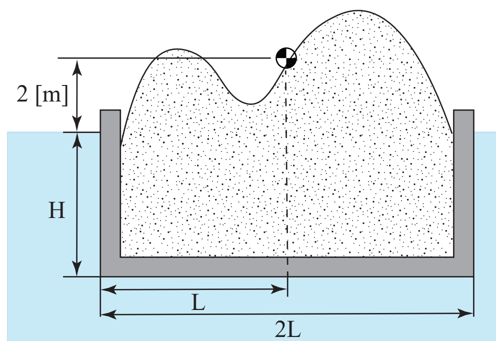
Homework #4

MEMS 0071 - Introduction to Fluid Mechanics

Assigned: September 28$^{\text{th}}$, 2019
Due: October 4$^{\text{th}}$, 2019

Problem #1

A coal barge that is 15 [m] wide (2L), that has a draft of 4 [m] (H), is loaded such that the center of gravity is 2 [m] above the waterline and 7.5 [m] (L) from either side. Assuming a unit length of the barge, determine if this configuration stable. Additionally, determine at what angle of θ that the barge become unstable.



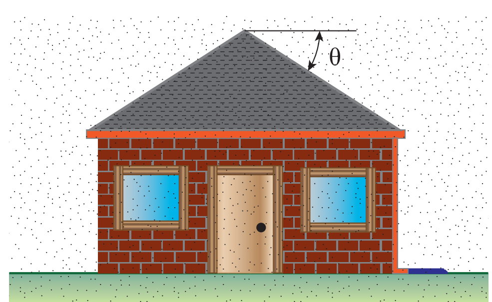
Problem #2

Fluid is flowing through a control surface with a velocity $\vec{V}$=⟨8, 1, −2⟩ [m/s]. The control surface is described via the cross product of the following two vectors: $\vec{a}$=⟨0.5, 10, −1⟩ [m] and $\vec{b}$=⟨0.75, 2, 6⟩ [m]. Determine the normal component of velocity with respect to the control surface in terms of its Cartesian components and magnitude.

Problem #3

A home, with a footprint of 10 [m] by 10 [m], and roof with an angle θ=35°, is experiencing heavy rainfall. If the rain is falling at a rate of 50 [mm/hr], and the single downspout has a cross-sectional flow area of 3,750 [mm^2], determine:

a) The velocity of the water exiting the downspout;
b) The volumetric flow rate through the downspout.



Problem #4

A circular balloon is being filled through the inlet, which has a cross-sectional area denoted as A_1. The air enters the balloon with a velocity V_1 and a density ρ_1. If the balloon radius is denoted as R, and the density within is taken as an average value ρ_{avg}, construct an expression for the time rate of change of mass within the balloon.

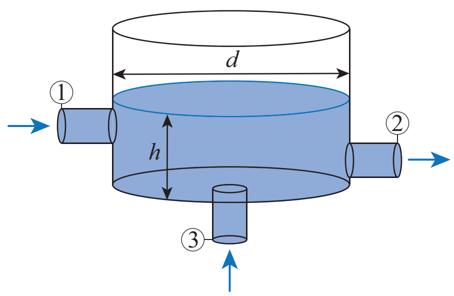
Problem #5

Water enters the tank with diameter d from 1 and 3 and exits at 2. The cross-sectional area of pipe 1 is 50 [mm^2], that of pipe 2 is 70 [mm^2]. The volumetric flow rate into the system through pipe 3 is 0.01 [m^3/s]. Determine the following:

a) An anlytical expression for the change of water height, dh/dt in terms of the volumetric flow rates $\dot{\forall}_1$, $\dot{\forall}_2$ and $\dot{\forall}_3$;

b) Once the system has reached steady-state, determine the exit velocity of the fluid at 2 given V_1=3 [m/s] and $\dot{\forall}_3$=0.01 [m^3/s];

c) The time it takes to reach steady-state, assuming the exit velocity obeys the Toricelli equation ($V = \sqrt{2gh}$)



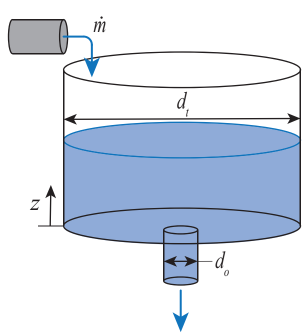
Problem #6

Water is entering a tank with diameter d_t with a constant mass flow rate of $\dot{m}$. The tank has an opening at the bottom with diameter d_o. Assume the exit velocity is $\sqrt{2gh}$. If the tank is empty, determine:

a) The maximum height the water can reach in the tank;

b) A relation for the water height, z, as a function of time;

c) The time it takes for the tank to reach steady-state.



Problem #7

Calculate the Reynolds number for the following scenarios and determine if the flow is laminar, transitional or turbulent:

a) Air (ρ=1.225 [kg/m^3], μ=16.82 [μPa-s]) flowing through a rectangular duct of 25 [cm] by 40 [cm] at a velocity of 1.5 [m/s]

b) Water (ρ=998 [kg/m^3], μ= $8.90 \cdot 10^{-4}$ [Pa-s]) flowing through a cylindrical spillway tunnel with a diameter of 15.24 [m] at 53.5 [m/s]

c) A Boeing 757 flying at 10,000 [m] through air (ρ=0.4135 [kg/m^3], μ=$1.458 \cdot 10^{-5}$ [Pa-s]) at a speed of 858 [km/hr] where the chord length of the wing is taken as 8.20 [m] at the fuselage and 1.73 [m] at the wingtip