# Quiz #4

MEMS 0051 - Introduction to Thermodynamics

Assigned: June 11$^{\text{th}}$, 2020
Due: June 12$^{\text{th}}$, 2020, 11:59 pm

## Problem #1

Consider a piston-cylinder that contains 1 [kg] of air at a pressure and temperature of 200 [kPa] and 300 [K], respectively. The air now undergoes a polytropic process until it reaches a final pressure of 2200 [kPa]. If the polytropic index is n = 1.3 for this process, determine the following:

a) the final volume and temperature of the air;
Let's look at our states and what we know:

| State 1: | → n = 1.3: → | State 2: |
|---|---|---|
| $P_1$ = 200 [kPa] | | $P_2$ = 2,200 [kPa] |
| $T_1$ = 300 [K] | | $T_2$ = ? |
| $\forall_1$ = ? | | $\forall_2$ = ? |
| $u_1$ = ? | | $u_2$ = ? |
| $m_1$ = 1 [kg] | | $m_2 = m_1$ |

The volume at state 1 can be found using the Ideal Gas Law:

$$\forall_1 = \frac{mRT_1}{P_1} = \frac{(1\ [\text{kg}])(0.287\ [\text{kJ/kg-K}])(300\ [\text{K}])}{(200\ [\text{kPa}])} = 0.4305\ [\text{m}^3]$$

Use the formula for a polytropic process to find the volume at state 2:

$$P_1\forall_1^n = P_2\forall_2^n \implies \forall_2 = \forall_1\left(\frac{P_1}{P_2}\right)^{\frac{1}{1.3}} = (0.4305\ [\text{m}^3])\left(\frac{200\ [\text{kPa}]}{2,200\ [\text{kPa}]}\right)^{\frac{1}{1.3}} = \boxed{0.06806\ [\text{m}^3]}$$

We can use the Ideal Gas Law to find the temperature at state 2:

$$T_2 = \frac{P_2\forall_2}{mR} = \frac{(2,200\ [\text{kPa}])(0.06806\ [\text{m}^3])}{(1\ [\text{kg}])(0.287\ [\text{kJ/kg-K}])} = \boxed{521.71\ [\text{K}]}$$

b) the work performed during the process;
The work can be found using the equation for work for a polytropic process:

$$W_{1\to2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = \frac{(2,200\ [\text{kPa}])(0.06806\ [\text{m}^3]) - (200\ [\text{kPa}])(0.4305\ [\text{m}^3])}{1-1.3} = \boxed{-212.11\ [\text{kJ}]}$$

c) the heat transferred during the process using Table A.5;
The heat transferred using constant-volume specific heat:

$$Q_{1\to2} = mC_{\forall0}(T_2-T_1)+W_{1\to2} = (1\ [\text{kg}])(0.717\ [\text{kJ/kg-K}])((521.71-300)\ [\text{K}])+(-212.11\ [\text{kJ}]) = \boxed{-53.14\ [\text{kJ}]}$$

d) the heat transferred during the process using Table A.7;
From Table A.7, we have for the specific internal energy:

$$u_1 = u_{@300\ [\mathrm{K}]} = 214.36\ [\mathrm{kJ/kg}] \qquad u_2 = u_{@521.71\ [\mathrm{K}]} = 376.01\ [\mathrm{kJ/kg}]$$

Calculating the heat transferred:

$$Q_{1\to 2} = m(u_2 - u_1) + W_{1\to 2} = (1\ [\mathrm{kg}])((376.01 - 214.36)\ [\mathrm{kJ/kg}]) + (-212.11\ [\mathrm{kJ}]) = \boxed{-50.46\ [\mathrm{kJ}]}$$

e) the percent error between the heat transfers using the answer determined from Table A.7 as the accepted value;
If the change in specific internal energy from Table A.7 is the accepted value, then the percent error is:

$$\%\ \mathrm{error} = \frac{Q_{\mathrm{A.5}} - Q_{\mathrm{A.7}}}{Q_{\mathrm{A.7}}} \cdot 100\% = \frac{(-53.14 - (-50.46))\ [\mathrm{kJ}]}{-50.46\ [\mathrm{kJ}]} \cdot 100\% = \boxed{5.3\%}$$

**Academic Integrity Statement**:

I hereby attest that I have received no assistance (from a friend, from another student, from an on-line resource, such as Chegg, etc.), and that I have provided no assistance to another student, during this examination. All the work presented within is solely my own work.

Signature: ______________________________________

Date: ________________________