Homework #1

MEMS 0031 - Electrical Circuits

Assigned January 11$^{\text{th}}$, 2019
Due: January 18$^{\text{th}}$, 2019

Problem #1

The total charge entering a circuit element is expressed as $q(t)$=6(3-e$^{\text{-5t}}$) for when $t \geq$0. When t<0, $q(t)$=0. Determine the current in the circuit element for $t \geq$0.

Solution:

Write down the relationship between current and charge.

$$i = \frac{dq}{qt}$$

Differentiate the charge function with respect to time to get the current.

$$i = \frac{d}{dt}[6(3 - e^{\text{-5t}})] = \boxed{30e^{\text{-5t}} \text{ [A]}}$$

Problem #2

The current in a circuit element is $i(t)$=2.5(1-e$^{\text{-14t}}$) [A] when $t \geq$0. When t<0, $i(t)$=0. Determine the total charge that has entered the circuit element for $t \geq$0.

Solution:

Write down the relationship between current and charge.

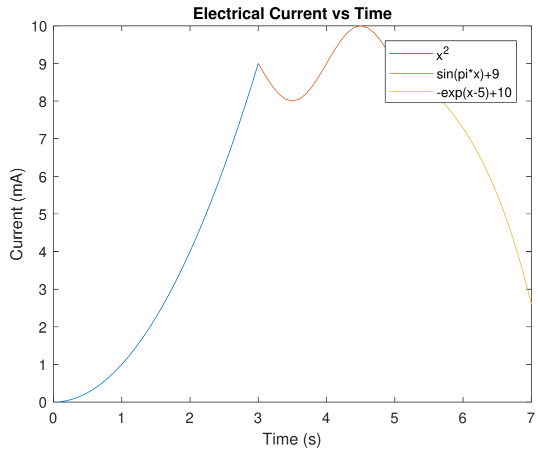
$$q(t) = q(0) + \int_0^t 2.5(1 - e^{\text{-14t}})dt$$

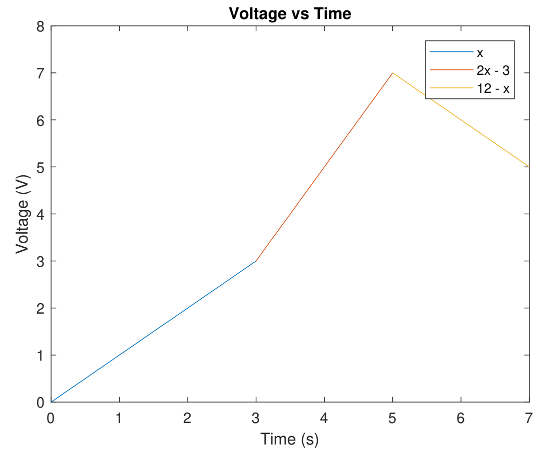
Integrate the current and evaluate from time 0 to t to get an expression for total charge accumulated over time.

$$\boxed{q(t) = 2.5t + \left(\frac{2.5}{14}\right)e^{-14t} \text{ [C]}}$$

Problem #3

The time variation of current and voltage through and across an electrical circuit element is depicted in the figure below. The element current and voltage adhere to the passive sign convention. Sketch the power delivered to the element for the time interval between 0 and 7 seconds. What is the total energy delivered to the element for the time interval between 0 and 7 seconds?





Solution:

Develop an expression for the total energy accumulated/dissipated over an interval of the time.

$$\Delta \mathrm{E} = \int_{x_1}^{x_2} P(x)dx$$

Electrical power is defined as:

$$P(x) = V(x)i(x)$$

Divide the piece-wise function into the intervals [0,3], [3,5] and [5,7] and integrate the product of the voltage and current accordingly.

$$\Delta \mathrm{E}_1 = \int_0^3 (x)(x^2)dx$$

$$\Delta \mathrm{E}_2 = \int_3^5 (2x-3)(\sin(\pi x)+9)dx$$

$$\Delta \mathrm{E}_3 = \int_5^7 (12-x)(-e^{(x-5)}+10)dx$$

The first energy accumulation/dissipation interval is a simple polynomial. The second and third interval requires integration by parts.

Interval 1:

$$\Delta \mathrm{E}_1 = \int_0^3 (x)(x^2)dx = \left(\frac{1}{4}\right)x^4\Big|_0^3 = \left(\frac{1}{4}\right)(3^4 - 0^4) = 20.250 \text{ [J]}$$

Interval 2:

Expanding the integral, we get that:

$$\Delta \mathrm{E}_2 = \int_3^5 (2x-3)(\sin(\pi x)+9)dx = \int_3^5 \Big[18x - 27 - 3\sin(\pi x)\Big]dx + \int_3^5 2x\sin(\pi x)dx$$

Using integration by parts, one can solve for the last term of the expanded integral.

$$\int udv = uv - \int vdu$$

$$u = 2x; \quad dv = \sin(\pi x)dx$$

$$du = 2dx; \quad v = \left(\frac{-1}{\pi}\right)\cos(\pi x)$$

Inserting the variables into the equation yields:

$$\int 2x\sin(\pi x)dx = 2x\left(\frac{-1}{\pi}\right)\cos(\pi x) - \int \left(\frac{-1}{\pi}\right)\cos(\pi x)2dx$$

Solving the integral for the other terms for the interval [3,5] yields:

$$\Delta \mathrm{E}_2 = \left[9x^2 - 27x + \left(\frac{3}{\pi}\right)\cos(\pi x) + \frac{2\sin(\pi x) - 2\pi x\cos(\pi x)}{\pi^2}\right]\Bigg|_3^5 = 91.273 \text{ [J]}$$

<u>Interval 3</u>:
Expanding the integral, we get that:

$$\Delta \mathrm{E}_3 = \int_5^7 (-e^{x-5} + 10)(12 - x)dx = \int_5^7 -12e^{x-5} + 120 - 10x + \int_5^7 xe^{x-5}dx$$

Integration by parts will be used again for the third interval of [5,7].

$$\int udv = uv - \int vdu$$

$$u = x; \quad dv = e^{x-5}dx$$

$$du = dx; \quad v = e^{x-5}$$

Inserting the variables into the equation yields:

$$\int xe^{x-5}dx = xe^{x-5} - \int e^{x-5}dx$$

Solving the integral for the other terms for the interval [5,7] yields:

$$\Delta \mathrm{E}_3 = \left[-13e^{x-5} - 5x^2 + 120x + xe^{x-5}\right]\Bigg|_5^7 = 83.666 \text{ [J]}$$

Summing the energy change for each interval, one finds that the total energy accumulated/dissipated from the time interval[0,7] is:

$$\Delta \mathrm{E} = \Delta \mathrm{E}_1 + \Delta \mathrm{E}_2 + \Delta \mathrm{E}_3 = 20.250 + 91.273 + 83.666 = \boxed{195.189 \text{ [J]}}$$

Problem #4

Find the total energy accumulated/dissipated supplied by the element shown below from t = 0 to 3 [s] when $v(t)$=cos(3t) [V] and $i(t)= -e^{-5t}$ [A]. Assume the element accumulated/dissipated 0 [J] for $t \leq 0$. Indicate whether the element is a power sink or power source.

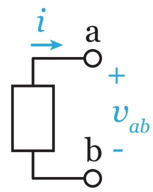


Figure 1: Schematic for Problem #4.

Solution:

Develop an expression for the total energy accumulated/dissipated over an interval of the time.

$$\Delta \text{E} = \int_{t_1}^{t_2} P(t)d\tau$$

Electrical power is defined as:

$$P(t) = V(t)i(t)$$

Integrate the function for power from time t = 0 to 3.

$$\Delta \text{E} = \int_0^3 \cos(3t)(-e^{3t})dx$$

Integration by parts will be used.

$$\int udv = uv - \int vdu$$

$$u = -e^{-5t}; \quad dv = \cos(3t)dx$$

$$du = 5e^{-5t}dx; \quad v = (\frac{1}{3})\sin(3t)$$

Inserting the variables into the equation yields:

$$\int -e^{-5t}\cos(3t)dx = -e^{-5t}(\frac{1}{3})\sin(3t) - \int (\frac{1}{3})\sin(3t)5e^{-5t}dx$$

Integration by parts must be used again for the second term on the right hand side of the equation.

$$\int \left(\frac{1}{3}\right)\sin(3t)5e^{-5t}dx$$

$$\int udv = uv - \int vdu$$

$$u = 5e^{-5t}; \quad dv = (\frac{1}{3})\sin(3t)dx$$

$$du = -25e^{-5t}dx; \quad v = \left(\frac{-1}{9}\right)\cos(3t)$$

Inserting the variables into the equation yields:

$$\int (\frac{1}{3})\sin(3t)5e^{-5t}dx = 5e^{-5t}\Big(\frac{-1}{9}\Big)\cos(3t) - \int \Big(\frac{-1}{9}\Big)\cos(3t)\Big(-25e^{-5t}dx\Big)$$

Inserting back into the top equation and combining like terms yields:

$$\Big(-1-\frac{25}{9}\Big)\int -e^{-5t}\cos(3t)dx = \Big[-e^{-5t}(\frac{1}{3})\sin(3t) + \frac{5}{9}e^{-5t}\cos(3t)\Big]$$

Now, simplify and evaluate the integral in its time domain

$$\int_0^3 -e^{-5t}\cos(3t)dx = \Big(\frac{-e^{-5t}}{34}\Big)\Big[(-3\sin(3t) + 5e^{-5t}\cos(3t)\Big]\Big|_0^3 = \boxed{-0.147\ [\mathrm{J}]}$$

Problem #5

A resistor manufacturer is prototyping with a new resistors made of copper. The length of the prototype is 10 [cm] and has an effective cross sectional area of 5e-9 [m^2]. The prototype is ohmic. Knowing this, answer the following:

- The electrical resistance of the element;
- The potential difference across the element with an induced electrical current of 20 [mA];
- Assuming the same current is induced through the element and the element dissipates energy, what is the power dissipated by the element;

Solution (a):
Develop an expression for the electrical resistance of the element.

$$R = \frac{(\rho_{cu})L}{A_c}$$

The electrical resistance for copper is $1.68 \cdot 10^{-8}$ [$\Omega \cdot$ m]. Therefore, the electrical resistance of the element is:

$$R = \frac{(1.68 \cdot 10^{-8}\ [\Omega \cdot \mathrm{m}])0.1[\mathrm{m}]}{(5 \cdot 10^{-9}[\mathrm{m}])} = \boxed{0.336\ [\Omega]}$$

Solution (b): Ohm's law shows the relationship between current, voltage and electrical resistance.

$$V = IR$$

Therefore the potential difference across the element is:

$$V = (20 \cdot 10^{-3}[\mathrm{A}])(0.336\ [\Omega]) = \boxed{6.72\ [\mathrm{mV}]}$$

Solution (c): The equation for power dissipation/accumulation of an electrical element is:

$$P = I^2R$$

Therefore the power dissipated by this element is:

$$P = (20 \cdot 10^{-3}\ [\mathrm{A}])^2(0.336\ [\Omega]) = \boxed{0.134\ \ [\mathrm{mW}]}$$

Problem #6

- There are prescribed currents through and voltages across circuits elements constituting a network as shown in the figure below. Determine if the voltages, currents and their reference directions are correct. Justify your answer. *Hint: apply the conservation of energy.*

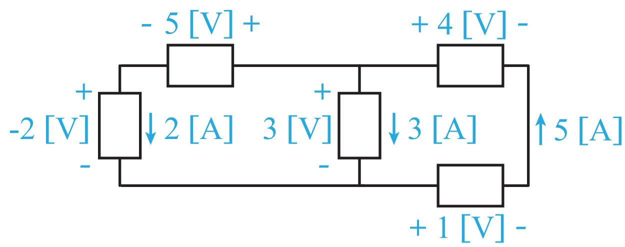


Figure 2: Schematic for Problem #6.

Solution (a):
For the voltages and current to be correct in value and direction, the power absorbed by the elements that accumulate energy must be offset by the power dissipated by the elements that produce energy. The equation for power is given by:

$$P = IV$$

The elements sum of the power supplied is:

$$P_{supplied} = (2\ [\mathrm{A}])(-2\ [\mathrm{V}]) + (5\ [\mathrm{A}])(-4\ [\mathrm{V}]) = -24\ [\mathrm{W}]$$

The elements sum of the power absorbed is:

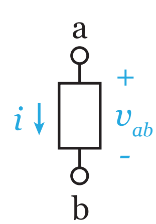
$$P_{absorbed} = (2\ [\mathrm{A}])(5\ [\mathrm{V}]) + (3\ [\mathrm{A}])(3\ [\mathrm{V}]) + (5\ [\mathrm{A}])(1\ [\mathrm{V}]) = 24\ [\mathrm{W}]$$

Since the magnitude of the power absorbed by elements is equal to the magnitude of the power supplied by elements, the voltages, currents and their reference directions are correct.

Problem #7

An electrical circuit element has voltage and current values as shown in the figure below. Determine if this particular circuit element is linear. If the element is linear, determine the voltage if a current of 40 [mA] runs through the element

v, V	i, mA
3.078	12
5.13	20
12.825	30



Solution:
For the element to be linear, the slope between the data points must be the same. It can be shown that the slope between the first two data points is:

$$a_1 = \frac{5.13\ [\mathrm{V}] - 3.078\ [\mathrm{V}]}{20\ [\mathrm{A}] - 12\ [\mathrm{A}]} = 0.257\ [\mathrm{V\ /\ A}]$$

It can be shown that the slope between the second two data points is:

$$a_2 = \frac{12.825 \text{ [V]} - 5.13 \text{ [V]}}{30 \text{ [A]} - 20 \text{ [A]}} = 0.769 \text{ [V / A]}$$

$$a_1 \neq a_2$$

The slopes made from the two sets of data are not equal; therefore, the element is not linear.

Problem #8

A current source and resistor are connected in series as shown in the figure below, which means the same current provided by the source is that which goes through the resistor. If the current source is 5 [A] and the voltage drop across the resistor is 22 [V], answer the following:

- Calculate the resistance R of the resistor and the power absorbed;
- If the element has a length of 10 cm and an effective cross sectional area of 9.09e-5 [cm^2], what material does it closely relate to.

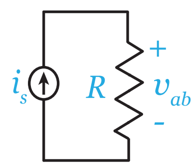


Figure 3: Schematic for Problem #8.

Solution (a):
Using Ohm's law, the electrical resistance of the element can be found to be:

$$R = \frac{V}{I} = \frac{22 \text{ [V]}}{5 \text{ [A]}} = \boxed{4.4 \text{ [}\Omega\text{]}}$$

Solution (b):
The equation for calculating electrical resistance with electrical resistivity will be useful to finding the material. Here, electrical resistance is given as:

$$R = \frac{\rho L}{A_c}$$

Solving for ρ yields:

$$\rho = \frac{(R)(A_c)}{L} = \frac{(4.4 \text{ [}\Omega\text{]})(9.09 \cdot 10^{-9}[\text{m}^2])}{0.1[\text{m}]} = 3.999 \cdot 10^{-7}[\Omega \cdot \text{m}]$$

Titanium has an electrical resistivity of $4.2 \cdot 10^{-7}[\Omega \cdot \text{m}]$ and is the material with the closest resistivity.

Problem #9

A voltage source and resistor are connected in series as shown in the figure below, which means the same voltage potential provided by the source is the same across the resistor. If the voltage source is 15 [V] and the resistance of the resistor is 6 [Ω], determine the current through the resistor and the power absorbed.

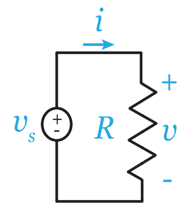


Figure 4: Schematic for Problem #9.

Solution:
Using Ohm's law, the electrical current running through the element can be found to be:

$$I = \frac{V}{R} = \frac{15\ [\text{V}]}{6\ [\Omega]} = \boxed{2.5\ [\text{A}]}$$

The power absorbed/dissipated by an electrical element is given as:

$$P = IV$$

As a result, the power absorbed by the element is:

$$P = (2.5\ [\text{A}])(15\ [\text{V}]) = \boxed{37.5\ [\text{W}]}$$