Chapter 2 - Circuit Elements

Lecture 2
Sections 2.1-2.5

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Student Learning Objectives

At the end of the lecture, students should be able to:

- Identify active and passive circuit elements
- Understand the constraints for an element to be considered linear
- Understand the formulation of electrical resistance and how it relates to resistors
- Understand and be able to apply Ohm's Law
- Calculate power dissipated in elements (Joule heating)
- Identify and verify ideal current and voltage source

Circuit Elements

- There are many circuit elements we will analyze in this course, and they are broken down into two categories:

Passive Elements

- Resistors (R)
- Inductors (L)
- Capacitors (C)
- Diodes

Active Elements

- Voltage sources
- Current sources
- Controlled sources
- Op-amps

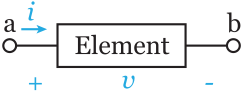
- We will first focus our attention on resistors, and voltage and current sources

Linear Circuit Elements

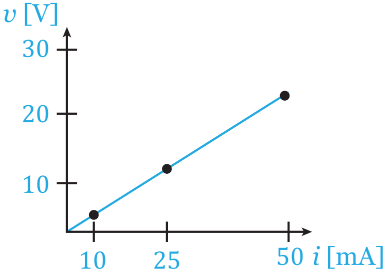
- **Linear circuit** elements satisfy the properties of **superposition** and **homogeneity**, and exhibit linear behavior
- **Superposition** states that the total response of a component with multiple stimuli is the sum of the responses of individual stimuli acting on the component
- **Homogeneity** states the dimensions of each contributing term are consistent

Linear Circuit Elements

- Is the element depicted below behaving linearly?



v [V]	i [mA]	v/i
4.5	10	0.45
11.25	25	0.45
22.5	50	0.45



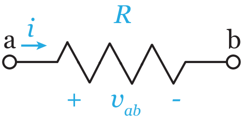
- What would non-linear behavior look like?

Resistance

- **Resistance** (R, [Ω], [V/A]) is the physical property of an element or device that impedes the flow of current
- R expressed in terms of the electrical resistivity ρ, which is dependent upon the valence electrons and free electrons, length of the element L, and cross-sectional area of the element A such that

$$\boxed{R = \frac{\rho L}{A}}$$

- The resistor will be represented by the following symbol

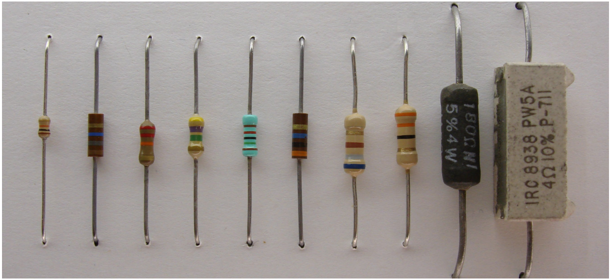


Example #1

- What is the resistance of a 30 [m] length of copper (ρ=1.68$\cdot$10^{-8} [Ω-m]) wire, if the wire were:
  - 12-gage (A=3.31 [mm^2])
  - 10-gage (A=5.26 [mm^2])

Resistors

- Resistors come in all shapes and sizes, and are made from a variety of material to satisfy design requirements



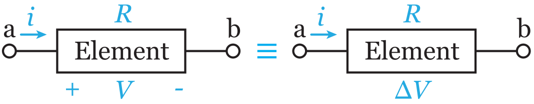
- Resistors have voltage and current ratings, and subsequently maximum power ratings
- The bands tell you the nominal resistance value and tolerance

Ohm's Law

- **<u>Ohm's law</u>** states the voltage potential across a resistor is proportional to the current through it, and is related by the constant of proportionality, resistance, such that

$$\boxed{V = iR}$$

- That is, a potential is needed across an element to drive a current through it:



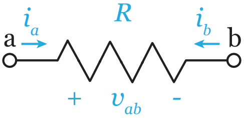
- This is commonly expressed as:

$$\Delta V = iR$$

- We will drop the ΔV convention, and indicate the potential via $+$ and $-$ signs

Ohm's Law

- Consider the resistor shown below, with a voltage potential v_{ab} driving i_a through R



- We can express i_a in terms of i_b

$$i_a = -i_b$$

- Applying Ohm's law consistent with the PSC:

$$V_{ab} = i_a R$$

- Substituting $-i_b$ for i_a

$$V_{ab} = -i_b R \implies V_{ba} = i_b R$$

- The potential is dependent upon the direction of i

Linear Resistors

- An important assumption made is that the resistor is operating within a region of linear behavior
- Linearity implies $v = f(i) = iR$, that is voltage will be a function of current, where R is independent of both voltage and current
- Given an element with resistance R, and currents i_1 and i_2 flowing through the element, with corresponding voltages $V_1{=}Ri_1$ and $V_2{=}Ri_2$ imposed across the element to drive currents i_1 and i_2, the total voltage applied would be:

$$V = R(i_1 + i_2) = V_1 + V_2$$

Power Dissipated

- The power dissipated by a resistor can be determined using Ohm's law. The potential is:

  $$V = iR$$

  We recall power is current times voltage

  $$P = Vi$$

  Substituting in V expressed by Ohm's law into the power equation

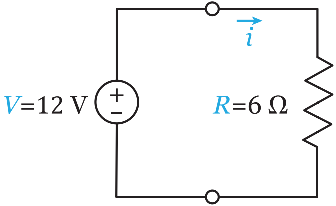
  $$P = (iR)i = i^2R$$

  This is known as **Joule heating**, which is also expressed as

  $$P = \frac{V^2}{R}$$

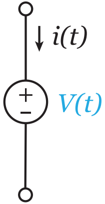
Example #2

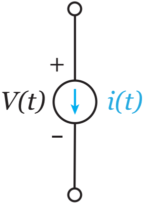
- Compute a) the current through the resistor and b) the power dissipated by the resistor



Independent Sources

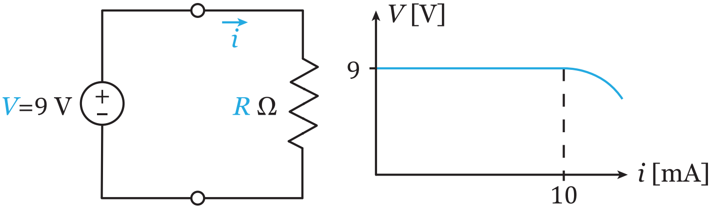
- There are two types of independent sources





Example #3

- Given the true power characteristics of a 9 [V] battery:



- If R=100 [Ω], does the voltage source behave ideally?
- If R=1,000 [Ω], does the voltage source behave ideally?