

This thesis presents a report on original research, published as joint work with Merschen and von Stengel in *Electronic Notes in Discrete Mathematics* (2010). Our result shows a polynomial time algorithm to find a Nash equilibrium for a particular class of games, which was previously used by Savani and von Stengel (2006) as an example of exponential time for the classical Lemke-Howson algorithm for bimatrix games (1964).

It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class PPAD (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou (1994) was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm. A PPAD-completeness proof of the games we consider would have provided a traceable proof of the Daskalakis, Goldberg and Papadimitriou (2005) and Chen and Deng (2009) results about the PPAD-completeness of every normal form game. Our result of polynomial-time solvability, on the other hand, indicates the existence of a special class of games, unless $\text{PPAD} = \text{P}$.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as a string of labels and an associated string of 0s and 1s satisfying some conditions, called *Gale conditions*, as seen in Savani and von Stengel (2006). The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, solved by Edmonds (1965).

Further results by Merschen (2012) and Vég  and von Stengel (2014) solved the open problem of the *sign* of the equilibrium found in polynomial time.