Abstract

IMPORTANT: at most 300 words! - this is more!

The pivoting technique employed by the classical Lemke-Howson algorithm for finding a Nash equilibrium of a bimatrix game has been a major motivation for the definition o the complexity class PPAD [**?**]. Daskalakis, Goldberg and Papaditrimiou [**?**] and Chen and Deng [**?**] later proved that finding a Nash equilibrium is PPAD-compete for any normal form game. The proof employed a long reduction of a discrete version of the search problem associated with Brouwer's fixed point theorem to a reduction to NASH for a particular class of games.

Savani and von Stengel [**?**] proved that the Lemke-Howson algorithm exhibits a exponential running time for a particular class of bimatrix games. These are games are characterised by a cyclic best response polytope. The best response polytope and the Nash equilibria of the game can also be represented by a string of labels and an associated string of 0s and 1s satisfying a set of conditions called the *Gale evenness condition*. This result led to conjecture that this particular class of games could be used for a more immediate proof of PPAD-completeness.

Our result, co-authored with Merschen and von Stengel [**?**], proves that NASH for these games is polynomially solvable, thus making the conjecture of PPAD-completeness less likely to be true. This is achieved exploiting the correspondence between the equilibria of the games and the Gale strings, and through a reduction to the PERFECT MATCHING problem, proved to be polynomially solvable by Edmonds [**?**].

All the pivoting results can be further analysed in the framework *Euler complexes*, or *oiks*, introduced by Edmonds [**?**]. Végh and von Stengel [**?**] have introduced a general definition of *pivoting system* to deal with *orientation* in oiks. An application to perfect matchings in Euler graphs allows to find in polynomial time a Nash equilibrium of given *sign* in the games corresponding to Gale Strings, a question left open by our article.