Homework #1

ME 2256/MEMS 1256 - Applications of Computational Heat and Mass Transfer

Assigned February 4^{th}, 2020
Due February 24^{th}, 2021

Problem #1

1. Consider one-dimensional, steady, source-free thermal diffusion through an insulated rod whose left and right ends are maintained at 300 and 750 [K], respectively. The half a meter- long rod is fabricated out of a composite material with an anisotropic thermal conductivity given by the following expression:

$$\lambda(x) = 3\mathrm{e}^{-6x}\ [\mathrm{W/m\text{-}K}]$$

with x=0 defined as the left end of the rod. The rod's cross-section is square in shape and has an area of 1 [cm] by 1 [cm].

   (a) Analytically determine the temperature distribution in the rod. Plot the temperature distribution as a function of rod length.
   (b) Using the finite-difference approach developed in class, modify your code to compute the temperature distribution in the rod. Using the explicit solver and 10 nodes, plot the results of the finite-difference solution after 1, 2, 5, and 10 sweeps against the analytical solution determined in (a).
   (c) Using the finite-volume approach developed in class, modify your code to compute the temperature distribution in the rod. Using 10 control volumes, evaluate the temperature using:
      i. Linearly interpolated interface thermal conductivity
      ii. Harmonic averaged interface thermal conductivity

   Plot the temperature and heat flux distributions in the rod for both cases against their corresponding analytically determined functions.
   (d) Calculate the temperature distribution using 10, 20, 40, 80, and 160 control volumes (using harmonic interface thermal conductivity) and plot each of the computed temperature distributions against the analytic solution.

2. Consider one-dimensional, steady, source-free thermal diffusion through an insulated conical rod. The temperature of the left face is maintained at a constant 300 [K] while the right face is subjected to a uniform heat flux of 500 [W/m^2]. The length of the conical rod is 10 [cm] and the radius of the cone is given by the following expression:

$$r(x) = 0.5x + 0.1\ [\mathrm{cm}]$$

The thermal conductivity of the rod is a constant 20 [W/m-K].

   (a) Analytically determine the temperature distribution in the rod. Plot the temperature distribution as a function of rod length.
   (b) Using 5 control volumes, set up the system of discretized equations in the form:

$$\mathbf{Ax} = \mathbf{b}$$

   Print out your **A** matrix and **b** vector.
   (c) Evaluate the temperature distribution in the rod using 5, 10, 20, 40, and 80 control volumes. Plot each solution against the analytic temperature distribution.

(d) For each of the five numerical solutions, compute the error at each control volume as:

$$\mathbf{e} = T_{\text{analytic}} - T_{\text{numeric}}$$

Compute the L2 norm of these five vectors using the following formula

$$l^2 \text{ of } \mathbf{e} = \|\mathbf{e}\|_2 = \sqrt{\sum_{i=1}^{n} |e_i|^2}$$

(e) On a log-log plot, plot the L2 norm of the five error vectors versus the control volume size (Δx). Report the slope of the line.