Chapter 4 - Energy Analysis for a Control Volume

Lecture 24
Sections 4.1-4.2

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Student Learning Objectives

At the end of the lecture, students should be able to:

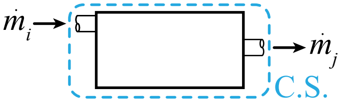
- Understand and apply the Conservation of Mass for an open system
- Apply the continuity equation to steady and transient problems
- Understand and apply the Conservation of Energy for an open system

Conservation of Mass

- For a closed system, the amount of mass within the system remained invariant with respect to

$$\frac{dm}{dt} = \dot{m} = 0$$

- For an open system, mass may do three things:
  1. remain constant with respect to time
  2. accumulate within the system with respect to time
  3. disperse from the system with respect to time



Conservation of Mass - Open System

- We can express the time rate of change of mass within the control volume as the rate of mass flow into the system less the rate of mass flow out of the system, i.e. the **continuity equation**

$$\boxed{\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \sum_{j=1}^{M} \dot{m}_j}$$

- We introduce the summation convention, for there can be multiple inlets and outlets associated with our systems
- If the mass flow into the system is the same as what exits the system:

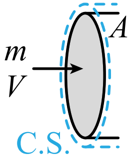
$$\frac{dm}{dt} = 0$$

Mass flow rate

- Mass is simply density times volume

$$m = \rho \forall$$

- On a time rate basis, i.e. how much mass is crossing a specified control surface, the **mass flow rate** is the density times cross-sectional flow area times velocity, or density times volumetric flow rate

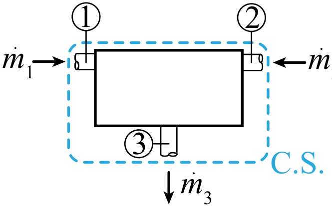


$$\boxed{\dot{m} = \rho A V = \rho \dot{\forall}\,[\mathrm{kg/s}]}$$

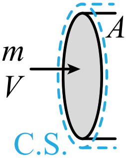
Example #1

- A feedwater heater is operating at steady state. There are two inlets and one exit. At inlet 1, water at a pressure of 700 [kPa] and temperature of 200 °C enters at a flow rate of 50 [kg/s]. At inlet 2, liquid water enters with a pressure of 700 [kPa] and a temperature of 40 °C through a 30 [cm^2] pipe. Saturated liquid exits (i.e. 3) with a volumetric flow rate of 0.07 [m^3/s].
- Determine the mass flow rates at inlet 2 and 3 and the velocity at 2.

Example #1



Example #1



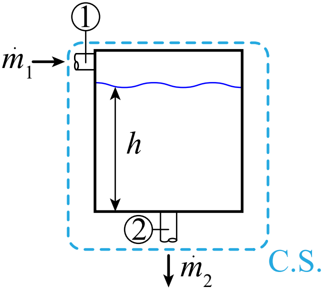
Example #2

- Water enters a tank at 1 with a mass flow rate of 3 [kg/s]. Water exits the tank at 2 proportional to the height of fluid within the tank such that:

$$\dot{m}_2 = 0.6h\,[\mathrm{kg/s}]$$

- h is the instantaneous height of fluid within the tank. If the tank diameter is 0.5 [m], and the density of water is assumed 1,000 [kg/m^3], and the tank is assumed initially empty, determine the variation of height of water within the tank with respect to time.

Example #2



Example #2

Example #2

Example #2

Conservation of Energy - Closed Systems

- Recall the Conservation of Energy equation for a closed system (i.e. no mass can cross the C.S.)

$$\frac{d\,E}{dt} = \dot{Q} - \dot{W}$$

- The LHS can be expressed as

$$\frac{d\,E}{dt} = \frac{d\,U}{dt} + \frac{d\,KE}{dt} + \frac{d\,PE}{dt}$$

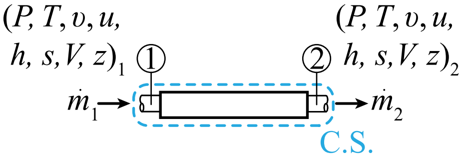
- When mass is able to cross the C.S., the system becomes an **open system**

Conservation of Energy - Open Systems

- The Conservation of Energy equation can be modified to consider energy flow into and out of the system

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) - \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + g z_j \right)$$

- The mass flow rates ($\dot{m}$) can bring/take energy into/out of the system



Energy Flux

- The third term on the RHS represents energy flux into our system associated with a mass flow rate into our system

$$\sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$

- The mass flow rate brings in energy in form of internal, pressure, kinetic and potential
- The internal and pressure energy are expressed in terms of enthalpy

Conservation of Energy

- The fourth term on the RHS represents energy flux out of our system associated with a mass flow rate out of our system

$$\sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + gz_j \right)$$

- The mass flow rate takes away in energy in form of internal, pressure, kinetic and potential

Conservation of Energy - Homogeneity

- Let us evaluate the units of the C.o.E. equation, starting with energy, heat and work

$$\frac{dE}{dt} \equiv \left[\frac{\text{kJ}}{\text{s}}\right]; \quad \dot{Q} \equiv \left[\frac{\text{kJ}}{\text{s}}\right]; \quad \dot{W} \equiv \left[\frac{\text{kJ}}{\text{s}}\right]$$

- Looking at each energy flux

$$\dot{m}h \equiv \left[\frac{\text{kg}}{\text{s}}\right]\left[\frac{\text{kJ}}{\text{kg}}\right] = \left[\frac{\text{kJ}}{\text{s}}\right]$$

- For the terms with velocity, gravity and elevation, recall 1 [N] = [kg-m/s^2]

$$\dot{m}V^2 \equiv \left[\frac{\text{kg}}{\text{s}}\right]\left[\frac{\text{m}^2}{\text{s}^2}\right] = \left[\frac{\text{Nm}}{\text{s}}\right] = \left[\frac{\text{J}}{\text{s}}\right]$$

$$\dot{m}gz \equiv \left[\frac{\text{kg}}{\text{s}}\right]\left[\frac{\text{m}}{\text{s}^2}\right][\text{m}] = \left[\frac{\text{Nm}}{\text{s}}\right] = \left[\frac{\text{J}}{\text{s}}\right]$$

Conservation of Energy - Steady State

- When our system is at steady-state, the LHS of the C.o.E. is identically equal to zero

$$\frac{d\,E}{dt} = \frac{d\,U}{dt} + \frac{d\,KE}{dt} + \frac{d\,PE}{dt} = 0$$

- Thus, we re-express the C.o.E. in terms of energy entering and exiting the system as

$$\dot{Q} + \textstyle\sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) = \dot{W} + \textstyle\sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + g z_j \right)$$

- There a multitude of steady-state devices we will analyze: pumps, compressors, boilers, turbines, condensers, throttles, traps, each with a unique C.o.E. equation describing its behavior

Conservation of Energy - Unsteady

- If the energy of the system varies as a function of time, the C.o.E. is termed "transient"
- The properties of the system evolve over time and must be formulated using a first-order differential equation

$$\frac{d\,E}{dt} = \frac{d\,U}{dt} + \frac{d\,KE}{dt} + \frac{d\,PE}{dt}$$
$$= \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$
$$- \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + g z_j \right)$$

Student Learning Objectives

At the end of the lecture, students should be able to:

- Understand and apply the Conservation of Mass for an open system
  - The continuity equation states the time rate of change of mass within a control volume is equal to the sum of the mass flows in less the mass flow out, expressed as

$$\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \sum_{j=1}^{M} \dot{m}_j$$

Student Learning Objectives

- Apply the continuity equation to steady and transient problems
  - For a steady-state problem, the sum of the mass flows into the system must be equal to the sum of the mass flows out of the system. Transient problems typically required the formulation of a first-order ordinary differential equation describing the rate equation.
- Understand and apply the Conservation of Energy for an open system

Student Learning Objectives

- Understand and apply the Conservation of Energy for an open system
  - The C.o.E. for open systems states that the change of energy within the control volume is equation to the rate heat is supplied less the rate at which work is done, plus the net energy flow into the system (enthalpy, and kinetic and potential energies) associated with the net mass influx. This is stated as

$$\frac{d\,E}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) - \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + g z_j \right)$$

Suggested Problems

- 4.11, 4.12, 4.13, 4.15, 4.17, 4.18, 4.19