Homework #5

MEMS 0051 - Introduction to Thermodynamics

Assigned June 19th, 2018
Due June 25th, 2018

Problem #1

Two rigid tanks are initially in thermodynamic equilibrium. The first tank is 0.5 [m] tall and sufficiently insulated, and contains 5 [kg] of carbon dioxide at 80 °C and a pressure of 200 [kPa]. The second tank is 0.25 [m] in diameter and not insulated, and contains 10 [kg] of argon at 25 °C and a pressure of 500 [kPa]. The two tanks are connected by a rigid piping system that contains a valve, which is then opened. After an extended period of time (assuming sufficiently long to achieve thermodynamic equilibrium internally and thermal equilibrium with the environment, which is 25°C), determine:

a) The final pressure (i.e. mechanical equilibrium pressure).
b) The heat transferred into our out of the tank system.

Solution: Initially, tank 1 has CO_2 at 80 °C and 200 [kPa] and tank 2 has Ar at 25 °C and 500 [kPa]. The tanks are then allowed to equilibrate in terms of pressure, as well as temperature, with the environment (25 °C). The Ideal Gas law would be applicable to solve for the final pressure, as well as state properties which can be used within the Conservation of Energy to solve for the heat removed from the tank system. We first must identify the critical temperatures and pressures of CO_2 and Ar, which will be used to evaluated the validity of the Ideal Gas law:

$$T_{cr,\mathrm{CO_2}} = 304.25\,[\mathrm{K}]; \qquad P_{cr,\mathrm{CO_2}} = 7.38\,[\mathrm{MPa}]$$

$$T_{cr,\mathrm{Ar}} = 151.15\,[\mathrm{K}]; \qquad P_{cr,\mathrm{Ar}} = 4.87\,[\mathrm{MPa}]$$

The final temperature of the system is 298.15 [K]. Checking the reduced temperature and pressure of each gas:

$$T_{r,\mathrm{CO_2}} = \frac{T_{min,\mathrm{CO_2}}}{T_{cr,\mathrm{CO_2}}} = \frac{298.15\,[\mathrm{K}]}{304.25\,[\mathrm{K}]} = 0.98 \not> 2$$

This value is not greater than 2, as required for the use of the Ideal Gas law. Therefore, we cannot proceed with the analysis.

Problem #2

For the following scenarios, determine the amount of heat transfer.

a) Heating a 2 [kg], 0.1 [m] long copper bar from 25°C to 100°C.

Solution: Copper is an incompressible solid, thus we can use the formulation of specific heat (Table A.3) times mass times change of temperature to determine the heat input. Note that the change of temperature has units of [K], but the temperature expressed in [°C] does not need to be converted to [K]:

$$Q = m\,C\,\Delta T = (2\,[\mathrm{kg}])(0.42\,[\mathrm{kJ/kg\text{-}K}])(100 - 25)\,[\mathrm{K}] = 63\,[\mathrm{kJ}]$$

b) 1,000 [kg] of asphalt cooling from 50°C to 20°C.

Solution:

$$Q = m\,C\,\Delta T = (1,000\,[\text{kg}])(0.92\,[\text{kJ/kg-K}])(20-50)\,[\text{K}] = -27,600\,[\text{kJ}]$$

c) The heating of 1 [kg] of oxygen in a mass-less piston-cylinder from 300 to 1,500 K.

Solution: A mass-less piston-cylinder will act as a constant pressure device. Thus, the heat input is merely the change of enthalpy. Using Table A.8 to obtain the enthalpy of oxygen at 1,500 and 300 [K]:

$$Q = m\,\Delta h = (1\,[\text{kg}])(1,540.23 - 273.15)\,[\text{kJ/kg}] = 1,267.08\,[\text{kJ}]$$

d) A piston-cylinder containing 0.1695 [kg] of nitrogen at 150 [kPa] and 25°C that is isothermally compressed to 1.0 [MPa], which requires 20 [kJ] of work.
Solution: To determine the amount of heat transferred, the Conservation of Energy must be evaluated. First, the volumes at States 1 and 2 need to be determined to evaluate the isothermal work. Using the Ideal Gas law:

$$\forall_1 = \frac{m\,R\,T}{P_1} = \frac{(0.1695\,[\text{kg}])(0.2968\,[\text{kJ/kg-K}])(298\,[\text{K}])}{150\,[\text{kPa}]} = 0.1\,[\text{m}^3]$$

The volume at State 2 is found recognizing that the temperature is constant:

$$\forall_1 = \frac{m\,R\,T}{P_2} = \frac{(0.1695\,[\text{kg}])(0.2968\,[\text{kJ/kg-K}])(298\,[\text{K}])}{1,000\,[\text{kPa}]} = 0.015\,[\text{m}^3]$$

The isothermal work is therefore:

$$W_{1\to 2} = P_1 \forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (150\,[\text{kPa}])(0.1\,[\text{m}^3])\ln\left(\frac{0.015\,[\text{m}^3]}{0.1\,[\text{m}^3]}\right) = -28.46\,[\text{kJ}]$$

Evaluating the Conservation of Energy, it is evident the change of internal energy is zero because the internal energy is a function of temperature, not pressure. Therefore:

$$Q_{1\to 2} = \cancelto{0}{m(u_2 - u_1)} + W_{1\to 2} = -28.46\,[\text{kJ}]$$

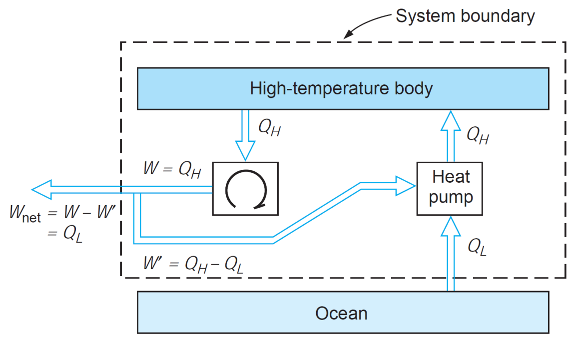
Verifying the validity of the Ideal Gas law:

$$T_{r,\mathrm{N_2}} = \frac{T_{min,\mathrm{N_2}}}{T_{cr,\mathrm{N_2}}} = \frac{298\,[\text{K}]}{126.2\,[\text{K}]} = 2.36$$

$$P_{r,\mathrm{N_2}} = \frac{P_{max,\mathrm{N_2}}}{P_{cr,\mathrm{N_2}}} = \frac{1,000\,[\text{kPa}]}{3,390\,[\text{kPa}]} = 0.29$$

Problem #3

Given the system below, where a heat pump transfers heat from a low-temperature infinite reservoir (the ocean) and transfers heat to a high-temperature infinite reservoir (say a room). The work powering the heat pump comes from a heat engine, which accepts heat from the same high-temperature reservoir. Is this configuration acceptable (i.e. does not violate the Kelvin-Planck and/or Clausius statements, nor is a perpetual motion machine of the 1st, 2nd and 3rd kind)?



Solution: This system is in violation of the Second law, in particular the Kelvin-Planck statement. The heat engine is not rejecting heat, therefore operating between only on temperature reservoir.

Problem #4

Two heat engines that operate in series (i.e. $\dot{Q}_H$ enters HE1 which rejects $\dot{Q}_M$, which is the input into HE2, which rejects $\dot{Q}_L$) between a high-temperature reservoir T_H and a low-temperature reservoir T_L. HE1 produces work $\dot{W}_1$ and HE2 produces work $\dot{W}_2$. Find the efficiency of each heat engine, and the efficiency of the overall system.

Solution: The efficiency of the first heat engine is:

$$\eta_{\text{H.E.1}} = \frac{\dot{W}_1}{\dot{Q}_H}$$

The efficiency of the second heat engine is:

$$\eta_{\text{H.E.2}} = \frac{\dot{W}_2}{\dot{Q}_M}$$

The efficiency of the overall system is:

$$\eta = \frac{\dot{W}_1 + \dot{W}_2}{\dot{Q}_H} = \frac{\dot{W}_1}{\dot{Q}_H} + \frac{\dot{W}_2}{\dot{Q}_H} = \eta_{\text{H.E.1}} + \frac{\dot{W}_2}{\dot{Q}_H}$$

Re-expressing the net work out of the second heat engine in terms of the heat input and efficiency:

$$\eta = \eta_{\text{H.E.1}} + \eta_{\text{H.E.2}} \frac{\dot{Q}_M}{\dot{Q}_H}$$

The quantity $\dot{Q}_M$ can be expressed in terms of the efficiency of the first heat engine:

$$\eta_{\text{H.E.1}} = \frac{\dot{W}_1}{\dot{Q}_H} = \frac{\dot{Q}_H - \dot{Q}_M}{\dot{Q}_H} \implies \dot{Q}_M = \dot{Q}_H(1 - \eta_{\text{H.E.1}})$$

Substituting this in for the expression for overall efficiency:

$$\eta = \eta_{\text{H.E.1}} + \eta_{\text{H.E.2}} \frac{\dot{Q}_H(1 - \eta_{\text{H.E.1}})}{\dot{Q}_H} = \eta_{\text{H.E.1}} + \eta_{\text{H.E.2}}(1 - \eta_{\text{H.E.1}})$$

Problem #5

A heat engine operates between a high-temperature reservoir T_{H1} and a low-temperature reservoir T_{ambient}. The work produced, $\dot{W}_1$, which is the difference of heat input $\dot{Q}_{H1}$ and heat rejected $\dot{Q}_{L1}$, powers a heat pump. Part of the work from the heat engine enters the heat pump $\dot{W}_2$, whereas the difference between $\dot{W}_1$ and $\dot{W}_2$ is designated as the net work, $\dot{W}_{\text{net}}$. The heat pump accepts heat $\dot{Q}_{L2}$ from the same low-temperature reservoir (T_{ambient}) and rejects heat $\dot{Q}_{H2}$ to a secondary high-temperature reservoir T_{H2}. Assuming T_{H1}=T_{H2} > T_{ambient}, determine, based upon the following cases (a-c), if this system satisfies the First Law and/or violates the Second Law. Then, assuming $T_{H1} > T_{H2} > T_{\text{ambient}}$, determine if this system satisfies the First Law and/or violates the Second Law.

	$\dot{Q}_{H1}$	$\dot{Q}_{L1}$	$\dot{W}_1$	$\dot{Q}_{H2}$	$\dot{Q}_{L2}$	$\dot{W}_2$
a	6	4	2	3	2	1
b	6	4	2	5	4	1
c	3	2	1	4	3	1

<u>Solution</u>: Assuming T_{H1}=T_{H2} > T_{ambient}:

a)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 2 = 6 - 4 \checkmark \text{ First Law}$$

$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 1 = 3 - 2 \checkmark \text{ First Law}$$

$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 1 > 0 \checkmark \text{ First \& Second Law}$$

$$\dot{Q}_{L1} > \dot{Q}_{L2} \,\&\, \dot{Q}_{H1} > \dot{Q}_{H2} \checkmark \text{ Second Law - agreement of K.P./Clausius statement}$$

b)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 2 = 6 - 4 \checkmark \text{ First Law}$$

$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 1 = 5 - 4 \checkmark \text{ First Law}$$

$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 1 > 0 \checkmark \text{ First \& Second Law}$$

$$\dot{Q}_{L1} = \dot{Q}_{L2} \,\&\, \dot{Q}_{H1} > \dot{Q}_{H2} \implies \times \text{ Second Law - violation of K.P. statement}$$

This is a violation of the Second Law for there is only a net exchange of heat with the high-temperature reservoirs, for the same amount of heat is rejected and pulled from the low-temperature reservoir.

c)

$$\dot{W}_1 = \dot{Q}_{H1} - \dot{Q}_{L1} \implies 1 = 3 - 2 \checkmark \text{ First Law}$$

$$\dot{W}_2 = \dot{Q}_{H2} - \dot{Q}_{L2} \implies 1 = 4 - 3 \checkmark \text{ First Law}$$

$$\dot{W}_{net} = \dot{W}_1 - \dot{W}_2 \implies 2 - 1 > 0 \checkmark \text{ First \& Second Law}$$

$$\dot{Q}_{L2} > \dot{Q}_{L1} \,\&\, \dot{Q}_{H1} < \dot{Q}_{H2} \implies \times \text{ Second Law - violation of Clausius statement}$$

This is violation of the Second law for, although the heat engine provides enough work to run the heat pump (unity), there is a net heat transference from the low-temperature to high-temperature reservoir.

Assuming $T_{H1} > T_{H2} > T_{\text{ambient}}$, solutions take the same form.