**MEMS 0051**
**Summer 2017**
**Midterm #1**
**6/26/2017**
**75 Minutes**

**Name (Print):** Solution

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This exam contains 5 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page. Please do all of your work in the provided blue examination book.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 20 | |
| 4 | 25 | |
| 5 | 35 | |
| Total: | 100 | |

**BONUS:**

5 pts: June $26^{\text{th}}$, 1945, marked the signing of what charter that established the Statute of the International Court of Justice?

## Basic Properties

1. (a) (5 points) Determine the quality of 2.0 [kg] of water having a volume of 0.5 [m$^3$] existing at 25 °C and a pressure of 3.169 [kPa].

Recall the definition for specific volume:

$$v = \frac{\forall}{m} = \frac{0.5\ [\text{m}^3]}{2.0\ [\text{kg}]} = 0.25\ [\text{m}^3/\text{kg}]$$

At 25°c and 3.169 [kPa], our specific volume is existing between the saturated liquid ($v_f$) and saturate vapor ($v_g$) specific volumes. Therefore, we can use the definition of quality:

$$\boxed{x = \frac{v - v_f}{v_g - v_f} = \frac{0.25 - 0.001003}{43.3583} = 0.574\%}$$

(b) (5 points) Determine the mass of saturated water vapor at 155 °C in a 40 [m$^3$] rigid tank. Since we know the specific volume of the saturated water vapor ($v_g$= 0.34676 [m$^3$/kg], the mass is simply found as:

$$\boxed{m = \frac{\forall}{v} = \frac{40\ [\text{m}^3]}{0.34676\ [\text{m}^3/\text{kg}]} = 115.354\ [\text{kg}]}$$

## Polytropic Process

2. (10 points) A piston-cylinder contains water at 500 °C and 3 [MPa]. It is cooled in a polytropic process to 200 °C and 1 [MPa]. Determine:

- the polytropic index $n$,
- the specific work in the process.

To determine the polytropic index $n$, we need the pressure and volume, or specific volume at the initial and final states. At State 1, we see the water is existing as a superheated vapor ($T_1 > T_{sat}$(3 [MPa]), therefore the specific volume is found to be 0.11619 [m$^3$/kg]. At State 2, the water is still existing as a superheated vapor ($T_2 > T_{sat}$(1 [MPa]), and the specific volume is found to be 0.20596 [m$^3$/kg]. Solving for the polytropic index:

$$\boxed{n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\frac{1\ [\text{MPa}]}{3\ [\text{MPa}]}\right)}{\ln\left(\frac{0.11619\ [\text{m}^3/\text{kg}]}{0.20596\ [\text{m}^3/\text{kg}]}\right)} = 1.919}$$

The specific work, or work per unit mass basis, is simply evaluated using our definition of work based upon the polytropic index:

$$\frac{W_{1\to2}}{m} = w_{1\to2} = \frac{P_2 v_2 - P_1 v_1}{1-n}$$

Evaluating the expression:

$$\boxed{w_{1\to2} = \frac{1,000\ [\text{kPa}] \cdot 0.20596 \left[\frac{\text{m}^3}{\text{kg}}\right] - 3,000\ [\text{kPa}] \cdot 0.11619 \left[\frac{\text{m}^3}{\text{kg}}\right]}{1 - 1.919} = 115.18\ [\text{kJ}]}$$

## Formulation of Work

3. (20 points) Water contained in a piston-cylinder assembly has an initial temperature of 150 °C, a quality of 50% and an initial volume of 0.05 m$^3$. The pressure of the process is given as a function of volume such that P($\forall$)=100 + C$\forall^{0.5}$ [kPa]. Heat is transferred to the piston-cylinder until the final pressure reaches 600 kPa. Determine:

   - the heat input.

   Hint: Solve for the constant C based upon givens at State 1.

| State 1: | $\longrightarrow$ | State 2: |
|---|---|---|
| $\text{T}_1$=150°C | | $\text{m}_2$=$\text{m}_1$ |
| $\text{x}_1$=0.50 | | $\text{P}_2$=600 [kPa] |
| $\text{P}_1=\text{P}_{\text{sat}}(\text{T}_1)$=475.9 [kPa] | | $v_2=\forall_2/\text{m}_2$=? |
| $\forall_1$=0.05 [m$^3$] | | |

To determine $Q_{1\to2}$, we need $m_1$, $u_1$, $u_2$ and $W_{1\to2}$. Solving for the mass at State 1 by using the volume and specific volume, the latter determined by the given quality:

$$m_1 = \frac{\forall_1}{v_1} = \frac{0.05\ [\text{m}^3]}{0.00109 + 0.5(0.39278 - 0.00109)[\text{m}^3/\text{kg}]} = 0.254\ [kg]$$

We can determine the internal energy at State 1 using quality:

$$u_1 = u_f + x(u_g - u_f) = 631.66 + 0.5(2,559.54 - 631.66)\ [\text{kJ/kg}] = 1,595.6\ [\text{kJ/kg}]$$

We can next determine the specific volume at State 2, $v_2$, which will be used to determine both the work and the internal energy. Using our relationship of pressure and volume applied to State 1, noting that the expression is explicitly given in terms of [kPa]:

$$P(\forall) = 100{+}\text{C}\forall^{0.5} = 475.9\ [\text{kPa}] = 100{+}\text{C}(0.05[\text{m}^3])^{0.5} \implies \text{C} = \frac{375.9\ [\text{kPa}]}{(0.05[\text{m}^3])^{0.5}} = 1,681 \left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]$$

Using the same formulation for pressure as function of volume, applied to State 2, with our solved for constant:

$$P_2 = 600[\text{kPa}] = 100 + \left(1,681\left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]\right)\forall_2^{0.5}$$

Solving for $\forall_2$

$$\forall_2 = \left(\frac{500[\text{kPa}]}{1,681\left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]}\right)^2 = 0.0885\ [\text{m}^3]$$

Therefore, the specific volume at State 2 is:

$$v_2 = \frac{\forall_2}{m} = \frac{0.0885\ [\text{m}^3]}{0.254\ [\text{kg}]} = 0.349\ [\text{m}^3/\text{kg}]$$

Given $v_2$ and $P_2$, we see the water at State 2 is in the superheated vapor region. Solving for $u_2$ via interpolation:

| v [m³/kg] | u [kJ/kg] |
|---|---|
| 0.31567 | 2,567.40 |
| 0.349 | $u_2$ |
| 0.35202 | 2,638.91 |

$$\implies u_2 = 2,632.97\ [\text{kJ/kg}]$$

Solving for work:

$$W_{1\to2} = \int P d\forall = \int (100 + 1,681.1\forall^{0.5}) d\forall = \left(100\forall + \frac{2}{3}\cdot 1,681.1\forall^{1.5}\right)\Big|_{0.05}^{0.0885} = 20.83\ [\text{kJ}]$$

Note, you can solve for work by determining the polytropic index ($n$=-0.405).

Bringing everything together to solve for the heat input:

$$\boxed{Q_{1\to2} = U_2 - U_1 + W_{1\to2} = 0.254\ [\text{kg}]\left(2,632.97 - 1,595.6\left[\frac{\text{kJ}}{\text{kg}}\right]\right) + 20.83\ [\text{kJ}] = 284.32\ [\text{kJ}]}$$

## Ideal Gas

4. (25 points) A frictionless piston-cylinder device contains 0.1 [kg] of air at 300 K at 100 [kPa]. The air is slowly compressed from its initial state in an isothermal process to a final pressure of 250 [kPa].

- Show the process on the P-v diagram,
- determine the work into the system,

- determine the heat rejected from the system.

| State 1: | $\longrightarrow$ | State 2: |
|---|---|---|
| $m_1$=1 [kg] | $n$=1 | $m_2$=$m_1$ |
| $T_1$=300 [K] | | $T_2$=$T_1$ |
| $P_1$=100 [kPa] | | $P_2$=250 [kPa] |

Solving for $\forall_1$ using the Idea Gas Law:

$$P_1\forall = mRT_1 \implies \forall_1 = \frac{mRT_1}{P_1} = \frac{0.1\text{ [kg]}\cdot 0.287\text{[kJ/kg-K]}\cdot 300\text{[K]}}{100\text{ [kPa]}} = 0.0861\text{ [m}^3\text{]}$$

Solving for $\forall_2$ using the Idea Gas Law:

$$P_2\forall = mRT_2 \implies \forall_2 = \frac{mRT_2}{P_2} = \frac{0.1\text{ [kg]}\cdot 0.287\text{[kJ/kg-K]}\cdot 300\text{[K]}}{250\text{ [kPa]}} = 0.0344\text{ [m}^3\text{]}$$

Since the process is isothermal, the work into the system is expressed as:

$$\boxed{W_{1\to 2} = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = 100\text{ [kPa]}\cdot 0.0861\text{ [m}^3\text{]}\cdot \ln\left(\frac{0.0344\text{ [m}^3\text{]}}{0.0861\text{ [m}^3\text{]}}\right) = -7.899\text{ [kJ]}}$$

Solving for the heat rejected from the system:

$$\boxed{Q_{1\to 2} = U_2 - U_1 + W_{1\to 2} = mC_\forall(T_2 - T_1) + W_{1\to 2} = W_{1\to 2} = -7.899\text{ [kJ]}}$$

Since there is no change of temperature between the initial and final state, there is no change in internal energy, and the heat input is the same as the work out.

### Multistep Process

5. (35 points) A frictionless piston-cylinder device has two sets of stops that constrain the piston. When the piston is at rest on the lower set of stops, the enclosed volume is 0.4 [m$^3$]. When the piston reaches the uppers tops, the enclosed volume is 0.6 [m$^3$]. The cylinder initially contains water at 100 [kPa] and a quality of 20%. It is heated until the water eventually exists as a saturated vapor. The mass of the piston is such that is requires 300 [kPa] generated within the piston-clyinder device to move. Determine:

   1. the final pressure in cylinder,
   2. the heat transfer during this process,
   3. the work for this process.

Hint: There are three processes going between 4 states.