MEMS 0071
Fall 2019
Midterm #1
9/27/2019

Name (Print): ____________________________

This exam contains 8 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

Problem	Points	Score
1	25	
2	45	
3	30	
Total:	100	

Bonus: This day, September 27th, 1779, who was elected to negotiate with the British over the American Revolutionary War peace terms?

John Adams

1. (25 points) A differential manometer is given below. Determine the pressure difference $P_A - P_B$, in [kPa], given the following:

- $\gamma_{\text{glycerine}}$=12,373 [N/m^3]
- $\text{SG}_{C_6H_{12}O_6}$=1.38
- ρ_{air}=1.225 [kg/m^3]
- SG_{Hg}=13.6
- ρ_{oil}=900 [kg/m^3]

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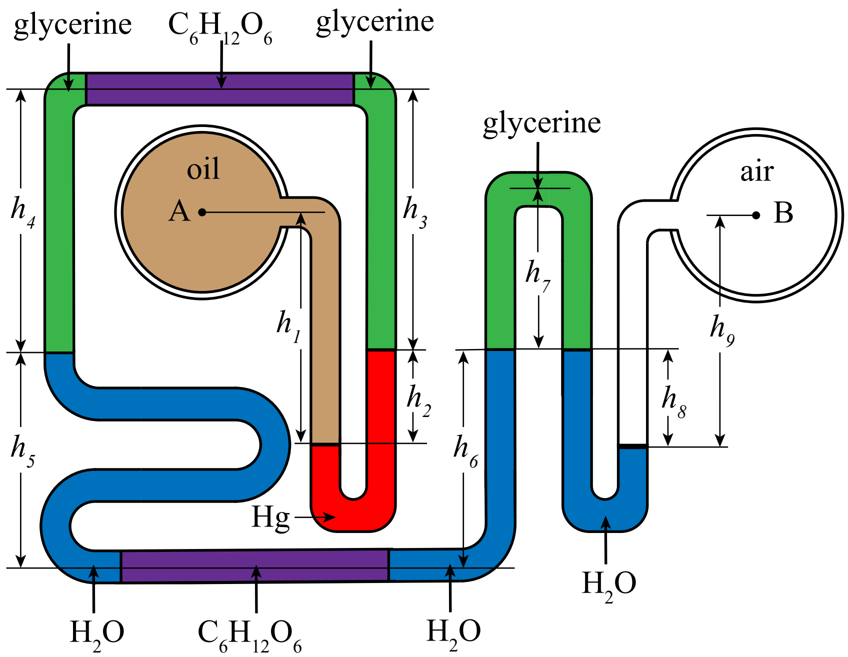
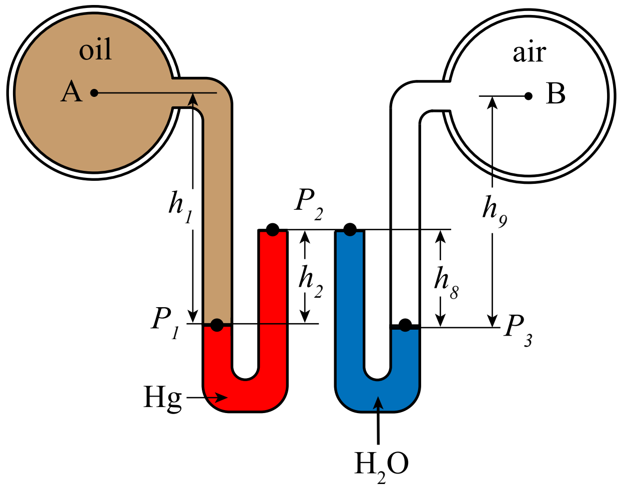
- ρ_{H_2O}=998 [kg/m^3]
- h_1=86.4 [mm]
- h_2=34.4 [mm]
- h_3=97.8 [mm]
- h_4=97.8 [mm]
- h_5=80.8 [mm]
- h_6=80.8 [mm]
- h_7=60.9 [mm]
- h_8=37.4 [mm]
- h_9=86.9 [mm]



The heights, h_3 and h_4 cancel out, as well as h_5 and h_6. Additionally, the glycerine and it's height h_7 are of no consequence. Thus, we are left with the following system:



We can construct the following system of equations:

$$P_1 = P_A + \rho_{\text{oil}} g h_1$$

$$P_2 = P_1 - \rho_{\text{Hg}} g h_2$$

$$P_3 = P_2 + \rho_{\text{H}_2\text{O}} g h_8$$

$$P_B = P_3 - \rho_{\text{air}} g h_9$$

Solving for the pressure difference between A and B:

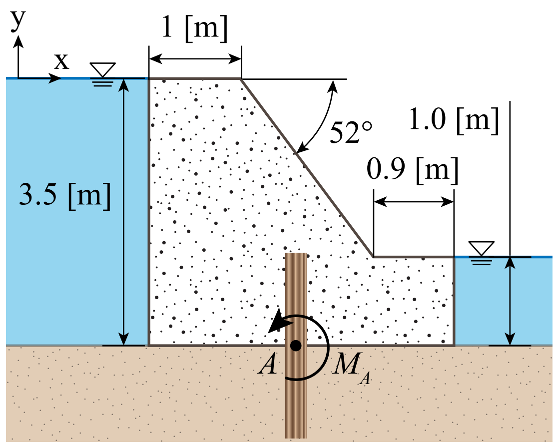
$$P_A - P_B = -g\{\rho_{\text{oil}} h_1 - \rho_{\text{Hg}} h_2 + \rho_{\text{H}_2\text{O}} h_8 - \rho_{\text{air}} h_9\}$$

$$= -(9.81\,[\text{m/s}^2])\{(900\,[\text{kg/m}^3])(0.0864\,[\text{m}]) - (13,600\,[\text{kg/m}^3])(0.0344\,[\text{m}]) + ...$$

$$... + (998\,[\text{kg/m}^3])(0.0374\,[\text{m}]) - (1.225\,[\text{kg/m}^3])(0.0869\,[\text{m}])\}$$

$$P_A - P_B = 3,461\,[\text{Pa}] = 3.46\,[\text{kPa}]$$

2. (45 points) A fixed-crest dam, such as Allegheny Dam 6, has a fixed height, and thus is unable to actively regulate river height. If the dam is 202 [m] long, the upstream river height is 3.5 [m] and the downstream river height is 1.0 [m], and the geometry of the dam is as annotated, determine the following:



a) (7.5 pts) The horizontal force acting on upstream face of the dam;

$$F_R = P_c A = \rho g h_c A = (998\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(1.75\,[\mathrm{m}])(3.5\,[\mathrm{m}])(202\,[\mathrm{m}]) = 12.11\,[\mathrm{MN}]$$

b) (7.5 pts) y′, for the resultant force acting on the upstream face of the dam;

$$y' = h_c + \frac{I_{\hat{x}\hat{x}}}{h_c A} = (1.75\,[\mathrm{m}]) + \frac{(202\,[\mathrm{m}])(3.5\,[\mathrm{m}])^3}{12(1.75\,[\mathrm{m}])(3.5\,[\mathrm{m}])(202\,[\mathrm{m}])} = 2.\bar{3}\,[\mathrm{m}]$$

By convention, this is measured from the surface of the upstream river.

c) (7.5 pts) The horizontal force acting on downstream face of the dam;

$$F_R = P_c A = \rho g h_c A = (998\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(0.5\,[\mathrm{m}])(1\,[\mathrm{m}])(202\,[\mathrm{m}]) = 989\,[\mathrm{kN}]$$

d) (7.5 pts) y′, for the resultant force acting on the downstream face of the dam;

$$y' = h_c + \frac{I_{\hat{x}\hat{x}}}{h_c A} = (0.5\,[\mathrm{m}]) + \frac{(202\,[\mathrm{m}])(1\,[\mathrm{m}])^3}{12(0.5\,[\mathrm{m}])(1\,[\mathrm{m}])(202\,[\mathrm{m}])} = 0.\bar{6}\,[\mathrm{m}]$$

By convention, this is measured from the surface of the upstream river.

e) (7.5 pts) The net reactionary force the timber pile must provide; The timber must exert a force in the direction from the downstream toward upstream (right to left) with a value of the difference of 12.11 [MN] and 989 [kN], i.e. 11.2 [MN]

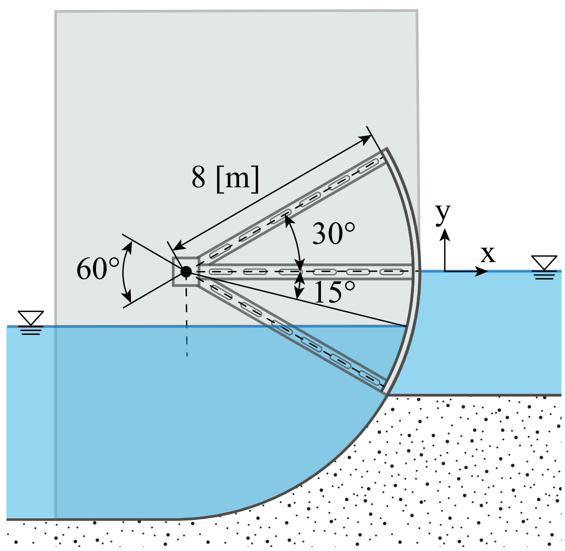
f) (7.5 pts) The moment about A the timber pile must provide for the dam to be static. We have 12.11 [MN] acting $1.1\bar{6}$ [m] above point A, causing a clockwise moment (we will denote this as a negative value), and 989 [kN] acting $0.\bar{3}$ [m] above point A, causing a counter-clockwise moment (we will denote this as a positive value). Thus, for the dam to be in equilibrium:

$$\sum M_A = 0 \implies -(12.11\,[\text{MN}])(1.1\bar{6}\,[\text{m}]) + (989\,[\text{kN}])(0.\bar{3}\,[\text{m}]) + M_R = 0$$

$$\implies M_R \approx 1.38 \cdot 10^8\,[\text{N-m}]$$

That is, a moment of that aforementioned magnitude needs to be applied in a counter-clockwise direction.

3. (30 points) A tainter gate is used to control the upstream river height. The tainter gate consists of two strut arms, each attached to pivot points on the dam structure, and a curved gate surface, as depicted below. The tainter gate is one-sixth of a circle, with struts arms (radius) of length 8 [m]. The angle between the strut arms, which are symmetric about the horizontal axis, is 60°. The gate is 30 [m] long. Given the geometry of the gate, the upstream and downstream water levels, and that the water is sea water, ρ=1,027 [kg/m^3], determine the following:



a) (10 pts) The net horizontal force acting on gate due to the water;

We will assume the strut arms and gate have infinite thickness. Looking at the upstream river level, we have to determine the height of the projected surface (dashed orange line), i.e. the height of the river, which we will denote as h_1. From trigonometry, knowing the hypotenuse of the blue triangle is 8 [m] and the angle between the base and hypotenuse is 30°:

$$h_1 = (8\,[\text{m}])(\sin(30^\circ)) = 4\,[\text{m}]$$

Thus, the force the upstream river is exerting in the negative x-direction on the gate is:

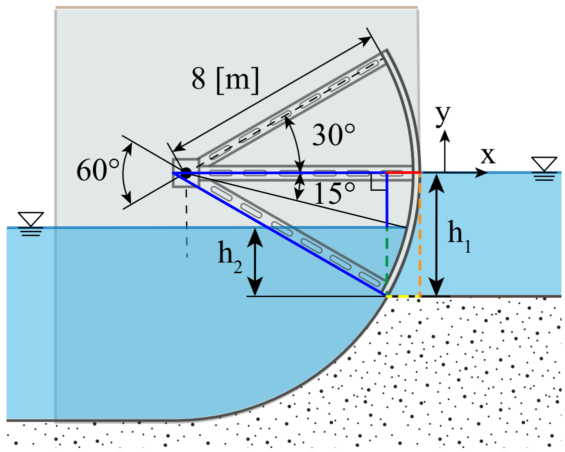
$$F_{R,-x} = P_c A = \rho g h_c A = (1,027\,[\text{kg/m}^3])(9.81\,[\text{m/s}^2])(2\,[\text{m}])(4\,[\text{m}])(30\,[\text{m}]) = 2.418\,[\text{MN}]$$

Similarly, at at the downstream river level, we have to determine the height of the projected surface, i.e. the height of the river, which we will denote as h_2. From trigonometry:

$$h_2 = (8\,[\text{m}])(\sin(30^\circ) - \sin(15^\circ)) = 1.93\,[\text{m}]$$

Thus, the net force the downstream river is exerting in the positive x-direction on the gate is:

$$F_{R,+x} = (1,027\,[\text{kg/m}^3])(9.81\,[\text{m/s}^2])(0.965\,[\text{m}])(1.93\,[\text{m}])(30\,[\text{m}]) = 0.563\,[\text{MN}]$$

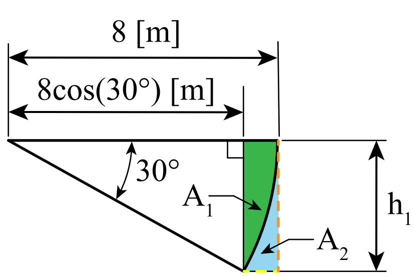


Thus, the net horizontal force acting on the gate due to the water is:

$$F_{x,net} = (-2.418 + 0.563)\,[\text{MN}] = -1.855\,[\text{MN}]$$

The minus sign indicates the force is acting the in the negative x-direction.

b) (10 pts) The net vertical force acting on the gate due to the water;
We first have to determine the pressure acting on the bottom of the upstream riverbed and multiply this by the projected area of the gate, i.e. the dashed yellow line, to give the vertical force acting in the positive y-direction.



Next, we need to determine the weight of the water above the projected horizontal surface, i.e. dashed yellow line. We know there is a rectangle with a base of 1.07 [m] (i.e. (8-8cos(30°) [m]) and a height of 4 [m]. Thus, the upstream fluid volume above the projected horizontal surface is simply A_2 times the length of the gate. A_2 is found by subtracting A_1 from the area of the box representing the projected surfaces:

$$A_1 = \left(\frac{30°}{360°}\right)\pi(8\,[\text{m}])^2 - \frac{(8\cos(30°)\,[\text{m}])(4\,[\text{m}])}{2} = 2.9\,[\text{m}^2]$$

$$A_2 = (1.07\,[\text{m}])(4\,[\text{m}]) - A_1 = 1.38\,[\text{m}^2]$$

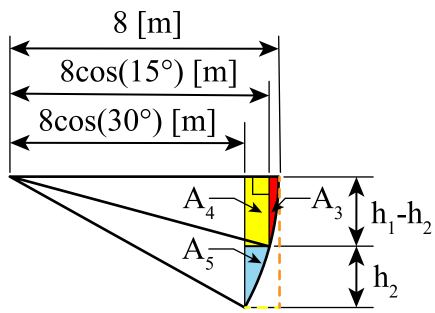
$$\forall_{\text{upstream}} = (1.38\,[\text{m}^2])(30\,[\text{m}]) = 41.4\,[\text{m}^3]$$

Thus,

$$W_{\text{upstream}} = \rho g \forall_{\text{upstream}} = (1{,}027\,[\text{kg/m}^3])(9.81\,[\text{m/s}^2])(41.4\,[\text{m}^3]) = 0.417\,[\text{MN}]$$

We can apply the same approach to calculating the weight of the fluid of the downstream river above the projected horizontal surface. Firstly, we calculate the area A_3 by taking the area of the arc made by a 15° angle less the area of the triangle with a height of h_1-h_2, i.e. 8sin(15° [m], i.e. 2.07 [m], and a base of 8cos(15°) [m]:

$$A_3 = \left(\frac{15°}{360°}\right)\pi(8\,[\text{m}])^2 - \frac{(8\cos(15°)\,[\text{m}])(2.07\,[\text{m}])}{2} = 0.38\,[\text{m}^2]$$



Then, A_5, the area of the water above the downstream gate, is simply A_1 less A_3 less A_4:

$$A_5 = 2.9\,[\text{m}^2] - 0.38\,[\text{m}^2] - ((8\cos(15°)\,[\text{m}] - (8\cos(30°)\,[\text{m}])(2.07\,[\text{m}]) = 0.87\,[\text{m}^2]$$

$$\forall_{\text{downstream}} = (0.87\,[\text{m}^2])(30\,[\text{m}]) = 25.97\,[\text{m}^3]$$

Thus,

$$W_{\text{downstream}} = \rho g \forall_{\text{downstream}} = (1,027\,[\text{kg/m}^3])(9.81\,[\text{m/s}^2])(25.97\,[\text{m}^3) = 0.262\,[\text{MN}]$$

The net force acting on the gate in the y-direction is:

$$F_{y,net} = (0.417 - 0.262)\,[\text{MN}] = 0.155\,[\text{MN}]$$

This, of course, is acting upwards.

c) (10 pts) The line of action, i.e. the angle, of the resultant force acting on the gate.
We can say the force is acting normal to the surface of the gate, with a magnitude of 1.86 [MN], acts at angle θ measured below the horizontal axis such that:

$$\theta = \tan^{-1}\left(\frac{F_{y,net}}{F_{x,net}}\right) = 4.77°$$