Chapter 5 - Differential Analysis of Fluid Motion

Lecture 26
Section 5.4

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
University of Pittsburgh

Student Learning Objectives

Students should be able to:

- Solve two-dimensional steady-state planar Poiseuille flow
- Solve two-dimensional steady-state multi-fluid Planar Couette flow

Newtonian Fluid - N.S. Equations

- Thus, the Navier-Stokes equations in x, y and z-directions

x-direction:

$$\rho\left(\frac{\delta u}{\delta t}+u\frac{\delta u}{\delta x}+v\frac{\delta u}{\delta y}+w\frac{\delta u}{\delta z}\right)=-\frac{\delta P}{\delta x}+\mu\left(\frac{\delta^2 u}{\delta x^2}+\frac{\delta^2 u}{\delta y^2}+\frac{\delta^2 u}{\delta z^2}\right)+\rho g_x$$

y-direction:

$$\rho\left(\frac{\delta v}{\delta t}+u\frac{\delta v}{\delta x}+v\frac{\delta v}{\delta y}+w\frac{\delta v}{\delta z}\right)=-\frac{\delta P}{\delta y}+\mu\left(\frac{\delta^2 v}{\delta x^2}+\frac{\delta^2 v}{\delta y^2}+\frac{\delta^2 v}{\delta z^2}\right)+\rho g_y$$

z-direction:

$$\rho\left(\frac{\delta w}{\delta t}+u\frac{\delta w}{\delta x}+v\frac{\delta w}{\delta y}+w\frac{\delta w}{\delta z}\right)=-\frac{\delta P}{\delta z}+\mu\left(\frac{\delta^2 w}{\delta x^2}+\frac{\delta^2 w}{\delta y^2}+\frac{\delta^2 w}{\delta z^2}\right)+\rho g_z$$

Plane Poiseuille Flow

- Consider a simple case of planar, pressure-driven flow. Assume the flow is steady-state, incompressible, and laminar. Assume the fluid is Newtonian.
- Calculate the velocity field

Plane Poiseuille Flow

Solution:

Plane Poiseuille Flow

Solution:

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Solution:

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Solution:

Plane Poiseuille Flow

Solution:

Plane Multi-Fluid Couette Flow

- Consider a Couette flow were there are two immiscible fluids existing between two parallel plates. The top plate moves with a velocity magnitude U in the x-direction while the bottom plate is stationary. Gravity acts in the negative z-direction. The interface between the two liquids is assumed parallel to the top and bottom plates.
- Assume the flow is steady-state, incompressible, and laminar. Assume the fluid is Newtonian.
- Calculate the velocity field

Plane Multi-Fluid Couette Flow

Schematic:

Plane Multi-Fluid Couette Flow

Solution:

Plane Multi-Fluid Couette Flow

Solution:

Plane Multi-Fluid Couette Flow

Solution:

Plane Multi-Fluid Couette Flow

Solution:

Plane Multi-Fluid Couette Flow

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