Chapter 7 - Energy Storage Elements

Lecture 22
Sections 7.2-7.4

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Student Learning Objectives

At the end of the lecture, students should be able to:

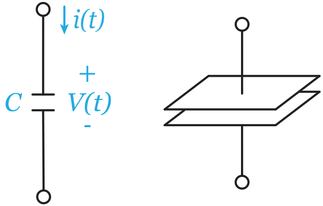
- Analyze the behavior of a capacitor as a function of time
- Construct an expression for the behavior of capacitors connected in series and parallel

Energy Storage Elements

- Inductors and Capacitors are “dynamic” devices that behave as a function of time, requiring either an integral or differential formulation
- In contrast to resistors, which are static devices that are modeled using algebraic equations, we must use ordinary differential equations to describe dynamics devices
- These differential equations gives us a time-history of the accumulation and/or discharge of energy from the device

Anatomy of Capacitors

- A conductor is schematically shown as follows:



Capacitor Constitutive Equation

- The charge stored in a capacitor is a function of C and $v(t)$

$$q(t) = Cv(t)$$

- Recalling the expression for current:

$$i = \frac{dq(t)}{dt} = C\frac{dv(t)}{dt}$$

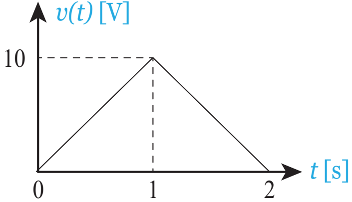
- Thus, the voltage accumulated/discharged is:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \implies v_0(t) + \frac{1}{C}\int_{t_0}^{t} i(\tau)d\tau$$

- It is noted that since $i(t)$ is bounded, $v(t)$ must be continuous

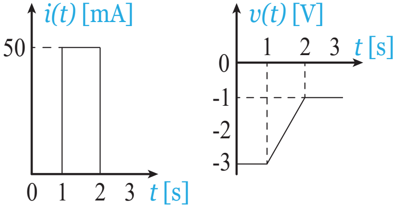
Example #1

- Given $v(t)$, determine the current as a function of time given C=1 [mF]



Example #2

- Given $i(t)$ and $v(t)$, determine the capacitance C:

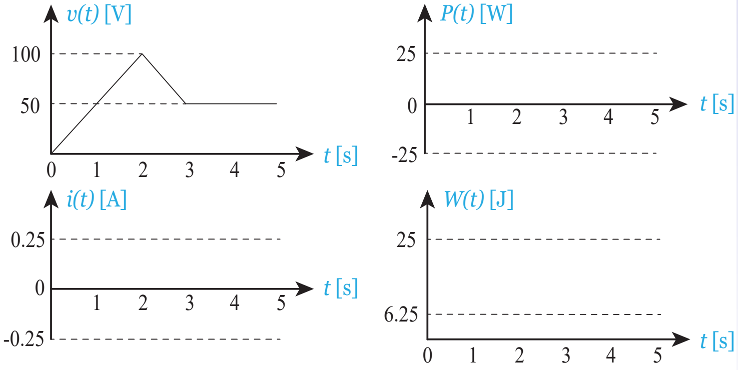


Energy Storage in Capacitor

- Construct an expression for the energy stored in a capacitor:

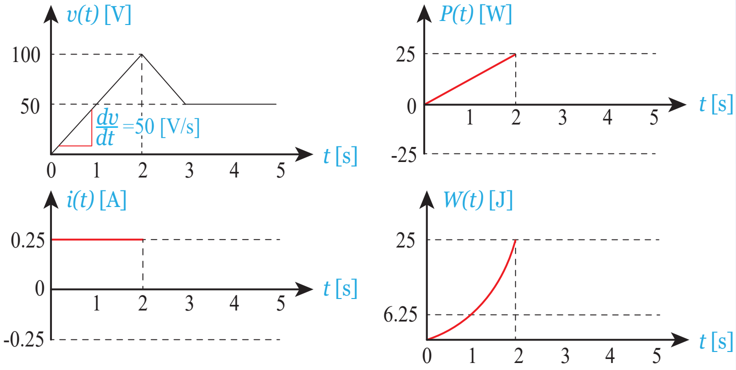
Example #3

- Given C=5 [mF] and $V(t)$, determine the current, power and energy as a function of time:



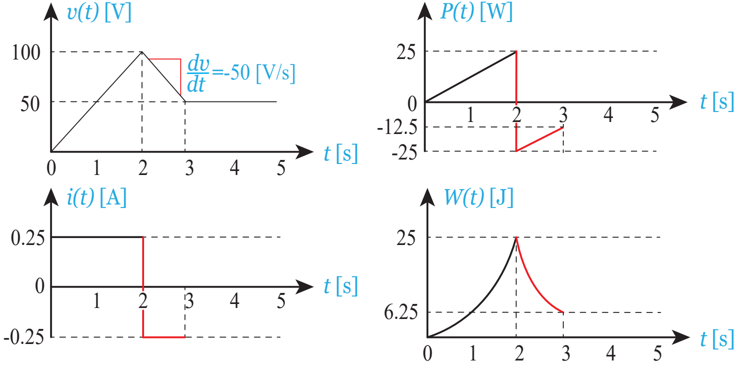
Example #3

- In the interval from 0 to 2 [s]:



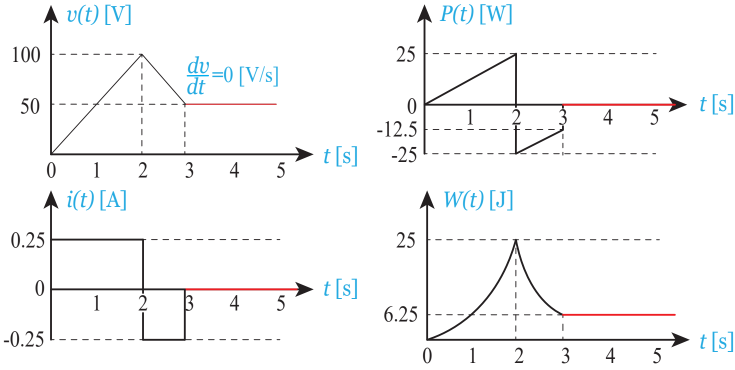
Example #3

- In the interval from 2 to 3 [s]:



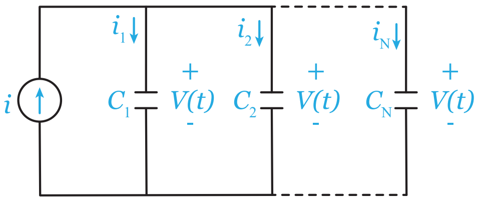
Example #3

- In the interval from 3 to ∞ [s]:



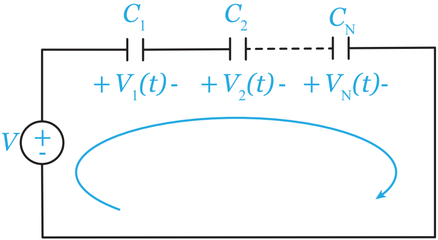
Parallel Capacitors

- Construct an expression for the equivalent capacitance:



Series Capacitors

- Construct an expression for the equivalent capacitance:



Student Learning Objectives

At the end of the lecture, students should be able to:

- Analyze the behavior of a capacitor as a function of time
  - The current into or out of a capacitor is expressed as the capacitance (ability to storage charge) times the time rate of change of potential across the capacitor
- Construct an expression for the behavior of capacitors connected in series and parallel
  - Capacitors in parallel sum arithmetically; the equivalent capacitance of capacitors in series is the reciprocal of the sum of the reciprocals of the individual capacitances

Suggested Problems

- 7.2-1, 7.2-2, 7.2-4, 7.2-7, 7.2-9, 7.2-11, 7.2-17, 7.3-2, 7.3-3, 7.3-5, 7.4-1, 7.4-3, 7.4-4, 7.4-5, 7.4-7