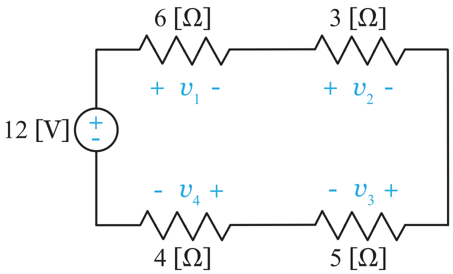
MEMS 0031 - Electrical Circuits
Quiz #3
May 29$^{\text{th}}$, 2019
90 points

Name: ______________________________

Problem #1

(15 pts.) Use voltage division for series resistors to determine the voltages v_1 through v_4 for the circuit shown below.



Recalling the formula for voltage division between resistors in a series formation, we can solve for v_1 through v_4 by multiplying the voltage source by the resistor whose voltage drop we are interested in divided by the equivalent resistance:

$$v_1 = (12\,[\text{V}])\left(\frac{6\,[\Omega]}{18\,[\Omega]}\right) = 4\,[\text{V}]$$

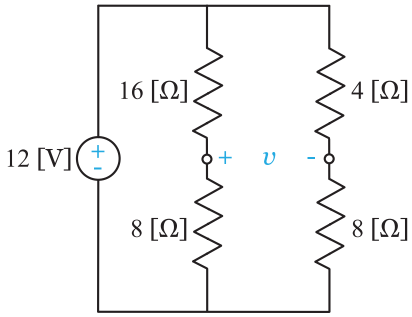
$$v_2 = (12\,[\text{V}])\left(\frac{3\,[\Omega]}{18\,[\Omega]}\right) = 2\,[\text{V}]$$

$$v_3 = (12\,[\text{V}])\left(\frac{5\,[\Omega]}{18\,[\Omega]}\right) = 3.33\,[\text{V}]$$

$$v_4 = (12\,[\text{V}])\left(\frac{4\,[\Omega]}{18\,[\Omega]}\right) = 2.67\,[\text{V}]$$

Problem #2

(15 pts.) Determine the voltage drop v across the circuit shown below.



In order to find the voltage potential across the circuit we need to determine the voltage drop after the 16 and 4 [Ω] resistors. We know that resistors in parallel have the same voltage and resistors in series split the supplied voltage. Therefore, we know the 16 and 8 [Ω] resistors share 12 [V] as do the 4 and 8 [Ω] resistors. So, using the same method used in Problem #1 we can find the voltage drop after the the 16 and 4 [Ω] resistors:

$$\mathrm{R}_{eq1} = 16\,[\Omega] + 8\,[\Omega] = 24\,[\Omega]$$

$$\mathrm{R}_{eq2} = 4\,[\Omega] + 8\,[\Omega] = 12\,[\Omega]$$

$$v_{16\,[\Omega]} = (12\,[\mathrm{V}])\left(\frac{16\,[\Omega]}{24\,[\Omega]}\right) = 8\,[\mathrm{V}]$$

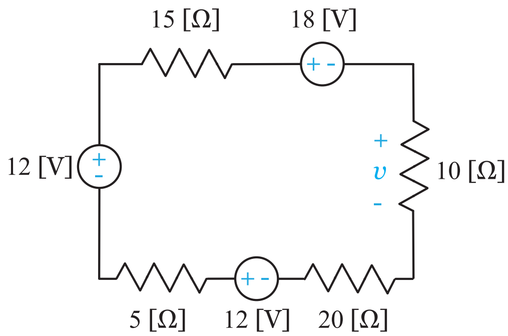
$$v_{4\,[\Omega]} = (12\,[\mathrm{V}])\left(\frac{4\,[\Omega]}{12\,[\Omega]}\right) = 4\,[\mathrm{V}]$$

Therefore, the voltage drop v is the difference between $v_{16\,[\Omega]}$ $v_{4\,[\Omega]}$:

$$v = (8 - 4)\,[\mathrm{V}] = 4\,[\mathrm{V}]$$

Problem #3

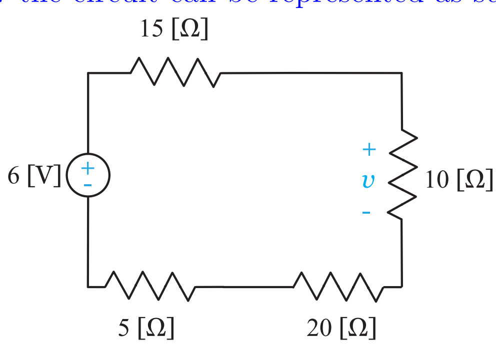
(15 pts.) Determine the voltage drop v across the 10 [Ω] resistor in the circuit shown below.



This problem is similar in nature to Problem #1. However, in this case we must add together the multiple voltage sources and represent it as a single source before we can do voltage division. Taking careful consideration of direction of the voltage sources and adding them up in a counter clockwise direction we end up with the following:

$$V_{total} = -12\,[\mathrm{V}] + 18\,[\mathrm{V}] - 12\,[\mathrm{V}] = -6\,[\mathrm{V}]$$

Now the circuit can be represented as such:

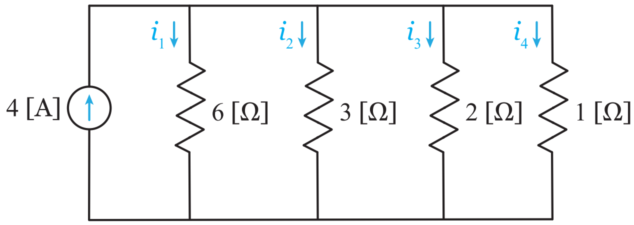


Finally, with the updated circuit representation, we can find the voltage across the 10 [Ω] resistor:

$$v = (6\,[\mathrm{V}])\left(\frac{10\,[\Omega]}{50\,[\Omega]}\right) = 1.2\,[\mathrm{V}]$$

Problem #4

(15 pts.) Determine the branch currents i_1 through i_4 in the circuit shown below.



Here, we see 4 [A] of current being split between four resistors in parallel. Recalling the formula for splitting current between parallel resistors (the current through one resistor is equal to the total current supplied to the circuit multiplied by the equivalent resistance over the resistor of interest):

$$\mathrm{R}_{eq} = \left(\frac{1}{6\,[\Omega]} + \frac{1}{3\,[\Omega]} + \frac{1}{2\,[\Omega]} + \frac{1}{1\,[\Omega]}\right)^{-1} = 0.5\,[\Omega]$$

$$i_1 = (4\,[\mathrm{A}])\left(\frac{0.5\,[\Omega]}{6\,[\Omega]}\right) = 0.333\,[\mathrm{A}]$$

$$i_2 = (4\,[\mathrm{A}])\left(\frac{0.5\,[\Omega]}{3\,[\Omega]}\right) = 0.667\,[\mathrm{A}]$$

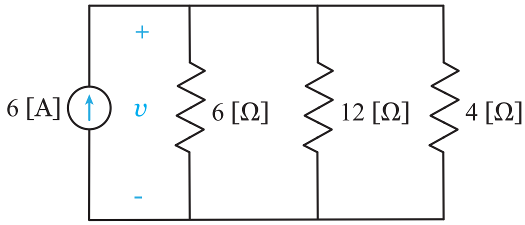
$$i_3 = (4\,[\mathrm{A}])\left(\frac{0.5\,[\Omega]}{2\,[\Omega]}\right) = 1\,[\mathrm{A}]$$

$$i_4 = (4\,[\mathrm{A}])\left(\frac{0.5\,[\Omega]}{1\,[\Omega]}\right) = 2\,[\mathrm{A}]$$

Finally, as a sanity check, we can see if all the currents add up to 4 [A]. They do!

Problem #5

(15 pts.) Determine the voltage drop v across resistor network in the circuit shown below.



This problem is similar to Problem #4 in that we need to find the current running through the parallel resistors. In this case, however, since resistors in parallel all have the same voltage potential, we only need to determine the current across one resistor:

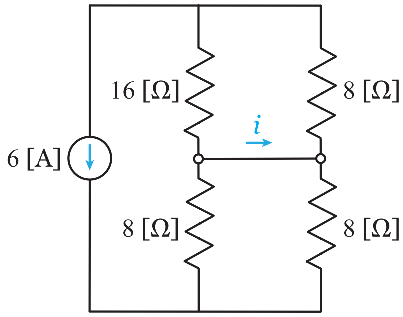
$$\mathrm{R}_{eq} = \left(\frac{1}{6\,[\Omega]} + \frac{1}{12\,[\Omega]} + \frac{1}{4\,[\Omega]}\right)^{-1} = 2\,[\Omega]$$

$$i_{6\,[\Omega]} = (6\,[\mathrm{A}])\left(\frac{2\,[\Omega]}{6\,[\Omega]}\right) = 2\,[\mathrm{A}]$$

$$v = (2\,[\mathrm{A}])(6\,[\Omega]) = 12\,[\mathrm{V}]$$

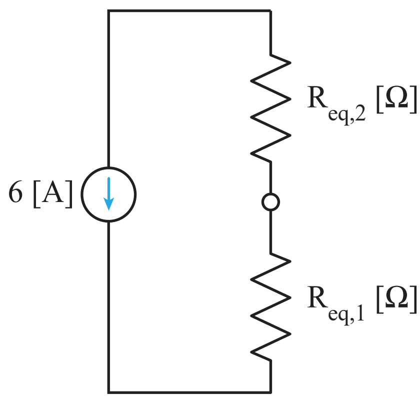
Problem #6

(15 pts.) Determine the current i in the circuit shown below.



We see that the bottom two 8 [Ω] resistors are in parallel with each other, and the top 16 [Ω] and 8 [Ω] resistors are in parallel with each other. Thus, the circuit can be simplified as shown where $R_{eq,1} = 4$ [Ω], and $R_{eq,2} = 5.33$ [Ω].

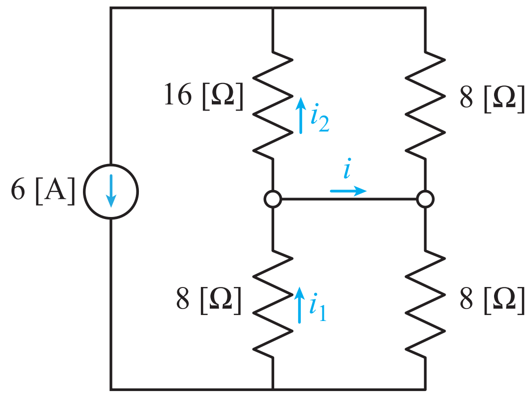
It is clear that 6 [A] flows through each equivalent resistance, and thus can be taken as the source current when using voltage division.



Expanding the circuit back out and using current division, we find currents i_1 and i_2 as follows:

$$i_1 = \left(\frac{(1/8)\,[\mathrm{S}]}{((1/8)+(1/8))\,[\mathrm{S}]}\right)(6\,[\mathrm{A}]) = 3\,[\mathrm{A}]$$

$$i_2 = \left(\frac{(1/16)\,[\mathrm{S}]}{((1/16)+(1/8))\,[\mathrm{S}]}\right)(6\,[\mathrm{A}]) = 2\,[\mathrm{A}]$$



Applying KCL at the node between the 16 [Ω] and 8 [Ω] resistor, we can solve for current i:

$$i_1 = i + i_2 \implies i = i_1 - i_2 = 1 \text{ [A]}$$