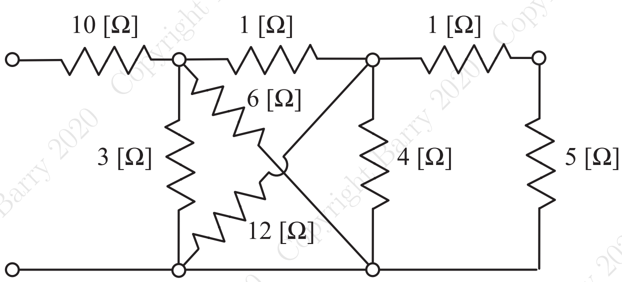
Homework #2

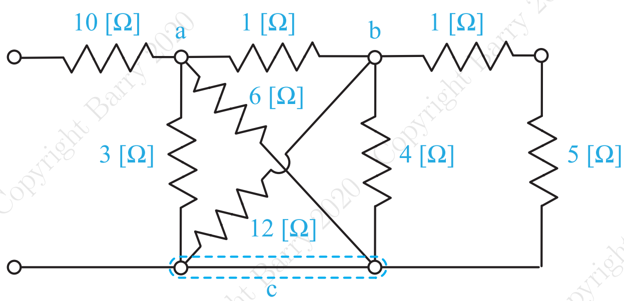
MEMS 0031 - Electrical Circuits

Assigned: May 15$^{\text{th}}$, 2020
Due: May 20$^{\text{th}}$, 2020 at 11:59 pm

Problem #1

Given the circuit below, determine the equivalent resistance, R_{eq}. Note the bridge.

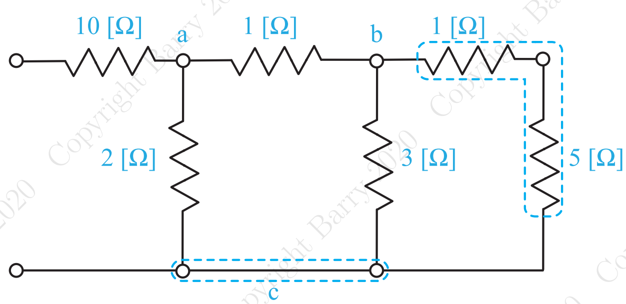




It is seen that the 3 and 6 [Ω] resistors, and the 4 and 12 [Ω] resistors, are in parallel, since they share the same nodes (a-c and b-c, respectively), meaning they have the same voltage potentials:

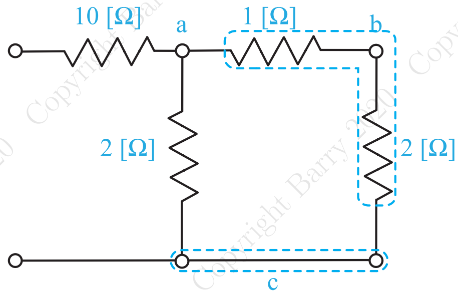
$$R_{eq,1} = \frac{(3\,[\Omega])(6\,[\Omega])}{(3+6)\,[\Omega]} = 2\,[\Omega]$$

$$R_{eq,1} = \frac{(4\,[\Omega])(12\,[\Omega])}{(4+12)\,[\Omega]} = 3\,[\Omega]$$



The 1 and 5 [Ω] resistors add in series to give an equivalent of 6 [Ω]. This is in parallel with the 3 [Ω] resistor, thus:

$$R_{eq,3} = \frac{(3\,[\Omega])(6\,[\Omega])}{(3+6)\,[\Omega]} = 2\,[\Omega]$$

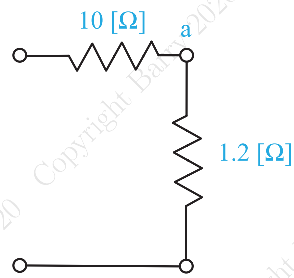


The 2 [Ω] and 1 [Ω] resistors are in series, for an equivalent of 3 [Ω], which is in parallel to the 2 [Ω] resistor, thus:

$$R_{eq,4} = \frac{(3\,[\Omega])(2\,[\Omega])}{(3+2)\,[\Omega]} = 1.2\,[\Omega]$$

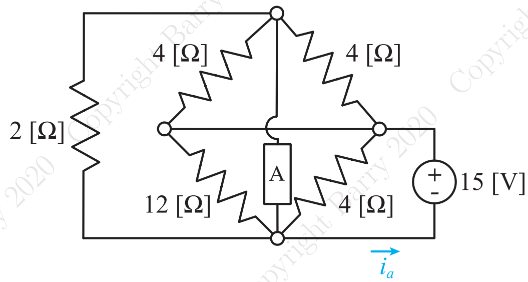
Lastly, the 10 [Ω] and 1.2 [Ω] are in series for, thus

$$\boxed{R_{eq,5} = 11.2\,[\Omega]}$$



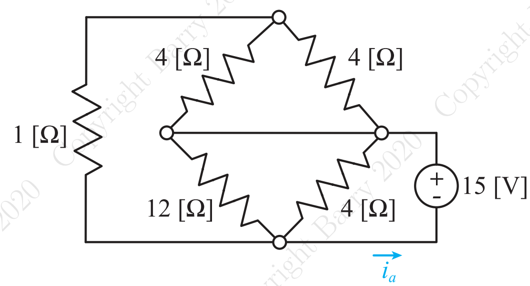
Problem #2

Given the circuit below, determine the current, i_a by using equivalent resistances. *Element A* has a resistance of 2 [Ω]. Note the bridge.



The two 2 [Ω] resistors are in a parallel configuration since they experience the same voltage potential, i.e. they are connected to the same node. Thus, the equivalent resistance of those to resistors is:

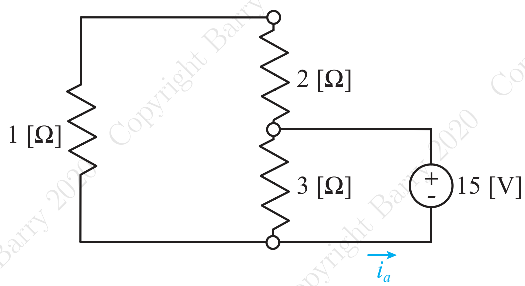
$$R_{eq,1} = \frac{(2\,[\Omega])(2\,[\Omega])}{(2+2)\,[\Omega]} = 1\,[\Omega]$$



The two 4 [Ω] resistors are in parallel, and the 12 [Ω] and 4 [Ω] resistor are also in parallel:

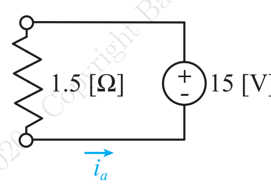
$$R_{eq,2} = \frac{(4\,[\Omega])(4\,[\Omega])}{(4+4)\,[\Omega]} = 2\,[\Omega]$$

$$R_{eq,3} = \frac{(12\,[\Omega])(4\,[\Omega])}{(12+4)\,[\Omega]} = 3\,[\Omega]$$



The 1 [Ω] and 2 [Ω] resistors are in series, thus they have an equivalence of 3 [Ω]. This equivalence is in parallel with the 3 [Ω] resistors:

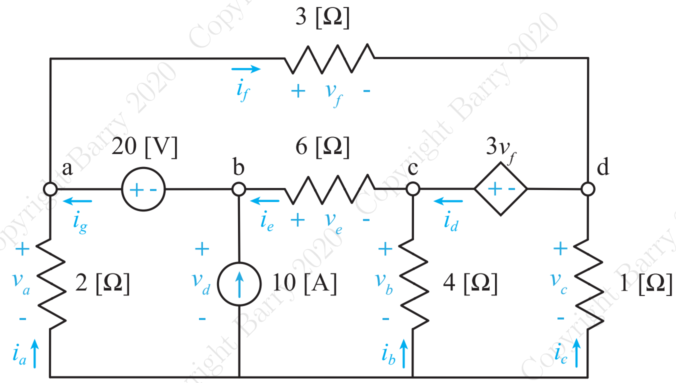
$$R_{eq,4} = \frac{(3\,[\Omega])(3\,[\Omega])}{(3+3)\,[\Omega]} = 1.5\,[\Omega]$$



Using Ohm's law, the current i_a is found to be 10 [A].

Problem #3

Using a combination of KCL and KVL, determine the currents i_a through i_g, and the voltage potentials v_a through v_f across each resistor and current source. Note: if a necessary current or voltage potential is not specified, please denote it with the next variable in alphabetical sequence.



We will start by applying KCL equations are our nodes. KCL at node a:

$$i_a + i_g = i_f \tag{1}$$

KCL at node b:

$$10\,[\mathrm{A}] + i_e = i_g \tag{2}$$

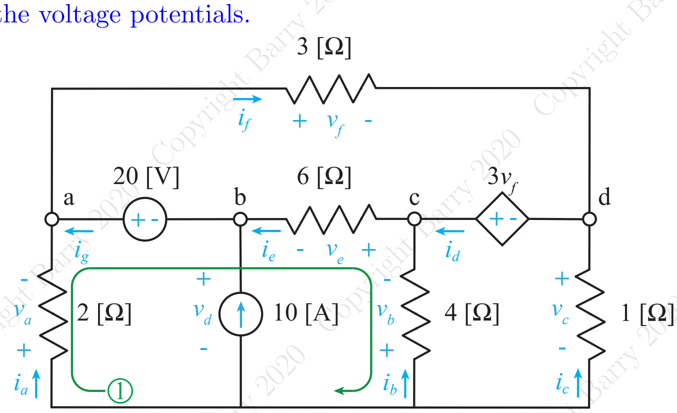
KCL at node c:

$$i_b + i_d = i_e \tag{3}$$

KCL at node d:

$$i_c + i_f = i_d \tag{4}$$

Currently, we have four equations and seven unknowns. Thus, we need three more independent equations. We will construct three KVL equations, relating our currents. We will apply KVL around loop 1. We have to note that our resistors must behave as passive elements. As they are currently described, they are active elements. Thus, we will flip the voltage potentials.

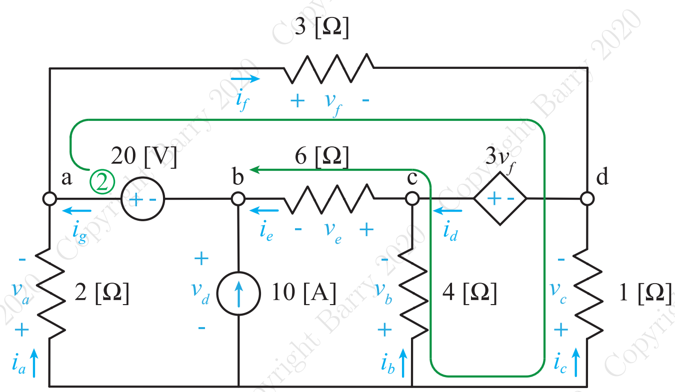


$$v_a + 20\,[\mathrm{V}] - v_e - v_b = 0 \tag{5}$$

Expressing the voltage potentials in terms of currents and resistances:

$$(2\,[\Omega])i_a - (6\,[\Omega])i_e - (4\,[\Omega])i_b = -20\,[\mathrm{V}] \tag{6}$$

Now, applying KVL around loop 2, which bypasses the VCCS, and ensuring the resistors obey the PSC:

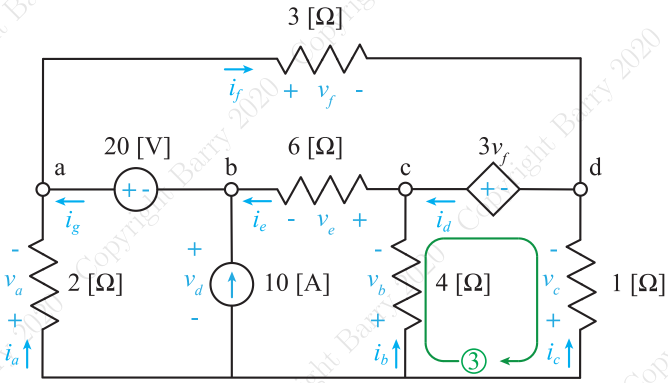


$$-20\,[\mathrm{V}] + v_f - v_c + v_b + v_e = 0 \tag{7}$$

Expressing the voltage potentials in terms of currents and resistances:

$$(3\,[\Omega])i_f - (1\,[\Omega])i_c + (4\,[\Omega])i_b + (6\,[\Omega])i_e = 20\,[\mathrm{V}] \tag{8}$$

Lastly, applying KVL around loop 3:



$$v_b + 3v_f - v_c = 0 \tag{9}$$

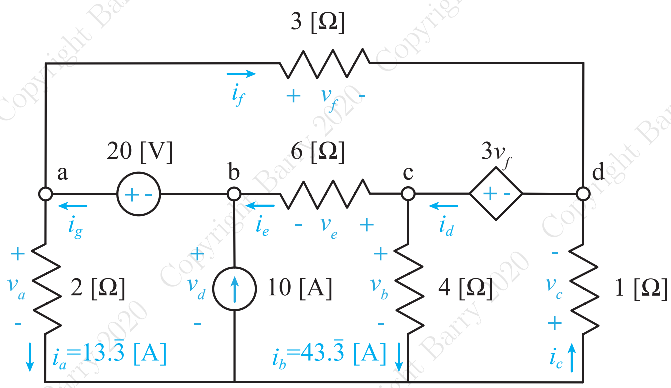
Expressing the voltage potentials in terms of currents and resistances:

$$(4\,[\Omega])i_b + 3(3\,[\Omega])i_f - (1\,[\Omega])i_c = 0 \tag{10}$$

Now we can put eqns. 1-4, 6, 8 and 10 in matrix form:

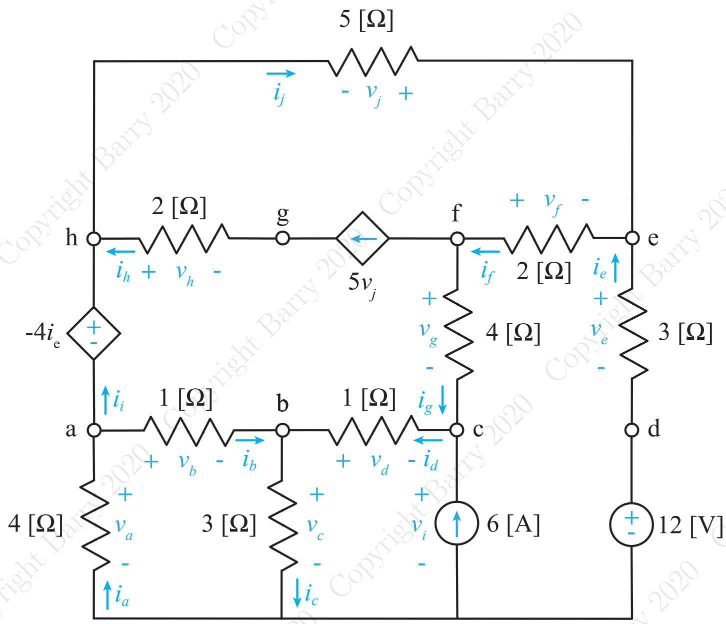
$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 & 0 \\ 2 & -4 & 0 & 0 & -6 & 0 & 0 \\ 0 & 4 & -1 & 0 & 6 & 3 & 0 \\ 0 & 4 & -1 & 0 & 0 & 9 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 0 \\ 0 \\ -20 \\ 20 \\ 0 \end{bmatrix} \Longrightarrow \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \end{bmatrix} = \begin{bmatrix} -13.\bar{3} \\ -43.\bar{3} \\ 46.\bar{6} \\ 71.\bar{1} \\ 27.\bar{7} \\ 24.\bar{4} \\ 37.\bar{7} \end{bmatrix}$$

The minus signs associated with i_a and i_b indicate that the assumed direction is incorrect. That is, we must flip the direction of these two currents, which is shown to the right. The voltages are now a trivial matter - we will apply Ohm's law to the resistors, and use KVL to determine the voltage potential across the current source. Thus, we have the following circuit, as shown to the right.



Problem #4

Using a combination of KCL and KVL, determine the currents i_a through i_j, and the voltage potentials v through v across each resistor and current source. Note: if a necessary current or voltage potential is not specified, please denote it with the next variable in alphabetical sequence.



We will begin by applying KCL to node a:

$$i_a = i_b + i_i \implies i_a - i_b - i_i = 0 \tag{11}$$

KCL at node b:

$$i_b + i_d = i_c \implies i_b - i_c + i_d = 0 \tag{12}$$

KCL at node c:

$$6\,[\mathrm{A}] + i_g = i_d \implies i_d - i_g = 6\,[\mathrm{A}] \tag{13}$$

KCL at node e:

$$i_e + i_j = i_f \implies i_e - i_f + i_j = 0 \tag{14}$$

KCL at node f:

$$i_f = i_g + i_h \implies i_f - i_g - i_h = 0 \tag{15}$$

KCL at node h:

$$i_i + i_h = i_j \implies i_h + i_i - i_j = 0 \tag{16}$$

We have six equations and ten unknowns. Thus, we must create KVL equations to related the currents through the elements to known resistances and voltages. Applying KVL around loop 1, noting the proper PSC associated with the resistors:

$$v_a + v_b + v_c = 0 \tag{17}$$

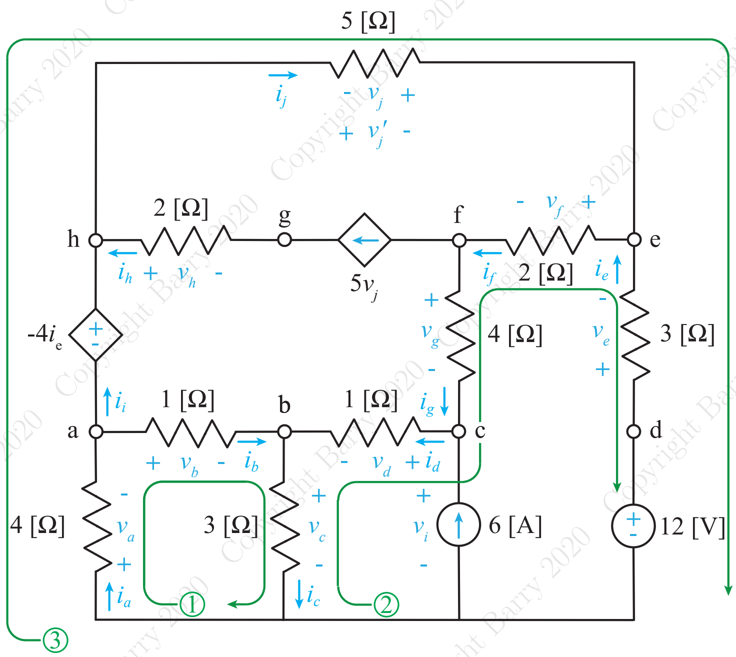
Applying Ohm's law:

$$(4\,[\Omega])i_a + (1\,[\Omega])i_b + (3\,[\Omega])i_c = 0 \tag{18}$$

Applying KVL around loop 2, noting the proper PSC associated with the resistors:

$$-v_c - v_d - v_g - v_f - v_e + 12\,[\text{V}] = 0 \tag{19}$$

Applying Ohm's law:



$$(3\,[\Omega])i_c + (1\,[\Omega])i_d + (3\,[\Omega])i_e + (2\,[\Omega])i_f + (4\,[\Omega])i_g = 12\,[\text{V}] \tag{20}$$

Applying KVL around loop 3, noting the proper PSC associated with the resistors, calling the proper formulation of v_j as v_j',:

$$v_a - (-4i_e) + v_j' - v_e + 12\,[\text{V}] = 0 \tag{21}$$

Applying Ohm's law:

$$(4\,[\Omega])i_a + 4i_e - (3\,[\Omega])i_e + (5\,[\Omega])i_j = -12\,[\text{V}] \tag{22}$$

Now, we cannot construct another independent loop. We must related the output of the CCCS to a nodal current, i.e. apply KCL at node g:

$$i_h = 5v_j \implies i_h = 5(5\,[\Omega])(-i_j) \implies i_h = -25i_j \implies i_h + 25i_j = 0 \tag{23}$$

Putting eqns. 11-16, 18, 20, 22 and 23 in matrix form:

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 \\ 4 & 1 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 1 & 3 & 2 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 25 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \\ i_h \\ i_i \\ i_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ 0 \\ 0 \\ 0 \\ 0 \\ 12 \\ -12 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_d \\ i_e \\ i_f \\ i_g \\ i_h \\ i_i \\ i_j \end{bmatrix} = \begin{bmatrix} -3.1225 \\ 0.3715 \\ 4.0395 \\ 3.6680 \\ 1.1621 \\ 1.0277 \\ -2.3320 \\ 3.3597 \\ -3.4941 \\ -0.1344 \end{bmatrix}$$

Once again, the minus signs associated with i_a, i_g, i_i and i_j indicate the assumed directions are incorrect, and we would flip the direction of those currents. This is shown in the circuit below. The voltage potentials across the resistors can be found using Ohm's law, and the voltage potentials across the current sources can be found via KVL - this exercise is left to the reader.