

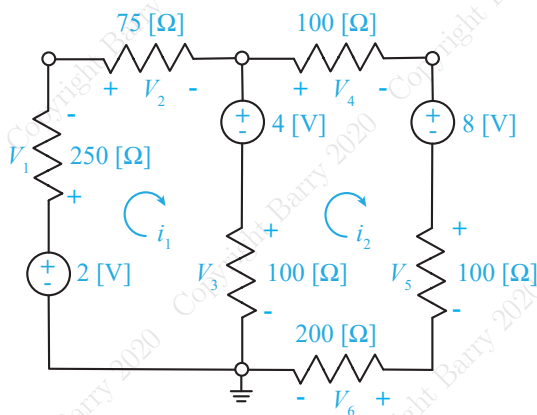
MCA Worksheet

MEMS 0031 - Electrical Circuits

June 4, 2020

Problem #1

Use MCA to find i_1 and i_2 in the circuit shown below.



Step 1: Construct N KVL loops. $N = 2$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Mesh Current 1:

$$-2 [\text{V}] + V_1 + V_2 + 4 [\text{V}] + V_3 = 0$$

Mesh Current 2:

$$-V_3 - 4 [\text{V}] + V_4 + 8 [\text{V}] + V_5 + V_6 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Mesh Current 1:

$$\begin{aligned} -2 [\text{V}] + (250 [\Omega])i_1 + (75 [\Omega])i_1 + 4 [\text{V}] + (100 [\Omega])(i_1 - i_2) &= 0 \\ \Rightarrow (425 [\Omega])i_1 - (100 [\Omega])i_2 &= -2 [\text{V}] \end{aligned}$$

Mesh Current 2:

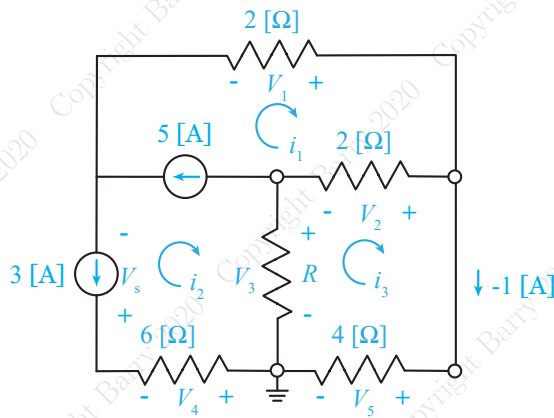
$$\begin{aligned} -(100 [\Omega])(i_1 - i_2) - 4 [\text{V}] + (100 [\Omega])i_2 + 8 [\text{V}] + (100 [\Omega])i_2 + (200 [\Omega])i_2 &= 0 \\ \Rightarrow -(100 [\Omega])i_1 + (500 [\Omega])i_2 &= -4 [\text{V}] \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 425 & -100 \\ -100 & 500 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -0.0069 \\ -0.0094 \end{bmatrix}$$

Problem #2

Use MCA to determine the resistance R in the circuit shown below.



Step 1: Construct N KVL loops. $N = 3$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Technically we need 1 KVL equation, however all mesh currents are specified. By inspection:

$$i_2 = -3 \text{ [A]}$$

And

$$i_3 = -1 \text{ [A]}$$

And we have a current source equation:

$$i_1 - i_2 = 5 \text{ [A]} \implies i_1 = 2 \text{ [A]}$$

Therefore the value of the resistance of the resistor can be found by applying KVL around the loop that contains Mesh Current 3:

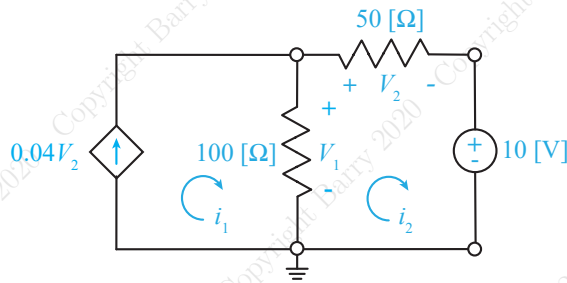
$$-V_3 - V_2 + V_5 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents:

$$\begin{aligned} -R(i_2 - i_3) - (2 \text{ [}\Omega\text{)})(i_1 - i_3) + (4 \text{ [}\Omega\text{)})i_3 &= 0 \\ \implies R &= \frac{-(2 \text{ [}\Omega\text{)})(3 \text{ [A]}) + (4 \text{ [}\Omega\text{)})(-1 \text{ [A]})}{(-2 \text{ [A]})} = 5 \text{ [}\Omega\text{]} \end{aligned}$$

Problem #3

Use MCA to determine the voltage drop across the $50\ [\Omega]$ resistor in the circuit shown below.



By inspection

$$i_1 = 0.04V_2$$

Mesh Current 2:

$$-V_1 + V_2 + 10\ [\text{V}] = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents:

Mesh Current 1:

$$i_1 = 0.04(50\ [\Omega])i_2 \implies i_1 - 2i_2 = 0$$

Mesh Current 2:

$$\begin{aligned} -(100\ [\Omega])(i_1 - i_2) + (50\ [\Omega])i_2 + 10\ [\text{V}] &= 0 \\ \implies -(100\ [\Omega])i_1 + (150\ [\Omega])i_2 &= -10\ [\text{V}] \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 1 & -2 \\ -100 & 150 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}$$

Step 1: Construct N KVL loops. $N = 2$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current: