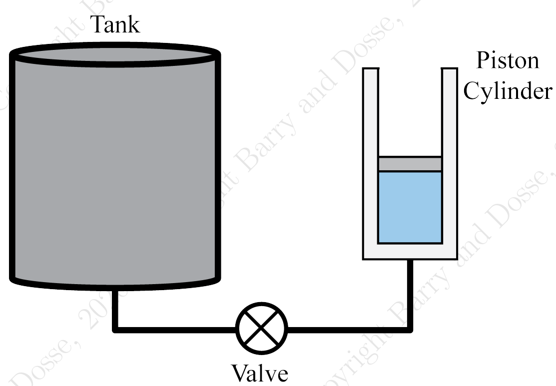
Quiz #2

MEMS 0051 - Introduction to Thermodynamics

Assigned: May 28th, 2020
Due: May 29th, 2020, 11:59 pm

Problem #1

A piston-cylinder device is connnected to a tank by a valve as shown in the figure below. The tank has a volume of 0.5 [m^3] and initially contains water at 1,000 [kPa] and 200 °C. The piston-cylinder is initially evacuated and the piston has an area of 0.2 [m^2] and requires 200 [kPa] to float above the bottom of the cylinder (i.e. any pressure greater than 200 [kPa] will cause the piston to rise upwards). The valve is now opened and water flows into the piston-cylinder until the piston rises to a height of 5.5 [m], where the valve is then closed. If the water in the tank is now saturated vapor/steam and is at a pressure of 500 [kPa], determine the following:



a) The final temperature of the water in the tank, $T_{2,T}$; Let's look at our states and what we know. We will denote the tank as T and the piston-cylinder as P:

State 1,Tank:
$P_{1,T}$ = 1,000 [kPa]
$T_{1,T}$ = 200 °C
$\nu_{1,T}$ = ?
$\forall_{1,T}$ = 0.5 [m^3]
$m_{1,T}$ = ?
x_1 = ?

State 2,Tank:
$P_{2,T}$ = 500 [kPa]
$T_{2,T}$ = ?
$\nu_{2.T}$ = ?
$\forall_{2,T}$ = 0.5 [m^3]
$m_{2,T}$ = ?
x_2 = 1

State 2,Piston:
$P_{2,P}$ = 200 [kPa]
$T_{2,P}$ = ?
$\nu_{2,P}$ = ?
$\forall_{2,P}$ = ?
$m_{2,P}$ = ?
x_3 = ?
A_P = 0.2 [m^2]
$h_{2,P}$ = 5.5 [m]

We know the water is a saturated vapor at state 2, and since we are given the pressure at state 2, we can find the final temperature since for every saturation pressure, there is only one allowable saturation temperature. So, looking at Table B.1.2, we see that:

$$\boxed{T_{2,T} = 151.86\ °\text{C}}$$

b) The final specific volume of the water in the piston-cylinder, $\nu_{2,P}$; We see from Table B.1.3 that the water in state 1 is a superheated vapor. We can find the specific volume:

$$\nu_1 = 0.205\,96\ [\text{m}^3/\text{kg}]$$

Now we can find the total mass in the system:

$$m_{1,T} = \frac{\forall_1}{\nu_1} = \frac{0.5\ [\text{m}^3]}{0.205\,96\ [\text{m}^3/\text{kg}]} = 2.428\ [\text{kg}]$$

The volume of the tank is unable to change, so $\forall_{2,T} = \forall_{1,T}$. From Table B.1.2, we see the specific volume for a pressure of 500 [kPa] and a quality of 1 is:

$$\nu_2 = 0.374\,89\ [\mathrm{m}^3/\mathrm{kg}]$$

We can calculate how much mass is left in the tank after the valve was opened and closed:

$$m_{2,T} = \frac{\forall_2}{\nu_2} = 1.334\ [\mathrm{kg}]$$

We can now move on to state 2 in the piston-cylinder. By the conservation of mass:

$$m_{2,P} = m_{1,T} - m_{2,T} = 2.428\ [\mathrm{kg}] - 1.334\ [\mathrm{kg}] = 1.094\ [\mathrm{kg}]$$

We can determine the specific volume of the piston-cylinder at state 2 by finding the volume of the water in the cylinder. So then:

$$\forall_{2,P} = A_P h_{2,P} = (0.2\ [\mathrm{m}^2])(5.5\ [\mathrm{m}]) = 1.1\ [\mathrm{m}^3]$$

The final specific volume of the water in the piston-cylinder then is:

$$\nu_{2,P} = \frac{\forall_{2,P}}{m_{2,P}} = \frac{1.1\ [\mathrm{m}^3]}{1.094\ [\mathrm{kg}]}$$

$$\boxed{\nu_{2,P} = 1.005\,48\ [\mathrm{m}^3/\mathrm{kg}]}$$

c) The final temperature of the water in the piston-cylinder, $\mathrm{T}_{2,\mathrm{P}}$; If the final pressure in the tank is 500 [kPa], then we know the final pressure in the piston-cylinder must be 200 [kPa]. If it were higher or lower, the piston would not be at rest and its height would have to increase or decrease to reach mechanical equilibrium. Since we have pressure and specific volume, we can use Table B.1.3 and interpolate between 150 °C and 200 °C under the 200 [kPa] entry to find the final temperature:

$$\frac{(\nu_{2,P} - \nu_{150\ ^\circ\mathrm{C}})\ [\mathrm{m}^3/\mathrm{kg}]}{(\nu_{200\ ^\circ\mathrm{C}} - \nu_{150\ ^\circ\mathrm{C}})\ [\mathrm{m}^3/\mathrm{kg}]} = \frac{(T_{2,P} - 150)\ ^\circ\mathrm{C}}{(200-150)\ ^\circ\mathrm{C}} \implies \frac{(1.005\,48 - 0.959\,64)\ [\mathrm{m}^3/\mathrm{kg}]}{(1.080\,34 - 0.959\,64)} = \frac{(T_{2,P} - 150)\ ^\circ\mathrm{C}}{(200-150)\ ^\circ\mathrm{C}}$$

$$\boxed{T_{2,P} = 168.99\ ^\circ\mathrm{C}}$$