Chapter 3

Thirds Spaces

The space developed in the last chapter was organized primarily in descending fifths, and works well for most tonal jazz. Motion by third, both major and minor, is also a common (though less frequent) occurrence, and will be the focus of this chapter. Harmonic motion by thirds is one of the main emphases of non-jazz transformational theorists; this chapter will allow the opportunity to explore connections between our approach to jazz harmony and the existing neo-Riemannian and transformational literature.

3.1 Minor-Third Substitutions

3.1.1 Formalism

The most common substitution for the dominant in jazz is undoubtedly the tritone substitution (discussed in Section 2.2), but the minor-third substitution is also relatively common, especially in the bebop era. By way of example, Figure 3.1 gives the changes to the opening five bars of Tadd Dameron's "Lady Bird." Normally the progression in mm. 3–4 would function as a ii–V in the key of E♭, but in m. 5 it resolves instead to C. What might have been a ii–V–I in C (Dm7–G7) does not appear;; the ii–V is transposed up a minor third to become Fm7–B♭7.

The identical tendency tones shared by tritone-substituted dominants makes them relatively easy to explain, but minor-third substitution is more difficult. Jazz harmony textbooks often do not provide an explanation for the phenomenon: Jerry Coker, for example, simply states that "the I

C^{maj7} | | F^{-7} | $B\flat^{7}$ |

C^{maj7}

Figure 3.1. Changes to "Lady Bird" (Tadd Dameron), mm. 1–5.

chord . . . is often preceded by IV-7 to ♭VII7, instead of the usual V7 chord."[1] The *Berklee Book of Jazz Harmony* places the ♭VII7 chord in its chapter on "modal interchange" (what might also be called modal mixture), and notes that its function is ambiguous: the chord has dominant quality but not dominant function, since it lacks the leading tone (of the following tonic).[2] Unlike the tritone substitution, there is no strong voice-leading rationale for the minor-third substitution; it is simply a progression that bebop players often used, and that we as analysts must now contend with.

There is, though, a certain similarity between the tritone and minor-third substitutions: just as the tritone evenly divides the octave, so too the minor third evenly divides the tritone. In the previous chapter, introducing tritone substitutes to ii–V space effectively divided the space in half, splitting the complete space into "front" and "back" sides (recall Figure 2.10). Repeating this process again results in a space that looks something like Figure 3.2 (which we will call "m3 space"). The introduction of minor thirds once again changes the topology of the space. This is somewhat easier to see by focusing again only on the centrally-located dominant seventh chords; while the tritone version of ii–V space is topologically equivalent to a Möbius strip, the minor-third version of the space is equivalent to a torus (see Figure 3.3).[3]

Figure 3.3 looks remarkably similar to a more familiar toroidal figure common in music theory: the neo-Riemannian Tonnetz. Despite the surface similarity though, the two are quite different. Like the Tonnetz, the dominant sevenths at the center of m3 space are arranged into axes of perfect fifths (verticals), minor thirds (horizontals), and major thirds (northwest–southeast diagonals, not shown in Figure 3.3).[4] Crucially though, the vertices in the m3-torus are dominant seventh chords (ordered triples), not individual notes; the m3-torus does not represent a

1. Jerry Coker, *Elements of the Jazz Language for the Developing Improvisor* (Miami: Belwin, 1991), 82.

2. Joe Mulholland and Tom Hojnacki, *The Berklee Book of Jazz Harmony* (Boston: Berklee Press, 2013), 123–24.

3. Turning Figure 3.3 into a torus involves gluing the top and bottom edges together and the left and right edges together; the dotted line representing perfect fifths then wraps around the surface of the torus in a continuous line (as though you had wrapped a barber's pole around a doughnut).

4. In fact, the graphs of the note-based Tonnetz and the m3-torus here are isomorphic. This fact introduces some tantalizing possibilities, but none turn out to be terribly interesting since, as Richard Cohn has shown, the consonant triad is unique among trichords in its capability for parsimonious voice leading; "Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations," *Journal of Music Theory* 41, no. 1 (Spring 1997): 1–7. As such, the m3-torus—made of (026) trichords—does not show common-tone relations, and the dual graph of the m3-torus gives vertices of set class (0134678T).

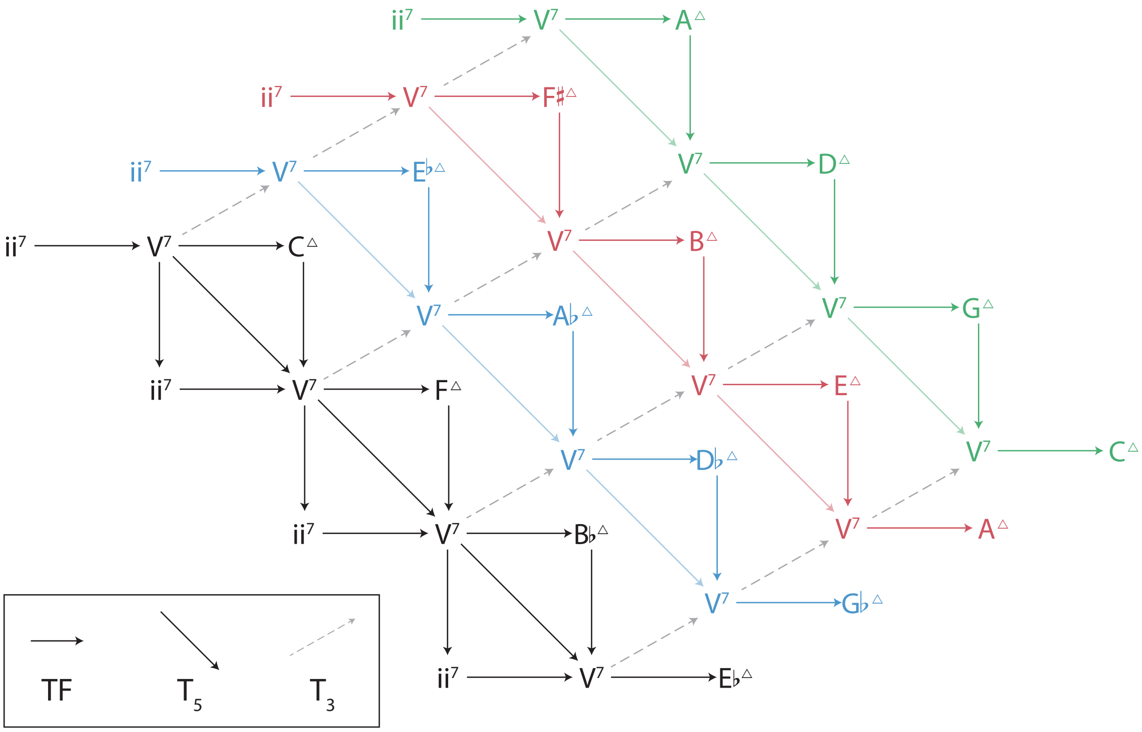

Figure 3.2. The complete minor-third representation of ii–V space (ii^7 chords omitted on rear levels for clarity), or m3 space.

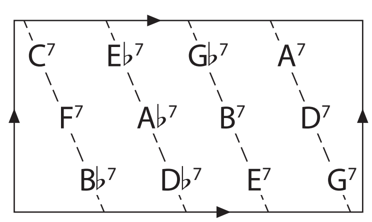

Figure 3.3. The toroidal center of minor-third space (the m3-torus).

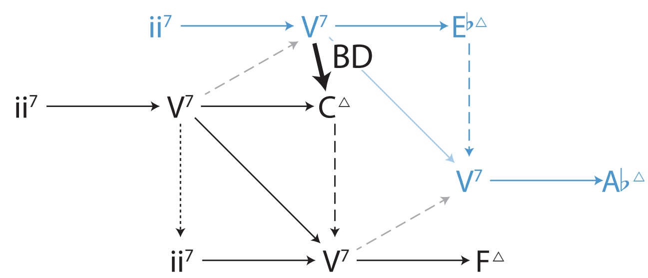

Figure 3.4. The BD transformation from B♭7 to CM7 in minor-third space.

parsimonious voice-leading space. The neo-Riemannian Tonnetz can also be drawn to represent triads instead of individual notes, but the resulting graph (Douthett and Steinbach's "chicken-wire torus") no longer resembles the m3-torus here.[5]

The minor-third arrangement of ii–V space makes it easy to define a transformation to represent the minor-third substitution. Because jazz musicians often refer to the minor-third substitution as the "backdoor substitution", we will call this transformation BD (for "backdoor"):

$$\mathrm{BD}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{dom}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 2, x_s - 4, x_t - 3) \in S_{\mathrm{maj}}$$

Musically, there is no compelling reason to define BD such that the third is calculated from the previous chord's seventh and vice versa; there is no voice-leading connection between them as there is in the TF transformation. Defining them this way, however, allows the same function to model both the transformation from minor to dominant seventh and from dominant to major seventh.[6] In the space, BD is represented as a diagonal line moving "frontward" between two layers; see Figure 3.4. With this definition, we can easily understand the progression in mm. 3–5 of "Lady Bird": Fm7 $\xrightarrow{\mathrm{TF}}$ B♭7 $\xrightarrow{\mathrm{BD}}$ CM7.

5. Jack Douthett and Peter Steinbach, "Parsimonious Graphs: A Study in Parsimony, Contextual Transformations, and Modes of Limited Transposition," *Journal of Music Theory* 42, no. 2 (1998): 246–48. The chicken-wire torus is the dual graph of the more common note-based Tonnetz; both are shown in Dmitri Tymoczko, "The Generalized Tonnetz," *Journal of Music Theory* 56, no. 1 (2012): Figure 1.

6. The definition of $S_{\mathrm{min}} \xrightarrow{\mathrm{BD}} S_{\mathrm{dom}}$ is not given, since it is relatively rare in jazz. It models a progression like iv^7–V^7, which is much more common in classical music.

3.1.2 Analytical Interlude: Joe Henderson, "Isotope"

Minor-third space, as it has been developed so far, may seem like merely another arrangement of ii–V space. One might reasonably ask why it merits a section in this chapter, instead of being merely an extension of the space like those explored in Section 2.3. In a situation parallel to that of 19th-century chromatic tonality, jazz after 1960 began to use more chromatic progressions—especially those built on thirds—in what still might be called tonal jazz. In these compositions, there is still a prevailing sense of key, but local harmonic progressions depart from the descending-fifths norm of earlier jazz.[7] Exploring minor-third space, and understanding why it merits special discussion, is easiest to do by using an example: Joe Henderson's composition "Isotope."

The solo changes for "Isotope" are given in Figure 3.5.[8] The tune is a modified 12-bar blues, and contains the typical lowered seventh of the blues dialect: all of the chords (even the tonic C chords) are major-minor sevenths. While we could analyze this set of changes in the "blues ii–V space" of Section 2.3.2, using the m3-torus of Figure 3.3 will help to highlight how minor-third space can be used in analysis.

An analysis of "Isotope" in the m3-torus is given in Figure 3.6. The solo changes begin with a tonic major-minor seventh chord for four bars, before moving to the IV chord in m. 5. Instead of moving directly from IV to I in m. 7 (which would be typical for a blues), Henderson moves first to B♭, resulting in a variant of the backdoor progression B♭7–C7 in mm. 6–7.[9] The tonic function of C7 is extended by moving to A7 in m. 8, a minor third away. What would be a string of T_5

7. Keith Waters has called this period "jazz's second practice" ("Chick Corea, Postbop Harmony, and Jazz's Second Practice" [paper presented at the annual meeting of the Society for Music Theory, Charlotte, NC, November 2013]). Waters, along with J. Kent Williams, has explored post-tonal jazz harmony using the Tonnetz and hyper-hexatonic systems familiar from classical theory; see "Modeling Diatonic, Acoustic, Hexatonic, and Octatonic Harmonies and Progressions in Two- and Three-Dimensional Pitch Spaces; or Jazz Harmony after 1960," *Music Theory Online* 16, no. 3 (August 2010).

8. "Isotope" was first recorded on Henderson's album *Inner Urge*, released in 1965. It uses a slightly different set of changes for the solos than it does for the head (often referred to simply as the "solo changes" and "head changes"). This is often the case when the head changes are complex, fast-moving, or contain unusual extensions to account for specific melody notes. These changes are taken from *The Real Book*; the C chord in m. 7 is played as either a major or a dominant seventh on the *Inner Urge* recording, so analyzing it as C7 here seems reasonable.

9. We could easily define a $\mathrm{BD}_{\mathrm{blues}}$ transformation, similar to $\mathrm{TF}_{\mathrm{blues}}$ from the last chapter; for now, we can simply understand this variant as BD $\odot$ 7th.

C^7			
F^7	$B\flat^7$	C^7	A^7
$A\flat^7$	G^7	C^7 A^7	$G\flat^7$ $E\flat^7$

Figure 3.5. Solo changes for "Isotope" (Joe Henderson).

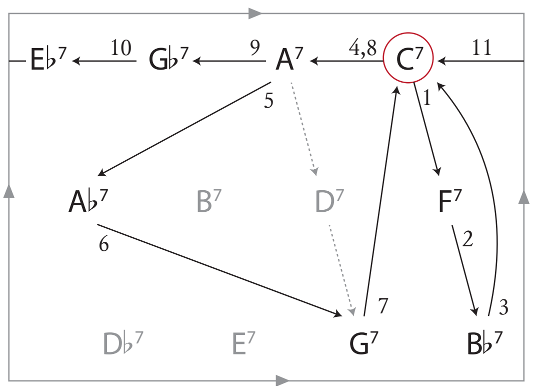

Figure 3.6. An analysis of "Isotope"(solo changes) in the m3-torus.

operations, A7–D7–G7–C7, is disrupted slightly by the tritone substitution of A♭7 for D7 (represented in the m3-torus by replacing a chord with the chord two spaces to its left or right). Once the progression returns to the tonic C7 in m. 11, Henderson uses a complete minor-third cycle as a turnaround, maintaining tonic function for two measures before beginning the next chorus.

The logic of the minor-third substitution means that the B♭7 chord in m. 6 is understood as a substitute for true dominant G7. Likewise, the A7 chord in m. 8 seems to substitute for the tonic in the same way; in an ordinary 12-bar blues, mm. 7–8 would both contain tonic. This correspondence suggests an interesting possibility: the top row of harmonies in Figure 3.6 all seem to have tonic function, while the two chords used in the bottom row both act as dominants. The

Figure 3.7. “Isotope,” head (Joe Henderson).

harmonies in the middle row, then, seem to serve as predominants (or subdominants), appearing in “Isotope” just before the G7 and B♭7 chords.[10]

This functional analysis helps to make sense of the unusual turnaround in the last two bars of the tune. Turnarounds are inherently prolongational structures, a way of providing harmonic interest between a chorus-concluding tonic and the next chorus-beginning tonic.[11] While most turnarounds use functional harmony (a ii–V–I progression or some variant), Henderson uses a non-functional minor-third cycle. Coming as it does at the end of the chorus, which makes liberal use of minor-third substitutions, we are primed to hear this cycle as a unique way of maintaining tonic function while avoiding the use of a functional harmonic progression.

This minor-third motion is seen first in the head changes, given in Figure 3.7. (Because the changes are altered in the head in order to fit the melody, this figure gives the melody as well.) Most of the alterations between the head and solo changes occur in the first four bars of the tune;

10. The question of the meaning of “function” is is a difficult one. Here I mean the term as Brian Hyer does: “it is not what a chord *does* that matters, but what it *is*: a functional designation names a chord’s *being*” (“What Is a Function?,” in *The Oxford Handbook of Neo-Riemannian Music Theories*, ed. Edward Gollin and Alexander Rehding [New York: Oxford University Press, 2011], 109, emphasis original).

11. The word “prolongational” in this sentence is admittedly problematic, given its Schenkerian implications. By using it here I mean only that at some deeper level the turnaround is harmonically superfluous, as it occurs after the main tonal conclusion of the chorus (an observation confirmed by the fact that the turnaround is usually omitted in the last head). I do not mean to imply that the turnaround in the last two bars of “Isotope” is harmonically uninteresting; indeed, it is the most distinctive feature of the piece.

the remaining differences are relatively insignificant.[12] While the solo changes give only C7 in the first four bars, the head changes elaborate this harmony with an alteration of a I–ii–V–I progression: the II^7 chord in m. 2 is preceded by E♭7. While it would be easy to write this chord off as an upper-neighbor harmony to the more structural II chord, doing so would minimize the important role of the minor-third substitution in the rest of the tune. What appears at first to be an inconsequential embellishment (substituting E♭7 for C7) gains in significance throughout the tune, first becoming realized in the backdoor progression in mm. 5–7 and reaching its fullest expression in the turnaround that ends the chorus.

Henderson's tour of the m3-torus in "Isotope" is interesting for a number of reasons. First, he is able to create a sense of tonal function and harmonic progression while using almost entirely major-minor ("dominant") seventh chords. This is a feature common to many blues tunes, but is stronger in "Isotope" given the pervasive use of minor-third substitutions. Second, this is a tune that does not seem to make much sense in the descending-fifths ii–V space of the previous chapter. While certainly some of the tune makes use of harmonic motion in fifths, the backdoor progression in mm. 6–7 and the final turnaround would appear as seemingly random, nonsensical jumps in ii–V space.

Finally, the progression of "Isotope" is not one that is easily explained using neo-Riemannian theory as it is usually applied to classical music. Constructing a Tonnetz usually relies on having two varieties of musical objects under consideration (commonly major and minor triads, or half-diminished and dominant seventh chords), while our m3-torus only uses major-minor sevenths. Neo-Riemannian theories often focus on smooth voice leading; measured in these terms, the B♭7 and C7 chords of mm. 5–6 are quite distant from one another, despite their functional equivalence to a V–I motion.[13] The turnaround, most unusual from a tonal perspective, is actually

12. The most obvious differences in mm. 5–12 of the tune are the "slash chords" in mm. 8–9. The chord symbol Em7/A indicates an E minor seventh chord played with an A in the bass; the resulting sound is an A7 chord with a suspended fourth (D replaces C♯). The older *Real Book* gives the same change as A7sus4. The only other slight alteration is the addition of the ii chord, Dm7, in m. 10.

13. Exactly how far apart B♭7 and C7 depends on how one chooses to measure voice-leading distance, and whether we consider major-minor sevenths in the usual way, as four-note chords, or in the way we have been doing so here, as ordered triples of root, third, and seventh. In Jack Douthett's Four-Cube trio, for example, B♭7 and C7 are

quite typical of patterns usually analyzed in neo-Riemannian terms: the major-minor sevenths found there are all minimal perturbations of a single diminished seventh chord, and each can be connected to the next with a minimal amount of voice-leading work (two semitones moving in opposite directions).[14]

3.2 Major-Third Spaces

3.2.1 Introduction: Coltrane Changes

Root motion by major third is one of the most difficult harmonic motions to explain using traditional tonal methods; it is with this kind of music that transformational methods have proven to be most useful.[15] The increasing use of these progressions in nineteenth-century harmony has a parallel in jazz, as Keith Waters has shown; in both, the harmonic vocabulary is familiar, but harmonic progressions are often unfamiliar.[16] Nonfunctional harmony is a defining feature of some post-bop jazz, including much of the music of Chick Corea, Herbie Hancock, Wayne Shorter, and others.[17] This dissertation, though, is interested specifically in tonal jazz, and we will remain careful during this discussion to avoid straying too far afield from this goal.

The locus classicus for root motion by major third in jazz is of course John Coltrane's "Giant Steps," first recorded in 1959 on the album of the same name.[18] Much of the use of nonfunctional

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maximally far apart—4 semitones; see Richard Cohn, *Audacious Euphony: Chromatic Harmony and the Triad's Second Nature* (New York: Oxford University Press, 2012), 157–58.

14. The notes in these four dominant sevenths form an octatonic collection, and have been studied fairly extensively in the literature. See, for example, ibid., 152–58; Douthett and Steinbach, "Parsimonious Graphs," 245–46; and Dmitri Tymoczko, *A Geometry of Music: Harmony and Counterpoint in the Extended Common Practice* (New York: Oxford University Press, 2011), 371.

15. The examples are too numerous to list here, but for an overview, see Matthew Bribitzer-Stull, "The A♭–C–E Complex: The Origin and Function of Chromatic Major Third Collections in Nineteenth-Century Music," *Music Theory Spectrum* 28, no. 2 (Fall 2006): 167–90.

16. Waters, "Chick Corea, Postbop Harmony, and Jazz's Second Practice."

17. Waters's work in particular has focused extensively on this music, though he is hardly alone. See, for example, Patricia Julien, "The Structural Function of Harmonic Relations in Wayne Shorter's Early Compositions: 1959–1963" (PhD diss., University of Maryland, 2003); Steven Strunk, "Notes on Harmony in Wayne Shorter's Compositions, 1964–67," *Journal of Music Theory* 49, no. 2 (Autumn 2005): 301–32; Keith Waters, "Modes, Scales, Functional Harmony, and Nonfunctional Harmony in the Compositions of Herbie Hancock," *Journal of Music Theory* 49, no. 2 (Fall 2005): 333–57; and Waters and Williams, "Jazz Harmony after 1960."

18. Many people have discussed "Giant Steps" in the literature; the most substantial work in this area is Matthew Santa's "Nonatonic Progressions in the Music of John Coltrane," *Annual Review of Jazz Studies* 13 (2003): 13–25. Guy

B^{maj7} D^{7}	G^{maj7} $B\flat^{7}$	$E\flat^{maj7}$	$A\text{-}^{7}$ D^{7}
G^{maj7} $B\flat^{7}$	$E\flat^{maj7}$ $F\sharp^{7}$	B^{maj7}	$F\text{-}^{7}$ $B\flat^{7}$
$E\flat^{maj7}$	$A\text{-}^{7}$ D^{7}	G^{maj7}	$C\sharp\text{-}^{7}$ $F\sharp^{7}$
B^{maj7}	$F\text{-}^{7}$ $B\flat^{7}$	$E\flat^{maj7}$	$C\sharp\text{-}^{7}$ $F\sharp^{7}$

Figure 3.8. Changes to "Giant Steps" (John Coltrane).

harmony in jazz that develops after 1960 can be traced back to "Giant Steps"; Keith Waters has outlined this lineage in selected compositions of Wayne Shorter, Bill Evans, and Herbie Hancock.[19] Given this influence, "Giant Steps" will serve here as a useful foil for major-third cycles in jazz more generally. Though we will delay a proper analysis of the tune until we have developed some formalism, a short overview will be useful at this point.

The changes to "Giant Steps" are given in Figure 3.8. The major-third construction of the tune is readily apparent: the three tonal centers are B, G, and E♭, as evidenced by the major seventh chords. These local tonics are all preceded by their dominants (mm. 1–3, 5–7) or by complete ii–V progressions (all other locations). Though the distance between the key centers is unusual, the individual progressions are not.[20] "Giant Steps" is not exactly tonal, but neither is it really atonal. When I listen to the piece, at least, the impression is not one of nonfunctional harmony, but rather of tonal harmony used in an unconventional way. This distinction is easiest to understand with a counterexample: Figure 3.9 gives the changes to Wayne Shorter's ballad "Infant

Capuzzo compares Santa's analysis to one done by Pat Martino in "Pat Martino's The Nature of the Guitar: An Intersection of Jazz Theory and Neo-Riemannian Theory," *Music Theory Online* 12, no. 1 (February 2006). See also David Demsey, "Chromatic Third Relations in the Music of John Coltrane," *Annual Review of Jazz Studies* 5 (1991): 145–80; and Matthew Goodheart, "The 'Giant Steps' Fragment," *Perspectives of New Music* 39, no. 2 (July 2001).

19. Keith Waters, "'Giant Steps' and the ic4 Legacy," *Intégral* 24 (2010): 135–62.

20. Frank Samarotto has suggested to me in connection with an unpublished paper of his that "Giant Steps" is chromatically coherent, while locally diatonic. As such, it represents an example of his "hypothetical" Type 4 coherence, "in which areas of diatony occur only in local isolation and in which some other (presumably post-tonal) coherence might be in effect." "Treading the Limits of Tonal Coherence: Transformation vs. Prolongation in Selected Works by Brahms" (paper presented at the annual meeting of the Society for Music Theory, Madison, WI, November 2003).

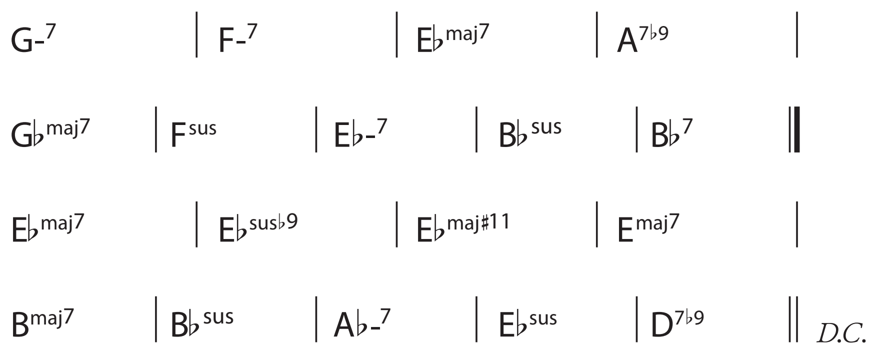


Figure 3.9. Changes to “Infant Eyes” (Wayne Shorter).

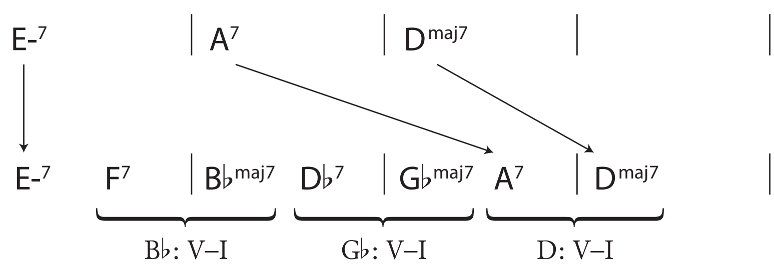


Figure 3.10. Coltrane’s major-third cycle as a substitution for a ii–V–I progression. (Adapted from Levine, *The Jazz Theory Book*, 359.)

Eyes”.[21] Here there are no ii–Vs, and the only V–I progressions occur across formal boundaries. Shorter’s use of harmony *does* seem nonfunctional, and gives the piece a floating quality that “Giant Steps” does not have. Rather, “Giant Steps” is strongly forward-directed: all of the dominant chords push toward their respective tonics, and although the global tonic may be in question, local tonic chords are crystal clear.[22]

The major-third cycle of “Giant Steps” is the most well-known example, but Coltrane first developed the progression as an elaborate substitution over a standard ii–V–I progression; this particular set of substitutions is often referred to as “Coltrane changes.”[23] Figure 3.10 shows how

21. These changes are from Mark Levine, *The Jazz Piano Book* (Petaluma, CA: Sher Music, 1989), 30. “Infant Eyes” appears on Shorter’s album *Speak No Evil* (1964).

22. Authors disagree on whether “Giant Steps” is in the key of B or E♭: David Demsey hears E♭ (“Chromatic Third Relations,” 171–72), while Andy Jaffe hears B (*Jazz Harmony* [Tübingen: Advance Music, 1996]). Given the organizing influence of the major-third cycle, I am not sure the question is so important; the tune uses tonal progressions, but may not be *in* a key.

23. Exactly how Coltrane devised this substitution set is difficult to say: authors have at various times pointed to classical sources—especially Nicolas Slonimsky’s *Thesaurus of Scales and Melodic Patterns*—as well as the music of Thelonious Monk, Dizzy Gillespie, and Tadd Dameron, among others. One source that is nearly always cited is the

"Tune Up"	$E-^{7}$	A^{7}	D^{maj7}	
"Countdown"	$E-^{7}$ F^{7}	$B\flat^{maj7}$ $D\flat^{7}$	$G\flat^{maj7}$ A^{7}	D^{maj7}
	$D-^{7}$	G^{7}	C^{maj7}	
	$D-^{7}$ $E\flat^{7}$	$A\flat^{maj7}$ B^{7}	E^{maj7} G^{7}	C^{maj7}
	$C-^{7}$	F^{7}	$B\flat^{maj7}$	
	$C-^{7}$ $D\flat^{7}$	$G\flat^{maj7}$ A^{7}	D^{maj7} F^{7}	$B\flat^{maj7}$
	$E-^{7}$	F^{7}	$B\flat^{maj7}$	$E\flat^{7}$
	$E-^{7}$	F^{7}	$B\flat^{maj7}$	$E\flat^{7}$

Figure 3.11. The changes to "Countdown" (Coltrane), compared with "Tune Up" (Miles Davis).

this process works: the goal of the progression (in this case, DM7) is shifted to the fourth bar; then, major seventh chords related by major third are placed on the downbeats (B♭M7 and G♭M7); finally, all of the major sevenths are preceded by their own dominants. This process can clearly be seen in Coltrane's composition "Countdown," which is based on the changes to Miles Davis's "Tune Up" (see Figure 3.11).[24] Coltrane changes can be superimposed over any four-measure ii–V–I progression, so they can be found not only in Coltrane's own compositions, but also in his improvisations on other tunes as well as his reharmonizations of standards (like "Body and Soul").[25]

tune "Have You Met Miss Jones?" (Richard Rodgers/Lorenz Hart), which we analyze in Section 3.2.3. For a review of these possible origins, see Demsey, "Chromatic Third Relations," 148–57; and Lewis Porter, *John Coltrane: His Life and Music* (Ann Arbor: University of Michigan Press, 1998), 145–47.

24. Nearly every discussion of Coltrane changes includes the "Countdown"/"Tune Up" pairing. See, for example, Demsey, "Chromatic Third Relations," 159–62; Mark Levine, *The Jazz Theory Book* (Petaluma, CA: Sher Music, 1995), 359–60; or many others. "Countdown" was also recorded on *Giant Steps*, and "Tune Up" can be heard on *Cookin' with the Miles Davis Quintet* (1957).

25. Demsey provides a list of third-relations in jazz tunes in an appendix to "Chromatic Third Relations," 179–80. Coltrane's famous take on "Body and Soul" is found on the album *Coltrane's Sound* (1960), which also features two original tunes that make prominent use of the major-third cycle: "Central Park West" and "Satellite."

3.2.2 Developing a Transformational System

Because Coltrane changes can be considered a ii–V variant, it is logical to include them in this study, even though we may be slightly pushing the limits of "tonal jazz." We now have a preliminary understanding of how the substitution works, but it still remains to incorporate it into the transformational system under development here. First, though, there is a bit of unfinished business to take care of: in Section 1.3, we touched on several transformational approaches only briefly, promising to return to them at a point when they would be more relevant. Given the central role of harmonic motion in thirds in many neo-Riemannian theories, it seems appropriate to fulfill that promise at this point.

Triads related by major third are important in many of these theories, but especially those of Richard Cohn. In both of Cohn's models of triadic space in *Audacious Euphony*—hexatonic cycles and Weitzmann regions—three M3-related major triads combine with three minor triads to form a six-chord system.[26] Thus, the three tonal centers of "Giant Steps" can be contained in a single hexatonic cycle or a single Weitzmann region.

Understanding the rest of "Giant Steps" in terms of one of these systems, though, leaves much to be desired. Cohn's systems, and others like them, are fundamentally triadic, which is clearly a problem for understanding jazz, with its saturation of seventh chords (and beyond). To account for this incongruity, we must either adapt the music to fit our analytical system, or adapt our analytical system to fit the music. The first option is clearly a nonstarter: Figure 3.12 gives a non-example of "Giant Steps" analyzed in Jack Douthett's "Cube Dance."[27] While this analysis makes the major-third cycle of the tonic chords clear, reducing the chords to triads loses the detail of the chord qualities (both major-seventh and dominant-seventh chords become major triads), as well as their functional relationships. Despite the prominent emphasis on M3-cycles in these theories,

26. Cohn, *Audacious Euphony*. Hexatonic cycles are discussed primarily in Cohn's chapter 2, and Weitzmann regions in chapter 4; chapter 5 combines these models into a single system which is then used throughout the rest of the book. Lewis Porter mentions Weitzmann's treatise on the augmented triad as a possible influence on Coltrane (*John Coltrane: His Life and Music*, 146).

27. In this figure "+" indicates major triads and "−" indicates minor triads. "Cube Dance" appears in Douthett and Steinbach, "Parsimonious Graphs," 254, and is one of Cohn's primary models of triadic space; see *Audacious Euphony*, 86–109 and following.

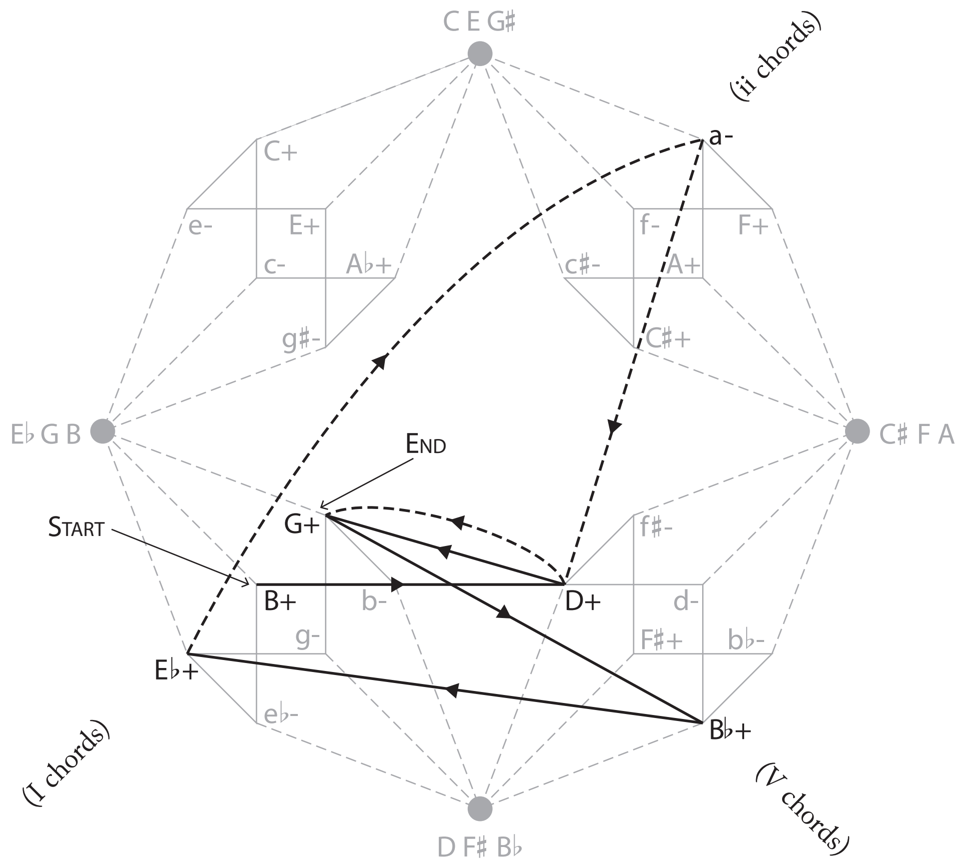


Figure 3.12. “Giant Steps,” mm. 1–5, analyzed in Douthett’s Cube Dance. Begin by following the solid arrows, then continue with the dashed arrows.

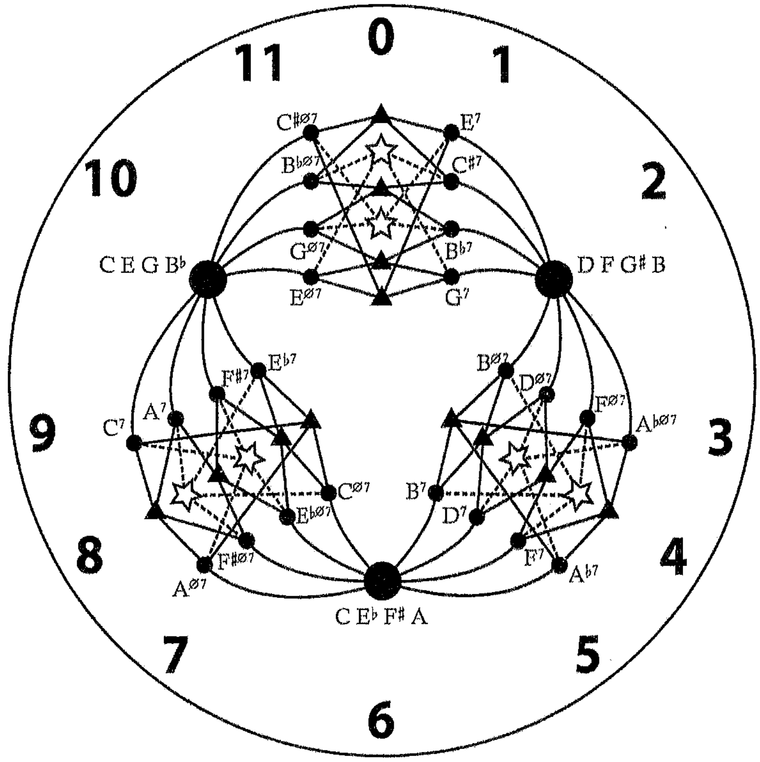
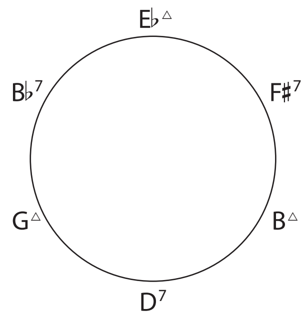


Figure 3.13. Cohn's "4-Cube Trio" (*Audacious Euphony*, Fig. 7.16, 158.) The black triangles indicate minor seventh chords, while the hollow stars indicate French sixth chords.

then, using these triadic systems as an analytical basis for our work here would not seem to be the answer.

There are of course neo-Riemannian theories involving seventh chords, but these turn out to be not so helpful for our purposes here either. Theories that include only the (0258) tetrachords—half-diminished and dominant sevenths—are obviously not suitable for analyzing the multiple seventh-chord types in jazz. The seventh-chord analogy of "Cube Dance" is what Cohn calls the "4-Cube Trio," which is shown in Figure 3.13.[28] As is readily apparent, 4-Cube Trio does not contain any major seventh chords, so it would also create problems if pressed into use for analyzing "Giant Steps."

28. This figure is taken from Cohn's book, and has several errors, the most important of which is that the C at the 10 o'clock position should be a C♯, forming a fully-diminished seventh chord. "4-Cube Trio" was originally devised by Jack Douthett, and is very similar to the "Power Towers" graphic in Douthett and Steinbach, "Parsimonious Graphs," 256 (which omits the French sixth chords). For more on its history, see Cohn, *Audacious Euphony*, 157n15.



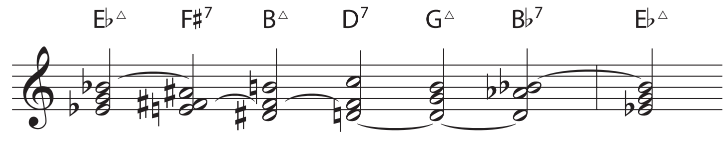


Figure 3.14. Matthew Santa's nonatonic cycles: the "Western" nonatonic cycle (left), and a three-voice parsimonious realization (right). (Adapted from "Nonatonic Progressions in Coltrane," 14.)

There is, though, a more fundamental problem with these neo-Riemannian theories of seventh chords, at least when approaching M3-cycles in jazz. As we have mentioned, many neo-Riemannian theories focus on efficient voice-leading, and parsimonious relationships among seventh chords can understood as minimal perturbations of fully-diminished seventh chords.[29] This does not generate major-third cycles (as in the triadic case), but rather partitions the octave into *minor* thirds. The three dominant sevenths appearing in "Giant Steps," for example, are in three different "towers" in the 4-Cube Trio, which does not reflect the organizing influence of major thirds in the way that the triadic Cube Dance does.

Some theorists have turned to neo-Riemannian theory to explain jazz progressions, though; closest to our intent here is Matthew Santa's nonatonic system for analyzing Coltrane.[30] In a parallel with Cohn's hexatonic systems, Santa draws from the nonatonic (or enneatonic) collection in order to explain "Giant Steps" in terms of parsimonious voice leading. Figure 3.14 shows one of Santa's cycles, along with three-voice parsimonious realization. (In this figure, note that the triangle indicates a major *triad*, not a major seventh chord.) This nonatonic system seems to be a convincing analysis of the opening of "Giant Steps," but it comes up a bit short as a general theoretical system. First, Santa considers only major triads and incomplete dominant seventh chords; all major seventh chords are reduced to triads, and minor sevenths—like the ii^7 chords of

29. This is a central thesis of Cohn's chapter 7 (see especially *Audacious Euphony*, 148–58), and figures prominently in Tymoczko's geometric theory (*A Geometry of Music*, 97–112).

30. Santa, "Nonatonic Progressions in Coltrane."

Major triads	Minor triads	Incomplete V^7
CM	Cm	C7
E♭M*	E♭m	B♭7*
EM	Em	E7
GM*	Gm	D7*
A♭M	A♭m	A♭7
BM*	Bm	F♯7*

Table 3.1. All possible consonant triads and incomplete dominant sevenths in the nonatonic collection {D, E♭, E, F♯, G, A♭, B♭, B, C}. Members of Santa's Western system are marked with a star.

"Giant Steps"—are simply ignored.[31] The cycle in Figure 3.14 is generated by the collection {D, E♭, E, F♯, G, A♭, B♭, B, C}, but it does not contain all of the triads or incomplete dominant sevenths in that collection (see Table 3.1). Santa's 4-cycle system, then, is somewhat misleading, since any triad or dominant seventh can be located in two different nonatonic collections.

Having brought up all of these approaches only to show their shortcomings, though, the question remains: what should a transformational system that includes major-third relations look like? Although the relationship of M3-cycles and smooth voice leading is valuable, so far in this study we have focused primarily on functional relationships, and it would seem foolish to abandon that approach here. As we noted above, "Giant Steps" does contain a major-third cycle, but within that cycle the progressions are functional: it is locally diatonic, but globally chromatic.

As it turns out, we can once again adapt ii–V space in order to show organization by major third, as shown in Figure 3.15. This figure looks very similar to the minor-third organization in Figure 3.2, but the relationships between layers have changed.[32] Here, the "layers" of the space are arranged in descending major thirds (T_8), while the descending fifths arrangement is otherwise unchanged. This arrangement means that all of "Giant Steps" happens in a single horizontal slice

31. Santa omits the fifth of dominant seventh chords, as we have been doing here. The reasons, though, are different: the stated reason is to keep the cardinalities of the chords the same, but he does not mention why he chooses not to use major seventh chords and complete dominant sevenths, for example. Santa notes later that including the fifth involves one of the three missing notes from the nonatonic collection, which is acceptable because "the fourth voice is not essential to the voice leading of the cycle" (Santa, "Nonatonic Progressions in Coltrane," 15).

32. The major-third figure represents a kind of cross-section of the minor-third torus: to see this clearly, locate the key areas C, A♭, and E on both Figure 3.2 and Figure 3.15.

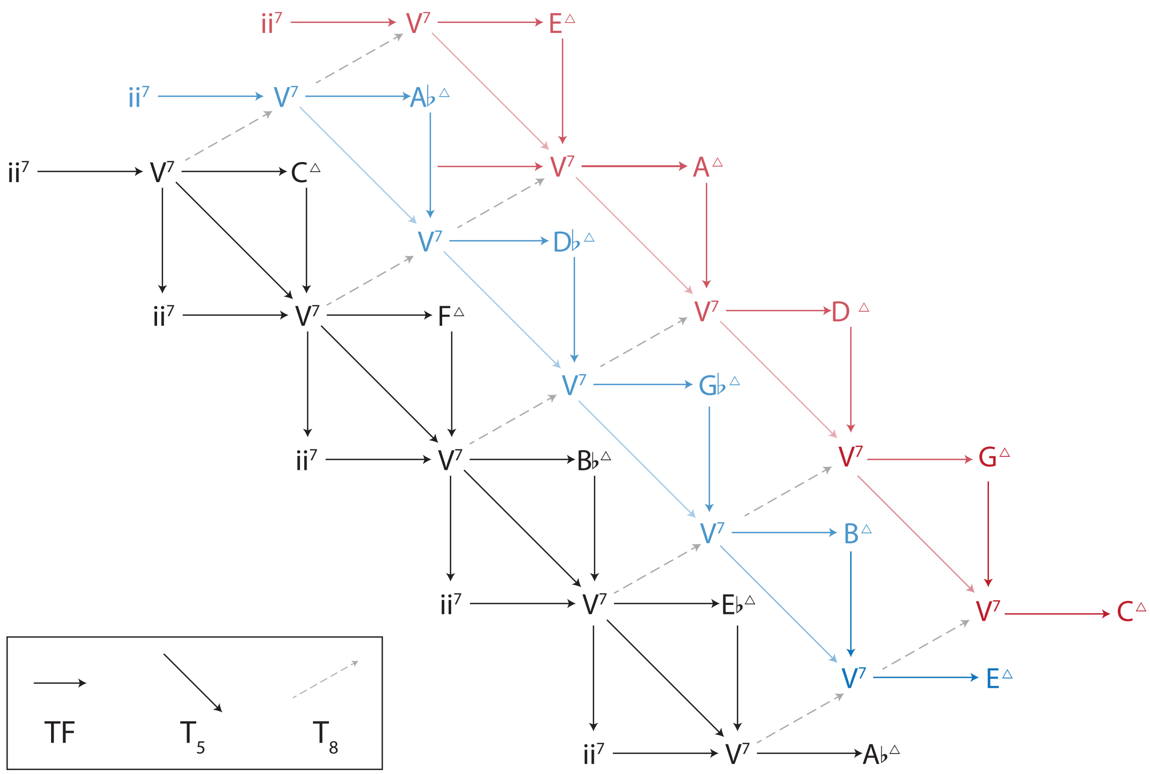


Figure 3.15. A major-third organization of ii–V space (ii^7 chords omitted on rear levels for clarity).

of the space.[33] This organization of ii–V space reflects our intuitions about the organization of this tune and others like it: by maintaining the integrity of the ii–V–I progressions and instead altering the relationships between them, we can keep both the local functional progressions important to improvising musicians *and* reflect the unusual chromatic organization of the tune itself.

The arrangement into T_8-related "layers" also helps to clarify in what sense Coltrane changes function as a substitution for a ii–V–I progression (see Figure 3.16). As usual, we could define a transformation to help explain this substitution. Though the T_8 between major seventh chords is certainly important, the most unusual surface feature in the substitution is the jump from a major-seventh chord to the dominant seventh whose root is a minor third higher; it is this harmonic move that gives the progression its forward momentum. We might call this

33. In fact, the rest of the figure is unnecessary for "Giant Steps"; the piece is easier to understand using a subgraph of the complete M3-space that contains only the ii–V–I progressions in B, G, and E♭.

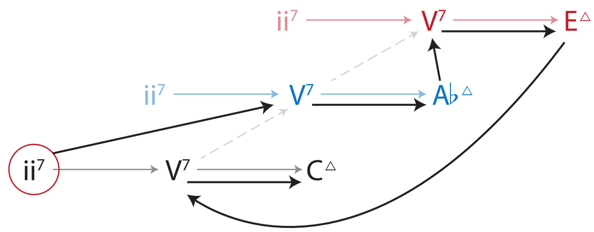


Figure 3.16. Coltrane changes as a ii–V–I substitution, shown in M3-space.

transformation CS (for "Coltrane Substitution"):

$$\mathrm{CS}(X) = Y, \text{ where } X = (x_r, x_t, x_s) \in S_{\mathrm{maj}} \text{ and } Y = (y_r, y_t, y_s) = (x_r + 3, x_t + 3, x_s + 2) \in S_{\mathrm{dom}}$$

With this transformation, we can understand the entire ii–V–I substitution as follows:

$$\mathrm{Dm7} \xrightarrow{\qquad\qquad\qquad\qquad\mathrm{TF}\qquad\qquad\qquad\qquad} \mathrm{G7} \xrightarrow{\mathrm{TF}} \mathrm{CM7}$$

becomes

$$\mathrm{Dm7} \xrightarrow{\mathrm{TF} \odot T_8} \mathrm{E\flat 7} \xrightarrow{\mathrm{TF}} \mathrm{A\flat M7} \xrightarrow{\mathrm{CS}} \mathrm{B7} \xrightarrow{\mathrm{TF}} \mathrm{EM7} \xrightarrow{\mathrm{CS}} \mathrm{G7} \xrightarrow{\mathrm{TF}} \mathrm{CM7}$$

This M3-space is well equipped to show the logic of major-third cycles, though other kinds of tonal relationships are more difficult to see: both tritone substitutes and minor-third substitutions are maximally far away in M3-space, for example. While this is indeed a limitation, the fact that ii–V space and its variants share the same essential features means that each can be substituted for another as needed for a given analytical situation. The spaces developed so far—ii–V space, its tritone-substituted variant, minor-third space, and now major-third space—all reflect the basic descending-fifths orientation of tonal jazz, but each prioritizes a particular secondary relationship.[34] Maintaining the same basic structure in our analytical apparatus allows us to understand a wide range of music as variations on a basic, functionally harmonic theme. There is no need for a great

34. These spaces represent all but one of the equal partitions of the octave: while we could easily construct a "whole-tone space," it would not have very many applications to tonal jazz. In cases where whole-tone relationships seem important, it is usually not problematic to consider a whole tone as a combination of two perfect fifths, which are readily shown in all of the other spaces.

switching of context from the logic of tritone substitutions to the logic of Coltrane changes, nor is there a need to invoke set classes of different cardinalities to justify dividing the octave into three or four equal parts. This presentation is developed in a rough parallel with the music itself: jazz musicians did not (and do not) discard everything they learned from bebop when approaching a progression like that of "Giant Steps"; rather, each is part of a single, coherent through line of tonal jazz.

3.2.3 Analytical Interlude: Richard Rodgers/Lorenz Hart, "Have You Met Miss Jones?"

Though we could turn to any number of Coltrane's middle-period compositions as analytical examples to illustrate major-third spaces, we will instead opt for the tune that is always cited as one of his influences: the Richard Rodgers and Lorenz Hart standard, "Have You Met Miss Jones?" While some of Coltrane's tunes (like "Giant Steps") use M3-cycles almost exclusively, "Miss Jones" will allow us the opportunity to see how organization by major thirds can participate in more typical, fifths-based jazz harmony. The changes for the tune are given in Figure 3.17; the analysis here will proceed in sections.[35]

The analysis of "Miss Jones" in M3-space is given in Figures 3.18a–3.18c. The A section (Figure 3.18a) is fairly typical, though it does include a fully-diminished seventh chord, which we have not yet seen in this study. This F♯°7 clearly functions as a passing chord, harmonizing the bass line F–F♯–G. The analysis in M3-space interprets this chord, as jazz musicians often do, as a D7♭9 without a root: functionally, the two chords are identical, with each leading to the following Gm7.[36] This Gm7 initiates a home-key ii–V in mm. 3–4 that resolves deceptively to Am7 in m. 5, at which point the piece begins a iii–vi–ii–V turnaround to return to FM7 for the repeat of the A section.

35. These changes are again taken from *The Real Book*, and are the standard changes for the tune; nearly all recordings agree with this set of changes.

36. Levine, *The Jazz Theory Book*, 85.

A	F^{maj7}	F♯^{o7}	G-^{7}	C^{7}
	A-^{7}	D-^{7}	G-^{7}	C^{7}
A′	F^{maj7}	F♯^{o7}	G-^{7}	C^{7}
	A-^{7}	D-^{7}	C-^{7}	F^{7}
B	B♭^{maj7}	A♭-^{7} D♭^{7}	G♭^{maj7}	E-^{7} A^{7}
	D^{maj7}	A♭-^{7} D♭^{7}	G♭^{maj7}	G-^{7} C^{7}
A″	F^{maj7}	F♯^{o7}	G-^{7}	C^{7} B♭^{7}
	A-^{7} D^{7}	G-^{7} C^{7}	F^{maj7}	

Figure 3.17. Changes to “Have You Met Miss Jones?” (Richard Rogers/Lorenz Hart).

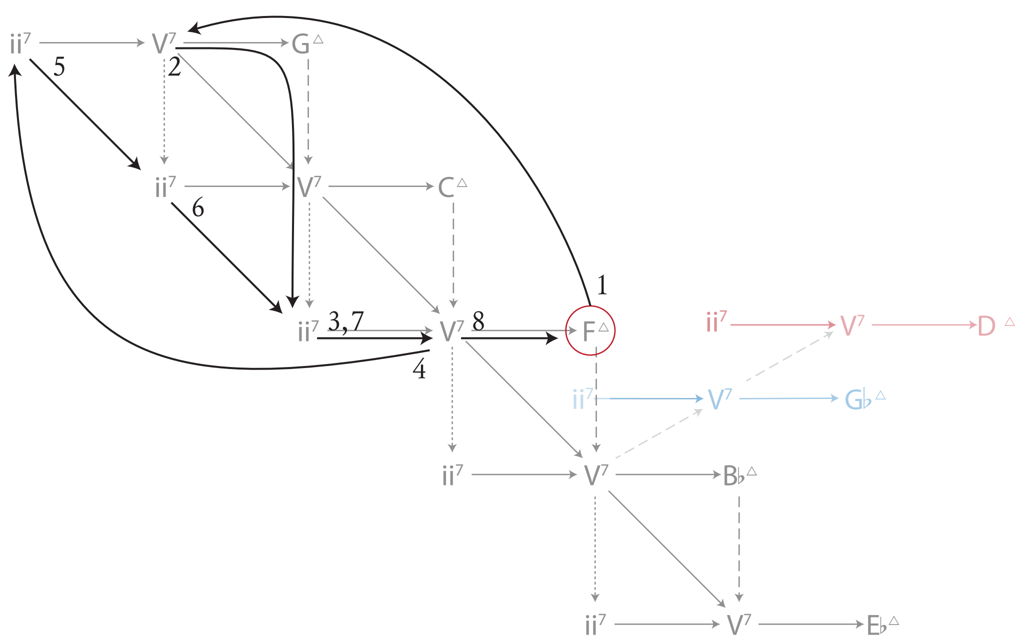


Figure 3.18a. “Miss Jones,” A section (mm. 1–8), analyzed in M3-space.

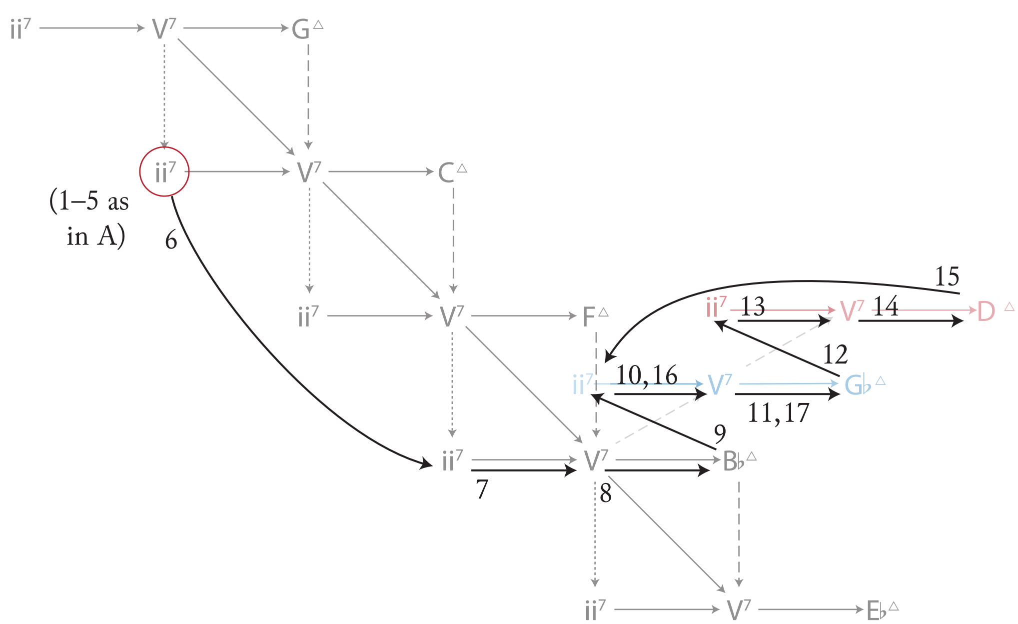


Figure 3.18b. “Miss Jones,” second A section and bridge (mm. 9–23).

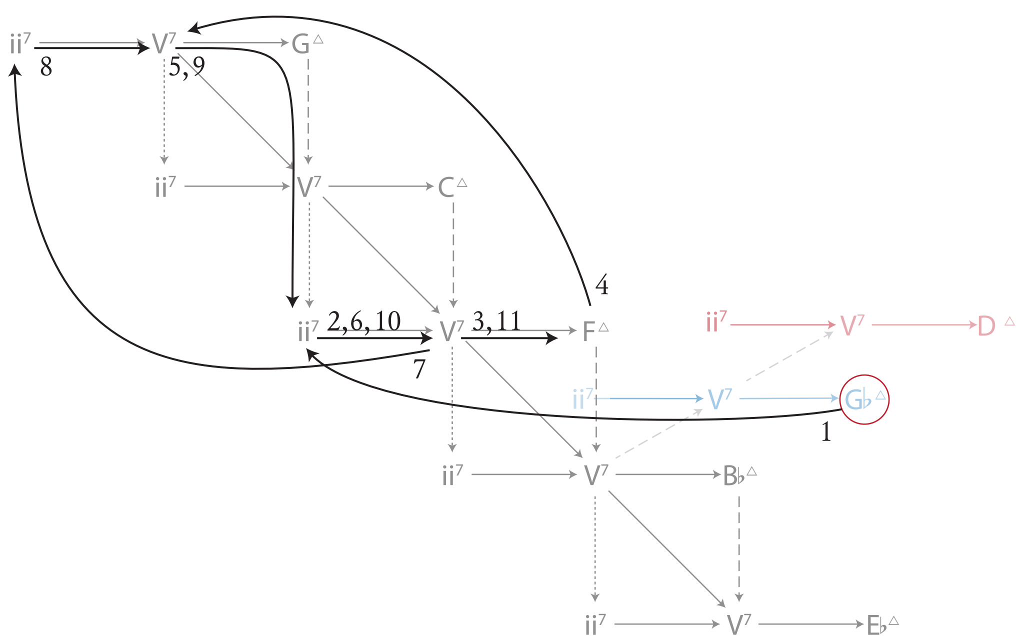


Figure 3.18c. “Miss Jones,” last two bars of bridge and final A section (mm. 23–32).

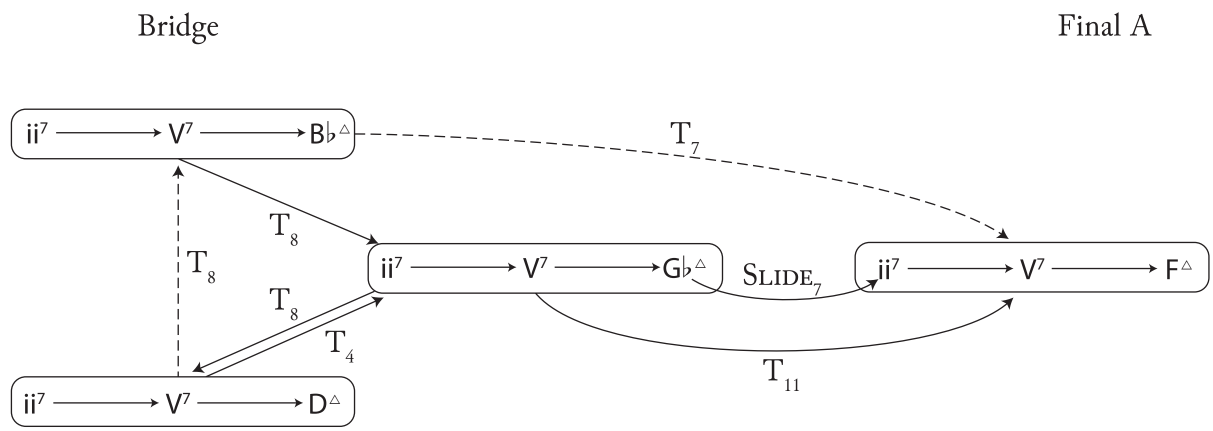


Figure 3.19. A transformation network for the bridge of "Miss Jones."

The second A section begins like the first, but the end is altered so that the bridge can begin on the subdominant, B♭ (Figure 3.18b). The bridge of this tune is its most well-known aspect, and contains the major-third cycle. This organization by major third is readily apparent in the space, as the music seems to break free of the descending fifths to elaborate the subdominant with a sequence that moves into the rear layers of the space and then returns. After arriving on B♭ in the first bar of the bridge, the tune moves to a ii–V–I in G♭ (a major third lower), followed by a ii–V–I in D (yet another major third lower). After making its way to the rear of the space, it begins to work its way back up the chain of thirds, finishing the bridge with a return to a ii–V–I in G♭. This G♭M7 chord moves via a SLIDE_7 transformation to a home-key ii–V–I, which returns to FM7 to begin the final A section. This final section (Figure 3.18c) is nearly the same as the first, but altered slightly in the last four bars to arrive more strongly on tonic in the penultimate bar of the form.

A more detailed transformation network for the bridge of "Miss Jones" is given in Figure 3.19.[37] In this network, transformations actualized in the music are shown as solid arrows, while others are shown with dotted arrows. (As usual, the unlabeled arrows are TF transformations.) A complete T_8-cycle is thwarted when the DM7 chord moves instead back to G♭, but the dotted

37. The "bubble notation" used in Figure 3.19 is first used in *GMIT*, 205–6. It is, as Lewin describes it, a "network-of-networks": each bubble here represents a single ii–V–I network (the unlabeled arrows are again TF transformations), and these networks are connected by larger-scale transpositions. In this figure, the SLIDE_7 transformations breaks through the bubble itself, and describes a transformation directly from G♭M7 to Gm7.

arrow shows that this move would have completed the cycle. It also clarifies the return to F major in the last A section: a larger-scale T_{11} from G♭ to F is accomplished via the SLIDE$_7$ transformation from G♭M7 to Gm7.

The analysis of "Have You Met Miss Jones" in M3-space demonstrates how the logic of major-third cycles in jazz is not independent from that of standard fifths-based harmony, but instead an extension of it. Any of our spaces would illustrate the A sections of "Miss Jones" equally well, but the construction of M3-space allows us to better understand the bridge. In normal ii–V space (Figure 2.10 on p. 50), the ii–V–I progressions related by major third are maximally far apart; analyzing the bridge in that space would make the T_8-related ii–V–Is seem like nonsensical harmonic motions. Rearranging the basic space as we have done in this section shows that the M3-cycle participates in a coherent way within the logic of the otherwise typical harmony of the tune.

3.3 Parsimonious Voice-Leading

Given the importance of parsimonious voice-leading in many current neo-Riemannian and transformational theories of harmony, it seems prudent to examine the transformational system we have been developing in the last two chapters in that light. Several of the transformations we have defined are indeed parsimonious, moving individual voices efficiently (the 3RD and 7TH transformations, among others) while others are less so (the transformation CS from this chapter, for example). Though there is indeed a great deal of literature on parsimonious voice-leading, all of our transformations are defined on ordered triples of the form (root, third, seventh), *not* on triads or seventh chords. As such, we will need to take a few steps in order to connect our work here with the literature that deals with these basic chord types directly.

For the moment, let us set aside the ordered-triple representation we have been using here and return to four-note seventh chords proper. Parsimonious relationships among seventh chords are shown in Douthett's "Four Cube Trio" (recall Figure 3.13); Figure 3.20 redraws a portion this

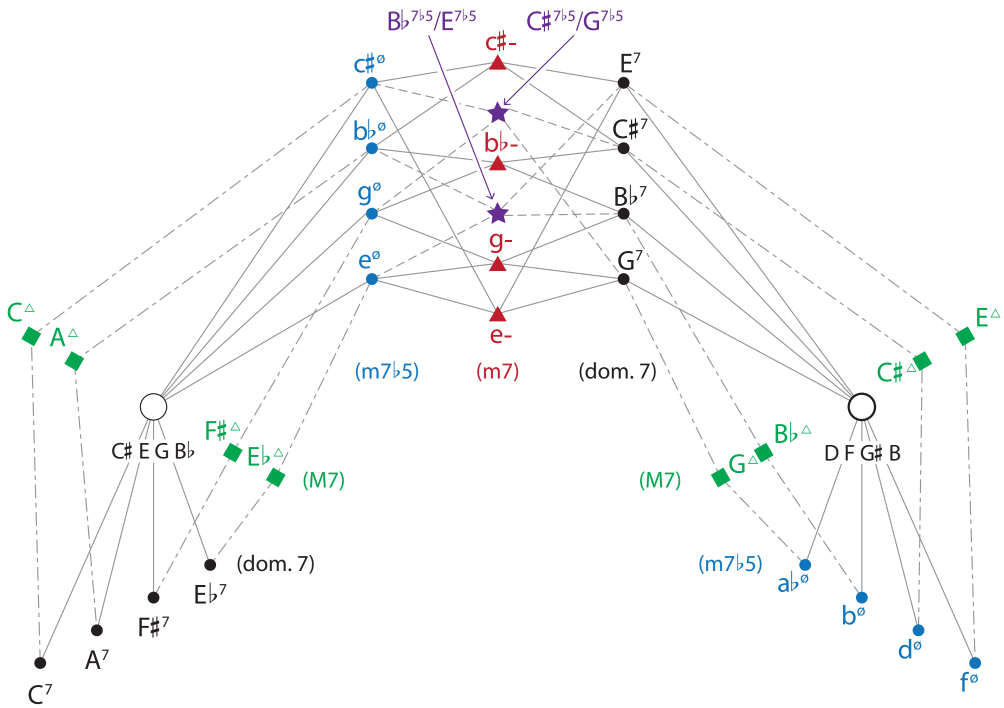


Figure 3.20. A portion of Four-Cube-Trio, redrawn to include major seventh chords. All lines, regardless of style, represent a single voice moving by half-step.

figure to include major seventh chords.[38] Because we are interested in this figure's application to jazz, the minor seventh chords have been labeled with root names, and the French sixth chords are labeled as dominant seventh chords with flatted fifths (a favorite chord of Thelonious Monk).[39]

By way of illustration, Figure 3.21 shows some of the transformations we have defined in the last two chapters in the Four-Cube Trio. The most important thing to note about this figure is that it is *not* a voice-leading graph of the transformations as they have been defined. Because all of our transformations are defined on ordered triples (ignoring the fifth of a seventh chord), mapping them into four-note space can be misleading. In our ordered triple representations, for example, there is no distinction between half-diminished and minor seventh chords, nor between the French sixths and the dominant sevenths (i.e., B♭7 is indistinguishable from B♭7♭5). This figure is

38. In addition to including the major seventh chords, this figure corrects some errors in Cohn's version (*Audacious Euphony*, Fig. 7.16, 158): in his version, the voice leading to and from the French sixth chords and minor seventh chords is incorrect. My thanks to Thomas Cooke-Dickens for helping to find many of these errors.

39. The French sixth chord, set class (0268), is of course invariant when transposed by tritone, so both roots are given in this figure.

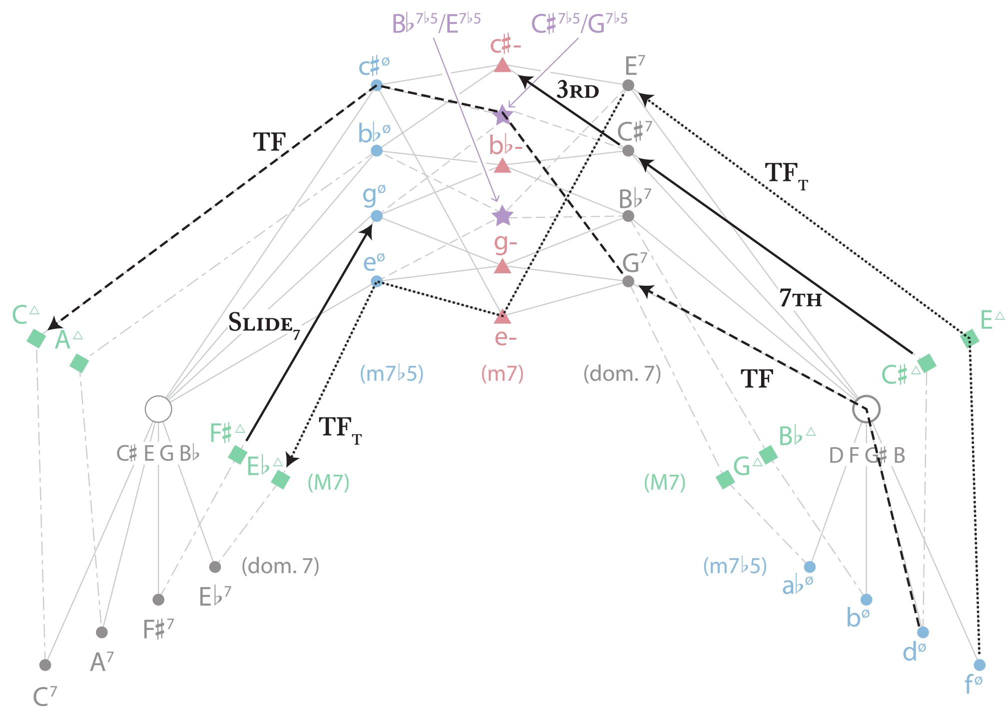


Figure 3.21. The transformations 3RD, 7TH, SLIDE$_7$, TF, and TF$_T$ in the Four-Cube-Trio.

included only to demonstrate that some of the transformations do represent single voice-leadings (the 3RD, 7TH, and SLIDE$_7$ transformations), while others do not. It is also worth noting that the SLIDE$_7$ transformation is the only one we have defined in which the voice-leading ascends (indicated by a clockwise motion in the figure).

The parsimonious picture that emerges after we collapse the chords which have the same ordered-triple representation is much less interesting.[40] Most damaging to the structure of the Four-Cube Trio is that the minor seventh chords become more discriminating: considered as an ordered triple, a minor seventh chord is only connected to one dominant seventh chord, not two (see Figure 3.22). This, plus the collapse of the French sixths into dominant sevenths, means that

40. This is not terribly surprising; our ordered triples are trichords of set classes (015), (016), and (026). Richard Cohn has shown ("Neo-Riemannian Operations, Parsimonious Trichords, and Their *Tonnetz* Representations") that the consonant triad, (037), is unique among trichords in its ability to form parsimonious relationships. Although his work there does not examine parsimonious relationships among members of different set classes, the fact that our three types are not nearly even means that we should not expect to find very many of these relationships.

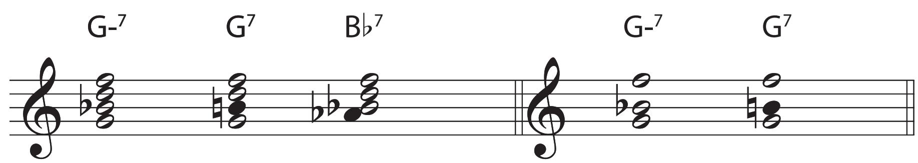


Figure 3.22. Parsimonious voice leading of the minor seventh chord, with its fifth (left) and without (right). Common tones are shown with hollow noteheads.

Starting Chord	Common Tones	Voice leading	Result	Transformation
CM7	r, t	$s - 1$	C7	7TH
CM7	r, s	$t - 1$	[CmM7]	[3RD]
CM7	t, s	$r + 1$	C♯m7	SLIDE_7
C7	r, t	$s + 1$	CM7	7TH^{-1}
C7	r, s	$t - 1$	Cm7	3RD
C7	t, s	–	–	–
Cm7	r, t	$s + 1$	[CmM7]	$[\text{7TH}^{-1}]$
Cm7	r, s	$t + 1$	C7	3RD^{-1}
Cm7	t, s	$r - 1$	BM7	SLIDE_7^{-1}

Table 3.2. Parsimonious voice-leading among members of S_{min}, S_{dom}, and S_{maj}. In this table, r, t, and s indicate the root, third, and seventh of a chord, respectively.

every chord is connected to exactly one other by single-half-step voice-leading. (For reference, a complete voice-leading roster for the ordered-triple representation is given in Table 3.2.)

We could generate the entire set of 36 ordered triples using the single-voice half-step transformations 7TH, 3RD, and SLIDE_7^{-1}:[41]

$$\text{CM7} \xrightarrow{\text{7TH}} \text{C7} \xrightarrow{\text{3RD}} \text{Cm7} \xrightarrow{\text{SLIDE}_7^{-1}} \text{BM7} \xrightarrow{\text{7TH}} \text{B7} \ldots \ \text{D}\flat\text{m7} \xrightarrow{\text{SLIDE}_7^{-1}} \text{CM7}$$

The resulting graph, however, is simply a circle, and does not mirror the rich voice-leading network of the Four-Cube Trio. We could redefine all of the other transformations in terms of these single voice-leadings (Dm7 $\xrightarrow{\text{TF}}$ G7 becomes Dm7 $\xrightarrow{\text{SLIDE}_7^{-1} \odot (\text{7TH} \odot \text{3RD} \odot \text{SLIDE}_7^{-1})^6 \odot \text{7TH}}$ G7),

41. The inverse is needed for SLIDE_7 because it was defined as a transformation from a major seventh chord to the minor seventh whose root is a half-step higher. Keeping the SLIDE_7 transformation intact in this circumstance would require inverses on both the 3RD and 7TH transformations.

but these decompositions would not seem to give much insight into tonal jazz—which is, after all, the aim of this study.

Throughout the course of the last two chapters, we have developed a fairly complete transformational system for jazz harmony. Taking ii–V space as our starting point, we have seen how it can altered in various ways to show different aspects of standard tonal jazz harmony. To this point, though, we have focused almost exclusively on chord symbols (via their abstraction into ordered triples). Of course, jazz harmony involves quite a bit more than relationships among three-note chords: these basic structures are altered in various ways by rhythm section members, and we have yet to say anything at all about the role harmony plays for an improvising musician. To do so, we'll need to expand our harmonic universe somewhat; the next chapter begins to take the first steps in that direction.