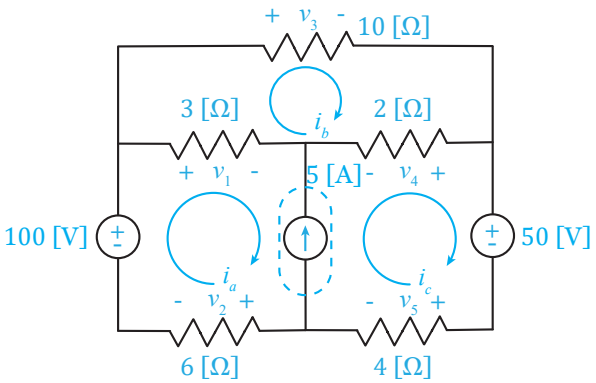
MEMS 0031 - Electrical Circuits
Quiz #5

Name: _____

Problem #1

Determine the mesh currents i_a, i_b and i_c using MCA.



Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-$#CS KVL equations, describing each mesh current:

Supermesh equation:

$$i_c - i_a = 5\,[\text{A}] \tag{1}$$

Supermesh:

$$-100\,[\text{V}] + V_1 - V_4 + 50\,[\text{V}] + V_5 + V_2 = 0$$

Mesh Current b:

$$V_3 + V_4 - V_1 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh :

$$-100\,[\text{V}] + (3\,[\Omega])(i_a - i_b) - (2\,[\Omega])(i_b - i_c) + 50\,[\text{V}] + (4\,[\Omega])i_c + (6\,[\Omega])i_a = 0$$

$$\implies 9i_a - 5i_b + 6i_c = 50\,[\text{V}] \tag{2}$$

Mesh Current b:

$$(10\,[\Omega])i_b + (2\,[\Omega])(i_b - i_c) - (3\,[\Omega])(i_a - i_b) = 0$$

$$\implies -3i_a + 15i_b - 2i_c = 0 \tag{3}$$

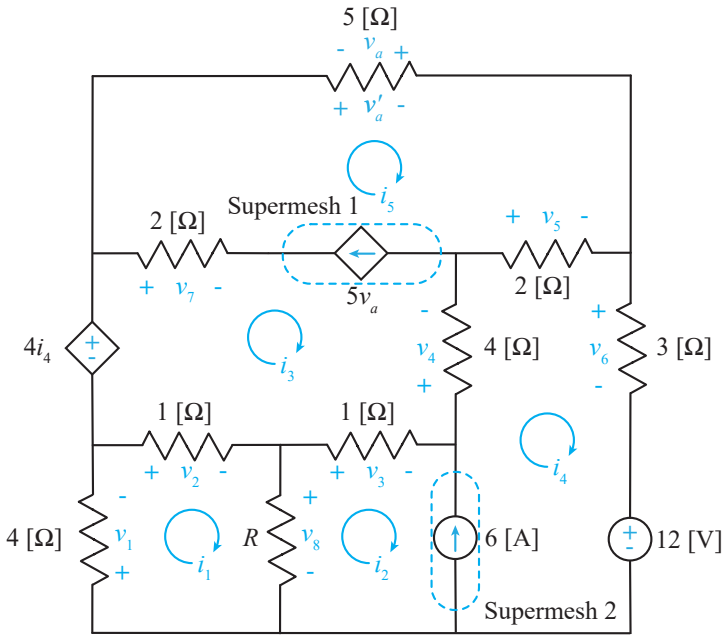
Putting eqns. 1 through 3 in matrix form:

$$\begin{bmatrix} -1 & 0 & 1 \\ 9 & -5 & 6 \\ -3 & 15 & -2 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 5 \\ 50 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} 1.75 \\ 1.25 \\ 6.75 \end{bmatrix}$$

Units are taken as [A].

Problem #2

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_5, given $R = 3\,[\Omega]$.



Step 1: Construct N KVL loops. $N = 5$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-$#CS KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_5 - i_3 = 5V_a$$

Supermesh 1, noting $V_a' = -V_a$:

$$-(4i_4) + V_a' - V_5 - V_4 - V_3 - V_2 = 0$$

Supermesh Equation 2:

$$i_4 - i_2 = 6\,[\text{A}]$$

Supermesh 2:

$$-V_8 + V_3 + V_4 + V_5 + V_6 + 12\,[\text{V}] = 0$$

Mesh Current 1:

$$V_1 + V_2 + V_8 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$-i_3 + i_5 - 5(5\,[\Omega])(-i_5) = 0 \implies -i_3 + 26i_5 = 0 \tag{4}$$

Supermesh 1:

$$-(4i_4) + (5\,[\Omega])i_5 - (2\,[\Omega])(i_4 - i_5) - (4\,[\Omega])(i_4 - i_3) - (1\,[\Omega])(i_2 - i_3) - (1\,[\Omega])(i_1 - i_3) = 0$$

$$\implies -i_1 - i_2 + 6i_3 - 10i_4 + 7i_5 = 0 \tag{5}$$

Supermesh Equation 2:

$$-i_2 + i_4 = 6\,[\text{A}] \tag{6}$$

Supermesh 2:

$$-(3\,[\Omega])(i_1 - i_2) + (1\,[\Omega])(i_2 - i_3) + (4\,[\Omega])(i_4 - i_3) + (2\,[\Omega])(i_4 - i_5) + (3\,[\Omega])i_4 = -12\,[\text{V}]$$

$$\implies -3i_1 + 4i_2 - 5i_3 + 9i_4 - 2i_5 = -12\,[\text{V}] \tag{7}$$

Mesh Current 1:

$$(4\,[\Omega])i_1 + (1\,[\Omega])(i_1 - i_3) + (3\,[\Omega])(i_1 - i_2) = 0 \implies 8i_1 - 3i_2 - i_3 = 0 \tag{8}$$

Putting eqns. 4 through 8 in matrix form:

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 26 \\ -1 & -1 & 6 & -10 & 7 \\ 0 & -1 & 0 & 1 & 0 \\ -3 & 4 & -5 & 9 & -2 \\ 8 & -3 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ -12 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -3.12 \\ -7.16 \\ -3.49 \\ -1.16 \\ -0.13 \end{bmatrix}$$

Units are taken as [A].