Chapter 3 - Fluid Statics

Lecture 7

Section 3.5

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
University of Pittsburgh

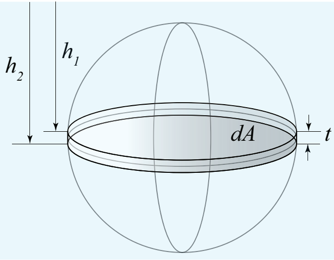
Student Learning Objectives

Students should be able to determine the:

- The buoyant force a fluid is exerting on an object
- If a body is completely submerged, neutrally buoyant or floating

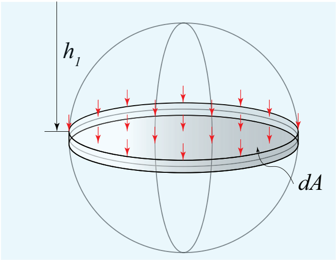
Submerged Differential Disc

- **Buoyancy** - the net vertical force acting on an object immersed in a liquid or floating on its surface
- Imagine a submerged sphere. Then look at a slice of the sphere with a differential area dA and thickness t=h_1-h_2.



Force Acting Downward

- There is a hydrostatic pressure on the top of the slice.

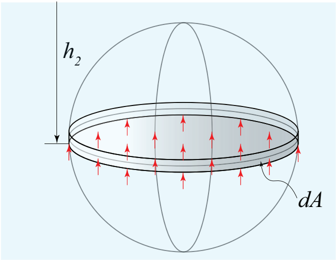


- The force on the top surface of the slice is:

$$F_{-y} = -(P_o + \rho g h_1)dA$$

Force Acting Upward

- There is a hydrostatic pressure on the bottom of the slice.



- The force on the bottom surface of the slice is:

$$F_{+y} = (P_o + \rho g h_2)dA$$

Vertical Force Balance

- The summation of forces in the y-direction yields

$$\sum F_y = 0 = (P_o + \rho g h_2)dA - (P_o + \rho g h_1)dA$$

- P_o acts on both sides, and recall the definition of volume

$$\implies \sum F_y = \rho g(h_2 - h_1)dA = \rho g d\forall$$

- Thus

$$\implies F_y = \int dF_y = \int_{\forall} \rho g d\forall = \rho g \forall$$

Buoyant Force

- Therefore, the buoyancy force is equal to the weight of the fluid displaced by the body

$$F_{buoy} = \rho g \forall$$

- This is the force the fluid is exerting upwards (+y) on the body
- Density is the main driving factor, not depth
- Fun fact - during the Siege of Syracuse (214-212 BC), the Romans were told not to kill Archimedes, a mathematician, scientist, physicist, engineer and astronomer who discovered the buoyancy force and many other great things such as infinitesimals, but they did

Classification of Submerged Bodies

- For a body to float, there either needs to be a net upward force, or the weight of the body must equal the weight of fluid displaced

$$\sum F_y = F_{buoy} - W = 0$$

- Thus

$$\rho_f g \forall_f = \rho_{body} g \forall_{body}$$

- Therefore

$$\frac{\forall_f}{\forall_{body}} = \frac{\rho_{body}}{\rho_f}$$

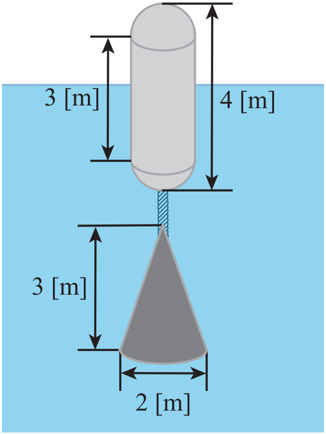
- If the RHS $\geq$1, the body is completely submerged, =1, the body is neutrally buoyant, $\leq$1, the body is floating

Demonstration

https://imgur.com/gallery/xeysEpd

Example #1

- A buoy is supporting a cone-shaped weight as shown in the figure below. The buoy has a specific weight of 3 [kN/m^3]. 1.5 [m] of the total height of the buoy must be above the surface of the water such that ships are able to see it. The specific weight of water is 9.79 [kN/m^3]. Calculate the minimum allowable weight of the cone.



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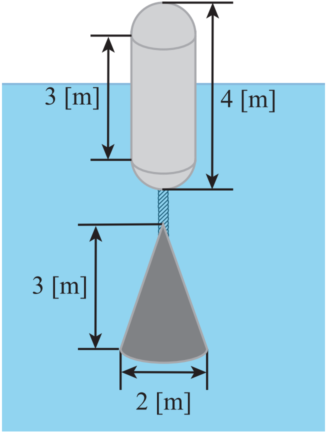

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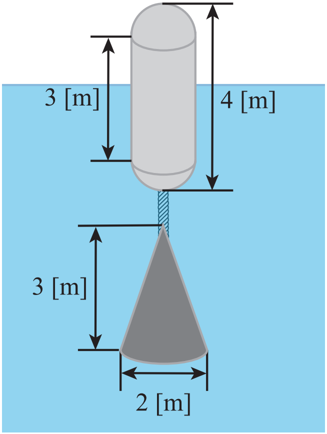

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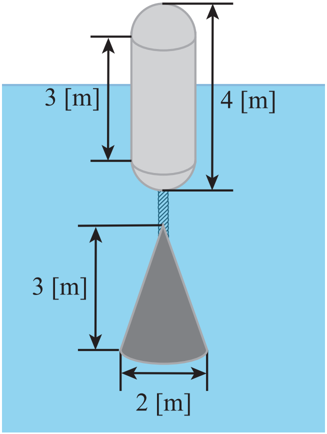

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