Source Transformation Worksheet

MEMS 0031 - Electrical Circuits

June 10th, 2020

Problem #1

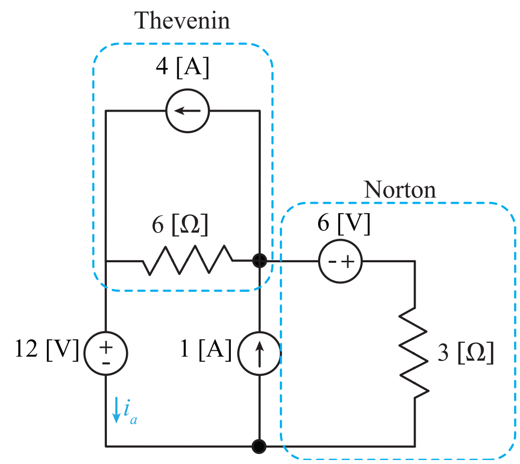
Use source transformation to find the current i_a in the circuit shown in the figure below.

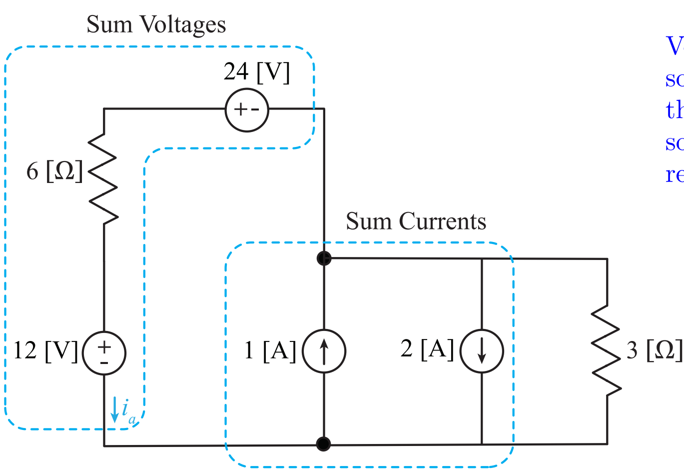
First, we transform the highlighted segments of the circuit into Thevenin and Norton equivalent circuits. The Thevenin equivalent is found as:

$$V_s = i_s R = (4\,[\text{A}])(6\,[\Omega]) = 24\,[\text{V}]$$

The Norton equivalent is found as:

$$i_s = \frac{V_s}{R} = \frac{6\,[\text{V}]}{3\,[\Omega]} = 2\,[\text{A}]$$

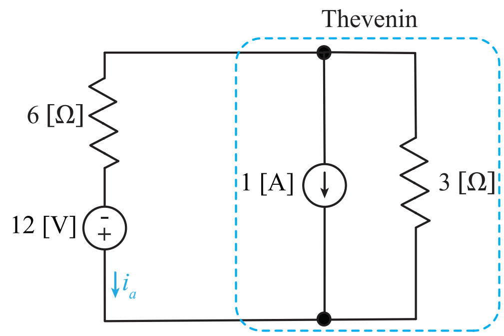


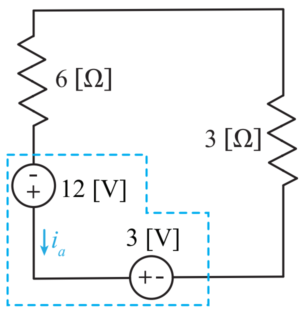


Voltage sources in series add, and current sources in parallel add. Thus, we can replace the 24 [V] and 12 [V] sources with a 12 [V] source (note the signs of the sources). The current sources yield a net current of 1 [A].

The current and resistor are then transformed into their Thevenin equivalent, such that we can sum up the independent voltage sources into one single independent source. The Thevenin equivalent is found as:

$$V_s = i_s R = (1\,[\text{A}])(3\,[\Omega]) = 3\,[\text{V}]$$

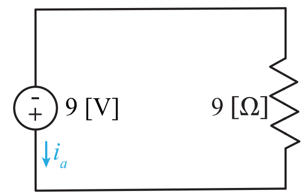




The voltage sources add (note the sign of the sources). The resistors in series add.

The resultant circuit appears as follows. The current i_a is found using Ohm's law:

$$i_a = \frac{V}{R} = \frac{9\,[\mathrm{V}]}{9\,[\Omega]} = 1\,[\mathrm{A}]$$



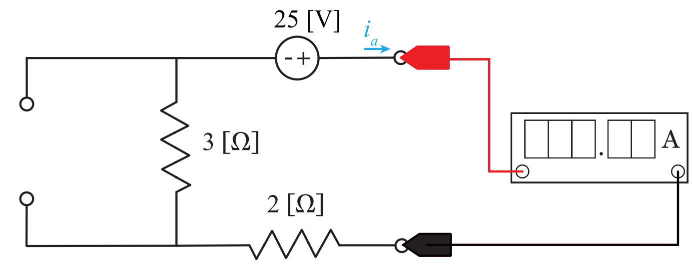
Problem #2

Use the principal of superposition to find the reading of the ammeter in the circuit shown in the figure below.

The principal of superposition is to find the contributions of independent and dependent sources on a variable of interest (i.e: current and voltage). In this circuit, there are two independent sources - one current and one voltage. Therefore, the circuit will be isolated so to have just one of these sources active at a time to find the current flowing through the ammeter.

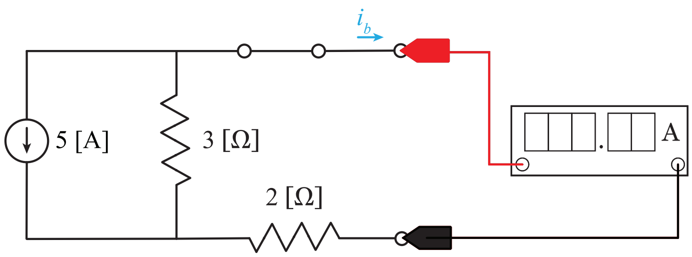
Voltage Source:
To isolate the voltage source, the independent current source will be opened. The resultant circuit is shown on the right. Current i_a can be found by finding the equivalent resistance of the circuit and then using Ohm's law to find the current flowing through it. The resultant current is found to be:

$$i_a = \frac{25\ [\mathrm{V}]}{5\ [\Omega]} = 5\ [\mathrm{A}]$$



Current Source:
To isolate the current source, the independent voltage source will be shorted. This simulates an absence of voltage gain/drop. The resultant circuit is shown on the right. Current i_b can be found via the law of conductance since it is an independent current source with resistors in parallel. The resultant current is found to be:

$$i_b = \frac{\frac{1}{2\ [\Omega]}}{\left(\frac{1}{3\ [\Omega]} + \frac{1}{2\ [\Omega]}\right)}(-5\ [\mathrm{A}]) = -3\ [\mathrm{A}]$$



The reason for the negative sign is to take into account the direction of the current which is determined by the polarity of the current source.
The summation of the currents is the current flowing through the ammeter from the original circuit. That is:

$$\mathrm{i} = i_a + i_b = 5 + -3\ [\mathrm{A}] = 2\ [\mathrm{A}]$$

Problem #3

Determine the Thevenin equivalent circuit for the circuit shown in the figure below.

There are three ways to determine the Thevenin equivalent resistance of a circuit.

1. Find Thevenin resistance (R_{th}) and short-circuit current (i_{sc}).
2. Find open-circuit voltage (V_{oc}) and short-circuit current (i_{sc}).
3. Find Thevenin resistance (R_{th}) and open-circuit voltage (V_{oc}) independently.

The first two methods use Ohm's law to determine the open-circuit voltage and Thevenin resistance independently.

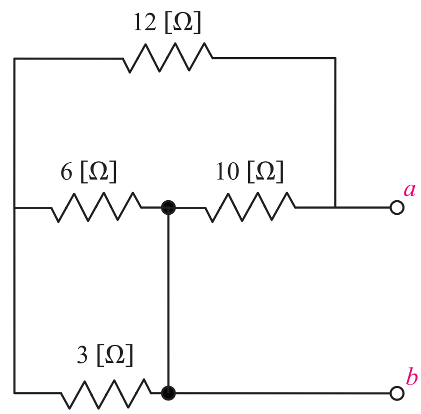
$$V_{oc} = i_{sc} R_{th}$$

No one method is generally better than the other. It all depends on the nature of the circuit being analyzed.

Thevenin Resistance:

The Thevenin resistance of a Thevenin equivalent circuit is the equivalent resistance of a circuit when all independent sources are deactivated. Such a circuit with all sources deactivated is shown to the right. Here, the equivalent resistor is 6 [Ω]. Thus, the Thevenin resistance is given as:

$$\mathrm{R}_{th} = 5.83\ [\Omega]$$



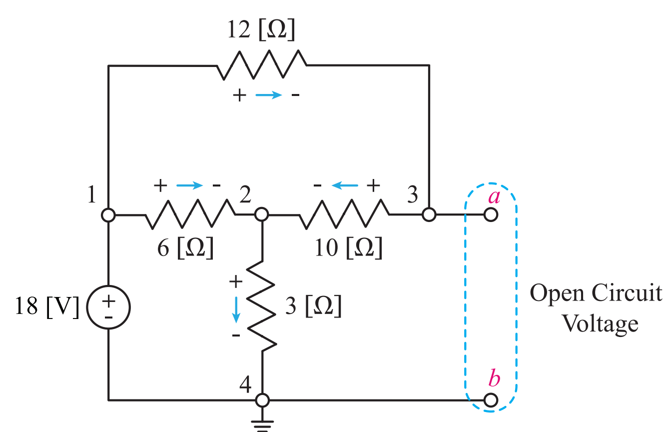
Open-circuit voltage:

The open-circuit voltage can be found using NVA to find the node voltages at a and b. Node b will be treated as a ground, therefore the node voltage at b will be 0 [V]. The open circuit voltage is gives as:

$$V_a - V_b = V_{oc}$$

Using NVA requires the derivation of KCL equations at the specified nodes. VS equation:

$$V_1 - V_4 = 18\ [\mathrm{V}]$$



KCL at node 2:

$$\frac{V_1 - V_2}{6\ [\Omega]} + \frac{V_3 - V_2}{10\ [\Omega]} = \frac{V_2 - V_4}{3\ [\Omega]}$$

KCL at node 3:

$$\frac{V_1 - V_3}{12\ [\Omega]} = \frac{V_3 - V_2}{10\ [\Omega]}$$

Solving the equations simultaneously via matrix inversion yields the following node voltages:

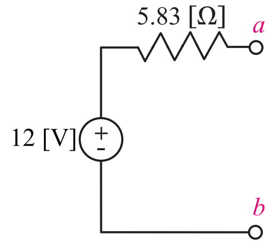
$$V_1 = 18\ [\mathrm{V}]; \quad V_2 = 7\ [\mathrm{V}]; \quad V_3 = 12\ [\mathrm{V}]$$

It is clear that the node voltage at node 3 is the same node voltage at node a. Knowing this, the open-circuit voltage can be found to be:

$$V_{oc} = 12 - 0 \text{ [V]} = 12 \text{ [V]}$$

The Thevenin equivalent circuit is:

The Thevenin resistance could have also been found by using Ohm's law with the open-circuit voltage and the short-circuit current. To find said current, MCA would need to be used to find the current that would flow through a short-wire connecting nodes a and b.



Problem #4

Determine the Norton equivalent circuit for the circuit shown in the figure below.

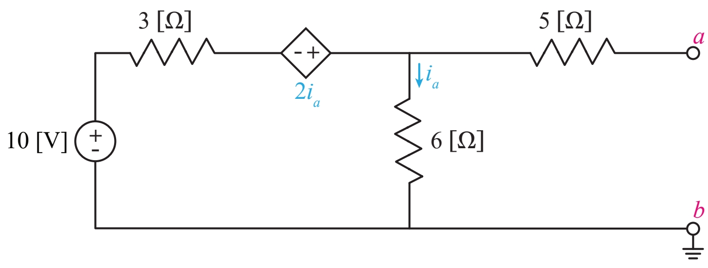


Figure 1: Schematic for Problem #4.

Solutions:

There are three ways to determine the Norton equivalent resistance of a circuit.

1. Find Norton resistance (R_n) and open-circuit voltage (V_{oc}).
2. Find open-circuit voltage (V_{oc}) and short-circuit current (i_{sc}).
3. Find Norton resistance (R_n) and short-circuit current (i_{sc}) independently.

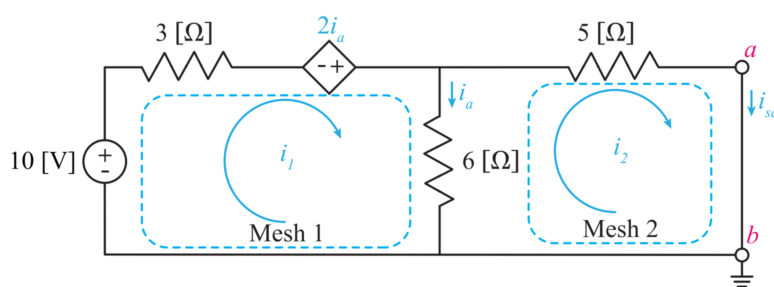
The first two methods use Ohm's law to determine the short-circuit current and Norton resistance independently. That is:

$$V_{oc} = i_{sc} R_n$$

No one method is generally better than the other. It all depends on the nature of the circuit being analyzed.

Short-circuit current:

The short-circuit current can be found using MCA to find the mech currents. The KVL equations for loops 1 and 2 are given as:



$$-10\ [\mathrm{V}] + 3i_1 - 2i_a + (i_1 - i_2)6\ [\Omega] = 0$$

$$5i_2 + (i_2 - i_1)6\ [\Omega] = 0$$

Current i_a, as defined in the image, is given as:

$$i_1 - i_2 = i_a$$

Solving the equations simultaneously or matrix inversion yields the following mesh currents:

$$i_1 = 2.075\ [\mathrm{A}]$$

$$i_2 = 1.132\ [\mathrm{A}]$$

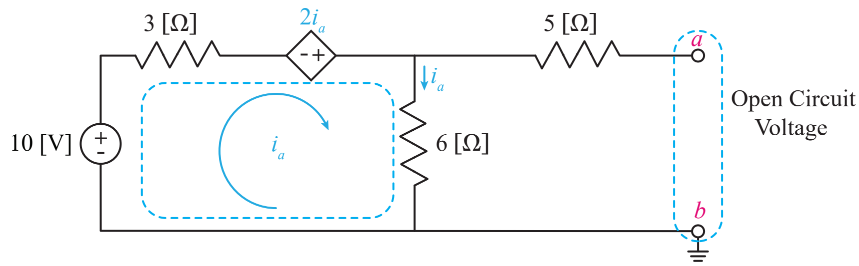
Here, i_2 is the short-circuit current. Therefore,

$$i_{sc} = 1.132\ [\mathrm{A}]$$

The dependent sources cannot be deactivated since they depend on other parameters throughout the circuit. Therefore, it is not feasible to find the equivalent resistance of a deactivated circuit. As a result, the open-circuit voltage will be found instead.

Open-circuit voltage:

The open-circuit voltage can be found by finding the voltage drop across the 6 [Ω] resistor. It is the same voltage drop from node a to be. Conservation of energy states that the summation of voltage drops around a loop is 0. The governing equation is given as:



$$-10 \text{ [V]} + (3 \text{ [}\Omega\text{]})i_a - 2i_a + (6 \text{ [}\Omega\text{]})i_a = 0$$

Solving for the current i_a yields:

$$i_a = 1.43 \text{ [A]}$$

The voltage drop across the 6 [Ω] resistor is given as:

$$\Delta V = (6 \text{ [}\Omega\text{]})i_a = 8.58 \text{ [V]}$$

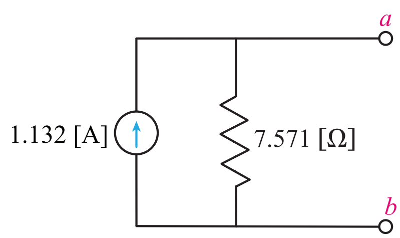
Therefore, the open-circuit voltage is given as:

$$V_{oc} = 8.58 \text{ [V]}$$

Using Ohm's law and the short-circuit current found earlier, the Norton resistance is found to be:

$$R_n = \frac{V_{oc}}{i_{sc}} = \frac{8.58 \text{ [V]}}{1.132 \text{ [A]}} = 7.58 \text{ [}\Omega\text{]}$$

The Norton equivalent circuit is:



Problem #5

Determine the maximum power that can be absorbed by a resistor, R, connected to the terminals a-b of the circuit shown in the figure below. Specify the value for R.

Solutions:
The maximum power that can be absorbed by a resistor, R, can be found after finding the Thevenin equivalent circuit. The Thevenin resistance from said circuit is the value of the resistor, R, to achieve maximum power absorbed/dissipated. The maximum power theorem states that the max power that can be delivered to a load resistance is when the load resistance is equal to the Thevenin resistance. That is:

$$\mathrm{R} = R_{th}$$

That power is given as:

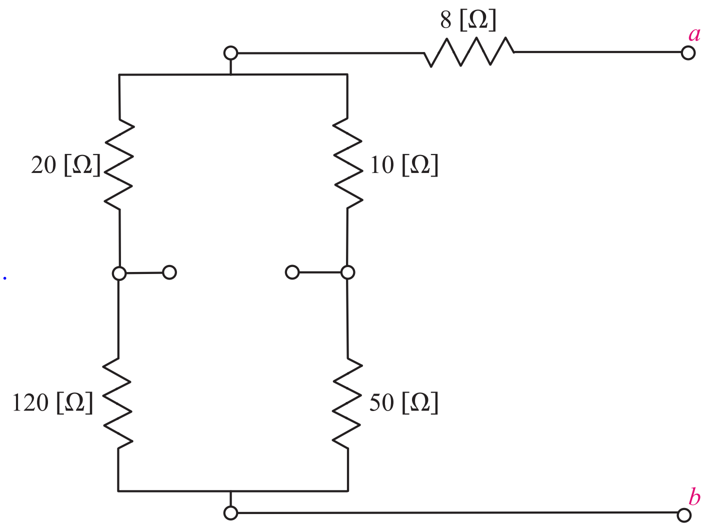
$$P_{max} = \frac{V_s^2}{4R_{th}}$$

Thevenin resistance:

The equivalent resistance is found after deactivating all independent sources. The circuit is shown to the right. The equivalent resistance is found to be:

$$\mathrm{R}_{th} = \left(\frac{1}{(120+20)\ [\Omega]}+\frac{1}{(10+50)\ [\Omega]}\right)^{-1}+\ldots$$

$$8\ [\Omega] = 50\ [\Omega]$$



Short-circuit current:

The short-circuit current can be found using MCA to find the mesh currents. A super mesh is created around the loops for i_1 and $\imath_2$. The KVL equations for the supermesh and mesh 2 is given as:

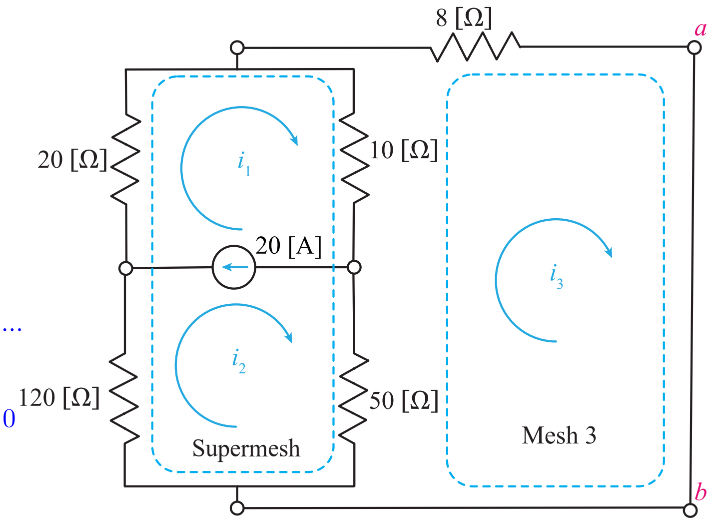
$$i_1 - i_2 = 20\ [\mathrm{A}]$$

$$i_1(20\ [\Omega])+(i_1-i_3)10\ [\Omega]+(i_2-i_3)50\ [\Omega]+\ldots$$

$$i_2(120\ [\Omega]) = 0$$

$$i_3(8\ [\Omega])+(i_3-i_1)10\ [\Omega]+(i_3-i_2)50\ [\Omega] = 0$$

Solving the equations simultaneously or matrix inversion yields the following mesh currents:



$$i_1 = 17.120\ [\mathrm{A}]$$

$$i_2 = -2.880\ [\mathrm{A}]$$

$$i_3 = 0.4 \text{ [A]}$$

Here, i_3 is the short-circuit current. Therefore,

$$i_{sc} = 0.4 \text{ [A]}$$

The open-circuit voltage is found using Ohm's law with the short-circuit current and the Thevenin resistance found earlier. This yields:

$$V_{oc} = i_{sc}R_{th} = (0.4 \text{ [A]})(50 \text{ [}\Omega\text{]}) = 20 \text{ [V]}$$

Using the max power theorem stated earlier, we can say:

$$P_{max} = \frac{(20 \text{ [V]})^2}{4(50 \text{ [}\Omega\text{]}}) = 2 \text{ [W]}$$

and that the load resistance to achieve that power is

$$R = 50 \text{ [}\Omega\text{]}$$