Estimation Design Document

1 Pseudorange Measurements

1.1 Physical Model

Let a single pseudorange be described by Equation 1.

$$
\begin{aligned}
p &= \sqrt{(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2} + ct_u \\
p &= \rho + ct_u
\end{aligned}
\tag{1}
$$

where:
p is the pseudorange observation [m]
(x, y, z) are user unknown coordinates [m]
t_u is unknown user clock bias [s], c is the speed of light,
(x_s, y_s, z_s) are satellite coordinates [m], and
ρ is the geometric range ($\sqrt{(x_s - x)^2 + (y_s - y)^2 + (z_s - z)^2}$).

The partial derivatives of p with respect to (x, y, z, ct_u) evaluate in Equation 2.

$$
\begin{aligned}
&\tfrac{\partial p}{\partial x} = -\tfrac{x_s - x}{\rho} = -a_x \\
&\tfrac{\partial p}{\partial y} = -\tfrac{y_s - y}{\rho} = -a_y \\
&\tfrac{\partial p}{\partial z} = -\tfrac{z_s - z}{\rho} = -a_z \\
&\tfrac{\partial p}{\partial ct_u} = 1
\end{aligned}
\tag{2}
$$

The terms a_x, a_y, and a_z are the direction cosines of the unit vector pointing from the user position to the satellite. The direction cosine vector is defined by Equation 3.

$$
\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \frac{x_s - x}{\rho} \\ \frac{y_s - y}{\rho} \\ \frac{z_s - z}{\rho} \end{bmatrix}
\tag{3}
$$

Thus, the linearized measurement equation is defined by Equation 4.

$$
\begin{aligned}
p &= p_0 - \tfrac{x_s - x}{\rho}\Delta x - \tfrac{y_s - y}{\rho}\Delta y - \tfrac{z_s - y}{\rho}\Delta z + \Delta ct_u \\
p &= p_0 - a_x\Delta x - a_y\Delta y - a_z\Delta z + \Delta ct_u
\end{aligned}
\tag{4}
$$

where:
p_0 is the approximate pseudorange (based on the best estimate of the user position and user time bias), and $\Delta x, \Delta y, \Delta z, \Delta ct_u$ are the linearized unknowns.

The pseudorange observation misclosure [1] is defined by Equation 5.

$$
v = p - \hat{p} = -a_x\Delta x - a_y\Delta y - a_z\Delta z + \Delta ct_u
\tag{5}
$$

[1] misclosure convention, misclosure = measured - estimated

For the case of four satellites, the equations for each satellite misclosure 'observation' can be put in the following matrix form.

$$\mathbf{v} = H\boldsymbol{\Delta}\mathbf{x}, \quad \mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}, \quad H = \begin{bmatrix} -a_{x1} & -a_{y1} & -a_{z1} & 1 \\ -a_{x2} & -a_{y2} & -a_{z2} & 1 \\ -a_{x3} & -a_{y3} & -a_{z3} & 1 \\ -a_{x4} & -a_{y4} & -a_{z4} & 1 \end{bmatrix}, \quad \boldsymbol{\Delta}\mathbf{x} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta ct_u \end{bmatrix} \tag{6}$$

1.2 Error Model

Let a single pseudorange observation be described by Equation 7.

$$p = \rho + d\rho + ct_u - ct_s + I + T + \varepsilon_p \tag{7}$$

where:
p, pseudorange measurement [m]
ρ, geometric range [m]
$d\rho$, geometric range error due to ephemeris error [m]
ct_u, receiver clock bias [m]
ct_s, satellite clock bias [m], (note that this correction is added to p, as defined in the GPS ICD)
I, ionospheric delay [m]
T, tropospheric delay [m]
ε_p, noise and multipath [m].

In practice, estimates of $d\rho$, ct_u, ct_s, I, and T are computed. Thus, the corrected pseudorange misclosure are best defined by Equation 8.

$$\begin{aligned} &v = p' - \hat{p} \\ &p' = p - d\rho + ct_s - I - T \\ &\hat{p} = \hat{\rho} + c\hat{t}_u \end{aligned} \tag{8}$$

2 Doppler Measurements

2.1 Physical Model

The Doppler shift [2] is defined by Equation 9.

$$\Delta f = f_T - f_R = f_t(\frac{\mathbf{v_s} - \mathbf{u_s} \cdot \mathbf{a}}{c}) \tag{9}$$

where:
f_T is the transmitted frequency [Hz],
f_R is the received frequency [Hz],
$\mathbf{v_s}$ is the velocity of the satellite (ECEF [m/s]),
$\mathbf{v_u}$ is the user velocity (ECEF [m/s]),
$\mathbf{a}$ is the direction cosine vector (the unit vector pointing from the user to the satellite),
and c is the speed of light [m/s].

The user to satellite range rate is represented by the dot product of the direction cosine unit vector (along the line of sight from the user to the satellite) and the relative velocity vector defined by Equation 10

$$\begin{aligned} \dot{\rho} &= \mathbf{v_r} \cdot \mathbf{a} \\ \mathbf{v_r} &= \mathbf{v_s} - \mathbf{v_u} \end{aligned} \tag{10}$$

where:
$\dot{\rho}$ is the geometric range rate [m/s], and
$\mathbf{v_r}$ is the relative velocity vector.

Let a single Doppler observation be described by Equation **??**.

To be completed...

[2] sign convention may vary, $\Delta f = f_T - f_R$ (NovAtel convention, increasing pseudorange means negative Doppler) vs $\Delta f = f_R - f_T$