Homework #3 Solutions

MEMS 0051 - Introduction to Thermodynamics

February 3, 2017

Problem #1

- A piston-cylinder device contains He gas initially at 150 kPa, 20 °C and 0.5 m^3. The He gas is compressed in a polytropic process to 400 kPa and 140 °C. Assume the He gas satisfies the ideal gas law. For He, R=2.0771 kJ/kg-K, C_P =5.2 kJ/kg-K, $C_\forall$ =3.116 kJ/kg-K.
- Determine the work into or out of the system during this process.
- Determine the heat loss or gain during this process.
- Hint: Determine n.

Solution:

State 1:	$\rightarrow$ Polytropic $\rightarrow$	State 2:
$\forall_1$=0.5 m^3		$\forall_2$=?
P_1=150 kPa		P_2=400 kPa
T_1=20 °C		T_2=140 °C

First, find the volume at State 2 using the ideal gas law. Since mass is constant and the gas constant R does not vary, we are able to express $\forall_2$ as the following:

$$\forall_2 \rightarrow \frac{P_1\forall_1}{T_1} = \frac{P_2\forall_2}{T_2} \rightarrow \forall_2 = \forall_1\left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right) = 0.5[\text{m}^3]\left(\frac{150[\text{kPa}]}{400[\text{kPa}]}\right)\left(\frac{413[\text{K}]}{293[\text{K}]}\right) = 0.264[\text{m}^3] \tag{1}$$

Next, find the polytropic index n by recalling $P\forall^n$=c:

$$\begin{gathered} P_1\forall_1^n = P_2\forall_2^n \rightarrow \ln(P_1\forall_1^n) = \ln(P_2\forall_2^n) \rightarrow \ln(P_1) + n\ln(\forall_1) = \ln(P_2) + n\ln(\forall_2) \\ \rightarrow \ln(P_2) - \ln(P_1) = n(\ln(\forall_1) - \ln(\forall_1)) \rightarrow n = \frac{\ln\left(\dfrac{P_2}{P_1}\right)}{\ln\left(\dfrac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\dfrac{400[\text{kPa}]}{150[\text{kPa}]}\right)}{\ln\left(\dfrac{0.5[\text{m}^3]}{0.264[\text{m}^3]}\right)} = 1.53 \end{gathered} \tag{2}$$

We have enough information to determine the work of the system, which will be used within the 1$^{\text{st}}$ Law to determine the heat loss or gain. The work is expressed using the following expression:

$$\boxed{W_{1\rightarrow2} = \frac{(P_2\forall_2 - P_1\forall_1)}{(1-n)} = \frac{(400[\text{kPa}]\cdot 0.264[\text{m}^3] - 150[\text{kPa}]\cdot 0.5[\text{m}^3])}{(1-1.53)} = -57.75[\text{kJ}]} \tag{3}$$

Recalling heat transferred through the 1$^{\text{st}}$ Law expressions and neglecting kinetic (KE) and potential (PE) energies, we have:

$$Q_{1\rightarrow2} = (E_2 - E_1) + W_{1\rightarrow2} = (U_2 - U_1) + W_{1\rightarrow2} = m(u_2 - u_1) + W_{1\rightarrow2} \tag{4}$$

Recall that the change in the internal energy is proportional to the constant-volume specific heat $C_\forall$ times the difference in temperature such that:

$$u_2 - u_1 = \int_{T_1}^{T_2} C_\forall dT \approx C_\forall (T_2 - T_1) \tag{5}$$

Additionally, within eqn. 4 we have to solve for mass m for the specific internal energy, which can be done through the ideal gas law evaluated at either State 1 or 2:

$$m = \frac{P\forall}{RT} = \frac{150[\text{kPa}] \cdot 0.5[\text{m}^3]}{2.0771[\text{kJ/kg-K}] \cdot 293[\text{K}]} = 0.123[\text{kg}] \tag{6}$$

Replacing eqn. 5 in for the specific internal energy in eqn. 4 we have:

$$\boxed{Q_{1\to2} = m \cdot C_\forall (T_2 - T_1) + W_{1\to2} = 0.123[\text{kg}] \cdot 3.116[\text{kJ/kg-K}](413[\text{K}] - 293[\text{K}]) - 57.75[\text{kJ}] = -11.76[\text{kJ}]} \tag{7}$$

The negative sign for the quantity of heat implies that heat is removed whereas the negative sign for the quantity of work means work is put into the system to compress the fluid (recall heat into a C.∀. is positive and work out of a C.∀. is positive).

Problem #2

- Water contained in a piston-cylinder assembly undergoes two processes in series from an initial state where the pressure is 1.0 MPa and the temperature is 400 °C. Process 1-2: The water is cooled as it is compressed at constant pressure to the saturated vapor state at 1.0 MPa. Process 2-3: The water is cooled at constant volume to 150 °C.
- Sketch both processes on T-v and P-v diagrams.
- For the overall process determine the work, in kJ/kg.
- For the overall process determine the heat transfer, in kJ/kg.

Solution:

State 1:	→ P=c →	State 2:	→ ∀=c →	State 3:
P_1 = 1,000 kPa		P_2=1,000 kPa		P_3=?
T_1=400 °		T_2=T_{sat}		T_3=150 °C

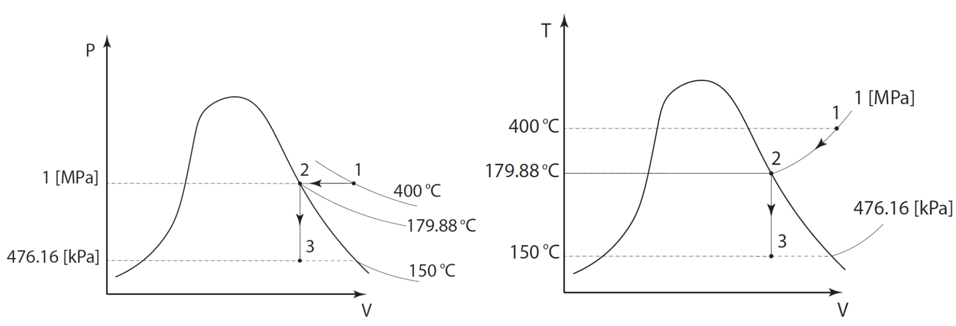

Figure 1: Figure 1: P-v and T-v diagrams from Problems #2

Since the piston is the only work mechanism:

$$W_{1\to3} = \int_{\forall_1}^{\forall_3} P d\forall = \int_{\forall_1}^{\forall_2} P d\forall + \int_{\forall_2}^{\forall_3} P d\forall \tag{8}$$

Since the process between States 2 and 3 is a constant volume process, the second term in the third expression of eqn. 8 is equal to zero. Additionally, we do not know the mass and are only asked to find the specific work. Therefore, we can express eqn. 8 as:

$$\frac{W}{m} = P(v_2 - v_1) \tag{9}$$

From the first part of the problem, we determine that the fluid is in the superheated region (T_{sat}=179.88 ° at P=1,000 [kPa], therefore T=400 °C > T_{sat}). Therefore, the specific volume and internal energy for State 1 can be pulled from the superheated steam tables such that:

$$v_1 = 0.30661[\text{m}^3/\text{kg}], \quad u_1 = 2,957.9[\text{kJ/kg}] \tag{10}$$

At State 2, the water is saturated vapor. The specific volume and internal energy can be pulled from the Saturated Water Pressure Table (P=1,000 [kPa]) such that:

$$v_2 = 0.19436[\text{m}^3/\text{kg}] \tag{11}$$

Therefore, the specific work (eqn. 9) can be evaluated as:

$$\boxed{w = \frac{W}{m} = P(v_2 - v_1) = 1,000[\text{kPa}](0.19436[\text{m}^3/\text{kg}] - 0.30661[\text{m}^3/\text{kg}]) = -112.25[\text{kJ/kg}]} \tag{12}$$

Determining the heat per unit mass, we apply the 1$^{\text{st}}$ Law, neglecting the change in potential and kinetic energy:

$$\frac{Q_{1\to3}}{m} = (u_3 - u_1) + w_{1\to3} \tag{13}$$

To find the specific internal energy at State 3, we have to determine the quality. We have T_3 and since the process from State 2 to 3 is constant volume, v_3=v_2. Therefore, looking at the Saturated Water Temperature Table (T=150 °C), we can use v_f and v_g to determine the quality at State 3:

$$x_3 = \frac{v_3 - v_{f,3}}{v_{g,3} - v_{f,3}} = \frac{0.19436[\text{m}^3/\text{kg}] - 0.001091[\text{m}^3/\text{kg}]}{0.39248[\text{m}^3/\text{kg}] - .001091[\text{m}^3/\text{kg}]} = 0.4938 \tag{14}$$

With quality, we can determine the specific internal energy:

$$u_3 = u_{f,3} + x(u_{g,3} - u_{f,3}) = 631.66[\text{kJ/kg}] - 0.4938(2,559.1[\text{kJ/kg}] - 631.66[\text{kJ/kg}]) = 1,583.43[\text{kJ/kg}] \tag{15}$$

Therefore, the heat per unit mass as expressed in eqn. 13 can finally be evaluated such that:

$$\boxed{\frac{Q_{1\to3}}{m} = (u_3 - u_1) + w_{1\to3} = (1,583.43[\text{kJ/kg}] - 2,957.9[\text{kJ/kg}]) - 112.25[\text{kJ/kg}] = -1,486.72[\text{kJ/kg}]} \tag{16}$$

The negative sign on the value of $q_{1\to3}$ indicates that heat is being removed from the C.∀.

Problem #3

- A piston-cylinder device contains a 1.5 kg of water at a pressure of 200 kPa and a temperature of 150 °C. It is heated in such a fashion that the pressure is linearly related to the volume, and reaches a final state where the pressure is 600 kPa and the temperature is 350 °C.
- Find the final volume of the system and the work required for this process.

Solution:

State 1:	$\rightarrow$ P=P($\forall$) $\rightarrow$	State 2:
P_1=200 kPa		P_2=600 kPa
T_1=150 °C		T_2=350 °C
m_1=1.5 kg		m_2=m_1=1.5 kg

Since the pressure is a linear function of volume, we can evaluate the work between States 1 and 2 using either the average pressure value or the integral formulation by keeping pressure as part of the integrand and evaluate it at our two volume bounds. The simplest method would be to use the average pressure. To evaluate the pressure term within the integrand, you must plot the two pressure versus volume points, fit a linear equation to those two points, integrate and evaluate said equation between the two volume bounds. Doing such will yield exactly the same value as if you averaged the pressure values. Thus, the work is:

$$W_{1\rightarrow 2} = \int_{\forall_1}^{\forall_2} P d\forall = \frac{m(P_1 + P_2)(v_2 - v_1)}{2} \tag{17}$$

Thus, we need to determine the specific volume at States 1 and 2. At State 1, we know the temperature is greater than the saturation (T_1=150 °C > T_{sat}(200 kPa)=120.23 °C):

$$v_1 = 0.95964[\text{m}^3/\text{kg}] \tag{18}$$

At State 2, the temperature is once again greater than the saturation temperature and we find the specific volume from the superheated steam tables:

$$v_2 = 0.47424[\text{m}^3/\text{kg}] \tag{19}$$

The final volume is simply the mass times the specific volume at State 2:

$$\boxed{\forall_2 = mv_2 = 1.5[\text{kg}] \cdot 0.47424[\text{m}^3/\text{kg}] = 0.7114[\text{m}^3]} \tag{20}$$

Lastly, eqn. 17 can be evaluated to determine the work to go from State 1 to 2:

$$\boxed{W_{1\rightarrow 2} = \frac{1.5[\text{kg}]}{2}(200[\text{kPa}] + 600[\text{kPa}])(0.47424[\text{m}^3/\text{kg}] - 0.95964[\text{m}^3/\text{kg}]) = -291.24[\text{kJ}]} \tag{21}$$

Problem #4

- A piston-cylinder device with a set of stops in the middle initially contains 3 kg of air at 200 kPa and 500 K. The air is compressed by lowering the frictionless piston in an isobaric process (n=0). When the piston hits the stops, the gas volume is 50% of the initial volume. Assume air behaves as an ideal gas (R=0.287 kJ/kg-K, C_P =1.004 kJ/kg-K, $C_\forall$ =0.717 kJ/kg-K. If the final temperature is 300 K, determine the total work done on the system.

Solution:

State 1:	$\rightarrow$ P=c $\rightarrow$	State 2:
m_1=3 kg		m_2=m_1=3 kg
P_1=200 kPa		P_2=P_1=200 kPa
T_1=500 K		T_2=300 K

Using the Ideal Gas Law for the volume at State 1:

$$\forall_1 = \frac{mRT_1}{P_1} = \frac{3[\text{kg}] \cdot 0.287[\text{kJ/kg-K}] \cdot 500[\text{K}]}{200[\text{kN-m}^2]} = 2.1525[\text{m}^3] \tag{22}$$

Similarly, the volume at State 2 is expressed as:

$$\forall_2 = \frac{mRT_2}{P_2} = \frac{3[\text{kg}] \cdot 0.287[\text{kJ/kg-K}] \cdot 300[\text{K}]}{200[\text{kN-m}^2]} = 1.2915[\text{m}^3] \tag{23}$$

It is evident $\forall_2 \neq \frac{1}{2}\forall_1 \neq 1.075$ [m^3], so the piston does not rest on the lower stops. Therefore, the work done on the system is:

$$\boxed{W_{1\to 2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = P_1(\forall_2 - \forall_1) = 200[\text{kPa}](1.2915[\text{m}^3] - 2.1525[\text{m}^3]) = -172.2[\text{kJ}]} \tag{24}$$

Problem #5

- A piston-cylinder device contains 1.5 kg of air at a temperature of 300 K and a pressure of 150 kPa. It is heated in a two-step process. The first step is a constant volume process where the final temperature is 1,000 K. The second step is a constant pressure process where the final temperature is 1,500 K.
- Determine the final volume and work required for this two step process.

Solution:

State 1:	$\to \forall$=c $\to$	State 2:	$\to$ P=c $\to$	State 3:
m_1=1.5 kg		m_2=m_1=1.5 kg		m_3=m_2=1.5 kg
P_1=150 kPa		P_2=?		P_3=?
T_1=300 K		T_2=1,000 kPa		T_3=1,500 kPa

The process from State 1 to 2 is ischoric, so no work is done. However, we need the volume at State 1 to determine the volume at State 2. Using the Ideal Gas Law:

$$\forall_1 = \frac{mRT_1}{P_1} = \frac{1.5[\text{kg}] \cdot 0.287[\text{kJ/kg-K}] \cdot 300[\text{K}]}{150[\text{kPa}]} = 0.861[\text{m}^3] \tag{25}$$

Thus, the volume at State 2 is equal to that at State 1. That allows us to find the pressure at State 2, which is the same as the pressure at State 3:

$$P_2 = P_1\left(\frac{T_2}{T_1}\right) = 150[\text{kPa}]\left(\frac{1,000[\text{K}]}{300[\text{K}]}\right) = 500[\text{kPa}] \tag{26}$$

Now that we have the pressure at State 3, we can determine the volume at State 3:

$$\boxed{\forall_3 = \forall_2\left(\frac{T_3}{T_3}\right) = 0.861[\text{m}^3]\left(\frac{1,500[\text{K}]}{1,000[\text{K}]}\right) = 1.2915[\text{m}^3]} \tag{27}$$

The work between States 1 and 3 is evaluated as a constant pressure process between States 2 and 3 since the process between States 1 and 2 is isochoric:

$$\boxed{W_{1\to 3} = P_3(\forall_3 - \forall_2) = 500[\text{kPa}](1.2915[\text{m}^3] - 0.8691[\text{m}^3]) = 215.3[\text{kJ}]} \tag{28}$$