MEMS 0071
Midterm #1
9/28/2018

Name (Print): Solutions

Version B

This exam contains 5 pages (including this cover page) and 3 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes. A calculator is permitted on this exam.

Do not write in the table to the right.

You are required to show your work on each problem on this exam. Please do all of your work in the blue book provided. The following rules apply:

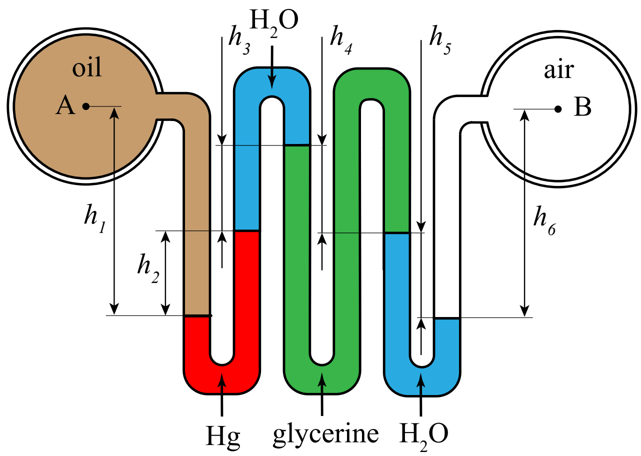
- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	30	
2	40	
3	30	
Total:	100	

- **BONUS** (5 pts): This day, September 28$^{\text{th}}$, 1939, marks the data when Nazi Germany and the Soviet Union agree on the division of which country?

Pressure Differential

1. (30 points) Given the manometer below, find the pressure difference $\Delta P = P_A - P_B$. The heights are $h_1 = h_6 = 86$ [mm], $h_2 = h_3 = h_4 = h_5 = 35$ [mm]. The fluid properties are $\rho_{H_2O} = 998$ [kg/m^3], $SG_{Hg} = 13.6$, $\rho_{air} = 1.225$ [kg/m^3], $\gamma_{glycerine} = 11{,}067$ [N/m^3] and $\rho_{oil} = 900$ [kg/m^3].



Creating intermediate pressures at the interfaces of the fluids, we have the following system of equations:

$$P_1 = P_A + \rho_{oil} g h_1$$
$$P_2 = P_1 - \rho_{Hg} g h_2$$
$$P_3 = P_2 - \rho_{H_2O} g h_3$$
$$P_4 = P_3 + \rho_{glyc} g h_4$$
$$P_5 = P_4 + \rho_{H_2O} g h_5$$
$$P_B = P_5 - \rho_{air} g h_6$$

Therefore:

$$\Delta P = P_A - P_B = -g(\rho_{oil} h_1 - \rho_{Hg} h_2 - \rho_{H_2O} h_3 + \rho_{glyc} h_4 + \rho_{H_2O} h_5 - \rho_{air} h_6)$$

The density of mercury is taken as:

$$\rho_{Hg} = SG_{Hg} \rho_{H_20,\,4^\circ C} = (13.6)(1,000\,[\mathrm{kg/m^3}]) = 13,600\,[\mathrm{kg/m^3}]$$

The density of glycerine is taken as:

$$\rho_{glyc} = \frac{\gamma_{glyc}}{g} = \frac{11,067\,[\mathrm{N/m^3}]}{9.81\,[\mathrm{m/s^2}]} = 1,128\,[\mathrm{kg/m^3}]$$

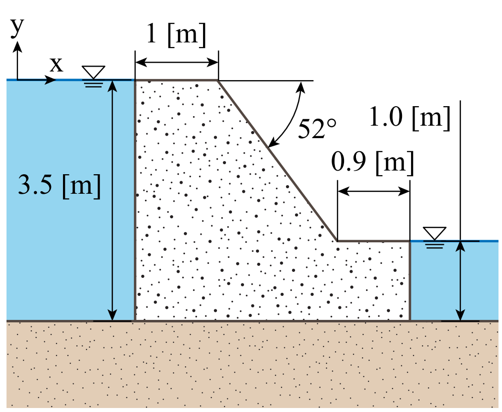
Substituting in the values:

$$\begin{aligned}\Delta P = P_A - P_B = -9.81\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]\Bigg\{&\left(900\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.086\,[\mathrm{m}]) - \left(13600\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.035\,[\mathrm{m}])\\ &- \left(998\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.035\,[\mathrm{m}]) + \left(1128\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.035\,[\mathrm{m}])\\ &+ \left(998\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.035\,[\mathrm{m}]) - \left(1.225\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.086\,[\mathrm{m}])\Bigg\}\\ = \boxed{3.52\,[\mathrm{kPa}]}\end{aligned}$$

Forces on a Planar Surface

2. (40 points) A fixed-crest dam, such as Allegheny Dam 6, has a fixed height, and thus is unable to actively regulate river height. If the dam is 202 [m] long, the upstream river height is 3.5 [m] and the downstream river height is 1.0 [m], and the geometry of the dam is as annotated, determine:

   a) (7.5 pts) The horizontal force acting on upstream face of the dam
   b) (10 pts) y′, for the resultant force acting on the upstream face of the dam
   c) (7.5 pts) The horizontal force acting on downstream face of the dam
   d) (10 pts) y′, for the resultant force acting on the downstream face of the dam
   e) (5 pts) The net horizontal force acting on the dam



a) The resultant force on the upstream face is found as:

$$F_{R+} = P_c A = (\rho g h_c)A = (998\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(1.75\,[\mathrm{m}])(3.5\,[\mathrm{m}])(202\,[\mathrm{m}]) = \boxed{12.113\,[\mathrm{MN}]}$$

b) The y-location, as measured from the surface of the upstream river, for where the force acts is:

$$y'_+ = h_c + \frac{I_{xx}}{h_c A} = (1.75\,[\mathrm{m}]) + \left(\frac{1}{12}\right)\frac{(202\,[\mathrm{m}])(3.5\,[\mathrm{m}])^3}{(1.75\,[\mathrm{m}])(3.5\,[\mathrm{m}])(202\,[\mathrm{m}])} = \boxed{2.\bar{3}\,[\mathrm{m}]\,(\text{downward})}$$

c) The resultant force on the downstream face is found as:

$$F_{R-} = P_c A = (\rho g h_c)A = (998\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(0.5\,[\mathrm{m}])(1.0\,[\mathrm{m}])(202\,[\mathrm{m}]) = \boxed{0.988\,[\mathrm{MN}]}$$

d) The y-location, as measured from the surface of the downstream river, for where the force acts is:

$$y'_{-} = h_c + \frac{I_{xx}}{h_c A} = (0.5\,[\mathrm{m}]) + \left(\frac{1}{12}\right)\frac{(202\,[\mathrm{m}])(1.0\,[\mathrm{m}])^3}{(0.5\,[\mathrm{m}])(1\,[\mathrm{m}])(202\,[\mathrm{m}])} = \boxed{0.\bar{6}\,[\mathrm{m}]\ (\text{downward})}$$

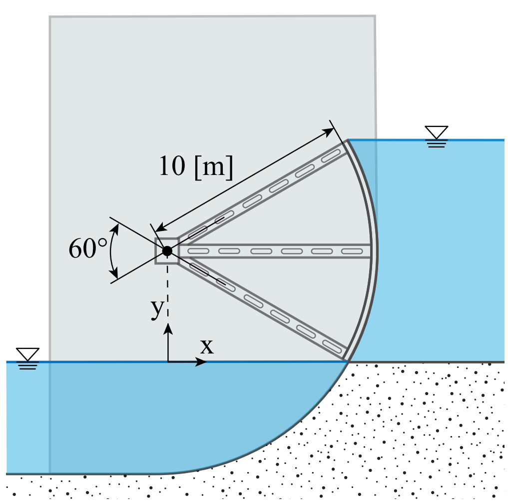
e) The net horizontal force, acting in the positive x-direction, acting on the dam is:

$$F_{\text{net}} = F_{R+} - F_{R-} = (12.113 - 0.988)\,[\mathrm{MN}] = \boxed{11.124\,[\mathrm{MN}]}$$

Forces on a Curved Surface

3. (30 points) A tainter gate is used to control the upstream river height. The tainter gate consists of two strut arms, each attached to pivot points on the dam structure, and a curved gate surface, as depicted below. The tainter gate is semi-circular, with struts arms of length 10 [m]. The angle between the strut arms is 60°. The gate is 30 [m] long. Given the geometry, and that the water is sea water, ρ=1,027 [kg/m^3], determine the following:

   a) (10 pts) The net horizontal force acting on gate
   b) (10 pts) The net vertical force acting on the gate
   c) (10 pts) The line of action of the resultant force acting on the gate



a) The net horizontal force acting on the gate would be that of a horizontal force acting on vertical plane for which the gate is projected:

$$F_H = P_c A = (\rho g h_c)A = (1027\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])(5\,[\mathrm{m}])(10\,[\mathrm{m}])(30\,[\mathrm{m}]) = \boxed{15.11\,[\mathrm{MN}]}$$

b) The net vertical force acting on the gate would be that of a vertical force acting on a horizontal place for which the gate is projected. That is, the vertical force is the density of the fluid times the gravitational acceleration times the volume of fluid displaced by the gate within the rectangular volume enclosed by the horizontal and vertical planes for which the curved surface is projected on. The area of the displaced fluid is one sixth of a circle with a 10 [m] radius minus the area of an equilateral triangle with sides of 10 [m].

$$F_V = \rho g \forall = (1027\,[\mathrm{kg/m^3}])(9.81\,[\mathrm{m/s^2}])\left(\frac{\pi(10\,[\mathrm{m}])^2}{6} - \frac{\sqrt{3}(10\,[\mathrm{m}])^2}{4}\right)(30\,[\mathrm{m}]) = \boxed{2.74\,[\mathrm{MN}]}$$

c) The line of action can be determined by calculating y', the total resultant force, as well as the angle for which the resultant acts, θ. Starting with y', which is taken in reference to the upstream river level:

$$y' = h_c + \frac{I_{xx}}{h_c A} = (5\,[\mathrm{m}]) + \left(\frac{1}{12}\right)\frac{(30\,[\mathrm{m}])(10\,[\mathrm{m}])^3}{(5\,[\mathrm{m}])(10\,[\mathrm{m}])(30\,[\mathrm{m}])} = \boxed{6.6\bar{6}\,[\mathrm{m}]\ (\mathrm{downward})}$$

The resultant is the square root of the sum of the squares of the horizontal and vertical forces:

$$R = \sqrt{F_H^2 + F_V^2} = \sqrt{(15.11\,[\mathrm{MN}])^2 + (2.74\,[\mathrm{MN}])^2} = \boxed{15.36\,[\mathrm{MN}]}$$

Lastly, the angle for which the resultant acts, at some depth y' below the river level, in reference to a horizontal plane, is:

$$\theta = \tan^{-1}\left(\frac{F_V}{F_H}\right) = \tan^{-1}\left(\frac{2.74}{15.11}\right) = \boxed{10.28^\circ}$$