Homework #3

MEMS 0051 - Introduction to Thermodynamics

Assigned: May 21$^{\text{st}}$, 2020
Due: May 28$^{\text{th}}$, 2020

Problem #1

A person decides they want a cup of tea. They bring 0.5 [L] of water to a boil using an electric tea kettle. The water is initially at a temperature and pressure of 20 °C and 101.3 [kPa]. If the average cost of electricity is 13 cents per kilowatt-hour, determine the following:

a) How much did it cost to heat the water to its boiling point? Assume the process is isobaric and that <u>no heat</u> is lost to the surroundings during heating.

Let's look at our states and what we know:

State 1:
$P_1 = 101.3$ [kPa]
$T_1 = 20$ °C
$\nu_1 = ?$
$u_1 = ?$
$x_1 = $ undefined
$\forall_1 = 0.0005$ [m^3]

State 2:
$P_2 = 101.3$ [kPa]
$T_2 = 100$ °C
$\nu_2 = ?$
$u_2 = ?$
$x_2 = 0$

From the First Law of Thermodynamics, we have:

$$U_2 - U_1 = Q_{1\to 2} - W_{1\to 2}$$

If we treat the electricity entering the system as electrical work, and we have no heat exchanged with the surroundings:

$$U_2 - U_1 = -W_{1\to 2} \implies W_{1\to 2} = m(u_1 - u_2)$$

So, we need to find the mass of the water in the kettle and the change in specific internal energy. At State 1, the fluid is compressed, so we can find the specific volume and specific internal energy from the compressed liquid water tables, i.e. Table B.1.4. Alternatively, since specific volume of water is strongly dependent on temperature and weakly dependent on pressure, we can take the specific volume of saturated liquid water at 20 °C from Table B.1.1: :

$$\nu_1 = \nu_{f@20\,°C} = 0.001\,002 \text{ [m}^3\text{/kg]} \qquad u_1 = u_{f@20\,°C} = 83.94 \text{ [kJ/kg]}$$

The mass then is equal to:

$$m = \frac{\forall_1}{\nu_1} = \frac{0.000\,5 \text{ [m}^3\text{]}}{0.001\,002 \text{ [m}^3\text{/kg]}} = 0.5 \text{ [kg]}$$

The specific internal energy at State 2 can also be found in Table B.1.1 using the boiling point of water:

$$u_2 = u_{f@100\,°C} = 418.91 \text{ [kJ/kg]}$$

Now we can find the work required to bring the water to a boil:

$$W_{1\to 2} = m(u_1 - u_2) = (0.5 \text{ [kg]})[(83.94 - 418.91) \text{ [kJ/kg]}] = -167.49 \text{ [kJ]}$$

Finally, we convert Joules to kilowatt-hours, and multiply by 13 cents per kWh to find the total cost:

$$\text{Cost} = (167.49 \text{ [kJ]})(2.7778 \cdot 10^{-4} \text{ [kWh/kJ]})(13 \text{ [cents/kWh]})$$

$$\boxed{\text{Cost} = 0.6 \text{ [cents]}}$$

We have accounted for the work required to change the internal energy of the water, but what about the work needed to change its volume? If your CV was drawn around the water (making it a control mass), then there would have been an amount of work done by the system equal to the pressure times the change in volume. Let us calculate this value. We see from Table B.1.1 for the specific volumes at 20 °C and 100 °C, we have:

$$\nu_1 = \nu_{f@20\,°C} = 0.001\,002\ [\text{m}^3/\text{kg}] \qquad\qquad \nu_2 = \nu_{f@100\,°C} = 0.001\,044\ [\text{m}^3/\text{kg}]$$

Calculating the work then:

$$W_{1\to2} = Pm(\nu_2 - \nu_1) = (101.3\ [\text{kPa}])(0.5\ [\text{kg}])((0.001\,044 - 0.001\,002)\ [\text{m}^3/\text{kg}]) = 0.002\ [\text{kJ}]$$

As we can see, the work for the moving boundary for a liquid is negligible.

b) Say this person forgets they were making tea and all of the water is turned to vapor. Starting at the boiling point, how much did it cost to transform all the water to saturated vapor? Assume again the process is isobaric and that <u>no heat</u> is lost to the surroundings during heating. Let's again look at our states and what we know:

State 2:
$P_2 = 101.3\ [\text{kPa}]$
$T_2 = 100\ °C$
$\nu_2 = 0.001\,044\ [\text{m}^3/\text{kg}]$
$u_2 = 418.91\ [\text{kJ/kg}]$
$x_2 = 0$
$m_2 = 0.5\ [\text{kg}]$

State 3:
$P_3 = 101.3\ [\text{kPa}]$
$T_3 = 100\ °C$
$\nu_3 = ?$
$u_3 = ?$
$x_3 = 1$

Assuming the CV is once again drawn around the water, the moving boundary work in this case will not be negligible and must be accounted for. Our formula for work will now be:

$$U_3 - U_2 = -W_{1\to2,\,\text{electric}} - W_{1\to2,\,\text{boundary}}$$

$$W_{1\to2,\,\text{e}} = m(u_2 - u_3) - Pm(\nu_3 - \nu_2)$$

We can find the specific volume and specific internal energy at state 3 by looking at Table B.1.1 and using the value for a temperature of 100 °C and a quality of $x = 1$:

$$\nu_3 = \nu_{g@100\,°C} = 1.672\,90\ [\text{m}^3/\text{kg}] \qquad\qquad u_3 = u_{g@100\,°C} = 2,506.50\ [\text{kJ/kg}]$$

Calculating the work then:

$$W_{2\to3} = (0.5\ [\text{kg}])[(418.91 - 2,506.50)\ [\text{kJ/kg}]] - (101.3\ [\text{kPa}])(0.5\ [\text{kg}])((1.672\,90 - 0.001\,044)\ [\text{m}^3/\text{kg}])$$

$$W_{2\to3} = -1,043.80\ [\text{kJ}] - 84.68\ [\text{kJ}] = -1,128.48\ [\text{kJ}]$$

Finally, we convert Joules to kilowatt-hours, and multiply by 13 cents per kWh to find the total cost:

$$\text{Cost} = (1,128.48\ [\text{kJ}])(2.777\,78 \cdot 10^{-4}\ [\text{kWh/kJ}])(13\ [\text{cents/kWh}])$$

$$\boxed{\text{Cost} = 4.075\ [\text{cents}]}$$

Note that there is a large difference between the two costs. This is because it takes a substantial amount of energy for a substance to undergo a phase change.

Problem #2

Consider a piston-cylinder device containing 5 [L] of air at a temperature and pressure of 50 °C and 150 [kPa]. The piston begins to rise upward, allowing the initial volume to double, and reducing the temperature of air to 15 °C. Assuming the air behaves as an ideal gas, determine the following:

a) The pressure at State 2, P_2;

Let's look at our states and what we know:

<div align="center">

State 1:
$P_1 = 150$ [kPa]
$T_1 = 50$ °C
$\nu_1 = ?$
$\forall_1 = 0.005$ [m³]

State 2:
$P_2 = ?$ [kPa]
$T_2 = 15$ °C
$\nu_2 = ?$
$\forall_2 = 0.010$ [m³]

</div>

We can find the pressure at state 2 using the Ideal Gas Law:

$$\frac{P_1\forall_1 = mRT_1}{P_2\forall_2 = mRT_2} \implies \frac{P_1 = T_1}{2P_2 = T_2}$$

$$P_2 = \frac{P_1}{2}\left(\frac{T_2}{T_1}\right) = \frac{150 \text{ [kPa]}}{2}\left(\frac{288.15 \text{ [K]}}{323.15 \text{ [K]}}\right) = \boxed{66.88 \text{ [kPa]}}$$

b) The work performed during the process. Assume a linear relationship between pressure and volume;

The work can be found from the following formula for a linear relationship:

$$W_{1\rightarrow 2} = \int P d\forall = \frac{(P_2 + P_1)(\forall_2 - \forall_1)}{2} = \frac{((150 + 66.88) \text{ [kPa]})((0.010 - 0.005) \text{ [m³]})}{2} = \boxed{0.5422 \text{ [kJ]}}$$

c) The amount of heat transferred into or out of the system given $u_1 = 231$ [kJ/kg] and $u_2 = 205.86$ [kJ/kg];

We need the mass to be able to calculate the heat transferred during the process:

$$m = \frac{P_1\forall_1}{RT_1} = \frac{(150 \text{ [kPa]})(0.005 \text{ [m³]})}{(0.287 \text{ [kJ/kgK]})(323.15 \text{ [K]})} = 0.008 \text{ [kg]}$$

We can now find the heat transferred using the conservation of energy:

$$U_2 - U_1 = Q_{1\rightarrow 2} - W_{1\rightarrow 2}$$

$$Q_{1\rightarrow 2} = m(u_2 - u_1) + W_{1\rightarrow 2} = (0.008 \text{ [kg]})((205.86 - 231) \text{ [kJ/kg]}) + 0.5422 \text{ [kJ]} = \boxed{0.3411 \text{ [kJ]}}$$

d) If the air should be treated as an ideal gas. Use $T_c = 132.41$ [K] and $P_c = 3,774$ [kPa];

We can verify if the air could be treated as an ideal gas by using the reduced pressure and temperature criteria:

$$P_r = \frac{P_1}{P_c} = \frac{150 \text{ [kPa]}}{3,774 \text{ [kPa]}} = 0.04 << 1 \qquad T_r = \frac{T_1}{T_c} = \frac{288.15 \text{ [K]}}{132.41 \text{ [K]}} = 2.18 > 2$$

Since the reduced pressure is significantly less than 1, and the reduced temperature is greater than 2, the air can be treated as an ideal gas.

Problem #3

A piston-cylinder device contains 10 [kg] of ammonia at 150 [kPa] and 40 °C. The system is first cooled at constant pressure, reducing the volume and causing the piston to move downwards until the piston comes to rest on some stops. At this point, the volume in the cylinder is only 25% of the original volume. The system is now cooled at constant volume until the temperature reaches -50 °C. Determine the following:

a) The quality, x, at States 1, 2, and 3, if applicable;

Let's look at our states and what we know:

State 1:	State 2:	State 3:
$P_1 = 150$ [kPa]	$P_2 = P_1 = 150$ [kPa]	$P_3 = ?$
$T_1 = 40$ °C	$T_2 = ?$	$T_3 = -50$ °C
$\nu_1 = ?$	$\nu_2 = ?$	$\nu_3 = ?$
$\forall_1 = ?$	$\forall_2 = 0.25\forall_1 = ?$	$\forall_3 = \forall_2 = ?$
$m = 10$ [kg]		

Starting with state 1, by looking at Table B.2.1, we see that $P_1 < P_{sat}$ for a temperature of 40 °C, which means the ammonia is a superheated vapor and therefore the quality at state 1 is undefined. We need another property to determine the quality at state 2. We can find the specific volume at state 1 by looking at Table B.2.2:

$$\nu_1 = \nu_{f@40\ °C} = 1.006\,2\ [\text{m}^3/\text{kg}]$$

Since this is a control mass, the specific volume will change in the same way the volume does. The specific volume at state 2 then is:

$$\nu_2 = 0.25\nu_1 = (0.25)(1.006\,2\ [\text{m}^3/\text{kg}]) = 0.251\,55\ [\text{m}^3/\text{kg}]$$

Looking at Table B.2.1, we see that we now have saturated ammonia. Table B.2.2. shows the saturation temperature for a pressure of 150 [kPa] is -25.22 °C. Interpolating for specific volume of ammonia as a saturated liquid and a saturated vapor:

$$\left[\frac{-25.22 - (-30)\ °C}{-25 - (-30)\ °C}\right] = \left[\frac{\nu_{f@-25.22\ °C} - 0.001476\ [\text{m}^3/\text{kg}]}{0.001\,490 - 0.001\,476\ [\text{m}^3/\text{kg}]}\right] \implies \nu_{f@-25.22\ °C} = 0.001\,489\ [\text{m}^3/\text{kg}]$$

$$\left[\frac{-25.22 - (-30)\ °C}{-25 - (-30)\ °C}\right] = \left[\frac{\nu_{g@-25.22\ °C} - 0.96339\ [\text{m}^3/\text{kg}]}{0.771\,19 - 0.963\,39\ [\text{m}^3/\text{kg}]}\right] \implies \nu_{g@-25.22\ °C} = 0.77965\ [\text{m}^3/\text{kg}]$$

Calculating the quality for state 2 then:

$$x_2 = \frac{\nu_2 - \nu_f}{\nu_g - \nu_f} = \frac{0.251\,55 - 0.001\,489\ [\text{m}^3/\text{kg}]}{0.779\,65 - 0.001\,489\ [\text{m}^3/\text{kg}]} = \boxed{0.321\,3}$$

We again need another property to calculate the quality at state 3. Since the process between states 2 and 3 is isochoric, we know that:

$$\nu_3 = \nu_2 = 0.251\,55\ [\text{m}^3/\text{kg}]$$

Looking at Table B.2.1., we see that we still have saturated ammonia. The quality can be found by taking the saturated liquid and saturated vapor specific volumes at -50 °C:

$$x_3 = \frac{\nu_3 - \nu_f}{\nu_g - \nu_f} = \frac{0.251\,55 - 0.001\,424\ [\text{m}^3/\text{kg}]}{2.627\,00 - 0.001\,424\ [\text{m}^3/\text{kg}]} = \boxed{0.095\,3}$$

b) The net work performed on or by the system moving from State 1 to 3; The process between states 1 and 2 occurs at constant pressure, so the work is calculated as:

$$W_{1\to2} = \int Pm\,d\nu = P_1 m(\nu_2 - \nu_1) = (150\ [\text{kPa}])(10\ [\text{kg}])((0.251\,55 - 1.006\,2)\ [\text{m}^3/\text{kg}]) = -1,131.98\ [\text{kJ}]$$

The process between states 2 and 3 is isochoric, meaning no change in volume occurred. Therefore the work performed is:

$$W_{2\to3} = 0\ [\text{kJ}]$$

The net work then is:

$$W_{net} = W_{1\to2} + W_{2\to3} = \boxed{-1,131.98\ [\text{kJ}]}$$

c) The net heat transferred into or out of the system moving from State 1 to 3; From the First Law of Thermo-dynamics for states 1 to 2:

$$U_2 - U_1 = Q_{1\rightarrow2} - W_{1\rightarrow2}$$

$$Q_{1\rightarrow2} = m(u_2 - u_1) + W_{1\rightarrow2}$$

We need to find values for u_1 and u_2. From Table B.2.2, we see that $u_1 = 1,406.0$ [kJ/kg]. u_2 can be found by interpolating for $u_{f@-25.22\,°C}$ and $u_{g@-25.22\,°C}$ and using the quality at state 2:

$$\left[\frac{(-25.22 - (-30))\,°C}{(-25 - (-30))\,°C}\right] = \left[\frac{u_{f@-25.22\,°C} - 44.08\,[\text{kJ/kg}]}{(66.36 - 44.08)\,[\text{kJ/kg}]}\right] \implies u_{f@-25.22\,°C} = 65.38\,[\text{kJ/kg}]$$

$$\left[\frac{(-25.22 - (-30))\,°C}{(-25 - (-30))\,°C}\right] = \left[\frac{u_{g@-25.22\,°C} - 1,288.9\,[\text{kJ/kg}]}{(1,294.3 - 1,288.9)\,[\text{kJ/kg}]}\right] \implies u_{g@-25.22\,°C} = 1,294.1\,[\text{kJ/kg}]$$

The value for u_2 then is:

$$u_2 = (1 - x_2)u_f + x_2 u_g = (0.678\,7)(65.38\,[\text{kJ/kg}]) + (0.321\,3)(1,294.1\,[\text{kJ/kg}]) = 460.17\,[\text{kJ/kg}]$$

Calculating the heat transferred from states 1 to 2:

$$Q_{1\rightarrow2} = (10\,[\text{kg}])((460.17 - 1,406.0)\,[\text{kJ/kg}]) - 1,131.98\,[\text{kJ}] = -10,590.28\,[\text{kJ}]$$

So, using the First Law of Thermodynamic for states 2 to 3:

$$U_3 - U_2 = Q_{2\rightarrow3} - W_{2\rightarrow3}$$

$$Q_{2\rightarrow3} = m(u_3 - u_2)$$

We need to find a value for u_3. Looking at Table B.2.1. and using the quality at state 3:

$$u_3 = (1 - x_3)u_f + x_3 u_g = (0.9047)(-43.82\,[\text{kJ/kg}]) + (0.095\,3)(1,265.2\,[\text{kJ/kg}]) = 80.93\,[\text{kJ/kg}]$$

Calculating the heat transferred from states 2 to 3:

$$Q_{2\rightarrow3} = (10\,[\text{kg}])((80.93 - 460.17)\,[\text{kJ/kg}]) = -3,792.4\,[\text{kJ}]$$

The net heat transferred <u>out</u> of the system then is:

$$Q_{net} = Q_{1\rightarrow2} + Q_{2\rightarrow3} = \boxed{-14,382.7\,[\text{kJ}]}$$

Alternatively, we could have reduced the number of steps in the solution by solving the following:

$$U_3 - U_1 = Q_{1\rightarrow3} - W_{1\rightarrow3} \implies Q_{1\rightarrow3} = m(u_3 - u_1) + W_{1\rightarrow3}$$

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Problem #4

Consider a well-insulated vessel that is divided into two sections by a thin liner, with each section containing water. The liner begins to fail and allows the two sections to mix until the vessel reaches thermodynamic equilibrium. All that is known about the first section before mixing occurred is that its initial pressure was 1,000 [kPa] and initial specific internal energy was 740.16 [kJ/kg]. The only thing known about the second section before mixing occurred is that it occupied most of the volume of the vessel at 4.982 [m^3]. After the vessel reaches equilibrium, the pressure is now 1,600 [kPa] and the specific internal energy is 2,692.26 [kJ/kg]. If no heat is transferred into or out of the system, and the total volume of the vessel is 5 [m^3], determine the following:

a) The specific volume, ν, specific internal energy, u, volume, $\forall$, and mass, m, for both sections before mixing (State 1) and for the vessel after mixing (State 2).

Let's look at our states and what we know. We will denote the two sections as A and B:

<u>State 1,A:</u>
$P_{1,A} = 1{,}000$ [kPa]
$T_{1,A} = ?$
$\nu_{1,A} = ?$
$u_{1,A} = 740.16$ [kJ/kg]
$\forall_{1,A} = ?$
$m_{1,A} = ?$

<u>State 1,B:</u>
$P_{1,B} = ?$
$T_{1,B} = ?$
$\nu_{1,B} = ?$
$u_{1,B} = ?$
$\forall_{1,B} = 4.982$ [m^3]
$m_{1,B} = ?$

<u>State 2:</u>
$P_2 = 1{,}600$ [kPa]
$T_2 = ?$
$\nu_2 = ?$
$u_2 = 2{,}692.26$ [kJ/kg]
$\forall_2 = 5$ [m^3]
$m_2 = ?$

Let's start at state 1,A. Looking at Table B.1.2., we see that for a pressure of 1,000 [kPa] that $u_{1,A} < u_{f@1000\,[\text{kPa}]}$. This means that the water in section A must be a <u>compressed/subcooled liquid</u>. Like specific volume, the specific internal energy of a compressed liquid is highly dependent on temperature, and minimally dependent on pressure. Looking at Table B.1.1, we see that the given specific internal energy corresponds to a temperature of 175 °C. The specific volume in section A is:

$$\boxed{\nu_{1,A} = 0.001\,121 \ [\text{m}^3/\text{kg}]}$$

The vessel is rigid, so we can find the volume of section A:

$$\forall_{1,A} = \forall_2 - \forall_{1,B} = (5 - 4.982) \ [\text{m}^3] = \boxed{0.018 \ [\text{m}^3]}$$

Now we can find the mass in section A:

$$m_{1,A} = \frac{\forall_1}{\nu_1} = \frac{0.018 \ [\text{m}^3]}{0.001\,121 \ [\text{m}^3/\text{kg}]} = \boxed{16.057 \ [\text{kg}]}$$

Moving on to section B, we see that there is insufficient data to find any properties.

State 2 contains enough information to find any property. Looking at Table B.1.3 on page 786, we see that the given pressure and specific internal energy correspond to a temperature of 250 °C and a specific volume of:

$$\boxed{\nu_2 = 0.141\,84 \ [\text{m}^3/\text{kg}]}$$

Since we know the total volume of the vessel, we can find the total mass:

$$m_2 = \frac{\forall_2}{\nu_2} = \frac{5 \ [\text{m}^3]}{0.141\,84 \ [\text{m}^3/\text{kg}]} = \boxed{35.251 \ [\text{kg}]}$$

Using the conservation of mass, we can determine the mass in section B:

$$m_{1,B} = m_2 - m_{1,A} = (35.251 - 16.057) \ [\text{kg}] = \boxed{19.194 \ [\text{kg}]}$$

Since we know the mass in section B, we can find the specific volume:

$$\nu_{1,B} = \frac{\forall_{1,B}}{m_{1,B}} = \frac{4.982 \ [\text{m}^3]}{19.194 \ [\text{kg}]} = \boxed{0.259\,56 \ [\text{m}^3/\text{kg}]}$$

To determine $u_{1,B}$, we need to use the First Law of Thermodynamics:

$$U_2 - U_1 = Q_{1 \to 2} - W_{1 \to 2}$$

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If only one control volume is drawn and it encompasses the contents of the tank, then no work is performed either on or by the system (the vessel is rigid - no change in volume):

$$W_{1\to2} = 0 \text{ [kJ]}$$

In addition, we know heat is not exchanged with the surroundings, so:

$$Q_{1\to2} = 0 \text{ [kJ]}$$

The conservation of energy now reduces to:

$$U_1 = U_2$$

$$m_{1,A}u_{1,A} + m_{1,B}u_{1,B} = m_2 u_2$$

Rearranging:

$$u_{1,B} = \frac{m_2 u_2 - m_{1,A}u_{1,A}}{m_{1,B}} = \frac{(35.251 \text{ [kg]})(2,692.26 \text{ [kJ/kg]}) - (16.057 \text{ [kg]})(740.16 \text{ [kJ/kg]})}{(19.194 \text{ [kg]})} = \boxed{4,325.32 \text{ [kJ/kg]}}$$

b) Is this system open, closed, or isolated? Explain why;
As no work is performed on or by the system, and no heat is exchanged with the surroundings, the vessel can be considered an isolated system.