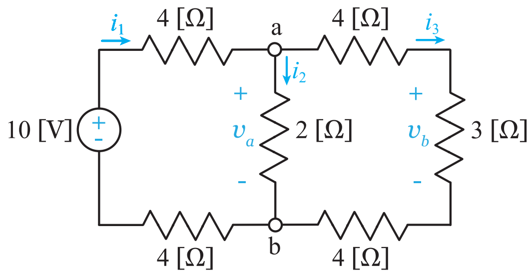
MEMS 0031 - Electrical Circuits
Quiz #2
May 22nd, 2019
90 points

Name: ______________________________

Problem #1

(25 pts.) Using only KVL/KCL/Ohm's law, determine the voltage drop across the 2 [Ω] resistor, V_a, the 3 [Ω] resistor, as well as the currents flowing through the 2 and 3 [Ω] resistors.

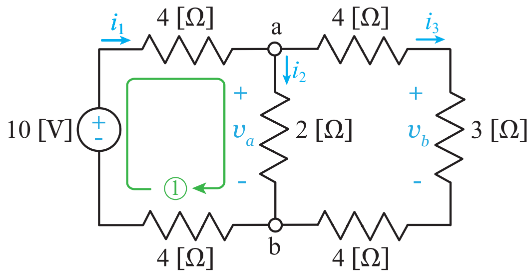
We begin by defining nodes a and b, and currents i_1, i_2, and i_3 as shown, obeying PSC.



Applying KCL at node a:

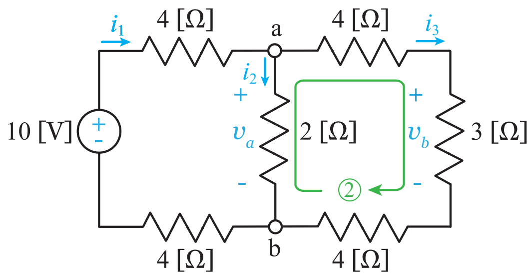
$$i_1 = i_2 + i_3 \tag{1}$$

Currently, we have one equation and three unknowns. Thus, we need two more independent equations. We will construct two KVL equations, relating our currents. We will apply KVL around loop 1.



$$-10\,[\mathrm{V}] + (4\,[\Omega])i_1 + (2\,[\Omega])i_2 + (4\,[\Omega])i_1 = 0 \implies (8\,[\Omega])i_1 + (2\,[\Omega])i_2 = 10\,[\mathrm{V}] \tag{2}$$

Now, applying KVL around loop 2:



$$-(2\,[\Omega])i_2 + (4\,[\Omega])i_3 + (3\,[\Omega])i_3 + (4\,[\Omega])i_3 = 0 \implies -(2\,[\Omega])i_2 + (11\,[\Omega])i_3 = 0 \tag{3}$$

Putting eqns. 1-3 in matrix form:

$$\begin{bmatrix} 1 & -1 & -1 \\ 8 & 2 & 0 \\ 0 & -2 & 11 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 1.032 \\ 0.873 \\ 0.159 \end{bmatrix}$$

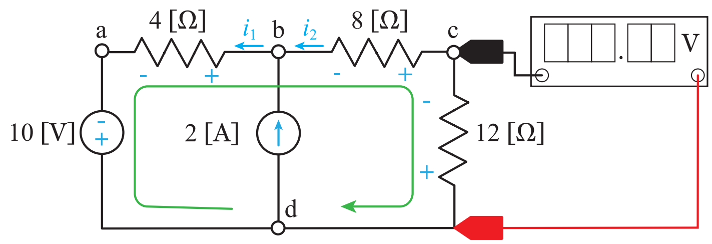
Units are taken as [A]. Having found the currents flowing through the 2 and 3 [Ω] resistors (i.e. i_2 and i_3 respectively), we can now determine the voltage drop V_a via Ohm's law:

$$V_a = i_2 R = (0.873\,[\text{A}])(2\,[\Omega]) = 1.746\,[\text{V}]$$

Problem #2

(30 pts.) Using only KVL/KCL/Ohm's law, determine the voltage measured by the voltmeter in the cicruit below.

We begin by defining i_1 and i_2 as shown, ensuring that i_2 obeys PSC since we know the red terminal of a voltmeter is the positive terminal and the black terminal is negative. We also define the voltage potentials across the resistors obeying PSC, and nodes a - d.



Applying KCL to node b:

$$i_1 = 2\,[\text{A}] + i_2 \tag{4}$$

Now, applying KVL around the loop shown:

$$10\,[\text{V}] - (4\,[\Omega])i_1 - (8\,[\Omega])i_2 - (12\,[\Omega])i_2 = 0 \tag{5}$$

Substituting i_1 from eq. 4 into eq. 5, it is found that $i_2 = 0.08\overline{3}$ [A].

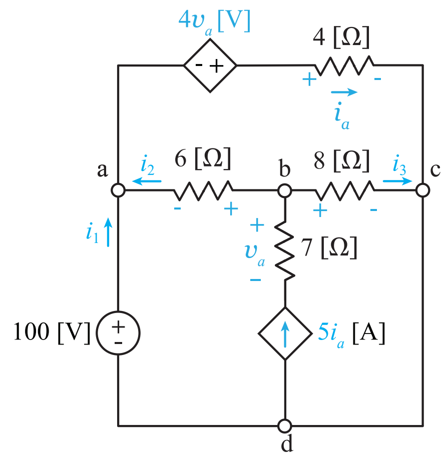
Now, we use Ohm's law to determine the reading of the voltmeter:

$$V = iR = (0.08\overline{3}\,[\text{A}])(12\,[\Omega]) = 1\ [\text{V}]$$

Problem #3

(35 pts.) Given the schematic below, using only KVL/KCL/Ohm's law, determine the numeric values of V_a and i_a.

We begin by defining nodes a through d, and currents i_1 through i_4 obeying PSC.



Applying KCL at node a:

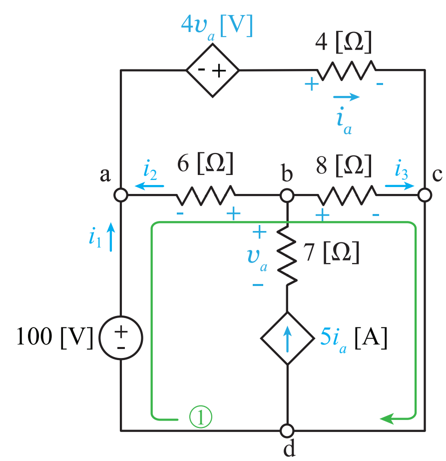
$$i_1 + i_2 = i_a \tag{6}$$

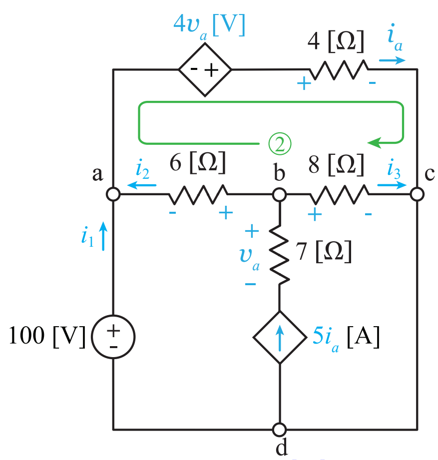
Now, applying KCL at node b:

$$5i_a = i_2 + i_3 \tag{7}$$

We currently have two equations and four unknowns. Thus, we need two more independent equations. We will construct two KVL equations relating our currents. Applying KVL around loop 1:

$$-100\,[\mathrm{V}] - (6\,[\Omega])i_2 + (8\,[\Omega])i_3 = 0 \tag{8}$$





Now, applying KVL around loop 2:

$$-4V_a + (4\,[\Omega])i_a - (8\,[\Omega])i_3 + (6\,[\Omega])i_2 = 0 \tag{9}$$

We see that in applying KVL around loop 2, we have introduced another variable, V_a. Thus, we need another equation, i.e. we will construct five equations to solve for five independent variables.

Applying Ohm's law to the 7 [Ω] resistor, recalling that a resistor must obey PSC:

$$V_a = -(5i_a)(7\,[\Omega]) = -35i_a\,[\mathrm{V}] \tag{10}$$

Putting eqns. 6-10 in matrix form:

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & -5 & 0 \\ 0 & -6 & 8 & 0 & 0 \\ 0 & 6 & -8 & 4 & -4 \\ 0 & 0 & 0 & 35 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_a \\ V_a \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_a \\ V_a \end{bmatrix} = \begin{bmatrix} 5.853 \\ -5.159 \\ 8.631 \\ 0.694 \\ -24.306 \end{bmatrix}$$

Units are taken as [A] for current, and [V] for voltage.