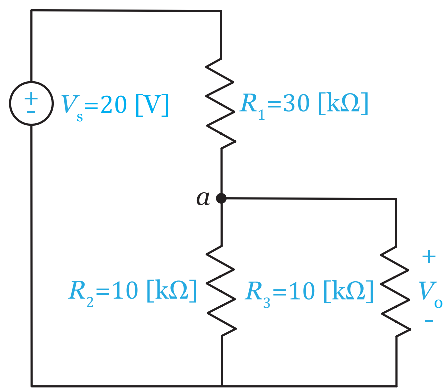
Series and Parallel Resistors and Sources

MEMS 0031 - Electrical Circuits

June 4, 2020

Problem #1 - Lectures 6 & 7

Given the potentiometer shown below, determine the output voltage V_o.



There are two 10 [kΩ] resistors in parallel, which creates an equivalent of

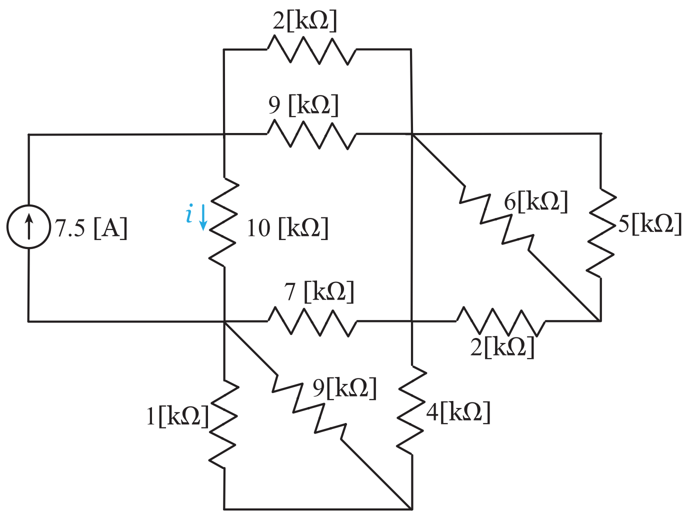
$$R_{eq,1} = \frac{(10\,[\text{k}\Omega])(10\,[\text{k}\Omega])}{(10\,[\text{k}\Omega]) + (10\,[\text{k}\Omega])} = 5\,[\text{k}\Omega]$$

Those two 10 [kΩ] resistors have the same voltage potential, therefore V_o is simply voltage division of the the 30 [kΩ] and $R_{eq,1}$ resistors

$$V_o = \left(\frac{5\,[\text{k}\Omega]}{30\,[\text{k}\Omega] + 5\,[\text{k}\Omega]}\right)(20\,[\text{V}]) = 2.86\,[\text{V}]$$

Problem #2 - Lectures 6 & 7

Find the voltage drop across the 10 [kΩ] resistor.



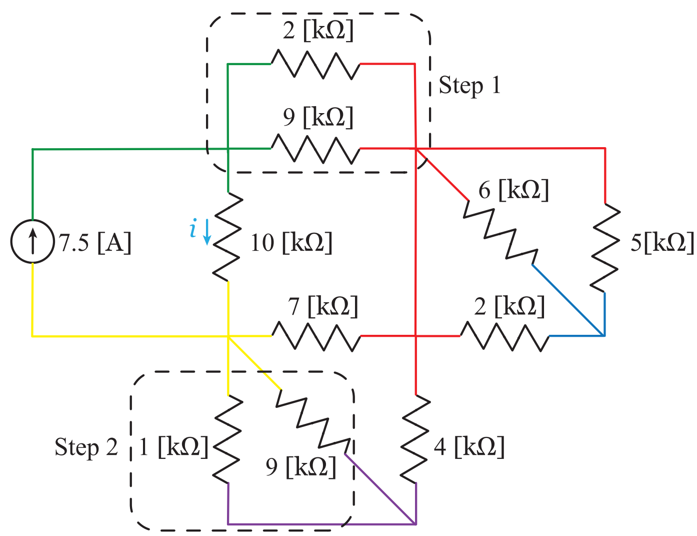
Let us start by color-coding the nodes. Our first step (Step 1) will be calculating the parallel resistance of the 2 and 9 [kΩ] resistors:

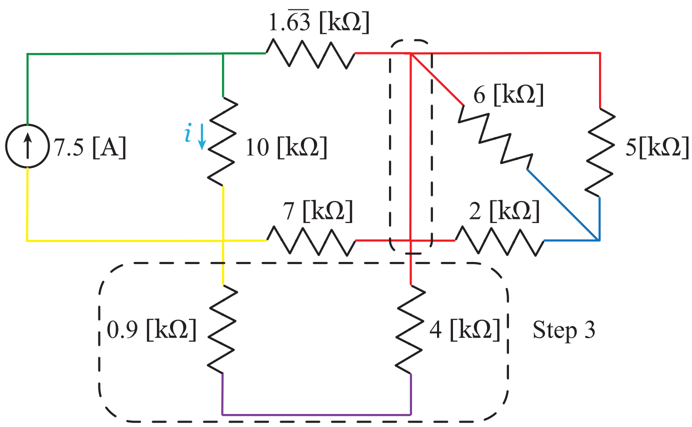
$$R_{eq} = \frac{(2\,[\text{k}\Omega])(9\,[\text{k}\Omega])}{(2+9)\,[\text{k}\Omega]} = 1.\overline{63}\,[\text{k}\Omega]$$

Our second step (Step 2) will be calculating the parallel resistance of the 1 and 9 [kΩ] resistors:

$$R_{eq} = \frac{(1\,[\text{k}\Omega])(9\,[\text{k}\Omega])}{(1+9)\,[\text{k}\Omega]} = 0.9\,[\text{k}\Omega]$$

Let us redraw the circuit to better see what is in parallel and what is in series.





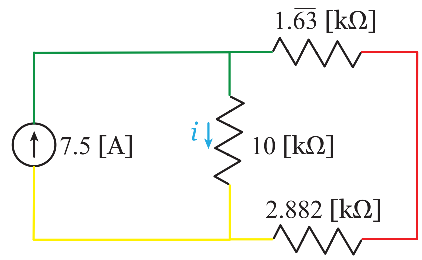
We note that the short, as highlighted, means all current will bypass the 2, 6 and 5 [kΩ] resistors. Although the 2, 6 and 5 [kΩ] resistors are in parallel, their equivalent resistance is of no consequence.

Next we can proceed to Step 3, where the 0.9 and 4 [kΩ resistors are in series, giving an equivalent of 4.9 [kΩ]. This equivalent is in parallel with te 7 [kΩ] resistor:

$$R_{eq} = \frac{(4.9\,[\text{k}\Omega])(7\,[\text{k}\Omega])}{(4.9+7)\,[\text{k}\Omega]} \approx 2.882\,[\text{k}\Omega]$$

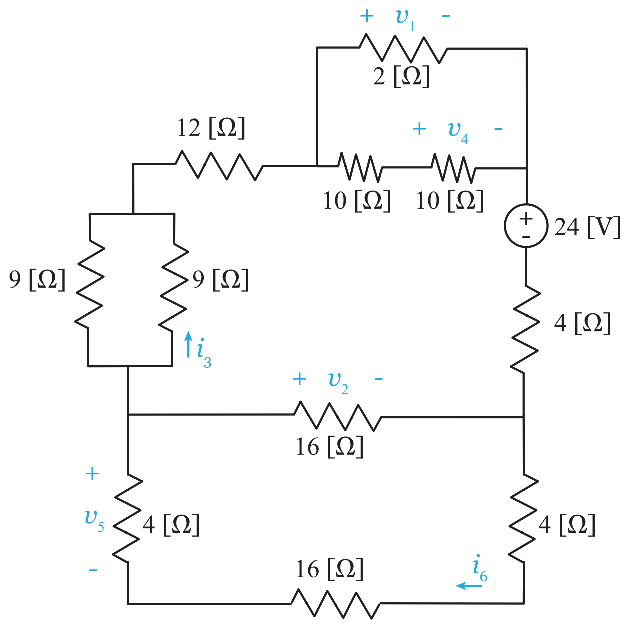
We wee the 2.88 [kΩ] and $1.\overline{63}$ resistors are in series, resulting in an equivalent of approximately 4.512 [kΩ]. Thus, the 10 and 4.512 [kΩ] resistors are in parallel, and the current through the 10 [kΩ] resistor is found via current division:

$$i = \left(\frac{4.512\,[\mathrm{k}\Omega]}{14.512\,[\mathrm{k}\Omega]}\right)(7.5\,[\mathrm{A}]) = 2.33\,[\mathrm{A}]$$



Problem #3 - Lectures 6 & 7

Determine the values of v_1, v_2, i_3, v_4, v_5, and i_6 in the circuit shown below.



Let us start by color-coding the nodes. Then, we will being to simply all the parallel and series resistors such that we can determine our source current from the 24 [V] source. We see in Step 1 the two 10 [Ω] resistors are in series, and their equivalent is in parallel with the 2 [Ω] resistor:

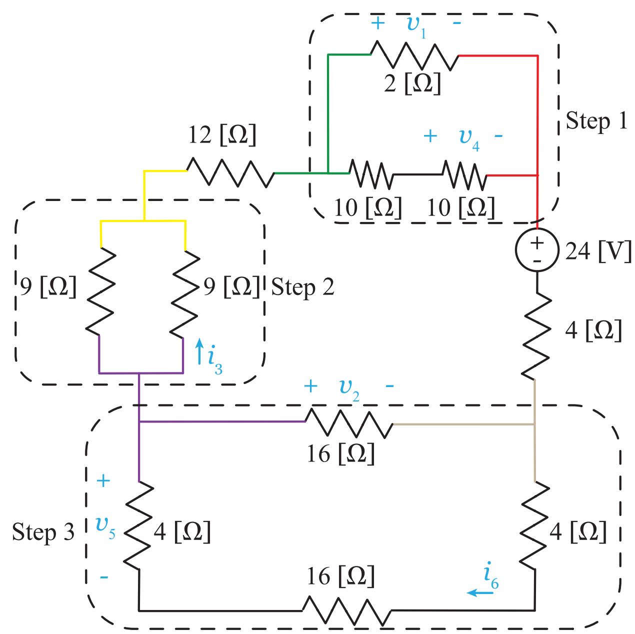
$$R_{eq} = \frac{(2\,[\Omega])(20\,[\Omega])}{(2+20)\,[\Omega]} = 1.\overline{81}\,[\Omega]$$

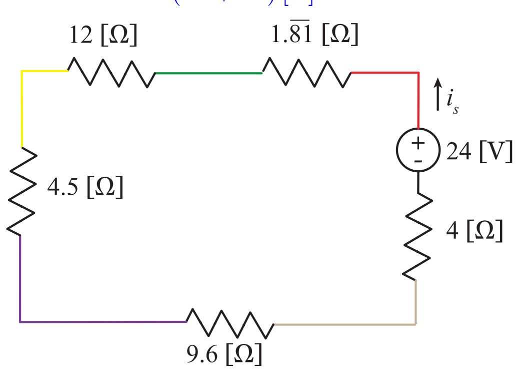
In Step 2, the two 9 [Ω] resistors are in parallel:

$$R_{eq} = \frac{(9\,[\Omega])(9\,[\Omega])}{(9+9)\,[\Omega]} = 4.5\,[\Omega]$$

In Step 3, the 4, 16 and 4 [Ω] resistors are in series, having an equivalent of 24 [Ω]. This equivalence is in parallel with the 16 [Ω] resistor:

$$R_{eq} = \frac{(16\,[\Omega])(24\,[\Omega])}{(16+24)\,[\Omega]} = 9.6\,[\Omega]$$





With the reduced circuit, we can calculate the source current:

$$i = \frac{24\,[\text{V}]}{31.91\,[\Omega]} = 0.752\,[\text{A}]$$

Now, we will look at the original circuit and use the concepts of voltage and current division to find the properties of interest.

To determine v_1, we recognize there is current division between the 2 and the two 10 [Ω] resistors:

$$i_{2\,[\Omega]} = \left(\frac{20\,[\Omega]}{22\,[\Omega]}\right)(0.752\,[\mathrm{A}]) \approx 0.684\,[\mathrm{A}]$$

Thus, the voltage drop across the 2 [Ω] resistor is:

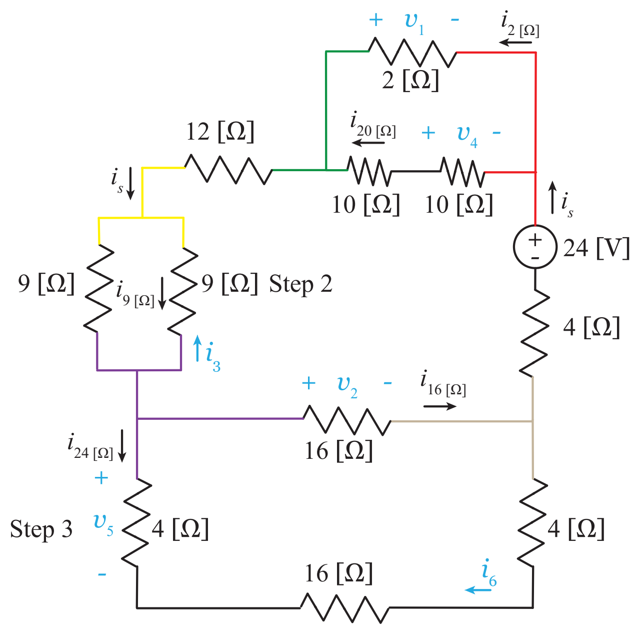
$$v_1 = -(0.684\,[\mathrm{A}])(2\,[\Omega]) = -1.368\,[\mathrm{V}]$$

The current running through the two 10 [Ω] resistors in series is:

$$i_{20\,[\Omega]} = \left(\frac{2\,[\Omega]}{22\,[\Omega]}\right)(0.752\,[\mathrm{A}]) \approx 0.0684\,[\mathrm{A}]$$

Thus, the voltage drop across the 10 [Ω] resistor is:

$$v_4 = -(0.0684\,[\mathrm{A}])(10\,[\Omega]) = -0.684\,[\mathrm{V}]$$



The current running through the 9 [Ω] resistor is found via current division:

$$i_{9\,[\Omega]} = \left(\frac{9\,[\Omega]}{18\,[\Omega]}\right)(0.752\,[\mathrm{A}]) \approx 0.376\,[\mathrm{A}]$$

The current i_3 is apposing the current running through the 9 [Ω] resistor:

$$i_3 = -0.376\,[\mathrm{A}]$$

The current running through the 16 [Ω] resistor is solved for via current division:

$$i_{16\,[\Omega]} = \left(\frac{24\,[\Omega]}{40\,[\Omega]}\right)(0.752\,[\mathrm{A}]) \approx 0.4512\,[\mathrm{A}]$$

Thus, the voltage drop across the 16 [Ω] resistor is:

$$v_2 = (0.4512\,[\mathrm{A}])(16\,[\Omega]) = 7.219\,[\mathrm{V}]$$

The current running through the 4, 16 and 4 [Ω] resistors in series is solved for via current division:

$$i_{16\,[\Omega]} = \left(\frac{16\,[\Omega]}{40\,[\Omega]}\right)(0.752\,[\mathrm{A}]) \approx 0.301\,[\mathrm{A}]$$

The current i_6 is opposing the current running through the 4, 16, and 4 [Ω] series resistors:

$$i_6 = -0.301\,[\mathrm{A}]$$

Thus, the voltage drop across the 4 [Ω] resistor is:

$$v_5 = (0.301\,[\mathrm{A}])(4\,[\Omega]) = 1.204\,[\mathrm{V}]$$