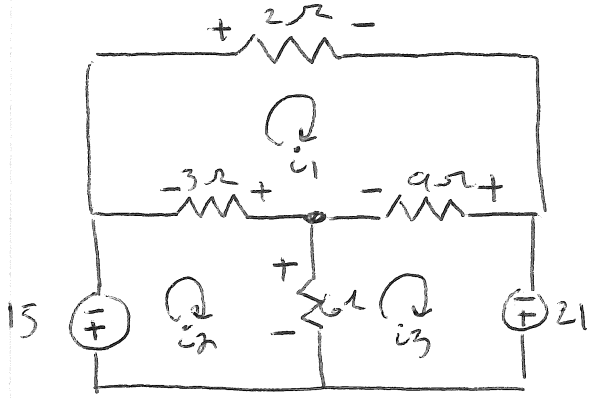
MEMS C031 - Electrical Circuits - HW #3 Solutions:

#1:



MCA to solve i_1, i_2 & i_3:

Mesh eqn 1: $2i_1 + 9(i_1 - i_3) + 3(i_1 - i_2) = 0$

$\rightarrow 14i_1 - 3i_2 - 9i_3 = 0$

Mesh eqn 2: $-3(i_1 - i_2) + 6(i_2 - i_3) + 15 = 0$

or $3(i_2 - i_1) + 6(i_2 - i_3) + 15 = 0$

$\rightarrow -3i_1 + 9i_2 - 6i_3 = -15$

Mesh eqn. 3: $-6(i_2 - i_3) - 9(i_1 - i_3) - 21 = 0$

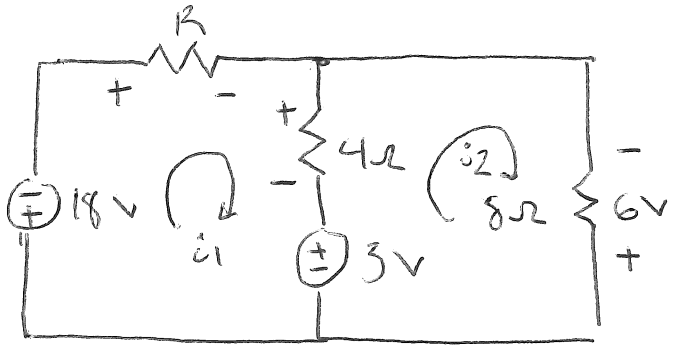
or $6(i_3 - i_2) + 9(i_3 - i_1) - 21 = 0$

$\rightarrow -9i_1 - 6i_2 + 15i_3 = 21$

Construct an array:

$$\begin{bmatrix} 14 & -3 & -9 \\ -3 & 9 & -6 \\ -9 & -6 & 15 \end{bmatrix} \begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -15 \\ 21 \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} i_1 \\ i_2 \\ i_3 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 2 \\ 4 \end{Bmatrix}}$$

#2:



MCA to solve for R

Mesh eqn 1: $18 + i_1 R + 4(i_1 - i_2) + 3 = 0$

$\rightarrow i_1(4+R) - 4i_2 = -21$

Mesh eqn 2: $-3 - 4(i_1 - i_2) - 6 = 0$

$\rightarrow -4i_1 + 4i_2 = 9$

But, $-6\,[V] = i_2 \cdot 8\,[\Omega]$ from Ohm's law

$\therefore i_2 = -6/8 = -0.75\,[A]$

$\therefore -4i_1 + 4(-3/4) = 9 \rightarrow -4i_1 - 3 = 9$

$i_1 = 12/-4 = -3\,[A]$

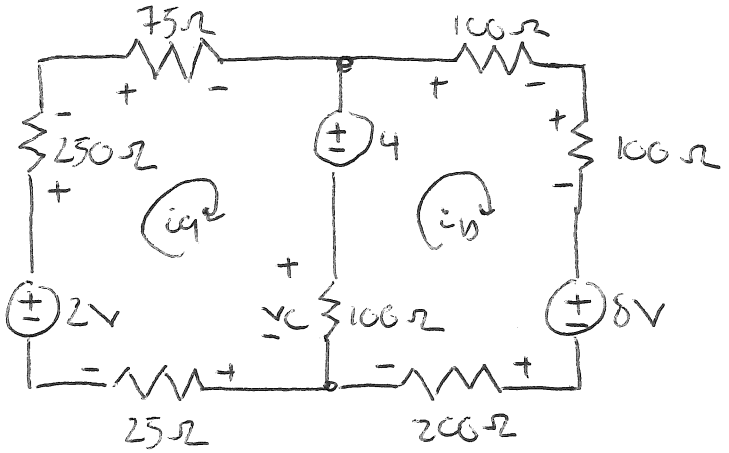
$\therefore -3(4+R) - 4(-3/4) = -21$

$-12 - 3R + 3 = -21$

$-3R = -12$

$\boxed{R = 4\,[\Omega]}$

#3.



MCA to find i_a, i_b & V_c

Mesh eqn a: $-2 + 250 i_a + 75 i_a + 4 + 100(i_a - i_b) + 25 i_a = 0$

$\rightarrow 450 i_a - 100 i_b = -2$

Mesh eqn b: $100(i_b - i_a) - 4 + 100 i_b + 100 i_b + 8 + 200 i_b = 0$

$\rightarrow -100 i_a + 500 i_b = -4$

Construct an array

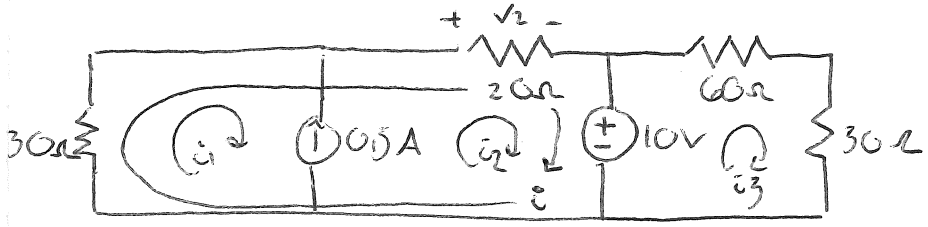
$$\begin{bmatrix} 450 & -100 \\ -100 & 500 \end{bmatrix} \begin{Bmatrix} i_a \\ i_b \end{Bmatrix} = \begin{Bmatrix} -2 \\ -4 \end{Bmatrix} \rightarrow \begin{Bmatrix} i_a \\ i_b \end{Bmatrix} = \begin{Bmatrix} -0.0063 \\ -0.0093 \end{Bmatrix}$$

$\therefore$ $V_c = 100(i_a - i_b)$

$= 100(-0.0065 + 0.0093)$

$\therefore V_c = 0.28\ [V]$

#4:



MCA to solve for V_2:

CS eqn: $0.5[A] = i_2 - i_1 \rightarrow i_1 = i_2 - 0.5$

Mesh eqn i: $30i_1 + 20i_2 + 10 = 0$

$\rightarrow 30i_1 + 20i_2 = -10$

$\therefore 30(i_2 - 0.5) + 20i_2 = -10$

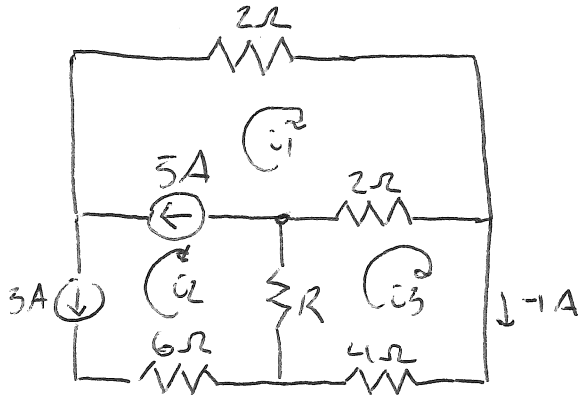
$30i_2 - 15 = -10$

$50i_2 = 5$

$i_2 = 0.1[A]$

$\therefore V_2 = i_2 \cdot 20\Omega = \boxed{2[V] = V_2}$

#5:



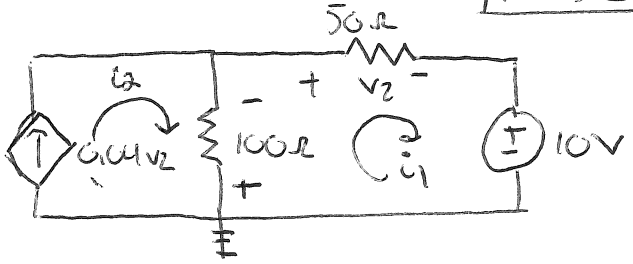
MCA to find R

CS eqn:
$5[A] = i_1 - i_2$

Given: $i_2 = -3\,[A]$
$i_3 = -1\,[A]$
$\therefore i_1 = 5[A] + i_2 = 2[A]$

Mesh eqn 3: $4i_3 + R(i_3 - i_2) + 2(i_3 - i_1) = 0$
$\rightarrow -4 + R(-1+3) + 2(-1-2) = 0$
$\rightarrow -4 + R(2) - 6 = 0$
$2R = 10$
$\boxed{R = 5\,[\Omega]}$

#6:



MCA to find v_2

Ohm's Law: $v_2 = 50\,i_1$
CS eqn: $i_2 = 0.04\,v_2$

Mesh eqn 1: $100(i_1 - i_2) + 50i_1 + 10 = 0$
$\rightarrow 100(i_1 - 0.04v_2) + 50\,i_1 = -10$
$\rightarrow 100(i_1 - 0.04(50\,i_1)) + 50\,i_1 = -10$
$\rightarrow 100\,i_1 - 200\,i_1 + 50\,i_1 = -10$
$\rightarrow -50\,i_1 = -10 \rightarrow \boxed{i_1 = 1/5 = 0.2\,[A]}$

$\therefore v_2 = 50\,i_1 = 50(1/5) = \boxed{10\,[V] = v_2}$