MEMS 0031 - HW1 Solutions

#1: $q(t) = 1(1-e^{-10t})$ for $t \geq 0$. Determine $i(t)$

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}\left[1(1-e^{-10t})\right] = \frac{d(1)}{dt}^{0} - \frac{d}{dt}\left(-e^{-10t}\right)$$

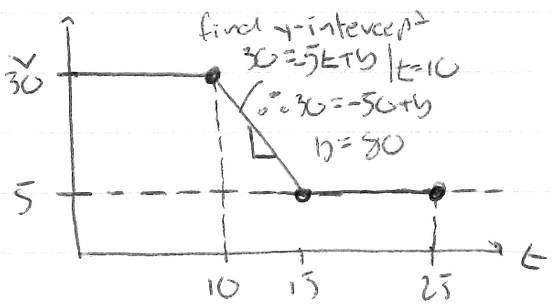
$$\therefore \boxed{i(t) = 10e^{-10t}\ [A]}$$

#2: $i(t) = 5(1-e^{-20t})\ [A]$ for $t \geq 0$. Determine $q(t)$

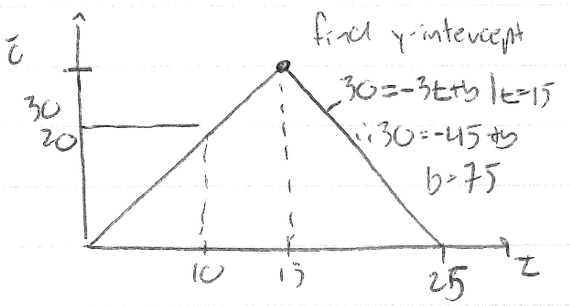
$$q(t) = \int_0^t i(t)\,d\tau + q(0)^{0} = \int_0^t 5\,d\tau - \int_0^t 5e^{-20t}\,d\tau$$

$$\boxed{q(t) = 5t + \frac{1}{4}e^{-20t} + C}$$

#3:



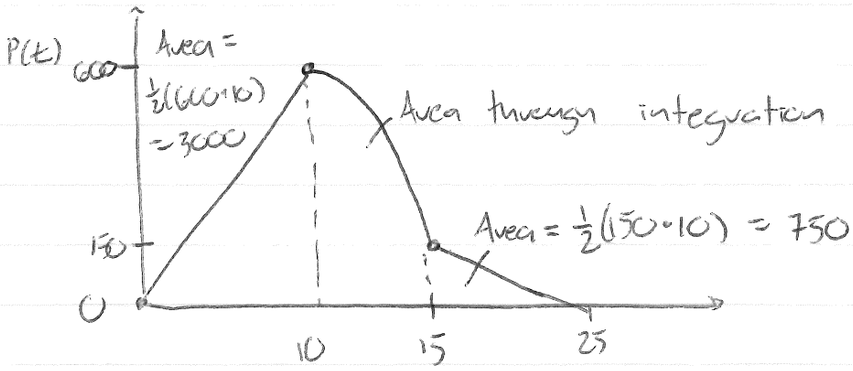
$$v = \begin{cases} 30 & 0 < t \leq 10 \\ -5t + b & 10 < t \leq 15 \\ 5 & 15 < t \leq 25 \end{cases}$$



$$i = \begin{cases} 2t & 0 < t \leq 10 \\ 2t & 10 < t \leq 15 \\ -3t + b & 15 < t \leq 25 \end{cases}$$

$$P(t) = \begin{cases} 30 \cdot 2t = \boxed{60t} & 0 < t \leq 10 \\ (-5t+80) \cdot 2t = \boxed{-10t^2 + 160t} & 10 < t \leq 15 \\ 5 \cdot (-3t+75) = \boxed{-15t + 375} & 15 < t \leq 25 \end{cases}$$

To determine the total energy, we need the area under the $P(t)$ vs. t curve.



$$E = \int_{10}^{15} (-10t^2 + 160t)\,dt = \int_{10}^{15} (-10t^2)\,dt + \int_{10}^{15} (160t)\,dt$$

$$= \frac{-10t^3}{3}\Big|_{10}^{15} + 80t^2\Big|_{10}^{15} = -\frac{23,750}{3} + 10,000$$

$$= 2083.33$$

$\therefore E = 3,000 + 2083.33 + 750 = \boxed{5,833.33\,[J] = E}$

#4: Given $v(t) = 4\cos(3t)$ [V] and $i(t) = \frac{\sin(3t)}{12}$ [A],

determine $P(t)$ and evaluate at $t = 0.5$ and 1 [s].

$$P(t) = v(t)\,i(t) = 4\cos(3t)\cdot\frac{\sin(3t)}{12} = \boxed{\frac{1}{3}\cos(3t)\sin(3t)}$$

Additionally, can be expressed as $\boxed{\frac{1}{6}\sin(6t)}$

Note: radians, not degrees!

$$P(t = 0.5\,[s]) = \frac{1}{3}\cos(1.5)\sin(1.5) = \boxed{0.0235\,[w] = P(t = 0.5)}$$

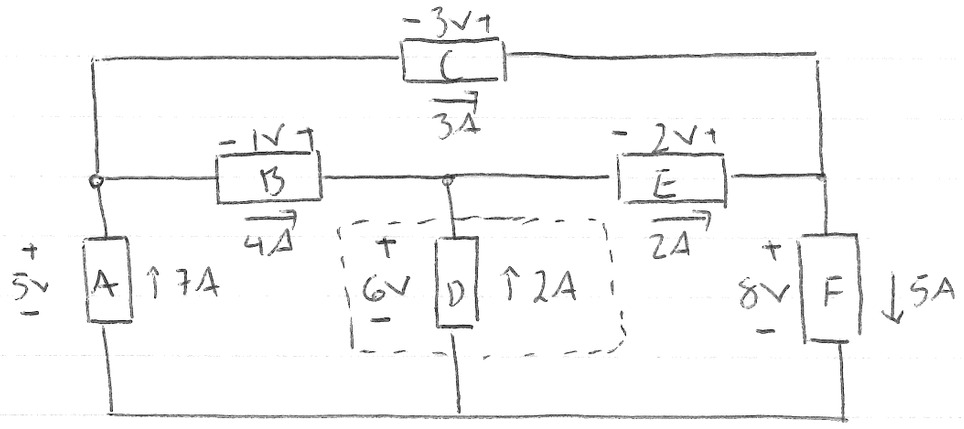
$$P(t = 10\,[s]) = \frac{1}{3}\cos(3)\sin(3) = \boxed{-0.0466\,[w] = P(t = 1)}$$

#5: Given $V = 9$ [V] and $i = 0.5$ [A]

$$P = V \cdot i = 9\,[V]\cdot 0.5\,[A] = \boxed{4.5\,[w] = P}$$

$$P(10\,\text{min}) = E(600\,[s]) = 4.5\,[w]\cdot 600\,[s] = \boxed{2,700\,[J] = E}$$

#6:



Pabs [W]	Element
40	F

Psupplied [W]	Element
35	A
4	B
9	C
12	D
4	E
64	

incorrect →

Element D has either an incorrect voltage or current reference direction

#7:

v [V]	i [A]	R = v/i
-3	-3	1
-4	-2	2
0	0	0
12	2	6
32	4	8
60	6	10

resistance is not constant - nonlinear

#8: Ohm's law: $V = iR \rightarrow R = \frac{V}{i} = \frac{24[V]}{3[A]} = 8[\Omega] = R$

$P_{absorbed} = i^2R = \frac{V^2}{R} = (3[A])^2 \cdot 8[\Omega] = \frac{(24[V])^2}{8[\Omega]} = 72[W] = P$

#9: Ohm's law: $V = iR \rightarrow i = \frac{V}{R} = \frac{15[V]}{5[\Omega]} = 3[A] = i$

$P_{absorbed} = i^2R = \frac{V^2}{R} = (3[A])^2 \cdot 5[\Omega] = \frac{(15[V])^2}{5[\Omega]} = 45[W] = P$