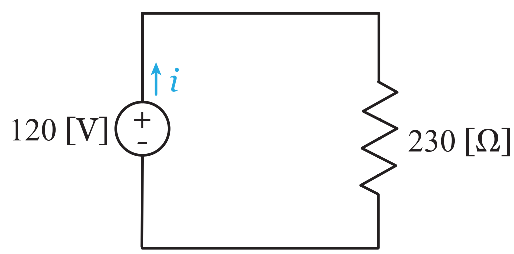
Homework #3

MEMS 0031 - Electrical Circuits

Assigned January 25th, 2019
Due February 1st, 2019

Problem #1

Find the voltage drop across each resistor, as shown in the circuit below, i.e. v_a, v_b, v_c and v_d. Also determine the current i flowing through the resistors.



Solutions:
The resistors in problem 1 are in series. Therefore, the circuit can reduced to a single 230 [Ω]. Using Ohm's law to find the current through said resistor:

$$v_{drop} = iR$$

$$i = \frac{v_{drop}}{R} = \frac{120[\mathrm{V}]}{230\,[\Omega]} = 0.522\,[\mathrm{A}]$$

$$v_a = (0.522\,[\mathrm{A}])(100\,[\Omega]) = 52.20\ [\mathrm{V}]$$

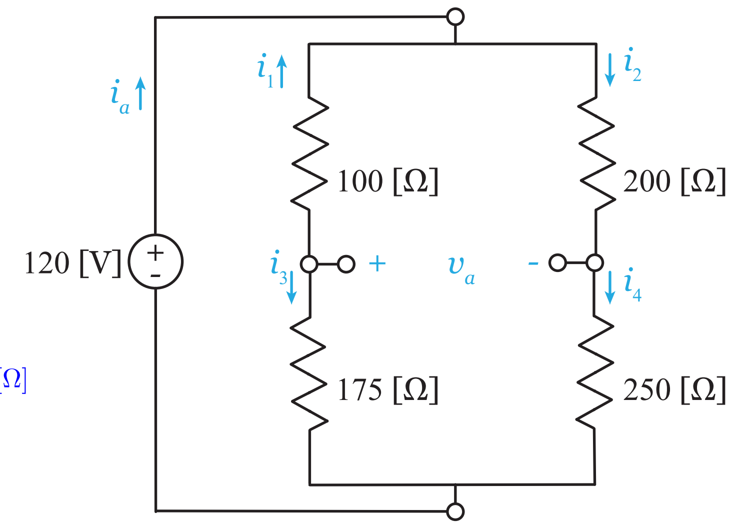
$$v_b = (0.522\,[\mathrm{A}])(70\,[\Omega]) = 36.52\ [\mathrm{V}]$$

$$v_c = (0.522\,[\mathrm{A}])(50\,[\Omega]) = 26.09\ [\mathrm{V}]$$

$$v_d = -(0.522\,[\mathrm{A}])(10\,[\Omega]) = -5.22\ [\mathrm{V}]$$

Problem #2

Find the voltage drop across each the open circuit, v_a, in the circuit shown below. Also determine the current flow through each segment of the circuit.



Solutions:
The circuit can be visualized as shown. Here, the 200 [Ω] and 250 [Ω] resistors are in series and 100 [Ω] and 175 [Ω] resistors are in series parallel to the other set of resistors. From this, we can find the current i_a using Ohm's law.

$$V = iR$$

Here, V is the potential difference supplied by the source and R is the equivalent resistance which is:

$$R = \frac{(200[\Omega] + 250[\Omega])(100[\Omega] + 175[\Omega])}{(200[\Omega] + 250[\Omega]) + (100[\Omega] + 175[\Omega])} = 170.689 \ [\Omega]$$

Therefore, the current i_a is calculated to be

$$i_a = \frac{V}{R} = \frac{120 \ [V]}{170.689 \ [\Omega]} = 0.703 \, [\text{A}]$$

The current through the upper and lower branch can be found using the same method. The only difference is the equivalent resistance.

$$i_{upper} = i_2 = i_4 = \frac{V}{R} = \frac{120 \ [V]}{450 \ [\Omega]} = 0.2667 \, [\text{A}]$$

$$i_{lower} = -i_1 = i_3 = \frac{V}{R} = \frac{120 \ [V]}{275 \ [\Omega]} = 0.4363 \, [\text{A}]$$

The voltage drop across the open circuit v_a can be found by finding the voltage drop after the 200 [Ω] and 100 [Ω] resistor.

$$\Delta V = (120 \ [\text{V}] - V_{100[\Omega]}) = iR \implies V_{100 \, [\Omega]} = 120 - (0.4363 \, [\text{A}])(100[\Omega]) = 76.364 \ [\text{V}]$$

$$\Delta V = (120 \ [\text{V}] - V_{200 \, [\Omega]}) = iR \implies V_{200 \, [\Omega]} = 120 - (0.2667 \, [\text{A}])(200[\Omega]) = 66.667 \ [\text{V}]$$

The voltage drop across the open circuit v_a is then found to be the difference between $V_{100[\Omega]}$ and $V_{200[\Omega]}$. Thus, it is found that:

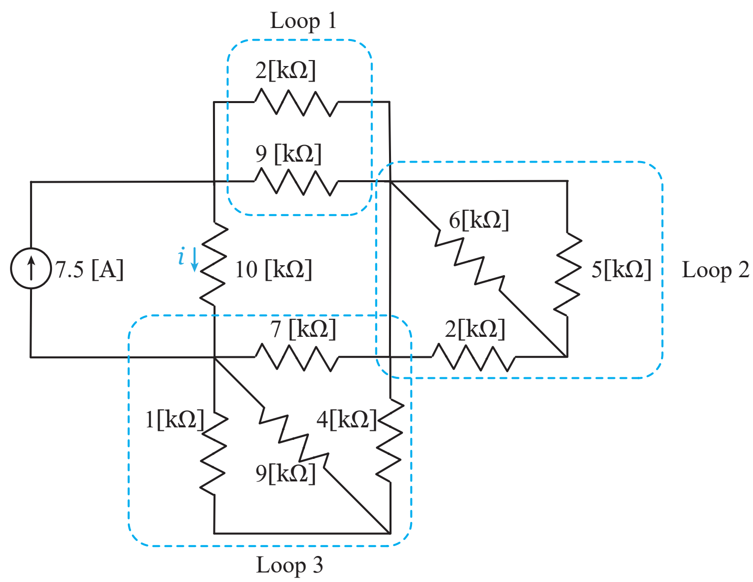
$$V_a = V_{100[\Omega]} - V_{200[\Omega]} = 76.364 \ [\text{V}] - 66.667 \ [\text{V}] \implies \boxed{9.697 \ [\text{V}]}$$

The voltage drop across each resistor can also be found using Ohm's law.

Problem #3

Find the voltage drop across the 10 [kΩ] resistor. Also determine the power dissipated by the circuit.

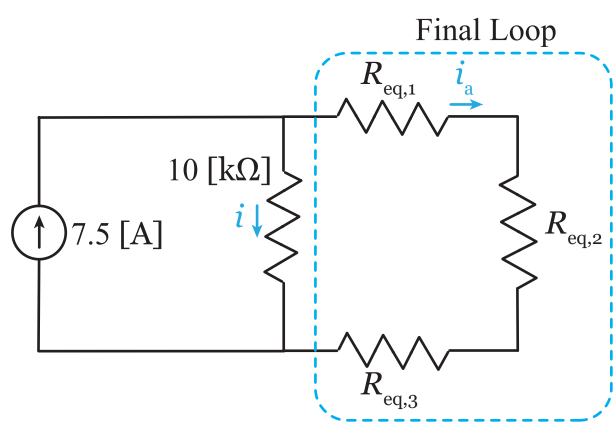
Solutions:
The circuit can be reduced as shown. The right loop is shorted by the short wire. The equivalent resistances are given as:



Loop 1:
Resistors are in parallel and have an equivalent resistance of this loop is $R_{eq,1} = 1.6364\,[\text{k}\Omega]$

Loop 2:
The 6 [kΩ] and 5 [kΩ] resistors are in parallel which are then in series with the 2 [kΩ]. However, the resultant equivalent resistor are in parallel with a short wire. Therefore, the equivalent resistance of this loop is $R_{eq,2} = 0\,[\text{k}\Omega]$

Loop 3:
The 1 [kΩ] and 9 [kΩ] resistors are in parallel which are then in series with the 4 [kΩ]. The resultant equivalent resistor is in parallel with the 7 [kΩ]. Therefore, the equivalent resistance of this loop is $R_{eq,3} = 2.8824\,[\text{k}\Omega]$



Since the voltage across the 10 [kΩ] resistor is the same as the equivalent resistances in series. The current supplied by the independent source is also the sum of the current across each branch. In other words:

$$V_{10[\text{k}\Omega]} = V_{R_{eq,1}} + V_{R_{eq,2}} + V_{R_{eq,3}} \implies$$

$$i(10\,[\text{k}\Omega]) = i_a R_{eq,1} + i_a R_{eq,2} + i_a R_{eq,3}$$

We also know that $7.5[\text{A}] = i + i_a$. From this, the two currents can be solve.

$$i = 2.334\,[\text{A}]$$

$$i_a = 5.166\,[\text{A}]$$

Knowing these currents, the voltage drop across the 10 [kΩ] resistor is given as:

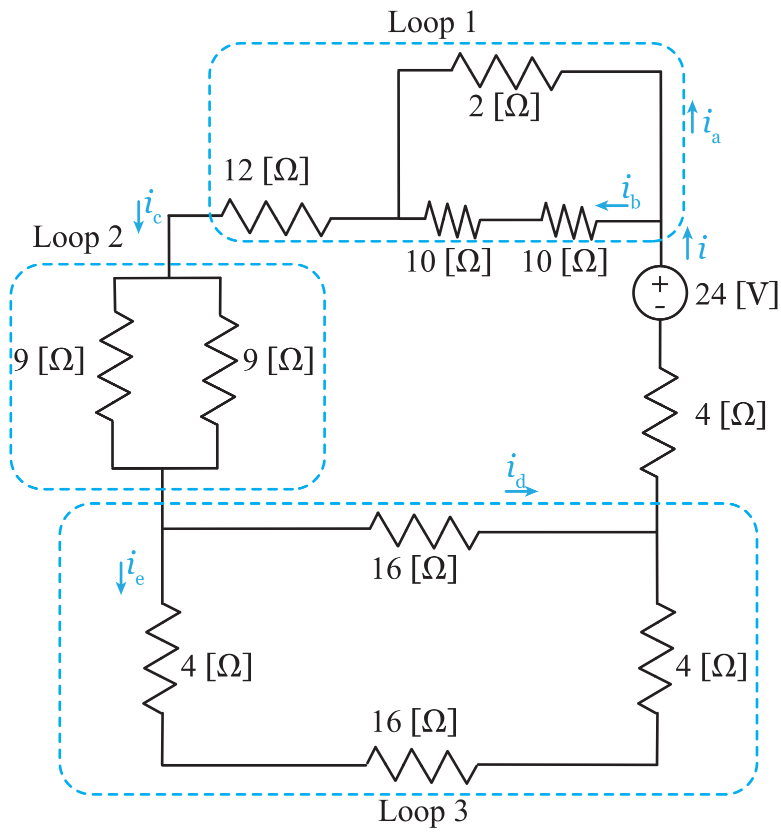
$$V_{10[\text{k}\Omega]} = i(10\,[\text{k}\Omega]) = 23.34\,[\text{kV}]$$

The total power dissipated is given as:

$$P = (i^2)(10[\text{k}\Omega]) + (i_a^2)(R_{eq,1} + R_{eq,2} + R_{eq,3}) = \boxed{175.0713\,[\text{kW}]}$$

Problem #4

Determine the values of v_1, v_2, i_3, v_4, v_5, and i_6 in the circuit shown below.



Solutions:
Consider the three loops highlighted to determine the current i running through the entire circuit.

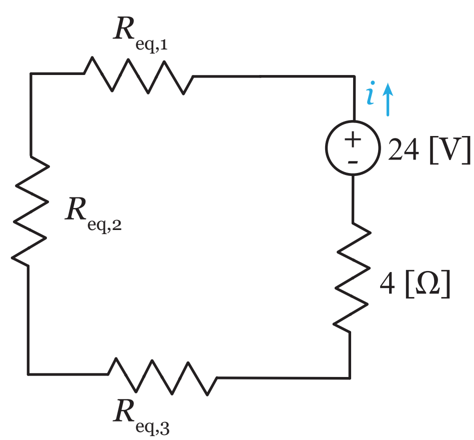
Loop 1:
Here the equivalent resistance is $R_{eq,1} = 13.8181\,[\Omega]$. It should also be noted that the sum of the voltages across the two $10\,[\Omega]$ resistors is the same as that of the $2\,[\Omega]$ resistor.

Loop 2:
Here the equivalent resistance is $R_{eq,2} = 4.50\,[\Omega]$.

Loop 3:
Here the equivalent resistance is $R_{eq,3} = 9.6\,[\Omega]$. The $4\,[\Omega]$, $16\,[\Omega]$ and $4\,[\Omega]$ are in series which are then in parallel with the second $16\,[\Omega]$ resistor.

The resultant circuit appears as follows:
The current i can be found to be:

$$i = \frac{24\,[\text{V}]}{R_{eq,1} + R_{eq,2} + R_{eq,3} + 4\,[\Omega]} = 0.751922\,[\text{A}]$$



Recall that the sum of the voltages across the two $10\,[\Omega]$ resistors is the same as that of the $2\,[\Omega]$ resistor. It should also be noted that the sum of currents i_a and i_b is equal to i. Therefore, it can be said that:

$$V_{2[\Omega]} = V_{10[\Omega]} + V_{10[\Omega]} \implies$$

$$(i_a)(2[\Omega]) = 2(i_b)(10[\Omega])$$

$$(i_a) + (i_b) = i$$

Solving for the currents, it shows that:

$$(i_a) = 0.68357\ [\text{A}]$$

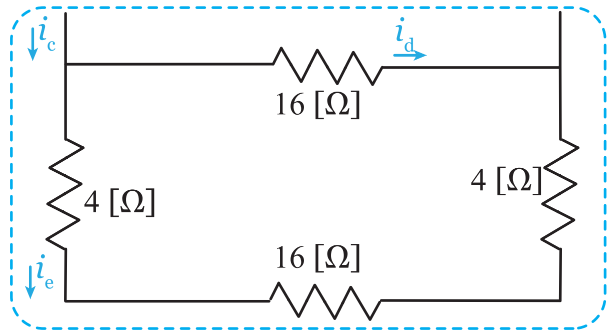
$$(i_b) = 0.06836\ [\text{A}]$$

The current i_c is clearly the sum of i_a and i_b. Since it is branch of into two resistors in the parallel with the same resistances, the current is split evenly. In other words:

$$i_c = \frac{i}{2} = 0.751922 \text{ [A]}$$

$$i_{9[\Omega]} = \frac{i}{2} = 0.37596 \text{ [A]}$$

Now take a look at loop 3. The current coming in is i_c which was found. It then diverges into i_d and i_e. Like loop 1, the voltage across the lone 16[Ω] equals that of the 16[Ω] and two 4[Ω] in series. In other words, it can be said that:



$$i_c = i_d + i_e$$

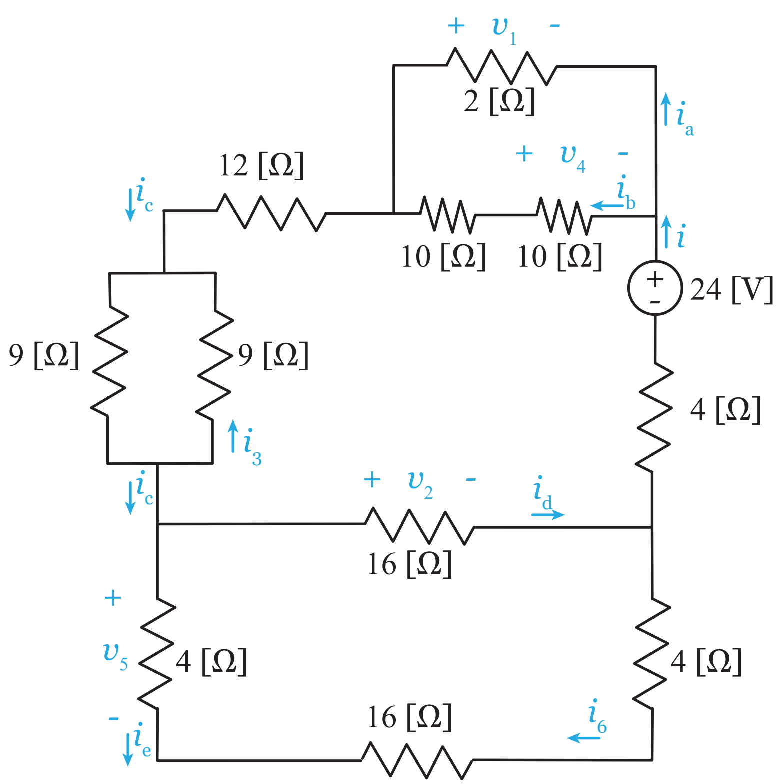
$$V_{16[\Omega]} = V_{4[\Omega]+4\,[\Omega]+16\,[\Omega]} \implies$$
$$i_d(16[\Omega]) = i_e(4\,[\Omega] + 4\,[\Omega] + 16\,[\Omega])$$

Solving for the currents, it shows that:

$$i_d = 0.451154 \text{ [A]}$$

$$i_e = 0.300769 \text{ [A]}$$

Now that the currents passing through each element have be found, the problem can be solved. Looking at the original problem and placing the currents found within, the circuit can be shown to be drawn as:



Following the sign convention, v_1 is solved to be:

$$v_1 = -i_a(2[\Omega]) = -1.367 \text{ [V]}$$

Take note of the negative sign given the sign convention direction illustrated in the problem statement. Once again, following the sign convention, v_2 is solved to be:

$$v_2 = i_d(16[\Omega]) = 7.218 \text{ [V]}$$

i_3 is the current flowing through the $9[\Omega]$ resistor. This was found earlier to be 0.37596 [A]. However, given the direction of the flow, the sign must be flipped.
Following the sign convention, v_4 is solved to be:

$$v_4 = -i_b(10[\Omega]) = -0.684 \text{ [V]}$$

Take note of the negative sign given the sign convention direction illustrated in the problem statement. Following the sign convention, v_5 is solved to be:

$$v_5 = i_e(4[\Omega]) = 1.203 \text{ [V]}$$

i_6 is the current i_e flowing through the $4[\Omega]$, $16[\Omega]$ and $4[\Omega]$ resistors in series. This was found earlier to be 0.300769 [A]. However, given the direction of the flow, the sign must be flipped.

As a result, the variables of interest are found to be:

$$\boxed{v_1 = -1.367 \text{ [V]}}$$

$$\boxed{v_2 = 7.218 \text{ [V]}}$$

$$\boxed{i_3 = -0.37596 \text{ [A]}}$$

$$\boxed{v_4 = -0.684 \text{ [V]}}$$

$$\boxed{v_5 = 1.203 \text{ [V]}}$$

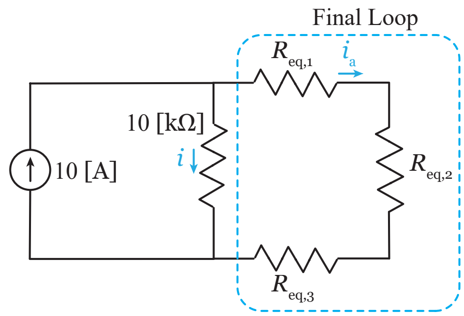
$$\boxed{i_6 = -0.300769 \text{ [A]}}$$

Problem #5

Assume no energy has been dissipated by the circuit prior to t=0 [s]. Determine the total energy dissipated by the circuit at t=5, 10 and 15 [s]. Assume the switch behavior is instantaneous at the times indicated.

Solutions:

To determine the total energy dissipated by the circuit at the specified times, it would be best to determine the equivalent resistances of the circuit at the time of a switch flipping.



Interval: t = 0[s] to t = 7[s]:

The circuit is setup exactly as that of problem 2. Therefore, the equivalent resistances are the same and given as:

$$R_{eq,1} = 1.6364\,[\text{k}\Omega]$$

$$R_{eq,2} = 0\,[\text{k}\Omega]$$

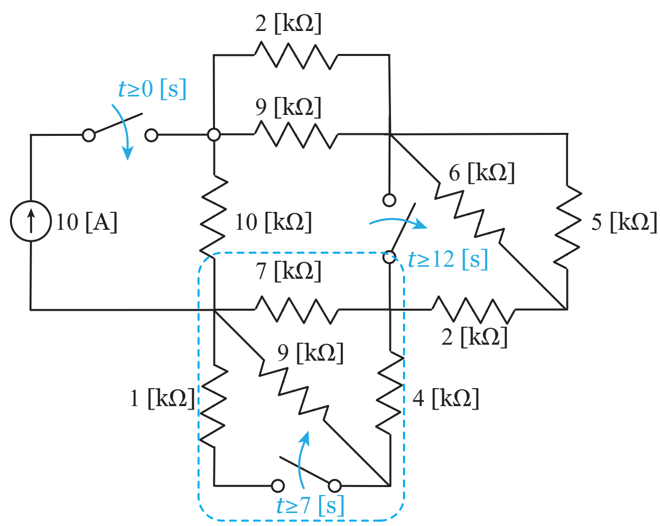
$$R_{eq,3} = 2.8824\,[\text{k}\Omega]$$

The equivalent resistors are in series and then in parallel with the 10[kΩ]. Thus, the final resistance has a value of:

$$R_{eq,f} = 3.1124\,[\text{k}\Omega]$$

The power dissipated by the circuit from 0[s] to 7[s] can be found to be:

$$P_{0\rightarrow 7[s]} = (10\,[\text{A}])^2 R_{eq,f} = 311.2400\,[\text{kW}]$$



Interval: t = 7[s] to t = 12[s]:

In this time interval, $R_{eq,1}$ and $R_{eq,2}$ are the same. $R_{eq,3}$ changes by omitting the 1[kΩ]. In the highlighted loop, the 9[kΩ] is in series with the 4[kΩ] which together is in parallel with the 7[kΩ]. The new $R_{eq,3}$ is found to be:

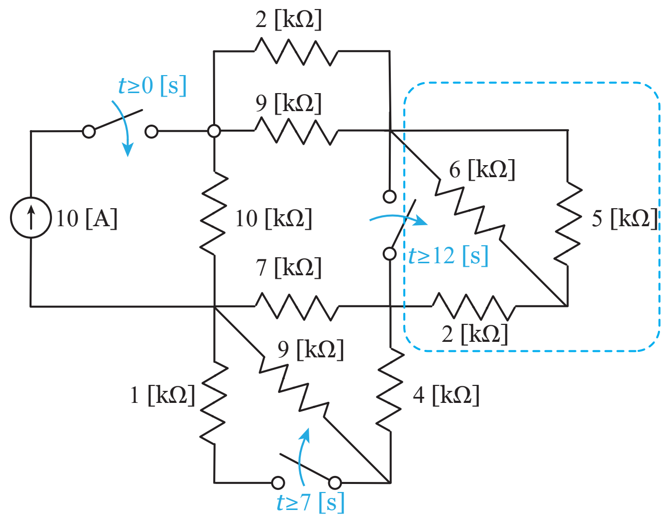
$$R_{eq,3} = 4.5500\,[\text{k}\Omega]$$

The equivalent resistors are in series and then in parallel with the 10[kΩ]. Thus, the final resistance has a value of:

$$R_{eq,f} = 3.8220\,[\text{k}\Omega]$$

The power dissipated by the circuit from 7[s] to 12[s] can be found to be:

$$P_{7\rightarrow 12[s]} = (10\,[\text{A}])^2 R_{eq,f} = 382.1974\,[\text{kW}]$$



Interval: t = 12[s] and on:

In this time interval, $R_{eq,1}$ and $R_{eq,3}$ are the same from the previous time interval. $R_{eq,2}$ changes by adding the 5[kΩ], 6[kΩ] and 2[kΩ] resistor. In the highlighted loop, the 5[kΩ] is in parallel with the 6[kΩ] which together is in series with the 2[kΩ]. The new $R_{eq,2}$ is found to be:

$$R_{eq,2} = 4.7273\,[\text{k}\Omega]$$

The equivalent resistors are in series and then in parallel with the 10[kΩ]. Thus, the final resistance has a value of:

$$R_{eq,f} = 5.2184\,[\text{k}\Omega]$$

The power dissipated by the circuit from 7[s] to 12[s] can be found to be:

$$P_{12[s]\to\infty} = (10[\text{A}])^2 R_{eq,f} = 521.8445\,[\text{kW}]$$

Given these powers, the total energy at 5[s], 10[s] and 15[s] can be calculated.

$$\Delta E_1 = \int_0^5 P_{0\to 7[s]}dt = 311.2400\text{t}\Big|_0^5 = 1,556.2[\text{kJ}]$$

$$\Delta E_2 = \int_0^7 P_{0\to 7[s]}dt + \int_7^{10} P_{7\to 12[s]}dt = 311.2400\text{t}\Big|_0^7 + 382.1974\text{t}\Big|_7^{10} = 3,325.3[\text{kJ}]$$

$$\Delta E_3 = \int_0^7 P_{0\to 7[s]}dt + \int_7^{12} P_{7\to 12[s]}dt + \int_{12}^{15} P_{12[s]\to\infty}dt = 311.2400\text{t}\Big|_0^7 + 382.1974\text{t}\Big|_7^{12} + 521.8445\text{t}\Big|_{12}^{15} = 5,655.2[\text{kJ}]$$

The total energy dissipated by the circuit at the specified times are:

$$t = 5[s] \implies \boxed{1,556.2\,[\text{kJ}]}$$

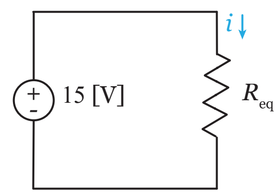
$$t = 10[s] \implies \boxed{3,325.3\,[\text{kJ}]}$$

$$t = 15[s] \implies \boxed{5,655.2\,[\text{kJ}]}$$

Problem #6

For the circuit shown below, determine the voltage v across the load resistance R_L when V_s=15 [V], R_x=100 [Ω], a=0.36 and R_L=150 [Ω]. Determine the power dissipated by each resistor.

Solutions:
The circuit can be reduced as shown. Here the equivalent resistance has a value of:

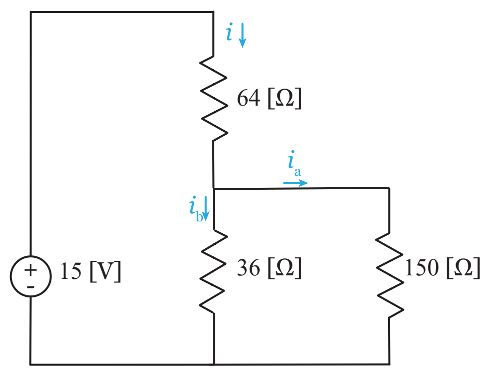


$$R_{eq} = 93.0323\,[\Omega]$$

Therefore, the current i is given as:

$$i = \frac{V}{R_{eq}} = \frac{15\,[\mathrm{V}]}{93.0323\,[\Omega]} = 0.1612\,[\mathrm{A}]$$

It should be noted that the voltage drop across the 36[Ω] and 150[Ω] is the same. Therefore:



$$i_a(150[\Omega]) = i_b(36\,[\Omega])$$

At the same time i_a and i_b sum to equal i. Thus, it can be said that:

$$i = i_a + i_b$$

Given the system of equations, it can be found that:

$$i_a = 0.0312\,[\mathrm{A}]$$

$$i_b = 0.1300\,[\mathrm{A}]$$

Now that the currents through each resistor are found, the power dissipated by each resistor can be found:

$$P_{64[\Omega]} = i^2(64\,[\Omega]) = \boxed{1.663\ [\mathrm{W}]}$$

$$P_{150[\Omega]} = i_a^2(150\,[\Omega]) = \boxed{0.146\ [\mathrm{W}]}$$

$$P_{36[\Omega]} = i_b^2(36\,[\Omega]) = \boxed{0.609\ [\mathrm{W}]}$$