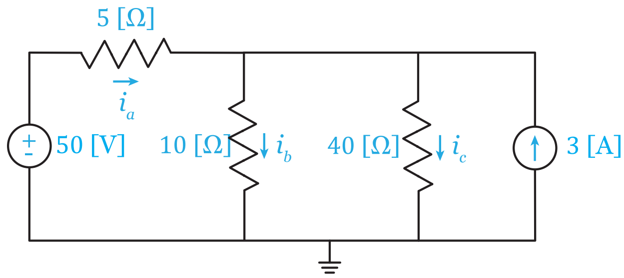
MEMS 0031 - Electrical Circuits
Quiz #4
June 5th, 2019
90 points

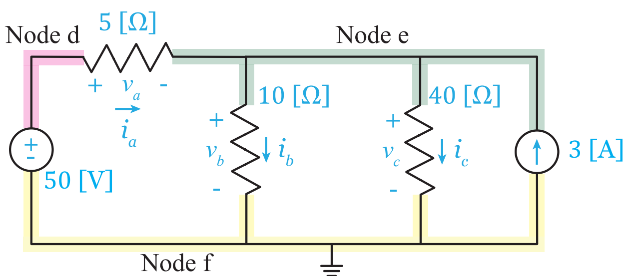
Name: ______________________________

Problem #1

(30 pts) Determine the branch currents i_a, i_b and i_c using NVA.



Step 1: Assign nodes (N) and leg currents to all branches/elements (notice how each node is highlighted in a different color, this makes it easier to distinguish between nodes and is an encouraged practice):
$N = 3$ and we define i_a, i_b and i_c as shown.



Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes:
We only need one KCL equation, which is applied at node e:

$$i_a + 3\,[\mathrm{A}] = i_b + i_c$$

Step 4: Apply Ohm's law in terms of node voltages:
KCL at node e:

$$\frac{V_d - V_e}{5\,[\Omega]} + 3\,[\mathrm{A}] = \frac{V_e - V_f}{10\,[\Omega]} + \frac{V_e - V_f}{40\,[\Omega]}$$

The voltage at node d is specified as 50 [V] and node f at 0 [V]. Thus, the KCL equation at node e becomes:

$$\frac{50 - V_e}{5\,[\Omega]} + 3\,[\mathrm{A}] = \frac{V_e - 0}{10\,[\Omega]} + \frac{V_e - 0}{40\,[\Omega]}$$

Therefore, solving the equation above the voltage at node e is:

$$V_e = 40\,[\mathrm{V}]$$

Therefore, we can determine the voltages a through c:

$$V_a = 10\,[\mathrm{V}], \quad V_b = 40\,[\mathrm{V}], \quad V_c = 40\,[\mathrm{V}]$$

Finally, we can determine the currents a through c:

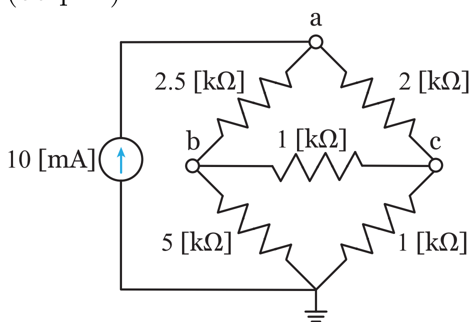
$$i_a = \frac{10\,[\mathrm{V}]}{5\,[\Omega]} = 2\,[\mathrm{A}]$$

$$i_b = \frac{40\,[\mathrm{V}]}{10\,[\Omega]} = 4\,[\mathrm{A}]$$

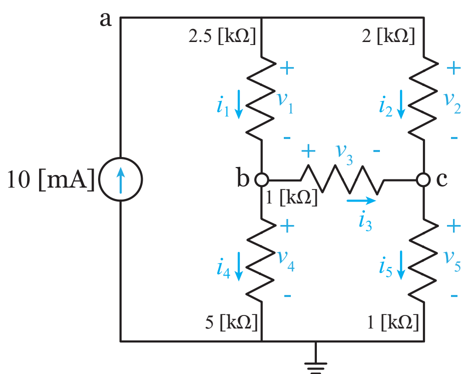
$$i_c = \frac{40\,[\mathrm{V}]}{40\,[\Omega]} = 1\,[\mathrm{A}]$$

Problem #2

(30 pts.) Determine the node voltages at a, b and c using NVA.



Step 1: Assign nodes (N) and leg currents to all branches/elements (the circuit has also been put into a more understandable format for ease of evaluation):
$N = 4$ and we define i_1, i_2, i_3, and i_4 as shown.



Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes:
We need 3 KCL equations applied at node a, b, and c:

$$\text{Node a} \implies 0.010\,[\text{A}] = i_1 + i_2$$

$$\text{Node b} \implies i_1 = i_3 + i_4$$

$$\text{Node c} \implies i_2 + i_3 = i_5$$

Step 4: Apply Ohm's law in terms of node voltages (note that the node connected to ground is referred to as node d:

KCL at node a:

$$0.010\,[\text{A}] = \frac{V_a - V_b}{2,500\,[\Omega]} + \frac{V_a - V_c}{2,000\,[\Omega]}$$

KCL at node b:

$$\frac{V_a - V_b}{2,500\,[\Omega]} = \frac{V_b - V_c}{1,000\,[\Omega]} + \frac{V_b - V_d}{5,000\,[\Omega]}$$

KCL at node c:

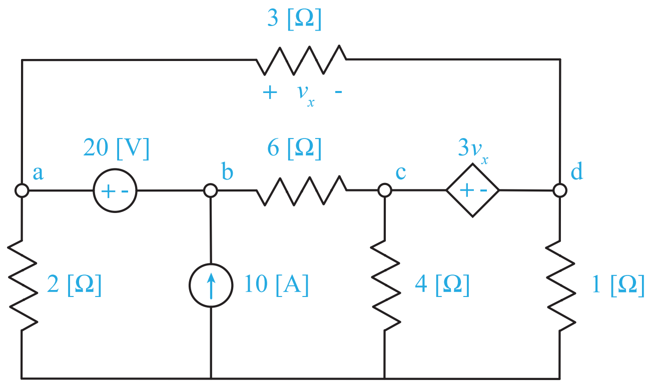
$$\frac{V_a - V_c}{2,000\,[\Omega]} + \frac{V_b - V_c}{1,000\,[\Omega]} = \frac{V_c - V_d}{1,000\,[\Omega]}$$

Remembering that V_d is equal to 0 [V] we can put the three previous equations into matrix form:

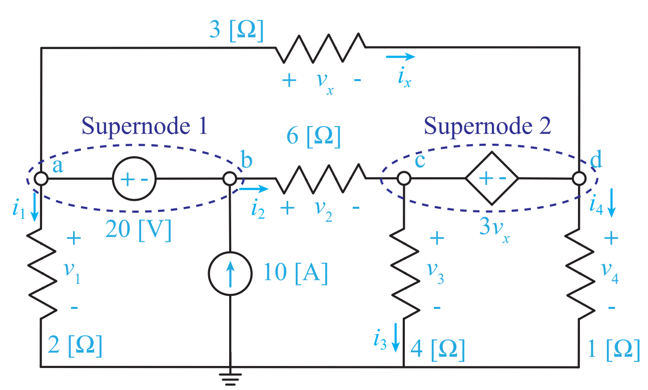
$$\begin{bmatrix} 0.0009 & -0.0004 & -0.0005 \\ 0.0004 & -0.0016 & 0.001 \\ 0.0005 & 0.001 & -0.0025 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 0.010 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 20 \\ 10 \\ 8 \end{bmatrix} \text{ [V]}$$

Problem #3

(30 pts.) Using Node Voltage Analysis, determine the node voltages at a, b, c and d.



Step 1: Assign nodes (N) and leg currents to all branches/elements (the circuit has also been put into a more understandable format for ease of evaluation):
$N = 5$ and we define i_1, i_2, i_3, i_4, and i_x as shown.



Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes:
We need 2 KCL equations applied at supernodes 1 and 2:

$$\text{Supernode 1} \implies 10\,[\text{A}] = i_x + i_1 + i_2$$

$$\text{Supernode 2} \implies i_2 + i_x = i_3 + i_4$$

Step 4: Apply Ohm's law in terms of node voltages (note that the two supernodes also provide us with 2 additional equations and V_e is equal to 0 [V]):
KCL at Supernode 1:

$$10\,[\text{A}] = \frac{V_a - V_d}{3\,[\Omega]} + \frac{V_a - V_e}{2\,[\Omega]} + \frac{V_b - V_c}{6\,[\Omega]}$$

KCL at Supernode 2:

$$\frac{V_b - V_c}{6\,[\Omega]} + \frac{V_a - V_d}{3\,[\Omega]} = \frac{V_c - V_e}{4\,[\Omega]} + \frac{V_d - V_e}{1\,[\Omega]}$$

We also have two additional equations from our supernodes. Supernode 1 equation:

$$V_a - V_b = 20\,[\mathrm{V}]$$

Supernode 2 equation:

$$V_c - V_d = (3)V_x$$

We also have an equation relating V_x to V_a and V_d

$$V_x = V_a - V_d$$

We can put the previous equations into matrix form:

$$\begin{bmatrix} (5/6) & (1/6) & (-1/6) & (-1/3) & 0 \\ (1/3) & (1/6) & (-5/12) & (-4/3) & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -3 \\ 1 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_x \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 20 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_x \end{bmatrix} = \begin{bmatrix} 26.67 \\ 6.67 \\ 173.33 \\ -46.67 \\ 73.33 \end{bmatrix} [\mathrm{V}]$$