Chapter 4 - Integral Form for a Control Volume
Introduction to Fluid Mechanics
Section 4.3

Mechanical Engineering and Materials Science
University of Pittsburgh

Student Learning Objectives

Students should be able to:

- Apply the Conservation of Energy to systems

Conservation of Energy

- Separating mechanical losses from irreversibilities, our general energy equation, based upon prior assumptions, can be expressed as

$$\boxed{\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1+h_{pump}=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2+h_{turbine}+h_{loss}}$$

  h stands for head [m], and we have pump head, turbine head and head losses

- The COE is a modification to the Bernoulli equation - mass and momentum (energy) must be conserved

Conservation of Energy

- Defining η as our mechanical efficiency, $0 \leq \eta \leq 1$
- Pump head is expressed as

$$h_{pump} = \frac{W_{pump}}{g} = \frac{\dot{W}_{pump}}{\dot{m}g} = \frac{\eta \dot{W}_{pump}}{\dot{m}g}$$

- Note that if η is less than 1, the pump produces less head, i.e. it can not pump a fluid as high as a more efficient pump

- Turbine head is expressed as

$$h_{turbine} = \frac{W_{turbine}}{g} = \frac{\dot{W}_{turbine}}{\dot{m}g} = \frac{\dot{W}_{turbine}}{\eta \dot{m}g}$$

- Since $h_{turbine}$ is the extracted head from the fluid, η less than 1 indicates a greater head needs to be provided to produce the same work as a more efficient turbine

Conservation of Energy

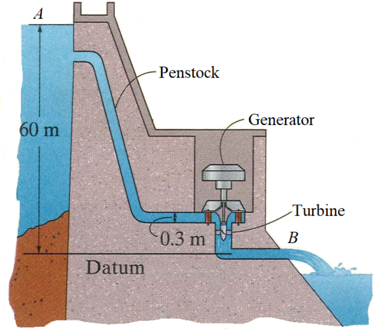
- Lastly, head losses are defined as such

$$h_{loss} = \frac{\dot{E}_{mech,loss}}{\dot{m}g}$$

- This notation is not convenient for evaluation. In MEMS 1071, you will evaluated k-factors and frictional losses in piping systems to populate this term.

Example #1

- Consider the hydroelectric power plant shown below. If the discharge at the exit of the dam is 1.7 [m^3/s], determine the power produced by the turbine assuming the frictional head loss through the penstock is 4 [m]. Also determine the power lost due to friction.



Example #1

- Solution:

Example #1

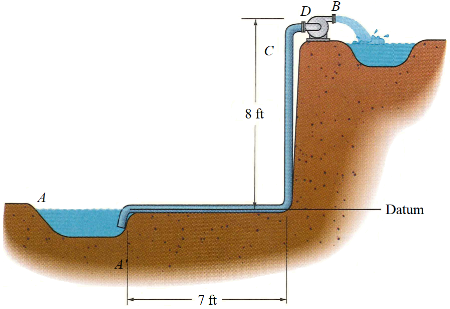
- Solution:

Example #1

- Solution:

Example #2

- Consider the pump system shown below. The pump supplies water to B at 2 [ft^3/s]. If the pipe is 6 in. diameter, determine the required horsepower to operate the pump, assuming the frictional head loss is 0.1 ft./ft.



Example #2

- Solution:

Example #2

- Solution:

Example #2

- Solution: