that are more than ten times the mass of the Sun, this process continues by burning carbon, oxygen, neon, and silicon and finally forming an iron core.




As you've probably guessed by now, this dataset aims to prove that stars follow a specific pattern in the sky, specifically called the Hertzsprung-Russell diagram, or simply the HR diagram. In the early 20th century, scientists discovered that the mass of a star is proportional to its luminosity, or the amount of light it produces. Both of these quantities are related to the temperature of the star. The mass and luminosity of the star are also related to its color. More massive stars are hotter and bluer, and those with less mass are cooler and redder. This perception led to the compilation of the HR diagram, a diagram of stars according to their brightness and color, which in turn is a reflection of their temperature [2].

# References

[1] Fariba Kafikang, Hassan Hassanabadi, Won Sang Chung, Filip Studnička. "Investigation of the white dwarfs based on deformed Lane–Emden equation", Annals of Physics, 2024

[2] Space.com: NASA, Space Exploration and Astronomy News