

# Chapter 2 - Circuit Elements

## Lecture 2

### Sections 2.1-2.5

Learning Objectives

2.1 Introduction

2.2 Engineering and  
Linear Models

2.4 Resistors

2.5 Independent  
Sources

## MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department  
University of Pittsburgh



# Student Learning Objectives

Chapter 2 - Circuit Elements

MEMS 0031

At the end of the lecture, students should be able to:

- ▶ Identify active and passive circuit elements
- ▶ Understand the constraints for an element to be considered linear
- ▶ Understand the formulation of electrical resistance and how it relates to resistors
- ▶ Understand and be able to apply Ohm's Law
- ▶ Calculate power dissipated in elements (Joule heating)
- ▶ Identify and verify ideal current and voltage source

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# Circuit Elements

Chapter 2 - Circuit  
Elements

MEMS 0031

- ▶ There are many circuit elements we will analyze in this course, and they are broken down into two categories:

## Passive Elements

- ▶ Resistors (R)
- ▶ Inductors (L)
- ▶ Capacitors (C)
- ▶ Diodes
- ▶ We will first focus our attention on resistors, and voltage and current sources

## Active Elements

- ▶ Voltage sources
- ▶ Current sources
- ▶ Controlled sources
- ▶ Op-amps

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- ▶ **Linear circuit** elements satisfy the properties of **superposition** and **homogeneity**, and exhibit linear behavior
- ▶ **Superposition** states that the total response of a component with multiple stimuli is the sum of the responses of individual stimuli acting on the component
- ▶ **Homogeneity** states the dimensions of each contributing term are consistent

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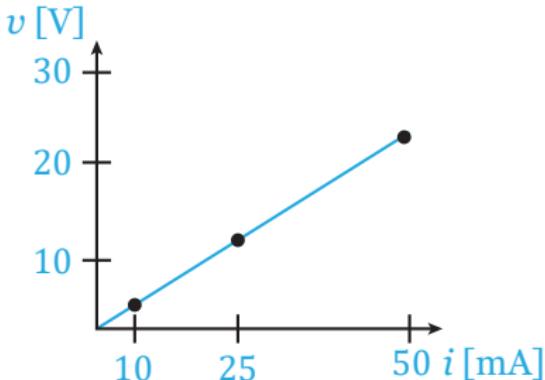
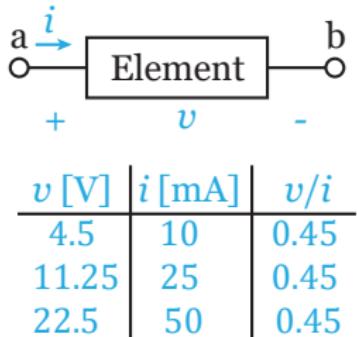
2.4 Resistors

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# Linear Circuit Elements

- ▶ Is the element depicted below behaving linearly?



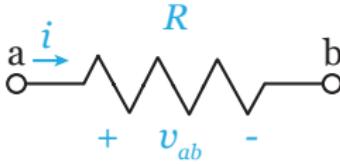
- ▶ What would non-linear behavior look like?



- ▶ **Resistance** ( $R$ , [ $\Omega$ ], [V/A]) is the physical property of an element or device that impedes the flow of current
- ▶  $R$  expressed in terms of the electrical resistivity  $\rho$ , which is dependent upon the valence electrons and free electrons, length of the element  $L$ , and cross-sectional area of the element  $A$  such that

$$R = \frac{\rho L}{A}$$

- ▶ The resistor will be represented by the following symbol



# Example #1

- ▶ What is the resistance of a 30 [m] length of copper ( $\rho=1.68 \cdot 10^{-8}$  [ $\Omega \cdot \text{m}$ ]) wire, if the wire were:
  - ▶ 12-gage ( $A=3.31$  [ $\text{mm}^2$ ])
  - ▶ 10-gage ( $A=5.26$  [ $\text{mm}^2$ ])

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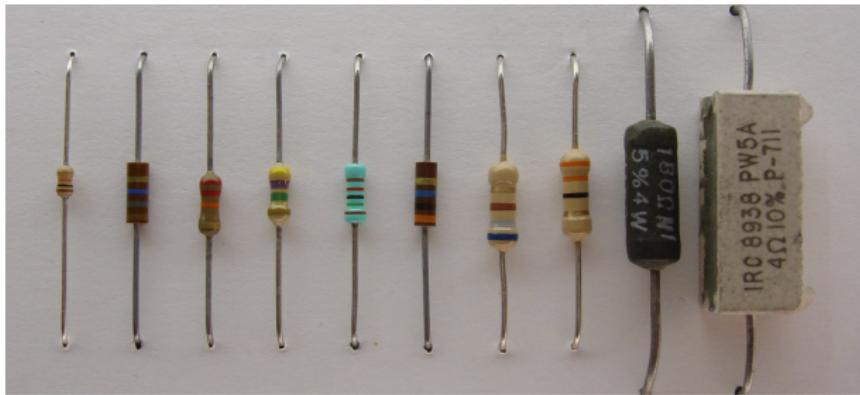
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# Resistors

- ▶ Resistors come in all shapes and sizes, and are made from a variety of material to satisfy design requirements



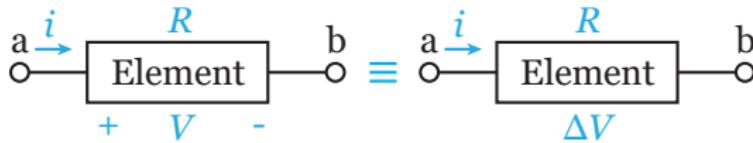
- ▶ Resistors have voltage and current ratings, and subsequently maximum power ratings
- ▶ The **bands** tell you the nominal resistance value and tolerance



- ▶ **Ohm's law** states the voltage potential across a resistor is proportional to the current through it, and is related by the constant of proportionality, resistance, such that

$$V = iR$$

- ▶ That is, a potential is needed across an element to drive a current through it:



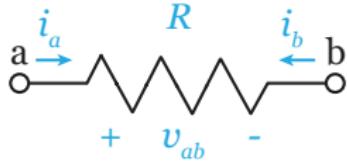
- ▶ This is commonly expressed as:

$$\Delta V = iR$$

- ▶ We will drop the  $\Delta V$  convention, and indicate the potential via + and - signs



- ▶ Consider the resistor shown below, with a voltage potential  $v_{ab}$  driving  $i_a$  through  $R$



- ▶ We can express  $i_a$  in terms of  $i_b$

$$i_a = -i_b$$

- ▶ Applying Ohm's law consistent with the PSC:

$$V_{ab} = i_a R$$

- ▶ Substituting  $-i_b$  for  $i_a$

$$V_{ab} = -i_b R \implies V_{ba} = i_b R$$

- ▶ The potential is dependent upon the direction of  $i$



# Linear Resistors

- ▶ An important assumption made is that the resistor is operating within a region of linear behavior
- ▶ Linearity implies  $v = f(i) = iR$ , that is voltage will be a function of current, where  $R$  is independent of both voltage and current
- ▶ Given an element with resistance  $R$ , and currents  $i_1$  and  $i_2$  flowing through the element, with corresponding voltages  $V_1=Ri_1$  and  $V_2=Ri_2$  imposed across the element to drive currents  $i_1$  and  $i_2$ , the total voltage applied would be:

$$V = R(i_1 + i_2) = V_1 + V_2$$

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# Power Dissipated

- ▶ The power dissipated by a resistor can be determined using Ohm's law. The potential is:

$$V = iR$$

We recall power is current times voltage

$$P = Vi$$

Substituting in  $V$  expressed by Ohm's law into the power equation

$$P = (iR)i = i^2 R$$

This is known as **Joule heating**, which is also expressed as

$$P = \frac{V^2}{R}$$

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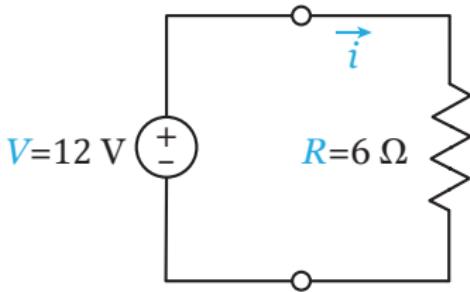
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# Example #2

- ▶ Compute a) the current through the resistor and  
b) the power dissipated by the resistor



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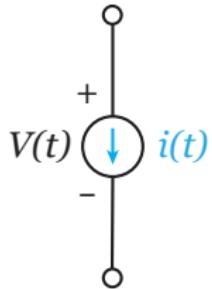
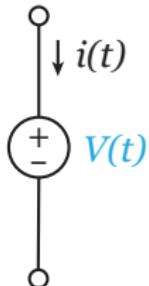
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# Independent Sources

- ▶ There are two types of independent sources



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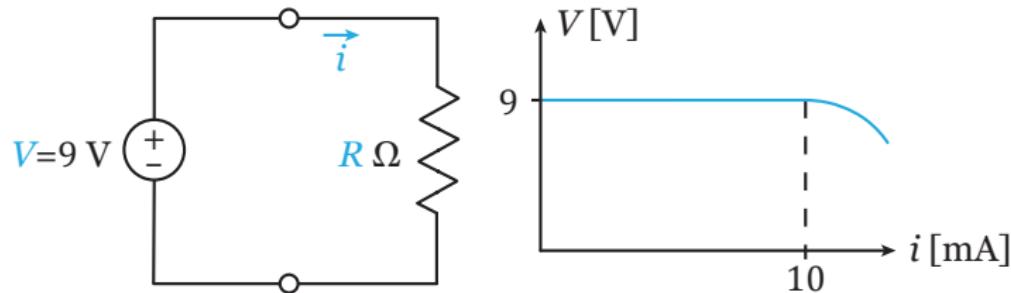
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# Example #3

- Given the true power characteristics of a 9 [V] battery:



- If  $R=100 \ [\Omega]$ , does the voltage source behave ideally?
- If  $R=1,000 \ [\Omega]$ , does the voltage source behave ideally?

