Chapter 9 - The Complete Response of Circuits with Two Energy Storage Elements

Lecture 29
Sections 9.1-9.6

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Student Learning Objectives

At the end of the lecture, students should be able to:

- Describe the behavior or RLC circuits using second-order, ordinary differential equations

Step Source

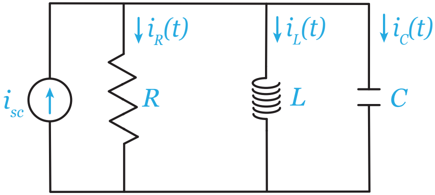
- We will work with the general second-order, ordinary differential equation

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

- $x(t)$ is the output of the circuit (i.e. $i(t)$ or $v(t)$)
- $f(t)$ is the input, either a constant or can vary with time

The RLC Circuit

- Consider the RLC circuit



- Constructing a KCL equation at the top node

$$i_{sc} = i_R + i_L + i_C$$

- Employing the constitutive equations for each component in terms of node voltages

$$i_{sc} = \frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v(\tau)\, d\tau + i_0 + C\frac{dv}{dt}$$

The RLC Circuit

- To eliminate the integral, we differentiate the equation
$$\frac{1}{R}\frac{dv}{dt} + \frac{v}{L} + C\frac{d^2v}{dt^2} = 0$$
- Dividing by C and arranging in order of derivatives
$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$
- Thus, we have the natural response for an RLC circuit

Natural Response of *RLC* Circuit

- The response is going to be that of the homogeneous

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

- The characteristic equation is

$$\lambda^2 + \frac{1}{RC}\lambda + \frac{1}{LC} = 0$$

- Using the quadratic equation and solving for λ

$$\lambda_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

- Thus, our solution takes the form

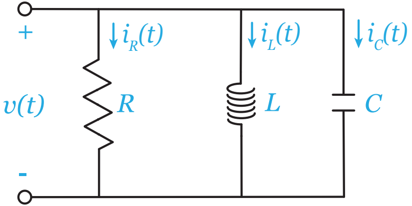
$$v(t) = A_1 \mathrm{e}^{\lambda_1 t} + A_2 \mathrm{e}^{\lambda_2 t}$$

Natural Response of RLC Circuit

- For simplicity, we will define α, the Neper frequency, as
$$\alpha = \frac{1}{2RC}$$
- We will also define ω_0, the resonant radian frequency, as
$$\omega_0 = \frac{1}{\sqrt{LC}}$$
- Thus, the roots becomes
$$\lambda_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$
- Thus, we are back at the original form
$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = f(t)$$

Example #1

- Consider an RLC circuit where $v(0^+)$=12 [V] and $i_L(0^+)$=30 [mA]



- Given C=0.2 [μF], L=50 [mH], R=200 [Ω], determine the initial current value in each branch and $v(t)$.

Example #1

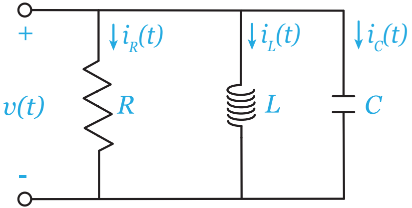
Example #1

Example #1

Example #1

Example #2

- Consider an RLC circuit where $v(0^+)$=12 [V] and $i_L(0^+)$=30 [mA]



- Given C=0.2 [μF], L=50 [mH], R=400 [Ω], determine the initial current value in each branch and $v(t)$.

Example #2

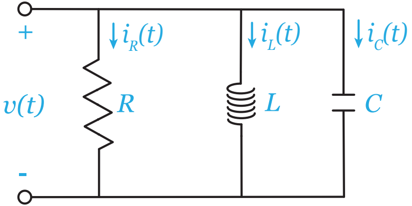
Example #2

Example #2

Example #2

Example #3

- Consider an RLC circuit where $v(0^+)$=12 [V] and $i_L(0^+)$=30 [mA]



- Given C=0.2 [μF], L=50 [mH], R=250 [Ω], determine the initial current value in each branch and $v(t)$.

Example #3

Example #3

Example #3

Example #3

Student Learning Objectives

At the end of the lecture, students should be able to:

- Describe the behavior or RLC circuits using second-order, ordinary differential equations
  - A second-order ordinary differential equation is used to described the behavior of parallel RLC circuits.
  - If $\alpha^2 > \omega_0^2$, the circuit is overdamped
  - If $\alpha^2 < \omega_0^2$, the circuit is underdamped
  - If $\alpha^2 = \omega_0^2$, the circuit is critically damped

Suggested Problems

- 9.2-1, 9.2-2, 9.2-5, 9.2-6, 9.2-7, 9.2-9, 9.2-10, 9.2-11, 9.2-15, 9.3-4, 9.4-5, 9.5-1, 9.5-3, 9.6-2