Lagrange's Equations:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot q_i}\right) - \frac{\partial L}{\partial q_i} = 0$$

$$L = T - V = \frac{1}{2} m l^2 \left[2\dot\phi_1^2 + \dot\phi_2^2 + 2\dot\phi_1\dot\phi_2\cos(\phi_1-\phi_2)\right] + mgl(2\cos\phi_1 + \cos\phi_2)$$

For ϕ_1: $\quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi_1}\right) - \frac{\partial L}{\partial \phi_1} = 0$

$$\frac{\partial L}{\partial \dot\phi_1} = 2ml^2\dot\phi_1 + ml^2\dot\phi_2\cos(\phi_1-\phi_2) \qquad \left(\dot\phi = \frac{d\phi}{dt} \therefore \frac{d}{dt}\left(\frac{d\phi}{dt}\right) = \frac{d^2\phi}{dt^2} = \ddot\phi\right.$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi_1}\right) = 2ml^2\ddot\phi_1 + ml^2\ddot\phi_2\cos(\phi_1-\phi_2) - ml^2\dot\phi_2\sin(\phi_1-\phi_2)\cdot(\dot\phi_1-\dot\phi_2)$$

$$\frac{\partial L}{\partial \phi_1} = -ml^2\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2) - 2mgl\sin\phi_1$$

$$\therefore 2ml^{\cancel{2}}\ddot\phi_1 + ml^{\cancel{2}}\ddot\phi_2\cos(\phi_1-\phi_2) - ml^{\cancel{2}}\dot\phi_2\sin(\phi_1-\phi_2)\cdot(\dot\phi_1-\dot\phi_2)$$
$$+ ml^{\cancel{2}}\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2) + 2mg\cancel{l}\sin\phi_1 = 0$$

$$\therefore 2ml\ddot\phi_1 + ml\ddot\phi_2\cos(\phi_1-\phi_2) - \cancel{ml\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2)} + ml\dot\phi_2^2\sin(\phi_1-\phi_2)$$
$$+ \cancel{ml\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2)} + 2mg\sin\phi_1 = 0$$

$$\therefore 2mL\ddot\phi_1 + mL\ddot\phi_2\cos(\phi_1-\phi_2) + mL\dot\phi_2\sin(\phi_1-\phi_2) + 2mg\sin\phi_1 = 0$$

$$L = T - V = \frac{1}{2} m l^2 \left[2\dot\phi_1^2 + \dot\phi_2^2 + 2\dot\phi_1\dot\phi_2\cos(\phi_1-\phi_2)\right] + mgl(2\cos\phi_1 + \cos\phi_2)$$

$$ml^2\dot\phi_1\dot\phi_2\cos(\phi_1-\phi_2)$$

For ϕ_2:

$$\frac{\partial L}{\partial \dot\phi_2} = ml^2\dot\phi_2 + ml^2\dot\phi_1\cos(\phi_1-\phi_2)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\phi_2}\right) = ml^2\ddot\phi_2 + ml^2\ddot\phi_1\cos(\phi_1-\phi_2) - ml^2\dot\phi_1\sin(\phi_1-\phi_2)(\dot\phi_1-\dot\phi_2)$$

$$\frac{\partial L}{\partial \phi_2} = ml^2\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2) - mgl\sin\phi_2$$

$$\therefore ml^2\ddot\phi_2 + ml^2\ddot\phi_1\cos(\phi_1-\phi_2) - ml^2\dot\phi_1\sin(\phi_1-\phi_2)(\dot\phi_1-\dot\phi_2)$$
$$- ml^2\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2) + mgl\sin\phi_2 = 0$$

$$\therefore ml^{\cancel{2}}\ddot\phi_2 + ml^{\cancel{2}}\ddot\phi_1\cos(\phi_1-\phi_2) - ml^{\cancel{2}}\dot\phi_1^2\sin(\phi_1-\phi_2) + \cancel{ml^2\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2)}$$
$$- \cancel{ml^2\dot\phi_1\dot\phi_2\sin(\phi_1-\phi_2)} + mg\cancel{l}\sin\phi_2 = 0$$

$$\therefore ml\ddot\phi_2 + ml\ddot\phi_1\cos(\phi_1-\phi_2) - ml\dot\phi_1^2\sin(\phi_1-\phi_2) + mg\sin\phi_2 = 0$$