

Chapter 5 - Second Law of Thermodynamics

Lecture 16 Sections 5.7-5.8

MEMS 0051 Introduction to Thermodynamics

Mechanical Engineering and Materials Science Department
University of Pittsburgh



Student Learning Objectives

Chapter 5 - Second
Law of
Thermodynamics

MEMS 0051

Learning Objectives

5.7 - The
Thermodynamic
Temperature Scale

5.8 - The Ideal Gas
Temperature Scale

Summary

At the end of the lecture, students should be able to:

- ▶ Define the absolute/thermodynamic temperature scale in general and for Ideal Gases



Absolute Temperature Scale

- ▶ In relation to the **Zeroth Law of Thermodynamics**, we can define an **absolute temperature scale** that is independent of any property of matter
- ▶ The **thermodynamic temperature** (a.k.a. absolute temperature scale) is the absolute measure of temperature (in SI units), as defined by the 2nd and **3rd Law of Thermodynamics**
- ▶ The 3rd Law states the entropy of a perfect crystal is zero at absolute zero (i.e. at absolute zero, a material is at its minimum energy)



Temperature and Carnot

- ▶ When we speak of the temperature of a system, we imply it is in thermal equilibrium
- ▶ When we analyze the Carnot cycle, the H.E. provides thermal communication between the high and low temperature reservoirs
- ▶ The H.E. directs \dot{Q}_H from T_H into the H.E., rejecting \dot{Q}_L to T_L



- ▶ The second proposition of the Carnot cycle states two reversible H.E. operating between the same temperature reservoirs T_H and T_L have the same efficiency

$$\eta_{\text{Carnot}} = 1 - \frac{Q_L}{Q_H}$$

- ▶ That is, the efficiency is therefore a function of only temperatures

$$\implies \eta_{\text{Carnot}} = 1 - \psi(T_L, T_H)$$

- ▶ ψ is a functional relation operator



Reversible Heat Engines

Learning Objectives

5.7 - The
Thermodynamic
Temperature Scale

5.8 - The Ideal Gas
Temperature Scale

Summary

- ▶ Imagine we have a reversible H.E. operating between T_1 and T_3 (hot and cold)
- ▶ Now we interject an intermediate temperature reservoir T_2 , where $T_1 > T_2 > T_3$, and two reversible H.E., one between T_1 and T_2 and the other between T_2 and T_3
- ▶ The efficiency of the H.E. between T_1 and T_3 would be the same as one consisting of two cycles between T_1 and T_2 and T_2 and T_3



- ▶ Thus, the efficiency of the first heat engine between T_1 and T_3 is

$$\eta_{\text{H.E.1}} = 1 - \frac{Q_3}{Q_1} \implies \frac{Q_3}{Q_1} = \psi(T_1, T_3)$$

- ▶ The efficiency of the second heat engine uses $|Q_2|$, the magnitude of heat transferred

$$\begin{aligned}\eta_{\text{H.E.2}} &= \frac{(Q_1 - |Q_2|) + (|Q_2| - Q_3)}{Q_1} \\ &= \left(1 - \frac{|Q_2|}{Q_1}\right) + \left(1 - \frac{Q_3}{|Q_2|}\right) \left(\frac{|Q_2|}{Q_1}\right) \\ &\implies \psi(T_2, T_3) \psi(T_1, T_2)\end{aligned}$$



- ▶ Since the efficiency of the H.E. are equal

$$\psi(T_1, T_3) = \psi(T_2, T_3)\psi(T_1, T_2)$$

- ▶ We notice the LHS is not a function of T_2 although the RHS is
- ▶ Therefore, the product of the two terms in the RHS must eliminate T_2 , and this is only achieved by using any monotonic function $g(T)=T$

$$\psi(T_2, T_3) = \frac{g(T_3)}{g(T_2)}; \quad \psi(T_1, T_2) = \frac{g(T_2)}{g(T_1)}$$

$$\psi(T_1, T_3) = \frac{g(T_3)}{g(T_1)}$$



- Therefore, expressing ψ in terms of $g(T)$, and choosing a reference T , we have an established temperature scale

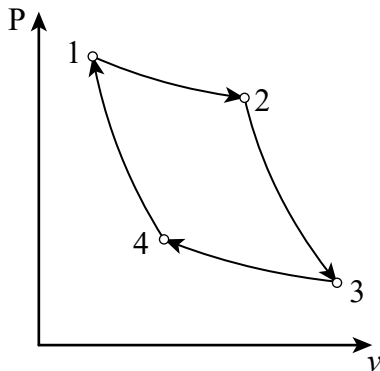
$$\frac{g(T_3)}{g(T_1)} = \frac{T_3}{T_1} = \frac{Q_3}{Q_1}$$

- Thus, the efficiency of the Carnot cycle can be expressed in terms of temperature

$$\eta_{\text{Carnot}} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$



- Consider the Carnot cycle as shown on the P - v diagram



- Recall the 4 individual processes (isothermal heat addition/rejection, reversible adiabatic compression/expansion)



- ▶ The reversible work is given in terms of an Ideal Gas is

$$\delta w = P d\nu = \frac{RT}{\nu} d\nu$$

- ▶ The change of internal energy can be expressed as

$$du = C_{VO}dT$$

- ▶ Thus, the heat added to the system is

$$\delta q = du + \delta w = C_{VO}dT + \frac{RT}{\nu}d\nu$$

- ▶ Let's apply these equations to each of the four processes



- ▶ Considering isothermal heat addition (1 to 2, $dT=0$)

$$\delta q = C_{VO}dT + \frac{RT}{\nu}d\nu \implies q_{1 \rightarrow 2} = RT_H \ln\left(\frac{\nu_2}{\nu_1}\right)$$

- ▶ Considering adiabatic expansion (2 to 3, $q=0$)

$$0 = C_{VO}dT + \frac{RT}{\nu}d\nu \implies 0 = \int_{T_H}^{T_L} \frac{C_{VO}}{T}dT + R \ln\left(\frac{\nu_3}{\nu_2}\right)$$

$$\implies \int_{T_L}^{T_H} \frac{C_{VO}}{T}dT = R \ln\left(\frac{\nu_3}{\nu_2}\right)$$



- ▶ Considering isothermal heat rejection (3 to 4, $dT=0$)

$$q_{3 \rightarrow 4} = -RT_L \ln\left(\frac{\nu_4}{\nu_3}\right) = RT_L \ln\left(\frac{\nu_3}{\nu_4}\right)$$

- ▶ Considering adiabatic compression (4 to 1, $q=0$)

$$0 = C_{\forall O} dT + \frac{RT}{\nu} d\nu \implies 0 = \int_{T_L}^{T_H} \frac{C_{\forall O}}{T} dT + R \ln\left(\frac{\nu_1}{\nu_4}\right)$$

$$\implies \int_{T_L}^{T_H} \frac{C_{\forall O}}{T} dT = -R \ln\left(\frac{\nu_1}{\nu_4}\right)$$



- ▶ We notice for the two adiabatic processes, we have the same integral (bounds and integrand)

$$\Rightarrow \int_{T_L}^{T_H} \frac{C_{VO}}{T} dT = -R \ln\left(\frac{\nu_1}{\nu_4}\right) = R \ln\left(\frac{\nu_3}{\nu_2}\right)$$

- ▶ Dividing by R and exponentiating, we obtain a useful relation of specific volumes

$$\frac{\nu_1}{\nu_4} = \frac{\nu_2}{\nu_3} \Rightarrow \frac{\nu_3}{\nu_4} = \frac{\nu_2}{\nu_1}$$



- ▶ Now, recalling the definition of the thermodynamic temperature scale, in particular, the ratio of heat rejected to input

$$\frac{Q_L}{Q_H} = \frac{q_{3 \rightarrow 4}}{q_{1 \rightarrow 2}} = \frac{R T_L \ln\left(\frac{\nu_3}{\nu_4}\right)}{R T_H \ln\left(\frac{\nu_2}{\nu_1}\right)} = \frac{T_L}{T_H}$$

- ▶ Thus, we have the same thermodynamic temperature scale for Ideal Gases.



Student Learning Objectives

At the end of the lecture, students should be able to:

- ▶ Define the absolute/thermodynamic temperature scale in general and for Ideal Gases
 - ▶ The absolute temperature scale, i.e. the thermodynamic temperatures scale, is that of Kelvin. Using this scale, we can replace the quantity Q_L/Q_H with T_L/T_H in our formulation for efficiency.



Suggested Problems

Chapter 5 - Second
Law of
Thermodynamics

MEMS 0051

► 5.44, 5.45, 5.46, 5.52, 5.60, 5.65, 5.97, 5.98, 5.99

Learning Objectives

5.7 - The
Thermodynamic
Temperature Scale

5.8 - The Ideal Gas
Temperature Scale

Summary

