A Short Summary of Tropospheric Math Models

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General Tropospheric Correction Models

$d_{trop} = d_h^z m_h(\varepsilon) + d_w^z m_w(\varepsilon)$

d_{trop}	Total tropospheric delay
d_h^z	Hydrostatic delay at zenith
ε	Elevation angle
$m_h(\varepsilon)$	Hydrostatic mapping function
d_w^z	'Wet' delay at zenith
$m_w(\varepsilon)$	'Wet' mapping function

Model implementations fall into two general categories.

(i) Meteorological data (Pressure, Temperature, Humidity) is available (geodetic and survey applications, i.e. carrier-phase DGPS for long baselines, residual differential tropospheric errors on are typically 1ppm or less with a maximum of 3ppm with baseline distance)

(ii) Meteorological data is not generally available (all other typical GPS users and even carrier-phase DGPS for short to medium baselines)

WAAS/ENGOS/MSAS Tropospheric Correction Guidelines

The WAAS guidelines recommend that a user applies a correction for tropospheric delay that is compliant with the International Civil Aviation Organization (ICAO) Standard and Recommended Practices (SARP's) for Satellite-Based Augmentation Systems (SBAS) (RTCA, 1999). These guidelines also cover the USA Wide Area Augmentation System (WAAS) and the Japanese Multi-functional Transport Satellite, MTSAT-based Satellite Augmentation System (MSAS). The recommended SBAS model provides an estimate of the zenith tropospheric delay which is dependent on empirical estimates of five meteorological parameters at the receiver - namely, pressure, temperature, water vapour pressure, temperature lapse rate and water vapour lapse rate. These estimates of the meteorological parameters are dependent on the receiver's height, latitude and day-of-year, and are interpolated from reference values for the yearly averages of the parameters and their associated seasonal variations, derived primarily from North American meteorological data. This is so that the model is consistent with the tropospheric model for the WAAS program detailed in (Collins and Langley, 1997) and also discussed in (Collins and Langley, 1998). The EGNOS guidelines then recommend mapping the zenith tropospheric delay estimate to the appropriate receiver-to-satellite elevation angle using an elevation angle-dependent mapping function.

Collins, J.P. and R.B. Langley (1997). A Tropospheric Delay Model for the User of the Wide Area Augmentation System. Final contract report prepared for Nav Canada, Department of Geodesy and Geomatics Engineering Technical Report No. 187, Univ. of New Brunswick, Federiction, N.B., Canada.

Collins, J.P. and R.B. Langley (1998). The residual tropospheric propagation delay: How bad can it get? Proceeding of ION GPS-98, The 11th International Meeting of the Satellite Division of The Institute of Navigation, Nashville, Tenn., September 15-18, pp. 729-738.

Selected Excerpts of the WAAS MOPS for an Applied Tropospheric Model

"2.1.1.2 GPS Signal Processing Requirements

GPS/WAAS equipment shall be designed to process the GPS signals and necessary data described in the latest GPS SPS Performance Standard, October 2001, and ICD-GPS-200C, "Navstar GPS Space Segment / Navigation User Interfaces", April 2000, under interference conditions described in Appendix C and under the minimum signal conditions defined in Section 2.1.1.10. If the ionospheric corrections provided by the WAAS are not applied to a pseudorange, then the equipment shall decode the ionospheric coefficients in the GPS navigation message and apply the ionospheric corrections described in the ICD-GPS- 200C, "Navstar GPS Space Segment / Navigation User Interfaces", April 2000. If the ionospheric corrections provided by WAAS are applied to a satellite pseudorange, the GPS ionospheric model shall not be used for that satellite. A tropospheric correction shall be applied (an acceptable algorithm is described in Appendix A, Section A.4.2.4)." (p. 23)

"2.1.1.3.3 WAAS Satellite Pseudorange Determination

The GPS/WAAS equipment shall determine the pseudorange to each WAAS satellite that is currently being used in the position computation. These pseudoranges shall be referenced to the same time base as that of the GPS satellites. The equipment shall account for earth rotation in determining the pseudorange. If the ionospheric corrections provided by the WAAS are not applied to the WAAS satellite pseudoranges, then the equipment shall decode the ionospheric coefficients in the GPS navigation message and apply the ionospheric corrections described in ICD-GPS-200C, "Navstar GPS Space Segment / Navigation User Interfaces", April 2000. A tropospheric correction shall be applied (an acceptable algorithm is described in Appendix A, Section A.4.2.4)." (p. 25)

"2.1.4.10 Application of Differential Correction Terms

The equipment shall meet the requirements specified in 2.1.1.4.12, except as modified below:

The equipment shall correct the pseudorange as:

$$PR_{i,corrected}(t) = PR_{i,measured}(t) + PRC_i\left(t_{i,of}\right) + RRC\left(t_{i,of}\right) \times \left(t - t_{i,of}\right) + TC_i + IC_i$$

where:

TC_i is the tropospheric model described in Section 2.1.4.10.3

IC_i is the ionospheric model described in Section 2.1.4.10.2.

" (p. 54)

“2.1.4.10.3 Application of Tropospheric Corrections

Equipment shall apply the tropospheric delay correction specified in Section A.4.2.4.” (p.54)

“A.4.2.4 Tropospheric Model

Because tropospheric refraction is a local phenomenon, all users will compute their own tropospheric delay correction.

The tropospheric delay estimate takes the form:

$$TC_i = -\left(d_{hyd} + d_{wet}\right) \cdot m(El_i) \qquad \text{[A-2]}$$

[d_{hyd}, d_{wet}] are calculated from the receiver’s height and estimates of five meteorological parameters: pressure [P (mbar)], temperature [T (K)], water vapor pressure [e (mbar)], temperature lapse rate [β (K/m)] and water vapor “lapse rate” [λ (dimensionless)].

Values of each of the five meteorological parameters, applicable to the receiver latitude [ϕ] and day-of-year [D] (starting with 1 January), are computed from the average and seasonal variation values given in Table A-2. Each parameter value [ξ] is computed as:

$$\xi(\phi, D) = \xi_0(\phi) - \Delta\xi(\phi) \cdot \cos\left(\frac{2\pi(D - D_{\min})}{365.25}\right) \qquad \text{[A-3]}$$

where D_{min}=28 for northern latitudes, D_{min}=211 for southern latitudes, and ξ_0, $\Delta\xi$ are the average and seasonal variation values for the particular parameter at the receiver’s latitude. For latitudes $|\phi| \le 15°$ and $|\phi| \ge 75°$, values for ξ_0 and $\Delta\xi$ are taken directly from Table A-2. For latitudes in the range $15° < |\phi| < 75°$, values for ξ_0 and $\Delta\xi$ at the receiver's latitude are each pre-calculated by linear interpolation between values for the two closest latitudes [ϕ_i, ϕ_{i+1}] in Table A-2:

$$\xi_0(\phi) = \xi_0(\phi_i) + \left[\xi_0(\phi_{i+1}) - \xi_0(\phi_i)\right] \cdot \frac{(\phi - \phi_i)}{(\phi_{i+1} - \phi_i)} \quad \text{[A-4]}$$

$$\Delta\xi(\phi) = \Delta\xi(\phi_i) + \left[\Delta\xi(\phi_{i+1}) - \Delta\xi(\phi_i)\right] \cdot \frac{(\phi - \phi_i)}{(\phi_{i+1} - \phi_i)} \quad \text{[A-5]}$$

TABLE A-2 METEOROLOGICAL PARAMETERS FOR TROPOSPHERIC DELAY

	Average				
Latitude (°)	**P_0 (mbar)**	**T_0 (K)**	**e_0 (mbar)**	**β_0 (K/m)**	**λ_0**
15° or less	1013.25	299.65	26.31	6.30e-3	2.77
30	1017.25	294.15	21.79	6.05e-3	3.15
45	1015.75	283.15	11.66	5.58e-3	2.57
60	1011.75	272.15	6.78	5.39e-3	1.81
75° or greater	1013.00	263.65	4.11	4.53e-3	1.55
	Seasonal Variation				
Latitude (°)	**ΔP (mbar)**	**ΔT (K)**	**Δe (mbar)**	**Db (K/m)**	**Dl**
15° or less	0.00	0.00	0.00	0.00e-3	0.00
30	-3.75	7.00	8.85	0.25e-3	0.33
45	-2.25	11.00	7.24	0.32e-3	0.46
60	-1.75	15.00	5.36	0.81e-3	0.74
75° or greater	-0.50	14.50	3.39	0.62e-3	0.30

Zero-altitude zenith delay terms [z_{hyd}, z_{wet}] are calculated as:

$$z_{hyd} = \frac{10^{-6} k_1 R_d P}{g_m} \quad \text{[A-6]}$$

$$z_{wet} = \frac{10^{-6} k_2 R_d}{g_m(\lambda + 1) - \beta R_d} \cdot \frac{e}{T} \quad \text{[A-7]}$$

where k_1 = 77.604 K/mbar, k_2 = 382000 K^2/mbar, R_d = 287.054 J/kg/K, and g_m = 9.784 m/s^2.

[d_{hyd}, d_{wet}] are calculated as:

$$d_{hyd} = \left(1 - \frac{\beta H}{T}\right)^{\frac{g}{R_d \beta}} \cdot z_{hyd} \qquad \text{[A-8]}$$

$$d_{wet} = \left(1 - \frac{\beta H}{T}\right)^{\frac{(\lambda+1)g}{R_d \beta} - 1} \cdot z_{wet} \qquad \text{[A-9]}$$

where g = 9.80665 m/s^2 and the receiver's height, [H] is expressed in units of meters above mean-sea-level.

The tropospheric correction mapping function for satellite elevation, $m(E_i)$, is calculated as:

$$m(El_i) = \frac{1.001}{\sqrt{0.002001 + \sin^2(El_i)}} \qquad \text{[A-10]}$$

This mapping function is valid for satellite elevation angles of not less than 5°.

A.4.2.5 Residual Tropospheric Error

The model for the residual error for the tropospheric delay estimate for satellite i is:

$$\sigma^2_{j,tropo} = (0.12 \cdot m(E))^2 \qquad \text{[A-11]}$$

where the tropospheric vertical error is σ_{TVE} = 0.12 m

” (p. A-8 to A-10)