Chapter 1
Introduction

1.1 Problems of Jazz Analysis

When compared to a score by Beethoven (for example), the jazz lead sheet appears strikingly bare. The Beethoven score specifies nearly everything one might need to know in order to perform it. Though the minor details—dynamics, articulations, phrasing marks, and the like—will differ from piece to piece, we can usually depend on the presence of some basic information. It is rare for traditional scores not to include the instrumentation, for example, and a score that did not include the number of measures or which notes to play in combination with which which other notes would be very unusual indeed.

And yet, this is the usual state affairs for the jazz lead sheet, which is probably the most common form of a "jazz score."[1] Most lead sheets only include the basic outline of a melody, along with a set of "changes" that prescribe the harmonic structure of a piece. Beyond this most basic instruction, every other aspect is left up to the performers. Of these two elements (melody and harmony), harmony has a much larger role in determining the course of a particular jazz performance, so it seems appropriate to focus our analytical attention on it.

Jazz is essentially a harmonic music. In a typical jazz performance, the melody of the piece is heard only twice (at the beginning and the end), while the harmonic structure is heard throughout, determining the structure of the performance. Each soloist typically plays one or more "choruses," where each chorus is understood as a single iteration of the piece's harmonic structure. In marked contrast to a Beethoven score, jazz compositions usually remain unspecified when it comes to their contrapuntal structure: performers will typically improvise counterpoint

1. This is not to say, of course, that there are not jazz compositions that do specify these minor details. These compositions are the exception, rather than the rule, in the music in which this study is interested.

that fits with the underlying harmonic framework. Harmony is the main restraining factor of a piece, and its primary method of coherence.

The word "jazz"—which has been used at various times to describe McKinney's Cotton Pickers, Benny Goodman, Sun Ra, John Zorn, Tito Puente, and Brad Mehldau, among many others—is inescapably vague, so it will be useful at this point to delimit the terms of this study somewhat. Here I am interested in in what might be called "tonal jazz," which begins in the swing area and continues through hard bop, covering roughly the years 1935–1965. In this music, functional harmonic progressions are the norm; "tonal jazz" is meant in opposition to "modal jazz," where the rate of harmonic change is slower and the harmony is mostly non-functional.[2] This includes much of the music that most people think of when they hear the word "jazz," including big-band swing (Count Basie, much of Duke Ellington's music), bebop (Charlie Parker, Dizzy Gillespie, Thelonious Monk), and the mainstream jazz that followed bebop, known variously as "hard bop" or "post-bop" (John Coltrane, Sonny Rollins, Bill Evans, and many others). I intend the dates to be flexible, especially on the later end; given the strong influence of the bebop tradition on jazz and jazz pedagogy, the hard bop style continued to exist well beyond 1965, and many players today still play in the style.[3]

Now that we have delineated "jazz," we should explain exactly what we mean by "harmony." Harmony is of course one of the oldest topics in music theory, and as such has been hotly contested throughout its history. It is often found in opposition to counterpoint; in this view, counterpoint is concerned with individual melodic voices, while harmony is concerned with individual verticalities. In other traditions (most notably the Schenkerian tradition), harmony is understood to be an outgrowth of counterpoint: verticalities arise primarily through contrapuntal

2. Miles Davis's "So What" is probably the most well-known modal jazz piece; it is a 32-bar tune in which the first chord, Dm, lasts 16 bars, moves to E♭m for 8 bars, and back to Dm for the final 8. The term also describes other similar pieces, including the rest of Davis's album *Kind of Blue*, John Coltrane's recording of "My Favorite Things," and Herbie Hancock's "Maiden Voyage."

3. As Scott DeVeaux puts it, bebop is "both the source of the present . . . and the prism through which we absorb the past. To understand jazz, one must understand bebop." *The Birth of Bebop: A Social and Musical History* (Berkeley: University of California Press, 1997), 3.

procedures. Furthermore, study of harmony is often broken down by genre: "tonal harmony" plays a different role than does "chromatic harmony" in both theoretical research and pedagogy.[4]

When jazz musicians refer to "harmony," they are typically referring to the changes themselves; that is, the chord symbols given on a lead sheet or arrangement. Even when they are not playing from sheet music, the chord symbol is the basic unit of harmonic understanding for jazz musicians. The reason for this is largely practical: a chord symbol is a concise way of referring to a particular sound, and improvising musicians must be able to understand this information quickly (when reading) and to recall it easily (when improvising).

Since the pioneering work of George Russell in the late 1950s, many jazz musicians conceive of an equivalence between a harmony (a chord symbol) and a scale.[5] The chord symbol Dm7 might imply a D dorian scale, for example, rather than simply the notes D–F–A–C. Because any of the notes of this scale will sound relatively consonant over a Dm7 chord, the chord symbol acts as a convenient shorthand for a particular "way of playing" for a jazz improvisor. This equivalence between chords and scales will be the focus of Chapter 4; for now it enough to note that understanding jazz harmony often involves more than understanding relationships between four-voice seventh chords.

When analyzing jazz harmony, it is often difficult to determine exactly what one should be analyzing. Lead sheets as circulated in fake books can be highly inaccurate, and often cannot be relied upon as a single source for any particular jazz performance, since it is rare that performers play directly from a lead sheet with no modifications.[6] In the case of jazz standards which may have

4. This is not meant to imply that "chromatic harmony" is not tonal; rather, studies that focus specifically on chromatic harmony often differentiate themselves from other tonal theoretical traditions.

5. George Russell, *The Lydian Chromatic Concept of Tonal Organization*, vol. 1, *The Art and Science of Tonal Gravity* (Brookline, MA: Concept, [1953] 2001).

6. Fake books are collections of lead sheets that traditionally were compiled anonymously and sold illegally, in order to avoid paying the copyright owners of the compositions they contained. The name "fake book" comes from the fact that with the melody and chord changes, jazz musicians can easily "fake" a tune they do not know. The most famous jazz fake book is ironically titled *The Real Book*, and was compiled in Boston in the early 1970s. In recent years, fake books have become mainstream, and most of them have now obtained proper copyright permissions. Hal Leonard now publishes the 6th edition of *The Real Book* (a nod to the five illegal editions); many of the notorious errors in the earlier editions have been corrected and it is now available for purchase legally. Further references to *The Real Book* in this document refer to this version unless otherwise noted. For a history of fake books, see Barry Kernfeld, *The Story of Fake Books: Bootlegging Songs to Musicians* (Lanham, MD: Scarecrow Press, 2006).

originated elsewhere, we might wonder whether should we analyze the original sources. In many cases, however, the "jazz standard" version may be significantly different from the original version, reflecting a history of adaptation by generations of jazz musicians.[7] To make matters worse for the hopeful academic, this knowledge is often secret knowledge, not written down and learned only from more experienced musicians.

Many published jazz analyses rely on transcriptions of particular performances as a way to avoid some of these issues. In general, this solution works well, and I will certainly make use of transcriptions from time to time. This study, however, is interested in harmony more generally, and transcriptions can confuse matters somewhat. The kinds of questions I am interested in answering are of the type "What can we say about harmony in the piece 'Autumn Leaves?'" and less often of the type "What can we say about Bill Evans's use of harmony in the recording of 'Autumn Leaves' from *Portrait in Jazz*?" Furthermore, even transcriptions are not definitive when it comes to harmony: the pianist and guitarist might not be playing the same chord; the soloist might have a different harmony in mind than the rhythm section; or the bass player might play a bass line in such a way that affects our perception of the chordal root. Even in the course of a single performance, a group might alter a tune's harmonic progression, perhaps preferring some substitutions during solos and others during the head.[8]

This is a problem without one clear solution, and it may make more sense to use one method or another depending on the situation. Some compositions have canonical recordings—Coleman Hawkins's recording of "Body and Soul," for example—and in those circumstances determining the changes is usually unproblematic. Other compositions are more fluid, and different choruses might alter the basic structure within the course of a single performance (substituting a V7♭9 for a V13♯11 chord, for example). In these cases, I am interested in what Henry Martin has called the

7. By the time they become standards, many non-jazz compositions (Tin Pan Alley songs or traditional songs like "Back Home Again in Indiana") have been adapted, typically to additional harmonic motion to provide interest during solos. Some examples are more radical: the tune "Alice in Wonderland" (analyzed below), for example, is known to jazz musicians as a jazz waltz with one chord per bar. The main title music in the 1951 film from which it was taken, however, is in 4/4 with a relatively slower harmonic rhythm.

8. The "head" is what jazz musicians call the statements of the melody in the course of a jazz performance. This melody is typically played once at the beginning of a performance and again at the end (often referred to as the "out head").

"ideal changes"; a hypothetical set of chords that we can use as a basis for understanding the many variations that might occur in actual performance.[9] These changes represent a sort of Platonic model of a composition; individual performances of "Autumn Leaves" can be seen as instances of some ideal AUTUMN LEAVES. Determining these ideal changes is often a process of mediating published lead sheets, recorded versions, and other sources; throughout this study I have tried to clarify exactly *what* harmony I am analyzing in any given example.[10]

In an attempt to answer some of these questions, this dissertation presents a transformational model of jazz harmony. While on the surface a transformational model may seem abstract and far-removed from the concerns of performing jazz musicians, harmony in jazz fits together nicely with David Lewin's famous "transformational attitude."[11] A jazz musician does not typically think of harmonies as a series of points in space, but rather as a series of "characteristic gestures" between them. Rather than focusing on an underlying tonality, a jazz musician often tries to "make the changes"—to fully engage with the sound of each individual harmony.

There is often quite a large gap between the way jazz is most commonly taught (in jazz studios and pedagogical books) and the way it has traditionally been understood by music theorists. Another goal of the present study is to use transformational methods in an attempt to narrow this gap, by bringing theoretical and mathematical rigor to materials that are often ignored by the theory community, and by applying established theoretical principles in a way that corresponds closely with the understanding of jazz musicians.

While we can never claim to know what jazz musicians think, we might get somewhat closer to an answer by examining jazz pedagogical materials. In the late 1960s, jazz began to be accepted into the academy, and many young jazz musicians began learned to play the music in schools, rather

9. Henry Martin, *Charlie Parker and Thematic Improvisation* (Lanham, MD: Scarecrow Press, 1996), 5–6. See also David J. Heyer, "Vocabulary, Voice Leading, and Motivic Coherence in Chet Baker's Improvisations" (PhD diss., University of Oregon, 2011), 36–39.

10. In places where I refer to a "tune" generically, I have provided at least two references to relatively straight-ahead recorded examples in the appendix.

11. David Lewin, *Generalized Musical Intervals and Transformations* (Oxford University Press, [1987] 2007), 159 (hereafter *GMIT).*

than exclusively from older musicians.[12] To supplement this teaching, a great deal of pedagogical material has appeared that aims to teach young musicians how to play jazz.

Unfortunately, there is little interaction between these pedagogical materials and music theoretical materials. Pedagogical materials, such as the recent *Berklee Book of Jazz Harmony*, often do not have bibliographies or mention recent work in the theoretical literature.[13] Likewise, most theoretical work does not refer to these harmonic handbooks which are staples of jazz education. Mark Levine's *Jazz Theory Book*, widely regarded in the jazz education world as *the* book on jazz theory, does not appear in any of the bibliographies in a special jazz issue of *Music Theory Online* (18.3), for example.[14]

In recent years the theory community has embraced pedagogical materials as a means of uncovering how historical musicians might have thought about their own music.[15] Though jazz pedagogues do not typically publish articles in music theory journals or otherwise consider themselves to be "music theorists," per se, the goal of their harmonic textbooks is quite similar to the goals of pedagogical books in music theory: to teach students how to think (or hear, or perform) in a particular style of music.

David Lewin points out in his introduction to transformational theory that when considering a particular musical passage, we often "conceptualize along with it, as one of its characteristic textural features, a family of directed measurements, distances, or motions of some sort."[16] I certainly hear these characteristic motions when I listen to jazz, and I think it is these motions that jazz pedagogues are emphasizing when they teach students to "make the changes." Despite its somewhat hostile mathematical appearance, transformational theory is an effective means of exploring these families of intuitions. Modeling sets of chord changes as transformations between

12. The Thelonious Monk Institute of Jazz maintains a web page on the history of jazz education in America at http://www.jazzinamerica.org/JazzResources/JazzEducation/Page.

13. A noted exception here is Andy Jaffe's *Jazz Harmony* (Tübingen: Advance Music, 1996), which features an extended bibliography that includes many theoretical works.

14. This issue of *Music Theory Online* is a Festschrift in memory of Steve Larson (September 2012), guest edited by Stephen Rodgers, Henry Martin, and Keith Waters.

15. See, for example, Robert Gjerdingen, *Music in the Galant Style* (New York: Oxford University Press, 2007); and Giorgio Sanguinetti, *The Art of Partimento: History, Theory, and Practice* (New York: Oxford University Press, 2012).

16. Lewin, *GMIT*, 16.

harmonic objects, for example, allows the theoretical discourse to draw on the ways in which jazz musicians teach harmony, and can bring these two disparate areas somewhat closer together.

1.2 Theoretical Approaches to Jazz Harmony

Studies of jazz harmony in recent years have primarily taken the form of Schenkerian analyses that seek to uncover large-scale voice-leading structures in order to define tonality. Schenkerian analysis has proven to be an extremely useful tool for analyzing tonal music, and Steve Larson's pioneering work in applying its methods to jazz has undoubtedly expanded the field of jazz studies and brought jazz analysis into the theoretical mainstream. While theorists may disagree on exactly *how* we should apply these Schenkerian techniques, hardly anyone seems to doubt that they are the best way to examine tonal structures in jazz.

The touchstone of the Schenkerian jazz literature is Steve Larson's *Analyzing Jazz: A Schenkerian Approach*, which is the culmination of his work of the previous decades.[17] In this and all of his work, Larson advocates what might be called an "orthodox" Schenkerian approach. He treats the extended tones common in jazz (sevenths, ninths, elevenths, etc.) as standing in for tonic members at some deeper structural level.[18] Steven Strunk's important article on linear intervallic patterns in jazz also uses an orthodox approach, as does the work of Daniel Arthurs, David Heyer, and Mark McFarland.[19]

17. Steve Larson, *Analyzing Jazz: A Schenkerian Approach* (Hillsdale, NY: Pendragon Press, 2009). This book makes use of much of his earlier work, including "Strict Use of Analytic Notation," *Journal of Music Theory Pedagogy* 10 (1996): 31–71; "Schenkerian Analysis of Modern Jazz: Questions about Method," *Music Theory Spectrum* 20, no. 2 (October 1998): 209–41; and "Composition versus Improvisation?," *Journal of Music Theory* 49, no. 2 (October 2005): 241–75.

18. Larson, *Analyzing Jazz*, 6.

19. Steven Strunk, "Linear Intervallic Patterns in Jazz Repertory," *Annual Review of Jazz Studies* 8 (1996): 63–115. See also, for example, Daniel Arthurs, "Reconstructing Tonal Principles in the Music of Brad Mehldau" (PhD diss., Indiana University, 2011); David J. Heyer, "Applying Schenkerian Theory to Mainstream Jazz: A Justification for an Orthodox Approach," *Music Theory Online* 18, no. 3 (September 2012); and Mark McFarland, "Schenker and the Tonal Jazz Repertory: A Response to Martin," *Music Theory Online* 18, no. 3 (September 2012). Arthurs's work on Mehldau is something of an outlier, for reasons explained further in note 38.

At levels close to the background, these orthodox analyses of jazz do not appear significantly different than analyses of classical music, and indeed that is part of their appeal.[20] Because most jazz is basically tonal music, these theorists are interested in showing its connection to the European classical tradition by using the same techniques to analyze both of them. This is especially important to note, since Schenkerian analysis has a well-known ethical component.[21] For Schenker, the compositions that were well-described by his theory were judged to be masterworks. By showing that jazz compositions can also be understood with these Schenkerian techniques, the implicit conclusion is that they too should be judged to be masterworks. Underlying these orthodox approaches, I think, is a desire to legitimize a place for jazz in academic music theory (I will return to this point in a moment).

An opposing group of theorists also supports the use of Schenkerian analysis, but argues that it should be adapted to account for tonal features specific to jazz. Principal among this group is Henry Martin, who advocates for the use of alternative *Ursätze* in jazz, including ascending or gapped *Urlinien*, non-triadic descents, and chromatic or neighbor-note *Urlinien*.[22] He argues that jazz pieces are "often influenced by a more African-American aesthetic that favors repetition and rhythmic interplay over voice-leading motion through descending linear progressions," and thus we should not feel obligated to adhere to traditional Schenkerian techniques.[23]

Martin draws in part on James McGowan's "dialects of consonance" in jazz, which describes different contextually stable notes of the tonic triad.[24] McGowan argues that consonance in jazz is

20. Throughout this document, the word "classical" used in this sense is used in its generic sense to stand for tonal music that participates in the Western art music tradition. This definition is imperfect, but is useful for distinguishing the music of Bach, Mozart, Beethoven, Schumann, and Wagner from that of Ellington, Basie, Parker, Monk, and Coltrane. I do not mean to imply that jazz cannot participate in the Western art music tradition; certainly at least some of it does.

21. Nicholas Cook, "Schenker's Theory of Music as Ethics," *The Journal of Musicology* 7, no. 4 (October 1989): 415–39.

22. Henry Martin, "Schenker and the Tonal Jazz Repertory," *Tijdschrift voor Muziektheorie* 16, no. 1 (2011): 16–17. This work draws on earlier work of his own as well as that of others, including Allen Forte, *The American Popular Ballad of the Golden Era, 1924–1950* (Princeton University Press, 1995); and David Neumeyer, "Thematic Reading, Proto-backgrounds, and Registral Transformations," *Music Theory Spectrum* 31, no. 2 (October 2009): 284–324.

23. Martin, "Schenker and the Tonal Jazz Repertory," 7.

24. See James McGowan, "Dynamic Consonance in Selected Piano Performances of Tonal Jazz" (PhD diss., Eastman School of Music, 2005); and "Psychoacoustic Foundations of Contextual Harmonic Stability in Jazz Piano Voicings," *Journal of Jazz Studies* 7, no. 2 (October 2011): 156–91.

stylistically defined, and that in certain styles we might hear extended tones as consonances, even though they would be dissonant in classical music. He describes three “principal dialects”: the added sixth, common in early jazz and Tin Pan Alley standards; the minor seventh, particular to the blues; and the major seventh, common in later jazz performance.[25] For Schenker, the background is derived from the consonant tonic triad; Martin is able use these stylistically determined definitions of consonance as support for his stylistically informed background structures.

McGowan’s work is among the minority in recent years that does not feature a Schenkerian bent; in addition to his dialects of consonance, he is interested in applying (paleo-) Riemannian functional analysis to jazz.[26] Despite this Riemannian focus, however, he is not interested in transformational analysis: David Lewin’s name appears nowhere in his bibliography. Closer to my own interests is John Bishop’s 2012 dissertation, which also turns to transformations and mathematical group theory to close the gap between jazz pedagogy and jazz theory.[27] Bishop focuses primarily on the interplay of pure triads in jazz, and connects harmony to chord-scales via these triads.

Other important non-Schenkerian models of jazz harmony are found in earlier works of Martin and Strunk. In his dissertation, Martin advocates a syntactic approach based on the circle of fifths, in which chains of descending fifths point towards a tonic pitch.[28] Steven Strunk’s early theory of jazz harmony is a layered approached that draws on the Schenkerian concept of analytical levels to uncover a single tonal center for a jazz tune.[29] These older models tend to fall more in line with how jazz musicians themselves discuss harmony, and we will return to them below.[30]

25. McGowan, “Dynamic Consonance,” 76–79 and throughout. The dialects are particularly clear in final tonic chords, especially in the case of the characteristic tonic major-minor seventh of the blues.

26. Ibid. On the distinction between “paleo-” and “neo-” Riemannian analysis, see Steven Rings, “Riemannian Analytical Values, Paleo- and Neo-,” in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Rehding (New York: Oxford University Press, 2011), 487–511.

27. John Bishop, “A Permutational Triadic Approach to Jazz Harmony and the Chord/Scale Relationship” (PhD diss., Louisiana State University, 2012).

28. Henry Martin, “Jazz Harmony” (PhD diss., Princeton University, 1980); see also his “Jazz Harmony: A Syntactic Background,” *Annual Review of Jazz Studies* 4 (1988): 9–30.

29. Steven Strunk, “The Harmony of Early Bop: A Layered Approach,” *Journal of Jazz Studies* 6 (1979): 4–53.

30. I have a suspicion that these earlier models of harmony tend reflect jazz musician’s intuitive understanding because they are not interested in legitimizing jazz analysis for the academy, as I suggest above. Both Strunk and Martin’s articles appear in the *Journal of Jazz Studies/Annual Review of Jazz Studies* (the journal was renamed in 1981), while more recent articles on jazz harmony have appeared in music theory journals: *Music Theory Spectrum*, *Journal of*

Given the prevalence of Schenkerian techniques in jazz analysis and its proven explanatory power in other tonal repertories, it is reasonable to ask why a different approach like transformational theory is useful or necessary. In order to answer this question, I would first like to problematize the Schenkerian focus of recent jazz analysis. Steve Larson's "Schenkerian Analysis of Modern Jazz: Questions about Method" will serve as a useful foil; it is one of the fundamental articles in the field, and its titular questions will help to guide the discussion here.[31]

In the article, Larson asks three main questions we must answer if we are to take seriously the suggestion that such Schenkerian analysis is appropriate for jazz:

1. Is it appropriate to apply to improvised music a method of analysis developed for the study of composed music?
2. Can features of jazz harmony (ninths, elevenths, and thirteenths) not appearing in the music Schenker analyzed be accounted for by Schenkerian analysis?
3. Do improvising musicians really intend to create the complex structures shown in Schenkerian analysis?[32]

Larson's answer to the first question is yes. Many of Schenker's own methods were of course developed for improvisatory music, and even if they had not been, they have proven to be explanatory for such music.[33] I agree with Larson on this point, and don't have anything in particular to add. Certainly we should not expect that our theories can prove useful only for the music for which they are designed; in fact as theorists we generally hope that the opposite is true, and that our theories have broader applications than originally intended!

The second question has generated much more discussion in the literature regarding exactly how we might apply Schenkerian methods to jazz, already discussed above. While this discussion is

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Music Theory, and the *Dutch Journal of Music Theory*, for example. As jazz research has moved from the fringes into the theoretical mainstream, I think it has grown more removed from jazz practice itself. As I mention above, one of the goals of the present study is to show how we might narrow this gap while still approaching the music with the necessary theoretical rigor.

31. This article appears with only slight changes as the second chapter of Larson's book, *Analyzing Jazz*. Garrett Michaelsen critiques Schenkerian analysis in similar ways in "Analyzing Musical Interaction in Jazz Improvisations of the 1960s" (PhD diss., Indiana University, 2013), 7–11.

32. Larson, "Schenkerian Analysis of Modern Jazz," 210.

33. In the opening of *Free Composition*, Schenker refers to improvisation as "the ability in which all creativity begins"; *Free Composition [Der freie Satz]*, ed. and trans. Ernst Oster (New York: Longman, [1935] 1979). Schenker's first *Erlauterungsausgabe* (explanatory edition) was indeed of an improvisatory work, Bach's *Chromatic Fantasy and Fugue*.

mostly one of the mechanics of analysis, it raises another point in which I am interested: is a jazz musician's conception of musical space (or musical structure) the same as a classical musician's? The orthodox Schenkerians argue that it is not, and that jazz is a fundamentally triadic music (since at some deep structural level all of the extended tones are reduced away). Those that favor a modified approach disagree with this characterization, but still agree that Schenkerian analysis is the best way to approach jazz harmony. My own reactions to this question overlap with my answer to Larson's third question, so I will return to it shortly.

Larson's third question regards compositional or improvisational intent. The argument is perhaps obvious: because Schenkerian analysis depends on uncovering long-range voice-leading plans, how could improvising musicians possibly hold such plans in their memory while playing? At some level, we might not even be interested in the answer to this question. Schenkerian analysis has proven explanatory, after all, for music that was doubtlessly composed without Schenkerian methods in mind (namely, music written before Schenker's birth). Nevertheless, Larson spends a great deal of time addressing this particular point, so we should see to it here as well.

After dismissing the possible intentional fallacy of this question, Larson turns to pianist Bill Evans as an example of how a jazz musician could produce complicated long-range voice-leading patterns while improvising. To do so, he relies heavily on an interview that Bill Evans gave on Marian McPartland's radio program, *Piano Jazz*.[34] Evans discusses how he always has a basic structure in mind while playing:

> McPartland: Well, when you say structure, you mean like, one chorus in a certain style, another . . .
>
> Evans: No, I'm talking about the abstract, architectural thing, like the theoretical thing.[35]

Evans goes on to demonstrate how he has certain structural features in mind (harmonic and melodic arrivals). Larson then shows how Evans's accompanying commentary can be understood as

34. The interview was recorded on November 6, 1978. The program is available online at http://www.npr.org/2010/10/08/92185496/bill-evans-on-piano-jazz, and was released on CD under Evans's name as well: *Marian McPartland's "Piano Jazz" Radio Broadcast* (The Jazz Alliance TJA-12038-2, 1993).

35. Larson, "Schenkerian Analysis of Modern Jazz," 219.

explaining voice-leading events like prolongation and linear progressions, and provides voice-leading analyses of his playing.[36]

I think that Larson's use of Schenker's methods to analyze the music of Bill Evans is well justified, but I am less sure of the extent to which they are applicable to jazz more generally. Larson anticipates this objection, noting that it might be offered on two grounds: "first, that Evans was unusually talented as an improviser; and second, that his way of thinking was radically different from that of other jazz musicians."[37] Certainly Evans was not a typical jazz musician: he was white, and studied at the Mannes College of Music, then and now a cradle of Schenkerian activity in this country.[38] Larson suggests that Evans was such an influential pianist that his Schenkerian improvisational tendencies might have influenced other musicians with whom he played. While this may be true, Evans was probably not seen as influential until after 1960, and this study is concerned primarily with music before that time (or at least, with musicians whose style was well established by that time).[39]

Larson allows that the first objection is justified: "That Evans was an unusually talented improviser—and that Schenkerian analysis can show this—is a principal argument of this article."[40] This statement is representative of the legitimizing enterprise of the application of Schenker's methods to jazz mentioned above. It also contains a dangerous implication, made explicit in Larson's closing paragraphs:

> Much jazz improvisation lacks the relationships that reward long-range hearing, and consists, as [André] Hodeir observes, of "disconnected bits of nonsense." . . . But the fact that jazz musicians often say that "a jazz improvisation should tell a story" suggests that many jazz musicians are concerned with creating and experiencing global relationships. That they do not always achieve this goal in performance is not surprising—the task is difficult. But there are exceptions.

36. See especially the table in Larson, "Schenkerian Analysis of Modern Jazz," 229.
37. Ibid., 239.
38. Brad Mehldau, who is the focus of Daniel Arthurs's work, is quite similar to Evans, in that he is a classically-trained white pianist with a strong acknowledged influence of the European classical tradition.
39. Bill Evans made his first recordings as a leader in 1958, but he was not widely known until his appearance on Miles Davis's *Kind of Blue* in 1959. Shortly after he left Davis, he won wide acclaim with his trio with Scott LaFaro and Paul Motian, whose groundbreaking first album was *Portrait in Jazz*, released in 1960.
40. Larson, "Schenkerian Analysis of Modern Jazz," 239.

> Is Schenkerian analysis applicable only to jazz performances that are exceptions? No, Schenkerian analysis may be applied to any jazz performance—and it may show the shortcomings of that performance.[41]

Far from "retaining Schenker's methods but not his epistemology, his specific insights into music but not the system of beliefs that supported them" (as Nicholas Cook suggests that modern Schenkerians often do), Larson seems to be using Schenkerian analysis to judge certain performances as "masterworks" and others as inferior, much in the way done by Schenker himself.[42] Performances by musicians who do not share Evans's interest in the "abstract, architectural thing" may well be excellent performances when judged by other value systems.[43] Furthermore, Schenkerian analyses of jazz often focus on what turns out to be the least interesting part of a jazz piece. Jazz is essentially tonal music, so it is not at all surprising (to me, at least) that it is often possible to reveal an *Ursatz* from a particular performance.

Whether or not jazz musicians are thinking in a Schenkerian manner is not really the point, however. As Steven Rings puts it in the introduction to his book, "any analytical act will . . . leave a surplus—a vast, unruly realm of musical experience that eludes the grasp of [a] single analytical model. Corners of that vast realm may nevertheless be illuminated via other analytical approaches, but those approaches will leave their own surpluses. And so on."[44] Schenkerian analysis typically focuses on long-range voice leading in order to reveal an underlying diatonic framework, while deemphasizing (some would say reducing) surface details, including much harmonic chromaticism.

This focus on harmony as a first-class object is something that is at the heart of much jazz pedagogical material, and might help constitute a different set of analytical values by which we can understand jazz. Rather than having to consign musicians who we cannot understand with Schenkerian analysis to a second tier of appreciation, we can instead try to understand them on

41. Larson, "Schenkerian Analysis of Modern Jazz," 240–41.

42. Cook, "Schenker's Theory of Music as Ethics," 439.

43. In particular, value systems that do not stem from the European classical tradition. Michaelsen, drawing on George Lewis's distinction between "Afrological" and "Eurological" modes of improvising, suggests that Schenkerian analysis is a particularly Eurological method of analysis; his own theory is designed to address interaction in improvisation, which is a more Afrological value. See "Analyzing Musical Interaction," 4–5, 11–12. Lewis first introduces the terms in "Improvised Music after 1950: Afrological and Eurological Perspectives," *Black Music Research Journal* 16, no. 1 (Spring 1996): 91–122.

44. Steven Rings, *Tonality and Transformation* (New York: Oxford University Press, 2011), 5.

something of their own terms. A Schenkerian analysis of Coltrane's "Giant Steps" solo might reveal that it is "lacking an artistically convincing relationship among structural levels," but it would likely be difficult to find a jazz musician who did not hold the composition up as an example of Coltrane's supreme mastery of the music.[45]

1.3 Transformational Theory

Given the current dominance of Schenkerian theory in the study of jazz harmony, we might ask what transformational thinking brings to the table. Transformational theory in recent years has focused on neo-Riemannian analysis, with particular emphasis on efficient voice leading and non-functional, chromatic progressions. Steven Rings has written that this focus on neo-Riemannian theory has "led to a view that some works are divvied up into some music that is tonal . . . and some that is transformational." Continuing, he argues that to do so "is to misconstrue the word transformational, treating it as a predicate for a certain kind of music, rather than as a predicate for a certain style of analytical and theoretical thought."[46] As he is right to point out, there is nothing about transformational theory that necessitates its restriction to this locally chromatic music; his book uses the theory to explain, as he says, "specifically *tonal* aspects of tonal music." It is this use of "transformational" that I wish to bring to bear on jazz, which is essentially tonal music.

Fundamental to the Schenkerian approach is the relatively equal balance of harmony and voice leading; for jazz musicians, though, this balance is heavily weighted toward the harmonic. Schenkerian analysis tends to deemphasize certain harmonies with the aim of exposing an underlying diatonic framework. This goal is in contrast with the typical goal of a performing jazz musician, for whom individual chords have first-class status.

Transformational theory too, often treats harmonies as first-class objects, and thus makes it especially appropriate for analyzing jazz. "Often," only because transformational theory can be used

45. Larson, "Schenkerian Analysis of Modern Jazz," 241. Though there are many published analyses of "Giant Steps," I am unaware of any that use Schenkerian methods.

46. Rings, *Tonality and Transformation*, 9.

for more than simply examining harmony: Lewin's Generalized Interval Systems (explained in detail below) only require a set of elements, a group of intervals, and a function mapping pairs of elements of the set into the group of intervals. Most commonly the elements of the set are harmonies, but they do not have to be.[47]

Mathematical music theories have become especially widespread in recent years.[48] Many of these models focus only on triads; while these models are valuable, nearly all chords in jazz are (at least) seventh chords. Because the neo-Riemannian literature is relatively well-known, it will be useful here to limit our focus to those theories that deal in some way with non-triadic music. This work falls basically into two categories: work that deals exclusively with a single type of chord, and work that deals with musical objects of different types.

Most of the studies dealing with a single type of chord are concerned with members of set class (0258), the half-diminished and dominant seventh chords. In a 1998 article, Adrian Childs develops a model for these chord types that is closely related to standard neo-Riemannian transformations on triads.[49] Edward Gollin's article in the same issue of the *Journal of Music Theory* explores three-dimensional Tonnetze in general, with special focus on the dominant and half-diminished seventh chords.[50]

In general, neo-Riemannian-type operations on the (0258) tetrachords turn out to be somewhat less useful than their triadic counterpoints, owing to the symmetry of set class (0258).[51] Any one tetrachordal Tonnetz can only show a subset of all of the (0258) tetrachords, while the

47. One of the main subjects of Ring's book is a GIS that describes "heard scale degrees," and does not include harmony at all; see *Tonality and Transformation*, 41–99, and throughout. Lewin provides several examples of non-harmonic GISes in *GMIT*, 16–24.

48. As evidenced in part by the number of books published in the last several years; in addition to Rings, there is Richard Cohn, *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature* (New York: Oxford University Press, 2012); and Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011).

49. Adrian P. Childs, "Moving Beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords," *Journal of Music Theory* 42, no. 2 (October 1998): 181–93.

50. Edward Gollin, "Some Aspects of Three-Dimensional 'Tonnetze,'" *Journal of Music Theory* 42, no. 2 (1998): 195–206. Because the terms are in common use in the transformational literature, "Tonnetz" and the plural "Tonnetze" are rendered without italics throughout this study.

51. Gollin refers to the differences between the two-dimensional triadic Tonnetz and his three-dimensional tetrachordal version as "degeneracies" (Ibid., 200). Child's cubic representation only shows 8 of the possible 24 (0258) tetrachords: those related by parsimonious voice leading to a single diminished seventh chord ("Beyond Neo-Riemannian Triads," 188).

familiar triadic Tonnetz of course shows all 24 major and minor triads. Recognizing this limitation, Jack Douthett and Peter Steinbach present a model that also includes minor sevenths and fully diminished seventh chords, using a digram they refer to as the "Power Towers."[52] While Douthett and Steinbach's description accounts for two of the three main types of seventh chords commonly used in jazz (it is missing the crucial dominant seventh), all of these neo-Riemannian models focus on parsimonious voice leading. While this focus is valuable, it will not prove to be terribly useful for the functional harmony in which this study is interested.

The other group of transformational models consists of what Julian Hook has termed "cross-type transformations": he extends Lewin's definition of a transformation network to allow for transformations between objects of different types.[53] This category of transformations contains the inclusion transformations (discussed by both Hook and Guy Capuzzo) which map a triad into the unique dominant or half-diminished seventh chord that contains it and vice versa.[54] Also included in this category are more general approaches for relating set classes of different cardinalities, including Joseph Straus's formulation of atonal voice leading and Clifton Callender's split and fuse operations.[55] Finally, Dmitri Tymoczko's continuous tetrachordal space can accommodate *all* four-note chords, but, as Hook notes, Tymoczko downplays (and sometimes ignores) the transformational aspects of his geometric models.[56]

Though I have mentioned these cross-type transformational works only in passing here, we will return to some of them in some detail below, where they will be more relevant. Because Hook

52. Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (1998): 255–56.

53. "Cross-Type Transformations and the Path Consistency Condition," *Music Theory Spectrum* 29, no. 1 (April 2007): 1–40. Lewin's definition of a transformational network is in *GMIT*, 193–97.

54. Julian Hook, "Uniform Triadic Transformations," *Journal of Music Theory* 46, nos. 1/2 (Spring–Autumn 2002): 57–126; Guy Capuzzo, "Neo-Riemannian Theory and the Analysis of Pop-Rock Music," *Music Theory Spectrum* 26, no. 2 (October 2004): 177–99.

55. Joseph Straus, "Uniformity, Balance, and Smoothness in Atonal Voice Leading," *Music Theory Spectrum* 25, no. 2 (October 2003): 305–52; Clifton Callender, "Voice-Leading Parsimony in the Music of Alexander Scriabin," *Journal of Music Theory* 42, no. 2 (1998): 219–33.

56. An explanation of this four-dimensional space is found in Tymoczko, *A Geometry of Music*, 93–112; much of this work is based on Dmitri Tymoczko, "The Geometry of Musical Chords," *Science* 313, no. 5783 (July 2006): 72–74; and Clifton Callender, Ian Quinn, and Dmitri Tymoczko, "Generalized Voice-Leading Spaces," *Science* 320, no. 5874 (April 2008): 346–48. Hook points out Tymoczko's relationship to transformational theory in his review of Tymoczko's book, ¶13–14.

does not strictly define what constitutes a "type" in a cross-type transformation, his formulation will allow us to apply transformational procedures rigorously in situations where we might wish to consider objects to be members of different types, even though they may be identical in some other typological system.[57]

1.4 Aside: Lead Sheet Notation

As mentioned in Section 1.1, jazz musicians often begin learning a particular tune with a lead sheet, and the changes found there form the harmonic foundation of a particular performance. Because this dissertation is interested in jazz harmony generally, lead sheets serve as a useful abstraction of the countless possible instantiations of any one tune, and considering them them briefly here will prove fruitful for the rest of this study.[58]

A lead sheet typically gives only a melody and a set of chord changes: Figure 1.1 gives the *Real Book* lead sheet for John Klenner and Sam Lewis's "Just Friends."[59] It is a very typical example, and nearly everything about the page is designed to make it easy for a jazz musician to "fake" a performance of the tune on the bandstand: the anonymous compilers of the book provide the composer and lyricist's names and a sample recording; the music is split into four-measure chunks to make the phrases and the form of the tune clear; and there is almost no extraneous information—even the key signature is omitted on all lines but the first. The melody is given for the head, and the changes are provided for the rhythm section (usually piano, bass, and drums, but sometimes other instruments like guitar) and for solos. All other aspects of the tune need to be negotiated prior to performance (how the tune will begin and end, for example).

57. In particular, it will be useful to consider the IIm7♭5 chord (a half-diminished seventh) as a different type than the V7 chord (a dominant seventh) given their different functional roles in jazz harmony, despite the fact that they are of the same set class.

58. It is worth mentioning that not all jazz musicians read music; jazz is largely an aural tradition, and many early jazz musicians did not read, instead learning the music by ear. For students learning jazz today, however, learning to read chord changes is an essential part of their training. For an ethnomusicological survey of jazz musicians' relationships with lead sheets, see Paul Berliner, *Thinking in Jazz: The Infinite Art of Improvisation* (Chicago: University of Chicago Press, 1994), 71–76.

59. This lead sheet is taken from the older, illegal *Real Book* (249). While the newer Hal Leonard edition maintains most of the original selections, in some cases they do not: "Just Friends" does not appear until vol. 4 of the Hal Leonard *Real Book*.

Figure 1.1. A sample lead sheet of "Just Friends" (John Klenner/Sam Lewis).

The Real Book uses standard conventions for labeling chords. A chord symbol consists of the chord root (referred to by a letter name), and a symbol indicating the quality: the most common of these are are the dominant seventh (simply "7"), minor seventh ("–7") and major seventh ("maj7").[60] Thus, the opening of "Just Friends" (G7–Cmaj7) consists of a G dominant seventh chord moving to a C major seventh chord. This abstraction is extremely useful for a performing musician, but leaves something to be desired if pressed into use as an analytical system. The chord symbols do not explicitly tell us, for example, that G7–Cmaj7 is a typical V^7–I^7 progression in C major; that kind of knowledge is implicit for experienced musicians and analysts.

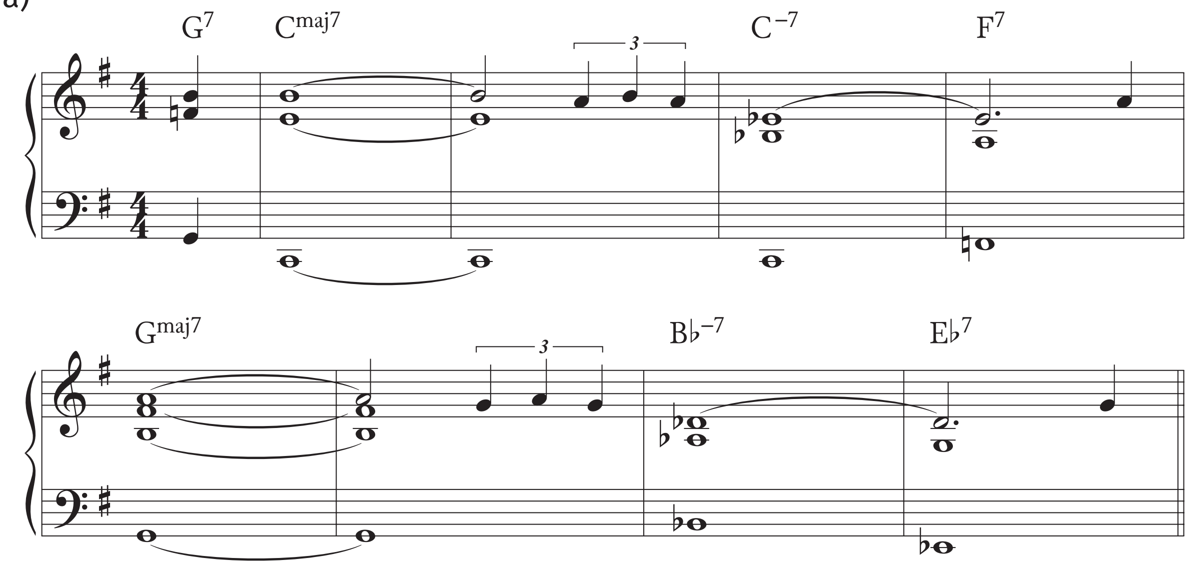
Complicating the problem somewhat is that chord symbols are imprecise by design. In most situations, jazz musicians do not want to be told exactly what notes they should play (if they did, they probably wouldn't have become jazz musicians); instead, they treat chord symbols only as guidelines. A G7 chord would certainly include the root, third, and seventh (G, B, and F), but might also include the ♯11 (C♯), ♭9 (A♭), or ♯5 (D♯), depending on the situation: the melody might suggest certain alterations, a performer might prefer some alterations over others, or an improvisor may work themselves into a dissonant portion of a solo where a bare dominant seventh in the piano would sound especially out of place.

To illustrate the flexibility of chord symbols, Figure 1.2 gives two realizations of the first eight measures of "Just Friends." The first is Mark Levine's, from early in his book on jazz piano; it uses what he calls "three-note voicings" (the root, third, and seventh).[61] The second realization is my own, and features many alterations to the basic outlines given by the chord symbols. Both of these realizations are valid interpretations of the given chord symbols, and are meant to reinforce the point that chord symbols, while only a guideline, indeed represent something important about a given harmonic progression. For all of their imprecision, chord symbols represent a reality for performing jazz musicians, and as such will be foundational for our work on harmony here.

60. Jamey Aebersold publishes a free book on his website that serves as an introduction to the very basics of jazz; the section titled "Nomenclature" is especially useful for deciphering chord symbols. *Jazz Handbook* (New Albany, IN: Jamey Aebersold Jazz, 2010), 15, http://www.jazzbooks.com/mm5/download/FQBK-handbook.pdf.

61. Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 21. Realizing chord symbols is perhaps most important for pianists and guitarists (who are most often charged with realizing them in performance), and any introductory text on these instruments will be filled with voicings to be used in different situations.

a)



b)

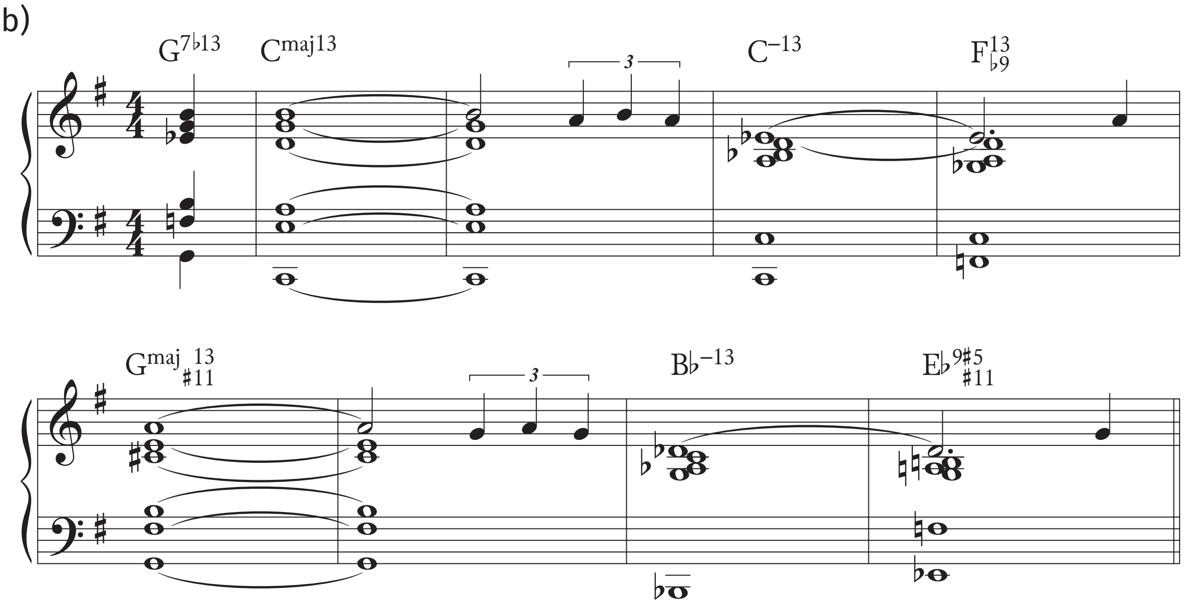


Figure 1.2. Two piano realizations of “Just Friends,” mm. 1–8.
a) Using three-note voicings (from Mark Levine, *The Jazz Piano Book*).
b) Using more alterations/extensions (chord symbols reflect voicings used).

1.5 Diatonic Chord Spaces

It will be easiest to introduce the transformational approach to jazz harmony developed in this study by way of an example. Much of this dissertation will be interested in the development of various musical spaces and motions that are possible within them. This kind of approach was first developed by David Lewin in *Generalized Musical Intervals and Transformations*, and a review of his approach will be useful before moving on to more involved examples.

1.5.1 Intervals and Transformations

Figure 1.3 shows the chord changes to the A section on the jazz standard "Autumn Leaves."[62] Jazz musicians sometimes refer to this progression as a "diatonic cycle": it uses only seventh-chords found in the G-minor diatonic collection.[63] As in classical music, the leading tone is raised in the dominant chord so that the resulting chord is D7, not Dm7. We can easily arrange this progression around the familiar circle of fifths, placing the tonic G minor at the top of the circle (see Figure 1.4).

C-^{7} | F^{7} | B♭^{maj7} | E♭^{maj7} |

A-$^{7\flat5}$ | D^{7} | G- | ||

Figure 1.3. The changes to "Autumn Leaves" (Joseph Kosma), A section.

While this arrangement around the diatonic circle of fifths makes intuitive sense, it can also represent what Lewin has called a Generalized Interval System (GIS). Generalized Interval Systems are Lewin's way of formalizing the "directed measurements, distances, or motions" that we often understand as "characteristic textural features" of a given musical space.[64] Though we will unpack

62. *The Real Book* gives these changes in E minor, but most recorded performances are in G minor. I have transposed the given changes to reflect the most common performance key. Steven Strunk analyzes both "Autumn Leaves" and "How My Heart Sings" (analyzed below) as examples of 10–7 linear intervallic patterns in "Linear Intervallic Patterns in Jazz Repertory," 96–97.

63. Emile De Cosmo's etude book titled simply *The Diatonic Cycle* (North Bergen, N.J.: EDC Publications, 1970) gives many different solo patterns possible over the diatonic cycle in all twelve major and minor keys.

64. Lewin, *GMIT*, 16.

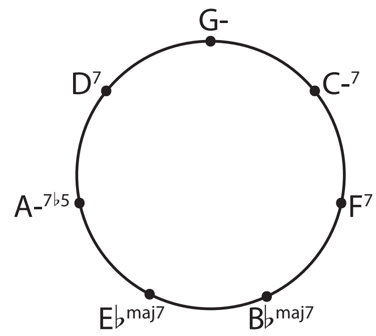


Figure 1.4. The changes to "Autumn Leaves," arranged around the diatonic circle of fifths.

this definition using the harmonies from the A section of "Autumn Leaves" as an example, Lewin's formal definition is as follows:

> A *Generalized Interval System* (GIS) is an ordered triple (S, IVLS, int), where S, the *space* of the GIS, is a family of elements, IVLS, the *group of intervals* for the GIS, is a mathematical group, and int is a function mapping S $\times$ S into IVLS, all subject to the two conditions (A) and (B) following.
> (A): For all r, s, and t in S, int(r,s)int(s,t) = int(r,t)
> (B): For every s in S and every i in IVLS, there is a unique t in S which lies the interval i from s, that is a unique t which satisfies the equation int(s,t) = i.[65]

The first element in a GIS is a family of elements, S, which Lewin also calls a musical space. Preceding this formal definition, he gives a number of examples of musical spaces, including the familiar diatonic and pitch and pitch-class spaces, along with less familiar musical spaces like frequency space, time point space, and various durational spaces.[66] In our "Autumn Leaves" example, we are interested in the (unordered) set of harmonies in the G-minor diatonic collection: $S =$ {Gm, Am7♭5, B♭M7, Cm7, D7, E♭M7, F7}.[67]

With the first element of a GIS satisfied, we must then define a group of intervals (IVLS). Though we could perhaps imagine a number of different ways to define intervals among elements of the set S (a point to which we will return later), the most obvious is to measure distances in

65. Lewin, *GMIT*, Definition 2.3.1 (26). *GMIT* contains many terms that Lewin renders in all capitals; I have rendered them here in small capitals (except when quoting directly) in order to reduce their typographical impact.

66. Ibid., 16–25.

67. The tonic harmony is given as it is in *The Real Book*, simply as Gm. In performance, a musician might choose to play this chord with a major seventh (GmM7), a sixth (Gm6), or even a minor seventh (Gm7; this is less likely since minor seventh chords are most commonly ii chords, not tonics).

diatonic steps between chord roots. Because we are interested in abstract chord roots and not the actual pitches played by some bass player or pianist's left hand, we will use diatonic pitch classes. This has the effect of modularizing the set of possible intervals, changing IVLS from $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ (as it would be in diatonic *pitch* space) to $\{0, 1, \ldots, 6\}$ (diatonic pitch *class* space).[68] Arithmetic in this group is mod-7, exactly in the way that arithmetic using the more familiar chromatic pitch class space, $\{0, 1, 2, \ldots, 11\}$, is mod-12.

Lewin specifies that IVLS must be a mathematical group, and we will take care here to show that IVLS = $\{0, 1, \ldots, 6\}$ is indeed such a group. A group is a set of elements, G, and a binary operation, $\otimes$, that satisfies the four group axioms:

- Closure: for $a, b \in G$, then $a \otimes b$ must be an element of G.
- Associativity: for $a, b, c \in G$, then the equation $(a \otimes b) \otimes c = a \otimes (b \otimes c)$ must be true.
- Identity element: There exists an element $e \in G$ such that for any element $a \in G$, $a \otimes e = e \otimes a = a$ is true.
- Inverses: For any element $a \in G$, there exists a unique element $a^{-1} \in G$ such that $a \otimes a^{-1} = a^{-1} \otimes a = e$ is true.[69]

To show that our IVLS is a group, it is sufficient to show that the set $\{0, 1, \ldots, 6\}$ under some binary operation satisfies the group axioms. The binary operation is simply addition mod-7 (which we will notate using the usual $+$ sign instead of the abstract $\otimes$ used above). We can then show that the set IVLS is closed: for any two elements $a, b \in \text{IVLS}$, $a + b$ is also an element of IVLS ($1 + 3 = 4; 4 + 4 = 1; 1 + 0 = 1$; and so on). Modular addition, like its non-modular counterpart, is associative: $(3 + 4) + 5 = 3 + (4 + 5)$, and likewise for any chosen elements of IVLS. The identity element for addition is 0, which combined with any element $a \in \text{IVLS}$ gives a itself. Inverses in the group are simply complements mod-7 (the number that when added to the given

68. In chromatic pitch space, we might say that the interval between C4 and A3 is -3, while the interval between C4 and A5 would be $+21$. In chromatic pitch class space, however, the interval is calculated mod-12 (because octaves are equivalent), and both of these intervals are equal to 9.

69. Lewin's definition of a mathematical group is spread over several pages in the first chapter of *GMIT* (4–6). For other definitions in works of music theory, see Rings, *Tonality and Transformation*, 12–13, and Julian Hook, *Exploring Musical Spaces* (New York: Oxford University Press, forthcoming), Ch. 5. Any introductory mathematical text on group theory will contain the group axioms; see, for example, Israel Grossman and Wilhelm Magnus, *Groups and Their Graphs* (Washington, D.C.: Mathematical Association of America, 1964), 10–14 and Nathan Carter, *Visual Group Theory* (Washington, D.C.: Mathematical Association of America, 2009), 51.

number gives 0, mod-7): $2^{-1} = 5; 1^{-1} = 6$; and so on. The integers modulo *n* are labeled $\mathbb{Z}_n$, so we may also refer to IVLS in our "Autumn Leaves" example as the group $\mathbb{Z}_7$.

The last element of a GIS is an interval function that maps $S \times S$ into IVLS. In other words, the interval from one element of S to another must be a member of the group $\mathbb{Z}_7$. In our "Autumn Leaves" example, the interval from one element of S to another is simply the number of steps in the G-minor diatonic collection (always counting upward) between the two elements. Thus, int(D7, Gm) = 3, since G, the root of the second chord, lies 3 diatonic steps above D, the root of the first. Likewise, int(Am7♭5, B♭M7) = 1; int(F7, E♭M7) = 6; and so on.

We now have all of the elements of a GIS, but we must still prove that Lewin's conditions A and B are satisfied, which we will do by example. Condition A states that for all r, s, and t in S, int(r,s)int(s,t) = int(r,t). In our "Autumn Leaves" example, int(Cm7, D7) and int(D7, E♭M7) (both interval 1) must combine to equal int(Cm7, E♭M7) (interval 2). Second, for every chord *s* in S and every interval *i* in $\mathbb{Z}_7$, there must be a unique chord *t* which satisfies the equation $\text{int}(s, t) = i$. For example, there must be exactly one chord that lies 2 units above Cm7: namely, E♭M7. It is easy to confirm that the two conditions are also true for any choice of $r, s, t \in$ S.

Before shifting our focus from generalized intervals to transformations, I want to return to the structure of IVLS, the group $\mathbb{Z}_7$. Above, we measured intervals by the number of G-minor diatonic steps between chord roots: a single step corresponded to the interval 1. We can also say that the group $\mathbb{Z}_7$ is a cyclic group, *generated* by the interval 1. (Cyclic groups are notated C_n, where *n* is the size of the group; we could therefore label the group in question C_7.) Counting diatonic steps is not the only way we might consider measuring intervals in the space, however. Given the ubiquitous descending fifths of "Autumn Leaves," we might instead like to measure distance by the number of descending fifths between chord roots. Though this generation has no noticeable effect on the abstract group structure of IVLS—the group is C_7 in either case—it does affect the last element of a GIS, the interval function.[70] As shown in Figure 1.5, int(Am7♭5, D7) = 3 when

70. The group C_7 can be generated by any of its members, since 7 is prime. In general, a cyclic group can be generated by a member only if the two are relatively prime. The group C_{12}, for example, can be generated only by 1, 5, 7, or 11; put another way, only the chromatic scale and circle of fifths (or fourths) cycle through all 12 pitches in the chromatic octave before returning to the starting point.

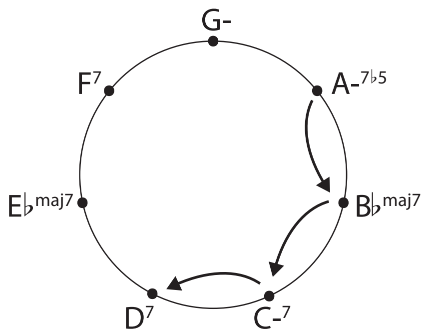


Figure 1.5. The "Autumn Leaves" GIS, generated by diatonic step (left) and descending fifth (right).

measured by diatonic steps (the left figure), but int(Am7♭5, D7) = 1 when measured by descending fifths (the right figure).[71]

Generalized Interval Systems are but one part of Lewin's project; we will now turn our attention to the "transformations" of the book's title. Much has been made of the "transformational attitude" that accompanies the shift from generalized intervals to transformations that occurs in the later chapters of *GMIT*. As Lewin has it, GIS thinking represents a Cartesian, observer-oriented position, examining musical objects as points in abstract space. This is in contrast to the transformational attitude, which Lewin describes as "much less Cartesian" in what is perhaps the most-cited portion of the book:

> Given locations s and t in our space, this attitude does not ask for some observed measure of extension between reified "points"; rather it asks: "If I am *at* s and wish to get to t, what characteristic gesture (e.g. member of STRANS) should I perform in order to arrive there?[72]

The GIS perspective outlined above is an intervallic perspective: we developed a system that allowed us to say that "the distance from Am7♭5 to D7 is 3." By replacing, as Lewin does, "the

71. To avoid confusion, we will use only the GIS that measures distance in diatonic steps from this point on.

72. Lewin, *GMIT*, 159. Henry Klumpenhouwer argues that the dichotomy between Cartesian GIS thinking and what he calls "anti-Cartesian" transformational thinking is the central theme of *GMIT*; see "In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's *Generalized Musical Intervals and Transformations*," *Music Theory Spectrum* 28, no. 2 (Fall 2006): 277–89. For more on the "transformational attitude," see Rings, *Tonality and Transformation*, 24–29; Ramon Satyendra, "An Informal Introduction to Some Formal Concepts from Lewin's Transformational Theory," *Journal of Music Theory* 48, no. 1 (Spring 2004): 102–3 and 116–17; and Julian Hook, "David Lewin and the Complexity of the Beautiful," *Intégral* 21 (2007): 172–77.

concept of interval-in-a-GIS" with "the concept of transposition-operation-on-a-space,"[73] we can convert this GIS statement into a transformational one: "transposing the root of Am7♭5 by three diatonic steps gives D7." The transformational statement is more active, replacing distance metrics with verbs like "transpose."[74]

Though there is certainly a difference in the language used in GIS statements and that used in transformational statements, Lewin takes care to note that the two attitudes are not diametrically opposed:

> we do not have to choose *either* interval-language *or* transposition-language; the generalizing power of transformational theory enables us to consider them as two aspects of one phenomenon, manifest in two different aspects of this musical composition.[75]

While we might prefer interval-language in some contexts and transformation-language in others, the two attitudes are quite closely related; any GIS statement can be converted into a transformational one by using the mechanism Lewin describes just before the famous passage in *GMIT*.

By combining the space S of a GIS with an operation-group on S, we can derive a transformational system. This operation-group must be simply transitive on S (hence the reference to STRANS in the quote above): "given any elements s and t of S, then there exists a unique member OP of STRANS such that OP(s) = t."[76] Lewin then states that in any GIS, "there is a unique transposition-operation T satisfying T(s) = T, namely $T = T_{\text{int}(s,t)}$."[77] In familiar chromatic pitch-class space, the unique transposition T_k is that operation where k is the interval in semitones between the two pitches: the interval between C and E♭ is 3, and the operator T_3 maps C onto E♭.

Converting our "Autumn Leaves" GIS into a transformational system, then, is only slightly more complicated than this chromatic pitch-class space example, since ordinary transposition will not work intuitively. To avoid confusion with the traditional transposition operator, we will instead

73. Lewin, *GMIT*, 157.

74. Ramon Satyendra describes the two attitudes as being noun-oriented (GISes) vs. verb-oriented (transformations); "An Informal Introduction," 102–3.

75. Lewin, *GMIT*, 160, emphasis original. Julian Hook clarifies this point in "David Lewin and the Complexity of the Beautiful," 172–77.

76. Lewin, *GMIT*, 157.

77. Ibid.

use the lowercase, t_k.[78] This diatonic transposition operator will be used in almost the same way, however: the operation t_k is that transposition which transposes the root of a chord k steps inside the G-minor diatonic collection. With this understanding, the conversion works as expected: the interval between Am7♭5 and D7 is 3, so the operator t_3 maps Am7♭5 onto D7 in the space $S = \{$Gm, Am7♭5, B♭M7, Cm7, D7, E♭M7, F7$\}$.[79] The same statement may also be written as Am7♭5 $\xrightarrow{t_3}$ D7, if we want to emphasize the idea of a transformation as a mapping operation.

Now that we have explored the underlying mathematics, we are in a position to reexamine the A section of "Autumn Leaves," (as given in Figure 1.3), as well as the circle-of-fifths arrangement in Figure 1.4. The progression is, quite simply, a series of t_3 operations within the G-minor diatonic space; each chord root descends by diatonic fifth. In a mod-12 chromatic space (like the ones we will begin developing in the next chapter) it can be difficult to make sense of this harmonic progression. The chord qualities can be confusing—two major sevenths in a row followed immediately by a half-diminished seventh—and it is sometimes hard to remember where the tritone in the bass falls in the progression, given the prevalence of perfect-fifth bass motion in chromatic space. Understood in the context of this diatonic space, however, the harmonic motion becomes much clearer.

It is reasonable to pause at this point and ask what advantages this transformational approach brings. After all, we began our discussion of "Autumn Leaves" by noting that jazz musicians sometimes refer to the progression of the A section as a "diatonic cycle," and it may seem as though we have gone through a great deal of mathematical rigmarole simply to arrive back at our starting point. There would seem to be very little difference between describing a progression as a "diatonic cycle" and describing it as "a series of t_3 transformations in the G-minor diatonic set of seventh chords." And yet, this is almost exactly the point. Transformational theory allows us a

78. The diatonic transposition operator t_k is described in Julian Hook, "Signature Transformations," in *Music Theory and Mathematics: Chords, Collections, and Transformations*, ed. Jack Douthett, Martha M. Hyde, and Charles J. Smith (Rochester: University of Rochester Press, 2008), 139–40.

79. The intervals chosen for measurement in the GIS do have an impact on the associated transformational system: if we had used the descending fifths generation as described above, the the operator t_1 (not t_3) would map Am7♭5 onto D7, since the interval in the underlying GIS would be different.

means to formalize what is often intuitive knowledge for jazz musicians, thereby narrowing the gap between the way jazz musicians discuss harmony and the way music theorists often do.

1.5.2 Analytical Applications

Before concluding this chapter, I want to examine "Autumn Leaves" in a bit more detail, then move on to a few other analytical examples. The full chord changes to "Autumn Leaves" are given in Figure 1.6. The opening A section, discussed at length above, can be understood as a series of t_3 operations in the G-minor diatonic set. The bridge modulates to the relative major, but we can still understand this passage using the same transformational system. After repeating the G-minor ii–V–I progression that concludes the A section in the first four measures of the bridge, the entire progression is transposed up a third (a larger-scale t_2) to repeat the ii–V–I in the key of B♭. Despite this modulation, the connections from chord to chord are all t_3 operations, continuing the chain that has been present since the beginning (see Figure 1.7).[80]

A	C-^{7}	F^{7}	B♭^{maj7}	E♭^{maj7}	
	A-$^{7\flat5}$	D^{7}	G-		:‖
B	A-$^{7\flat5}$	D^{7}	G-		
	C-^{7}	F^{7}	B♭^{maj7}		‖
C	A-$^{7\flat5}$	D^{7}	G-^{7} C^{7}	F-^{7} B♭^{7}	
	A-$^{7\flat5}$	D^{7}	G-		‖

Figure 1.6. The complete changes to "Autumn Leaves."

There are three passages in this piece that are not simply t_3 operations: the connection from Gm to Am7♭5 that begins the bridge; the connection from B♭M7 at the end of the bridge to Am7♭5 that follows; and the third and fourth bars of the final section, Gm7–C7–Fm7–B♭7. The

80. Lewin's definition of a transformation network appears is at *GMIT*, 196. We will delay an in-depth explanation until the next chapter, at which point it will be more relevant.

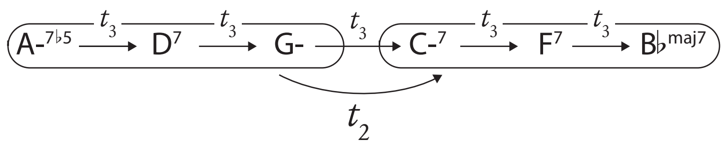


Figure 1.7. A transformation network for "Autumn Leaves," bridge.

first of these, Gm–Am7♭5, is a t_1 transformation. The bridge begins by retracing the same harmonic ground as the last four bars of the A section (a ii–V–I progression in G minor); the connection to the bridge, then, can be understood as reversing the two descending fifths that ended the A section. This observation can be represented algebraically ($t_3^{-1} \odot t_3^{-1} = t_4 \odot t_4 = t_1$) or graphically (by taking two steps counterclockwise in the circle of fifths in Figure 1.4).[81]

The next transformation that breaks the series of t_3 operations is the t_6 from B♭M7 to Am7♭5 at the end of the bridge. This t_6 is easily understood as a combination of two t_3 operations, by imagining that there is a "missing" E♭M7 chord in the last bar of the bridge.[82] Interpolating this chord allows us to hear the progression (beginning on Cm7) as identical to that of the first eight bars, now displaced to span a formal boundary, as shown in Figure 1.8.[83]

The only four chords in "Autumn Leaves" that cannot be understood in the GIS developed above are those in the progression Gm7–C7–Fm7–B♭7. While the chord roots belong to the G-minor collection, the qualities are incorrect. This progression consists of two ii^7–V^7 progressions, the first in F and the second in E♭ (a tonic that, like the one at the end of the bridge, does not actually appear in the music). These ii–V progressions are best situated in chromatic, rather than diatonic, space; for now, we will pass over this progression until we have developed such a space.

81. Recall the discussion in the preface (page ix): the symbol ⊙ here represents function composition from left-to-right.

82. This observation has not gone unrecognized by jazz musicians: at least one recording (Vince Guaraldi, on the album *A Flower is a Lovesome Thing*) includes the VImaj7 chord in this measure, though most do not.

83. Hearing this parallelism also requires hearing the Gm7 in the third bar of the final A section as equivalent to the cadential Gm in the first. The chord symbol Gm is ambiguous by nature; if a performer wanted to bring out this parallelism, she could perhaps play the tonic chords in the first A section as minor seventh chords.

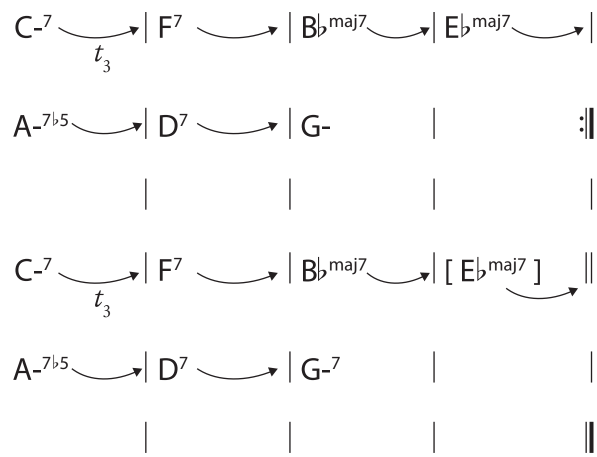


Figure 1.8. The diatonic cycle of "Autumn Leaves," with a hypermetrically displaced copy spanning the formal boundary at the end of the bridge.

It is relatively rare for tunes to be as systematically diatonic as "Autumn Leaves" (though Bart Howard's "Fly Me to the Moon" comes close); instead, pieces often make use of diatonic cycles in only a portion of a piece before moving on to contrasting music. A relatively straightforward example of this is Sammy Fain and Bob Hilliard's "Alice in Wonderland," the changes to which are shown in Figure 1.9.[84] Like "Autumn Leaves," "Alice in Wonderland" begins with a minor diatonic cycle in mm. 1–7 (also beginning on a iv chord), shown here in A minor. We can modify our GIS from above simply by changing the set (*S* is now the set {Am, Bm7♭5, CM7, Dm7, E7, FM7, G7}); the group IVLS and the interval function are identical.[85] The harmony in m. 7 is an Am7 chord, but because the quality of the tonic seventh chord is ambiguous in minor keys, this chord is easily understood as tonic.

The second eight measures are all diatonic in the key of C major; the linking E♭7 chord signals a shift between diatonic collections.[86] This move to the relative major is a common one, and the only difference between the two diatonic sets is the shift of E7 (in A minor) to Em7 (in C

84. The most well-known recording of "Alice in Wonderland" is probably on Bill Evans's *Portrait in Jazz*. These changes are as given in *The Real Book*, and as played by Evans. This figure gives the changes to the second ending of the first sixteen measures (the first ending contains a ii–V in D minor to return to the opening).

85. The interval functions are identical, with the caveat that diatonic steps are to be counted in the A-minor collection, not the G-minor.

86. The E♭7 chord is a tritone substitute for A7, which is of course the dominant of the following Dm chord. This harmonic motion is not easily understood in either the A-minor or C-major diatonic spaces; we will return to tritone substitutes in the next chapter.

D-^{7}	G^7	C^{maj7}	F^{maj7}	
B-$^{7\flat5}$	E^7	A-	E$\flat^7$	
D-^{7}	G^7	E-^{7}	A-^{7}	
D-^{7}	G^7	C^{maj7}		‖

Figure 1.9. Changes to "Alice in Wonderland" (Sammy Fain/Bob Hilliard), mm. 1–16.

major). The progression here is not as systematic as the first eight (see the annotations in Figure 1.10). After a C-major ii–V in mm. 9–10, the harmony moves up a step to Em7–Am7.[87] The progression G7–Em7 seems to move backwards in the cycle, almost as if realizing the phrase is in danger of arriving at C major too soon. We can capture this intuition by choosing to understand the progression not as a forward-directed t_5, but as an algebraically-equivalent combination of three ascending fifths: $t_3{}^{-1} \odot t_3{}^{-1} \odot t_3{}^{-1}$ (which we could also write as $t_3{}^{-1}$ raised to the third power: $(t_3{}^{-1})^3$).

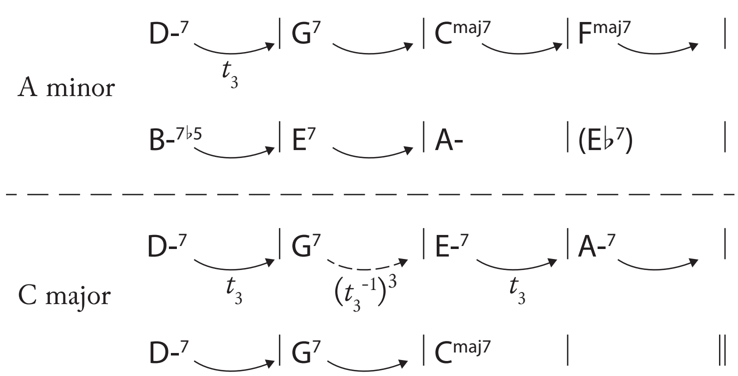


Figure 1.10. The changes to "Alice in Wonderland," with transformational labels between harmonies.

Throughout *GMIT*, Lewin is clear that transformational theory is a means of expressing our "intuitions" about a musical passage in a mathematically rigorous way.[88] As he puts it: "If I want to change Gestalt 1 into Gestalt 2 . . . , what sorts of admissible transformations in my space

87. It is this Am7 that confirms we are in a C-major diatonic space; it would be more typical to transpose the ii–V progression exactly to Em7–A7.

88. Steven Rings refers to Lewin's "intuitions" as "apperceptions"; see *Tonality and Transformation*, 17–21.

(members of STRANS or otherwise) will do the best job?"[89] Our explication of diatonic seventh-chord spaces may appear to stem from the desire to label everything a t_3 and move on, confident that we could justify our labels mathematically if called upon to do so. This, though, could not be further from the truth; developing the space allows us a powerful means to capture intuitions (or apperceptions) like the one above. Though the operation t_5 does map G7 to Em7 in the C-major diatonic space, hearing this connection as $t_3^{-1} \odot t_3^{-1} \odot t_3^{-1}$ instead represents the idea of stepping backwards through the circle of descending fifths (an observation that may not be easy to show using other methods of analysis).

Both examples of outright diatonic cycles we have seen thus far have been in minor keys. In general, minor-key cycles are easier to use than those in major keys (a C-major diatonic cycle is given for reference in Figure 1.11). In minor, the raised leading tone in the V chord means that the cycle consists of a ii–V–I in the relative major and a ii–V–I in the tonic joined by the VImaj7 chord (as we saw in the second cycle in Autumn Leaves, Figure 1.8). In a major-key cycle, the lone half-diminished seventh chord is followed immediately by three minor seventh chords (in C major, Bm7♭5–Em7–Am7–Dm7), which by comparison is a relatively unusable tonal progression.

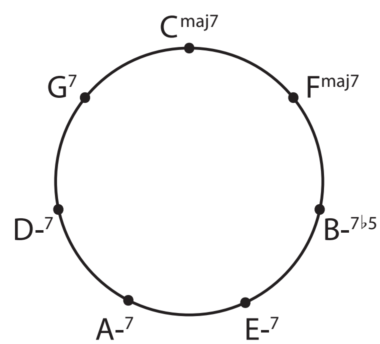


Figure 1.11. A diatonic cycle in C major.

Nevertheless, Earl Zindars's standard "How My Heart Sings" (the changes to which are given in Figure 1.12) does indeed contain a major-key cycle.[90] Unlike the previous two examples, this

89. Lewin, *GMIT*, 159.

90. The canonical recording of this tune is again by Bill Evans, on the album *How My Heart Sings!* Evans seems to have a propensity towards tunes with diatonic cycles: *Portrait in Jazz* also contains a very well-known recording of "Autumn Leaves."

tune begins on the iii chord, at which point it begins the t_3 cycle. Once again, it is the Am7 chord that alerts us that this progression takes place in diatonic, rather than chromatic, space (A7 would seem to make more harmonic sense, as the dominant of the following D minor). After reaching the tonic in m. 5, the t_3s continue making their way to A minor, four measures later.

E-^{7}	A-^{7}	D-^{7}	G^{7}
C^{maj7}	F^{maj7}	B-$^{7\flat5}$	E$^{7\flat9}$
A-^{7}	A$\flat^{\circ7}$	A-^{7}/G	F♯-$^{7\flat5}$

Figure 1.12. Changes to "How My Heart Sings" (Earl Zindars), mm. 1–12.

Notably, the chord in m. 8 is an E7♭9—this chord belongs not to C major, but rather to A minor. At some point, then, the progression shifts from taking place in a C-major cycle (the E chord is a minor seventh in the opening bar) to an A-minor cycle. Unlike the abrupt shift to the relative major in "Alice in Wonderland" (signalled by E♭7), this modulation is gradual, using a traditional pivot chord.[91] In a transformational reading, the progression maintains the t_3 sequence throughout the first nine measures, but the underlying diatonic set changes almost imperceptibly from C major to A minor. Zindars uses this modulation in order to negotiate the unusual succession of chord qualities in the major diatonic cycle: by beginning the progression on iii in a major key and modulating to the relative minor before returning to it, he is able to have his cake and eat it too—the chord root E appears first as a minor seventh and again later as a dominant of the relative minor.

One more example will suffice to conclude our discussion of diatonic cycles: Jerome Kern's "All the Things You Are" (the changes are given in Figure 1.13).[92] Though this tune does not

91. Exactly which chord functions as the pivot is of little importance, so long as it happens before the E dominant seventh. I am inclined to hear the FM7 as a pivot (functioning simultaneously as IV and VI), in order to keep both ii–V–I progressions (in C major and A minor) intact.

92. These changes are again taken from *The Real Book*. The Am7♭5 chord in the sixth bar of the A′ section is not included in some charts of this tune, so I have put it in parentheses here. (The older, illegal *Real Book* as well as another illegal fake book called simply *The Book* both omit this chord.)

contain a cycle as explicit as the examples we have seen so far, viewing this piece through the lens of diatonic seventh chord space reveals relationships that may otherwise go unnoticed.

A	F-^{7}	B♭-^{7}	E♭^{7}	A♭^{maj7}
	D♭^{maj7}	G^{7}	C^{maj7}	‖
A′	C-^{7}	F-^{7}	B♭^{7}	E♭^{maj7}
	A♭^{maj7}	(A-$^{7\flat5}$) D^{7}	G^{maj7}	E$^{7\sharp9}$ ‖
B	A-^{7}	D^{7}	G^{maj7}	
	F♯-$^{7\flat5}$	B^{7}	E^{maj7}	C$^{7\sharp5}$ ‖
A″	F-^{7}	B♭-^{7}	E♭^{7}	A♭^{maj7}
	D♭^{maj7}	G♭^{7}	C-^{7}	B^{o7}
	B♭-^{7}	E♭^{7}	A♭^{maj7}	‖

Figure 1.13. Changes to "All the Things You Are" (Jerome Kern).

"All the Things You Are" begins in the key of F minor, and progresses through a diatonic cycle (a chain of t_3 operations in an F-minor diatonic seventh-chord GIS) until arriving on D♭M7 in m. 5.[93] At this point, the chord roots continue to descend by diatonic fifth in the key of F minor, but the qualities of the chords rooted on G and C have been altered, from Gm7♭5 and C7 (as they would be in the F-minor GIS) to G7 and CM7. This arrival on a C major chord, rather than a C dominant seventh, has the effect of a half cadence in the prevailing key of F minor. The half-cadential C-major chord also serves as a linking chord to the next phrase, which contains a diatonic cycle in the key of C minor. Like the first A section, this phrase also veers away from the cycle to end in a half cadence on G major.

93. I tend to hear the opening of "All the Things You Are" in F minor, though it is certainly possible to hear it in A♭ major instead. In either case, the diatonic GIS is nearly identical; the only difference is the quality of the C chord (Cm7 in A♭, and C7 in F minor). Readers who prefer to hear the opening in A♭ can easily make the necessary alterations to the commentary here.

The bridge of this tune is usually described as being made up of two ii–V–I progressions, the first in G major and the second in E major. While this is true, we might also understand this progression as an alteration of a diatonic cycle in E minor. The bridge begins on a iv chord, which initiates a ii–V–I in the relative major. After two bars of GM7, a t_6 takes us to F♯m7♭5. Just as in the bridge to "Autumn Leaves," we can understand this t_6 as a combination of two t_3 operations, interpolating a missing CM7 chord.[94] Though the cycle of the bridge (and its associated GIS) is in the key of E minor, the final chord is an E *major* seventh, a kind of Picardy third; in this way the bridge, like the A sections, can end on a major chord.

The final A section of "All the Things You Are" is an expanded version of the first, now ending in the overall tonic of A♭. It begins, after a linking C7♯5 chord, by outlining a series of t_3 operations that lead to D♭M7.[95] At this point we might expect the t_3s to continue, leading to Gm7♭5–C7–Fm (the tonic of the cycle), but instead we see G♭7–Cm7–B°7. This progression is clearly not diatonic, so we cannot say much about it at this point. It is interesting to note, however, that the harmony four bars from the end is a B♭m7, which is exactly where the chain of t_3s would have arrived, had it continued (see Figure 1.14). After this phrase expansion, the piece closes with a ii–V–I in the key of A♭.

Again, we might pause to ask what is gained by hearing "All the Things You Are" in diatonic, rather than chromatic, space. After all, there is never a clear statement of a diatonic cycle in the manner of "Autumn Leaves" or "Alice in Wonderland," and music in chromatic space (as we will begin to see in the next chapter) still tends to descend by fifth. Without the guiding influence of F minor, though, the succession of chord qualities at the beginning is difficult to make sense of: two minor chords, followed by a dominant seventh, then two major chords, all seemingly unrelated to

94. Again, this implicit diatonicism has not gone unnoticed; Ahmad Jamal arrives emphatically on CM7 in the fourth bar of the bridge on *Jamal at the Pershing, Vol. 2*.

95. If the linking dominant did not appear at the end of the bridge, we would see the succession EM7–Fm7. This succession is remarkably similar to Lewin's SLIDE operation: retaining the third of the chord while moving the root and fifth by half-step; see *GMIT*, 178. Here, the seventh is also retained as a common tone; we will return to this operation (which we will call SLIDE^7) in the next chapter.

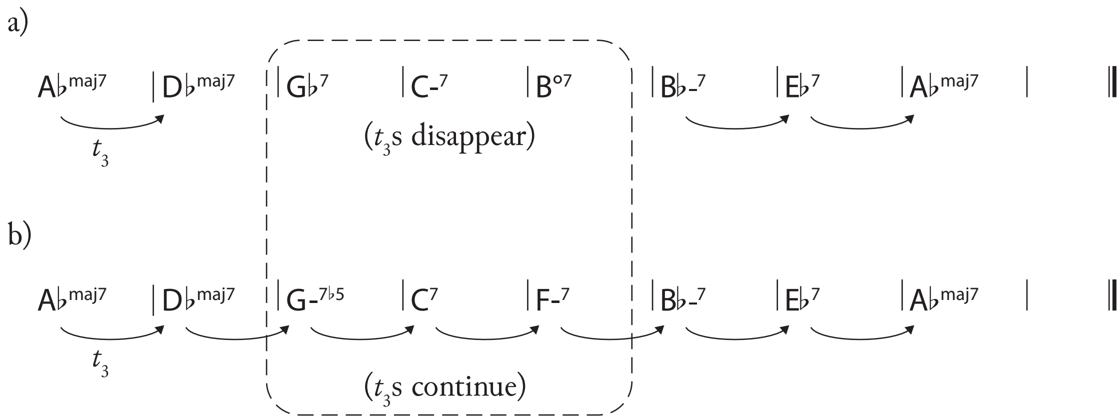


Figure 1.14. The final nine bars of "All the Things You Are."
a) The changes as written.
b) A hypothetical version that continues the t_3 cycle in F minor/A♭ major.

the phrase-concluding tonic C major.[96] Understanding the first 8 bars as diatonic, ending with a tonicized half cadence, makes sense of the chord qualities, and eliminates the difficult-to-explain third relation A♭M–CM that results from a desire to hear both V–I progressions in the A section as tonic-defining.[97]

Hearing "All the Things You Are" diatonically allows us to listen to sections of music at once: the first eight bars are in F minor, the next eight in C minor, the bridge in E minor, and the last twelve return to F minor before shifting to the relative major, A♭, for the final cadence. When we hear the tune as a chain of t_3 operations in shifting diatonic spaces, our attention is drawn to the connections between the spaces—key areas—themselves, rather than the (comparatively boring) series of descending diatonic fifths that occur within them. While transformational analyses are often accused of privileging chord-to-chord connections to the detriment of long-range hearing, in this case the GIS framework developed above allows us to hear over longer distances where

96. Henry Martin analyzes the A section simply as a series of descending fifths (taking note of the aberrant root tritone D♭–G) ending on C major. His analysis assumes a chromatic space, in which the harmonic progressions are more difficult to make sense of: he notes that "a certain tonal ambiguity pervades this piece" ("Jazz Harmony," 15–19.).

97. Hearing the third-relations does have the nice side effect, absent from our analysis here, that all of the phrase-ending major-sevenths in the first sixteen bars (A♭, C, E♭, G) spell out the tonic major-seventh chord; ibid., 19 makes this observation.

chord-to-chord connections may fail to do so (while, crucially, still recognizing the importance of chord-to-chord connections for a jazz musician aiming to "make the changes").

Though they do appear occasionally, most jazz tunes do not contain diatonic cycles, and thus we will need to expand the transformational approach introduced here to account for a larger portion of jazz practice. Chapter 2 will outline an approach to chromatic space that will help to understand the ii–V–I progressions that we passed over in our discussion of diatonic space. Transformational approaches, and neo-Riemannian theories in particular, have flourished partly because of their ability to explain non-functional progressions that contain primarily root motion by thirds. Chapter 3 will draw upon this literature to approach jazz, especially common after bebop, that is more dependent on thirds than fifths for structure. Though harmony is crucially important to performing jazz musicians, much of the jazz pedagogical literature equates chords with scales: a Dm7 chord is functionally equivalent to a D dorian scale, for example. Chapter 4 will develop a transformational approach for these "chord-scales," treating scales as first-class harmonic objects. Finally, Chapter 5 will bring the theoretical work of the early chapters to a close, by taking a close analytic look at tunes based on George Gershwin's "I Got Rhythm."