Winters 1.0

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1 Winters' smoothing and forecasting method

Winters generalized Holt's method for working with data with seasonal variations.

This package uses Holt-Winters' triple exponential smoothing method to obtain an estimation of the systematic component of a series, which may have a trend and seasonality changing locally, and to obtain forecasts for the series. The model used is:

$$Y_t = T_t + u_t$$

where T_t is the systematic component and u_t is a random error. Two different formulations may be used: additive and multiplicative. The estimation of the systematic component for period $t+\ell$ (with $\ell > 0$), or the forecast of the observation $Y_{t+\ell}$, based on information in t, in the **additive formulation** is:

$$\hat{T}_t(\ell) = \hat{Y}_t(\ell) = S_t + \ell\, b_t + I_{t+\ell-m}$$

being m the seasonal periodicity and

$$\begin{aligned} S_t &= \delta_1(Y_t - I_{t-m}) + (1-\delta_1)(S_{t-1} + b_{t-1}) \\ b_t &= \delta_2(S_t - S_{t-1}) + (1-\delta_2)b_{t-1} \\ I_t &= \delta_3(Y_t - S_{t-1} - b_{t-1}) + (1-\delta_3)I_{t-m} \end{aligned}$$

with $0 < \delta_1 < 1$, $0 < \delta_2 < 1$ and $0 < \delta_3 < 1$.

The **multiplicative formulation** is:

$$\hat{T}_t(\ell) = \hat{Y}_t(\ell) = (S_t + \ell\, b_t) \cdot I_{t+\ell-m}$$

being

$$\begin{aligned} S_t &= \delta_1(Y_t/I_{t-m}) + (1-\delta_1)(S_{t-1} + b_{t-1}) \\ b_t &= \delta_2(S_t - S_{t-1}) + (1-\delta_2)b_{t-1} \\ I_t &= \delta_3(Y_t/S_t) + (1-\delta_3)I_{t-m} \end{aligned}$$

with $0 < \delta_1 < 1$, $0 < \delta_2 < 1$ and $0 < \delta_3 < 1$.

There are several alternatives for determining the initial values for S_t, b_t and I_t in both formulations. In this package, initial values for S_t are the average of the first year of data. Initial values of b_t are set to the average of the slopes for each period in the first two years. Initial values for I_t are set as $I_i = Y_i - S_0$ for additive seasonality and $I_i = Y_i/S_0$ for multiplicative seasonality, where $i = 1, \ldots, m$ and S_0 is the initial value of S_t.

2 Command-line use

The function available for command-line and scripting use is named Winters. It returns a bundle (to be described below) and takes the following arguments:

name	*type*	*comment*	*default value*
y	series	the dependent variable	-
formu	boolean	Formulation (0: Additive, 1: Multiplic.)	1
δ_1	scalar	smoothing parameter for the level	0.3
δ_2	scalar	smoothing parameter for the slope	0.1
δ_3	scalar	smoothing parameter for the seasonality	0.7
pred	integer	number of final observations for evaluating the quality of the predictions	0

We illustrate two calls to this function by reference to the sample script included in the package. This script begins with

```
include Winters.gfn
open data9-3.gdt
```

The data file `data9-3.gdt` Quarterly data from 1972.2 through 1993.4 for San Diego Gas and Electric Company.

The next command uses Winters multiplicative method to estimate a systematic component for the series *reskwh* (kilowatt-hour sales to residential customers (millions)), with $\delta_1 = 0.5$, $\delta_2 = 0.1$, $\delta_3 = 0.7$ and forecast the last four observations of the sample. This time we use *pred=4* which implies that the Winters function is used with the sample from 1972.2 to 1992.4. The last four observations (from 1993.1 to 1993.4) are forecasted and the function reports the Mean Square Error (MSE) and the Mean Absolute Error (MAE) of the forecasts.

```
bundle BW = Winters(reskwh, 1, 0.5, 0.1, 0.7, 4)
```

The next two lines of the script use Winters' method to estimate a systematic (trend + seasonal) component and forecast 4 observations out of the sample. The first one allocates space at the end of the sample to put the new values, so extending the dataset up to 1994.4. The values of the variables for these new observations are initially assigned as 'missing observations', (with value NA).

```
dataset addobs +4
```

The next line invokes the Winters function with parameters $Y = reskwh$, $formu = 0$ (additive formulation), $\delta_1 = 0.3$, $\delta_2 = 0.1$, $\delta_3 = 0.7$ and save the result to bundle BW2. Not writing any value for *pred*, this is left at its default value [0] so, no intra-sample predictions are made. The resulting component will have values forecasted out of the sample. (In spite of the added observations, if we write a value of *pred* different from zero this implies that the forecasts are made only for the values in the sample).

```
bundle BW2 = Winters(reskwh, 0)
```

3 GUI use

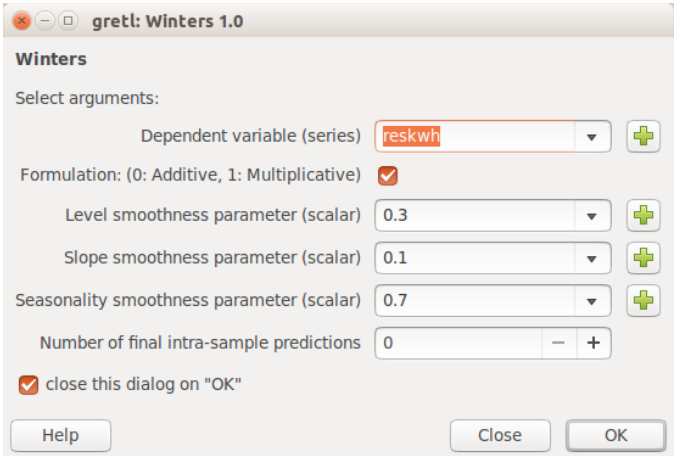

Figure 1: Winters package dialog box

This package may be invoked by means of the graphical user interface (GUI) of *gretl*. This may be done through the gretl menu `/Tools/Function packages`. When the package is executed, a dialog box as in Figure 1 emerges. All the parameters appearing in this window are auto-explicative, except, perhaps, the last one *Number of final intra-sample predictions* which corresponds to the parameter *pred* mentioned above, i.e. is the number of

observations at the end of the sample used to calculate MSE and MAE of the forecasts.

The output window of the package may be seen in Figure 2. It presents the values of the original series and the estimated systematic component and (optionally) the MSE and MAE of the forecasts. By clicking on the second icon of the window (starting from the left side), the estimated component may be saved to the gretl dataset and clicking on the last icon a graph of the original series and the estimated component is displayed.

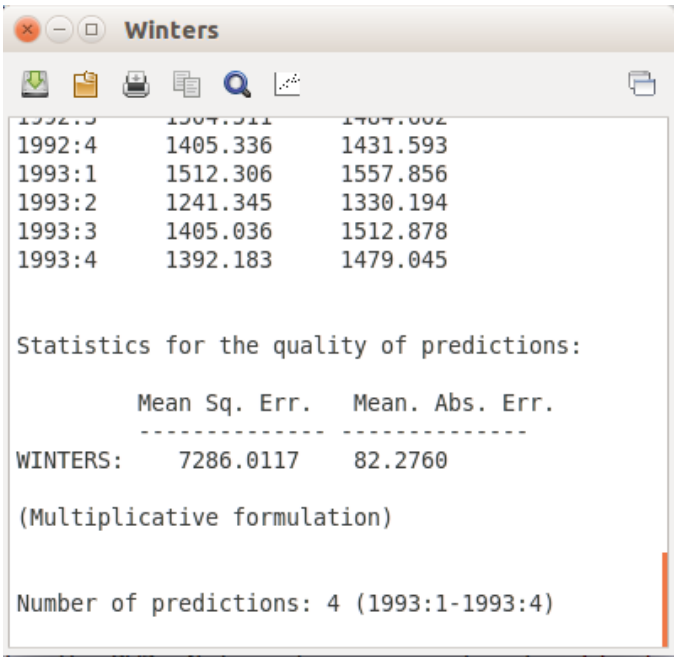

Figure 2: Output of the Winters package

Notes:

- IMPORTANT: If you want to make some forecasts out of the sample, you need to extend the sample before hand in the number of observations that you want. For example, in a console or in a *hansl* script:

  ```
  dataset addobs +4
  ```

  or in the GUI: `/Data/Add obs.`

- This method is suitable for series with periodicity less than a year (quarterly, monthly, etc) containing seasonal cycles. For making forecast with series of this type without seasonality, or with annual series, you should use the Holt method (see Holt package).

Reference:
Econometría. Series temporales y predicción, (1993). Jose M. Otero. Ed: AC. Madrid.