Chapter 1 - Electric Circuit Variables

Lecture 1
Sections 1.2-1.5

MEMS 0031 Electrical Circuits

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Note's

- The following scheme will be used within our notes:
  1. **Bold** words are definitions
  2. *Italic* words or letters represent variables
  3. Underlined words or phrases represent a law, theorem and/or hypothesis
- Each section (number and title) is displayed on the right navigation pane, following our text
- Important equations will be boxed

Student Learning Objectives

At the end of the lecture, students should be able to:

- Understand the concept and formulation of charge and current
- Understand the concept and formulation of voltage
- Recognize and understand active and passive sign conventions
- Understand and apply the formulation of power
- Understand and apply the conservation of energy

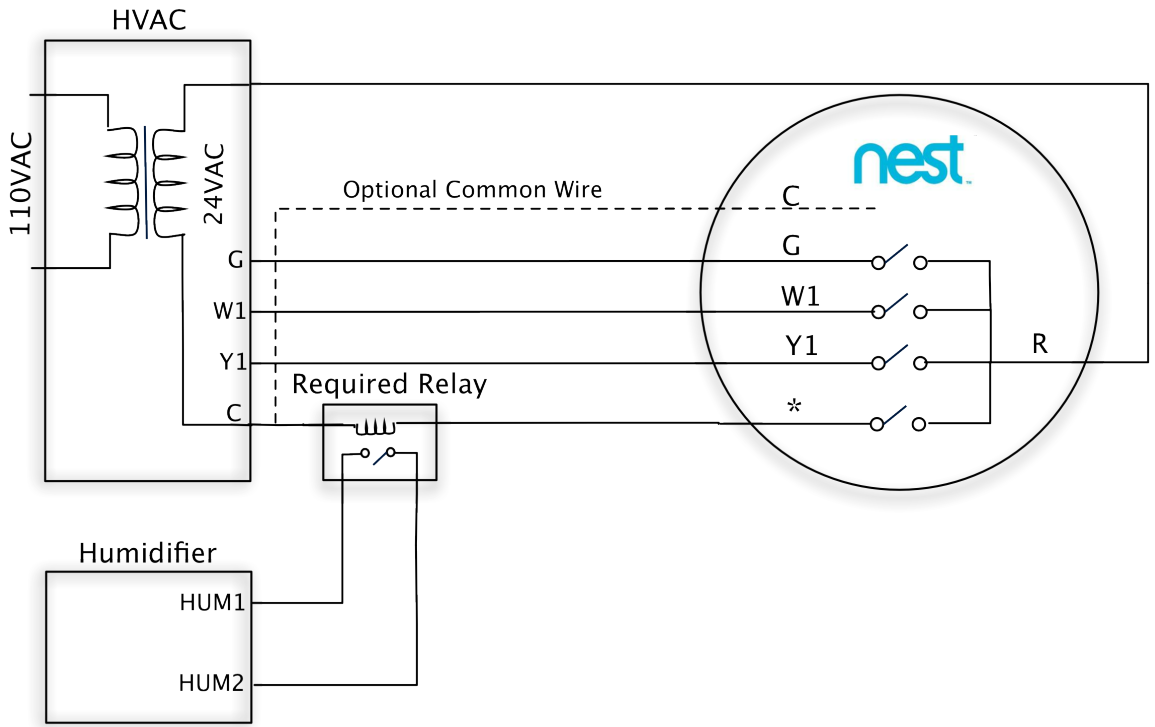
Units

- The units we will use in this course are as follows:

Quantity	Name	Formula	Symbol
Time	second		s
Electric Current	Ampere		A
Frequency	Hertz	s^{-1}	Hz
Energy or Work	Joule	N-m	J
Power	Watt	J/s	W
Electric Charge	Coulomb	A-s	C
Electric Potential	Volt	W/A	V
Electric Resistance	Ohm	V/A	Ω
Electric Conductance	Siemens	A/V	S
Electric Capacitance	Farad	C/V	F
Magnetic Flux	Weber	V-s	Wb
Inductance	Henry	Wb/A	H

Electric Circuit

- An **electric circuit**, or electric network, is an interconnection of electrical elements linked together in a closed path so that electric current may flow



Charge and Current

- **Charge** (q, [C] (Coulomb), [A-s]) is the "quantity" of electricity, i.e. electrons, responsible for electrical phenomena

$$q(t) = \int_{-\infty}^{t} i\, d\tau = q(0) + \int_{0}^{t} i\, d\tau$$

- **Current** (i, [A], [C/s]) is the time rate of flow of charge past a point

$$i = \frac{d\, q}{dt}$$

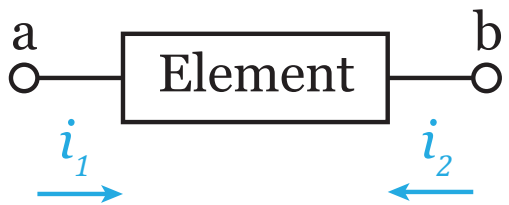
- Current must be conserved! Charge carriers cannot be created nor destroyed.

Example #1

- For $t<0$, no charge exists at the terminal of a device. At $t=0$, a 5 [A] current begins to flow into the device.
- Determine:
  1. The charge accumulated for $t>0$
  2. If the current stopped after 10 [s], how much charge has accumulated?
- Solution:

Current in an Element

- **Current** has direction and magnitude - it is a vector - and can exist in, or through, an element
- The direction is indicated by either $\rightarrow$, $\leftarrow$, $\uparrow$ or $\downarrow$, and the magnitude is represented by a variable i_1, i_2, etc. The units are assumed [A] unless otherwise noted.
- Consider a circuit element that exists between nodes a and b, carrying current i_1



- Based upon the opposing direction of i_1 and i_2, we can say:

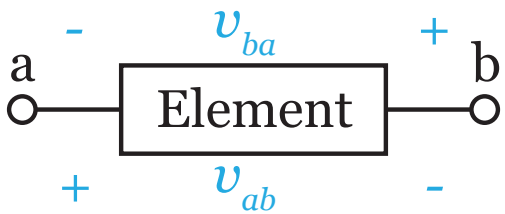
$$i_1 = -i_2$$

Voltage in an Element

- **Voltage** (V, [V]) is a potential that drives a current across or through an element
- The voltage across an element, ΔV, is the work (w) required to move a unit positive charge (q) from the $-$ to the $+$ terminal

$$V = \frac{dw}{dq}$$

- Note: our unit charge in this convention is positive, although an electron has a negative charge (Thanks Ben Franklin)

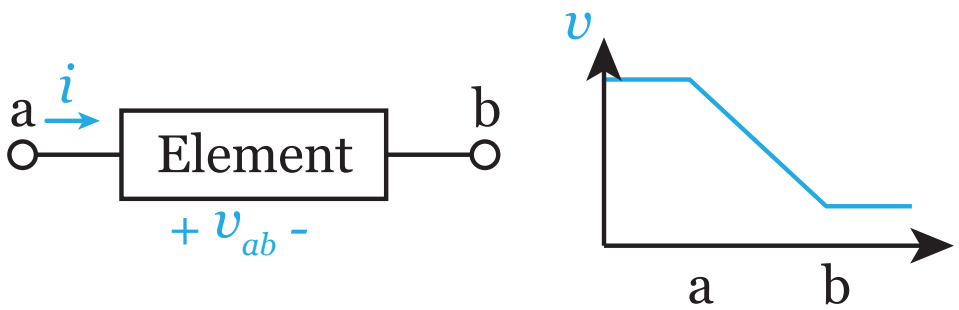


Convention

- We first have to define a consistent convention based on the direction of current through, and direction of a voltage potential across, an element
- The **Passive Sign Convention** refers to an element that is a **Power Sink**
- The **Active Sign Convention** refers to an element that is a **Power Source**
- Let us look at each of these conventions individually

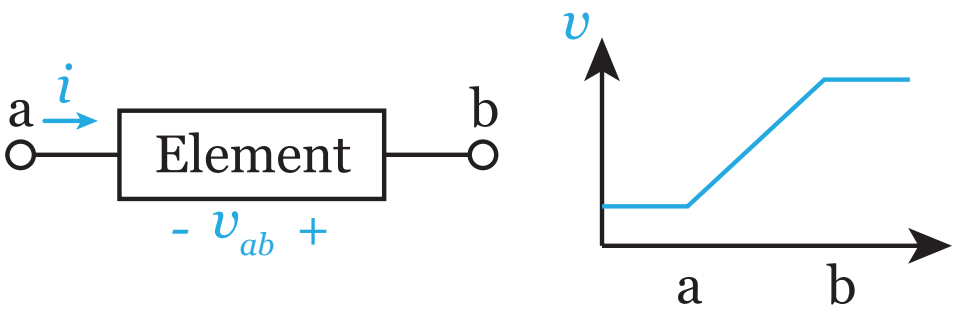
Passive Sign Convection

- the **Passive Sign Convention** (PSC) means the circuit component is dissipating power within the circuit - if current is conserved, a decrease in power manifests through a decrease in potential
- Therefore, a passive component is decreasing the potential, i.e. voltage, of the charge carries
- The PSC is depicted as follows:



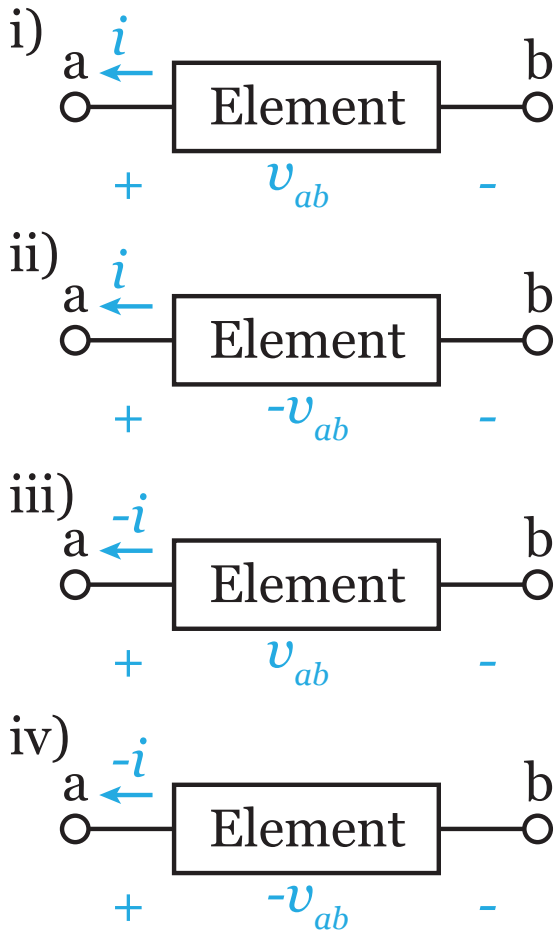
Active Sign Convention

- The **Active Sign Convention** (ASC) means the circuit component is providing energy to the circuit - if current is conserved, an increase in power manifests through an increase in potential
- Therefore, an active component is increasing the potential, i.e. voltage, of the charge carries
- The ASC is depicted as follows:



Example #2

- For the four circuits shown, determine if they are active (i.e. a source), or passive (i.e. a sink):
  - i)
  - ii)
  - iii)
  - iv)



Power

- **Power** (P, [W]) is the time rate of expending or absorbing energy (work)

$$P = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = Vi$$

- To determine the work of an element (either dissipated or supplied)

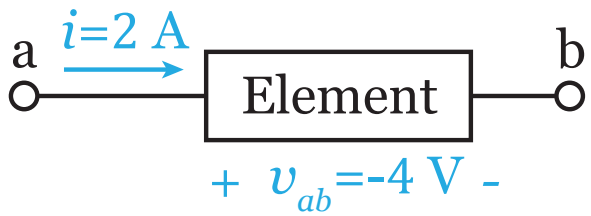
$$w = \int_{t_1}^{t_2} P\, d\tau$$

Example #3

- Assume the current into the terminal of an element is $20e^{-5{,}000t}$ [A] and the voltage across the element is $10e^{-5{,}000t}$ [kV] for $t \geq 0$
- Determine
  1. The power supplied to the element at 1 [ms]
  2. The energy delivered to the element
- Solution:

Example #4

- Given the element below, determine
  1. The power dissipated
  2. The power supplied
  3. If this is a source or sink



- Solution:

Conservation of Power

- The **Conservation of Energy**, or **Conservation of Electrical Power**, states the power dissipated must be equal to the power supplied:

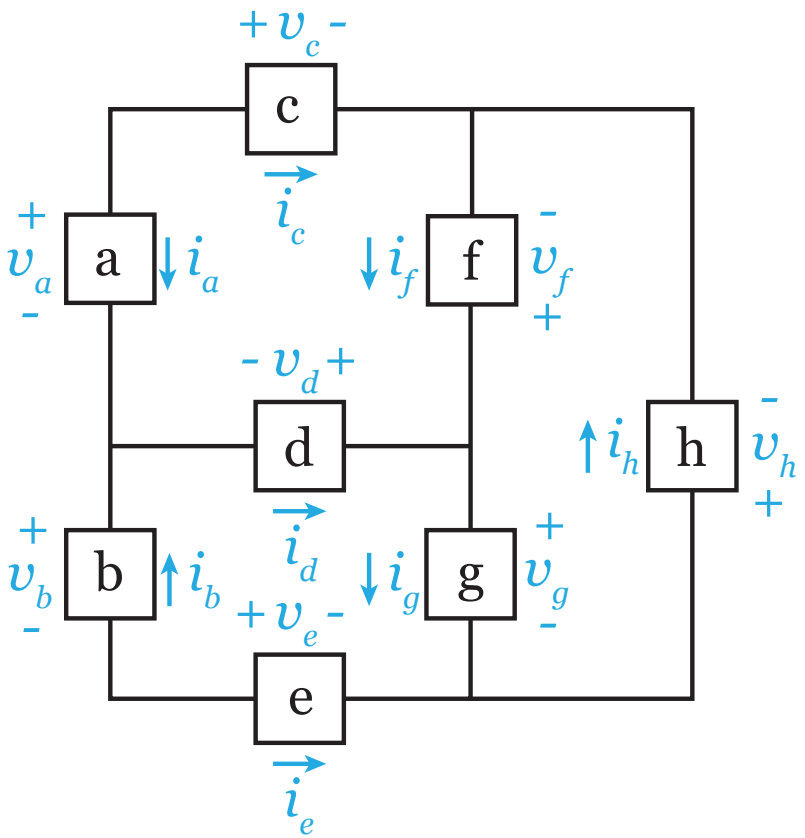
$$P_{\text{supplied}} = P_{\text{dissipated}}$$

- The circuit cannot supply more energy that it is dissipating, nor can it dissipate more energy that is supplied

Example #5

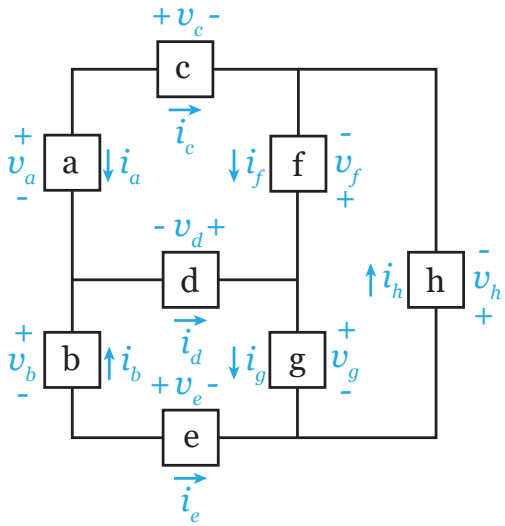
- Using the passive sign convention, determine if the circuit obeys **conservation of energy**

	[V]	[A]
a	120	-10
b	120	9
c	10	10
d	10	1
e	-10	-9
f	-100	5
g	120	4
h	-220	-5



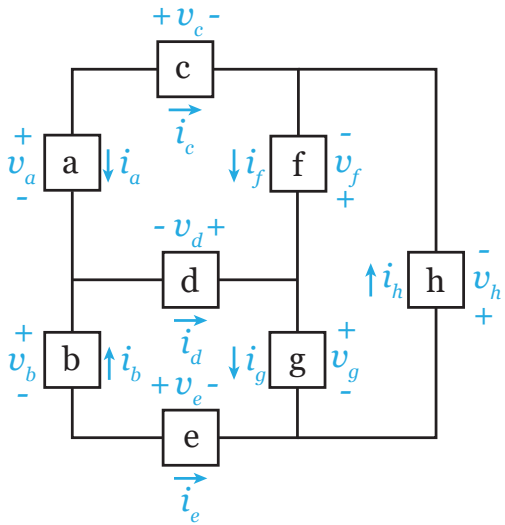
Example #5

- Solution:



Example #5

- Solution:



Student Learning Objectives

At the end of the lecture, students should be able to:

- Understand the concept and formulation of charge and current
  - Electric charge is the time-integral of electric current. Current is the flow of charge carriers (i.e. electrons) per unit time.
- Understand the concept and formulation of voltage
  - Voltage is a potential that is able to drive current through an element.
- Recognize and understand active and passive sign conventions
  - An active element is supplying power to the circuit - the potential is increasing as current runs through the device. The opposite for a passive element.

Student Learning Objectives

- Understand and apply the formulation of power
  - Power is the time-rate of supplying or dissipating energy. Power is simply the voltage potential across an element times the current running through it.
- Understand and apply the conservation of energy
  - Energy can neither be created nor destroyed. The amount of energy supplied to a circuit is equal to the amount of energy dissipated by a circuit.

Suggested Problems

- 1.2-3, 1.2-5, 1.3-4, 1.5-1, 1.5-2, 1.7-3