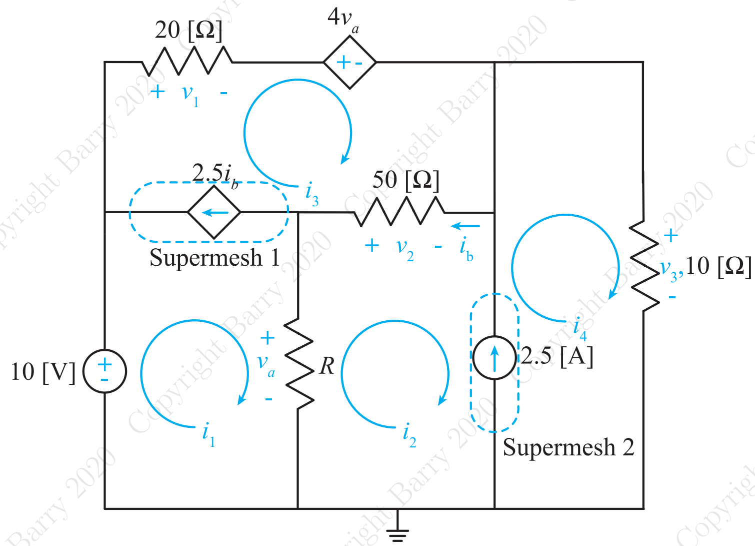
Homework #5

MEMS 0031 - Electrical Circuits

Assigned: June 5[th], 2020
Due: June 10[th], 2020 at 11:59 pm

Problem #1

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_4, given $R = 20\,[\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 4$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

Supermesh 1 equation:

$$i_3 - i_1 = 2.5 i_b$$

Supermesh 1:

$$-10\,[\mathrm{V}] + V_1 + 4V_a - (50\,[\Omega])(i_2 - i_3) + (20\,[\Omega])(i_1 - i_2) = 0$$

Supermesh equation 2:

$$i_4 - i_2 = 2.5\,[\mathrm{A}]$$

Supermesh 2:

$$-(20\,[\Omega])(i_1 - i_2) + (50\,[\Omega])(i_2 - i_3) + V_3 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$i_3 - i_1 = 2.5(i_3 - i_2) \implies i_1 - 2.5i_2 + 1.5i_3 = 0 \qquad (1)$$

Supermesh 1:

$$-10\,[\mathrm{V}] + (20\,[\Omega])i_3 + 4(20\,[\Omega])(i_1 - i_2) - (50\,[\Omega])(i_2 - i_3) + (20\,[\Omega])(i_1 - i_2) = 0$$

$$\implies 100i_1 - 150i_2 + 70i_3 = 10 \tag{2}$$

Supermesh 2 equation:

$$i_4 - i_2 = 2.5\,[\mathrm{A}] \tag{3}$$

Supermesh 2:

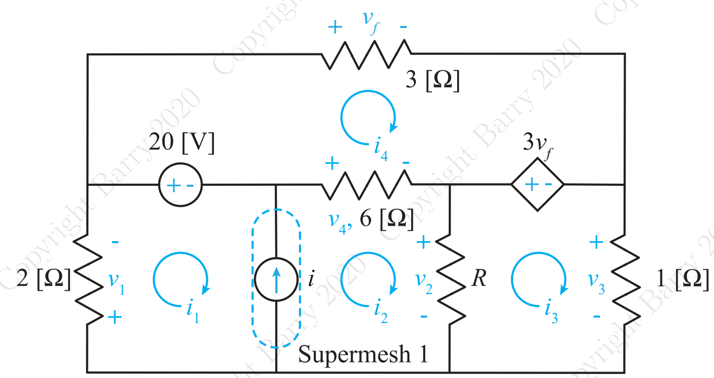
$$-(20\,[\Omega])(i_1 - i_2) + (50\,[\Omega])(i_2 - i_3) + (10\,[\Omega])i_4 = 0 \implies -20i_1 + 70i_2 - 50i_3 + 10i_4 = 0 \tag{4}$$

Putting eqns. 1 through 4 in matrix form:

$$\begin{bmatrix} 1 & -2.5 & 1.5 & 0 \\ 100 & -150 & 70 & 0 \\ 0 & -1 & 0 & 1 \\ -20 & 70 & -50 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 2.5 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -3.25 \\ -5.5 \\ -7 \\ -3 \end{bmatrix}$$

Problem #2

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_4, given $i = 10\,[\text{A}]$ and $R = 4\,[\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 4$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_2 - i_1 = 10\,[\text{A}]$$

Supermesh 1:

$$V_1 + 20\,[\text{V}] + V_4 + V_2 = 0$$

Mesh Current 3

$$-V_2 + 3V_f + V_3 = 0$$

Mesh Current 4

$$V_f - 3V_f - V_4 = 20\,[\text{V}]$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$-i_1 + i_2 = 10 \tag{5}$$

Supermesh 1:

$$(2\,[\Omega])i_1 + (6\,[\Omega])(i_2 - i_4) + (4\,[\Omega])(i_2 - i_3) = -20\,[\text{V}] \implies 2i_1 + 10i_2 - 4i_3 - 6i_4 = -20 \tag{6}$$

Mesh Current 3

$$-(4\,[\Omega])(i_2 - i_3) + 3(3\,[\Omega])i_4 + (1\,[\Omega])i_3 = 0 \implies -4i_2 + 5i_3 + 9i_4 = 0 \tag{7}$$

Mesh Current 4:

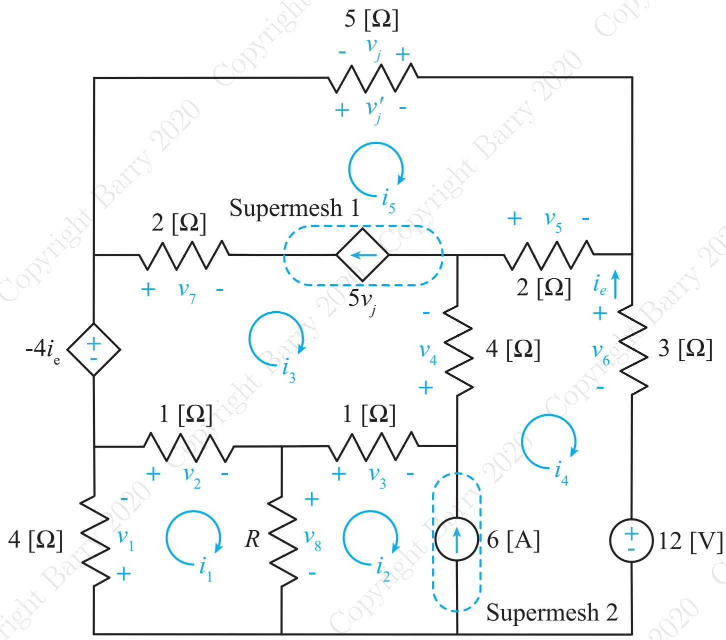
$$-2(3\,[\Omega])i_4 - (6\,[\Omega])(i_2 - i_4) = 20\,[\text{V}] \implies -6i_2 = 20 \tag{8}$$

Putting eqns. 5 through 8 in matrix form:

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 2 & 10 & -4 & -6 \\ 0 & -4 & 5 & 9 \\ 0 & -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \\ 0 \\ 20 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -13.33 \\ -3.33 \\ -46.67 \\ 24.44 \end{bmatrix}$$

Problem #3

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_5, given $R = 3\,[\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 5$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_5 - i_3 = 5V_j$$

Supermesh 1, noting $V_j' = -V_j$:

$$-(-4i_e) + V_j' - V_5 - V_4 - V_3 - V_2 = 0$$

Supermesh Equation 2:

$$i_4 - i_2 = 6\,[\text{A}]$$

Supermesh 2:

$$-V_8 + V_3 + V_4 + V_5 + V_6 + 12\,[\text{V}] = 0$$

Mesh Current 1:

$$V_1 + V_2 + V_8 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$-i_3 + i_5 - 5(5\,[\Omega])(-i_5) = 0 \implies -i_3 + 26i_5 = 0 \tag{9}$$

Supermesh 1:

$$4(-i_4) + (5\,[\Omega])i_5 - (2\,[\Omega])(i_4 - i_5) - (4\,[\Omega])(i_4 - i_3) - (1\,[\Omega])(i_2 - i_3) - (1\,[\Omega])(i_1 - i_3) = 0$$

$$\implies -i_1 - i_2 + 6i_3 - 10i_4 + 7i_5 = 0 \tag{10}$$

Supermesh Equation 2:

$$-i_2 + i_4 = 6\,[\text{A}] \tag{11}$$

Supermesh 2:

$$-(3\,[\Omega])(i_1 - i_2) + (1\,[\Omega])(i_2 - i_3) + (4\,[\Omega])(i_4 - i_3) + (2\,[\Omega])(i_4 - i_5) + (3\,[\Omega])i_4 = -12\,[\mathrm{V}]$$
$$\implies -3i_1 + 4i_2 - 5i_3 + 9i_4 - 2i_5 = -12\,[\mathrm{V}] \tag{12}$$

Mesh Current 1:

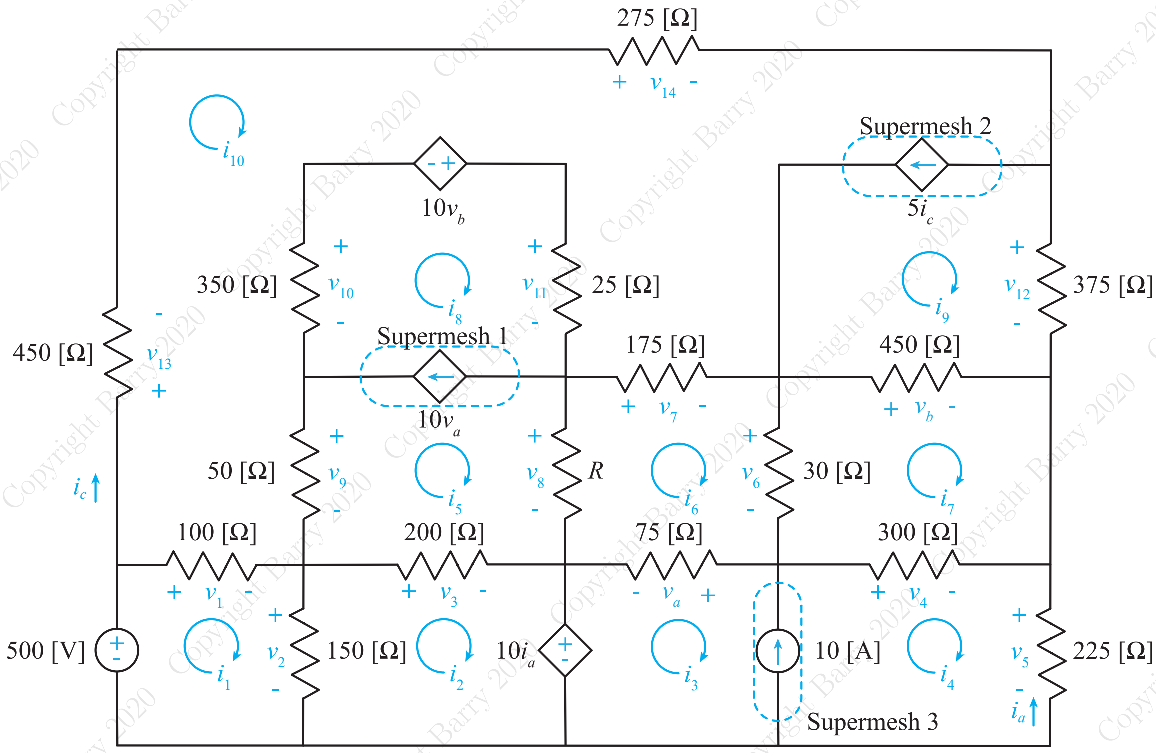
$$(4\,[\Omega])i_1 + (1\,[\Omega])(i_1 - i_3) + (3\,[\Omega])(i_1 - i_2) = 0 \implies 8i_1 - 3i_2 - i_3 = 0 \tag{13}$$

Putting eqns. 9 through 13 in matrix form:

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 26 \\ -1 & -1 & 6 & -10 & 7 \\ 0 & -1 & 0 & 1 & 0 \\ -3 & 4 & -5 & 9 & -2 \\ 8 & -3 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ -12 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -3.12 \\ -7.16 \\ -3.49 \\ -1.16 \\ -0.13 \end{bmatrix}$$

Problem #4

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_{10}, given $R = 125\,[\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 10$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N-\#$CS KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_8 - i_5 = 10V_a$$

Supermesh 1:

$$-V_9 - V_{10} - 10V_b + V_{11} + V_8 - V_3 = 0$$

Supermesh Equation 2:

$$i_{10} - i_9 = 5i_c$$

Supermesh 2:

$$V_{13} + V_{14} + V_{12} - V_b - V_7 - V_{11} + 10V_b + V_{10} + V_9 - V_1 = 0$$

Supermesh Equation 3:

$$i_4 - i_3 = 10\,[\mathrm{A}]$$

Supermesh 3:

$$-10i_a - V_a + V_4 + V_5 = 0$$

Mesh Current 1:

$$-500\,[\mathrm{V}] + V_1 + V_2 = 0$$

Mesh Current 2:

$$-V_2 + V_3 + 10i_a = 0$$

Mesh Current 6:

$$-V_8 + V_7 + V_6 + V_a = 0$$

Mesh Current 7:

$$-V_6 + V_b - V_4 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

We will denote the following variables:

$$i_a = -i_4$$

$$i_c = i_{10}$$

$$V_a = (75\,[\Omega])(i_6 - i_3) \implies V_a = -75i_3 + 75i_6$$

$$V_b = (450\,[\Omega])(i_7 - i_9) \implies V_b = 450i_7 - 450i_9$$

$$V_1 = (100\,[\Omega])(i_1 - i_{10}) \implies V_1 = 100i_i - 100i_{10}$$

$$V_2 = (150\,[\Omega])(i_1 - i_2) \implies V_2 = 150i_1 - 150i_2$$

$$V_3 = (200\,[\Omega])(i_2 - i_5) \implies V_3 = 200i_2 - 200i_5$$

$$V_4 = (300\,[\Omega])(i_4 - i_7) \implies V_4 = 300i_4 - 300i_7$$

$$V_5 = (225\,[\Omega])i_4 \implies V_5 = 225i_4$$

$$V_6 = (30\,[\Omega])(i_6 - i_7) \implies V_6 = 30i_6 - 30i_7$$

$$V_7 = (175\,[\Omega])(i_6 - i_{10}) \implies V_7 = 175i_6 - 175i_{10}$$

$$V_8 = (125\,[\Omega])(i_5 - i_6) \implies V_8 = 125i_5 - 125i_6$$

$$V_9 = (50\,[\Omega])(i_{10} - i_5) \implies V_9 = -50i_5 + 50i_{10}$$

$$V_{10} = (350\,[\Omega])(i_{10} - i_8) \implies V_{10} = -350i_8 + 350i_{10}$$

$$V_{11} = (25\,[\Omega])(i_8 - i_{10}) \implies V_{11} = 25i_8 - 25i_{10}$$

$$V_{12} = (375\,[\Omega])i_9 \implies V_{12} = 375i_9$$

$$V_{13} = (450\,[\Omega])i_{10} \implies V_{13} = 450i_{10}$$

$$V_{14} = (275\,[\Omega])i_{10} \implies V_{14} = 275i_{10}$$

Supermesh Equation 1:

$$750i_3 - i_5 - 750i_6 + i_8 = 0 \tag{14}$$

Supermesh 1:

$$-200i_2 + 375i_5 - 125i_6 - 4,500i_7 + 375i_8 + 4,500i_9 - 425i_{10} = 0 \tag{15}$$

Supermesh Equation 2:

$$-i_9 - 4i_{10} = 0 \tag{16}$$

Supermesh 2:

$$-100i_i - 50i_5 - 175i_6 + 4,050i_7 - 375i_8 - 3,675i_9 + 1,425i_{10} = 0 \tag{17}$$

Supermesh Equation 3:

$$-i_3 + i_4 = 10 \tag{18}$$

Supermesh 3:

$$75i_3 + 535i_4 - 75i_6 - 300i_7 = 0 \tag{19}$$

Mesh Current 1:

$$250i_i - 150i_2 - 100i_{10} = 500 \tag{20}$$

Mesh Current 2:

$$-150i_1 + 350i_2 - 10i_4 - 200i_5 = 0 \tag{21}$$

Mesh Current 6:

$$-75i_3 - 125i_5 + 405i_6 - 30i_7 - 175i_{10} = 0 \tag{22}$$

Mesh Current 7:

$$-300i_4 - 30i_6 + 780i_7 - 450i_9 = 0 \tag{23}$$

Putting eqns. 14 through 23 in matrix form:

$$\begin{bmatrix} 0 & 0 & 750 & 0 & -1 & -750 & 0 & 1 & 0 & 0 \\ 0 & -200 & 0 & 0 & 375 & -125 & -4,500 & 375 & 4,500 & -425 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -4 \\ -100 & 0 & 0 & 0 & -50 & -175 & 4,050 & -375 & -3,675 & 1,425 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 75 & 535 & 0 & -75 & -300 & 0 & 0 & 0 \\ 250 & -150 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -100 \\ -150 & 350 & 0 & -10 & -200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -75 & 0 & -125 & 405 & -30 & 0 & 0 & -175 \\ 0 & 0 & 0 & -300 & 0 & -30 & 780 & 0 & -450 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \\ 0 \\ 500 \\ 0 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix} = \begin{bmatrix} -8.8187 \\ -17.7072 \\ -9.4516 \\ 0.5484 \\ -24.4010 \\ -9.4196 \\ 0.9700 \\ -0.4180 \\ 1.9438 \\ -0.4859 \end{bmatrix}$$