Homework #9

MEMS 0051 - Introduction to Thermodynamics

Assigned March 29$^{\text{th}}$, 2019
Due April 5$^{\text{th}}$, 2019

Starting with the Conservation of Energy equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + gz_j \right),$$

and the Continuity equation:

$$\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \sum_{j=1}^{M} \dot{m}_j,$$

Solve the following:

Problem #1

Steam at 0.6 [MPa] and 200 °C enters an insulated nozzle with a velocity of 50 [m/s]. It leaves at a pressure of 0.15 [MPa] and a velocity of 600 [m/s]. Determine the final temperature if the steam is superheated in the final state and the quality if it saturated.

Solution:

Inlet:	$\rightarrow$	Outlet:
P=600 [kPa]		P=150 [kPa]
V = 50 [m/s]		V = 600 [m/s]
T=200 °C		

Consider the designated controlled volume to be that of the nozzles outer layer and its inlet and outlet face. Because the the device is a nozzle, no internal work is being done on the working fluid. At the same time, the nozzle is insulated. Therefore, there is negligible heat transfer occurring into the control volume of the nozzle. The height change in a nozzle system is also assumed negligible

$$\dot{W}_{C.\forall} = 0$$

$$\dot{Q}_{C.\forall} = 0$$

$$\text{P.E}_o = \text{P.E}_i$$

The conservation of energy equation simplifies to:

$$\cancel{\dot{Q}_{C.\forall}} - \cancel{\dot{W}_{C.\forall}} = \dot{E}_e - \dot{E}_i \implies$$

Looking at the saturation tables for saturated mixtures in Table B.1.1, it can be seen that the working fluid is in a superheated state based off its inlet conditions. Looking at Table B.1.3, the specific enthalpy is found to be:

$$\text{h}_i = 2,850.1 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

Since there is no change in mass of the working fluid within the controlled volume, based on the conservation of mass equation:

$$\dot{m}_e = \dot{m}_i$$

Simplifying the equation yields:

$$0 = \cancel{\dot{m}_e}\left(h_e + \frac{V_e^2}{2} + \cancel{gz_e}\right) - \cancel{\dot{m}_i}\left(h_i + \frac{V_i^2}{2} + \cancel{gz_i}\right) \implies$$

$$h_e = h_i + \frac{V_i^2}{2} - \frac{V_e^2}{2}$$

Solving for the specific enthalpy at the outlet, h_e, yields:

$$h_e = 2,850.1\left[\frac{\text{kJ}}{\text{kg-K}}\right] + \frac{(50)^2}{2(1000)} - \frac{(600)^2}{2(1000)} = 2,671.4\left[\frac{\text{kJ}}{\text{kg-K}}\right]$$

The kinetic energy is divided by 1000 so as to equate with the units for specific enthalpy.
Looking at Table B.1.2, we see that the specific enthalpy exist between the specific enthalpy at the saturated vapor and saturated mixture state. That is:

$$\text{h}_f(150[\text{kPa}]) < \text{h}_e < \text{h}_g(150[\text{kPa}])$$

Therefore the working fluid is in the saturated mixture state. Using the quality equation with specific enthalpy, the quality can be found.

$$\text{h}_e = \text{h}_f(150[\text{kPa}]) + x(\text{h}_g(150[\text{kPa}]) - \text{h}_f(150[\text{kPa}])) \implies$$

$$\boxed{x = 0.99}$$

Problem #2

A device has one inlet with a cross-sectional flow area of 0.6 [m^2] in which steam enters with a velocity of 50 [m/s], a pressure of 1,000 [kPa] and a temperature of 400 °C. There are two outlets. One outlet has saturated liquid exiting through a 0.018 [m^2] pipe with a mass flow rate of 50 [kg/s] at a pressure of 150 [kPa]. Determine:

a) the mass flow rate at the inlet;
b) the mass flow rate of the second outlet.

<u>Solution</u>: State 1 exists as a superheated vapor, and the specific volume is directly read from the table as 0.30659 [m^3/kg]. Thus, the mass flow rate at the inlet is determined by:

$$\dot{m} = \frac{A_1 V_1}{\nu_1} = \frac{(0.6\,[\text{m}^2])(50\,[\text{m/s}])}{0.30659\,[\text{m}^3/\text{kg}]} = 97.85\,[\text{kg/s}]$$

The mass flow rate at the second outlet is found via the Continuity equation:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \implies \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = (97.85 - 50)\,[\text{kg/s}] = 47.85\,[\text{kg/s}]$$

Problem #3

Air enters a device at 1,000 [kPa] and 580 [K] and leaves with a volumetric flow rate of 1.8 [m^3/s] at 100 [kPa] and 500 [K]. Heat is transferred from the device to the surroundings at 347 [kJ] per kilogram of air entering the device. Determine:

a) the power developed by the device;
b) the the volumetric flow rate at the inlet.

Solution: The volumetric flow rate rate the inlet is known for the mass flow rate through the device must be constant. The mass flow rate, at the exit, in terms of the volumetric flow rate is:

$$\dot{m} = \rho \dot{\forall} = \frac{\dot{\forall}}{\nu}$$

The specific volume is found via the Ideal Gas law:

$$P \forall = m R T \implies P \nu = R T \implies \nu = \frac{R T}{P} = \frac{(0.287\,[\text{kJ/kg-K}])(500\,[\text{K}])}{100\,[\text{kPa}]} = 1.435\,[\text{m}^3/\text{kg}]$$

Thus, the mass flow rate at the inlet is:

$$\dot{m} = \frac{1.8\,[\text{m}^3/\text{s}]}{1.435\,[\text{m}^3/\text{kg}]} = 1.254\,[\text{kg/s}]$$

Now, applying the Conservation of Energy equation, assuming steady-state and ignoring changes in kinetic and potential energy, and expressing in terms of work developed:

$$\dot{W} = \dot{Q} + \dot{m}(h_1 - h_2)$$

The enthalpy at State 1 and 2 are found from Table A.7.1:

$$h_1 = 586.35\,[\text{kJ/kg}]$$

$$h_2 = 503.65\,[\text{kJ/kg}]$$

Therefore, the work developed is:

$$\dot{W} = -347\,[\text{kJ/kg}] + (1.254\,[\text{kg/s}])(586.35 - 503.65)\,[\text{kJ/kg}] = -242.93\,[\text{kJ/kg}]$$

Can we use the Ideal Gas law?

Problem #4

Air flows through a diffuser with a mass flow rate of 0.5 [kg/s] from an inlet condition of 300 [kPa], 290 [K] and 400 [m/s] to an exit condition of 1,4000 [kPa] and 40 [m/s]. Determine:
a) the exit temperature of the air;
b) the inlet cross-sectional flow area.

Solution: The inlet cross-sectional flow area is determined from the definition of mass flow rate:

$$\dot{m} = \frac{A V}{\nu} \implies A = \frac{\dot{m}\,\nu}{V} = \frac{(0.5\,[\text{kg/s}])\,\nu}{400\,[\text{m/s}]}$$

The specific volume is determined from the Ideal Gas law:

$$nu = \frac{R T}{P} = \frac{(0.287\,[\text{kJ/kg-K}])(290\,[\text{K}])}{300\,[\text{kPa}]} = 0.277\,[\text{m}^3/\text{kg}]$$

Thus, the inlet cross-sectional flow area is:

$$A = \frac{\dot{m}\,\nu}{V} = \frac{(0.5\,[\text{kg/s}])(0.277\,[\text{m}^3/\text{kg}])}{400\,[\text{m/s}]} = 0.000\,346\,[\text{m}^2]\ \text{(i.e. about half an inch sqared)}$$

To determine the exit temperature of the air, we look to the Conservation of Energy equation for a diffuser, assuming steady-state, ignoring heat and work and potential energy:

$$0 = h_1 - h_2 + \frac{V_i^1 - V_2^2}{2}$$

Solving for the enthalpy at State 2, knowing the enthalpy at State 1 is from Table A.7.1:

$$h_2 = h_1 + \frac{V_i^1 - V_2^2}{2} = 290.43\,[\mathrm{kJ/kg}] - \frac{((400-40)\,[\mathrm{m/s}])^2}{2\cdot 1000} = 355.23\,[\mathrm{kJ/kg}]$$

Interpolating between 340 and 360 K on Table A.7.1:

$$\frac{(T_2 - 340)\,[\mathrm{K}]}{(360-340)\,[\mathrm{K}]} = \frac{(355.23 - 340.70)\,[\mathrm{kJ/kg}]}{(360.86 - 340.70)\,[\mathrm{kJ/kg}]} \implies T_2 = 354.4\,[\mathrm{K}]$$

Problem #5

A turbine with sufficient insulation accepts steam at the rate of 85 [m^3/min] at 3,000 [kPa] and 400 °C. A portion of the steam is siphoned from the turbine at a pressure of 500 [kPa], a temperature of 180 °C at a velocity of 20 [m/s]. The remainder of the steam, with a mass flow rate of 40,000 [kg/hr] expands to a pressure of 6 [kPa] with a quality of 90%. Determine:

a) the power developed by the turbine;
b) the diameter of the siphon.

Solution: Setting up the Continuity equation:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \implies \frac{\dot{\forall}_1}{\nu_1} = \frac{A_2 V_2}{\nu_2} + (40,000\,[\mathrm{kg/hr}])(1/3,600\,[\mathrm{hr/s}])$$

The specific volume at State 1, a superheated vapor, and State 2, also a superheated vapor, are found as:

$$\nu_1 = 0.09936\,[\mathrm{m^3/kg}]$$

$$\frac{(180-151.86)\,[^\circ\mathrm{C}]}{(200-151.86)\,[^\circ\mathrm{C}]} = \frac{(\nu - 0.37489)\,[\mathrm{m^3/kg}]}{(0.42492 - 0.37489)\,[\mathrm{m^3/kg}]} \implies \nu_2 = 0.40413\,[\mathrm{m^3/kg}]$$

Solving for the exit cross-sectional flow area at State 2:

$$A_2 = \left(\frac{(85\,[\mathrm{m^3/min}])(1/60\,[\mathrm{min/s}])}{0.09936\,[\mathrm{m^3/kg}]} - (40,000\,[\mathrm{kg/hr}])(1/3,600\,[\mathrm{hr/s}])\right)\left(\frac{0.40413\,[\mathrm{m^3/kg}]}{20\,[\mathrm{m/s}]}\right) = 0.0636\,[\mathrm{m^2}]$$

Applying the Conservation of Energy, ignoring the velocities at State 1 and 3, all potential energies and heat transfer:

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2\left(h_2 + \frac{V_2^2}{2}\right) - \dot{m}_3 h_3$$

The enthalpy at State 1 comes from the superheated steam tables:

$$h_1 = 3,230.82\,[\mathrm{kJ/kg}]$$

The enthalpy at State 2 comes the superheated steam tables:

$$\frac{(180-151.86)\,[^\circ\mathrm{C}]}{(200-151.86)\,[^\circ\mathrm{C}]} = \frac{(h_2 - 2,748.67)\,[\mathrm{kJ/kg}]}{(2,855.37 - 2,748.67)\,[\mathrm{kJ/kg}]} \implies h_2 = 2,811.04\,[\mathrm{kJ/kg}]$$

The enthalpy at State 3 comes the saturated water tables:

$$\frac{(6-5)\,[\mathrm{kPa}]}{(7.5-5)\,[\mathrm{kPa}]} = \frac{(h_{3,f} - 137.79)\,[\mathrm{kJ/kg}]}{(168.77 - 137.79)\,[\mathrm{kJ/kg}]} \implies h_{3,f} = 150.18\,[\mathrm{kJ/kg}]$$

$$\frac{(6-5)\,[\mathrm{kPa}]}{(7.5-5)\,[\mathrm{kPa}]} = \frac{(h_{3,g} - 2,561.45)\,[\mathrm{kJ/kg}]}{(2,574.79 - 2,561.45)\,[\mathrm{kJ/kg}]} \implies h_{3,g} = 2,566.79\,[\mathrm{kJ/kg}]$$

$$h_3 = h_{3,f} + x_3(h_{3,g} - h_{3,f}) = (150.18\,[\mathrm{kJ/kg}]) + 0.9(2,566.79 - 150.18)\,[\mathrm{kJ/kg}] = 2,325.13\,[\mathrm{kJ/kg}]$$

The mass flow rate at State 1 is:

$$\dot{m}_1 = \frac{(85\,[\mathrm{m}^3/\mathrm{min}])(1/60\,[\mathrm{min/s}])}{0.09936\,[\mathrm{m}^3/\mathrm{kg}]} = 14.26\,[\mathrm{kg/s}]$$

The mass flow rate at State 3 is:

$$\dot{m}_3 = \frac{40,000\,[\mathrm{kg/hr}]}{3,600\,[\mathrm{s/hr}]} = 11.1\,[\mathrm{kg/s}]$$

The mass flow rate at State 2 is the difference:

$$\dot{m}_2 = 3.16\,[\mathrm{kg/s}]$$

Thus,

$$\dot{W} = (14.26\,[\mathrm{kg/s}])(3,230.82\,[\mathrm{kJ/kg}]) - (3.16\,[\mathrm{kg/s}])\left((2,811.04\,[\mathrm{kJ/kg}]) + \frac{(20\,[\mathrm{m/s}])^2}{2\cdot 1000}\right) - (11.1\,[\mathrm{kg/s}])(2,325.13\,[\mathrm{kJ/kg}])$$

$$\dot{W} = 11,379.03\,[\mathrm{kJ/kg}]$$