Homework #3

MEMS 0051 - Introduction to Thermodynamics

Assigned January 26th, 2018
Due: February 2nd, 2018

Problem #1

Please refer to the Saturated Water tables.

a) Determine the quality of water at the following states:

i.) T=115 °C, ν=0.4 [m^3/kg]

$$x = \frac{\nu - \nu_f}{\nu_g - \nu_f} = \frac{(0.4 - 0.001056)\ [\text{m}^3/\text{kg}]}{(1.03658 - 0.001056)\ [\text{m}^3/\text{kg}]} = 0.385$$

ii.) P=250 [kPa], ν=0.25 [m^3/kg]

$$x = \frac{\nu - \nu_f}{\nu_g - \nu_f} = \frac{(0.5 - 0.001067)\ [\text{m}^3/\text{kg}]}{(0.71871 - 0.001067)\ [\text{m}^3/\text{kg}]} = 0.695$$

b) Determine the specific volume of water at the following states:

i.) P=100 [kPa], x=0.7

$$\nu = \nu_f + x(\nu_g - \nu_f) = 0.001403\ [\text{m}^3/\text{kg}] + (0.7)(1.69400 - 0.001043)\ [\text{m}^3/\text{kg}] = 1.186115\,[\text{m}^3/\text{kg}]$$

ii.) T=90 °C, x=0.4

$$\nu = \nu_f + x(\nu_g - \nu_f) = 0.001036\ [\text{m}^3/\text{kg}] + (0.4)(2.36056 - 0.001036)\ [\text{m}^3/\text{kg}] = 0.94485\,[\text{m}^3/\text{kg}]$$

Problem #2

a) 100 [kg] of CO_2 is contained in a 2 [m^3] vessel at 25 °C.

i.) Calculate the gas constant, R, for CO_2 based on its molecular mass listed in Table A.2.

$$R = \frac{\bar{R}}{M} = \frac{8.3145\,[\text{kJ/kmol-K}]}{44.01\,[\text{kg/kmol}]} = 0.1889\,[\text{kJ/(kg-K)}]$$

ii.) Determine how many moles, n, of CO_2 are in the vessel.

$$n = \frac{m}{M} = \frac{100\,[\text{kg}]}{44.01\,[\text{kg/kmol}]} = 2.272\,[\text{kmol}]$$

iii.) What is the pressure of CO_2 in the vessel?

$$P\forall = mRT \implies P = \frac{mRT}{\forall} = \frac{(100\,[\text{kg}])(0.1889\,[\text{kJ/kg-K}])(298.15\,[\text{K}])}{2\,[\text{m}^3]} = 2816.0\,[\text{kPa}]$$

b) Heat is now added to the vessel until it reaches a temperature of 200 °C.

i.) Is the specific volume of CO_2 constant during this process? Why or why not?
Yes, because the specific volume is simply the ratio of volume to mass, and both of those properties are constant because it's a closed system in a rigid vessel.

ii.) What is the final pressure in the vessel?
Volume, the gas constant, and mass are constant for this process, so we can develop the following relationship from the remaining variables in the Ideal Gas Law:

$$\frac{P_2}{T_2} = \frac{P_1}{T_1} \implies P_2 = P_1\left(\frac{T_2}{T_1}\right) = 2816.0\,[\text{kPa}]\left(\frac{473.15\,[\text{K}]}{298.15\,[\text{K}]}\right) = 4468.9\,[\text{kPa}]$$

iii.) Calculate the reduced pressure, P_r, of CO_2 after being heated.

$$P_r = \frac{P_2}{P_c} = \frac{4468.9\,[\text{kPa}]}{7,380\,[\text{kPa}]} = 0.606$$

Note that you are asked to use the pressure after heating because it is higher than the initial pressure, and is therefore closer to the critical pressure.

iv.) Calculate the reduced temperature, T_r, of CO_2 before being heated.

$$T_r = \frac{T}{T_c} = \frac{298.15\,[\text{K}]}{304.1\,[\text{K}]} = 0.9799$$

Note that you are asked to use the temperature before heating because it is lower than the final temperature, and is therefore closer to the critical temperature.

v.) Can we assume that CO_2 behaved like an ideal gas throughout this process? Why or why not?
We cannot assume ideal gas behavior for this process because it does not meet either of the following criteria: $P_r \ll 1$ and $T_r > 2$.

Problem #3

a) Consider 1 [kg] of saturated water vapor contained in a piston-cylinder apparatus. The vapor temperature is 100 °C.

i.) What is the total internal energy, U, of the water vapor at this state?

$$U_1 = mu_g = (1\,[\text{kg}])(2,506.5\,[\text{kJ/kg}]) = 2,506.5\,[\text{kJ}]$$

ii.) What is the pressure of water vapor at this state?

$$P_1 = P_{\text{sat}}(100°\text{C}) = 101.3\,[\text{kPa}]$$

iii.) What volume is occupied by the water vapor? (*Hint: look up the specific volume*)

$$\forall_1 = m\nu_g = (1\,[\text{kg}])(1.6729\,[\text{m}^3/\text{kg}]) = 1.6729\,[\text{m}^3]$$

b) An external force now pushes down on the piston, compressing the vapor isobarically until it reaches a final volume of 1.2 [m^3].

i.) What is the specific volume of the saturated water mixture now?

$$\nu_2 = \frac{\forall_2}{m} = \frac{1.2\,[\text{m}^3]}{1\,\text{kg}]} = 1.2\,[\text{m}^3/\text{kg}]$$

ii.) What is the quality of the saturated water mixture?

$$x = \frac{\nu_2 - \nu_f}{\nu_g - \nu_f} = \frac{(1.2 - 0.001044)\ [\text{m}^3/\text{kg}]}{(1.67290 - 0.001044)\ [\text{m}^3/\text{kg}]} = 0.717$$

iii.) What is the total internal energy of both phases (U_f+U_g) in this final state?

$$U_2 = U_f + U_g = m_f u_f + m_g u_g = (1-x)mu_f + xmu_g = ...$$
$$... = (1 - 0.717)(1\,[\text{kg}])(418.91\,[\text{kJ/kg}]) + 0.717(1\,[\text{kg}])(2,506.5\,[\text{kJ/kg}]) = 1,916\,[\text{kJ}]$$

iv.) How much work was **done by** the piston in this process? (Note: W=P($\forall_2$-$\forall_1$) for constant pressure processes).

$$W_{1\to 2} = P(\forall_2 - \forall_1) = (101.3\,[\text{kPa}])(1.2 - 1.6729\,[\text{m}^3]) = -47.9\,[\text{kJ}]$$

Sign check: we get a negative value for work done by the piston, which makes sense because compression work was done on the piston.

v.) How much heat was **transferred into** the saturated water during this process?

$$Q_{1\to 2} = U_2 - U_1 + W_{1\to 2} = (1,916 - 2,506.5\,[\text{kJ}]) - 47.9\,[\text{kJ}] = -638.4\,[\text{kJ}]$$

Sign check: we get a negative value for heat transferred into the system, which means that heat is actually transferred out of the system during the compression process.