MEMS 0071
Fall 2019
Midterm #2
10/25/2019

Name (Print): ______________________________

This exam contains 6 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or notes. Calculators are permitted on this exam.

The following rules apply:

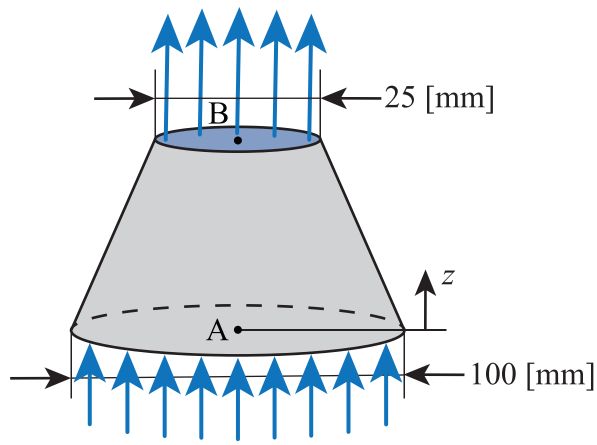
- All work must be done in the blue testing book. Any work done on the exam question sheet will not be graded.
- All work must be substantiated. A result with no methodology and mathematics will not be graded.
- Do not write in the table to the right.

Problem	Points	Score
1	25	
2	25	
3	25	
4	25	
Total:	100	

Bonus: This day, October 25th, 1917, marked the start of the Bolshevik Revolution, which brought which party to pre-eminence in Soviet Russia?

Communist

1. (25 points) If a nozzle is attached to the end of a pipe, and the pressure at point A is 200 [kPa], determine the reactionary force needed to hold the nozzle in place.



We start by applying the Conservation of Linear Momentum:

$$\sum \vec{F} = \frac{\partial}{\partial t}\int_{\forall} \rho \vec{V}\, d\forall + \int_{C.S.} \rho \vec{V}(\vec{V}\cdot\vec{n})\, dA = 0$$

Assuming steady state, the external forces acting on the nozzle are both the reactionary (-z direction) and a surface force (+z direction) due to the pressure on the bottom face, and the flow is normal to the control surface defining the top and bottom outlet and inlet, respectively, the Conservation of Linear Momentum reduces to:

$$\vec{F}_s - \vec{F}_R = \rho(V_B^2 A_B - V_A^2 A_A) = (998\,[\mathrm{kg/m^3}])\left(V_B^2\left(\frac{\pi(0.025\,[\mathrm{m}])^2}{4}\right) - V_A^2\left(\frac{\pi(0.1\,[\mathrm{m}])^2}{4}\right)\right)$$

From the continuity equation:

$$\dot{m}_{in} = \dot{m}_{out} \implies \rho V_A A_A = \rho V_B A_B \implies V_B = 16 V_A$$

To determine the velocities at A and B, we can apply the Bernoulli equation. Assuming the change of height (i.e. z) is negligible, and the fluid is water:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + \cancelto{0}{z_A} = \frac{P_B}{\rho g} + \frac{V_B^2}{2g} + \cancelto{0}{z_B} \implies V_B^2 - V_A^2 = \frac{2\cancelto{200\,[\mathrm{kPa}]}{(P_A - P_B)}}{\rho}$$

With two equations and two unknowns:

$$V_A = 1.254\,[\mathrm{m/s}]; \quad V_B = 20.06\,[\mathrm{m/s}]$$

Thus, the reactionary force is:

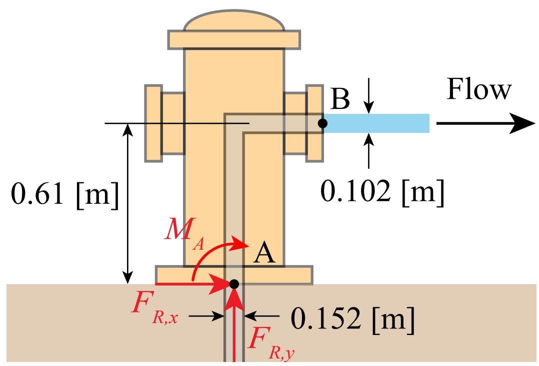
$\vec{F}_R =$

$(200\,[\text{kPa}])\left(\dfrac{\pi(0.1\,[\text{m}])^2}{4}\right) - (998\,[\text{kg/m}^3])\left((20.06\,[\text{m/s}])^2\left(\dfrac{\pi(0.025\,[\text{m}])^2}{4}\right) - (1.254\,[\text{m/s}])^2\left(\dfrac{\pi(0.1\,[\text{m}])^2}{4}\right)\right)$

$= 1.39\,[\text{kN}]$

This force would act in the negative z-direction.

2. (25 points) Consider a fire hydrant that weights 27.22 [kg]. If water is flowing through the hydrant at a rate of 0.057 [m^3/s], determine the reactionary forces and moments at the base of the hydrant.



We start by applying the Conservation of Linear Momentum to the principal directions, assuming steady-state. Starting with the x-direction:

$$\sum \vec{F}_x = \cancelto{0}{\frac{\partial}{\partial t}\int_{\forall} \rho V_x \, d\forall} + \int_{C.S.} \rho V_x^2 \, dA$$

$$\implies \vec{F}_{R,x} = \rho V_x^2 A$$

To solve for the velocity of the flow exiting the hydrant in the x-direction, we apply the definition of volumetric flow:

$$0.057\,[\text{m}^3/\text{s}] = V_x A \implies V_x = \frac{0.057\,[\text{m}^3/\text{s}]}{\left(\dfrac{\pi(0.102\,[\text{m}])^2}{4}\right)} = 6.98\,[\text{m/s}]$$

Substituting this back into the Conservation of Linear Momentum for the x-direction:

$$\vec{F}_{R,x} = (998\,[\text{kg/m}^3])(6.98\,[\text{m/s}])^2)\left(\frac{\pi(0.102\,[\text{m}])^2}{4}\right) = 397.3\,[\text{N}]$$

There is no pressure force acting on the exit of the fluid from the hydrant because it is at atmosphere. Continuing with the y-direction:

$$\sum \vec{F}_x = \cancelto{0}{\frac{\partial}{\partial t}\int_{\forall} \rho V_x \, d\forall} + \int_{C.S.} \rho V_x^2 \, dA$$

$$\implies \vec{F}_{R,y} = \vec{W} + \rho V_y^2 A - \vec{F}_{P,y}$$

The pressure, linear momentum and reactionary force act in the positive y-direction while the weight acts in the negative y-direction. Solving for the velocity in the y-direction from the volumetric flow:

$$0.057\,[\mathrm{m}^3/\mathrm{s}] = V_y A \implies V_y = \frac{0.057\,[\mathrm{m}^3/\mathrm{s}]}{\left(\frac{\pi(0.152\,[\mathrm{m}])^2}{4}\right)} = 3.14\,[\mathrm{m/s}]$$

The pressure acting on the inlet of the hydrant is found via applying the Bernoulli equation between points A and B. The pressure at B is atmospheric (zero gage). Thus:

$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + \cancelto{0}{z_A} = \cancelto{0}{\frac{P_B}{\rho g}} + \frac{V_B^2}{2g} + z_B \implies P_A = \rho\left\{\left(\frac{V_B^2 - V_A^2}{2}\right) + g z_B\right\}$$

$$P_A = (998\,[\mathrm{kg/m}^3])\left\{\left(\frac{(6.98\,[\mathrm{m/s}])^2 - (3.14\,[\mathrm{m/s}])^2}{2}\right) + 9.81\,[\mathrm{m/s}^2])(0.61\,[\mathrm{m}]\right\} = 25.36\,[\mathrm{kPa}]$$

Thus, the Conservation of Linear Momentum in the y-direction is:

$$\vec{F}_{R,y} =$$

$$(27.22\,[\mathrm{kg}])(9.81\,[\mathrm{m/s}^2]) - (998\,[\mathrm{kg/m}^3])(3.14\,[\mathrm{m/s}])^2)\left(\frac{\pi(0.152\,[\mathrm{m}])^2}{4}\right) - (25.36\,[\mathrm{kPa}])\left(\frac{\pi(0.152\,[\mathrm{m}])^2}{4}\right)$$

$$= -371\,[\mathrm{N}]$$

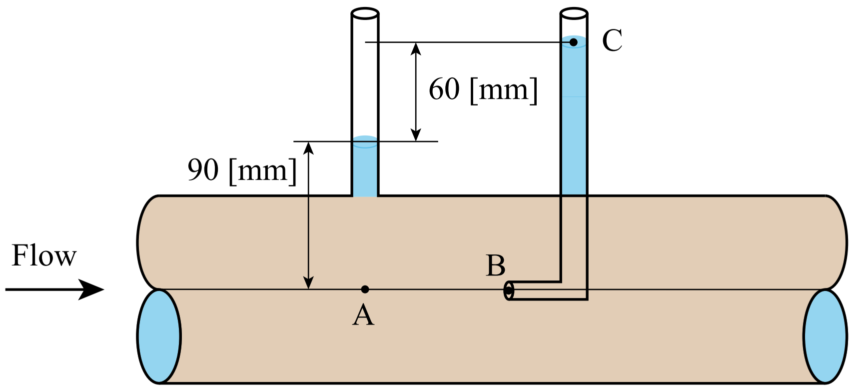
Thus, as drawn, the reactionary force in the y-direction should be acting downward with a magnitude of 371 [N]. Lastly, to solve the for the Conservation of Angular Momentum, we simply determine the reactionary moment:

$$\sum \vec{M}_A = \cancelto{0}{\frac{\partial}{\partial t}\int_{\forall} \rho(\vec{V}\times\vec{r}\, d\forall} + \int_{C.S.} \rho(\vec{V}\times\vec{r})\cancelto{\vec{V}_x}{(\vec{V}\cdot\vec{n})}\, dA$$

The cross product is simply $\vec{V}_x$ multiplied by the moment arm of 0.61 [m]. Thus:

$$\vec{M}_A = (998\,[\mathrm{kg/m}^3])(6.98\,[\mathrm{m/s}])^2(0.61\,[\mathrm{m}])\left(\frac{\pi(0.102\,[\mathrm{m}])^2}{4}\right) = 242.4\,[\mathrm{N/m}]$$

3. (25 points) Given the following static and dynamic pressure port set-up, determine the average velocity of water flowing through a pipe.



The pressure difference between points A and B is simply the hydrostatic pressure as measured between the static and dynamic pressure port:

$$P_B - P_A = \rho g h = (998\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(0.060\,[\mathrm{m}]) = 587.4\,[\mathrm{Pa}]$$

That makes physical sense, for the pressure at B is higher, for all kinetic energy of the fluid has been converted into pressure energy, pushing the fluid column higher than at point A. All that remains is applying the Bernoulli equations between points A and B:

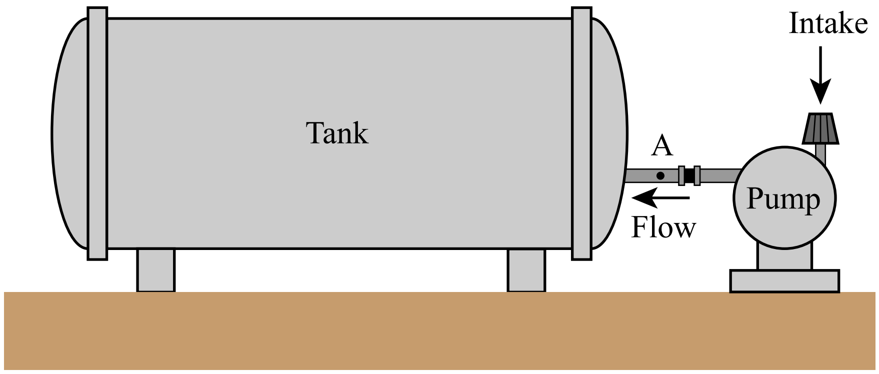
$$\frac{P_A}{\rho g} + \frac{V_A^2}{2g} + \cancelto{0}{z_A} = \frac{P_B}{\rho g} + \cancelto{0}{\frac{V_B^2}{2g}} + \cancelto{0}{z_B} \Longrightarrow V_A = \sqrt{2\left(\frac{P_B - P_A}{\rho}\right)}$$

$$V_A = \sqrt{2\left(\frac{587.4\,[\mathrm{Pa}]}{998\,[\mathrm{kg/m^2}]}\right)} = 1.08\,[\mathrm{m/s}]$$

4. (25 points) A tank, with a volume of 1.5 [m^3], is being filled with air. The air enters the tank (point A) with a velocity of 8 [m/s], through a pipe with a diameter of 10 [mm]. The air is entering the tank at a temperature of 303 [K] and a pressure of 500 [kPa]. Determine the rate at which the density within the tank changing at this instant. The gas constant is taken as 0.287 [kJ/kg-K].

We start by applying the Conservation of Mass:

$$\frac{dm_{sys}}{dt} = \frac{\partial}{\partial t}\int_{\forall} \rho\, d\forall + \int_{C.S.} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$



Thus, the time rate of change of density is merely the negative of the mass flux into the system per the volume of the system. Assuming the flow is turbulent and normal to the control surface, and determining the density of air entering the control volume through the Ideal gas law:

$$\frac{\partial \rho}{\partial T} = -\frac{1}{\forall}\int_{C.S.} \rho(\vec{V}\cdot\vec{n})\,dA = -\frac{1}{1.5\,[\mathrm{m}^3]}\left(\frac{500\,[\mathrm{kPa}]}{(0.287\,[\mathrm{kJ/kg\text{-}K}])(303\,[\mathrm{K}])}\right)(8\,[\mathrm{m/s}])\left(\frac{\pi(0.01\,[\mathrm{m}])^2}{4}\right)$$

$$= 2.41\cdot 10^{-3}\,[\mathrm{kg/m^3\text{-}s}]$$