

The HIP package

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Abstract

The HIP package is a collection of `gretl` scripts to estimate probit models which may feature endogenous regressors and/or heteroskedasticity. Estimation is done via maximum likelihood under the assumption of multivariate normality.

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1 Introduction

The HIP package is a collection of `gretl` scripts to estimate probit models which may feature endogenous regressors and/or heteroskedasticity. Estimation is done via maximum likelihood under the assumption of multivariate normality.

Most other packages provide similar facilities separately. However, the additional computational complexity of handling, at the same time, endogeneity and the special form of conditional heteroskedasticity we deal with here is minimal, so we give a command which naturally nests the two special cases but can just as easily handle the general one.

2 The model

The model which HIP handles can be thought of as the union of the familiar IV-probit model and the heteroskedastic probit model, that is models that can be written in the following form:

$$y_i^* = \mathbf{Y}_i' \boldsymbol{\beta}_1 + \mathbf{X}_{1i}' \boldsymbol{\beta}_2 + \varepsilon_i = \mathbf{Z}_i' \boldsymbol{\beta} + \varepsilon_i \quad (1)$$

$$\mathbf{Y}_i = \boldsymbol{\Pi}_1' \mathbf{X}_{1i} + \boldsymbol{\Pi}_2' \mathbf{X}_{2i} + \mathbf{u}_i = \boldsymbol{\Pi}' \mathbf{X}_i + \mathbf{u}_i \quad (2)$$

$$\left(\begin{array}{c} \varepsilon_i \\ \mathbf{u}_i \end{array} \middle| \mathbf{X}_i, \mathbf{W}_i \right) \sim N \left[\left(\begin{array}{c} 0 \\ \mathbf{0} \end{array} \right), \left(\begin{array}{cc} \sigma_i^2 & \sigma_i \boldsymbol{\lambda}' \\ \sigma_i \boldsymbol{\lambda} & \boldsymbol{\Sigma} \end{array} \right) \right] \quad (3)$$

$$\sigma_i = \exp \{ \mathbf{W}_i' \boldsymbol{\alpha} \} \quad (4)$$

The variable y_i^* is assumed to be unobservable; what is observable is $y_i = \mathbb{1}(y_i^* > 0)$. \mathbf{Y}_i is a vector of p endogenous continuous variables and \mathbf{X}_{1i} is a k_1 -vector of exogenous variables; equation (2) is the reduced form for the endogenous regressors in (1), and also includes a k_2 -vector of instrument \mathbf{X}_{2i} .

The notable feature of equation (3) (apart from the customary normality assumption) is the fact that ε_i is allowed to be conditionally heteroskedastic, with variance given by equation (4), where \mathbf{W}_i is a vector of q exogenous variables. Of course, the elements of \mathbf{W}_i may also be elements of \mathbf{X}_i . For identification purposes, though, \mathbf{W}_i should not include a constant term or equivalent variables, such as for example a complete set of dummies.

Note that the familiar IV-probit model arises as a special case of the above under the constraint $\boldsymbol{\alpha} = \mathbf{0}$ whereas, in a parallel fashion, the so-called “heteroskedastic probit model” corresponds to the above model under the constraint $\boldsymbol{\lambda} = \mathbf{0}$, in which case obviously the parameters the two equations (1) and (2) become independent and can be estimated separately.

3 A few examples

3.1 IV probit — through a script

To begin with, we’ll apply IV probit to a time-honoured problem, that is female labour force participation.¹ We’ll use the immortal dataset used in Mroz (1987), supplied among gretl’s example datasets. We will exemplify HIP through a script first, and then we’ll take a look at the GUI hook that HIP provides. Of course, in both examples we’ll assume HIP has correctly been installed.

The script can be very simple:

¹Examples like the one presented here are quite common in several other software packages. Go check.

```
include HIP.gfn
open mroz87.gdt --quiet
```

```
list X1 = const WE KL6
series other_inc = (FAMINC - WW*WHRS) / 1000
HIP(LFP, X1, other_inc, HE)
```

which yields:

```
Probit model with endogenous regressors
ML, using observations 1-500
Dependent Variable: y
Instrumented: other_inc
Instruments: const, fem_educ, kids, male_educ
Parameter covariance matrix: OPG
```

	coefficient	std. error	z	p-value
const	0.367208	0.460541	0.7973	0.4253
fem_educ	0.211111	0.0290285	7.273	3.53e-13 ***
kids	-0.182093	0.0484024	-3.762	0.0002 ***
other_inc	-0.0542756	0.00608741	-8.916	4.83e-19 ***

Log-likelihood	-2368.2062	Akaike criterion	4756.4124
Schwarz criterion	4798.5585	Hannan-Quinn	4772.9505
Conditional ll	-252.045293	Cragg-Donald stat.	101.060

```
Overall test (Wald) = 154.702 (3 df, p-value = 0.0000)
Endogeneity test (Wald) = 6.84499 (1 df, p-value = 0.0089)
```

In this case we used the function HIP, which takes as arguments

1. the dependent variable
2. the exogenous explanatory variables (normally as a list)
3. the endogenous explanatory variables (as a list or, like in this this case, as a single variable name)
4. the instruments (as a list or, like in this this case, as a single variable name)

The function HIP accepts in fact more arguments than this, but we'll leave that for later. In fact, it must also be said that the function HIP produces a gretl bundle in output, although in this example the function is called in such a way that the bundle is not stored anywhere. If you had wanted to store the estimated model in a bundle called "Bonham", you should have called the HIP function like this:

```
Bonham = HIP(LFP, X1, other_inc, HE)
```

The estimate you get for standard errors uses OPG (Outer Product of Gradients) as the standard method for computing the covariance matrix of the estimates. This choice was made for the sake of performance but, as will be shown below, other methods are readily available.

The auxiliary statistics HIP reports are the usual likelihood-based criteria (besides the total likelihood, the maximized value for its conditional component only is also reported — see section A.1 in the appendix for details) and the Cragg-Donald statistic as a way to monitor for weak instrument. The endogeneity test is a test for $\lambda = 0$, the overall test is a test for $\beta = 0$ (apart from the intercept).

3.2 Heteroskedastic probit

Here, we'll replicate the example given in William Greene's textbook (5th edition), which also uses Mroz's dataset. The script goes like this:

```
include HIP.gfn

open mroz87.gdt -q
series WA2 = WA^2
series KIDS = (KL6 + K618)>0
income = FAMINC /10000

list X = const WA WA2 income KIDS WE
list Z = income KIDS

list endo = WE
list inst = WMED WFED CIT

Mitchell = HIP_setup(LFP, X, null, null, Z)

set stopwatch
HIP_estimate(&Mitchell)
printf "Elapsed time = %g seconds\n", $stopwatch

HIP_printout(&Mitchell, 2)
```

Note that in this case we did not use the `HIP` function, but instead we split its workload between three separate functions:

HIP_setup Sets up the model: basically, it has the same parameters as the all-rounder `HIP` function seen above. Returns a bundle.

HIP_estimate Estimates the model: takes as argument the bundle address, plus an optional scalar for the verbosity

HIP_printout Prints out the results contained in the bundle.

This may be convenient, at times, because it gives you a finer control on “what happens if”. For example, the Cragg-Donald statistics gets computed during the initialisation of the bundle. You may want to decide whether to proceed with estimation or not depending on how strong your instruments are.

Of course, the same can be accomplished through the GUI.

3.3 Let's get HIP. Heteroskedasticity and endogeneity at the same time.

The script goes

```

set echo off
set messages off
include HIP.gfn

```

```

open mroz87.gdt -q

```

```

list EXOG = const WA CIT K618
list ENDOG = WE
list ADDIN = WMED WFED
list HETVAR = HW

```

```

Paice = HIP(LFP, EXOG, ENDOG, ADDIN, HETVAR, 2)

```

In this case, the “2” tells HIP to be moderately verbose: don’t print out all the iterations, but show us the “first stage” coefficients. The output is as follows:

```

Heteroskedastic probit model with endogenous regressors
ML, using observations 1-753
Dependent Variable: y
Instrumented: WE
Instruments: const, WA, CIT, K618, WMED, WFED
Parameter covariance matrix: OPG

```

	coefficient	std. error	z	p-value	
const	-0.551804	1.36344	-0.4047	0.6857	
WA	-0.0304390	0.0172559	-1.764	0.0777	*
CIT	-0.0242784	0.208991	-0.1162	0.9075	
K618	-0.0927252	0.0896549	-1.034	0.3010	
WE	0.199646	0.101330	1.970	0.0488	**

Variance

	coefficient	std. error	z	p-value	
HW	0.117934	0.0571806	2.062	0.0392	**

"First-stage" regressions

	coefficient	std. error	z	p-value	
const	9.68554	0.586171	16.52	2.49e-61	***
WA	-0.0159435	0.0104384	-1.527	0.1267	
CIT	0.495907	0.152627	3.249	0.0012	***
K618	-0.136765	0.0612498	-2.233	0.0256	**
WMED	0.180089	0.0265972	6.771	1.28e-11	***
WFED	0.168085	0.0253072	6.642	3.10e-11	***

```

Log-likelihood      -2069.9119 Akaike criterion      4167.8239
Schwarz criterion   4232.5608 Hannan-Quinn        4192.7637
Conditional ll      -494.848818 Cragg-Donald stat.  103.337

```

Overall test (Wald) = 6.36207 (4 df, p-value = 0.1737)

Endogeneity test (Wald) = 0.509859 (1 df, p-value = 0.4752)

Test for overidentifying restrictions (LM) = 9.15786 (1 df, p-value = 0.0025)
Heteroskedasticity test (Wald) = 4.25379 (1 df, p-value = 0.0392)

3.4 Through the GUI

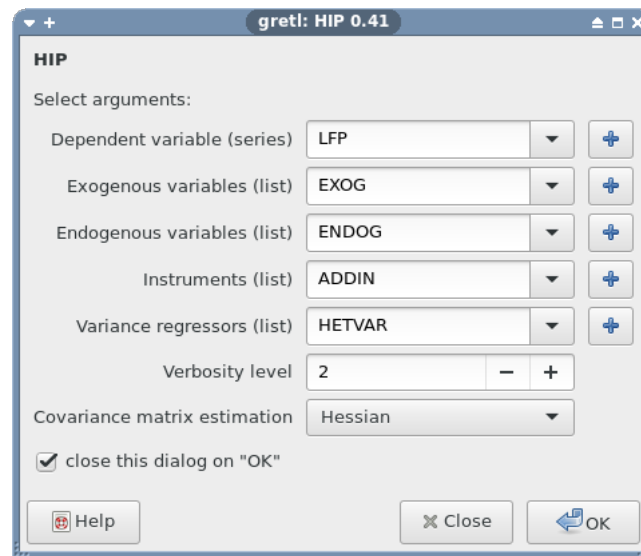


Figure 1: HIP GUI hook

Note that in this case we changed the default value of “Verbosity” from 1 to 2 and the default value of “Covariance matrix estimation” from 0 to 1. This will have the effect of showing us the first stage equation as well, and to use the Hessian instead of the OPG as the method for computing standard errors. All this is apparent in Figure 2.

By using the *Save* menu, you can choose the individual elements of the bundle to store away for later use if you want. Alternatively, you can save the bundle as a model via the *File > Save to session as icon* menu entry. If you do, assuming that you called your bundle “Bonham” again, then it will show in the “Icon view” gretl window, together with other session elements you want to keep (see Figure 3).

4 Computational details

HIP uses the analytical score and BFGS as the preferred optimisation method. The analytical Hessian is not implemented yet, but may be in the future.

Numerical instability can have several causes: ...

- Checking what happens during maximization is always very informative; set the verbosity parameter to 3.
- scaling of the variables (especially \mathbf{Y}_i): we do our best, but hey, give us a hand

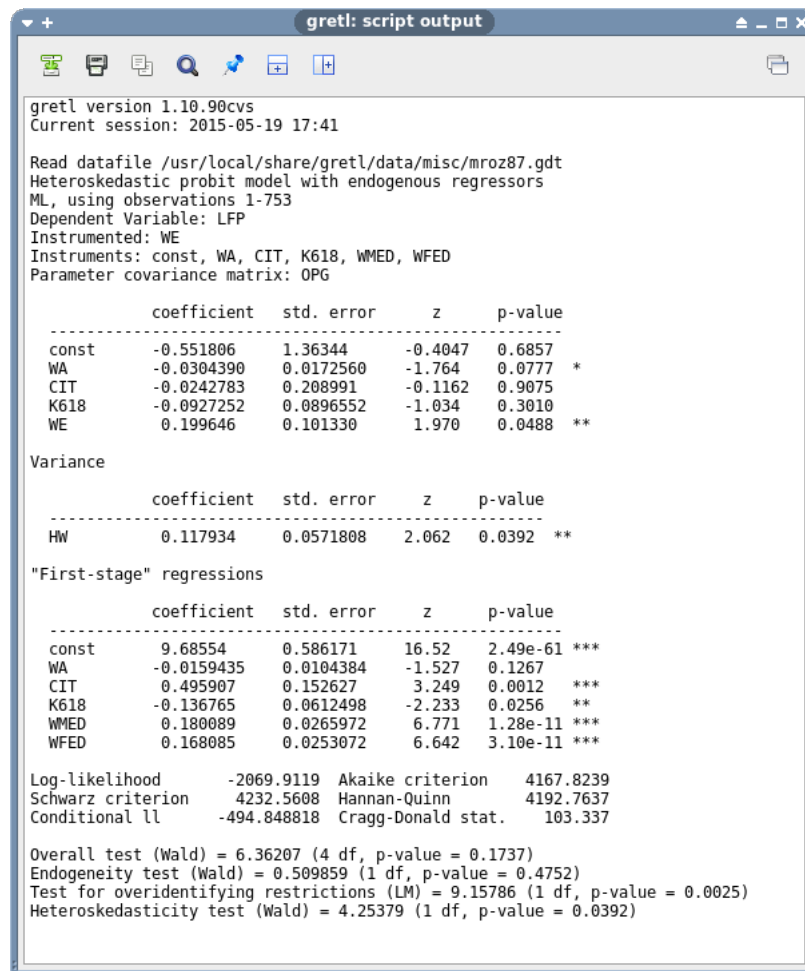


Figure 2: HIP output

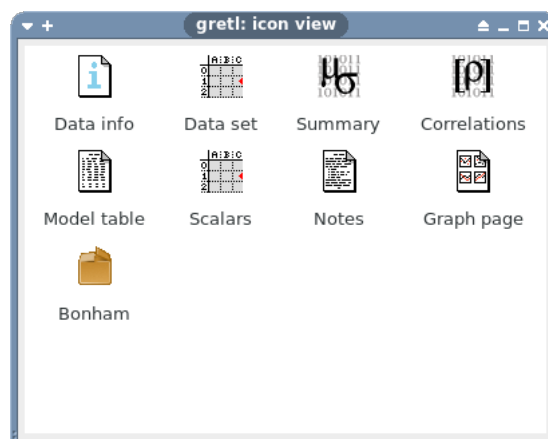


Figure 3: Icon view with a HIP bundle

- weak instruments: in some cases, there's little that can be done; see for example the artificially-generated dataset contained in the example file `weak.gdt` in the examples directory. We do some heuristics, but we're not omnipotent

References

- Chesher, A. & Irish, M. (1987). Residual analysis in the grouped and censored normal linear model. *Journal of Econometrics*, 34, 33–61.
- Cragg, J. G. & Donald, S. G. (1993). Testing identifiability and specification in instrumental variable models. *Econometric Theory*, 9, 222–240.
- Mroz, T. (1987). The Sensitivity of an Empirical Model of Married Women's hours of work to economic and statistical assumptions. *Econometrica*, 55, 765–799.
- Rivers, D. & Vuong, Q. H. (1988). Limited information estimators and exogeneity tests for simultaneous probit models. *Journal of Econometrics*, 39(3), 347–366.

A The boring stuff

For computational purposes, we reparametrize the model using the Cholesky decomposition $\Sigma^{-1} = CC'$. Moreover, by defining the quantities below it is possible to reparametrise the joint density in a computationally convenient way:

$$\begin{aligned}\nu_i &= \frac{1}{\sqrt{1 - \psi' \psi}} \left(\frac{\mathbf{Z}_i' \boldsymbol{\beta}}{\sigma_i} + \boldsymbol{\omega}_i' \boldsymbol{\psi} \right) \\ \boldsymbol{\psi} &= C' \boldsymbol{\lambda} \\ \boldsymbol{\omega}_i &= C' (\mathbf{Y}_i - \Pi \mathbf{X}_i) \\ \boldsymbol{\pi} &= \text{vec}(\Pi) \\ \mathbf{c} &= \text{vech}(C)\end{aligned}$$

The estimable parameters are $\theta' = [\boldsymbol{\beta}', \boldsymbol{\alpha}', \boldsymbol{\pi}', \boldsymbol{\psi}', \mathbf{c}']$

A.1 The loglikelihood

As usual in these models, we divide the loglikelihood for one observation in a marginal and a conditional component:

$$\begin{aligned}\ell_i &= \ell_i^m + \ell_i^c \\ \ell_i^c &= \ln P(y_i | \mathbf{X}_i, \mathbf{W}_i, \mathbf{u}_i) \\ \ell_i^m &= \ln f(\mathbf{u}_i | \mathbf{X}_i, \mathbf{W}_i)\end{aligned}$$

The marginal component is nothing but an ordinary Gaussian loglikelihood:

$$\ell_i^m = -\frac{p}{2} \ln(2\pi) + \sum_{j=1}^p \ln c_{jj} - \frac{1}{2} \boldsymbol{\omega}_i' \boldsymbol{\omega}_i$$

The conditional component is itself rather simple:

$$\ell_i^c = y_i \ln \Phi(\nu_i) + (1 - y_i) \ln [1 - \Phi(\nu_i)] \quad (5)$$

The only feature that sets ℓ_i^c apart from an ordinary probit loglikelihood is that the index function depends non-linearly on some of the parameters of the model, unless $\boldsymbol{\alpha}$ and $\boldsymbol{\psi}$ are both zero.

A.2 The score

The analytical score will be derived in steps: first the marginal component, then the conditional component. Of course, the chain rule will be very useful.

Note first that the marginal component only depends on $\boldsymbol{\pi}$ (through $\boldsymbol{\omega}_i$) and \mathbf{c} . Hence,

$$\frac{\partial \ell_i^m}{\partial \boldsymbol{\pi}} = \frac{\partial \ell_i^m}{\partial \boldsymbol{\omega}_i} \frac{\partial \boldsymbol{\omega}_i}{\partial \boldsymbol{\pi}} = \boldsymbol{\omega}_i' (\mathbf{X}_i' \otimes C') = \mathbf{X}_i' \otimes (C \boldsymbol{\omega}_i)'$$

and

$$\frac{\partial \ell_i^m}{\partial \mathbf{c}} = \tilde{\mathbf{c}}' - \boldsymbol{\omega}_i' \frac{\partial \boldsymbol{\omega}_i}{\partial \mathbf{c}} = \tilde{\mathbf{c}}' - [\boldsymbol{\omega}_i' \otimes (\mathbf{Y}_i - \Pi \mathbf{X}_i)'] S$$

where $\tilde{\mathbf{c}}$ is defined as $\text{vech}[(I \odot C)^{-1}]$ and S is a selection matrix $S = \frac{\partial \text{vec}(C)}{\partial \text{vech}(C)}$.

For the purpose of computing the score for the conditional component, note that ℓ_i^c depends on the parameters only through the index function ν_i , so $\frac{\partial \ell_i^c}{\partial \theta}$ can be evaluated as

$$\frac{\partial \ell_i^c}{\partial \theta} = \frac{\partial \ell_i^c}{\partial \nu_i} \frac{\partial \nu_i}{\partial \theta};$$

define $\mu(\nu_i)$ as

$$\mu(\nu_i) = \frac{\partial \ell_i^c}{\partial \nu_i} = y_i \frac{\phi(\nu_i)}{\Phi(\nu_i)} - (1 - y_i) \frac{\phi(\nu_i)}{1 - \Phi(\nu_i)}$$

which is the customary (signed) inverse Mill's ratio. Then,

$$\begin{aligned} \frac{\partial \nu_i}{\partial \beta} &= \frac{1}{\sigma_i \sqrt{1 - \psi' \psi}} \mathbf{Z}_i' \\ \frac{\partial \nu_i}{\partial \alpha} &= \frac{\partial \nu_i}{\partial \sigma_i} \frac{\partial \sigma_i}{\partial \alpha} = \left[-\frac{\mathbf{Z}_i' \beta}{\sigma_i^2 \sqrt{1 - \psi' \psi}} \right] \sigma_i \mathbf{W}_i' = - \left(\frac{\mathbf{Z}_i' \beta}{\sigma_i \sqrt{1 - \psi' \psi}} \right) \mathbf{W}_i' \\ \frac{\partial \nu_i}{\partial \psi} &= \frac{1}{\sigma_i^2 (1 - \psi' \psi)} \left[\sigma_i^2 \sqrt{1 - \psi' \psi} \omega_i' - \frac{\sigma_i^2}{2} \nu_i (-2 \cdot \psi') \right] = \frac{\omega_i'}{\sqrt{1 - \psi' \psi}} + \frac{\nu_i \psi'}{1 - \psi' \psi} \\ \frac{\partial \nu_i}{\partial \mathbf{c}} &= \frac{\psi'}{\sqrt{1 - \psi' \psi}} \frac{\partial \omega_i}{\partial \mathbf{c}} \\ \frac{\partial \nu_i}{\partial \pi} &= \frac{\psi'}{\sqrt{1 - \psi' \psi}} \frac{\partial \omega_i}{\partial \pi} = -\frac{\psi'}{\sqrt{1 - \psi' \psi}} (\mathbf{X}_i' \otimes C') = \frac{1}{\sqrt{1 - \psi' \psi}} [\mathbf{X}_i' \otimes (C\psi)'] \end{aligned}$$

As a consequence, the score with respect to \mathbf{c} and π may be written as

$$\begin{aligned} \frac{\partial \ell_i}{\partial \mathbf{c}} = \frac{\partial \ell_i^m}{\partial \mathbf{c}} + \frac{\partial \ell_i^c}{\partial \mathbf{c}} &= \tilde{\mathbf{c}}' + \left(\frac{\psi'}{\sqrt{1 - \psi' \psi}} - \omega_i' \right) \frac{\partial \omega_i}{\partial \mathbf{c}} = \\ &= \tilde{\mathbf{c}}' + \left(\frac{\psi'}{\sqrt{1 - \psi' \psi}} - \omega_i' \right) [I \otimes (\mathbf{Y}_i - \Pi \mathbf{X}_i)'] = \\ &= \tilde{\mathbf{c}}' + \left[\left(\frac{\psi'}{\sqrt{1 - \psi' \psi}} - \omega_i' \right) \otimes (\mathbf{Y}_i - \Pi \mathbf{X}_i)' \right] \end{aligned}$$

B List of functions

B.1 Model setup

FIXME: THIS IS OUTDATED AT THE MOMENT AND NEEDS TO BE FIXED.

```
HIP_setup(series y, list EXOG, list ENDOG[null], list ADDIN[null],
list HETVAR[null], int s[0:2:0])
```

1. a series containing y_i , the dependent binary variable (**required**)
2. a list containing the exogenous variables \mathbf{X}_{1i} in \mathbf{X}_i in equation (1) (**required**)

3. a list containing the exogenous variables \mathbf{Y}_i in equations (1)–(2)
4. a list containing the additional instruments \mathbf{X}_{2i} in \mathbf{X}_i in equation (2)
5. a list containing the variables \mathbf{W}_i of the skedastic function in equation (1)
6. a scalar, acting as an integer, for the choice of the covariance matrix estimation method

B.2 Estimation

```
HIP_estimate(bundle *b)
```

General estimation function. Its argument is:

1. the address of a model bundle created via `HIP_setup` (**required**)

B.3 Output

Note: these functions assume that the bundle they refer to contain a model that has already been estimated. No checks are performed.

```
HIP_printout(bundle *b, int verbose[0:1:3])
```

Prints out a model. Its arguments are

1. the address of a model bundle filled via `HIP_estimate` (**required**)
2. a scalar, acting as a integer, to choose the verbosity: 0 = quiet, 1 = main equation, 2 = first stages, 3 = mle verbose.

B.4 GUI wrapper

```
function bundle HIP(series y, list EXOG, list ENDOG[null], list ADDIN[null],  
list HETVAR[null], int v[0:3:1], int s[0:2:0])
```

Using the same argument descriptions as `HIP_setup`, after checking the rank condition (if estimating instrumental variables probit), it calls:

1. `HIP_setup`
2. `HIP_estimate`
3. `HIP_printout`

The parameter `v` controls the verbosity level: 0 = quiet, 1 = main equation only, 2 = first stages, 3 = mle verbose.

C Bundle elements

Name	Type	Purpose
Model descriptors		
n	scalar	number of observations
het	scalar	acting as a Boolean switch, Heteroskedastic probit
iv	scalar	acting as a Boolean switch, Instrumental Variables probit
T	scalar	number of observations used
t1	scalar	first observation used
t2	scalar	last observation used
Data		
depvar	series	dependent variable
mEXOG	matrix	exogenous regressors
mk1	matrix	number of exogenous regressors
mENDOg	matrix	endogenous regressors
mp	matrix	number of endogenous regressors
mADDIN	matrix	additional instruments
mk2	matrix	number of additional instruments
mHETVAR	matrix	variance regressors
mq	matrix	number of variance regressors
mZ	matrix	total regressors
mh	matrix	number of total regressors
mX	matrix	total instruments
mk	matrix	number of total instruments
Strings		
depvarname	string	dependent variable name
mEXOGnames	string	exogenous regressors names
mENDOgnames	string	endogenous regressors names
mADDINnames	string	additional instruments names
mHETVARnames	string	variance regressors names
mZnames	string	total regressors names
mXnames	string	total instruments names
Estimation parameters		
vcvtype	scalar	acting as an integer, method for estimating the covariance matrix: 0 = OPG (default), 1 = empirical Hessian, 2 = Sandwich

Estimation results		
errcode	scalar	error code from catch
uhat	series	first stage residuals (Rivers & Vuong, 1988)
rescale	matrix	square root of the diagonal elements of first stage residuals covariance matrix
lnl0	scalar	second stage log-likelihood (Rivers & Vuong, 1988)
theta	matrix	coefficients
VCVtheta	matrix	covariance matrix
lnl1	scalar	log-likelihood
lnl1m	scalar	marginal log-likelihood (if iv)
lnl1c	scalar	conditional log-likelihood (if iv)
llt	series	log-likelihood
SCORE	matrix	score matrix by observation
infocrit	matrix	information criteria
Diagnostics ²		
WaldAll	matrix	Wald overall test
WaldEnd	matrix	Wald endogeneity test
LMOverid	matrix	LM test for overidentifying restrictions
HETtest	matrix	if iv Wald test, else LR test of Heteroskedasticity
CraggDondald	scalar	Cragg & Donald (1993) statistic for weak instruments
normtest	matrix	Conditional moment test for normality of ε_i Chesher & Irish (1987)