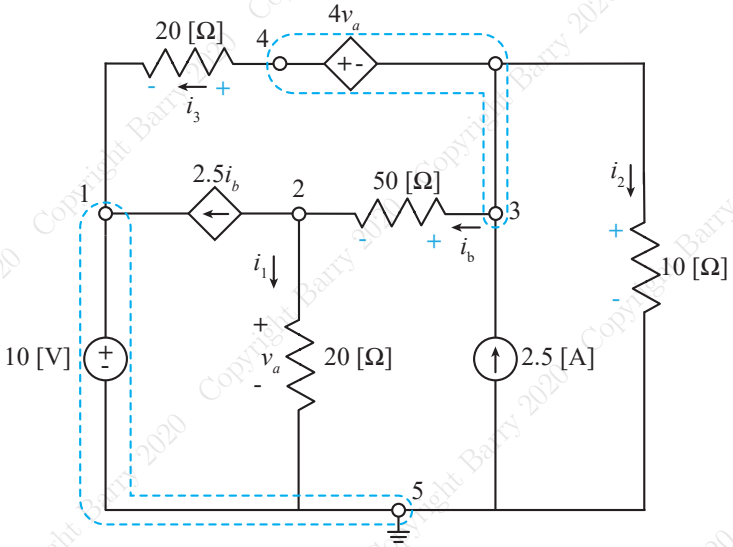
MEMS 0031 - Electrical Circuits
Quiz #3
Assigned: June 4th, 2020
Due: June 7th, 2020, 9:00 pm

Name: _____

Problem #1

Using Node Voltage Analysis, determine the branch currents i_1 through i_3, and i_b.



Step 1: Assign nodes (N) and leg currents to all branches/elements: $N = 5$ and all currents are defined as shown.

Step 2: Assign voltage potential consistent with PSC: Voltage potentials assigned as shown.

Step 3: $N - 1 -$ #VS KCL equations, applied at non-zero nodes:

Supernode equation:

$$V_4 - V_3 = 4V_a \implies 4V_2 + V_3 - V_4 = 0 \tag{1}$$

Independent VS:

$$V_1 = 10\,[\text{V}] \tag{2}$$

KCL at supernode:

$$i_b + i_2 + i_3 = 2.5\,[\text{A}]$$

KCL at node 2:

$$i_1 + 1.5i_b = 0$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at supernode:

$$\frac{V_3 - V_2}{50\,[\Omega]} + \frac{V_3}{10\,[\Omega]} + \frac{V_4 - V_1}{20\,[\Omega]} = 2.5\,[\text{A}] \implies -5V_1 - 2V_2 + 12V_3 + 5V_4 = 250 \tag{3}$$

KCL at node 2:

$$\frac{V_2}{20\,[\Omega]} + 1.5\left(\frac{V_3 - V_2}{50\,[\Omega]}\right) = 0 \implies 2V_2 + 3V_3 = 0 \tag{4}$$

Putting eqns. 1-4 in matrix form, with solution units taken as [V] and [A].:

$$\begin{bmatrix} 0 & 4 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ -5 & -2 & 12 & 5 \\ 0 & 2 & 3 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 250 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 45 \\ -30 \\ 150 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_b \end{bmatrix} = \begin{bmatrix} 2.25 \\ -3 \\ 7 \\ -1.5 \end{bmatrix}$$