

Homework #7

MEMS 0051 - Introduction to Thermodynamics

Assigned July 11th, 2018

Due July 16th, 2018

Problem #1

A high-temperature reservoir at 800 K dissipated 2,000 [kJ] of heat to a low-temperature thermal reservoir of a) 500 K and b) 750 K. Determine which of these two processes is the least reversible, i.e. which has the greatest change of entropy.

Solution: Recalling the definition of entropy:

$$dS \equiv \left(\frac{\delta Q}{T} \right)$$

We define the change of entropy, i.e. how much the entropy changes as heat is transferred across the boundary of our temperature reservoir, as:

$$\Delta S = \int \left(\frac{\delta Q}{T} \right)$$

There are two changes of entropy happening. Entropy is decreasing as heat is leaving our high-temperature reservoir (i.e. ΔS_1), and entropy is increasing as heat is entering our low-temperature reservoir (i.e. ΔS_2):

$$\Delta S_{\text{net}} = \oint \left(\frac{\delta Q}{T} \right)_b \implies \Delta S_{\text{net}} = \int \left(\frac{\delta Q}{T} \right)_h + \int \left(\frac{\delta Q}{T} \right)_c = \Delta S_1 + \Delta S_2$$

For case a), recalling the directionality of heat transfer (negative if leaving C.V., positive if entering C.V.):

$$\Delta S_{\text{net}} = \frac{\delta Q}{T_h} + \frac{\delta Q}{T_c} = \frac{-2,000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{2,000 \text{ [kJ]}}{500 \text{ [K]}} = -2.5 \text{ [kJ/K]} + 4.0 \text{ [kJ/K]} = 1.5 \text{ [kJ/K]}$$

For case b):

$$\Delta S_{\text{net}} = \frac{\delta Q}{T_h} + \frac{\delta Q}{T_c} = \frac{-2,000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{2,000 \text{ [kJ]}}{750 \text{ [K]}} = -2.5 \text{ [kJ/K]} + 2.7 \text{ [kJ/K]} = 0.2 \text{ [kJ/K]}$$

The total change of entropy of case a) is greater than that of case b), i.e. there is more entropy produced.

Problem #2

A rigid tank contains R-134a in the quantity of 5 [kg] initially at 293 K and 140 [kPa]. The refrigerant is cooled in a process until the pressure within the tank reaches 100 [kPa]. Determine the change of entropy during this process.

Solution:

Given the temperature and pressure at State 1, we recognize the refrigerant exists as a superheated vapor. We must also recognize this is a constant-volume process. Therefore, we are interested in the specific entropy and specific volume at States 1 and 2. For the quantities of interest at State 1, we must interpolate between pressure at the given temperature:

$$\frac{(140 - 100) \text{ [kPa]}}{(150 - 100) \text{ [kPa]}} = \frac{(s_1 - 1.8869) \text{ [kJ/kg-K]}}{(1.8515 - 1.8869) \text{ [kJ/kg-K]}} = \frac{(\nu_1 - 0.23392) \text{ [m}^3\text{/kg]}}{(0.15424 - 0.23392) \text{ [m}^3\text{/kg]}}$$

$$\Rightarrow s_1 = 1.858\,58 \text{ [kJ/kg-K]}$$

$$\Rightarrow \nu_1 = 0.170\,176 \text{ [m}^3\text{/kg]}$$

At State 2, $\nu_2 = \nu_1$ whereas $P_2 = 100 \text{ [kPa]}$, which means it exists within the vapor dome. Determining the specific entropy at State 2 by use of quality:

$$\frac{(100 - 85.1) \text{ [kPa]}}{(10.1.3 - 85.1) \text{ [kPa]}} = \frac{(\nu_f - 0.000\,722) \text{ [m}^3\text{/kg]}}{(0.000\,728 - 0.000\,722) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_f = 0.000\,728 \text{ [m}^3\text{/kg]}$$

$$\frac{(100 - 85.1) \text{ [kPa]}}{(10.1.3 - 85.1) \text{ [kPa]}} = \frac{(\nu_g - 0.224\,02) \text{ [m}^3\text{/kg]}}{(0.190\,02 - 0.224\,02) \text{ [m}^3\text{/kg]}} \Rightarrow \nu_g = 0.192\,75 \text{ [m}^3\text{/kg]}$$

Therefore, the quality is

$$x_2 = \frac{\nu_2 - \nu_f}{\nu_g - \nu_f} = \frac{(0.170\,176 - 0.000\,728) \text{ [m}^3\text{/kg]}}{(0.192\,75 - 0.000\,728) \text{ [m}^3\text{/kg]}} = 0.88$$

Determining the saturated liquid and saturated vapor specific entropy values:

$$\frac{(100 - 85.1) \text{ [kPa]}}{(10.1.3 - 85.1) \text{ [kPa]}} = \frac{(s_f - 0.849\,9) \text{ [kJ/kg-K]}}{(0.869 - 0.849\,9) \text{ [kJ/kg-K]}} \Rightarrow s_f = 0.862 \text{ [kJ/kg-K]}$$

$$\frac{(100 - 85.1) \text{ [kPa]}}{(10.1.3 - 85.1) \text{ [kPa]}} = \frac{(s_g - 1.749\,3) \text{ [kJ/kg-K]}}{(1.745\,3 - 1.749\,3) \text{ [kJ/kg-K]}} \Rightarrow s_g = 1.746 \text{ [kJ/kg-K]}$$

Therefore, the specific entropy at State 2 is:

$$s_2 = s_f + x_2(s_g - s_f) = 0.862 \text{ [kJ/kg-K]} + 0.88(1.746 - 0.862) \text{ [kJ/kg-K]} = 1.64 \text{ [kJ/kg-K]}$$

Lastly, the change of entropy during this process is the difference between the specific entropy values at the final and initial state times mass:

$$\Delta S = m(s_2 - s_1) = (5 \text{ [kg]})(1.64 - 1.858\,58) \text{ [kJ/kg-K]} = -1.093 \text{ [kJ/K]}$$

Problem #3

A mass-less piston-cylinder device contains 1.36 [kg] of water at 138 [kPa] and 21 °C. 3,700 [kJ] of thermal energy is supplied to the water. Determine the change of entropy of the water during this process.

Solution:

Since this is a mass-less piston-cylinder device, the water expands isobarically. State 1 exists as a compressed liquid (treat it as saturated liquid water at 21 °C) and will proceed to State 2, as determined by the Conservation of Energy, which is expressed in terms of enthalpy:

$$Q_{1 \rightarrow 2} = m(h_2 - h_1) \Rightarrow h_2 = \frac{Q_{1 \rightarrow 2}}{m} + h_1$$

The enthalpy at State 1 is found via interpolation:

$$\frac{(21 - 20) \text{ [°C]}}{(25 - 20) \text{ [°C]}} = \frac{(h_1 - 83.94) \text{ [kJ/kg]}}{(104.87 - 83.94) \text{ [kJ/kg]}} \Rightarrow h_1 = 88.126 \text{ [kJ/kg]}$$

Since you are already at the Saturated Water Temperature Table, it would be prudent to calculate the specific entropy at State 1:

$$\frac{(21 - 20) \text{ [°C]}}{(25 - 20) \text{ [°C]}} = \frac{(s_1 - 0.296\,6) \text{ [kJ/kg-K]}}{(0.367\,3 - 0.296\,6) \text{ [kJ/kg-K]}} \Rightarrow s_1 = 0.310\,74 \text{ [kJ/kg-K]}$$

The enthalpy at State 2 is found to be:

$$h_2 = \frac{3,700 \text{ [kJ]}}{1.36 \text{ [kg]}} + 88.126 \text{ [kJ/kg]} = 2,808.7 \text{ [kJ/kg]}$$

The enthalpy at State 2 is greater than the saturated vapor enthalpy at 21 °C, indicating it is a superheated vapor. The enthalpy entry exists between the 100 and 200 [kPa] sub-tables and the 150 to 200 °C temperature entries. Starting with creating a 138 [kPa] entry at 150 °C:

$$\frac{(138 - 100) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} = \frac{(h_{150[^\circ\text{C}], 138 \text{ [kPa]}} - 2,776.38) \text{ [kJ/kg]}}{(2,768.8 - 2,776.38) \text{ [kJ/kg]}} \implies h_{150[^\circ\text{C}], 138 \text{ [kPa]}} = 2,773.5 \text{ [kJ/kg]}$$

Next, for 300°C, 138 [kPa] entry:

$$\frac{(138 - 100) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} = \frac{(h_{200[^\circ\text{C}], 138 \text{ [kPa]}} - 2,875.27) \text{ [kJ/kg]}}{(2,870.46 - 2,875.27) \text{ [kJ/kg]}} \implies h_{200[^\circ\text{C}], 138 \text{ [kPa]}} = 2,873.44 \text{ [kJ/kg]}$$

We are interested in determining the specific entropy at 138 [kPa] and an enthalpy value of 2,808.7 [kJ/kg]. Therefore, the entropy is found via triple interpolation, first by creating an entry at 150 °C:

$$\frac{(138 - 100) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} = \frac{(s_{150[^\circ\text{C}], 138 \text{ [kPa]}} - 7.6133) \text{ [kJ/kg-K]}}{(7.2795 - 7.6133) \text{ [kJ/kg-K]}} \implies s_{150[^\circ\text{C}], 138 \text{ [kPa]}} = 7.486 \text{ [kJ/kg-K]}$$

Next, at 200 °C:

$$\frac{(138 - 100) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} = \frac{(s_{200[^\circ\text{C}], 138 \text{ [kPa]}} - 7.8342) \text{ [kJ/kg-K]}}{(7.5066 - 7.8342) \text{ [kJ/kg-K]}} \implies s_{200[^\circ\text{C}], 138 \text{ [kPa]}} = 7.71 \text{ [kJ/kg-K]}$$

Lastly, interpolation between enthalpy at 138 [kPa]:

$$\frac{(2,808.7 - 2,773.5) \text{ [kJ/kg]}}{(2,873.44 - 2,773.5) \text{ [kJ/kg]}} = \frac{(s_2 - 7.486) \text{ [kJ/kg-K]}}{(7.71 - 7.486) \text{ [kJ/kg-K]}} \implies s_2 = 7.565 \text{ [kJ/kg-K]}$$

Therefore, the increase of entropy is the mass times difference of entropy between the final and initial state:

$$\Delta S = m(s_2 - s_1) = (1.36 \text{ [kg]})(7.565 - 0.31074) \text{ [kJ/kg-K]} = 9.86 \text{ [kJ/K]}$$

Problem #4

Liquid methane is the main undergoes a process from 100 K and 1,000 [kPa] to 120 K and 5,000 [kPa]. Determine the change of entropy for this process using:

- tabulated values;
- Tds equations for incompressible substances.

Solution:

Using tabulated values (Table B.7.1), at 100 [K] and 1,000 [kPa], i.e. State 1, liquid methane is existing as a compressed liquid. Therefore, taking the specific entropy as that of the saturated liquid specific entropy evaluated at 100 [K],

$$s_1 = 4.5538 \text{ [kJ/kg-K]}$$

State 2 is also a compressed liquid, and the entropy is taking as the saturated liquid specific entropy at 120 [K]:

$$s_2 = 5.1867 \text{ [kJ/kg-K]}$$

The change of entropy is therefore:

$$\Delta s = s_2 - s_1 = (5.1867 - 4.5338) \text{ [kJ/kg-K]} = 0.6529 \text{ [kJ/kg-K]}$$

Using the Tds equation for incompressible substances:

$$\Delta s = C \ln\left(\frac{T_2}{T_1}\right)$$

finding C as C_{sat} taken at the average temperature of 105 [K] from [here](#) to be 3.42 [kJ/kg-K]:

$$\frac{(105 - 102.574) \text{ [K]}}{(106.891 - 102.574) \text{ [K]}} = \frac{(C - 54.896) \text{ [J/mol-K]}}{(55.602 - 54.896) \text{ [J/mol-K]}} \implies C = 55.293 \text{ [J/mol-K]}$$

$$\left(55.293 \left[\frac{\text{J}}{\text{mol-K}}\right]\right) \left(\frac{1 \text{ [kJ]}}{1,000 \text{ [J]}}\right) \left(\frac{1 \text{ [mol]}}{0.01604 \text{ [kg]}}\right) = 3.45 \text{ [kJ/kg-K]}$$

Therefore

$$\Delta s = (3.45 \text{ [kJ/kg-K]}) \ln\left(\frac{120 \text{ [K]}}{100 \text{ [K]}}\right) = 0.629 \text{ [kJ/kg-K]}$$

Problem #5

Air at STSP conditions is compressed to 600 [kPa] and 300 K. Determine the change of entropy using:

- tabulated values;
- Tds equations for compressible substances.

Solution: Using the Tds equations:

$$\Delta s = s_2 - s_1 = C_{P0} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = -0.508 \text{ [kJ/kg-K]}$$

Since the temperature increase is mild, we can treat C_{P0} as a constant taken at 298 [K]. Therefore:

$$s_2 - s_1 = (1.004 \text{ [kJ/kg-K]}) \ln\left(\frac{300 \text{ [K]}}{298 \text{ [K]}}\right) - (0.287 \text{ [kJ/kg-K]}) \ln\left(\frac{600 \text{ [kPa]}}{100 \text{ [kPa]}}\right)$$

Using Table A.7.1:

$$\Delta s = s_2^o - s_1^o - R \ln\left(\frac{P_2}{P_1}\right) = (6.86926 - 6.86305) \text{ [kJ/kg-K]} - (0.287 \text{ [kJ/kg-K]}) \ln\left(\frac{600 \text{ [kPa]}}{100 \text{ [kPa]}}\right) = -0.508 \text{ [kJ/kg-K]}$$

Problem #6

Consider an automobile engine with a compression ratio of 8, i.e. $v_1/v_2=8$. If State 1 exists at STSP, determine the final temperature of the air.

Solution: Using the relative volume formulation (Table A.7.2):

$$\frac{v_2}{v_1} = \frac{1}{8} = \frac{v_{r2}}{v_{r1}} \implies v_{r2} = \frac{v_{r1}}{8} = \frac{182.29}{8} = 22.786$$

This value exists between 660 and 680 [K]:

$$\frac{(T - 660) \text{ [K]}}{(680 - 660) \text{ [K]}} = \frac{22.786 - 23.662}{21.818 - 23.662} \implies T = 669.5 \text{ [K]}$$