

Homework #7

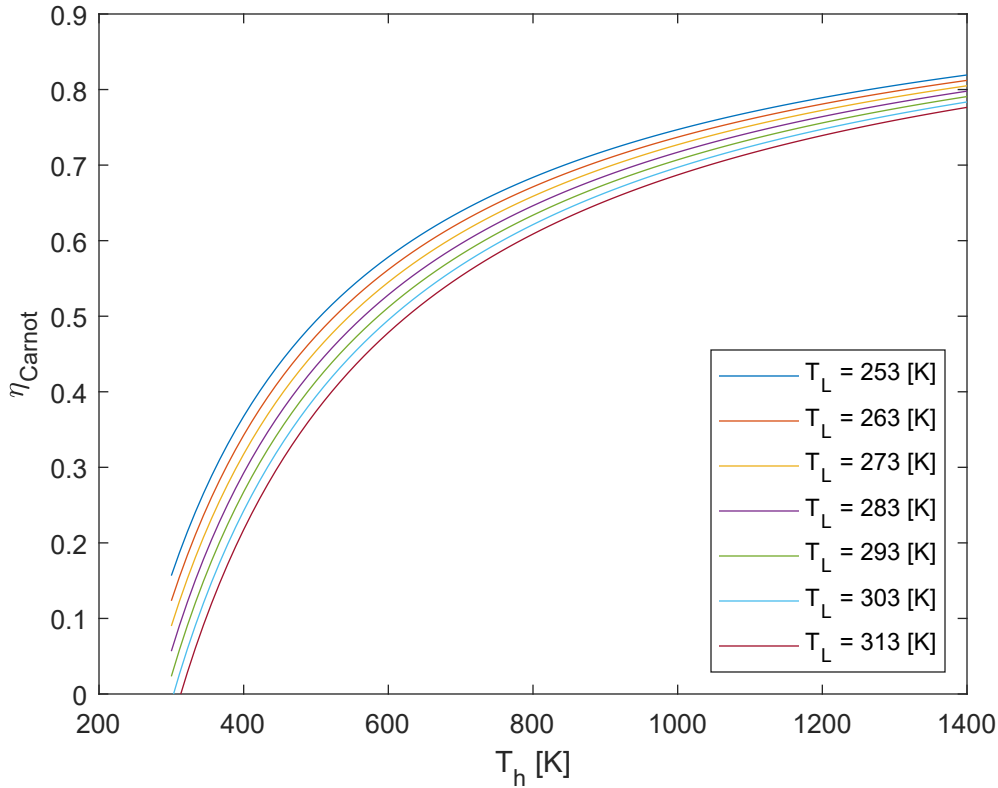
MEMS 0051 - Introduction to Thermodynamics

Assigned February 28th, 2020
Due March 6th, 2020

Problem #1

In Matlab, plot the Carnot efficiency of a heat engine as a function of low- and high-temperature reservoir temperatures. The high-temperature reservoir temperature varies between 300 [K] and 1,400 [K]. The low-temperature reservoir has discrete temperatures of 253, 263, 273, 283, 293, 303, and 313 [K].

```
1 clear all
2 close all
3 clc
4
5 T_H = linspace(300,1400,100);
6 T_L = [253 263 273 283 293 303 313];
7
8 for i = 1:length(T_L)
9     for j = 1:length(T_H)
10         eta_carnot(j) = 1 - (T_L(i)/T_H(j));
11     end
12     plot(T_H, eta_carnot);
13     hold on
14 end
15
16 xlabel('T_h [K]');
17 ylabel('\eta_{Carnot}');
18 ylim([0 0.9])
19 set(gcf, 'color', 'white')
20 legend('T_L = 253 [K]', 'T_L = 263 [K]', 'T_L = 273 [K]', 'T_L = 283 [K]', '
        T_L = 293 [K]', 'T_L = 303 [K]', 'T_L = 313 [K]', 'Location', 'southeast');
```



Problem #2

A piston-cylinder device contains 1 [kg] of water, which is undergoing a Carnot cycle. The water starts the isothermal expansion process with a quality of 25% and a pressure of 1,500 [kPa], and ends as a saturated vapor. The adiabatic expansion process is completed when the pressure reaches 100 [kPa] and the water exists at a quality of 84.9%. Determine:

- The heat for each of the four processes;
- the work for each of the four processes;
- the thermal efficiency of the cycle.

We need to determine our state variables. Using EES:

State 1:	$T = c$	State 2:	$S = c$	State 3:	$T = c$	State 4:
$x_1=0.25$		$x_2=1.0$		$x_3=0.849$		$x_4=0.34$
$P_1=1,500$ [kPa]		$P_2=P_1$		$P_3=100$ [kPa]		$P_4=P_3$
$T_1=T_{\text{sat}}(P_1)=198.32$ °C		$T_2=T_1$		$T_3=99.6$ °C		$T_4=T_3$
$\nu_1=0.0338$ [m ³ /kg]		$\nu_2=0.1318$ [m ³ /kg]		$\nu_3=1.4386$ [m ³ /kg]		$\nu_4=0.5726$ [m ³ /kg]
$u_1=1,280.8$ [kJ/kg]		$u_2=2,593.9$ [kJ/kg]		$u_3=2,190.4$ [kJ/kg]		$u_4=1,122.3$ [kJ/kg]
$s_1=3.347$ [kJ/kg-K]		$s_2=s_1$		$s_3=6.444$ [kJ/kg-K]		$s_4=s_1$

- The heat for each of the processes is found via the Conservation of Energy:

$$Q_{1 \rightarrow 2} = (U_2 - U_1) + W_{1 \rightarrow 2} = (1 \text{ [kg]}) (2,593.9 - 1,280.8) \text{ [kJ/kg]} + (1,500 \text{ [kPa]}) (1 \text{ [kg]}) (0.1318 - 0.0338) \text{ [m}^3\text{/kg]} = 1,460.1 \text{ [kJ]}$$

$$Q_{2 \rightarrow 3} = 0 \text{ [kJ]}$$

$$Q_{3 \rightarrow 4} = (U_4 - U_3) + W_{3 \rightarrow 4} = (1 \text{ [kg]}) (1,122.3 - 2,190.4) \text{ [kJ/kg]} + (100 \text{ [kPa]}) (1 \text{ [kg]}) (0.5726 - 1.4386) \text{ [m}^3\text{/kg]} = -1,154.7 \text{ [kJ]}$$

$$Q_{4 \rightarrow 1} = 0 \text{ [kJ]}$$

b) The work for each of the four processes was integral to part a):

$$W_{1 \rightarrow 2} = (1,500 \text{ [kPa]})(1 \text{ [kg]})(0.1318 - 0.0338) \text{ [m}^3/\text{kg}] = 147 \text{ [kJ]}$$

$$W_{2 \rightarrow 3} = -(U_3 - U_2) = -(1 \text{ [kg]})(2,190.4 - 2,593.9) \text{ [kJ/kg]} = 403.5 \text{ [kJ]}$$

$$W_{3 \rightarrow 4} = (100 \text{ [kPa]})(1 \text{ [kg]})(0.5726 - 1.4386) \text{ [m}^3/\text{kg}] = -86.6 \text{ [kJ]}$$

$$W_{4 \rightarrow 1} = -(U_1 - U_4) = -(1 \text{ [kg]})(1,280.8 - 1,122.3) \text{ [kJ/kg]} = -158.5 \text{ [kJ]}$$

c) The thermal efficiency is can be found based up on the temperature differences, our the power output per heat input:

$$\eta = 1 - \frac{T_L}{T_H} = 1 - \frac{372.75 \text{ [K]}}{471.47 \text{ [K]}} = 0.209$$

In terms of the net work output per heat input:

$$\eta = \frac{P_o}{Q_H} = \frac{(147 + 403.5 - 86.6 - 158.5) \text{ [kJ]}}{1,460.1 \text{ [kJ]}} = 0.209$$

In terms of the difference of heat input and output, per heat input:

$$\eta = \frac{Q_H - Q_L}{Q_H} = \frac{(1,460.1 - 1,153.7) \text{ [kJ]}}{1,460.1 \text{ [kJ]}} = 0.209$$

Problem #3

A Carnot cycle, that uses 1 [kg] of air as the working fluid, has a thermal efficiency of 50%. If the heat transferred to the air during the isothermal expansion process is 50 [kJ], and the pressure at the beginning of this process is 700 [kPa], and the volume at the beginning of this process is 0.12 [m³], determine:

- The maximum and minimum temperatures of the cycle;
- the volume at the end of the isothermal heat addition process;
- the work for each of the four processes;
- the heat for each of the four processes;

This problem is to be completed considering the specific heat of air as a i) constant value taken from Table A.5, ii) evaluated using the average temperate temperature, which is the arithmetic mean of the maximum and minimum temperature, from Table A.6 and iii) evaluated using the integral average of the specific heat using Table A.6.

a) We are given pressure and volume at State 1: $P_1 = 700 \text{ [kPa]}$ and $V_1 = 0.12 \text{ [m}^3\text{]}$. Using the ideal gas law, we can solve for temperature, noting that $R = 0.287 \text{ [kJ/kg-K]}$

$$T_1 = \frac{P_1 V_1}{R} = \frac{(700 \text{ [kPa]})(0.12 \text{ [m}^3\text{]})}{(0.287 \text{ [kJ/kg-K]})} = 292.68 \text{ [K]}$$

Since this is the temperature at the beginning of our isothermal expansion, it will also be our maximum temperature. Thus:

$$T_{max} = 292.68 \text{ [K]}$$

Since we know our Carnot efficiency, we can use T_{max} to find T_{min} ,

$$\eta_{Carnot} = 1 - \frac{T_{min}}{T_{max}} = 0.5$$

$$\Rightarrow T_{min} = 146.34 \text{ [K]}$$

b) For the isothermal expansion process from State 1 to 2, we know the heat supplied (50 [kJ]). Applying the conservation of energy to this process:

$$U_2 - U_1 = Q_{1 \rightarrow 2} - W_{1 \rightarrow 2}$$

Since $T_1 = T_2$:

$$dU = C_{v0}(T_2 - T_1) = 0$$

Thus:

$$Q_{1 \rightarrow 2} = W_{1 \rightarrow 2} = 50 \text{ [kJ]}$$

We can now solve for the volume at State 2, recognizing that for an isothermal polytropic process, $n = 1$:

$$\begin{aligned} W_{1 \rightarrow 2} &= P_1 v_1 \ln\left(\frac{v_2}{v_1}\right) = 50 \text{ [kJ]} \implies \ln\left(\frac{v_2}{v_1}\right) = \frac{50 \text{ [kJ]}}{P_1 v_1} \\ \implies v_2 &= \exp\left(\frac{50 \text{ [kJ]}}{P_1 v_1}\right) v_1 = \exp\left(\frac{50 \text{ [kJ]}}{(700 \text{ [kPa]})(0.12 \text{ [m}^3\text{]})}\right) (0.12 \text{ [m}^3\text{]}) = 0.217 \text{ [m}^3\text{]} \end{aligned}$$

c) To determine the amount of work done for each process, we need the remainder of our state variables. Using the Ideal Gas Law and the temperature and volume at State 2, we can solve for pressure at State 2;

$$P_2 = \frac{RT_2}{v_2} = \frac{(0.287 \text{ [kJ/kg-K]})(292.68 \text{ [K]})}{(0.217 \text{ [m}^3\text{]})} = 386.09 \text{ [kPa]}$$

We know the work and heat from State 1 to 2, i.e. are both equal to 50 [kJ]. From State 2 to 3, we need the pressure at State 3. This is found by recalling the change of entropy between State 2 and 3 is zero:

$$S_3 - S_2 = C_{P0} \ln\left(\frac{T_3}{T_2}\right) - R \ln\left(\frac{P_3}{P_2}\right) = 0 \implies P_3 = P_2 \exp\left(\frac{C_{P0}}{R} \ln\left(\frac{T_3}{T_2}\right)\right)$$

To evaluate this expression, we can calculate our constant pressure specific heats. From Table A.5, $C_{P0, \text{constant}} = 1.004 \text{ [kJ/kg-K]}$ (case i)), and using the mean temperature:

$$T_{avg} = \frac{(292.68 + 146.34) \text{ [K]}}{2} = 219.51 \text{ [K]}$$

$$\implies C_{P0, \text{average}} = 1.05 - 0.365(.2195) + 0.85(.2195)^2 - 0.39(.2195)^3 = 1.0067 \text{ [kJ/kg-K]}$$

Using the integral-average specific heat (case iii)):

$$C_{P0, \text{int}} = \frac{1}{\Delta\theta} \int_{\theta_1}^{\theta_2} f(\theta) d\theta = \frac{1}{0.14634} \left(1.05(\theta) - \frac{0.365}{2}(\theta)^2 + \frac{0.85}{3}(\theta)^3 - \frac{0.39}{4}(\theta)^4 \right) \bigg|_{0.14634}^{0.29268} = 1.0077 \text{ [kJ/kg-K]}$$

Thus, the pressure at State 3 can take three values, depending on how we evaluated C_{P0} :

$$P_3 = \begin{cases} (386.09 \text{ [kPa]}) \exp\left(\frac{(1.004 \text{ [kJ/kg-K]})}{(0.287 \text{ [kJ/kg-K]})} \ln\left(\frac{(146.34 \text{ [K]})}{(292.68 \text{ [K]})}\right)\right) = 34.17 \text{ [kPa]}, \text{ for i)} \\ (386.09 \text{ [kPa]}) \exp\left(\frac{(1.0067 \text{ [kJ/kg-K]})}{(0.287 \text{ [kJ/kg-K]})} \ln\left(\frac{(146.34 \text{ [K]})}{(292.68 \text{ [K]})}\right)\right) = 33.95 \text{ [kPa]}, \text{ for ii)} \\ (386.09 \text{ [kPa]}) \exp\left(\frac{(1.0077 \text{ [kJ/kg-K]})}{(0.287 \text{ [kJ/kg-K]})} \ln\left(\frac{(146.34 \text{ [K]})}{(292.68 \text{ [K]})}\right)\right) = 33.86 \text{ [kPa]}, \text{ for iii)} \end{cases}$$

The volume at State 3 is found via the Ideal Gas law:

$$v_3 = \frac{RT_3}{P_3} = \begin{cases} \frac{(0.287 \text{ [kJ/kg-K]})(146.34 \text{ [K]})}{34.17 \text{ [kPa]}} = 1.229 \text{ [m}^3\text{]}, \text{ for i)} \\ \frac{(0.287 \text{ [kJ/kg-K]})(146.34 \text{ [K]})}{33.95 \text{ [kPa]}} = 1.237 \text{ [m}^3\text{]}, \text{ for ii)} \\ \frac{(0.287 \text{ [kJ/kg-K]})(146.34 \text{ [K]})}{33.86 \text{ [kPa]}} = 1.240 \text{ [m}^3\text{]}, \text{ for iii)} \end{cases}$$

Since the process for State 2 to 3 is adiabatic, $Q_{2\rightarrow3}=0$. Thus, the work is found as:

$$W_{2\rightarrow3} = -(U_3 - U_2) = -mC_{v0}(T_3 - T_2)$$

Recall $C_{v0} = C_{P0} - R$, we have the following:

$$W_{2\rightarrow3} = \begin{cases} (1 \text{ [kg]})(1.004 - 0.287) \text{ [kJ/kg-K]}(146.34 - 292.68) \text{ [K]} = 104.93 \text{ [kJ]}, \text{ for i)} \\ (1 \text{ [kg]})(1.0067 - 0.287) \text{ [kJ/kg-K]}(146.34 - 292.68) \text{ [K]} = 105.32 \text{ [kJ]}, \text{ for ii)} \\ (1 \text{ [kg]})(1.0077 - 0.287) \text{ [kJ/kg-K]}(146.34 - 292.68) \text{ [K]} = 105.47 \text{ [kJ]}, \text{ for iii)} \end{cases}$$

The heat rejected from States 3 to 4 is found via the efficiency equation:

$$\eta = 1 - \frac{Q_{3\rightarrow4}}{Q_{1\rightarrow2}} = 1 - \frac{Q_{3\rightarrow4}}{50 \text{ [kJ]}} = 0.5 \implies Q_{3\rightarrow4} = 25 \text{ [kJ]}$$

Since the process from State 3 to 4 is isothermal, the change of internal energy. Thus, from the Conservation of Energy, and taking into account directionality, $W_{3\rightarrow4} = -Q_{3\rightarrow4} = 25 \text{ [kJ]}$. Lastly, the heat supplied/rejected from State 4 to 1 is zero (adiabatic process). The work is found by calculating the change of internal energy:

$$W_{4\rightarrow1} = -(U_4 - U_1) = -mC_{v0}(T_1 - T_4)$$

Recall $C_{v0} = C_{P0} - R$, we have the following:

$$W_{2\rightarrow3} = \begin{cases} (1 \text{ [kg]})(1.004 - 0.287) \text{ [kJ/kg-K]}(292.68 - 146.34) \text{ [K]} = -104.93 \text{ [kJ]}, \text{ for i)} \\ (1 \text{ [kg]})(1.0067 - 0.287) \text{ [kJ/kg-K]}(292.68 - 146.34) \text{ [K]} = -105.32 \text{ [kJ]}, \text{ for ii)} \\ (1 \text{ [kg]})(1.0077 - 0.287) \text{ [kJ/kg-K]}(292.68 - 146.34) \text{ [K]} = -105.47 \text{ [kJ]}, \text{ for iii)} \end{cases}$$

Finally, our reduced temperature criteria is not met ($T_r = 1.105$), thus this will all for naught.

Problem #4

Water vapor, which exists at 100 [kPa] and 400 [K], undergoes a process to a final state where the pressure is 500 [kPa] and the temperature is 900 [K]. Using Matlab, determine the change in specific entropy via the following:

- Using the superheated vapor tables;
- using the specific heat listed in Table A.5;
- using the specific heat, evaluated using an average temperature, as listed in Table A.6;
- using the specific heat, evaluated via the integral average, as listed in Table A.6.

```
1 clear all
2 close all
3 clc
4
5 % State 1:
6 P_1 = 100; %[kPa]
7 T_1 = 400; %[K]
8
9
10 % State 2:
11 P_2 = 500; %[kPa]
12 T_2 = 900; %[K]
```

```

13
14 % ----- Part a) ----- %
15 s_1 = 7.501; %[kJ/kg-K]
16 s_2 = 8.420; %[kJ/kg-K]
17 ds = s_2 - s_1;
18 fprintf('The change in specific entropy using the superheated steam tables is:
        ds=%f [kJ/kg-K]\n',ds)
19
20 % ----- Part b) ----- %
21 C_P0 = 1.872; %[kJ/kg-K]
22 R = 0.4615; %[kJ/kg-K]
23 ds = C_P0*log(T_2/T_1) - R*log(P_2/P_1);
24 fprintf('The change in specific entropy using Table A.5 is: ds=%f [kJ/kg-K]\n'
        ,ds)
25
26 % ----- Part c) ----- %
27 C_0 = 1.79;
28 C_1 = 0.107;
29 C_2 = 0.586;
30 C_3 = -0.20;
31 T_avg = (T_1 + T_2)/2;
32 theta = T_avg/1000;
33 C_P0 = C_0 + C_1*theta + C_2*theta^2 + C_3*theta^3;
34 ds = C_P0*log(T_2/T_1) - R*log(P_2/P_1);
35 fprintf('The change in specific entropy using Table A.6 for an average C_P0 is
        : ds=%f [kJ/kg-K]\n',ds)
36
37 % ----- Part d) ----- %
38 CP0_int = @(x) 1000*( C_0.*x + (C_1/2).*x.^2 + (C_2/3).*x.^3 + (C_3/4).*x.^4
        );
39 C_P0 = (CP0_int(T_2/1000) - CP0_int(T_1/1000))/(T_2 - T_1);
40 ds = C_P0*log(T_2/T_1) - R*log(P_2/P_1);
41 fprintf('The change in specific entropy using Table A.6 for an integral-
        average C_P0 is: ds=%f [kJ/kg-K]\n',ds)

```

Problem #5

Using Matlab, calculate the change of specific entropy for a working fluid that enters a condenser as a saturated liquid and exits as a saturated vapor, at a temperature of -6.8 °C. Additionally, calculate the quantity of heat removed. The working fluids to be considered are:

- a) Ammonia;
- b) R-410a;
- c) R-12;
- d) R-134a using Table A.5;
- e) R-134a using Table A.6;
- f) R-134a using Table B.5.1.

EES was used to populate all pertinent values for parts a) through c).

```

1 clear all
2 close all
3 clc
4
5 % ----- Part a) ----- %
6 s_1 = 0.884; %[kJ/kg-K]

```

```

7  u_1 = 168.1; %[kJ/kg]
8
9  s_2 = 5.712; %[kJ/kg-K]
10 u_2 = 1332.1; %[kJ/kg]
11
12 q = u_2 - u_1;
13 ds = s_2 - s_1;
14
15 fprintf('The change in specific entropy for ammonia is: ds=%4f [kJ/kg-K]\n',
    ds)
16 fprintf('The heat removed from the ammonia is: q=%2f [kJ/kg]\n\n',q)
17
18 % ----- Part b) ----- %
19 s_1 = 0.963; %[kJ/kg-K]
20 u_1 = 189.3; %[kJ/kg]
21
22 s_2 = 1.824; %[kJ/kg-K]
23 u_2 = 393.1; %[kJ/kg]
24
25 q = u_2 - u_1;
26 ds = s_2 - s_1;
27
28 fprintf('The change in specific entropy for R-410a is: ds=%4f [kJ/kg-K]\n',ds
    )
29 fprintf('The heat removed from the R-410a is: q=%2f [kJ/kg]\n\n',q)
30
31 % ----- Part c) ----- %
32 s_1 = 0.119; %[kJ/kg-K]
33 u_1 = 29.6; %[kJ/kg]
34
35 s_2 = 0.700; %[kJ/kg-K]
36 u_2 = 167.7; %[kJ/kg]
37
38 q = u_2 - u_1;
39 ds = s_2 - s_1;
40
41 fprintf('The change in specific entropy for R-12 is: ds=%4f [kJ/kg-K]\n',ds)
42 fprintf('The heat removed from the R-12 is: q=%2f [kJ/kg]\n\n',q)

```

For parts d) and e), we get zero values, for since we are changing phase, we have no formulation for the change. Furthermore, the heat rejected during a constant pressure process needs a change of temperature. Thus, we have to rely on table values for phase-change processes.

Problem #6

You are quenching a 2,000 [kg] 304 stainless steel (SS) bar, which is initially at 1,120 °C. You are placing the SS bar in a tank of liquid, which has a volume of 8 [m³] and an initial temperature of 300 [K]. Using Matlab, and Table A.3, determine the net change of entropy of the SS bar and liquid, considering the following liquids:

- a) Water;
- b) light oil;
- c) glycerine.

```

1  clear all
2  close all

```

```

3  clc
4
5  m_steel = 2000;
6  C_steel = 0.46;
7  T_i_steel = 1393;
8  T_liq = 300;
9
10 %———— Part a. —————%
11
12 rho_h2o = 997;
13 C_h2o = 4.18;
14 m_h2o = rho_h2o*8;
15
16 T_f = (m_h2o*C_h2o*T_liq + m_steel*C_steel*T_i_steel)/(m_h2o*C_h2o + m_steel*
    C_steel);
17
18 del_s_steel = m_steel*C_steel*log(T_f/T_i_steel);
19 des_s_h2o = m_h2o*C_h2o*log(T_f/T_liq);
20
21 dS_h2o = des_s_h2o - del_s_steel;
22
23 fprintf('Net chance of entropy using water: dS = %f [kJ/k]\n',dS_h2o);
24
25 %———— Part b. —————%
26
27 rho_oil = 910;
28 C_oil = 1.8;
29 m_oil = rho_oil*8;
30
31 T_f = (m_oil*C_oil*T_liq + m_steel*C_steel*T_i_steel)/(m_oil*C_oil + m_steel*
    C_steel);
32
33 del_s_steel = m_steel*C_steel*log(T_f/T_i_steel);
34 des_s_oil = m_oil*C_oil*log(T_f/T_liq);
35
36 dS_oil = des_s_oil - del_s_steel;
37
38 fprintf('Net chance of entropy using oil: dS = %f [kJ/K]\n',dS_oil);
39
40 %———— Part c. —————%
41
42 rho_gly = 1260;
43 C_gly = 2.42;
44 m_gly = rho_gly*8;
45
46 T_f = (m_gly*C_gly*T_liq + m_steel*C_steel*T_i_steel)/(m_gly*C_gly + m_steel*
    C_steel);
47
48 del_s_steel = m_steel*C_steel*log(T_f/T_i_steel);
49 des_s_gly = m_gly*C_gly*log(T_f/T_liq);
50
51 dS_gly = des_s_gly - del_s_steel;
52
53 fprintf('Net chance of entropy using glycerine: dS = %f [kJ/K]\n',dS_gly);

```