Homework #9

MEMS 0051 - Introduction to Thermodynamics

Assigned April 11$^{\text{th}}$, 2020
Due April 18$^{\text{th}}$, 2020

Problem #1

Steam enters a turbine at 6,000 [kPa], 400 °C, 10 [m/s] at a rate of 4,600 [kg/hr]. The turbine produces 1,000 [kW] of mechanical power. The fluid then exits the turbine at 10 [kPa] at 50 [m/s] with a quality of 90%. Calculate the rate of heat transfer between the turbine and surrounding in [kW].
Applying the steady-state C.o.E. equation:

$$0 = \dot{Q} - 1,000\,[\text{kW}] + \left(\frac{4,600\,[\text{kg/hr}]}{3,600\,[\text{s/hr}]}\right)\left(h_1 - h_2 + \frac{\{(10\,[\text{m/s}])^2 - (50\,[\text{m/s}])^2\}\,[\text{kJ/kg}]}{2,000\,[\text{m}^2/\text{s}^2]}\right)$$

The enthalpy at State 1 is found via the temperature and pressure (using EES) to be 3,177.0 [kJ/kg]. The enthalpy at State 2 is found via the pressure and quality (using EES) to be 2,344.6 [kJ/kg]. Thus, the heat is found as:

$$\dot{Q} = 1,000\,[\text{kW}] + \left(\frac{4,600\,[\text{kg/hr}]}{3,600\,[\text{s/hr}]}\right)\left((2,344.6 - 3,177.0)\,[\text{kJ/kg}] - \frac{\{(10\,[\text{m/s}])^2 - (50\,[\text{m/s}])^2\}\,[\text{kJ/kg}]}{2,000\,[\text{m}^2/\text{s}^2]}\right)$$

$$\therefore \dot{Q} = -62.1\,[\text{kW}]$$

Problem #2

An air compressor draws air at 100 [kPa] and 290 [K] through a 0.1 [m^2] opening at a velocity of 6 [m/s]. The air then exits the compressor at 700 [kPa] and a temperature of 450 [K] with a velocity of 2 [m/s]. The compressor rejects heat to the surroundings at a rate of 180 [kJ/min]. Calculate the necessary power input, in [kW].
Applying the steady-state C.o.E equation:

$$0 = -\frac{180\,[\text{kJ/min}]}{60\,[\text{s/min}]} - \dot{W} + \dot{m}\left(h_1 - h_2 + \frac{\{(6\,[\text{m/s}])^2 - (2\,[\text{m/s}])^2\}\,[\text{kJ/kg}]}{2,000\,[\text{m}^2/\text{s}^2]}\right)$$

The enthalpies at States 1 and 2 are found via temperature and pressure (using EES) to be 290.3 [kJ/kg] and 451.8 [kJ/kg], respectively. The mass flow rate is found as:

$$\dot{m} = \frac{AV}{\nu} = \frac{(0.1\,[\text{m}^2])(6\,[\text{m/s}])}{0.8325\,[\text{m}^2/\text{kg}]} = 0.72\,[\text{kg/s}]$$

Therefore, the work into the system is:

$$\dot{W} = -\frac{180\,[\text{kJ/min}]}{60\,[\text{s/min}]} + (0.72\,[\text{kg/s}])\left((290.3 - 451.8)\,[\text{kJ/kg}] + \frac{\{(6\,[\text{m/s}])^2 - (2\,[\text{m/s}])^2\}\,[\text{kJ/kg}]}{2,000\,[\text{m}^2/\text{s}^2]}\right)$$

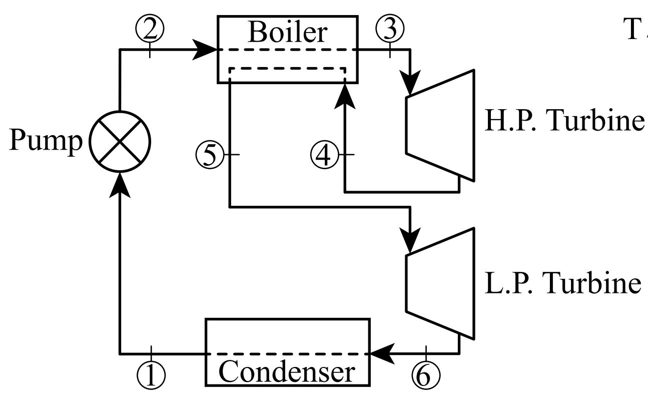
$$\therefore \dot{W} = -119.3\,[\text{kW}]$$

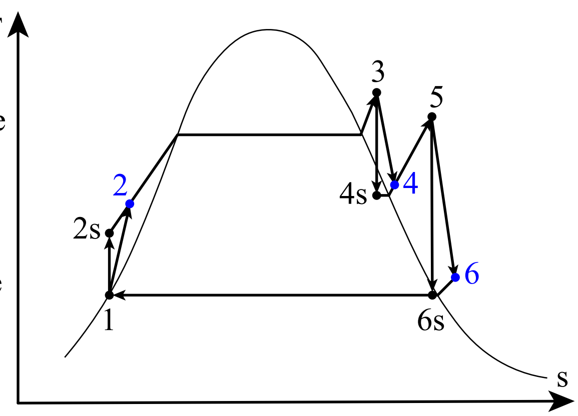
Problem #3

Steam enters the turbine at 10 [MPa] and 500 °C and expands to 500 [kPa]. It is then reheated to 450 °C before entering a second turbine, where it expands to 15 [kPa]. Saturated liquid exits the condenser at 15 [kPa]. The net power produced is 1,000 [MW]. If the efficiency of the turbines is 85%, and that of the pump is 95%, determine:

a) The thermal efficiency (net work per heat input)
b) The backwork ratio (work of the pump per turbine)
c) The mass flow rate of steam
d) The rate of heat supplied to the boiler
e) The rate of heat rejected from the condenser

State 1 will be the inlet to the pump. State 2s will be the exit of the pump, assuming an isentropic process between States 1 and 2s. State 2 will be the actual outlet of the pump, considering the isentropic efficiency of the pump. State 3 will be the inlet to the high-pressure turbine. State 4s will the be outlet of the high-pressure turbine, assuming an isentropic process between States 3 and 4s. State 4 will be the actual outlet of the high-pressure turbine, considering the isentropic efficiency of the turbine. State 5 will be the inlet to the low-pressure turbine. State 6s will be the outlet of the low pressure turbine, assuming an isentropic process between State 5 and 6s. State 6 will be the actual outlet of the low-pressure turbine, considering the isentropic efficiency of the turbine. This is shown in the figure below, both schematically and as a $T-s$ diagram. Note: images not drawn to scale. EES is used to solve all properties, which populate the property table.





State 1:	State 2s:	State 2:	State 3:
P_1=15 [kPa]	P_{2s}=10 [MPa]	P_2=P_{2s}	P_3=10 [MPa]
T_1=54 °C	T_{2s}=54.4 °C	T_2=54.5 °C	T_3=500 °C
x_1=0	x_{2s}=undef.	x_2=undef.	x_3=undef.
h_1=226.0 [kJ/kg]	h_{2s}=236.1 [kJ/kg]	h_2=236.63 [kJ/kg]	h_3=3,374 [kJ/kg]
s_1=0.755 [kJ/kg-K]	s_{2s}=s_1	s_2=0.757 [kJ/kg-K]	s_3=6.597 [kJ/kg-K]
State 4s:	State 4:	State 5:	State 6s:
P_{4s}=500 [kPa]	P_4=P_{4s}	P_5=P_4	P_{6s}=P_1
T_{4s}=151.9 °C	T_4=157.3 °C	T_5=450 °C	T_{6s}=54.0 °C
x_{4s}=0.95	x_4=undef.	x_5=undef.	x_{6s}=0.99
h_{4s}=2,653.2 [kJ/kg]	h_4=2,761.3 [kJ/kg]	h_5=3,377.2 [kJ/kg]	h_{6s}=2,578.1 [kJ/kg]
s_{4s}=s_3 [kJ/kg-K]	s_4=6.850 [kJ/kg-K]	s_5=7.945 [kJ/kg-K]	s_{6s}=s_5

State 6:
P_6=P_1
T_6=106.1 °C
x_6=undef.
h_6=2,698 [kJ/kg]
s_6=8.290 [kJ/kg-K]

Solving for the enthalpy and entropy at State 1 given a pressure and quality, we can determine the enthalpy at State 2s, since $s_{2s} = s_1$. Then, the enthalpy at State 2 can be found via the isentropic efficiency of a pump such that:

$$\eta_{s,\text{pump}} = \frac{W_s}{W_a} = \frac{h_{2s} - h_1}{h_2 - h_1} = 0.95 \implies h_2 = \frac{h_{2s} - h_1}{0.95} + h_1 = \frac{(236.1 - 226.0)\,[\text{kJ/kg}]}{0.95} + 226.0\,[\text{kJ/kg}]$$

$$\therefore h_2 = 236.63\,[\text{kJ/kg}]$$

Since the pressure and temperature at State 3 are given, the enthalpy and entropy can be directly found. The enthalpy at State 4s is found via pressure and entropy, since $s_{4s} = s_3$. Then, the enthalpy at State 4 can be found via the isentropic efficiency of a turbine such that:

$$\eta_{s,\text{turbine}} = \frac{W_a}{W_s} = \frac{h_3 - h_4}{h_3 - h_{4s}} = 0.85 \implies h_4 = h_3 - 0.85(h_3 - h_{4s})$$

$$\therefore h_4 = 2,761.3\,[\text{kJ/kg}]$$

Solving for the enthalpy and entropy at State 5 given a pressure and temperature, we can determine the enthalpy at State 6s, since $s_{6s} = s_5$. Then, the enthalpy at State 6 can be found via the isentropic efficiency of a turbine such that:

$$\eta_{s,\text{turbine}} = \frac{W_a}{W_s} = \frac{h_5 - h_6}{h_5 - h_{6s}} = 0.85 \implies h_6 = h_5 - 0.85(h_5 - h_{6s})$$

$$\therefore h_6 = 2,698.0\,[\text{kJ/kg}]$$

a) The thermal efficiency is the net work per heat input:

$$\eta = \frac{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)}{(h_3 - h_2) + (h_5 - h_4))} = 34.14\%$$

b) The back-work ratio, or pump work per turbine work, is:

$$BWR = \frac{h_2 - h_1}{(h_3 - h_4) + (h_5 - h_6)} = 8.22\text{E} - 3$$

c) The mass flow rate of the steam is found as the stated net work per net work in terms of enthalpies:

$$\dot{m} = \frac{1,000\,[\text{MW}]}{(h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)} = 780\,[\text{kg/s}]$$

d) The rate of heat supplied to the boiler is the mass flow rate times the change of enthalpies between States 3 and 2 and States 5 and 4:

$$\dot{Q}_h = \dot{m}((h_3 - h_2) + (h_5 - h_4)) = 2.93\,[\text{MW}]$$

e) The rate of heat rejected from the condenser is the mass flow rate times the change of enthalpies between States 6 and 1:

$$\dot{Q}_l = \dot{m}(h_6 - h_1) = 1.93\,[\text{MW}]$$

Problem #4

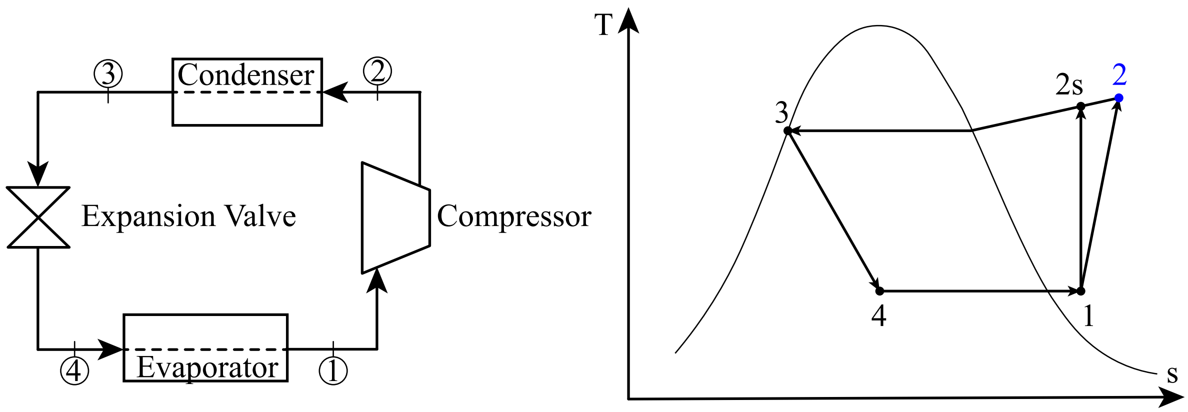
Refrigerant R-134a is the working fluid for a refrigeration cycle. The mass flow rate of the refrigerant is 1 [kg/s]. The compressor has an isentropic efficiency of 95%, and valve operate adiabatically, and can be modeled as a throttle device. The following state data are known.

State 1:	State 2:	State 3:	State 4:
P_1=150 [kPa]	P_2=500 [kPa]	x_3=0	T_4=-10 °C
T_1=-10 °C			x_4=0.5

Determine:

a) The coefficient of performance of this cycle.
b) The rate of entropy generation through the expansion valve (i.e. throttle).

State 1 is the inlet of the compressor. State 2s is the outlet of the compressor, and inlet of the condenser, assuming an isentropic process between States 1 and 2s. State 2 is the actual outlet of the compressor, considering the isentropic efficiency of the compressor. State 3 is the outlet of the condenser, and State 4 is the inlet to evaporator. This is shown in the figure below, both schematically and as a $T-s$ diagram. Note: images not drawn to scale. EES is used to solve all properties, which populate the property table.



State 1:	State 2s:	State 2:	State 3:
P_1=150 [kPa]	P_{2s}=500 [kPa]	P_2=P_{2s}	P_3=P_2
T_1=-10 °C	T_{2s}=28.6 °C	T_2=30.1 °C	T_3=15.7 °C
x_1=undef.	x_{2s}=undef.	x_2=undef.	x_3=0
h_1=246.1 [kJ/kg]	h_{2s}=271.7 [kJ/kg]	h_2=273.05 [kJ/kg]	h_3=73.3 [kJ/kg]
s_1=0.966 [kJ/kg-K]	s_{2s}=s_1	s_2=0.971 [kJ/kg-K]	s_3=0.280 [kJ/kg-K]

State 4:
P_4=P_1 T_4=-10 °C
x_4=0.5
h_4=141.5 [kJ/kg]
s_4=0.546 [kJ/kg-K]

The enthalpy and entropy at State 1 is found via the temperature and pressure. The enthalpy at State 2s is found since $s_{2s} = s_1$. Then, the enthalpy at State 2 is found via the isentropic efficiency of the compressor such that:

$$\eta_{s,\text{compressor}} = \frac{W_s}{W_a} = \frac{h_{2s}-h_1}{h_2-h_1} = 0.95 \implies h_2 = \frac{h_{2s}-h_1}{0.95} + h_1 = \frac{(271.7-246.1)\,[\text{kJ/kg}]}{0.95} + 246.1\,[\text{kJ/kg}]$$

$$\therefore h_2 = 273.05\,[\text{kJ/kg}]$$

The enthalpy and entropy at State 3 are found via quality and pressure. The enthalpy and entropy at State 4 are found via temperature and quality. Note: the enthalpy at State 4 is not equal to that at State 3, for the density is not constant between these two states.

a) The coefficient of performance is found as the heat removed from the refrigerated space per the work input:

$$\beta = \frac{Q_l}{W} = \frac{h_1-h_4}{h_2-h_1} = 3.88$$

b) The rate of entropy generated through the expansion valve is found be employing the Entropy Rate Balance equation. The system is steady-state, and the expansion valve is perfectly insulated (i.e. adiabatic expansion). Thus, the E.R.B. becomes:

$$\cancelto{0}{\frac{dS_{C.\forall.}}{dt}} = \dot{m}(s_3 - s_4) + \cancelto{0}{\frac{\delta Q}{T}} + \dot{\sigma} \implies \dot{\sigma} = \dot{m}(s_4 - s_3) = (1\,[\text{kg/s}])(0.546 - 0.280)\,[\text{kJ/kg-K}]$$

$$\therefore \dot{\sigma} = 0.266\,[\text{kW/K}]$$