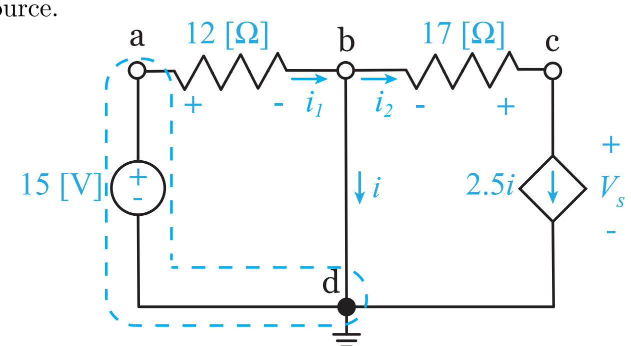
Homework #5

MEMS 0031 - Electrical Circuits

Assigned Due February 8$^{\text{th}}$, 2019
Due February 15$^{\text{th}}$, 2019

Problem #1

Use Node Voltage Analysis (NVA) to determine the voltage potential V_s across the current controlled current source.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 3$ since node b is connected to ground by a shorted wire, i.e. $V_b = V_d = 0$. We define i_1 and i_2 as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 - \#\text{VS}$ KCL equations, applied at non-zero nodes:

Independent VS:

$$V_a = 15\,[\text{V}] \tag{1}$$

KCL at node b:

$$i_1 = i + i_2$$

KCL at node c:

$$i_2 = 2.5i$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at node b:

$$\frac{V_a}{12[\Omega]} = i - \frac{V_c}{17[\Omega]} \tag{2}$$

KCL at node c:

$$-\frac{V_c}{17[\Omega]} = 2.5i \tag{3}$$

Solving eqns. 2 and 3 for current i and setting them equal to each other yields:

$$-\left(\frac{1}{2.5}\right)\frac{V_c}{17[\Omega]} = \frac{V_a}{12[\Omega]} + \frac{V_c}{17[\Omega]} \implies -\left(\frac{1}{2.5}\right)\frac{V_c}{17[\Omega]} = \frac{15\,[\text{V}]}{12[\Omega]} + \frac{V_c}{17[\Omega]} \tag{4}$$

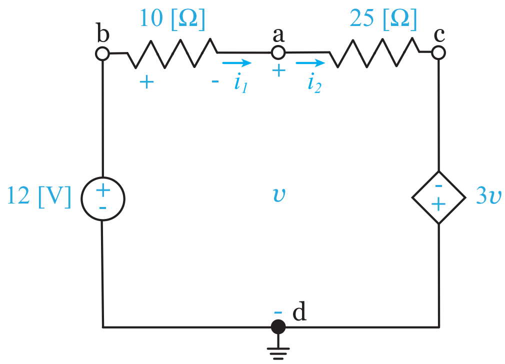
$$V_c = -15.179\,[\text{V}]$$

Finally, solving for V_s:

$$V_s = V_c - \cancelto{0}{V_d} = -15.179\,[\text{V}]$$

Problem #2

Use Node Voltage Analysis (NVA) to determine the node voltage at a.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 4$ and we define i_1 and i_2 as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1$ KCL equations, applied at non-zero nodes:

KCL at node a:

$$i_1 = i_2$$

12 [V] source:

$$V_b = 12\,[\mathrm{V}] \tag{5}$$

VCVS:

$$3v = \cancelto{0}{V_d} - V_c \implies 3(V_a - \cancelto{0}{V_d}) = -V_c \implies 3V_a + V_c = 0 \tag{6}$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at node a:

$$\frac{V_b - V_a}{10\,[\Omega]} = \frac{V_a - V_c}{25\,[\Omega]} \implies -7V_a + 5V_b + 2V_c = 0 \tag{7}$$

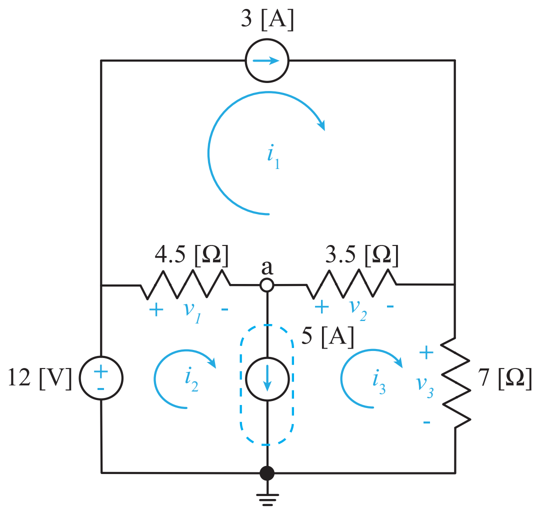
Putting eqns. 5-7 in matrix form:

$$\begin{bmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ -7 & 5 & 2 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 4.615 \\ 12 \\ -13.846 \end{bmatrix}$$

Units are taken as [V].

Problem #3

Determine currents i_1, i_2, and i_3 using Mesh Current Analysis (MCA).



Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

3 [A] current source:

$$i_1 = 3\,[\mathrm{A}] \tag{8}$$

Supermesh equation:

$$i_2 - i_3 = 5\,[\mathrm{A}] \tag{9}$$

Supermesh 1:

$$-12\,[\mathrm{V}] + V_1 + V_2 + V_3 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh:

$$-12\,[\mathrm{V}] + (4.5\,[\Omega])(i_2 - i_1) + (3.5\,[\Omega])(i_3 - i_1) + (7\,[\Omega])i_3 = 0$$

$$\implies -8i_1 + 4.5i_2 + 10.5i_3 = 12 \tag{10}$$

Putting eqns. 8 through 10 in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -8 & 4.5 & 10.5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 5.9 \\ 0.9 \end{bmatrix}$$

Units are taken as [A].

Problem #4

Use Mesh Current Analysis (MCA) to find the total power developed in the circuit in Fig. 1

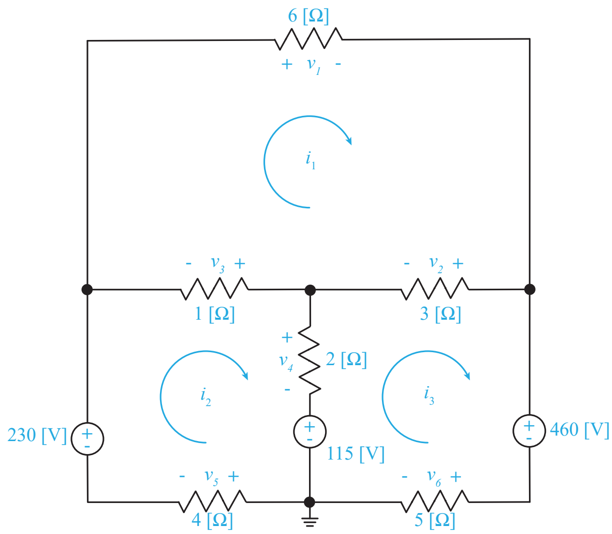


Figure 1: Schematic for Problem #4.

Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#\text{CS}$ KVL equations, describing each mesh current:

Mesh current 1:

$$V_1 + V_2 + V_3 = 0$$

Mesh current 2:

$$-230\,[\text{V}] - V_3 + V_4 + 115\,[\text{V}] + V_5 = 0$$

Mesh current 3:

$$-115\,[\text{V}] - V_4 - V_2 + 460\,[\text{V}] + V_6 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Mesh current 1:

$$(6\,[\Omega])i_1 + (3\,[\Omega])(i_1 - i_3) + (1\,[\Omega])(i_1 - i_2)$$

$$\implies 10i_1 - i_2 - 3i_3 = 0 \tag{11}$$

Mesh current 2:

$$-(1\,[\Omega])(i_1 - i_2) + (2\,[\Omega])(i_2 - i_3) + (4\,[\Omega])i_2 = 115\,[\text{V}]$$

$$\implies -i_1 + 7i_2 - 2i_3 = 115 \tag{12}$$

Mesh current 3:

$$-(2\,[\Omega])(i_2 - i_3) - (3\,[\Omega])(i_1 - i_3) + (5\,[\Omega])i_3 = -345\,[\text{V}]$$

$$\implies -3i_1 - 2i_2 + 10i_3 = -345 \tag{13}$$

Putting eqns. 11 through 13 in matrix form:

$$\begin{bmatrix} 10 & -1 & -3 \\ -1 & 7 & -2 \\ -3 & -2 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 115 \\ -345 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -10.6 \\ 4.4 \\ -36.8 \end{bmatrix}$$

Units are taken as [A].

Now that the mesh currents are found flowing through every resistor in the circuit, the power dissipated by them can be determined:

$$\mathrm{P} = \sum_{j=1}^{6} (i_j)^2 (R_j) = (i_1)^2(6[\Omega]) + (i_2 - i_1)^2(1[\Omega]) + (i_2 - i_3)^2(2[\Omega]) + (i_2)^2(4[\Omega]) + (i_3 - i_1)^2(3[\Omega]) + (i_3)^2(5[\Omega]) \implies$$

$$\mathrm{P} = \sum_{j=1}^{6} (i_j)^2 (R_j) = (-10.6\ [\mathrm{A}])^2(6[\Omega]) + (4.4\ [\mathrm{A}] - -10.6\ [\mathrm{A}])^2(1[\Omega]) + (4.4\ [\mathrm{A}] - -36.8\ [\mathrm{A}])^2(2[\Omega]) + ...$$

$$(4.4\ [\mathrm{A}])^2(4[\Omega]) + (-36.8\ [\mathrm{A}] - -10.6\ [\mathrm{A}])^2(3[\Omega]) + (-36.8\ [\mathrm{A}])^2(5[\Omega]) \implies$$

$$P = \boxed{13.202\ [\mathrm{kW}]}$$

Problem #5

Use Mesh Current Analysis (MCA) to find the power dissipated in the 1 [kΩ] resistor.

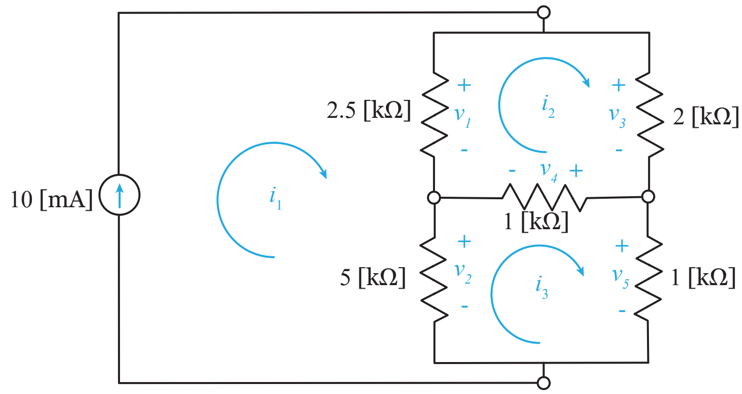


Figure 2: Schematic for Problem #5.

Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

10 [mA] current source:

$$i_1 = 10\,[\text{mA}] \tag{14}$$

Mesh current 2:

$$-V_1 + V_3 + V_4 = 0$$

Mesh current 3:

$$-V_2 - V_4 + V_5 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.
Mesh current 2:

$$-(2.5\,[\text{k}\Omega])(i_1 - i_2) + (2\,[\text{k}\Omega])i_2 + (1\,[\text{k}\Omega])(i_2 - i_3) = 0$$
$$\implies -2.5i_1 + 5.5i_2 - i_3 = 0 \tag{15}$$

Mesh current 3:

$$-(5\,[\text{k}\Omega])(i_1 - i_3) - (1\,[\text{k}\Omega])(i_2 - i_3) + (1\,[\text{k}\Omega])i_3 = 0$$
$$\implies -5i_1 - i_2 + 7i_3 = 0 \tag{16}$$

Putting eqns. 14 through 16 in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 \\ -2.5 & 5.5 & -1 \\ -5 & -1 & 7 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 8 \end{bmatrix}$$

Units are taken as [mA].

To determine the power dissipated by the middle 1 [kΩ] resistor, the current flowing through it must be determined. This current is the difference of the mesh currents from mesh 3 and 2 respectively. This yields:

$$P_{1[\text{k}\Omega]} = (i_3 - i_2)^2(1[\text{k}\Omega]) = \boxed{4\ [\text{mW}]}$$