Chapter 3 - First Law of Thermodynamics and Energy

Lecture 9
Section 3.4

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh

Learning Objectives

3.4 Work - Moving Boundary

Summary

Student Learning Objectives

At the end of the lecture, students should be able to:

- Formulate an expression for work for moving boundary systems
- Calculate and apply the polytropic index to moving boundary systems

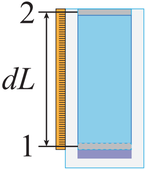
Learning Objectives

3.4 Work - Moving Boundary

Summary

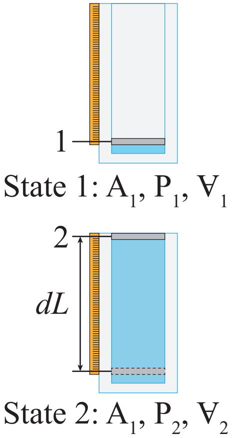
Moving Boundary System

- Moving boundary systems include piston-cylinders, compressors, etc.
- Consider a piston-cylinder, in which the cylinder is raised a distance of dL from state 1 to 2:



- Considering this a quasi-equilibrium process, we can calculate the work done onto/by the system.

Moving Boundary System



- By definition,

$$\delta W = F dx$$

- Force is pressure times area, and the change in distance is dL

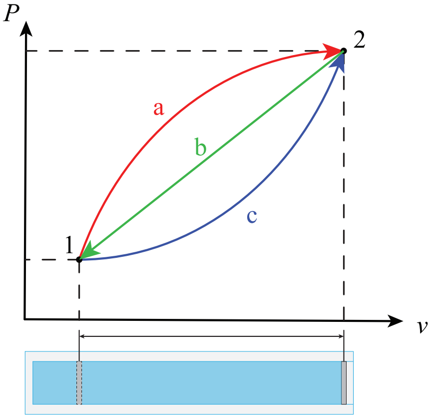
$$\delta W = PAdL \rightarrow Pd\forall$$

- Therefore

$$\boxed{W_{1\rightarrow 2} = \int_1^2 \delta W = \int_1^2 P(\forall) d\forall}$$

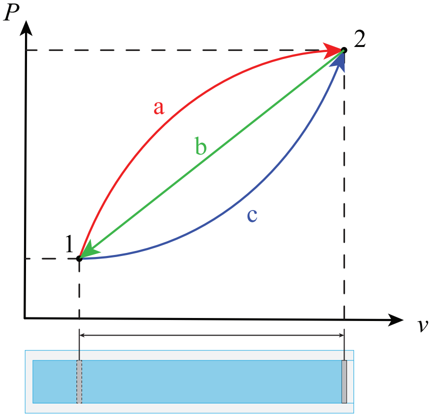
Moving Boundary System

- Recall integration is calculating the area under a curve - as the piston moves between the two states, P can have varied dependence on $\forall$, i.e. path dependent



Moving Boundary System

- We use the inexact differential δW because W is a path function (i.e. it is path dependent) - we use the exact differential $d\forall$ because $\forall$ only depends on the initial and final states



Polytropic Process

- To quantify the path dependence of work, we must consider a polytropic process, a process pertaining to a function which has different values for one variable.
- A polytropic process is defined as:

$$P\forall^n = c$$

- n is the polytropic index, $n \in \mathbb{R}$, $-\infty \leq n \leq \infty$.
  - n=0 $\implies$ Isobaric (P=c)
  - n=1 $\implies$ Isothermal (T=c)
  - n=γ $\implies$ Isentropic (S=c)
  - n=∞ $\implies$ Isochoric ($\forall$=c)

Polytropic Index

- The polytropic index can be visualized on a P-$\forall$ diagram, and can take any real value between $-\infty$ and ∞!
- When n=0:

$$P(\forall^n) = c \implies P(\forall^0) = c \implies P(1) = c$$

- When $n \to \infty$:

$$P(\forall^\infty) = c \implies \forall = c$$

- So why is this of use? Given any value for the polytropic index, we can now calculate the path-dependent work of a system.

Work via Polytropic Index

- Taking the equation for work:

$$W_{1\to 2} = \int_1^2 \delta W = \int_1^2 P d\forall$$

- If we define $P\forall^n$=c=$P_1\forall_1^n$=$P_2\forall_2^n$ for States 1 and 2, and rearrange our expression for P:

$$P = \frac{P_1\forall_1^n}{\forall^n} = \frac{P_2\forall_2^n}{\forall^n} = \frac{c}{\forall^n}$$

- Reinserting this for P within our expression for work:

$$W_{1\to 2} = \int_1^2 P d\forall = c\int_1^2 \frac{1}{\forall^n} d\forall$$

Work via Polytropic Index

- The integrand can be expressed as:

$$\frac{1}{\forall^n} = \forall^{-n}$$

- Inserting this into the integral expression for work:

$$W_{1\to 2} = c\int_1^2 \forall^{-n} d\forall = \frac{c}{1-n}\forall^{1-n}\Big|_1^2$$

- Evaluating this:

$$W_{1\to 2} = \frac{c}{1-n}(\forall_2^{1-n} - \forall_1^{1-n})$$

- Recalling $P\forall^n$=c=$P_1\forall_1^n$=$P_2\forall_2^n$, substitute in the respective values for the constant

Work via Polytropic Index

- Thus:

$$W_{1\to 2} = \left(\frac{c}{1-n}\right)\forall_2^{1-n} - \left(\frac{c}{1-n}\right)\forall_1^{1-n} =$$

$$= \frac{P_2\forall_2^n\forall_2^{1-n} - P_1\forall_1^n\forall_1^{1-n}}{1-n}$$

- Recalling $x^n x^{1-n}{=}x$, we are left with an expression for work based upon the pressures and volumes of States 1 and 2, the polytropic index n:

$$\boxed{W_{1\to 2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n}}$$

Work via Polytropic Index

- Taking the limiting case n=1:

$$P\forall^n = c = P_1\forall_1$$

- Rearranging for P:

$$P = \frac{P_1\forall_1}{\forall^n} = \frac{P_1\forall_1}{\forall}$$

- Substitute P in for the definition of work:

$$W_{1\rightarrow 2} = \int_1^2 P d\forall = P_1\forall_1 \int_1^2 \frac{d\forall}{\forall}$$

- Notice how P_1 and $\forall_1$ are constants - we are not integrating w.r.t. $\forall_1$, rather $\forall$. Thus:

$$\boxed{W_{1\rightarrow 2} = P_1\forall_1 \ln(\forall)\Big|_1^2 = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right)}$$

Example #1

- A piston-cylinder undergoes an expansion process from an initial pressure of 3 [bar] and volume of 0.1 [m^3] to a final volume of 0.2 [m^3]. Determine the final work for the process in [kJ] for:
  1. n=1.5
  2. n=1.0
  3. n=0

Solution:

Example #1

Solution:

Example #1

Solution:

Example #2

- Construct an expression for the polytropic index n based upon pressure and volume between two states

Solution:

Student Learning Objectives

At the end of the lecture, students should be able to:

- Formulate an expression for work for moving boundary systems
  - Work for a moving boundary system is formulated based upon pressure and volume:

$$W_{1\to 2} = \int P(\forall)d\forall$$

  - We can formulate this in terms of the polytropic index:

$$W_{1\to 2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n}$$

  - When $n = 1$:

$$W_{1\to 2} = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right)$$

Student Learning Objectives

- Calculate and apply the polytropic index to moving boundary systems
  - n is calculated as:

$$n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\frac{P_1}{P_2}\right)}{\ln\left(\frac{\forall_2}{\forall_1}\right)}$$

Suggested Problems

- 3.37, 3.43, 3.45, 3.46, 3.49, 3.52, 3.56