

Homework #8

MEMS 0051 - Introduction to Thermodynamics

Assigned March 24th, 2019
Due: March 29th, 2019

Problem #1

In a reversible process, N₂ is compressed in a cylinder from 100 [kPa] and 20 [°C] to 500 [kPa]. During this compression process, the relation between pressure and volume is $PV^{1.3} = \text{constant}$. Calculate the work and heat transfer per kilogram.

$$\begin{array}{ccc} \text{State 1:} & \rightarrow & \text{State 2:} \\ T_1=20 \text{ }^\circ\text{C} & & P_2=500 \text{ [kPa]} \\ P_1=100 \text{ [kPa]} & & \end{array}$$

Since the pressure volume relationship is given as:

$$PV^{1.3} = \text{Constant}$$

the relationship between the first and second state can be given as:

$$P_1 V_1^{1.3} = P_2 V_2^{1.3}$$

Since the process is reversible, the change in entropy is 0. Therefore, the process is isentropic. Based on the ideal gas relationship for isentropic process:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^{1.3}$$

it is concluded that the specific heat ratio, k, is 1.3 and the polytropic index, n, for the process is 1.3. The relationship between temperature and pressure for isentropic processes is:

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}} \Rightarrow \\ \frac{T_2}{293.2 \text{ [K]}} &= \left(\frac{500 \text{ [kPa]}}{100 \text{ [kPa]}} \right)^{\frac{1.3-1}{1.3}} \Rightarrow T_2 = 425 \text{ [K]} \end{aligned}$$

The work from state 1 to state 2 is:

$$W_{1 \rightarrow 2} = \int_1^2 P dV = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n}$$

Dividing both sides of the equation by the mass sets the equation up to solve for the specific work. Using the temperature at state 2 just found, it can be found that the specific work is:

$$w_{1 \rightarrow 2} = \frac{R(T_2 - T_1)}{1 - n} = \frac{(0.2968 \left[\frac{\text{kJ}}{\text{kg-K}} \right])(425 - 293.2 \text{ [K]})}{1 - 1.3} = \boxed{-130.4 \text{ [kJ/kg]}}$$

The conservation of energy equation yields:

$$q_{1 \rightarrow 2} = \Delta u + w_{1 \rightarrow 2}$$

The change in internal energy is given as:

$$\Delta u = C_{vo}(T_2 - T_1)$$

The specific heat for the nitrogen is $0.745 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$. Solving for the change in internal energy yields:

$$\Delta u = C_{vo}(T_2 - T_1) = 0.745(425 - 293.2) = 98.191 \text{ [kJ/kg]}$$

Putting the values back into the conservation of energy equation yields:

$$q_{1 \rightarrow 2} = 98.191 \text{ [kJ/kg]} - 130.4 \text{ [kJ/kg]} = \boxed{-32.2 \text{ [kJ/kg]}}$$

These equations required the assumption that nitrogen acts as an ideal gas throughout the process; therefore, it is necessary to prove this using reduced pressure and temperature. Obtaining the values for the critical pressure and temperature from Table A.2 yields:

$$\frac{P_{max}}{P_c} = \frac{500 \text{ [kPa]}}{3390 \text{ [kPa]}} = 0.1475 \ll 1$$

$$\frac{T_{min}}{T_c} = \frac{293.2 \text{ [K]}}{126.2 \text{ [K]}} = 2.323 > 2$$

Since the conditions were met, it can be concluded that nitrogen acts as an ideal gas.

Problem #2

A hydrogen gas in a piston/cylinder assembly at 280 [K], 100 [kPa] with a volume of 0.1 [m³] is now compressed to a volume of 0.01 [m³] in a reversible adiabatic process. What is the new temperature and how much work is required.

$$\begin{array}{ccc} \text{State 1:} & \rightarrow & \text{State 2:} \\ T_1=280 \text{ [K]} & & T_2=? \text{ [K]} \\ P_1=100 \text{ [kPa]} & & P_2=? \text{ [kPa]} \\ V_1=0.1 \text{ [m}^3\text{]} & & V_2=0.01 \text{ [m}^3\text{]} \end{array}$$

The conservation of energy equation yields:

$$q_{1 \rightarrow 2} = \Delta u + w_{1 \rightarrow 2}$$

For a reversible process, the change in entropy is 0.

$$S_2 - S_1 = \oint \frac{Q}{T} + S_{gen} = 0$$

For an ideal gas undergoing an isentropic process:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1}$$

The specific heat ratio, k, for hydrogen is found in Table A.5. Its value is **1.409**. This is also the value for the polytropic index. Inputting the values for volume, the initial temperature and the specific heat ratio yields:

$$\frac{T_2}{280 \text{ [K]}} = \left(\frac{0.1 \text{ [m}^3\text{]}}{0.01 \text{ [m}^3\text{]}} \right)^{1.409-1} \Rightarrow$$

$$\boxed{T_2 = 718 \text{ [K]}}$$

For an ideal gas undergoing a reversible process, the specific work from state 1 to state 2 is

$$w_{1 \rightarrow 2} = \frac{P_1 V_1 - P_2 V_2}{1 - n} = \frac{R(T_2 - T_1)}{1 - n} = \frac{(0.2968 \left[\frac{\text{kJ}}{\text{kg-K}} \right])(718 - 280 \text{ [K]})}{1 - 1.409} = \boxed{-317.845 \text{ [kJ/kg]}}$$

Problem #3

Liquid R-134a refrigerant undergoes a process from 0 [°C] and 1,000 [kPa] to 20 [°C] and 5,000 [kPa]. Determine the change in specific entropy for this process using:

- tabulated values.
- Tds equations for in-compressible substances.

Part a:

$$\begin{array}{ccc} \text{State 1:} & \rightarrow & \text{State 2:} \\ T_1=0 \text{ [°C]} & & T_2=20 \text{ [°C]} \\ P_1=1 \text{ [MPa]} & & P_2=5 \text{ [MPa]} \end{array}$$

From Table B.5.1, it can be seen that the pressures at state 1 and 2 are greater than those of the saturation pressures. Therefore, the refrigerant is in the compressed liquid waves. Due to this, we can approximate the property values being those at the saturated liquid state. Those specific entropy values are:

$$s_1 = 1.000 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$
$$s_2 = 1.0963 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

The change in specific entropy is found to be:

$$\Delta s = s_2 - s_1 = 1.0963 - 1.000 \left[\frac{\text{kJ}}{\text{kg-K}} \right] = 0.0963 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

Part b:

For an in-compressible fluid, the change in specific entropy is given as:

$$ds \approx C \ln \left(\frac{T_2}{T_1} \right)$$

The specific heat for the refrigerant is found in Table A.4. Its value is $1.43 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$. Inputting the values for temperature and specific heat yields:

$$ds \approx \left(1.43 \left[\frac{\text{kJ}}{\text{kg-K}} \right] \right) \ln \left(\frac{293.15 \text{ [K]}}{273.15 \text{ [K]}} \right) = 0.1010 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

Problem #4

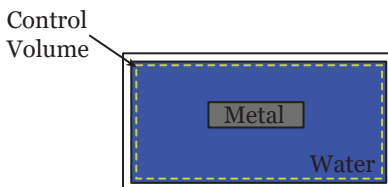
A 0.5 [kg] metal bar initially at 1200 [K] is removed from an oven and quenched by immersing it in a closed tank containing 0.9 [kg] of water initially at 300 [K]. Each substance can be assumed to be in-compressible. An appropriate constant specific heat for the water is $4.2 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$ and an appropriate constant specific heat for the metal is $0.42 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$. Assume the tank is well insulated (e.g no heat entering or leaving the tank). Determine the following:

- The final equilibrium temperature of the metal and the water
- Entropy generated

Part a:

First, a control volume is created around the tank containing the water and the metal bar. From here, the conservation of energy equation is:

$$\Delta U = Q - W$$



The tank is well insulated, therefore there is no heat entering or leaving. There is also no additional work being done on the system. The change in internal energy is that of the metal bar and the water. As a result, the conservation of energy equation turns into:

$$\Delta U_m + \Delta U_w = 0$$

Since the two objects, the metal and water, are in-compressible, their respective change in internal energy is:

$$\Delta U_m = m_m C_m (T_{final} - T_{initial,m})$$

$$\Delta U_w = m_w C_w (T_{final} - T_{initial,w})$$

Solving for T_{final} yields:

$$\begin{aligned} T_{final} &= \frac{m_w (C_w/C_m) T_{initial,w} + m_m T_{initial,m}}{m_w (C_w/C_m) + m_m} \Rightarrow \\ &= \frac{(0.9[\text{kg}])(4.2/0.42)(300 [\text{K}]) + (0.5[\text{kg}])(1200 [\text{K}])}{(0.9[\text{kg}])(4.2/0.42) + (0.5[\text{kg}])} \Rightarrow \\ T_{final} &= \boxed{347.368 [\text{K}]} \end{aligned}$$

Part b:

The entropy generated is given by the following equation:

$$\Delta S = \Delta S_m + \Delta S_w = \oint \frac{\mathcal{Q}}{T} + S_{gen} = 0$$

Since there is no heat entering or leaving the system, the cyclic integral term is canceled. Since the objects are in-compressible, their respective change in entropy is:

$$\begin{aligned} \Delta S_w &= m_w C_w \ln\left(\frac{T_{final}}{T_{initial,w}}\right) = (0.9 [\text{kg}])(4.2 \left[\frac{\text{kJ}}{\text{kg-K}}\right]) \ln\left(\frac{347.368 [\text{K}]}{300 [\text{K}]}\right) = 0.5542 \left[\frac{\text{kJ}}{\text{K}}\right] \\ \Delta S_m &= m_m C_m \ln\left(\frac{T_{final}}{T_{initial,m}}\right) = (0.5 [\text{kg}])(0.42 \left[\frac{\text{kJ}}{\text{kg-K}}\right]) \ln\left(\frac{347.368 [\text{K}]}{1200 [\text{K}]}\right) = -0.2603 \left[\frac{\text{kJ}}{\text{K}}\right] \\ \Delta S &= \Delta S_m + \Delta S_w = -0.2603 + 0.5542 \left[\frac{\text{kJ}}{\text{K}}\right] = 0.2939 \left[\frac{\text{kJ}}{\text{K}}\right] \end{aligned}$$

Therefore, the entropy generated is:

$$\boxed{0.2939 \left[\frac{\text{kJ}}{\text{K}}\right]}$$

Problem #5

One kilogram of O_2 modeled as an ideal gas undergoes a process from 300 [K], 2 bar to 1500 [K], 1.5 bar. Determine the change in specific entropy using:

1. Constant specific heats in Table A.5
2. Standard entropy in Table A.8
3. Specific heat as a function of temperature in Table A.6 evaluated at 900 [K]

The equation for change in specific entropy for an ideal gas is:

$$\Delta s = C_{po} \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

For O_2 , the gas constant is $0.2598 \left[\frac{\text{kJ}}{\text{kg-K}}\right]$.

Part 1:

Table A.5 shows the specific heat for oxygen is $0.922 \left[\frac{\text{kJ}}{\text{kg-K}}\right]$. Therefore, the change in specific entropy is:

$$\Delta s = 0.922 \left[\frac{\text{kJ}}{\text{kg-K}}\right] \ln\left(\frac{1500 [\text{K}]}{300 [\text{K}]}\right) - 0.2598 \left[\frac{\text{kJ}}{\text{kg-K}}\right] \ln\left(\frac{1.5 \text{ bar}}{2.0 \text{ bar}}\right) = \boxed{1.5586 \left[\frac{\text{kJ}}{\text{kg-K}}\right]}$$

Part 2:

To find the change in specific entropy using standard entropy, the following equation is used:

$$\Delta s = \left(s^\circ(T_2) - s^\circ(T_1) \right) - R \ln \left(\frac{P_2}{P_1} \right)$$

Table A.8 provides the standard entropy for for ideal gases.

$$s^\circ(T_1) = s^\circ(300 \text{ [K]}) = 6.4168 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

$$s^\circ(T_2) = s^\circ(1500 \text{ [K]}) = 8.0649 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

$$\Delta s = \left(8.0649 - 6.4168 \left[\frac{\text{kJ}}{\text{kg-K}} \right] \right) - 0.2598 \left[\frac{\text{kJ}}{\text{kg-K}} \right] \ln \left(\frac{1.5 \text{ bar}}{2.0 \text{ bar}} \right) = \boxed{1.7228 \left[\frac{\text{kJ}}{\text{kg-K}} \right]}$$

Part 3:

To find the change in specific entropy using specific heat as a function of temperature, we evaluate the specific heat using the equation in Table A.6. The coefficients for the specific heat for oxygen are composed together to develop the equation:

$$C_{po} = 0.88 - 0.0001\theta + 0.54\theta^2 - 0.33\theta^3$$

Here, θ is defined as:

$$\theta = \frac{\text{Temperature}}{1000}$$

where temperature is measured in Kelvin. Evaluating the values for specific heat at 900 [K] yields:

$$C_{po} = 1.0767 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$$

Now that the specific heat is found, the change in specific entropy is evaluated to be:

$$\Delta s = 1.0767 \left[\frac{\text{kJ}}{\text{kg-K}} \right] \ln \left(\frac{1500 \text{ [K]}}{300 \text{ [K]}} \right) - 0.2598 \left[\frac{\text{kJ}}{\text{kg-K}} \right] \ln \left(\frac{1.5 \text{ bar}}{2.0 \text{ bar}} \right) = \boxed{1.8077 \left[\frac{\text{kJ}}{\text{kg-K}} \right]}$$

Problem #6

Hot combustion air at 1500 [K] expands in a polytropic process to a volume six times as large with $n = 1.5$. Find the specific boundary work and the specific heat transfer.

$$\begin{array}{ccc} \text{State 1:} & \rightarrow & \text{State 2:} \\ T_1 = 1500 \text{ [K]} & & T_2 = 6T_1 \\ V_1 = V_1 & & V_2 = 6V_1 \end{array}$$

For a polytropic process, the specific work from state 1 to state 2 for an ideal gas is given as:

$$w_{1 \rightarrow 2} = \frac{P_1 V_1 - P_2 V_2}{1 - n} = \frac{R(T_2 - T_1)}{1 - n}$$

The gas constant for air is $0.287 \left[\frac{\text{kJ}}{\text{kg-K}} \right]$. The temperature at state 2 can be evaluated for an ideal gas using the following relationship:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{k-1}$$

Here, k is the specific heats ratio which is also the polytropic index, 1.5. Solving for T_2 yields:

$$T_2 = T_1 \left(\frac{V_1}{6V_1} \right)^{1.5-1} = 1500 \text{ [K]} \left(\frac{1}{6} \right)^{0.5} \Rightarrow$$

$$T_2 = 612.4 \text{ [K]}$$

Inserting the temperature, gas constant and polytropic index into the specific work equation yields:

$$w_{1 \rightarrow 2} = \frac{(0.287 \left[\frac{\text{kJ}}{\text{kg-K}} \right])(612.4 - 1500 \text{ [K]})}{1 - 1.5} = \boxed{509.5 \text{ [kJ/kg]}}$$

The conservation of energy equation yields:

$$q_{1 \rightarrow 2} = \Delta u + w_{1 \rightarrow 2}$$

The specific internal energy for air modeled as an ideal gas is tabulated in Table A7.1 for various temperatures.

$$u_1 = 1205.25 \text{ [kJ/kg]}$$

$$u_2 = 444.6 \text{ [kJ/kg]}$$

Solving for specific heat transfer from state 1 to 2 yields:

$$q_{1 \rightarrow 2} = (444.6 - 1205.25 \text{ [kJ/kg]}) + 509.5 \text{ [kJ/kg]} = \boxed{-251.168 \text{ [kJ/kg]}}$$