Diagonalization of the Measurement Variance-Covariance Matrix

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Abstract

In Kalman filtering, it is very useful to be able to process measurements sequentially (i.e. one at a time). This method eliminates the need for matrix inversion operations since sequential processing uses scalar measurement updates [1]. The method requires that measurements are uncorrelated. When this is not the case, diagonalizeation of the measurement variance-covariance matrix and a transformation that is a linear combination of the original measurements into a new measurement set is performed.

1 Methods

1.1 Best References

The book by Brown and Hwang [1] provides excellent reference information for these method (refer to pages 250-252 for the method description, problem 6.2 on page 283 introduces measurement variance-covariance diagonalization, and pages 367-370 discuss U-D factorization).

The book by Grewel and Andrews [2] discusses decorrelated measurement noise (pages 221-225). Table 6.6 of this book provides a good summarized procedure for measurement decorrelation.

1.2 Diagonalization Example

Problem 6.2 [1]:

"Consider the measurment to be a 2-tuple $[z_1, z_2]^T$, and assume that the measurment errors are correlated such that the $\mathbf{R}$ matrix is of the form

$$\mathbf{R} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \tag{1}$$

(a) Form a new measurment pair, z_1' and z_2', as a linear comination of the orginal pair such that the errors in the new pair are uncorrelated. (Hint: First, let $z_1' = z_1$ and then assume $z_2' = c_1 z_1 + c_2 z_2$ and choose the constants c_1 and c_2 such that the new measurement errors are uncorrelated.)" [1pp.283]

Answer:

(a) Alternatively let $z_2' = z_2$ and assume $z_1' = c_1 z_1 + c_2 z_2$:

$$z' = \begin{bmatrix} c_1 z_1 + c_2 z_2 \\ z_2 \end{bmatrix} \tag{2}$$

Using variance-covariance propagation law:

$$\begin{gathered} y = f(z) \\ C_y = \tfrac{\delta f}{\delta z} C_z \tfrac{\delta f}{\delta z}^T \end{gathered} \tag{3}$$

Thus $\frac{\delta f}{\delta z}$ is:

$$\frac{\delta f}{\delta z} = \begin{bmatrix} c_1 & c_2 \\ 0 & 1 \end{bmatrix} \tag{4}$$

Thus $R' = \frac{\delta f}{\delta z} R \frac{\delta f}{\delta z}^T$ is:

$$\begin{aligned} R' &= \begin{bmatrix} c_1 & c_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} c_1 & 0 \\ c_2 & 1 \end{bmatrix} \\ &= \begin{bmatrix} c_1 r_{11} + c_2 r_{12} & c_1 r_{12} + c_2 r_{22} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} c_1 & 0 \\ c_2 & 1 \end{bmatrix} \end{aligned} \tag{5}$$

Results in:

$$R' = \begin{bmatrix} c_1^2 r_{11} + 2c_1c_2r_{12} + c_2^2 r_{22} & c_1r_{11} + c_2r_{22} \\ c_1r_{11} + c_2r_{22} & r_{22} \end{bmatrix} \tag{6}$$

Choose $c_1 = 1$ and thus $c2 = -r_{11}/r_{22}$ to make the 'new' measurements uncorrelated. This result in:

$$R' = \begin{bmatrix} r_{11} - 2\frac{r_{11}}{r_{22}}r_{12} + \frac{r_{11}^2}{r_{22}} & 0 \\ 0 & r_{22} \end{bmatrix} \tag{7}$$

1.3 Measurement Decorrelation Procedure

> "The vector-valued measurement $z = Hx + v$, with correlated components of the measurement error $E[vV^T] = R$, is transformed to the measurement $z' = H'x + v'$ with uncorrelated components of the measurment error, $v'(E[v'v'^T] = D$, a diagonal matrix), by overwriting H with $H' = U^{-1}H$ and $zwithz' = U^{-1}z$, after decomposing R to UDU^T, overwriting the diagonal of R with D."[2.pp.224]

2 References

1. Brown, R. G., P. Y. C Hwang (1997). Introduction to Random Signals and Applied Kalman Filtering. Third Edition. John Wiley and Sons Inc. pp. 250-252, 283, 367-370.

2. Grewal, M. S., A. P. Andrews (2001). Kalman Filtering Theory and Practice Using Matlab. Second Edition. John Wiley and Sons Inc.pp. 221-225.