Abstract

This thesis presents a report on original research, published as joint work with Merschen and von Stengel in *Electronic Notes in Discrete Mathematics* [1]. Our result shows a polynomial time algorithm to solve two problems related to labeled Gale strings, a combinatorial structure consisting a string of labels and a bitstring satisfying certain conditions introduced by Gale in [5].

Gale strings can be used in the representation of a particular class of games that Savani and von Stengel [10] used as an example of exponential running time for the classical Lemke-Howson algorithm to find a Nash equilibrium of a bimatrix game [7]. It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class PPAD (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou [9] was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm.

Our result, on the contrary, sets apart this class of games as a case for which there is a polynomial-time algorithm to find a Nash equilibrium. Since Daskalakis, Goldberg and Papaditrimiou [3] and Chen and Deng [2] proved the PPAD-completeness of finding a Nash equilibrium in general normal-form games, we have a special class of games, unless PPAD = P.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as Gale strings, as seen in Savani and von Stengel [10]. The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, proven by Edmonds [4].

Further results by Merschen [6] and Végh and von Stengel [11] will be mentioned.

An appendix relates an amendment to the proof of the PPAD-completeness result by Daskalakis, Goldberg and Papaditrimiou [3].

1 Introduction

2 Complexity, Games, Polytopes and Gale Strings

2.1 The Complexity Classes P and PPAD

2.2 Normal Form Games and Nash Equilibria

file: background-subsection

2.3 Bimatrix Games and Best Response Polytopes

file: polytopes-subsection

2.4 Cyclic Polytopes and Gale Strings

2.5 Labeling and the Problem Another Gale

file: gale-def-subsection

3 Algorithmic and Complexity Results

3.1 Pivoting

3.2 The Lemke-Howson Algorithm and Parity

file: pivoting-LH-subsection

3.3 The Complexity of Gale and Another Gale

file: main-result-subsection - done!

4 Further results

Appendix A: A result about PPAD completeness of Nash

Appendix B: Notation

References

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