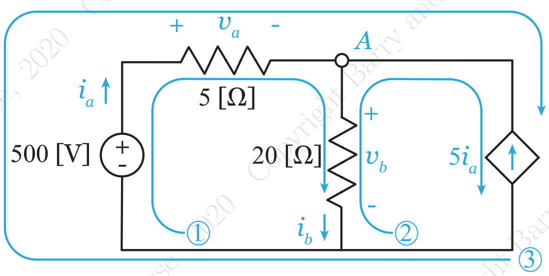
Kirchoff's Laws Worksheet

MEMS 0031 - Electrical Circuits

May 13, 2020

Problem #1 - Lecture 3

For the circuit shown below, determine the currents i_a and i_b and voltage potentials v_a and v_b. What is the voltage potential across the CCCS?



a) We can solve this using KVl and KCL. To start, we will apply KVL around loop 1:

$$-500\,[\text{V}] + (5\,[\Omega])i_a + (20\,[\Omega])i_b = 0 \tag{1}$$

Applying KCL at node A:

$$i_a + 5i_a = i_b \tag{2}$$

Applying KVL around loop 3:

$$500\,[\text{V}] + 5\,[\Omega])i_a + v_s = 0 \tag{3}$$

Putting these equations in matrix form:

$$\begin{bmatrix} 5 & 20 \\ 6 & -1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 500 \\ 0 \end{bmatrix}$$

Thus, the currents, in units of [A], are:

$$\begin{bmatrix} i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 4 \\ 24 \end{bmatrix}$$

The voltage potentials are found via Ohm's law:

$$v_a = (5\,[\Omega])i_a = 20\,[\text{V}]$$

$$v_b = (20\,[\Omega])i_b = 480\,[\text{V}]$$

The voltage potential across the CCCS can be found be applying KVL to loop 2 or 3. Applying KVL around loop 2:

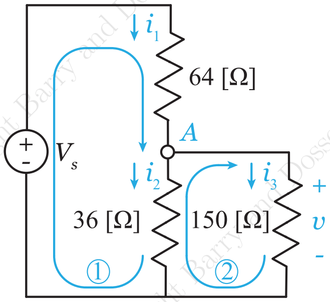
$$-v_b + v_{CCCS} = 0 \implies v_{CCCS} = 480\,[\text{V}]$$

Applying KVL around loop 3:

$$-500\,[\text{V}] + v_a + v_{CCCS} = 0 \implies v_{CCCS} = 500\,[\text{V}] - v_a = 480\,[\text{V}]$$

Problem #2 - Lecture 4

For the circuit shown below, determine the current through each resistor and the voltage v across the load resistance R_L when V_s=15 [V], R_x=100 [Ω], a=0.36 and R_L=150 [Ω].



We need to determine either the voltage potential across, or the current running through, each resistor. Applying KCL at node A:

$$i_1 = i_2 + i_3 \tag{4}$$

Since we have one equation and three unknowns, we need to construct two more equations. Applying KVL around loop 1:

$$-V_s + (64\,[\Omega])i_1 + (36\,[\Omega])i_2 = 0 \tag{5}$$

Applying KVL around loop 2:

$$-(36\,[\Omega])i_2 + (150\,[\Omega])i_3 = 0 \tag{6}$$

Putting the system of equations in matrix form:

$$\begin{bmatrix} 1 & -1 & -1 \\ 64 & 36 & 0 \\ 0 & -36 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ V_s \\ 0 \end{bmatrix}$$

Solving for the currents, with units of [A]:

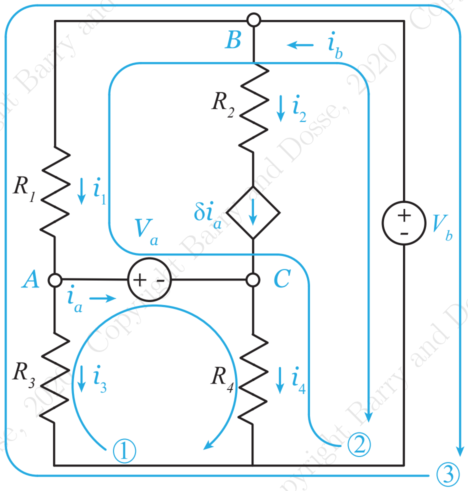
$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0.1612 \\ 0.1300 \\ 0.0312 \end{bmatrix}$$

The voltage drop across the load resistance is found using Ohm's law:

$$v = i_3 R = (\ [\mathrm{A}])(150\,[\Omega]) = 4.68\,[\mathrm{V}]$$

Problem #3 - Lecture 5

In the circuit below, using KCL and KVL, construct a system of equations that allows you to solve for the current flowing through each element. Additionally, solve for i_a symbolically, in terms of all other circuit variables



We recognize we have two independent sources, V_a and V_b, and one dependent source. The CCCS has an output δi_a, which is proportional to i_a, the current flowing through V_a. To proceed in determining the current flowing through each element, we will employ KCL at the nodes labeled A, B and C. At node A:

$$i_1 = i_a + i_3 \tag{7}$$

At node B:

$$i_b = i_1 + i_2 \tag{8}$$

Lastly, at node C, recognizing $i_2 = \delta i_a$:

$$i_2 + i_a = i_4 \implies i_a(\delta + 1) = i_4 \tag{9}$$

We currently have three equations and six unknowns. We need three more independent equations. To do such, we will employ KVL around three loops. Applying KVL around loop 1:

$$-i_3R_3 + V_a + i_4R_4 = 0 \tag{10}$$

Applying KVL around loop 2:

$$-i_4R_4 - V_a - i_1R_1 + V_b = 0 \tag{11}$$

Applying KVL around loop 3:

$$-i_3R_3 - i_1R_1 + V_b = 0 \tag{12}$$

Putting our equations in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & -1 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & (\delta+1) & 0 \\ 0 & 0 & -R_3 & R_4 & 0 & 0 \\ -R_1 & 0 & 0 & -R_4 & 0 & 0 \\ -R_1 & 0 & -R_3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_a \\ i_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -V_a \\ V_a - V_b \\ -V_b \end{bmatrix}$$