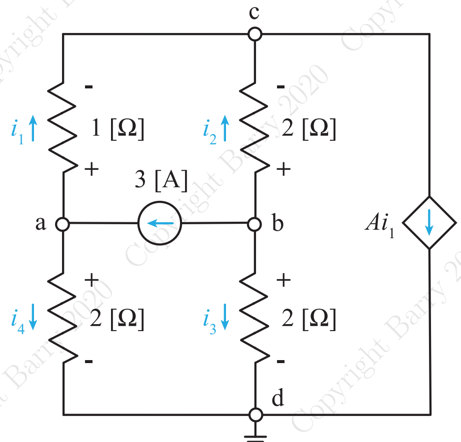
Homework #4

MEMS 0031 - Electrical Circuits

Assigned: May 28$^{\text{th}}$, 2020
Due: June 3$^{\text{rd}}$, 2020 at 11:59 pm

Problem #1

Using Node Voltage Analysis (NVA), determine the currents i_1 and i_2, and the voltages at nodes a, b and c, given $A = 4$. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 4$ and we define i_3 and i_4 as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N-1$ KCL equations, applied at non-zero nodes:

KCL at node a:

$$i_1 + i_4 = 3\,[\text{A}]$$

KCL at node b:

$$i_2 + i_3 = -3\,[\text{A}]$$

KCL at node c:

$$i_1 + i_2 = 4i_1 \implies -3i_1 + i_2 = 0$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at node a:

$$\frac{V_a - V_c}{1\,[\Omega]} + \frac{V_a}{2\,[\Omega]} = 3\,[\text{A}] \implies 3V_a - 2V_c = 6\,[\text{A}] \tag{1}$$

KCL at node b:

$$\frac{V_b - V_c}{2\,[\Omega]} + \frac{V_b}{2\,[\Omega]} = -3\,[\text{A}] \implies 2V_b - V_c = -6\,[\text{A}] \tag{2}$$

KCL at node c:

$$-3\left(\frac{V_a - V_c}{1\,[\Omega]}\right) + \frac{V_b - V_c}{2\,[\Omega]} = 0 \implies -6V_a + V_b + 5V_b = 0 \tag{3}$$

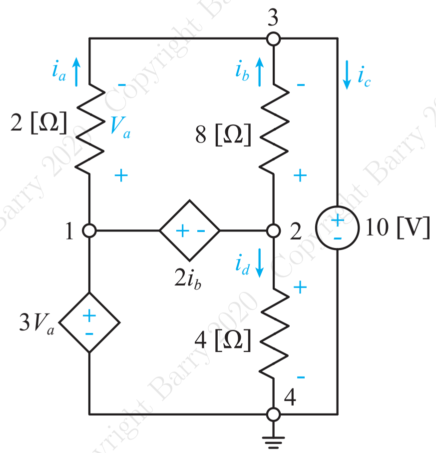
Putting eqns. 1-3 in matrix form:

$$\begin{bmatrix} 3 & 0 & -2 \\ 0 & 2 & -1 \\ -6 & 1 & 5 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 8.\overline{6} \\ 2 \\ 10 \end{bmatrix}$$

Units are taken as [V]. The calculation of the currents is provided in the MATLAB script.

Problem #2

Using Node Voltage Analysis (NVA), determine the currents i_b and i_c, and the voltages potential V_a. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 4$ and we define i_a through i_d as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1-\#$VS KCL equations: We need zero KCL equations. The voltages sources provide enough information to solve the problem. We will apply Step 4 as we formulate our VS expressions in terms of node voltages:

VCCS:

$$V_1 = 3V_a \implies V_1 = 3(V_1 - V_3) \implies 2V_1 - 3V_3 = 0 \tag{4}$$

CCVS:

$$V_1 - V_2 = 2i_b \implies V_1 - V_2 = 2\left(\frac{V_2 - V_3}{8\,[\Omega]}\right) \implies 4V_1 - 5V_2 + V_3 = 0 \tag{5}$$

Independent VS:

$$V_3 = 10\,[\mathrm{V}] \tag{6}$$

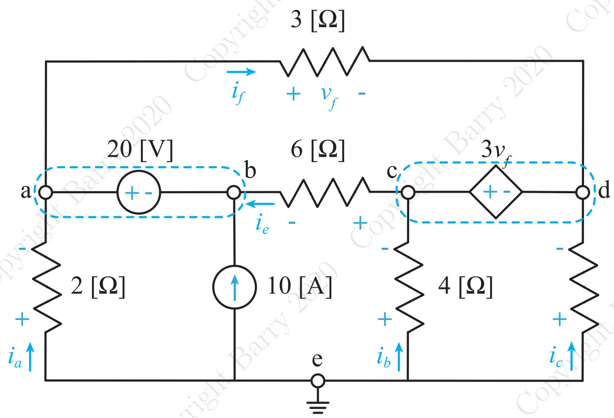
Putting eqns. 4-6 in matrix form:

$$\begin{bmatrix} 2 & 0 & -3 \\ 4 & -5 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \implies \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 14 \\ 10 \end{bmatrix}$$

Units are taken as [V]. The calculation of the currents is provided in the MATLAB script.

Problem #3

Using Node Voltage Analysis (NVA), determine the currents i_a through i_g. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 5$ and we have our branch currents already defined as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 - \#\text{VS}$ KCL equations, applied at non-zero nodes:

Left supernode equation:

$$V_a - V_b = 20 \tag{7}$$

Right supernode equation:

$$V_c - V_d = 3V_f \implies V_c - V_d - 3(V_a - V_d) = 0 \implies 3V_a - V_c - 2V_d = 0 \tag{8}$$

KCL at left supernode:

$$i_a + 10\,[\text{A}] + i_e - i_f = 0$$

KCl at right supernode:

$$i_b + i_c + i_f - i_e = 0$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at left supernode:

$$\frac{0 - V_a}{2\,[\Omega]} + \frac{V_c - V_b}{6\,[\Omega]} - \frac{V_a - V_d}{3\,[\Omega]} = -10\,[\text{A}] \tag{9}$$

KCL at right supernode:

$$\frac{0 - V_c}{4\,[\Omega]} + \frac{0 - V_d}{1\,[\Omega]} + \frac{V_a - V_d}{3\,[\Omega]} - \frac{V_c - V_b}{6\,[\Omega]} = 0 \tag{10}$$

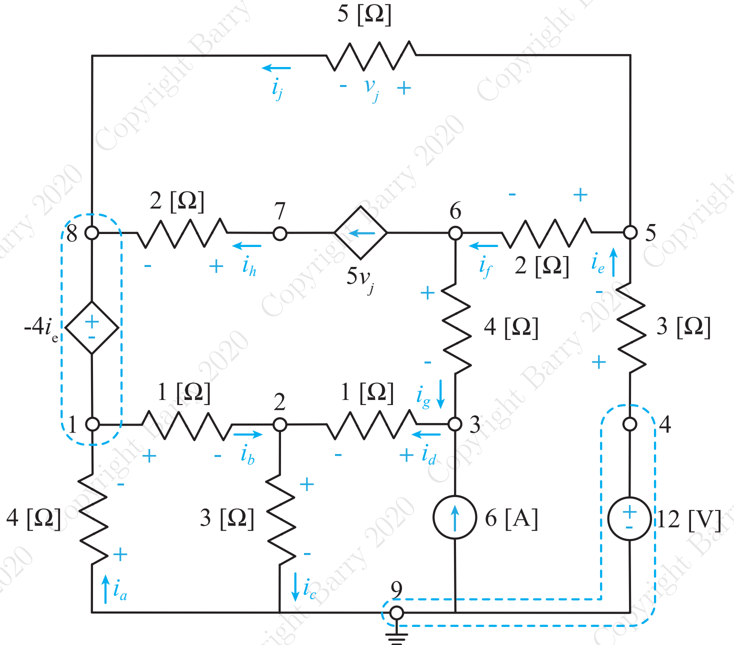
Putting eqns. 7-10 in matrix form:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 3 & 0 & -1 & -2 \\ -5/6 & -1/6 & 1/6 & 1/3 \\ 1/3 & 1/6 & -5/12 & --4/3 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ -10 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 26.6667 \\ 6.6667 \\ 173.3333 \\ -46.6667 \end{bmatrix}$$

Units are taken as [V]. The calculation of the currents is provided in the MATLAB script.

Problem #4

Using Node Voltage Analysis (NVA), determine the currents i_a through i_j. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 9$ and we have our branch currents already defined as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 - \#\text{VS}$ KCL equations, applied at non-zero nodes:

Supernode:

$$V_8 - V_1 = -4i_e$$

Independent VS:

$$V_4 = 12\,[\text{V}]$$

KCL at supernode:

$$i_a - i_b + i_h + i_j = 0$$

KCL at node 2:

$$i_b - i_c + i_d = 0$$

KCL at node 3:

$$-i_d + i_g = -6\,[\text{A}]$$

KCL at node 5:

$$i_e - i_f - i_j = 0$$

KCL at node 6:

$$i_f - i_g - 5v_j = 0$$

KCL at node 7:

$$5v_j - i_h = 0$$

Step 4: Apply Ohm's law in terms of node voltages:

Supernode:

$$V_8 - V_1 = -4\left(\frac{V_4 - V_5}{3}\right) \implies 3V_1 - 4V_4 + 4V_5 - 3V_8 = 0 \tag{11}$$

Independent VS:

$$V_4 = 12 \tag{12}$$

KCL at supernode:

$$\frac{0 - V_1}{4\,[\Omega]} - \frac{V_1 - V_2}{1\,[\Omega]} + \frac{V_7 - V_8}{2\,[\Omega]} + \frac{V_5 - V_8}{5\,[\Omega]} = 0 \implies -25V_1 + 20V_2 + 4V_5 + 10V_7 - 14V_8 = 0 \tag{13}$$

KCL at node 2:

$$\frac{V_1 - V_2}{1\,[\Omega]} - \frac{V_2}{3\,[\Omega]} + \frac{V_3 - V_2}{1\,[\Omega]} = 0 \implies 3V_1 - 7V_2 + 3V_3 = 0 \tag{14}$$

KCL at node 3:

$$-\frac{V_3 - V_2}{1\,[\Omega]} + \frac{V_6 - V_3}{4\,[\Omega]} = -6\,[\text{A}] \implies 4V_2 - 5V_3 + V_6 = -24 \tag{15}$$

KCL at node 5:

$$\frac{V_4 - V_5}{3\,[\Omega]} - \frac{V_5 - V_6}{2\,[\Omega]} - \frac{V_5 - V_8}{5\,[\Omega]} = 0 \implies 10V_4 - 31V_5 + 15V_6 + 6V_8 = 0 \tag{16}$$

KCL at node 6:

$$\frac{V_5 - V_6}{2\,[\Omega]} - \frac{V_6 - V_3}{4\,[\Omega]} - 5(V_5 - V_8) = 0 \implies V_3 - 18V_5 - 3V_6 + 20V_8 = 0 \tag{17}$$

KCL at node 7:

$$5(V_5 - V_8) - \frac{V_7 - V_8}{2\,[\Omega]} = 0 \implies 10V_5 - V_7 - 9V_8 = 0 \tag{18}$$

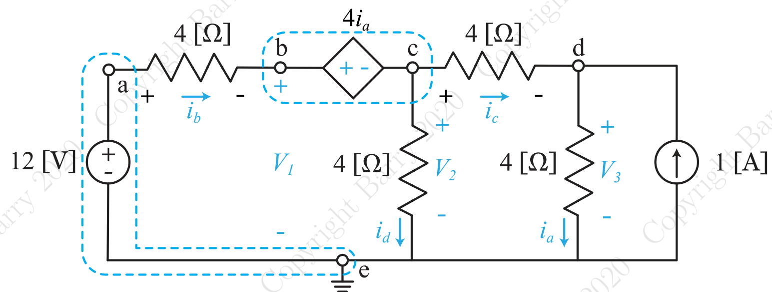
Putting eqns. 11-18 in matrix form:

$$\begin{bmatrix} 3 & 0 & 0 & -4 & 4 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -25 & 20 & 0 & 0 & 4 & 0 & 10 & -14 \\ 3 & -7 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & -5 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 10 & -31 & 15 & 0 & 6 \\ 0 & 0 & 1 & 0 & -18 & -3 & 0 & 20 \\ 0 & 0 & 0 & 0 & 10 & 0 & -1 & -9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ 0 \\ 0 \\ -24 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Units are taken as [V]. The calculation of the currents is provided in the MATLAB script.

Problem #5

Using Node Voltage Analysis (NVA), determine the voltage potentials V_1 through V_3. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 5$ and we have our branch currents already defined as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 - \#\text{VS}$ KCL equations, applied at non-zero nodes:

Independent VS:

$$V_a = 12\,[\text{V}] \tag{19}$$

Supernode:

$$V_b - V_c = 4i_a$$

KCL at supernode:

$$i_b - i_c - i_d = 0$$

KCL at node d:

$$i_c - i_a = -1\,[\text{A}]$$

Step 4: Apply Ohm's law in terms of node voltages:

Supernode:

$$V_b - V_c - 4\left(\frac{V_d}{4}\right) = 0 \implies V_b - V_c - V_d = 0 \tag{20}$$

KCL at supernode:

$$\frac{V_a - V_b}{4\,[\Omega]} - \frac{V_c - V_d}{4\,[\Omega]} - \frac{V_c}{4\,[\Omega]} = 0 \implies V_a - V_b - 2V_c + V_d = 0 \tag{21}$$

KCL at node d:

$$\frac{V_c - V_d}{4\,[\Omega]} - \frac{V_d}{4\,[\Omega]} = -1\,[\text{A}] \implies V_c - 2V_d = -4 \tag{22}$$

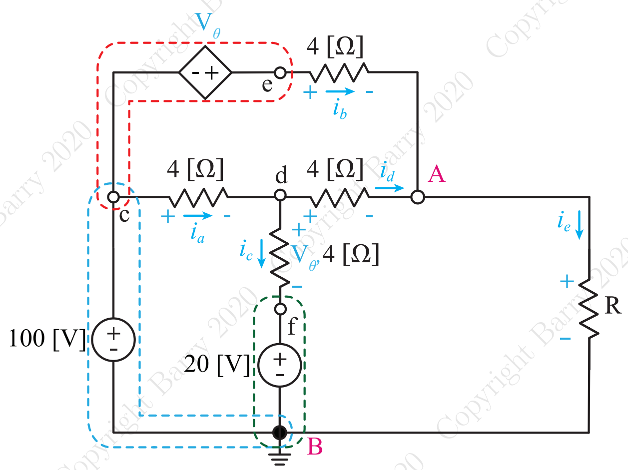
Putting eqns. 19-22 in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 \\ 1 & -1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \\ -4 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} 12 \\ 8 \\ 4 \\ 4 \end{bmatrix}$$

Units are taken as [V]. The calculation of the currents is provided in the MATLAB script.

Problem #6

Using Node Voltage Analysis (NVA), determine the voltage potential V_θ given R= 6 [Ω]. Note: if you use any other method than NVA to determine branch currents, i.e. currents through the resistors, your answer will be marked incorrect.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 6$ and we have our branch currents already defined as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes. Note: the combination of the 100 [V] source and VCVS consolidates to one VS equation, and thus we will apply KCL at two non-source nodes.

Independent 100 [V] VS:

$$V_c = 100\,[\mathrm{V}] \tag{23}$$

Independent 20 [V] VS:

$$V_f = 20\,[\mathrm{V}] \tag{24}$$

VCVS:

$$V_e - V_c = V_\theta$$

KCL at node d:

$$i_a - i_c - i_d = 0$$

KCL at node a:

$$i_b + i_d - i_e = 0$$

Step 4: Apply Ohm's law in terms of node voltages:

VCVS:

$$V_e - V_c = V_d - V_f \implies V_c + V_d - V_e - V_f = 0 \tag{25}$$

KCL at node d:

$$\frac{V_c - V_d}{4\,[\Omega]} - \frac{V_d - V_f}{4\,[\Omega]} - \frac{V_d - V_a}{4\,[\Omega]} = 0 \implies V_a + V_c - 3V_d + V_f = 0 \tag{26}$$

KCL at node a:

$$\frac{V_e - V_a}{4\,[\Omega]} + \frac{V_d - V_a}{4\,[\Omega]} - \frac{V_a}{6\,[\Omega]} = 0 \implies -8V_a + 3V_d + 3V_e = 0 \tag{27}$$

Putting eqns. 23-27 in matrix form:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & -3 & 0 & 1 \\ -8 & 0 & 3 & 3 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix} = \begin{bmatrix} 100 \\ 20 \\ 0 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix} = [$$