# Midterm #2

MEMS 0031 - Electrical Circuits
Spring 2021

Assigned: August $3^{\text{rd}}$, 2022
Due: August $11^{\text{th}}$, 2022, 11:59 pm via Gradescope

**Rules**

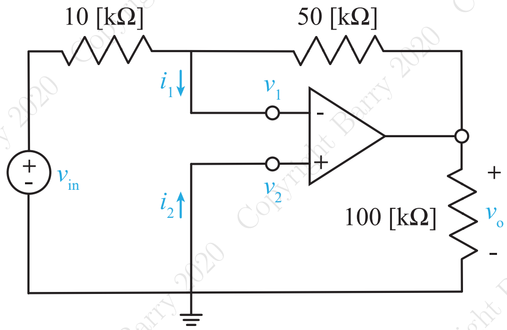
The following listed rules, in addition, but not limited to, those listed on the syllabus, those listed on page 11, and those outlined by Pitt's Academic Integrity Policy, apply to this examination:

1. This test is open notes, open book, open lecture videos, open homework and homework solutions, open quiz and quiz solutions, and you are able to reference previous assessment materials posted on GitHub;
2. You are to complete all **3** problems.
3. You can direct general questions to Dr. Barry via email. A general question constitutes a point of clarification with a question, for example "Why is electrical circuits the coolest subject ever?" Specific questions about how to solve a problem, pertinent equations, related to general guidance, etc., are *not* permitted;
4. You are *not* to communicate with *any* other student about this exam. Period;
5. You are *not* to use any online resources, such as Chegg, Quora, etc.. Seeking external assistance in the form of posting this exam, posting pictures/screenshots/images of this exam, posting questions from this exam, asking questions pertaining to the problems within the exam, etc., are in direction violation of the Academic Integrity policy and will result in immediate failure of this exam;
6. Unsubstantiated results will be marked incorrect. A result where the mathematics do not substantiate the final result will be marked wrong;
7. The work for your exam must be submitted on the exam sheets. Failure to do so will result in your exam not being graded. Your work must be neat, legible, and follow a clear and logical progression. Your final answer must be boxed with proper units. As outlined in the course syllabus, an answer without units will be marked incorrect;
8. You must complete the Academic Integrity Statement and include it with your exam submission for your exam to be graded.

## Problem #1

(40 pts.) Given a TL051C operational amplifier as shown in the figure directly below, determine the ratio of the voltage output to input via the following methods:

a) Treating the op-amp as ideal;

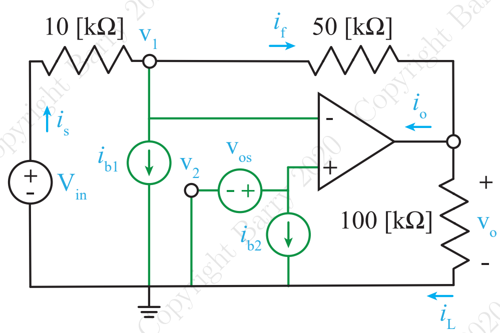



Constructing KCL at the node associated with the inverting terminal:

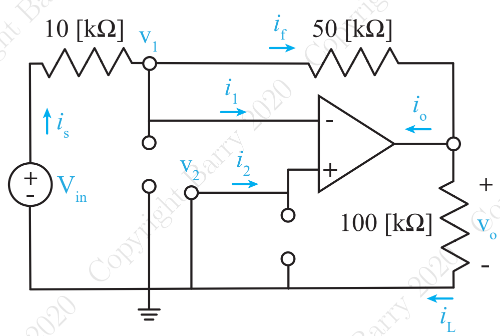
$$\frac{V_{\text{in}} - \cancelto{0}{V_1}}{10\,[\text{k}\Omega]} = \cancelto{0}{i_1} + \frac{\cancelto{0}{V_1} - V_{\text{o}}}{50\,[\text{k}\Omega]} \implies \frac{V_{\text{o}}}{V_{\text{in}}} = -5$$

b) Modeling the op-amp using the Offset Model;

Modeling the op-amp using the Offset Model, we have the following depiction:

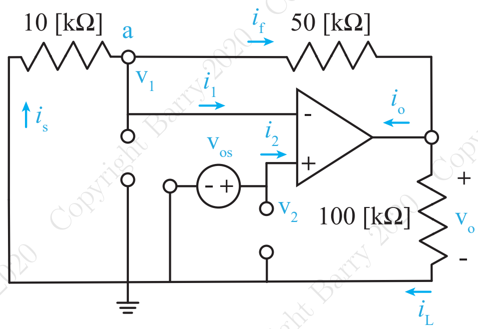



We have four independent sources. We can proceed by using superposition (short voltage, open current sources). Starting with $V_{\text{in}}$, we recognize this is the same configuration as an ideal op-amp:




$$\implies \frac{V_{\text{o}}}{V_{\text{in}}} = -5$$

For $V_{\text{os}}$, we recognize this is a non-inverting op-amp:




$$i_1 = i_2 = 0; \quad V_1 = V_2 = V_{\text{os}}$$

Applying KCL at a:

$$i_s = \cancelto{0}{i_1} + i_f$$

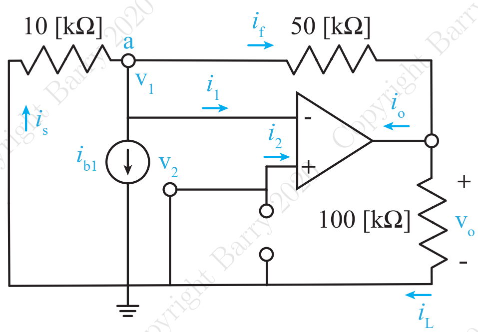
Applying Ohm's law:

$$\frac{0 - V_1}{10\,[\text{k}\Omega]} = \frac{V_1 - V_{\text{o}}}{50\,[\text{k}\Omega]}$$

$$\frac{V_{\text{o}}}{50\,[\text{k}\Omega]} = \frac{V_1}{50\,[\text{k}\Omega]} + \frac{V_1}{10\,[\text{k}\Omega]}$$

$$\implies V_{\text{o}} = 6V_{\text{os}}$$

Now considering $i_{\text{b1}}$:




Applying KCL at a:

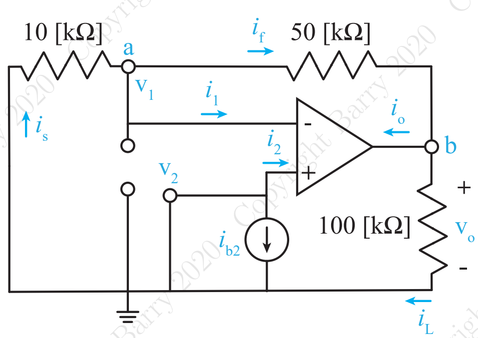
$$i_s = \cancelto{0}{i_1} + i_{\text{b1}} + i_f$$

Applying Ohm's law:

$$\frac{0 - \cancelto{0}{V_1}}{10\,[\text{k}\Omega]} = i_{\text{b1}} + \cancelto{0}{i_1} + \frac{\cancelto{0}{V_1} - V_{\text{o}}}{50\,[\text{k}\Omega]}$$

$$\implies V_o = i_{\text{b1}}(50\,[\text{k}\Omega])$$

Now considering $i_{\text{b2}}$:



Applying KCL at a:

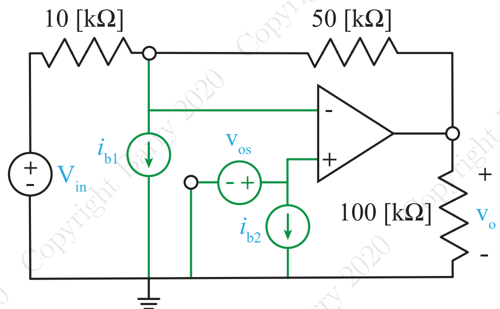
$$i_s = \cancelto{0}{i_1} + i_f$$

Applying Ohm's law:

$$\frac{0 - \cancelto{0}{V_1}}{10\,[\mathrm{k\Omega}]} = \cancelto{0}{i_1} + \frac{\cancelto{0}{V_1} - V_\mathrm{o}}{50\,[\mathrm{k\Omega}]}$$

$$\implies V_\mathrm{o} = 0$$

Summing terms:



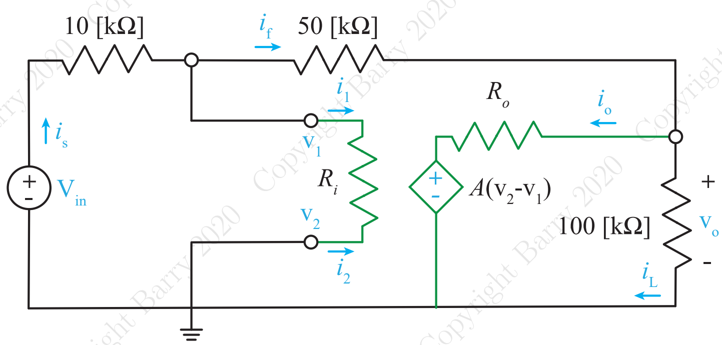

$$V_\mathrm{o} = -5V_\mathrm{in} + 6V_\mathrm{os} + i_\mathrm{b1}(50\,[\mathrm{k\Omega}])$$

Given a TL051C op-amp (consult Table. 6.7-1 on page 228), the predicted output, is:

$$V_\mathrm{o} = -5V_\mathrm{in} + 6(0.59\,[\mathrm{mV}]) + (0.03\,[\mathrm{nA}])(50\,[\mathrm{k\Omega}]) = -5V_\mathrm{in} + 0.003\,541\,5\,[\mathrm{V}]$$

c) Modeling the op-amp using the Finite Gain Model;

We wills start by applying KCL node a:




$$i_s = i_1 + i_f$$

Applying Ohm's law:

$$\frac{V_\mathrm{in} - V_1}{10\,[\mathrm{k\Omega}]} = \frac{V_1 - V_2}{R_i} + \frac{V_1 - V_\mathrm{o}}{50\,[\mathrm{k\Omega}]}$$

$$\implies V_\mathrm{o} = (50\,[\mathrm{k\Omega}])\left(-\frac{V_\mathrm{in}}{10\,[\mathrm{k\Omega}]} + V_1\left(\frac{1}{R_i} + \frac{1}{10\,[\mathrm{k\Omega}]} + \frac{1}{50\,[\mathrm{k\Omega}]}\right) + \cancelto{0}{V_2}\left(\frac{1}{R_i}\right)\right) \implies V_\mathrm{o} = -5V_\mathrm{in} + 6.000\,000\,05V_1$$

To solve for $V_1$, apply KCL to the right node:

$$i_f = i_o + i_L$$

Applying Ohm's law:

$$\frac{V_1 - V_\mathrm{o}}{50\,[\mathrm{k\Omega}]} = \frac{V_o - A(\cancelto{0}{V_2} - V_1)}{R_o} + \frac{V_o}{100\,[\mathrm{k\Omega}]}$$

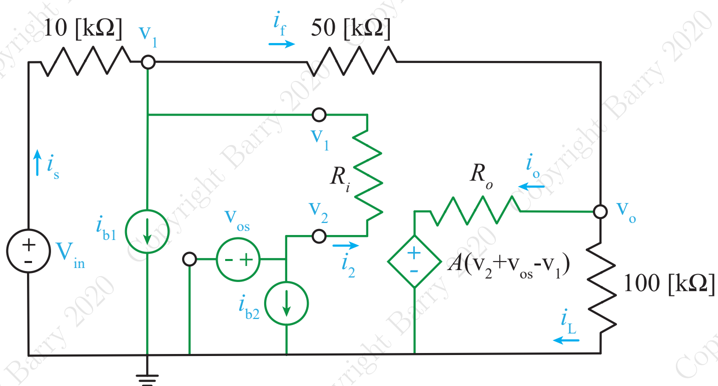
$$V_1\left(\frac{1}{50\,[\mathrm{k}\Omega]}-\frac{A}{R_o}\right)=V_\mathrm{o}\left(\frac{1}{R_o}+\frac{1}{50\,[\mathrm{k}\Omega]}+\frac{1}{100\,[\mathrm{k}\Omega]}\right)\implies V_1=-9.595\cdot 10^{-6}V_\mathrm{o}$$

Substituting this into the first KCL equation:

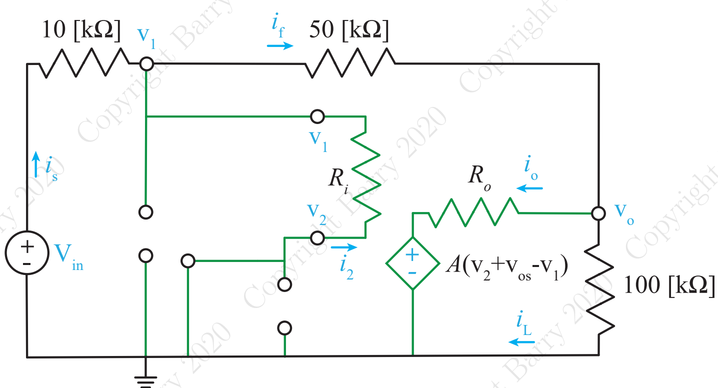
$$V_\mathrm{o}=-5V_\mathrm{in}+(6.000\,000\,05)(-9.595\cdot 10^{-6})V_\mathrm{o}\implies 1.00005757V_\mathrm{o}=-5V_\mathrm{in}\implies \frac{V_\mathrm{o}}{V_\mathrm{in}}=-4.999\,712$$

d) Modeling the op-amp using both the Offset and Finite Gain Models.

We will construct the following model:




Once again, we have found independent sources. We will use superposition considering each source. Starting with $V_\mathrm{in}$:

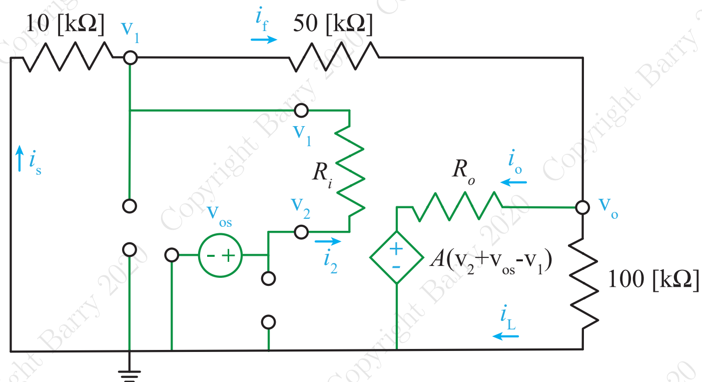



$$i_s+\cancelto{0}{i_2}=i_f$$

Applying Ohm's law:

$$\frac{V_\mathrm{in}-\cancelto{0}{V_1}}{10\,[\mathrm{k}\Omega]}=\frac{\cancelto{0}{V_1}-V_\mathrm{o}}{50\,[\mathrm{k}\Omega]}\implies \frac{V_\mathrm{o}}{V_\mathrm{in}}=-5$$

Now considering $V_\mathrm{os}$, we will apply KCL at the top node:




$$i_s = \cancelto{0}{i_2} + i_f$$

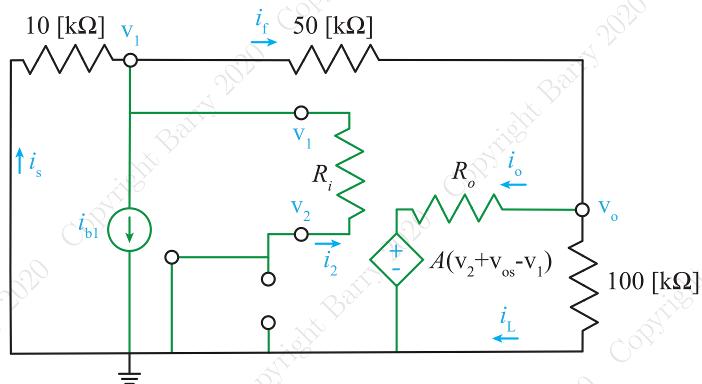
Applying Ohm's law:

$$\frac{0 - V_1}{10\,[\mathrm{k}\Omega]} + \cancelto{0}{i_2} = \frac{V_1 - V_\mathrm{o}}{50\,[\mathrm{k}\Omega]}$$

Knowing $V_1 = V_2 = V_\mathrm{os}$, there is no output from the VCVS, thus $V_\mathrm{o}$ is no factor:

$$V_\mathrm{o} = 6V_\mathrm{os}$$

Consider $i_{\mathrm{b}1}$, applying KCL to the top node:

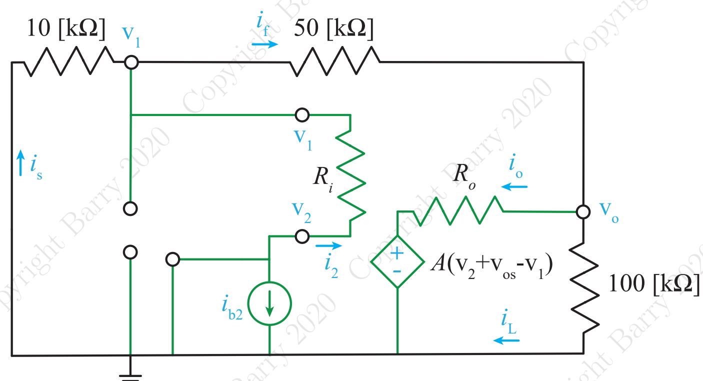



$$\cancelto{0}{i_s} + \cancelto{0}{i_2} = i_{\mathrm{b}1} + i_f$$

Applying Ohm's law:

$$i_{\mathrm{b}1} + \frac{\cancelto{0}{V_1} - V_\mathrm{o}}{50\,[\mathrm{k}\Omega]} = 0 \implies V_\mathrm{o} = (50\,[\mathrm{k}\Omega])i_{\mathrm{b}1}$$

Lastly, considering $i_{\mathrm{b}2}$, we will once again apply KCL to the top node:



$$\overset{0}{\cancel{i_s}} + \overset{0}{\cancel{i_2}} = i_f$$

We note that $i_2$ is zero, and the bias current source $i_{\mathrm{b2}}$ is shorted to ground. Thus $V_{\mathrm{o}}$ is zero. Combining all results:

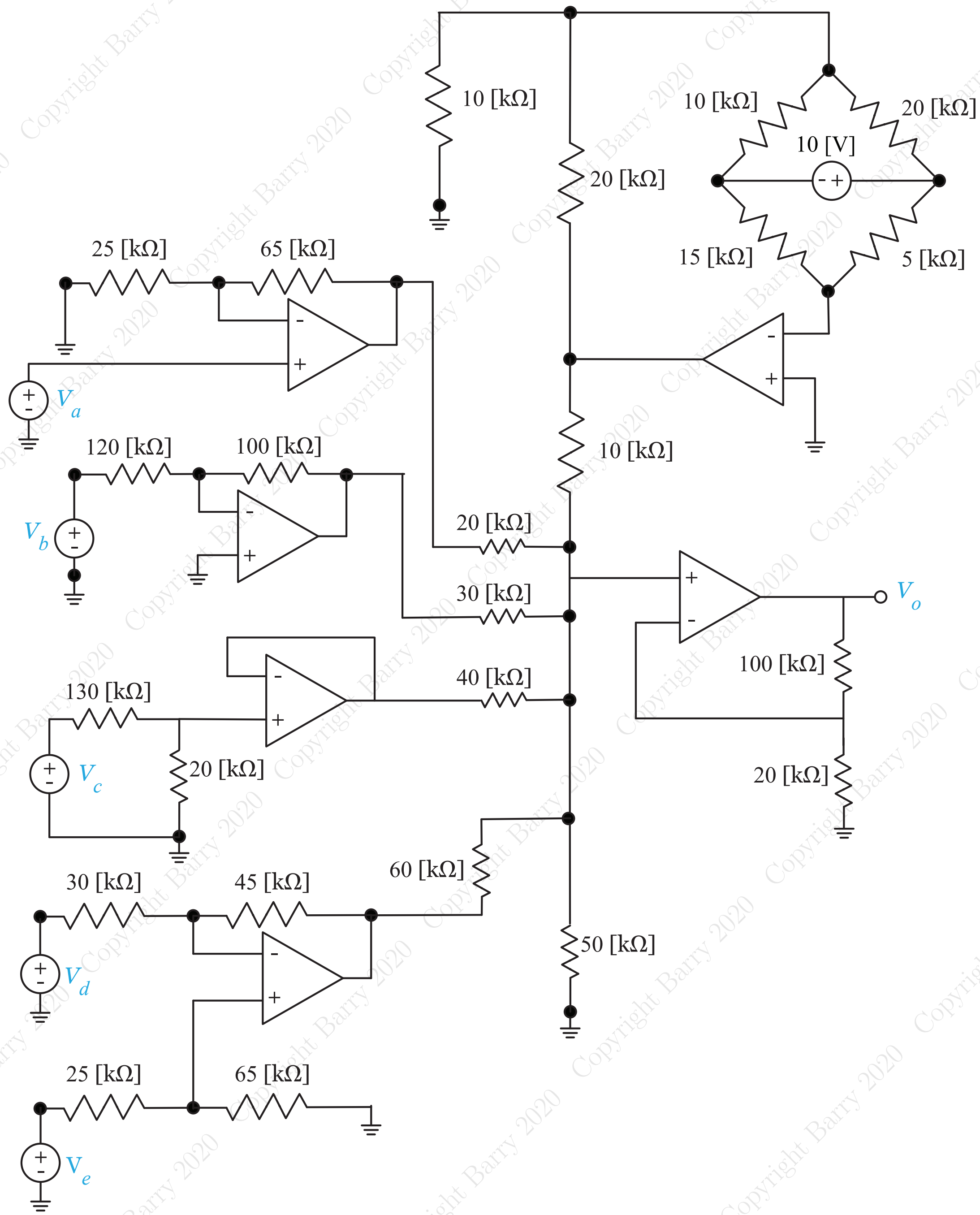
$$V_{\mathrm{o}} = -5V_{\mathrm{in}} + 6V_{\mathrm{os}} + (50\,[\mathrm{k\Omega}])i_{\mathrm{b2}} = -5V_{\mathrm{in}} + 0.003\,541\,5\,[\mathrm{V}]$$

This is the same as solely using the Offset Model. If the current running through $R_i$ is considered, the result changes such that:

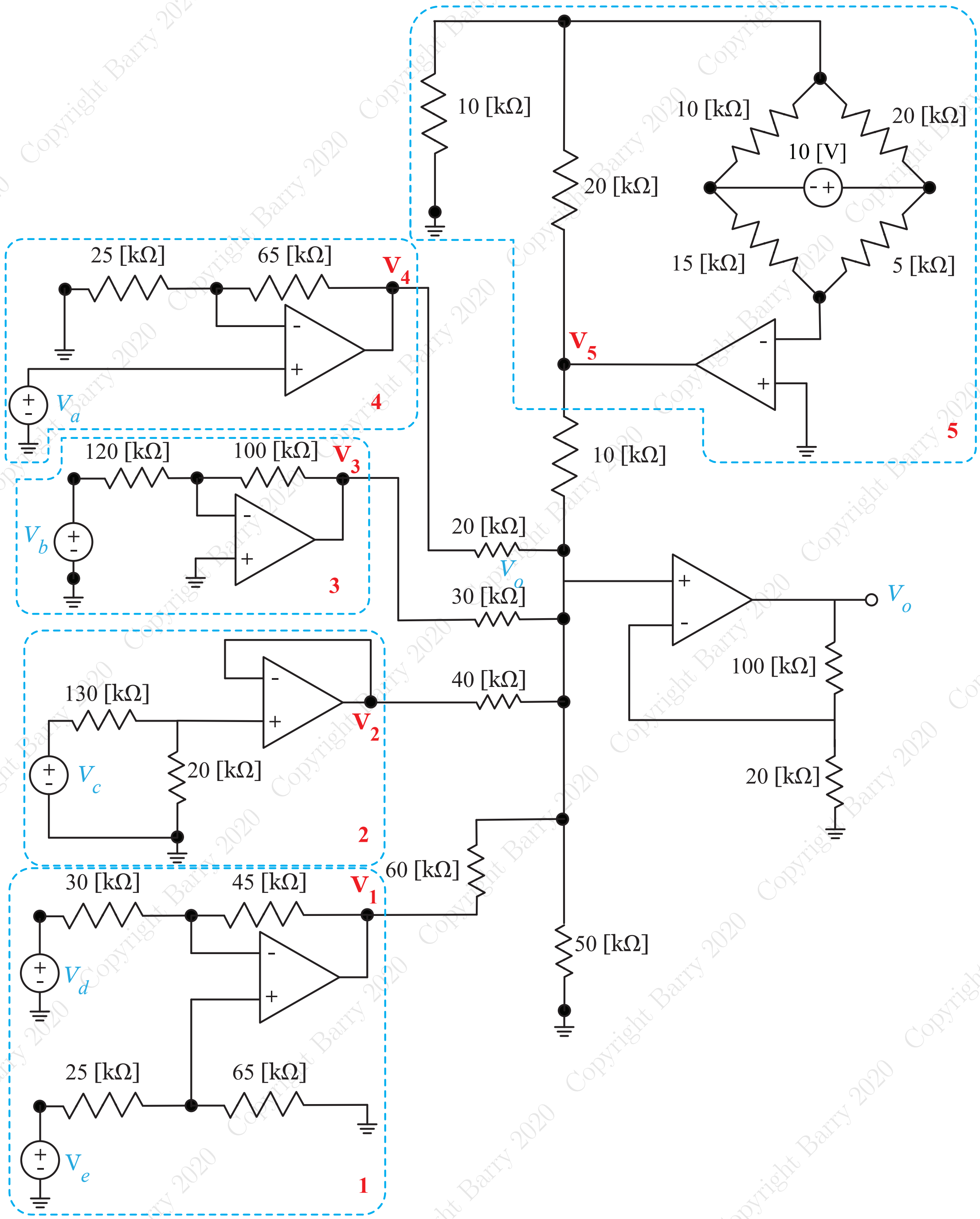
$$V_{\mathrm{o}} = -4.999\,712V_{\mathrm{in}} + 0.003\,541\,5\,[\mathrm{V}]$$

## Problem #2

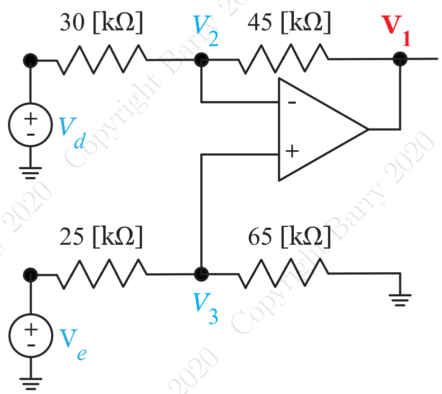
(40 pts.) Given the following system of op-amps, determine the output $V_o$ as a function of all inputs.



We will isolate each op-amp (1-5), and determine the output of each. Using Fig. 6.5-1 on pg. 217, where applicable, we can determine the following:



Starting with op-amp **1**, we can apply KCL at the non-inverting terminal node:

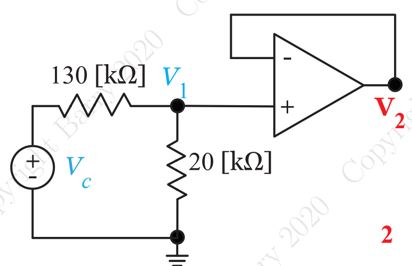



$$\frac{V_e - V_3}{25\,[\mathrm{k}\Omega]} = \frac{V_3}{65\,[\mathrm{k}\Omega]} \implies V_3 = V_2 = \frac{13}{18}V_e$$

we can apply KCL at the inverting terminal node, in terms of node voltages:

$$\frac{V_d - V_2}{30\,[\mathrm{k}\Omega]} = \frac{V_2 - \mathbf{V}_1}{45\,[\mathrm{k}\Omega]} \implies \mathbf{V}_1 = 2.5V_2 - 1.5V_d \implies \mathbf{V}_1 = \frac{65}{36}V_e - 1.5V_d$$

Looking at op-amp **2**, we have to determine $V_1$ via voltage division:




$$V_1 = (20\,[\mathrm{k}\Omega])\left(\frac{V_c}{150\,[\mathrm{k}\Omega]}\right) = \frac{2}{15}V_c$$

$$\mathbf{V}_2 = \frac{2}{15}V_c$$

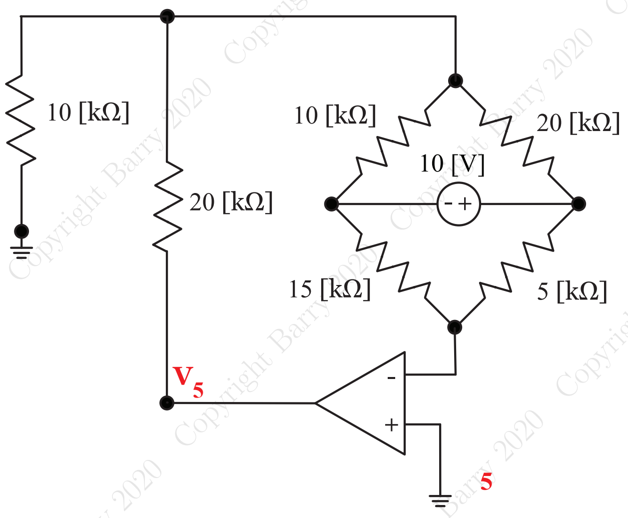
Looking at op-amp **3**, we recognize this is an inverting op-amp. Therefore:

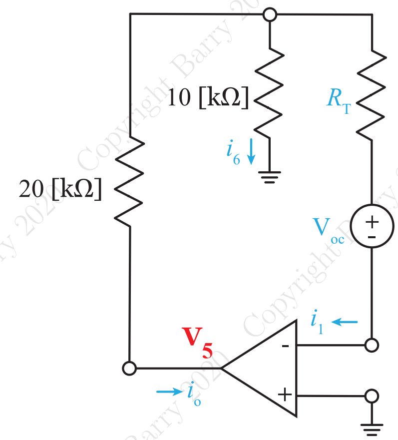
$$\mathbf{V}_3 = -\frac{5}{6}V_b$$

Looking at op-amp **4**, we will apply KCl at the top node to determine $V_1$:

$$\frac{0 - V_a}{25\,[\mathrm{k}\Omega]} = \frac{V_a - \mathbf{V}_4}{65\,[\mathrm{k}\Omega]} \implies \mathbf{V}_4 = \frac{18}{5}V_a$$

Looking at op-amp **5**, we will have to determine the Thevenin resistance of the bridge by shorting the voltage source, and then determine the open-circuit voltage, such that we can simplify the circuit as shown. The 10 and 20 [kΩ] resistors are in parallel, with the equivalence of being in series of the equivalence of the 15 and 5 [kΩ] resistors existing in parallel. Thus, $R_T$=125/12 [kΩ]. The open circuit voltage is found via the method in Lecture 19, i.e. doing KVL loops and calculating the voltage drop across the 20 and 5 [kΩ] resistors. Thus, $V_{\mathrm{oc}}$=-25/6 [V].



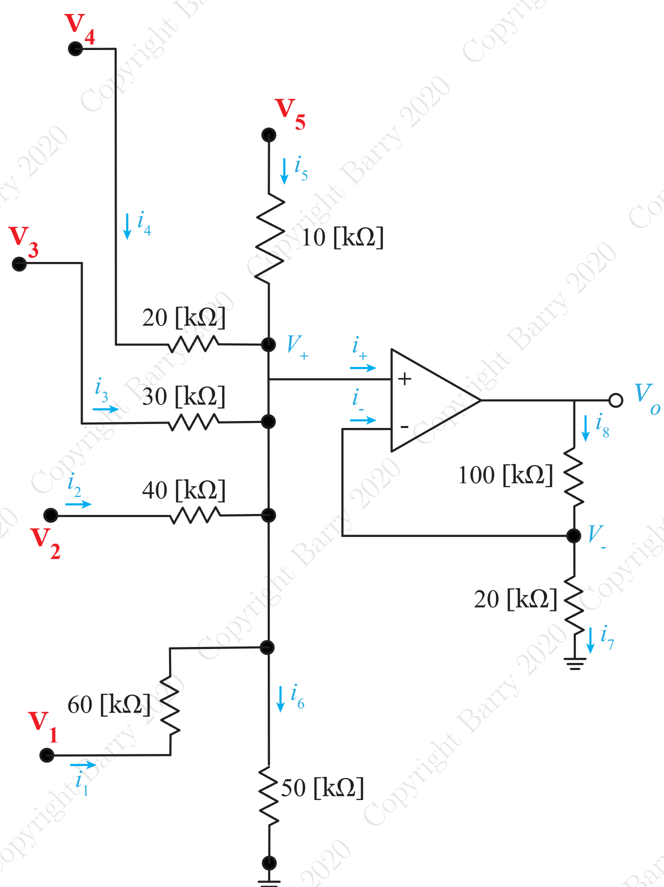

Applying KCL at the top node of the reduced circuit:

$$0 = \overset{0}{\cancel{i_1}} + i_6 + i_o$$

Applying Ohm's law:

$$\frac{-25/6\,[\mathrm{V}]}{10\,[\mathrm{k}\Omega]} + \frac{(-25/6\,[\mathrm{V}]) - \mathbf{V}_5}{20\,[\mathrm{k}\Omega]} = 0 \implies \mathbf{V}_5 = -\frac{75}{6}\,[\mathrm{V}]$$

Now we can apply these inputs to the summing op-amp:



Applying KCL at the non-inverting termminal:

$$i_1 + i_2 + i_3 + i_4 + i_5 = i_6 + \cancelto{0}{i_+}$$

Applying Ohm's law:

$$\frac{\mathbf{V}_1 - V_+}{60\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_2 - V_+}{40\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_3 - V_+}{30\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_4 - V_+}{20\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_5 - V_+}{10\,[\mathrm{k}\Omega]} = \frac{V_+}{50\,[\mathrm{k}\Omega]}$$

$$\implies V_+ = \frac{\frac{\mathbf{V}_1}{60\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_2}{40\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_3}{30\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_4}{20\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_5}{10\,[\mathrm{k}\Omega]}}{\left(\frac{1}{10\,[\mathrm{k}\Omega]} + \frac{1}{20\,[\mathrm{k}\Omega]} + \frac{1}{30\,[\mathrm{k}\Omega]} + \frac{1}{40\,[\mathrm{k}\Omega]} + \frac{1}{50\,[\mathrm{k}\Omega]} + \frac{1}{60\,[\mathrm{k}\Omega]}\right)}$$

$$\implies V_+ = \frac{200}{49}\left(\frac{\mathbf{V}_1}{60\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_2}{40\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_3}{30\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_4}{20\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_5}{10\,[\mathrm{k}\Omega]}\right)$$

Applying KCL at the inverting terminal:

$$i_8 = \cancelto{0}{i_-} + i_7$$

Applying Ohm's law:

$$\frac{V_o - V_-}{100\,[\mathrm{k}\Omega]} = \frac{V_-}{20\,[\mathrm{k}\Omega]} \implies V_o = 6V_-$$

Recognizing $V_- = V_+$,

$$V_o = \frac{1,200}{49}\left(\frac{\mathbf{V}_1}{60\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_2}{40\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_3}{30\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_4}{20\,[\mathrm{k}\Omega]} + \frac{\mathbf{V}_5}{10\,[\mathrm{k}\Omega]}\right)$$

Substituting in knowns:

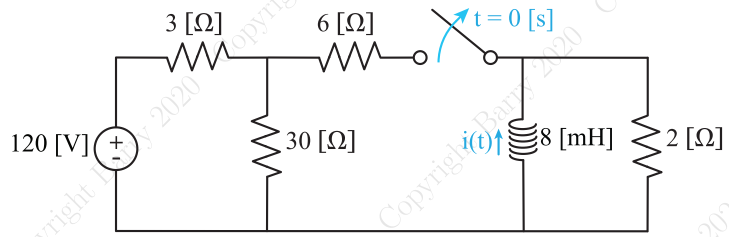
$$V_o = \frac{216}{49}V_a - \frac{100}{147}V_b + \frac{4}{49}V_c - \frac{30}{49}V_d + \frac{325}{441}V_e - \frac{1,500}{49}$$

Or in decimal format:

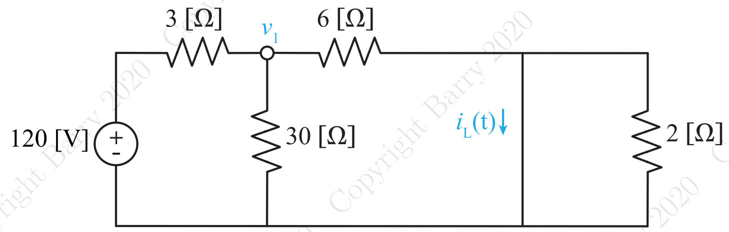
$$V_o = 4.408V_a - 0.680V_b + 0.082V_c - 06.12V_d + 0.737V_e - 30.612$$

## Problem #3

(20 pts.) The switch in the circuit shown below has been closed for a long time and is opened at $t = 0$ [s]. Determine the natural response of the circuit $i(t)$ and the initial energy stored in the inductor.




With the switch closed, a steady-state solution is achieved, and the inductor is behaving as a short circuit. Thus, the current running through the inductor is found via applying KCL at the node with a voltage marked $V_1$. Said equation, expressed via Ohm's law:




$$\frac{120 - V_1}{3\,[\Omega]} = \frac{V_1}{30\,[\Omega]} + \frac{V_1}{6\,[\Omega]} \implies V_1 = 75\,[\mathrm{V}]$$

Therefore, the current running through the inductor, noting the directionality is:

$$i_L(0) = \frac{75\,[\mathrm{V}]}{6\,[\Omega]} = 12.5\,[\mathrm{A}]$$

Once the switch opens, the inductor will dissipate power to the 2 [Ω] resistor - there is no source forcing the response of the system. Thus:

$$i_L(t) = \cancelto{0}{i_{SC}} + (i_L(0) + \cancelto{0}{i_{SC}})\exp(-t/\tau)$$

The time constant is found as:

$$\tau = \frac{L}{R} = \frac{8\,[\mathrm{mH}]}{2\,[\Omega]} = 0.004\,[\mathrm{s}^{-1}]$$

Therefore:

$$i_L(t) = -12.5\exp(-250t)$$

The initial energy stored in the inductor is calculated as:

$$E = \frac{Li^2(t)}{2} = \frac{(0.008\,[\mathrm{H}])(-12.5\,[\mathrm{A}])^2}{2} = 0.625\,[\mathrm{J}]$$

**Academic Integrity Statement**:

I hereby attest that I have received no assistance (from a friend, from another student, from an on-line resource, such as Chegg, etc.), and that I have provided no assistance to another student, during this exam. All the work presented within is solely my own work.

Signature: ______________________________________________

Date: ____________________________