Homework #7

MEMS 0051 - Introduction to Thermodynamics

Assigned March 2$^{\text{nd}}$, 2019
Due: March 8$^{\text{th}}$, 2019

Problem #1

A high-temperature reservoir at 800 [K] dissipates 1 [MJ] of heat into a heat engine. The heat engine then dissipates 6.5 [kJ] into a cold-temperature reservoir. For the following cases of the cold-temperature reservoir, determine the change in specific entropy. Be sure to comment on the reversibility or irreversibility of the processes.
The change in specific entropy, or entropy, is given by the equation:

$$\Delta S = \oint \frac{Q}{T}$$

For this process,

$$\Delta S = \oint \frac{Q}{T} = \frac{Q_H}{T_H} + \frac{Q_L}{T_L}$$

The irreversibility/reversibility of a process is given by the following conditions:
1. $\Delta S < 0$: Irreversible, Possible
2. $\Delta S = 0$: Reversible, Possible
3. $\Delta S > 0$: Reversible, Not Possible

The change in specific entropy, or entropy, for real and ideal processes obey conditions 1 and 2 respectively, never condition 3.

a) T_L=300 [K]
$$\Delta S = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = \frac{1000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{-(6.5 \text{ [kJ]})}{300 \text{ [K]}} = \boxed{1.2283 \text{ [kJ/(kg K)]} : \text{Reversible, Not Possible}}$$

b) T_L=450 [K]
$$\Delta S = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = \frac{1000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{-(6.5 \text{ [kJ]})}{450 \text{ [K]}} = \boxed{1.2356 \text{ [kJ/(kg K)]} : \text{Reversible, Not Possible}}$$

c) T_L=100 [K]
$$\Delta S = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = \frac{1000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{-(6.5 \text{ [kJ]})}{100 \text{ [K]}} = \boxed{1.1850 \text{ [kJ/(kg K)]} : \text{Reversible, Not Possible}}$$

d) T_L=500 [K]
$$\Delta S = \frac{Q_H}{T_H} + \frac{Q_L}{T_L} = \frac{1000 \text{ [kJ]}}{800 \text{ [K]}} + \frac{-(6.5 \text{ [kJ]})}{500 \text{ [K]}} = \boxed{1.2370 \text{ [kJ/(kg K)]} : \text{Reversible, Not Possible}}$$

Problem #2

A rigid tank contains 3 [kg] of R-134a at 293.15 [K] and 175 [kPa]. The refrigerant is then cooled until the pressure within the tank reaches 105 [kPa]. Determine the change of entropy during the process.

$$\begin{array}{ccc} \underline{\text{State 1:}} & \rightarrow & \underline{\text{State 2:}} \\ \text{T}_1\text{=293.15 [K]} & & \text{P}_2\text{=105 [kPa]} \\ \text{P}_1\text{=175 [kPa]} & & \\ \text{m}_1\text{=3 [kg]} & & \end{array}$$

Looking at the saturation tables for R-134a in table B.5.1 shows that the initial state of the refrigerant is in the super-heated state. Going to table B.5.2, it can be seen that values need to be interpolated between pressures 150

[kPa] and 200 [kPa] at temperature 20 [°C]. From this, it can be found that:

$$v_1 = 0.1343 \ [\text{m}^3/\text{kg}]$$

$$s_1 = 1.8387 \ [\text{kJ}/(\text{kg K})]$$

Since the tank is rigid, the mass and volume does not change throughout the process. Therefore, the specific volume at state one is the same as in state 2.

$$v_1 = v_2 = 0.1343 \ [\text{m}^3/\text{kg}]$$

Knowing this and looking at the values for specific volume at the most extreme cases between 100 [kPa] and 150 [kPa] in table B.5.2 shows that the second state can either be in the saturated state or compressed liquid state. Looking at the saturation tables for R-134a in Tables B.5.1, the values for v_g and v_f are not tabulated for a pressure of 105 [kPa]. Interpolating between pressures 101.3 [kPa] and 107.2 [kPa] yields:

$$v_g(105[\text{kPa}]) = 0.183\,991\,52 \ [\text{m}^3/\text{kg}]$$

$$v_f(105[\text{kPa}]) = 0.000\,729\,25 \ [\text{m}^3/\text{kg}]$$

After finding these values, it can be seen that the specific volume at state 2 lays within v_f the range of v_g. Therefore, state 2 of the refrigerant is in the saturated mixture phase.
Knowing this, the quality at this state can easily be found using the following:

$$v = v_f(1 - x) + xv_g \implies$$

$$x = 0.728850$$

Now that the quality is known, the specific entropy, as well as other properties, at state 2 can be found. The values for s_g and s_f are not tabulated for a pressure of 105 [kPa]. Interpolating between pressures 101.3 [kPa] and 107.2 [kPa] yields:

$$s_g(105[\text{kPa}]) = 1.744\,547\,46 \ [\text{kJ}/(\text{kg K})]$$

$$s_f(105[\text{kPa}]) = 0.873\,013\,56 \ [\text{kJ}/(\text{kg K})]$$

After finding these values, the specific entropy at state 2 is found to be:

$$s_2 = s_f(1 - x) + xs_g \implies$$

$$s_2 = 1.508\,231\,04 \ [\text{kJ}/(\text{kg K})]$$

Lastly, we can find the change in specific entropy to be:

$$\Delta s = s_2 - s_1 = \boxed{-0.330\,469 \ [\text{kJ}/(\text{kg K})]}$$

Seeing that the change in specific entropy is less than 0, it is clear that the process is <u>irreversible</u>.

Problem #3

A piston cylinder containing ammonia at 100 [kPa] and -10 [°C] is cooled in an isothermal reaction until the height of the piston is a fifth of what it was before cooling. What is the work per unit mass done by the piston cylinder?

We should first note that the pistons cylinder cross section can not change. Therefore a change in height by a factor of 1/5 means the volume at state 2 is 1/5 of the original volume. We write down the states as:

$$
\begin{array}{ccc}
\underline{\text{State 1:}} & \rightarrow & \underline{\text{State 2:}} \\
T_1 = 263.15 \ [\text{K}] & & T_2 = 263.15 \\
P_1 = 100 \ [\text{kPa}] & & \forall_2 = (1/5)\forall_1 \\
\forall_1 = \forall_1 & &
\end{array}
$$

Looking at the saturation table of ammonia on Table B.5.1, it can be seen that the pressure at state 1 is less than that of the saturation pressure at -10 [°C]. Therefore, the ammonia is in the super-heated state. Looking at Table B.5.2, we can see that the specific volume, enthalpy, internal energy and entropy are:

$$v_1 = 1.2621 \ [\text{m}^3/\text{kg}]$$

$$u_1 = 1324.6 \ [\text{kJ/kg}]$$

$$s_1 = 6.0477 \ [\text{kJ/(kg K)}]$$

Given this, we can determine the specific volume at state 2 is one fifth of the state 1. That is:

$$v_2 = (1/5)v_1 = 0.2524 \ [\text{m}^3/\text{kg}]$$

Since the specific volume is non-existent at the same temperature in the other sections of the super-heated table, we look to the saturated tables on table B.2.1. Here, we see the specific volume at state 2 lays within v_f the range of v_g at -10 [°C]. That is:

$$v_g(-10[°\text{C}]) = 0.418\,08 \ [\text{m}^3/\text{kg}]$$

$$v_f(-10[°\text{C}]) = 0.001\,534 \ [\text{m}^3/\text{kg}]$$

Therefore, state 2 of the refrigerant is in the saturated mixture phase. Knowing this, the quality at this state can easily be found using the following:

$$v = v_f(1-x) + xv_g \implies$$

$$x = 0.6023$$

Using the quality as well as the values for specific internal energy and entropy at the liquid and vapor state, the properties of ammonia can be found to be:

$$u_2 = 841.7839 \ [\text{kJ/kg}]$$

$$s_2 = 3.5080 \ [\text{kJ/(kg K)}]$$

Now that states are known, we can find the specific work from state 1 to 2 using the conservation of energy equation. That is:

$$w_{1\to2} = q_{1\to2} - \Delta u_{1\to2}$$

The heat transferred can be found using the cyclic integral of heat per quantity of temperature.

$$\Delta s = \frac{q_{1\to2}}{T} \implies$$

$$q_{1\to2} = (\Delta s)(T) = (3.5080 - 6.0477 \ [\text{kJ/(kg K)}])(263.15 \ [\text{K}]) = -668.3221 \ [\text{kJ/kg}]$$

The change in internal energy is:

$$\Delta u_{1\to2} = u_2 - u_1 = 841.7839 - 1324.6 \ [\text{kJ/kg}] = -482.8161 \ [\text{kJ/kg}]$$

Finally, plugging in these values into the conservation of energy equation yields that the specific work for this process is:

$$w_{1\to2} = q_{1\to2} - \Delta u_{1\to2} = -668.3221 \ [\text{kJ/kg}] - (-482.8161 \ [\text{kJ/kg}]) \implies$$

$$\boxed{-185.5060 \ [\text{kJ/kg}]}$$

Problem #4

An experiment is being performed where a 13.5 [L] balloon containing 300 [g] of super-heated CO_2 at 1 [MPa] is cooled in an isothermal process by removing 170 [kJ/kg] of heat from the system. Determine the volume change of the balloon. Do not treat as an ideal gas.

We should first note that the process is isothermal. Therefore the temperature does not change from state 1 to state 2. We write down the states as:

$$\underline{\text{State 1:}} \qquad \rightarrow \qquad \underline{\text{State 2:}}$$
$$P_1 = 1 \text{ [MPa]} \qquad\qquad T_2 = T_1$$
$$m_1 = 300 \text{ [g]}$$
$$\forall_1 = 13.5 \text{ [L]} = 0.0135 \text{ [m}^3\text{]}$$

The specific volume at state 1 is the quotient of the volume and mass respectively:

$$v_1 = \frac{\forall_1}{m_1} = 0.045 \text{ [m}^3\text{/kg]}$$

Looking at the specific volumes and pressure values in table B.3.1, it can be seen that the specific volume is greater than that of the approximated specific volume of 100% saturated vapor at 1 [MPa]. Therefore, the CO_2 is in the superheated state.

Looking at Table B.3.2, the values for temperature and specific entropy can be calculated by interpolating between the specific volumes at -20 [°C] and 0 [°C] at the 1000 [kPa] pressure entry. This yields:

$$T_1 = -13.0853[°\text{C}] = 260.0647 \text{ [K]}$$

$$s_1 = 1.4903 \text{ [kJ/(kg K)]}$$

Since the process of heat removal is isothermal, the temperature at state 2 is given as:

$$T_2 = -13.0853[°\text{C}] = 260.0647 \text{ [K]}$$

The change in specific entropy can be found using the cyclic integral for heat removal/addition across the temperature boundaries. Here, q_{rem} is the specific heat **removed** and T is the boundary temperature it is being **removed** at. That is:

$$\Delta s = \oint \frac{q}{T} = \frac{q_{rem}}{T} = \frac{-170 \text{ [kJ/kg]}}{260.0647 \text{ [K]}} = -0.653\,683 \text{ [kJ/(kg K)]}$$

Using the change in specific entropy and the specific entropy at state 1, the property at state 2 can be found to be:

$$s_2 = s_1 + \Delta s = 0.836\,617 \text{ [kJ/(kg K)]}$$

Looking at the specific entropy values at the temperature at state 2 across the superheated table, it can be seen that the specific entropy at state 2 is too low for any of the values in the superheated tables. Therefore, we look to Table B.3.1.

It can be seen that the specific entropy at state 2 exists between the approximated specific entropy values for saturated liquid and saturated vapor states at T_2. Therefore, the CO_2 is in the saturated mixture state.

Looking at the saturation tables for CO_2 in Table B.3.1, the values for v_g, v_f, s_g and s_f can be found for a temperature of -13.0853 [°C] by interpolating between -12 [°C] and -14 [°C]. This yields:

$$v_g(-13.0853[°\text{C}]) = 0.015\,501\,797 \text{ [m}^3\text{/kg]}$$

$$v_f(-13.0853[°\text{C}]) = 0.001\,001\,574 \text{ [m}^3\text{/kg]}$$

$$s_g(-13.0853[°\text{C}]) = 1.248\,480\,765 \text{ [kJ/(kg K)]}$$

$$s_f(-13.0853[°\text{C}]) = 0.224\,446\,275 \text{ [kJ/(kg K)]}$$

The specific entropy at state 2 exists between the two values for specific entropy. Knowing this, the quality at this state can easily be found using the following:

$$s = s_f(1 - x) + x v_s \implies$$

$$x = 0.597\,786\,238$$

Using the quality as well as the values for specific volume at the liquid and vapor state, the specific volume at state 2 can be found as:

$$v_2 = 0.009\,669\,608 \ [\mathrm{m}^3/\mathrm{kg}]$$

Finally, the change in volume throughout the process can be found to be:

$$\Delta \forall = (m)\Delta v = (0.3[\mathrm{kg}])(v_2 - v_1) = (0.3[\mathrm{kg}])(0.009\,669\,608 - 0.045 \ [\mathrm{m}^3/\mathrm{kg}]) \implies$$

$$\boxed{\Delta \forall = -0.010\,599\,[\mathrm{m}^3]}$$

Problem #5

The condenser in a refrigeration process takes in R-410a refrigerant at 300 [kPa] and 35 [°C]. The condenser then cools the refrigerant until the pressure within the tank reaches 138.8[kPa]. The heat rejected from the condenser is emitted into the ambient environment at a constant temperature of 20 [°C]. Assuming the condenser acts like a rigid container, determine the following:

1. Heat per unit mass rejected (**q**)
2. Work per unit mass done by the condenser to perform this process (**w**)
3. Coefficient of performance (β)

$$\underline{\text{State 1:}} \qquad \rightarrow \qquad \underline{\text{State 2:}}$$
$$T_1 = 308.15 \text{ [K]} \qquad P_2 = 138.8 \text{ [kPa]}$$
$$P_1 = 300 \text{ [kPa]}$$

Since the condenser acts like a rigid container, the mass and volume of the refrigerant does not change. As a result, the specific volume at state 2 is the same as that of state 1.

$$v_1 = v_2$$

Looking at the saturation tables for R-410a in table B.4.1 shows that the initial state of the refrigerant is in the super-heated state. Going to table B.4.2, it can be seen that values need to be interpolated between temperatures 20 [°C] and 40 [°C] at pressure 300 [kPa]. From this, it can be found that:

$$v_1 = 0.1135 \text{ [m}^3\text{/kg]}$$

$$s_1 = 1.2912 \text{ [kJ/(kg K)]}$$

$$u_1 = 288.85 \text{ [kJ/kg]}$$

Knowing this and looking at the values for specific volume at the most extreme cases between 100 [kPa] and 150 [kPa] in table B.4.2 shows that the second state can either be in the saturated state or compressed liquid state. Looking at the saturation tables for R-410a in Table B.4.1, the values for v_g and v_f are tabulated for a pressure of 138.8 [kPa].

$$v_g(138.8[\text{kPa}]) = 0.178\,04 \text{ [m}^3\text{/kg]}$$

$$v_f(138.8[\text{kPa}]) = 0.000\,752 \text{ [m}^3\text{/kg]}$$

The specific volume in state 2 exists between these two values. Therefore, state 2 of the refrigerant is in the saturated mixture phase. Knowing this, the quality at this state can easily be found using the following:

$$v = v_f(1 - x) + xv_g \implies$$

$$x = 0.6360$$

Using the quality as well as the values for specific internal energy and entropy at the liquid and vapor state, the properties of the refrigerant can be found to be:

$$u_2 = 147.308 \text{ [kJ/kg]}$$

$$s_2 = 0.715 \text{ [kJ/(kg K)]}$$

The heat transferred can be found using the cyclic integral of heat per quantity of temperature.

$$\Delta s = \frac{q_{1\rightarrow2}}{T_{amb}} \implies$$

$$q_{1\rightarrow2} = (\Delta s)(T_{amb}) = (0.715 - 1.2912 \text{ [kJ/(kg K)]})(293.15 \text{ [K]}) \implies$$

$$q_{1\rightarrow2} = \boxed{\mathbf{q} = -168.9130 \text{ [kJ/kg]}}$$

Now that the heat per unit mass (specific heat transferred) is known, we can find the specific work done by the condenser from state 1 to 2 using the conservation of energy equation. That is:

$$w_{1\rightarrow2} = q_{1\rightarrow2} - \Delta u_{1\rightarrow2}$$

The change in internal energy is:

$$\Delta u_{1 \to 2} = u_2 - u_1 = 147.308 - 288.85 \text{ [kJ/kg]} = -141.5420 \text{ [kJ/kg]}$$

Plugging this value into the conservation of energy equation yields that the specific work for this process is:

$$w_{1 \to 2} = q_{1 \to 2} - \Delta u_{1 \to 2} = -168.9130 \text{ [kJ/kg]} - (-141.5420 \text{ [kJ/kg]}) \implies$$

$$\boxed{\mathbf{w} = -27.371 \text{ [kJ/kg]}}$$

In other words, the condenser put in 27.371 [kJ/kg] of specific work to make this process happen.
The condensers sole purpose is to extract energy, in the form of heat, from the refrigerant by using some input energy to perform this process. Therefore, the coefficient of performance β is equivalent to that of a refrigerator. That is

$$\beta = \frac{Q_L}{W} = \frac{\dot{Q}_L}{\dot{W}} = \frac{q_L}{w}$$

The specific heat being extracted was found to be -168.913 [kJ/kg]. The specific work input was found to be -27.371 [kJ/kg]. In conclusion, we can find the coefficient of performance to be:

$$\beta = \frac{q_L}{w} = \frac{-168.913 \text{ [kJ/kg]}}{-27.371 \text{ [kJ/kg]}} \implies$$

$$\boxed{\beta = 6.171}$$