Chapter 2 - Fundamental Concepts

Lecture 20
Sections 6.3 and 2.6

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
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Student Learning Objectives

Students should be able to:

- Determine the relationship between strain rate and shear stress in terms of viscosity

Strain Rate Tensor

- Recall the **strain rate tensor**:

$$\underline{\underline{\epsilon}}_{ij} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{bmatrix}$$

- For all intents and purposes, quantifying the strain generated within a fluid is difficult. What if we convert strain to stress, and stress to force? Thus, we can see how an applied force causes changes in velocity.

Stress Fields

- Recall from statics that it is important to look at strain, as well as stress, within the material.
- Strain is a result of stress, which can be in the form of **surface forces** and **body forces**.
- These two types of forces lead to two types of stress:
  1. Normal stress - component of force acting normal to the surface, and can be defined as:

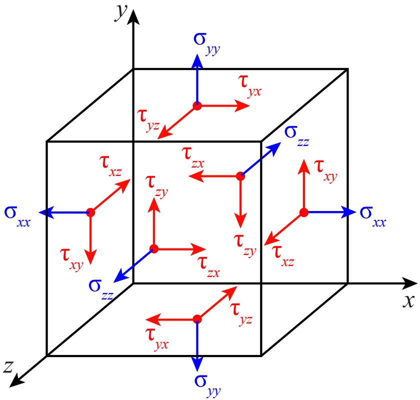
$$\sigma_n = \lim_{\partial A \to 0} \frac{\partial F_n}{\partial A} = \frac{F_n}{A}$$

  2. Shear stress - component of force acting perpendicular to the surface, and can be defined as:

$$\tau = \lim_{\partial A \to 0} \frac{\partial F_t}{\partial A} = \frac{F_t}{A_n}$$

Stress Fields

- Each face of a differential volume is experiencing three stresses: one normal and two shear.



Stress Fields

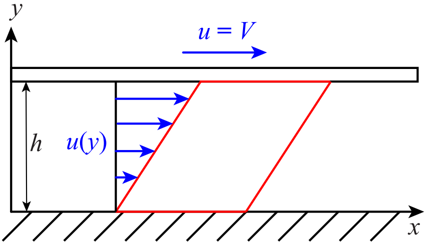
- The first subscript is the direction in which the face of the volume is located; the second subscript is the direction in which the stress is acting. For example, τ_{xy} means the shear stress acting on a plane that is mutually orthogonal to the x-direction, and the stress is in the y-direction.
- Therefore, we can define all stress acting on a point of interest in tensor form: notation:

$$\underline{\underline{\boldsymbol{\sigma}}} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

- The first, second, and third rows are all the forces acting on the plane that is mutually orthogonal to the x-, y-, and z-directions, respectively.
- The first, second, and third columns are the directions in which these stress are acting: x, y, and z.

Viscosity and Shear Stress

- We have to define the shear stress that is acting on the differential volume. Imagine the following situation, where we have a fluid constrained between a moving top plate that has some velocity magnitude V, and an immovable bottom plate.
- In this situation, we are imparting a shear stress onto this fluid. We call this is a shear driven flow, or **Couette flow**.

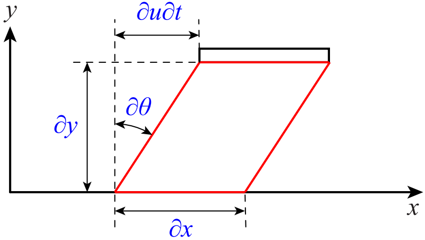


Viscosity and Shear Stress

- We can define the shear stress as:

$$\tau_{yx} = \lim_{\partial A_y \to 0} \frac{\partial F_x}{\partial A_y} = \frac{dF_x}{dA_y}$$

- To look at the deformation over some time interval:



- The shear stress, from a classical mechanics perspective is:

$$\tau_{yx} \approx \partial \theta = \frac{\partial u \partial t}{\partial y}$$

Viscosity and Shear Stress

- During some time interval dt, $d\theta$ is the angle between the normal in the y-direction and the line representing the velocity gradient. We can express our deformation rate as as:

$$\frac{\partial\theta}{\partial t} = \frac{\partial u}{\partial y} \implies \frac{d\theta}{dt} = \frac{du}{dy}$$

- Shearing will generate motion in the x-direction, and that motion will vary in the y-direction, i.e. $u = u(y)$.
- We can conclude that the rate of deformation of a fluid is equivalent to the velocity gradient, and it can be shown that said deformation is proportional to the velocity gradient:

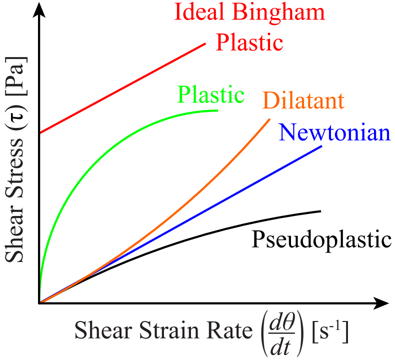
$$\tau_{yx} \propto \frac{du}{dy}$$

Classifications of Fluids

- The constant of proportionality that relates shear stress to the velocity gradient is called dynamic viscosity (μ) [Pa-s]:

$$\tau = \mu \frac{du}{dy}$$

- Fluids exhibit various behaviors, all attributed to how μ varies with respect to the $\dot{\gamma}$.
- Fluids where $\dot{\gamma}$ is linearly proportional to τ are referred to as **Newtonian fluids**.



Non-Newtonian Fluids

- When shear stress is not directly proportional to the rate of deformation, the fluid is **non-Newtonian**.
- The **power law** commonly describes non-Newtonian fluids:

$$\tau_{yx} = k\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy} = \eta\frac{du}{dy}$$

- The first term on the RHS of the second term of the power law equation is the apparent viscosity
- If $\eta < 1$, shear thinning (pseudoplastic).
- If $\eta > 1$, shear thickening (dilatant).

Non-Newtonian Fluids

- **Dilatant** - shear thickening fluid that exhibits and increase in resistance to flow at higher strain rates, i.e. corn starch solution.
- **Pseudoplastic** - shear thinning fluid that exhibits less resistance to flow at higher strain rates, e.g. latex paint, blood plasma.
- **Plastic** - a shear thinning fluid that exhibits very strong thinning
- **Bingham plastic** - limiting case of a plastic fluid that requires a finite yield stress before flow can be initiated, e.g. drilling fluids, toothpaste, ketchup $\rightarrow$ $\tau_{yx} = \tau_y + \mu_p du/dy$

Example #1

- Given a Newtonian fluid, determine the velocity profile $u(y)$ of the fluid under Couette flow conditions. Express your answer in terms of the top plate velocity V, the height of the channel h and the distance from the bottom stationary plate y.

Example #1