MEMS 0051
Spring 2017
Midterm #1
2/15/2017

Name (Print): Solution

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	15	
3	20	
4	25	
5	30	
Total:	100	

Written Problem #1

1. (a) (5 points) Determine the quality of the water existing at 180 °C, 0.1400 [m^3/kg]

   Solution: At 180 °C, the saturated liquid and saturated vapor specific volumes can be found from the saturated water temperature tables: v_f=0.001127[$m^3 \cdot kg^{-1}$] v_g=0.19405 [$m^3 \cdot kg^{-1}$] Therefore, the quality can be found as:

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.1400 - 0.001127}{0.19405 - 0.001127} = \boxed{71.98\%} \tag{1}$$

   (b) (5 points) Determine the mass of saturated vapor at 155 °C in a 40 [m^3] rigid tank.

   Solution: Saturated water vapor has a saturated vapor specific volume of 0.34676 [$m^3 \cdot kg^{-1}$] at 155 °C per the saturated water temperature tables. Therefore:

$$m = \frac{\forall}{v_g} = \frac{40\ [\text{m}^3]}{0.34676\ [\text{m}^3 \cdot \text{kg}^{-1}]} = \boxed{115.35\ [\text{kg}]} \tag{2}$$

Written Problem #2

2. (15 points) **What is the pressure** of 3 [kg] of air contained in a closed rigid room of 25 [m^3] if the temperature is 25 °C? **What is the new pressure** of the room if the temperature is double to 50 °C?

Solution:
Using the Ideal Gas Law, $P\forall = mRT$, we can re-arrange to solve for the pressure of the system such that:

$$P = \frac{3\ [\text{kg}] \cdot 0.287\ [\text{kJ/kg-K}] \cdot 298.15\ [\text{K}]}{25\ [\text{m}^3]} = \boxed{10.29\ [\text{kPa}]} \tag{3}$$

When the temperature is increased to 50 °C:

$$P = \frac{3\ [\text{kg}] \cdot 0.287\ [\text{kJ/kg-K}] \cdot 323.15\ [\text{K}]}{25\ [\text{m}^3]} = \boxed{11.13\ [\text{kPa}]} \tag{4}$$

Written Problem #3

3. (20 points) **Determine the work** done by a piston-cylinder arrangement *filled with air* initially having a volume of 4 [m^3] and a pressure of 200 [kPa]. At the final state, the volume of the control mass is 25 [m^3], while the pressure drops to 80 [kPa]. **Determine the final temperature** if the initial temperature is 200 K.

Solution:

State 1:	$\rightarrow$	State 2:
T$_1$=200 [K]		$\forall_2$=25 [m^3]
$\forall_1$=4 [m^3]		P$_2$=80 [kPa]
P$_1$=200 [kPa		

Since the working fluid is air, we can apply the Ideal Gas Law. We can verify at the end of the problem that our reduced pressure is much less than the critical pressure (3,771 [kPa]) and that the final temperature is at least two times greater than our critical temperature (132.41 [K]). The Ideal Gas Law states the the values for $P\forall{=}mRT$ must be constant between states. In this situation, it is a control-mass problem and the gas constant R is independent of States, thus:

$$\frac{P_1\forall_1}{T_1} = \frac{P_2\forall_2}{T_2} \rightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)\left(\frac{\forall_2}{\forall_1}\right) = 200\text{ [K]}\left(\frac{80[\text{kPa}]}{200[\text{kPa}]}\right)\left(\frac{25[\text{m}^3]}{4[\text{m}^3]}\right) = \boxed{500\text{ [K]}} \tag{5}$$

To determine the work between States 1 and 2, we need to find the polytropic process. The process is neither constant temperature, pressure or volume. The polytropic index is found as:

$$n = \frac{\ln\left(\dfrac{P_1}{P_2}\right)}{\ln\left(\dfrac{\forall_2}{\forall_1}\right)} = \frac{\ln\left(\dfrac{200[\text{kPa}]}{80[\text{kPa}]}\right)}{\ln\left(\dfrac{25[\text{m}^3]}{4[\text{m}^3]}\right)} = 0.5 \tag{6}$$

Thus, the work can be taken as expressed in terms of the polytropic index:

$$W_{1\rightarrow2} = \frac{P_2\forall_2 - P_1\forall_1}{1-n} = \frac{(80\text{ [kPa]}\cdot 25\text{ [m}^3]) - (200\text{ [kPa]}\cdot 4\text{ [m}^3])}{1-0.5} = \boxed{2,400\text{ [kJ]}} \tag{7}$$

Written Problem #4

4. (25 points) A piston-cylinder contains 2 [kg] of water with a volume of 0.1 [m^3] at 212.42°C. The piston is then expanded isothermally until the pressure drops to 1,000 [kPa]. **Find the total process work**. Note: this is a two-step process and only the second process is purely isothermal.

Solution:

State 1:	→ T=c →	State 2:	→ T=c →	State 3:
m_1=2 [kg]	→ P=c →	$m_2=m_1$		$m_3=m_2$
$\forall_1$=0.1 [m^3]		$v_2=v_g$		P_3=1,000 [kPa]
T_1=212.42 °C		$T_2=T_1$		$T_3=T_2$
		$P_2=P_1$		

At State 1, the pressure can be found by looking at the saturated water pressure tables and looking at the saturation temperature. A temperature of 212.42 °C has a saturation pressure of 2,000 [kPa]. The specific volume can be found since the mass and volume are given:

$$v_1 = \frac{\forall_1}{m_1} = \frac{0.1\ [\text{m}^3]}{2\ [\text{kg}]} = 0.05[\text{m}^3 \cdot \text{kg}] \tag{8}$$

We see the specific volume at State 1 exists between the saturated liquid (v_f=0.001177 [m$^3\cdot$kg^{-1}]) and saturated vapor (v_g=0.09963 [m$^3\cdot$kg^{-1}]) specific volumes. As the piston is expanded at a constant temperature to a pressure of 1,000 [kPa], we have a constant-volume and constant-temperature process as the fluid moves from State 1 to State 2 as seen in Fig. 1.

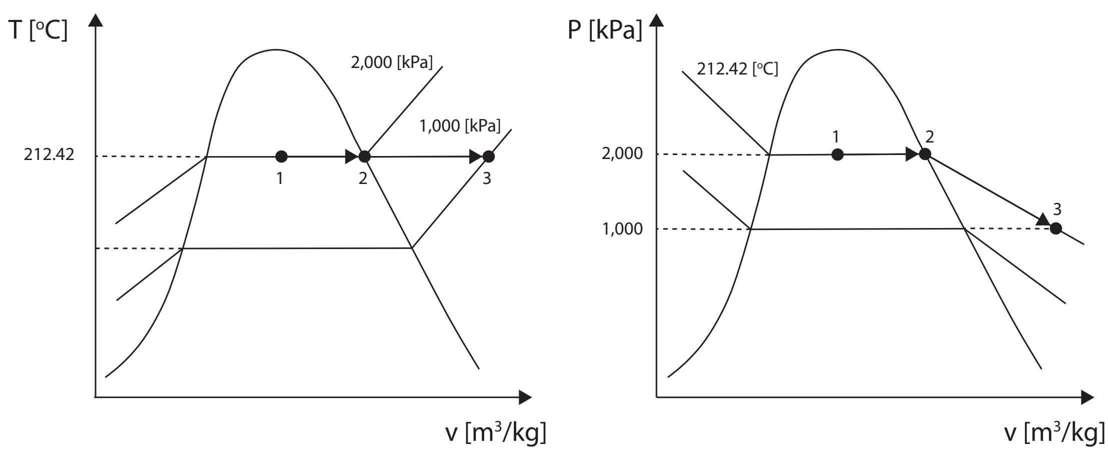

Figure 1: T-v and P-v for Written Problem #4.

Thus, as the piston expands at a constant-temperature and constant-pressure, State 2 is the saturated vapor taken at 2,000 [kPa]. Thus v_2=0.09963 [m$^3\cdot$kg^{-1}]. Note that the polytropic index for an isobaric process is zero while the polytropic index for an isothermal process is unity. Since there are two processes happening going from State 1 to State 2 with two different polytropic indices, we have to discern which process is the appropriate one. By the strict definition of work, we have the integral of pressure with respect to volume. Since the pressure is constant, we are left with a constant pressure process. Furthermore, you could calculate the polytropic index for this problem and determine that it is indeed zero, indicating it is a constant-pressure process. Therefore, the work from State 1 to 2 can be taken as a constant pressure process since the volume is expanding:

$$W_{1\rightarrow 2} = \int_{\forall_1}^{\forall_2} P d\forall = (2\,[\text{kg}])(2,000\,[\text{kPa}])(0.09963 - 0.05\,[\text{m}^3 \cdot \text{kg}^{-1}]) = 198.52\,[\text{kJ}] \tag{9}$$

Note that the process is not purely isothermal, thus we can not use the formulation of work equation when the polytropic index is equal to unity. Next, at State 3, the fluid is existing at 212.42 °C and a pressure of 1,000 [kPa], placing the fluid in the superheated vapor region. From the superheated vapor table, the specific volume can be interpolated:

$$\frac{(212.42 - 200)\,[^\circ\text{C}]}{(250 - 200)\,[^\circ\text{C}]} = \frac{(v_3 - 0.20596)\,[\text{m}^3 \cdot \text{kg}^{-1}]}{(0.23268 - .20596)\,[\text{m}^3 \cdot \text{kg}^{-1}]} \rightarrow v_3 = 0.21260[\text{m}^3 \cdot \text{kg}^{-1}] \tag{10}$$

Thus, the work from States 2 to 3 can be evaluated as an isothermal process:

$$W_{2\rightarrow 3} = P_2 \forall_2 \ln\left(\frac{\forall_3}{\forall_2}\right) = (2,000[\text{kPa}])(2[\text{kg}])(0.09663\,[\text{m}^3{\cdot}\text{kg}^{-1}])\ln\left(\frac{0.21260\,[\text{m}^3 \cdot \text{kg}^{-1}]}{0.09963\,[\text{m}^3 \cdot \text{kg}^{-1}]}\right) = 302.06[\text{kJ}] \tag{11}$$

Thus, the total work of the system is the summation of the two works of the individual processes:

$$W_{1\rightarrow 3} = W_{1\rightarrow 2} + W_{2\rightarrow 3} = 198.52\,[\text{kJ}] + 302.06\,[\text{kJ}] = \boxed{500.58\,[\text{kJ}]} \tag{12}$$

Written Problem #5

5. (30 points) 10 [kg] of water is contained within a piston-cylinder with a constant pressure occupies 0.633 [m^3] at a temperature of 450 °C. The system is then cooled to 20 °C. **Determine the work** done on the system **and the heat transfer** from the system.

Solution:

State 1:	→ P=c →	State 2:
m_1= 10 [kg]		m_2=m_1
$\forall_1$=0.633 [m^3]		P_2=P_1
T_1=450 °C		T_2=20 °C

First, we have to determine where State 1 lies. To do such, we find the specific volume:

$$v_1 = \frac{\forall_1}{m_1} = \frac{0.633\,[\text{m}^3]}{10\,[\text{kg}]} = 0.0633\,[\text{m}^3/\text{kg}] \tag{13}$$

The combination of specific volume and temperature clearly indicates the water is existing as a superheated vapor, that is, $v_1 > v_g(450\ [°\text{C}])$. The superheated vapor exists at a pressure of 5,000 [kPa] as found from the superheated steam tables. The specific internal energy at State 1 is 2,999.64 [kJ/kg]. We next need to determine where State 2 lies. The temperature is given as 20 [°C] and the pressure is the same as State 1. Thus, the substance exists as a compressed liquid ($P > P_{\text{sat}}$(5,000 [kPa])). Using the compressed liquid water tables, the specific volume and specific internal energy at State 2 are found to be 0.001002 [m^3/kg] and 83.91 [kJ/kg]. You may also take the saturated liquid water values evaluated at 20 [°C] since water is incompressible, and notice the specific internal energy changes by only 0.03 [kJ/kg]. The work can be evaluated as a constant-pressure process:

$$W_{1\to2} = P \cdot m(v_2 - v_1) = (5,000\,[\text{kPa}])(10\,[\text{kg}])(0.001002 - 0.0633)\,[\text{m}^3/\text{kg}] = \boxed{-3,114.9\,[\text{kJ}]} \tag{14}$$

Then, to determine the heat transfer, we apply the Conservation of Energy equation:

$$Q_{1\to2} = m(u_2 - u_1) + W_{1\to2} = (10[\text{kg}])(83.91 - 2,999.64)[\text{kJ/kg}] - 3,114.9\,[\text{kJ}] = \boxed{-32,272.2\,[\text{kJ}]} \tag{15}$$