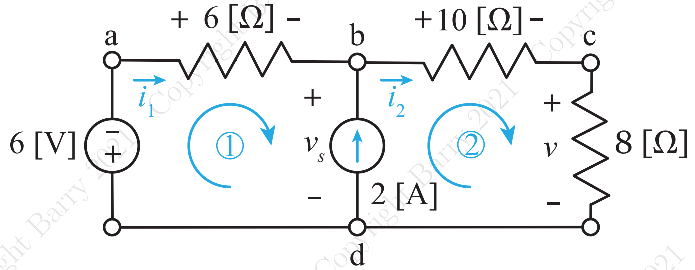
MEMS 0031 - Electrical Circuits
Quiz #2
Assigned: February 15th, 2021
Due: February 19th, 2021, 9:00 pm

Name: ______________________________

Intermediate Problem #1

(1.5 pts) Using Kirchhoff's Voltage and Current Laws, determine the voltage drop, v across the 8 [Ω] resistor:



We will construct two KVL loops, 1 and 2. Starting with Loop 1:

$$6\,[\text{V}] + V_{6\,[\Omega]} + V_s = 0$$

Proceeding to Loop 2:

$$-V_s + V_{10\,[\Omega]} + V = 0$$

Combining the KVL equations for Loops 1 and 2 via V_s:

$$-6\,[\text{V}] - V_{6\,[\Omega]} = V_{10\,[\Omega]} + V$$

Next we will apply KCL at node b:

$$i_1 + 2\,[\text{A}] = i_2$$

We two equations and four unknowns. Let us use Ohm's law to transform the voltage potentials from our combined KVL equation into expressions considering currents described by the KCL equation and the provided resistances:

$$-6\,[\text{V}] - i_1(6\,[\Omega]) = i_2(10+8)\,[\Omega]$$

We have two equations and two unknowns. Solving for i_1 and i_2:

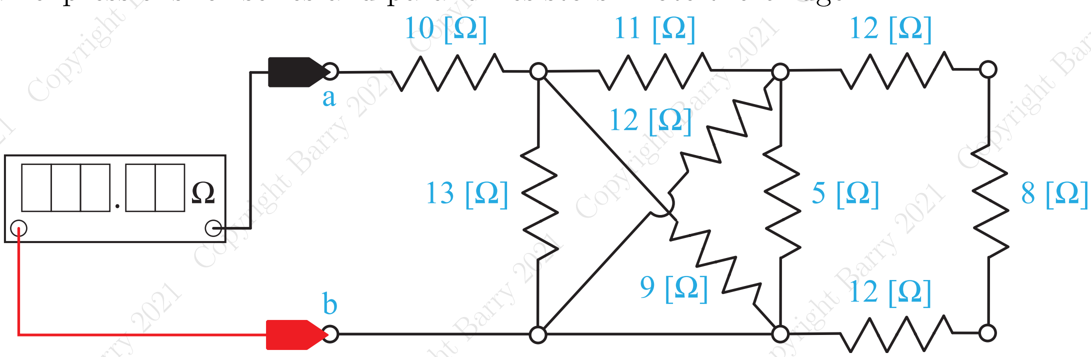
$$\begin{bmatrix} 1 & -1 \\ 6 & 18 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \left[-1.75\,[\text{A}] 0.25\,[\text{A}]\right]$$

Thus the voltage drop across the 8 [Ω] resistor is found via Ohm's law:

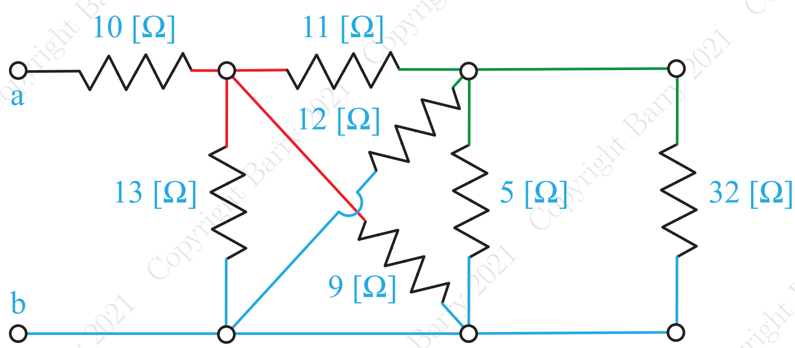
$$V = i_2(8\,[\Omega]) = (0.25\,[\text{A}])(8\,[\Omega]) = 2\,[\text{V}]$$

Intermediate Problem #2

(1.5 pts) Determine the equivalent resistance of the circuit as measured by the multimeter. Recall expressions for series and parallel resistors. Note the bridge.

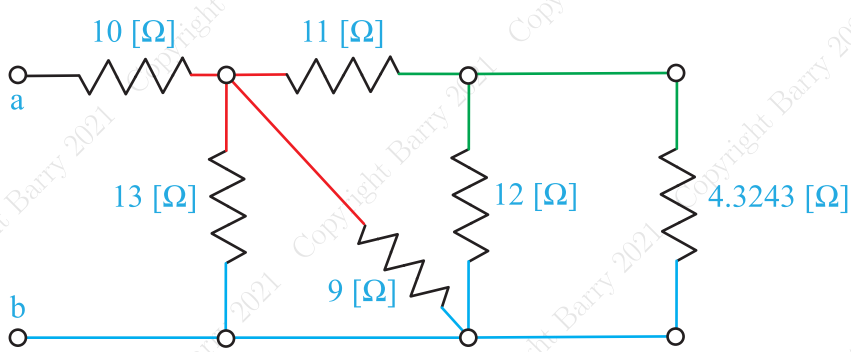


Nodes are color-coded. We will sequentially work through the problem. There is a quicker path using the color-coded nodes which will be done at the end of the problem. Starting on the right side of the circuit, we see the 12, 8 and 12 [Ω] resistors are in series, that form an equivalent resistance of 32 [Ω]. This 32 [Ω] resistor is in parallel with the 5 [Ω] resistor:



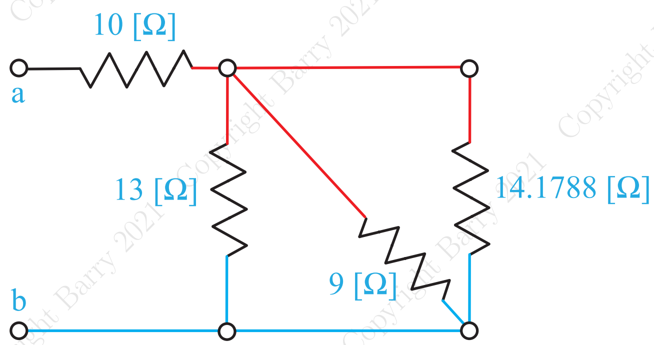
$$R_{eq} = \frac{(5\,[\Omega])(32\,[\Omega])}{(5+32)\,[\Omega]} = 4.3243\,[\Omega]$$

We see the diagonal 12 [Ω] resistor is in parallel with the 4.3243 [Ω] resistors:



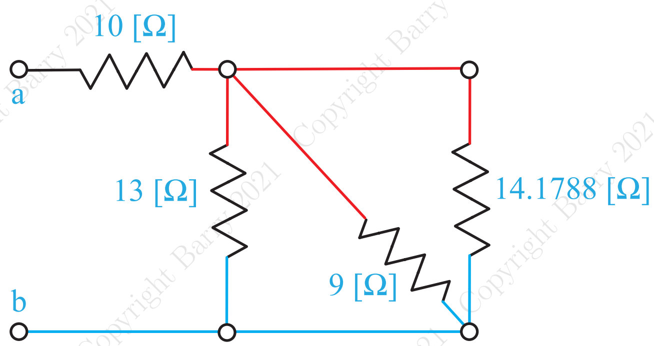
$$R_{eq} = \frac{(4.3243\,[\Omega])(12\,[\Omega])}{(4.3243+12)\,[\Omega]} = 3.1788\,[\Omega]$$

The 11 [Ω] resistor is in series with the 3.1788 [Ω] resistor, with this result (14.1788 [Ω]) being in parallel with the 9 [Ω] resistor.



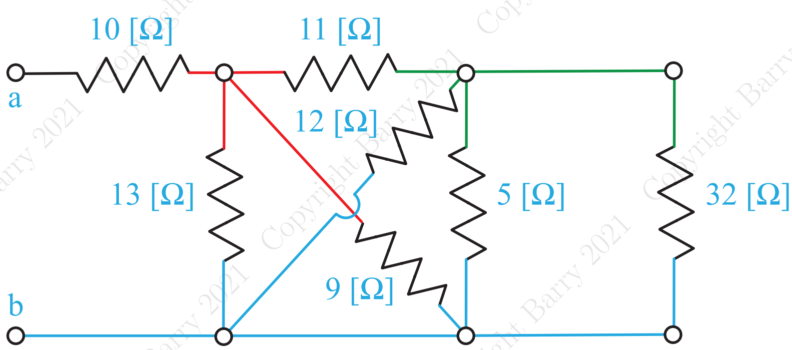
$$R_{eq} = \frac{(9\,[\Omega])(14.1788\,[\Omega])}{(9 + 14.1788)\,[\Omega]} = 5.5054\,[\Omega]$$

This equivalent resistance is in parallel with the 13 [Ω] resistor:



$$R_{eq} \frac{(13\,[\Omega])(5.5054\,[\Omega])}{(13 + 5.5054)\,[\Omega]} = 3.8675\,[\Omega]$$

Lastly, this equivalent resistance is in series with the 10 [Ω] resistor, and the circuit thus have a resistance of 13.8675 [Ω]. Now, to do this problem in more efficiently, we will refer to the first color-coded diagram:



We see the 12, 5 and 32 [Ω] resistors are in parallel, and their equivalent is in series with the 11 [Ω] resistor.

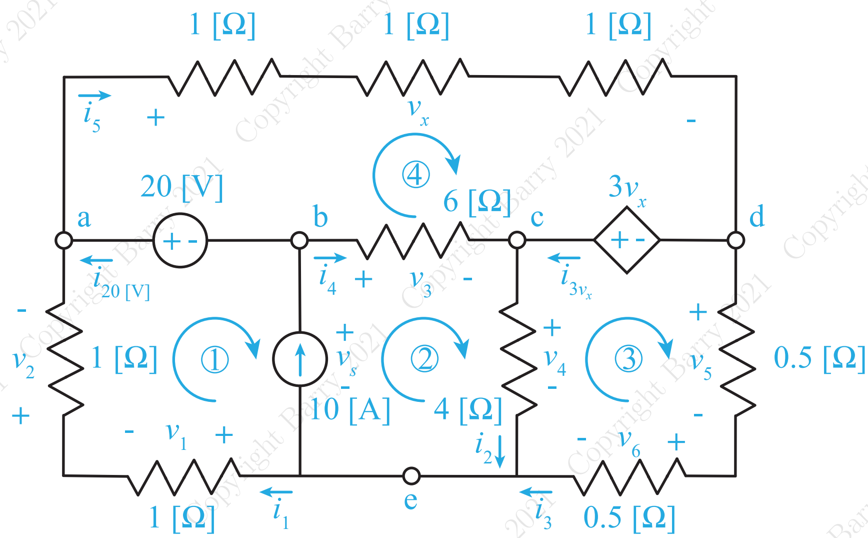
$$R_{eq} = \left(\frac{1}{12\,[\Omega]} + \frac{1}{5\,[\Omega]} + \frac{1}{32\,[\Omega]}\right)^{-1} + 11\,[\Omega] = 14.1788\,[\Omega]$$

This equivalence is in parallel with the 9 and 13 [Ω] resistors, with the equivalence being in parallel with the 10 [Ω] resistor. Thus:

$$R_{eq} = \left(\frac{1}{14.1788\,[\Omega]} + \frac{1}{9\,[\Omega]} + \frac{1}{13\,[\Omega]}\right)^{-1} + 10\,[\Omega] = 13.8675\,[\Omega]$$

Challenge Problem #1

(2 pts) Using Kirchhoff's Current and Voltage Laws, determine the voltage V_x.
We will denote our KVL loops and branch currents as follows:



We will start by constructing expressions for each KVL loop, starting with Loop 1:

$$V_2 + 20\,[\text{V}] + V_s + V_1 = 0$$

Proceeding to Loop 2:

$$-V_s + V_3 + V_4 = 0$$

We can equate the expressions for V_s from our Loop 1 and 2 equations:

$$V_1 + V_2 + V_3 + V_4 = -20\,[\text{V}] \tag{1}$$

Proceeding to Loop 3:

$$-V_4 + 3V_x + V_5 + V_6 = 0 \tag{2}$$

Proceeding to Loop 4:

$$V_x - 3V_x - V_3 - 20\,[\text{V}] = 0 \tag{3}$$

Currently we have seven unknowns (V_1 through V_6 and V_x) and only three equations. We will convert our voltage expressions in terms of branch currents and resistances before casting any KCL equations. Note our branch currents are defined through the resistors such that the resistors obey the PSC. Starting with Eq. 1:

$$i_1(1\,[\Omega]) + i_1(1\,[\Omega]) + i_4(6\,[\Omega]) + i_2(4\,[\Omega]) = -20\,[\mathrm{V}] \implies i_1(2\,[\Omega]) + i_2(4\,[\Omega]) + i_4(6\,[\Omega]) = -20\,[\mathrm{V}] \tag{4}$$

Proceeding to Eq. 2:

$$-i_2(4\,[\Omega]) + 3(i_5(3\,[\Omega])) + i_3(1\,[\Omega]) = 0 \implies -i_2(4\,[\Omega]) + i_3(1\,[\Omega]) + i_5(9\,[\Omega]) = 0 \tag{5}$$

Proceeding to Eq. 3

$$-i_4(6\,[\Omega]) - 2(i_5(3\,[\Omega])) = 20\,[\mathrm{V}] \tag{6}$$

We see we have five unknowns (i_1 through i_5) and three equations. Thus, we need only two KCL equations. We will apply our first KCL equation at node e:

$$-i_1 + i_2 + i_3 = 10\,[\mathrm{A}] \tag{7}$$

There is no other node that we can apply KCL to that doesn't introduce a new variable. Thus, we will need two apply two more KCL equations. The first will be at node a:

$$i_1 + i_{20\,[\mathrm{V}]} = i_5$$

The second will be at node b:

$$10\,[\mathrm{A}] = i_4 + i_{20\,[\mathrm{V}]}$$

Equating the expressions for $i_{20\,[\mathrm{V}]}$:

$$-i_1 + i_4 + i_5 = 10\,[\mathrm{A}] \tag{8}$$

Now we can cast Eqs. 4 through 8 in matrix form:

$$\begin{bmatrix} 2 & 4 & 0 & 6 & 0 \\ 0 & -4 & 1 & 0 & 9 \\ 0 & 0 & 0 & -6 & -6 \\ -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -20 \\ 0 \\ 20 \\ 10 \\ 10 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -13.\bar{3} \\ 43.\bar{3} \\ -46.\bar{6} \\ -27.\bar{7} \\ 24.\bar{4} \end{bmatrix}$$

With i_5 known, we can solve for V_x:

$$V_x = i_5(3\,[\Omega]) = (24.\bar{4}\,[\mathrm{A}])(3\,[\Omega]) = 73.\bar{3}\,[\mathrm{V}]$$