

# Homework #3

MEMS 0051 - Introduction to Thermodynamics

Assigned January 25<sup>th</sup>, 2019  
Due: February 1<sup>st</sup>, 2019

## Problem #1

Please refer to the Saturated Water tables.

a) Determine the quality of water at the following states:

i.)  $T=150$  [°C],  $\nu=0.285$  [m<sup>3</sup>/kg]

Using Table B.1.1 on pg. 776:

$$x = \frac{\nu - \nu_f}{\nu_g - \nu_f} = \frac{(0.285 - 0.001090) \text{ [m}^3\text{/kg]}}{(0.39278 - 0.001090) \text{ [m}^3\text{/kg]}} = \boxed{0.725}$$

ii.)  $P=500$  [kPa],  $\nu=0.285$  [m<sup>3</sup>/kg]

Using Table B.1.2 on pg. 780:

$$x = \frac{\nu - \nu_f}{\nu_g - \nu_f} = \frac{(0.285 - 0.001093) \text{ [m}^3\text{/kg]}}{(0.37489 - 0.001093) \text{ [m}^3\text{/kg]}} = \boxed{0.760}$$

iii.)  $P=800$  [kPa],  $\nu=0.61813$  [m<sup>3</sup>/kg]

Using Table B.1.2 on pg. 780, it can be seen that the specific volume is greater than that of the saturated vapor state. Therefore, the quality of the water sample is undefined because it's in the super-heated vapor state.

iv.)  $T=100$  [°C],  $\nu=0.001020$  [m<sup>3</sup>/kg]

Using Table B.1.1 on pg. 776, it can be seen that the specific volume is lower than that of the saturated liquid state. Therefore, the quality of the water sample is undefined because it's in the compressed liquid/sub-cooled liquid state.

b) Determine the specific volume and specific internal energy of water at the following states:

Whenever the state of a sample exists between the saturated liquid and saturated vapor region, the specific internal energy and the specific volume can be found using the following:

$$u = xu_g + (1 - x)u_f$$

$$\nu = x\nu_g + (1 - x)\nu_f$$

i.)  $P=200$  [kPa],  $x=0.5$

Using Table B.1.2 on pg. 780, it can be found that:

$$\nu = (0.5)(0.88573 \text{ [m}^3\text{/kg]}) + (1 - 0.5)(0.001061 \text{ [m}^3\text{/kg]}) = \boxed{0.44340 \text{ [m}^3\text{/kg]}}$$

$$u = (0.5)(2529.49 \text{ [kJ/kg]}) + (1 - 0.5)(504.47 \text{ [kJ/kg]}) = \boxed{1,516.98 \text{ [kJ/kg]}}$$

ii.)  $T=300$  [°C],  $x=0.5$

Using Table B.1.2 on pg. 778, it can be found that:

$$\nu = (0.5)(0.02167 \text{ [m}^3\text{/kg]}) + (1 - 0.5)(0.001404 \text{ [m}^3\text{/kg]}) = \boxed{0.01154 \text{ [m}^3\text{/kg]}}$$

$$u = (0.5)(2562.96 \text{ [kJ/kg]}) + (1 - 0.5)(1331.97 \text{ [kJ/kg]}) = \boxed{1,947.47 \text{ [kJ/kg]}}$$

iii.)  $P=523 \text{ [kPa]}$ ,  $x=0.5$

The properties for the saturated liquid and vapor state must be found using interpolation between the 500 [kPa] and 550 [kPa] state. Using Table B.1.2 on pg. 778, it can be found that:

$$\frac{(523 - 500) \text{ [kPa]}}{(550 - 500) \text{ [kPa]}} = \frac{(\nu_g - 0.37489) \text{ [m}^3\text{/kg]}}{(0.34268 - 0.37489) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_g = 0.360073 \text{ [m}^3\text{/kg]}$$

$$\frac{(523 - 500) \text{ [kPa]}}{(550 - 500) \text{ [kPa]}} = \frac{(\nu_f - 0.001093) \text{ [m}^3\text{/kg]}}{(0.001097 - 0.001093) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_f = 0.001095 \text{ [m}^3\text{/kg]}$$

$$\frac{(523 - 500) \text{ [kPa]}}{(550 - 500) \text{ [kPa]}} = \frac{(u_g - 2,561.23) \text{ [kJ/kg]}}{(2,564.47 - 2,561.23) \text{ [kJ/kg]}} \Rightarrow u_g = 2,562.72 \text{ [kJ/kg]}$$

$$\frac{(523 - 500) \text{ [kPa]}}{(550 - 500) \text{ [kPa]}} = \frac{(u_f - 639.66) \text{ [kJ/kg]}}{(655.30 - 639.66) \text{ [kJ/kg]}} \Rightarrow u_f = 646.85 \text{ [kJ/kg]}$$

The specific volume and specific internal energy is found to be:

$$\nu = (0.5)(0.360073 \text{ [m}^3\text{/kg]}) + (1 - 0.5)(0.001095 \text{ [m}^3\text{/kg]}) = \boxed{0.177686 \text{ [m}^3\text{/kg]}}$$

$$u = (0.5)(2,562.72 \text{ [kJ/kg]}) + (1 - 0.5)(646.85 \text{ [kJ/kg]}) = \boxed{1,604.785 \text{ [kJ/kg]}}$$

iv.)  $T=72 \text{ [}^\circ\text{C]}$ ,  $x=0.5$

The properties for the saturated liquid and vapor state must be found using interpolation between the 70 [°C] and 75 [°C] state. Using Table B.1.1 on pg. 776, it can be found that:

$$\frac{(72 - 70) \text{ [}^\circ\text{C]}}{(75 - 70) \text{ [}^\circ\text{C]}} = \frac{(\nu_g - 5.04217) \text{ [m}^3\text{/kg]}}{(4.13123 - 5.04217) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_g = 4.67779 \text{ [m}^3\text{/kg]}$$

$$\frac{(72 - 70) \text{ [}^\circ\text{C]}}{(75 - 70) \text{ [}^\circ\text{C]}} = \frac{(\nu_f - 0.001023) \text{ [m}^3\text{/kg]}}{(0.001026 - 0.001023) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_f = 0.001024 \text{ [m}^3\text{/kg]}$$

$$\frac{(72 - 70) \text{ [}^\circ\text{C]}}{(75 - 70) \text{ [}^\circ\text{C]}} = \frac{(u_g - 2,469.55) \text{ [kJ/kg]}}{(2,475.91 - 2,469.55) \text{ [kJ/kg]}} \Rightarrow u_g = 2,472.09 \text{ [kJ/kg]}$$

$$\frac{(72 - 70) \text{ [}^\circ\text{C]}}{(75 - 70) \text{ [}^\circ\text{C]}} = \frac{(u_f - 292.93) \text{ [kJ/kg]}}{(313.87 - 292.93) \text{ [kJ/kg]}} \Rightarrow u_f = 301.31 \text{ [kJ/kg]}$$

The specific volume and specific internal energy is found to be:

$$\nu = (0.5)(4.67779 \text{ [m}^3\text{/kg]}) + (1 - 0.5)(0.001024 \text{ [m}^3\text{/kg]}) = \boxed{2.33941 \text{ [m}^3\text{/kg]}}$$

$$u = (0.5)(2,472.09 \text{ [kJ/kg]}) + (1 - 0.5)(301.31 \text{ [kJ/kg]}) = \boxed{1,386.7 \text{ [kJ/kg]}}$$

c) Determine the specific internal energy of water for the states listed in part a.

i.)  $T=150 \text{ [}^\circ\text{C]}$ ,  $\nu=0.285 \text{ [m}^3\text{/kg]}$

Using Table B.1.1 on pg. 776:

$$u = (0.725)(2,559.54 \text{ [kJ/kg]}) + (1 - 0.725)(631.66 \text{ [kJ/kg]}) = \boxed{2,029.37 \text{ [kJ/kg]}}$$

ii.)  $P=500 \text{ [kPa]}$ ,  $\nu=0.285 \text{ [m}^3\text{/kg]}$

Using Table B.1.2 on pg. 780:

$$u = (0.760)(2,561.23 \text{ [kJ/kg]}) + (1 - 0.760)(639.66 \text{ [kJ/kg]}) = \boxed{2,100.05 \text{ [kJ/kg]}}$$

- iii.)  $P=800$  [kPa],  $\nu=0.61813$  [m<sup>3</sup>/kg]

The state of the sample of water is in the super-heated region. Using Table B.1.3 on pg. 786, it is shown that this state occurs at a temperature of 800 [°C] with a corresponding specific internal energy of 3,661.14 [kJ/kg]

- iv.)  $T=100$  [°C],  $\nu=0.001020$  [m<sup>3</sup>/kg]

The state of the sample of water is in the compressed liquid region. Using Table B.1.4 on pg. 791, it is shown that this state occurs at a pressure of 50,000 [kPa], with a corresponding specific internal energy of 405.86 [kJ/kg]

## Problem #2

- a) 100 [kg] of C<sub>2</sub>H<sub>4</sub> is contained in a 3 [m<sup>3</sup>] vessel at 300 [K].

- i.) Calculate the gas constant,  $R$ , for C<sub>2</sub>H<sub>4</sub> based on its molecular mass listed in Table A.5.

Using Table A.5 on pg. 760, the molar mass is found to be 28.054[kg/kmol]. The universal gas constant is 8.3145 [kJ / (kmol-K)]. Therefore, the gas constant of ethylene is found to be:

$$R = \frac{\bar{R}}{M} = \frac{8.3145 \left[ \frac{\text{kJ}}{\text{kg-K}} \right]}{28.054 \left[ \frac{\text{kg}}{\text{kmol}} \right]} = \boxed{0.296375 \left[ \frac{\text{kJ}}{\text{kg-K}} \right]}$$

- ii.) Determine how many moles,  $n$ , of C<sub>2</sub>H<sub>4</sub> are in the vessel.

The number of moles is found to be:

$$n = \frac{m}{M} = \frac{100 \text{ [kg]}}{28.054 \left[ \frac{\text{kg}}{\text{kmol}} \right]} = \boxed{n = 3.565 \text{ [kmol]}}$$

- iii.) What is the pressure of C<sub>2</sub>H<sub>4</sub> in the vessel?

Using an ideal gas assumption, it can be found that:

$$P = \frac{mRT}{\forall} = \frac{(100 \text{ [kg]})(0.2964 \left[ \frac{\text{kJ}}{\text{kmol-K}} \right])(300 \text{ [K]})}{3 \text{ [m}^3\text{]}} = \boxed{P = 2.964 \text{ [MPa]}}$$

- b) Heat is now added to the vessel until it reaches a temperature of 500 K.

- i.) Is the specific volume of C<sub>2</sub>H<sub>4</sub> constant during this process? Why or why not?

Because the volume is fixed and no mass is escaping, the specific volume of the sample of ethylene cannot change.

- ii.) What is the final pressure in the vessel?

Using the ideal gas assumption, one can show that:

$$\frac{P_1}{T_1} = \frac{m_1 R_1}{\forall_1}$$

$$\frac{P_2}{T_2} = \frac{m_2 R_2}{\forall_2}$$

However, because the volume, mass and gas constant of the ethylene does not change during the process, it can be said that:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \implies P_2 = \frac{P_1 T_2}{T_1}$$

Therefore, the final pressure of the ethylene is given as:

$$P_2 = \frac{P_1 T_2}{T_1} = \frac{(2.964 \text{ [MPa]})(500 \text{ [K]})}{(300 \text{ [K]})} \implies \boxed{P_2 = 4.94 \text{ [MPa]}}$$

- iii.) Calculate the reduced pressure,  $P_r$ , of  $C_2H_4$  after being heated.

Using Table A.2, the critical pressure can be found to be  $P_c = 5.04$  [MPa]. The reduced pressure after the process can then be calculated as:

$$P_r = \frac{P_2}{P_c} = \frac{4.94 \text{ [MPa]}}{5.04 \text{ [MPa]}} = \boxed{P_r = 0.98}$$

- iv.) Calculate the reduced temperature,  $C_2H_4$   $T_r$ , of before being heated.

Using Table A.2, the critical temperature can be found to be  $T_c = 282.4$  [K]. The reduced temperature before the process can then be calculated as:

$$T_r = \frac{T_1}{T_c} = \frac{300 \text{ [K]}}{282.4 \text{ [K]}} = \boxed{T_r = 1.06}$$

- v.) Can we assume that  $C_2H_4$  behaved like an ideal gas throughout this process? Why or why not? For a system to be treated as an ideal gas,  $P_r \ll 1$  and  $T_r > 2$ . Both the reduced pressure and temperature do not remotely adhere to these conditions. Therefore, the ideal gas does not behave like an ideal gas throughout this process.

### Problem #3

- a) Consider 2 [kg] of saturated R-134a vapor contained in a piston-cylinder apparatus. The vapor temperature is -52 [°C].

- i.) What is the total internal energy,  $U$ , of the R-134a vapor at this state?

The properties for the saturated R-134a vapor state must be found using interpolation between the -55 [°C] and -50 [°C] state. Using Table B.5 on pg. 810, it can be found that:

$$\frac{(-52 - -55) \text{ [°C]}}{(-50 - -55) \text{ [°C]}} = \frac{(u_g - 346.50) \text{ [kJ/kg]}}{(349.31 - 346.50) \text{ [kJ/kg]}} \implies u_g(-52 \text{ [°C]}) = 348.186 \text{ [kJ/kg]}$$

The total internal energy is the product of the samples mass and internal energy. Therefore:

$$U = mu_g = (2 \text{ [kg]})(348.186 \text{ [kJ/kg]}) = \boxed{696.372 \text{ [kJ]}}$$

- ii.) What is the pressure of R-134a vapor at this state?

$$\frac{(-52 - -55) \text{ [°C]}}{(-50 - -55) \text{ [°C]}} = \frac{(P_{sat} - 22.9 \text{ [kPa]})}{(29.9 - 22.2) \text{ [kPa]}} \implies \boxed{P_{sat}(-52 \text{ [°C]}) = 26.82 \text{ [kPa]}}$$

- iii.) What volume is occupied by the R-134a vapor? (*Hint: look up the specific volume*)

$$\frac{(-52 - -55) \text{ [°C]}}{(-50 - -55) \text{ [°C]}} = \frac{(\nu_g - 0.78678) \text{ [m}^3\text{/kg]}}{(0.59657 - 0.78678) \text{ [m}^3\text{/kg]}} \implies \nu_g(-52 \text{ [°C]}) = 0.6727 \text{ [m}^3\text{/kg]}$$

The total volume is the product of the samples mass and specific volume. Therefore:

$$V = m\nu_g = (2 \text{ [kg]})(0.6727 \text{ [m}^3\text{/kg]}) = \boxed{1.3454 \text{ [m}^3\text{/kg]}}$$

- b) An external force now pushes down on the piston, compressing the vapor isobarically until it reaches a final volume of 1.0 [m<sup>3</sup>].

- i.) What is the specific volume of the saturated R-134a mixture now?

$$\nu_2 = \frac{V_2}{m_2} = \frac{1.0 \text{ [m}^3\text{]}}{2 \text{ [kg]}} = \boxed{0.5 \text{ [m}^3\text{/kg]}}$$

- ii.) What is the quality of the saturated R-134a mixture?

The properties for the saturated R-134a liquid state must be found using interpolation between the -55 [°C] and -50 [°C] state. Using Table B.5 on pg. 810, it can be found that:

$$\frac{(-52 - -55) [^{\circ}\text{C}]}{(-50 - -55) [^{\circ}\text{C}]} = \frac{(\nu_f - 0.000689) [\text{m}^3/\text{kg}]}{(0.000695 - 0.000689) [\text{m}^3/\text{kg}]} \implies \nu_f(-52 [^{\circ}\text{C}]) = 0.0006926 [\text{m}^3/\text{kg}]$$

Taking the specific volume of saturated R-134a vapor from part a), the quality is defined as:

$$x = \frac{\nu - \nu_f}{\nu_g - \nu_f} = \frac{(0.5 - 0.0006926) [\text{m}^3/\text{kg}]}{(0.6727 - 0.0006926) [\text{m}^3/\text{kg}]} = \boxed{0.743}$$

- iii.) What is the total internal energy of both phases ( $U_f + U_g$ ) in this final state?

First,  $u_f(-52 [^{\circ}\text{C}])$  must be found.

$$\frac{(-52 - -55) [^{\circ}\text{C}]}{(-50 - -55) [^{\circ}\text{C}]} = \frac{(u_f - 132.36) [\text{kJ/kg}]}{(137.60 - 132.36) [\text{kJ/kg}]} \implies u_f(-52 [^{\circ}\text{C}]) = 135.504 [\text{kJ/kg}]$$

Taking  $u_g(52 [^{\circ}\text{C}])$  from part a), the total energy is defined as:

$$U_{tot} = m((1-x)u_f + xu_g) = (2 [\text{kg}]((1-0.743)(135.504 [\text{kJ/kg}]) + (0.743)(348.186 [\text{kJ/kg}]))) = \boxed{587.054 [\text{kJ/kg}]}$$

- iv.) How much work was **done by** the piston in this process? (Note:  $W = P(\forall_2 - \forall_1)$  for constant pressure processes).

The work done by a piston on a control volume is defined as:

$$W_{1 \rightarrow 2} = P(\forall_2 - \forall_1)$$

The process is isobaric, so the pressure remains constant at  $P_{sat} = 26.82 [\text{kPa}]$ . Therefore, the work is:

$$W_{1 \rightarrow 2} = 26.82 [\text{kPa}](1 [\text{m}^3] - 1.3454 [\text{m}^3]) = \boxed{-9.264 [\text{kJ}]}$$

The negative sign associated with the work indicates that 9.264 [kJ] of work was done on the system.

- v.) How much heat was **transferred into** the saturated R-134a during this process?

According to the first law of thermodynamics:

$$Q_{1 \rightarrow 2} = U_2 - U_1 + W_{1 \rightarrow 2} = (587.054 [\text{kJ}] - 696.372 [\text{kJ}]) - 9.261 [\text{kJ}] = \boxed{-118.582 [\text{kJ}]}$$

In other words, 118.582 [kJ] of heat is transferred out of the system.