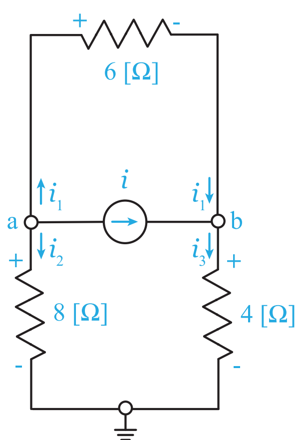
Node Voltage Analysis

MEMS 0031 - Electrical Circuits

May 27, 2020

Problem #1

Use NVA to solve for the nodal voltage given a source current of 1 [A].



Step 1: Assign nodes (N) and leg currents to all branches/elements:
$N = 3$ and we define i_1, i_2 and i_3 as shown.

Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1$ KCL equations, applied at non-zero nodes:
Applying KCL at node a:

$$i_1 + i_2 = -1\,[\text{A}]$$

Applying KCL at node b:

$$1\,[\text{A}] + i_1 = i_3 \implies i_3 - i_1 = 1\,[\text{A}]$$

Step 4: Apply Ohm's law in terms of node voltages:

KCL at node a:

$$\frac{V_a - V_b}{6\,[\Omega]} + \frac{V_a}{8\,[\Omega]} = -1\,[\text{A}] \implies V_a\left(\frac{7}{24}\right) - V_b\left(\frac{1}{6}\right) = -1$$

KCL at node b:

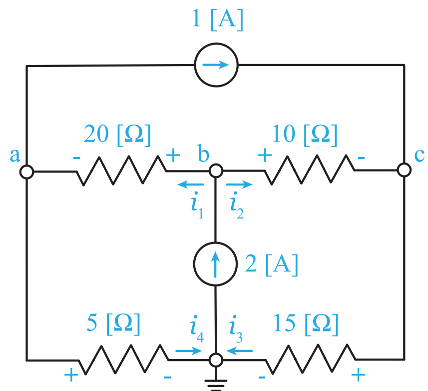
$$\frac{V_b}{4\,[\Omega]} - \frac{V_a - V_b}{6\,[\Omega]} = 1\,[\text{A}] \implies -V_a\left(\frac{1}{6}\right) + V_b\left(\frac{5}{12}\right) = 1$$

In matrix form:

$$\begin{bmatrix} 7/24 & -1/6 \\ -1/6 & 5/12 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} -2.6667 \\ 1.3333 \end{bmatrix}$$

Problem #2

Use NVA to solve for v_1, v_2 and v_3 in the circuit shown below.



Step 1: Assign nodes (N) and leg currents to all branches/elements:
$N = 4$ and we define i_1, i_2, i_3 and i_4 as shown.

Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N-1$ KCL equations, applied at non-zero nodes:
KCL at node a:

$$i_1 = i_4 + 1\,[\mathrm{A}] \implies i_1 - i_4 = 1\,[\mathrm{A}]$$

KCL at node b:

$$i_1 + i_2 = 2\,[\mathrm{A}]$$

KCL at node c:

$$1\,[\mathrm{A}] + i_2 = i_3 \implies i_2 - i_3 = -1\,[\mathrm{A}]$$

Step 4: Apply Ohm's law in terms of node voltages:
KCL at node a:

$$\frac{V_b - V_a}{20\,[\Omega]} - \frac{V_a}{5\,[\Omega]} = 1\,[\mathrm{A}]$$

KCL at node b:

$$\frac{V_b - V_a}{20\,[\Omega]} + \frac{V_b - V_c}{10\,[\Omega]} = 2\,[\mathrm{A}]$$

KCL at node c:

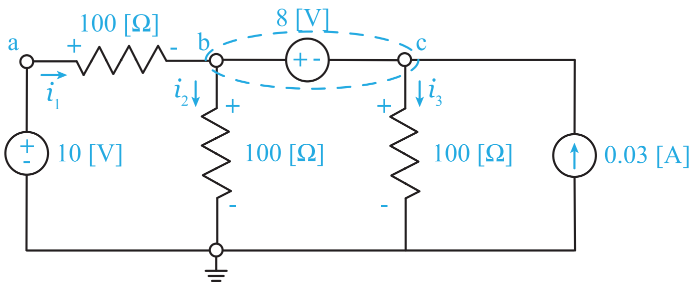
$$\frac{V_b - V_c}{10\,[\Omega]} - \frac{V_c}{15\,[\Omega]} = -1\,[\mathrm{A}]$$

In matrix form:

$$\begin{bmatrix} -1/4 & 1/20 & 0 \\ -1/20 & 3/20 & -1/10 \\ 0 & 1/10 & -1/6 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \implies \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 2 \\ 30 \\ 24 \end{bmatrix}$$

Problem #3

Use NVA to find the node voltages in the circuit shown below.



Step 1: Assign nodes (N) and leg currents to all branches/elements:
$N = 4$ and we define i_1, i_2 and i_3 as shown.

Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes:
We only need one KCL equation, which is applied at the supernode:

$$i_1 + 0.03\,[\text{A}] = i_2 + i_3$$

Step 4: Apply Ohm's law in terms of node voltages:
KCL at supernode:

$$\frac{V_a - V_b}{100\,[\Omega]} + 0.03\,[\text{A}] = \frac{V_b}{100\,[\Omega]} + \frac{V_c}{100\,[\Omega]}$$

The voltage at node a is specified as 10 [V]. Thus, the KCL equation at the supernode becomes:

$$0.1 - \frac{V_b}{100} + 0.03 = \frac{V_b}{100} + \frac{V_c}{100}$$

The supernode provides us with one additional equation:

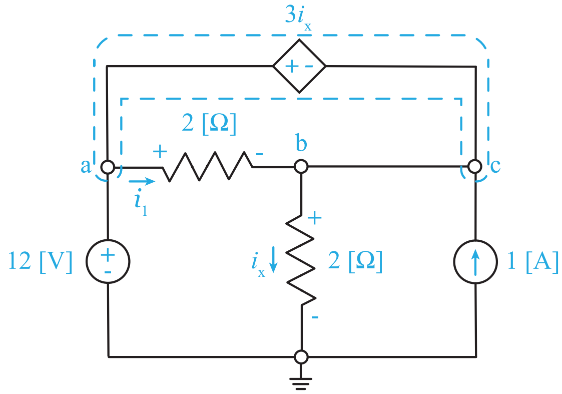
$$V_b - V_c = 8\,[\text{V}]$$

In matrix form:

$$\begin{bmatrix} ^2/_{100} & ^1/_{100} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 0.13 \\ 8 \end{bmatrix} \implies \begin{bmatrix} V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

Problem #4

Use NVA to find i_x in the circuit shown below.



Step 1: Assign nodes (N) and leg currents to all branches/elements:
$N = 3$ and we define i_1 as shown.

Step 2: Assign voltage potential consistent with PSC:
Voltage potentials assigned as shown.

Step 3: $N - 1 - \#VS$ KCL equations, applied at non-zero nodes:
We see we do not need any KCL equations. The voltage sources provide enough equations.

The supernode provides the following equation:

$$V_a - V_b = 3i_x \implies V_a - V_b = 3\left(\frac{V_b}{2\,[\Omega]}\right)$$

The voltage at node a is specified as 12 [V]. Thus, the supernode equation can be solved for in terms of V_b:

$$V_b\left(\frac{3}{2}+1\right) = 12 \implies V_b = 4.8\,[\mathrm{V}]$$

The leg current i_x can be found via Ohm's law:

$$i_x = \frac{4.8\,[\mathrm{V}]}{2\,[\Omega]} = 2.4\,[\mathrm{A}]$$