# Chapter 6 - Entropy

Lecture 18

Section 6.1

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh

# Student Learning Objectives



At the end of the lecture, students should be able to:

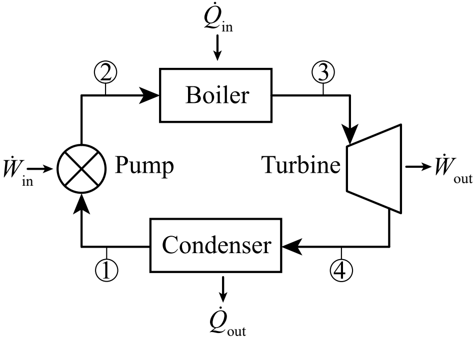
- Apply the Clausius Inequality to quantify the amount of irreversibility within a system

# Clausius Inequality

- The Clausius Inequality is mathematically stated as
$$\oint \left(\frac{\delta Q}{T}\right)_b \leq 0$$
- $\oint$ represents the cyclic integral
- The term $\delta Q$ represents the heat transfer at a part of the system boundary during a part of the cycle
- $T$ is the absolute temperature of the location on the boundary, $b$ at which $\delta Q$ is transferred
- The cyclic integral of heat transfer is expressed as
$$\oint \delta Q = Q_H - Q_L > 0$$

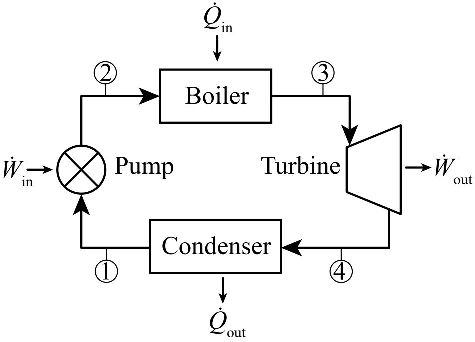
# Irreversible Cycles

- Consider an irreversible cycle, an irreversible Rankine cycle



# Irreversible Cycles

- The inlet to the boiler is sat. liquid at 700 [kPa]
- The outlet of the boiler is sat. vapor
- The output of the turbine is a fluid with a quality of 0.9 and a pressure of 15 [kPa]
- The outlet of the condenser is a fluid with a quality of 0.1



# Irreversible Cycles



- The heat into the boiler, on a per mass basis

$$q_{2\rightarrow 3} = h_3 - h_2 = 2,066.3\,[\text{kJ/kg}]$$

- The heat is added at a saturation temperature of 164.97 °C

- A condenser is also a constant pressure, constant temperature process (going from a high to low quality), thus the heat rejected at a saturation temperature of 53.97 °C is

$$q_{4\rightarrow 1} = h_1 - h_4 = -1,898.4\,[\text{kJ/kg}]$$

# Irreversible Cycles



- Evaluating the cyclic integral, which is the sum of the heat transfer processes at their respective boundary temperatures

$$\oint \frac{\delta Q}{T} = \int_2^3 \left( \frac{\delta Q}{T} \right)_{T_H} + \int_4^1 \left( \frac{\delta Q}{T} \right)_{T_L}$$

- The boundary temperatures are constant

$$\oint \frac{\delta Q}{T} = \frac{1}{T_H} \int_2^3 \delta Q_H + \frac{1}{T_L} \int_4^1 \delta Q_L$$

- Thus, on a per mass basis

$$\oint \frac{\delta q}{T} = \frac{q_{2\rightarrow 3}}{T_H} + \frac{q_{4\rightarrow 1}}{T_L}$$

# Irreversible Cycles



- Substituting in the values

$$\oint \frac{\delta q}{T} = \frac{2,066.3\,[\text{kJ/kg}]}{438.12\,[\text{K}]} - \frac{1,898.4\,[\text{kJ/kg}]}{337.12\,[\text{K}]}$$

$$= 4.716 - 5.803 = -1.807\,[\text{kJ/kg-K}]$$

- We conclude that for an irreversible cycle that the heat rejected per boundary temperature will be greater than the heat accepted per boundary temperature

$$\boxed{\oint \left(\frac{\delta Q}{T}\right)_b < 0}$$

# Irreversible Cycles

- That is, as irreversibities increase to $\infty$

$$W = Q_H - Q_L \to 0$$

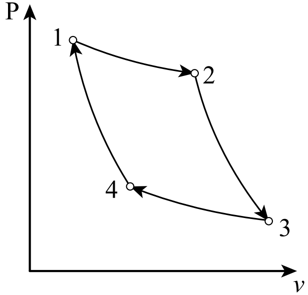
- Which means

$$Q_H \to Q_L \implies \oint \delta Q = 0,$$

- Since $T_L < T_H$, $\oint \dfrac{\delta Q}{T}$ becomes more negative
- Increasing negativity of $\oint \dfrac{\delta Q}{T}$ indicates greater irreversibilities are present within the system
- But what happens when $\oint \dfrac{\delta Q}{T}$ becomes less negative?

# Reversible Cycle

- Taking the Carnot cycle (a reversible heat engine) for example




- There are two heat transfer processes (1→2, 3→4)

# Reversible Cycles



- The cyclic integral of heat transfer is expressed as

$$\oint \delta Q = Q_H - Q_L > 0$$

- That is, as irreversibilities approach 0, $W \rightarrow$ a maximum values, which means $Q_H$ must be greater than $Q_L$
- Therefore, the Clausius Inequality would be expressed as

$$\oint \left(\frac{\delta Q}{T}\right)_b = \int \left(\frac{\delta Q}{T}\right)_{T_H} - \int \left(\frac{\delta Q}{T}\right)_{T_L}$$

# Reversible Cycles



- Since the boundary temperatures are constant

$$= \frac{1}{T_H}\int \delta Q_H - \frac{1}{T_L}\int \delta Q_L = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

- As $T_H \rightarrow T_L$, the cyclic integral of heat $\oint \delta Q$ approaches zero

- That is, the cyclic integral of heat per boundary temperature must also tend to zero

$$\boxed{\oint \left(\frac{\delta Q}{T}\right)_b = 0}$$

# Student Learning Objectives



At the end of the lecture, students should be able to:

- Apply the Clausius Inequality to quantify the amount of irreversibility within a system
  - The Clausis Inequality states that the cyclic integral of heat transfer per the respective boundary temperature must be less than or equal to zero - it is equal to zero when the cycle is reversible, and less than zero when the cycle is irreversible.

# Suggested Problems

- 6.17, 6.18, 6.19, 6.20, 6.21, 6.22, 6.23