Chapter 3 - Fluid Statics

Lecture 6

Section 3.4

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
University of Pittsburgh

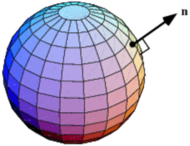
Student Learning Objectives

Students should be able to determine the:

- magnitude of a force acting on a submerged curved surface;
- direction of a force acting on a submerged curved surface;
- the line of action of a force acting on a submerged curved surface.

Hydrostatic Force on Submerged Curved Surfaces

- The only difference in deriving the formulation for the resultant force F_R acting on a curved surface is that our pressure force is acting per unit normal at every infinitesimally small dA
- Looking at a sphere, each normal vector $\vec{n}$ will vary slightly as you move along the surface.



Hydrostatic Force on Submerged Curved Surfaces

- To rectify this, we integrate over $d\vec{A}$ instead of dA, and it is also helpful to break F_R down into x- and y- components, F_x and F_y

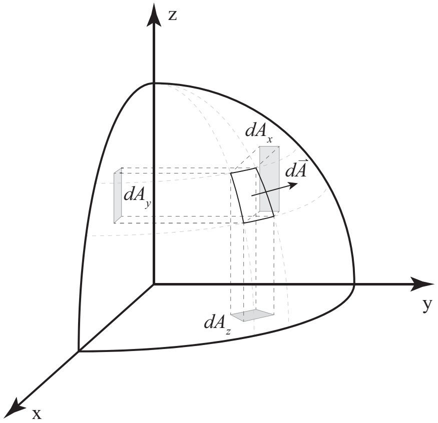
- Therefore, replacing dA with $d\vec{A}$, F_R now becomes a vector expression:

$$d\vec{F}_R = -Pd\vec{A}$$

- Note the minus sign - when we previously evaluated F_R, it was a magnitude (hence no sign), and now it acts against $\vec{n}$

Hydrostatic Force on Submerged Curved Surfaces

- Notice how we *project* each component of the differential area on the x-y, y-z and z-x axes.



Hydrostatic Force on Submerged Curved Surfaces

- Since $\vec{F}_R$ is a vector, we can write it in terms of the x-,y- and z- components (magnitude and direction):

$$\vec{F}_R = \vec{F}_x + \vec{F}_y + \vec{F}_z = F_{Rx}\hat{i} + F_{Ry}\hat{j} + F_{Rz}\hat{k}$$

- Integrating our force vector:

$$\int d\vec{F}_R = -P\int d\vec{A} \implies \vec{F}_R = -P\int_A d\vec{A}$$

- We know the force vector has three components (x,y,z) and that the area has three components (A_x,A_y,A_z)

Hydrostatic Force on Submerged Curved Surfaces

- If we multiply our force vector by the unit normal vector (vector dot product):

$$\begin{aligned} x: &\quad F_{Rx} = \vec{F}_R \cdot \hat{i} \\ y: &\quad F_{Ry} = \vec{F}_R \cdot \hat{j} \\ z: &\quad F_{Rz} = \vec{F}_R \cdot \hat{k} \end{aligned}$$

Hydrostatic Force on Submerged Curved Surfaces

- Integrating each direction of the force vector:

$$x: \quad F_{Rx} = -\int_A P d\vec{A} \cdot \hat{i} = -\int_A P dA_x$$

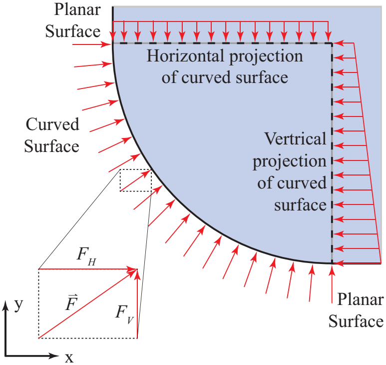
$$y: \quad F_{Ry} = -\int_A P d\vec{A} \cdot \hat{j} = -\int_A P dA_y$$

$$z: \quad F_{Rz} = -\int_A P d\vec{A} \cdot \hat{k} = -\int_A P dA_z$$

- Note how the x-component of the force is the pressure acting on the x-component of the differential area (i.e. projection of dA onto the y-z plane), and the like for the remaining components

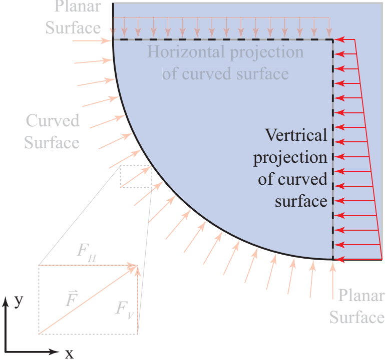
Hydrostatic Force on Submerged Curved Surfaces

- If we consider a purely 2-D case (x,y), lets look at the x-component of force:



Hydrostatic Force on Submerged Curved Surfaces

- If we project $d\vec{A}$ onto the vertical (y-z) plane, we see that the force (magnitude and line of action) is equal to the hydrostatic force as would be acting on a purely vertical plate (projection)



Hydrostatic Force on Submerged Curved Surfaces

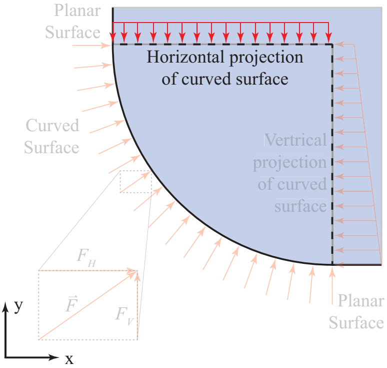
- This means (using H to designate horizontal force):

$$\boxed{F_{Rx} = F_H = P_c A}$$

- i.e. the horizontal force acting on a curved surface is equivalent to the force on a vertically submerged planar surface

Hydrostatic Force on Submerged Curved Surfaces

- Similarly, the vertical force, $F_{Ry}=F_V$, ignoring atmospheric pressure (assuming it is acting on the free surface and other side of the surface of interest not in contact with the fluid)



Hydrostatic Force on Submerged Curved Surfaces

- F_V is merely the weight of the fluid above the surface ($P{=}\rho g h$, $h dA_z{=}d\forall$):

$$F_{Ry} = F_v = \int_A P dA_z = \int_A \rho g h dA_z = \int_\forall \rho g d\forall = \boxed{\rho g \forall = F_V}$$

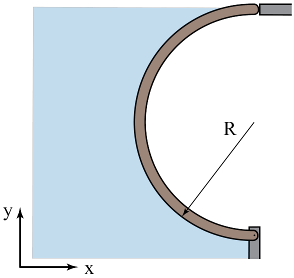
Hydrostatic Force on Submerged Curved Surfaces

- Therefore, we have the following equations to use on a curved surface in 2-D space:

$$F_{Rx} = F_H = P_c A$$
$$R_{Ry} = F_V = \rho g \forall$$
$$F_R = \sqrt{F_H^2 + F_V^2}$$

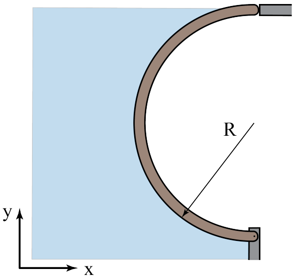
Example #1

- There is semi-circular sluice gate as depicted below. The semi-circle has a radius of 3 [m], hinged at the bottom, and it 4 [m] long. Determine:
  1. $\vec{F}_R$
  2. the force on the pin, as well as the top of the gate.



Example #1

- Solution:



Example #1

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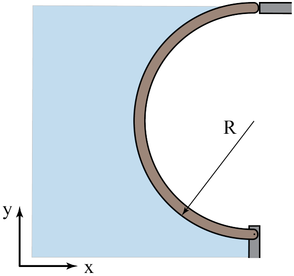

Example #1

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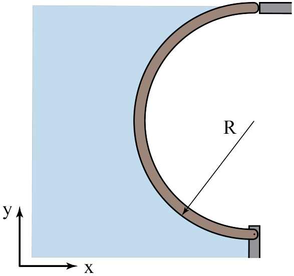

Example #1

- Solution: