Chapter 9 - Differential Analysis of Fluid Flow

Lecture 24 Sections 9.5 and 9.6

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science University of Pittsburgh Chapter 9 Differential
Analysis of Fluid
Flow

MEMS 0071

Student Learning Objectives

Payler-Stokes
Equations
0.6 Differential



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Equations
0.6 Differential

analysis of Fluid low Problems

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Students should be able to:

➤ Solve two-dimensional steady-state Navier-Stokes in cylindrical coordinates.

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9.5 The Navier-Stokes Equations

0.6 Differential Analysis of Fluid Flow Problems

▶ The Navier-Stokes equations in r, θ and z-directions: r-direction:

$$\rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) = -\frac{\partial P}{\partial r} + \dots$$

$$\dots \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial (ru_r)}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right) + \rho g_r$$

 $\underline{\theta}$ -direction:

$$\begin{split} \rho \bigg(\frac{\partial u_{\theta}}{\partial t} + u_{r} \frac{\partial u_{\theta}}{\partial r} + \frac{u_{\theta}}{r} \frac{\partial u_{\theta}}{\partial \theta} + \frac{u_{r} u_{\theta}}{r} + u_{z} \frac{\partial u_{\theta}}{\partial z} \bigg) &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \dots \\ \dots \mu \bigg(\frac{\partial}{\partial r} \bigg(\frac{1}{r} \frac{\partial (r u_{\theta})}{\partial r} \bigg) + \frac{1}{r^{2}} \frac{\partial^{2} u_{\theta}}{\partial \theta^{2}} + \frac{2}{r^{2}} \frac{\partial u_{r}}{\partial \theta} + \frac{\partial^{2} u_{\theta}}{\partial z^{2}} \bigg) + \rho g_{\theta} \end{split}$$



Newtonian Fluid - N.S. Equations

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$\underline{z\text{-direction}}$:

$$\rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) = -\frac{\partial P}{\partial z} + \dots$$
$$\dots \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right) + \rho g_z$$

Continuity:

$$\frac{1}{r}\frac{\partial(ru_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} = 0$$



Let us look at a basic example of Poiseuille flow between concentric cylinders with radii r_a and r_b . Chapter 9 -Differential Analysis of Fluid Flow

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lavier-Stokes equations



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5 The avier-Stokes quations



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