

Dynamics:

$$Ml^2 \ddot{\theta} - Mgs \sin \theta = \tau$$

$$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \rightarrow \dot{x} = \begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{l} \sin(x_1) + \frac{\tau}{Ml^2} \end{bmatrix}$$



Forward Euler:  $\frac{x_{k+1} - x_k}{\Delta t} = f(x_k)$

$$\rightarrow x_{k+1} = x_k + \Delta t \cdot f(x_k)$$

$$= x_k + \Delta t \cdot \begin{bmatrix} x_2 \\ -\frac{g}{l} \sin(x_1) + \frac{\tau}{Ml^2} \end{bmatrix}$$

$$= \begin{bmatrix} x_1 + \Delta t \cdot x_2 \\ x_2 + \Delta t \cdot \left( -\frac{g}{l} \sin(x_1) + \frac{\tau}{Ml^2} \right) \end{bmatrix}$$

Jacobian:  $F = \frac{\partial f}{\partial x} = \begin{bmatrix} 1 & \Delta t \\ -\frac{g}{l} \cos(x_1) \cdot \Delta t & 1 \end{bmatrix} \bigg|_{x_1 = x_{k-1}}$

$$\hat{x}_{k|k-1} = f(x_{k-1|k-1}, u_{k-1})$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_{k-1}$$

} Predict

$$\hat{y}_k = z_k - h(\hat{x}_{k|k-1})$$

$$K_k = \frac{P_{k|k-1} H_k^T}{H_k P_{k|k-1} H_k^T + R_k}$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \hat{y}_k$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

} Update