Homework #4

MEMS 0051 - Introduction to Thermodynamics

Assigned February 2nd, 2018
Due: February 9th, 2018

Problem #1

a) You push a lawnmower with a total force of 50 [N] at a constant velocity of 0.1 [m/s] for a total of 10 seconds. The lawnmower has a mass of 20 [kg].

i.) How much power ($\dot{W}$) are you transferring to the lawnmower [W]?

$$\dot{W} = FV = (50\,[\text{N}])(0.1\,[\text{m/s}]) = 5\,[\text{W}]$$

ii.) How much work (W) do you transfer to the lawnmower during those 10 seconds [J]?

$$W = \dot{W}\Delta t = (5\,[\text{W}])(10\,[\text{s}]) = 50\,[\text{J}]$$

iii.) How much specific work (w) do you transfer to the lawnmower [J/kg]?

$$w = \frac{W}{m} = \frac{50\,[\text{J}]}{20\,[\text{kg}]} = 2.5\,[\text{J/kg}]$$

b) Now assume that heat is leaving the lawnmower at a rate of 2 [W] throughout the 10 seconds.

i.) How much total heat (Q) leaves the lawnmower during those 10 seconds [J]?

$$Q = \dot{Q}\Delta t = (2\,[\text{W}])(10\,[\text{s}]) = 20\,[\text{J}]$$

ii.) How much specific heat (q) leaves the lawnmower during those 10 seconds [kJ/kg]?

$$q = \frac{Q}{m} = \frac{20\,[\text{J}]}{20\,[\text{kg}]} = 1\,[\text{J/kg}]$$

c) Now consider the lawnmower as a thermodynamic system with a control surface drawn all around it. Make sure you use the 1st Law sign conventions for heat and work.

i.) Is work positive or negative for the lawnmower system? Explain.
Negative, because work is being done to the lawnmower, and work is positive when it is done by a system.

ii.) Is heat positive or negative for the lawnmower system? Explain.
Negative, because heat is being transferred out of the lawnmower system, and heat is positive when it goes into a system.

Problem #2

a) A piston cylinder is initially filled with air occupying 0.5 [m^3]at a pressure of 100 [kPa]. The piston then compresses the air to a new volume of 0.2 [m^3]. The compression process is governed by the polytropic equation: $P_1\forall_1^n$=$P_2\forall_2^n$. Determine how much work is done by the gas for the following polytropic indices:

i.) n=∞
Work is zero for an isochoric process because the volume doesn't change.

ii.) n=0

For an isobaric process, the pressure is constant

$$W_{1\to2} = P(\forall_2 - \forall_1) = (100\,[\text{kPa}])(0.2 - 0.5)\,[\text{m}^3] = -30\,[\text{kJ}]$$

iii.) n=1

$$W_{1\to2} - P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = (100\,[\text{kPa}])(0.5\,[\text{m}^3])\ln\left(\frac{0.2}{0.5}\right) = -45.8\,[\text{kJ}]$$

iv.) n=1.4

Using the general expression for work

$$W_{1\to2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n}$$

To solve for P_2

$$P_2 = P_1\left(\frac{\forall_1}{\forall_2}\right)^n = 100\,[\text{kPa}]\left(\frac{0.5\ [\text{m}^3]}{0.2\ [\text{m}^3]}\right)^{1.4} = 360.7\,[\text{kPa}]$$

Therefore

$$W_{1\to2} - \frac{(360.7\,[\text{kPa}])(0.2\,[\text{m}^3]) - (100\,[\text{kPa}])(0.5\,[\text{m}^3])}{1-1.4} = -55.3\,[\text{kJ}]$$

b) Now assume that we don't know the polytropic index, but we do know that the final pressure is 200 kPa. Solve for the polytropic index, n

Solving for n

$$n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\frac{200}{100}\right)}{\ln\left(\frac{0.5}{0.2}\right)} = 0.756$$

Problem #3

a) Determine the change in internal energy, (U_2-U_1), for each of the following ideal gas cases. (Hint: refer to Table A.5 to look up $C_\forall$ for each of these gases.)

i.) 1 [kg] of argon going from 300 K to 400 K

$$U_2 - U_1 = mC_\forall(T_2 - T_1) = (1\,[\text{kg}])(0.312\,[\text{kJ/kg-K}])(400 - 300)\,[\text{K}] = 31.2\,[\text{kJ}]$$

ii.) 2.5 [kg] of carbon dioxide going from 500 K to 300 K

$$U_2 - U_1 = mC_\forall(T_2 - T_1) = (2.5\,[\text{kg}])(0.653\,[\text{kJ/kg-K}])(300 - 500)\,[\text{K}] = -326.5\,[\text{kJ}]$$

iii.) 2 [kg] of nitrogen going from 400 K to 300 K

$$U_2 - U_1 = mC_\forall(T_2 - T_1) = (2\,[\text{kg}])(0.745\,[\text{kJ/kg-K}])(300 - 400\,[\text{K}]) = -149\,[\text{kJ}]$$

Problem #4

a) A piston-cylinder device contains air at 100 [kPa], 25 °C and 0.2 [m^3]. The air then undergoes an isobaric, polytropic expansion to a volume of 0.4 [m^3]. (R=0.287 [kJ/kg-K] for air).

i.) What is the mass of air contained in the piston-cylinder?
Using the Ideal Gas Law:

$$m = \frac{P\forall}{RT} = \frac{(100\,[\text{kPa}])(0.2\,[\text{m}^3])}{(0.287\,[\text{kJ/kg-K}])(298\,[\text{K}])} = 0.234\,[\text{kg}]$$

ii.) What is the final temperature after expansion?
m, R and P are constants, therefore the Ideal Gas Law reduces to

$$\frac{\forall_1}{T_1} = \frac{\forall_2}{T_2} \implies T_2 = T_1\left(\frac{\forall_2}{\forall_1}\right) = 298\,[\text{K}]\left(\frac{0.4}{0.2}\right) = 596\,[\text{K}]$$

iii.) How much work is done by the gas during this expansion?

$$W_{1\to2} = P(\forall_2 - \forall_1) = (100\,[\text{kPa}])(0.4 - 0.2)\,[\text{m}^3] = 20\,[\text{kJ}]$$

iv.) What is the change in internal energy, (U_2-U_1), during this expansion?

$$U_2 - U_1 = mC_\forall(T_2 - T_1) = (0.234\,[\text{kg}])(0.717\,[\text{kJ/kg-K}])(596 - 298)\,[\text{K}] = 50\,[\text{kJ}]$$

v.) How much heat is transferred into the gas during this expansion?

$$Q_{1\to2} = (U_2 - U_1) + W_{1\to2} = 50\,[\text{kJ}] + 20\,[\text{kJ}] = 70\,[\text{kJ}]$$

Problem #5

a) Water contained in a piston-cylinder assembly has an initial temperature of 150 °C, a quality of 50% and an initial volume of 0.05 [m^3]. The pressure of the process is given as $P(\forall)=100 + C\forall^{0.5}$ [kPa], where C is a constant. Heat is transferred to the piston-cylinder until the final pressure reaches 600 kPa.
   i.) Determine the heat input. (Hint, you need to determine C.)

State 1:	→	State 2:
T_1=150 °C		P_2=600 [kPa]
x_1=0.5		
$\forall_1$=0.05 [m^3]		

To determine $Q_{1\to2}$, we need m_1, u_1, u_2 and $W_{1\to2}$. At State 1, since there is quality, we known $P_1=P_{sat}(T_1)$=475.9 [kPa], which allows us to pull ν_f and ν_g from the steam Tables.

$$\nu_1 = \nu_f + x(\nu_g - \nu_f) \implies \nu_1 = 0.001090\ [\text{m}^3/\text{kg}] + (0.5)(0.39278 - 0.001090)\ [\text{m}^3/\text{kg}] = 0.196935\,[\text{m}^3/\text{kg}]$$

Thus, we are able to determine the mass at State 1:

$$m_1 = \frac{\forall_1}{\nu_1} = \frac{0.05\,[\text{m}^3]}{0.196935\,[\text{m}^3/\text{kg}]} = 0.254\,[\text{kg}]$$

Additionally, since we have quality at State 1, we can determine specific internal energy:

$$u_1 = u_f + x(u_g - u_f) \implies u_1 = 631.66\ [\text{kJ/kg}] + (0.5)(2,559.54 - 631.66)\ [\text{kJ/kg}] = 1,595.6\,[\text{kJ/kg}]$$

We only have the pressure at State 2 - one more independent property must be known before the specific internal energy at State 2 can be determined. We do have a relationship between pressure and volume. Applying this relationship to State 1 to determine the constant C, and then using our pressure at State 2 to determine the volume at State 2, and knowing mass is constant, we can determine the specific volume at State 2. Evaluating $P(\forall)$ at State 1:

$$P(\forall) = 100 + C(\forall)^{0.5} \implies C = \frac{475.9 - 100}{0.05^{1/2}} = 1,681.1$$

Therefore, as State 2:

$$\forall_2 = \left(\frac{P - 100}{C}\right)^2 \implies \forall_2 = 0.0885\,[\mathrm{m}^3]$$

The specific volume at State 2 is found as:

$$\nu_2 = \frac{\forall_2}{m} = \frac{0.0885\,[\mathrm{m}^3]}{0.254\,[\mathrm{kg}]} = 0.34843\,[\mathrm{m}^3/\mathrm{kg}]$$

Knowing P_2 and ν_2, it is evident the water now exists as a superheated vapor, and the internal energy is found from interpolation:

ν [m^3/kg]	u [kJ/kg]
0.31567	2,567.40
0.34843	u_2
0.35202	2,638.91

Therefore, u_2=2,631.85 [kJ/kg]. Lastly, the work from State 1 to 2 can be found through 1.) the integration of our pressure as a function of volume equation or 2.) the evaluation of the polytropic index. Solving via integration:

$$W_{1\to2} = \int_{\forall_1}^{\forall_2} \mathrm{P}(\forall)\,\mathrm{d}\forall = 100\forall + \frac{2 \cdot 1,681.1}{3}^{3/2}\Bigg|_{0.05}^{0.0885} = 20.83\,[\mathrm{kJ}]$$

The polytropic index is found as:

$$n = \frac{\left(\dfrac{P_2}{P_1}\right)}{\left(\dfrac{\forall_1}{\forall_2}\right)} = \frac{\left(\dfrac{600}{475.9}\right)}{\left(\dfrac{.05}{.0885}\right)} = -0.405832$$

Therefore, the work is:

$$W_{1\to2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = \frac{(600\mathrm{kPa})(0.0885\ [\mathrm{m}^3]) - (475.9\mathrm{kPa})(0.05\ [\mathrm{m}^3])}{1 - -0.405832} = 20.85\,[\mathrm{kJ}]$$

Lastly, the heat input is:

$$Q_{1\to2} = m(u_2 - u_1) + W_{1\to2} = (0.254\,[\mathrm{kg}])(2,631.85 - 1.595.6)\ [\mathrm{kJ/kg}] + 20.85[\mathrm{kJ}] = 284.06\,[\mathrm{kJ}]$$