

Chapter 6 - Entropy

Lecture 21

Sections 6.7-6.8

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
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Student Learning Objectives

At the end of the lecture, students should be able to:

- ▶ Evaluate the relation between P , T , ν and k for an Ideal Gas undergoing an isentropic process



Entropy Change for Ideal Gases - C_{V0} & C_{P0}

- Recall, the change of entropy of an Ideal Gas can be expressed as

$$ds = C_{V0} \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{\nu_2}{\nu_1}\right)$$

- Additionally, it can be expressed as

$$ds = C_{P0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

- For isentropic processes, the change of entropy is zero



Ideal Gas Isentropic Process

- ▶ When an Ideal Gas undergoes a reversible process

$$0 = C_{P0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

- ▶ Rearranging in terms of temperature

$$\ln\left(\frac{T_2}{T_1}\right) = \frac{R}{C_{P0}} \ln\left(\frac{P_2}{P_1}\right)$$

- ▶ Exponentiating

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{C_{P0}}}$$



Ideal Gas Isentropic Process

- ▶ Recall the expression for the gas constant

$$R = C_{P0} - C_{V0}$$

- ▶ The exponent then becomes

$$\frac{R}{C_{P0}} = \frac{C_{P0} - C_{V0}}{C_{P0}} = \frac{k - 1}{k}$$

- ▶ k is defined at the ratio of constant pressure to constant volume specific heat
- ▶ Thus, for an Ideal Gas undergoing an isentropic process

$$\boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}} \quad (1)$$



Ideal Gas Isentropic Process

- ▶ The pressure of an Ideal Gas can be expressed as

$$P = \frac{RT}{\nu}$$

- ▶ Substituting this expression into the RHS

$$\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left\{ \left(\frac{RT_2}{\nu_2}\right) \left(\frac{\nu_1}{RT_1}\right) \right\}^{\frac{k-1}{k}} = \left\{ \left(\frac{T_2}{T_1}\right) \left(\frac{\nu_1}{\nu_2}\right) \right\}^{\frac{k-1}{k}}$$

- ▶ Recall Eq. 1

$$\frac{T_2}{T_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k-1}{k}} \left(\frac{\nu_1}{\nu_2}\right)^{\frac{k-1}{k}} \Rightarrow \left(\frac{T_2}{T_1}\right)^{\frac{1}{k}} = \left(\frac{\nu_1}{\nu_2}\right)^{\frac{k-1}{k}}$$

- ▶ Exponentiating each side to k

$$\boxed{\frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1}} \quad (2)$$



Ideal Gas Isentropic Process

- ▶ Expressing the Ideal Gas law in terms of temperature

$$T = \frac{P \nu}{R}$$

- ▶ Substituting this into the LHS of Eq. 1:

$$\left(\frac{P_2 \nu_2}{R} \right) \left(\frac{R}{P_1 \nu_1} \right) = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

- ▶ Grouping like terms

$$\frac{\nu_2}{\nu_1} = \left(\frac{P_2}{P_1} \right)^{\frac{-1}{k}}$$

- ▶ Rearranging in terms of pressure and exponentiating to $-k$

$$\frac{P_2}{P_1} = \left(\frac{\nu_2}{\nu_1} \right)^{-k} \implies \boxed{\frac{P_2}{P_1} = \left(\frac{\nu_1}{\nu_2} \right)^k} \quad (3)$$



- ▶ It is evident Eq. 3 is a familiar formulation

$$\frac{P_2}{P_1} = \left(\frac{\nu_1}{\nu_2} \right)^k \implies P_2 \nu_2^k = P_1 \nu_1^k$$

- ▶ That is

$$P \nu^k = \text{constant}$$

- ▶ Expressing the change of entropy for an Ideal Gas undergoing an isentropic process, the pressure times (specific) volume between the initial and final states must remain constant, and the constant of proportionality is the polytropic index, $n=k$
- ▶ If isentropic, we use k from Table A.5, which is also known as γ
- ▶ Otherwise, we use n



- ▶ Since $n=k$ for an Ideal Gas undergoing an isentropic process, we can re-express the previous formulations in terms of the polytropic index n

$$\frac{P_2}{P_1} = \left(\frac{\nu_1}{\nu_2} \right)^n$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \left(\frac{\nu_1}{\nu_2} \right)^{n-1}$$

- ▶ Recall the polytropic index could be used to determine the quantity of work supplied or delivered by a system



- ▶ The expression for work for a non-isothermal process in terms of the polytropic index is

$$W_{1 \rightarrow 2} = \frac{P_2 \forall_2 - P_1 \forall_1}{1 - n}$$

- ▶ Expressing P and \forall in terms of the Ideal Gas law, the work for a reversible polytropic process is

$$W_{1 \rightarrow 2} = \frac{m R (T_2 - T_1)}{1 - n}$$



- Recall for an isothermal process, $n=1$, thus the evaluation of work was executed through the integration of the change of volume

$$W_{1 \rightarrow 2} = \int_1^2 P dV = P_1 V_1 \int_1^2 \frac{dV}{V} = P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

- Re-expressing state 1 in terms of the Ideal Gas law, recalling $T_1=T_2=T$, and the ratio of volumes in terms pressure, the work for a reversible isothermal process is

$$W_{1 \rightarrow 2} = m R T \ln\left(\frac{P_1}{P_2}\right)$$



- ▶ Recall u and h are dependent only upon T , therefore, in an isothermal process, neither changes and heat must equal work

$$\delta Q = \delta W$$

- ▶ Additionally, the heat added is equal to the temperature times change of entropy

$$\delta Q = T dS$$

- ▶ Therefore

$$Q_{1 \rightarrow 2} = W_{1 \rightarrow 2} = T(S_2 - S_1) = m R T \ln\left(\frac{P_1}{P_2}\right)$$



Example #1

- ▶ Air is compressed from an initial pressure and temperature of 1 [bar] and 20 °C, respectively, to a pressure of 5 [bar] in a reversible process. Determine the work and heat transfer per unit mass, and change of entropy, if the polytropic index is known as 1.3. How does this compare if the process is isentropic? Use EES.



Example #1

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Learning Objectives

6.7 - Entropy
Change of an Ideal
Gas

6.8 - Reversible
Polytropic Process
for an Ideal Gas

Summary



Example #1



Student Learning Objectives

At the end of the lecture, students should be able to:

- ▶ Evaluate the relation between P , T , ν and k for an Ideal Gas undergoing an isentropic process
 - ▶ We can relate P , T , and ν using the ratio of constant pressure to constant volume specific heat, k , such that

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \quad \frac{T_2}{T_1} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1} \quad \frac{P_2}{P_1} = \left(\frac{\nu_1}{\nu_2}\right)^k$$



Suggested Problems

► 6.100, 6.102, 6.105, 6.108, 6.111, 6.114

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► Solution to Example #1 using Tables:

For $n=1.3$:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{1}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = 293 \text{ [K]} \left(\frac{5 \text{ [bar]}}{1 \text{ [bar]}}\right)^{\frac{1.3-1}{1.3}} = 425 \text{ [K]}$$

Therefore, work per mass basis

$$\frac{W_{1 \rightarrow 2}}{m} = \frac{R(T_2 - T_1)}{1 - n} = \frac{0.287 \text{ [kJ/kg-K]}(425 - 293) \text{ [K]}}{1 - 1.3}$$

$$\frac{W_{1 \rightarrow 2}}{m} = -126.29 \text{ [kJ/kg]}$$

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Summary



The heat added comes from the 1st Law:

$$\begin{aligned}\frac{Q_{1 \rightarrow 2}}{m} &= (u_2 - u_1) + \frac{W_{1 \rightarrow 2}}{m} = C_{v0}(T_2 - T_1) + \frac{W_{1 \rightarrow 2}}{m} \\ &= 0.717 \text{ [kJ/kg-K]}(425 - 293) \text{ [K]} - 126.29 \text{ [kJ/kg]} \\ \frac{Q_{1 \rightarrow 2}}{m} &= -31.64 \text{ [kJ/kg]}\end{aligned}$$

The change of entropy is calculated as:

$$\begin{aligned}ds &= C_{P0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \\ &= (1.004 \text{ [kJ/kg-K]}) \ln\left(\frac{425 \text{ [K]}}{293 \text{ [K]}}\right) - \dots \\ \dots &- (0.287 \text{ [kJ/kg-K]}) \ln\left(\frac{5 \text{ [bar]}}{1 \text{ [bar]}}\right) = -0.088 \text{ [kJ/kg-K]}\end{aligned}$$



► Solution to Example #1 using Tables:

For $n=1.4$:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{1}} \Rightarrow T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = 293 \text{ [K]} \left(\frac{5 \text{ [bar]}}{1 \text{ [bar]}}\right)^{\frac{1.4-1}{1.4}} = 464.1 \text{ [K]}$$

Therefore, work per mass basis

$$\frac{W_{1 \rightarrow 2}}{m} = \frac{R(T_2 - T_1)}{1 - n} = \frac{0.287 \text{ [kJ/kg-K]}(464.1 - 293) \text{ [K]}}{1 - 1.4}$$

$$\frac{W_{1 \rightarrow 2}}{m} = -121.5 \text{ [kJ/kg]}$$

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Summary



The heat added comes from the 1st Law:

$$\begin{aligned}\frac{Q_{1 \rightarrow 2}}{m} &= (u_2 - u_1) + \frac{W_{1 \rightarrow 2}}{m} = C_{v0}(T_2 - T_1) + \frac{W_{1 \rightarrow 2}}{m} \\ &= 0.717 \text{ [kJ/kg-K]}(464.1 - 293) \text{ [K]} 1.2 \text{ [kJ/kg]} \\ \frac{Q_{1 \rightarrow 2}}{m} &= -31.64 \text{ [kJ/kg]}\end{aligned}$$

The change of entropy is calculated as:

$$\begin{aligned}ds &= C_{P0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) \\ &= (1.004 \text{ [kJ/kg-K]}) \ln\left(\frac{464.1 \text{ [K]}}{293 \text{ [K]}}\right) - \dots \\ \dots &- (0.287 \text{ [kJ/kg-K]}) \ln\left(\frac{5 \text{ [bar]}}{1 \text{ [bar]}}\right) = -1 \cdot 10^{-4} \text{ [kJ/kg-K]}\end{aligned}$$

