Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

Lecture 29 Sections 9.1-9.6

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department University of Pittsburgh

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response

9.6 - Under-damped Response

0.5 - Critically Damped Response



Student Learning Objectives

At the end of the lecture, students should be able to:

▶ Describe the behavior or *RLC* circuits using second-order, ordinary differential equations

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.6 - Under-damped

Response

9.5 - Critically Damped Response



Step Source

► We will work with the general second-order, ordinary differential equation

$$\frac{d^2x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

- ightharpoonup x(t) is the output of the circuit (i.e. i(t) or v(t))
- ightharpoonup f(t) is the input, either a constant or can vary with time

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction

RLC Circuits

9.3 - Soln. of 2nd Order Diff. Eqn.

Response 9.6 - Under-damped

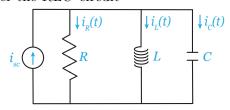
Response

.5 - Critically Damped Response



The RLC Circuit

Consider the *RLC* circuit



► Constructing a KCL equation at the top node

$$i_{sc} = i_R + i_L + i_C$$

► Employing the constitutive equations for each component in terms of node voltages

$$i_{sc} = \frac{v}{R} + \frac{1}{L} \int_{t_0}^t v(\tau) d\tau + i_0 + C \frac{dv}{dt}$$

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction

9.2 - Diff. Eqn. for RLC Circuits

Order Diff. Eqn.

Response

9.6 - Under-damped

Response

Damped Response

Summary



Slide 4 of 24

The RLC Circuit

➤ To eliminate the integral, we differentiate the equation

$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0$$

▶ Dividing by C and arranging in order of derivatives

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

► Thus, we have the natural response for an *RLC* circuit

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction9.2 - Diff. Eqn. for

RLC Circuits

9.3 - Soln. of 2nd Order Diff. Eqn.

9.6 - Under-damped

9.6 - Under-damped Response

9.5 - Critically Damped Response



Natural Response of *RLC* Circuit

► The response is going to be that of the homogeneous

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

► The characteristic equation is

$$\lambda^2 + \frac{1}{RC}\lambda + \frac{1}{LC} = 0$$

▶ Using the quadratic equation and solving for λ

$$\lambda_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

► Thus, our solution takes the form

$$v(t) = A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t}$$

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

.1 - Introduction

RLC Circuits

9.3 - Soln. of 2nd

Order Diff. Eqn.

9.6 - Under-damped

Response

Damped Respons

Summary



Slide 6 of 24

Natural Response of *RLC* Circuit

▶ For simplicity, we will define α , the Neper frequency, as

$$\alpha = \frac{1}{2RC}$$

• We will also define ω_0 , the resonant radian frequency, as

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

► Thus, the roots becomes

$$\lambda_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

▶ Thus, we are back at the original form

$$\frac{d^2x(t)}{dt^2} + 2\alpha \frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

.1 - Introduction

RLC Circuits

9.3 - Soln. of 2nd

Order Diff. Eqn.

9.6 - Under-damped

Response

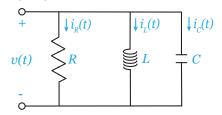
9.5 - Critically Damped Response

Summary



Slide 7 of 24 ©

Consider an *RLC* circuit where $v(0^+)=12$ [V] and $i_L(0^+)=30$ [mA]



• Given C=0.2 [μ F], L=50 [mH], R=200 [Ω], determine the initial current value in each branch and v(t).

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.4 - Over-damped Response

9.6 - Under-damped Response

0.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.4 - Over-damped Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.4 - Over-damped Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.4 - Over-damped Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

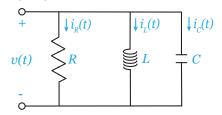
9.4 - Over-damped Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



▶ Consider an *RLC* circuit where $v(0^+)=12$ [V] and $i_L(0^+)=30 \text{ [mA]}$



• Given $C=0.2 [\mu F]$, L=50 [mH], $R=400 [\Omega]$, determine the initial current value in each branch and v(t).

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

9.6 - Under-damped

Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response 9.6 - Under-damped

9.5 - Critically Damped Response

ummary

Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response 9.6 - Under-damped

9.5 - Critically Damped Response

Summary

Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

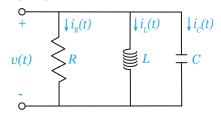
Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Consider an *RLC* circuit where $v(0^+)=12$ [V] and $i_L(0^+)=30$ [mA]



• Given C=0.2 [μ F], L=50 [mH], R=250 [Ω], determine the initial current value in each branch and v(t).

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response

9.6 - Under-damped Response

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response) 6 - Under-damped

Response

9.5 - Critically

Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

nesponse 9.6 - Under-damped

9.5 - Critically Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response) 6 - Under-damped

9.5 - Critically
Damped Response



Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

earning Objectives

9.1 - Introduction

RLC Circuits

Order Diff. Eqn.

Response) 6 - Under-damped

9.5 - Critically Damped Response



Student Learning Objectives

At the end of the lecture, students should be able to:

- ▶ Describe the behavior or *RLC* circuits using second-order, ordinary differential equations
 - ► A second-order ordinary differential equation is used to described the behavior of parallel *RLC* circuits.
 - ▶ If $\alpha^2 > \omega_0^2$, the circuit is overdamped
 - ▶ If $\alpha^2 < \omega_0^2$, the circuit is underdamped
 - If $\alpha^2 = \omega_0^2$, the circuit is critically damped

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

.1 - Introduction

RLC Circuits

Order Diff. Eqn.

9.4 - Over-damped

9.6 - Under-damped

Response

.5 - Critically Damped Response



Suggested Problems

▶ 9.2-1, 9.2-2, 9.2-5, 9.2-6, 9.2-7, 9.2-9, 9.2-10, 9.2-11, 9.2-15, 9.3-4, 9.4-5, 9.5-1, 9.5-3, 9.6-2

Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

MEMS 0031

Learning Objectives

1 - Introduction

RLC Circuits

Order Diff. Eqn.

tesponse) 6 - Under-damped

9.5 - Critically

