Chapter 2 - Properties of a Pure Substance

Lecture 6
Sections 2.8-2.9

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Chapter 2 -
Properties of a Pure
Substance

MEMS 0051

Learning Objectives

2.8 The Ideal Gas
States

2.9 The
Compressibility
Factor

Summary

Student Learning Objectives

At the end of the lecture, students should be able to:

- ▶ Identify an Ideal Gas and apply the Ideal Gas law

- ▶ Determine when the Ideal Gas Law is applicable based upon reduced temperature and pressure

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Ideal Gas State

▶ Our discussion of an ideal gas begins with valence bond theory - tntermolecular potential energy, $E(r)$, exists between molecules - Mie Model

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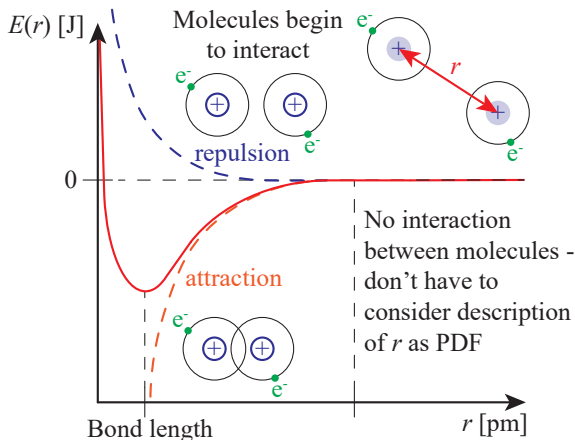
Ideal Gas State

▶ There exist situations where $E(r)$ is at a global minimum and zero:

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Ideal Gas State
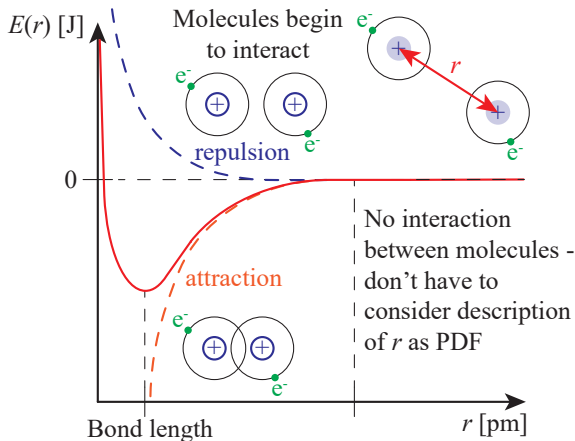
▶ At low to medium densities, r is very large and $E(r)=0$, which means the molecules are independent of one another, and we don't need a description of each molecule's position

▶ Thus, the gas behaves in accordance to the **Ideal Gas Law**:

$$P\forall = n\bar{R}T; \quad P\bar{\nu} = \bar{R}T$$

▶ Recall n is the number of particles within the gas (i.e. moles), expressed as:

$$n = \frac{m}{M}$$

▶ Recall $\bar{R}$ is the universal gas constant:

$$\bar{R} = 8.3145 \left[\frac{\text{kJ}}{\text{kmol-K}} \right]$$

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Ideal Gas State

▶ The gas constant R is related to $\bar{R}$ by the molecular mass M

$$R = \frac{\bar{R}}{M}$$

▶ Substituting n into the Ideal Gas Law

$$\boxed{P\forall = mRT; \quad P\nu = RT}$$

▶ Note T must be in the absolute temperature scale!

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Ideal Gas State

▶ Table A.5 provides the properties M and R for various Ideal Gases

TABLE A.5
Properties of Various Ideal Gases at 25° C, 100 kPa (SI Units)*

Gas	Chemical Formula	Molecular Mass (kg/kmol)	R (kJ/kg-K)	ρ (kg/m³)	C_{p0} (kJ/kg-K)	C_{v0} (kJ/kg-K)	$k = \dfrac{C_p}{C_v}$
Steam	H_2O	18.015	0.4615	0.0231	1.872	1.410	1.327
Acetylene	C_2H_2	26.038	0.3193	1.05	1.699	1.380	1.231
Air	—	28.97	0.287	1.169	1.004	0.717	1.400
Ammonia	NH_3	17.031	0.4882	0.694	2.130	1.642	1.297
Argon	Ar	39.948	0.2081	1.613	0.520	0.312	1.667
Butane	C_4H_{10}	58.124	0.1430	2.407	1.716	1.573	1.091

Image taken from Borgnakke & Sonntage, 8e

▶ Note the properties are taken at STSP. Can they be used for a pressure greater than 100 [kPa]?

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Ideal Gas State

▶ You can always calculate R is you recall $\bar{R}$ and look up M by using Table A.2.

TABLE A.2
Critical Constants

Substance	Formula	Molec. Mass	Temp. (K)	Press. (MPa)	Vol. (m^3/kg)
Ammonia	NH_3	17.031	405.5	11.35	0.00426
Argon	Ar	39.948	150.8	4.87	0.00188
Bromine	Br_2	159.808	588	10.30	0.000796
Carbon dioxide	CO_2	44.01	304.1	7.38	0.00212
Carbon monoxide	CO	28.01	132.9	3.50	0.00333
Chlorine	Cl_2	70.906	416.9	7.98	0.00175

Image taken from Borgnakke & Sonntage, 8e

▶ Table A.2 also provides the critical constants (i.e. temperature and pressure at the critical point on the $P - T$ diagram).

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Example #1

▶ If the system if a C.M., i.e. $m_1 = m_2$ where 1 and 2 correspond to States 1 and 2, determine the pressure at State 2 in terms of P_1, T_1, $\forall_1$, T_2 and $\forall_2$

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Compressibility

▶ We will focus on the concept of reduced pressure and reduced temperature as to define the applicability of the Ideal Gas Law

▶ **Reduced pressure** is defined as the pressure of the state over the critical pressure

$$P_r = \frac{P}{P_c}$$
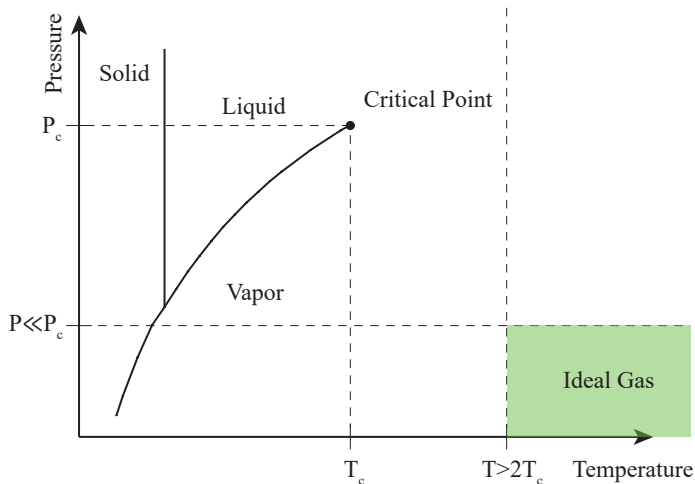
▶ **Reduced temperature** is defined as the temperature of the state over the critical temperature

$$T_r = \frac{T}{T_c}$$

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Compressibility

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Summary

▶ For a substance to be treated as an Ideal Gas, both $P_r \ll 1$ and $T_r > 2$ have to be satisfied

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Compressibility

▶ This can also be seen graphically on Figure D.1 on page 829 of the steam tables:

$$Z = \frac{P\nu}{RT}$$

▶ This still requires knowledge of T_r and P_r, therefore we will forego the use of this graph
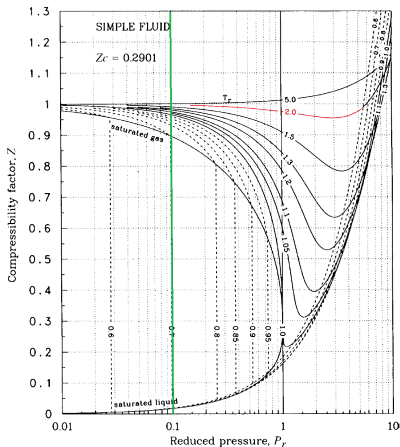

Image taken from Borgnakke & Sonntage, 8e

▶ Note: we <u>will not</u> treat water vapor as an Ideal Gas - we have the steam tables that are more accurate than an idealized model

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Example #2

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2.8 The Ideal Gas States

2.9 The Compressibility Factor

Summary

▶ Imagine you are a Formula 1 driver. If your tyres heat up to 800 °C during hard cornering and acceleration, and we assume the temperature of the air within the tyre is the same as the tyre itself, how much has the pressure within the tyre increased assuming that before the race the pressure was 241.3 [kPa] and 298.15 [K]? Can we treat air as an ideal gas under this situation? Take T_c=132.41 [K] and P_c=3,774 [kPa].

Solution:

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Example #2

Solution:

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Student Learning Objectives

At the end of the lecture, students should be able to:

▶ Identify an Ideal Gas and apply the Ideal Gas law

    ▶ An Ideal Gas is one where the pressure is low enough such that intermolecular interaction, i.e. the attraction and repulsion of individual molecules does not need to be consider. The Ideal Gas law is stated mathematically:

$$P \forall = mRT$$

▶ Determine when the Ideal Gas Law is applicable based upon reduced temperature and pressure

    ▶ The following two inequalities must be satisfied:

$$P_r = \frac{P}{P_c} \ll 1; \quad T_r = \frac{T}{T_c} > 2$$

Suggested Problems

▶ 2.65, 2.66, 2.68, 2.69, 2.74, 2.75, 2.76, 2.80, 2.84

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