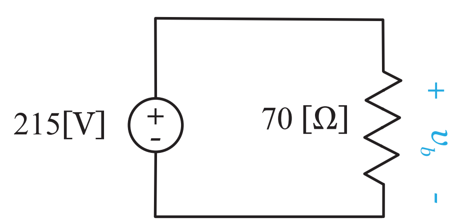
Homework #4

MEMS 0031 - Electrical Circuits

Assigned Due February 1$^{\text{st}}$, 2019
Due February 8$^{\text{th}}$, 2019

Problem #1

- Determine the power dissipated by the 70 [Ω] resistor.



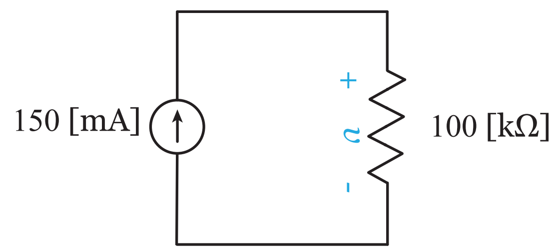
Solution:
The independent voltage sources, when added correctly create an individual voltage source of 215 [V] with the given polarity shown in the figure. Therefore, the power the circuit dissipates is given as:

$$\text{P} = \frac{V^2}{R} = \frac{(215\ [\text{V}])^2}{70[\Omega]} = \boxed{660.357\ [\text{W}]}$$

Problem #2

- Determine the potential difference v across the 100 [kΩ] resistor. Also, determine the power it dissipates.

Solution:
The independent current sources, when added correctly create an individual current source of 150 [mA] with the given current flow shown in the figure. Therefore, the potential difference across the resistor, v, is given as:



$V = iR = (150\ [\text{mA}])(100[\text{k}\Omega]) = \boxed{15\ [\text{kV}]}$

The power dissipated is solved to be:

$\text{P} = (i^2)(R) = (0.150\ [\text{mA}])^2(100[\text{k}\Omega]) = \boxed{2.25\ [\text{kW}]}$

Problem #3

- Use nodal voltage analysis (NVA) to find the node voltages at a and b.

Solutions:

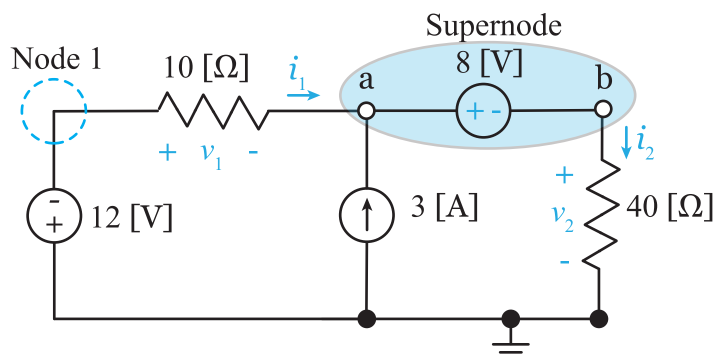
The supernode highlighted around the 8 [V] indicates that the difference between the voltages at nodes a and b is 8 [V]. Therefore, it can be said that:

$$V_a - V_b = 8 \text{ [V]}$$

Node 1, indicated by the blue loop, has a voltage value of -12 [V] given that there is no resistor between it and the ground that would reduce the voltage potential.

Using Kirchhoff's Current Law (KCL) at node a, the summation of the currents must be zero. This yields:

$$i_1 + 3 \text{ [A]} = i_2$$



The currents i_1 and i_2 can be put in terms of the node voltages bounding them and the resistors between them. This yields:

$$i_1 = \frac{V_{Node1} - V_a}{10[\Omega]} = \frac{-12 - V_a}{10[\Omega]}$$

$$i_2 = \frac{V_b - V_{ground}}{40[\Omega]} = \frac{V_b - 0}{40[\Omega]}$$

Inserting these values into KCL equation just derived yields:

$$\frac{-12 - V_a}{10[\Omega]} + 3 \text{ [A]} = \frac{V_b - 0}{40[\Omega]}$$

Using this equation and the supernode equation found in the beginning, a system of equations can been developed to solve for the values of V_a and V_b.

$$V_a = \boxed{16 \text{ [V]}}$$

$$V_b = \boxed{8 \text{ [V]}}$$

Problem #4

- The resistor R in Fig. 1 has a resistance of 200 [kΩ]. Determine the voltage values at a,b and c. Also determine the power dissipated/accumulated by all the resistors.

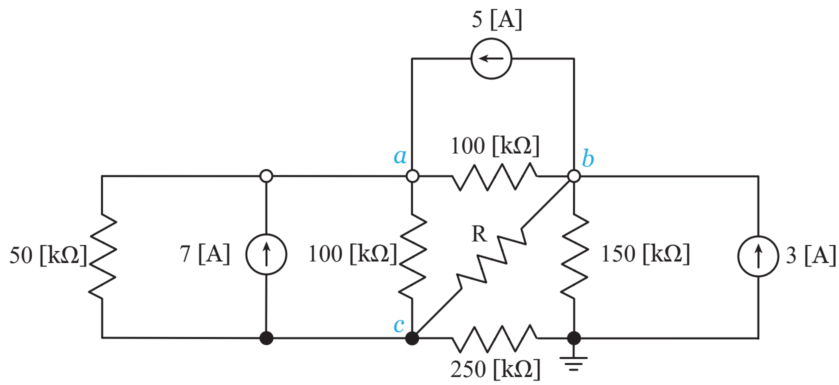
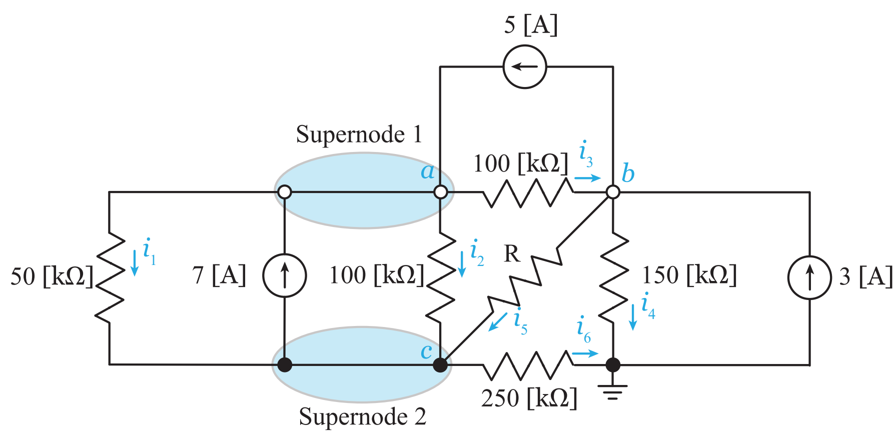


Figure 1: Schematic for Problem #4.

Solutions:
Several currents have been drawn so as to create KCL equations about nodes of interest. They are shown in the figure to the right. Take note of the supernodes about nodes a and c. The KCL equations about the two supernodes and node b, given the directions of the current, yields:



Supernode 1:

$$i_1 + i_2 + i_3 = 7 \text{ [A]} + 5 \text{ [A]}$$

Supernode 2:

$$i_1 + i_2 + i_5 = i_6 + 7 \text{ [A]}$$

Node b:

$$i_3 + 3 \text{ [A]} = i_4 + i_5 + 5 \text{ [A]}$$

The currents i_1, i_2, i_3, i_4, i_5 and i_6 can be put in terms of the node voltages bounding them and the resistors between them. This yields:

$$i_1 = \frac{V_a - V_c}{50\text{k}[\Omega]} \quad i_2 = \frac{V_a - V_c}{100\text{k}[\Omega]}$$

$$i_3 = \frac{V_a - V_b}{100\text{k}[\Omega]} \quad i_4 = \frac{V_b - V_{ground}}{150\text{k}[\Omega]}$$

$$i_5 = \frac{V_b - V_c}{200\text{k}[\Omega]} \quad i_6 = \frac{V_c - V_{ground}}{250\text{k}[\Omega]}$$

Inserting these values into KCL equation just derived and taking into account that V_{ground} is 0 yields:

$$\frac{V_a - V_c}{50\text{k}[\Omega]} + \frac{V_a - V_c}{100\text{k}[\Omega]} + \frac{V_a - V_b}{100\text{k}[\Omega]} = 7\text{ [A]} + 5\text{ [A]}$$

$$\frac{V_a - V_c}{50\text{k}[\Omega]} + \frac{V_a - V_c}{100\text{k}[\Omega]} + \frac{V_b - V_c}{200\text{k}[\Omega]} = \frac{V_c - V_{ground}}{250\text{k}[\Omega]} + 7\text{ [A]}$$

$$\frac{V_a - V_b}{100\text{k}[\Omega]} + 3\text{ [A]} = \frac{V_b - V_{ground}}{150\text{k}[\Omega]} + \frac{V_b - V_c}{200\text{k}[\Omega]} + 5\text{ [A]}$$

Given the system of equations, the nodal voltages at a, b and c can be found to be:

$$V_a = 603.125\text{ [kV]} \quad V_b = 259.375\text{ [kV]} \quad V_c = 317.708\text{ [kV]}$$

Now that the node voltages are found between every resistor in the circuit, the power dissipated by it can be determined:

$$\text{P} = \sum_{i=1}^{6} \frac{(\Delta V_i)^2}{R_i} = \frac{(V_a - V_c)^2}{50[\text{k}\Omega]} + \frac{(V_a - V_c)^2}{100[\text{k}\Omega]} + \frac{(V_a - V_b)^2}{100[\text{k}\Omega]} + \frac{(V_b - V_{ground})^2}{150[\text{k}\Omega]} + \frac{(V_b - V_c)^2}{200[\text{k}\Omega]} + \frac{(V_c - V_{ground})^2}{250[\text{k}\Omega]} \implies$$

$$\frac{(603.125\text{ [kV]} - 317.708\text{ [kV]})^2}{50[\text{k}\Omega]} + \frac{(603.125\text{ [kV]} - 317.708\text{ [kV]})^2}{100[\text{k}\Omega]} + \frac{(603.125\text{ [kV]} - 259.375\text{ [kV]})^2}{100[\text{k}\Omega]} + \ldots$$

$$\frac{(259.375\text{ [kV]} - 0)^2}{150[\text{k}\Omega]} + \frac{(259.375\text{ [kV]} - 317.708\text{ [kV]})^2}{200[\text{k}\Omega]} + \frac{(317.708\text{ [kV]} - 0)^2}{250[\text{k}\Omega]} \implies$$

$$P = \boxed{4,494.792\text{ [kW] dissipated}}$$

Problem #5

- Determine the power dissipated/accumulated by each resistor in Fig. 2

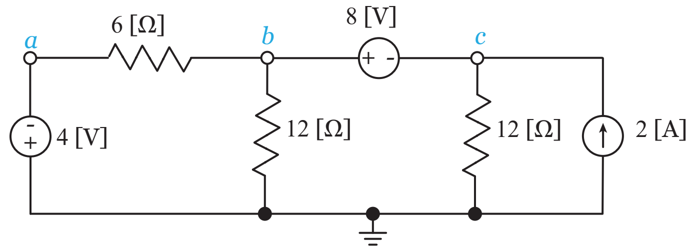


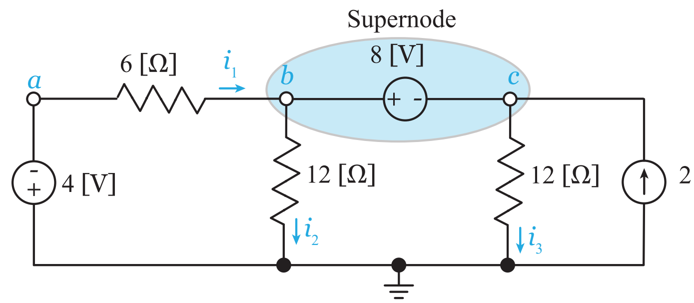
Figure 2: Schematic for Problem #5.

Solutions:

The supernode highlighted around the 8 [V] indicates that the difference between the voltages at nodes b and c is 8 [V]. Therefore, it can be said that:

$$V_b - V_c = 8 \text{ [V]}$$

Node a has a voltage value of -4 [V] given that there is no resistor between it and the ground that reduces the voltage potential.



Using Kirchhoff's Current Law (KCL) at the supernode, the summation of the currents must be zero. This yields:

$$i_1 + 2 \text{ [A]} = i_2 + i_3$$

The currents i_1, i_2 and i_3 can be put in terms of the node voltages bounding them and the resistors between them. This yields:

$$i_1 = \frac{V_a - V_b}{6[\Omega]} = \frac{-4 - V_b}{6[\Omega]}$$

$$i_2 = \frac{V_b - V_{ground}}{12[\Omega]} = \frac{V_b - 0}{12[\Omega]}$$

$$i_3 = \frac{V_c - V_{ground}}{12[\Omega]} = \frac{V_c - 0}{12[\Omega]}$$

Inserting these values into KCL equation just derived yields:

$$\frac{-4 - V_b}{6[\Omega]} + 2 \text{ [A]} = \frac{V_b - 0}{12[\Omega]} + \frac{V_c - 0}{12[\Omega]}$$

Using this equation and the supernode equation found in the beginning, a system of equations have been developed to solve for the values of V_b and V_c.

$$V_b = 6 \text{ [V]} \quad V_c = -2 \text{ [V]}$$

Now that the node voltages are found between every resistor in the circuit, the power dissipated by it can be determined:

$$\text{P} = \sum_{i=1}^{3} \frac{(\Delta V_i)^2}{R_i} = \frac{(V_b - V_a)^2}{6[\Omega]} + \frac{(V_b - V_{ground})^2}{12[\Omega]} + \frac{(V_c - V_{ground})^2}{12[\Omega]} \implies$$

$$\frac{(6\ [\text{V}] - -4\ [\text{V}])^2}{6[\Omega]} + \frac{(6\ [\text{V}] - 0)^2}{12[\Omega]} + \frac{(-2\ [\text{V}] - 0)^2}{12[\Omega]} \implies$$

$$P = \boxed{20\ [\text{W}]\ \text{dissipated}}$$