I. THEORY

A. Engineering of phonon-modes with optical tweezers

We consider a setup of N ions of charge e and mass m in harmonic trap with trapping frequencies ω_α, $\alpha = x, y, z$. The potential energy of the system can be written as:

$$V(\vec{\rho}) = \frac{1}{2}\sum_{\alpha,i} m\omega_\alpha^2 (\rho_\alpha^{(i)})^2 + \frac{1}{2}\sum_{i\neq j} \frac{e^2}{4\pi\epsilon_0 |\vec{\rho}^{(i)} - \vec{\rho}^{(j)}|} \quad (1)$$

with ϵ_0 the vacuum permitivity and $\vec{\rho}^{(i)} = (\rho_x^{(i)}, \rho_y^{(i)}, \rho_z^{(i)})$ the position of ion i. The equilibrium positions $\vec{R}^{(i)}$ are the solutions to $\nabla V = 0$. We obtain the phonon spectrum by expanding the Coulomb interaction term for small deviations from the equilibrium position $\vec{\rho} = \vec{R} + \vec{r}$. This results in the Lagrangian:

$$\begin{aligned}\mathbf{L} =& \frac{m}{2}\Bigg(\sum_i\sum_\alpha \left(\dot{r}_\alpha^{(i)}\right)^2 \\ &-\frac{1}{2}\sum_{i,j}\sum_{\alpha,\beta} r_\alpha^{(i)} r_\beta^{(j)} \left(\frac{d^2V}{r_\alpha^{(i)} r_\beta^{(j)}}\right)_{r_\alpha^{(i)}, r_\beta^{(i)} \to 0}\Bigg) \\ =& \frac{m}{2}\left(\sum_i\sum_\alpha \left(\dot{r}_\alpha^{(i)}\right)^2 - \frac{1}{2}\sum_{i,j}\sum_{\alpha,\beta} r_\alpha^{(i)} r_\beta^{(j)} A_{\alpha,\beta}^{(i,j)}\right) \quad (2)\end{aligned}$$

We find the normalized eigenvectors $\vec{b}_k$ and eigenvalues $\lambda_k = \omega_k^2$ of the modes $k = 1, .., 3N$ by diagonalization of $\mathbf{A}$. The eigenvectors $\vec{b}_k$ describe the amplitude of motion of each ion in each eigenmode and have $3N$ entries. In case of a 1-dimensional ion crystal, the eigenmodes separate in 3 subclasses, corresponding to the directions of motion x, y, z.

We now consider a situation in which particular ions are pinned to their equilibrium position by an additional optical tweezer. This leads to an additional local confinement that we denote with the 3×3 trapfrequency matrices $\mathbf{\Omega}_i^2$ for each ion i. We then write the modified potential energy as:

$$\begin{aligned}V(\vec{\rho}) =& \sum_i\sum_\alpha m\omega_\alpha^2 \left(\rho_\alpha^{(i)}\right)^2 + \frac{1}{2}\sum_i\sum_{\alpha,\beta} m\left(\Omega_{\alpha,\beta}^{(i)}\right)^2 r_\alpha^{(i)} r_\beta^{(i)} \\ &+ \frac{1}{2}\sum_{i\neq j}\frac{e^2}{4\pi\epsilon_0|\vec{\rho}^{(i)} - \vec{\rho}^{(j)}|} \quad (3)\end{aligned}$$

This does not change the equilibrium positions of the ions. The new eigenvalues $\tilde{\lambda}_k$ and eigenvectors $\tilde{b}_k$ can be found by diagonalization of $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{\Omega}^2$. We note that $\mathbf{\Omega}^2$ has N blocks of dimension 3×3 on its diagonal. In the most general case, $5N$ parameters are needed to specify $\mathbf{\Omega}^2$ – 3 independent trapfrequencies and two angles specifying the orientation of the tweezer on each ion. However, in most practical cases fewer parameters would be needed as described below.

B. Effective Ising interaction

These modified phonon modes can be coupled to the electronic state of the individual ions, by using a state-dependent force generated with two non-copropagating bichromatic lasers. The frequency of the laser differs slightly and leads to the state dependent force $F_i = (\Omega/2)e^{-i\mu t}\hat{\sigma}_z^{(i)}$, where μ is the beatnote frequency and Ω the interaction strength of the laser with the ions. The laser-ion interaction Hamiltonian is $\hat{H}_{\text{q-ph}} = \sum_i F_i \exp\left(i\vec{k}\cdot\hat{\vec{r}}^{(i)}\right) + \text{h.c.}$ with $\vec{k}$ the resulting wavevector of the interfering laser fields. By expressing the ion positions in term of the phonon modes of the crystal, we can rewrite this Hamiltonian as follows:

$$\hat{H}_{\text{q-ph}} = \frac{\Omega}{2}\sum_i \left(e^{i\sum_m \eta_m^{(i)}\left(\hat{a}_m^\dagger + \hat{a}_m\right) - i\mu t} + \text{h.c.}\right)\hat{\sigma}_z^{(i)}, \quad (4)$$

where the creation and annihilation operators for the m-th phonon mode are denoted by $\hat{a}_m^\dagger$ and $\hat{a}_m$. The Lamb-Dicke parameter $\eta_m^{(i)}$ is scaled with the motion amplitude of the i-th ion on the m-th phonon mode ($\vec{b}_m^{(i)}$), i.e. $\eta_m^{(i)} = \vec{b}_m^{(i)} \cdot \vec{k}\sqrt{\hbar/(2M\omega_m)}$ with M the ion mass and ω_m the phonon mode frequency.

Add Magnus or dressed picture transformation.

1. *Higher order couplings*

II. NON-NATIVE SPIN-SPIN INTERACTIONS

We are interested on controlling the strength of the couplings to engineer the Ising Hamiltonian of our system. These can be controlled by modifying both the frequency of the laser beatnote and the phonon modes of the crystal using optical tweezers. To obtain a target Ising coupling matrix $\mathbf{J}$, we need the set of tweezer trap frequencies $\{\Omega_n^{(i)}\}$ and beatnote frequency μ which minimizes the error:

$$\epsilon = \left\|\mathbf{J} - \mathbf{J}'(\{\Omega_n^{(i)}\},\ \mu)\right\| \quad (5)$$

where $\mathbf{J}'$ is the resulting coupling matrix of the pinned crystal. We use a gradient descent optimization algorithm to find the optimal values of $\{\Omega_n^{(i)}\}$ and μ. To speed up the convergence of the algorithm, we start with seed values for which the initial error $\epsilon_{\text{seed}} < \epsilon_0$ where ϵ_0 is a threshold error with a typical value of XX. The seed values are drawn randomly and must satisfy $|\mu| < \mu_{\min}$ and $\sum_{n,i}\left(\Omega_n^{(i)}\right)^2 < \Omega^2_{\max}$, the former guaranteeing dispersive coupling between the laser and the phonon modes and the latter considering the maximum laser power available to generate the tweezers.

A. Homogenization of interaction

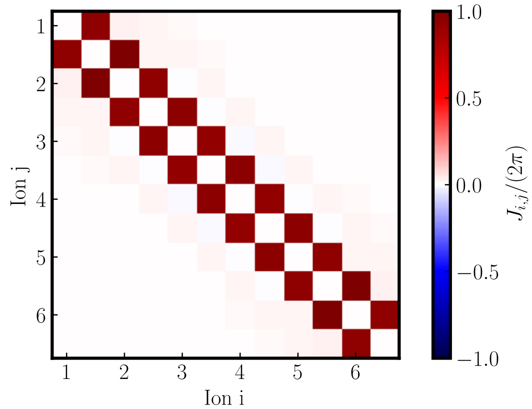

FIG. 1. Near-neighbor homogeneous coupling

B. Control of interaction strength

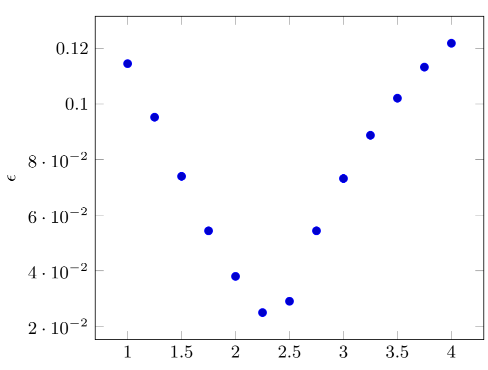

FIG. 2. Error $\epsilon = \frac{\|\mathbf{J}-\mathbf{J}'\|}{\|\mathbf{J}\|}$ of coupling matrix for pinned crystals

Move to intro:A powerful feature of our approach is the capability of controlling the decay strength of power law interactions $1/|\rho|^\alpha$ in a linear ion chain. This will allow quantum dynamical processes such as MBL, Scrambling, ...

We first consider a linear ion crystal with almost homogeneous inter-ion distance. In absence of optical tweezers, the interaction between ions will approximately follow a Coulomb-type decay law $J^{(i,j)} \propto 1/|\rho|$. To obtain dipole-dipole or higher order type of interactions, we include optical traps along one radial direction of the crystal and calculate the resulting interaction on the second radial direction. The resulting coupling matrices after optimization of the tweezer array strength are shown in Fig. 3 with the respective error from the ideal case in Fig. 2

C. Arbitrary connectivity graphs

III. EXPERIMENTAL IMPLEMENTATION

A. ac-Stark shift and off-resonant scattering

Each optical tweezer will introduce a differential ac-Stark shift between the qubit states of the pinned ions and lead off-resonant scattering. Both processes can lead to uncontrollable phase shifts [**2?**] on the state of the system which affect the performance of quantum operations [1]. In the case of $^{171}\mathrm{Yb}^+$, we are concerned about both effects on the qubit states encoded on the hyperfine levels $|\mathrm{F}=0, m_\mathrm{F}=0\rangle \equiv |0\rangle$ and $|\mathrm{F}=1, m_\mathrm{F}=0\rangle \equiv |1\rangle$ of the $^2\mathrm{S}_{1/2}$ ground state. For a strongly confining, red-detuned tweezer ($\Omega_\mathrm{p} = 1.34$ MHz, $\lambda = 1024$ nm), the scattering rate is $\ll 1$ Hz (Fig. 4(a)) meaning that for quantum operations faster than 1 s the effect of off-resonant scattering is negligible.

The main contribution to the differential Stark shift arises from off-resonant dipole couplings between the qubit states and hyperfine levels of the $^2\mathrm{P}_{1/2}$ and $^2\mathrm{P}_{3/2}$ states. Due to the large detuning of the tweezer and the small hyperfine splittings, at the highest power Δ_AC 2 kHz (Fig. 4(b)). This will limit the fidelity of any operation, in particular for process with durations of few microseconds or longer. For certain ionic species as Ca^+, the Stark shift can be eliminated by using tweezers operating at a magic wavelength or polarizability, however for the ground state qubit states of $^{171}\mathrm{Yb}^{+1}$ none of them exist. An alternative is to use pairs of blue and red detuned tweezers such that the combined Stark shift becomes zero. In Fig. 5 we show an example using tweezers operating at 532 and 224 nm.

[1] Erhard, A., Wallman, J. J., Postler, L., Meth, M., Stricker, R., Martinez, E. A., Schindler, P., Monz, T., Emerson, J., and Blatt, R. (2019). Characterizing large-scale quantum computers via cycle benchmarking. *Nature Communica-*

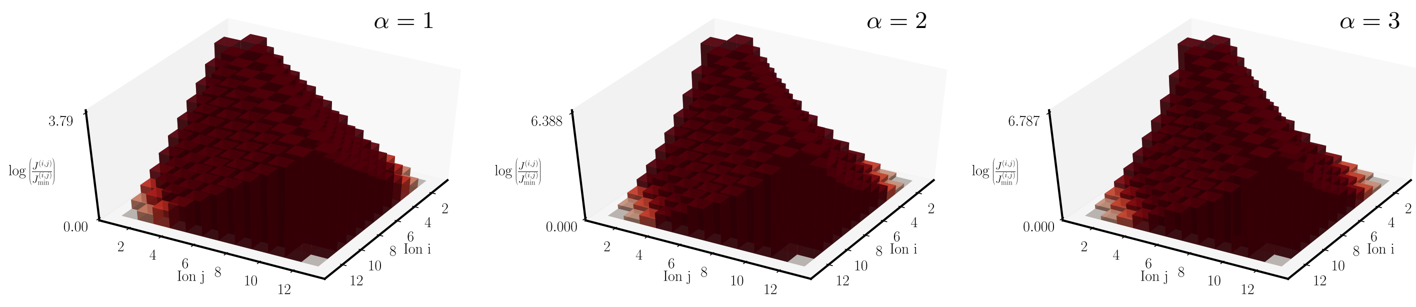

FIG. 3. Power-law coupling $r^{-\alpha}$

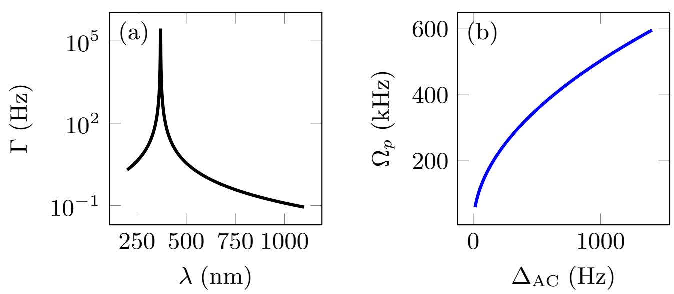

FIG. 4. (a) Scattering rate of Yb^{+1}, (b) Stark shifts and pinning frequencies from a tightly focused tweezer ($w_0 = 2$ μm) at 1070 nm for a laser power of (a) 1 W and (b) 10 to 1000 mW and linear polarization

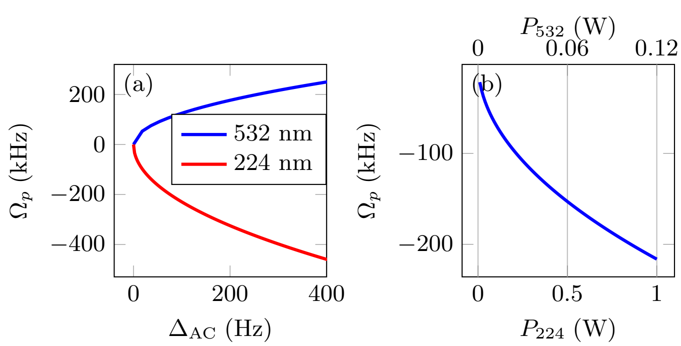

FIG. 5. (a) Stark shifts and pinning frequencies from a tightly focused tweezer ($w_0 = 2$ μm) at 248 and 532 nm for a laser power of 1 W and (b) combined pinning frequency for tweezer pair with zero ac-Stark shift.

tions, 10(1):5347.

[2] Uys, H., Biercuk, M. J., VanDevender, A. P., Ospelkaus, C., Meiser, D., Ozeri, R., and Bollinger, J. J. (2010). Decoherence due to Elastic Rayleigh Scattering. *Physical Review Letters*, 105(20):200401.