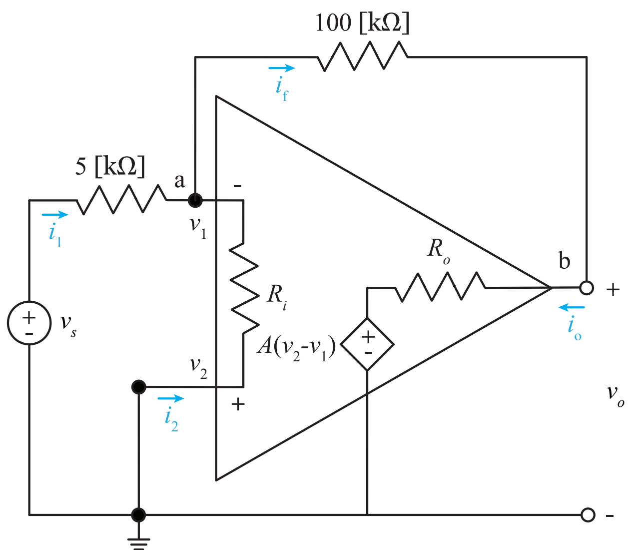
# Op-amps, Inductors and Capacitors Worksheet

MEMS 0031 - Electrical Circuits

June 24$^{\text{th}}$, 2020

## Problem #1

Given the Op-amp shown below, and using the Finite Gain model, determine the ratio of the output to input voltages, given an input resistance of 500 [kΩ], an output resistance of 5 [kΩ] and a gain of 300,000. How does this compare to an ideal op-amp?




Let us start by constructing and equation that relates the input to output voltages, i.e. KCL at node a:

$$i_i + i_2 = i_f$$

We will denote the 5 [kΩ] resistor as $R_s$ and the 100 [kΩ] resistor as $R_f$. Applying Ohm's law

$$\frac{V_s - V_a}{R_s} + \frac{\cancelto{0}{V_2} - V_a}{R_i} = \frac{V_a - V_o}{R_f} \tag{1}$$

We need a second KCL equation to account for the VCCS - we will apply KCL at node b:

$$i_f = i_o$$

Applying Ohm's law to express this in terms of node voltages:

$$\frac{V_a - V_o}{R_f} = \frac{V_o - A(-V_a)}{R_o} \tag{2}$$

We have two equations and three unknowns, but recall we are interested in the quantity $V_o/V_s$, which consolidates two unknowns into one. Therefore, we will express eqn. 1 as:

$$V_a \underbrace{\left(\frac{1}{R_s} + \frac{1}{R_i} + \frac{1}{R_f}\right)}_{\text{term 1}} = V_s \underbrace{\frac{1}{R_s}}_{\text{term 2}} + V_o \underbrace{\frac{1}{R_f}}_{\text{term 3}} \tag{3}$$

Eqn. 2 can be expressed as:

$$V_a \underbrace{\left(\frac{1}{R_f} - \frac{A}{R_o}\right)}_{\text{term 4}} = V_o \underbrace{\left(\frac{1}{R_o} + \frac{1}{R_f}\right)}_{\text{term 5}} \tag{4}$$

Equating $V_a$ to solve for the ratio of $V_o/V_s$:

$$\frac{V_o}{V_s} = \frac{\text{term 2}}{\left(\frac{(\text{term 5})(\text{term 1})}{\text{term 4}}\right) - \text{term 3}} = -19.9985$$

To treat this as an ideal op-amp, we would apply KCL at node a, not consider the current running through the internal resistor:

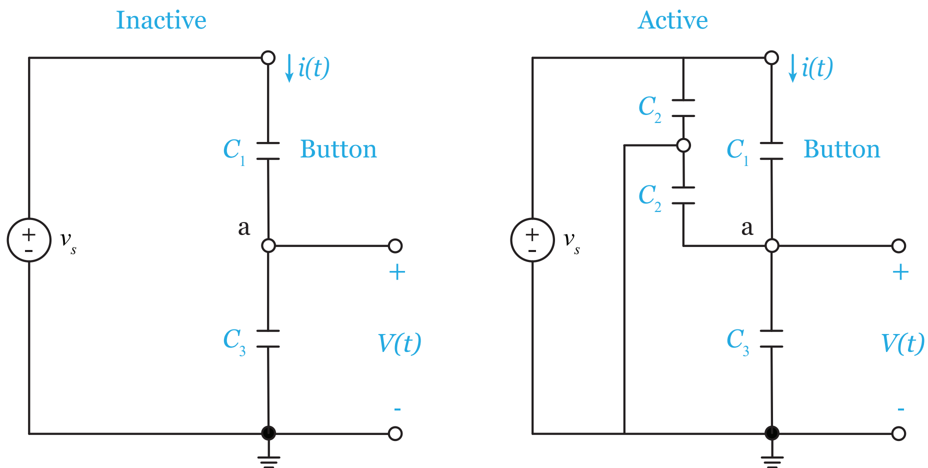
$$i_1 = i_f$$

In terms of node voltages:

$$\frac{V_s - \cancelto{0}{V_a}}{5\,[\text{k}\Omega]} = \frac{\cancelto{0}{V_a} - V_o}{100\,[\text{k}\Omega]} \implies \frac{V_o}{V_s} = -\frac{100\,[\text{k}\Omega]}{5\,[\text{k}\Omega]} = -20$$

# Problem #2

There are various devices that use capacitors are proximity switches. When the switch is activated, the button can be represented as a set of parallel capacitors; two capacitors in series, in parallel with the original capacitor. The capacitors in series are connected to a node that is connected to ground. Given the system below, where $C = 25$ [pF] for all capacitors, determine the output voltage when the button is and is not activated.




Let us consider the first situation when the button is inactive. The same current $i(t)$ is running through both currents, so if we apply KCL at node a, and expressing the currents in terms of capacitance and time rate of change of voltage:

$$C_1 \frac{d(V_s - V(t))}{dt} = C_3 \frac{dV(t)}{dt}$$

Solving for the output voltage:

$$\frac{dV(t)}{dt} = \left(\frac{C_1}{C_1 + C_3}\right)\frac{dV_s}{dt}$$

Integrate with respect to time:

$$V(t) = \left(\frac{C_1}{C_1 + C_3}\right)V_s + V_o(t)$$

The leading coefficient of $V_s$ is one-half. The initial voltage $V_o(t)$ would be zero for steady-state operation. Therefore:

$$V(t) = 0.5V_s$$

Now, when the switch is activated, current has the ability to run through $C_2$ and to ground, as well as $C_1$ and $C_3$. Applying KCL at node a, and expressing the currents in terms of capacitance and time rate of change of voltage:

$$C_2 \frac{d(-V(t))}{dt} + C_1 \frac{d(V_s - V(t))}{dt} = C_3 \frac{dV(t)}{dt}$$

Solving for the output voltage:

$$\frac{dV(t)}{dt} = \left(\frac{C_1}{C_1 + C_2 + C_3}\right)\frac{dV_s}{dt}$$

Integrate with respect to time:

$$V(t) = \left(\frac{C_1}{C_1 + C_2 + C_3}\right)V_s + V_o(t)$$

The leading coefficient of $V_s$ is one-third. The initial voltage $V_o(t)$ would be zero for steady-state operation. Therefore:

$$V(t) = 0.\bar{3}V_s$$