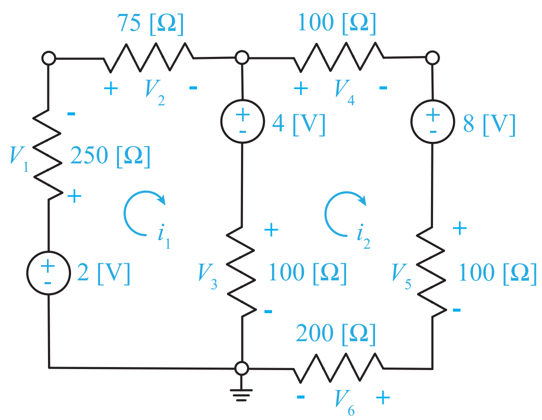
MCA Worksheet

MEMS 0031 - Electrical Circuits

June 3, 2020

Problem #1

Use MCA to find i_1 and i_2 in the circuit shown below.



Step 1: Construct N KVL loops. $N = 2$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

Mesh Current 1:

$$-2\,[\mathrm{V}] + V_1 + V_2 + 4\,[\mathrm{V}] + V_3 = 0$$

Mesh Current 2:

$$-V_3 - 4\,[\mathrm{V}] + V_4 + 8\,[\mathrm{V}] + V_5 + V_6 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Mesh Current 1:

$$-2\,[\mathrm{V}] + (250\,[\Omega])i_1 + (75\,[\Omega])i_1 + 4\,[\mathrm{V}] + (100\,[\Omega])(i_1 - i_2) = 0$$

$$\implies (425\,[\Omega])i_1 - (100\,[\Omega])i_2 = -2\,[\mathrm{V}]$$

Mesh Current 2:

$$-(100\,[\Omega])(i_1 - i_2) - 4\,[\mathrm{V}] + (100\,[\Omega])i_2 + 8\,[\mathrm{V}] + (100\,[\Omega])i_2 + (200\,[\Omega])i_2 = 0$$

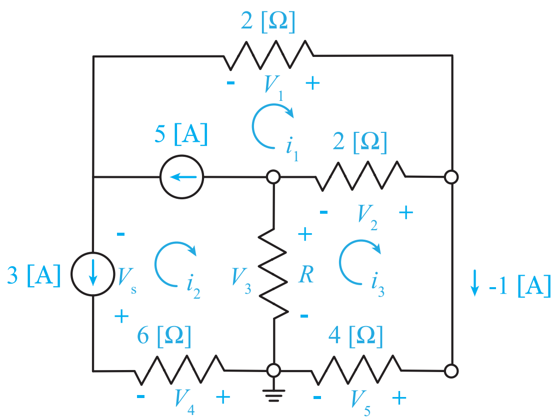
$$\implies -(100\,[\Omega])i_1 + (500\,[\Omega])i_2 = -4\,[\mathrm{V}]$$

In matrix form:

$$\begin{bmatrix} 425 & -100 \\ -100 & 500 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -0.0069 \\ -0.0094 \end{bmatrix}$$

Problem #2

Use MCA to determine the resistance R in the circuit shown below.



Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-$#CS KVL equations, describing each mesh current:

Technically we need 1 KVL equation, however all mesh currents are specified. By inspection:

$$i_2 = -3\,[\text{A}]$$

And

$$i_3 = -1\,[\text{A}]$$

And we have a current source equation:

$$i_1 - i_2 = 5\,[\text{A}] \implies i_1 = 2\,[\text{A}]$$

Therefore the value of the resistance of the resistor can be found by applying KVL around the loop that contains Mesh Current 3:

$$-V_3 - V_2 + V_5 = 0$$

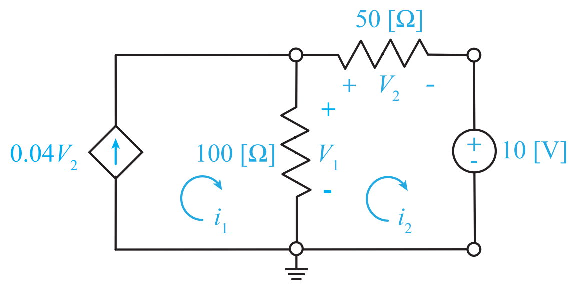
Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents:

$$-R(i_2 - i_3) - (2\,[\Omega])(i_1 - i_3) + (4\,[\Omega])i_3 = 0$$

$$\implies R = \frac{-(2\,[\Omega])(3\,[\text{A}]) + (4\,[\Omega])(-1\,[\text{A}])}{(-2\,[\text{A}])} = 5\,[\Omega]$$

Problem #3

Use MCA to determine the voltage drop across the 50 [Ω] resistor in the circuit shown below.



Step 1: Construct N KVL loops. $N = 2$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

By inspection

$$i_1 = 0.04V_2$$

Mesh Current 2:

$$-V_1 + V_2 + 10\,[\text{V}] = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents:

Mesh Current 1:

$$i_1 = 0.04(50\,[\Omega])i_2 \implies i_1 - 2i_2 = 0$$

Mesh Current 2:

$$-(100\,[\Omega])(i_1 - i_2) + (50\,[\Omega])i_2 + 10\,[\text{V}] = 0$$

$$\implies -(100\,[\Omega])i_1 + (150\,[\Omega])i_2 = -10\,[\text{V}]$$

In matrix form:

$$\begin{bmatrix} 1 & -2 \\ -100 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}$$