**Abstract**

This thesis presents a report on original research, published as joint work with Merschen and von Stengel in *Electronic Notes in Discrete Mathematics* [1]. Our result shows a polynomial time algorithm to solve two problems related to labeled Gale strings, a combinatorial structure consisting a string of labels and a bitstring satisfying certain conditions introduced by Gale in [5].

Gale strings can be used in the representation of a particular class of games that Savani and von Stengel [9] used as an example of exponential running time for the classical Lemke-Howson algorithm to find a Nash equilibrium of a bimatrix game [7]. It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class PPAD (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou [8] was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm.

Our result, on the contrary, sets apart this class of games as a case for which there is a polynomial-time algorithm to find a Nash equilibrium. Since Daskalakis, Goldberg and Papaditrimiou [3] and Chen and Deng [2] proved the PPAD-completeness of finding a Nash equilibrium in general normal-form games, we have a special class of games, unless PPAD = P.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as Gale strings, as seen in Savani and von Stengel [9]. The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, proven by Edmonds [4].

Further results by Merschen [6] and Végh and von Stengel [10] will be mentioned.

An appendix relates an amendment to the proof of the PPAD-completeness result by Daskalakis, Goldberg and Papaditrimiou [3].

# 1 Introduction

# 2 Complexity, Games, Polytopes and Gale Strings

## 2.1 The Complexity Classes P and PPAD

## 2.2 Normal Form Games and Nash Equilibria

file: background-subsection

## 2.3 Bimatrix Games and Best Response Polytopes

file: polytopes-subsection

## 2.4 Cyclic Polytopes and Gale Strings

## 2.5 Labeling and the Problem Another Gale

file: gale-def-subsection

# 3 Algorithmic and Complexity Results

## 3.1 Pivoting

## 3.2 The Lemke-Howson Algorithm and Parity

file: pivoting-LH-subsection

## 3.3 The Complexity of Gale and Another Gale

file: main-result-subsection - done!

## 4 Further results

## Appendix A: A result about PPAD completeness of Nash

## Appendix B: Notation

## References


[1] M. M. Casetti, J. Merschen, B. von Stengel (2010). Finding Gale Strings. *Electronic Notes in Discrete Mathematics* issue, pp. n–m.

[2] X. Chen, X. Deng (2006). Settling the complexity of two-player Nash equilibrium. *Proc. 47th FOCS*, pp. 261–272.

[3] C. Daskalakis, P. W. Goldberg, C. H. Papadimitriou (2006). The complexity of computing a Nash equilibrium. *Proc. Ann. 38th STOC*, pp. 71–78 change ref to econometrica(?)

[4] J. Edmonds (1965). Paths, trees, and flowers. *Canad. J. Math.* 17, pp. 449–467.

[5] D. Gale (1963), Neighborly and cyclic polytopes. *Convexity, Proc. Symposia in Pure Math.*, Vol. 7, ed. V. Klee, American Math. Soc., Providence, Rhode Island, pp. 225–232 check if right typography

[6] J. Merschen (2012) thesis

[7] C. E. Lemke, J. T. Howson, Jr. (1964). Equilibrium points of bimatrix games. *J. Soc. Indust. Appl. Mathematics* 12, pp. 413–423.

[8] C. H. Papadimitriou (1994). On the complexity of the parity argument and other inefficient proofs of existence. *J. Comput. System Sci.* 48, pp. 498–532.

[9] R. Savani, B. von Stengel (2006). Hard-to-solve bimatrix games. *Econometrica* 74, pp. 397–429.

[10] L. Végh, B. von Stengel ref