Homework #5

MEMS 0051 - Introduction to Thermodynamics

Assigned: June 11th, 2020
Due: June 18th, 2020

Problem #1

A heat engine is operating at a thermal efficiency of $\eta_{th} = 40\%$ and produces 1,000 [kJ/hr]. The power produced is used to drive a refrigerator. The refrigerator removes the same amount of heat, $\dot{Q}_L$, that is rejected by the heat engine. The heat rejected by the refrigerator is then supplied to the heat engine.

a) What are $\dot{Q}_L$, $\dot{Q}_H$, and the COP of the refrigerator?

We first solve $\dot{Q}_H$ for the heat engine:

$$\dot{Q}_H = \frac{\dot{W}}{\eta_{th}} = \frac{1,000 \text{ [kJ/hr]}}{0.40} = 2,500 \text{ [kJ/hr]}$$

Now, we can solve $\dot{Q}_L$ for the heat engine, which is the amount of heat removed by the refrigerator:

$$\dot{Q}_L = \dot{Q}_H - \dot{W} = (2,500 - 1,000) \text{ [kJ/hr]} = \boxed{1,500 \text{ [kJ/hr]}}$$

Since the work produced by the heat engine is the work into the refrigerator, $\dot{Q}_H$ of the refrigerator can be found by:

$$\dot{Q}_H = \dot{W} + \dot{Q}_L = (1,000 + 1,500) \text{ [kJ/hr]} = \boxed{2,500 \text{ [kJ/hr]}}$$

Finally, the COP of the refrigerator can now be determined:

$$\beta = \frac{Q_L}{W} = \frac{1,500 \text{ [kJ/hr]}}{1,000 \text{ [kJ/hr]}} = \boxed{1.5}$$

b) Is this system realistic? Why or why not? No, this system is not realistic. The combination of the heat engine and refrigerator appears to run forever without needing any further heat or energy from their surroundings. This implies that there are no irreversibilities in the system, which for real systems, is not possible, and makes this a PMM of the 3rd kind.

c) Allow Q_L and/or Q_H to be any number. At what value are the thermal efficiency, η_{th}, and the COP, β, equal to one another? To determine values of Q_H, Q_L, and when $\eta = \beta$, start by equating the formulas for efficiency:

$$\eta = \beta \quad \Rightarrow \quad \frac{W}{Q_H} = \frac{Q_L}{W} \quad \Rightarrow \quad Q_H Q_L = W^2 \quad \Rightarrow \quad Q_H Q_L = (Q_H - Q_L)^2 \quad \Rightarrow \quad Q_H^2 - 3Q_H Q_L + Q_L^2 = 0$$

Solving the quadratic will lead to a required relationship between Q_H and Q_L, which is:

$$Q_H = \frac{3Q_L + \sqrt{5Q_L^2}}{2}$$

Having Q_H and Q_L follow this relationship causes $\eta = \beta$, and the value at which they are equal is:

$$\eta = \beta = \boxed{0.618}$$

Problem #2

Consider 2 [kg] of nitrogen, N_2, in a piston-cylinder undergoing a Carnot cycle between temperature reservoirs at 300 [K] and 1,100 [K]. The isothermal heat addition process results in the volume expanding from an initial value of 2.0 [m^3] to 6.0 [m^3]. The nitrogen (N_2) may be treated as an ideal gas. Use the tabulated values in Table A.8 to determine changes in internal energy (because u_2-u_1=$C_{\forall O}(T_2$-$T_1)$ is no longer a good approximation at temperatures as high as 1,100 [K].)

a) Calculate the Carnot efficiency of this cycle.

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\,[\text{K}]}{1,100\,[\text{K}]} = \boxed{0.73}$$

b) Determine the heat added during the isothermal expansion (Q_H) and the heat rejected during the isothermal compression (Q_L).
As derived in lecture (by applying the 1^{st} Law to these two processes):

$$Q_H = mRT_H \ln\left(\frac{\nu_2}{\nu_1}\right), \quad Q_L = mRT_L \ln\left(\frac{\nu_3}{\nu_4}\right)$$

Recall that for the ideal gas Carnot cycle, ν_3/ν_4=ν_2/ν_1. Subbing in values: ν_3/ν_4=ν_2/ν_1=6.0/2.0 = 3
Now solving for the heat transfers:

$$Q_H = (2\ [\text{kg}])(0.2968\,[\text{kJ/kg-K}])(1,100\,[\text{K}])\ln(3) = \boxed{717.35\,[\text{kJ/kg}]}$$

$$Q_L = (2\ [\text{kg}])(0.2968\,[\text{kJ/kg-K}])(300\,[\text{K}])\ln(3) = \boxed{195.64\,[\text{kJ/kg}]}$$

c) Determine the changes in internal energy for the adiabatic expansion and the adiabatic compression processes;
From Table A.8, we have for specific internal energy:

$$u_2 = u_1 = u_{@1,100\,[\text{K}]} = 867.14\ [\text{kJ/kg}] \qquad u_4 = u_3 = u_{@300\,[\text{K}]} = 222.63\ [\text{kJ/kg}]$$

For the adiabatic expansion:

$$U_3 - U_2 = m(u_3 - u_2) = (2\ [\text{kg}])((222.63 - 867.14)\ [\text{kJ/kg}]) = \boxed{-1,289.02\ [\text{kJ}]}$$

The change in internal energy for the adiabatic compression process must be opposite sign but same magnitude as that for the adiabatic expansion. So:

$$U_1 - U_4 = m(u_1 - u_4) = (2\ [\text{kg}])((867.14 - 222.63)\ [\text{kJ/kg}]) = \boxed{1,289.02\ [\text{kJ}]}$$

d) Determine the net work done by this cycle; The net work done by this cycle is simply the heat in minus the heat out:

$$W_{net} = Q_H - Q_L = (717.35 - 195.64)\ [\text{kJ/kg}] = \boxed{521.71\ [\text{kJ}]}$$

Problem #3

A powerplant is operating in a cycle with the following information given:

- Boiler: 1 [MW] of heat added to the water
- Turbine: ? [MW] of power produced by the turbine
- Condenser: 0.58 [MW] of heat removed from the water
- Pump: 0.02 [MW] of power required to operate the pump

With this information, answer the following:

a) What is the thermal efficiency, η_{th}?
The thermal efficiency can be calculated by using the known values of $\dot{Q}_H$ and $\dot{Q}_L$. Solving then:

$$\eta_{th} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = \frac{(1 - 0.58)\ [\text{MW}]}{1\ [\text{MW}]} = \boxed{0.42}$$

Note that the net power produced by the cycle is:

$$\dot{W}_{net} = \dot{Q}_H - \dot{Q}_L = (1 - 0.58)\ [\text{MW}] = 0.42\ [\text{MW}]$$

b) What is the power produced by the turbine?
The cyclic integral of work is equal to the cyclic integral of heat. We can write out the existing parts of the cycle to find the power produced by the turbine:

$$\dot{Q}_H - \dot{Q}_L = \dot{W}_{turb} - \dot{W}_{pump}$$

Rearranging to find the turbine power:

$$\dot{W}_{turb} = (1 - 0.58 + 0.02)\ [\text{MW}] = \boxed{0.44\ [\text{MW}]}$$

c) If the process was reversed and the system could be operated as a refrigerator, what would the COP value be?
The COP value is simply:

$$\beta = \frac{\dot{Q}_L}{\dot{Q}_H - \dot{Q}_L} = \frac{0.58\ [\text{MW}]}{(1 - 0.58)\ [\text{MW}]} = \boxed{1.381}$$

Problem #4

A heat engine operates between a high-temperature reservoir T_{H1} and a low-temperature reservoir T_{ambient}. The work produced, $\dot{W}_1$, which is the difference of heat input $\dot{Q}_{H1}$ and heat rejected $\dot{Q}_{L1}$, powers a heat pump. Part of the work from the heat engine enters the heat pump $\dot{W}_2$, whereas the difference between $\dot{W}_1$ and $\dot{W}_2$ is designated as the net work, $\dot{W}_{\text{net}}$. The heat pump accepts heat $\dot{Q}_{L2}$ from the same low-temperature reservoir (T_{ambient}) and rejects heat $\dot{Q}_{H2}$ to a secondary high-temperature reservoir T_{H2}. Assuming $T_{H1} > T_{H2} > T_{\text{ambient}}$, determine, based upon the following cases (a-f), if this system satisfies the First Law and/or violates the Second Law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	5	3	2	3	2	1
b	6	4	2	5	4	1
c	5	4	1	3	1	2
d	3	2	1	4	3	1
e	5	3	2	2	2	0
f	6	7	1	5	4	1

With $T_{H1} > T_{H2} > T_{\text{ambient}}$:

a)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 5 - 3 = 2 \checkmark \text{ First Law}$$
$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 3 - 2 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 1 = 1 \geq 0 \checkmark \text{ First Law}$$
$$\dot{Q}_{L1} > \dot{Q}_{L2}\ \&\ \dot{Q}_{H1} > \dot{Q}_{H2} \checkmark \text{ Second Law - agreement of K.P./Clausius statement}$$

b)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 6 - 4 = 2 \checkmark \text{ First Law}$$
$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 5 - 4 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 1 = 1 \geq 0 \checkmark \text{ First Law}$$
$$\dot{Q}_{L1} = \dot{Q}_{L2}\ \&\ \dot{Q}_{H1} > \dot{Q}_{H2} \implies \times\ \text{Second Law - violation of K.P. statement}$$

This is a violation of the Second Law for there is only a net exchange of heat with the high-temperature reservoirs, for the same amount of heat is rejected and pulled from the low-temperature reservoir.

c)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 5 - 4 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 3 - 1 = 2 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 1 - 2 = -1 < 0 \times \text{ First Law}$$

This is a violation of the First law as since the system can only produce a zero or positive net work, energy must be created to power the heat pump.

$$\dot{Q}_{L2} > \dot{Q}_{L1} \,\&\, \dot{Q}_{H1} > \dot{Q}_{H2} \implies \checkmark \text{ Second Law - agreement of K.P./Clausius statement}$$

d)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 3 - 2 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 4 - 3 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 1 - 1 = 0 \geq 0 \checkmark \text{ First Law}$$
$$\dot{Q}_{L1} < \dot{Q}_{L2} \,\&\, \dot{Q}_{H1} < \dot{Q}_{H2} \implies \times \text{ Second Law - violation of Clausius statement}$$

This is violation of the Second law for, although the heat engine provides enough work to run the heat pump (unity), there is a net heat transference from the low-temperature to high-temperature reservoir.

e)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 5 - 3 = 2 \checkmark \text{ First Law}$$
$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 2 - 2 = 0 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 0 = 0 \geq 0 \checkmark \text{ First Law}$$
$$\dot{Q}_{L1} > \dot{Q}_{L2} \,\&\, \dot{Q}_{H1} > \dot{Q}_{H2} \implies \times \text{ Second Law - violation of Clausius statement}$$

This is violation of the Second law for heat is moving from the cold temperature reservoir to the hot temperature reservoir with zero work input.

f)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 6 - 7 = -1 \times \text{ First Law}$$

This is a violation of the First Law as energy must be created from nothing to equal the listed work output ($\dot{W}_1$) from the heat engine.

$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 5 - 4 = 1 \checkmark \text{ First Law}$$
$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 1 - 1 = 0 \geq 0 \checkmark \text{ First Law}$$
$$\dot{Q}_{L1} > \dot{Q}_{L2} \,\&\, \dot{Q}_{H1} > \dot{Q}_{H2} \implies \times \text{ Second Law - agreement of K.P./Clausius statement}$$

This is violation of the Second law for heat is moving from the cold temperature reservoir to the hot temperature reservoir and even producing work.