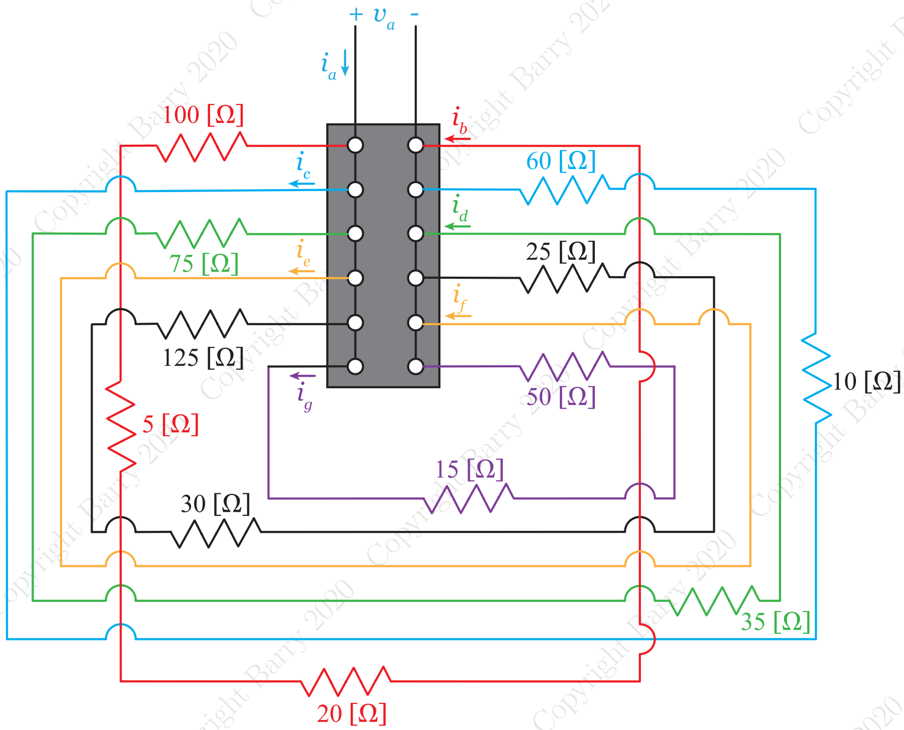
Homework #3

MEMS 0031 - Electrical Circuits

Assigned: May 22nd, 2020
Due: May 27th, 2020 at 11:59 pm

Problem #1

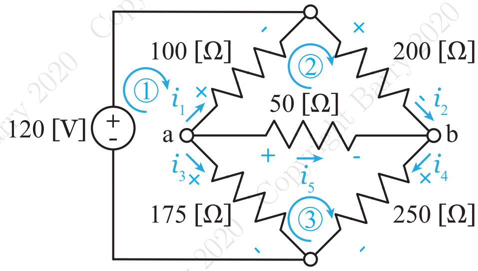
Given the circuit below, determine the currents i_a through i_g, and the voltage drop across each resistor, using the concepts of series and parallel resistors, given $v_a = 240$ [V]. The voltage across each resistor should be denoted in the form of $v_{\mathrm{X}\,[\Omega]}$, where X indicates the numeric value of the resistance.



If we color-code each trace, we see the yellow wire carrying i_e from the positive terminal, which is re-labeled as i_f at the negative terminal, is a short. Thus, no current flows through any of the resistors. Since it is a short, the current flowing through cannot be determined.

Problem #2

Given the circuit below, determine the currents i_1 through i_5, and the voltage drop across each resistor, using the concepts of series and parallel resistors, given a 120 [V] source. The voltage across each resistor should be denoted in the form of $v_{\text{X}\,[\Omega]}$, where X indicates the numeric value of the resistance.



We note that no resistor exists in either a series or parallel configuration with any other resistor. Thus, we must default to applying KCL and KVL to our circuit. Applying KCL at nodes a and b:

$$i_1 + i_3 + i_5 = 0$$

$$i_2 - i_4 + i_5 = 0$$

Applying KVL around loops 1 through 3:

$$-120\,[\text{V}] - (100\,[\Omega])i_1 + (175\,[\Omega])i_3 = 0$$

$$(100\,[\Omega])i_1 + (200\,[\Omega])i_2 - (50\,[\Omega])i_5 = 0$$

$$(-175\,[\Omega])i_3 + (50\,[\Omega])i_5 + (250\,[\Omega])i_4 = 0$$

In matrix form:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 1 \\ -100 & 0 & 175 & 0 & 0 \\ 100 & 200 & 0 & 0 & -50 \\ 0 & 0 & -175 & 250 & 50 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 120 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -0.4638 \\ 0.2427 \\ 0.4207 \\ 0.2858 \\ 0.0431 \end{bmatrix}$$

The minus sign associated with i_1 indicates the current should be going in the opposite direction. Making that adjustment, we can solve for the voltage potentials across each resistor using Ohm's law:

$$V_{100\,[\Omega]} = (100\,[\Omega])(0.4638\,[\text{A}]) = 43.68\,[\text{V}]$$

$$V_{200\,[\Omega]} = (200\,[\Omega])(0.2427\,[\text{A}]) = 48.54\,[\text{V}]$$

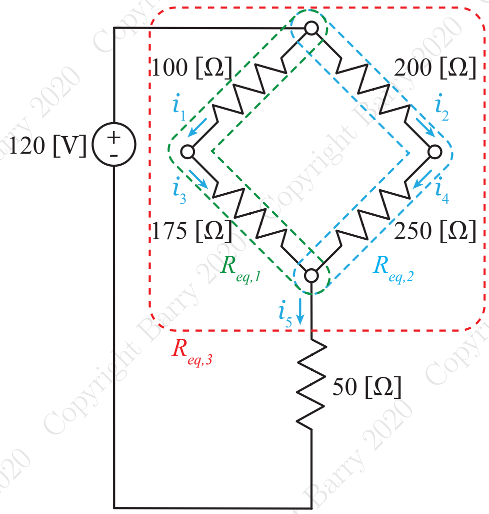
$$V_{175\,[\Omega]} = (175\,[\Omega])(0.4207\,[\text{A}]) = 73.6225\,[\text{V}]$$

$$V_{250\,[\Omega]} = (250\,[\Omega])(0.2858\,[\text{A}]) = 71.45\,[\text{V}]$$

$$V_{50\,[\Omega]} = (50\,[\Omega])(0.0431\,[\text{A}]) = 2.155\,[\text{V}]$$

Problem #3

Given the circuit below, determine the currents i_1 through i_3, and the voltage drop across each resistor, using the concepts of series and parallel resistors, given a 120 [V] source. The voltage across each resistor should be denoted in the form of $v_{X\,[\Omega]}$, where X indicates the numeric value of the resistance.



We recognize the 100 and 175 [Ω] resistors are in series, and the 200 and 250 [Ω] resistors are also in series. Thus, their respective equivalent resistances are:

$$R_{eq,1} = (100 + 175)\,[\Omega] = 275\,[\Omega]$$

$$R_{eq,2} = (200 + 250)\,[\Omega] = 450\,[\Omega]$$

These two equivalent resistors are in parallel, yielding and equivalent resistance of:

$$R_{eq,3} = \frac{(275\,[\Omega])(450\,[\Omega])}{(275 + 450)\,[\Omega]} = 170.7\,[\Omega]$$

This equivalent resistance, $R_{eq,3}$, is in series with the 50 [Ω] resistor, giving a total equivalent resistance of 220.7 [Ω]. Thus, i_5 can be directly solved for via Ohm's law:

$$i_5 = \frac{120\,[\mathrm{V}]}{220.7\,[\Omega]} = 0.544\,[\mathrm{A}]$$

Now, we can use current division to solve for i_1 through i_4. Note the change of direction of i_1 in the figure above. Also note $i_1 = i_3$ and $i_2 = i_4$. Solving for i_1:

$$i_1 = \left(\frac{450\,[\Omega]}{(275 + 450)\,[\Omega]}\right)(0.544\,[\mathrm{A}]) = 0.338\,[\mathrm{A}]$$

Solving for i_2:

$$i_2 = \left(\frac{275\,[\Omega]}{(275 + 450)\,[\Omega]}\right)(0.544\,[\mathrm{A}]) = 0.206\,[\mathrm{A}]$$

Now, solving for the voltage potentials across each resistor, we will use Ohm's law. Note the potential solved for is consistent with the passive sign convention:

$$V_{100\,[\Omega]} = (100\,[\Omega])(0.338\,[\mathrm{A}]) = 33.76\,[\mathrm{V}]$$

$$V_{175\,[\Omega]} = (175\,[\Omega])(0.338\,[\mathrm{A}]) = 59.15\,[\mathrm{V}]$$

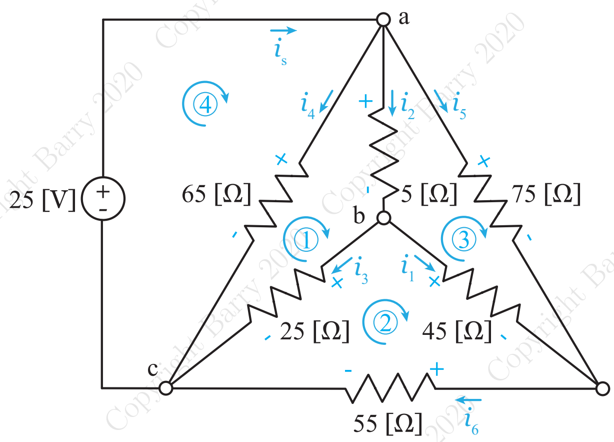
$$V_{200\,[\Omega]} = (200\,[\Omega])(0.206\,[\mathrm{A}]) = 41.2\,[\mathrm{V}]$$

$$V_{250\,[\Omega]} = (250\,[\Omega])(0.206\,[\mathrm{A}]) = 51.5\,[\mathrm{V}]$$

$$V_{50\,[\Omega]} = (50\,[\Omega])(0.544\,[\mathrm{A}]) = 27.2\,[\mathrm{V}]$$

Problem #4

Given the circuit below, determine the currents i_1 through i_6, and the voltage drop across each resistor, using the concepts of series and parallel resistors, given a 25 [V] source. The voltage across each resistor should be denoted in the form of $v_{\text{X [}\Omega\text{]}}$, where X indicates the numeric value of the resistance.



We note that no resistor exists in either a series or parallel configuration with any other resistor. Thus, we must default to applying KCL and KVL to our circuit. Applying KCL at nodes a, b and c:

$$i_s - i_2 - i_4 - i_5 = 0$$

$$-i_1 + i_2 - i_3 = 0$$

$$i_3 + i_4 + i_6 - i_s = 0$$

Since we have seven unknowns and only three equations, we need four more independent equations. Applying KVL to loops 1 through 4:

$$-(65\,[\Omega])i_4 + (5\,[\Omega])i_2 + (25\,[\Omega])i_3 = 0$$

$$(55\,[\Omega])i_6 - (25\,[\Omega])i_3 + (45\,[\Omega])i_1 = 0$$

$$-(45\,[\Omega])i_1 - (5\,[\Omega])i_2 + (75\,[\Omega])i_5 = 0$$

$$-25\,[\text{V}] + (65\,[\Omega])i_4 = 0$$

In matrix form:

$$\begin{bmatrix} 0 & -1 & 0 & -1 & -1 & 0 & 1 \\ -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & -1 \\ 0 & 5 & 25 & -65 & 0 & 0 & 0 \\ 45 & 0 & -25 & 0 & 0 & 55 & 0 \\ -45 & -5 & 0 & 0 & 75 & 0 & 0 \\ 0 & 0 & 0 & 65 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_s \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 25 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_s \end{bmatrix} = \begin{bmatrix} 0.1268 \\ 0.9390 \\ 0.8122 \\ 0.3846 \\ 0.1387 \\ 0.2655 \\ 1.4623 \end{bmatrix}$$

Now, solving for the voltage potentials across each resistor, we will use Ohm's law (all computation was done in MATLAB). Note the potential solved for is consistent with the passive sign convention:

$$\begin{bmatrix} V_{45\,[\Omega]} \\ V_{5\,[\Omega]} \\ V_{25\,[\Omega]} \\ V_{65\,[\Omega]} \\ V_{75\,[\Omega]} \\ V_{55\,[\Omega]} \end{bmatrix} = \begin{bmatrix} 5.7052 \\ 4.6949 \\ 20.3051 \\ 25.0000 \\ 10.4002 \\ 14.5998 \end{bmatrix}$$