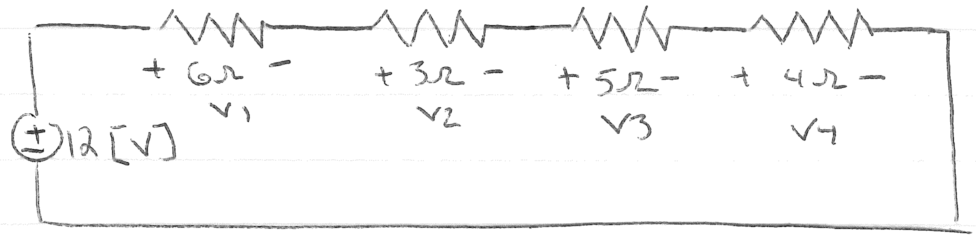
MEMS 0031 - Electrical Circuits - Homework #3 Solutions

#1:



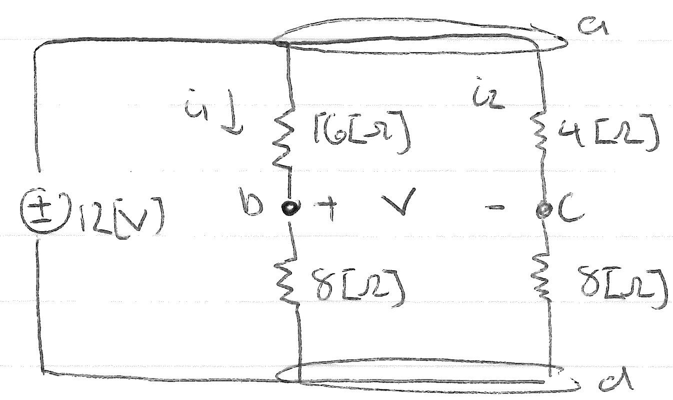
$$v_1 = \left(\frac{6\,[\Omega]}{6+3+5+4\,[\Omega]}\right) 12\,[V] = 4\,[V] = v_1$$

$$v_2 = \left(\frac{3\,[\Omega]}{6+3+5+4\,[\Omega]}\right) 12\,[V] = 2\,[V] = v_2$$

$$v_3 = \left(\frac{5\,[\Omega]}{6+3+5+4\,[\Omega]}\right) 12\,[V] = 3\tfrac{1}{3}\,[V] = v_3$$

$$v_4 = \left(\frac{4\,[\Omega]}{6+3+5+4\,[\Omega]}\right) 12\,[V] = 2\tfrac{2}{3}\,[V] = v_4$$

#2:



Two approaches, we will take the simplest of KCL

i_1: $\dfrac{V_a - V_b}{16\Omega} = \dfrac{V_b - V_d}{8\Omega}$ i_2: $\dfrac{V_a - V_c}{4\Omega} = \dfrac{V_c - V_d}{8\Omega}$

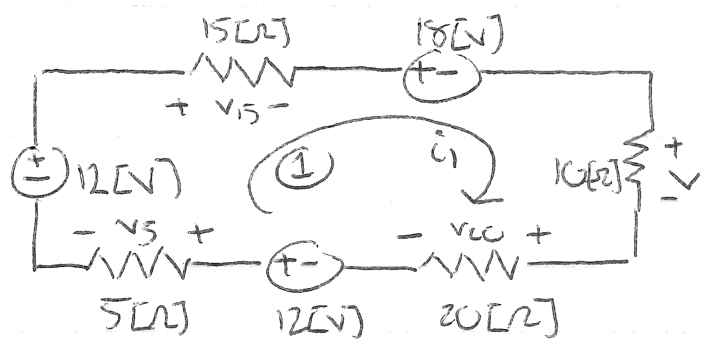
$V_A = 12\,[V]$, $V_d = 0\,[V]$

$$\frac{12\,[V]}{16\,[\Omega]} - \frac{V_b}{16\,[\Omega]} = \frac{V_b}{8\,[\Omega]} \qquad \frac{12\,[V]}{4\,[\Omega]} - \frac{V_c}{4\,[\Omega]} = \frac{V_c}{8\,[\Omega]}$$

$$\rightarrow \frac{12\,[V]}{16\,[\Omega]} = V_b\left(\frac{3}{16\,[\Omega]}\right) \qquad \rightarrow 3\left[\frac{V}{\Omega}\right] = V_c\left[\frac{3}{8\,[\Omega]}\right] \rightarrow \boxed{V_c = 8\,[V]}$$

$$\boxed{V_b = 4\,[V]} \qquad \boxed{\therefore V = V_b - V_c = -4\,[V]}$$

#3:



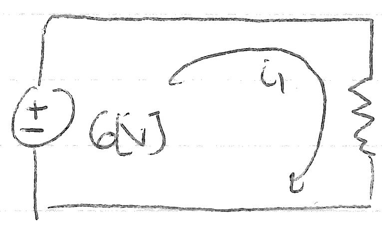
KVL:

$-12V + v_{15} + 18V + v_{10}$
$+ v_{20} - 12V + v_5 = 0$

Need v_{10}:

Create equivalent circuit to determine i_1:

Vs add in series, so do resistors:

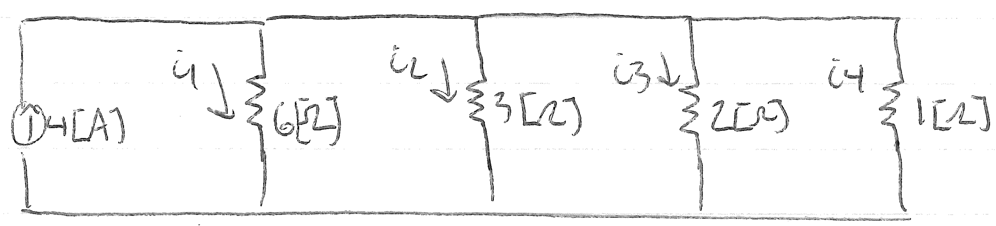


$R_{eq} = (15+10+20+5\,[\Omega]) = 50\,[\Omega]$

$\therefore i_1 = \frac{V_s}{R_{eq}} = \frac{6\,[V]}{50\,[\Omega]} = 0.12\,[A]$

$\therefore v_{10} = i_1 \cdot R = 0.12[A] \cdot 10[\Omega] = \boxed{1.2[V] = v_{10}}$

#4:



Use conductances since more than two resistors in ∥:

$G_1 = 1/6\,[S]$, $G_2 = 1/3\,[S]$, $G_3 = 1/2\,[S]$, $G_4 = 1\,[S]$

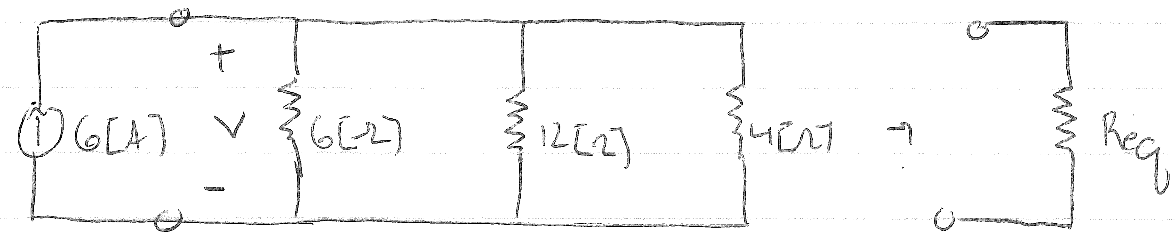
$$i_1 = \left(\frac{G_1}{G_1+G_2+G_3+G_4}\right)\cdot i_S = \left(\frac{1/6}{1/6+1/3+1/2+1}\right)\cdot 4[A] = \boxed{1/3\,[A] = i_1}$$

$$i_2 = \left(\frac{G_2}{G_1+G_2+G_3+G_4}\right)\cdot i_S = \left(\frac{1/3}{1/6+1/3+1/2+1}\right)\cdot 4[A] = \boxed{2/3\,[A] = i_2}$$

Saving some paper and using the same formulation:

$\boxed{i_3 = 1[A]}$ $\boxed{i_4 = 2[A]}$

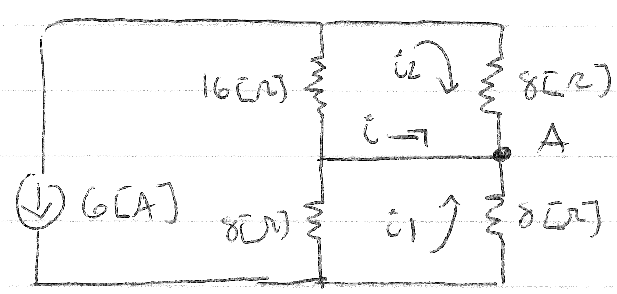
#5:



$$R_{eq} = \frac{4\cdot 12}{4+12} = 3 \rightarrow R_{eq} = \frac{3\cdot 6}{3+6} = \boxed{2\,[\Omega] = R_{eq}}$$

$$v = iR = 6[A]\cdot 2[\Omega] = \boxed{12[V] = v}$$

#6:



Current division b/t two parallel portions;

KCL @ Node A: $i_1 + i_2 + i = 0$

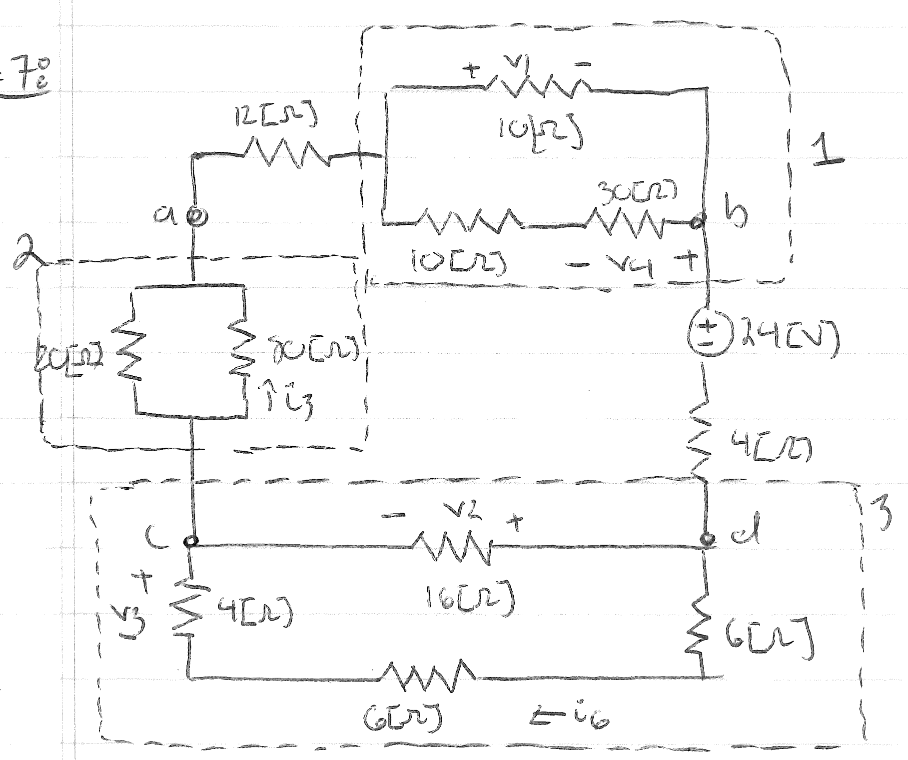
$$i_1 = \left(\frac{8\,[\Omega]}{8+8[\Omega]}\right)\cdot 6[A] = 3[A]$$

$$i_2 = \left(\frac{16\,[\Omega]}{(8+16\,[\Omega])}\right)(-6[A]) = -4[A]$$

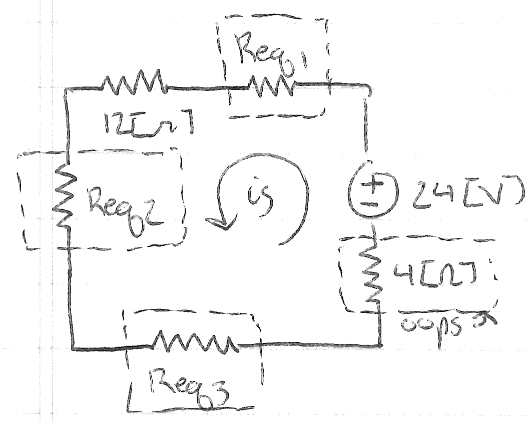
$i + 3[A] - 4[A] = 0$

$\boxed{\therefore\ i = 1\,[A]}$

#7:



Create equivalents for regions 1, 2 & 3



$$R_{eq1} = \frac{10 \cdot 40}{10+40} = 8[\Omega]$$

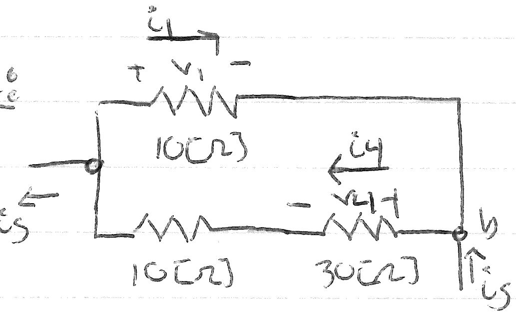
$$R_{eq2} = \frac{80 \cdot 20}{80+20} = 16[\Omega]$$

$$R_{eq3} = \frac{16 \cdot 16}{16+16} = 8[\Omega]$$

$$R_{eq} = R_{eq1} + R_{eq2} + R_{eq3} + 12 + 4 = 48[\Omega]$$

$$\therefore\ i_s = \frac{V_s}{R_{eq}} = \frac{24[V]}{48[\Omega]} = \boxed{0.5[A] = i_s}$$

Region 1:



$$i_4 = \left(\frac{10}{10+10+30}\right) i_s = 0.1[A]$$

$$i_1 = \frac{(30+10)}{(10+10+30)} i_s = 0.4[A]$$

KCL @ b:

$i_s + i_1 = i_4$

$0.5[A] + (0.4[A]) = 0.1[A]$

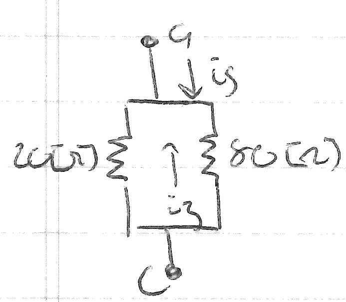
↳ change direction

$$\therefore\ i_1 = -0.4[A)$$

$$\therefore\ V_1 = i_1 \cdot R = -0.4[A] \cdot 10[\Omega] = \boxed{-4[V] = v_1}$$

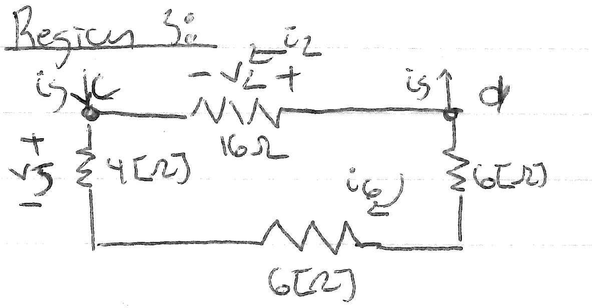
$$\therefore\ V_4 = i_4 \cdot R = 0.1[A] \cdot 30[\Omega] = \boxed{3[V] = v_4}$$

Region 2:



$$i_3 = \left(\frac{20[\Omega]}{20+80[\Omega]}\right)(-0.5[A]) = \boxed{-.1[A] = i_3}$$

Region 3:



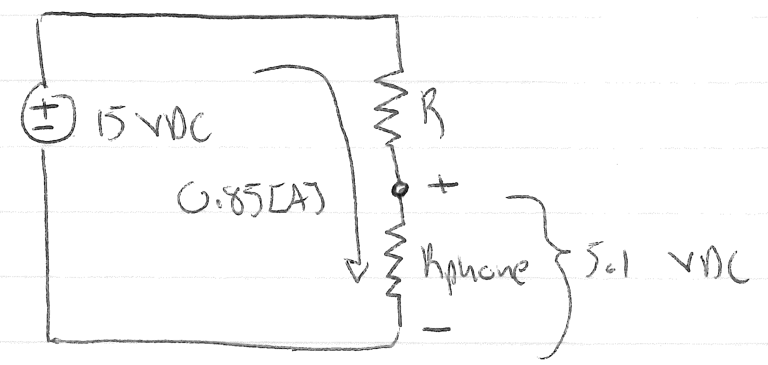
$$i_2 = i_6 = -i_s/2$$

$$\boxed{i_6 = -0.25[A]}$$

$$v_2 = -0.25[A] \cdot 16[\Omega] = \boxed{-4[V] = v_2}$$

$$v_5 = -0.25[A] \cdot 4[\Omega] = \boxed{-1[V] = v_5}$$

#8:



$$R_{eq} = R + R_{phone} = \frac{V_s}{i} = \frac{15\,[VDC]}{0.85\,[A]} = 17.6471\,[\Omega]$$

Apply voltage division

$$V_{phone} = \left(\frac{R_{phone}}{R_{eq}}\right) \cdot V_s = \left(\frac{R_{phone}}{17.6471\,[\Omega]}\right) 15\,[VDC] = 5.1\,[V]$$

$$\boxed{\begin{array}{l} \therefore R_{phone} = 6\,[\Omega] \\ \therefore R = 11.647\,[\Omega] \end{array}}$$