

**Due: 3/29/2020 at 9:00 pm through Courseweb**

**MEMS 0051  
Spring 2020  
Midterm #2  
3/27/2020**

**Name (Print):** \_\_\_\_\_

This exam contains 10 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials and the date on the top of every page, in case the pages become separated. The following rules apply:

- You may use your notes, your book, EES and MATLAB. You may also use online thermodynamic property calculators, however EES is recommended.
- All work must be done on the test sheets. If you need more room, please note that you have added an additional sheet. Number that additional sheet, initial and date.
- If you use EES for determination of properties, please note so. However, if the values are wrong, you will receive no credit.
- If you use MATLAB to solve for a system of equations, please email your script to mmb49@pitt.edu with the subject "MEMS 0051 Midterm 2 Script".
- All work must be substantiated by the work preceding it. A result with no methodology and no mathematics will be marked incorrect.
- Do not write in the table to the right.

Problem	Points	Score
1	15	
2	20	
3	25	
4	40	
Total:	100	

**Academic Integrity Statement:**

I hereby attest that I have received no assistance (from a friend, from an on-line resource, etc.), and that I have provided no assistance, during this exam. All the work presented within is solely my own work.

Signature: \_\_\_\_\_

Date: \_\_\_\_\_

**Problem #1**

1. (15 points) An inventor published the following data from an experiment of a power cycle, where the hot-side temperature reservoir was 527 °C, and the cold-side temperature reservoir was 27 °C. For each of the following scenarios, determine if there is a violation of the 2<sup>nd</sup> Law, and if so, identify which statement:

(a) (5 pts.)  $Q_H=700$  [kJ],  $Q_L=300$  [kJ],  $W=400$  [kJ]

The power output is equal to the difference of the heat supplied and heat rejected. There is no direct violation of the Clausius or Kelvin-Planck statements. The efficiency of the cycle is:

$$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{400 \text{ [kJ]}}{700 \text{ [kJ]}} = 0.57$$

This value is below the Carnot efficiency, which is:

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ [K]}}{800 \text{ [K]}} = 0.625$$

2. (5 pts.)  $Q_H=640$  [kJ],  $Q_L=240$  [kJ],  $W=400$  [kJ]

The power output is equal to the difference of the heat supplied and heat rejected. There is no direct violation of the Clausius or Kelvin-Planck statements. The efficiency of the cycle is:

$$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{400 \text{ [kJ]}}{640 \text{ [kJ]}} = 0.625$$

This value is equal to the Carnot efficiency, which is:

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ [K]}}{800 \text{ [K]}} = 0.625$$

3. (5 pts.)  $Q_H=640$  [kJ],  $Q_L=200$  [kJ],  $W=400$  [kJ]

The power output is less than the difference of the heat supplied and heat rejected. The actual power output should be 440 [kJ]. There is no direct violation of the Clausius or Kelvin-Planck statements. However, the Conservation of Energy, i.e. First Law, is violated, since energy is not conserved. The efficiency of the cycle is:

$$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{400 \text{ [kJ]}}{640 \text{ [kJ]}} = 0.625$$

This value is equal to the Carnot efficiency, which is:

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ [K]}}{800 \text{ [K]}} = 0.625$$

**Problem #2**

2. (20 points) A refrigerator with  $\beta=4.5$  receives 0.8 [kW] of electrical power. The heat rejected from the refrigerated space occurs within the condenser; the condensation process occurs at 28 °C. The temperature of the surroundings is of 20 °C. Determine the following:

(a) (7.5 pts.) The rate at which energy is rejected to the surroundings;

By definition of the coefficient of performance for a refrigerator, in rate form:

$$\beta = \frac{\dot{Q}_L}{\dot{W}} = 4.5 \implies Q_L = 4.5(0.8 \text{ [kW]}) = 3.6 \text{ [kW]}$$

The heat rejected to the surroundings is found via the definition of work:

$$W = Q_H - Q_L \implies Q_H = (0.8 + 4.6) \text{ [kW]} = 4.4 \text{ [kW]}$$

- (b) (7.5 pts.) The lowest theoretical temperature of the cold-side temperature reservoir (i.e. the refrigerated space);

Assuming we have a Carnot refrigeration cycle, the efficiency can be expressed in terms of heats and temperature such that:

$$\eta_{\text{Carnot}} = \frac{T_L}{T_H - T_L} = \frac{Q_L}{W} \implies T_L = T_H \left( \frac{Q_L}{W} \right) \left( 1 + \frac{Q_L}{W} \right)^{-1}$$

$$T_L = (293 \text{ [K]}) \left( \frac{3.6 \text{ [kW]}}{0.8 \text{ [kW]}} \right) \left( 1 + \frac{3.6 \text{ [kW]}}{0.8 \text{ [kW]}} \right)^{-1} = 239.73 \text{ [K]}$$

- (c) (5 pts.) The maximum theoretical power (in [kW]) that a waste-heat recovery device could produce when operating between the condenser and surroundings.

Constructing a heat engine between the condenser and surroundings:

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{293 \text{ [K]}}{301 \text{ [K]}} = 0.027$$

This efficiency of this heat engine would be equal to the power output per heat supplied, with the later being the heat rejected from part a):

$$\eta_{\text{Carnot}} = \frac{W}{Q_H} \implies W = 0.027 Q_H = 0.027(4.4 \text{ [kW]}) = 0.119 \text{ [kW]}$$

**Problem #3**

3. (25 points) A Carnot refrigeration cycle, using 0.1 [kg] of air as the working fluid, removes heat from the low-temperature reservoir at a temperature of -23 °C. The quantity of heat removed during this process is 3.4 [kJ]. Heat is then rejected to the high-temperature reservoir, which exists at a temperature of 27 °C. The volume at the end of the heat rejection process is 0.01 [m<sup>3</sup>]. Determine/complete the following:

- (a) (10 pts.) Draw the  $P-\nu$  and  $T-s$  diagram of this cycle, labeling each state (with proper  $P$ ,  $\nu$ ,  $T$  and  $s$  values) and process between states;

Our givens include the mass and temperature at each state.

State 1:	State 2:	State 3:	State 4:
$m_1=0.1$ [kg]	$m_2=m_1$	$m_3=m_1$	$m_4=m_1$
$T_1=250$ [K]	$T_2=250$ [K]	$T_3=300$ [K]	$T_4=300$ [K]
$P_1=455.6$ [kPa]	$P_2=285.4$ [kPa]	$P_3=538.5$ [kPa]	$P_4=859.9$ [kPa]
$\forall_1=0.01569$ [m <sup>3</sup> ]	$\forall_2=0.0251$ [m <sup>3</sup> ]	$\forall_3=0.01597$ [m <sup>3</sup> ]	$\forall_4=0.01$ [m <sup>3</sup> ]
$s_1=s_4$	$s_2=6.383$ [kJ/kg-K]	$s_3=s_2$	$s_4=6.247$ [kJ/kg-K]

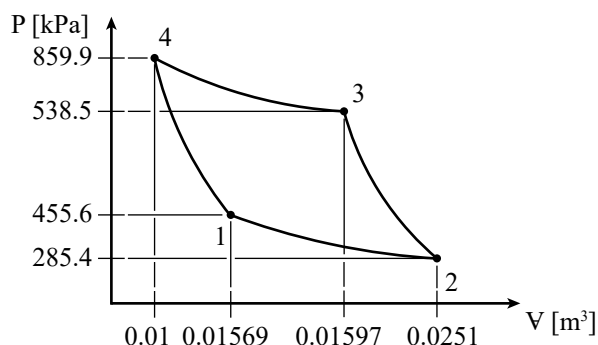
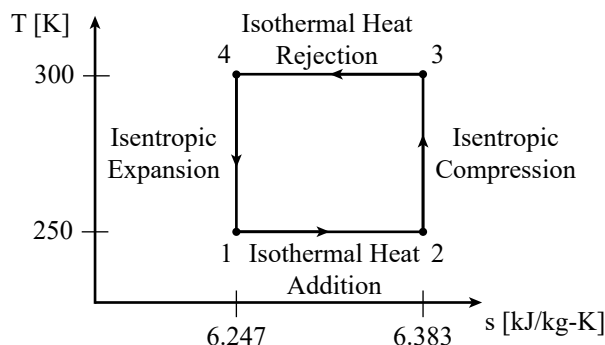
The pressure at State 4 is found via the Ideal Gas law (or EES):

$$P_4 = \frac{m_4 R T_4}{\forall_4} = \frac{(0.1 \text{ [kg]}) (0.287 \text{ [kJ/kg-K]}) (300 \text{ [K]})}{0.01 \text{ [m}^3\text{]}} = 861 \text{ [kPa]}$$

The entropy at State 4 is found using EES to be 6.247 [kJ/kg-K] (since pressure and specific volume are known). Thus, the pressure and volume at State 1 can be found via temperature and entropy using EES. From State 1 to 2, we are adding 3.4 [kJ] of heat under a constant-temperature process. Thus, the specific entropy value is found through through:

$$Q_{1 \rightarrow 2} = mT(s_2 - s_1) \implies s_2 = \frac{Q_{1 \rightarrow 2}}{mT} + s_1 = \frac{3.4 \text{ [kJ]}}{(0.1 \text{ [kg]}) (250 \text{ [K]})} + 6.247 \text{ [kJ/kg-K]} = 6.383 \text{ [kJ/kg-K]}$$

The pressure and volume can be found via the specific entropy and temperature using EES. Since the temperature and entropy are known at State 3, the pressure and volume can be found via EES. Note, any property determined from EES can be also determined from the relations described in Lecture 21. Thus, we can plot and label these processes and points.



- (b) (15 pts.) The work associated with each of the four processes. Units to be reported in [kJ].

The work for the two isothermal processes is equal to the heat, such that:

$$W_{1 \rightarrow 2} = Q_{1 \rightarrow 2} = 3.4 \text{ [kJ]}$$

$$W_{3 \rightarrow 4} = mT(s_4 - s_3) = (0.1 \text{ [kg]})(300 \text{ [K]})(6.247 - 6.383) \text{ [kJ/kg-K]} = -4.08 \text{ [kJ]}$$

The work for the isentropic compression process from State 2 to 3 is:

$$W_{2 \rightarrow 3} = \frac{mR(T_3 - T_2)}{1 - n}$$

where  $n$  is replaced with  $k$ , which is equal to 1.4 for air. We can verify  $n$  is indeed equal to 1.4:

$$n = \frac{\ln\left(\frac{P_3}{P_2}\right)}{\ln\left(\frac{v_2}{v_3}\right)} = 1.42$$

This is within the tolerance of values given by EES. Thus:

$$W_{2 \rightarrow 3} = \frac{(0.1 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})(300 - 250) \text{ [K]}}{1 - 1.4} = -3.59 \text{ [kJ]}$$

For the isentropic expansion process between States 4 and 1:

$$W_{4 \rightarrow 1} = \frac{(0.1 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})(250 - 300) \text{ [K]}}{1 - 1.4} = 3.59 \text{ [kJ]}$$

**Problem #4**

4. (40 points) Consider 1 [kg] of air, which is initially treated as an Ideal gas. The air undergoes a reversible process from an initial state, where the pressure is 100 [kPa] and the temperature is 298.15 [K], to a final state where the pressure is 1,200 [kPa]. If the work put into the system is 200 [kJ], determine the following:

- (a) (20 pts.) The final temperature at State 2;

<u>State 1:</u> $m_1=1$ [kg] $P_1=100$ [kPa] $T_1=298.15$ [K]	<u>State 2:</u> $m_2=m_1$ $P_2=1,200$ [kPa]
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For air undergoing a reversible process, the work into the system is:

$$W_{1 \rightarrow 2} = \frac{mR(T_2 - T_1)}{n}$$

Or,

$$T_2 = \frac{(1 - n)W_{1 \rightarrow 2}}{mR} + T_1$$

Additionally, we can use the following relation:

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

Or,

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

Setting the expressions for  $T_2$  equal:

$$T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = \frac{(1 - n)W_{1 \rightarrow 2}}{mR} + T_1$$

Substituting in values:

$$(298.15 \text{ [K]}) \left( \frac{1,200 \text{ [kPa]}}{100 \text{ [kPa]}} \right)^{\frac{n-1}{n}} = \frac{(1 - n)(-200 \text{ [kJ]})}{(1 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})} + 298.15 \text{ [K]}$$

Solving for  $n$ , we get three values. One is unity, which makes no physical sense; the temperature must increase as we increase the pressure. The other two require us to re-evaluate our expression for  $T_2$  based upon pressures and  $n$ . In doing such, we see one value of  $n = 0.821$  gives us a temperature that is less than the initial (i.e. 173.736 [K]). This also doesn't make physical sense. Thus, we conclude that  $n = 2.377$  is the correct value, for :

$$T_2 = (298.15 \text{ [K]}) \left( \frac{1,200 \text{ [kPa]}}{100 \text{ [kPa]}} \right)^{\frac{2.377-1}{2.377}} = 1,257 \text{ [K]}$$



- (b) (10 pts.) The change of entropy between the final and initial state, using Table A.6; using the following formulation:

$$ds = \int_{T_1}^{T_2} \frac{C_{P0}}{T} dT - R \ln\left(\frac{P_2}{P_1}\right)$$

A function can be created to evaluate the integral of the specific heat per temperature, with respect to temperature, such that:

$$\int \frac{C_{P0}(T)}{T} dT = 1.05 \ln(T) - \frac{(0.365T)}{1,000} + \frac{(0.85T^2)}{2(1,000)^2} - \frac{0.39T^3}{3(1,000)^3}$$

Evaluating this between the two bounds of  $T_1$  and  $T_2$ , we have:

$$ds = (1.5408 - 0.7132) [\text{kJ/kg-K}] = 0.8276 [\text{kJ/kg-K}]$$

(b) (10 pts.) The change of entropy between the final and initial state, using Table A.7.1:

Using Table A.7.1:

$$s_T^0(T_1) = 6.86305 \text{ [kJ/kg-K]}$$

$$s_T^0(T_2) = 8.34596 \text{ [kJ/kg-K]}$$

The change of the standard entropy values replaced the integral, such that

$$ds = (8.34596 - 6.86305 - 0.7132) \text{ [kJ/kg-K]} = 0.770 \text{ [kJ/kg-K]}$$