Chapter 8 - The Complete Response of RL and RC Circuits

Lecture 28
Sections 8.6 & 8.7

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Student Learning Objectives

At the end of the lecture, students should be able to:

- Solve for the response of RL and RC circuits with step-function sources

Step Source

- Consider the voltage output of the system taking the form

$$v(t) = \begin{cases} 1 & t \leq t_0 \\ 1 & t > t_0 \end{cases}$$

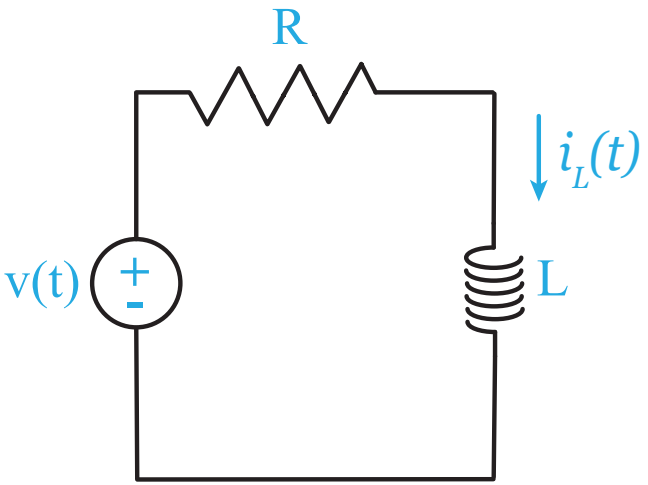
- We will call $v(t)$=$u(t - t_0)$, i.e. the unit step forcing function
- This allows us to define the voltage as

$$v(t) = V_0\, u(t - t_0)$$

- V_0 is the voltage at t_0

Step Source RL

- Consider the RL circuit



- Applying KVL

$$v = iR + L\frac{di}{dt}$$

- Separating terms

$$\frac{di}{dt} - \frac{-iR + v}{L}$$

- Grouping i and t

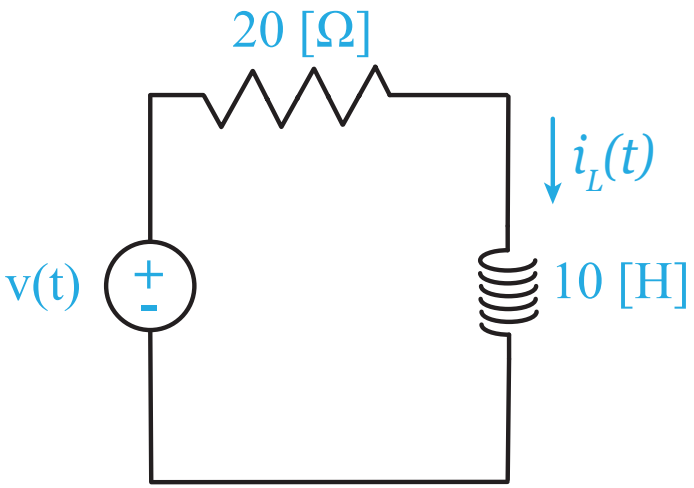
$$\frac{di}{i - (v/R)} = \frac{-R}{L}dt$$

- Integrating

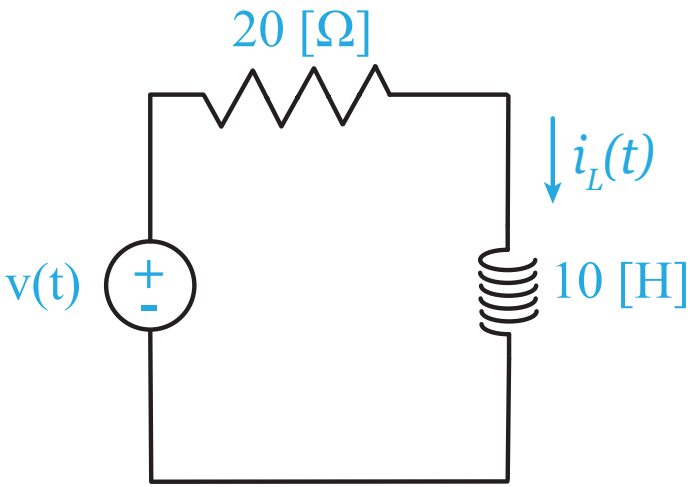
$$\frac{i(t) - (v/R)}{i_0 - (v/R)} = \mathrm{e}^{-\frac{t}{\tau}} \implies i(t) = \frac{v}{R} + \left(i_0 - \frac{v}{R}\right)\mathrm{e}^{-\frac{t}{\tau}}$$

Example #1

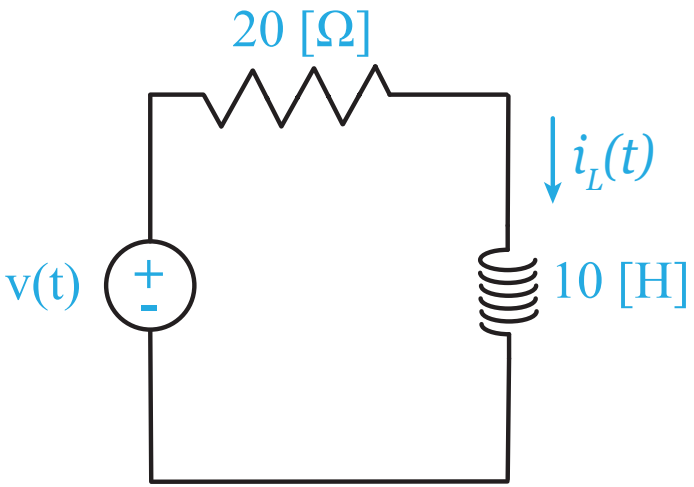
- If $v(t)$=4-8$u(t)$ [V], determine the current through the inductor as a function of time



Example #1

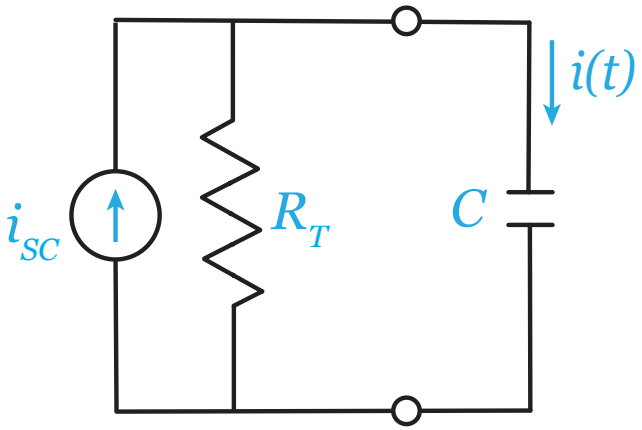


Example #1



Step Source RC

- Consider the RC circuit



- Applying KCL

$$i_{sc} = \frac{v}{R} + C\frac{dv}{dt}$$

- Dividing by C

$$\frac{dv}{dt} + \frac{v}{RC} = \frac{i_{sc}}{C}$$

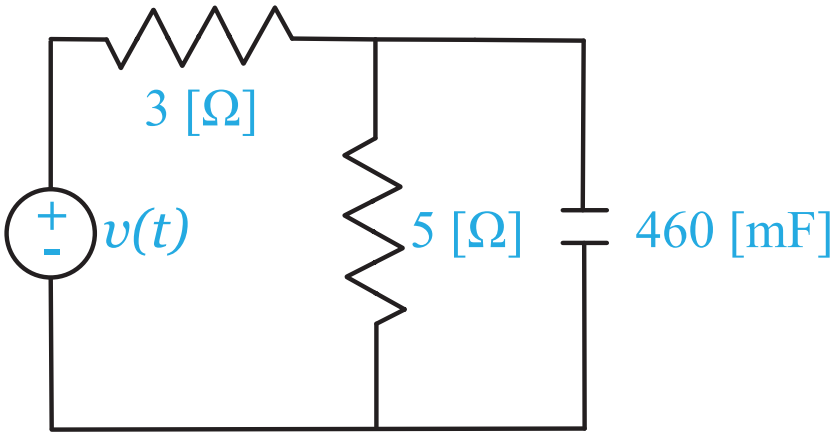
- This solution is of the same form as we saw with the RL circuit, thus

$$v(t) = i_{sc}R + (v_0 - i_{sc}R)\mathrm{e}^{-\frac{t}{\tau}}$$

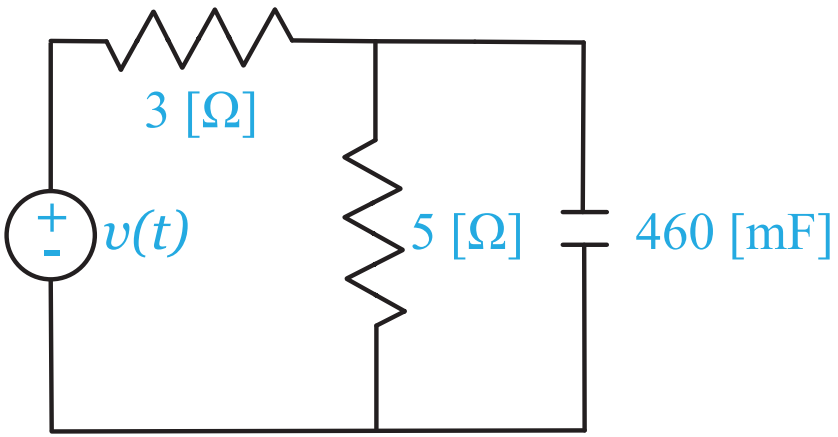
- Thus, we have the same solution methodology for RC circuits as RL

Example #2

- If $v(t)$=7-14$u(t)$ [V], determine the voltage across the capacitor as a function of time



Example #2



General Form

- The reader is referred to page 335 of the text; we shall not cover this.

Student Learning Objectives

At the end of the lecture, students should be able to:

- Solve for the response of RL and RC circuits with step-function sources
  - The unit step-function allows us to solve for the current in an RL circuit and voltages in a RC circuit, which differed from our conventional representation. This method is comparable to sequential switching with constant sources.

Suggested Problems

- 8.6-1, 8.6-2, 8.6-5, 8.6-6, 8.6-9, 8.7-1, 8.7-3, 8.7-5