Chapter 6 - Entropy

Lecture 19
Sections 6.2-6.4

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
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Student Learning Objectives

At the end of the lecture, students should be able to:

- Formulate the definition of entropy
- Calculate the entropy of a substance given two independent properties
- Calculate the change of entropy for a reversible process

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Entropy

- The Clausius Inequality indicates that the more negative the cyclic integral of heat transfer per surface temperature, the more irreversibilities generated within the cycle

$$\oint \left(\frac{\delta Q}{T}\right)_b \leq 0$$

- The definition of entropy follows the path-independence formulation of work

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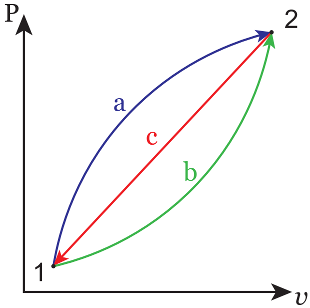
Entropy of a Substance

- Let us consider a C.M. undergoing a reversible cycle between an initial and final state



- Consider the cycle between 1 and 2 via a and c

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_a + \int_2^1 \left(\frac{\delta Q}{T}\right)_c$$

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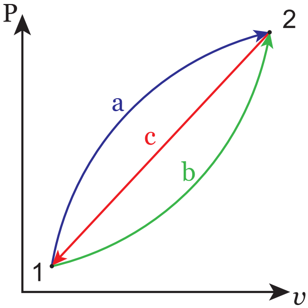
Entropy of a Substance

- Now consider the same C.M. undergoing a reversible cycle between 1 and 2 via b and c



- The Clausius Inequality is expressed as

$$\oint \frac{\delta Q}{T} = \int_1^2 \left(\frac{\delta Q}{T}\right)_b + \int_2^1 \left(\frac{\delta Q}{T}\right)_c$$

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Entropy of a Substance

- The cyclic integral from 1 to 2 via a and c, and b and c, are equal

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_{\mathrm{a}} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\mathrm{c}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\mathrm{b}} + \int_2^1 \left(\frac{\delta Q}{T}\right)_{\mathrm{c}}$$

- We note between 2 and 1 via c is present on both sides, therefore

$$\int_1^2 \left(\frac{\delta Q}{T}\right)_{\mathrm{a}} = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\mathrm{b}}$$

- Thus, the cyclic integral of $\delta Q/T$ is the same for all paths between 1 and 2 for all reversible processes, i.e. the quantity is independent of path

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Definition of Entropy

- **Entropy** (S, s) [kJ/kg-K] for a reversible process is defined in the following form

$$dS \equiv \left(\frac{\delta Q}{T}\right)_{\text{rev}}$$

- We define the change of entropy between state 1 and 2 as

$$\boxed{S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{rev}}}$$

- Since entropy is an extensive property, the change of entropy between two states is the same for <u>all</u> processes, both reversible and irreversible

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Entropy of a Substance

- Entropy follows the same formulations as did specific volume, specific internal energy and specific enthalpy

$$s = (1-x)s_f + x\, s_g$$

$$s = s_f + x(s_g - s_f)$$

- Entropy is taken in reference to a specified temperature - for our steam tables, s=0 [kJ/kg-K] at T=0.01 °C - it is irrelevant since the difference of entropy is our concern

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Entropy Change - Carnot Cycle

- Consider the change of entropy within the Carnot Cycle

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{rev}} = \frac{1}{T_H} \int_1^2 \delta Q + \frac{1}{T_L} \int_3^4 \delta Q$$

$$= \frac{Q_{1\to2}}{T_H} + \frac{Q_{3\to4}}{T_L} = (S_2 - S_1) + (S_4 - S_3)$$

- We can depict this process on a T-s diagram to illustrate the four difference processes within the cycle

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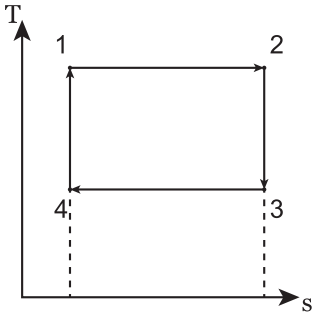
T-s - Carnot Cycle

- Depicting the Carnot Cycle on a T-s diagram, we can identify unique features



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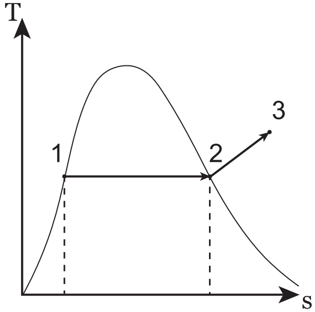
Internally Reversible Processes

- Considering a system where we change from a saturated liquid to saturated vapor



- The change of entropy between states 1 and 2 is simply

$$s_2 - s_1 = \frac{1}{mT}\int_1^2 \delta Q = \frac{q_{1\to 2}}{T} = \frac{h_g - h_f}{T}$$

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Internally Reversible Processes

- If we wanted to determine how much heat is transferred to go from state 2 to 3

$$q_{2\to 3} = \frac{1}{m}\int_2^3 \delta Q$$

- Recalling from the Clausius Inequality

$$dS = \frac{\delta Q}{T} \implies \delta Q = T\,dS$$

- Therefore

$$q_{2\to 3} = \int_2^3 T\,ds$$

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Example #1

- A piston-cylinder contains 1 [L] of saturated liquid R-410 A at 20 °C and slowly expands in a constant-temperature process to a final pressure of 400 [kPa]. Assuming this process is reversible, calculate the work and heat transfer of this process.

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Example #1

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Example #1

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Student Learning Objectives

At the end of the lecture, students should be able to:

- Formulate the definition of entropy
  - Entropy is a quantification of the irreversibility associated with a heat transfer process occurring at a boundary temperature.
- Calculate the entropy of a substance given two independent properties
  - The change of entropy between two states is the integration of the heat transferred per boundary temperature between the initial and final state.
- Calculate the change of entropy for a reversible process
  - A reversible process can have a change of entropy. However, evaluate over a cycle, the net change of entropy will be zero.

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Suggested Problems

- 6.24, 6.25, 6.26, 6.27 6.28, 6.30, 6.33, 6.35, 6.36, 6.38, 6.41, 6.46, 6.56, 6.59

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