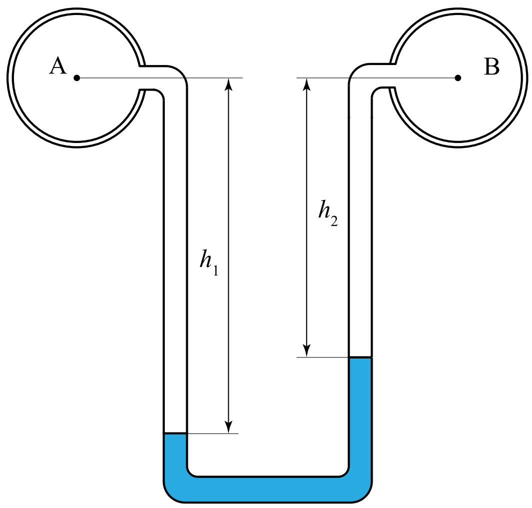
Homework #2

MEMS 0071 - Introduction to Fluid Mechanics

Assigned: September 7$^{\text{th}}$, 2019
Due: September 13$^{\text{th}}$, 2019

Problem #1

Consider the following differential manometer. The fluid represented by the white is air, that by blue is water. Given h_1=96 [mm], h_2=74 [mm], determine the pressure difference ΔP=P_{A}-P_{B} in [kPa].



Creating intermediate pressures at the interfaces of the fluids, starting from left to right, we have the following system of equations:

$P_1 = P_A + \rho_{\text{air}} g h_1$
$P_2 = P_1 - \rho_{\text{H}_2\text{O}} g(h_1 - h_2)$
$P_B = P_2 - \rho_{\text{air}} g h_2$

Thus:

$$P_B = P_A + g(\rho_{\text{air}} h_1 - \rho_{\text{H}_2\text{O}}(h_1 - h_2) - \rho_{\text{air}} h_2)$$

Solving for ΔP=P_{A}-P_{B}:

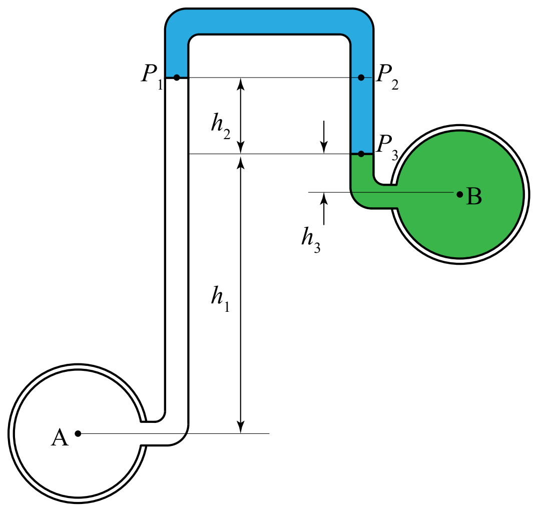
$$\begin{aligned}\Delta P = P_A - P_B &= -g(\rho_{\text{air}} h_1 - \rho_{\text{H}_2\text{O}}(h_1 - h_2) - \rho_{\text{air}} h_2)\\ &= \left(-9.81\left[\frac{\text{m}}{\text{s}^2}\right]\right)\left\{\left(1.225\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.096\,[\text{m}]) - \left(998\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.096 - 0.074)\,[\text{m}]) - \left(1.225\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.074\,[\text{m}])\right\}\\ &= 215.12\,[\text{Pa}]\end{aligned}$$

If we neglect air:

$$\begin{aligned}\Delta P = P_A - P_B = -g(-\rho_{\text{H}_2\text{O}}(h_1 - h_2)) &= \left(-9.81\left[\frac{\text{m}}{\text{s}^2}\right]\right) - \left(998\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.096 - 0.074)\,[\text{m}])\\ &= 215.39\,[\text{Pa}]\end{aligned}$$

Problem #2

Consider the following differential manometer. The fluid represented by the white is air, that by blue is water and that by green is oil, with a specific gravity SG=0.83. Given h_1=130 [mm], h_2=36 [mm] and h_3=18 [mm], determine the pressure difference ΔP=P_A-P_B in [kPa].



Starting at Point A, the system of equations is as follows:

$$P_1 = P_A - \rho_{\mathrm{air}} g(h_1 + h_2)$$
$$P_2 = P_1$$
$$P_3 = P_2 + \rho_{\mathrm{H_2O}} g h_2$$
$$P_B = P_3 + \rho_{\mathrm{oil}} g h_3$$

Thus

$$P_B = P_A - \rho_{\mathrm{air}} g h_1 + \rho_{\mathrm{H_2O}} g h_2 + \rho_{\mathrm{oil}} g h_3$$

Solving for ΔP=P_A-P_B:

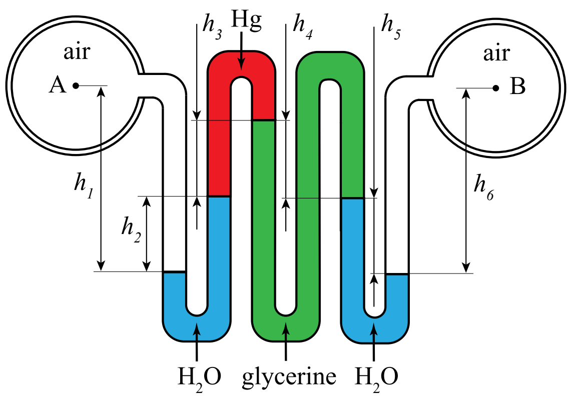
$$\Delta P = P_A - P_B = g(\rho_{\mathrm{air}}(h_1 + h_2) - \rho_{\mathrm{H_2O}} h_2 - \rho_{\mathrm{oil}} h_3)$$

$$\Delta P = \left(9.81\left[\frac{\mathrm{m}}{\mathrm{s}^2}\right]\right)\left\{\left(1.225\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.166\,[\mathrm{m}]) - \left(998\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.036\,[\mathrm{m}]) - \left(0.83\cdot 1,000\left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]\right)(0.018\,[\mathrm{m}])\right\}$$

$$\Delta P = -497.06\left[\frac{\mathrm{kg}}{\mathrm{m\text{-}s}^2}\right] = -0.497\,[\mathrm{kPa}]$$

Problem #3

Given the manometer below, find the pressure difference $\Delta P = P_A - P_B$. The heights are $h_1 = h_6 = 86$ [mm], $h_2 = h_3 = h_4 = h_5 = 35$ [mm]. The fluid properties are $\rho_{H_2O} = 998$ [kg/m^3], $SG_{Hg} = 13.6$, $\rho_{air} = 1.225$ [kg/m^3], $\gamma_{glycerine} = 11{,}067$ [N/m^3] and $\rho_{oil} = 900$ [kg/m^3].



Creating intermediate pressures at the interfaces of the fluids, starting from left to right, we have the following system of equations:

$$P_1 = P_A + \rho_{\text{air}} g h_1$$
$$P_2 = P_1 - \rho_{\text{H}_2\text{O}} g h_2$$
$$P_3 = P_2 - \rho_{\text{Hg}} g h_3$$
$$P_4 = P_3 + \rho_{\text{glyc}} g h_4$$
$$P_5 = P_4 + \rho_{\text{H}_2\text{O}} g h_5$$
$$P_B = P_5 - \rho_{\text{air}} g h_6$$

Therefore:

$$\Delta P = P_A - P_B = -g(\rho_{\text{air}} h_1 - \rho_{\text{H}_2\text{O}} h_2 - \rho_{\text{Hg}} h_3 + \rho_{\text{glyc}} h_4 + \rho_{\text{H}_2\text{O}} h_5 - \rho_{\text{air}} h_6)$$

The density of mercury is taken as:

$$\rho_{\text{Hg}} = SG_{\text{Hg}} \rho_{\text{H}_2\text{O}(4^\circ\text{c})} = (13.6)(1,000\,[\text{kg/m}^2]) = 13,600\,[\text{kg/m}^3]$$

The density of glycerine is taken as:

$$\rho_{\text{glyc}} = \frac{\gamma_{glyc}}{g} = \frac{11,067\,[\text{N/m}^3]}{9.81\,[\text{m/s}^2]} = 1,128\,[\text{kg/m}^3]$$

Substituting in known values:

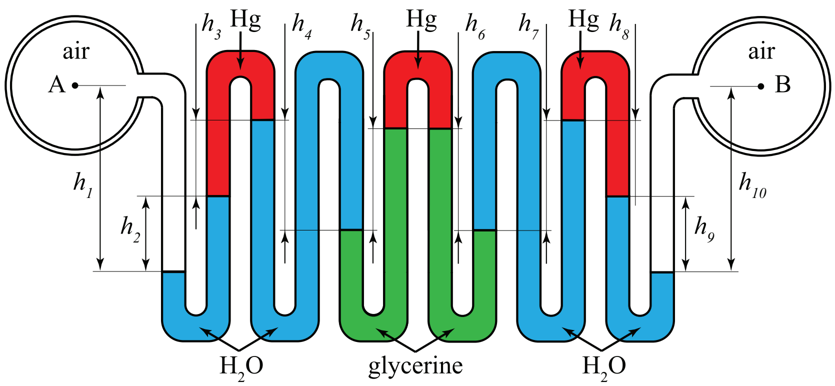
$$\begin{aligned}\Delta P = P_A - P_B = -9.81\left[\frac{\text{m}}{\text{s}^2}\right]\Bigg\{&\left(1.225\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.086\,[\text{m}]) - \left(998\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.035\,[\text{m}]) \\ &- \left(13,600\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.035\,[\text{m}]) + \left(1,128\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.035\,[\text{m}]) \\ &+ \left(998\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.035\,[\text{m}]) - \left(1.225\left[\frac{\text{kg}}{\text{m}^3}\right]\right)(0.086\,[\text{m}])\Bigg\} \\ = \boxed{4.28\,[\text{kPa}]}\end{aligned}$$

Problem #4

Given the manometer below, find the pressure difference $\Delta P = P_A - P_B$. The density of the fluids used within the manometer, as well as the heights of each fluid level, are given below:

- ρ_{H_2O}=1,000 [kg/m^3]
- SG_{Hg}=13.6
- ρ_{air}=1.225 [kg/m^3]
- γ_{glyc}=11,067 [N/m^3]

- h_1=h_{10}=86 [mm]
- h_2=h_9=35 [mm]
- h_3=h_8=35 [mm]
- h_4=h_7=51 [mm]
- h_5=h_6=47 [mm]



All fluid heights are the same, and the fluid levels and fluids are symmetric about the glycerine/mercury interfaces in the center of the tube. Thus $\Delta P = 0$ [kPa]. The proof is left up to the reader.

Problem #5

Given the expression for the resultant force acting on y' of a a submerged plate as:

$$y' F_R = P_o y_c A + \rho g \sin(\theta)(I_{\hat{x}\hat{x}} + y_c^2 A)$$

prove that y' is equal to the following:

$$y' = y_c + \frac{\rho g \sin(\theta) I_{\hat{x}\hat{x}}}{F_R}$$

Recall that resultant forces F_R is expressed as:

$$F_R = (P_o + \rho g \sin(\theta) y_c) A$$

Expanding the RHS of $y' F_R$:

$$y' F_R = P_o y_c A + \rho g \sin(\theta) I_{\hat{x}\hat{x}} + \rho g \sin(\theta) y_c^2 A$$

Rearranging the order of terms:

$$y' F_R = P_o y_c A + \rho g \sin(\theta) y_c^2 A + \rho g \sin(\theta) I_{\hat{x}\hat{x}}$$

Dividing terms by F_R, within the second term expressed in terms of $(P_o + \rho g \sin(\theta) y_c) A$:

$$y' = \frac{P_o y_c A + \rho g \sin\theta y_c^2 A}{(P_o + \rho g \sin(\theta) y_c) A} + \frac{\rho g \sin(\theta) I_{\hat{x}\hat{x}}}{F_R}$$

Factoring out y_c and A from the second term:

$$y' = \frac{y_c (P_o + \rho g \sin(\theta) y_c) A}{(P_o + \rho g \sin(\theta) y_c) A} + \frac{\rho g \sin(\theta) I_{\hat{x}\hat{x}}}{F_R} = y_c + \frac{\rho g \sin(\theta) I_{\hat{x}\hat{x}}}{F_R}$$

Problem #6

Given the expression for the resultant force acting on y' of a a submerged plate as:

$$y'F_R = P_o y_c A + \rho g \sin(\theta)(I_{\hat{x}\hat{x}} + y_c^2 A)$$

prove that y', when ambient pressure is neglected, is equal to the following:

$$y' = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}$$

Recall that resultant forces F_R is expressed as:

$$F_R = \rho g \sin(\theta) y_c A$$

Removing P_o from the RHS of $y'F_R$:

$$y'F_R = \rho g \sin(\theta)(I_{\hat{x}\hat{x}} + y_c^2 A)$$

Multiplying through:

$$y'F_R = \rho g \sin(\theta) I_{\hat{x}\hat{x}} + \rho g \sin(\theta) y_c^2 A$$

Dividing terms by F_R, within the second term expressed in terms of $\rho g \sin(\theta) y_c A$:

$$y' = \frac{\rho g \sin(\theta) I_{\hat{x}\hat{x}}}{\rho g \sin(\theta) y_c A} + \frac{\rho g \sin(\theta) y_c^2 A}{\rho g \sin(\theta) y_c A} = \frac{I_{\hat{x}\hat{x}}}{y_c A} + y_c = y_c + \frac{I_{\hat{x}\hat{x}}}{A y_c}$$