Chapter 6 - Entropy

Lecture 22

MEMS 0051 Introduction to Thermodynamics

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Student Learning Objectives

At the end of the lecture, students should be able to:

- Derive the relationship for relative pressure and relative volume for istentropic processes for Ideal Gases

Entropy Change for Ideal Gases

- Recall, the change of entropy of an Ideal Gas undergoing an isentropic can be expressed as

$$0 = s^o(T_2) - s^o(T_1) - R\ln\left(\frac{P_2}{P_1}\right)$$

- Saw we know the temperature at State 1 and pressures at States 1 and 2

$$s^o(T_2) = s^o(T_1) + R\ln\left(\frac{P_2}{P_1}\right)$$

- Since T_1 is known, it follows $s^o(T_1)$ could be found via interpolation, then $s^o(T_2)$ could be calculated, and then T_2 found via interpolation

Relative Pressure Formulation

- Alternatively, re-arranging our first expression in terms of pressure

$$P_2 = P_1 \exp\left(\frac{s^o(T_2) - s^o(T_1)}{R}\right)$$

- Re-expressing in terms of the ratio of pressures

$$\frac{P_2}{P_1} = \exp\left(\frac{s^o(T_2)/R}{s^o(T_1)/R}\right)$$

Relative Pressure

- The numerator and denominator of within the exponential are solely a function of temperature, and each quantity is termed **relative pressure**, $P_r(T)$, not to be confused with reduced pressure

$$\boxed{\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}}$$

- This is only valid for an isentropic process
- Relative pressure values are provided in Table A.7.2

Relative Volume Formation

- We can also derive a relation between specific volume and temperature
- Recall for an Ideal gas, the ratio of specific volumes between States 1 and 2

$$\frac{\nu_2}{\nu_1} = \left(\frac{RT_2}{P_2}\right)\left(\frac{P_1}{RT_1}\right)$$

- States 1 and 2 have the same entropy, therefore

$$\frac{\nu_2}{\nu_1} = \left(\frac{RT_2}{P_r(T_2)}\right)\left(\frac{P_r(T_1)}{RT_1}\right)$$

Relative Volume

- The ratio of the gas constant times temperature per relative pressure is referred to as **relative volume**, $\nu_r(T)$

$$\boxed{\frac{\nu_2}{\nu_1} = \frac{\nu_{r2}}{\nu_{r1}}}$$

- These two relations can bed used in conjunction with our previous expressions

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{\nu_1}{\nu_2}\right)^{k-1}$$

- The benefit is the polytropic/isentropic index does not need to be known (although k=$C_{P0}/C_{\forall 0}$, but said specific heats are evaluated at STSP)

Example #1

- A rigid, well-insulated tank initially contains 5.0 [kg] of air at a pressure of 500 [kPa] and a temperature of 500 [K]. A leak develops in the tank, and air slowly escapes until the pressure within the tank reaches atmospheric. Determine the total amount of mass remaining in the tank and it's final temperature.

Example #1

Example #1

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Student Learning Objectives

At the end of the lecture, students should be able to:

- Derive the relationship for relative pressure and relative volume for istentropic processes for Ideal Gases
  - The following relations can be used to determine properties of interest:

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}}; \quad \frac{\nu_2}{\nu_1} = \frac{\nu_{r2}}{\nu_{r1}}$$

Suggested Problems

- Repeat the suggested problems from Lecture 21, and use the relative pressure and volume formulations to determine pertinent system properties, where applicable.