MEMS 0051
Spring 2018
Midterm #1
2/16/2018

Name (Print): ____________________________

This exam contains 4 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

Problem	Points	Score
1	10	
2	30	
3	30	
4	30	
Total:	100	

BONUS (5 pts):

This date, February 16$^{\text{th}}$, 1923, marks the opening of this pharaoh's tomb by archaeologist Howard Carter.

Written Problem #1

1. (10 points) Determine the polytropic index for a process where air, initially in a piston cylinder with a pressure of 1,000 [kPa] and a volume of $8{\cdot}10^{-5}$ [m^3], is expanded to a pressure of 350 [kPa] and a volume of $8{\cdot}10^{-4}$ [m^3].

Solution:

$$n = \frac{\ln\left(\dfrac{P_2}{P_1}\right)}{\ln\left(\dfrac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\dfrac{350\,[\text{kPa}]}{1,000\,[\text{kPa}]}\right)}{\ln\left(\dfrac{8\cdot 10^{-5}\,[\text{m}^3]}{8\cdot 10^{-4}\,[\text{m}^3]}\right)} \implies \boxed{n = 0.456} \qquad 10\,(\text{pts})$$

Written Problem #2

2. (30 points) Water vapor at 3,000 [kPa] and 300°C is contained within a piston-cylinder. The water is cooled in a constant volume process until the temperature reaches 200°C. The water is then compressed in a constant temperature process until the pressure is 2,500 [kPa]. **Determine the following**:

   1. The specific volumes at States 1, 2 and 3.
   2. The quality at State 2.

Solution:

1. At State 1, $T{>}T_{\text{sat}}$ at 3,000 [kPa], which means this is a superheated vapor. Using Table B.1.3 on page 787,

$$\boxed{\nu_1 = 0.08114\,[\text{m}^3/\text{kg}]} \qquad 7\,(\text{pts})$$

At State 2, the specific volume is the same as State 1 because it is cooled in a *constant volume* process:

$$\boxed{\nu_2 = 0.08114\,[\text{m}^3/\text{kg}]} \qquad 7\,(\text{pts})$$

At State 3, $P{>}P_{\text{sat}}$ at 200°C, thus is exists as a compressed/subcooled liquid. Using either Table B.1.4 and interpolating, or recognizing $\nu{=}\nu(T)$ and $\nu \neq \nu(P)$, we can use Table B.1.1 on page 778,

$$\boxed{\nu_3 = 0.001156\,[\text{m}^3/\text{kg}]} \qquad 7\,(\text{pts})$$

2. To determine the quality at State 2, we recognize $\nu_f{<}\nu_2{<}\nu_g$. Therefore,

$$x_2 = \frac{\nu - \nu_f}{\nu_g - \nu_f} - \frac{0.08114 - 0.001156\,[\text{m}^3/\text{kg}]}{0.12736 - 0.001156\,[\text{m}^3/\text{kg}]} \implies \boxed{x_2 = 0.634} \qquad 9\,(\text{pts})$$

Written Problem #3

3. (30 points) One kilogram of air undergoes a thermodynamic cycle consisting of three processes:

   1. From the initial state, it proceeds to State 2 via a constant volume process
   2. From State 2, it proceeds to State 3 via a constant temperature process
   3. From State 3, it proceeds back to the initial state via a constant pressure process

   At State 1, the temperature 300 [K] and the pressure is 100 [kPa]. At State 2, the pressure is 200 [kPa]. The critical pressure of air is 3.786 [MPa] and the critical temperature of air is 132.63 [K]. Using the Ideal Gas law, **determine the following**:

   1. The temperature at State 2.
   2. The specific volume at State 3.
   3. If this process satisfies the requirements to treat air as an Ideal Gas.

   Solution:
   To determine the temperature at State 2, use the Ideal Gas law, noting the volume, mass and gas constant are all constant,

$$\frac{P_1\forall}{mRT_1} = \frac{P_2\forall}{mRT_2} \implies T_2 = \left(\frac{P_2}{P_1}\right)T_1 = \left(\frac{200\,[\text{kPa}]}{100\,[\text{kPa}]}\right)(300\,[\text{K}]) \implies \boxed{T_2 = 600\,[\text{K}]} \qquad 10\,(\text{pts})$$

   The specific volume at State 3 is found by using the Ideal Gas law. The temperature at State 3 is the same as State 2, and the pressure at State 3 is the same as State 1. The gas constant is found from Table A.5 on page 760,

$$\nu_3 = \frac{RT_3}{P_3} = \frac{RT_2}{P_1} = \frac{(0.287\,[\text{kJ/kg-K}])(600\,[\text{K}])}{100\,[\text{kPa}]} \implies \boxed{\nu_3 = 1.722\,[\text{m}^3/\text{kg}]} \qquad 10\,(\text{pts})$$

   To verify the reduced temperature, we take our minimum temperature,

$$T_r = \frac{300\,[\text{K}]}{132.63\,[\text{K}]} \implies \boxed{T_r = 2.26 > 2} \qquad 5\,(\text{pts})$$

   To verify the reduced pressure, we take the maximum pressure,

$$P_r = \frac{P}{P_c} = \frac{200\,[\text{kPa}]}{3,786\,[\text{kPa}]} \implies \boxed{P_r = 0.053 \ll 1} \qquad 5\,(\text{pts})$$

Written Problem #4

4. (30 points) Water is contained within a piston-cylinder device and undergoes a series of processes from an initial state with a pressure of 1,000 [kPa] and a temperature of 400°C, to a final state. The processes are as follows:

   1. The water vapor is cooled in a constant pressure process to saturated vapor.
   2. The water is then cooled in a constant volume process to $150°C$.

   Determine the following:

   1. Determine the work into the system, in [kJ/kg].

2. Determine the heat out of the system, in [kJ/kg].

Solution:
The work into the system is the sum of the work of the two individual processes:

$$W_{1\to 3} = W_{1\to 2} + W_{2\to 3}$$

We know State 2 to State 3 is constant volume, such that $W_{2\to 3}$=0. Therefore, all that is needed is the specific volume at States 1 and 2. At State 1, T>T_{sat} at 1,000 [kPa], which means it is a superheated vapor. Therefore, the specific volume is found from Table B.1.3 from page 785,

$$\nu_1 = 0.30659\,[\text{m}^3/\text{kg}] \qquad 5\,(\text{pts})$$

At State 2, the water exists as a saturated vapor at 1,000 [kPa]. Thus, using Table. B.1.2 on page 782,

$$\nu_2 = 0.19444\,[\text{m}^3/\text{kg}] \qquad 5\,(\text{pts})$$

Thus, the total work per unit mass is the work from State 1 to State 2, which is a constant pressure process:

$$\frac{W_{1\to 3}}{m} = 1,000[\text{kPa}](0.001127 - 0.30659\,[\text{m}^3/\text{kg}]) \implies \boxed{\frac{W_{1\to 3}}{m} = -112.15\,[\text{kJ/kg}]} \qquad 5\,(\text{pts})$$

The heat per unit mass into the system is the difference of internal energies between State 3 and 1 plus the work into the system:

$$\frac{Q_{1\to 3}}{m} = \{(u_2 - u_1) + (u_3 - u_2)\} + \frac{W_{1\to 3}}{m}$$

The specific internal energy at State 1 is found from Table B.1.3 on page 785,

$$u_1 = 2,957.29\,[\text{kJ/kg}] \qquad 5\,(\text{pts})$$

At State 3, ν_3=ν_2, which is between ν_f and ν_g at 150°C. Interpolating between specific volume and specific internal energy,

$$\frac{\nu_3 - \nu_f(\text{T})}{\nu_g(\text{T}) - \nu_f(\text{T})} = \frac{u_3 - u_f(\text{T})}{u_g(\text{T}) - u_f(\text{T})}$$

$$\implies u_3 = \left(\frac{0.19444 - 0.001090\,[\text{m}^3/\text{kg}]}{0.39278 - 0.001090\,[\text{m}^3/\text{kg}]}\right)(2,559.54 - 631.66\,[\text{kJ/kg}]) + 631.66\,[\text{kJ/kg}]$$

$$\implies u_3 = 1,583.32\,[\text{kJ/kg}] \qquad 5\,(\text{pts})$$

Therefore, the heat per unit mass into the system is

$$\frac{Q_{1\to 3}}{m} = (1,583.32 - 2,957.29\,[\text{kJ/kg}] - 112.15\,[\text{kJ/kg}] \implies \boxed{\frac{Q_{1\to 3}}{m} = -1,489.12\,[\text{kJ/kg}]} \qquad 5\,(\text{pts})$$