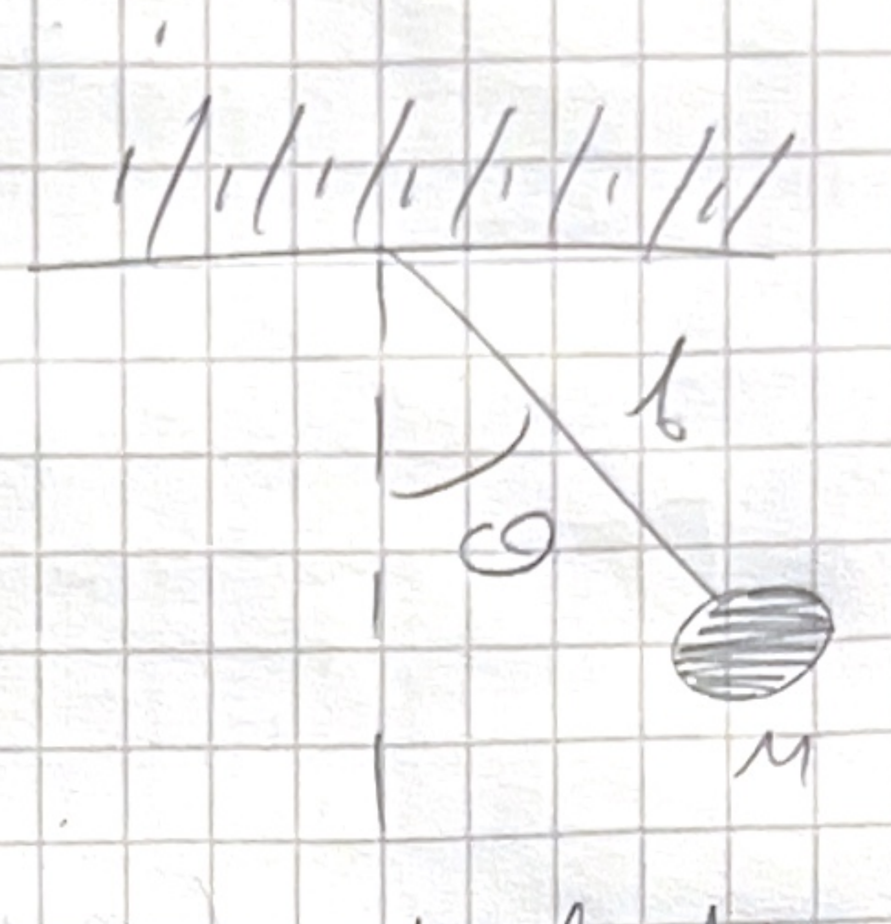
Kalman filter:



$m=1,\ l=1/2$

$ml\ddot{\theta} = -mgl\sin\theta$

$x = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \rightarrow \dot{x} = \begin{bmatrix} x_2 \\ -\frac{g}{l}\sin\theta \end{bmatrix} = f(x)$

$F = \frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos\theta & 0 \end{bmatrix}$

(I) Linearization + Kalman:

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(II) or EKF:

$\dot{x} = f(x)$

$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Predict:

If the model is linear then this is a simple state space (I)

$x_{k+1} = f(x_k, u_k)$

$P_{k|k-1} = F \cdot P_{k-1|k-1} \cdot F^T + Q_k$

Update:

$K_k = \frac{P_{k|k-1} \cdot H_k^T}{(H_k \cdot P_{k|k-1} \cdot H_k^T + R_k)}$

$\hat{x}_{k+1} = x_{k+1} + K_k (z_k - H_k \cdot x_{k+1})$

$P_{k|k} = (I - K_k \cdot H_k) P_{k|k-1}$

(*) The simulation is discrete so:

$\frac{x_{k+1} - x_k}{\Delta t} = \dot{x} \rightarrow \boxed{x_{k+1} = x_k + \Delta t \cdot \dot{x}}$

Simulation time

(*) The prediction guys dont use u as the control input like us control, it's a dynamic term.