MEMS 0051
Summer 2018
Midterm #1
6/18/2018

Name (Print): ____________________________

This exam contains 7 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	20	
3	30	
4	40	
Total:	100	

BONUS (5 pts):

This date, June 18$^{\text{th}}$, 1940, marks the speech *whom* gave to the House of Commons stating that although the Battle of France may be over in the eyes of the Germans, "the Battle of Britain is about to begin." This speech is commonly referred to as "This was their finest hour."

Written Problem #1

1. (10 points) Determine the specific volume for water existing at 1,823 [kPa] and 462 °C.

Solution:

Water is existing as a superheated vapor. Referencing table B.1.3 on page 789, we will interpolate to find the specific volume of water existing at 462 °C and pressures of 1,800 and 2,000 [kPa]:

$$\frac{(462-400)\,[^\circ\mathrm{C}]}{(500-400)\,[^\circ\mathrm{C}]} = \frac{(\nu(1,800\,[\mathrm{kPa}]) - 0.16847)\,[\mathrm{m}^3/\mathrm{kg}]}{(0.19550-0.16847)\,[\mathrm{m}^3/\mathrm{kg}]} \implies \nu(1,800\,[\mathrm{kPa}]) = 0.18523\,[\mathrm{m}^3/\mathrm{kg}]$$

$$\frac{(462-450)\,[^\circ\mathrm{C}]}{(500-450)\,[^\circ\mathrm{C}]} = \frac{(\nu(2,000\,[\mathrm{kPa}]) - 0.16353)\,[\mathrm{m}^3/\mathrm{kg}]}{(0.17568-0.16353)\,[\mathrm{m}^3/\mathrm{kg}]} \implies \nu(2,000\,[\mathrm{kPa}]) = 0.16644\,[\mathrm{m}^3/\mathrm{kg}]$$

Now we can interpolate between pressure at our given temperature:

$$\frac{(1,823-1,800)\,[\mathrm{kPa}]}{(2,000-1,800)\,[\mathrm{kPa}]} = \frac{(\nu(1,823\,[\mathrm{kPa}]) - 0.18523)\,[\mathrm{m}^3/\mathrm{kg}]}{(0.16644-0.18523)\,[\mathrm{m}^3/\mathrm{kg}]} \implies \nu(1,823\,[\mathrm{kPa}]) = 0.18307\,[\mathrm{m}^3/\mathrm{kg}]$$

Written Problem #2

2. (20 points) 1 [kg] of water initially at 20 °C and 100 [kPa] is contained within a mass-less piston-cylinder device. Heat is added until the temperature reaches 110 °C. Determine:

   1. The quantity of heat required for the process.
   2. The work produced.

Solution:

Applying the Conservation of Energy to a closed system:

$$dE = \delta Q - \delta W \implies U_{\text{final}} - U_{\text{initial}} = Q_{\text{initial}\to\text{final}} - W_{\text{initial}\to\text{final}}$$

Thus, we need to determine the total change of internal energy between the initial and final states, as well as the work between said states. At State 1, the water exists as a compressed liquid, and the specific volume and specific internal energy is found using the Saturated Water Tables, Table B.1.1 on page 776:

$$\nu_1 = \nu_f(20\,[^\circ\mathrm{C}]) = 0.001002\,[\mathrm{m}^3/\mathrm{kg}]$$
$$u_1 = u_f(20\,[^\circ\mathrm{C}]) = 83.94\,[\mathrm{kJ/kg}]$$

The volume can be found based upon the specific volume and mass:

$$\forall_1 = \nu_1\, m = (0.001002\,[\mathrm{m}^3/\mathrm{kg}])(1\,[\mathrm{kg}]) = 0.001002\,[\mathrm{m}^3]$$

As the system proceeds in a constant-pressure process, it will intersect the vapor dome at the saturated liquid line (State 2). No quantities are needed. Then, as the system changes state from saturated liquid to saturated vapor, it will proceed in a constant-temperature, constant-pressure process to the saturated vapor line (State 3). No quantities needed. Lastly, as the system proceeds in a constant pressure process to a temperature of 110 °C, $T > T_{sat}$ which places the final state in the superheated vapor region (State 4). Using Table B.1.3 on page 784, the specific volume and specific specific internal energy are found via interpolation:

$$\frac{(110-99.6)\,[^\circ\text{C}]}{(150-99.6)\,[^\circ\text{C}]} = \frac{(\nu_4 - 1.69400)\,[\text{m}^3/\text{kg}]}{(1.93636-1.69400)\,[\text{m}^3/\text{kg}]} \implies \nu_4 = 1.7440\,[\text{m}^3/\text{kg}]$$

$$\frac{(110-99.6)\,[^\circ\text{C}]}{(150-99.6)\,[^\circ\text{C}]} = \frac{(u_4 - 2,506.06)\,[\text{kJ/kg}]}{(2,582.75-2,506.06)\,[\text{kJ/kg}]} \implies \nu_4 = 2,521.88\,[\text{kJ/kg}]$$

The volume at State 4 is simply the mass times specific volume:

$$\forall_4 = \nu_4\, m = (1.7440\,[\text{m}^3/\text{kg}])(1\,[\text{kg}]) = 1.7440\,[\text{m}^3]$$

Since the process is constant process (even through the vapor dome), the work is simply the pressure times change in volume:

$$W_{1\to4} = P(\forall_4 - \forall_1) = (100\,[\text{kPa}])(1.7440 - 0.001002)\,[\text{m}^3] = 174.2998\,[\text{kJ}]$$

Note the positive value indicates work done by the system. The change of internal energy between the final and initial states is the mass times the change of specific internal energy between the final and initial states:

$$U_{\text{final}} - U_{\text{initial}} = m(u_4 - u_1) = (1\,[\text{kg}])(2,521.88 - 83.94)\,[\text{kJ/kg}] = 2,437.94\,[\text{kJ}]$$

Lastly, the heat input is simply the change of internal energy plus work:

$$Q_{1\to4} = (174.2998 + 2,437.94)\,[\text{kJ}] = 2,612.24\,[\text{kJ}]$$

Written Problem #3

3. (30 points) A piston-cylinder contains air at Standard Temperature Standard Pressure (25 °C and 100 [kPa]) and has an initial volume of 0.855 [m^3]. The air is compressed to where the final temperature is 200 °C and a pressure of 750 [kPa]. Determine:

   1. The work required to compress the piston-cylinder.
   2. The heat removed from the piston-cylinder.
   3. Whether, given P_c=3.78 [MPa] and T_c=132.63 [K], if the Ideal Gas law is valid for this analysis.

Solution:

State 1:	$\to$	State 2:
T_1=298 K		T_2= 473 K
P_1=100 [kPa]		P_2=700 [kPa]
$\forall_1$=0.855 [m^3]		

To determine the work needed to compress the piston-cylinder, we need the final volume as well as the mass. The mass is found from using the Ideal Gas law at State 1:

$$m = \frac{P_1 \forall_1}{RT_1} = \frac{(100\,[\text{kPa}])(0.855\,[\text{m}^3])}{(0.287\,[\text{kJ/kg-K}])(298\,[\text{K}])} = 1.0\,[\text{kg}]$$

The volume at State 2 is found through the Ideal Gas law:

$$\forall_2 = \frac{mRT_2}{P_2} = \frac{(1.0\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])(473\,[\text{K}])}{750\,[\text{kPa}]} = 0.181\,[\text{m}^3]$$

The polytropic index n is solved for as:

$$n = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{\forall_2}{\forall_1}\right)} = \frac{\ln\left(\frac{100\,[\text{kPa}]}{750\,[\text{kPa}]}\right)}{\ln\left(\frac{0.181\,[\text{m}^3]}{0.855\,[\text{m}^3]}\right)} = 1.3$$

Therefore, the work from State 1 to 2 is:

$$W_{1\rightarrow 2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = \frac{((750\,[\text{kPa}])(0.181\,[\text{m}^3]) - (100\,[\text{kPa}])(0.855\,[\text{m}^3]}{1-1.3} = -167.5\,[\text{kJ}]$$

The heat removed from the piston-cylinder is found vi the Conservation of Energy:

$$Q_{1\rightarrow 2} = m(u_2 - u_1) + W_{1\rightarrow 2}$$

The internal energy of air at State 1 and State 2 is found from Table A.7.1 on page 762:

$$u_1 = 213.04\,[\text{kJ/kg}]$$

$$\frac{(473-460)\,[\text{K}]}{(480-460)\,[\text{K}]} = \frac{(u_2 - 330.31)\,[\text{kJ/kg}]}{(345.04-330.31)\,[\text{kJ/kg}]} \implies u_2 = 339.88\,[\text{kJ/kg}]$$

Therefore:

$$Q_{1\rightarrow 2} = (1.0\,[\text{kg}])((339.88-213.04)\,[\text{kJ/kg}]) - 167.5\,[\text{kJ}] = -40.66\,[\text{kJ}]$$

To determine if the Ideal Gas law was suitable for use, take the maximum pressure and minimum temperature to calculated the reduced values;

$$P_r = \frac{P_{\text{max}}}{P_c} = \frac{750\,[\text{kPa}]}{3,780\,[\text{kPa}]} = 0.198 \ll 1$$

$$T_r = \frac{T_{\text{min}}}{T_c} = \frac{298\,[\text{K}]}{132.63\,[\text{K}]} = 2.25 > 2$$

Written Problem #4

4. (40 points) A system contains 1 [kg] of water and undergoes a power cycle, which is comprised of the following processes:

- *Process 1 → 2* - Constant pressure heating at 1,000 [kPa] from saturated vapor.

- *Process 2 → 3* - Constant volume cooling to P_3 = 500 [kPa], T_3= 160 °C
- *Process 3 → 4* - Isothermal compression with $Q_{3\to4}$ = -815.8 [kJ].
- *Process 4 → 1* - Constant volume heating

Determine the following:

1. This process schematically on P-ν and T-ν diagrams.
2. Determine the heat supplied to the system in *Process 1 → 2.*
3. Determine the work done by the system in *Process 1 → 2.*
4. Determine the heat rejected in *Process 2 → 3.*
5. Determine the work done onto the system in *Process 3 → 4.*
6. Determine the heat supplied in *Process 4 → 1.*

Solution:

The first process is constant-pressure heating from saturated vapor at P_1=P_2=1,000 [kPa] to an elevated temperature, where the specific volume at State 2 is equal to that at State 3 (P_3=500 [kPa], T_3=160 °C), i.e. ν_2=ν_3. Thus, the heat supplied in the process from State 1 to 2 is merely the change of enthalpy:

$$Q_{1\to2} = m(h_2 - h_1)$$

The enthalpy at State 1 is h_g(1,000 [kPa])=2,778.08 [kJ/kg] from Table B.1.2. To determine the enthalpy at State 2, the specific volume at State 3 must be known. The specific volume for State 3 is determined from interpolation from Table B.1.3:

$$\frac{(160 - 151.86)\,[^\circ\text{C}]}{(200 - 151.86)\,[^\circ\text{C}]} = \frac{(\nu_3 - 0.37489)\,[\text{m}^3/\text{kg}]}{(0.42492 - 0.37489)\,[\text{m}^3/\text{kg}]} \implies \nu_3 = \nu_2 = 0.38335\,[\text{m}^3/\text{kg}]$$

To determine the enthalpy at State 2, reference the 1,000 [kPa] entry on Table B.1.3 and interpolate between the specific volume at State 2, i.e. between the 500 and 600 °C entries:

$$\frac{(0.38335 - 0.35411)\,[\text{m}^3/\text{kg}]}{(0.40109 - 0.35411)\,[\text{m}^3/\text{kg}]} = \frac{(h_2 - 3,478.44)\,[\text{kJ/kg}]}{(3,697.85 - 3,478.44)\,[\text{kJ/kg}]} \implies h_2 = 3,615.0\,[\text{kJ/kg}]$$

Therefore, the heat required for the first process is:

$$Q_{1\to2} = m(h_2 - h_1) = (1\,\text{kg}])(3,615.0 - 2,788.08)\,[\text{kJ/kg}] = 836.92\,[\text{kJ}]$$

The work done in the process from State 1 to 2 is simply the mass times pressure times change of specific volume. The specific volume at State 1 is ν_1=ν_g(1,000 [kPa])=0.19444 [m^3/kg]. The specific volume at State 2 is the same as State 3. Therefore, the work from State 1 to 2 is:

$$W_{1\to2} = mP(\nu_2 - \nu_1) = (1\,[\text{kg}])(1,000\,[\text{kPa}])(0.38335 - 0.19444)\,[\text{m}^3/\text{kg}] = 188.95\,[\text{kJ}]$$

To determine the heat required from State 2 to 3, it is evident that only a change of internal energy needs to be calculated, since there is no work in an isochoric process. The internal energy at State 2 is found via interpolation between specific volume (Table B.1.3, 1,000 [kPa] entry):

$$\frac{(0.38335 - 0.35411)\,[\text{m}^3/\text{kg}]}{(0.40109 - 0.35411)\,[\text{m}^3/\text{kg}]} = \frac{(u_2 - 3,124.34)\,[\text{kJ/kg}]}{(3,296.76 - 3,124.34)\,[\text{kJ/kg}]} \implies u_2 = 3,231.65\,[\text{kJ/kg}]$$

The internal energy at State 3 is found in the same manner (i.e. interpolation) as specific volume such that:

$$\frac{(160-151.86)\,[^{\circ}\text{C}]}{(200-151.86)\,[^{\circ}\text{C}]}=\frac{(u_3-2{,}561.23)\,[\text{kJ/kg}]}{(2{,}642.91-2{,}561.23)\,[\text{kJ/kg}]}\implies u_3=2{,}575.04\,[\text{kJ/kg}]$$

Therefore, the heat required from State 2 to 3 is:

$$Q_{2\to3}=m(u_3-u_2)=(1\,[\text{kg}])(2{,}575.04-3{,}231.65)\,[\text{kJ/kg}]=-656.61\,[\text{kJ}]$$

To determine the work done in the process of going from State 3 to 4, we recognize the process is isothermal only outside the vapor dome; inside the vapor dome, the process is constant pressure when evaluating work. We will break this down into two steps, the first being isothermal between State 3 and the saturated vapor state at 160 °C, and the second being isobaric between the saturated vapor state at 160 °C and State 4:

$$W_{3\to4}=P_3\forall_3\ln\left(\frac{V_g(160\,[^{\circ}\text{C}])}{V_3}\right)+P_4(\forall_g(160\,[^{\circ}\text{C}])-\forall_4)$$

$$W_{3\to4}=(500\,[\text{kPa}])(0.38335\,[\text{m}^3])\ln\left(\frac{0.30706\,[\text{m}^3]}{0.38335\,[\text{m}^3]}\right)+(617.8\,[\text{kPa}])(0.19444-0.30706)\,[\text{m}^3]=-112.14\,[\text{kJ}]$$

Lastly, the heat required for the process from State 4 to 1 is equal to the mass of the system times the change of internal energy. The internal energy at State 1 is u_1=u_g(1,000 [kPa])=2,583.64 [kJ/kg]. The internal energy at State 4 is found via the determination of quality of saturated water existing at 160 °C based upon the known specific volume:

$$x=\frac{\nu_4-\nu_f(160\,^{\circ}\text{C})}{\nu_g(160\,^{\circ}\text{C})-\nu_f(160\,^{\circ}\text{C})}=\frac{u_4-u_f(160\,^{\circ}\text{C})}{u_g(160\,^{\circ}\text{C})-u_f(160\,^{\circ}\text{C})}$$

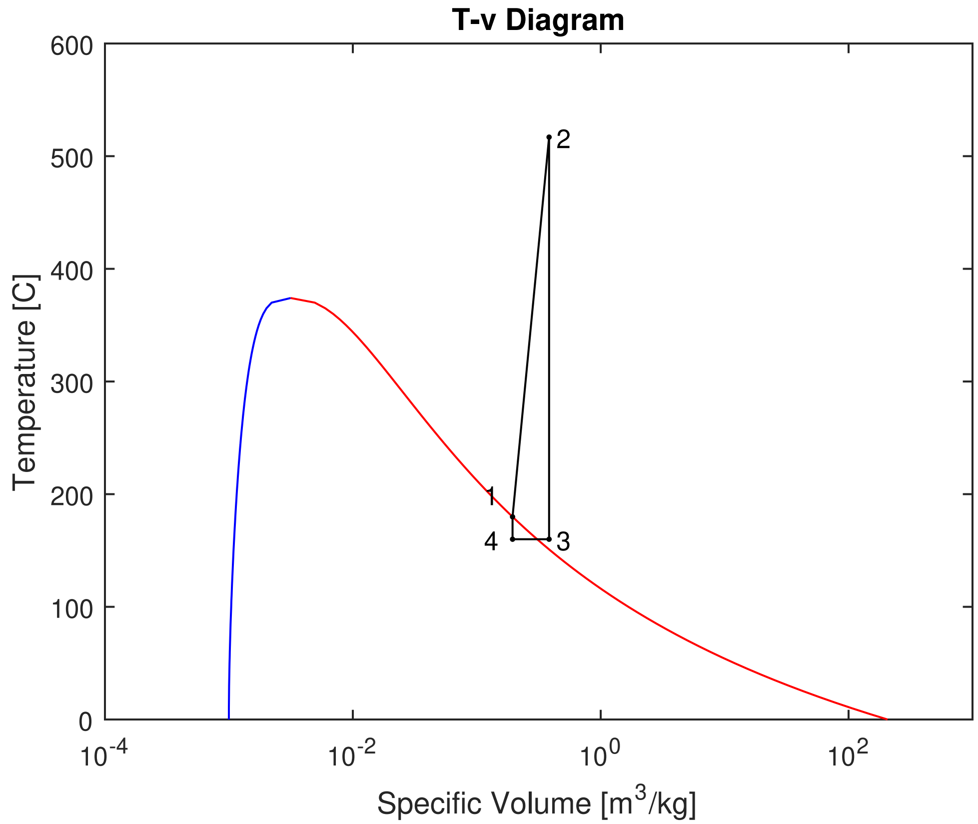
$$\implies u_4=\left(\frac{(0.19444-0.001102)\,[\text{m}^3/\text{kg}]}{(0.30596)\,[\text{m}^3/\text{kg}]}\right)(1{,}893.52\,[\text{kJ/kg})+(674.85\,[\text{kJ/kg}])=1{,}871.38\,[\text{kJ}]$$

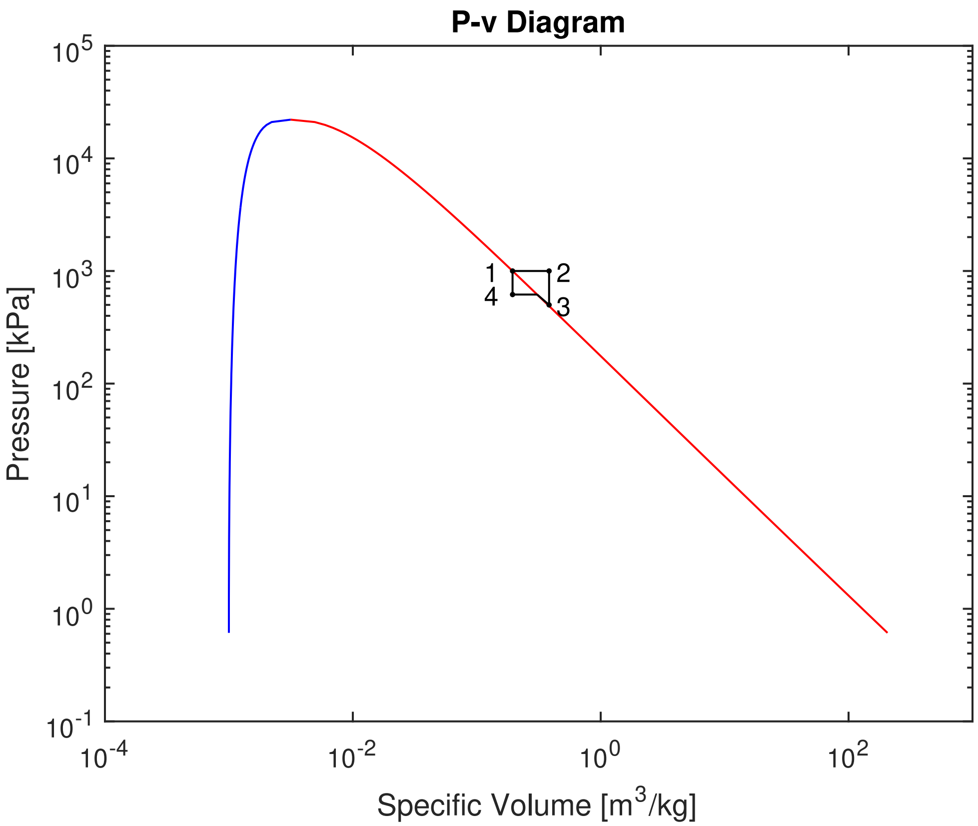
Therefore,

$$Q_{4\to1}=m(u_1-u_4)=(1\,[\text{kg}])(2{,}583.64-1{,}871.38)\,[\text{kJ}]=712.26\,[\text{kJ}]$$

To make the T-ν diagram, we would like to know T_2, which is found via interpolation:

$$\frac{(T_2-500)\,[^{\circ}\text{C}]}{(600-500)\,[^{\circ}\text{C}]}=\frac{(0.38335-0.37489)\,[\text{m}^3/\text{kg}]}{(0.42492-0.37489)\,[\text{m}^3/\text{kg}]}\implies T_2=516.9\,[^{\circ}\text{C}]$$


T-v Diagram
Temperature [C]
Specific Volume [m^3/kg]
600
500
400
300
200
100
0
10^-4
10^-2
10^0
10^2
1
2
3
4


P-v Diagram
Pressure [kPa]
Specific Volume [m^3/kg]
10^5
10^4
10^3
10^2
10^1
10^0
10^-1
10^-4
10^-2
10^0
10^2
1
2
3
4