# Homework #4

MEMS 0051 - Introduction to Thermodynamics

Assigned February 1st, 2020
Due February 7th, 2020

## Problem #1

A piston-cylinder device contains air at a pressure of 600 [kPa] and a temperature of 290 [K]. The initial volume is 0.1 [m$^3$]. A constant-pressure process produces 55 [kJ] of work done by the system.
(a) Determine the final temperature of the air.
(b) Determine the heat input.
Solution:

| State 1: | $\rightarrow$P=c, W=55 [kJ]$\rightarrow$ | State 2: |
|---|---|---|
| $P_1$=600 [kPa] | | $P_2$=$P_1$ |
| $T_1$=290 K | | $T_2$=? |
| $\forall_1$=0.1 [m$^3$] | | |

To determine the final temperature, we need the final volume and mass of the system. The mass is found through the Ideal Gas law applied to State 1:

$$m = \frac{P_1 \forall_1}{RT_1} = \frac{(600\,[\text{kPa}])(0.1\,[\text{m}^3])}{(0.287\,[\text{kJ/kg-K}])(290\,[\text{K}])} = 0.721\,[\text{kg}]$$

The final volume is found through the definition of work for a constant-pressure process:

$$W_{1\to2} = P(\forall_1 - \forall_2) \implies \forall_2 = \frac{W_{1\to2}}{P} + \forall_1 = \frac{55\,[\text{kJ}]}{600\,[\text{kPa}]} + 0.1\,[\text{m}^3] = 0.192\,[\text{m}^3]$$

Therefore, the final temperature is found by applying the Ideal Gas law at State 2:

$$T_2 = \frac{P_2 \forall_2}{mR} = \frac{(600\,[\text{kPa}])(0.192\,[\text{m}^3])}{(0.721\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])} = 556.7\,[\text{K}]$$

To evaluate the heat input, we apply the Conservation of Energy:

$$Q_{1\to2} = (U_2 - U_1) + W_{1\to2} = m(u_2 - u_1) + W_{1\to2} = mC_\forall(T_2 - T_1) + W_{1\to2} = mC_P(T_2 - T_1) = m(h_2 - h_1)$$

Using constant-volume specific heat to solve for the change of internal energy, since we are given work:

$$Q_{1\to2} = (0.721\,[\text{kg}])(0.717\,[\text{kJ/kg-K}])((556.7 - 290)[\text{K}]) + 55\,[\text{kJ}] - 192.9\,[\text{kJ}]$$

Using Table A.7.1, the most accurate method:

$$u_1 = 207.19\,[\text{kJ/kg}]$$
$$u_2 = 402.25\,[\text{kJ/kg}]$$

Therefore,

$$Q_{1\to2} = (0.721\,[\text{kg}])((402.25 - 207.19)[\text{kJ/kg}]) + 55\,[\text{kJ/kg}] = 195.6\,[\text{kJ}]$$

Note: you could use enthalpy or $C_P$ and ignore $W_{1\to2}$ and arrive at the same solution.

# Problem #2

Water contained in a piston-cylinder assembly has an initial temperature of 150 °C, a quality of 50% and an initial volume of 0.05 [m$^3$]. The pressure of the process is given as P($\forall$)=100 + C$\forall^{0.5}$ kPa. Heat is transferred to the piston-cylinder until the final pressure reaches 600 [kPa].

(a) Determine the heat input.

(b) Plot this process on your $P$-$\nu$ and $T$-$\nu$ diagrams in Matlab.

Solution:

| State 1: | $\rightarrow$ | State 2: |
|---|---|---|
| $T_1$=150 °C | | $P_2$=600 [kPa] |
| $P_1$=$P_{\text{sat}}(T_1)$=475.9 [kPa] | | |
| $x_1$=0.50 | | |
| $\forall_1$=0.05 [m$^3$] | | |

To determine $Q_{1\to2}$, we need $m_1$, $u_1$, $u_2$, $\nu_2$ and $W_{1\to2}$. Determining the mass at State 1 based upon volume and specific volume, with the latter determined via quality:

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{0.05\,[\text{m}^3]}{(0.00109 + 0.5(0.39278 - 0.00109)\,[\text{m}^3/\text{kg}]} = 0.254\,[\text{kg}]$$

The internal energy at State 1 is determined via quality:

$$u_1 = u_f(150\,[^\circ\text{C}]) + x(u_g(150\,[^\circ\text{C}]) - u_f(150\,[^\circ\text{C}])) = (631.66 + 0.5(2.559.54 - 631.66))\,[\text{kJ/kg}] = 1,595.6\,[\text{kJ/kg}]$$

To get the specific volume at State 2, we can apply our pressure versus volume relation to solve for the constant C, then solve for the volume at State 2 based upon the given pressure:

$$P_1 = 100 + C\forall_1^{0.5} \implies 475.9\,[\text{kPa}] = 100\,[\text{kPa}] + C(0.05[\text{m}^3])^{0.5} \implies C = \frac{375.9\,[\text{kPa}]}{\sqrt{0.05\,[\text{m}^3]}} = 1,681.1\,[\text{kPa/m}^{1.5}]$$

Therefore, at State 2:

$$P_2 = 100\,[\text{kPa}] + (1,681.1\,[\text{kPa/m}^{1.5}])\forall_2^{0.5} \implies \forall_2 = 0.0855\,[\text{m}^3]$$

Thus, the specific volume at State 2:

$$\nu_2 = \frac{\forall_2}{m} = \frac{0.0885\,[\text{m}^3]}{0.254\,[\text{kg}]} = 0349\,[\text{m}^3/\text{kg}]$$

Based upon $P_2$ and $\nu_2$, it is evident State 2 exists in the superheated vapor region. Interpolating between the saturation temperature of 600 [kPa] and the 200 °C entry:

$$\frac{(0.349 - 0.31567)\,[\text{m}^3/\text{kg}]}{(0.35202 - 0.31567)\,[\text{m}^3/\text{kg}]} = \frac{(u_2 - 2,567.4)\,[\text{kJ/kg}]}{(2,638.91 - 2,567.4)\,[\text{kJ/kg}]} \implies u_2 = 2,632.97\,[\text{kJ/kg}]$$

To determine $W_{1\to2}$, we can integrate $P(\forall)$:

$$W_{1\to2} = \int_{\nu_1}^{\nu_2} P(\forall)\,d\forall = \int_{\nu_1}^{\nu_2} (100 + 1,681.1\forall^{0.5})\,d\forall = 100\forall + \frac{2}{3}1,681.1\forall^{1.5}\Big|_{0.05}^{0.0885} = 20.83\,[\text{kJ}]$$

Lastly, applying the Conservation of Energy to solve for $Q_{1\to2}$:

$$Q_{1\to2} = m(u_2 - u_1) + W_{1\to2} = (0.254\,[\text{kg}])(2,632.97 - 1,595.6)\,[\text{kJ/kg}] + 20.83\,[\text{kJ}] = 284.32\,[\text{kJ}]$$

# Problem #3

A piston-cylinder device has two stops; a lower set and an upper set, that constrain the cylinder. When the piston is on the lower stops, the volume is 0.4 Liters. When the piston reaches the upper stops, the volume is 0.6 Liters. The piston-cylinder initially contains water at 100 [kPa] and a quality of 20%. The water is heated until it is completely transformed to steam. Additionally, the mass of the piston requires a pressure of 300 [kPa] to raise it. When the piston hits the upper stops:

(a) Determine the final pressure in the cylinder.
(b) Determine the heat input.
(c) Determine the work for the overall process.
(d) Plot this process on your $P$-$\nu$ and $T$-$\nu$ diagram in Matlab.
(e) Hint: there are 4 states. Think about the processes involved.

Solution:

| | | | | | | |
|---|---|---|---|---|---|---|
| State 1: | $\rightarrow \forall$=c $\rightarrow$ | State 2: | $\rightarrow$ P=c $\rightarrow$ | State 3: | $\rightarrow \forall$=c $\rightarrow$ | State 4: |
| $\forall_1 = 4\cdot10^{-4}$ [m$^3$] | | $\forall_2 = \forall_1$ | | $\forall_3 = 6\cdot10^{-4}$ [m$^3$] | | $\forall_4 = \forall_3$ |
| $P_1 = 100$ [kPa] | | $P_2 = 300$ [kPa] | | $P_3 = P_2$ | | $x_4 = 1.0$ |
| $x_1 = 0.2$ | | | | | | |

State 1 variable, as determined from the saturation pressure and quality:

$$\nu_1 = \nu_f(100\,[\text{kPa}]) + x_1(\nu_g(100\,[\text{kPa}]) - \nu_f(100\,[\text{kPa}])) = (0.001043 + 0.2(1.69296))\,[\text{m}^3/\text{kg}] = 0.3396\,[\text{m}^3/\text{kg}]$$

$$u_1 = u_f(100\,[\text{kPa}]) + x_1(u_g(100\,[\text{kPa}]) - u_f(100\,[\text{kPa}])) = (417.33 + 0.2(2,088.72))\,[\text{kJ/kg}] = 835.074\,[\text{kJ/kg}]$$

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{4\cdot10^{-4}\,[\text{m}^3]}{0.3396\,[\text{m}^3/\text{kg}]} = 0.001178\,[\text{kg}]$$

From State 1 to 2, the volume is constant ($\nu_2$=$\nu_1$) until the pressure reaches 300 [kPa], in which the piston is able to move, i.e. the volume can increase. At 300 [kPa], we have to determine the specific volume to see if either a) we have reached 100% vapor or b) we have hit the upper stops.

$$\nu_3 = \frac{\forall_3}{m} = \frac{6\cdot10^{-4}\,[\text{m}^3]}{0.001178\,[\text{kg}]} = 0.509338\,[\text{m}^3/\text{kg}] < \nu_g(300\,[\text{kPa}])$$

Therefore, scenario b) is what has happened, and there is quality associated with the water at State 3:

$$x_3 = \frac{\nu_3 - \nu_f(300\,[\text{kPa}])}{\nu_g(300\,[\text{kPa}]) - \nu_f(300\,[\text{kPa}])} = \frac{(0.509338 - 0.001073)\,[\text{m}^3/\text{kg}]}{(0.60582 - 0.001073)\,[\text{m}^3/\text{kg}]}$$

Thus, we proceed from State 3 to State 4 in an isochoric fashion (i.e. $\nu_4$=$\nu_3$). Additionally, the quality is $x_4$=1.0 Looking at the Saturated Water Pressure entry table, State 4 lies between 350 and 375 [kPa]. Thus, the pressure at State 4 is found via interpolation:

$$\frac{(P_4 - 350)\,[\text{kPa}]}{(375 - 350)\,[\text{kPa}]} = \frac{(0.509338 - 0.52425)\,[\text{m}^3/\text{kg}]}{(0.49137 - 0.52425)\,[\text{m}^3/\text{kg}]} \implies P_4 = 361.31\,[\text{kPa}]$$

The internal energy at State 4 is found via interpolation:

$$\frac{(0.509338 - 0.52425)\,[\text{m}^3/\text{kg}]}{(0.49137 - 0.52425)\,[\text{m}^3/\text{kg}]} = \frac{(u_4 - 2,548.92)\,[\text{kJ/kg}]}{(2,551.31 - 2,548.92)\,[\text{kJ/kg}]} \implies u_4 = 2,550.0\,[\text{kJ/kg}]$$

Evaluating the work for the process:

$$W_{1\to4} = \cancelto{0}{\int_{\forall_1}^{\forall_2} P\,d\forall} + \int_{\forall_2}^{\forall_3} P\,d\forall + \cancelto{0}{\int_{\forall_3}^{\forall_4} P\,d\forall} = P_2(\forall_3 - \forall_2) = (300\,[\text{kPa}])(6-4)\cdot10^{-4}\,[\text{m}^3] = 0.06\,[\text{kJ}]$$

The heat input is found through the Conservation of Energy:

$$Q_{1\to4} = m\big((U_2 - U_1) + (U_3 - U_2) + (U_4 - U_3)\big) + W_{1\to4} = m(u_4 - u_1) + W_{1\to4}$$
$$= (0.001178\,[\text{kg}])(2,550 - 835.074)\,[\text{kJ/kg}] + 0.06\,[\text{kJ}] = 2.08\,[\text{kJ}]$$

# Problem #4

A piston-cylinder device contains 0.1 [kg] of air at a pressure of 100 [kPa] and a temperature of 400 [K] that undergoes an expansion process. The volume of the piston-cylinder device expands from 1 [$m^3$] to 3 [$m^3$] at a constant pressure of 2,000 [kPa]. Then, as the piston-cylinder device expands from 3 [$m^3$] to 5 [$m^3$], the pressure linearly decreases from 2,000 to 1,000 [kPa].

(a) Determine the heat input.

(b) Determine the work for the overall process.

Solution:

| State 0: | → | State 1: | → | State 2: | → | State 3: |
|---|---|---|---|---|---|---|
| $m_0$=1 [kg] | | $\forall_1$=1 [$m^3$] | | $\forall_1$=3 [$m^3$] | | $\forall_1$=5 [$m^3$] |
| $P_0$=100 [kPa] | | $P_1$=2,000 [kPa] | | $P_2$=$P_1$ | | $P_3$=1,000 [kPa] |
| $T_0$=400 [K] | | | | | | |

Determining the volume at State 0 from the Ideal Gas law:

$$\forall_0 = \frac{mrT_0}{P_0} = \frac{(0.1\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])(400\,[\text{K}])}{100\,[\text{kPa}]} = 0.1148\,[\text{m}^3]$$

Moving to State 1, we can determine the temperature, which is used to populate the internal energy:

$$T_1 = \frac{P_1\forall_1}{mR} = \frac{(2,000\,[\text{kPa}])(1\,[\text{m}^3])}{(0.1\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])} = 69,686\,[\text{K}]$$

At this point in time, we should recognize the temperature at State 1 is rather large. Although the constraints for validity of the Ideal Gas law requires $T_r$>2, however, $T_1$ is greater than reasonable limits, for this temperature is greater than what is required to create a plasma (≈55,000 [K]) out of the nitrogen molecules (main components of air). You can see that the reduce pressure requirement at State 1 is not satisfied either. We therefore conclude that the use of the Ideal Gas law is not valid. This problem illustrates engineering judgment. Yes, you can use an equation to get a value, but does the value make sense?

# Problem #5

For the following scenarios, determine the amount of heat transfer.

(a) Heating a 2 [kg], 0.1 [m] long copper bar from 25°C to 100°C.

Solution: Copper is an incompressible solid, thus we can use the formulation of specific heat (Table A.3) times mass times change of temperature to determine the heat input. Note that the change of temperature has units of [K], but the temperature expressed in [°C] does not need to be converted to [K]:

$$Q = m\,C\,\Delta T = (2\,[\text{kg}])(0.42\,[\text{kJ/kg-K}])(100 - 25)\,[\text{K}] = 6.3\,[\text{kJ}]$$

(b) 1,000 [kg] of asphalt cooling from 50°C to 20°C.

Solution:

$$Q = m\,C\,\Delta T = (1,000\,[\text{kg}])(0.92\,[\text{kJ/kg-K}])(50 - 25)\,[\text{K}] = -23,000\,[\text{kJ}]$$

(c) The heating of 1 [kg] of oxygen in a mass-less piston-cylinder from 300 to 1,500 K.

Solution: A mass-less piston-cylinder will act as a constant pressure device. Thus, the heat input is merely the change of enthalpy. Using Table A.8 to obtain the enthalpy of oxygen at 1,500 and 300 [K]:

$$Q = m\,\Delta h = (1\,[\text{kg}])(1,540.23 - 273.15)\,[\text{kJ/kg}] = 1,267.08\,[\text{kJ}]$$

(d) A piston-cylinder containing 0.1695 [kg] of nitrogen at 150 [kPa] and 25°C that is isothermally compressed to 1.0 [MPa], which requires 20 [kJ] of work.
Solution: To determine the amount of heat transferred, the Conservation of Energy must be evaluated. First, the volumes at States 1 and 2 need to be determined to evaluate the isothermal work. Using the Ideal Gas law:

$$\forall_1 = \frac{m\,R\,T}{P_1} = \frac{(0.1695\,[\text{kg}])(0.2968\,[\text{kJ/kg-K}])(298\,[\text{K}])}{150\,[\text{kPa}]} = 0.1\,[\text{m}^3]$$

The volume at State 2 is found recognizing that the temperature is constant:

$$\forall_1 = \frac{m\,R\,T}{P_2} = \frac{(0.1695\,[\text{kg}])(0.2968\,[\text{kJ/kg-K}])(298\,[\text{K}])}{1,000\,[\text{kPa}]} = 0.015\,[\text{m}^3]$$

The isothermal work is therefore:

$$W_{1\to 2} = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (150\,[\text{kPa}])(0.1\,[\text{m}^3])\ln\left(\frac{0.015\,[\text{m}^3]}{0.1\,[\text{m}^3]}\right) = -28.46\,[\text{kJ}]$$

Evaluating the Conservation of Energy, it is evident the change of internal energy is zero because the internal energy is a function of temperature, not pressure. Therefore:

$$Q_{1\to 2} = \cancelto{0}{m(u_2 - u_1)} + W_{1\to 2} = -28.46\,[\text{kJ}]$$

Verifying the validity of the Ideal Gas law:

$$T_{r,\text{N}_2} = \frac{T_{min,\text{N}_2}}{T_{cr,\text{N}_2}} = \frac{298\,[\text{K}]}{126.2\,[\text{K}]} = 2.36$$

$$P_{r,\text{N}_2} = \frac{P_{max,\text{N}_2}}{P_{cr,\text{N}_2}} = \frac{1,000\,[\text{kPa}]}{3,390\,[\text{kPa}]} = 0.29$$