Homework #8

MEMS 0051 - Introduction to Thermodynamics

Assigned July 26th, 2018
Due July 30th, 2018

Starting with the Conservation of Energy equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + gz_j \right),$$

and the Continuity equation:

$$\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \sum_{j=1}^{M} \dot{m}_j,$$

Solve the following:

Problem #1

Steam enters a turbine through a pipe with a diameter of 0.2 [m]. The steam enters with a velocity of 100 [m/s], a pressure of 14,000 [kPa] and a temperature of 600 °C. The steam is exhausted through a pipe with a diameter of 0.8 [m], a pressure of 500 [kPa] and a temperature of 180 °C. Determine:

a) the exit velocity of the steam;
b) the mass flow rate of the steam.

b) The Conservation of Energy need not be applied. The mass flow rate into the system must be that out of the system. The inlet mass flow rate can be found from the inlet conditions:

$$\dot{m} = \frac{A_1 V_1}{\nu} = \frac{\pi\,(0.2\,[\mathrm{m}])^2\,(100\,[\mathrm{m/s}])}{4\,\nu}$$

The specific volume at the inlet state is found via interpolation:

$$\frac{(14,000 - 10,000)\,[\mathrm{kPa}]}{(15,000 - 10,000)\,[\mathrm{kPa}]} = \frac{(\nu - 0.03837)\,[\mathrm{m^3/kg}]}{(0.02491 - 0.03837)\,[\mathrm{m^3/kg}]} \implies \nu = 0.027602\,[\mathrm{m^3/kg}]$$

Therefore, the mass flow rate is:

$$\dot{m} = \frac{AV}{\nu} = \frac{\pi\,(0.2\,[\mathrm{m}])^2\,(100\,[\mathrm{m/s}])}{4\,0.027602\,[\mathrm{m^3/kg}]} = 113.818\,[\mathrm{kg/s}]$$

a) The exit velocity can now be solved for:

$$V_2 = \frac{\dot{m}\,\nu_2}{A_2} = \frac{(113.818\,[\mathrm{kg/s}])\,\pi\,(0.8\,[\mathrm{m}])^2}{4\,\nu_2}$$

The specific volume at the outlet is solved for based upon given state properties via interpolation:

$$\frac{(180 - 151.86)\,[^\circ\mathrm{C}]}{(200 - 151.86)\,[^\circ\mathrm{C}]} = \frac{(\nu - 0.37489)\,[\mathrm{m^3/kg}]}{(0.42492 - 0.37489)\,[\mathrm{m^3/kg}]} \implies \nu_2 = 0.40413\,[\mathrm{m^3/kg}]$$

Thus, the exit velocity is:

$$V_2 = \frac{(113.818\,[\mathrm{kg/s}])\,\pi\,(0.8\,[\mathrm{m}])^2}{4\,(0.40413\,[\mathrm{m^3/kg}])} = 141.57\,[\mathrm{m/s}]$$

Problem #2

An open feedwater heater (OFWH) accepts liquid water at 1,000 [kPa] and a temperature of 50 °C. The OFWH also accepts water with a mass flow rate per that of inlet one, i.e. $\dot{m}_2/\dot{m}_1$=0.22. Saturated liquid water exits the OFWH. Determine:

a) the temperature of the second incoming stream, if superheated;
b) the quality of the second incoming stream, if saturated.

Applying the Continuity equation to the system:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

We can express $\dot{m}_2$ as $0.22\dot{m}_1$. Therefore:

$$1.22\dot{m}_1 = \dot{m}_3$$

Applying the Conservation of Energy equation to the OFWH, assuming steady-state, ignoring heat and work, and kinetic and potential energy:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \implies \dot{m}_1 h_1 + 0.22\dot{m}_1 h_2 = 1.22\dot{m}_1 h_3 \implies h_1 + 0.22h_2 = 1.22h_3$$

We know the enthalpy at State 3 is that of saturated water at 1,000 [kPa], i.e h_3=762.79 [kJ/kg]. The enthapy of State 1 is that of compressed water at 1,000 [kPa] and 50 °C, i.e. h_f(50 °C)=209.31 [kJ/kg]. Substituting in these values:

$$209.31\,[\text{kJ/kg}] + 0.22h_2 = 1.22(762.79\,[\text{kJ/kg}]) \implies h_2 = 3,278.61\,[\text{kJ/kg}]$$

At 1,000 [kPa], this puts State 2 in the superheated vapor region, existing between 400 and 500 °C:

$$\frac{(T_2 - 400)\,[^\circ\text{C}]}{(500 - 400)\,[^\circ\text{C}]} = \frac{(3,278.61 - 3,263.88)\,[\text{kJ/kg}]}{(3,4788.44 - 3,263.88)\,[\text{kJ/kg}]} \implies T_2 = 406.87\,[^\circ\text{C}]$$

Problem #3

Steam enters a nozzle operating at a pressure of 30 [bar] and a temperature of 320 °C with negligible velocity. The steam exits the nozzle at a pressure of 15 [bar] and a velocity of 10 [m/s]. The mass flow rate is 2.5 [kg/s]. Assume the nozzle is well insulated.

a) Determine the exit temperature of the steam.

Applying the steady-state C.o.E. equations ignoring heat, work , changes of potential energy, and inlet velocity:

$$h_2 = h_1 - \frac{V_2^2}{2}$$

The enthalpy at State 1 is found, via EES, to be 3,042.6 [kJ/kg]. Thus, the enthalpy at State 2 is found as:

$$h_2 = (3,024.6\,[\text{kJ/kg}]) + \frac{(10\,[\text{m/s}])^2\,[\text{kJ/kg}]}{2,000\,[\text{m}^2/\text{s}^2]} = 3,024.65\,[\text{kJ/kg}]$$

The temperature is found via the enthalpy and pressure using EES to be 302.7 °C.

Problem #4

Air expands through a turbine with a mass flow rate of 10 [kg/s] from a pressure of 5 [bar] to 1 [bar] The temperature of the air at the inlet is 900 [K] where it is 600 [K] at the outlet. The inlet velocity is negligible, but the exit velocity is 100 [m/s]. All heat transfer and potential energy changes can be neglected.

a) Determine the power output of the turbine in [kW].
b) Determine the exit cross-sectional flow area in [m^2].

a) Applying the steady-state C.o.E. equation:

$$\dot{W} = \dot{m}\left(h_1 - h_2 - \frac{V_2^2}{2}\right)$$

The enthalpies at State 1 and 2 are found via EES, such that:

$$\dot{W} = (10\,[\text{kg/s}])\left((933.6 - 607.4)\,[\text{kJ/kg}] - \frac{(100\,[\text{m/s}])^2\,[\text{kJ/kg}]}{2{,}000\,[\text{m}^2/\text{s}^2]}\right) = 3{,}212\,[\text{kW}]$$

b) The cross-sectional flow area at the outlet is found using the continuity equation, where the specific volume is found via EES, such that:

$$A_2 = \frac{\dot{m}\nu}{V} = \frac{(10\,[\text{kg/s}])(1.7231\,[\text{m}^3/\text{kg}])}{100\,[\text{m/s}]} = 0.172\,[\text{m}^2]$$

Problem #5

An air-conditioner's cooling coil is a heat exchanger that extracts heat from air and it is picked up by the working fluid, R-134A (Table B.5, page 180). Air is passed over the heat exchanger coils. Air enters the heat exchanger with a volumetric flow rate of 40 [m^3/min], a temperature of 40 °C and a pressure of 1 [bar] and exits at a temperature of 20 °C. The R-134A enters the heat exchanger with a quality of 40% at a temperature of 10 °C and exits as a saturated vapor at the same temperature.

a) Determine the mass flow rate of the R-134A.

b) Determine the rate of energy transfer, in [kJ/min], form the air to the refrigerant.

a) Applying the steady-state C.o.E. to the entirety of the heat exchanger, recognizing there is no heat exchange between the heat exchanger and ambient, nor work, changes of kinetic or potential energies, labeling the air inlet and exit as States 1 and 2, and the R-134a inlet and exit as States 3 and 4:

$$0 = \dot{m}_{\text{air}}(h_1 - h_2) - \dot{m}_{\text{R-134a}}(h_3 - h_4) \implies \dot{m}_{\text{R-134a}} = \dot{m}_{\text{air}}\frac{h_1 - h_2}{h_4 - h_3}$$

The enthalpies can be pulled from EES give the temperature and pressure of air, as well s the quality and temperature of R-134a such that. The mass flow rate of the air is based upon the Ideal Gas law. Thus:

$$\dot{m}_{\text{R-134a}} = \left(\frac{P\dot{\forall}}{RT}\frac{h_1 - h_2}{h_4 - h_3} = \frac{(100\,[\text{kPa}])(0.\bar{6}\,[\text{m}^3/\text{s}])}{(0.287\,[\text{kJ/kg-K}])(313.15\,[\text{K}])}\right)\left(\frac{(313.5 - 293.3)\,[\text{kJ/kg}]}{(256.2 - 141.7)\,[\text{kJ/kg}]}\right) = 0.131\,[\text{kg/s}]$$

b) To determine the rate of energy transfer from the air to the refrigerant, we apply the C.o.E. to the air domain only. Assuming steady-state, no work, and no change in kinetic or potential energies:

$$0 = \dot{Q} + \dot{m}_{\text{air}}(h_1 - h_2) \implies \dot{Q} = \dot{m}_{\text{air}}(h_2 - h_1)$$

Substituting in values:

$$\dot{Q} = \left(\frac{(100\,[\text{kPa}])(0.\bar{6}\,[\text{m}^3/\text{s}])}{(0.287\,[\text{kJ/kg-K}])(313.15\,[\text{K}])}\right)(293.3 - 313.5)\,[\text{kJ/kg}] = -14.99\,[\text{kW}] = -899\,[\text{kJ/min}]$$

Problem #6

A pump delivers water at a volumetric flow rate of 0.05 [m^3/s] through an 18 [cm] diameter pipe located 100 [m] above the pump inlet, which has a diameter of 15 [cm]. The pressure at the inlet and exit of the pump is equal to 1 [bar], and the temperature can be assumed constant at 20 °C.

a) Determine the power input into the pump in [kW].

Applying the steady-state C.o.E. equation between States 1 and 2, ignoring heat transfer and enthalpies:

$$\dot{W} = \dot{m}\left(\frac{V_1^2}{2} - \frac{V_2^2}{2} + g(z_1 - z_2)\right)$$

The mass flow rate of the fluid is found from the volumetric flow rate and specific volume:

$$\dot{m} = \frac{\dot{\forall}}{\nu} = \frac{0.05\,[\mathrm{m}^2/\mathrm{s}]}{0.001002\,[\mathrm{m}^3/\mathrm{kg}]} = 49.9\,[\mathrm{kg/s}]$$

The velocity at State 1 and 2 is found via:

$$V_1 = \frac{\dot{\forall}}{A} = \frac{0.05\,[\mathrm{m}^3/\mathrm{s}]}{\pi\left(\frac{(0.18\,[\mathrm{m}])^2}{4}\right)} = 1.965\,[\mathrm{m/s}]$$

$$V_1 = \frac{\dot{\forall}}{A} = \frac{0.05\,[\mathrm{m}^3/\mathrm{s}]}{\pi\left(\frac{(0.15\,[\mathrm{m}])^2}{4}\right)} = 2.829\,[\mathrm{m/s}]$$

Thus, the pump work is:

$$\dot{W} = (49.9\,[\mathrm{kg/s}])\left(\frac{\{(1.965\,[\mathrm{m/s}])^2 - (2.829\,[\mathrm{m/s}])^2\}\,[\mathrm{kJ/kg}]}{2,000\,[\mathrm{m}^2/\mathrm{s}^2]}\right) - \frac{(9.81\,[\mathrm{m/s}^2])(100\,[\mathrm{m}])\,[\mathrm{kJ/kg}]}{1,000\,[\mathrm{m}^2/\mathrm{s}^2]} = -49.06\,[\mathrm{kW}]$$

Problem #7

A tank contains 2 [ft^3] of air with an initial pressure of 50 [psi] and an initial temperature of 70 °F. The tank is punctured and air flows out at a constant mass flow rate of 0.01 [lb$_\mathrm{m}$/s]. Using the ideal gas law, determine:

a) the rate of change of mass within the tank the moment the leak occurs;

b) the mass of air in the tank as a function of time (plot).

The rate of change of mass within the tank is:

$$\frac{dm_{\mathrm{C.\forall.}}}{dt} = -0.01\,[\mathrm{lb_m/s}]$$

The initial mass in the tank is:

$$m_{\mathrm{init}} = \frac{P\forall}{RT} = \frac{(344.74\,[\mathrm{kPa}])(0.0566\,[\mathrm{m}^3])}{(0.287\,[\mathrm{kJ/kg\text{-}K}])(294.3\,[\mathrm{K}])} = 0.224\,[\mathrm{lb_m}] = 0.102\,[\mathrm{kg}]$$

The final mass in the tank is when the pressure and temperature at atmospheric:

$$m_{\mathrm{init}} = \frac{P\forall}{RT} = \frac{(101.3\,[\mathrm{kPa}])(0.0566\,[\mathrm{m}^3])}{(0.287\,[\mathrm{kJ/kg\text{-}K}])(293.15\,[\mathrm{K}])} = 0.068\,[\mathrm{lb_m}] = 0.031\,[\mathrm{kg}]$$

Thus, the mass linearly decreases from 0.102 [lb$_\mathrm{m}$] to 0.031 [lb$_\mathrm{m}$] in the period of 7.1 [s], thereafter, it remains at a constant value of 0.031 [lb$_\mathrm{m}$].

Problem #8

A very well insulated chamber with volume of 1 [m^3] contains air initially at 101 [kPa] and 37.8 °C. Supply and discharge lines are connected to the chamber. The supply air is at 200 [kPa] and 93.3 °C. The discharge line is connected to atmosphere. At time t=0, the valves are simultaneously opened, allowing air to flow at a constant rate of 1 [kg/min] through both lines. The air within the chamber can be assumed well mixed, i.e. a uniform temperature and pressure at each time step. Determine:

a) the temperature, in °C, as a function of time;

b) the pressure, in [kPa], as a function of time.

a) Applying the C.o.E. equation, ignoring changes of kinetic and potential energies, as well as work and heat:

$$\frac{dU}{dt} = \dot{m}(h_1 - h_2)$$

The LHS can be expressed as mu, where du is then expressed as the constant-volume specific heat times the change of temperature. The RHS can be expressed as the mass flow rate times the constant-pressure specific heat times the change of temperature. The inlet temperature, i.e. State 1, is a constant. The outlet temperature, i.e. State 2, varies with time. Thus:

$$mC_{\forall}\frac{dT(t)}{dt} = \dot{m}C_P(T_1 - T(t))$$

Recalling k is the ratio of the constant-pressure to constant-volume specific heat, and dropping the dependence of t:

$$\frac{dT}{dt} + \frac{\dot{m}k}{m}T = \frac{\dot{m}k}{m}T_1$$

Determining the homogeneous solution by setting the RHS equal to zero, the characteristic equation is:

$$\lambda + \frac{\dot{m}k}{m} = 0 \implies \lambda = -\frac{\dot{m}k}{m}$$

Thus, the homogeneous solution takes the form:

$$T_h(t) = C_1 e^{\lambda t} = C_1 e^{\frac{-\dot{m}k}{m}t}$$

The particular solution takes the form of the RHS, i.e. a constant:

$$T_p(t) = \alpha \implies \frac{\dot{m}k}{m}\alpha = \frac{\dot{m}k}{m}T_1 \implies \alpha = T_1$$

Thus:

$$T(t) = T_h(t) + T_p(t) = C_1 e^{\frac{-\dot{m}k}{m}t} + T_1$$

Solving for the constant C_1 by providing the initial condition:

$$T(0) = T_{\text{init}} = C_1 + T_1 \implies C_1 = T_{\text{init}} - T_1$$

Therefore

$$T(t) = (T_{\text{init}} - T_1)e^{\frac{-\dot{m}k}{m}t} + T_1$$

b) The pressure as a function of time is simply found via the Ideal Gas law:

$$P(t) = \frac{mRT(t)}{\forall}$$