Homework #1

MEMS 0031 - Electrical Circuits

Assigned: May 7^{th}, 2020
Due: May 13^{th}, 2020 at 11:59 pm

Problem #1

The total charge entering a circuit element is expressed as $q(t)$=1(t−e^{-10t}) for when t ≥0. When t<0, $q(t)$=5 [C]. Determine the current in the circuit element for t ≥0.

Current is the time-rate-of-change of charge:

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(t - e^{-10t}\right) = 1 + 10e^{-10t} \text{ [A]}$$

Problem #2

The current in a circuit element is $i(t)$=5(t−e$^{-20t^2}$) [A] when t ≥0. When t<0, $i(t)$=1.5 [A]. Determine the total charge that has entered the circuit element for t ≥0.

The charge is the time-integral of current:

$$q(t) = \int_{-\infty}^{t} i(\tau)\,d\tau = q(0) + \int_{0}^{t} i(\tau)\,d\tau$$

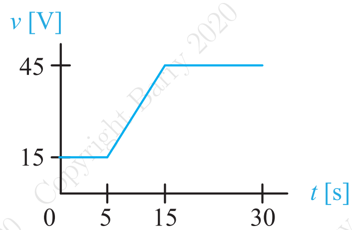
Since we are only interested in the time period t>0, we will disregard the initial current. The initial current integrated between the bounds of negative infinity and zero would be infinite. Thus, the charge in the time interval of interest is:

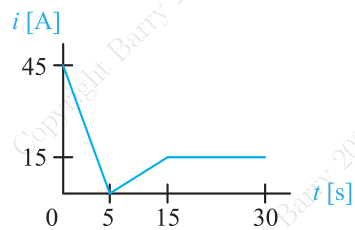
$$q(t) = \int_{0}^{t} 5t - 5e^{-20t^2}\,dt = \frac{5t^2}{2} - 5\left(\frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{20}}\text{erf}(\sqrt{20}t)\right) \text{ [C]}$$

Although the erf asymptotically approaches a value of 0.9908..., the t-squared term dominates and the charge entering the circuit approaches infinity as time approaches infinity.

Problem #3

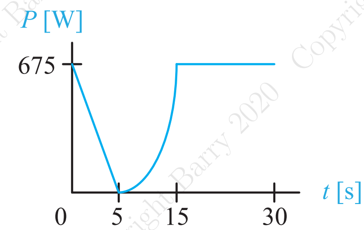
The time variation of current and voltage through and across an electrical circuit element is depicted in the figure below. Sketch the power delivered to the element for the time interval between 0 and 30 seconds. What is the total energy delivered to the element for the time interval between 0 and 30 seconds?





The power as a function of time is simply the voltage times the current, both of which are functions of time. Expressing this in our time intervals:

$$P(t) = \begin{cases} (15\,[\mathrm{V}])(-9t+45)\,[\mathrm{A}] = (-135t+675)\,[\mathrm{W}], 0 \leq \ t\,[\mathrm{s}]<5 \\ (3t\,[\mathrm{V}])(1.5t-7.5)\,[\mathrm{A}] = (4.5t^2-22.5t)\,[\mathrm{W}],\ 5 \leq t\,[\mathrm{s}] \leq 15 \\ (45\,[\mathrm{V}])(15\,[\mathrm{A}]) = 675\,[\mathrm{W}],\ 15< t\,[\mathrm{s}] \leq 30 \end{cases}$$



To determine the energy delivered to the element, we must integrate each power expression over their respective time intervals:

$$w(t) = \begin{cases} (-67.5t^2+675t)\,[\mathrm{J}],\ 0 \leq t\,[\mathrm{s}]<5 \\ (1.5t^3-11.25t^2)\,[\mathrm{J}],\ 5 \leq t\,[\mathrm{s}] \leq 15 \\ 675t\,[\mathrm{J}],\ 15< t\,[\mathrm{s}] \leq 30 \end{cases}$$

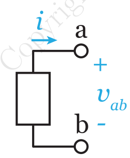
Evaluating this integrals over the bounds:

$$w(t) = \begin{cases} 1,687.5\,[\mathrm{J}],\ 0 \leq t\,[\mathrm{s}]<5 \\ 2,625\,[\mathrm{J}],\ 5 \leq t\,[\mathrm{s}] \leq 15 \\ 10,125\,[\mathrm{J}],\ 15< t\,[\mathrm{s}] \leq 30 \end{cases}$$

Thus, the energy delivered is 14,437.5 [J].

Problem #4

Find the power, $P(t)$, supplied by the element shown in the figure to the right when $v(t)$=4cos(3t) [V] and $i(t)$=sin(3t)/12 [A]. Determine $P(t)$ for when t=0.75 and 1.2 [s].



The power is the voltage times the current such that:

$$P(t) = (4\cos(3t)\,[\mathrm{V}])\left(\frac{\sin(3t)}{12}\,[\mathrm{A}]\right) = \frac{1}{3}\sin(3t)\cos(3t)\,[\mathrm{W}] = \frac{\sin(6t)}{6}\,[\mathrm{W}]$$

Evaluating at the given times:

$$P(t=0.75) = \frac{\sin(4.5)}{6}\,[\mathrm{W}] = -0.163\,[\mathrm{W}]$$

$$P(t=1.2) = \frac{\sin(7.2)}{6}\,[\mathrm{W}] = 0.132\,[\mathrm{W}]$$

Problem #5

A car uses a 14.7 [V] battery to power the starter motor. If the starter motor draws 250 [A], what is the power supplied by the battery? What is the power supplied by the battery over a 1 second period?

The power supplied by the battery is the voltage times the current:

$$P_{\text{supplied}} = iV = (250\,[\mathrm{A}])(14.7\,[\mathrm{V}]) = 3,675\,[\mathrm{W}]$$

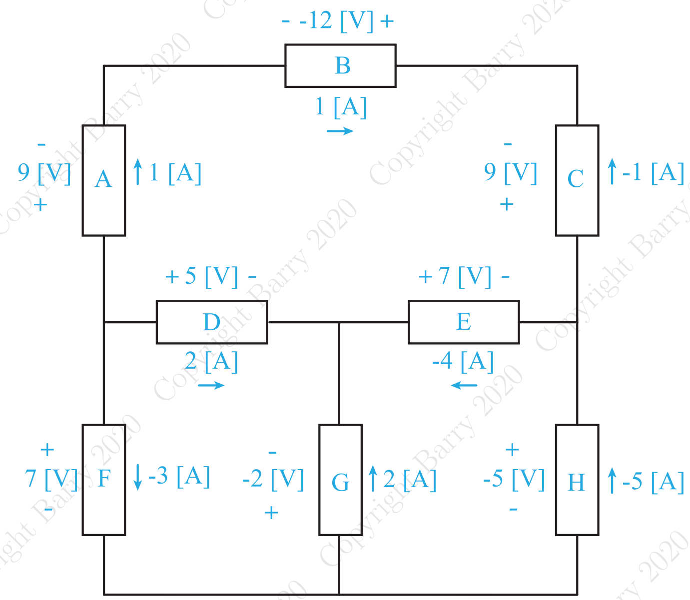
The power supplied by the battery over a 1 second period is the energy delivered:

$$w = \int_0^1 P(t)\,dt = 3,675\,[\mathrm{J}]$$

Problem #6

Given the circuit below, determine:

a) The power supplied;
b) The power dissipated;
c) If the circuit obeys the Conservation of Energy.



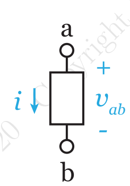
Creating a table, with labeling each element as shown above, we can determine the power supplied or power dissipated:

Element	Power Supplied [W]	Power Dissipated [W]
A	-	9
B	-	12
C	9	-
D	-	10
E	-	28
F	21	-
G	4	-
H	25	-
Total	59	59

It is clear this circuits obeys the Conservation of Energy.

Problem #7

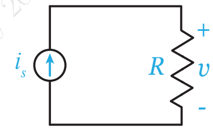
An electrical circuit element has voltage and current values as shown in the figure to the right. Determine if this particular circuit element is linear or non-linear.
If we calculate the resistance, v/i, we can see after 1.5 [V] and 1.5 [V], the resistance is no longer constant, and thus we have non-linear behavior.



v [V]	i [A]	v/i
-2	-2	1
0	0	undef.
1.5	1.5	1
3.78	3.76	1.005
4.2	4.1	1.024
5	4.8	1.042

Problem #8

A current source and resistor are connected in series as shown in the figure to the right. If the current source is 5 [A] and the voltage drop across the resistor is 18.3 [V], calculate a) the resistance R of the resistor, and b) the power dissipated by the resistor.



a) Using Ohm's law, the resistance R is the voltage potential v per the current i such that:

$$R = \frac{v}{i} = \frac{18.3\,[\mathrm{V}]}{5\,[\mathrm{A}]} = 3.66\,[\Omega]$$

b) The power dissipated by the resistor can be found via two formulations:

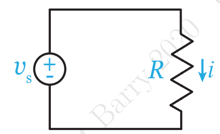
$$P_{\text{diss}} = i^2R = (5\,[\mathrm{A}])^2(3.66\,[\Omega]) = 91.5\,[\mathrm{W}]$$

Alternatively:

$$P_{\text{diss}} = \frac{V^2}{R} = \frac{(18.3\,[\mathrm{V}])^2}{3.66\,[\Omega]} = 91.5\,[\mathrm{W}]$$

Problem #9

A voltage source and resistor are connected in series as shown in the figure to the right. If the voltage source is 16 [V] and the resistance of the resistor is 4.2 [Ω], determine a) the current i drawn from the voltage source, and b) the power dissipated by the resistor.



a) Using Ohm's law, the current is the voltage potential v per the resistance R such that:

$$i = \frac{v}{R} = \frac{16\,[\mathrm{V}]}{4.2\,[\Omega]} = 3.81\,[\mathrm{A}]$$

b) The power dissipated by the resistor can be found via two formulations:

$$P_{\text{diss}} = i^2R = (3.81\,[\mathrm{A}])^2(4.2\,[\Omega]) = 60.97\,[\mathrm{W}]$$

Alternatively:

$$P_{\text{diss}} = \frac{V^2}{R} = \frac{(16\,[\mathrm{V}])^2}{4.2\,[\Omega]} = 60.95\,[\mathrm{W}]$$