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# The expectation hypothesis of interest rates and network theory: The case of Brazil

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ABSTRACT

This paper investigates the topological properties of the Brazilian term structure of interest rates network. We build the minimum spanning tree (MST), which is based on the concept of ultrametricity, using the correlation matrix for interest rates of different maturities. We show that the short-term interest rate is the most important within the interest rates network, which is in line with the Expectation Hypothesis of interest rates. Furthermore, we find that the Brazilian interest rates network forms clusters by maturity.


## 1. Introduction

A large number of researchers have studied the structure of finance complex networks and found that it can be used to map the data-structures onto graphs [1–8]. These studies have relied mainly on stock market data and have provided important empirical results, which suggest that complex network theory may be very helpful in explaining the dynamic characteristics of stock markets.

In this paper, we extend this analysis to the Brazilian term structure of interest rates market, by studying a network generated by a variety of interest rates maturities.[1] The main objective is to characterize the topology and taxonomy of interest rates networks and study the dynamics of this network.

The Brazilian interest rates network is an interesting case study for a number of reasons. First, it is one of the most important in Latin America. Second, it has a well developed bond market. Finally, there are a large number of interest rates for different maturities, which allows testing for the Expectation Hypothesis of interest rates, which is one of the main theories for explaining interest rates dynamics.[2]

This paper contributes to the literature by examining whether the Expectation Hypothesis for interest rates holds for the Brazilian term structure of interest rates. To the best of our knowledge this is the first paper that discusses this issue, and presents novel results and important insights. Our results imply that the Selic interest rates has the largest domination power within the network, about twice as much as any other interest rate for different maturities, which suggests that it is

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1 The term structure of interest rates can be defined as the relation between interest rates and the time to maturity.

2 The expectation hypothesis of interest rates states that long term interest rates are basically formed as expectations of future short-term interest rates.

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the benchmark interest rate and is favorable to the Expectation Hypothesis. Furthermore, short-term interest rates are the most important in the network, followed by long-term interest rates. Finally, interest rates tend to form clusters according to maturity.

The remainder of the paper is structured as follows: The next section discusses the literature review for interest rates networks, while Section 3 introduces the methodology and the sampling procedures. Section 4 shows the data and Section 5 presents empirical results. Finally, Section 6 provides some final considerations.

## 2. Literature review

Many researchers have recently shown that it is possible to detect the formation of clusters within networks, using the MST based on ultrametricity [1–10]. Their researches have shown the topological properties of networks, which has associated a meaningful economy taxonomy.

A seminal paper presented by Mantegna [2], shows that the minimum spanning tree (MST) and the associated subdominant ultrametric tree, obtained from a distance matrix, selects a topological space for the stocks of a portfolio traded in a financial market. This topology is useful to describe the financial markets and, in the search of economic common factors affecting specific groups of stocks.

Bonanno et al. [3] compare the topological properties of the minimum spanning tree (MST) of empirical data recorded at the New York Stock Exchange (NYSE) with MSTs obtained from simple models of the portfolio dynamics. They found that the empirical tree has features of a complex network that cannot be reproduced, even as a first approximation, by a random market model and by the widespread one-factor model.

Jung et al. [7] presents a network structure of the Korean stock market with its minimum spanning tree through the correlation matrix. They found that the Korean stock market does not form clusters of the business sector or of industry categories.

Onnela et al. [5] have studied the dynamics of asset trees and applied it to portfolio analysis. They have shown that the tree evolves over time and have found that the normalized tree length decreases and remains low during a crash. On another work, Onnela et al. [6] give a demonstration of how the prominent 1987 stock market crash, which culminated in Black Monday, may be viewed from the perspective of dynamic asset trees.

Coelho et al. [8] study the extent and evolution of interdependence between world equity markets using the MST approach. They found that there is a strong tendency for markets to organize by geographical location. They also found that the mean correlations show a tendency to increase over the period as whole, while mean distances in the MST and the mean occupation layers have been trending downward. Finally, they show that while clusters of any given period may be homogeneous, the likelihood of these remaining stable over a reasonable portfolio period is small.

Di Matteo et al. [11] investigate weekly data of interest rates in money and capital markets, for a period of 16 years. They found that the correlations inside the clusters are strong in any part of the analyzed period. On another work, Di Mateo et al. [12] propose a general method to study the hierarchical organization of financial data by embedding the structure of their correlations in metric graphs in multidimensional spaces. The resulting graphs contain the MST as sub-graph and they preserve its hierarchical structure. They also found that this embedding procedure can be extended to surfaces of higher genus constructing in this way networks with different degree of complexity and tunable information content.

Despite a large body of evidence that suggests that network theory is useful in analyzing stock markets there are only a very few studies that focus on fixed income market such as the work of Di Matteo et al. [11,12]. Nonetheless, these works have not studied whether the expectation hypothesis of interest rates hold. In this paper we will characterize the Brazilian fixed income network and address whether the expectation hypothesis hold.

The expectations hypothesis (EH), which states that the long-term interest rate is an average of expected future short-term rates plus a time independent risk premia, is a convenient way to deal with the term structure. The EH has been studied by many researchers [13–21], however the empirical evidence varies from one study to the next depending on the precise implication tested, the segment of the yield curve examined or the period under study.

Longstaff [17] tested the expectations hypothesis at the extreme short end of the term structure using short-term repurchase rates. Seo [13] shows the effect of the transaction costs on the predictability of the term structure of interest rates and find that the short-term interest rate responds to the past term spread[3] only if the term spread exceeds the threshold parameters. He also suggests the threshold error correction model,[4] which is consistent with the stylized fact of the term structure. The threshold cointegration model outperforms the linear cointegration model and the random walk model, and thus it can be used in the evaluation of the monetary policy and economic forecasting.

Jondeau and Ricart [14] propose an alternative approach to testing the EH, which takes account of the potential non-stationarity of interest rates in an error correction model(ECM) framework. They also tested the EH for the US, German, French and UK Euro-rates over the period from 1975 to 1997. They found that they almost never reject the theory for French and UK rates, whereas they generally reject the theory for US and German rates. Christiansen [20] tested the EH using the the

[3] Term spreads are long-term minus short-term interest rates.

[4] An Error Correction Model (ECM) is a dynamic model in which the movement of the variables in any period is related to the previous period's gap from long-run equilibrium.

forward-rate regression. He found that the EH is close to being accepted(numerically). For a long data set the EH is rejected statistically for maturities between 1 and 25 years and for a shorter data set we are unable to reject the EH. He shows that the results are invariant to the maturity of the forward rates.[5]

Brown et al. [22] have found that the expectations hold when rates are less volatile and/or that we may be entering a period of lower volatility. Gerlach [16] found that term spreads contain no information about future short-term rates and the EH is also soundly rejected by the data. He implemented a modified version of the EH that incorporates time-varying term premia, which he is unable to reject. Moreover, the hypothesis that the parameters are constant before and after the onset of the Asian crisis in July, 1997 is not rejected.

Esteve [23] has tested for threshold cointegration in VEC models to the Spanish term structure of interest rates during the period 1980–2002. He used the expectations hypothesis in the context of cointegration theory, which suggests that the long and short interest rates are linked through a long-run relationship with parameters $(1, -1)$, i.e., that the interest rate spread is mean-reverting. He found that nonlinear cointegration between long and short interest rates is rejected, meaning that a linear cointegration model would provide an adequate empirical description for the Spanish term structure of interest rate. Camarero and Tamarit [15] have applied several tests using as an example the expectations hypothesis of the term structure of interest rates. The results are consistent with the existence of cointegration between the long and short run Spanish interest rates, with a vector$(1, -1)$, as predicted by the theory.

Tabak [19] tested the expectations hypothesis (EH) using cointegration techniques, for maturities ranging from 1 to 12 months, for the Brazilian market. He found evidence suggesting that, for the period 1995–2006, the cointegration implication generally seems to hold. He also found strong evidence supporting causality from short to long rates and also in the opposite direction. Empirical evidence supports the expectations theory of the term structure of interest rates.

Cuthbertson et al. [21] have tested the expectations hypothesis of the term structure of interest rates for the German money market at the short end of the maturity spectrum using a variety of metrics. They used the VAR methodology to forecast the future interest rates. The VAR methodology allows explicit consideration of potential non-stationarity in the the data as do their tests based on the cointegration literature. They found that the results tend to support the hypothesis and the EH may adequately characterize the German data because interest rates have been reasonably volatile under money supply targeting but not extremely volatile, owing to the credible long term anti-inflation stance of the Bundesbank.

In this paper we will employ recently developed methods within statistical physics to study whether the expectation hypothesis hold and characterize the fixed income network.

## 3. Interest rates networks

We use the cross-correlation in interest rate changes from December 5, 1997 to March 11, 2008. The dataset is daily closure changes in maturities of term structure of interest rates, and the correlation matrix should represent long-term trends. The log-change of the interest rates is defined as $Si(t) = \ln(Yi(t)) - \ln(Yi(t-1))$, where $Yi(t)$ is the interest rate of maturity $i$ at time $t$. Using this variable we calculate the cross-correlation function as:

$$\rho_{i,j} = \frac{\langle S_i \cdot S_j\rangle - \langle S_i\rangle\langle S_j\rangle}{\sqrt{(\langle S_i^2\rangle - \langle S_i\rangle^2)(\langle S_j^2\rangle - \langle S_j\rangle^2)}}, \tag{1}$$

where $\langle S_i\rangle$ represents the statistical average of $S_{i,t}$ for a given time period. All cross-correlations range from $-1$ to 1.[6] The matrix $\rho_{i,j}$ has $n \times n$ order if there are $n$ rates in the sample and is symmetric as $\rho_{i,j} = \rho_{j,i}$.

### 3.1. Minimum spanning tree

We would like to build a MST to study the topology of this network. However, the MST requires the use of a variable that can be interpreted as distance, satisfying the three axioms of Euclidian distance. Therefore, we transform this matrix in order to build a distance matrix. To build the interest rates network we employ the metric distance $d_{i,j} = \sqrt{2(1-\rho_{i,j})}$ proposed by Mantegna [1], where $\rho_{i,j}$ is the correlation between changes in maturities $i$ and $j$.[7]

The MST is a graph that connects all the $n$ nodes of the graph with $n-1$ edges, such that the sum of all edge weights $\sum_{i,j\in D} d_{i,j}$ is a minimum, where $D$ is the distance matrix. The MST extracts significant information from the distance matrix and it reduces the information space from $\frac{n\times(n-1)}{2}$ correlations to $n-1$ tree edges. It is the spanning tree of the shortest length using the Kruskal algorithm of the $d_{i,j}$ and is a graph without cycles connecting all nodes with links.[8]

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5 Forward rate is a projection of future interest rates calculated from either spot rates or the yield curve.

6 A value of $\rho_{i,j} = 1$ implies that interest rates have a perfect correlation between them, whereas a value of $\rho_{i,j} = -1$ suggests that these interest rates are perfectly anti-correlated.

7 This metric satisfies the three axioms of Euclidian distance: (i) $d_{i,j} = 0$ if and only if $i = j$, (ii) $d_{i,j} = d_{j,i}$, and (iii) $d_{i,j} \le d_{i,k} + d_{k,j}$.

8 The Kruskal algorithm has the following steps: (1). Choose a pair of maturities with the nearest distance and connect with a line proportional to this distance, (2). Connect a pair with second nearest distance, (3). Connect the nearest pair that is not connected by the same tree, and (4). Repeat step three until all maturities are connected in one tree.

Define the maximal distance $d^*_{i,j}$ between two successive maturities when moving from maturity $i$ to maturity $j$ over the shortest path of the MST connecting these two maturities.[9] The distance $d^*_{i,j}$ satisfies the above axioms of Euclidian distance and also the following ultrametric inequality:

$$d_{i,j} \leq \max[d_{i,k}, d_{k,j}]. \tag{2}$$

To construct the hierarchical tree we used the subdominant ultrametric distance $d^*_{i,j}$ in clustering algorithms. For a better interpretation of the hierarchical tree of interest rates network, we apply two clustering algorithms: single linkage clustering method and complete linkage clustering method. The single linkage(also known as the nearest neighbor) method is one of the simplest agglomerative hierarchical clustering method. The defining feature of the method is that the distance considered is defined as the distance between the closest pair of nodes. In the single linkage method, the distance $D_s$ can be defined as:

$$D_s = \mathrm{Min}(d_{i,j}). \tag{3}$$

Here the distance between every possible node pair $(i, j)$ is computed. The minimum value of these distances is said to be the distance between nodes $i$ and $j$. In other words, the distance between two nodes is given by the value of the shortest link between the nodes. At each stage of hierarchical clustering, the nodes $i$ and $j$, for which $d_{i,j}$ is minimum, are merged.

The complete linkage(also known as the farthest neighbor) method is the opposite of single linkage clustering method. Distance between nodes is now defined as the distance between the most distant pair of nodes. In the single linkage method, the distance $D_c$ can be defined as:

$$D_c = \mathrm{Max}(d_{i,j}). \tag{4}$$

Here the distance between every possible node pair $(i, j)$ is computed. The maximum value of these distances is said to be the distance between nodes $i$ and $j$. In other words, the distance between two nodes is given by the value of the longest link between the nodes.

We have studied both the single linkage and the complete linkage clustering methods, and found that the complete linkage is more adequate in detecting clusters in an interest rate network. The results are in accordance with Di Matteo (2002) [24].

### 3.2. Weighted networks measures

We also estimate a variety of network measures to characterize the interest rates network. In order to study the evolution of the networks parameters over time we estimate them using a moving window of fixed size (1008 observations, approximately four years of data). Therefore, we estimate the parameters for the sample that comprises observation 1 to 1008, 2 to 1009, and so forth until we use all the sample. This dynamic approach allows studying the evolution of the network over time.

An important network measure is defined as disparity [25]. For a given vertex $i$ with connectivity $k_i$ and strength $s_i$ all weights $w_{i,j}$ can be of the same order if $s_i = k_i$. However, if one weight dominates (or a small number of vertexes) over the others we may have an heterogenous case. The disparity of vertex $i$ can be measured as:

$$D(i) = \sum_{j \in v(i)} \left[ \frac{w_{i,j}}{s_i} \right]^2, \tag{5}$$

where $v(i)$ is the set of neighbors of $i$.

We also study closeness centrality [26], which proxies for the proximity to the rest of vertices in the network. The higher its value, the closer that vertex is to the others (on average). Given a vertex $k$ and a graph $G$, it can be defined as:

$$C(k) = \frac{1}{\sum_{h \in G} d_G(k, h)}, \tag{6}$$

where $d_G(k, h)$ is the minimum distance from vertex $k$ to vertex $h$. This measures the influence of a vertex in a graph.

Domination power measures of individual nodes are able to find the centrality in a network that takes the direction and the weight of the relations into account. Van den Brink and Gilles [27] have developed the degree based domination measure called $\beta$-measure as described below:

$$\beta(i) = \sum_{j=1}^{n} \frac{w_{i,j}}{\lambda(j)}, \tag{7}$$

[9] This distance is called subdominant ultrametric distance and a space connected by these distances provides a topological space that has associated a unique indexed hierarchy.

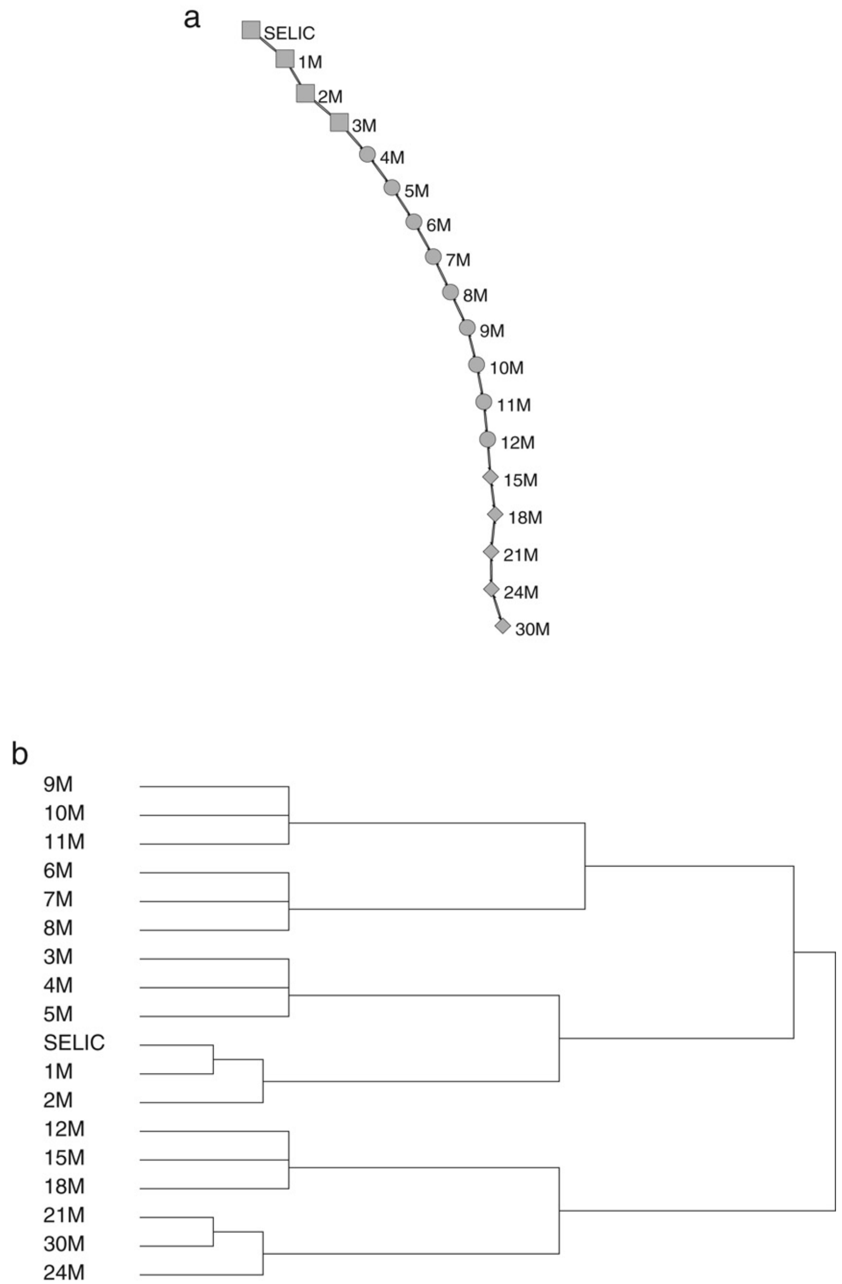


**Fig. 1.** (a) Plot of the MST of a network connecting the full sample of interest rates for the period from December 5, 1997 to March 11, 2008. The node shape is based on their terms, in months: (Square) for short-term, (Ellipse) for medium-term and (Diamond) for long-term rates. (b) Plot of the Taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST of the full sample of interest rates, using the simple linkage clustering method.

where $\lambda_{i,j}$ is the dominance weight of node $j$ given by

$$\lambda(j) = \sum_{i=1}^{n} w_{i,j}. \tag{8}$$

We use the weighted clustering coefficient proposed by Onnela et al. [28], which reflects how large triangle weights are compared to network maximum. The weighted clustering coefficient can be measured as:

$$\tilde{C}_{i,O} = \frac{1}{k_i(k_i-1)} \sum_{j,k} (\hat{w}_{i,j}\hat{w}_{i,k}\hat{w}_{j,k})^{1/3}, \tag{9}$$

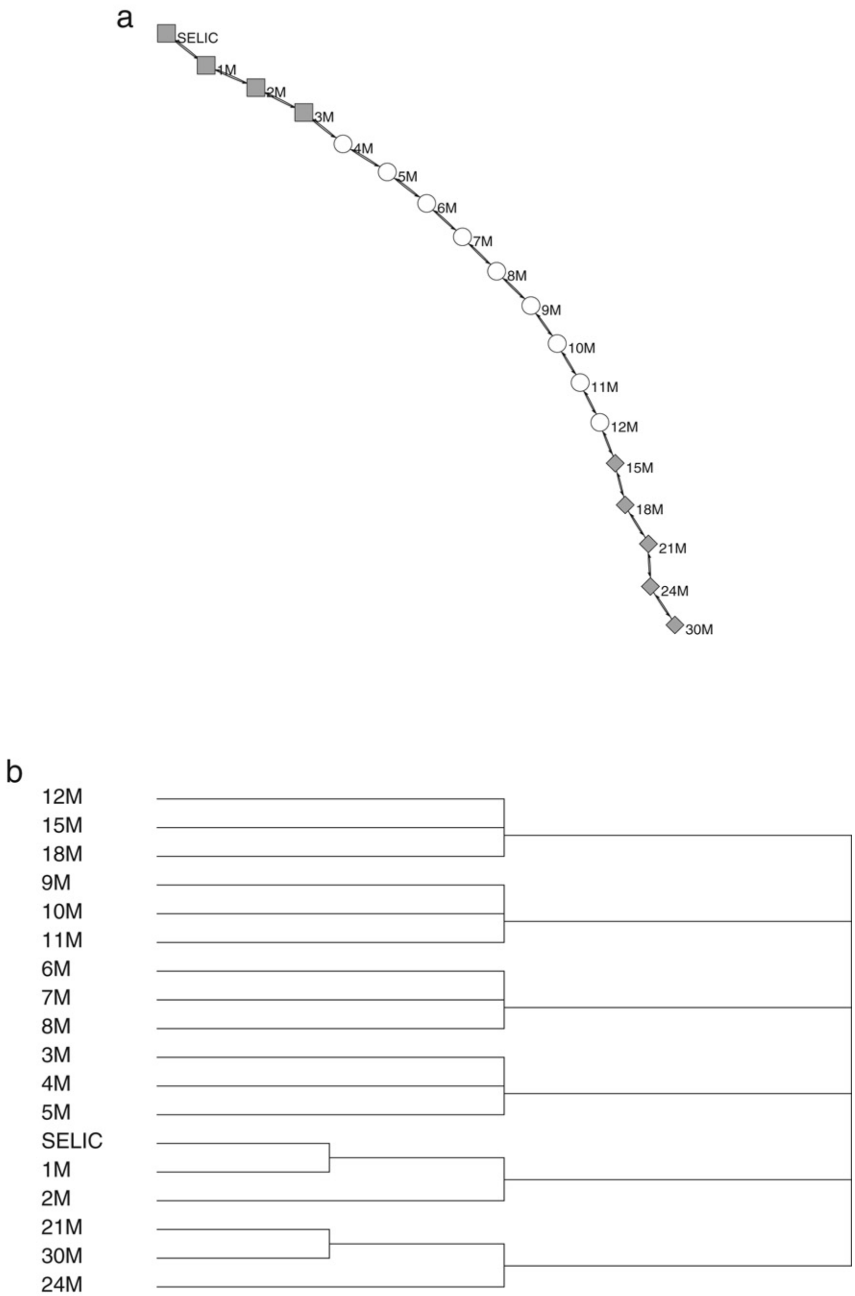


**Fig. 2.** (a) Plot of the MST of a network connecting the full sample of interest rates for the period from December 5, 1997 to March 11, 2008. The node shape is based on their terms, in months: (Square) for short-term, (Ellipse) for medium-term and (Diamond) for long-term rates. (b) Plot of the Taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST of the full sample of interest rates, using the complete linkage clustering method.

where they use weights scaled by the largest weight in the network, $\hat{w}_{i,j} = \frac{w_{i,j}}{\max(w_{i,j})}$ and the contribution of each triangle depends on all of its edge weights.

We also use the weighted clustering coefficient proposed by Zhang et al. [29] which can be measured as:

$$\tilde{C}_{i,Z} = \frac{\sum_{j,k}(\hat{w}_{i,j}\hat{w}_{i,k}\hat{w}_{j,k})}{(\sum_k \hat{w}_{i,k})^2 - \sum_k \hat{w}_{i,k}^2} \tag{10}$$

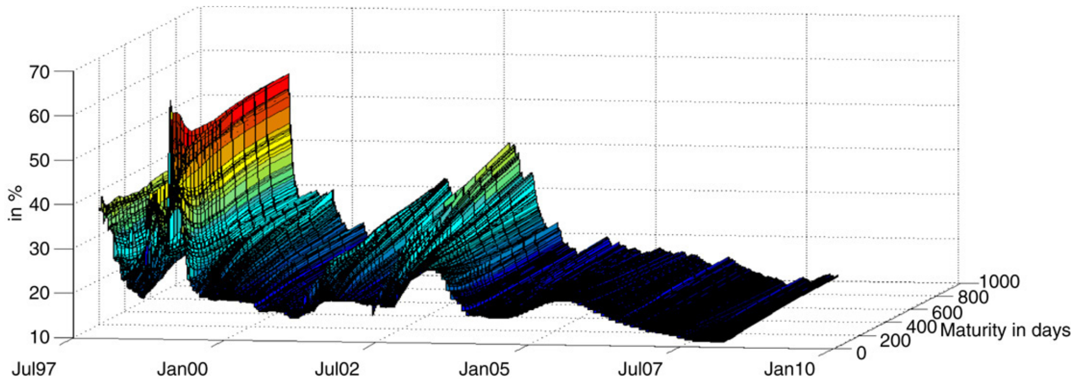


**Fig. 3.** Plot of the evolution of interest rates for different maturities for the period from December 5, 1997 to March 11, 2008. For the color version, see the online paper.

where the weights have been normalized by $\max(w_{i,j})$. This weighted clustering coefficient is insensitive to additive noise which may result in appearance of "false positive" edges with small weights. An overview of these methods may be found in Ref. [30].

We calculate the vertex strength of the network [25,31], which is given by:

$$s_i = \sum_{j \in \Gamma(i)} w_{i,j} \tag{11}$$

where $w_{i,j}$ is the weight of links connected to vertex $i$ and $\Gamma(i)$ is the nearest neighbors of $i$. The strength measure gives the information about node's connectivity and the importance of the weights of its links.

Another important measure we calculate, is the entropy of the network [26], which is given by:

$$H = -\sum_k P(k)\log(P(k)). \tag{12}$$

The maximum value of entropy is given for a uniform degree distribution, whereas the minimum is zero (all vertices have the same degree).

The term structure of interest rates is varying over time and depends on expectations about future monetary policy. In general, interest rates for long-term maturities are higher than for short-term maturities. However, if the Central Bank raises interest rates and this is perceived as a temporary increase due to an increase in short-term inflation, agents may expect short-term interest rates to decrease in the future. In this case long-term interest rates may be lower than short-term interest rates for a while, which implies an inverted yield curve (term structure). This inverted yield curve provides important information: agents expect future short-term interest rates to decrease. Furthermore, the yield curve could be flat or hump-shaped.

The entropy and disparity measures are useful to evaluate the degree of heterogeneity of the network and how it varies over time. If the degree of entropy and disparity is increasing over time then one should expect a stronger change in the shape of the yield curve.

The dominance and strength measures are useful to assess the relative importance of each node in the network. Therefore, they can be used to check whether the Selic interest rate is the most important node in the interest rates network. If this is the case then our results are favorable to the Expectations Hypothesis.

## 4. Data

The data has 2530 observations of interest rates for a variety of maturities, from Selic (Overnight) to 30 months. The data is sampled daily, beginning on December 5, 1997 and ending on March 11, 2008. We build the network using correlation distances between the maturities in the sample following Mantegna [2].[10] We use interest rates changes because the level is non-stationary. We have 18 nodes in our network comprised of interest rates of different maturities.

We study the different maturities of the interest rates in the network built using the previous MST, based on the concept of ultrametricity.

We also evaluate the dynamics of the evolution of network measures over time. We employ a recursive approach in which we use the first 1008 observations to estimate closeness centrality, disparity, entropy, and clustering coefficient. To calculate these measures we use the correlations matrix for each period of time, recursively.

10 In order to check whether results depend on the specific sample the 2530 observations were divided in two samples, with 1265 for the first set and 1265 for the second.

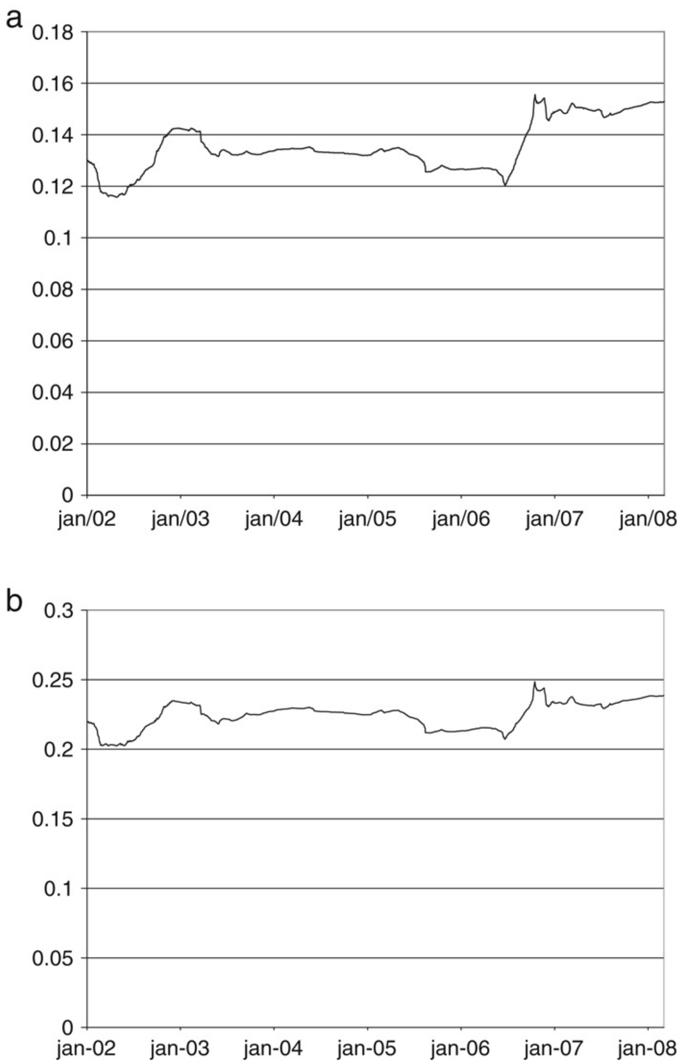


**Fig. 4.** (a) Plot of the clustering coefficient $\tilde{C}_{i,O}$, based on the concept of subgraph intensity proposed by Onnela et al., for interest rates. (b) Plot of the clustering coefficient $\tilde{C}_{i,Z}$ proposed by Zhang et al., where the weights have been normalized by $\max(w)$, for interest rates.

## 5. Empirical results

Networks analysis have many properties that help researchers to understand the interactions between agents in a complex system. The networks property of hierarchy is useful to observe that the networks often have structure in which vertices cluster together into groups that then join to form groups of groups, from the lowest levels of organization up to the level of the entire network.

The ultrametric hierarchical tree with the MST provides useful information to investigate the number and nature of the common factors that affect the interest rates. The analysis shows us that the influence of economics factors for interest rates is specific to each maturity ˘.

The interest rates with high values of the distances $d_{i,j}$ are influenced by factors, which are specific to these rates. For low values of $d_{i,j}$, the interest rates are affected either by factors which are common to all maturities and by factors which are specific to the considered maturity of the interest rates. The length of the segments observed for each group, quantifies the relative relevance of these factors.

The patterns that we can observe are that, if the distances of $d_{i,j}$ are low, the interest rates tend to form strong clusters and for high values of $d_{i,j}$ these links tend to be weak. For our full sample we identify values of $d_{i,j}$ between 0 and 1.4 indicating that the Brazilian term structure of interest rates is homogeneous with respect to maturity and forms strong clusters.

Fig. 1a–b presents the MST and the taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST, for a network based on the interest rates distances of the full sample (December 5, 1997 to March 11, 2008). We present

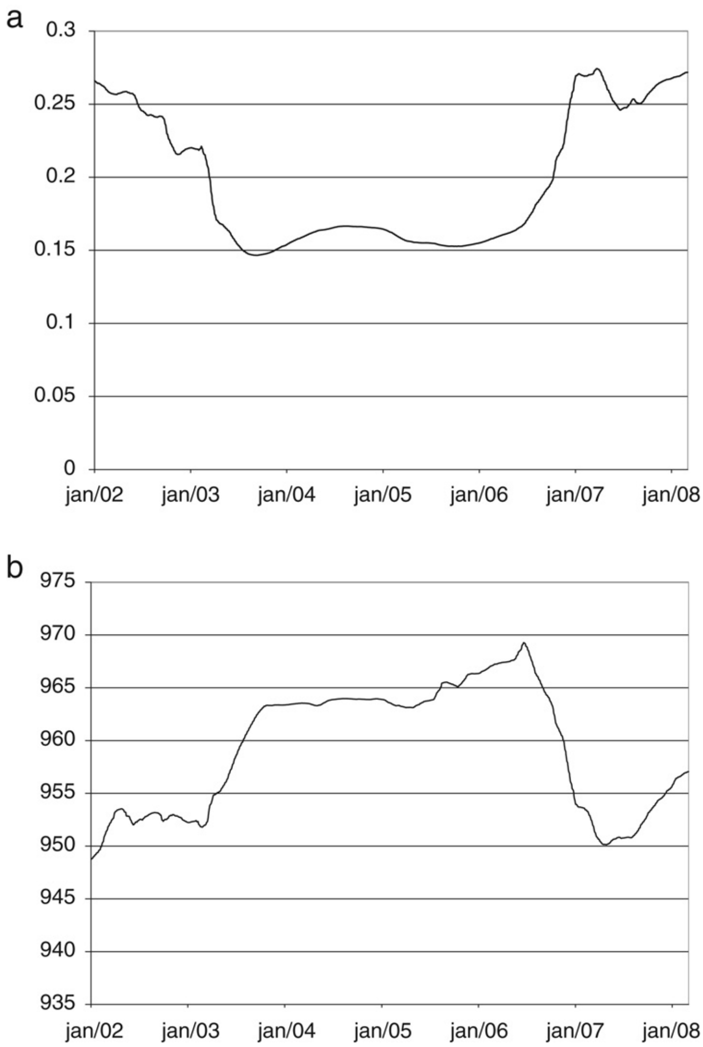


**Fig. 5.** (a) Plot of the average closeness centrality for interest rates, (b) Plot of the entropy for the entire network over time. We employ 1008 observations for each calculation and estimate these measures recursively on a moving average basis.

the network based on the maturity of each interest rate.[11] The Fig. 1b shows the taxonomy Hierarchical tree based on the simple linkage clustering method. The MST and taxonomy tree presented in Fig. 1 suggest the emergence of clusters and we can see that these interest rates are clustered by maturity.[12]

Fig. 2a–b presents the MST and the taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST, for a network based on the interest rates distances of the full sample (December 5, 1997 to March 11, 2008), but Fig. 1b shows the taxonomy Hierarchical tree based on the complete linkage clustering method. We present the network based on the maturity of each interest rate.

Fig. 3 presents the evolution of interest rates for different maturities for the period from December 1997 to March 2008. As we can see the dispersion of interest rates was much higher in the period prior to 1999, in which the Inflation Targeting framework was implemented. Furthermore, we can see how the value of these interest rates are decreasing over time.

Fig. 4a–b presents the evolution of average weighted clustering coefficients, $\tilde{C}_{i,O}$ (proposed by Onnela et al.) and $\tilde{C}_{i,Z}$ (proposed by Zhang et al.), for the interest rates network. The figures these coefficients present low degree of heterogeneity over time.

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11 We also split the sample in two time periods and plot both the MST and the taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST and find qualitatively the same results.

12 We also build the same graphs for two time periods, dividing the sample before and after the Inflation Targeting regime and results are qualitatively the same.

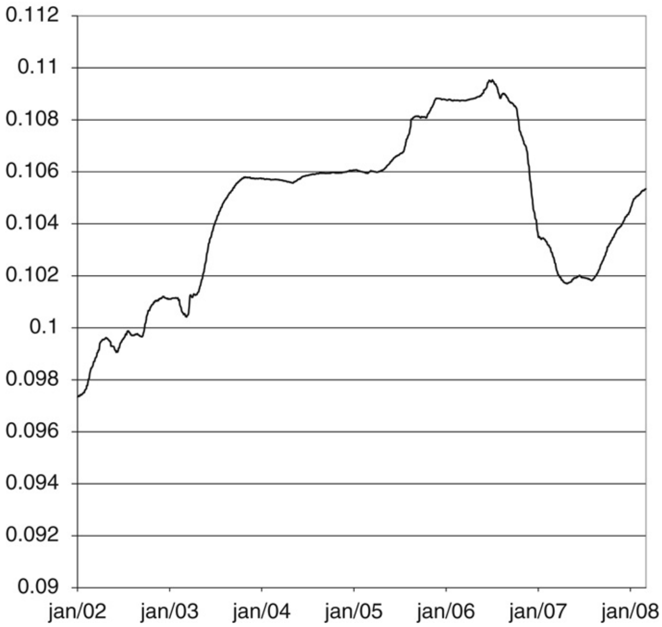


**Fig. 6.** Plot of the average disparity for interest rates. We employ 1008 observations for each calculation and estimate these measures recursively on a moving average basis.

Fig. 5a–b present the evolution of closeness centrality and entropy. The average closeness centrality fall within the (0.1465–0.2743) range. This measure decrease in the 2001–2003 period, was stable in the subsequent period of 2004–2006, and increase in the recent years. Finally, the entropy measures imply an increase in network heterogeneity in the 2003–2006 period, with a subsequent fall in the 2006–2007 period. Therefore, the degree of heterogeneity seems to be fluctuating over time.

Fig. 6 presents the evolution of average disparity. The average disparity measures fall within the (0.0973–0.1095) range. Overall, from the entropy and disparity measures we can observe that the degree of heterogeneity seems to be increasing over time, which implies changes in the information provided in the term structure of interest rates. It is important to notice that these results imply that changes in interest rates for different maturities have not been proportional. It is worth mentioning that these changes are relatively small, approximately 12.5% and 2.1% for the disparity and entropy measures, respectively.

We also estimate a vector error correction (VEC) model for the interest rates in our sample. The choice of lags in the model was based on the Akaike information criteria. In this modeling we allow interest rates to cointegrate, which implies they have a common trend, which is true for our sample. Therefore, we perform the previous analysis to this filtered data. The MST and the taxonomy tree for the VEC model are presented in Fig. 7a–b. From this figure we can infer that clusters seem to change slightly. Furthermore, the results for network measures are qualitatively the same.

Table 1 presents the strength, graph centrality, dominance and closeness centrality for each maturity. We present results for the full sample (Panel A) and the pre Inflation Targeting period (Panel B) and post Inflation Targeting period (Panel C). From this table we can infer that the Selic interest rate is the most important within the network. It has a dominance about as twice as any other interest rates (measured by the $\beta$) in the full sample and continues at high level in the Pre-Inflation Target and Post-Inflation Target samples. Furthermore, short-term interest rates seem to be the most important within the network, followed by long-term interest rates.

These results together imply that the Expectation Hypothesis of interest rates seem to hold in Brazil, since the most important node in the network is the base interest rate (Selic). We also estimate a vector error correction (VEC) model and include an error correction term for the interest rates in our sample. By doing so we test whether the distance matrix of estimated endogenous variables within the VEC system share the same characteristics of the raw data, which seems to be the case. Therefore, our results are robust to estimating a VEC model for interest rates and allowing for long-term relationships between these interest rates.

## 6. Conclusions

This paper shows that the MST and the ultrametric hierarchical tree can be used to analyze the Brazilian term structure network. The evidence suggests that the Brazilian term structure forms clusters and is homogeneous. The results also

**Table 1**
Network measures for Brazilian interest rates.

| Maturity | Strength | Graph centrality | Dominance (Beta) | Closeness centrality |
|---|---|---|---|---|
| PANEL A—Full sample | | | | |
| SELIC | 23.80 | 0.70 | 2.69 | 0.04 |
| 1M | 12.81 | 0.73 | 1.38 | 0.08 |
| 2M | 10.93 | 0.73 | 1.15 | 0.09 |
| 3M | 9.89 | 0.71 | 1.02 | 0.10 |
| 4M | 9.24 | 0.72 | 0.93 | 0.11 |
| 5M | 9.26 | 0.72 | 0.92 | 0.11 |
| 6M | 7.90 | 0.72 | 0.74 | 0.13 |
| 7M | 7.72 | 0.72 | 0.72 | 0.13 |
| 8M | 7.58 | 0.71 | 0.70 | 0.13 |
| 9M | 7.54 | 0.72 | 0.69 | 0.13 |
| 10M | 7.56 | 0.72 | 0.69 | 0.13 |
| 11M | 7.65 | 0.72 | 0.70 | 0.13 |
| 12M | 7.82 | 0.71 | 0.73 | 0.13 |
| 15M | 8.10 | 0.71 | 0.78 | 0.12 |
| 18M | 8.96 | 0.71 | 0.90 | 0.11 |
| 21M | 9.87 | 0.70 | 1.01 | 0.10 |
| 24M | 10.68 | 0.70 | 1.11 | 0.09 |
| 30M | 11.08 | 0.70 | 1.16 | 0.09 |
| PANEL B—Pre-inflation target | | | | |
| SELIC | 14.43 | 0.95 | 2.51 | 0.07 |
| 1M | 10.08 | 1.23 | 1.74 | 0.10 |
| 2M | 8.39 | 1.41 | 1.43 | 0.12 |
| 3M | 7.19 | 1.61 | 1.19 | 0.14 |
| 4M | 6.02 | 1.44 | 0.94 | 0.16 |
| 5M | 5.52 | 1.34 | 0.82 | 0.18 |
| 6M | 5.12 | 1.25 | 0.73 | 0.19 |
| 7M | 4.87 | 1.19 | 0.67 | 0.20 |
| 8M | 4.82 | 1.13 | 0.65 | 0.21 |
| 9M | 4.90 | 1.09 | 0.65 | 0.20 |
| 10M | 5.03 | 1.06 | 0.67 | 0.20 |
| 11M | 5.16 | 1.04 | 0.69 | 0.19 |
| 12M | 5.31 | 1.03 | 0.72 | 0.19 |
| 15M | 5.33 | 1.02 | 0.73 | 0.18 |
| 18M | 5.80 | 0.99 | 0.82 | 0.17 |
| 21M | 6.40 | 0.97 | 0.94 | 0.15 |
| 24M | 6.94 | 0.95 | 1.03 | 0.14 |
| 30M | 6.99 | 0.95 | 1.04 | 0.14 |
| PANEL C—Post-inflation target | | | | |
| SELIC | 9.98 | 1.22 | 2.22 | 0.10 |
| 1M | 7.80 | 1.42 | 1.73 | 0.12 |
| 2M | 6.61 | 1.59 | 1.45 | 0.15 |
| 3M | 5.63 | 1.79 | 1.20 | 0.18 |
| 4M | 4.74 | 2.08 | 0.95 | 0.21 |
| 5M | 4.25 | 2.03 | 0.81 | 0.23 |
| 6M | 3.97 | 1.86 | 0.73 | 0.25 |
| 7M | 3.80 | 1.75 | 0.67 | 0.26 |
| 8M | 3.73 | 1.65 | 0.64 | 0.27 |
| 9M | 3.72 | 1.59 | 0.64 | 0.27 |
| 10M | 3.77 | 1.53 | 0.64 | 0.26 |
| 11M | 3.85 | 1.49 | 0.66 | 0.26 |
| 12M | 3.98 | 1.45 | 0.69 | 0.25 |
| 15M | 4.39 | 1.37 | 0.79 | 0.23 |
| 18M | 4.76 | 1.31 | 0.88 | 0.21 |
| 21M | 5.12 | 1.28 | 0.97 | 0.19 |
| 24M | 5.45 | 1.25 | 1.04 | 0.18 |
| 30M | 6.10 | 1.22 | 1.19 | 0.16 |

The table shows important measures for the full sample and for Pre-IT and Post-IT data, for different maturities. We find that the short-term interest rate is the most important within the interest rate network followed by the long-term interest rate and medium-term interest rate, in all the panels. It is possible to see that in the table, observing the strength and the dominance measures. We also apply the VEC model to the full sample and to the Pre-IT and Post-IT samples and find qualitatively the same results.

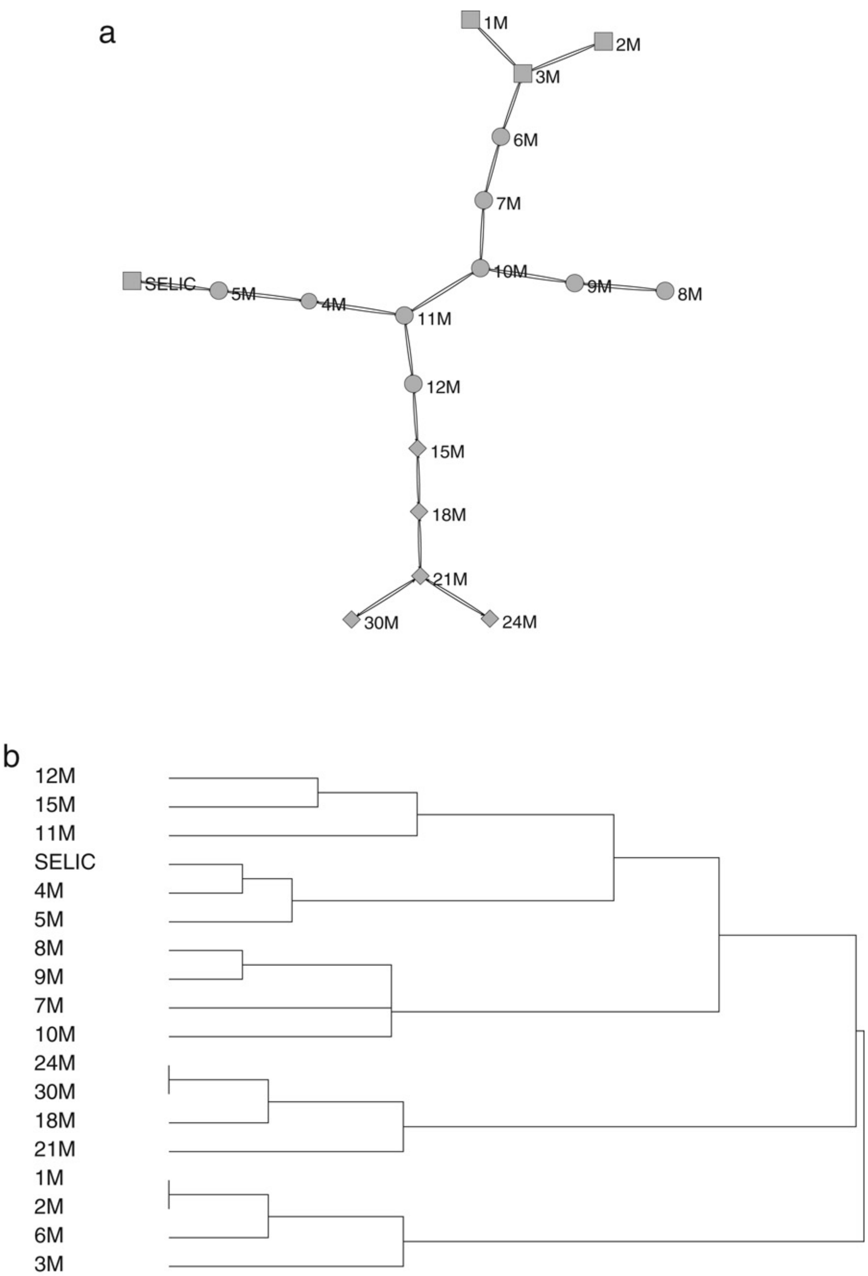


**Fig. 7.** (a) Plot of the MST, for the VEC model, of a network connecting the full sample of interest rates for the period from December 5, 1997 to March 11, 2008. The node shape is based on their terms, in months: (Square) for short-term, (Ellipse) for medium-term and (Diamond) for long-term rates. (b) Plot of the Taxonomy Hierarchical tree of the subdominant ultrametric associated to the MST of the full sample of interest rates.

provide an indication that the sector is vulnerable to the influence of external risks and tends to create complete and robust clusters. These clusters are defined by maturity, i.e, we have short, medium and long-term interest rates clusters.

We also develop a dynamic analysis of the interest rates network using weighted network measures: closeness centrality, disparity, entropy, dominance and clustering. From the analysis we infer that the Selic interest rates is the most important node in the network, which is favorable to the Expectation Hypothesis of interest rates.

This paper has shown a novel approach to analyze the evolution of interest rates network. Further research could focus on Global interest rates to evaluate the linkages between different countries. Furthermore, when one evaluates a variety of countries the development of multiple dimension graphs could be useful to analyze interest rates and exchange rates networks simultaneously. Finally, analyzing forward and spot interest rates could provide useful insights about price discovery. These are important topics that should be in our research agenda.

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