MEMS 0051
Spring 2019
Midterm #1
2/15/2017

Name (Print): ____________________________

This exam contains 6 pages (including this cover page) and 2 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.

Do not write in the table to the right.

Problem	Points	Score
1	50	
2	50	
Total:	100	

Bonus:

Written Problem #1

1. (50 points) A piston-cylinder device initially contains 0.4 [m^3] of helium at 100 [kPa] and a temperature of 80 °C. The air is then compressed to a volume of 0.1 [m^3] and a pressure of 80 [kPa]. Determine the heat removed from the system, as well as the work required to compress the piston. Ensure that helium is able to be treated as an Ideal gas during this process.
Solution: We can construct our table of states as follows:

State 1:	$\rightarrow T$=c $\rightarrow$	State 2:
T_1=353.15 K		
$\forall_1$=0.4 [m^3]		$\forall_2$=0.1 [m^3]
P_1=100 [kPa]		P_2=80 [kPa]

To evaluate the quantities of work and heat, we must apply the conservation of energy such that:

$$m(u_2 - u_1) = Q_{1\to 2} - W_{1\to 2}$$

To evaluate the change of internal energy, as well as work, we must determine the mass of the system. Using the Ideal gas law at State 1, and Table A.5, we find the mass as

$$P_1\forall_1 = mRT_1 \implies m = \frac{P_1\forall_1}{RT_1} = \frac{(100\,[\text{kPa}])(0.4\,[\text{m}^3])}{(2.077\,1\,[\text{kJ/kg-K}])(353.15\,[\text{K}])} = 0.054\,5\,[\text{kg}] \qquad (6.25\,\text{pts.})$$

Next, we must determine the temperature at State 2. Using the Ideal gas law, for a constant m and R:

$$\frac{P_1\forall_1}{T_1} = \frac{P_2\forall_2}{T_2} \implies T_2 = T_1\left(\frac{P_2}{P_1}\right)\left(\frac{\forall_2}{\forall_1}\right) = (353.15\,[\text{K}])\left(\frac{80\,[\text{kPa}]}{100\,[\text{kPa}]}\right)\left(\frac{0.1\,[\text{m}^3]}{0.4\,[\text{m}^3]}\right)$$

$$\implies T_2 = 70.63\,[\text{K}] \qquad (6.25\,\text{pts.})$$

We are able to determine the change of internal energy using via the constant volume specific heat:

$$du = C_\forall dT \implies u_2 - u_1 = (3.116\,[\text{kJ/kg-K}])(70.63 - 353.15)\,[\text{K}] = -880.33\,[\text{kJ/kg}]$$

Thus, the total change in internal energy is:

$$dU = mdu = (0.054\,5\,[\text{kg}])(-550.21\,[\text{kJ/kg}]) = -47.98\,[\text{kJ}] \qquad (6.25\,\text{pts.})$$

To determine the work, we have to determine the polytropic index n:

$$n = \frac{\ln\left(\dfrac{P_1}{P_2}\right)}{\ln\left(\dfrac{\forall_2}{\forall_1}\right)} = -0.161 \qquad (6.25\,\text{pts.})$$

Thus, the work from State 1 to 2 (negative indicates it is work done onto the system) is:

$$W_{1\to 2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = \frac{(80\,[\text{kPa}])(0.1\,[\text{m}^3]) - (100\,[\text{kPa}])(0.4\,[\text{m}^3])}{1-(-0.161)} - 27.56\,[\text{kJ}] \qquad (6.25\,\text{pts.})$$

Therefore, the heat from State 1 to 2 (negative indicates heat is removed from the system) is:

$$Q_{1\to2} = dU + W_{1\to2} = (-47.98\,[\text{kJ}]) + (-27.56\,[\text{kJ}]) = -75.54\,[\text{kJ}] \qquad (6.25\,\text{pts.})$$

To ensure we can use the Ideal gas law, we have to compute the reduced temperature and pressure, using the critical constants on Table A.2:

$$P_r = \frac{P_{max}}{P_c} = \frac{100\,[\text{kPa}]}{227\,[\text{kPa}]} = 0.44 < 1 \qquad (6.25\,\text{pts.})$$

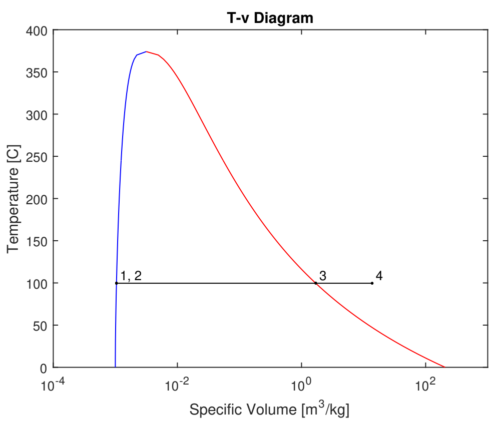
$$T_r = \frac{T_{min}}{T_c} = \frac{70.63\,[\text{K}]}{5.19\,[\text{K}]} = 13.61 \gg 2 \qquad (6.25\,\text{pts.})$$

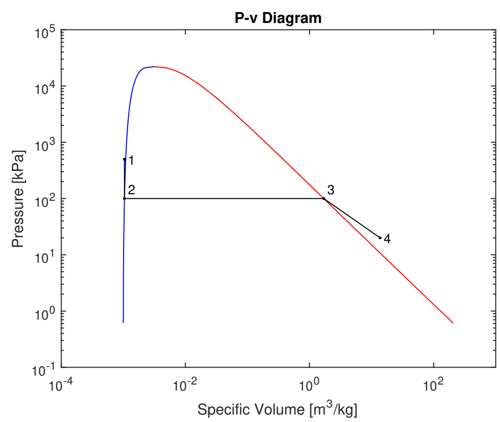
Written Problem #2

2. (50 points) 10 [kg] of water in a piston-cylinder device goes from an initial state of 99.62 °C and 500 [kPa] to a final state of 99.62 °C and 20 [kPa] in an isothermal process. Determine the amount of heat supplied to and the work done by the system. *Hint: this process is not purely isothermal.*

Solution:

Looking at the T-ν and P-ν diagrams (Figs. 1 and 2), we recognize we start in the subcooled liquid region and progress to the saturated liquid line of the vapor dome in a constant temperature processed. Once we reach the saturated liquid line, we progress to the saturated vapor line in a constant temperature and constant pressure process. Then, we progress to the superheated vapor region in a constant pressure process.





We can construct and populate our table of states as follows:

State 1:	$\rightarrow T$=c $\rightarrow$	State 2:	$\rightarrow T$=c $\rightarrow$
T_1=99.62 °C		T_2= 99.62 °C	$\rightarrow T$=c $\rightarrow$
P_1=500 [kPa]		P_2=P_{sat}(99.62 °C)=100 [kPa]	
$\nu_1 \approx \nu_f(T_1)$=0.001 043 [m^3/kg]	(5 pts.)	ν_2=$\nu_f(T_2,P_2)$=0.001 043 [m^3/kg]	
		u_2=$u_f(T_2, P_2)$=417.33 [kJ/kg]	

State 3:	$\rightarrow T$=c $\rightarrow$	State 4:
T_3= 99.62 °C		T_4= 99.62 °C
P_3=P_{sat}(99.62 °C=100 [kPa])		P_4=20 [kPa]
ν_3=$\nu_g(T_2,P_2)$=1.694 00 [m^3/kg]		
u_3=$u_g(T_3, P_3)$=2,506.06 [kJ/kg]		

To determine the amount of heat supplied, as well as the done by the system, we have to evaluate the conservation of energy equation. Thus, the only properties we need to solve for will be the specific volume at State 4 and specific internal energy at States 1 and 4. Solving for the specific internal energy at State 1:

	P=500 [kPa]	
T °C	u [kJ/kg]	
80	334.73	
99.62	417.19	(5 pts.)
100	418.80	

It is noted that you can can assume $u_1 \approx u_f(99.62\,°\text{C})$, for the specific internal energy is more strongly dependent on temperature than pressure.

To determine these quantities at State 4, we have to construct a 20 [kPa] pressure entry table, with said properties evaluated at 99.62 °C. Constructing our pressure entry tables:

	P=10 [kPa]	
T °C	ν [m^3/kg]	u [kJ/kg]
50	14.869 20	2,443.87
99.62	17.177	2,514.93
100	17.195 61	2,515.50

	P=50 [kPa]	
T °C	ν [m^3/kg]	u [kJ/kg]
81.33	3.240 34	2,483.85
99.62	3.414 5	2,511.02
100	3.418 33	2,511.61

Now, interpolating between pressure entries for the given temperature:

		T=99.62 °C		
P [kPa]	ν [m^3/kg]		u [kJ/kg]	
10	17.177 34		2,514.93	
20	13.736 4	(5 pts.)	2,513.95	(5 pts.)
50	3.414 5		2,511.02	

It is noted you can assume the properties at 99.62 °C are close to those at 100 °C and only have to interpolate once between pressures (10 and 50 [kPa]), i.e. $\nu_4 \approx 13.751\,29$ [m^3/kg] and $u_4 \approx 2{,}514.527\,5$ [kJ/kg].

We can now evaluate the conservation of energy equation. We will apply the equation between each state, then sum the total response. From State 1 to 2:

$$m(u_2 - u_1) = Q_{1\to2} + W_{1\to2}$$

The formulation for work for an isothermal process is:

$$W_{1\to2} = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (500\,[\text{kPa}])(10\,[\text{kg}])(0.001\,043\,[\text{m}^3/\text{kg}])\ln\left(\frac{0.001\,043\,[\text{m}^3/\text{kg}]}{0.001\,043\,[\text{m}^3/\text{kg}]}\right)$$
$$\implies W_{1\to2} = 0\,[\text{kJ}] \qquad (5\,\text{pts.})$$

We immediately see this term is zero, for $\nu_2 = \nu_1$. Thus,

$$Q_{1\to2} = m(u_2 - u_1) = (10\,[\text{kg}])(417.33 - 417.19)\,[\text{kJ/kg}] = 1.4\,[\text{kJ}] \qquad (5\,\text{pts.})$$

If you assume $u_1 \approx u_f(99.62\,°\text{C})$, then $Q_{1\to2}$ is identically equal to zero.

Evaluating the conservation of energy between States 2 and 3:

$$m(u_3 - u_2) = Q_{2\to3} - W_{2\to3}$$

The work from States 2 to 3 is treated as isobaric work. Recall the integral form of the definition of work. Thus:

$$W_{2\to3} = P\int_2^3 d\forall = (100\,[\text{kPa}])(10\,[\text{kg}])(1.694\,00 - 0.001\,043)\,[\text{m}^3/\text{kg}])$$

$$\implies W_{2\to3} = 1,692.957\,[\text{kJ}] \qquad (5\,\text{pts.})$$

The heat from States 2 to 3 is then:

$$Q_{2\to3} = m(u_3 - u_2) + W_{2\to3} = (10\,[\text{kg}])(2,506.06 - 417.33)\,[\text{kJ/kg}] + 1,692.957\,[\text{kJ}]$$

$$\implies Q_{2\to3} = 22,570.257\,[\text{kJ}] \qquad (5\,\text{pts.})$$

Lastly, applying the conservation of energy equation between States 3 and 4:

$$m(u_4 - u_3) = Q_{3\to4} - W_{3\to4}$$

The formulation for work for an isothermal process is:

$$W_{3\to4} = P_3\forall_3 \ln\left(\frac{\forall_4}{\forall_3}\right) = (100\,[\text{kPa}])(10\,[\text{kg}])(1.694\,00\,[\text{m}^3/\text{kg}])\ln\left(\frac{13.736\,4\,[\text{m}^3/\text{kg}]}{1.694\,00\,[\text{m}^3/\text{kg}]}\right)$$
$$\implies W_{3\to4} = 3,545.47\,[\text{kJ}] \qquad (5\,\text{pts.})$$

The heat from States 3 to 4 is then:

$$Q_{3\to4} = m(u_4 - u_3) + W_{3\to4} = (10\,[\text{kg}])(2,513.95 - 2,506.06)\,[\text{kJ/kg}] + 3,545.47\,[\text{kJ}]$$

$$Q_{3\rightarrow 4} = 3,642.37\,[\text{kJ}] \qquad (5\,\text{pts.})$$

Now, the heat supplied and work done between States 1 and 4 is the sum of the individual processed:

$$Q_{1\rightarrow 4} = 26,214.03\,[\text{kJ}]; \qquad W_{1\rightarrow 4} = 5,238.427\,[\text{kJ}]$$