Homework #6

MEMS 0051 - Introduction to Thermodynamics

Assigned February 23rd, 2018
Due: March 2nd, 2018

Problem #1

1. Calculate the Carnot efficiency of a heat engine operating between the following reservoir temperature combinations:

   a) T_H=500 [K]. T_L=250 [K]

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{250\,[\text{K}]}{500\,[\text{K}]} = \boxed{0.5}$$

   b) T_H=1,000[K]. T_L=300 [K]

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300\,[\text{K}]}{1000\,[\text{K}]} = \boxed{0.7}$$

   c) T_H=600 [K]. T_L=50 [K]

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{50\,[\text{K}]}{600\,[\text{K}]} = \boxed{0.9167}$$

   d) T_H=205 [K]. T_L=105 [K]

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{105\,[\text{K}]}{205\,[\text{K}]} = \boxed{0.4878}$$

2. Based on your previous calculations, how is Carnot efficiency affected by raising T_H? What about raising T_L?

Increasing the T_H will increase the efficiency while increasing the T_L will decrease the efficiency. However, it should be noted that a modest temperature is needed to draw work from the heat coming from the hot temperature reservoir.

Problem #2

Assume a cyclic heat engine extracts energy ($\dot{Q}_H$) from a hot temperature reservoir having a temperature of 350 [°C] and the machine. If heat is rejected from the same machine and a cold temperature reservoir ($\dot{Q}_L$) having a temperature of 30 [°C], what can be said about the possibility of the machine for the following cases? First find the Carnot cycle efficiency. This will determine whether the heat engine is possible or not.

$$\eta_{\text{H.E}} \leq \eta_{\text{Carnot}}$$

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{303.15\,[\text{K}]}{623.15\,[\text{K}]} = 0.5135$$

a) $\dot{Q}_H$= 12 [kW], $\dot{Q}_L$= 6 [kW]

$$\eta_{\text{H.E}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 0.5$$

$\boxed{\text{The heat engine is possible}}$

b) $\dot{Q}_H$= 45 [kW], $\dot{Q}_L$= 13 [kW]

$$\eta_{\text{H.E}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 0.7111$$

$\boxed{\text{The heat engine is impossible}}$

c) $\dot{Q}_H = 30$ [kW], $\dot{Q}_L = 5$ [kW]

$$\eta_{\text{H.E}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 0.8333$$

$\boxed{\text{The heat engine is impossible}}$

d) $\dot{Q}_H = 2$ [kW], $\dot{Q}_L = 1$ [kW]

$$\eta_{\text{H.E}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 0.5$$

$\boxed{\text{The heat engine is possible}}$

e) $\dot{Q}_H = 21$ [kW], $\dot{Q}_L = 8$ [kW]

$$\eta_{\text{H.E}} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 0.619$$

$\boxed{\text{The heat engine is impossible}}$

Problem #3

200 [g] of liquid water at 35 [°C] is put into ice trays and left in a refrigerator at atmospheric pressure. Said refrigerator works in a Carnot cycle between a cold temperature reservoir at -8 [°C] and a hot temperature reservoir at 35 [°C]. The refrigerator motor-compressor has a power rating of 800 [W]. Assuming the water is the only substance being cooled, how many seconds will it take to bring the water to 0 [°C]? Assume a constant specific heat of 4.1855 [kJ/(kg K)].

First determine the coefficient of performance (COP) of the refrigerator:

$$\beta = \beta_{carnot} = \frac{T_L}{T_H - T_L} = \frac{-8[°\text{C}] + 273.15}{35[°\text{C}] + 273.15 - 8[°\text{C}] - 273.15} = 6.1663$$

The COP can then be used to determine the rate of cooling using the input work provided.

$$\beta = \frac{\dot{Q}_L}{\dot{W}} \implies$$

$$\beta\dot{W} = (6.1663)(-800[\text{W}]) = \dot{Q}_L = -4.93[\text{kW}]$$

Next, determine the change in internal energy using the specific heat equation.

$$\Delta U = mC_v\Delta T = (0.2[\text{kg}])\left(4.1855\left[\frac{\text{kJ}}{\text{kg-K}}\right]\right)\Big((0[°\text{C}] + 273.15) - (35[°\text{C}] + 273.15)\Big) = -29.2985[\text{kJ}]$$

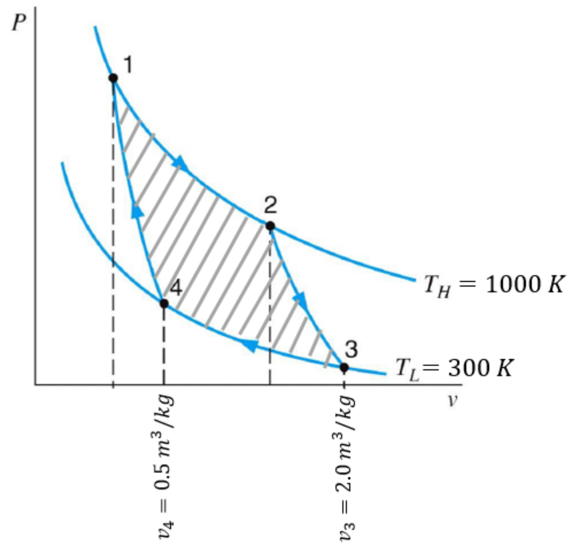
Lastly, we can determine the total time of cooling by determining the quotient of the change in internal energy and the rate of cooling.

$$t_{total} = \frac{\Delta U}{\dot{Q}_L} = \frac{-29.2985[\text{kJ}]}{-4.93[\text{kW}]} = \boxed{5.94\,[\text{s}]}$$

Problem #4

Consider N_2 in a piston-cylinder undergoing a Carnot cycle between temperature reservoirs at 300 [K] and 1,000 [K]. The isothermal compression process results in the specific volume shrinking from 2.0 [m^3/kg] to 0.5 [m^3/kg]. R=0.2968 [kJ/kg-K] for N_2, and you may treat it as an ideal gas. Use the tabulated values in Table A.8 to determine changes in internal energy (because u_2-u_1=$C_{\forall O}(T_2$-$T_1)$ isn't a good approximation at temperatures as high as 1,000 [K].)

a) Draw this cycle on a P-ν diagram. Label the low and high temperature isotherms, and also label the two given specific volumes.



b) Calculate the Carnot efficiency of this cycle.

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300}{1,000} = 0.7$$

c) Determine the heat added during the isothermal expansion (q_H) and the heat rejected during the isothermal compression (q_L).
As derived in lecture (by applying the 1$^{\text{st}}$ Law to these two processes):

$$q_H = RT_H \ln\left(\frac{\nu_2}{\nu_1}\right), \quad q_L = RT_L \ln\left(\frac{\nu_3}{\nu_4}\right)$$

Recall that for the ideal gas Carnot cycle, nu_2/ν_1=ν_3/ν_4. Subbing in values: ν_2/ν_1=ν_3/ν_4=2.0/0.5=4.
Now solving for the heat transfers:

$$q_H = (0.2968\,[\text{kJ/kg-K}])(1000\,[\text{K}])\ln(4) = 411.452\,[\text{kJ/kg}]$$

$$q_L = (0.2968\,[\text{kJ/kg-K}])(300\,[\text{K}])\ln(4) = 123.436\,[\text{kJ/kg}]$$

d) Determine the changes in internal energy for the adiabatic expansion and the adiabatic compression processes.
Because the high temperature reservoir is too high to use the following relationship with change in internal energy:

$$u_2 - u_1 = C_{\forall O}(T_2 - T_1)$$

Tabulated values must be taken from the table. Here, we get:

$$u_1 = u_2 = 779.11\ [\text{kJ/kg}]$$

$$u_3 = u_4 = 222.63\ [\text{kJ/kg}]$$

We find that the expansion takes place from state 2 to state 3 while the compression takes place at state 4 to state 1. Therefore, it can be said that:

$$q_{\text{expansion}} = u_2 - u_3 = 779.11 - 222.63 = 556.48\,[\text{kJ/kg}]$$

$$q_{\text{compression}} = u_4 - u_1 = 222.63 - 779.11 = -556.48\,[\text{kJ/kg}]$$

e) Determine the net work done by this cycle. Shade the net work on your P-ν diagram. The net work done by this cycle is simply the heat in minus the heat out:

$$w_{net} = q_H - q_L = 411.452 - 123.436 = 288.017\,[\text{kJ/kg}]$$

The net work (the enclosed area of 1-2-3-4-1) is shaded in the P-ν diagram above.