

Homework #5

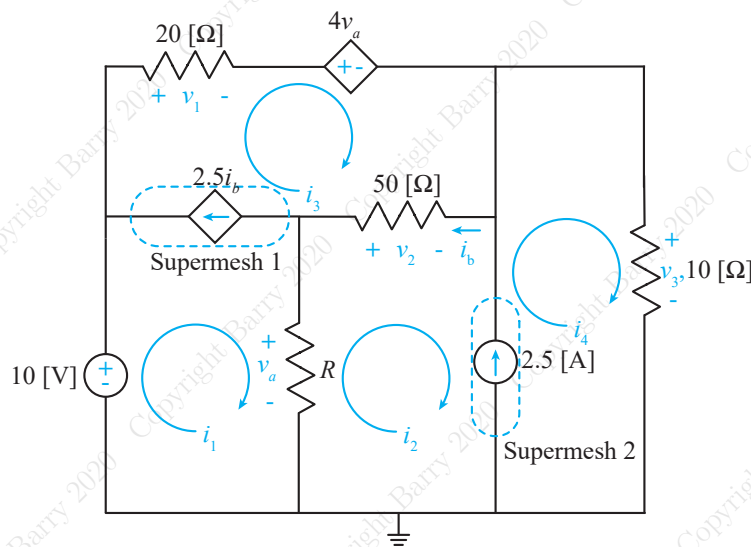
MEMS 0031 - Electrical Circuits

Assigned: June 5th, 2020

Due: June 10th, 2020 at 11:59 pm

Problem #1

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_4 , given $R = 20 [\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 4$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Supermesh 1 equation:

$$i_3 - i_1 = 2.5i_b$$

Supermesh 1:

$$-10 [\text{V}] + V_1 + 4V_a - (50 [\Omega])(i_2 - i_3) + (20 [\Omega])(i_1 - i_2) = 0$$

Supermesh equation 2:

$$i_4 - i_2 = 2.5 [\text{A}]$$

Supermesh 2:

$$-(20 [\Omega])(i_1 - i_2) + (50 [\Omega])(i_2 - i_3) + V_3 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$i_3 - i_1 = 2.5(i_3 - i_2) \implies i_1 - 2.5i_2 + 1.5i_3 = 0 \quad (1)$$

Supermesh 1:

$$\begin{aligned} -10[V] + (20[\Omega])i_3 + 4(20[\Omega])(i_1 - i_2) - (50[\Omega])(i_2 - i_3) + (20[\Omega])(i_1 - i_2) &= 0 \\ \Rightarrow 100i_1 - 150i_2 + 70i_3 &= 10 \end{aligned} \quad (2)$$

Supermesh 2 equation:

$$i_4 - i_2 = 2.5[A] \quad (3)$$

Supermesh 2:

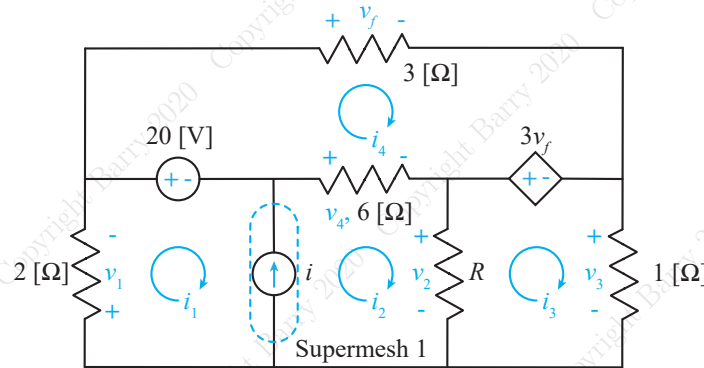
$$-(20[\Omega])(i_1 - i_2) + (50[\Omega])(i_2 - i_3) + (10[\Omega])i_4 = 0 \Rightarrow -20i_1 + 70i_2 - 50i_3 + 10i_4 = 0 \quad (4)$$

Putting eqns. 1 through 4 in matrix form:

$$\begin{bmatrix} 1 & -2.5 & 1.5 & 0 \\ 100 & -150 & 70 & 0 \\ 0 & -1 & 0 & 1 \\ -20 & 70 & -50 & 10 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 2.5 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -3.25 \\ -5.5 \\ -7 \\ -3 \end{bmatrix}$$

Problem #2

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_4 , given $i = 10\text{ [A]}$ and $R = 4\text{ [}\Omega\text{]}$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 4$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Supernode Equation 1:

$$i_2 - i_1 = 10\text{ [A]}$$

Supernode 1:

$$V_1 + 20\text{ [V]} + V_4 + V_2 = 0$$

Mesh Current 3

$$-V_2 + 3V_f + V_3 = 0$$

Mesh Current 4

$$V_f - 3V_f - V_4 = 20\text{ [V]}$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supernode 1 equation:

$$-i_1 + i_2 = 10 \quad (5)$$

Supernode 1:

$$(2\text{ [}\Omega\text{)})i_1 + (6\text{ [}\Omega\text{)})(i_2 - i_4) + (4\text{ [}\Omega\text{)})(i_2 - i_3) = -20\text{ [V]} \Rightarrow 2i_1 + 10i_2 - 4i_3 - 6i_4 = -20 \quad (6)$$

Mesh Current 3

$$-(4\text{ [}\Omega\text{)})(i_2 - i_3) + 3(3\text{ [}\Omega\text{)})i_4 + (1\text{ [}\Omega\text{)})i_3 = 0 \Rightarrow -4i_2 + 5i_3 + 9i_4 = 0 \quad (7)$$

Mesh Current 4:

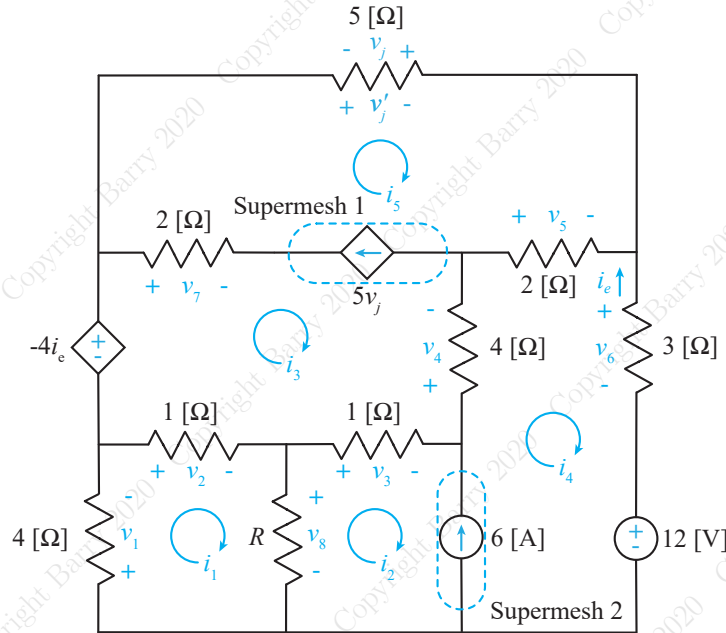
$$-2(3\text{ [}\Omega\text{)})i_4 - (6\text{ [}\Omega\text{)})(i_2 - i_4) = 20\text{ [V]} \Rightarrow -6i_2 = 20 \quad (8)$$

Putting eqns. 5 through 8 in matrix form:

$$\begin{bmatrix} -1 & 1 & 0 & 0 \\ 2 & 10 & -4 & -6 \\ 0 & -4 & 5 & 9 \\ 0 & -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 10 \\ -20 \\ 0 \\ 20 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} -13.33 \\ -3.33 \\ -46.67 \\ 24.44 \end{bmatrix}$$

Problem #3

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_5 , given $R = 3\ [\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 5$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_5 - i_3 = 5V_j$$

Supermesh 1, noting $V'_j = -V_j$:

$$-(-4i_e) + V'_j - V_5 - V_4 - V_3 - V_2 = 0$$

Supermesh Equation 2:

$$i_4 - i_2 = 6\text{ [A]}$$

Supermesh 2:

$$-V_8 + V_3 + V_4 + V_5 + V_6 + 12\text{ [V]} = 0$$

Mesh Current 1:

$$V_1 + V_2 + V_8 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

Supermesh 1 equation:

$$-i_3 + i_5 - 5(5\text{ [Ω]})(-i_5) = 0 \implies -i_3 + 26i_5 = 0 \quad (9)$$

Supermesh 1:

$$\begin{aligned} 4(-i_4) + (5\text{ [Ω]})i_5 - (2\text{ [Ω]})(i_4 - i_5) - (4\text{ [Ω]})(i_4 - i_3) - (1\text{ [Ω]})(i_2 - i_3) - (1\text{ [Ω]})(i_1 - i_3) &= 0 \\ \implies -i_1 - i_2 + 6i_3 - 10i_4 + 7i_5 &= 0 \end{aligned} \quad (10)$$

Supermesh Equation 2:

$$-i_2 + i_4 = 6\text{ [A]} \quad (11)$$

Supermesh 2:

$$\begin{aligned} -(3[\Omega])(i_1 - i_2) + (1[\Omega])(i_2 - i_3) + (4[\Omega])(i_4 - i_3) + (2[\Omega])(i_4 - i_5) + (3[\Omega])i_4 &= -12[\text{V}] \\ \implies -3i_1 + 4i_2 - 5i_3 + 9i_4 - 2i_5 &= -12[\text{V}] \end{aligned} \quad (12)$$

Mesh Current 1:

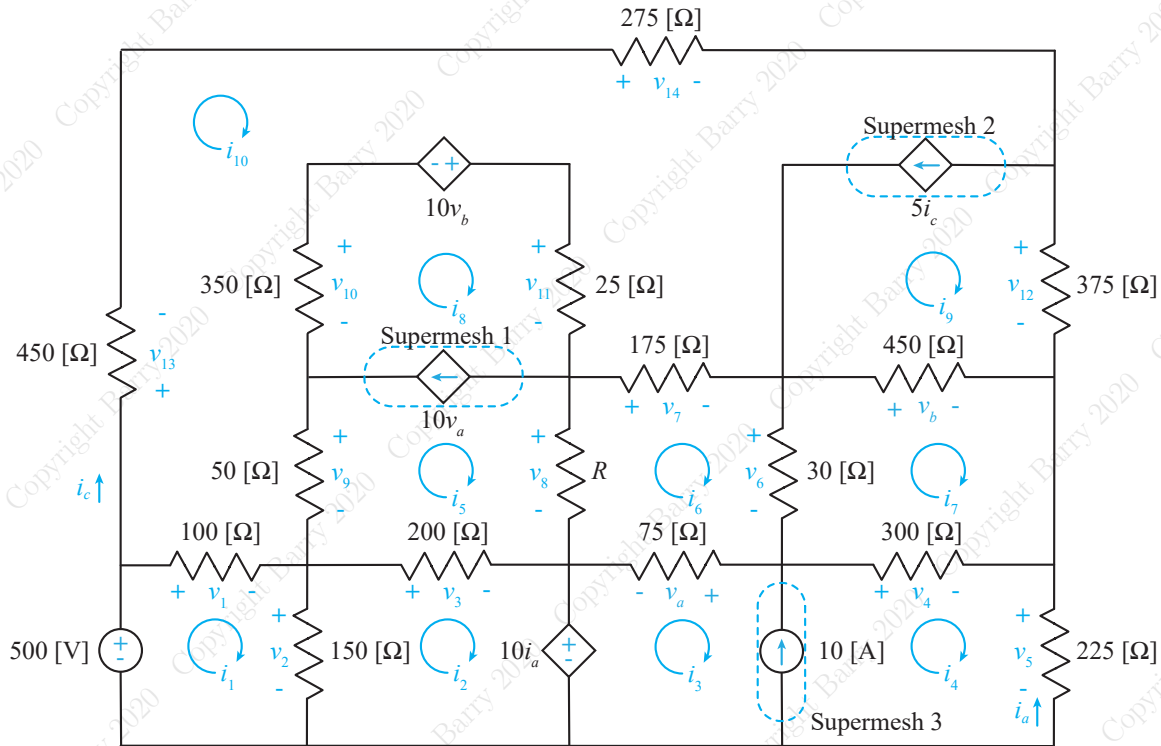
$$(4[\Omega])i_1 + (1[\Omega])(i_1 - i_3) + (3[\Omega])(i_1 - i_2) = 0 \implies 8i_1 - 3i_2 - i_3 = 0 \quad (13)$$

Putting eqns. 9 through 13 in matrix form:

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 26 \\ -1 & -1 & 6 & -10 & 7 \\ 0 & -1 & 0 & 1 & 0 \\ -3 & 4 & -5 & 9 & -2 \\ 8 & -3 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 6 \\ -12 \\ 0 \end{bmatrix} \implies \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \end{bmatrix} = \begin{bmatrix} -3.12 \\ -7.16 \\ -3.49 \\ -1.16 \\ -0.13 \end{bmatrix}$$

Problem #4

Using Mesh Current Analysis (MCA), determine the mesh currents i_1 through i_{10} , given $R = 125 [\Omega]$. Note: if you use any other method than MCA to determine the mesh currents, your answer will be marked incorrect.



Step 1: Construct N KVL loops. $N = 10$

Step 2: Assign voltage potentials across resistors/current sources consistent PSC. Note - do not dually label shared elements!

Step 3: Construct $N - \#CS$ KVL equations, describing each mesh current:

Supermesh Equation 1:

$$i_8 - i_5 = 10V_a$$

Supermesh 1:

$$-V_9 - V_{10} - 10V_b + V_{11} + V_8 - V_3 = 0$$

Supermesh Equation 2:

$$i_{10} - i_9 = 5i_c$$

Supermesh 2:

$$V_{13} + V_{14} + V_{12} - V_b - V_7 - V_{11} + 10V_b + V_{10} + V_9 - V_1 = 0$$

Supermesh Equation 3:

$$i_4 - i_3 = 10 [\text{A}]$$

Supermesh 3:

$$-10i_a - V_a + V_4 + V_5 = 0$$

Mesh Current 1:

$$-500 [\text{V}] + V_1 + V_2 = 0$$

Mesh Current 2:

$$-V_2 + V_3 + 10i_a = 0$$

Mesh Current 6:

$$-V_8 + V_7 + V_6 + V_a = 0$$

Mesh Current 7:

$$-V_6 + V_b - V_4 = 0$$

Step 4: Apply Ohm's law to express voltage potentials in terms of mesh currents.

We will denote the following variables:

$$\begin{aligned} i_a &= -i_4 & V_6 &= (30 [\Omega])(i_6 - i_7) \Rightarrow V_6 = 30i_6 - 30i_7 \\ i_c &= i_{10} & V_7 &= (175 [\Omega])(i_6 - i_{10}) \Rightarrow V_7 = 175i_6 - 175i_{10} \\ V_a &= (75 [\Omega])(i_6 - i_3) \Rightarrow V_a = -75i_3 + 75i_6 & V_8 &= (125 [\Omega])(i_5 - i_6) \Rightarrow V_8 = 125i_5 - 125i_6 \\ V_b &= (450 [\Omega])(i_7 - i_9) \Rightarrow V_b = 450i_7 - 450i_9 & V_9 &= (50 [\Omega])(i_{10} - i_5) \Rightarrow V_9 = -50i_5 + 50i_{10} \\ V_1 &= (100 [\Omega])(i_1 - i_{10}) \Rightarrow V_1 = 100i_1 - 100i_{10} & V_{10} &= (350 [\Omega])(i_{10} - i_8) \Rightarrow V_{10} = -350i_8 + 350i_{10} \\ V_2 &= (150 [\Omega])(i_1 - i_2) \Rightarrow V_2 = 150i_1 - 150i_2 & V_{11} &= (25 [\Omega])(i_8 - i_{10}) \Rightarrow V_{11} = 25i_8 - 25i_{10} \\ V_3 &= (200 [\Omega])(i_2 - i_5) \Rightarrow V_3 = 200i_2 - 200i_5 & V_{12} &= (375 [\Omega])i_9 \Rightarrow V_{12} = 375i_9 \\ V_4 &= (300 [\Omega])(i_4 - i_7) \Rightarrow V_4 = 300i_4 - 300i_7 & V_{13} &= (450 [\Omega])i_{10} \Rightarrow V_{13} = 450i_{10} \\ V_5 &= (225 [\Omega])i_4 \Rightarrow V_5 = 225i_4 & V_{14} &= (275 [\Omega])i_{10} \Rightarrow V_{14} = 275i_{10} \end{aligned}$$

Supernode Equation 1:

$$750i_3 - i_5 - 750i_6 + i_8 = 0 \quad (14)$$

Supernode 1:

$$-200i_2 + 375i_5 - 125i_6 - 4,500i_7 + 375i_8 + 4,500i_9 - 425i_{10} = 0 \quad (15)$$

Supernode Equation 2:

$$-i_9 - 4i_{10} = 0 \quad (16)$$

Supernode 2:

$$-100i_1 - 50i_5 - 175i_6 + 4,050i_7 - 375i_8 - 3,675i_9 + 1,425i_{10} = 0 \quad (17)$$

Supernode Equation 3:

$$-i_3 + i_4 = 10 \quad (18)$$

Supernode 3:

$$75i_3 + 535i_4 - 75i_6 - 300i_7 = 0 \quad (19)$$

Mesh Current 1:

$$250i_1 - 150i_2 - 100i_{10} = 500 \quad (20)$$

Mesh Current 2:

$$-150i_1 + 350i_2 - 10i_4 - 200i_5 = 0 \quad (21)$$

Mesh Current 6:

$$-75i_3 - 125i_5 + 405i_6 - 30i_7 - 175i_{10} = 0 \quad (22)$$

Mesh Current 7:

$$-300i_4 - 30i_6 + 780i_7 - 450i_9 = 0 \quad (23)$$

Putting eqns. 14 through 23 in matrix form:

$$\begin{bmatrix} 0 & 0 & 750 & 0 & -1 & -750 & 0 & 1 & 0 & 0 \\ 0 & -200 & 0 & 0 & 375 & -125 & -4,500 & 375 & 4,500 & -425 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -4 \\ -100 & 0 & 0 & 0 & -50 & -175 & 4,050 & -375 & -3,675 & 1,425 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 75 & 535 & 0 & -75 & -300 & 0 & 0 & 0 \\ 250 & -150 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -100 \\ -150 & 350 & 0 & -10 & -200 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -75 & 0 & -125 & 405 & -30 & 0 & 0 & -175 \\ 0 & 0 & 0 & -300 & 0 & -30 & 780 & 0 & -450 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 10 \\ 0 \\ 500 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \\ i_9 \\ i_{10} \end{bmatrix} = \begin{bmatrix} -8.8187 \\ -17.7072 \\ -9.4516 \\ 0.5484 \\ -24.4010 \\ -9.4196 \\ 0.9700 \\ -0.4180 \\ 1.9438 \\ -0.4859 \end{bmatrix}$$