Homework #8

MEMS 0071 - Introduction to Fluid Mechanics

Assigned: November 3rd, 2019
Due: November 8th, 2019

Problem #1

If the x- and y-components of a steady velocity field of a incompressible fluid are give as:

$$u = 2y; \qquad v = 4x$$

Determine the stream function describing this velocity field.

Problem #2

Consider the following stream function:

$$\psi(x, y) = y^2 - x$$

Determine the velocity of the flow for the position y=1 [m] and along the streamline $\psi(x, y)$=2 [m^2/s]. Furthermore, show the stream function satisfies continuity.

Problem #3

If a uniform flow field is described by the following velocity field

$$u = U\sin(\theta); \qquad v = -U\cos(\theta)$$

where U is a constant, determine the stream function for this flow, and verify that it satisfies continuity.

Problem #4

If the streamline for a fluid flowing around a corner is given as

$$\psi(x, y) = 5xy\,[\mathrm{m}^2/\mathrm{s}]$$

Determine the x- and y-components of velocity at the point (2, 3)

Problem #5

If the velocity field for a flow is given as

$$\vec{V} = 6y\hat{\imath} + 3x\hat{\jmath}\,[\mathrm{m/s}]$$

Determine the equation of the streamline, and evaluate said streamline for the point (1, 2)

Problem #6

If an inviscid fluid is steadily flowing around a sphere with radius r, where the velocity profile is given as

$$\vec{V} = V_o\left(1 + \frac{r^3}{x^3}\right)$$

where V_o is the free-stream velocity. Considering the flow in the x-direction only, where the flow can be broken down into three regions:

$$\vec{V} = \begin{cases} V_o\hat{\imath} & x < -10 \\ V\hat{\imath} & -10 \leq x < r \\ 0 & x = -r \end{cases}$$

Determine the pressure variation along the streamline in the x-direction from $-\infty$ to $-r$. Hint: energy must be conserved along a streamline.

Problem #7

Given the following velocity field

$$u = a(x^2 - y^2); \qquad v = -2axy$$

Determine the stream function describing it.