

## Midterm #2

### MEMS 0051 - Introduction to Thermodynamics Summer 2020

Assigned: July 9<sup>th</sup>, 2020

Due: July 16<sup>th</sup>, 2020, 3:55 pm via Gradescope (for written) and Canvs for MATLAB code

#### Rules

The following listed rules, in addition, but not limited to, those listed on the syllabus, those listed on page 3, and those outlined by Pitt's Academic Integrity Policy, apply to this examination:

1. This test is open notes, open book, open lecture videos, open homework and homework solutions, open quiz and quiz solutions, and you are able to reference previous assessment materials;
2. You can direct general questions to either the MEMS 0051 Slack, or email Dr. Barry or Mr. Dosse. A general question constitutes a point of clarification with a question, for example "Why is there entropy?" Specific questions about how to solve a problem, pertinent equations, related to general guidance, etc., are *not* permitted;
3. You are *not* to communicate with *any* other student about this exam. Period;
4. An answer without units will be marked incorrect;
5. If you use EES and/or MATLAB, submit your script(s) to Canvas, titled as your username, e.g. mmb49, and the appropriate file type, eg. .txt or .m. Include your name and the problem number in the comments in the script correspond.
6. You are *not* to use any online resources, such as Chegg, Quora, etc., or any form of thermodynamic property calculator. Seeking external assistance in the form of posting this exam, posting questions from this exam, asking questions pertaining to the problems within the exam, etc., is in direction violation of the Academic Integrity policy.
7. Unsubstantiated results will be marked incorrect;
8. You must complete the Academic Integrity Statement and include it with your exam submission for your exam to be graded.

## Problem #1

(30 pts.) Consider an insulated tank that initially contains oxygen in two different sections (A and B) separated by an insulated wall. Section A contains 1 [kg] of oxygen at a pressure and temperature of 3,000 [kPa] and 75 [K]. Section B contains 1.5 [kg] of oxygen at a pressure and temperature of 3,000 [kPa] and 125 [K]. The insulation on the wall is now removed and the oxygen in the two sections exchange heat until the system reaches equilibrium. Use EES to determine the properties of the oxygen as needed. Determine the following:

- the value of the specific heat evaluated at the final temperature;
- plot entropy generation vs. temperature for this process using MATLAB. Your plot should have a title and the  $x$  and  $y$ -axes labeled with units. The  $x$ -axis should span from 75 [K] to 125 [K]. In addition, your entropy-versus-temperature plot should use at least 500 evenly-spaced data points spanning the temperature range.
- the final temperature of the tank using the principle of maximum entropy;
- the entropy generated during the process.

## Problem #2

(30 pts.) A piston-cylinder device contains air initially at a pressure and temperature of 100 [kPa] and 300 [K]. First, the air undergoes an isochoric process until it reaches a temperature of 450 [K]. Next, the air undergoes an isothermal process. The air then undergoes an isentropic process until it reaches a pressure of 400 [kPa]. Finally, the air undergoes a polytropic process with  $n = 1.2$ , returning the air to the initial state, thus completing a cycle. It is also known that the change in entropy from States 1 to 2 (the isochoric process) plus the change in entropy from States 2 to 3 (the isothermal process) is:

$$(S_2 - S_1) + (S_3 - S_2) = -1 \text{ [kJ/K]}$$

You may use values from the Tables if desired, but all work should be done in EES, i.e. your deliverable should be an EES script saved as a text file. The air can be treated as an Ideal gas, but the specific heats,  $C_P$  and  $C_V$ , cannot be treated as constants. If the cycle is internally reversible, determine or answer the following:

- the pressure, temperature, and volume (*not* specific volume) of the air at each state;
- the net work and heat transfer, *not* on a per mass basis, for the cycle;
- Based on your results, did the cycle generate entropy? Explain your answer.

### Problem #3

(40 pts.) A piston-cylinder device contains 10 [kg] of air, initially at 105 [kPa] and a temperature of 26.85 °C. Then, the device undergoes a quasi-equilibrium process where the pressure increases to 1,250 [kPa]. If 2,000 [kJ] of work was required to compress the substance, determine:

- a) the temperature of the final state;

<u>State 1:</u>	<u>State 2:</u>
$m_1=10$ [kg]	$m_2=m_1$
$P_1=105$ [kPa]	$P_2=1,250$ [kPa]
$T_1=300$ [K]	

For air undergoing a reversible process, the work into the system is:

$$W_{1 \rightarrow 2} = \frac{mR(T_2 - T_1)}{1 - n}$$

Or,

$$T_2 = \frac{(1 - n)W_{1 \rightarrow 2}}{mR} + T_1$$

Additionally, we can use the following relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Or,

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$

Setting the expressions for  $T_2$  equal:

$$T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \frac{(1 - n)W_{1 \rightarrow 2}}{mR} + T_1$$

Substituting in values:

$$(300 \text{ [K]}) \left(\frac{1,250 \text{ [kPa]}}{105 \text{ [kPa]}}\right)^{\frac{n-1}{n}} = \frac{(1 - n)(-2,000 \text{ [kJ]})}{(10 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})} + 300 \text{ [K]}$$

Solving for  $n$ , we get three values. To determine which value is correct, let us ensure all polytropic expressions are satisfied. We will need the volumes at States 1 and 2. Using the Ideal Gas Law:

$$v_1 = \frac{mRT_1}{P_1} = \frac{(10 \text{ [kg]})(0.287 \text{ [kJ/kg-K]})(300 \text{ [K]})}{105 \text{ [kPa]}} = 8.2 \text{ [m}^3\text{]}$$

$n$	$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$ [K]	$v_2 = \frac{mRT_2}{P_2}$ [m <sup>3</sup> ]	$W_{1 \rightarrow 2} = \frac{mR(T_2 - T_1)}{1 - n}$ [kJ]	$W_{1 \rightarrow 2} = P_1 v_1 \ln\left(\frac{v_2}{v_1}\right)$ [kJ]
0.813	169.888	0.3901	-2,000	-
1	300	0.6888	-	-2,132.64
2.378	1,260.442	2.8940	-2,000	-

We see that an isothermal process does not yield the same value for work into our system, thus  $n = 1$  is not the correct polytropic index. We have to decide between 0.813 and 2.378. Ensure the relations between pressure, temperature and volume are satisfied:

$n$	$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$	$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{n}{n-1}}$
0.813	11.905 = 11.905	11.905 = 11.905
2.378	11.905 = 11.905	11.905 = 11.905

At this point, you may say both 0.813 and 2.378 are valid values for the polytropic index. For us to treat this system as an Ideal Gas, we must ensure if the criteria for the reduced pressure and reduced temperature are met. The critical pressure for air is 3,785.8 [kPa], and thus  $P_r=0.3302$  for both values of  $n$ . The critical temperature for air is 132.63 [K]. When  $n = 0.813$ ,  $T_r = 1.2809$ , which is clearly not greater than 2. When  $n = 2.378$ ,  $T_r = 9.504$ , which satisfies our requirement. Thus,  $n = 2.378$  is the correct value, and conclude:

$$T_2 = (300 \text{ [K]}) \left( \frac{1,250 \text{ [kPa]}}{105 \text{ [kPa]}} \right)^{\frac{2.378-1}{2.378}} = 1,260.442 \text{ [K]}$$

- b) the quantity of heat removed from the piston-cylinder during this process using the **integral-averaged temperature-dependent constant-volume specific heat** for the change of internal energy;

This was not a typographical error, nor did I want you to integrate  $C_{V0}$  from Table A.5. Recalling  $R = C_{P0}(T) - C_{V0}$ , we can express  $C_{V0}$  in terms of variables that are a function of temperature, which means  $C_{V0}$  itself is a function of temperature:

$$C_{V0}(T) = C_{P0}(T) - R$$

Using Table A.6, we can express  $C_{P0}$  as a function of temperature, which is denoted by the variable  $T$ , recalling  $\theta = T/1000$ :

$$C_{P0}(T) = C_0 + C_1 \frac{T}{1,000} + C_2 \frac{T^2}{(1,000)^2} + C_3 \frac{T^3}{(1,000)^3}$$

Therefore

$$C_{V0}(T) = C_0 + C_1 \frac{T}{1,000} + C_2 \frac{T^2}{(1,000)^2} + C_3 \frac{T^3}{(1,000)^3} - R$$

Evaluating the strict definition of integral-average of  $C_{V0}$  (yes, I know the  $\Delta T$  terms cancel out when evaluating the change of internal energy):

$$\tilde{C}_{V0} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} C_{V0} dT = \left( \frac{1}{T_2 - T_1} \right) \left( C_0 T + C_1 \frac{T^2}{2,000} + C_2 \frac{T^3}{3(1,000)^2} + C_3 \frac{T^4}{4(1,000)^3} - RT \right) \Bigg|_{T_1}^{T_2}$$

Evaluating this between  $T_2$  and  $T_1$ :

$$\begin{aligned} \tilde{C}_{V0} = & \left( 1.05(1,260.442 \text{ [K]}) - (0.365) \frac{(1,260.442 \text{ [K]})^2}{2,000} + (0.85) \frac{(1,260.442 \text{ [K]})^3}{3(1,000)^2} - (0.39) \frac{(1,260.442 \text{ [K]})^4}{4(1,000)^3} - \dots \right. \\ & \dots - (0.287)(1,260.442 \text{ [K]}) - \left( 1.05(300 \text{ [K]}) - (0.365) \frac{(300 \text{ [K]})^2}{2,000} + (0.85) \frac{(300 \text{ [K]})^3}{3(1,000)^2} - (0.39) \frac{(300 \text{ [K]})^4}{4(1,000)^3} - \dots \right. \\ & \left. \left. \dots - (0.287)(300 \text{ [K]}) \right) \right) \left( \frac{1}{960.442 \text{ [K]}} \right) = 0.8056 \text{ [kJ/kg-K]} \end{aligned}$$

To determine the quantity of heat removed, we will apply the Conservation of Energy equation for a closed system:

$$dU = Q_{1 \rightarrow 2} - W_{1 \rightarrow 2} \implies Q_{1 \rightarrow 2} = dU + W_{1 \rightarrow 2}$$

Recalling the expression for the change of internal energy based upon specific heat:

$$Q_{1 \rightarrow 2} = m \tilde{C}_{V0} \Delta T + W_{1 \rightarrow 2} = (10 \text{ [kg]}) (0.8056 \text{ [kJ/kg-K]}) (960.442 \text{ [K]}) + (-2,000 \text{ [kJ]}) = 5,737.2 \text{ [kJ]}$$

The positive sign means heat was supplied to the system, due to the long-standing convention we have adopted in the formulation of the Conservation of Energy equation. Thus, the quantity of heat rejected is the negative of heat supplied:

$$Q_{1 \rightarrow 2, \text{rejected}} = -5,737.2 \text{ [kJ]}$$

- c) the quantity of heat removed from the piston-cylinder during this process using temperature-dependent specific internal energy;

Using Table A.7.1, we can determine the specific internal energy as a function of temperature:

$$u_1 = 214.36 \text{ [kJ/kg-K]}$$

$$\frac{(1,260.442 - 1,250) \text{ [K]}}{(1,300 - 1,250) \text{ [K]}} = \frac{(u_2 - 977.89) \text{ [kJ/kg]}}{(1,022.75 - 977.89) \text{ [kJ/kg]}} \Rightarrow u_2 = 987.2586 \text{ [kJ/kg]}$$

Applying the Conservation of Energy equation for a close system:

$$Q_{1 \rightarrow 2} = m(u_2 - u_1 + W_{1 \rightarrow 2}) = (10 \text{ [kg]})(987.2586 - 214.36) \text{ [kJ/kg]} + (-2,000 \text{ [kJ]}) = 5,728.986 \text{ [kJ]}$$

Once again, we have to take the negative of this, for the Conservation of Energy equation is telling us we are supplying heat during this process. Therefore,

$$Q_{1 \rightarrow 2, \text{rejected}} = -5,728.986 \text{ [kJ]}$$

Note the percent difference of values obtained by part b) and c) is 0.143%. We would expect these values to be near identical, which a percent difference of 0.07% confirms.

- d) the change of entropy using Gibb's equation, evaluating the **integral-average of the specific heat per temperature term using the temperature-dependent specific heat** given on Table A.6;

The change of entropy, using Gibb's equation, is expressed as

$$ds = \int_{T_1}^{T_2} \frac{C_{P0}}{T} dT - R \ln \left( \frac{P_2}{P_1} \right)$$

We have already constructed an expression for  $C_{P0}(T)$  in part b). Now, we have to divide it by  $T$  and integrate with respect to  $T$ :

$$\frac{C_{P0}(T)}{T} = \frac{C_0}{T} + \frac{C_1}{1,000} + C_2 \frac{T}{(1,000)^2} + C_3 \frac{T^2}{(1,000)^3}$$

Integrating:

$$\begin{aligned} \int_{T_1}^{T_2} \frac{C_{P0}(T)}{T} dT &= \left( C_0 \ln(T) + C_1 \frac{T}{1,000} + C_2 \frac{T^2}{2(1,000)^2} + C_3 \frac{T^3}{3(1,000)^3} \right) \Big|_{T_1}^{T_2} = \dots \\ &= \left( 1.05 \ln(1,260.442 \text{ [K]}) - (0.365) \frac{(1,260.442 \text{ [K]})}{1,000} + (0.85) \frac{(1,260.442 \text{ [K]})^2}{2(1,000)^2} - (0.39) \frac{(1,260.442)^3}{3(1,000)^3} \right) - \dots \\ &= \left( 1.05 \ln(300 \text{ [K]}) - (0.365) \frac{(300 \text{ [K]})}{1,000} + (0.85) \frac{(300 \text{ [K]})^2}{2(1,000)^2} - (0.39) \frac{(300)^3}{3(1,000)^3} \right) = 1.5368 \text{ [kJ/kg-K]} \end{aligned}$$

Therefore:

$$dS = (10 \text{ [kg]}) \left( (1.5368 \text{ [kJ/kg-K]}) - (0.287 \text{ [kJ/kg-K]}) \ln \left( \frac{1,250 \text{ [kPa]}}{105 \text{ [kPa]}} \right) \right) = 8.259 \text{ [kJ/K]}$$

- e) the change of entropy using Gibb's equation, evaluating the integral of the specific heat per temperature term using standard entropy.

The change of entropy using standard entropy is expressed as:

$$ds = (s_T^0(T_2) - s_T^0(T_1)) - R \ln \left( \frac{P_2}{P_1} \right)$$

Using Table A.7.1, we have the following standard entropy values:

$$s_T^0(T_1) = 6.86926$$

$$\frac{(1,260.442 - 1,250) \text{ [K]}}{(1,300 - 1,250) \text{ [K]}} = \frac{(s_T^0(T_2) - 8.39402) \text{ [kJ/kg-K]}}{(8.44046 - 8.39402) \text{ [kJ/kg-K]}} \Rightarrow s_T^0(T_2) = 8.4037 \text{ [kJ/kg-K]}$$

Therefore, the change of entropy is:

$$dS = (10 \text{ [kg]}) \left( (8.4037 - 6.86926) \text{ [kJ/kg-K]} - (0.287 \text{ [kJ/kg-K]}) \ln \left( \frac{1,250 \text{ [kPa]}}{105 \text{ [kPa]}} \right) \right) = 8.236 \text{ [kJ/K]}$$

The percent difference between solutions for part d) and e) is 0.282%, a value within acceptable limits.

**Academic Integrity Statement:**

I hereby attest that I have received no assistance (from a friend, from another student, from an on-line resource, such as Chegg, etc.), and that I have provided no assistance to another student, during this exam. All the work presented within is solely my own work.

Signature: \_\_\_\_\_

Date: \_\_\_\_\_