

Midterm #1

MEMS 0051 - Introduction to Thermodynamics
Spring 2021

Assigned: March 1st, 2021

Due: March 12th, 2021, 11:59 pm via Gradescope

Rules

The following listed rules, in addition, but not limited to, those listed on the syllabus, those listed on page 11, and those outlined by Pitt's Academic Integrity Policy, apply to this examination:

1. This test is open notes, open book, open lecture videos, open homework and homework solutions, open quiz and quiz solutions, and you are able to reference previous assessment materials posted on GitHub;
2. You are to complete all **3** problems.
3. You can direct general questions to Dr. Barry via email. A general question constitutes a point of clarification with a question, for example "Why is thermodynamics the coolest subject ever?" Specific questions about how to solve a problem, pertinent equations, related to general guidance, etc., are *not* permitted;
4. You are *not* to communicate with *any* other student about this exam. Period;
5. You are *not* to use any online resources, such as Chegg, Quora, etc.. Seeking external assistance in the form of posting this exam, posting pictures/screenshots/images of this exam, posting questions from this exam, asking questions pertaining to the problems within the exam, etc., are in direction violation of the Academic Integrity policy and will result in immediate failure of this exam;
6. Unsubstantiated results will be marked incorrect. A result where the mathematics do not substantiated the final result will be marked wrong;
7. The work for your exam must be submitted on the exam sheets. Failure to do so will result in your exam not being graded. Your work must be neat, legible, and follow a clear and logical progression. Your final answer must be boxed with proper units, and significant digits. As outlined in the course syllabus, an answer without units will be marked incorrect;
8. You must complete the Academic Integrity Statement and include it with your exam submission for your exam to be graded.

Problem #1

(20 pts.) A rigid, non-deformable tank contains R-410a, which is existing as a saturated vapor at -3°C . The tank is then cooled to a temperature of -43.7°C . Determine the amount of specific heat transferred (units taken as $[\text{kJ/kg}]$).

This problem is 3.91 from the 8th edition, or 5.35 from the 7th edition, and the solution is presented in the accompanying 7th edition solution manual. We will elaborate upon the solution and provide a robust method to solving it. The key to the problem is recognizing a non-deformable tank is control mass, and the volume is invariant. Thus there is no work. The specific heat transferred is merely the difference of specific internal energies between the final and initial states. Therefore, we can list our state variables as follows:

State 1:	State 2:
$T_1 = -3^\circ\text{C}$	$T_2 = -43.7^\circ\text{C}$
$\nu_1 = \nu_g(T_1)$	$\nu_2 = \nu_1$
$m_1 = ?$	$m_2 = m_1$

Given State 1 is existing as saturated vapor, we are able to determine the specific volume and specific internal energy (which we will need to populate the Conservation of Energy equation later) at said state:

$$\frac{(-3 - (-5))^\circ\text{C}}{(0 - (-5))^\circ\text{C}} = \frac{(\nu_1 - 0.03848) [\text{m}^3/\text{kg}]}{(0.03267 - 0.03848) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_1 = 0.036156 [\text{m}^3/\text{kg}]$$

$$\frac{(-3 - (-5))^\circ\text{C}}{(0 - (-5))^\circ\text{C}} = \frac{(u_1 - 251.41) [\text{kJ/kg}]}{(253.02 - 251.41) [\text{kJ/kg}]}$$

$$\Rightarrow u_1 = 252.054 [\text{kJ/kg}]$$

We know the specific volume at State 2 is that at State 1, and given the temperature at State 2, we see ν_2 is greater than the saturated liquid and less than the saturated vapor specific volumes corresponding to T_2 , which means the substance is existing as a saturated mixture. Thus, we will create a -43.7°C temperature entry for ν_f and ν_g :

$$\frac{(-43.7 - (-45))^\circ\text{C}}{(-40 - (-45))^\circ\text{C}} = \frac{(\nu_{f, -43.7^\circ\text{C}} - 0.000752) [\text{m}^3/\text{kg}]}{(0.000762 - 0.000752) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_{f, -43.7^\circ\text{C}} = 0.000755 [\text{m}^3/\text{kg}]$$

$$\frac{(-43.7 - (-45))^\circ\text{C}}{(-40 - (-45))^\circ\text{C}} = \frac{(\nu_{g, -43.7^\circ\text{C}} - 0.17804) [\text{m}^3/\text{kg}]}{(0.14291 - 0.17804) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_{g, -43.7^\circ\text{C}} = 0.168906 [\text{m}^3/\text{kg}]$$

Thus the quality at State 2 is:

$$x_2 = \frac{\nu_2 - \nu_{f, -43.7^\circ\text{C}}}{\nu_{g, -43.7^\circ\text{C}} - \nu_{f, -43.7^\circ\text{C}}} = \frac{(0.036156 - 0.000755) [\text{m}^3/\text{kg}]}{(0.168906 - 0.000755) [\text{m}^3/\text{kg}]} = 0.2105$$

To determine the specific internal energy at State 2, we need to determine u_f and u_g existing at T_2 :

$$\frac{(-43.7 - (-45))^\circ\text{C}}{(-40 - (-45))^\circ\text{C}} = \frac{(u_{f, -43.7^\circ\text{C}} - (-7.02)) [\text{kJ/kg}]}{((-0.13) - (-7.02)) [\text{kJ/kg}]}$$

$$\Rightarrow u_{f, -43.7^\circ\text{C}} = -5.2286 [\text{kJ/kg}]$$

$$\frac{(-43.7 - (-45))^\circ\text{C}}{(-40 - (-45))^\circ\text{C}} = \frac{(u_{g, -43.7^\circ\text{C}} - 235.64) [\text{kJ/kg}]}{(237.81 - 235.64) [\text{kJ/kg}]}$$

$$\Rightarrow u_{g, -43.7^\circ\text{C}} = 236.2042 [\text{kJ/kg}]$$

Thus the specific internal energy at State 2 is:

$$u_2 = u_{f, -43.7^\circ\text{C}} + x_2(u_{g, -43.7^\circ\text{C}} - u_{f, -43.7^\circ\text{C}}) = -(5.2286 \text{ [kJ/kg]}) + 0.2105(236.2042 - (-5.2286)) \text{ [kJ/kg]}$$

$$\Rightarrow u_2 = 45.383 \text{ [kJ/kg]}$$

Therefore, we have all our necessary state variables to evaluate the Conservation of Energy, on a per-mass basis, noting that specific work is zero:

$$q_{1 \rightarrow 2} = u_2 - u_1 = (45.383 - 252.054) \text{ [kJ/kg]} = -206.7 \text{ [kJ/kg]}$$

EES Solution

"State 1"

T[1] = -3 [C]

x[1] = 1.0

v[1] = volume(R410A, T=T[1], x=x[1])

u[1] = intenergy(R410A, T=T[1], x=x[1])

"State 2"

T[2] = -43.7 [C]

v[2] = v[1]

x[2] = quality(R410A, v=v[2], T=T[2])

u[2] = intenergy(R410A, v=v[2], T=T[2])

q12 = u[2] - u[1]

"Code was verified to the solution of Problem 5.35, and was used to verify the presented solution."

Problem #2

(30 pts.) A tank contains 0.488 [kg] of water initially at 247.3 [kPa] with a quality of 9.5%. This tank is connected to a piston-cylinder by a valve, that is initially closed. The piston-cylinder contains 0.502 [kg] of water initially at 198 [kPa] and a temperature of 406 °C. The valve connecting the tank and piston-cylinder is then opened, allowing thermodynamic equilibrium to be achieved. Additionally, heat transfer is allowed to take place between the system and surroundings, until a final temperature of 148.5 °C is reached. Determine the following:

- The final pressure of the system;
- The volume of the system;
- The work associated with this process;
- The heat transfer associated with this process.

This problem is taken from the 8th edition text, and is denoted as 3.213. This problem is the same as 5.63 from the 7th edition text, of which we have the solution manual, albeit the state variables are slightly different. To begin, let us collect our state variables. State 1 will reflect the conditions within the tank (valve closed). State 2 will reflect the conditions within the piston-cylinder (valve closed). State 3 will reflect the total system (tank and piston-cylinder with the valve open):

State 1:	State 2:	State 3:
$m_1=0.488$ [kg]	$m_2=0.502$ [kg]	$m_3=m_1+m_2=0.99$ [kg]
$P_1=247.3$ [kPa]	$P_2=298$ [kPa]	$P_3=?$
$x_1=0.095$	$x_2=?$	$x_3=?$
$T_1=T_{sat}(P_1)$	$T_2=406$ °C	$T_3=148.5$ °C
Phase: Sat. Water	Phase: S.H.V	Phase: ?

To begin, we know the final mass of the system (0.99 [kg]), and we can easily determine the specific volumes and specific internal energies of each state (State 1 through 3). Thus, we can apply the Conservation of Energy between the initial states and final state to determine both the heat transfer and work. Thus, we will begin by finding the specific volumes and specific internal energies of each state. Starting with State 1:

$$\frac{(247.3 - 225) \text{ [kPa]}}{(250 - 225) \text{ [kPa]}} = \frac{(\nu_{f,247.3 \text{ [kPa]}} - 0.001064) \text{ [m}^3/\text{kg]}}{(0.001067 - 0.001064) \text{ [m}^3/\text{kg]}} \Rightarrow \nu_{f,247.3 \text{ [kPa]}} = 0.001067 \text{ [m}^3/\text{kg]}$$

$$\frac{(247.3 - 225) \text{ [kPa]}}{(250 - 225) \text{ [kPa]}} = \frac{(\nu_{g,247.3 \text{ [kPa]}} - 0.79325) \text{ [m}^3/\text{kg]}}{(0.71871 - 0.79325) \text{ [m}^3/\text{kg]}} \Rightarrow \nu_{g,247.3 \text{ [kPa]}} = 0.726760 \text{ [m}^3/\text{kg]}$$

$$\nu_1 = \nu_{f,247.3 \text{ [kPa]}} + x_1(\nu_{g,247.3 \text{ [kPa]}} - \nu_{f,247.3 \text{ [kPa]}}) = 0.001067 \text{ [m}^3/\text{kg]} + (0.095)(0.726760 - 0.001067) \text{ [m}^3/\text{kg]}$$

$$\Rightarrow \nu_1 = 0.070008 \text{ [m}^3/\text{kg]}$$

The volume at State 1 is the mass times the specific volume:

$$V_1 = m_1 \nu_1 = (0.488 \text{ [kg]})(0.070008 \text{ [m}^3/\text{kg]}) = 0.034164 \text{ [m}^3]$$

Next we can find the specific internal energy:

$$\frac{(247.3 - 225) \text{ [kPa]}}{(250 - 225) \text{ [kPa]}} = \frac{(u_{f,247.3 \text{ [kPa]}} - 520.45) \text{ [kJ/kg]}}{(535.08 - 520.45) \text{ [kJ/kg]}} \Rightarrow u_{f,247.3 \text{ [kPa]}} = 533.50 \text{ [kJ/kg]}$$

$$\frac{(247.3 - 225) \text{ [kPa]}}{(250 - 225) \text{ [kPa]}} = \frac{(u_{g,247.3 \text{ [kPa]}} - 2,533.56) \text{ [kJ/kg]}}{(2,537.21 - 2,533.56) \text{ [kJ/kg]}} \Rightarrow u_{g,247.3 \text{ [kPa]}} = 2,536.816 \text{ [kJ/kg]}$$

$$u_1 = u_{f,247.3 \text{ [kPa]}} + x_1(u_{g,247.3 \text{ [kPa]}} - u_{f,247.3 \text{ [kPa]}}) = 533.50 \text{ [kJ/kg]} + (0.095)(2,536.816 - 533.50) \text{ [kJ/kg]}$$

$$\Rightarrow u_1 = 723.815 \text{ [kJ/kg]}$$

Continuing to State 2, which is a superheated vapor, we need to construct an intermediate pressure table for 198 [kPa] evaluated between 400 °C and 500 °C:

$$\left. \frac{(200 - 198) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} \right|_{400^\circ\text{C}} = \frac{(1.54930 - \nu_{198 \text{ [kPa], } 400^\circ\text{C}}) \text{ [m}^3/\text{kg]}}{(1.54930 - 3.10263) \text{ [m}^3/\text{kg]}}$$

$$\Rightarrow \nu_{198 \text{ [kPa]}, 400^\circ\text{C}} = 1.580367 \text{ [m}^3/\text{kg]}$$

Constructing the 500 °C temperature entry for the 198 [kPa] table:

$$\left. \frac{(200 - 198) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} \right|_{500^\circ\text{C}} = \frac{(1.78139 - \nu_{198 \text{ [kPa]}, 500^\circ\text{C}}) \text{ [m}^3/\text{kg]}}{(1.78139 - 3.56547) \text{ [m}^3/\text{kg]}}$$

$$\Rightarrow \nu_{198 \text{ [kPa]}, 500^\circ\text{C}} = 1.817072 \text{ [m}^3/\text{kg]}$$

Now we can interpolate between temperature:

$$\left. \frac{(406 - 400)^\circ\text{C}}{(500 - 400)^\circ\text{C}} \right|_{198 \text{ [kPa]}} = \frac{(\nu_{198 \text{ [kPa]}, 406^\circ\text{C}} - 1.580367) \text{ [m}^3/\text{kg]}}{(1.817072 - 1.580367) \text{ [m}^3/\text{kg]}}$$

$$\Rightarrow \nu_2 = \nu_{198 \text{ [kPa]}, 406^\circ\text{C}} = 1.59457 \text{ [m}^3/\text{kg]}$$

Thus, the volume at State 2 is:

$$V_2 = m_2 \nu_2 = (0.502 \text{ [kg]})(1.59457 \text{ [m}^3/\text{kg]}) = 0.800474 \text{ [m}^3]$$

The same approach is taken to find the specific internal energy at State 2:

$$\left. \frac{(200 - 198) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} \right|_{400^\circ\text{C}} = \frac{(2,966.69 - u_{198 \text{ [kPa]}, 400^\circ\text{C}}) \text{ [kJ/kg]}}{(2,966.69 - 2,967.85) \text{ [kJ/kg]}}$$

$$\Rightarrow u_{198 \text{ [kPa]}, 400^\circ\text{C}} = 2,966.7132 \text{ [kJ/kg]}$$

Constructing the 500 °C temperature entry for the 198 [kPa] table:

$$\left. \frac{(200 - 198) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} \right|_{500^\circ\text{C}} = \frac{(3,130.75 - u_{198 \text{ [kPa]}, 500^\circ\text{C}}) \text{ [kJ/kg]}}{(3,130.75 - 3,131.54) \text{ [kJ/kg]}}$$

$$\Rightarrow u_{198 \text{ [kPa]}, 500^\circ\text{C}} = 3,130.7658 \text{ [kJ/kg]}$$

Now we can interpolate between temperature:

$$\left. \frac{(406 - 400)^\circ\text{C}}{(500 - 400)^\circ\text{C}} \right|_{198 \text{ [kPa]}} = \frac{(u_{198 \text{ [kPa]}, 406^\circ\text{C}} - 2,966.7132) \text{ [kJ/kg]}}{(3,130.7658 - 2,966.7132) \text{ [kJ/kg]}}$$

$$\Rightarrow u_2 = u_{198 \text{ [kPa]}, 406^\circ\text{C}} = 2,976.556 \text{ [kJ/kg]}$$

Dr. Barry's Conjecture

Recall the piston is floating at a pressure of 198 [kPa]. Once the valve is opened, the pressure within the tank will be expended to raise the piston, until an equilibrium pressure of 198 [kPa] is reached with the system, while heat is simultaneously removed/added (unless too much heat is removed and the pressure then decreases). That is to say, the pressure within the tank will be used to generate work, and that the pressure should remain constant (at a minimum value) as the temperature reaches 148.5 °C. To check this hypothesis (which the book does in a very hand-wavy manner and with no rigor), let's consider the piston is fixed, and we will open the valve between the tank and piston-cylinder. In doing such, the volume of the system is fixed, and since it is a constant mass, we can say:

$$V_3 = V_1 + V_2 = (0.034164 + 0.800474) \text{ [m}^3] = 0.834638 \text{ [m}^3]$$

The total mass of the system is the sum of States 1 and 3. Thus, the specific volume of State 3 is found as:

$$\nu_3 = \frac{V_3}{m_3} = \frac{0.834638 \text{ [m}^3]}{(0.488 + 0.502) \text{ [kg]}} = 0.843069 \text{ [m}^3/\text{kg]}$$

We are given $T_3=148.5^\circ\text{C}$. Consulting Table B.1.1, we see ν_3 is greater than the saturated vapor specific volume, which means this system is existing as a superheated vapor. Consulting Table B.1.3, we see this state is existing

between a pressure of 200 [kPa] and 300 [kPa]. Thus, we will first make a temperature entry for 148.5 °C on the 200 [kPa] and 300 [kPa] pressure-entry tables, and then interpolate between specific volume to determine the pressure at State 3. Starting with the 200 [kPa] pressure-entry table:

$$\left. \frac{(148.5 - 120.23) ^\circ\text{C}}{(150 - 120.23) ^\circ\text{C}} \right|_{200 \text{ [kPa]}} = \frac{(\nu_{200 \text{ [kPa]}, 148.5^\circ \text{C}} - 0.88573) [\text{m}^3/\text{kg}]}{(0.95964 - 0.88573) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_{200 \text{ [kPa]}, 148.5^\circ \text{C}} = 0.95592 [\text{m}^3/\text{kg}]$$

Proceeding to the 300 [kPa] pressure-entry table:

$$\left. \frac{(148.5 - 133.55) ^\circ\text{C}}{(150 - 133.55) ^\circ\text{C}} \right|_{300 \text{ [kPa]}} = \frac{(\nu_{300 \text{ [kPa]}, 148.5^\circ \text{C}} - 0.60582) [\text{m}^3/\text{kg}]}{(0.63388 - 0.60582) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_{300 \text{ [kPa]}, 148.5^\circ \text{C}} = 0.63132 [\text{m}^3/\text{kg}]$$

Now we can interpolate between specific volumes at a temperature of 148.5 °C to determine the pressure at the final state:

$$\left. \frac{(P_3 - 200) [\text{kPa}]}{(300 - 200) [\text{kPa}]} \right|_{148.5^\circ \text{C}} = \frac{(0.843069 - 0.95592) [\text{m}^3/\text{kg}]}{(0.63132 - 0.95592) [\text{m}^3/\text{kg}]}$$

Thus,

$$\Rightarrow P_3 = 234.77 [\text{kPa}]$$

Thus, there is still enough energy within the system, even after the temperature has reached 148.5 °C, with a fixed volume, to maintain a pressure greater than 198 [kPa]. Therefore, our hypothesis is correct!

$$\Rightarrow \boxed{P_3 = 198 [\text{kPa}]}$$

At State 3, we know the temperature is 148.5 °C, and the pressure is 198 [kPa]. This places State 3 as a superheated vapor. We already know the specific volume at a temperature of 148.5 °C and a pressure of 200 [kPa] ($\nu_{200 \text{ [kPa]}, 148.5^\circ \text{C}} = 0.95592 [\text{m}^3/\text{kg}]$), but we have to find the specific volume at 148.5 °C and a pressure of 100 [kPa], and then interpolate between our pressure-entry tables to obtain a value at 148.5 °C and a pressure of 198 [kPa]. Constructing the 100 [kPa] pressure-entry table:

$$\left. \frac{(148.5 - 99.62) ^\circ\text{C}}{(150 - 99.62) ^\circ\text{C}} \right|_{100 \text{ [kPa]}} = \frac{(\nu_{100 \text{ [kPa]}, 148.5^\circ \text{C}} - 1.69400) [\text{m}^3/\text{kg}]}{(1.93636 - 1.69400) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_{100 \text{ [kPa]}, 148.5^\circ \text{C}} = 1.92914 [\text{m}^3/\text{kg}]$$

Now we can interpolate between pressure-entry tables evaluated at a temperature of 148.5 °C:

$$\left. \frac{(198 - 100) [\text{kPa}]}{(200 - 100) [\text{kPa}]} \right|_{148.5^\circ \text{C}} = \frac{(\nu_3 - 1.92914) [\text{m}^3/\text{kg}]}{(0.95592 - 1.92914) [\text{m}^3/\text{kg}]}$$

$$\Rightarrow \nu_3 = 0.97538 [\text{m}^3/\text{kg}]$$

We also have to find the specific internal energy at State 3. Starting with the 200 [kPa] pressure-entry table:

$$\left. \frac{(148.5 - 120.23) ^\circ\text{C}}{(150 - 120.23) ^\circ\text{C}} \right|_{200 \text{ [kPa]}} = \frac{(u_{200 \text{ [kPa]}, 148.5^\circ \text{C}} - 2,529.49) [\text{kJ/kg}]}{(2,576.87 - 2,529.49) [\text{kJ/kg}]}$$

$$\Rightarrow u_{200 \text{ [kPa]}, 148.5^\circ \text{C}} = 2,574.48 [\text{kJ/kg}]$$

Proceeding to the 100 [kPa] pressure-entry table:

$$\left. \frac{(148.5 - 99.62)^\circ\text{C}}{(150 - 99.62)^\circ\text{C}} \right|_{100 \text{ [kPa]}} = \frac{(u_{100 \text{ [kPa]}, 148.5^\circ\text{C}} - 2,506.06) \text{ [kJ/kg]}}{(2,582.75 - 2,506.06) \text{ [kJ/kg]}}$$

$$\Rightarrow u_{100 \text{ [kPa]}, 148.5^\circ\text{C}} = 2,580.47 \text{ [kJ/kg]}$$

Interpolating between pressures to determine the specific internal energy at State 3:

$$\left. \frac{(198 - 100) \text{ [kPa]}}{(200 - 100) \text{ [kPa]}} \right|_{148.5^\circ\text{C}} = \frac{(u_3 - 2,580.47) \text{ [kJ/kg]}}{(2,574.48 - 2,580.47) \text{ [kJ/kg]}}$$

$$\Rightarrow u_3 = 2,574.60 \text{ [kJ/kg]}$$

Now we want to find the work associated with this process, and the amount of heat transferred. Writing out the Conservation of Energy equation:

$$m_3 u_3 - (m_1 u_1 + m_2 u_2) = Q_{1 \rightarrow 3} - W_{1 \rightarrow 3}$$

We know all masses and specific internal energies. Thus, all that is left to solve is work. We know the pressure was constant during the work process, and we can calculate the final volume of the system at State 3 as:

$$V_3 = \nu_3 m_3 = (0.97538 \text{ [m}^3/\text{kg]})(0.99 \text{ [m}^3/\text{kg]}) = 0.96562 \text{ [m}^3\text{]}$$

Thus:

$$\text{c) } W_{1 \rightarrow 2} = P_2 (V_3 - (V_1 + V_2)) = (198 \text{ [kPa]})(0.96562 - 0.834638) \text{ [m}^3\text{]} = 25.94 \text{ [kJ]}$$

Lastly, we can calculate the heat as:

$$Q_{1 \rightarrow 3} = m_3 u_3 - (m_1 u_1 + m_2 u_2) + W_{1 \rightarrow 3}$$

$$= (0.99 \text{ [kg]})(2,574.60 \text{ [kJ/kg]}) - ((0.488 \text{ [kg]})(723.815 \text{ [kJ/kg]}) + (0.502 \text{ [kg]})(2,976.556 \text{ [kJ/kg]})) + 25.94 \text{ [kJ]}$$

$$\Rightarrow \text{d) } Q_{1 \rightarrow 3} = 727.34 \text{ [kJ]}$$

EES Solution

"State 1"

```
P[1] = 247.3 [kPa]
x[1] = 0.095
m[1] = 0.488 [kg]
v[1] = volume(Water,P=P[1],x=x[1])
T[1] = temperature(Water,P=P[1],x=x[1])
Vol[1] = m[1]*v[1]
```

```
u[1] = intenergy(Water,P=P[1],x=x[1])
Phase1$=phase$(Water,P=P[1],x=x[1])
```

"State 2"

```
P[2] = 198 [kPa]
m[2] = 0.502 [kg]
T[2] = 406 [C]
u[2] = intenergy(Water,T=T[2],P=P[2])
v[2] = volume(Water,T=T[2],P=P[2])
x[2] = quality(Water,T=T[2],P=P[2])
Vol[2] = m[2]*v[2]
Phase2$=phase$(Water,P=P[2],T=T[2])
```

"State 3"

```
m[3] = m[1] + m[2]
```

```

P[3] = P[2]
T[3] = 148.5 [C]
v[3] = volume(Water,P=P[3],T=T[3])
Vol[3] = v[3]*m[3]
x[3] = quality(Water,T=T[3],P=P[3])
u[3] = intenergy(Water,T=T[3],v=v[3])
Phase3$=phase$(Water,T=T[3],P=P[3])

"Calculation of work"
W13 = P[3]*(Vol[3] - (Vol[1] + Vol[2]))

"Calculation of heat"
Q13 = m[3]*u[3] - ( m[1]*u[1] + m[2]*u[2]) + W13

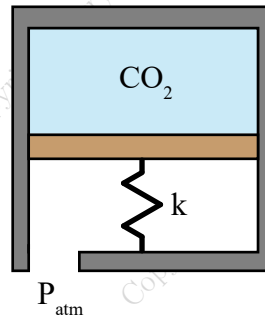
"Code was verified to the solution of Problem 5.63 - simply change P[1], x[1], m[1], P[2], m[2]
T[2] and T[3] to the values in the text"

```


Problem #3

(50 pts.) A piston-cylinder device is connected to a spring, which has linear behavior (i.e. the pressure within the piston cylinder linearly changes with volume - we literally constructed this expression in Lecture 8, Example #1 part b)). Within the piston-cylinder device, there exists 2,000 [g] of CO₂ existing at a pressure of 502 [kPa] and a temperature of 1,173 [K]. Then, the system is then cooled to a temperature of 609 [K], and in doing such, the pressure is 302 [kPa]. Determine the following:

- The work associated with this process;
- The heat transfer associated with this process assuming a constant specific heat taken from Table A.5;
- The heat transfer associated with this process assuming a specific heat, based upon the average of the initial and final temperatures, from Table A.6;
- The heat transfer associated with this process assuming a specific heat, based upon the integral-averaged value using the initial and final temperatures, from Table A.6;
- The heat transfer associated with this process assuming a specific heat taken from Table A.8;
- The applicability of the Ideal Gas Law.



Note: figure is not drawn to scale

This is problem 5.114 from the 7th edition, which has a solution in the corresponding solution manual. This problem is similar to 3.49 from the 8th edition, which was a suggested problem from Lecture 7. The problem was modified (parts b) through e)) based upon a similar procedure done in Quiz 3. To begin, we will list our state variables:

State 1:	State 2:
$m_1=2$ [kg]	$m_2=m_1$
$P_1=502$ [kPa]	$P_2=302$ [kPa]
$T_1=1,173$ [K]	$T_2=609$ [K]

What is unique about this problem is we are given temperature and pressures at the final state. Thus, we can see if the Ideal Gas law is applicable before starting the problem. Checking the reduced pressure and reduced temperature:

$$P_r = \frac{P_{max}}{P_c} = \frac{502 \text{ [kPa]}}{7,380 \text{ [kPa]}} = 0.068 \ll 1$$

$$T_r = \frac{T_{min}}{T_c} = \frac{609 \text{ [K]}}{304.1 \text{ [K]}} = 2.003 > 2$$

Thus the Ideal Gas law is applicable, and part f) is completed. Now, we know the pressure varies linearly with volume. Thus, we can use the arithmetic mean of pressure when evaluating work from State 1 to 2. All we need is the volumes at States 1 and 2. Using the Ideal Gas law:

$$v_1 = \frac{m_1 R T_1}{P_1} = \frac{(2 \text{ [kg]})(0.1889 \text{ [kJ/kg-K]})(1,173 \text{ [K]})}{(502 \text{ [kPa]})} = 0.883 \text{ [m}^3\text{]}$$

$$v_2 = \frac{m_2 R T_2}{P_2} = \frac{(2 \text{ [kg]})(0.1889 \text{ [kJ/kg-K]})(609 \text{ [K]})}{(302 \text{ [kPa]})} = 0.762 \text{ [m}^3\text{]}$$

Evaluating work using an arithmetic mean for the pressure:

$$\text{a) } W_{1 \rightarrow 2} = \left(\frac{P_1 + P_2}{2} \right) (v_2 - v_1) = \left(\frac{(502 + 302) \text{ [kPa]}}{2} \right) (0.762 - 0.883) \text{ [m}^3\text{]} = -48.642 \text{ [kJ]}$$

To evaluate the heat transferred using a constant specific heat taken from Table A.5, we must recall both the Conservation of Energy equation, as well as the expression for the change of internal energy based upon the constant volume specific heat. We note that C_{P0} is a function of T , and thus C_{v0} is a function of T , and that the constant volume specific heat of an Ideal Gas is equal to the constant pressure specific heat less the gas constant. That is:

$$m(u_2 - u_1) = Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} \implies Q_{1 \rightarrow 2} = mC_{v0}(T)(T_2 - T_1) + W_{1 \rightarrow 2}$$

For part b):

$$Q_{1 \rightarrow 2} = (2 \text{ [kg]})(0.653 \text{ [kJ/kg-K]})(609 - 1,173) \text{ [K]} - 48.642 \text{ [kJ]}$$

$$\boxed{\text{b)} \implies Q_{1 \rightarrow 2} = -785.23 \text{ [kJ]}}$$

For part c), we are asked to evaluate the constant volume specific heat, based upon the average of the initial and final temperatures. That is, using Table A.6, we can evaluate C_{P0} using an average temperature, and then calculate C_{v0} . Using Table A.6 to calculate C_{P0} using the average of the temperatures:

$$T_{avg} = \frac{T_1 + T_2}{2} = \frac{(1,173 + 609) \text{ [K]}}{2} = 891 \text{ [K]}$$

$$\theta = \frac{T_{avg}}{1,000} = 0.891$$

$$C_{P0} = C_0 + C_1\theta + C_2\theta^2 + C_3\theta^3 = 0.45 + (1.67)(0.891) - (1.27)(0.891)^2 + (0.39)(0.891)^3 = 1.206 \text{ [kJ/kg-K]}$$

$$C_{v0} = C_{P0} - R = (1.206 - 0.1889) \text{ [kJ/kg-K]} = 1.0171 \text{ [kJ/kg-K]}$$

$$Q_{1 \rightarrow 2} = (2 \text{ [kg]})(1.0171 \text{ [kJ/kg-K]})(609 - 1,173) \text{ [K]} - 48.642 \text{ [kJ]}$$

$$\boxed{\text{c)} \implies Q_{1 \rightarrow 2} = -1,195.93 \text{ [kJ]}}$$

For part d), we want to use the integral-averaged specific heat evaluated between the final and initial temperatures.

$$\theta_1 = \frac{T_1}{1,000} = \frac{1,173 \text{ [K]}}{1,000} = 1.173$$

$$\theta_2 = \frac{T_2}{1,000} = \frac{609 \text{ [K]}}{1,000} = 0.609$$

$$\tilde{C}_{P0} = \frac{1}{\theta_2 - \theta_1} \left(0.45\theta + \frac{1.67\theta^2}{2} - \frac{1.27\theta^3}{3} + \frac{0.39\theta^4}{4} \right) \Big|_{\theta_1}^{\theta_2}$$

$$\tilde{C}_{P0} = \frac{1}{1.173 - 0.609} \left(\left(0.45(1.173) + \frac{1.67(1.173)^2}{2} - \frac{1.27(1.173)^3}{3} + \frac{0.39(1.173)^4}{4} \right) - \dots \right. \\ \left. \dots - \left(0.45(0.609) + \frac{1.67(0.609)^2}{2} - \frac{1.27(0.609)^3}{3} + \frac{0.39(0.609)^4}{4} \right) \right) = 1.200 \text{ [kJ/kg-K]}$$

$$C_{v0} = C_{P0} - R = (1.200 - 0.1889) \text{ [kJ/kg-K]} = 1.0111 \text{ [kJ/kg-K]}$$

$$Q_{1 \rightarrow 2} = (2 \text{ [kg]})(1.0111 \text{ [kJ/kg-K]})(609 - 1,173) \text{ [K]} - 48.642 \text{ [kJ]}$$

$$\boxed{\text{d)} \implies Q_{1 \rightarrow 2} = -1,189.16 \text{ [kJ]}}$$

Lastly, we will use Table A.8 to pull our specific internal energies. For State 1:

$$\frac{(1,173 - 1,100) \text{ [K]}}{(1,200 - 1,100) \text{ [K]}} = \frac{(u_1 - 888.55) \text{ [kJ/kg]}}{(996.64 - 888.55) \text{ [kJ/kg]}}$$

$$\Rightarrow u_1 = 967.46 \text{ [kJ/kg]}$$

The specific internal energy at State 2:

$$\frac{(609 - 600) \text{ [K]}}{(650 - 600) \text{ [K]}} = \frac{(u_1 - 392.72) \text{ [kJ/kg]}}{(437.71 - 392.72) \text{ [kJ/kg]}}$$

$$\Rightarrow u_2 = 400.82 \text{ [kJ/kg]}$$

Therefore:

$$Q_{1 \rightarrow 2} = (2 \text{ [kg]})(400.82 - 967.46) \text{ [kJ/kg]} - 48.642 \text{ [kJ]}$$

$$\boxed{\text{d) } \Rightarrow Q_{1 \rightarrow 2} = -1,181.92 \text{ [kJ]}}$$

This reflects a 0.61% difference in solutions using the integral-averaged constant pressure specific heat, a 1.18% difference in solutions using the constant pressure specific heat based on an average temperature, and a 40.33% difference in solutions treating the constant volume specific heat as a constant.