Homework #2

MEMS 0031 - Electrical Circuits

Assigned January 21^{st}, 2019
Due January 25^{th}, 2019

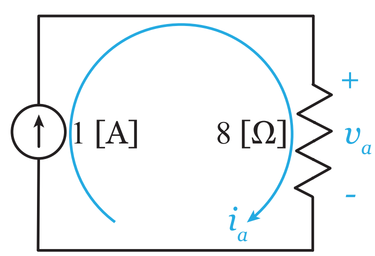
Problem #1

- In the circuits shown below, (a)-(e), determine:
  a) The unknown value(s) of i and/or v.
  b) The power dissipated by each resistor.
- For the circuit shown in (f), develop a system of equations that would allow you to determine the current in each element, and solve for the current i_a symbolically in terms of all other circuit variables.

Solutions:

Part (a):
Using Ohm's law to find the voltage drop across the 8 [Ω] resistor:

(a)



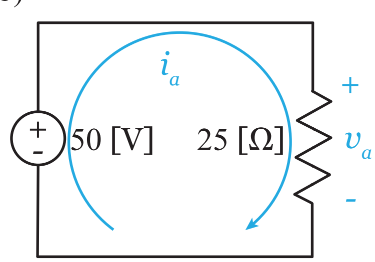
$$v_a = iR = (1\,[\text{A}])(8\,[\Omega]) = \boxed{8\,[\text{V}]}$$

Note the resistor is in accordance with the passive sign convention. The power dissipated is calculated as either a function of current and resistance, or voltage and current, and is found as:

$$P_{dis} = i^2R = iV = (1\,[\text{A}])^2(8\,[\Omega]) = (1\,[\text{A}])(8\,[\text{V}]) = \boxed{8\,[\text{W}]}$$

Part (b):
Using Ohm's law to find the current through the 25 [Ω] resistor:

(b)

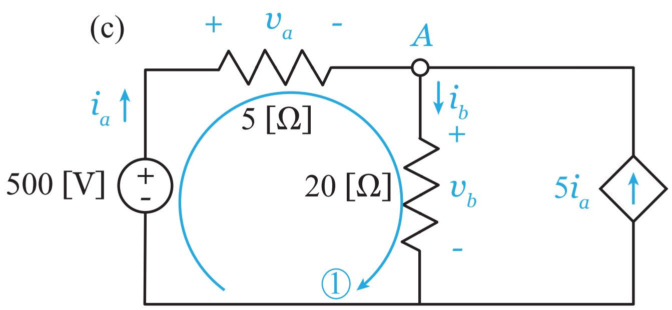


$$v_a = i_aR \implies i_a = \frac{v_a}{R} = \frac{50\,[\text{V}]}{25\,[\Omega]} = \boxed{2\,[\text{A}]}$$

Note the resistor is in accordance with the passive sign convention. The power dissipated is calculated as either a function of current and resistance, or voltage and current, and is found as:

$$P_{dis} = i^2R = iV = (2\,[\text{A}])^2(25\,[\Omega]) = (2\,[\text{A}])(50\,[\text{V}]) = \boxed{100\,[\text{W}]}$$

Part (c):
We have a variety of unknowns within the system: v_a, v_b, and most importantly, i_a. To generate an equation that relates i_a to the two sources, we can use KVL around ①, expressing the voltage drop across the resistors using Ohm's law, denoting the current through the 20 [Ω] resistor as i_b:



$$\text{KVL around Loop ①}: \; -500\,[\text{V}] + v_a + v_b = 0$$

Expressing v_a and v_b using Ohm's law:

$$\implies -500\,[\text{V}] + i_a(5\,[\Omega]) + i_b(20\,[\Omega]) = 0$$

$$\implies i_a(5\,[\Omega]) + i_b(20\,[\Omega]) = 500\,[\text{V}]$$

Now, we have one equation and two unknowns. Defining node A at the junction of the two resistors and CCCS, we can apply KCL at said node:

$$\text{KCL at Node } A: \; i_a + 5i_a = i_b \implies i_b = 6i_a$$

We can substitute this back into the KVL equation such that:

$$i_a(5\,[\Omega]) + (6i_a)(20\,[\Omega]) = 500\,[\text{V}] \implies i_a(125\,[\Omega]) = 500[V]$$

$$\therefore \boxed{i_a = 4\,[\text{A}]}$$

Thus, we can solve for i_b from the KCL equation such that:

$$i_b = 6i_a \implies \boxed{i_b = 24\,[\text{A}]}$$

The voltages across each resistor can be found via Ohm's law, which was expressed in the original KVL equation:

$$v_a = i_a(5\,[\Omega]) = (4\,[\text{A}])(5\,[\Omega]) \implies \boxed{v_a = 20\,[\text{V}]}$$

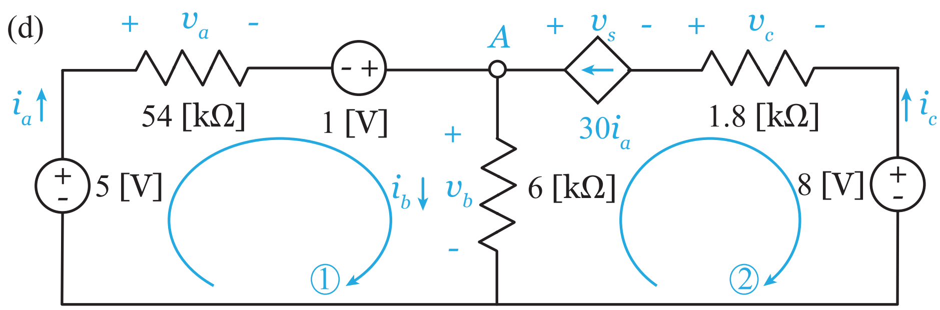
$$v_b = i_b(20\,[\Omega]) = (24\,[\text{A}])(20\,[\Omega]) \implies \boxed{v_b = 480\,[\text{V}]}$$

The power dissipated by each resistor is found as a function of current and resistance or current and voltage. Note, the resistors are already represented in PSC.

$$P_{dis,\,5\,[\Omega]} = (i_a)^2(5\,[\Omega]) = i_a v_a = (4\,[\text{A}])^2(5\,[\Omega]) = (4\,[\text{A}])(20\,[\text{V}]) \implies \boxed{P_{dis,\,5\,[\Omega]} = 80\,[\text{W}]}$$

$$P_{dis,\,20\,[\Omega]} = (i_b)^2(20\,[\Omega]) = i_b v_b = (24\,[\text{A}])^2(20\,[\Omega]) = (24\,[\text{A}])(480\,[\text{V}]) \implies \boxed{P_{dis,\,20\,[\Omega]} = 11,520\,[\text{W}]}$$

Part (d):
We recognize there are two independent loops in the circuit, ① and ②. We can apply two separate KVL equation around these loops to determine pertinent system parameters.



Starting with the first loop:

$$-5\,[\mathrm{V}] + v_a - 1\,[\mathrm{V}] + v_b = 0$$

Applying Ohm's law:

$$-5\,[\mathrm{V}] + i_a(54\,[\mathrm{k\Omega}]) - 1\,[\mathrm{V}] + i_b(6\,[\mathrm{k\Omega}]) = 0$$

We have two unknowns, i_a and i_b. We will relate these two by applying KCL at node A:

$$i_a + 30i_a = i_b \implies i_b = 31i_a$$

We have two equations for two unknowns. Solving for i_a:

$$\boxed{i_a = 25\,[\mu\mathrm{A}]}$$

Solving for i_b:

$$\boxed{i_b = 775\,[\mu\mathrm{A}]}$$

The voltages across each resistor can be found via Ohm's law, which was expressed in the original KVL equation:

$$v_a = i_a(54\,[\mathrm{k\Omega}]) = (25\,[\mu\mathrm{A}])(54\,[\mathrm{k\Omega}]) \implies \boxed{v_a = 1.35\,[\mathrm{V}]}$$

$$v_b = i_b(6\,[\mathrm{k\Omega}]) = (775\,[\mu\mathrm{A}])(6\,[\mathrm{k\Omega}]) \implies \boxed{v_b = 4.65\,[\mathrm{V}]}$$

The power dissipated by the two resistors encircled by the first loop is found as:

$$P_{dis,\,54\,[\mathrm{k\Omega}]} = (i_a)^2(54\,[\mathrm{k\Omega}]) = (25\,[\mu\mathrm{A}])^2(54\,[\mathrm{k\Omega}]) \implies \boxed{P_{dis,\,54\,[\mathrm{k\Omega}]} = 33.75\,[\mu\mathrm{W}]}$$

$$P_{dis,\,6\,[\mathrm{k\Omega}]} = (i_b)^2(6\,[\mathrm{k\Omega}]) = (775\,[\mu\mathrm{A}])^2(6\,[\mathrm{k\Omega}]) \implies \boxed{P_{dis,\,6\,[\mathrm{k\Omega}]} = 3,603.75\,[\mu\mathrm{W}]}$$

Moving to the second loop, we recognize that i_c must be equal to i_b - i_a, or $30i_a$. Solving for i_c:

$$i_c = 30i_a \implies \boxed{i_c = 750\,[\mu\mathrm{A}]}$$

The voltage across the 1.8 [kΩ] is found via Ohm's law. It is noted that the resistor is depicted having an active sign convention. Solving for ν_c as it is represented:

$$\nu_c = i_c(1.8[\mathrm{k\Omega}]) = (750\,[\mu\mathrm{A}])(1.8[\mathrm{k\Omega}]) \implies \boxed{\nu_c = -1.35\,[\mathrm{V}]}$$

Applying KVL and recognizing there exists a voltage drop across the CCCS:

$$-v_b + v_s + v_c + 8\,[\mathrm{V}] = 0$$

Applying Ohm's law, and recognizing the 1.8 [kΩ] resistor is depicted having an active sign convention:

$$-i_b(6\,[\text{k}\Omega]) + v_s + (30i_a)(1.8\,[\text{k}\Omega]) + 8 = 0$$

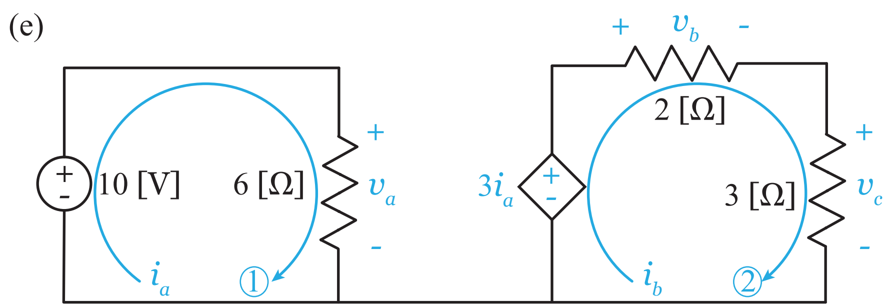
$$\Longrightarrow\; v_s = (6\,[\text{k}\Omega])(775\,[\mu\text{A}]) + (1.8\,[\text{k}\Omega])(30(25\,[\mu\text{A}])) - 8\,[\text{V}] = \Longrightarrow \boxed{\nu_s = -2\,[\text{V}]}$$

The power dissipated in the 1.8 [kΩ] resistor is found as:

$$P_{dis,\,1.8\,[\text{k}\Omega]} = (i_c)^2(1.8\,[\text{k}\Omega]) = (750\,[\mu\text{A}])^2(1.8\,[\text{k}\Omega]) \;\Longrightarrow\; \boxed{P_{dis,\,1.8\,[\text{k}\Omega]} = 1{,}012.5\,[\mu\text{W}]}$$

Part (e):

We recognize there are two independent loops in the circuit, ① and ②. We can apply two separate KVL equation around these loops to determine pertinent system parameters.



Starting with the first loop:

$$\text{KVL around Loop ①}: \; -10\,[\text{V}] + v_a = 0$$

Applying Ohm's law:

$$i_a(6\,[\Omega]) = 10\,[\text{V}] \;\Longrightarrow\; i_a = \frac{10\,[\text{V}]}{6\,[\Omega]} \;\Longrightarrow\; \boxed{i_a = 1.\bar{6}\,[\text{A}]}$$

Therefore, v_a can be determined via Ohm's law or the previous KVL equation:

$$\therefore \boxed{v_a = 10\,[\text{V}]}$$

Continuing with the second loop, applying KVL:

$$-3i_a + v_b + v_c = 0$$

Applying the value for i_a and Ohm's law:

$$-3(1.\bar{6}\,[\text{V}]) + i_b(5\,[\Omega]) = 0 \;\Longrightarrow\; \boxed{i_b = 1\,[\text{A}]}$$

Thus, the voltage across the 2 and 3 [Ω] resistors is found as:

$$v_b = i_bR = (1\,[\text{A}])(2\,[\Omega]) = \Longrightarrow \boxed{v_b = 2\,[\text{V}]}$$

$$v_c = i_bR = (1\,[\text{A}])(3\,[\Omega]) \;\Longrightarrow\; \boxed{v_c = 3\,[\text{V}]}$$

The power dissipated by each resistor is as follows:

$$P_{dis,\,6\,[\Omega]} = (i_a)^2(6\,[\Omega]) = i_a v_a = (1.\bar{6}\,[\text{A}])^2(6\,[\Omega]) = (1.\bar{6}\,[\text{A}])(10\,[\text{V}]) \;\Longrightarrow\; \boxed{P_{dis,\,6\,[\Omega]} = 16.\bar{6}\,[\text{W}]}$$

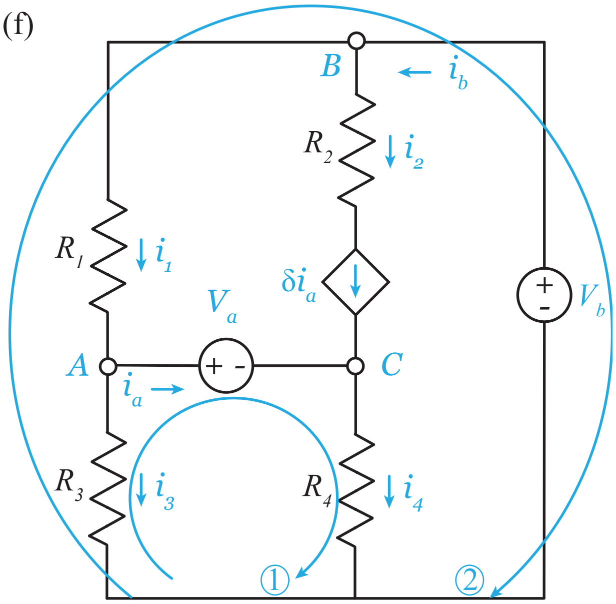
$$P_{dis,\,2\,[\Omega]} = (i_b)^2(2\,[\Omega]) = i_b v_b = (1\,[\text{A}])^2(2\,[\Omega]) = (1\,[\text{A}])(2\,[\text{V}]) \;\Longrightarrow\; \boxed{P_{dis,\,2\,[\Omega]} = 2\,[\text{W}]}$$

$$P_{dis,\,3\,[\Omega]} = (i_b)^2(3\,[\Omega]) = i_b v_c = (1\,[\text{A}])^2(3\,[\Omega]) = (1\,[\text{A}])(3\,[\text{V}]) \;\Longrightarrow\; \boxed{P_{dis,\,3\,[\Omega]} = 3\,[\text{W}]}$$

Part (f):

We want to put the currents in terms of system variables, i.e. resistances and sources. The first step is to apply KCL at nodes A, B and C.

(f)



Starting at node A:

$$i_1 = i_a + i_3$$

On to node B:

$$i_b = i_1 + i_2$$

On to node C, recognizing i_2=δi_a:

$$i_2 + i_a = i_4 \implies i_a(\delta + 1) = i_4$$

Thus, we have the current in relation to one another, however, not in terms of the sources. We will select two loops that do not contain a dependent source, 1 and 2. Around the first loop:

$$-i_3R_3 + V_a + i_4R_4 = 0$$

Around the second loop:

$$-i_3R_3 - i_1R_1 + V_b = 0$$

Thus, we have enough independent equations to solve each current, as well as i_b.