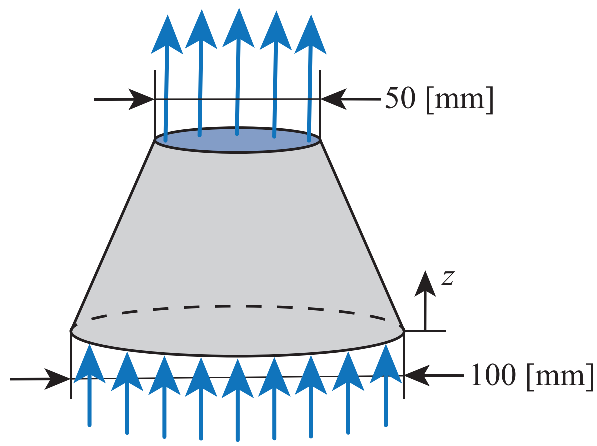
Homework #6

MEMS 0071 - Introduction to Fluid Mechanics

Assigned: October 12$^{\text{th}}$, 2019
Due: October 18$^{\text{th}}$, 2019

Problem #1

Water enters a nozzle with a velocity of 1.5 [m/s]. Determine:
a) the exit velocity;
b) the force in the z-direction required to hold the nozzle into position.



The mass flow rate into the system is found as:

$$\dot{m} = \rho AV = (998\,[\text{kg/m}^3])\left(\frac{\pi(0.1\,[\text{m}])^2}{4}\right)(1.5\,[\text{m/s}]) = 11.76\,[\text{kg/s}]$$

Thus, the fluid is exiting the system with the following velocity:

$$V = \frac{\dot{m}}{\rho A} = \frac{11.76\,[\text{kg/s}]}{(998\,[\text{kg/m}^3])\left(\frac{\pi(0.05\,[\text{m}])^2}{4}\right)} = 6\,[\text{m/s}]$$

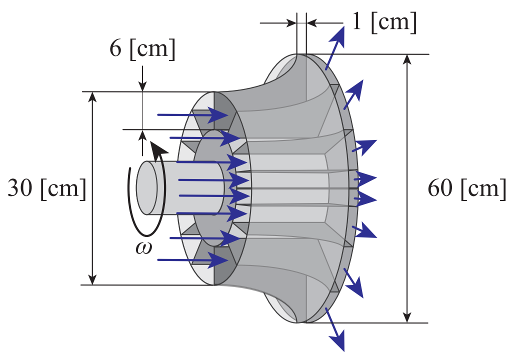
The Conservation of Linear momentum, under steady state, with the reactionary force existing in the positive z-direction, reduces to the following:

$$F_r = \int_{C.S} \vec{V}\rho(\vec{V}\cdot\vec{n})dA = \dot{m}V_{\text{out}} - \dot{m}V_{\text{in}} = (11.76\,[\text{kg/s}])((6-1.5)\,[\text{m/s}]) = 52.92\,[\text{N}]$$

Problem #2

The impeller of a centrifugal blower has a diameter of 30 [cm] and a blade width of 6 [cm] at the inlet, and a diameter of 60 [cm] and a blade with of 1 [cm] at the outlet. The blower increases the pressure of atmospheric air by 125 [kPa], whereas the temperature can be assumed invariant at 20°C. Disregarding mechanical losses, and assuming the air velocity at the inlet and outlet are equal to the impeller velocities at the inlet and outlet, determine:

1. The volumetric flow rate of when when the impeller is operated at 3,000 rpm and the power consumption of the blower is 450 [W].
2. The normal component of velocity at the inlet and outlet of the impeller.



Expressing the conservation of angular momentum, assuming steady-state, no surface or body torques:

$$\sum M = \cancelto{0}{\vec{T}_s} + \cancelto{0}{\vec{T}_b} + \vec{T}_{shaft} = \cancelto{0}{\frac{\delta}{\delta t}\int_{C.\forall.} (\vec{r} \times \vec{V}\rho d\forall} + \int_{C.S.} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n})dA$$

$$\implies \vec{T}_{shaft} = \dot{m}(r_2V_2 - r_1V_1) = \dot{m}\omega(r_2^2 - r_1^2)$$

To determine the mass flow rate one method is to determine the density and volumetric flow rate. Or one could determine the mass flow rate given the density, cross-sectional flow area and exit velocity. The density is found through the Ideal Gas Law:

$$\rho = \frac{P}{RT} = \frac{(226.325\,[\text{kPa}])}{(0.287\,[\text{kJ/kg-K}])(293\,[\text{K}])} = 2.691\,[\text{kg/m}^3]$$

The volumetric flow rate, which is density times mass flow rate, is found as follows:

$$\dot{W} = \omega\vec{T}_{shasft} = \rho\dot{\forall}\omega^2(r_2^2 - r_1^2)$$

$$\implies \dot{\forall} = \frac{\dot{W}}{\rho\omega^2(r_2^2 - r_1^2)} = \frac{450\,[\text{W}]}{(2.691\,[\text{kg/m}^3])\left(\frac{2\pi 3,000}{60}\left[\frac{\text{rad}}{\text{s}}\right]\right)^2((0.3\,[\text{m}])^2 - (0.15\,[\text{m}])^2)} = 0.025\,[\text{m}^3\text{s}]$$

The normal component of the velocity at the inlet is:

$$V_{in} = \frac{\dot{\forall}}{A_{in}} = \frac{0.025\,[\text{m}^3\text{s}]}{\left(\frac{\pi(0.3\,[\text{m}])^2}{4}\right) - \left(\frac{\pi(0.18\,[\text{m}])^2}{4}\right)} = 0.55\,[\text{m/s}]$$

The normal component of velocity at the outlet is:

$$V_{out} = \frac{\dot{\forall}}{A_{out}} = \frac{0.025\,[\text{m}^3\text{s}]}{\pi(0.6\,[\text{m}])(0.01\,[\text{m}])} = 1.36\,[\text{m/s}]$$

Problem #3

A sprinkler is depicted in the figure below. Water enters the bottom of the sprinkler with a pressure of 20 [kPa] gage with a volumetric flow rate of 12.5 [L/min]. The sprinkler rotates at 60 [rpm]. Given that the diameter of the sprinkler nozzle is 4 [mm], and the nozzle is oriented 45° above the horizontal plane, calculate the jet speed *relative* to the nozzle. That is, the water exiting the nozzle should have a greater speed that what the sprinkler rotates at. Also determine the total torque, $\vec{T}$, that the sprinkler produces under steady-state operation.

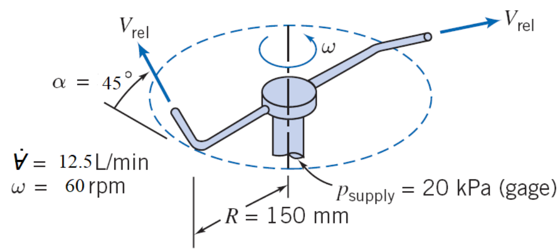


Figure 1: Schematic for Problem #4.

Expressing the conservation of angular momentum, assuming steady-state, no surface or body torques:

$$\sum M = \cancelto{0}{\vec{T}_s} + \cancelto{0}{\vec{T}_b} + \vec{T}_{shaft} = \cancelto{0}{\frac{\delta}{\delta t}\int_{C.\forall.}(\vec{r}\times\vec{V})\rho d\forall} + \int_{C.S.}(\vec{r}\times\vec{V})\rho(\vec{V}\cdot\vec{n})dA$$

$$\implies \vec{T}_{shaft} = \sum_{out}(\vec{r}\times\dot{m}\vec{V})$$

In scalar form, we need to solve for the relative velocity. The velocity leaving the jet is:

$$V_{jet} = \frac{\dot{\forall}}{2A_{jet}} = \frac{2.08\bar{3}\cdot 10^{-4}\,[\mathrm{m^3 s}]}{2\left(\pi(0.004\,[\mathrm{m}])^2\frac{}{4}\right)} = 8.29\,[\mathrm{m/s}]$$

The only contribution to the moment is the velocity component in-plane with the sprinkler arm:

$$V = V_{jet}\cos(45^\circ) = 5.86\,[\mathrm{m/s}]$$

Thus, the velocity, relative to the moving C.S. is:

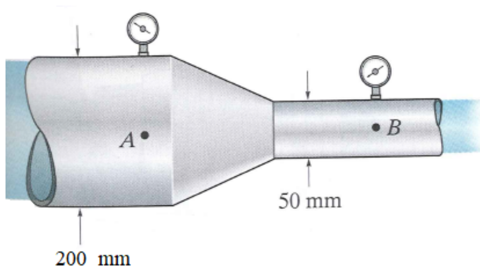
$$V_{rel} = V - \omega r = (3.51\,[\mathrm{m/s}]) - (2\pi\,[\mathrm{rad/s}])(0.15\,[\mathrm{m}]) = 4.92\,[\mathrm{m/s}]$$

Thus,

$$\vec{T}_{shaft} = \sum_{out} r\rho\dot{\forall}V_{rel} = (0.15\,[\mathrm{m}])(4.92\,[\mathrm{m/s}])(998\,[\mathrm{kg/m^3}])(12.5\,[\mathrm{L/min}])(0.001\,[\mathrm{m^3/L}])(1/60\,[\mathrm{min/s}]) = 0.153\,[\mathrm{N/m}]$$

Problem #4

Air enters A with a velocity of 6.5 [m/s], a pressure of 32 [kPa] gage and a temperature of 40 °C. Determine the pressure of the fluid at B.



At location A, the density of air is:

$$\rho = \frac{P}{RT} = \frac{(133.325\,[\text{kPa}])}{(0.287\,[\text{kJ/kg-K}])(313\,[\text{K}])} = 1.484\,[\text{kg/m}^3]$$

Applying the Bernoulli equation between locations A and B, assuming there is no change of elevation:

$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + \cancelto{0}{gz_A} = \frac{P_B}{\rho} + \frac{V_B^2}{2} + \cancelto{0}{gz_B}$$

Solving for P_B:

$$P_B = P_A + \rho\left(\frac{V_A^2 - V_B^2}{2}\right)$$

To determine the velocity at B, we apply the conservation of mass:

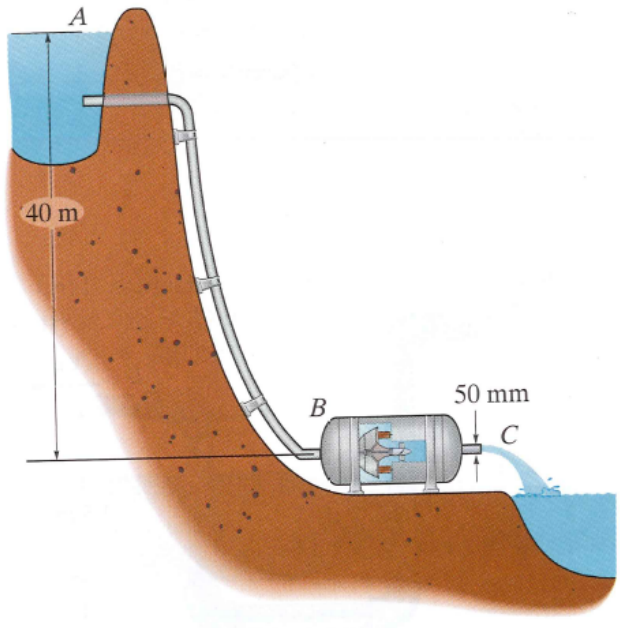
$$V_B = V_A\left(\frac{A_A}{A_B}\right) = (6.5\,[\text{m/s}])\left(\frac{(0.2\,[\text{m}])^2}{(0.05\,[\text{m}])^2}\right) = 104\,[\text{m/s}]$$

Thus,

$$P_B = 133.325\,[\text{kPa}]) + (1.484\,[\text{kg/m}^3])\left(\frac{(6.5\,[\text{m/s}])^2 - (104\,[\text{m/s}])^2}{2}\right) = 125.33\,[\text{kPa}]$$

Problem #5

Water from the upper reservoir flows through a 75 [m] long, 50 [mm] diameter pipe into the turbine at B. If the head loss is 0.1 [m] per 100 [m] of length, and the water exits the turbine at C with a velocity of 10 [m/s], determine the power output of the turbine assuming η=0.80.



Applying the Conservation of Energy between A and C:

$$\cancelto{0}{\frac{P_A}{\rho}} + \cancelto{0}{\frac{V_A^2}{2}} + gz_A + \cancelto{0}{h_{pump}} = \cancelto{0}{\frac{P_C}{\rho}} + \frac{V_C^2}{2} + \cancelto{0}{gz_C} + h_{turb} + h_{loss}$$

Thus, the turbine head is:

$$h_{turb} = z_A - \frac{V_C^2}{2g} - h_{loss} = (40\,[\text{m}]) - \frac{(10\,\text{m/s}])^2}{2(9.81\,[\text{m/s}])} - (75\,[\text{m}])\left(\frac{0.1\,[\text{m}]}{100\,[\text{m}]}\right) = 34.83\,[\text{m}]$$

The work of the turbine is expressed as:

$$\dot{W} = \dot{m} g h_{turb} \eta$$

Determining the mass flow rate:

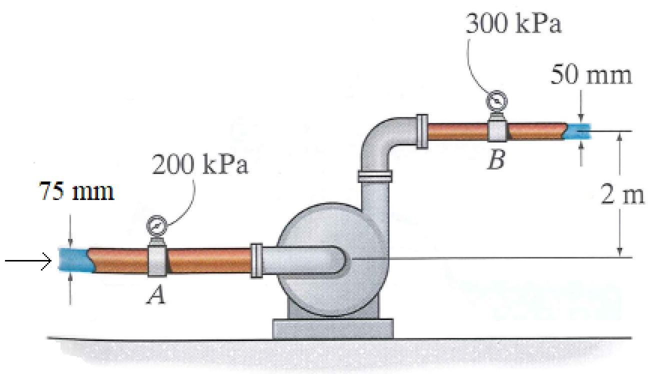
$$\dot{m} = (998\,[\text{kg/m}^3])(10\,[\text{m/s}])\left(\frac{\pi(0.05\,[\text{m}])^2}{4}\right) = 19.6\,[\text{kg/s}]$$

Therefore,

$$\dot{W} = (19.6\,[\text{kg/s}])(9.81\,[\text{m/s}^2])(34.83\,[\text{m}])(0.8) = 5.36\,[\text{kW}]$$

Problem #6

Given the water pressures at the inlet and exit of the pump as shown in the figure below, assuming the volumetric flow rate is 0.1 [m^3/s], determine the required pumping power.



Applying the Conservation of Energy between A and B:

$$\frac{P_A}{\rho} + \frac{V_A^2}{2} + gz_A + h_{pump} = \frac{P_B}{\rho} + \frac{V_B^2}{2} + gz_B + \cancelto{0}{h_{turb}} + \cancelto{0}{h_{loss}}$$

Therefore:

$$h_{pump} = \frac{P_A - P_B}{\rho g} + \frac{V_B^2 - V_A^2}{2g} + z_B$$

The velocity at A is determined from the Conservation of mass:

$$V_A = \frac{\dot{\forall}}{A_A} = \frac{(0.1\,[\mathrm{m}^3/\mathrm{s}])}{\left(\dfrac{\pi(0.075\,[\mathrm{m}])^2}{4}\right)} 22.64\,[\mathrm{m/s}]$$

The velocity at B is determined from the Conservation of mass:

$$V_B = \frac{\dot{\forall}}{A_B} = \frac{(0.1\,[\mathrm{m}^3/\mathrm{s}])}{\left(\dfrac{\pi(0.05\,[\mathrm{m}])^2}{4}\right)} 50.93\,[\mathrm{m/s}]$$

Therefore:

$$h_{pump} = \frac{(300,000 - 200,000)\,[\mathrm{Pa}]}{(998\,[\mathrm{kg/m}^3])(9.81\,[\mathrm{m/s}^2])} + \frac{(50.93\,[\mathrm{m/s}])^2 - (22.64\,[\mathrm{m/s}])^2}{(998\,[\mathrm{kg/m}^3])(9.81\,[\mathrm{m/s}^2])} + (2\,[\mathrm{m}]) = 118.3\,[\mathrm{m}]$$

The pump work is expressed as:

$$\dot{W} = \dot{m} g h_{pump} = (998\,[\mathrm{kg/m}^3])(0.1\,[\mathrm{m}^2/\mathrm{s}])(9.81\,[\mathrm{m/s}^2])(118.3\,[\mathrm{m}]) = 16\,[\mathrm{kW}]$$

Problem #7

There is a large tank, with the 1 open to atmosphere, that is filled with water to a height of h, from the center of the the outlet pipe 2. The valve on the outlet pipe is opened, allowing the water to flow out. Assume 2 is exposed to atmospheric pressure. Using the Bernoulli equation, derive the Toricelli equation, i.e. the expression for the outlet velocity as a function of height.

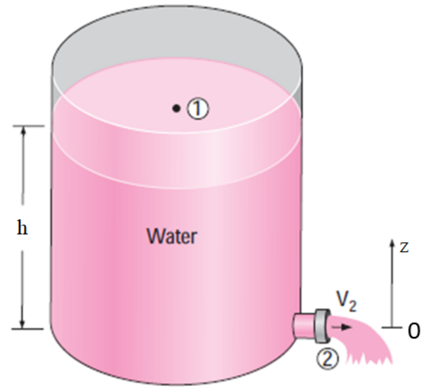


Figure 2: Schematic for Problem #1.

Starting with the Bernoulli Equation, and assuming the pressure at 1 and 2 are the same, and that the velocity at 1 is minimal:

$$\cancelto{0}{\frac{P_1}{\rho}} + \cancelto{0}{\frac{V_1^2}{2}} + gz_1 = \cancelto{0}{\frac{P_2}{\rho}} + \frac{V_2^2}{2} + gz_2$$

Therefore, defining $z_1 - z_2 = h$:

$$\boxed{V_2 = \sqrt{2gh}}$$

Problem #8

In Figure 2 provided below, the fountains at the Bellagio can shoot water as high as 140 [m] (distance between 1 and 2). The fountains uses pumps that pressurize the water to 250 [psi] (1,723.7 [kPa]) which is <u>*then*</u> sent through a nozzle with an exit diameter of 6.35 [cm]. Determine the exit velocity of the water from the nozzle at 1. Hint: be very discerning about the location of where you apply the pressure.

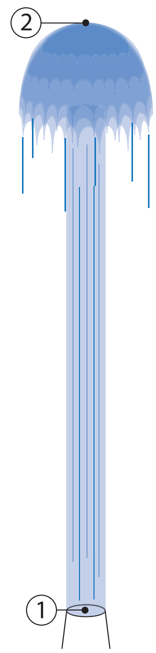


Figure 3: Schematic for Problem #2.

Starting with the Bernoulli Equation, assuming the pressures at 1 and 2 are atmospheric, that the velocity at 2 is zero, and that z_1 is the reference elevation:

$$\cancelto{0}{\frac{P_1}{\rho}} + \frac{V_1^2}{2} + \cancelto{0}{gz_1} = \cancelto{0}{\frac{P_2}{\rho}} + \cancelto{0}{\frac{V_2^2}{2}} + gz_2$$

Thus,

$$V_1 = \sqrt{2gz_2} = \sqrt{(2)(9.81\,[\mathrm{m/s^2}])(140\,[\mathrm{m}])}$$

$$\therefore \boxed{V_1 = 52.4\,[\mathrm{m/s}]}$$

Problem #9

In Figure 3 provided below, a nozzle forces air into the compressor of a jet engine. The jet is flying at 200 [m/s], which is the velocity that enters the engine at 1. The ratio of inlet to exit area of the inlet nozzle is 1.65. Determine the velocity of the air entering the compressor at location 2. If the mass flow rate through the jet engine is 192 [kg/s], and the ratio of nozzle inlet area 1 to nozzle exit area 3 is 2.875, assuming the density of air is still 1.225 [kg/m^3], determine the velocity and pressure at 3, as well as the pressures at 1 and 2. Hint: apply 0 at a location with known pressure and velocity.

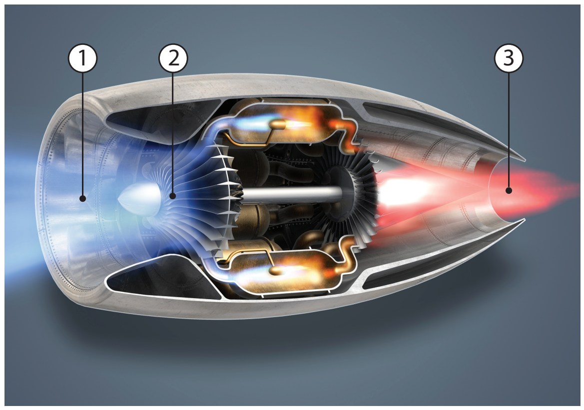


Figure 4: Schematic for Problem #3.

Placing 0 sufficiently far in front of the jet engine, assuming a zero-velocity flow at that point, and the same elevation as 1, 2 and 3, we have the following equation:

$$\frac{P_0}{\rho} + \cancelto{0}{\frac{V_0^2}{2}} + \cancelto{0}{gz_0} = \frac{P_1}{\rho} + \frac{V_1^2}{2} + \cancelto{0}{gz_1} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + \cancelto{0}{gz_2} = \frac{P_3}{\rho} + \frac{V_3^2}{2} + \cancelto{0}{gz_3}$$

Solving for P_1:

$$P_1 = P_0 - \rho\frac{V_1^2}{2} = 101,325\,[\text{Pa}] - (1.225\,[\text{kg/m}^3])\left(\frac{(200\,[\text{m/s}])^2}{2}\right)$$

$$\therefore \boxed{P_1 = 76.85\,[\text{kPa}]}$$

The equation for P_2 becomes:

$$P_2 = P_1 + \rho\left(\frac{V_1^2 - V_2^2}{2}\right)$$

Solving for V_2 based upon the continuity equation:

$$\dot{m}_1 = \dot{m}_2 \implies \rho A_1 V_1 = \rho A_2 V_2 \implies V_2 = \left(\frac{A_1}{A_2}\right) V_2$$

$$\therefore \boxed{V_2 = 330\,[\text{m/s}]}$$

$$\implies P_2 = 76,825\,[\text{Pa}] + (1.225\,[\text{kg/m}^2])\left(\frac{(200\,[\text{m/s}])^2 - (330\,[\text{m/s}])^2}{2}\right)$$

$$\therefore \boxed{P_2 = 34.623\,[\text{kPa}]}$$

Any determination of properties at 3 can not be done via the Bernoulli Equation due to the introduction/extraction of mechanical work and heat.