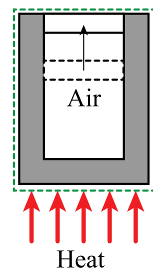
Homework #1

MEMS 0051 - Introduction to Thermodynamics

Assigned January 11$^{\text{th}}$, 2019
Due: January 18$^{\text{th}}$, 2019

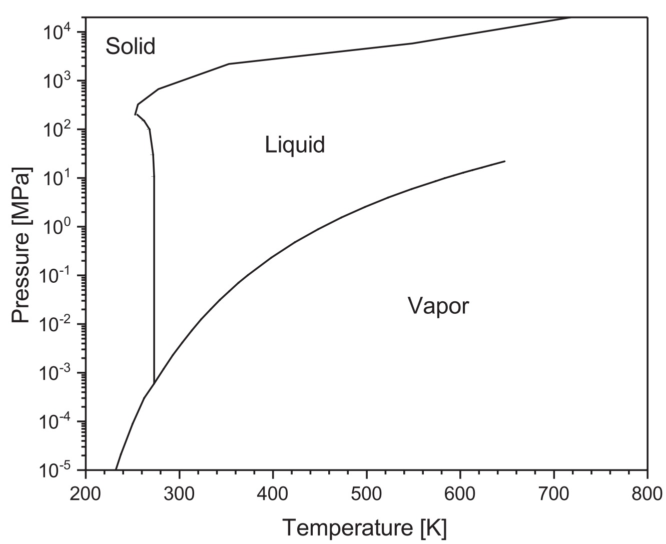
Problem #1



Consider a piston cylinder shown to the right. The green dashed line is the control surface (C.S.) Answer the following questions.
(a) Is the control volume (C.∀.) an open or closed system? Closed System
(b) Is the C.∀. that of an isolated system? No
(c) Are the contents of the C.∀. undergoing a process or cycle? Process

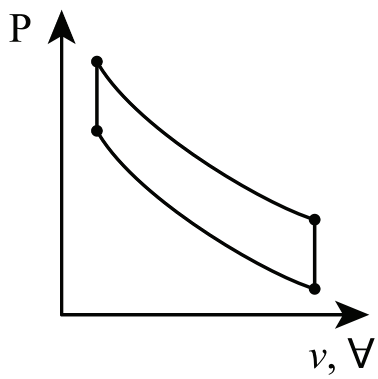
Problem #2

Given the phase diagram of water below, determine the phase based upon the following properties:
(a) 200 [K] and 1 [MPa] solid
(b) 220 [K] and 0.1 [kPa] solid
(c) 400 [K] and 10,000 [MPa] solid
(d) 326.85 [C] and 0.01 [MPa] vapor
(e) 126.85 [C] and 0.1 [kPa] vapor
(f) 400 [K] and 100 [MPa] liquid
(g) 700 [K] and 100 [MPa] undefined



Problem #3

Given the $P - \nu$ diagram for the Otto cycle below, identify the two processes that comprise this cycle.



Solution:
The two process that make up the Otto cycle are the isochoric process and isothermal process. The vertical lines are the isochoric processes and the sloped curves are the isothermal process.

Problem #4

Given the following list of properties, determine if they are intensive or extensive (i.e. write "intensive" or "extensive" next the corresponding number on the homework submission sheet). You may have to research what a given property is.

(a) Temperature Intensive
(b) Thermal conductivity Intensive
(c) Density Intensive
(d) Thermal diffusivity Intensive
(e) Total energy Extensive
(f) Kinematic viscosity Intensive
(g) Volume Extensive
(h) Specific Heat Capacity Intensive
(i) Magnetic permeability Intensive
(j) Coefficient of thermal expansion Intensive

Problem #5

Water vapor at 350 [kPa], which has a specific volume of 0.52425 [m^3/kg], is contained in a piston-cylinder device. At this initial state, the piston is 0.2 [m] from the bottom of the cylinder. The water vapor is then cooled in a constant pressure process such that final volume occupies half the initial. Determine:

(a) the final specific volume;
(b) the final mass;
(c) specify if this system is a open or closed, a control mass and/or isolated.

Solution (a):
Develop an expression for the volume of the system in its initial and final state.

$$\forall_1 = \text{A}_{piston}(0.2\ \ [\text{m}])$$

$$\forall_2 = \text{A}_{piston}(0.1\ \ [\text{m}])$$

Create an expression for the mass of the system at states 1 and 2.

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{\text{A}_{piston}(0.2\ \ [\text{m}])}{0.52425\ \ [\text{m}^3/\text{kg}]}$$

$$m_2 = \frac{\forall_2}{\nu_2} = \frac{\text{A}_{piston}(0.1\ \ [\text{m}])}{\nu_2\ \ [\text{m}^3/\text{kg}]}$$

Mass is conserved since the system is closed.

$$m_1 = m_2$$

Combine the equations to solve for specific volume at state 2.

$$\nu_2 = \frac{(\forall_2)(\nu_1)}{\forall_1} = \frac{\text{A}_{piston}(0.1\ \ [\text{m}])(0.52425\ \ [\text{m}^3/\text{kg}])}{(\text{A}_{piston})(0.2\ \ [\text{m}])}$$

Therefore, the final specific volume is:

$$\boxed{\nu_2 = 0.262\ [\text{m}^3/\text{kg}]}$$

Solution (b):
The mass of the system is the quotient of the volume to the specific volume:

$$\frac{\forall_1}{\nu_1} = \frac{\forall_2}{\nu_2}$$

Solution (c):
The system is closed, a controlled mass system and is not isolated.

Problem #6

There exists a container with a volume of 10 [m^3]. This container is filled with 7 [m^3] of coarse stone, which has a density of 1,575 [kg/m^3], 1 [m^3] of sand, which has a density of 1,482 [kg/m^3], and the rest is filled with water, which has a density of 998 [kg/m^3]. Determine:

(a) the average specific volume;
(b) the average density.

Solution (a):
Find the total mass and total volume of the system.

$$m_{total} = (\forall_{stone}\rho_{stone} + \forall_{sand}\rho_{sand} + \forall_{water}\rho_{water})$$

$$= (7\ [\text{m}^3])(1,575\ [\text{kg/m}^3]) + (1\ [\text{m}^3])(1,482\ [\text{kg/m}^3]) + (2\ [\text{m}^3])(998\ [\text{kg/m}^3]) = 14,503\ [\text{kg}]$$

$$\forall_{total} = \sum_{i=1}^{n} \forall_i = (\forall_{stone} + \forall_{sand} + \forall_{water}) = (7\ [\text{m}^3] + 1\ [\text{m}^3] + 2\ [\text{m}^3]) = 10\ [\text{m}^3]$$

The average specific volume will be the quotient of the total volume to the total mass.

$$\nu_{average} = \frac{\forall_{total}}{m_{total}} = \frac{10\ [\text{m}^3]}{14,503\ [\text{kg}]} = \boxed{6.895 \cdot 10^{-4}\ \ [\text{m}^3/\text{kg}]}$$

Solution (b):
The average density is simply the reciprocal of the average specific volume.

$$\rho_{average} = \frac{1}{\nu_{average}} = \frac{1}{6.895 \times 10^{-4}[\text{m}^3/\text{kg}]} = \boxed{1.450 \cdot 10^3\ [\text{kg/m}^3]}$$

Problem #7

A pipe feeds water 20 [m] from the ground to a local reservoir. The water flows with a velocity of approximately 5 [m/s]. It also has a specific internal energy of 100 [kJ/kg]. What is the total specific energy of the system with respect to the ground?

Solution:
The total specific energy a controlled volume is defined as:

$$e = u + \frac{V^2}{2} + gz$$

Here, u is the specific internal energy, V is the velocity of the substance, g is the gravitational constant and z is the height with respect to some reference point. All components of energy have units of [kJ/kg]. Therefore, the total specific energy of the fluid at the specified height is:

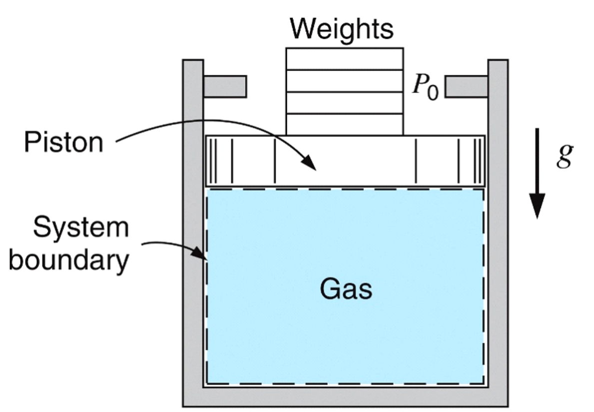
$$e = 100\ [\text{kJ/kg}] + \frac{(5\ [\text{m/s}])^2}{2} + (9.81\ [\text{m/s}^2])(20\ [\text{m}])$$

$$\boxed{e = 100.21\ [\text{kJ/kg}]}$$

Problem #8

Consider the piston-cylinder assembly shown below. Assume that the fluid has been given sufficient time to expand, and that the piston has been stationary for awhile. Weights are then placed on the piston. Given that the total mass of the piston and the attached weights is 10 [kg] and the piston has a diameter of 5 [cm], answer the following:

(a) Determine the internal pressure of the fluid to reach static equilibrium;
(b) What happens if the diameter of the piston-cylinder assembly is halved?
(c) If air has a density 1.225 [kg/m^3] and water has a density of 998 [kg/m^3], which fluid will experience greater internal pressure if used in the piston-cylinder assembly?
(d) Can you assume quasi-static equilibrium throughout the fluid during compression? Explain.
(e) Did the fluid undergo a process or a cycle? Explain.
(f) Is the fluid an isolated system during this compression? Explain.



Solution (a):
The internal pressure is the ratio of the force acting normal to an area of interest.

$$P = \frac{F}{A}$$

The area on which the force acts normal to is the cross-sectional area of the piston. The force itself is the gravitational force of the weights

$$A_{piston} = \frac{(\pi)(0.05\ [\mathrm{m}])^2}{4} = 0.002\ [\mathrm{m}^2]$$

$$F = mg = (10\ [\mathrm{kg}])(9.81\ [\mathrm{m/s}^2]) = 98.1\ [\mathrm{N}]$$

Therefore, the internal pressure of the fluid is:

$$P = \frac{98.1\ [\mathrm{N}]}{0.002\ [\mathrm{m}^2]} = \boxed{49.962\ [\mathrm{kPa}]}$$

Solution (b):
Decreasing the diameter by a factor of 2 reduces the cross-sectional area by a factor of 4. Since the weight does not change and since the internal pressure is inversely proportional to the cross-sectional area, it will increase by a factor of 4.

$$P = \frac{F}{(0.25)(A_{piston})} = \frac{98.1\ [\mathrm{N}]}{(0.25)(0.002\ [\mathrm{m}^2])} = \boxed{199.848\ [\mathrm{kPa}]}$$

The area on which the force acts normal to is the cross-sectional area of the piston. The force itself is the gravitational force of the weights

$$A_{piston} = \frac{(\pi)(0.05\ [\mathrm{m}])^2}{4} = 0.002\ [\mathrm{m}^2]$$

$$F = mg = (10\ [\mathrm{kg}])(9.81\ [\mathrm{m/s}^2]) = 98.1\ [\mathrm{N}]$$

Therefore, the internal pressure of the fluid is:

$$P = \frac{98.1\ [\mathrm{N}]}{0.002\ [\mathrm{m}^2]} = 199.848\ [\mathrm{kPa}]$$

Solution (c):
Internal pressure on a fluid is not dependent on the fluid's density. It is merely the ratio of the force acting normal to an area of interest. Therefore, both fluids will experience the same internal pressure in the same setup.

Solution (d):
Quasi-static equilibrium can not be assumed throughout the compression process. This is because all the weights were placed on the piston at once.

Solution (e):
The fluid underwent a process. This is because there is no subsequent process to bring the piston back to its original state.

Solution (f):
The fluid is not isolated during the compression. This is because external work from the weights are compressing the fluid from outside the system's boundary.