Chapter 2

ii–V Space

Because most jazz is not purely diatonic, we need to expand our transformational system to account for more chromatic examples. This chapter will begin that work, taking the very common ii–V–I progression as its basis. Most of the spaces we will develop in this chapter will fall under the broad category of "fifths spaces," but at the end of the chapter we will have occasion to return to the diatonic space of Chapter 1 to see how they might be enhanced with the chromatic spaces introduced here.

2.1 A Descending Fifths Arrangement

2.1.1 Formalism

The most common harmonic progression in jazz is undoubtedly the ii^7–V^7–I^7 progression (hereafter, simply ii–V–I, or often just ii–V). It is the first progression taught in most jazz method books, and the only small-scale harmonic progression to have an entire Aebersold play-along volume dedicated to it.[1] The progression is so prevalent that many jazz musicians describe tunes in terms of their constituent ii–Vs; a musician might describe the bridge of "All the Things You Are" (shown in Figure 2.1) as being "ii–V to G, ii–V to E, then V–I in F."[2] Given the importance of this

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1. Jamey Aebersold, *The II–V⁷–I Progression*, Jamey Aebersold Play-A-Long Series, vol. 3 (New Albany, IN: Jamey Aebersold Jazz, 1974). The Aebersold play-along series is a staple of jazz pedagogues; most contain a selection of tunes, along with a CD of a rhythm section so that students can practice with a recording. The ii–V volume is number three of well over 100, and includes the phrase "the most important musical sequence in jazz!" on the cover.

2. Our diatonic analysis of "All the Things You Are" in the previous chapter notwithstanding, the ubiquity of ii–V–I progressions means that many jazz musicians are apt to hear the progression as successions of ii–Vs, even in cases where a diatonic pattern may be present.

$A\text{-}^7$	D^7	G^{maj7}		
$F\sharp\text{-}^{7\flat5}$	B^7	E^{maj7}	$C^{7\sharp5}$	‖
$F\text{-}^7$				

Figure 2.1. The bridge of "All the Things You Are" (Jerome Kern).

$$\mathrm{ii}^7 \xrightarrow{\mathrm{TF}} \mathrm{V}^7 \xrightarrow{\mathrm{TF}} \mathrm{C}^{\triangle}$$

Figure 2.2. A transformation network for a ii–V–I in C major: Dm7–G7–CM7.

progression for improvising jazz musicians, it seems natural to use it as the basis for developing a more general transformational model of jazz harmony.

Figure 2.2 shows a transformation network for a single ii–V–I progression; we will begin by developing the formal apparatus for this progression, after which we can begin to combine ii–V–I progressions to form a larger musical space.[3] This figure, with its combination of general Roman numerals and specific key centers, is designed to reflect how jazz musicians tend to talk about harmony; we might read this network as "a ii–V–I in C." The combination of Roman numerals and key areas bears some similarity to Fred Lerdahl's chordal-regional space, but Figure 2.2 is a transformation network, while chordal-regional space is strictly a spatial metaphor.[4]

We have encountered one transformation network already (Figure 1.7 in the previous chapter), but we have yet to define the concept formally. Transformation networks are a major part of David Lewin's project in *GMIT*, and have been thoroughly covered in the literature, so we will need to consider the formalism only briefly here.[5] A transformation network consists of objects of some kind (here, they are chords) represented as vertices in a graph, along with some relations

3. The triangle on the C chord in this figure indicates a major seventh. The triangle (instead of "maj7" or "M7") is intended to save space and reduce clutter in the graphical representations.

4. Fred Lerdahl, *Tonal Pitch Space* (New York: Oxford University Press, 2004), 96–97.

5. Lewin's definition of a transformation network is in *Generalized Musical Intervals and Transformations* (Oxford University Press, [1987] 2007), 196. For a relatively concise summary, see Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 110–16.

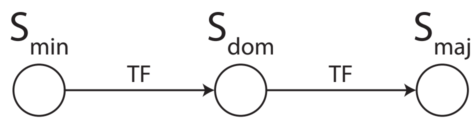

Figure 2.3. The underlying transformation graph for a single ii–V–I progression.

(transformations) between them, represented as arrows. In Lewin's definition, all of the objects in a transformation network must be members of a single set *S*, and the transformations must be functions from *S* into *S* itself.[6] The transformations in Figure 1.7 are indeed Lewinnian transformations (mappings in the G-minor diatonic set), but the ii–V–I transformation network is more complex.

The transformation TF in Figure 2.2 is in fact a *cross-type* transformation, as defined by Julian Hook.[7] Hook expands Lewin's definition of a transformation network to include objects of different types, necessary to define transformations in the ii–V–I progression. The progression contains musical objects of three types of diatonic seventh chords: minor, dominant, and major sevenths (in the key of C major, the progression is Dm7–G7–CM7). Using Hook's relaxed definition, we are free to define transformations from any set of objects to any other; to understand the figure above, we need to define the transformation TF such that it maps ii^7 chords to V^7 chords, and V^7 chords to I^7 chords.

Before defining the transformations, however, we first need to define the sets themselves. To help with this, Figure 2.3 shows the underlying transformation graph of the transformation network in Figure 2.2. Throughout *GMIT*, Lewin is careful to distinguish transformation graphs from transformation networks: a graph is an abstract structure, showing only relations (transformations) between unspecified set members, while a network realizes a graph, specifying the actual musical objects under consideration.[8] Because cross-type transformation graphs contain objects of different types, a node in a cross-type transformation graph must be labeled with the set

6. *GMIT*, Definitions 9.3.1 (196) and 1.3.1 (3).
7. Julian Hook, "Cross-Type Transformations and the Path Consistency Condition," *Music Theory Spectrum* 29, no. 1 (April 2007): 1–40.
8. *GMIT*, 195–96 and throughout. See also Hook, "Cross-Type Transformations," 6–8.

from which the node contents may be drawn (even in the abstract transformation graph).[9] In Figure 2.3, the nodes are labeled simply S_{min}, S_{dom}, and S_{maj}, which we can understand as the sets of minor, dominant, and major seventh chords, respectively.

While at its core the ii–V–I progression contains three types of seventh chords, in reality a jazz musician might add any number of extensions or alterations to this basic structure. Given this practice, defining the archetypal progression as being composed of four-note set classes (seventh chords) seems unnecessarily restrictive. In order to allow for some freedom in the chord qualities, we will consider only chordal roots, thirds, and sevenths; these pitches are sufficient to distinguish the three chord qualities in a ii–V–I.[10]

In this chapter, we will represent a chord with an ordered triple $X = (x_r, x_t, x_s)$, where x_r is the root of the chord, x_t the third, and x_s the seventh. The definitions of the three sets are as follows:[11]

$$S_{\text{min}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 3; x_s - x_r = 10\} \qquad \text{ii}^7 \text{ chords}$$

$$S_{\text{dom}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 10\} \qquad \text{V}^7 \text{ chords}$$

$$S_{\text{maj}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 11\} \qquad \text{I}^7 \text{ chords}$$

The definitions are intuitive, and have clear musical relevance: ii^7 chords have a minor third (interval 3) and minor seventh (interval 10), V^7 chords have a major third and minor seventh (intervals 4 and 10), and I^7 chords have a major third and major seventh (intervals 4 and 11). Defining the chords this way rather than as four-note set classes offers the great advantage of flexibility. Using the ordered-triple representation, the progressions Dm7–G7–Cmaj7 and Dm9(♭5)–G7♭13♯9♭9–Cmaj7♯11 are understood as equivalent, since the roots, thirds, and sevenths are the same: both progressions are represented $(2, 5, 0)$–$(7, 11, 5)$–$(0, 4, 11)$. Because the sets are defined in pitch-class space, the three sets all have cardinality 12: each pitch class is the root of exactly one ii^7, V^7, and I^7 chord.

9. Hook, “Cross-Type Transformations,” 7.

10. In fact, many jazz piano texts begin with “three-note” or “shell” voicings, consisting only of chordal roots, thirds, and sevenths; see, for example Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 17–22; and Joe Mulholland and Tom Hojnacki, *The Berklee Book of Jazz Harmony* (Boston: Berklee Press, 2013), 211–12.

11. Here and throughout this chapter, pitch classes are represented as mod-12 integers, with C $= 0$; all calculations are performed mod-12.

With the space of the nodes defined, we can now formulate the transformation representing a ii–V–I, which we will call simply "TF":

$$\mathrm{TF}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{min}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 1, x_t) \in S_{\mathrm{dom}}$$

$$\mathrm{TF}(Y) = Z, \text{ where } Y = (y_r, y_t, y_s) \in S_{\mathrm{dom}} \text{ and } Z = (z_r, z_t, z_s) = (y_r + 5, y_s - 1, y_t) \in S_{\mathrm{maj}}$$

Again, these definitions are designed to be musically relevant; the voice-leading diagram in Figure 2.4 illustrates this more clearly.[12] The root of the second chord is a fifth below the root of the first ($y_r = x_r + 5$), the third of the second chord is a semitone below the seventh of the first ($y_t = x_s - 1$), and the seventh of the second chord is a common tone with the third of the first ($y_s = x_t$). In Lewin's transformational language, if a jazz musician is "at a ii^7 chord" and wishes to "get to a V^7 chord," the transformation that will do the best job is TF: "move the root down a fifth and the seventh down a semitone to become the new third." (Recall that we may also write ii$^7 \xrightarrow{\mathrm{TF}}$ V^7, rather than TF(ii^7) = V^7.) Note that the transformation TF is also valid between V^7 and I^7 as well (the second equation above, involving sets S_{dom} and S_{maj}). TF is both one-to-one and onto for sets of ordered triples; it maps each ii^7 to a unique V^7, and each V^7 to a unique I^7. As such, its inverse (TF^{-1}) is well defined, and allows motion backwards along the arrows shown in the transformation graph in Figure 2.3.

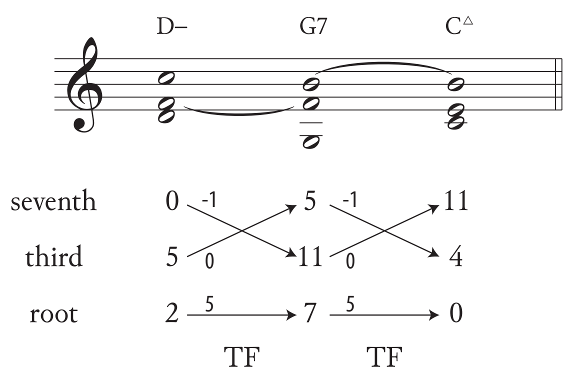

Figure 2.4. Voice leading in the ii–V–I progression.

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12. This figure represents what Joseph Straus calls "transformational voice leadings" in his study of atonal voice leading; "Uniformity, Balance, and Smoothness in Atonal Voice Leading," *Music Theory Spectrum* 25, no. 2 (October 2003): 305–52.

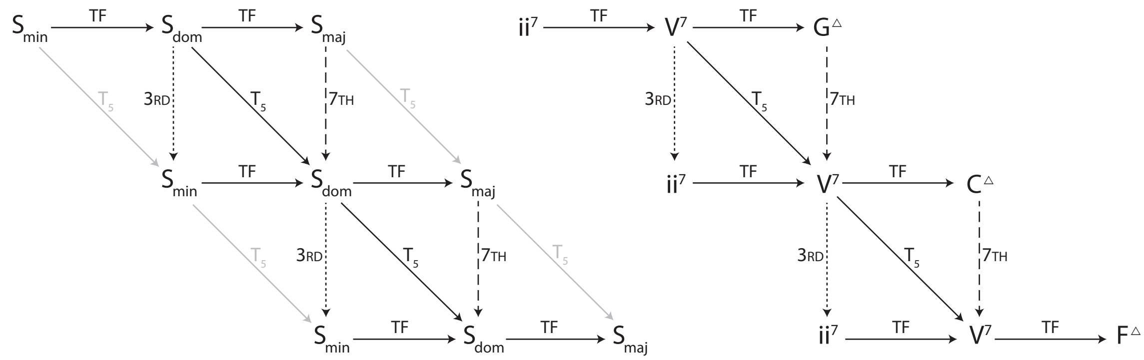

Figure 2.5. A transformation graph (left) and transformation network (right) for a small portion of ii–V space.

It is worth mentioning here that TF and $\mathrm{TF_T}$ (which we will define in the next section) are well-defined operations for any ordered triple of members of the integers mod-12 (i.e., a member of the set $\mathbb{Z}_{12} \times \mathbb{Z}_{12} \times \mathbb{Z}_{12}$). There is nothing mathematically incorrect about the statement $(0, 1, 2) \xrightarrow{\mathrm{TF}} (5, 1, 1) \xrightarrow{\mathrm{TF}} (10, 0, 1)$, for example, but this succession has little musical relevance for the applications under consideration here. Because Hook does not formally define what he means by a "type," the formulation allows for situations like this one, in which the three types are all members of a single larger set.[13] The advantage for defining TF as a cross-type transformation is that the content of a single node in the transformation graph is restricted to members of a 12-element set of specific ordered-triple configurations.

With this understanding of the transformations involved in a single ii–V–I progression, we can continue to see how we might connect multiple progressions in order to form a larger ii–V space. Because root motion by descending fifth is extremely common in jazz, we might consider connecting ii–V–I progressions by descending fifth; Figure 2.5 illustrates this arrangement both as a transformation graph and a transformation network. This descending fifths arrangement means

13. Hook himself makes this clear, noting that for any two sets S and T it is possible to define a single-type transformation in the union set $S \cup T$. He also notes that even when a single-type transformation is possible, "the cross-type approach is often simpler and more natural," which certainly seems to be the case here. "Cross-Type Transformations," 5n8.

that all of the chords sharing a root are aligned vertically (directly below GM7 is G7, which is itself above Gm7). This arrangement allows us to define two more transformations, which we will call simply 7TH and 3RD:

$$\text{7TH}(L) = M, \text{ where } L = (l_r, l_t, l_s) \in S_{\text{maj}} \text{ and } M = (m_r, m_t, m_s) = (l_r, l_t, l_s - 1) \in S_{\text{dom}}$$

$$\text{3RD}(M) = N, \text{ where } M = (m_r, m_t, m_s) \in S_{\text{dom}} \text{ and } N = (n_r, n_t, n_s) = (m_r, m_t - 1, m_s) \in S_{\text{min}}$$

Like the TF transformation, the 7TH and 3RD transformations have clear musical relevance: each lowers the given note by a semitone. Although adjacent progressions are connected by descending fifth, the T_5 labels connecting adjacent ii^7 chords and I^7 chords are shown in gray in the graph (and omitted in the network, and in later examples), since these chords are not often directly connected in jazz.

By extending the network of Figure 2.5, we arrive at the entirety of ii–V space, as shown in Figure 2.6. Because ii–V space includes cross-type transformations, it does not easily form a Lewinnian GIS.[14] Considered more generally, though, it is easy to see that by considering a single ii–V–I progression as a unit, ii–V space maps cleanly onto ordinary pitch-class space. As Figure 2.6 makes clear, we can consider the ii–V–I in C as being three perfect fifths above the ii–V–I in E♭ (or put transformationally, the T_3 operation transforms a ii–V–I in C to one in E♭). This formulation does not allow us a means to say, for example, that “the ii chord in C is x units away from the V chord in E♭,” but because ii–V–Is are rarely split up, falling back on normal pitch-class distance is sufficient in most situations.[15]

14. It would be possible to form a GIS by considering all ordered triples as the group, as suggested above. While this is possible, defining an interval function in this group is much more difficult: such a function would need to account for the 36 ordered triples in ii–V space (ii^7, V^7, and I^7 chords) as well as the many more (1692) that are not included in the space. Such a function is conceivable, but would not in any case reflect the musical realities ii–V space is interested in portraying.

15. ii–V space is a directed graph, so in circumstances where the pitch-class distance metric is somehow not sufficient, we can instead rely on the standard way of measuring distance in a directed graph: by counting the number of edges in the shortest path between two chords. The distance from ii^7 of C to V^7 of E♭ is then 4: ii to V in C (1 edge), then 3 T_5s to V of E♭.

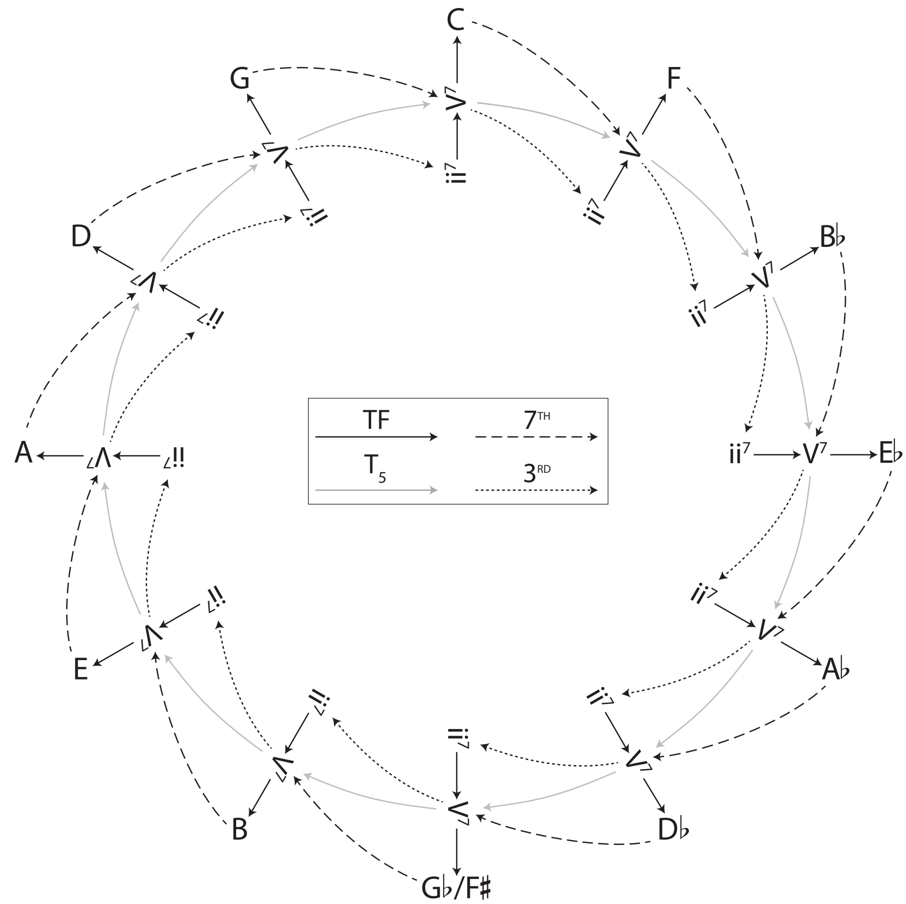

Figure 2.6. The complete ii–V space, arranged around the circle of fifths.

2.1.2 Analytical Interlude: Lee Morgan, "Ceora"

Though we will return to the formalism a bit later, we have defined enough of ii–V space at this point to see how it might be useful in analysis. To do so, we will examine Lee Morgan's composition "Ceora," first recorded on the 1965 album *Cornbread*. The changes for the A section are given in Figure 2.7, and the accompanying moves in ii–V space are shown in Figure 2.8.[16] "Ceora" is in the key of A♭ major, and begins with the progression I–ii–V–I in the first three bars, staying within a single horizontal slice of ii–V space. This is followed immediately by a ii–V–I progression in D♭, a fifth lower (mm. 4–5).

16. These changes are taken from *The Real Book*, and reflect what is played on the *Cornbread* recording. In this figure, the circle indicates the tonic, while the numbers on the labels indicate the order of transformations.

$A\flat^{maj7}$	$B\flat{-}^{7}$ $E\flat^{7}$	$A\flat^{maj7}$	$E\flat{-}^{7}$ $A\flat^{7}$
$D\flat^{maj7}$	$D{-}^{7}$ G^{7}	$C{-}^{7}$	$F^{7\sharp9}$
$B\flat{-}^{7}$	$E\flat^{7}$	$C{-}^{7}$	F^{7}
$D{-}^{7}$	G^{7}	$C{-}^{7}$ F^{7}	$B\flat{-}^{7}$ $E\flat^{7}$ ‖

Figure 2.7. Changes for the A section of “Ceora” (Lee Morgan).

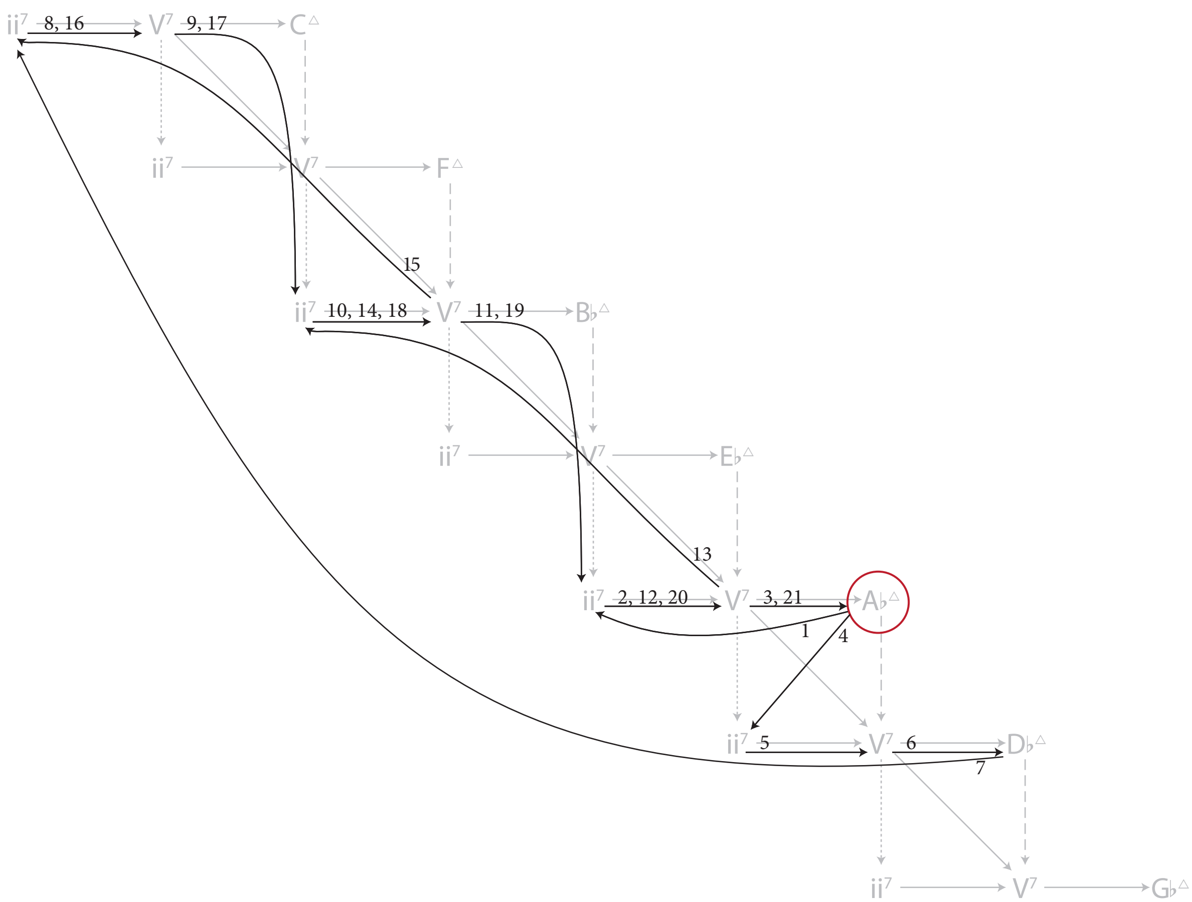

Figure 2.8. The A section of “Ceora” in ii–V space.

At this point, we might expect the ii–Vs to continue in descending fifths, but the potential ii^7 chord in G♭ is substituted with Dm7, its tritone substitute, which then resolves as a ii–V in C.[17] Instead of resolving to C major in m. 7, this ii–V resolves instead to C *minor*: both the seventh and third of the expected CM7 are lowered to become Cm7. (This progression is extremely common, and is one of the principal means of maintaining harmonic motion in the course of a jazz tune.)

A similar progression in B♭ follows, leading to a B♭m7 chord in m. 9. We then hear a ii–V progression in the tonic in mm. 9–10, but the expected A♭M7 does not materialize; the E♭7 chord moves instead to Cm7 as ii of B♭ (a northwesterly move in the space). This progression repeats in mm. 11–13, leading once again to the Dm7 chord first heard in m. 5. The repeated upward motions in the space have the effect of ramping up the tonal tension in the passage; not only do the dominants fail to resolve as expected, but their stepwise rising motion take the music far away from the tonic A♭. To release this harmonic tension, the ii–V in C resolves at m. 15 to C minor, at which point the harmonic rhythm doubles and the progression follows the normal descending fifths pattern to reach the tonic that begins the B section in m. 17.

The B section of "Ceora" (shown in Figure 2.9) follows much of the same trajectory as the A section until the last four bars; the only differences are the addition of the ♭5 in the Cm7 and the ♯9 in the F7 in mm. 11–12 of the section. Because we have defined chords and transformations only in terms of chordal roots, thirds, and sevenths, neither of these changes affect our transformational reading of the passage. Instead of ramping up to ii^7 of C as in the A section, the ii–V in B♭ resolves to B♭m7 in m. 12. This B♭m7 becomes the ii chord of a ii–V–I in tonic, resolving in m. 15 of the section. A final ii–V in the last measure provides additional harmonic interest, and functions as a turnaround to lead smoothly back to A♭M7 to begin the next chorus.

At this point, we have successfully mapped all of the chords in "Ceora" to their associated locations in ii–V space; it is reasonable to ask, though, whether this mapping of chords to space locations should even count as "analysis." After all, ii–V space contains each minor, dominant, and major seventh chord exactly once, so we did not even need to make any decisions as to where in the

17. We will return to the concept of tritone substitutes in the next section.

$A\flat^{maj7}$	$B\flat{-}^{7}$ $E\flat^{7}$	$A\flat^{maj7}$	$E\flat{-}^{7}$ $A\flat^{7}$
$D\flat^{maj7}$	$D{-}^{7}$ G^{7}	$C{-}^{7}$	$F^{7\sharp9}$
$B\flat{-}^{7}$	$E\flat^{7}$	$C{-}^{7\flat5}$	$F^{7\sharp9}$
$B\flat{-}^{7}$	F^{7}	$A\flat^{maj7}$	$B\flat{-}^{7}$ $E\flat^{7}$

Figure 2.9. Changes for the B section of "Ceora" (Lee Morgan).

space a particular chord should go. Have we, in fact, learned anything about "Ceora" from our exploration of ii–V space?

The answer, I think, is yes. Although we could criticize ii–V space for being simply a particular arrangement of common harmonic progressions in jazz, similar arrangements have proven themselves useful in many areas of music theory: the circle of fifths, the neo-Riemannian Tonnetz, the pitch-class "clock face," and countless others.[18] One of the benefits of ii–V space is that it allows us to easily visualize common harmonic motions in jazz. The succession of chord symbols that make up the changes to "Ceora" may make immediate sense to an experienced jazz musician, but ii–V space allows others to to make sense of these relationships more clearly. The fact that our analysis in ii–V space may seem obvious is in fact a feature, not a bug; such a criticism reveals that ii–V space, with all its mathematical formalism, can clarify information that may otherwise remain hidden in the raw data of the chord symbols.[19]

18. For a study of many different musical spaces (and a defense of their use), see Julian Hook, *Exploring Musical Spaces* (New York: Oxford University Press, forthcoming), Chapter 1 and throughout.

19. Music analysis is in many ways similar to the field of data visualization: both often involve revealing the underlying structure of what might otherwise seem like an undifferentiated stream of data. As Edward Tufte tells us, visualizations are often "more precise and revealing" than other, mathematical means of analyzing data. *The Visual Display of Quantitative Information*, 2nd ed. (Cheshire, CT: Graphics Press, 2001), 13–14.

2.2 Tritone Substitutions

2.2.1 Formalism

There is an important aspect of jazz harmony that has not yet been considered in our discussion of ii–V space. Crucial to harmony beginning in the bebop era is the tritone substitution: substituting a dominant seventh chord for the dominant seventh whose root is a tritone away.[20] Because tritone-substituted dominants are functionally equivalent, both the progressions Dm7–G7–Cmaj7 and Dm7–D♭7–Cmaj7 may be analyzed as ii–V–I progressions in the key of C.

This functional equivalence means that a tritone-substituted dominant can act as a shortcut to an otherwise distant portion of ii–V space. In the circle-of-fifths arrangement of Figure 2.6, keys related by tritone are maximally far apart (diametrically opposed on the circle), but in jazz practice, G7 and D♭7 are functionally identical (both dominant-function chords in C major). To account for this progression in our space, we need to somehow bring these chords closer together; one solution is to connect two segments of the space by T_6 in a sort of "third dimension," as shown in Figure 2.10. The topology of this space is more complicated than the ordinary circle of fifths, however. Once a progression reaches the bottom of the "front" side of the figure, it reappears at the top of the "back" side (G♭ at the bottom is listed again as F♯ at the top); likewise, progressions disappearing off the bottom of the back side reappear at the top of the front side (C major is given in both locations).

This arrangement of key centers is topologically equivalent to a Möbius strip, which is somewhat easier to see by focusing only on the dominant seventh chords, as shown in Figure 2.11.[21] By wrapping this figure into a circle and gluing the ends together with a half-twist (so that the two G7 chords and the D♭7/C♯7 match up), we arrive at the desired Möbius strip. Though the

20. The tritone substitution has been discussed extensively in the literature, so we will not discuss it at any length here. See, for example, Nicole Biamonte, "Augmented-Sixth Chords vs. Tritone Substitutes," *Music Theory Online* 14, no. 2 (June 2008); Henry Martin, "Jazz Harmony: A Syntactic Background," *Annual Review of Jazz Studies* 4 (1988): 11; and Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 360–65.

21. A similar diagram can be found in Werner Pöhlert, *Basic Harmony*, trans. Jürgen Krohn and Norman Bowie (Werner Pöhlert Publications, 1989), 5, and can be seen implicitly in Figure 1-1 of Martin, "Jazz Harmony."

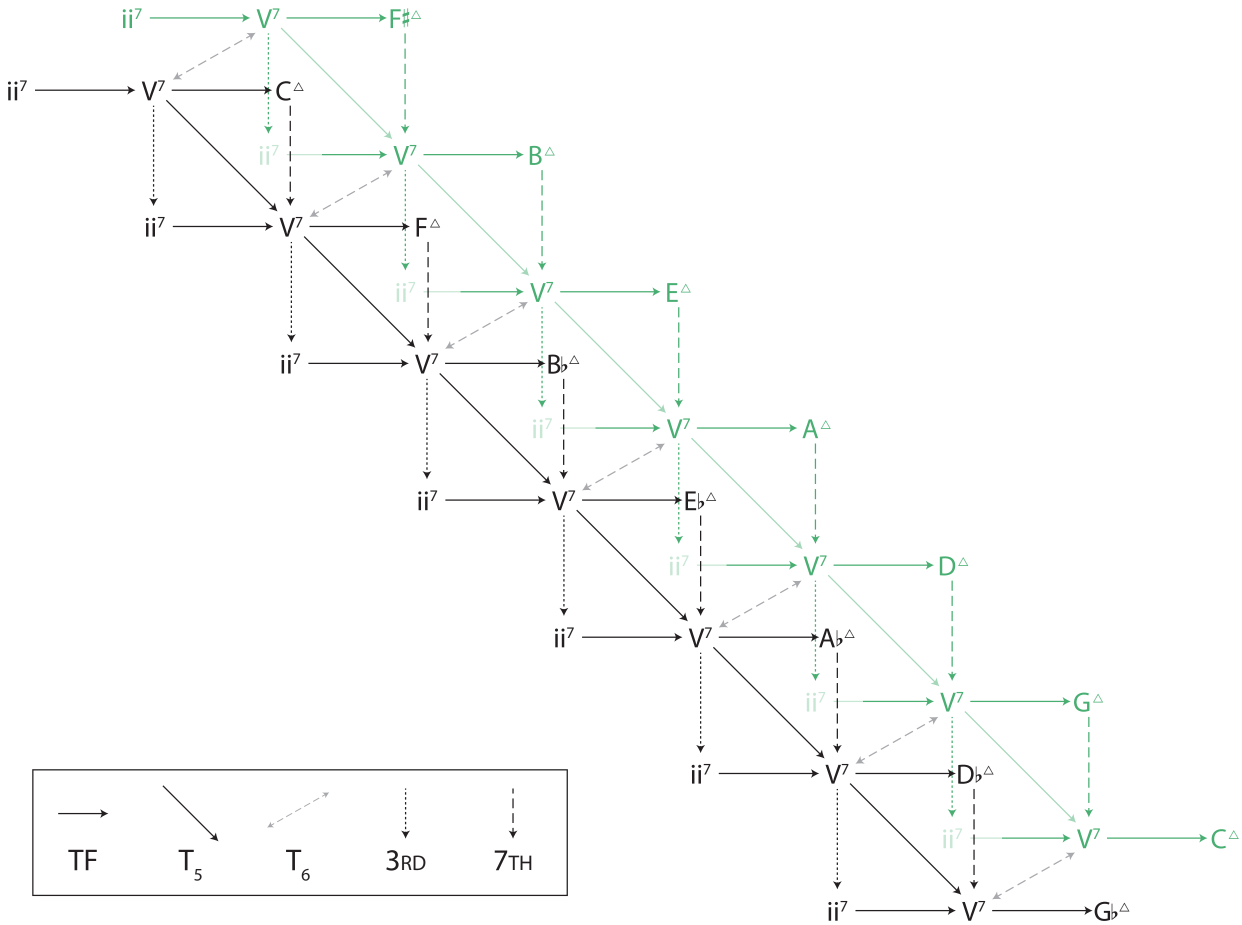

Figure 2.10. The complete ii–V space, showing tritone substitutions.

underlying topology is easier to visualize this way, it is difficult to include all of the other progressions (the ii–Vs themselves) in this diagram, so we will continue to use the "three-dimensional" version of Figure 2.10, with the understanding that this topology remains in effect. In any case, the arrangement of keys into the front and back sides is arbitrary, and may be repositioned as necessary; it is often convenient to have the tonic key (when there is one) centrally located at the front of the space.

While we could navigate this space using only the transformations TF and T_6, it is convenient to define another transformation to help with a common progression like Dm7–D♭7–Cmaj7. We

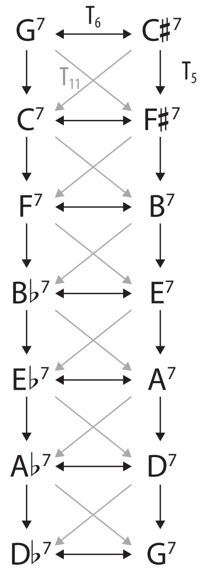

Figure 2.11. The Möbius strip at the center of ii–V space.

will call this transformation $\mathrm{TF_T}$, to highlight its relationship to the more normative TF:

$$\mathrm{TF_T}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{min}} \text{ and } Y = (y_r, y_t, y_s) = (x_r - 1, x_s + 5, x_t + 6) \in S_{\mathrm{dom}}$$

$$\mathrm{TF_T}(Y) = Z, \text{ where } Y = (y_r, y_t, y_s) \in S_{\mathrm{dom}} \text{ and } Z = (z_r, z_t, z_s) = (y_r - 1, y_s + 5, y_t + 6) \in S_{\mathrm{maj}}$$

The $\mathrm{TF_T}$ transformation represents a tritone substitution, but it transforms bass motion by fifth into bass motion by semitone (*not* bass motion by tritone); the voice-leading diagram in Figure 2.12 clarifies the relationship with the ordinary TF. Because TF and T_6 commute, $\mathrm{TF_T}$ can be considered as either TF followed by T_6, or vice versa. With this new transformation, we can understand the progression A♭m7–D♭7–CM7 as a substituted ii–V–I in C: A♭m7 $\xrightarrow{\mathrm{TF}}$ D♭7 $\xrightarrow{\mathrm{TF_T}}$ CM7.

The introduction of tritone substitutes complicates the space somewhat; Figure 2.13 shows a transformation network of the same portion of the space as in Figure 2.5, but with some chords replaced with their tritone substitutes (shown in green).[22] The relationship between a substituted

22. Determining the structure of the underlying transformation graph of this network is straightforward, so I have not included a figure of it here.

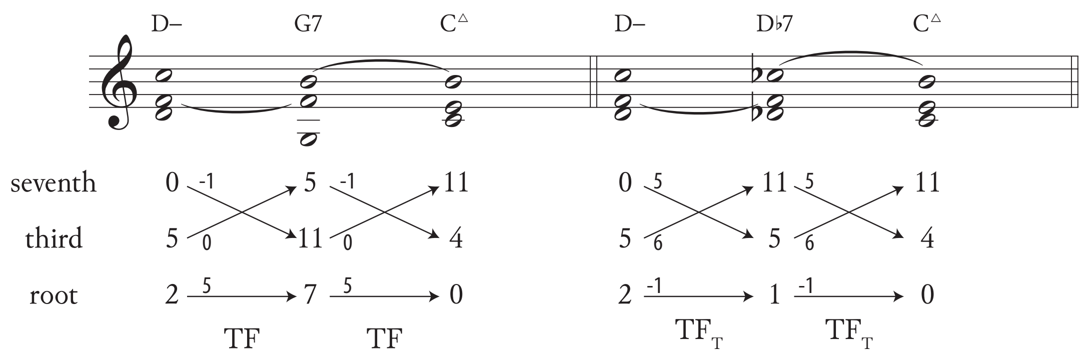

Figure 2.12. Voice leading in the TF and TF$_T$ transformations, compared.

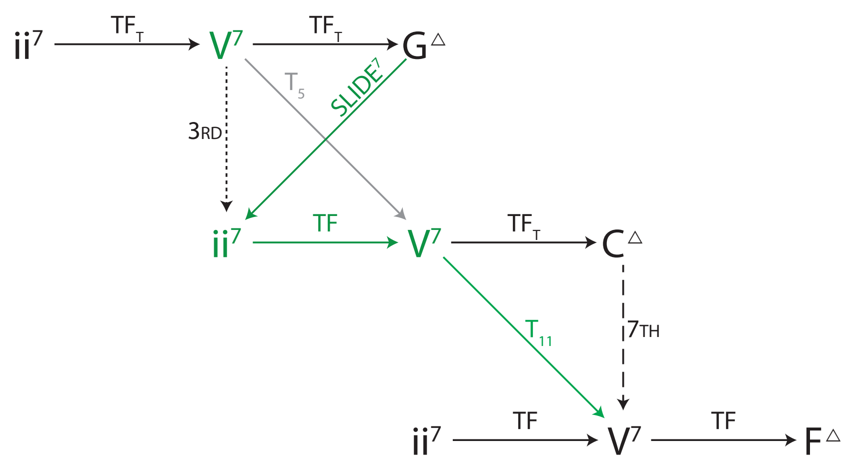

Figure 2.13. A transformation network for a small portion of ii–V space, with tritone substitutions.

dominant in G major (A♭7) and the unsubstituted ii^7 chord in C (A♭m7) is still, of course, a 3RD transformation. The substituted V^7 in C moving to the diatonic V^7 in F changes the transposition from a descending fifth to a descending half-step, as indicated by the T_{11} arrow.

Perhaps most interesting in this tritone-substituted space is the new relationship between a major seventh chord and the substituted ii^7 in the progression a fifth below (in this figure, between GM7 and A♭m7). Normally there is no voice-leading connection between these two chords, but with the substituted ii^7, the third and seventh are both held as common tones, and the root and fifth of the chord both ascend by semitone (from G–B–D–F♯ to A♭–C♭–E♭–G♭).[23] Because of its similarity to the standard SLIDE transformation, with the addition of the common tone seventh, we will call this transformation SLIDE⁷:[24]

$$\text{SLIDE}^7(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\text{maj}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 1, x_t, x_s) \in S_{\text{min}}$$

In jazz this progression occurs frequently when moving between key centers related by half step, though it is uncommon in classical music.[25] We have encountered this transformation once already: the motion from D♭M7 to Dm7 in mm. 5–6 of "Ceora" is indeed a typical SLIDE⁷ transformation (see Figure 2.14).

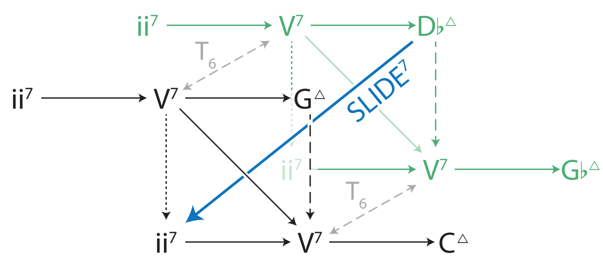

Figure 2.14. The SLIDE⁷ transformation from D♭M7 to Dm7 in mm. 5–6 of "Ceora."

2.2.2 Analytical Interlude: Charlie Parker, "Blues for Alice"

Equipped with these new tritone-substitution transformations, we can now analyze somewhat more complicated music; Charlie Parker's "Blues for Alice" will serve as a useful first example (the

23. Though we are defining chords as ordered triples in this chapter, I have included the fifth in this description to highlight the relationship to the triadic SLIDE, which maintains the root and fifth of a triad while changing the quality of the third.

24. The SLIDE transformation was introduced by David Lewin (*GMIT*, 178), but has since become a part of of the standard set of Neo-Riemannian transformations. SLIDE⁷ is defined here only as a transformation from I^7 chords to ii^7 chords, but of course the triadic SLIDE is an involution (two successive applications of SLIDE to any triad will result in the same triad). We can define SLIDE⁷ so that it acts in a similar way, but we will delay that possibility until Section ??.

25. The SLIDE⁷ transformation can be found in a chromatic sequence in the second movement of the Fauré string quartet, mm. 36–39. (I am thankful to Julian Hook for pointing this example out to me.)

F^{maj7}	$E\text{–}^{7\flat5}$ A^7	$D\text{–}^7$ G^7	$C\text{–}^7$ F^7
$B\flat^7$	$B\flat\text{–}^7$ $E\flat^7$	$A\text{–}^7$ D^7	$A\flat\text{–}^7$ $D\flat^7$
$G\text{–}^7$	C^7	F^{maj7} D^7	$G\text{–}^7$ C^7

Figure 2.15. Changes to “Blues for Alice” (Charlie Parker).

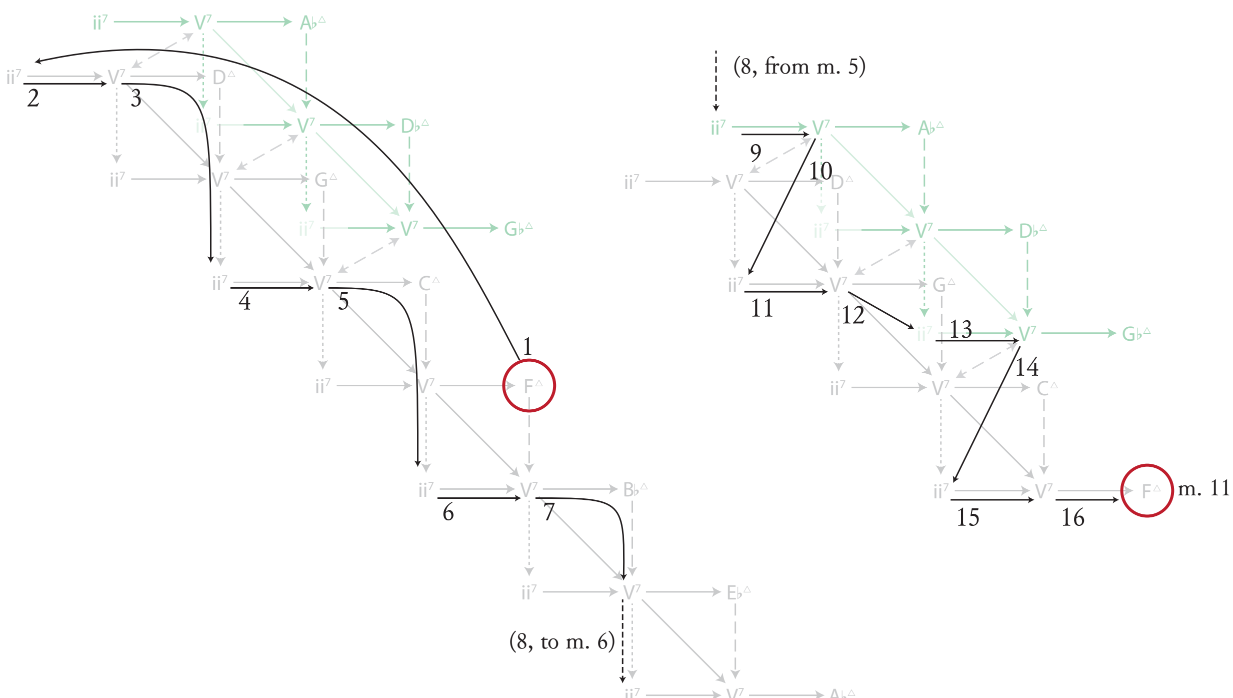

Figure 2.16. An analysis of “Blues for Alice” in ii–V space: mm. 1–5 (left), mm. 6–11 (right).

changes are given in Figure 2.15).[26] The essential structure of the blues is present: the tune arrives on a subdominant in m. 5, and on a home-key ii–V in m. 9 of the twelve-bar form.

Parker elaborates this basic structure with a series of stepwise descending ii–V progressions (see Figure 2.16). The first of these is a diatonic descent: m. 2 jumps from the tonic F major to a ii–V in D, which resolves (via the 7TH and 3RD transformations) to a Dm7 chord as the ii^7 of C major. We first saw this progression in "Ceora" (mm. 6–7), where we noted that it was a very common way of maintaining harmonic motion; instead of a ii–V resolving to its tonic, it resolves to the minor seventh chord with the same root. Because this progression is so common, it is useful to define it as its own transformation, which we will call EC (for "evaded cadence"). Unlike TF, EC is only useful as a transformation from V^7 chords to ii^7 chords:

$$\mathrm{EC}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{dom}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 2, x_t - 1) \in S_{\mathrm{min}}$$

EC is of course equivalent to TF $\odot$ 7TH $\odot$ 3RD, but only when the starting chord is a V^7 chord (a member of S_{dom}). In ii–V space, EC can be represented by starting on a dominant, then following one arrow to the right and two arrows downward. The structure of the space immediately shows that EC is impossible beginning on a ii^7 chord; we can follow a single arrow to the right, but there is only one downward arrow from a V^7 chord.[27]

This pattern of stepwise descending ii–Vs continues until arriving at the subdominant B♭ in m. 5, which includes the standard blues alteration of the lowered seventh.[28] The intuition that this

26. This progression is often known as the "Bird Blues," though Mark Levine calls it the "descending blues" in *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 228. Like many sets of Parker changes, several different versions exist; the changes here represent a mediation of these sources. *The Real Book* gives Am7 (a vi chord) instead of FM7 in m. 11; Levine's *Jazz Theory Book* gives D♭7 (a tritone substitute) instead of G7 in the second half of m. 3; and the *Charlie Parker Omnibook* omits both D7 chords (the first in the second half of m. 7, the second in m. 11) and the tonic in m. 11 is an F7. Most of these differences are minor, and over the course of a recorded performance the changes might vary among all of these versions. Other compositions that contain this progression include Parker's own "Confirmation," Sonny Stitt's "Jack Spratt," and Toots Thielemans's "Bluesette."

27. At least, impossible if we want to stay within the three sets we are studying in this chapter. Like the other transformations we have defined, EC is an admissible transformation on the set of all mod-12 ordered triples: if we begin with a Dm7 chord, $(2, 5, 0)$, EC gives us the triple $(7, 10, 4)$, which of course is not a major, minor, or dominant seventh chord.

28. The major-minor seventh chord as a stable chord is characteristic of the blues; see, for example, James McGowan, "Psychoacoustic Foundations of Contextual Harmonic Stability in Jazz Piano Voicings," *Journal of Jazz Studies* 7, no. 2 (October 2011): 158–59 and throughout. This fact is somewhat obscured in ii–V space, since major-minor sevenths appear in the space only as V^7 chords; we will return to this limitation later in Section 2.3.2.

B♭7 is in fact a stable harmony, rather than a descending-fifth transposition of F7, can be captured somewhat in our transformational labels. Instead of labeling this progression F7 $\xrightarrow{T_5}$ B♭7, we might instead label it as F7 $\xrightarrow{\mathrm{TF} \odot \textsc{7th}}$ B♭7; this designation expresses the notion that the B♭7 chord is heard as a resolution to a stable chord (the TF transformation) that has merely been inflected with the lowered seventh (the 7TH transformation). Combining this with the rest of mm. 2–5, it is easy to construct a transformation network:

$$\mathrm{Em7}\flat 5 \xrightarrow{\mathrm{TF}} \mathrm{A7} \xrightarrow{\mathrm{EC}} \mathrm{Dm7} \xrightarrow{\mathrm{TF}} \mathrm{G7} \xrightarrow{\mathrm{EC}} \mathrm{Cm7} \xrightarrow{\mathrm{TF}} \mathrm{F7} \xrightarrow{\mathrm{TF} \odot \textsc{7th}} \mathrm{B}\flat 7$$

After this B♭7 chord, Parker uses a *chromatic* stepwise pattern of ii–Vs (mm. 5–9), which we can understand as a tritone-substituted version of the earlier descending fifths pattern:

$$\mathrm{B}\flat\mathrm{m7} \xrightarrow{\mathrm{TF}} \mathrm{E}\flat 7 \xrightarrow{\mathrm{EC_T}} \mathrm{Am7} \xrightarrow{\mathrm{TF}} \mathrm{D7} \xrightarrow{\mathrm{EC_T}} \mathrm{A}\flat\mathrm{m7} \xrightarrow{\mathrm{TF}} \mathrm{D}\flat 7 \xrightarrow{\mathrm{EC_T}} \mathrm{Gm7}$$

(Here, $\mathrm{EC_T}$ is the tritone-substituted variant of EC, equivalent to $\mathrm{TF_T} \odot \textsc{7th} \odot \textsc{3rd}$, applied to a dominant seventh chord.) Once this sequence arrives on Gm7 as the ii chord of the tonic F, there is a ii–V–I progression in the home key. After the resolution in m. 11, the progression moves backwards through fifths space to begin a VI–ii–V turnaround to F major to begin the next chorus.[29]

So far, we have not said very much about the first two chords in “Blues for Alice”: FM7–Em7. In ii–V space, these chords are relatively far apart (4 edges): FM7 $\xrightarrow{\textsc{7th}}$ F7 $\xrightarrow{T_6}$ B7 $\xrightarrow{T_5}$ E7 $\xrightarrow{\textsc{3rd}}$ Em7. Because ii–V space prioritizes functional relationships, chord progressions that are close in terms of voice leading often appear quite distant in the space. In reality, a musician would probably *not* think of this move as being distant, since the two chords are so close to one another in pitch space: to rephrase again in Lewinnian terms, the “characteristic motion” that does the best job in taking us from FM7 to Em7 is something like “move the root down a half step and both the third and seventh down a whole step.” This transformation is easy enough to define, but would not

29. A “turnaround” is what jazz musicians call a short progression that leads from a chord (often a tonic chord) back to itself. They appear most commonly at the ends of forms, and provide harmonic interest during solos, when a player might play several choruses in a row. The ii–V appears frequently in this formal location, as do many of its variants: vi–ii–V, iii–VI–ii–V, iii–♭III7–ii–♭II7, etc.

be part of ii–V space proper; inevitably, the space cannot tell us everything we want to know about jazz harmony. The space is designed to show typical harmonic motions, so progressions that do not seem to lie well in ii–V space demand other explanations (and indeed, often they are voice-leading explanations).

2.3 A Few Extensions

2.3.1 Minor Tonic Chords

As it has been developed thus far, ii–V space has a glaring omission: it requires that all tonic chords be major sevenths. Certainly there are jazz tunes in minor keys, and thus there is a need to account for the tonic minor. We have already seen ii–V progressions that resolve to minor chords—we called that transformation EC in the previous section—but the only minor chords in the space are ii^7 chords, not tonics. One of the advantages of ii–V space is that it is easily extended to account for harmonic features specific to particular situations; in this section, we will do just that to allow for stable minor tonic chords.

The minor ii–V–i progression is usually played as ii7♭5–V7♯9–i mM7.[30] Because we are working with ordered triples of only roots, thirds, and sevenths in this chapter, the alteration of the fifth and ninth have no effect on our definitions of S_{min} and S_{dom} (defined in Section 2.1 above). We do, however, need to formally define the set of minor-major seventh chords (chords with a minor third and major seventh):

$$S_{\text{mM7}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 3; x_s - x_r = 11\}$$

30. The extensions used for the dominant chord in a minor ii–V is quite flexible: Aebersold's *II–V^7–I Progression* gives the quality as 7♯9, but Mark Levine usually gives the chord symbol simply as "alt." Levine includes the minor ii–V in the category of "melodic minor scale harmony," and "alt." is short for the "altered scale" (the seventh mode of melodic minor). The G altered scale is G–A♭–B♭–B♮–C♯–E♭–F♮–G, and is sometimes called the "diminished whole-tone" scale, since it begins as an octatonic scale and ends as a whole-tone scale, or the "super-locrian" scale, the locrian mode with a flatted fourth. This sound could be expressed with a number of different chord symbols—G7(♭9♯9♯11♭13) or G7(♭5♯5♭9♯9), for example—so jazz musicians typically write "alt." See *The Jazz Theory Book*, 70–77. We will return to this equivalence between chords and scales in Chapter 4.

With this definition in place, we can explore how this set interacts with the three sets we have seen already in this chapter. The 3RD transformation works intuitively, and transforms a major seventh chord to a minor-major seventh with the same root:

$$3\text{RD}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\text{maj}} \text{ and } Y = (y_r, y_t, y_s) = (x_r, x_t - 1, x_s) \in S_{\text{mM7}}$$

Likewise, the 7TH transformation transforms a minor-major seventh into a minor-minor seventh with the same root:

$$7\text{TH}(Y) = Z, \text{ where } Y = (y_r, y_t, y_s) \in S_{\text{mM7}} \text{ and } Z = (z_r, z_t, z_s) = (y_r, y_t, y_s - 1) \in S_{\text{min}}$$

It will also be useful to define versions of the TF and TF_T transformations that transform a dominant seventh into a *minor* tonic, equivalent to TF $\odot$ 3RD or $\text{TF}_\text{T} \odot$ 3RD. We will call them simply "tf" and "tf_T" (the lowercase here is meant to parallel the use of lowercase letters to indicate minor triads):

$$\text{tf}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\text{dom}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 5, x_s - 2, x_t) \in S_{\text{mM7}}$$
$$\text{tf}_\text{T}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\text{dom}} \text{ and } Y = (y_r, y_t, y_s) = (x_r - 1, x_s + 4, x_t + 6) \in S_{\text{mM7}}$$

Note that unlike the standard TF and TF_T transformations, tf and tf_T only transform V^7 chords to I^7 chords; the same transformations do *not* hold for ii^7 to V^7.

Figure 2.17 shows a small portion of ii–V space that includes minor tonic chords. Because most jazz tunes do not contain exclusively minor chords, this figure gives both major and minor tonic chords in every key. The transformations defined in the previous paragraph are readily apparent in the space, with the exception of the 7TH transformation from a minor-major seventh to a minor seventh—GmM7 moving to Gm7 as ii^7 of F major, for example. Though we will not do so here, determining how to fill in the figure with tritone substitutions, or to conform it around the circle of fifths in the manner of Figure 2.6, is easy enough to imagine (if not to draw, given the added complexity of the minor-major sevenths).

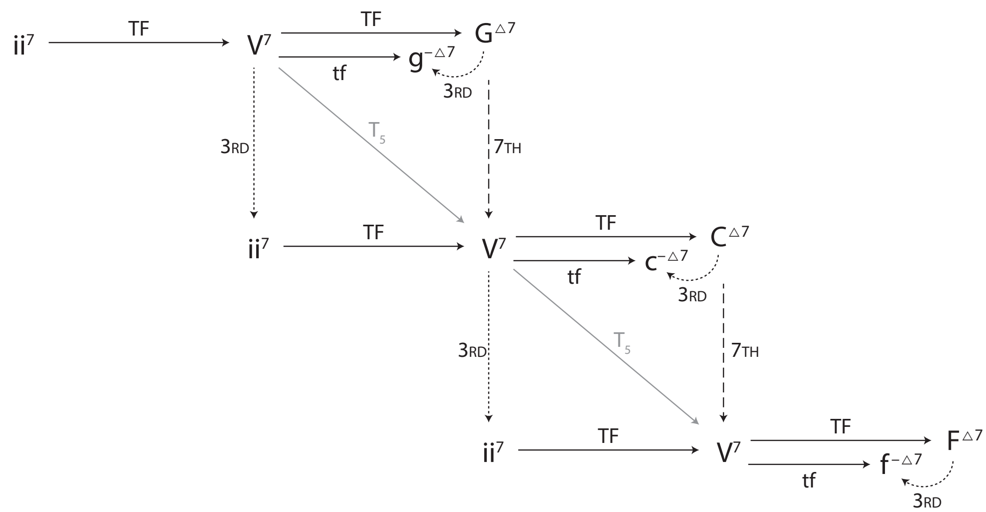

Figure 2.17. A small portion of ii–V space, including minor tonic chords.

By way of a brief example, Figure 2.18 gives the changes for Miles Davis's "Solar." This tune is in C minor, though that is not immediately apparent from the changes themselves; in the canonical recording of this piece (from Davis's own *Walkin'*), the C minor chords are played as minor-major sevenths, and the piece ends on a CmM7 chord. The fact that the only tonic chord appears in the opening bar of the form gives performances of this tune even more of cyclical quality than is usual in jazz. By not arriving on a tonic at the end of the short 12-bar form, Davis achieves a formal overlap at every chorus: the opening tonic serves simultaneously as the harmonic resolution of the previous chorus and the formal beginning of the next.

The analysis in ii–V space is mostly unremarkable, but it is given in Figure 2.19. Note that this figure has replaced CM7 at the top of the space with a minor tonic, CmM7, and as such there is no arrow given between the C-minor tonic and V^7/F. This analysis, though, is not possible in the ii–V space of the previous section, since the C-minor harmony of the first bar is decidedly *not* a ii^7 chord (it would be ii of B♭, and there is no B♭ major harmony in the piece at all).

C–		G–^{7}	C^{7}
F^{maj7}		F–^{7}	B♭^{7}
E♭^{maj7}	E♭–^{7} A♭^{7}	D♭^{maj7}	D–$^{7\flat5}$ G$^{7\flat9}$
C–$^{[maj7]}$			

Figure 2.18. Changes to “Solar” (Miles Davis).

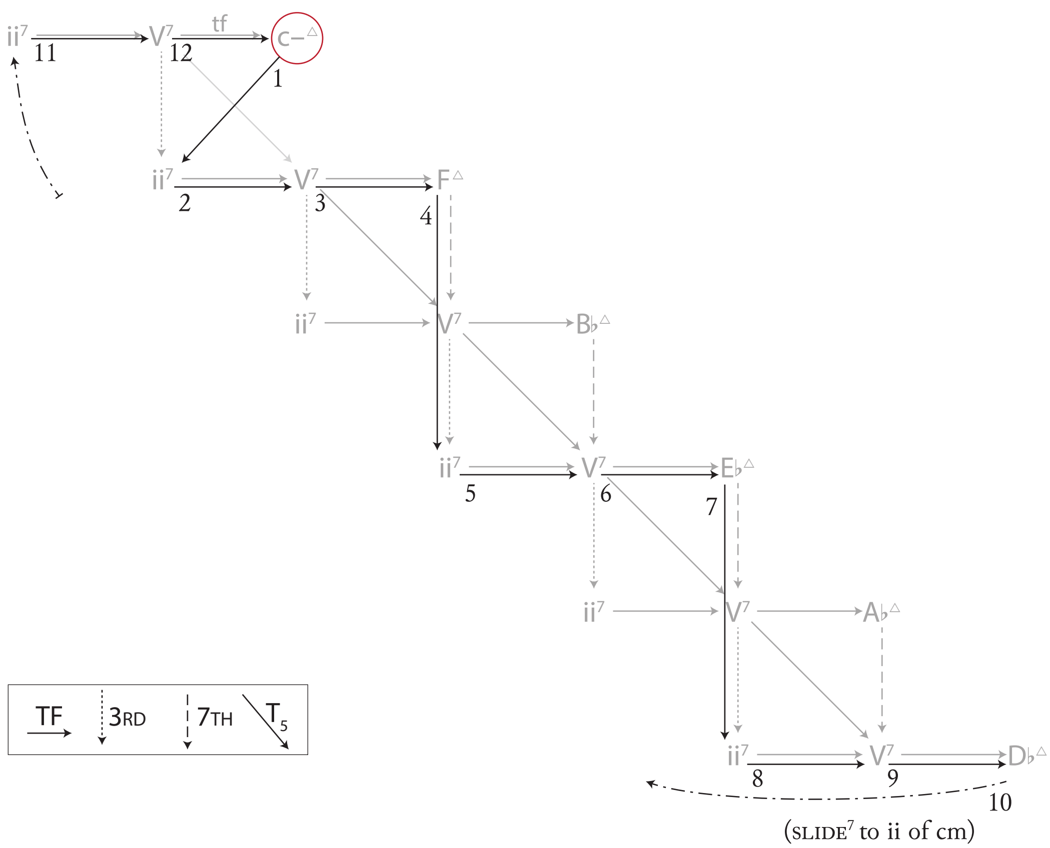

Figure 2.19. An analysis of “Solar” in ii–V space, with C minor tonic.

2.3.2 Other Kinds of Tonic Chords

We have now solved the problem of tonic chords that happen to be minor-major seventh chords, but in fact the problem is more general: it would be nice to have some way of allowing for any kind of tonic chord we might find in real music. As mentioned in Section 1.2, James McGowan has argued for what he calls three "dialects of consonance" in jazz (extended tones we might consider consonant): the added sixth, the minor seventh, and the major seventh.[31] Both of the approaches in this chapter so far have focused only on the major-seventh dialect, when it appears atop both major and minor triads. Many Tin Pan Alley tunes end with tonic add-6 chords (which appear nowhere in ii–V space), and as we noted in our discussion of "Blues for Alice", it is very common for a blues tonic to be a major-minor seventh chord (which appear only as V^7 chords in the space).

The solution to this shortcoming of the space is to introduce some general transformation (which we might call "ResI") that could be redefined as needed for each style.[32] The generic space would then appear as it does in Figure 2.20. This space is still arranged in perfect fifths, and the basic shape of the ii–V–I progressions is still present, but the quality of the tonic chords is unspecified. Before using this space in analysis, of course, we must actually define what we mean by a "tonic chord" in a given situation. Because ii–V space contains cross-type transformations, this means we need to define both the *set* of tonic chords and the transformation ResI, from S_{dom} to the set of tonics.[33] (By defining ResI to be equivalent to TF and defining tonic chords to be members of S_{maj}, for example, the generic space here becomes the specific layout of ii–V space first presented in Figure 2.10.)

Again, it will be easiest to demonstrate exactly how this generic space can be actualized by means of an example. In our analysis of "Blues for Alice" in Section 2.2.2 above, we noted that the

31. James McGowan, "Dynamic Consonance in Selected Piano Performances of Tonal Jazz" (PhD diss., Eastman School of Music, 2005), 76–79.

32. The name of the transformation ResI is inspired by Steven Rings's use of the transformation "ResC" in the first chapter of *Tonality and Transformation*, 25-27.

33. In practice, ResI will almost always be a transformation that moves the root of a V^7 down a perfect fifth. In theory, however, there is no limitation on the definition of ResI. It is possible, for example, to construct a space where tritone substitutes are normative by defining ResI to be equal to TF_{T}; in this case, the gray arrows in Figure 2.20 would represent the transformation $\text{7TH} \odot T_6$.

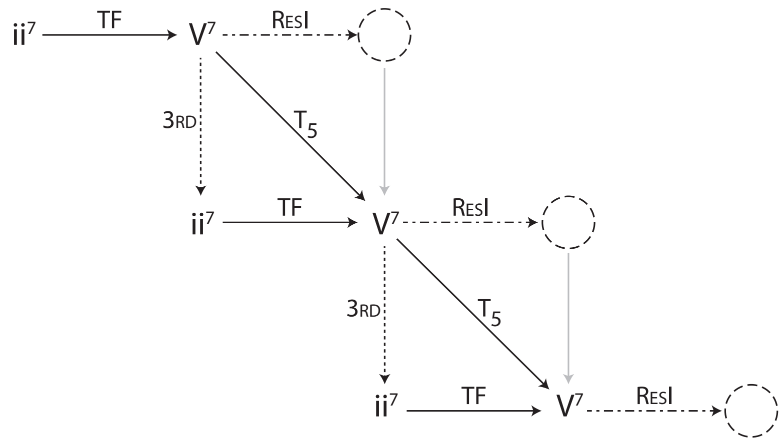

Figure 2.20. A generic version of ii–V space, with unspecified tonic chords.

B♭7 chord in m. 5 served as the resolution of the ii–V in the preceding bar, but contained the lowered seventh, which is typical for the blues. There, we tried to capture the intuition that the B♭7 was stable by labeling the transformation as TF $\odot$ 7TH: a resolution merely inflected with the lowered seventh. This transformation, though, still results in the B♭7 as a dominant seventh chord (it appears in the space only as V^7 of F).

The generic ResI transformation offers a better solution, in that we can define a "blues TF," which resolves a V^7 to a tonic major-minor seventh:

$$S_{\mathrm{IMm7}} = \{(x_r, x_t, x_s) \mid x_t - x_r = 4; x_s - x_r = 10\}$$

$$\mathrm{TF}_{\mathrm{blues}}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{dom}} \text{ and } Y = (y_r, y_t, y_s)$$
$$= (x_r + 5, x_s - 1, x_t - 1) \in S_{\mathrm{IMm7}}$$

Note that $\mathrm{TF}_{\mathrm{blues}}$ is equivalent to T_5, but is defined in a way to demonstrate its similarity to TF (see the voice leading in Figure 2.21; $\mathrm{TF}_{\mathrm{blues}}$ is undefined on ii^7 chords). We have also defined the set S_{IMm7}, the set of tonic major-minor seventh chords; this seems intuitive, but is somewhat complicated. S_{IMm7} is exactly equivalent to S_{dom}—in the language of set theory, they are the same set. The difference between them is not structural, but interpretive: S_{IMm7} is the set of *tonic*

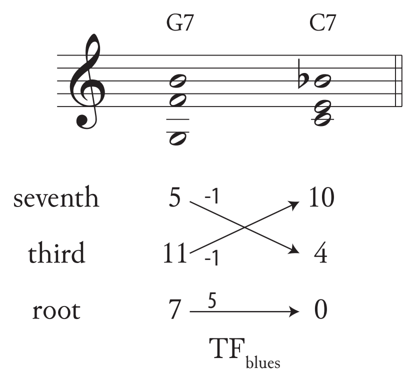

Figure 2.21. Voice leading in the $\mathrm{TF}_{\mathrm{blues}}$ transformation.

major-minor seventh chords, while S_{dom} is the set of *dominant* major-minor sevenths. This distinction allows us to capture the difference between B♭7 as a stable resolution (as it is in m. 5 of "Blues for Alice") and B♭7 as V^7 of E♭ (as in m. 8 of "Solar", for example).

This sort of interpretive analysis lies at the heart of Steven Rings's work in *Tonality and Transformation*; the GISes he develops there are designed to capture the intuitions that collections of pitches can be heard (or experienced) differently in different contexts. We can adapt this work slightly to capture the intuition that tonic major-minor sevenths are experienced differently than dominant major-minor sevenths; Rings would say that the two sets have different *qualia*.[34]

The tonal GIS Rings develops in his second chapter consists of ordered pairs of the form (scale degree, acoustic signal); as he has it, "the notation $(\hat{7}, x)$ . . . represents the apperception: 'scale degree seven inheres in acoustic signal *x*.' "[35] Rings goes on to describe sets of these ordered pairs, which we will use to capture our intuitions about the varying roles of the B♭7 chord, as shown below:[36]

$$\underset{\text{B}\flat\text{7 as dominant}}{\left\{\begin{matrix}(\hat{4}, \mathrm{A}\flat) \\ (\hat{7}, \mathrm{D}) \\ (\hat{5}, \mathrm{B}\flat)\end{matrix}\right\}} \qquad \underset{\text{B}\flat\text{7 as tonic}}{\left\{\begin{matrix}(\flat\hat{7}, \mathrm{A}\flat) \\ (\hat{3}, \mathrm{D}) \\ (\hat{1}, \mathrm{B}\flat)\end{matrix}\right\}}$$

34. Rings, *Tonality and Transformation*, 41–43 (and throughout).
35. Ibid., 44.
36. Ibid., 55.

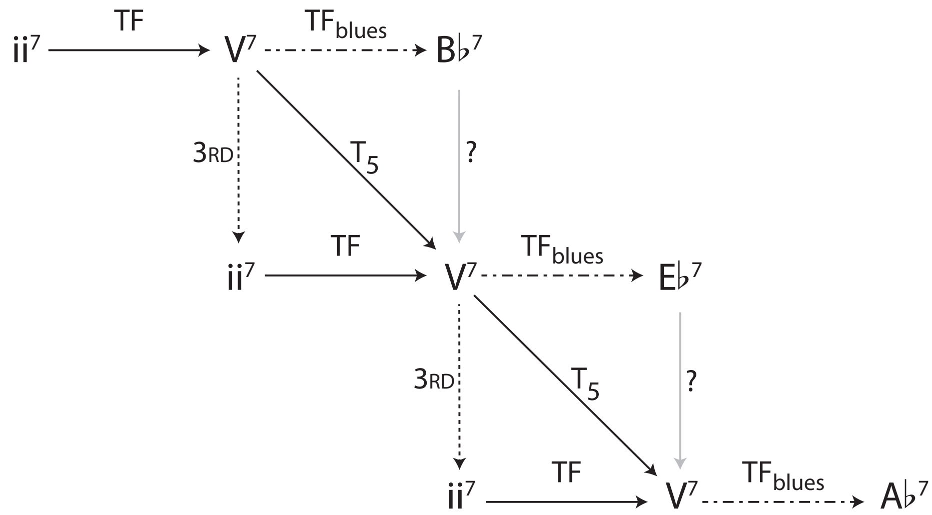

Figure 2.22. A small portion of "blues ii–V space."

Here, the left figure (read bottom to top as root, third, seventh) represents B♭7 as a dominant seventh of E♭ (i.e., with $\hat{5}$, $\hat{7}$, and $\hat{4}$) while the right figure represents the same three pitch classes as a tonic major-minor seventh (with $\hat{1}$, $\hat{3}$, and $\flat\hat{7}$).[37] Rings's system of heard scale degrees allows us to distinguish between the sets S_{IMm7} and S_{dom}: the B♭7 in m. 5 of "Blues for Alice" is a member of S_{IMm7}, while the B♭7 in m. 8 of "Solar" is a member of S_{dom}.

With the distinction between tonic and dominant minor-major sevenths worked out, we can now specify the generic space of Figure 2.20 to create what we might call a "blues ii–V space"; a small portion of this space is shown in Figure 2.22. This space, though, presents another complication: the top arrow marked with a question mark represents a transformation from B♭7 as tonic to B♭7 as dominant. The pitch classes remain the same, but the *quale* of the chord changes from tonic to dominant, so this transformation is not simply the identity (T_0).

Because this transformation is one of *quale*, we can turn to Rings's tonal GIS for an explanation. Intervals in this GIS are measured with ordered pairs, like the elements themselves: the first element is a scale-degree interval (measured upward), and the second is a pitch-class

37. In fact, we could make similar statements for all of the sets developed in this chapter: S_{min} would then become (speaking loosely) "the set of minor-minor seventh chords acting as $\hat{2}$, $\hat{4}$, and $\hat{1}$ in some key." In most cases, however, this level of precision is unnecessary, since the quality of the chord uniquely identifies its function.

interval.[38] "Pivot intervals" are those intervals where the second element of the pair is 0.[39] In the situation here, we have what Rings would call a "pivot fifth" between the two B♭7 chords:

$$\left\{\begin{matrix} (\flat\hat{7}, \mathrm{A}\flat) \\ (\hat{3}, \mathrm{D}) \\ (\hat{1}, \mathrm{B}\flat) \end{matrix}\right\} \xrightarrow[\text{“pivot fifth”}]{(5\text{th},0)} \left\{\begin{matrix} (\hat{4}, \mathrm{A}\flat) \\ (\hat{7}, \mathrm{D}) \\ (\hat{5}, \mathrm{B}\flat) \end{matrix}\right\}$$

The pitch-class interval here is 0, since both chords contain B♭, D, and A♭, and the scale-degree interval is a 5th ($\hat{1}$ to $\hat{5}$, $\hat{3}$ to $\hat{7}$, and $\flat\hat{7}$ to $\hat{4}$). With this transformation defined, we can now more fully realize our intuitions about the short passage in "Blues for Alice" (mm. 4–6):

$$\ldots\ \mathrm{Cm7} \xrightarrow{\mathrm{TF}} \mathrm{F7} \xrightarrow{\mathrm{TF}_{\mathrm{blues}}} \mathrm{B}\flat 7 \xrightarrow{\text{pivot 5th} \odot \text{3RD}} \mathrm{B}\flat\mathrm{m7} \xrightarrow{\mathrm{TF}} \mathrm{E}\flat 7\ \ldots$$

2.3.3 Interaction with Diatonic Spaces

The preceding consideration of other kinds of tonic chords has taken us relatively far afield from the starting point of this chapter, and indeed these extensions are not necessary to understand most tonal jazz. For many purposes, the conventional space developed in Sections 2.1 and 2.2 will be sufficient. What is missing in our treatment so far, though, is the concept of a global tonic. This dissertation, after all, is interested in tonal jazz, and most of this music (and certainly all of the examples in this chapter) is in a key. To this point, we have acknowledged this fact only by mentioning the key of a particular tune in our analytical commentary, or circling the tonic chord in a representation of the space. Defining ii–V space as a fully chromatic space has many advantages: it is rare that every chord in a tune can be understood in a single key, and it is convenient not to have to switch continually between diatonic collections. Moreover, chromatic spaces are much more regular than their diatonic counterparts: chromatic step size is consistent, while diatonic step size varies between one and two half steps.[40]

38. Rings, *Tonality and Transformation*, 46–48.

39. Ibid., 58–66.

40. For more on diatonic systems generally, see John Clough and Gerald Myerson, "Variety and Multiplicity in Diatonic Systems," *Journal of Music Theory* 29, no. 2 (Autumn 1985): 249–70.

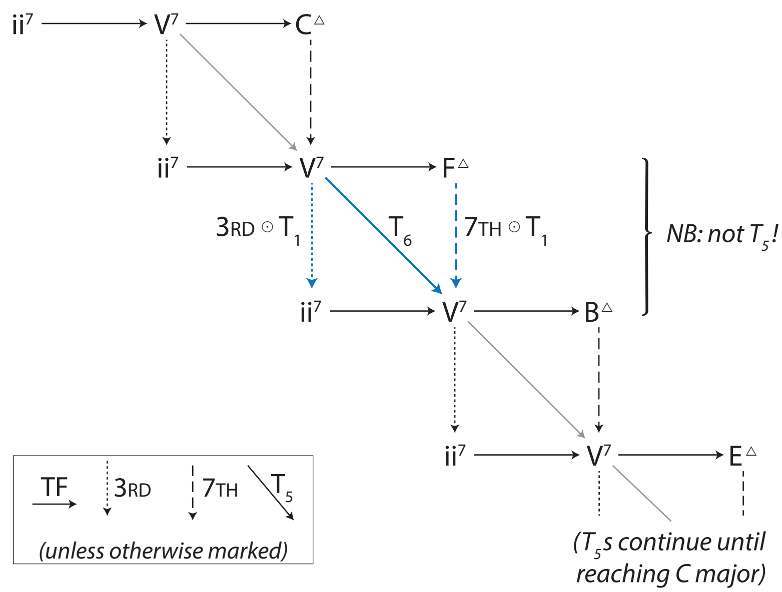

Figure 2.23. A portion of ii–V space, conformed to the white-key diatonic circle of fifths.

Still, given the exploration of diatonic transformational systems in Section 1.5, it seems wise to consider what a diatonic ii–V space might look like. We first made the space chromatic by arranging individual ii–V–I progressions in descending fifths (recall Figure 2.6). We could instead arrange the space according to the *diatonic* circle of fifths, as shown in Figure 2.23. This space looks much like the chromatic space, with the exception of the diminished fifth between $\hat{4}$ and $\hat{7}$, where the regular transformational structure of the chromatic space breaks down. The change of the descending perfect fifth (T_5) to a diminished fifth (T_6) means that all of the transformations linking these two key areas must all be combined with T_1: C7 $\xrightarrow{\text{3RD} \odot T_1}$ C♯m7, C7 $\xrightarrow{T_6}$ F♯7, and FM7 $\xrightarrow{\text{7TH} \odot T_1}$ F♯7.[41]

As noted in Chapter 1, diatonic space (the cyclic group $\mathcal{C}_7$) can be generated by any of its members, while chromatic space ($\mathcal{C}_{12}$) can only be generated by the members 1, 5, 7, or 11 (half-steps and perfect fourths/fifths). Diatonic ii–V space, then, offers the interesting possibility of departing from the fifths-based space used so in this chapter, in favor of some other organization

41. These transformations are all relatively parsimonious, and seem in some way related to the SLIDE7 transformation introduced above. We will delay a discussion of these parsimonious aspects of these transformations more generally until the next chapter.

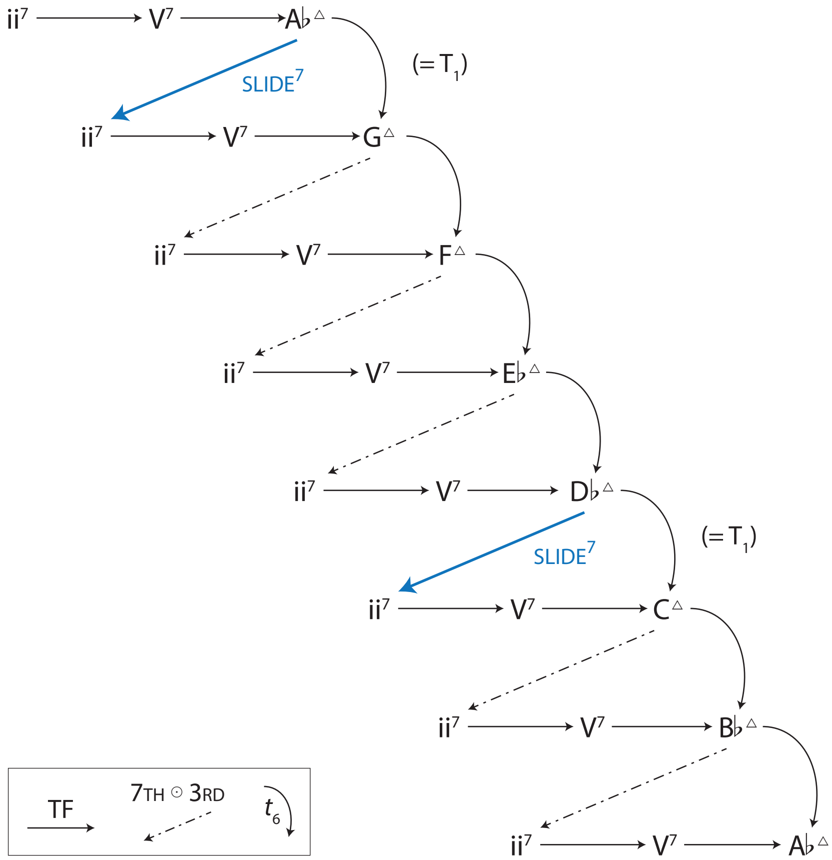

Figure 2.24. An A♭-major diatonic ii–V space, arranged in descending steps.

of the space (since any interval we might choose will generate the entire space). To see how such an organization might allow us to capture different kinds of analytical insights, I want to return to briefly to Lee Morgan's "Ceora."

In the analysis of "Ceora" in Section 2.1.2, we saw that the whole tune takes place in four key areas: D♭, C, B♭, and the tonic A♭. Given this organization, we might consider arranging ii–V space in descending diatonic steps, as shown in Figure 2.24. (The entire figure could be wrapped around a circle so that the identical ii–V–I progressions in A♭ at the top and bottom of the figure line up.) This arrangement into steps means that the key areas used in the tune are adjacent in the space; in the chromatic space of Figure 2.8, they were separated by an intervening fifth.

This figure is structurally a bit different than the other spaces we have been exploring in this chapter, so it will be helpful to examine it in some detail before returning to "Ceora." The

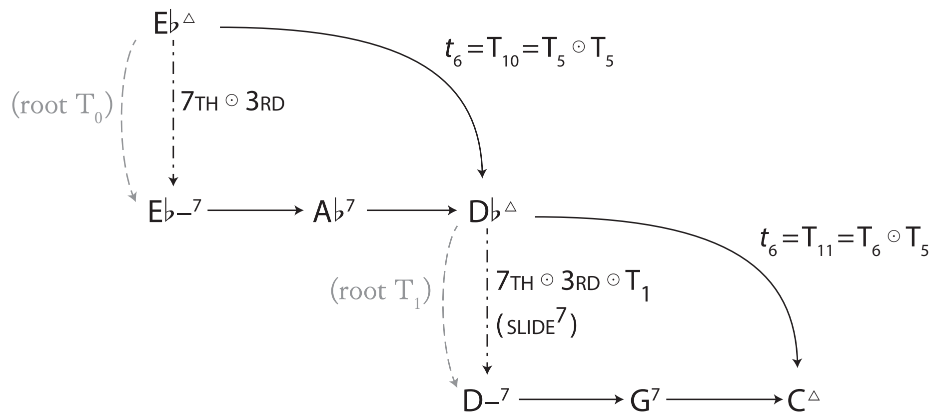

Figure 2.25. Detail of diatonic ii–V space, showing the SLIDE^7 transformation between key centers related by half-step.

arrangement into descending steps means that we can no longer align all of the seventh chords sharing a root; only the major and minor seventh chords sharing a root are adjacent in the space.[42] The GM7 and Gm7 (as ii^7/F) chords are close to one another, for example, but G7 (V^7/C) is farther removed. The key areas in this figure are not connected by T_5, but instead by t_6: all of the roots of the major seventh chords (reading down the right side of the figure) are members of the 4-flat diatonic collection.[43] Though the diatonic distance between key areas is consistent, the chromatic distance varies: there are two points in the space connected by half steps rather than whole steps.

As we saw above, the tritone appearing in diatonic space alters the transformational structure somewhat: transformations spanning this tritone must must be combined with T_1. Here, the relationship between most I^7 and ii^7 chords is the transformation 7TH ⊙ 3RD, but between the keys of D♭ and C (as well as A♭ and G), it is 7TH ⊙ 3RD ⊙ T_1. This transformation is in fact equivalent to the transformation SLIDE^7 (see the detail in Figure 2.25); this diatonic origin is one of the reasons the SLIDE^7 transformation is so common in tonal jazz.

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42. This figure has been skewed somewhat to conserve space on the page. If it were drawn in a manner parallel with standard ii–V space, a ii^7 chord would be directly below the I^7 chord with the same root. As is the case throughout this study, the particular visual representation chosen for a given space does not affect the abstract structure of the space itself.

43. In our discussion of the diatonic transposition operator in Section 1.5.1, we defined t_k to be the operation that transposes the root of a chord k steps inside a particular diatonic collection. There, we were interested in a particular 7-element set of chords (with their attendant qualities); here, the root is still transposed in a particular diatonic collection, but the resulting chord remains a major seventh chord.

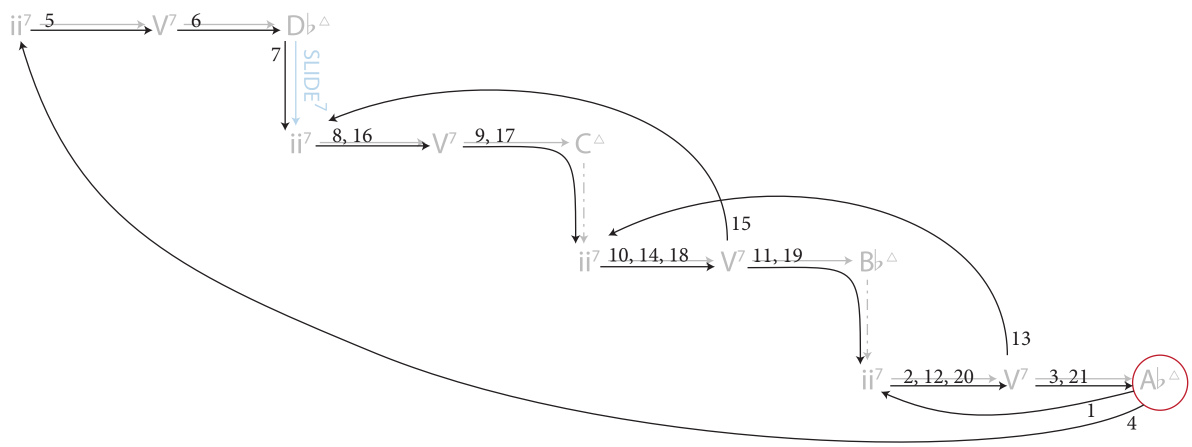

Figure 2.26. An analysis of "Ceora" in diatonic ii–V space.

All of "Ceora" takes place in a relatively small portion of diatonic ii–V space; Figure 2.26 gives an analysis of the A section in this space (the changes can be found in Figure 2.7).[44] The analysis of course looks very similar to our analysis in Section 2.1.2, but the stepwise arrangement of the space helps us to show different analytical insights. Moves in the space that are relatively close in the chromatic fifths arrangement in Figure 2.8 appear much larger in this arrangement (the move from A♭M7 to E♭m7, marked "4" in this figure), and vice versa (the SLIDE^7 from D♭M7 to Dm7, marked "7").

It is worth noting at this point that although we have adapted ii–V space to show aspects of diatonicism, ii–V space is still chromatic. The transformations are still defined on ordered triples of mod-12 (not mod-7) integers, and nothing in the ii–V–I progressions themselves has changed. We used the guiding influence of a diatonic collection in this section only to choose the key centers we showed in particular representation of the space. This use reflects the construction of jazz itself; tunes are often globally diatonic (in a key), while locally chromatic, using ii–V–I progressions to tonicize other key areas to a much greater extent than is usually seen in classical music. The reason for this is largely practical. The head-solos-head form of most jazz means that we hear the same

44. "Ceora" is perhaps even more diatonic than this space implies, since CM7 and B♭M7 never appear in the music, while D♭M7 does. Thus the chord qualities strongly suggest A♭ major: I and IV (A♭ and D♭) both appear as major sevenths, while ii and iii (B♭ and C) appear only as minor seventh chords (unstable ii^7 chords).

progression repeated many times (Morgan's recording of "Ceora" runs about 6 ½ minutes, for example), and using only pitches from the A♭-major diatonic collection would quickly become boring.

This arrangement of ii–V space, combining chromatic and diatonic operations, is mathematically complicated. As Steven Rings notes, transformation networks involving both chromatic and diatonic operations violate Lewin's formal definition of a transformation network, since they act on different sets.[45] The underlying transformation graph is not path consistent, since two different paths in the space can combine to form t_6: SLIDE[7] ⊙ TF ⊙ TF (A♭M7 to GM7, for example), and (7TH ⊙ 3RD) ⊙ TF ⊙ TF (GM7 to FM7).[46] Put another way, putting GM7 in the top row of Figure 2.24 while leaving the transformational labels unchanged does not work: obeying the t_6 arrow requires FM7 to occupy the row below, but following the other path would give F♯M7. The graph is, however, realizable: it is possible, as Figure 2.24 attests, to fill in the nodes such that the arrows *do* make sense.[47] Both Rings and Hook have shown that transformation networks that are not path consistent—like the diatonic ii–V space developed in this section—can nevertheless be analytically productive.

2.3.4 Summary

By using the ii–V–I as the basis of the transformational spaces developed in this chapter, we can now understand a large swath of tonal jazz harmony. Because this progression is so ubiquitous, many jazz tunes can be well understood using only the spaces here (perhaps with some adaptations, as suggested in this final section). Treating chords as ordered triples of root, third, and seventh allowed us to define transformations in a way that is still valid when the actual form of a chord might differ greatly from instance to instance. This is a useful abstraction, and we will continue to use it in the next chapter, where we will also consider relationships among our sets of ordered triples more generally.

45. Rings, *Tonality and Transformation*, 98–99.
46. Path consistency is described in Hook, "Cross-Type Transformations," 25–28.
47. Ibid., 29.