**MEMS 0051**
**Summer 2017**
**Midterm #1**
**6/26/2017**
**75 Minutes**

**Name (Print):** Solution

---

This exam contains 8 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page. Please do all of your work in the provided blue examination book.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

| Problem | Points | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 20 | |
| 4 | 25 | |
| 5 | 35 | |
| Total: | 100 | |

**BONUS:**

5 pts: June 26$^{\text{th}}$, 1945, marked the signing of what charter that established the Statute of the International Court of Justice?

## Basic Properties

1. (a) (5 points) Determine the quality of 2.0 [kg] of water having a volume of 0.5 [$m^3$] existing at 25 °C and a pressure of 3.169 [kPa].

Recall the definition for specific volume:

$$v = \frac{\forall}{m} = \frac{0.5\ [\text{m}^3]}{2.0\ [\text{kg}]} = 0.25\ [\text{m}^3/\text{kg}]$$

At 25°c and 3.169 [kPa], our specific volume is existing between the saturated liquid ($v_f$) and saturate vapor ($v_g$) specific volumes. Therefore, we can use the definition of quality:

$$\boxed{x = \frac{v - v_f}{v_g - v_f} = \frac{0.25 - 0.001003}{43.3583} = 0.574\%}$$

(b) (5 points) Determine the mass of saturated water vapor at 155 °C in a 40 [$m^3$] rigid tank. Since we know the specific volume of the saturated water vapor ($v_g$= 0.34676 [$m^3$/kg], the mass is simply found as:

$$\boxed{m = \frac{\forall}{v} = \frac{40\ [\text{m}^3]}{0.34676\ [\text{m}^3/\text{kg}]} = 115.354\ [\text{kg}]}$$

## Polytropic Process

2. (10 points) A piston-cylinder contains water at 500 °C and 3 [MPa]. It is cooled in a polytropic process to 200 °C and 1 [MPa]. Determine:

- the polytropic index $n$,
- the specific work in the process.

To determine the polytropic index $n$, we need the pressure and volume, or specific volume at the initial and final states. At State 1, we see the water is existing as a superheated vapor ($T_1 > T_{sat}$(3 [MPa]), therefore the specific volume is found to be 0.11619 [$m^3$/kg]. At State 2, the water is still existing as a superheated vapor ($T_2 > T_{sat}$(1 [MPa]), and the specific volume is found to be 0.20596 [$m^3$/kg]. Solving for the polytropic index:

$$\boxed{n = \frac{\ln\left(\frac{P_2}{P_1}\right)}{\ln\left(\frac{\forall_1}{\forall_2}\right)} = \frac{\ln\left(\frac{1\ [\text{MPa}]}{3\ [\text{MPa}]}\right)}{\ln\left(\frac{0.11619\ [\text{m}^3/\text{kg}]}{0.20596\ [\text{m}^3/\text{kg}]}\right)} = 1.919}$$

The specific work, or work per unit mass basis, is simply evaluated using our definition of work based upon the polytropic index:

$$\frac{W_{1\to2}}{m} = w_{1\to2} = \frac{P_2v_2 - P_1v_1}{1-n}$$

Evaluating the expression:

$$\boxed{w_{1\to2} = \frac{1,000\,[\text{kPa}]\cdot 0.20596\left[\frac{\text{m}^3}{\text{kg}}\right] - 3,000\,[\text{kPa}]\cdot 0.11619\left[\frac{\text{m}^3}{\text{kg}}\right]}{1-1.919} = 115.18\,[\text{kJ}]}$$

### Formulation of Work

3. (20 points) Water contained in a piston-cylinder assembly has an initial temperature of 150 °C, a quality of 50% and an initial volume of 0.05 m$^3$. The pressure of the process is given as a function of volume such that P($\forall$)=100 + C$\forall^{0.5}$ [kPa]. Heat is transferred to the piston-cylinder until the final pressure reaches 600 kPa. Determine:

   - the heat input.

   Hint: Solve for the constant C based upon givens at State 1.

| State 1: | $\longrightarrow$ | State 2: |
|---|---|---|
| $\text{T}_1$=150°C | • | $\text{m}_2$=$\text{m}_1$ |
| $\text{x}_1$=0.50 | • | $\text{P}_2$=600 [kPa] |
| $\text{P}_1$=$\text{P}_{\text{sat}}(\text{T}_1)$=475.9 [kPa] | • | $v_2$=$\forall_2/\text{m}_2$=? |
| $\forall_1$=0.05 [m$^3$] | • | • |

To determine $Q_{1\to2}$, we need $m_1$, $u_1$, $u_2$ and $W_{1\to2}$. Solving for the mass at State 1 by using the volume and specific volume, the latter determined by the given quality:

$$m_1 = \frac{\forall_1}{v_1} = \frac{0.05\,[\text{m}^3]}{0.00109 + 0.5(0.39278 - 0.00109)[\text{m}^3/\text{kg}]} = 0.254\,[kg]$$

We can determine the internal energy at State 1 using quality:

$$u_1 = u_f + x(u_g - u_f) = 631.66 + 0.5(2,559.54 - 631.66)\,[\text{kJ/kg}] = 1,595.6\,[\text{kJ/kg}]$$

We can next determine the specific volume at State 2, $v_2$, which will be used to determine both the work and the internal energy. Using our relationship of pressure and volume applied to State 1, noting that the expression is explicitly given in terms of [kPa]:

$$P(\forall) = 100{+}\text{C}\forall^{0.5} = 475.9\,[\text{kPa}] = 100{+}\text{C}(0.05[\text{m}^3])^{0.5} \implies \text{C} = \frac{375.9\,[\text{kPa}]}{(0.05[\text{m}^3])^{0.5}} = 1,681\left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]$$

Using the same formulation for pressure as function of volume, applied to State 2, with our solved for constant:

$$P_2 = 600[\text{kPa}] = 100 + \left(1,681\left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]\right)\forall_2^{0.5}$$

Solving for $\forall_2$

$$\forall_2 = \left(\frac{500[\text{kPa}]}{1,681\left[\frac{\text{kPa}}{\sqrt{\text{m}^3}}\right]}\right)^2 = 0.0885\ [\text{m}^3]$$

Therefore, the specific volume at State 2 is:

$$v_2 = \frac{\forall_2}{m} = \frac{0.0885\ [\text{m}^3]}{0.254\ [\text{kg}]} = 0.349\ [\text{m}^3/\text{kg}]$$

Given $v_2$ and $P_2$, we see the water at State 2 is in the superheated vapor region. Solving for $u_2$ via interpolation:

| v [m$^3$/kg] | u [kJ/kg] |
|---|---|
| 0.31567 | 2,567.40 |
| 0.349 | $u_2$ |
| 0.35202 | 2,638.91 |

$$\implies u_2 = 2,632.97\ [\text{kJ/kg}]$$

Solving for work:

$$W_{1\to 2} = \int P d\forall = \int (100 + 1,681.1\forall^{0.5}) d\forall = \left(100\forall + \frac{2}{3}\cdot 1,681.1\forall^{1.5}\right)\Bigg|_{0.05}^{0.0885} = 20.83\ [\text{kJ}]$$

Note, you can solve for work by determining the polytropic index ($n$=-0.405).

Bringing everything together to solve for the heat input:

$$\boxed{Q_{1\to 2} = U_2 - U_1 + W_{1\to 2} = 0.254\ [\text{kg}]\left(2,632.97 - 1,595.6\left[\frac{\text{kJ}}{\text{kg}}\right]\right) + 20.83\ [\text{kJ}] = 284.32\ [\text{kJ}]}$$

## Ideal Gas

4. (25 points) A frictionless piston-cylinder device contains 0.1 [kg] of air at 300 K at 100 [kPa]. The air is slowly compressed from its initial state in an isothermal process to a final pressure of 250 [kPa].

- Show the process on the P-v diagram,
- determine the work into the system,

- determine the heat rejected from the system.

| State 1: | $\longrightarrow$ | State 2: |
|---|---|---|
| $m_1$=1 [kg] | $n$=1 | $m_2$=$m_1$ |
| $T_1$=300 [K] | • | $T_2$=$T_1$ |
| $P_1$=100 [kPa] | • | $P_2$=250 [kPa] |

Solving for $\forall_1$ using the Idea Gas Law:

$$P_1\forall = mRT_1 \implies \forall_1 = \frac{mRT_1}{P_1} = \frac{0.1\text{ [kg]} \cdot 0.287\text{[kJ/kg-K]} \cdot 300\text{[K]}}{100\text{ [kPa]}} = 0.0861\text{ [m}^3\text{]}$$

Solving for $\forall_2$ using the Idea Gas Law:

$$P_2\forall = mRT_2 \implies \forall_2 = \frac{mRT_2}{P_2} = \frac{0.1\text{ [kg]} \cdot 0.287\text{[kJ/kg-K]} \cdot 300\text{[K]}}{250\text{ [kPa]}} = 0.0344\text{ [m}^3\text{]}$$

Since the process is isothermal, the work into the system is expressed as:

$$\boxed{W_{1\to2} = P_1\forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right) = 100\text{ [kPa]} \cdot 0.0861\text{ [m}^3\text{]} \cdot \ln\left(\frac{0.0344\text{ [m}^3\text{]}}{0.0861\text{ [m}^3\text{]}}\right) = -7.899\text{ [kJ]}}$$

Solving for the heat rejected from the system:

$$\boxed{Q_{1\to2} = U_2 - U_1 + W_{1\to2} = mC_\forall(T_2 - T_1) + W_{1\to2} = W_{1\to2} = -7.899\text{ [kJ]}}$$

Since there is no change of temperature between the initial and final state, there is no change in internal energy, and the heat input is the same as the work out.

## Multistep Process

5. (35 points) A frictionless piston-cylinder device has two sets of stops that constrain the piston. When the piston is at rest on the lower set of stops, the enclosed volume is 0.4 [m$^3$]. When the piston reaches the uppers tops, the enclosed volume is 0.6 [m$^3$]. The cylinder initially contains water at 100 [kPa] and a quality of 20%. It is heated until the water eventually exists as a saturated vapor. The mass of the piston is such that is requires 300 [kPa] generated within the piston-clyinder device to move. Determine:

   1. the final pressure in cylinder,
   2. the heat transfer during this process,
   3. the work for this process.

   Hint: There are three processes going between 4 states.

1. the final pressure in the cylinder;

Let's look at our states and what we know:

| State 1: | State 2: | State 3: | State 4: |
|---|---|---|---|
| $P_1 = 100$ [kPa] | $P_2 = 300$ [kPa] | $P_3 = 300$ [kPa] | $P_4 = ?$ |
| $\forall_1 = 0.4$ [m$^3$] | $\forall_2 = 0.4$ [m$^3$] | $\forall_3 = 0.6$ [m$^3$] | $\forall_4 = 0.6$ [m$^3$] |
| $x_1 = 0.2$ | $x_2 = ?$ | $x_3 = ?$ | $x_4 = 1$ |
| $\nu_1 = ?$ | $\nu_2 = ?$ | $\nu_3 = ?$ | $\nu_4 = ?$ |
| $u_1 = ?$ | $u_2 = ?$ | $u_3 = ?$ | $u_4 = ?$ |
| $m_{sys} = ?$ | | | |

The difficulty of this problem stems primarily from identifying the necessary properties at each state. We are told as a hint that there will be three processes between four states. Based on this information, the process between states 1 and 2 must be isochoric heating since the piston cannot rise upwards until the pressure has reached 300 [kPa]. The process between states 2 and 3 must be heating at constant pressure. As more heat is added, the piston will continue to rise upwards until it makes contact with the upper stops, preventing further motion. Finally, the process between states 3 and 4 will be that once again of isochoric heating since the quality is not yet 100% (we can verify this as we move through the states). This will push the pressure in the cylinder to some higher, unknown pressure.

Beginning at state 1:

$$u_1 = u_f + x_1 u_{fg} = (417.33 + 0.2(2088.72)) \text{ [kJ/kg]} = 835.07 \text{ [kJ/kg]}$$

$$\nu_1 = \nu_f + x_1 \nu_{fg} = (0.001043 + 0.2(1.69296)) \text{ [m}^3\text{/kg]} = 0.33964 \text{ [m}^3\text{/kg]}$$

Now that we have the specific volume, we can find the mass of the system:

$$m_{sys} = \frac{\forall_1}{\nu_1} = \frac{0.4 \text{ [m}^3]}{0.33964 \text{ [m}^3\text{/kg]}} = 1.178 \text{ [kg]}$$

Since we know the pressure and the volume at state 2, we can solve for the remaining properties. Note that as the process from states 1 to 2 is isochoric, we can say:

$$\nu_2 = \nu_1 = 0.33964 \text{ [m}^3\text{/kg]}$$

From this, we can find quality and specific internal energy:

$$x_2 = \frac{\nu_2 - \nu_f}{\nu_{fg}} = \frac{(0.33964 - 0.001073) \text{ [m}^3\text{/kg]}}{(0.60475) \text{ [m}^3\text{/kg]}} = 0.56$$

$$u_2 = u_f + x_2 u_{fg} = (561.13 + 0.56(1982.43)) \text{ [kJ/kg]} = 1671.29 \text{ [kJ/kg]}$$

Since we have the mass and the pressure and volume at state 3, we can solve for the remaining properties, starting with specific volume:

$$\nu_3 = \frac{\forall_3}{m_{sys}} = \frac{0.6 \text{ [m}^3]}{1.178 \text{ [kg]}} = 0.50934 \text{ [m}^3\text{/kg]}$$

$$x_3 = \frac{\nu_3 - \nu_f}{\nu_{fg}} = \frac{(0.50934 - 0.001073)\ [\text{m}^3/\text{kg}]}{(0.60475)\ [\text{m}^3/\text{kg}]} = 0.84$$

$$u_3 = u_f + x_3 u_{fg} = (561.13 + 0.84(1982.43))\ [\text{kJ/kg}] = 2226.37\ [\text{kJ/kg}]$$

Because the process between states 3 and 4 is again isochoric, we can say:

$$\nu_4 = \nu_3 = 0.50934\ [\text{m}^3/\text{kg}]$$

We may not know the pressure at state 4 like the other cases, but we know the process ends when the quality reaches 100%. Knowing this, we can see that as $\nu_4$ must be equal to $\nu_g$, the final pressure must lie between 350 [kPa] and 375 [kPa]. Interpolating, we find:

$$P_4 = 350\ [\text{kPa}] + (0.50934 - 0.52425)\ [\text{m}^3/\text{kg}] \cdot \frac{(375 - 350)\ [\text{kPa}]}{(0.49137 - 52425)\ [\text{m}^3/\text{kg}]} = \boxed{361.34\ [\text{kPa}]}$$

Finally, we can also find the specific internal energy by interpolating:

$$u_4 = 2548.92\ [\text{kJ/kg}] + (361.34 - 350)\ [\text{kPa}] \cdot \frac{(2551.31 - 2548.92)\ [\text{kJ/kg}]}{(375 - 350)\ [\text{kPa}]} = 2550\ [\text{kJ/kg}]$$

2. the heat transfer during this process;

The total heat transferred during this process is the summation of the heat transferred between each state. Using the First Law and moving from state 1 to 2:

$$Q_{1\to2} = m(u_2 - u_1) + W_{1\to2} = (1.178\ [\text{kg}])((1671.29 - 835.07)\ [\text{kJ/kg}]) + 0\ [\text{kJ}] = 985.07\ [\text{kJ}]$$

From state 2 to 3:

$$Q_{2\to3} = m(u_3 - u_2) + W_{2\to3} = (1.178\ [\text{kg}])((2226.37 - 1671.29)\ [\text{kJ/kg}]) + 60\ [\text{kJ}] = 713.88\ [\text{kJ}]$$

From state 3 to 4:

$$Q_{3\to4} = m(u_4 - u_3) + W_{3\to4} = (1.178\ [\text{kg}])((2550 - 2226.37)\ [\text{kJ/kg}] + 0\ [\text{kJ}] = 381.24\ [\text{kJ}]$$

The total heat transfer then is:

$$Q_{tot} = Q_{1\to2} + Q_{2\to3} + Q_{3\to4} = 985.07\ [\text{kJ}] + 713.88\ [\text{kJ}] + 381.24\ [\text{kJ}] = \boxed{2080.19\ [\text{kJ}]}$$

Conversely, if we recognize the the final and initial states are what matter, we can simply say:

$$Q_{1\to4} = m(u_4 - u_1) + W_{1\to4} = (1.178\ [\text{kg}])((2550 - 835.07)\ [\text{kJ/kg}]) + 60\ [\text{kJ}] = \boxed{2080.19\ [\text{kJ}]}$$

3. the work for this process;

The only work occurring in this process is due to the moving boundary from the piston rising upwards. As there is no change in volume moving from states 1 to 2, and 3 to 4, we can say:

$$W_{1\to2} = W_{3\to4} = 0\ [\text{kJ}]$$

The work moving from states 2 to 3 is found by:

$$W_{2\to 3} = P_2(\forall_3 - \forall_2) = (300 \text{ [kPa]})((0.6 - 0.4) \text{ [m}^3\text{]}) = 60 \text{ [kJ]}$$

Therefore, the total work for this process is:

$$W_{1\to 4} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} = \boxed{60 \text{ [kJ]}}$$