Chapter 3 - Fluid Statics

Lecture 8
Section 3.5

Introduction to Fluid Mechanics

Mechanical Engineering and Materials Science
University of Pittsburgh

Student Learning Objectives

Students should be able to determine the:

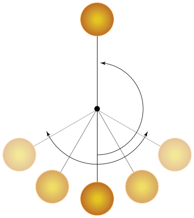
- The metacenter of a floating object
- If a floating object is stable within a fluid system

Stability of Buoyant Objects

- How stable are buoyant objects?

Vertical Stability

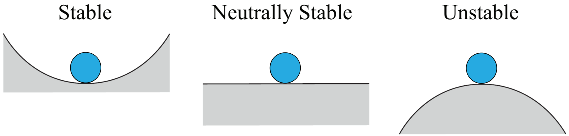
- Buoyant objects have *vertical stability* - same line of action for both the buoyant force and weight
- Think of a pendulum - mathematically, there are two critical points, one at the top and one at the bottom

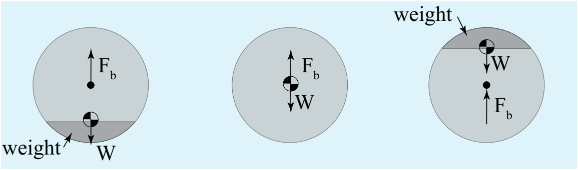


- However, the top is mathematically unstable - there is no restorative force when the pendulum is perturbed

Stability - Center of Buoyancy

- This can be expressed in terms of **rotational stability** - related to the location of the **center of gravity**, CG (or just G, or ◐), and **center of buoyancy**, B, where B is the centroid of the displaced volume.



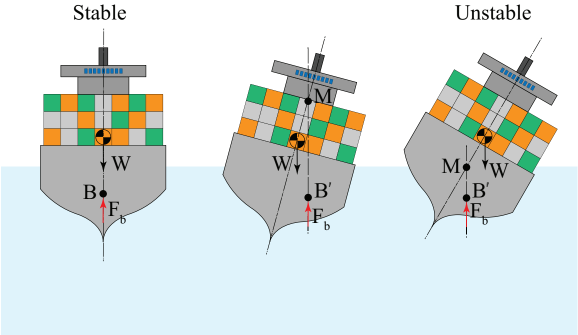


Stability - Center of Gravity

- The following criteria are applied to the stability of an immersed object
  1. if ⊕ below B, immersed body is stable
  2. if ⊕ above B, immersed body is unstable
  3. if ⊕=B, immersed body is neutrally stable

Stability of Floating Objects

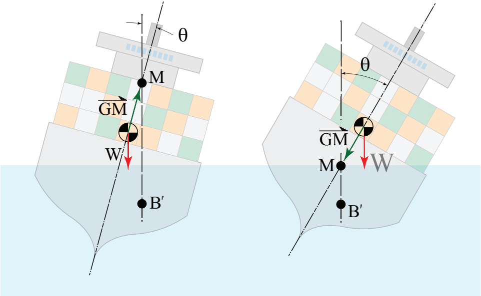
- For floating bodies, the picture changes
  1. If ◒ below B, floating body is stable
  2. If ◒ above B, floating body can be stable



Metacentric Height

- Stability depends on the **metacenter**, M, which is the intersection of the $\oplus$ and shifted center of buoyancy, B′, and **metacentric height**

$$\overrightarrow{\mathrm{GM}} = \overrightarrow{\mathrm{M}} - \overrightarrow{\oplus}$$

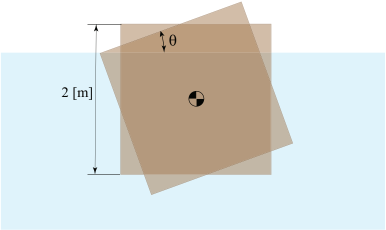


Stability and Metacentric Height

- Stability is dependent upon M and ⊕:
  1. if M above ⊕, GM is positive - stable
  2. if M below ⊕, GM is negative - unstable
- The larger |GM| of a positive $\overrightarrow{\mathrm{GM}}$, the more stable the object is
- The small disturbance angle, also known as the rolling angle θ, is typically limited to 20°
- In practical applications, a keel can counteract lateral forces/shifts in weight that may cause a high rolling angle

Example #1

- Imagine we have a wooden block that is 0.2 [m] per side (L). A vertical force is applied to the left side of the block, which causes the block to tilt positive 20° from the x-axis. Determine the buoyant force on the block, and show the block will be in stable equilibrium after the force is removed.



Example #1

- Solution:

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- Solution:

Review of Hydrostatics

- What constitutes a fluid
- No-slip boundary condition
- The Continuum Assumption
- Application of control surfaces and control volumes (open and closed)
- Distinguish between intensive and extensive properties
- Know the relationship of specific gravity and density
- Distinguish between gage, absolute and atmospheric pressure
- Determine the pressure variation in fluid as a function of height

Review of Hydrostatics

- Understand Pascal's Paradox
- Understand how to apply the variation of pressure as a function of depth to manometers
- Determine the magnitude of a force acting on a submerged planar surface
- Determine the direction of a force acting on a submerged planar surface
- Determine the the line of action of a force acting on a submerged planar surface
- Determine the magnitude of a force acting on a submerged curved surface
- Determine the direction of a force acting on a submerged curved surface

Review of Hydrostatics

- Determine the the line of action of a force acting on a submerged curved surface
- Determine the buoyant force a fluid is exerting on an object
- Determine if a body is completely submerged, neutrally buoyant or floating
- Determine the location of the center of gravity and center of buoyancy
- Determine if an object is stable within a fluid system
- Determine the metacenter of a floating object
- Determine if a floating object is stable within a fluid system