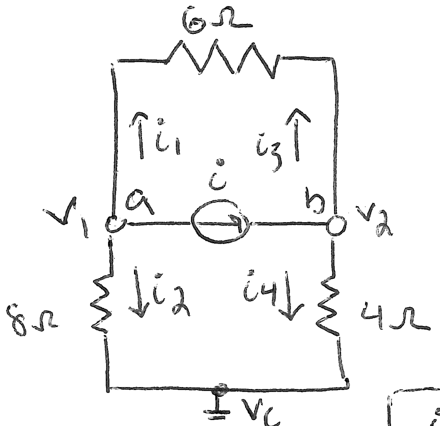
MEMS 0031 - Electrical Circuits - HW #4 Solutions:

#1:



NVA to find i:

KCL @ node a:

$i + i_1 + i_2 = 0$

$\rightarrow i = -i_1 - i_2$ ← Apply Ohm's Law

$$\boxed{i = -\frac{(v_a - v_b)}{6} - \frac{(v_1 - \cancel{v_c}^{0})}{8}}$$

where $v_a = v_1$, $v_b = v_2$

or apply KCL @ node b:

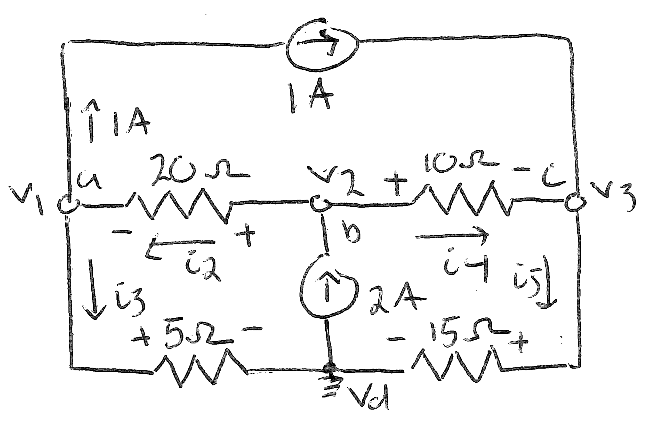
$i = i_3 + i_4$ ← Apply Ohm's Law

$$\boxed{i = \frac{(v_b - v_a)}{6} + \frac{(v_b - \cancel{v_c}^{0})}{4}}$$

where $v_a = v_1$, $v_b = v_2$

We have to leave i in terms of v_1 & v_2 because we do not have any information about those voltage values.

#2:



NVA to find v_1, v_2 & v_3

KCL @ node a:

$i_2 = 1[A] + i_3$ ← Apply Ohm's Law

$$\frac{V_2 - V_1}{20} = 1[A] + \frac{V_1 - \cancel{V_4}^{\,0}}{5} \Rightarrow \left(\frac{V_2}{20}\right) \underbrace{- \left(\frac{V_1}{20}\right) - \left(\frac{V_1}{5}\right)}_{-1/4\, V_1} = 1[A]$$

like terms

KCL @ node b:

$2[A] = i_2 + i_4$ ← Apply Ohm's Law

$$\frac{V_2 - V_1}{20} + \frac{V_2 - V_3}{10} = 2[A] \Rightarrow \left(\frac{V_2}{20} + \frac{V_2}{10}\right) - \frac{V_1}{20} - \frac{V_3}{10} = 2[A]$$

KCL @ node c:

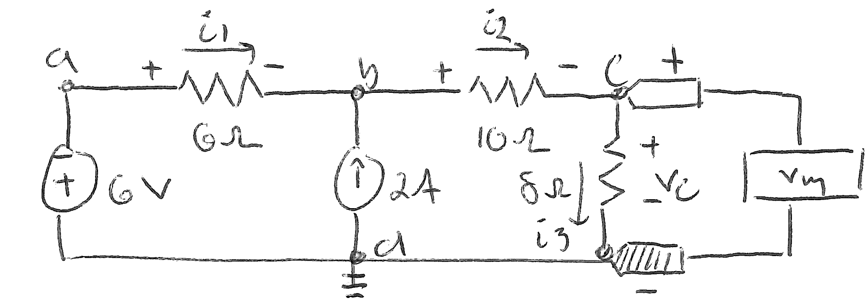
$1[A] + i_4 = i_5$ ← Apply Ohm's Law

$$1[A] + \frac{V_2 - V_3}{10} = \frac{V_3 - \cancel{V_4}^{\,0}}{15} \Rightarrow \left(\frac{V_3}{15} + \frac{V_3}{10}\right) - \frac{V_2}{10} = 1[A]$$

Construct an array:

$$\begin{bmatrix} -1/4 & 1/20 & 0 \\ -1/20 & 3/20 & -1/10 \\ 0 & -1/10 & 1/6 \end{bmatrix} \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} \Rightarrow \boxed{\begin{Bmatrix} V_1 \\ V_2 \\ V_3 \end{Bmatrix} = \begin{Bmatrix} 2 \\ 30 \\ 24 \end{Bmatrix}}$$

#3:



NVA to find v_a, v_b, v_c and v_m

KCL @ node b:

$i_1 + 2[A] = i_2$ ← Apply Ohm's Law

$$\frac{v_a - v_b}{6} - \left(\frac{v_b - v_c}{10}\right) = -2[A]$$ $\boxed{v_a = -6[V] \quad \text{VS eqn}}$

$$\Rightarrow -1 - \frac{v_b}{6} - \frac{v_b}{10} + \frac{v_c}{10} = -2[A] \Rightarrow -v_b\left(\frac{1}{6} + \frac{1}{10}\right) + v_c\left(\frac{1}{10}\right) = -1$$

KCL @ node c:

$i_2 = i_3$ ← Apply Ohm's Law

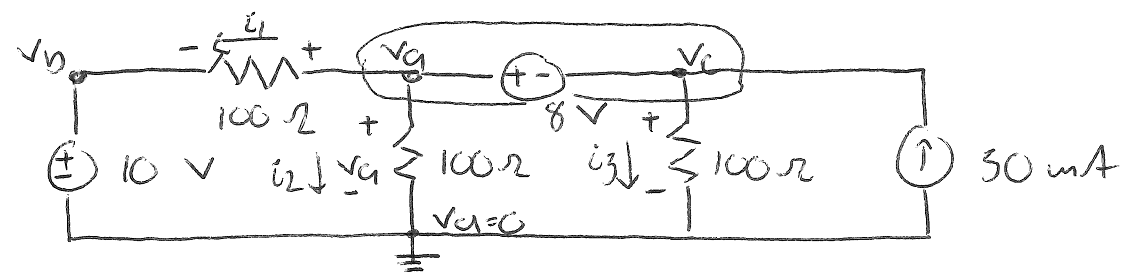
$$\frac{v_b - v_c}{10} = \frac{v_c - \cancel{v_d}^{\,0}}{8} \Rightarrow \frac{v_b}{10} - \frac{v_c}{10} - \frac{v_c}{8} = 0 \Rightarrow -v_c\left(\frac{1}{10} + \frac{1}{8}\right) + \frac{v_b}{10} = 0$$

Construct array:

$$\begin{bmatrix} -4/15 & 1/10 \\ +1/40 & -9/40 \end{bmatrix} \begin{Bmatrix} v_b \\ v_c \end{Bmatrix} = \begin{Bmatrix} -1 \\ 0 \end{Bmatrix} = \boxed{\begin{Bmatrix} v_b \\ v_c \end{Bmatrix} = \begin{Bmatrix} 4.5 \\ 2 \end{Bmatrix}}$$

$\boxed{\therefore v_m = v_c = 2[V]}$

#4:



NVA to find va:

VS equ: $V_b = 10\,[V]$ & $8\,[V] = V_a - V_c$

Apply KCL to supernode:

$0.030\,[A] = i_1 + i_2 + i_3$ ← Apply Ohm's Law

$$0.030\,[A] = \frac{V_a - \cancel{V_b}^{\,10}}{100} + \frac{V_a - \cancel{V_d}^{\,0}}{100} + \frac{V_c - \cancel{V_d}^{\,0}}{100}$$

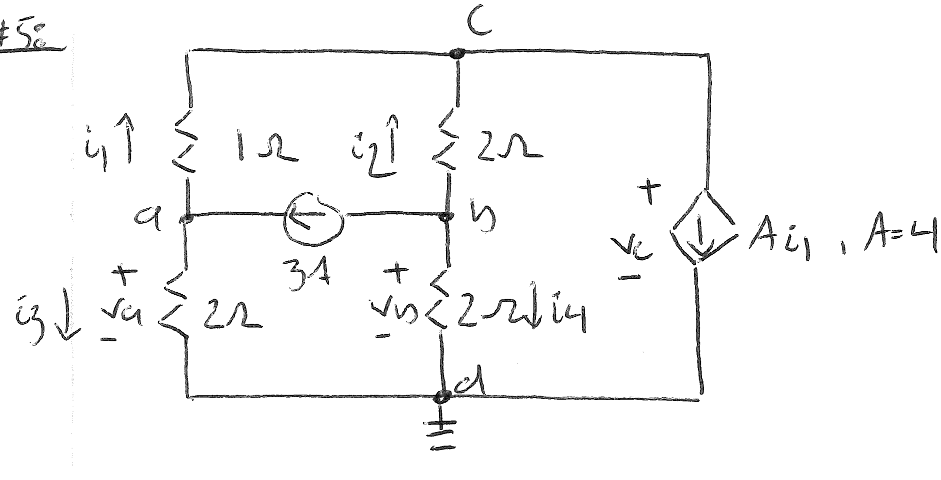
$$\rightarrow 0.030\,[A] = \frac{V_a}{100} - \frac{10}{100} + \frac{V_a}{100} + \frac{V_c}{100}$$

Therefore, we have 2 equs & 2 unknowns

$$\left.\begin{aligned} V_a\left(\tfrac{1}{50}\right) + V_c\left(\tfrac{1}{100}\right) &= 0.13 \\ V_a - V_c &= 8 \end{aligned}\right\}$$ construct an array

$$\begin{bmatrix} 1/50 & 1/100 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} V_a \\ V_c \end{Bmatrix} = \begin{Bmatrix} 0.13 \\ 8 \end{Bmatrix} \rightarrow \boxed{\begin{Bmatrix} V_a \\ V_c \end{Bmatrix} = \begin{Bmatrix} 7 \\ -1 \end{Bmatrix}}$$

#5:



NVA to find i_1 & i_2

KCL @ node a:

$3[A] = i_1 + i_3$ ← Apply Ohm's Law

$$3[A] = \frac{v_a - v_c}{1} + \frac{v_a - \cancel{v_d}^{\,0}}{2} \Rightarrow v_a\left(1+\frac{1}{2}\right) - v_c = 3[A]$$

KCL @ node b:

$i_2 + i_4 = -3[A]$ ← Apply Ohm's Law

$$\frac{v_b - v_c}{2} + \frac{v_b - \cancel{v_d}^{\,0}}{2} = -3[A] \Rightarrow v_b\left(\frac{1}{2}+\frac{1}{2}\right) - v_c\left(\frac{1}{2}\right) = -3[A]$$

KCL @ node c:

$i_1 + i_2 = 4i_1$ ← Apply Ohms Law

$$-3\left(\frac{v_a - v_c}{1}\right) + \frac{v_b - v_c}{2} = 0 \Rightarrow -3v_a + \frac{1}{2}v_b + v_c\left(3 - \frac{1}{2}\right) = 0$$

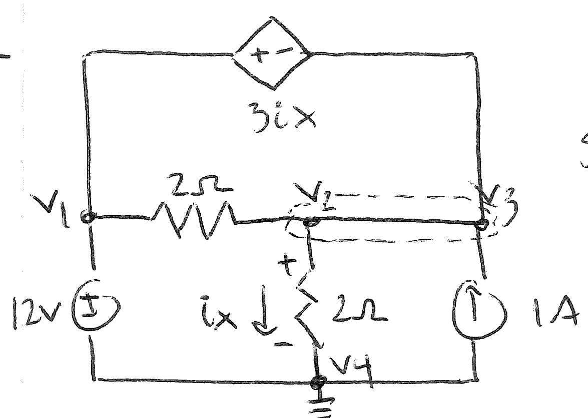
Create an array:

$$\begin{bmatrix} 3/2 & 0 & -1 \\ 0 & 1 & -1/2 \\ -3 & 1/2 & 5/2 \end{bmatrix} \begin{Bmatrix} V_A \\ V_b \\ V_c \end{Bmatrix} = \begin{Bmatrix} 3 \\ -3 \\ 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} V_A \\ V_b \\ V_c \end{Bmatrix} = \begin{Bmatrix} 8\tfrac{2}{3} \\ 2 \\ 10 \end{Bmatrix}$$

$$\therefore i_1 = \frac{V_A - V_c}{1} = 8\tfrac{2}{3} - 10 = \boxed{-4/3\,[A] = i_1}$$

$$\therefore i_2 = \frac{V_b - V_c}{2} = \frac{2-10}{2} = \boxed{-4\,[A] = i_2}$$

#6:



NVA to find ix:

Starting:

$$i_x = \frac{V_2 - V_4^{\,0}}{2} = \frac{V_2}{2}$$

Indication we need to find V_2 which equals V_3

VS eqns: $V_1 = 12\,[V]$ & $3i_x = V_1 - V_3$

Our Vs eqn from the supernode with $V_3 = V_2$:

$$3i_x = V_1^{\,12} - V_2 = 3i_x = \underbrace{3\left(\frac{V_2}{2}\right)}_{\text{current through } 2\Omega}$$

$$\therefore 12 - V_2 = \frac{3}{2}V_2 \rightarrow 12 = \frac{5}{2}V_2 \rightarrow V_2 = \frac{24}{5} = 4.8$$

$$\therefore i_x = \frac{V_2}{2} = \frac{4.8}{2} = \boxed{2.4\,[A] = i_x}$$