

Homework #4

MEMS 0051 - Introduction to Thermodynamics

Assigned February 1st, 2019

Due: February 8th, 2019

Problem #1

- a) You push a lawnmower with a total force of 100 [N] at a constant velocity of 0.5 [m/s] for a total of 35 seconds. The lawnmower has a mass of 20 [kg].

- i.) How much power (\dot{W}) are you transferring to the lawnmower [W]?

$$\dot{W} = FV = (50 \text{ [N]})(0.5 \text{ [m/s]}) = 50 \text{ [W]}$$

- ii.) How much work (W) do you transfer to the lawnmower during those 10 seconds [J]?

$$W = \dot{W}\Delta t = (50 \text{ [W]})(35 \text{ [s]}) = 1,750 \text{ [J]}$$

- iii.) How much specific work (w) do you transfer to the lawnmower [J/kg]?

$$w = \frac{W}{m} = \frac{1,750 \text{ [J]}}{20 \text{ [kg]}} = 87.5 \text{ [J/kg]}$$

- b) Now assume that heat is leaving the lawnmower at a rate of 2 [W] throughout the 35 seconds.

- i.) How much total heat (Q) leaves the lawnmower during those 35 seconds [J]?

$$Q = \dot{Q}\Delta t = (2 \text{ [W]})(35 \text{ [s]}) = 70 \text{ [J]}$$

- ii.) How much specific heat (q) leaves the lawnmower during those 35 seconds [kJ/kg]?

$$q = \frac{Q}{m} = \frac{70 \text{ [J]}}{20 \text{ [kg]}} = 3.5 \text{ [J/kg]}$$

- c) Now consider the lawnmower as a thermodynamic system with a control surface drawn all around it. Make sure you use the 1st Law sign conventions for heat and work.

- i.) Is work positive or negative for the lawnmower system? Explain.

Negative, because work is being done to the lawnmower, and work is positive when it is done by a system.

- ii.) Is heat positive or negative for the lawnmower system? Explain.

Negative, because heat is being transferred out of the lawnmower system, and heat is positive when it goes into a system.

Problem #2

- a) A piston cylinder is initially filled with air occupying 0.5 [m³] at a pressure of 50 [kPa]. The piston then compresses the air to a new volume of 0.2 [m³]. The compression process is governed by the polytropic equation: $P_1 V_1^n = P_2 V_2^n$. Determine how much work is done by the gas for the following polytropic indices:

- i.) $n = \infty$

Work is zero for an isochoric process because the volume doesn't change.

ii.) $n=0$

For an isobaric process, the pressure is constant

$$W_{1 \rightarrow 2} = P(\forall_2 - \forall_1) = (50 \text{ [kPa]})(0.2 - 0.5 \text{ [m}^3]) = \boxed{-15 \text{ [kJ]}}$$

iii.) $n=1$

$$W_{1 \rightarrow 2} - P_1 \forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (50 \text{ [kPa]})(0.5 \text{ [m}^3]) \ln\left(\frac{0.2}{0.5}\right) = \boxed{-22.9 \text{ [kJ]}}$$

iv.) $n=1.2$

Using the general expression for work

$$W_{1 \rightarrow 2} = \frac{P_2 \forall_2 - P_1 \forall_1}{1 - n}$$

To solve for P_2

$$P_2 = P_1 \left(\frac{\forall_1}{\forall_2}\right)^n = 150.1 \text{ [kPa]}$$

Therefore

$$W_{1 \rightarrow 2} = \frac{(150.1 \text{ [kPa]})(0.2 \text{ [m}^3]) - (50 \text{ [kPa]})(0.5 \text{ [m}^3])}{1 - 1.2} = \boxed{-25.1 \text{ [kJ]}}$$

- b) Now assume that we don't know the polytropic index, but we do know that the final pressure is 235 [kPa]. Solve for the polytropic index, n .

Solving for n

$$n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\frac{235 \text{ [kPa]}}{50 \text{ [kPa]}}\right)}{\ln\left(\frac{0.5 \text{ [m}^3]}{0.2 \text{ [m}^3]}\right)} = \boxed{n = 1.69}$$

Problem #3

- a) Determine the change in internal energy, $(U_2 - U_1)$, for each of the following ideal gas cases. (Hint: refer to Table A.5 to look up C_V for each of these gases.)
- i.) 1 [kg] of Neon gas going from 300 K to 500 K

$$U_2 - U_1 = mC_V(T_2 - T_1) = (1 \text{ [kg]})(0.618 \text{ [kJ/kg-K]})(500 - 300 \text{ [K]}) = \boxed{123.6 \text{ [kJ]}}$$

- ii.) 2.5 [kg] of R-12 refrigerant going from 500 K to 300 K

$$U_2 - U_1 = mC_V(T_2 - T_1) = (2.5 \text{ [kg]})(0.547 \text{ [kJ/kg-K]})(300 - 500 \text{ [K]}) = \boxed{-273.5 \text{ [kJ]}}$$

- iii.) 2 [kg] of Acetylene going from 400 K to 300 K

$$U_2 - U_1 = mC_V(T_2 - T_1) = (2 \text{ [kg]})(1.380 \text{ [kJ/kg-K]})(300 - 400 \text{ [K]}) = \boxed{-276.0 \text{ [kJ]}}$$

Problem #4

- a) A piston-cylinder device contains nitrogen at 100 [kPa], 25 °C and 0.2 [m³]. The air then undergoes an isobaric, polytropic expansion to a volume of 0.5 [m³].

i.) What is the mass of air contained in the piston-cylinder?

$$m = \frac{P\forall}{RT} = \frac{(100 \text{ [kPa]})(0.2 \text{ [m}^3\text{]})}{(0.2968 \text{ [kJ/kg-K]})(298 \text{ [K]})} = \boxed{0.226 \text{ [kg]}}$$

ii.) What is the final temperature after expansion?

m , R and P are constants, therefore the Ideal Gas law reduces to

$$\frac{\forall_1}{T_1} = \frac{\forall_2}{T_2} \implies T_2 = T_1 \left(\frac{\forall_2}{\forall_1} \right) = 298 \text{ [K]} \left(\frac{0.5}{0.2} \right) = \boxed{745 \text{ [K]}}$$

iii.) How much work is done by the gas during this expansion?

$$W_{1 \rightarrow 2} = P(\forall_2 - \forall_1) = (100 \text{ [kPa]})(0.5 - 0.2 \text{ [m}^3\text{]}) = \boxed{30 \text{ [kJ]}}$$

iv.) What is the change in internal energy, $(U_2 - U_1)$, during this expansion?

$$U_2 - U_1 = mC_v(T_2 - T_1) = (0.226 \text{ [kg]})(0.745 \text{ [kJ/kg-K]})(745 - 298 \text{ [K]}) = \boxed{75.261 \text{ [kJ]}}$$

v.) How much heat is transferred into the gas during this expansion?

$$Q_{1 \rightarrow 2} = (U_2 - U_1) + W_{1 \rightarrow 2} = \boxed{105.26 \text{ [kJ]}}$$

Problem #5

a) Water contained in a piston-cylinder assembly has an initial temperature of 200 °C, a quality of 50% and an initial volume of 0.05 m³. The pressure of the process is given as $P(\forall) = 100 + C\forall^{0.5}$ [kPa], where C is a constant. Heat is transferred to the piston-cylinder until the final pressure reaches 600 kPa.

i.) Determine the heat input. (Hint, you need to determine C .)

State 1:	\rightarrow	State 2:
$T_1 = 200 \text{ }^\circ\text{C}$		$P_2 = 600 \text{ [kPa]}$
$x_1 = 0.5$		
$\forall_1 = 0.05 \text{ [m}^3\text{]}$		

To determine $Q_{1 \rightarrow 2}$, need m_1 , u_1 , u_2 and $W_{1 \rightarrow 2}$. At State 1, since there is quality, we know $P_1 = P_{\text{sat}}(T_1) = 1,553.8$ [kPa], which allows us to pull ν_f and ν_g from the steam Tables.

$$\nu_1 = 0.001156 + 0.5(0.12620) = 0.0643 \text{ [m}^3\text{/kg]}$$

Thus, we are able to determine the mass at State 1:

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{0.05 \text{ [m}^3\text{]}}{0.0643 \text{ [m}^3\text{/kg]}} = 0.7781 \text{ [kg]}$$

Additionally, since we have quality at State 1, we can determine specific internal energy:

$$u_1 = 850.64 + 0.5(1744.6) = 1,722.94 \text{ [kJ/kg]}$$

We only have the pressure at State 2 - one more independent property must be known before the specific internal energy at State 2 can be determined. We do have a relationship between pressure and volume. Applying this relationship to State 1 to determine the constant C , and then using our pressure at State 2 to determine the volume at State 2, and knowing mass is constant, we can determine the specific volume at State 2. Evaluating $P(\forall)$ at State 1:

$$P(\forall) = 100 + C(\forall)^{0.5} \implies C = \frac{1,553.8}{0.05^{1/2}} = 6,501.6$$

Therefore, as State 2:

$$\forall_2 = \left(\frac{P - 100}{C} \right)^2 \Rightarrow \forall_2 = 0.00591 [\text{m}^3]$$

The specific volume at State 2 is found as:

$$\nu_2 = \frac{\forall_2}{m} = \frac{0.00591 [\text{m}^3]}{0.7781 [\text{kg}]} = 0.0076 [\text{m}^3/\text{kg}]$$

Knowing P_2 and ν_2 , it is evident the water exists as a saturated mixture, and the internal energy is found after finding the new quality.

$$x_2 = \frac{\nu_2 - \nu_f}{\nu_g - \nu_f} = \frac{0.0076 - 0.001101}{0.31567 - 0.001101} = .021$$

$$u = 669.88 + 1,897.52(0.021) = 709.728 [\text{kJ/kg}]$$

Lastly, the work from State 1 to 2 can be found through 1.) the integration of our pressure as a function of volume equation or 2.) the evaluation of the polytropic index. Solving via integration:

$$W_{1 \rightarrow 2} = \int_{\forall_1}^{\forall_2} P d\forall = 100\forall + \frac{2 \cdot 6,501.6}{3} \forall^{3/2} \bigg|_{0.05}^{0.00591} = -50.90 [\text{kJ}]$$

The polytropic index is found as:

$$n = \frac{\log\left(\frac{P_2}{P_1}\right)}{\log\left(\frac{\forall_1}{\forall_2}\right)} = -0.4456024$$

Therefore, the work is:

$$W_{1 \rightarrow 2} = \frac{P_2 \forall_2 - P_1 \forall_1}{1 - n} = -51.29 [\text{kJ}]$$

Take note that the percentage difference between the work calculated from integration and the polytropic equation is approximately 0.76%

Lastly, the heat input is:

$$Q_{1 \rightarrow 2} = m(u_2 - u_1) + W_{1 \rightarrow 2} = -839.67 [\text{kJ}]$$

In other words, 839.67 [kJ] is being rejected from the system.