Homework #4

MEMS 0051 - Introduction to Thermodynamics

June 19th, 2018

Problem #1

A piston-cylinder device contains air at a pressure of 600 [kPa] and a temperature of 290 [K]. The initial volume is 0.1 [m^3]. A constant-pressure process produces 55 [kJ] of work done by the system.

- Determine the final temperature of the air.
- Determine the heat input.

Solution:

State 1:	$\rightarrow$P=c, W=55 [kJ]$\rightarrow$	State 2:
P_1=600 [kPa]		P_2=P_1
T_1=290 K		T_2=?
$\forall_1$=0.1 [m^3]		

To determine the final temperature, we need the final volume and mass of the system. The mass is found through the Ideal Gas law applied to State 1:

$$m = \frac{P_1 \forall_1}{RT_1} = \frac{(600\,[\text{kPa}])(0.1\,[\text{m}^3])}{(0.287\,[\text{kJ/kg-K}])(290\,[\text{K}])} = 0.721\,[\text{kg}]$$

The final volume is found through the definition of work for a constant-pressure process:

$$W_{1\rightarrow 2} = P(\forall_1 - \forall_2) \implies \forall_2 = \frac{W_{1\rightarrow 2}}{P} + \forall_1 = \frac{55\,[\text{kJ}]}{600\,[\text{kPa}]} + 0.1\,[\text{m}^3] = 0.192\,[\text{m}^3]$$

Therefore, the final temperature is found by applying the Ideal Gas law at State 2:

$$T_2 = \frac{P_2 \forall_2}{mR} = \frac{(600\,[\text{kPa}])(0.192\,[\text{m}^3])}{(0.721\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])} = 556.7\,[\text{K}]$$

To evaluate the heat input, we apply the Conservation of Energy:

$$Q_{1\rightarrow 2} = (U_2 - U_1) + W_{1\rightarrow 2} = m(u_2 - u_1) + W_{1\rightarrow 2} = mC_{\forall}(T_2 - T_1) + W_{1\rightarrow 2} = mC_P(T_2 - T_1) = m(h_2 - h_1)$$

Using constant-volume specific heat to solve for the change of internal energy, since we are given work:

$$Q_{1\rightarrow 2} = (0.721\,[\text{kg}])(0.717\,[\text{kJ/kg-K}])((556.7 - 290)[\text{K}]) + 55\,[\text{kJ}] - 192.9\,[\text{kJ}]$$

Using Table A.7.1, the most accurate method:

$$u_1 = 207.19\,[\text{kJ/kg}]$$
$$u_2 = 402.25\,[\text{kJ/kg}]$$

Therefore,

$$Q_{1\rightarrow 2} = (0.721\,[\text{kg}])((402.25 - 207.19)[\text{kJ/kg}]) + 55\,[\text{kJ/kg}] = 195.6\,[\text{kJ}]$$

Note: you could use enthalpy or C_P and ignore $W_{1\rightarrow 2}$ and arrive at the same solution.

Problem #2

Water contained in a piston-cylinder assembly has an initial temperature of 150 °C, a quality of 50% and an initial volume of [0.05 m^3]. The pressure of the process is given as P($\forall$)=100 + C$\forall^{0.5}$ kPa. Heat is transferred to the piston-cylinder until the final pressure reaches 600 kPa.

- Determine the heat input.
- Plot this process on your P-ν and T-ν diagrams in Matlab.

Solution:

State 1:	$\rightarrow$	State 2:
T_1=150 °C		P_2=600 [kPa]
P_1=$P_{\text{sat}}(T_1)$=475.9 [kPa]		
x_1=0.50		
$\forall_1$=0.05 [m^3]		

To determine $Q_{1\rightarrow 2}$, we need m_1, u_1, u_2, ν_2 and $W_{1\rightarrow 2}$. Determining the mass at State 1 based upon volume and specific volume, with the latter determined via quality:

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{0.05\,[\text{m}^3]}{(0.00109 + 0.5(0.39278 - 0.00109)\,[\text{m}^3/\text{kg}]} = 0.254\,[\text{kg}]$$

The internal energy at State 1 is determined via quality:

$$u_1 = u_f(150\,[^\circ\text{C}]) + x(u_g(150\,[^\circ\text{C}]) - u_f(150\,[^\circ\text{C}])) = (631.66 + 0.5(2.559.54 - 631.66))\,[\text{kJ/kg}] = 1,595.6\,[\text{kJ/kg}]$$

To get the specific volume at State 2, we can apply our pressure versus volume relation to solve for the constant C, then solve for the volume at State 2 based upon the given pressure:

$$P_1 = 100 + C\forall_1^{0.5} \implies 475.9\,[\text{kPa}] = 100\,[\text{kPa}] + C(0.05[\text{m}^3])^{0.5} \implies C = \frac{375.9\,[\text{kPa}]}{\sqrt{0.05\,[\text{m}^3]}} = 1,681.1\,[\text{kPa/m}^{1.5}]$$

Therefore, at State 2:

$$P_2 = 100\,[\text{kPa}] + (1,681.1\,[\text{kPa/m}^{1.5}])\forall_2^{0.5} \implies \forall_2 = 0.0855\,[\text{m}^3]$$

Thus, the specific volume at State 2:

$$\nu_2 = \frac{\forall_2}{m} = \frac{0.0885\,[\text{m}^3]}{0.254\,[\text{kg}]} = 0349\,[\text{m}^3/\text{kg}]$$

Based upon P_2 and ν_2, it is evident State 2 exists in the superheated vapor region. Interpolating between the saturation temperature of 600 [kPa] and the 200 °C entry:

$$\frac{(0.349 - 0.31567)\,[\text{m}^3/\text{kg}]}{(0.35202 - 0.31567)\,[\text{m}^3/\text{kg}]} = \frac{(u_2 - 2,567.4)\,[\text{kJ/kg}]}{(2,638.91 - 2,567.4)\,[\text{kJ/kg}]} \implies u_2 = 2,632.97\,[\text{kJ/kg}]$$

To determine $W_{1\rightarrow 2}$, we can integrate $P(\forall)$:

$$W_{1\rightarrow 2} = \int_{\nu_1}^{\nu_2} P(\forall)\,d\forall = \int_{\nu_1}^{\nu_2} (100 + 1,681.1\forall^{0.5})\,d\forall = 100\forall + \frac{2}{3}1,681.1\forall^{1.5}\bigg|_{0.05}^{0.0885} = 20.83\,[\text{kJ}]$$

Lastly, applying the Conservation of Energy to solve for $Q_{1\rightarrow 2}$:

$$Q_{1\rightarrow 2} = m(u_2 - u_1) + W_{1\rightarrow 2} = (0.254\,[\text{kg}])(2,632.97 - 1,595.6)\,[\text{kJ/kg}] + 20.83\,[\text{kJ}] = 284.32\,[\text{kJ}]$$

Problem #3

A piston-cylinder device has two stops; a lower set and an upper set, that constrain the cylinder. When the piston is on the lower stops, the volume is 0.4 Liters. When the piston reaches the upper stops, the volume is 0.6 Liters. The piston-cylinder initially contains water at 100 [kPa] and a quality of 20%. The water is heated until it is completely transformed to steam. Additionally, the mass of the piston requires a pressure of 300 [kPa] to raise it. When the piston hits the upper stops:

- Determine the final pressure in the cylinder.
- Determine the heat input.
- Determine the work for the overall process.
- Plot this process on your P-ν and T-ν diagram in Matlab.
- Hint: there are 4 states. Think about the processes involved.

Solution:

State 1:	$\rightarrow \forall$=c $\rightarrow$	State 2:	$\rightarrow$ P=c $\rightarrow$	State 3:	$\rightarrow \forall$=c $\rightarrow$	State 4:
$\forall_1$=4$\cdot$10^{-4} [m^3]		$\forall_2$=$\forall_1$		$\forall_3$=6$\cdot$10^{-4} [m^3]		$\forall_4$=$\forall_3$
P_1=100 [kPa]		P_2=300 [kPa]		P_3=P_2		x_4=1.0
x_1=0.2						

State 1 variable, as determined from the saturation pressure and quality:

$$\nu_1 = \nu_f(100\,[\text{kPa}]) + x_1(\nu_g(100\,[\text{kPa}]) - \nu_f(100\,[\text{kPa}])) = (0.001043 + 0.2(1.69296))\,[\text{m}^3/\text{kg}] = 0.3396\,[\text{m}^3/\text{kg}]$$

$$u_1 = u_f(100\,[\text{kPa}]) + x_1(u_g(100\,[\text{kPa}]) - u_f(100\,[\text{kPa}])) = (417.33 + 0.2(2,088.72))\,[\text{kJ/kg}] = 835.074\,[\text{kJ/kg}]$$

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{4\cdot 10^{-4}\,[\text{m}^3]}{0.3396\,[\text{m}^3/\text{kg}]} = 0.001178\,[\text{kg}]$$

From State 1 to 2, the volume is constant (ν_2=ν_1) until the pressure reaches 300 [kPa], in which the piston is able to move, i.e. the volume can increase. At 300 [kPa], we have to determine the specific volume to see if either a) we have reached 100% vapor or b) we have hit the upper stops.

$$\nu_3 = \frac{\forall_3}{m} = \frac{6\cdot 10^{-4}\,[\text{m}^3]}{0.001178\,[\text{kg}]} = 0.509338\,[\text{m}^3/\text{kg}] < \nu_g(300\,[\text{kPa}])$$

Therefore, scenario b) is what has happened, and there is quality associated with the water at State 3:

$$x_3 = \frac{\nu_3 - \nu_f(300\,[\text{kPa}])}{\nu_g(300\,[\text{kPa}]) - \nu_f(300\,[\text{kPa}])} = \frac{(0.509338 - 0.001073)\,[\text{m}^3/\text{kg}]}{(0.60582 - 0.001073)\,[\text{m}^3/\text{kg}]}$$

Thus, we proceed from State 3 to State 4 in an isochoric fashion (i.e. ν_4=ν_3). Additionally, the quality is x_4=1.0 Looking at the Saturated Water Pressure entry table, State 4 lies between 350 and 375 [kPa]. Thus, the pressure at State 4 is found via interpolation:

$$\frac{(P_4 - 350)\,[\text{kPa}]}{(375 - 350)\,[\text{kPa}]} = \frac{(0.509338 - 0.52425)\,[\text{m}^3/\text{kg}]}{(0.49137 - 0.52425)\,[\text{m}^3/\text{kg}]} \implies P_4 = 361.31\,[\text{kPa}]$$

The internal energy at State 4 is found via interpolation:

$$\frac{(0.509338 - 0.52425)\,[\text{m}^3/\text{kg}]}{(0.49137 - 0.52425)\,[\text{m}^3/\text{kg}]} = \frac{(u_4 - 2,548.92)\,[\text{kJ/kg}]}{(2,551.31 - 2,548.92)\,[\text{kJ/kg}]} \implies u_4 = 2,550.0\,[\text{kJ/kg}]$$

Evaluating the work for the process:

$$W_{1\rightarrow 4} = \cancelto{0}{\int_{\forall_1}^{\forall_2} P\,d\forall} + \int_{\forall_2}^{\forall_3} P\,d\forall + \cancelto{0}{\int_{\forall_3}^{\forall_4} P\,d\forall} = P_2(\forall_3 - \forall_2) = (300\,[\text{kPa}])(6-4)\cdot 10^{-4}\,[\text{m}^3] = 0.06\,[\text{kJ}]$$

The heat input is found through the Conservation of Energy:

$$Q_{1\to 4} = m\big((U_2 - U_1) + (U_3 - U_2) + (U_4 - U_3)\big) + W_{1\to 4} = m(u_4 - u_1) + W_{1\to 4}$$

$$= (0.001178\,[\text{kg}])(2,550 - 835.074)\,[\text{kJ/kg}] + 0.06\,[\text{kJ}] = 2.08\,[\text{kJ}]$$

Problem #4

A piston-cylinder device contains 0.1 [kg] of air at a pressure of 100 [kPa] and a temperature of 400 [K] that undergoes an expansion process. The volume of the piston-cylinder device expands from 1 [m^3] to 3 [m^3] at a constant pressure of 2,000 [kPa]. Then, as the piston-cylinder device expands from 3 [m^3] to 5 [m^3], the pressure linearly decreases from 2,000 to 1,000 [kPa].

- Determine the heat input.
- Determine the work for the overall process.

Solution:

State 0:	→	State 1:	→	State 2:	→	State 3:
m_0=1 [kg]		$\forall_1$=1 [m^3]		$\forall_1$=3 [m^3]		$\forall_1$=5 [m^3]
P_0=100 [kPa]		P_1=2,000 [kPa]		P_2=P_1		P_3=1,000 [kPa]
T_0=400 [K]						

Determining the volume at State 0 from the Ideal Gas law:

$$\forall_0 = \frac{mrT_0}{P_0} = \frac{(0.1\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])(400\,[\text{K}])}{100\,[\text{kPa}]} = 0.1148\,[\text{m}^3]$$

Moving to State 1, we can determine the temperature, which is used to populate the internal energy:

$$T_1 = \frac{P_1\forall_1}{mR} = \frac{(2,000\,[\text{kPa}])(1\,[\text{m}^3]}{(0.1\,[\text{kg}])(0.287\,[\text{kJ/kg-K}])} = 69,686\,[\text{K}]$$

At this point in time, we should recognize the temperature at State 1 is rather large. Although the constraints for validity of the Ideal Gas law requires T_r>2, however, T_1 is greater than reasonable limits, for this temperature is greater than what is required to create a plasma (≈55,000 [K]) out of the nitrogen molecules (main components of air). We therefore conclude that the use of the Ideal Gas law is not valid. This problem illustrates engineering judgment. Yes, you can use an equation to get a value, but does the value make sense?