Homework #4

MEMS 0051 - Introduction to Thermodynamics

Assigned: May 28th, 2020
Due: June 4th, 2020

Problem #1

Calculate the change in specific internal energy, u_2-u_1, for the following items. Assume the change occurs at constant pressure. Note that the change in temperature, T_2 - T_1, is the same in both Celsius, °C, and Kelvin, [K].

a) Aluminum at 100 [kPa] cooled from 200 °C to 80 °C; From Table A.3:

$$u_2 - u_1 = C(T_2 - T_1) = (0.90 \text{ [kJ/kg-K]})((80 - 200) \text{ °C}) = \boxed{-108 \text{ [kJ/kg]}}$$

Note that for solids and liquids, $C_\forall = C_P = C$

b) Ammonia at 100 [kPa] cooled from -35 °C to -50 °C; The ammonia must be a liquid at these temperatures and pressure, so using Table A.4:

$$u_2 - u_1 = C(T_2 - T_1) = (4.84 \text{ [kJ/kg-K]})((-50 - (-35)) \text{ °C}) = \boxed{-72.6 \text{ [kJ/kg]}}$$

If we want to be more accurate, we can also use the saturated liquid values for internal energy from Table B.2.1:

$$u_2 - u_1 = (-43.82 - 21.93) \text{ [kJ/kg]} = \boxed{-65.75 \text{ [kJ/kg]}}$$

c) Carbon monoxide at 100 [kPa] heated from -15 °C to 45 °C; From Table A.5:

$$u_2 - u_1 = C_\forall(T_2 - T_1) = (0.744 \text{ [kJ/kg-K]})((45 - (-15)) \text{ °C}) = \boxed{44.64 \text{ [kJ/kg]}}$$

d) R-12 at 100 [kPa] heated from 0 °C to 105 °C; From Table A.5:

$$u_2 - u_1 = C_\forall(T_2 - T_1) = (0.547 \text{ [kJ/kg-K]})((105 - 0) \text{ °C}) = \boxed{57.44 \text{ [kJ/kg]}}$$

Note that the given temperature range does not meet the criteria for the R-12 to be considered an ideal gas, and therefore caution should be used when using this formula.

e) R-134a at 101.3 [kPa] cooled from a quality of 0.6 to a saturated liquid; From Table B.5.1 using a pressure of 101.3 [kPa]:

$$u_2 - u_1 = u_f - u_{x=0.6} = (165.73 - 284.03) \text{ [kJ/kg]} = \boxed{-118.3 \text{ [kJ/kg]}}$$

Calculate the change in specific enthalpy, h_2-h_1, for the following items. Assume the change occurs at constant pressure. Note that the change in temperature, T_2 - T_1, is the same in both Celsius, °C, and Kelvin, [K].

a) Aluminum at 100 [kPa] cooled from 200 °C to 80 °C; From Table A.3:

$$h_2 - h_1 = C(T_2 - T_1) = (0.90 \text{ [kJ/kg-K]})((80 - 200) \text{ °C}) = \boxed{-108 \text{ [kJ/kg]}}$$

Note that for solids and liquids, $C_\forall = C_P = C$

b) Water at 100 [kPa] heated from 10 °C to 95 °C; From Table A.4:

$$h_2 - h_1 = C(T_2 - T_1) = (4.18\ [\text{kJ/kg-K}])((95 - 10)\ °\text{C}) = \boxed{355.3\ [\text{kJ/kg}]}$$

If we want to be more accurate, from Table B.1.1, we have:

$$h_2 - h_1 = h_{@95\ °\text{C}} - h_{@10\ °\text{C}} = (397.94 - 41.99)\ [\text{kJ/kg}] = \boxed{355.95\ [\text{kJ/kg}]}$$

c) Carbon monoxide at 100 [kPa] heated from -15 °C to 45 °C; From Table A.5:

$$h_2 - h_1 = C_P(T_2 - T_1) = (1.041\ [\text{kJ/kg-K}])((45 - (-15))\ °\text{C}) = \boxed{62.46\ [\text{kJ/kg}]}$$

d) R-22 at 100 [kPa] cooled from 30 °C to -41.03 °C; From Table A.5:

$$h_2 - h_1 = C_P(T_2 - T_1) = (0.658\ [\text{kJ/kg-K}])((-41.03 - 30)\ °\text{C}) = \boxed{-46.74\ [\text{kJ/kg}]}$$

Note that the given temperature range does not meet the criteria for the R-22 to be considered an ideal gas, and therefore caution should be used when using this formula.

e) Nitrogen (N_2) at 101.3 [kPa] heated from a quality of 0.4 to a saturated vapor; From Table B.6.1:

$$h_2 - h_1 = h_g - h_{x=0.4} = (76.69 - (-42.614))\ [\text{kJ/kg}] = \boxed{119.30\ [\text{kJ/kg}]}$$

Problem #2

Consider a piston-cylinder device where the piston is putting pressure on water due to a linear spring, i.e. a spring that follows Hooke's Law, attached to the other side of the piston. The saturated water is at a pressure of 400 [kPa] and a quality of 0.8. Heat is now added such that the spring is further compressed and the height of the piston increases by 0.5 [m]. If the area of the piston is 0.1 [m^2] and the spring constant, k, is 80 [kN/m], determine the following:

a) The height of the piston at state 1; Let's look at our states and what we know:

State 1:
P_1 = 400 [kPa]
T_1 = ?
ν_1 = ?
u_1 = ?
x_1 = 0.8
L_1 = ?
m = ?

State 2:
P_2 = ?
T_2 = ?
ν_2 = ?
u_2 = ?
x_2 = ?
$L_2 = L_1 + 0.5$ [m]
A_p = 0.1 [m^2]
k = 80 [kN/m]

State 1 has two independent properties. The remaining properties at state 1 are:

$$T_1 = 143.63\ °\text{C} \qquad \nu_1 = 0.37019\ [\text{m}^3/\text{kg}] \qquad u_1 = 2163.70\ [\text{kJ/kg}]$$

If the spring is in equilibrium, we can rearrange Hooke's Law to solve for the spring deflection, i.e. the height of the piston from the bottom of the cylinder:

$$F_1 = kL_1, \text{ but } F_1 = P_1 A_p, \text{ so,}$$

$$P_1 A_p = kL_1$$

Solving for L_1:

$$L_1 = \frac{P_1 A_p}{k} = \frac{(400\ [\text{kPa}])(0.1\ [\text{m}^2])}{80\ [\text{kN/m}]} = \boxed{0.5\ [\text{m}]}$$

b) The work done by the system; Since no external pressure is acting on the piston, the work done by the system is equivalent to the energy stored inside the spring. We know the height of the piston at state 1 is 0.5 [m], so the height of the piston at state 2 must be 1 [m]. Solving for work:

$$W_{1\to 2} = \int_{0.5\ [\mathrm{m}]}^{1.0\ [\mathrm{m}]} kL\,dL = \frac{1}{2}kL^2\Big|_{0.5\ [\mathrm{m}]}^{1.0\ [\mathrm{m}]} = \frac{1}{2}(80\ [\mathrm{kN/m}])(((1.0)^2 - (0.5)^2)\ [\mathrm{m}^2]) = \boxed{30\ [\mathrm{kJ}]}$$

c) The heat transferred into the system;
Using the conservation of energy:

$$U_2 - U_1 = Q_{1\to 2} - W_{1\to 2}$$
$$Q_{1\to 2} = m(u_2 - u_1) + W_{1\to 2}$$

We need the mass of the system. Using the values from state 1:

$$m = \frac{\forall_1}{\nu_1} = \frac{A_p L_1}{\nu_1} = \frac{(0.1\ [\mathrm{m}^2])(0.5\ [\mathrm{m}])}{0.37019\ [\mathrm{m}^3/\mathrm{kg}]} = 0.1351\ [\mathrm{kg}]$$

We need to find u_2. To do this, we need to find two independent properties at state 2. We can find the specific volume at state 2 since we know the height of the piston and the mass:

$$\nu_2 = \frac{\forall_2}{m} = \frac{(0.1\ [\mathrm{m}^2])(1.0\ [\mathrm{m}])}{0.1351\ [\mathrm{kg}]} = 0.74019\ [\mathrm{m}^3/\mathrm{kg}]$$

We can also find the pressure using Hooke's Law:

$$P_2 = \frac{F_2}{A_p} = \frac{kL_2}{A_p} = \frac{(80\ [\mathrm{kN/m}])(1.0\ [\mathrm{m}])}{0.1\ [\mathrm{m}^2]} = 800\ [\mathrm{kPa}]$$

Based on the pressure and specific volume, the water at state 2 is a superheated vapor. We can find u_2 by interpolating between 1,000 °C and 1,100 °C under the 800 [kPa] entry. The value then is:

$$u_2 = 4,072.85\ [\mathrm{kJ/kg}]$$

Finally, we can solve for the heat transferred into the system:

$$Q_{1\to 2} = (0.1351\ [\mathrm{kg}])((4,072.85 - 2,163.70)\ [\mathrm{kJ/kg}]) + 30\ [\mathrm{kJ}] = \boxed{287.93\ [\mathrm{kJ}]}$$

Problem #3

A piston-cylinder device contains Helium (He) at 100 [kPa] and -40 °C. The helium now undergoes isobaric heating until it reaches a temperature of 90 °C. If the change in internal energy is $U_2 - U_1 = 202.54$ [kJ] and the helium can be treated as an ideal gas, determine the following:

a) The work performed; Let's look at our states and what we know:

State 1:	$\longrightarrow$ P = c $\longrightarrow$	State 2:
P_1 = 100 [kPa]		P_2 = 100 [kPa]
T_1 = -40 °C		T_2 = 90 °C
$\forall_1$ = ?		$\forall_2$ = ?
ν_1 = ?		ν_2 = ?
u_1 = ?	$U_2 - U_1$ = 202.54 [kJ]	u_2 = ?
h_1 = ?		h_2 = ?

The first property we need is the mass of the system. Since we have the change in internal energy, if we have the change in specific internal energy, we can find the mass. Assuming the helium is an ideal gas:

$$u_2 - u_1 = C_\forall(T_2 - T_1)$$

From Table A.5, we have the specific heat at constant volume, so solving for the change in specific internal energy:

$$u_2 - u_1 = (3.116\ [\mathrm{kJ/kg\text{-}K}])((90 - (-40))\ ^\circ\mathrm{C}) = 405.08\ [\mathrm{kJ/kg}]$$

Solving for the mass:

$$m = \frac{\Delta U}{\Delta u} = \frac{202.54\ [\text{kJ}]}{405.08\ [\text{kJ/kg}]} = 0.5\ [\text{kg}]$$

Solving for the volume at state 1:

$$\forall_1 = \frac{mRT_1}{P} = \frac{(0.5\ [\text{kg}])(2.0771\ [\text{kJ/kg-K}])(233.15\ [\text{K}])}{100\ [\text{kPa}]} = 2.421\ [\text{m}^3]$$

Solving for the volume at state 2:

$$\forall_2 = \frac{mRT_2}{P} = \frac{(0.5\ [\text{kg}])(2.0771\ [\text{kJ/kg-K}])(363.15\ [\text{K}])}{100\ [\text{kPa}]} = 3.771\ [\text{m}^3]$$

Solving for the work performed then:

$$W_{1\to2} = P(\forall_2 - \forall_1) = (100\ [\text{kPa}])((3.771 - 2.421)\ [\text{m}^3]) = \boxed{135\ [\text{kJ}]}$$

b) The heat transferred using internal energy, U;
From the conservation of energy:

$$U_2 - U_1 = Q_{1\to2} - W_{1\to2}$$
$$Q_{1\to2} = m(u_2 - u_1) + W_{1\to2}$$

We have all these values, so:

$$Q_{1\to2} = (0.5\ [\text{kg}])((405.08\ [\text{kJ/kg}]) + 135\ [\text{kJ}] = \boxed{337.54\ [\text{kJ}]}$$

c) The heat transferred using enthalpy, H;
Since this is a constant pressure process, we can use the constant pressure specific heat from Table A.5:

$$Q_{1\to2} = mC_P(T_2 - T_1) = (0.5\ [\text{kg}])(5.193\ [\text{kJ/kg-K}])((90 - (-40))\ ^\circ\text{C}) = \boxed{337.55\ [\text{kJ}]}$$

Problem #4

A piston-cylinder device contains air at 100 [kPa] and 25 °C. At this state, the specific enthalpy, h_1, is also known, and is given as 298.62 [kJ/kg]. From this state, the air is first heated at constant volume to a temperature of 400 °C. The air next undergoes a polytropic process where the polytropic index is unknown to a volume of 0.1 [m^3]. The air then undergoes isothermal expansion until its specific enthalpy is $h_4 = 399.02$ [kJ/kg]. The air is now finally cooled at constant pressure until it has returned to its initial state, experiencing a reduction in internal energy of 16.763 [kJ]. Treating the air as an ideal gas, determine the following:

a) The pressure, P, temperature, T, and volume, $\forall$ for each state (*Hint:* there are four states); Let's look at our states and what we know:

State 1:	State 2:	State 3:	State 4:
$P_1 = 100$ [kPa]	$P_2 = ?$	$P_3 = ?$	$P_4 = P_1 = 100$ [kPa]
$T_1 = 25$ °C	$T_2 = 400$ °C	$T_3 = ?$	$T_4 = T_3 = ?$
$\forall_1 = ?$	$\forall_2 = \forall_1 = ?$	$\forall_3 = 0.1$ [m^3]	$\forall_4 = ?$
$\nu_1 = ?$	$\nu_2 = ?$	$\nu_3 = ?$	$\nu_4 = ?$
$u_1 = ?$	$u_2 = ?$	$u_3 = ?$	$u_4 = ?$
$h_1 = 298.62$ [kJ/kg]	$h_2 = ?$	$h_3 = ?$	$h_4 = 399.02$ [kJ/kg]
$U_1 - U_4 = -16.763$ [kJ]			

There are a couple different ways to begin finding the unknown properties at each state. Let us start by determining the specific volume at state 1. Using the Ideal Gas Law:

$$\nu_1 = \frac{RT_1}{P_1} = \frac{(0.287 \text{ [kJ/kg-K]})(298.15 \text{ [K]})}{100 \text{ [kPa]}} = 0.85569 \text{ [m}^3\text{/kg]}$$

Since the mass is constant within the piston-cylinder, we know that:

$$\nu_2 = \nu_1 = 0.85569 \text{ [m}^3\text{/kg]}$$

We can now find the pressure at state 2 using the Ideal Gas Law:

$$P_2 = \frac{RT_2}{\nu_2} = \frac{(0.287 \text{ [kJ/kg-K]})(673.15 \text{ [K]})}{0.85569 \text{ [m}^3\text{/kg]}} = \boxed{225.78 \text{ [kPa]}}$$

Let us take a look at states 1 and 4. We are given the specific enthalpy at both states. We can use this to find the temperature at state 4:

$$h_4 - h_1 = C_P(T_4 - T_1)$$

$$T_4 = T_1 + \frac{h_4 - h_1}{C_P} = 25 \text{ °C} + \frac{(399.02 - 298.62) \text{ [kJ/kg]}}{1.004 \text{ [kJ/kg-K]}} = \boxed{125 \text{ °C}}$$

Since the process is isothermal between states 3 and 4, the temperature at state 3 then is:

$$T_3 = T_4 = \boxed{125 \text{ °C}}$$

We can also find the specific volume at state 4:

$$\nu_4 = \frac{RT_4}{P_4} = \frac{(0.287 \text{ [kJ/kg-K]})(398.15 \text{ [K]})}{100 \text{ [kPa]}} = 1.14269 \text{ [m}^3\text{/kg]}$$

To finish defining the properties at each state, we need to find the mass of the system. This can be done using First Law of Thermodynamics between states 4 and 1:

$$U_1 - U_4 = Q_{4\to1} - W_{4\to1}$$

Since the process is isobaric, we know that the heat transffered out of the system is equivalent to the change in enthalpy. Subbing in:

$$m(u_1 - u_4) = m(h_1 - h_4) - Pm(\nu_1 - \nu_4)$$

Crossing out the mass, we can find the change in specific internal energy:

$$u_1 - u_4 = (298.62 - 399.02) \text{ [kJ/kg]} - (100 \text{ [kPa]})((0.85569 - 1.14269) \text{ [m}^3\text{/kg]}) = -71.7 \text{ [kJ/kg]}$$

We can now find the mass since we know the change in internal energy as shown:

$$m = \frac{U_1 - U_4}{u_1 - u_4} = \frac{-16.763 \text{ [kJ]}}{71.7 \text{ [kJ/kg]}} = 0.2338 \text{ [kg]}$$

Now we can the volumes:

$$\forall_1 = m\nu_1 = (0.2338 \text{ [kg]})(0.85569 \text{ [m}^3\text{/kg]}) = \boxed{0.2 \text{ [m}^3\text{]}}$$

Since the process is isochoric between states 1 and 2, we have:

$$\forall_2 = \forall_1 = \boxed{0.2 \text{ [m}^3\text{]}}$$

The volume at state 4 is:

$$\forall_4 = m\nu_4 = (0.2338 \text{ [kg]})(1.14269 \text{ [m}^3\text{/kg]}) = \boxed{0.267 \text{ [m}^3\text{]}}$$

Finally, we can find the pressure at state 3:

$$P_3 = \frac{mRT_3}{\forall_3} = \frac{(0.2338 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})(398.15 \text{ [K]})}{0.1 \text{ [m}^3\text{]}} = \boxed{267.16 \text{ [kPa]}}$$

b) The unknown polytropic index; Using the formula for polytropic index:

$$n = \frac{\ln\left(\frac{P_3}{P_2}\right)}{\ln\left(\frac{\forall_2}{\forall_3}\right)} = \frac{\ln\left(\frac{267.16 \text{ [kPa]}}{225.78 \text{ [kPa]}}\right)}{\ln\left(\frac{0.2 \text{ [m}^3\text{]}}{0.1 \text{ [m}^3\text{]}}\right)} = \boxed{0.2428}$$