

## Abstract

This thesis presents a report on original research, published as conjoint work with Merschen and von Stengel in ENDM (2010). Our result shows a polynomial time algorithm to find a Nash equilibrium for a particular class of games, which was previously used by Savani and von Stengel (2006) as an example of exponential time for the classical Lemke-Howson algorithm for bimatrix games (1964).

It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class **PPAD** (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou (1994) was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm. A **PPAD**-completeness proof of the games we consider would have provided a traceable proof of the Daskalakis, Goldberg and Papadimitriou (2005) and Chen and Deng (2009) results about the **PPAD**-completeness of every normal form game. Our result of polynomial-time solvability, on the other hand, indicates the existence of a special class of games, unless **PPAD** = **P**.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as a string of labels and an associated string of 0s and 1s satisfying some conditions, called *Gale conditions*, as seen in Savani and von Stengel (2006). The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, solved by Edmonds (1965).

Further results by Merschen (2012) and Végh and von Stengel (2014) solved the open problem of the *sign* of the equilibrium found in polynomial time.

# 1 Introduction

What, why

An appendix collects the main notation used throughout the thesis.

A final appendix presents our correction to an error in the proof of the **PPAD**-completeness of NASH in ??.

## 2 Definitions and context

### 2.1 Gale strings

A *Gale string*

### 2.2 Bimatrix games

[basic on games, NE]

### 2.3 Cyclic polytopes

### 2.4 The Lemke-Howson algorithm

The Lemke-Howson algorithm  
for Gale

### 2.5 Pivoting and the class PPAD

touch on pivoting as one of the reasons to introduce PPA(D). \*just give the def of directed\*, the idea of pivoting + sign will be discussed in "further results" section.  
The focus is "why the main result is relevant"

mention oiks, so you can later mention that EulG - as the ones used for MAIN are oik. Again: not too much.

## 3 The complexity of COMPLETELY LABELED GALE STRING and ANOTHER COMPLETELY LABELED GALE STRING

Note: why not call them GALE and ANOTHER GALE? It would make it more readable.

\*\*Main result!\*\* - the reduction to Perfect matching; both GALE and ANOTHER GALE are in P, we're happy.

## 4 Further results

The framework provided by our result led to further questions, related to the issue of the \*sign\* of an index - and so on (Merschen, VvS)

Open problems (?)

## Appendix A: Notation

For a matrix  $A$  we denote its transpose with  $A^T$ . We treat vectors  $u, v$  in  $\mathbb{R}^d$  as column vectors, so  $u^T v$  is their scalar product. By  $\mathbf{0}$  we denote a vector of all 0's, of suitable dimension, by  $\mathbf{1}$  a vector of all 1's. A unit vector, which has a 1 in its  $i$ th component and 0 otherwise, is denoted by  $e_i$ . Inequalities like  $u \geq \mathbf{0}$  hold for all components. For a set of points  $S$  we denote its convex hull by  $\text{conv } S$ .

## Appendix B: A result about PPAD completeness of NASH

## References