# Chapter 2 - Properties of a Pure Substance

## Lecture 4
## Sections 2.4, 2.6-2.7

### MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh

# Student Learning Objectives

At the end of the lecture, students should be able to:

▶ Understand how to use the thermodynamics tables

▶ Be able to identify the compressed/subcooled and saturated liquid regions and superheated vapor region on a $T$-$v$ diagram, as well as in the steam tables

▶ Be able to interpolate between two published values

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# Thermodynamic Tables

▶ A **steam table** has tabulated values of properties based upon temperature and pressure, reflecting the liquid and vapor phases

▶ A helpful graph, shown below, shows which table to use if you have a subcooled, compressed or saturated liquid or saturated or superheated vapor

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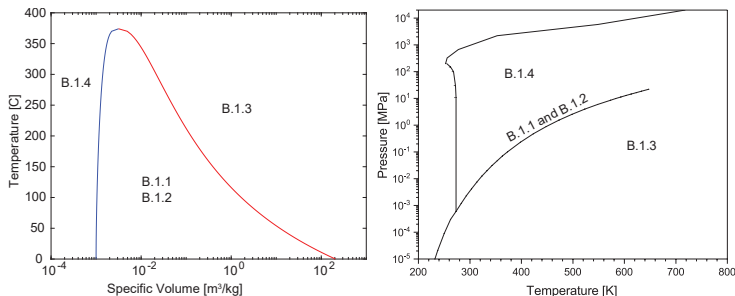
# Saturated Water

▶ The saturated water temperature-entry table, B.1.1, starts on page 776

**776** ■ APPENDIX B SI UNITS: THERMODYNAMIC TABLES

TABLE B.1
*Thermodynamic Properties of Water*

TABLE B.1.1
*Saturated Water*

| Temp. ($^\circ$C) | Press. (kPa) | Specific Volume, m³/kg | | | Internal Energy, kJ/kg | | |
|---|---|---|---|---|---|---|---|
| | | Sat. Liquid $v_f$ | Evap. $v_{fg}$ | Sat. Vapor $v_g$ | Sat. Liquid $u_f$ | Evap. $u_{fg}$ | Sat. Vapor $u_g$ |
| 0.01 | 0.6113 | 0.001000 | 206.131 | 206.132 | 0 | 2375.33 | 2375.33 |
| 5 | 0.8721 | 0.001000 | 147.117 | 147.118 | 20.97 | 2361.27 | 2382.24 |
| 10 | 1.2276 | 0.001000 | 106.376 | 106.377 | 41.99 | 2347.16 | 2389.15 |

Image taken from Borgnakke & Sonntage, 8e

▶ The **saturated liquid** values of $v$, $u$, etc. are denoted by a subscript $f$

▶ The **saturated vapor** values of $v$, $u$, etc. are denoted by a subscript $g$

▶ The difference between $g$ and $f$ is the subscript $fg$

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# Saturated Water

▶ The saturated water, pressure-entry table, B.1.2, starts on page 780

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TABLE B.1.2
*Saturated Water Pressure Entry*

| Press. (kPa) | Temp. (°C) | Specific Volume, m³/kg | | | Internal Energy, kJ/kg | | |
|---|---|---|---|---|---|---|---|
| | | Sat. Liquid $v_f$ | Evap. $u_{fg}$ | Sat. Vapor $v_g$ | Sat. Liquid $u_f$ | Evap. $u_{fg}$ | Sat. Vapor $u_g$ |
| 0.6113 | 0.01 | 0.001000 | 206.131 | 206.132 | 0 | 2375.3 | 2375.3 |
| 1 | 6.98 | 0.001000 | 129.20702 | 129.20802 | 29.29 | 2355.69 | 2384.98 |
| 1.5 | 13.03 | 0.001001 | 87.97913 | 87.98013 | 54.70 | 2338.63 | 2393.32 |
| 2 | 17.50 | 0.001001 | 67.00285 | 67.00385 | 73.47 | 2326.02 | 2399.48 |
| 2.5 | 21.08 | 0.001002 | 54.25285 | 54.25385 | 88.47 | 2315.93 | 2404.40 |
| 3 | 24.08 | 0.001003 | 45.66402 | 45.66502 | 101.03 | 2307.48 | 2408.51 |

Image taken from Borgnakke & Sonntage, 8e

▶ Note: you should consult both tables if the substance is given as saturated

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# Example #1

▶ Determine the saturated vapor specific volume $v_g$ of water existing at 225 [kPa] and 124.00 °C.

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# Compressed Liquid

▶ It is noted a **compressed liquid** refers both to the **compressed liquid** state and the **subcooled liquid** state

▶ What you notice is that the specific volume is strongly dependent on temperature and weakly dependent on pressure, such that if you are looking for $\nu$ at an elevated pressure and know the temperature, you can use the saturated water temperature-entry table using $T$:

$$\nu \approx \nu(T) = \nu_f$$

▶ This is because water is an incompressible substance:

$$\nu \not\approx \nu(P)$$

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# Compressed Liquid

► The compressed liquid water table, B.1.4, starts on page 790

**790** ■ APPENDIX B SI UNITS: THERMODYNAMIC TABLES

TABLE B.1.4
*Compressed Liquid Water*

| Temp. (°C) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) |
|---|---|---|---|---|---|---|---|---|
| | 500 kPa (151.86°C) | | | | 2000 kPa (212.42°C) | | | |
| Sat. | 0.001093 | 639.66 | 640.21 | 1.8606 | 0.001177 | 906.42 | 908.77 | 2.4473 |
| 0.01 | 0.000999 | 0.01 | 0.51 | 0.0000 | 0.000999 | 0.03 | 2.03 | 0.0001 |
| 20 | 0.001002 | 83.91 | 84.41 | 0.2965 | 0.001001 | 83.82 | 85.82 | .2962 |
| 40 | 0.001008 | 167.47 | 167.98 | 0.5722 | 0.001007 | 167.29 | 169.30 | .5716 |
| 60 | 0.001017 | 251.00 | 251.51 | 0.8308 | 0.001016 | 250.73 | 252.77 | .8300 |
| 80 | 0.001029 | 334.73 | 335.24 | 1.0749 | 0.001028 | 334.38 | 336.44 | 1.0739 |

Image taken from Borgnakke & Sonntage, 8e

► Note the sub-pressure entry tables

► Note there are no saturated states, i.e. $f$ or $g$

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# Example #2

▶ Determine the specific volume $v$ of water existing at 10,000 [kPa] and 60 °C using the compressed liquid water table B.1.4.

▶ How does this compare to the value of $v_f(60 \text{ °C})$ published on the saturated water temperature-entry table B.1.1.?

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# Superheated Vapor

► A **superheated vapor** state is when $T$ is greater than $T_{sat}$ for a given $P_{sat}$

► Alternatively, a superheated vapor occurs when $P$ is less than $P_{sat}$ for a given $T_{sat}$

► The superheated vapor water table, B.1.3, starts on page 784

**784** ■ APPENDIX B SI UNITS: THERMODYNAMIC TABLES

TABLE B.1.3
*Superheated Vapor Water*

| Temp. (°C) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) | $v$ (m³/kg) | $u$ (kJ/kg) | $h$ (kJ/kg) | $s$ (kJ/kg-K) |
|---|---|---|---|---|---|---|---|---|
| | $P = 10$ kPa (45.81°C) | | | | $P = 50$ kPa (81.33°C) | | | |
| Sat. | 14.67355 | 2437.89 | 2584.63 | 8.1501 | 3.24034 | 2483.85 | 2645.87 | 7.5939 |
| 50 | 14.86920 | 2443.87 | 2592.56 | 8.1749 | — | — | — | — |
| 100 | 17.19561 | 2515.50 | 2687.46 | 8.4479 | 3.41833 | 2511.61 | 2682.52 | 7.6947 |
| 150 | 19.51251 | 2587.86 | 2782.99 | 8.6881 | 3.88937 | 2585.61 | 2780.08 | 7.9400 |
| 200 | 21.82507 | 2661.27 | 2879.52 | 8.9037 | 4.35595 | 2659.85 | 2877.64 | 8.1579 |
| 250 | 24.13559 | 2735.95 | 2977.31 | 9.1002 | 4.82045 | 2734.97 | 2975.99 | 8.3555 |

Image taken from Borgnakke & Sonntage, 8e

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# Example #3

▶ Suppose you are looking for the specific volume of water at 400 °C and a pressure of 22,089 [kPa]. Is this possible?

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# Logic

▶ A helpful interpretation of the $T$-$v$ diagram in terms of $T_{\text{sat}}$ and $P_{\text{sat}}$ helps direct which table you should consult:

| $T$ | $P$ | State | Table |
|---|---|---|---|
| $T=T_{\text{sat}}$ | $P=P_{\text{sat}}$ | Saturated Water | B.1.1-B.1.2 |
| $T<T_{\text{sat}}$ | $P=P_{\text{sat}}$ | Subcooled Liquid | B.1.4, B.1.1 |
| $T=T_{\text{sat}}$ | $P>P_{\text{sat}}$ | Compressed Liquid | B.1.4, B.1.1 |
| $T>T_{\text{sat}}$ | $P=P_{\text{sat}}$ | Superheated Vapor | B.1.3 |
| $T=T_{\text{sat}}$ | $P<P_{\text{sat}}$ | | |

▶ **IF THE VALUE IS NOT ON THE TABLE, YOU ARE DOING SOMETHING WRONG** - these tables are the backbone of all thermodynamics cycles - if they were wrong, modern civilization would not exist

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# Linear Interpolation

- ▶ What happens when we are interested in a value between published values?

- ▶ **Linear interpolation** assumes the slope between two known points is linear, and therefore any point on that line in reference to a published value will create a line of the same slope of the two published values

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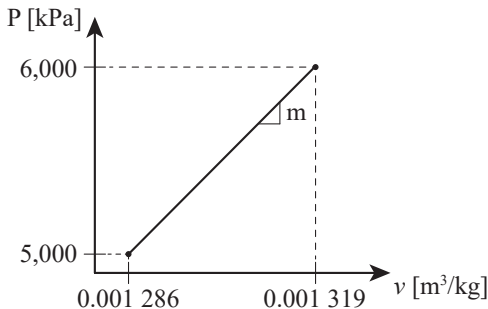# Linear Interpolation

▶ Let us consider a representative example. Say we want to determine the saturated liquid specific volume at a pressure of 5,328 [kPa].

▶ We see this pressure is bounded between 5,000 and 6,000 [kPa], which have corresponding $\nu_f$ of 0.001 286 and 0.001 319 [m$^3$/kg], respectively

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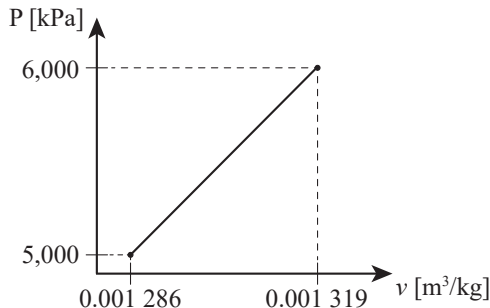
# Linear Interpolation

▶ The slope m is the rise per run:

$$m = \frac{(6{,}000 - 5{,}000)\,[\text{kPa}]}{(0.001\,319 - 0.001\,286)\,[\text{m}^3/\text{kg}]}$$

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# Linear Interpolation

▶ To find the specific volume at a pressure of 5,328 [kPa], we will plot the pressure value on the line existing between 5,000 and 6,000 [kPa], recognizing m′ is equal to m
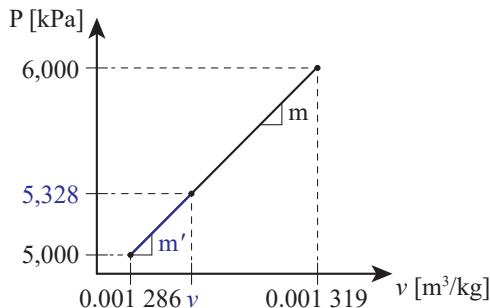


$$m' = \frac{(5,328 - 5,000)\,[\text{kPa}]}{(\nu - 0.001\,286)\,[\text{m}^3/\text{kg}]}$$

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# Linear Interpolation

▶ Setting the RHS of the expression of m and m′ equal:

$$\frac{(6,000 - 5,000)\,[\text{kPa}]}{(0.001\,319 - 0.001\,286)\,[\text{m}^3/\text{kg}]} = \frac{(5,328 - 5,000)\,[\text{kPa}]}{(\nu - 0.001\,286)\,[\text{m}^3/\text{kg}]}$$

▶ Or in terms of ratios of rises and runs:

$$\underbrace{\frac{(5,328 - 5,000)\,[\text{kPa}]}{(6,000 - 5,000)\,[\text{kPa}]}}_{0.328\,[\,]} = \frac{(\nu - 0.001\,286)\,[\text{m}^3/\text{kg}]}{(0.001\,319 - 0.001\,286)\,[\text{m}^3/\text{kg}]}$$

▶ Therefore:

$$\nu = 0.328(0.001\,319 - 0.001\,286)\,[\text{m}^3/\text{kg}] + 0.001\,286\,[\text{m}^3/\text{kg}]$$
$$= 0.001\,297\,[\text{m}^3/\text{kg}]$$

▶ Check you answer is between the upper and lower bounds of the value on the x-axis

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# Example #4

▶ Find $\nu_g(T=32.5\ °C)$ from the saturated water table.

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# Example #5

▶ Find $\nu(T=400\ °C, P=350\ [kPa])$ from the superheated vapor water table.

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# Student Learning Objectives

At the end of the lecture, students should be able to:

▶ Understand how to use the thermodynamics tables
  ▶ The thermodynamics tables are structured such that if you have two independent properties (temperature and pressure), you can pull pertinent values, such as specific volume, from the tables.

▶ Be able to identify the compressed/subcooled and saturated liquid regions and superheated vapor region on a $T$-$v$ diagram, as well as in the steam tables
  ▶ Tables B.1.1 and B.1.2 are used for saturated water, B.1.3 for superheated vapor, and B.1.4 for compressed/subcooled liquids.

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# Student Learning Objectives

▶ Be able to interpolate between two published values

    ▶ Given two published points, we can construct an equation describing the slope. If we have an intermediate value, i.e. between the two published points, we will assume the slope created by the intermediate point and either of the published points is equal to the slope of the two published points.

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# Suggested Problems

▶ Our text does not have problems focusing purely on interpolation.

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