Chapter 4 - Integral Form for a Control Volume

Lecture 15
Section 4.3

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
University of Pittsburgh

Student Learning Objectives

Students should be able to:

- Understand the formulation of the Conservation of Angular Momentum equation in an RTT framework

Reynolds Transport Theorem

- Recall RTT

$$\left.\frac{dB}{dt}\right)_{sys} = \frac{\delta}{\delta t}\int_{C.\forall.} b\rho d\forall + \int_{\text{C.S.}} b\rho \overrightarrow{V}\cdot d\overrightarrow{A}$$

- The LHS is the rate of change of the extensive property of system
- The first term on the RHS is the rate of change of the extensive property of the system within the C.∀.
- The second term on the RHS is the rate at which the extensive property of the system is exiting the C.∀. through the C.S.

Definition of Torque

- Torque, $\vec{T}$, is the time rate of change of angular momentum, $\vec{H}$

$$\vec{T} = \frac{d\vec{H}}{dt}$$

- Torque is simply force times distance (force times position vector), which is applied to the surface of a fluid volume

$$\vec{T}_s = \vec{r} \times \vec{F}_s$$

Definition of Torque

- There is also a body torque (think of an object where the ◕ overhangs a support)

$$\vec{T}_b = \int_m \vec{r} \times \vec{g} dm$$

- Lastly, there can be an applied external torque introduced into our system from a shaft (say an impeller in a liquid), $\vec{T}_{shaft}$
- Therefore, the externally applied torque on our system is the sum of these three

$$\vec{T} = \vec{T}_s + \vec{T}_b + \vec{T} shaft$$

Cross Product

- When defining torque, we introduce a new operator, the cross product $\times$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- The cross product is determinant of the above matrix

$$\vec{r} \times \vec{F} = (r_y F_z - r_z F_y)\hat{\imath} - (r_x F_z - r_z F_x)\hat{\jmath} + (r_x F_y - r_y F_x)\hat{k}$$

Conservation of Angular Momentum

- If we define $\vec{H}$ as our extensive system property and $\vec{r} \times \vec{V}$ as our intensive system property, we have the following

$$\vec{T} = \frac{d\vec{H}}{dt}\Bigg)_{sys} = \frac{\delta}{\delta t}\int_{C.\forall.}(\vec{r} \times \vec{V})\rho d\forall + \int_{C.S.}(\vec{r} \times \vec{V})\rho(\vec{V} \cdot d\vec{A})$$

- The LHS is all external torques acting on our system

Conservation of Angular Momentum

- The angular momentum flux across the C.S. can be simplified, assuming steady state

$$\left.\frac{d\vec{H}}{dt}\right)_s = \int_{C.S.} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n})dA$$

- The term $\int_A \rho(\vec{V} \cdot \vec{n})dA$ is $\dot{m}$

$$\boxed{\begin{aligned}\left.\frac{d\vec{H}}{dt}\right)_s &= \int_{C.S.} (\vec{r} \times \vec{V})\rho(\vec{V} \cdot \vec{n})dA = \\ &\sum_{out} \vec{r} \times \dot{m}\vec{V} - \sum_{in} \vec{r} \times \dot{m}\vec{V}\end{aligned}}$$

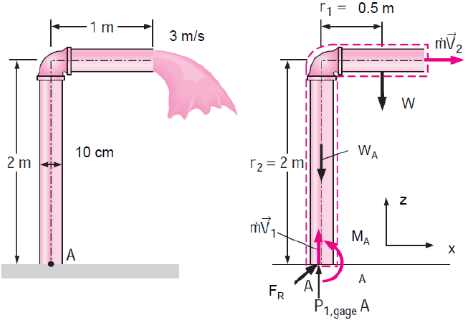
Conservation of Angular Momentum

- If the forces and momentum flows are in the same plane, thus the generated moments are in the same plane about the same axis, we can use the scalar form (for steady-state conditions)

$$\boxed{\left.\frac{d\overrightarrow{H}}{dt}\right)_s = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V}$$

Example #1

- Water is pumped through a 10 [cm] diameter pipe that is 2 [m] tall and 1 [m] long. Water is discharged to the atmosphere with an average velocity of 3 [m/s]. The mass of the horizontal portion of the pipe, including the water, is 12 [kg].



Example #1

- Determine the bending moment, M_A and the required length of the horizontal duct that would reduce M_A to zero

Example #1 Solution:

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