Chapter 7 - Energy Storage Elements

Lecture 23

Sections 7.5-7.7

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

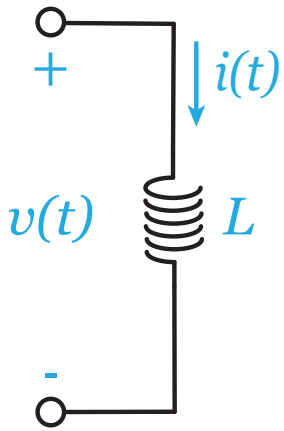
Student Learning Objectives

At the end of the lecture, students should be able to:

- Analyze the behavior of a inductor as a function of time
- Construct an expression for the behavior of inductors connected in series and parallel

Anatomy of Inductors

- A conductor is schematically shown as follows:



  - An insulated wire is wrapped around a magnetic core
  - Energy is stored in the generated magnetic field

- The strength of the magnetic field is proportional to N - the number of turns of the wire
- The inductance L has units of a henry [H]

Inductor Constitutive Equation

- The voltage potential across the inductor is a function of L and $i(t)$

$$v(t) = L\frac{d\,i(t)}{dt}$$

- Thus, the current flowing through an inductor is

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)\,d\tau$$

- For the time interval $-\infty < t \leq 0$

$$i(t) = i(t_0) + \frac{1}{L}\int_{0}^{t} v(\tau)\,d\tau$$

- It is noted that since $i(t)$ is bounded, $v(t)$ must be continuous

Comparison to Capacitors

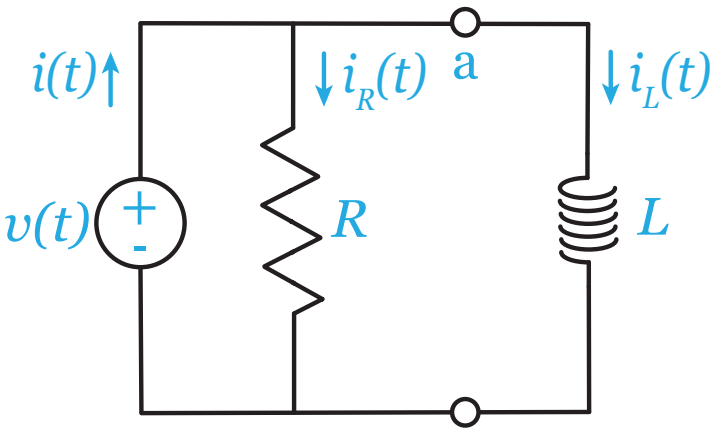
- If we compare the constitutive equations for a capacitor and inductor:

$$i(t) = C\frac{d}{dt}v(t) \qquad\qquad v(t) = L\frac{d}{dt}i(t)$$

- These equations, coupled with Ohm's law, gives a complete description of the passive devices covered in this course

Example #1

- The input to the circuit is $v(t)$=4exp(-20t) [V] for $t > 0$. The output current is $i(t)$=-1.2exp(-20t)-1.5 [A] for $t > 0$. The initial inductor current is $i_L(0)$=-3.5 [A]. Determine the inductance L and resistance R of the circuit.



Example #1

Example #1

Energy Stored in an Inductor

- Recall energy is the time integral of power. Power is expressed as

$$P(t) = v(t)i(t)$$

- Substituting in $v(t)$ for an inductor

$$P(t) = \left(L\frac{d\,i(t)}{dt}\right)i(t)$$

- The energy in an inductor is then

$$E_I = \int_{t_0}^{t} P(\tau)\,d\tau = L\int_{t_0}^{t} \frac{d\,i(\tau)}{d\tau} i(\tau)\,d\tau$$

$$= L\int_{t_0}^{t} i(\tau)d\,i(\tau) = \frac{L}{2}\left(i^2(t) - i^2(t_0)\right)$$

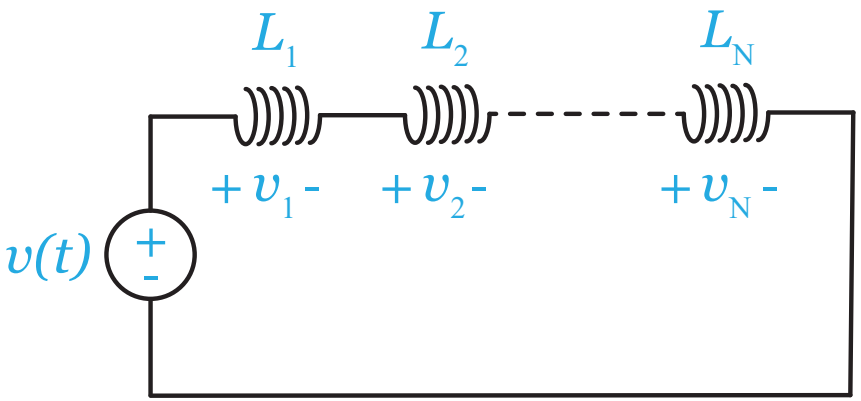
$$\therefore E_I(t) = \frac{Li^2(t)}{2}$$

Example #2

- Find the power and energy stored in a 0.1 [H] inductor given $i(t)$=20(t)exp(-2t) [A] and $v(t)$=2exp(-2t)(1-2t) [V] for $t \geq 0$, if $i(t)$=0 for $t < 0$

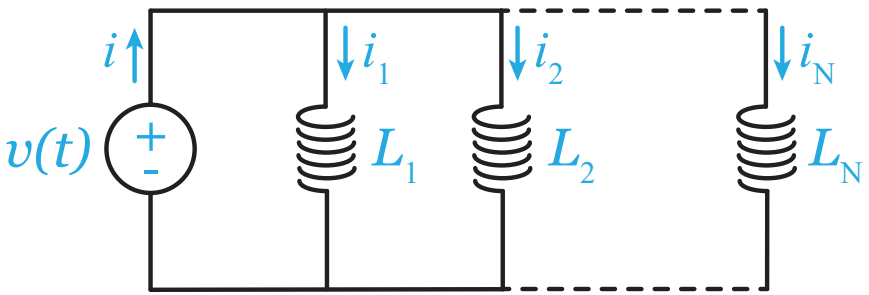
Inductors in Series

- Consider the following. What is the equivalent inductance?



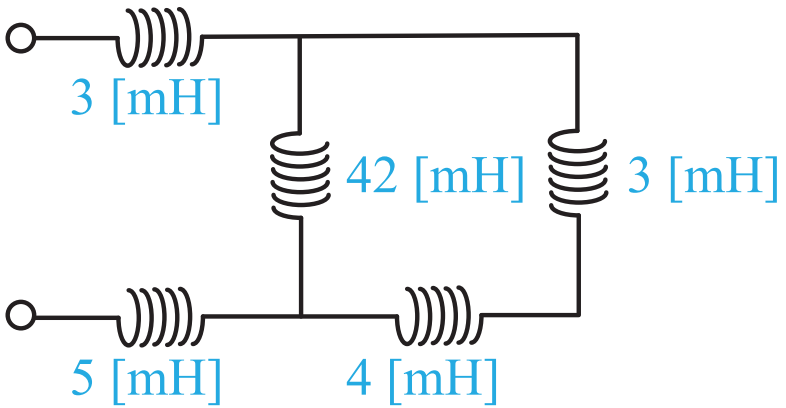
Inductors in Parallel

- Consider the following. What is the equivalent inductance?



Example #3

- Determine the equivalent inductance



Student Learning Objectives

At the end of the lecture, students should be able to:

- Analyze the behavior of a inductor as a function of time
  - The voltage potential across an inductor is the inductance (ability to store charge) times the time rate of change of current running through the inductor
- Construct an expression for the behavior of inductors connected in series and parallel
  - Inductors in series sum arithmetically; an equivalent inductance of inductors in parallel is the reciprocal of the sum of the reciprocals of the individual inductances

Suggested Problems

- 7.5-1, 7.5-2, 7.5-3, 7.5-4, 7.5-6, 7.5-12, 7.5-13, 7.5-14, 7.6-1, 7.6-2, 7.7-1, 7.7-3, 7.7-5, 7.7-7, 7.7-10