

**MEMS 0051**  
**Spring 2020**  
**Midterm #1**  
**2/14/2020**

**Name (Print):** \_\_\_\_\_

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This exam contains 7 pages (including this cover page) and 2 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

Problem	Points	Score
1	50	
2	50	
Total:	100	

Bonus: This day, February 14<sup>th</sup>, 1876, marks the day Alexander Graham Bell filed a patent for what device?

**Written Problem #1**

1. (50 points) Two rigid tanks are connected by a ball valves. In one tank, there exists 200 [kg] of carbon dioxide at 100 °C and 425 [kPa]. The other tank, with a volume of 25 [m<sup>3</sup>], contains nitrogen at 125 °C and 125 [kPa]. The valve is then opened, and the gases are allowed to mix. If the final equilibrium temperature is 25 °C, determine:

- The final equilibrium pressure within the system;
- The total heat transferred during this process;
- Ensure the gases can be treated as Ideal gases only after items a) and b) have been completed. Failure to complete items a) and b) before item c) will result in a substantial loss of points.

a) The final pressure, denoted  $P_f$ , of the system must be the same for the carbon dioxide and nitrogen such that:

$$P_f = P_{\text{CO}_2} = P_{\text{N}_2} \quad (1)$$

The pressure of the carbon dioxide is expressed as:

$$P_{\text{CO}_2} = \frac{m_{\text{CO}_2} R_{\text{CO}_2} T_f}{V_{\text{CO}_2}} \quad (2)$$

The pressure of the nitrogen is expressed as:

$$P_{\text{N}_2} = \frac{m_{\text{N}_2} R_{\text{N}_2} T_f}{V_{\text{N}_2}} \quad (3)$$

These pressures must be equal to the final pressure (i.e. equations eqns. 1-3) such that:

$$P_f = \frac{m_f R_f T_f}{V} = \frac{m_{\text{CO}_2} R_{\text{CO}_2} T_f}{V_{\text{app,CO}_2}} = \frac{m_{\text{N}_2} R_{\text{N}_2} T_f}{V_{\text{app,N}_2}} \quad (4)$$

where  $m_f$  is the sum of the masses,  $R_f$  is the mass-weighted gas constant,  $T_f$  is the final temperature (given as 298.15 [K]),  $V$  is the final volume of the gas mixture, and the subscripts “app” indicates the apparent volume that gas would occupy given the final temperature. In an incremental fashion, we will piece together each of the needed variables. Starting with the tank containing the carbon dioxide, we will determine the volume using the Ideal gas law:

$$V_{\text{CO}_2} = \frac{m_{\text{CO}_2} R_{\text{CO}_2} T_{\text{init,CO}_2}}{P} = \frac{(200 \text{ [kg]})(0.1889 \text{ [kJ/kg-K]})(373.15 \text{ [K]})}{(425 \text{ [kPa]})} = 33.17 \text{ [m}^3] \quad (5)$$

Thus, the total volume is 58.17 [m<sup>3</sup>]. Next, we will determine the mass of the nitrogen using the Ideal gas law:

$$m_{\text{N}_2} = \frac{P V_{\text{N}_2}}{R_{\text{N}_2} T_{\text{init,N}_2}} = \frac{(125 \text{ [kPa]})(25 \text{ [m}^3])}{(0.2968 \text{ [kJ/kg-K]})(398.15 \text{ [K]})} = 26.44 \text{ [kg]} \quad (6)$$

Thus, the total mass is 226.44 [kg]. All that remains to solve eqn. 3 is the gas constant  $R_f$ .

Weighting the gas constant by mass:

$$R_f = \frac{m_{N_2} R_{N_2} + m_{CO_2} R_{CO_2}}{m_f} = \frac{(26.44 \text{ [kg]})(0.2968 \text{ [kJ/kg-K]}) + (200 \text{ [kg]})(0.1889 \text{ [kJ/kg-K]})}{(226.44 \text{ [kg]})} = 0.2015 \text{ [kJ/kg-K]} \quad (7)$$

Thus, the final pressure is calculated from the second term in eqn. 4 such that:

$$P_f = \frac{m_f R_f T_f}{V} = \frac{(226.44 \text{ [kg]})(0.2015 \text{ [kJ/kg-K]})(298.15 \text{ [K]})}{(58.17 \text{ [m}^3\text{]})} = 233.87 \text{ [kPa]} \quad (8)$$

To verify that the mass-averaging of the gas constant was indeed the correct approach, we can calculate the apparent volumes of the carbon dioxide and nitrogen at the final state and ensure they sum to the total volume of the system (i.e. the third and fourth terms in eqn. 4):

$$V_{\text{app},CO_2} = \frac{m_{CO_2} R_{CO_2} T_f}{P_f} = \frac{(200 \text{ [kg]})(0.1889 \text{ [kJ/kg-K]})(298.15 \text{ [K]})}{(233.87 \text{ [kPa]})} = 48.16 \text{ [m}^3\text{]} \quad (9)$$

$$V_{\text{app},N_2} = \frac{m_{N_2} R_{N_2} T_f}{P_f} = \frac{(26.44 \text{ [kg]})(0.2968 \text{ [kJ/kg-K]})(298.15 \text{ [K]})}{(233.87 \text{ [kPa]})} = 10.01 \text{ [m}^3\text{]} \quad (10)$$

These clearly sum to 58.17 [m<sup>3</sup>], thus the mass-weighting of the gas constant was correct. It is noted that if you do not mass-weight the gas constant, there are three approaches you can take to solving for  $P_f$  in eqn. 4. The first would be to guess values for both  $V_{\text{app},CO_2}$  and  $V_{\text{app},N_2}$  until the third and fourth terms in eqn. 4 are equal. This is not recommended. The second is to recognize that  $V_{\text{app},CO_2}$  and  $V_{\text{app},N_2}$  must sum to  $V_f$ , and thus you have provided a second equation (two equations, two unknowns). Thus, you can solve for either apparent volume and obtain the final pressure. The third is to use the concept of partial pressures. In eqn. 4, you can replace the apparent volumes with the final volume and sum the third and fourth terms.

b) To determine the heat transferred during this process, we have to apply the conservation of energy, recognizing there is no work done onto or by the system. Thus, the conservation of energy equation is formulated as:

$$Q_{1 \rightarrow 2} = U_2 - U_1 = m_{CO_2}(u_{CO_2,2} - u_{CO_2,1}) + m_{N_2}(u_{N_2,2} - u_{N_2,1}) \approx m_{CO_2} C_{V0,CO_2}(T_f - \text{init},CO_2) + m_{N_2} C_{V0,N_2}(T_f - \text{init},N_2) \quad (11)$$

The specific internal energies for carbon dioxide and nitrogen at the final (designated with a subscript 2) and initial (designated with a subscript 1) can be found from Table A.8. For nitrogen, using Table A.8 on page 764, we have to interpolate between temperatures for our initial and final internal energies:

$$\frac{(398.15 - 350) \text{ [K]}}{(400 - 350) \text{ [K]}} = \frac{(u_{N_2,1} - 259.80) \text{ [kJ/kg]}}{(297.09 - 259.80) \text{ [kJ/kg]}} \implies u_{N_2,1} = 295.71 \text{ [kJ/kg]} \quad (12)$$

$$\frac{(298.15 - 250) \text{ [K]}}{(300 - 250) \text{ [K]}} = \frac{(u_{N_2,2} - 185.50) \text{ [kJ/kg]}}{(222.63 - 185.50) \text{ [kJ/kg]}} \implies u_{N_2,2} = 221.26 \text{ [kJ/kg]} \quad (13)$$

For carbon dioxide, the initial and final internal energies are found through interpolation of

Table A.8 on page 765:

$$\frac{(373.15 - 350) \text{ [K]}}{(400 - 350) \text{ [K]}} = \frac{(u_{\text{CO}_2,1} - 191.78) \text{ [kJ/kg]}}{(228.19 - 191.78) \text{ [kJ/kg]}} \Rightarrow u_{\text{CO}_2,1} = 208.64 \text{ [kJ/kg]} \quad (14)$$

$$\frac{(298.15 - 250) \text{ [K]}}{(300 - 250) \text{ [K]}} = \frac{(u_{\text{CO}_2,2} - 126.21) \text{ [kJ/kg]}}{(157.70 - 126.21) \text{ [kJ/kg]}} \Rightarrow u_{\text{CO}_2,2} = 156.53 \text{ [kJ/kg]} \quad (15)$$

Thus, substituting the values from eqns. 12-15 into eqn. 11:

$$\begin{aligned} Q_{1 \rightarrow 2} &= (200 \text{ [kg]})(156.53 - 208.64) \text{ [kJ/kg]} + (26.44 \text{ [kg]})(221.26 - 295.71) \text{ [kJ/kg]} \\ &= -12.39 \cdot 10^3 \text{ [kJ]} \end{aligned} \quad (16)$$

Now, if you used the specific heat values provided in Table A.5, the fourth term of eqn. 11 can be evaluated as:

$$\begin{aligned} Q_{1 \rightarrow 2} &= m_{\text{CO}_2} C_{v0,\text{CO}_2} (T_f - T_{\text{init},\text{CO}_2}) + m_{\text{N}_2} C_{v0,\text{N}_2} (T_f - T_{\text{init},\text{N}_2}) \\ &= (200 \text{ [kg]})(0.653 \text{ [kJ/kg-K]})(298.15 - 373.15) + (26.44 \text{ [kg]})(0.745 \text{ [kJ/kg-K]})(298.15 - 398.15) \\ &= -11.77 \cdot 10^3 \text{ [kJ]} \end{aligned} \quad (17)$$

Thus, there is only a 5.17% difference between the solutions through using the specific internal energies or constant-volume specific heats to calculate the heat rejected.

c) To determine whether or not the gases can be treated as Ideal gases, we evaluate the reduced pressure and temperature of each. Using the maximum pressure and minimum temperature of each, as well as the critical constants found in Table A.2 on page 758, we have the following:

$$P_{r,\text{CO}_2} = \frac{P_{\text{max},\text{CO}_2}}{P_c} = \frac{425 \text{ [kPa]}}{7,380 \text{ [kPa]}} = 0.06 \ll 1 \quad (18)$$

$$P_{r,\text{N}_2} = \frac{P_{\text{max},\text{N}_2}}{P_c} = \frac{233.87 \text{ [kPa]}}{3,390 \text{ [kPa]}} = 0.07 \ll 1 \quad (19)$$

$$T_{r,\text{CO}_2} = \frac{T_{\text{min},\text{CO}_2}}{T_c} = \frac{298.15 \text{ [K]}}{304.1 \text{ [K]}} = 0.98 \not> 2 \quad (20)$$

$$T_{r,\text{N}_2} = \frac{T_{\text{min},\text{N}_2}}{T_c} = \frac{298.15 \text{ [K]}}{126.2 \text{ [K]}} = 2.36 > 2 \quad (21)$$

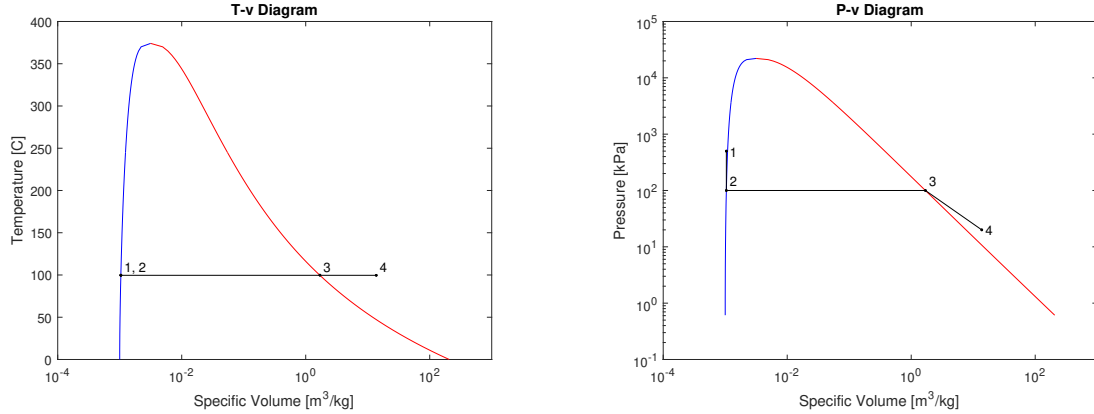
The reduced temperature for carbon dioxide is met.

## Written Problem #2

- (50 points) 1 [kg] of *ammonia* in a piston-cylinder device goes from an initial state of 49.37 °C and 3,000 [kPa] to a final state of 49.37 °C and 1,723 [kPa] in an isothermal process. Determine the amount of heat supplied to and the work done by the system. *Hint: this process is not purely isothermal.*

Looking at the  $T$ - $\nu$  and  $P$ - $\nu$  diagrams below (note these are a representation of the diagrams and do not reflect actual values, for I do not want to construct said diagrams for ammonia and

used those of water instead), we recognize we start in the subcooled liquid region and progress to the saturated liquid line of the vapor dome in a constant temperature process. Once we reach the saturated liquid line, we progress to the saturated vapor line in a constant temperature and constant pressure process. Then, we progress to the superheated vapor region in a constant temperature process. Our pressure decreases from State 1 to 2, and from 3 to 4.



We can construct and populate our table of states as follows:

<u>State 1:</u>	<u><math>T = \zeta</math></u>	<u>State 2:</u>	<u><math>T = \zeta</math></u>	<u>State 3:</u>	<u><math>T = \zeta</math></u>	<u>State 4:</u>
$T_1 = 49.37 \text{ }^\circ\text{C}$		$T_2 = T_1$	<u><math>P = \zeta</math></u>	$T_3 = T_1$		$T_4 = T_1$
$P_1 = 3,000 \text{ [kPa]}$		$P_2 = P_{sat}(T_2)$		$P_3 = P_{sat}(T_3)$		$P_4 = 1,723 \text{ [kPa]}$
$\nu_1 \approx \nu_f(T_1)$		$\nu_2 = \nu_f(T_2, P_2)$		$\nu_3 = \nu_g(T_3, P_3)$		
$u_1 \approx u_f(T_1)$		$u_2 = u_f(T_2, P_2)$		$u_3 = u_g(T_3, P_3)$		

To determine the amount of heat supplied, as well as the done by the system, we have to evaluate the conservation of energy equation. Thus, the only properties we need to solve for will be the specific volume at States 1, 3 and 4, as well as the specific internal energies at States 1, 3 and 4. Solving for the specific volume at State 1 by assuming  $\nu_1 \approx \nu_f(49.37^\circ\text{C})$ , for the specific volume is not strongly a function of pressure:

$$\frac{(49.37 - 45)[^\circ\text{C}]}{(50 - 45)[^\circ\text{C}]} = \frac{(\nu_1 - 0.001750) [\text{m}^3/\text{kg}]}{(0.001777 - 0.001750) [\text{m}^3/\text{kg}]} \Rightarrow \nu_1 = 0.001767 [\text{m}^3/\text{kg}] \quad (22)$$

Solving for the specific internal energy at State 1 by assuming  $u_1 \approx u_f(49.37^\circ\text{C})$ , for the specific internal energy is more strongly dependent on temperature than pressure, we are left with:

$$\frac{(49.37 - 45)[^\circ\text{C}]}{(50 - 45)[^\circ\text{C}]} = \frac{(u_1 - 393.19) [\text{kJ/kg-K}]}{(417.87 - 393.13) [\text{kJ/kg-K}]} \Rightarrow u_1 = 414.76 [\text{kJ/kg}] \quad (23)$$

The specific volume at State 3, which is the same as the saturated vapor specific volume at  $49.37^\circ\text{C}$ , is found via interpolation such that:

$$\frac{(49.37 - 45)[^\circ\text{C}]}{(50 - 45)[^\circ\text{C}]} = \frac{(\nu_3 - 0.07248) [\text{m}^3/\text{kg}]}{(0.06337 - 0.07248) [\text{m}^3/\text{kg}]} \Rightarrow \nu_3 = 0.064518 [\text{m}^3/\text{kg}] \quad (24)$$

Solving for the specific internal energy at State 3, which is the same as the saturated vapor specific internal energy at 49.37 °C, is found via interpolation such that:

$$\frac{(49.37 - 45)[^{\circ}\text{C}]}{(50 - 45)[^{\circ}\text{C}]} = \frac{(u_3 - 1,342.1) [\text{kJ/kg-K}]}{(1,342.7 - 1,342.1) [\text{kJ/kg-K}]} \implies u_3 = 1,342.62 [\text{kJ/kg}] \quad (25)$$

To determine the necessary quantities at State 4, we have to construct a 1,723 [kPa] pressure entry table, with said properties evaluated at 49.37 °C. It is noted we first have to construct a 49.37 °C entry for the 1,600 [kPa] pressure entry sub-table first for both specific volume and specific internal energy. In doing such:

$$\frac{(49.37 - 41.03)[^{\circ}\text{C}]}{(50 - 41.03)[^{\circ}\text{C}]} = \frac{(\nu_{1,600 [\text{kPa}]} - 0.08079) [\text{m}^3/\text{kg}]}{(0.08506 - 0.08079) [\text{m}^3/\text{kg}]} \implies \nu_{1,600 [\text{kPa}]} = 0.084760 [\text{m}^3/\text{kg}] \quad (26)$$

$$\frac{(49.37 - 41.03)[^{\circ}\text{C}]}{(50 - 41.03)[^{\circ}\text{C}]} = \frac{(u_{1,600 [\text{kPa}]} - 1,342.2) [\text{kJ/kg-K}]}{(1,364.9 - 1,342.2) [\text{kJ/kg-K}]} \implies u_{1,600 [\text{kPa}]} = 1,363.24 [\text{kJ/kg}] \quad (27)$$

Thus, our modified 1,600 [kPa] pressure entry table would like:

$T$ °C	$P=1,600$ [kPa]	
	$\nu$ [m <sup>3</sup> /kg]	$u$ [kJ/kg]
41.03	0.08079	1,341.2
49.37	0.084760	1,363.24
50	0.08506	1,364.9

Now we can interpolate between pressures for specific volume and specific internal energy at a temperature of 49.37 °C:

$$\frac{(1,723 - 1,600) [\text{kPa}]}{(2,000 - 1,600) [\text{kPa}]} = \frac{(\nu_{1,723 [\text{kPa}]} - 0.084760) [\text{m}^3/\text{kg}]}{(0.06444 - 0.084760) [\text{m}^3/\text{kg}]} \implies \nu_{1,723 [\text{kPa}]} = 0.078512 [\text{m}^3/\text{kg}] \quad (28)$$

$$\frac{(1,723 - 1,600) [\text{kPa}]}{(2,000 - 1,600) [\text{kPa}]} = \frac{(u_{1,723 [\text{kPa}]} - 1,363.24) [\text{kJ/kg-K}]}{(1,342.6 - 1,363.24) [\text{kJ/kg-K}]} \implies u_{1,723 [\text{kPa}]} = 1,356.89 [\text{kJ/kg}] \quad (29)$$

Now we can now evaluate the conservation of energy equation. We will apply the equation between each state, then sum the total response. From State 1 to 2:

$$m(u_2 - u_1) = Q_{1 \rightarrow 2} + W_{1 \rightarrow 2} \quad (30)$$

The formulation for work for an isothermal process is:

$$W_{1 \rightarrow 2} = P_1 \forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (2,000 [\text{kPa}])(0.001767 [\text{m}^3]) \ln\left(\frac{0.001767 [\text{m}^3]}{0.001767 [\text{m}^3]}\right) = 0 [\text{kJ}] \quad (31)$$

The heat input from State 1 to 2 is zero, for the specific internal energy at States 1 and 2 are taken as the saturated liquid specific internal energy evaluated at 49.37 °C (i.e.  $u_1 = u_2$ ):

$$Q_{1 \rightarrow 2} = 0 \text{ [kJ]} \quad (32)$$

Evaluating the conservation of energy between States 2 and 3:

$$m(u_3 - u_2) = Q_{2 \rightarrow 3} - W_{2 \rightarrow 3} \quad (33)$$

The work from States 2 to 3 is treated as isobaric work. Thus we need the pressure at State 2 (which is the same as State 3), which is found via interpolation:

$$\frac{(49.37 - 41.03)[^\circ\text{C}]}{(50 - 41.03)[^\circ\text{C}]} = \frac{(P_2 - 1,782.0) \text{ [kPa]}}{(2,033.1 - 1,782.0) \text{ [kPa]}} \implies P_2 = 2,001.46 \text{ [kPa]} \quad (34)$$

Evaluating the work from State 2 to 3:

$$W_{2 \rightarrow 3} = P_2 \int_2^3 dV = (2,001.46 \text{ [kPa]})(1 \text{ [kg]})(0.064518 - 0.001767) \text{ [m}^3\text{/kg]} = 125.59 \text{ [kJ]} \quad (35)$$

The heat from States 2 to 3 is then:

$$Q_{2 \rightarrow 3} = m(u_3 - u_2) + W_{2 \rightarrow 3} = (1 \text{ [kg]})(1,342.62 - 414.76) \text{ [kJ/kg]} + 125.59 \text{ [kJ]} = 1,053.46 \text{ [kJ]} \quad (36)$$

Lastly, applying the conservation of energy equation between States 3 and 4:

$$m(u_4 - u_3) = Q_{3 \rightarrow 4} - W_{3 \rightarrow 4} \quad (37)$$

The formulation for work for an isothermal process is:

$$W_{3 \rightarrow 4} = P_3 V_3 \ln\left(\frac{V_4}{V_3}\right) = (2,001.46 \text{ [kPa]})(0.064518 \text{ [m}^3\text{]}) \ln\left(\frac{0.078512 \text{ [m}^3\text{]}}{0.064518 \text{ [m}^3\text{]}}\right) = 25.35 \text{ [kJ]} \quad (38)$$

The heat from States 3 to 4 is then:

$$Q_{3 \rightarrow 4} = m(u_4 - u_3) + W_{3 \rightarrow 4} = (1 \text{ [kg]})(1,356.89 - 1,342.62) \text{ [kJ/kg]} + 25.35 \text{ [kJ]} = 39.61 \text{ [kJ]} \quad (39)$$

Now, the heat supplied and work done between States 1 and 4 is the sum of the individual processed:

$$Q_{1 \rightarrow 4} = 1,093.07 \text{ [kJ]}; \quad W_{1 \rightarrow 4} = 150.94 \text{ [kJ]} \quad (40)$$