This thesis presents a report on original research, published as joint work with Merschen and von Stengel in *Electronic Notes in Discrete Mathematics* (2010). Our result shows a polynomial time algorithm to solve two problems related to labeled Gale strings, a combinatorial structure consisting a string of labels and a bitstring satisfying certain conditions, called *Gale conditions*.

Gale strings can be used in the representation of a particular class of games, that Savani and von Stengel (2006) used as an example of exponential running time for the classical Lemke-Howson algorithm to find a Nash equilibrium of bimatrix games (1964). It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class PPAD (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou (1994) was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm.

Our result, on the contrary, sets apart this class of games as a case for which there is a polynomial-time algorithm to find a Nash equilibrium. Since Daskalakis, Goldberg and Papaditrimiou (2005) and Chen and Deng (2009) proved the PPAD-completeness of finding a Nash equilibrium in general normal-form games, we have a special class of games, unless PPAD = P.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as a string of labels and a bitstring, as seen in Savani and von Stengel (2006). The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, solved by Edmonds (1965).

Further results by Merschen (2012) and Végh and von Stengel (2014) solved the open problem of the *sign* of the equilibrium found in polynomial time.