Homework #8

MEMS 0051 - Introduction to Thermodynamics

Assigned July 26th, 2018
Due July 30th, 2018

Starting with the Conservation of Energy equation:

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \sum_{i=1}^{N} \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right) - \sum_{j=1}^{M} \dot{m}_j \left(h_j + \frac{V_j^2}{2} + g z_j \right),$$

and the Continuity equation:

$$\frac{dm}{dt} = \sum_{i=1}^{N} \dot{m}_i - \sum_{j=1}^{M} \dot{m}_j,$$

Solve the following:

Problem #1

Steam enters a turbine through a pipe with a diameter of 0.2 [m]. The steam enters with a velocity of 100 [m/s], a pressure of 14,000 [kPa] and a temperature of 600 °C. The steam is exhausted through a pipe with a diameter of 0.8 [m], a pressure of 500 [kPa] and a temperature of 180 °C. Determine:

a) the exit velocity of the steam;
b) the mass flow rate of the steam.

Solution: The Conservation of Energy need not be applied. The mass flow rate into the system must be that out of the system. The inlet mass flow rate can be found from the inlet conditions:

$$\dot{m} = \frac{A_1 V_1}{\nu} = \frac{\pi \, (0.2 \, [\text{m}])^2 \, (100 \, [\text{m/s}])}{4 \, \nu}$$

The specific volume at the inlet state is found via interpolation:

$$\frac{(14,000 - 10,000) \, [\text{kPa}]}{(15,000 - 10,000) \, [\text{kPa}]} = \frac{(\nu - 0.03837) \, [\text{m}^3/\text{kg}]}{(0.02491 - 0.03837) \, [\text{m}^3/\text{kg}]} \implies \nu = 0.027602 \, [\text{m}^3/\text{kg}]$$

Therefore, the mass flow rate is:

$$\dot{m} = \frac{AV}{\nu} = \frac{\pi \, (0.2 \, [\text{m}])^2 \, (100 \, [\text{m/s}])}{4 \, 0.027602 \, [\text{m}^3/\text{kg}]} = 113.818 \, [\text{kg/s}]$$

The exit velocity can now be solved for:

$$V_2 = \frac{\dot{m} \, \nu_2}{A_2} = \frac{(113.818 \, [\text{kg/s}]) \, \pi \, (0.8 \, [\text{m}])^2}{4 \, \nu_2}$$

The specific volume at the outlet is solved for based upon given state properties via interpolation:

$$\frac{(180 - 151.86) \, [^\circ\text{C}]}{(200 - 151.86) \, [^\circ\text{C}]} = \frac{(\nu - 0.37489) \, [\text{m}^3/\text{kg}]}{(0.42492 - 0.37489) \, [\text{m}^3/\text{kg}]} \implies \nu_2 = 0.40413 \, [\text{m}^3/\text{kg}]$$

Thus, the exit velocity is:

$$V_2 = \frac{(113.818 \, [\text{kg/s}]) \, \pi \, (0.8 \, [\text{m}])^2}{4 \, (0.40413 \, [\text{m}^3/\text{kg}])} = 141.57 \, [\text{m/s}]$$

Problem #2

A device has one inlet with a cross-sectional flow area of 0.6 [m^2] in which steam enters with a velocity of 50 [m/s], a pressure of 1,000 [kPa] and a temperature of 400 °C. There are two outlets. One outlet has saturated liquid exiting through a 0.018 [m^2] pipe with a mass flow rate of 50 [kg/s] at a pressure of 150 [kPa]. Determine:

a) the mass flow rate at the inlet;

b) the mass flow rate of the second outlet.

Solution: State 1 exists as a superheated vapor, and the specific volume is directly read from the table as 0.30659 [m^3/kg]. Thus, the mass flow rate at the inlet is determined by:

$$\dot{m} = \frac{A_1 V_1}{\nu_1} = \frac{(0.6\,[\mathrm{m}^2])(50\,[\mathrm{m/s}])}{0.30659\,[\mathrm{m}^3/\mathrm{kg}]} = 97.85\,[\mathrm{kg/s}]$$

The mass flow rate at the second outlet is found via the Continuity equation:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \implies \dot{m}_3 = \dot{m}_1 - \dot{m}_2 = (97.85 - 50)\,[\mathrm{kg/s}] = 47.85\,[\mathrm{kg/s}]$$

Problem #3

Air enters a device at 1,000 [kPa] and 580 [K] and leaves with a volumetric flow rate of 1.8 [m^3/s] at 100 [kPa] and 500 [K]. Heat is transferred from the device to the surroundings at 347 [kJ] per kilogram of air entering the device. Determine:

a) the power developed by the device;

b) the the volumetric flow rate at the inlet.

Solution: The volumetric flow rate rate the inlet is known for the mass flow rate through the device must be constant. The mass flow rate, at the exit, in terms of the volumetric flow rate is:

$$\dot{m} = \rho\dot{\forall} = \frac{\dot{\forall}}{\nu}$$

The specific volume is found via the Ideal Gas law:

$$P\forall = mRT \implies P\nu = RT \implies \nu = \frac{RT}{P} = \frac{(0.287\,[\mathrm{kJ/kg\text{-}K}])(500\,[\mathrm{K}])}{100\,[\mathrm{kPa}]} = 1.435\,[\mathrm{m}^3/\mathrm{kg}]$$

Thus, the mass flow rate at the inlet is:

$$\dot{m} = \frac{1.8\,[\mathrm{m}^3/\mathrm{s}]}{1.435\,[\mathrm{m}^3/\mathrm{kg}]} = 1.254\,[\mathrm{kg/s}]$$

Now, applying the Conservation of Energy equation, assuming steady-state and ignoring changes in kinetic and potential energy, and expressing in terms of work developed:

$$\dot{W} = \dot{Q} + \dot{m}(h_1 - h_2)$$

The enthalpy at State 1 and 2 are found from Table A.7.1:

$$h_1 = 586.35\,[\mathrm{kJ/kg}]$$

$$h_2 = 503.65\,[\mathrm{kJ/kg}]$$

Therefore, the work developed is:

$$\dot{W} = -347\,[\mathrm{kJ/kg}] + (1.254\,[\mathrm{kg/s}])(586.35 - 503.65)\,[\mathrm{kJ/kg}] = -242.93\,[\mathrm{kJ/kg}]$$

Can we use the Ideal Gas law?

Problem #4

Air flows through a diffuser with a mass flow rate of 0.5 [kg/s] from an inlet condition of 300 [kPa], 290 [K] and 400 [m/s] to an exit condition of 1,4000 [kPa] and 40 [m/s]. Determine:

a) the exit temperature of the air;
b) the inlet cross-sectional flow area.

Solution: The inlet cross-sectional flow area is determined from the definition of mass flow rate:

$$\dot{m} = \frac{A\,V}{\nu} \implies A = \frac{\dot{m}\,\nu}{V} = \frac{(0.5\,[\text{kg/s}])\,\nu}{400\,[\text{m/s}]}$$

The specific volume is determined from the Ideal Gas law:

$$nu = \frac{R\,T}{P} = \frac{(0.287\,[\text{kJ/kg-K}])(290\,[\text{K}])}{300\,[\text{kPa}]} = 0.277\,[\text{m}^3/\text{kg}]$$

Thus, the inlet cross-sectional flow area is:

$$A = \frac{\dot{m}\,\nu}{V} = \frac{(0.5\,[\text{kg/s}])(0.277\,[\text{m}^3/\text{kg}])}{400\,[\text{m/s}]} = 0.000\,346\,[\text{m}^2]\text{ (i.e. about half an inch sqared)}$$

To determine the exit temperature of the air, we look to the Conservation of Energy equation for a diffuser, assuming steady-state, ignoring heat and work and potential energy:

$$0 = h_1 - h_2 + \frac{V_i^1 - V_2^2}{2}$$

Solving for the enthalpy at State 2, knowing the enthalpy at State 1 is from Table A.7.1:

$$h_2 = h_1 + \frac{V_i^1 - V_2^2}{2} = 290.43\,[\text{kJ/kg}] - \frac{((400-40)\,[\text{m/s}])^2}{2 \cdot 1000} = 355.23\,[\text{kJ/kg}]$$

Interpolating between 340 and 360 K on Table A.7.1:

$$\frac{(T_2 - 340)\,[\text{K}]}{(360-340)\,[\text{K}]} = \frac{(355.23 - 340.70)\,[\text{kJ/kg}]}{(360.86 - 340.70)\,[\text{kJ/kg}]} \implies T_2 = 354.4\,[\text{K}]$$

Problem #5

A turbine with sufficient insulation accepts steam at the rate of 85 [m^3/min] at 3,000 [kPa] and 400 °C. A portion of the steam is siphoned from the turbine at a pressure of 500 [kPa], a temperature of 180 °C at a velocity of 20 [m/s]. The remainder of the steam, with a mass flow rate of 40,000 [kg/hr] expands to a pressure of 6 [kPa] with a quality of 90%. Determine:

a) the power developed by the turbine;
b) the diameter of the siphon.

Solution: Setting up the Continuity equation:

$$\dot{m}_1 = \dot{m}_2 + \dot{m}_3 \implies \frac{\dot{\forall}_1}{\nu_1} = \frac{A_2 V_2}{\nu_2} + (40,000\,[\text{kg/hr}])(1/3,600\,[\text{hr/s}])$$

The specific volume at State 1, a superheated vapor, and State 2, also a superheated vapor, are found as:

$$\nu_1 = 0.09936\,[\text{m}^3/\text{kg}]$$

$$\frac{(180 - 151.86)\,[^\circ\text{C}]}{(200 - 151.86)\,[^\circ\text{C}]} = \frac{(\nu - 0.37489)\,[\text{m}^3/\text{kg}]}{(0.42492 - 0.37489)\,[\text{m}^3/\text{kg}]} \implies \nu_2 = 0.40413\,[\text{m}^3/\text{kg}]$$

Solving for the exit cross-sectional flow area at State 2:

$$A_2 = \left(\frac{(85\,[\mathrm{m}^3/\mathrm{min}])(1/60\,[\mathrm{min/s}])}{0.09936\,[\mathrm{m}^3/\mathrm{kg}]} - (40,000\,[\mathrm{kg/hr}])(1/3,600\,[\mathrm{hr/s}])\right)\left(\frac{0.40413\,[\mathrm{m}^3/\mathrm{kg}]}{20\,[\mathrm{m/s}]}\right) = 0.0636\,[\mathrm{m}^2]$$

Applying the Conservation of Energy, ignoring the velocities at State 1 and 3, all potential energies and heat transfer:

$$\dot{W} = \dot{m}_1 h_1 - \dot{m}_2\left(h_2 + \frac{V_2^2}{2}\right) - \dot{m}_3 h_3$$

The enthalpy at State 1 comes from the superheated steam tables:

$$h_1 = 3,230.82\,[\mathrm{kJ/kg}]$$

The enthalpy at State 2 comes the superheated steam tables:

$$\frac{(180 - 151.86)\,[^\circ\mathrm{C}]}{(200 - 151.86)\,[^\circ\mathrm{C}]} = \frac{(h_2 - 2,748.67)\,[\mathrm{kJ/kg}]}{(2,855.37 - 2,748.67)\,[\mathrm{kJ/kg}]} \implies h_2 = 2,811.04\,[\mathrm{kJ/kg}]$$

The enthalpy at State 3 comes the saturated water tables:

$$\frac{(6 - 5)\,[\mathrm{kPa}]}{(7.5 - 5)\,[\mathrm{kPa}]} = \frac{(h_{3,f} - 137.79)\,[\mathrm{kJ/kg}]}{(168.77 - 137.79)\,[\mathrm{kJ/kg}]} \implies h_{3,f} = 150.18\,[\mathrm{kJ/kg}]$$

$$\frac{(6 - 5)\,[\mathrm{kPa}]}{(7.5 - 5)\,[\mathrm{kPa}]} = \frac{(h_{3,g} - 2,561.45)\,[\mathrm{kJ/kg}]}{(2,574.79 - 2,561.45)\,[\mathrm{kJ/kg}]} \implies h_{3,g} = 2,566.79\,[\mathrm{kJ/kg}]$$

$$h_3 = h_{3,f} + x_3(h_{3,g} - h_{3,f}) = (150.18\,[\mathrm{kJ/kg}]) + 0.9(2,566.79 - 150.18)\,[\mathrm{kJ/kg}] = 2,325.13\,[\mathrm{kJ/kg}]$$

The mass flow rate at State 1 is:

$$\dot{m}_1 = \frac{(85\,[\mathrm{m}^3/\mathrm{min}])(1/60\,[\mathrm{min/s}])}{0.09936\,[\mathrm{m}^3/\mathrm{kg}]} = 14.26\,[\mathrm{kg/s}]$$

The mass flow rate at State 3 is:

$$\dot{m}_3 = \frac{40,000\,[\mathrm{kg/hr}]}{3,600\,[\mathrm{s/hr}]} = 11.1\,[\mathrm{kg/s}]$$

The mass flow rate at State 2 is the difference:

$$\dot{m}_2 = 3.16\,[\mathrm{kg/s}]$$

Thus,

$$\dot{W} = (14.26\,[\mathrm{kg/s}])(3,230.82\,[\mathrm{kJ/kg}]) - (3.16\,[\mathrm{kg/s}])\left((2,811.04\,[\mathrm{kJ/kg}]) + \frac{(20\,[\mathrm{m/s}])^2}{2\cdot 1000}\right) - (11.1\,[\mathrm{kg/s}])(2,325.13\,[\mathrm{kJ/kg}])$$

$$\dot{W} = 11,379.03\,[\mathrm{kJ/kg}]$$

Problem #6

An open feedwater heater (OFWH) accepts liquid water at 1,000 [kPa] and a temperature of 50 °C. The OFWH also accepts water with a mass flow rate per that of inlet one, i.e. $\dot{m}_2/\dot{m}_1$=0.22. Saturated liquid water exits the OFWH. Determine:

a) the temperature of the second incoming stream, if superheated;
b) the quality of the second incoming stream, if saturated.

Solution: Applying the Continuity equation to the system:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

We can express $\dot{m}_2$ as $0.22\dot{m}_1$. Therefore:

$$1.22\dot{m}_1 = \dot{m}_3$$

Applying the Conservation of Energy equation to the OFWH, assuming steady-state, ignoring heat and work, and kinetic and potential energy:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \implies \dot{m}_1 h_1 + 0.22\dot{m}_1 h_2 = 1.22\dot{m}_1 h_3 \implies h_1 + 0.22h_2 = 1.22h_3$$

We know the enthalpy at State 3 is that of saturated water at 1,000 [kPa], i.e h_3=762.79 [kJ/kg]. The enthapy of State 1 is that of compressed water at 1,000 [kPa] and 50 °C, i.e. h_f(50 °C)=209.31 [kJ/kg]. Substituting in these values:

$$209.31\,[\text{kJ/kg}] + 0.22h_2 = 1.22(762.79\,[\text{kJ/kg}]) \implies h_2 = 3,278.61\,[\text{kJ/kg}]$$

At 1,000 [kPa], this puts State 2 in the superheated vapor region, existing between 400 and 500 °C:

$$\frac{(T_2 - 400)\,[^\circ\text{C}]}{(500 - 400)\,[^\circ\text{C}]} = \frac{(3,278.61 - 3,263.88)\,[\text{kJ/kg}]}{(3,4788.44 - 3,263.88)\,[\text{kJ/kg}]} \implies T_2 = 406.87\,[^\circ\text{C}]$$