**Abstract**

This thesis presents a report on original research, published as conjoint work with Merschen and von Stengel in ENDM (2010). Our result shows a polynomial time algorithm to find a Nash equilibrium for a particular class of games, which was previously used by Savani and von Stengel (2006) as an example of exponential time for the classical Lemke-Howson algorithm for bimatrix games (1964).

It was conjectured that solving these games via the Lemke-Howson algorithm was complete in the class **PPAD** (Proof by Parity Argument, Directed version). A major motivation for the definition of this class by Papadimitriou (1994) was, in turn, to capture the pivoting technique of many results related to the Nash equilibrium, including the Lemke-Howson algorithm. A **PPAD**-completness proof of the games we consider would have provided a traceable proof of the Daskalakis, Goldberg and Papaditrimiou (2005) and Chen and Deng (2009) results about the **PPAD**-completeness of every normal form game. Our result of polynomial-time solvability, on the other hand, indicates the existence of a special class of games, unless **PPAD** = **P**.

Our proof exploits two results. The first one is the representation of the Nash equilibria of these games as a string of labels and an associated string of 0s and 1s satisfying some conditions, called *Gale conditions*, as seen in Savani and von Stengel (2006). The second one is the polynomial-time solvability of the problem of finding a perfect matching in a graph, solved by Edmonds (1965).

Further results by Merschen (2012) and Végh and von Stengel (2014) solved the open problem of the *sign* of the equilibrium found in polynomial time.

# 1 Introduction

What, why and overview of tools used

# 2 Definitions

## 2.1 Gale strings

Definition

## 2.2 Bimatrix games

[basic on games, NE]

## 2.3 Cyclic polytopes

## 2.4 The Lemke-Howson algorithm

The Lemke-Howson algorithm
    for Gale

## 2.5 Pivoting and the class PPAD

touch on pivoting as one of the reasons to introduce PPA(D). *just give the def of directed*, the idea of pivoting + sign will be discussed in "further results" section. The focus is "why the main result is relevant"

mention oiks, so you can later mention that EulG - as the ones used for MAIN are oik. Again: not too much.

# 3 The complexity of COMPLETELY LABELED GALE STRING and ANOTHER COMPLETELY LABELED GALE STRING

Note: why not call them GALE and ANOTHER GALE? It would make it more readable.

**Main result!** - the reduction to Perfect matching; both GALE and ANOTHER GALE are in P, we're happy.

# 4 Further results

The framework provided by our result led to further questions, related to the issue of the *sign* of an index - and so on (Merschen, VvS)

Open problems (?)

# Appendix A: Notation

For a matrix $A$ we denote its transpose with $A^T$. We treat vectors $u, v$ in $\mathbb{R}^d$ as column vectors, so $u^T v$ is their scalar product. By $\mathbf{0}$ we denote a vector of all 0's, of suitable dimension, by $\mathbf{1}$ a vector of all 1's. A unit vector, which has a 1 in its $i$th component and 0 otherwise, is denoted by $e_i$. Inequalities like $u \geq \mathbf{0}$ hold for all components. For a set of points $S$ we denote its convex hull by $\operatorname{conv} S$.

# Appendix B: A result about PPAD completeness of NASH

# References