HW5 KEY

1.

a. **The 3 chemicals were randomly selected from a large group/population of chemicals. So the effect of chemicals will be random (depending on the chemicals selected). So the random-effects model is appropriate here. σ_A^2 here measures the differences between chemicals in the population.** $\otimes$

b. **4 steps in hypothesis test** $\otimes$
(1) $H_0\colon \sigma_A^2 = 0$ versus $H_a\colon \sigma_A^2 \neq 0$, α=0.10
(2) F=3.781, df = (2,14)
(3) P-value = 0.0486 < α=0.10 (BTW: Rejection Region = $\{F > F_{.10,2,14} = 2.726\}$)
(4) Reject H_0 at the 0.10 level.

c. **Show some calculations.** $\otimes$

$$n = 6 + 6 + 5 = 17, \qquad r = \frac{17 - \dfrac{6^2 + 6^2 + 5^2}{17}}{3 - 1} = 5.6471$$

$$\hat{\sigma}_A^2 = \frac{168.02 - 44.44}{5.6471} = 21.8838 \ \otimes$$

d. Estimate of total variance in a single pulp brightness value = MSE + $\hat{\sigma}_A^2$ = 44.44+21.8838 = **66.3238.** $\otimes$

e. Proportion of total variation in a single pulp brightness value is attributed to differences among bleaching chemicals = $\frac{21.8838}{66.3238} \times 100\% = 33.00\%$ $\otimes$

2.

(a) The interaction plot shows reasonably parallel curves for Roller Brands 2 and 3. Roller Brand 1 seems to go against the parallel trend because of the responses at Paint Brands 2 and 3. This suggests that the effects of the Roller Brand and Paint Brand may not be additive. However, note that each treatment combination was not replicated (only one response per treatment). So the evidence against additivity of factor effects may not be strong/convincing. Replicating the treatments will provide more convincing information.

(b)

$$\textit{Factor}\ \text{A means: } \bar{x}_{1.} = 445.75, \bar{x}_{2.} = 442.25, \bar{x}_{3.} = 446.25$$

$$\textit{Factor}\ \text{B means: } \bar{x}_{.1} = 1351/3 = 450.33, \bar{x}_{.2} = 1337/3 = 445.67,$$

$$\bar{x}_{.3} = 1325/3 = 441.67, \bar{x}_{.4} = 1324/3 = 441.33$$

$$\bar{x}_{..} = 444.75, \mathrm{I} = 3, \mathrm{J} = 4 \qquad \otimes$$

$$SSA = 4 \times \left[(445.75 - 444.75)^2 + (442.25 - 444.75)^2 + (446.25 - 444.75)^2\right] = 38$$

$$SSB = 3 \times \left[\left(1351/3 - 444.75\right)^2 + \left(1337/3 - 444.75\right)^2 + \left(1325/3 - 444.75\right)^2 + \left(1324/3 - 444.75\right)^2\right]$$
$$= 159.583$$

(c)
(1) H_0: $\beta_1=\beta_2=\beta_3=\beta_4=0$ versus H_a:{H_0 is false.} $\otimes$
(2) F=7.85 $\otimes$
(3) P-value=0.017 $\otimes$
(4) Reject H_0 at the α=.05 level. $\otimes$

Coverage means differ among paint brands.

(d)

(1) H_0: $\alpha_1=\alpha_2=\alpha_3$ versus H_a: {H_0 is false.} ⊗

(2) F=2.80 ⊗

(3) P-value=0.138 ⊗

(4) Retain H_0 at the α=.05 level. ⊗

Coverage means do not significantly differ among roller brands.

(f)

i) Residuals versus fitted: The plot does not show strong trends suggesting that variance changes with treatment/fitted value. No evidence that constant variance assumption is violated.

ii) Normal plot of residuals: The plot is very linear. Anderson-Darling test does not reject normality assumption (P-value is quite high). So no evidence that normality is violated.

3.

(a)

I=3, J=5, SSA=11.7, SSB=113.5, SSE=25.6

A df = 3-1 = 2, MSA=11.7/2=5.85

Error df = (3-1)(5-1) = 8, MSE=25.6/8=3.2

Factor A: F=5.85/3.2 = 1.83

(1) H_0: $\alpha_1=\alpha_2=\alpha_3=0$ versus H_a: {H_0 is false.} ⊗

(2) F=1.83 ⊗

(3) RR = {$F \geq F_{.05,2,8}$ = 4.46} ⊗

(4) Retain H_0. ⊗

(b) **A block design with block=house is used because houses are not expected to be similar. Differences between houses may affect the way assessors assess a house. So a completely randomized design using 15 houses will confound house differences with assessor differences. In this case, our ability to detect assessor differences is weakened.** ⊗