

Introduction

The most common epidemiological models are compartmental models, in which a system of differential equations describes how parts of the population move between different disease states, or *compartments*. The canonical example of such a model is the SIR model, which has three compartments: Susceptible, Infected, and Recovered. The dynamics of a population modeled by this framework are shown in figure 1, with associated system of differential equations 2.

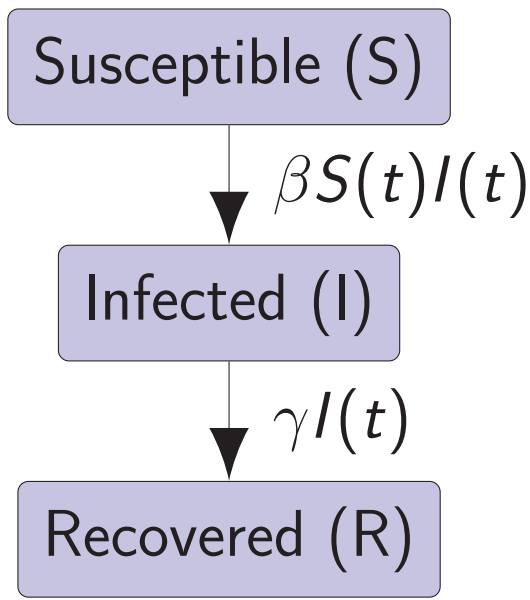


Figure : This flowchart depicts the standard SIR epidemiological model. It is accompanied by the system of differential equations 2.

$$\begin{aligned}\frac{dS}{dt} &= -\beta S(t)I(t) \\ \frac{dI}{dt} &= \beta S(t)I(t) - \gamma I(t) \\ \frac{dR}{dt} &= \gamma I(t) \\ N &= S(t) + I(t) + R(t)\end{aligned}$$

Figure : The system of differential equations governing the SIR model.

The SIR system, though very informative about basic disease dynamics, fails to capture the intricacies of more complicated diseases. This project examines the spread of tuberculosis (TB) in the United States (US) via compartmental models and stochastic, agent-based models. These explorations allow one to explore the impact, both epidemiological and economic, of various intervention strategies. These models are based on a 2012 compartmental model of TB in the US created by A. N. Hill, J. E. Becerra, and K. G. Castro.

The Basic Hill Model

The Hill model is another compartmental model, illustrated in figure 3.

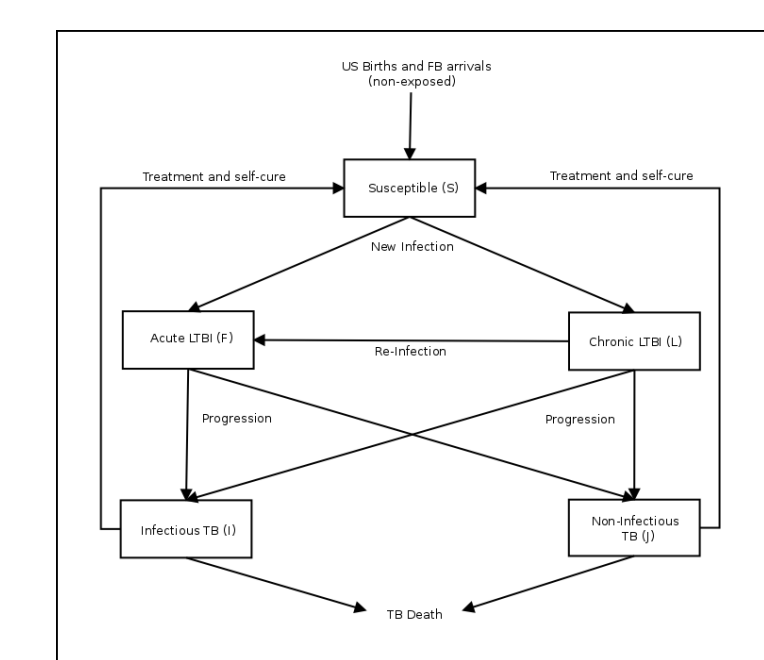


Figure : A flow chart representing the compartments of the Hill Model.

The illustration of this model is not full, as there are two subpopulations in the Hill Model. TB dynamics in the US differ radically between US born (USB) individuals and foreign born (FB) individuals, so this model maintains two distinct populations in which the disease can spread. There is a near identical diagram to describe the dynamics of the model for the FB population. Further, each compartment in this model also loses people to do natural death. The system of differential equations for this model is naturally more complex and is provided in an appendix. In order to implement this model, Hill, Becerra, and Castro used the R programming language, with the `lsoda` routine to numerically solve the differential equations.

For this implementation, various numerical solvers were implemented, including an Eulerian Method, a Quadratic Method, a Fourth-order runge kutta method, and the `lsoda` routine. Once the basic Hill model was implemented, further tracking capabilities were added as well as economic modeling. This implementation tracks total active TB cases, entering cases of LTBI, and treatment costs for active and latent disease for all populations. Tracking was used to evaluate various interventions explored in Hill, Becerra, and Castro's work, as well as interventions exploring the possibility of treating incoming LTBI cases.

TB Reduction Strategies for the United States

From the Hill model, we

Intervention Analysis

Intervention Analysis

incidencePlotRedEnLTBI.pdf

Figure : A graph of the projected incidence levels per million in the US-born, Foreign-born, and Total US population, given that LTBI rates in Foreign-born arrivals are reduced by 0%, 50%, 75%, and 100%.

Economic Modeling

We extended our basic implementation of the Hill model to incorporate economic data of treatment costs for Active and Latent Tuberculosis. In our model, we estimate the average health care costs to be \$14,014.90 and \$403.45 to treat a single case of Active or Latent TB respectively. Given these costs and the average treatment rates for Active and Latent TB in the USA, we modeled the expected economic burden of Tuberculosis projected over the next hundred years.

EnLTBIRedGroupCost.pdf

Figure : A graph of the cumulative implementation costs, savings, and net US health care system costs of reducing rates of LTBI in Foreign-born arrivals by 50%, 75%, and 100%. To determine the implementation cost for each intervention, the average cost to identify and treat entering Foreign-born individuals with LTBI was set to be \$600, \$800, and \$1000 for the 50%, 75%, and 100% reduction strategies respectively.

An Agent Based Implementation

In addition to the extended Hill model (implemented in R), we also created a stochastic agent-based counterpart in both NetLogo and C++. Stochastic models add notions of uncertainty into the model structure, while agent-based models simulate global behavior of the system by describing local behavior of individual agents in the population.

Figure 6 shows the incidence rate over time for both the R and NetLogo models. Although the NetLogo model is stochastic, its incidence rates roughly coincide with those given by the R model. With the C++ model (figure 7), we obtain even better convergence.

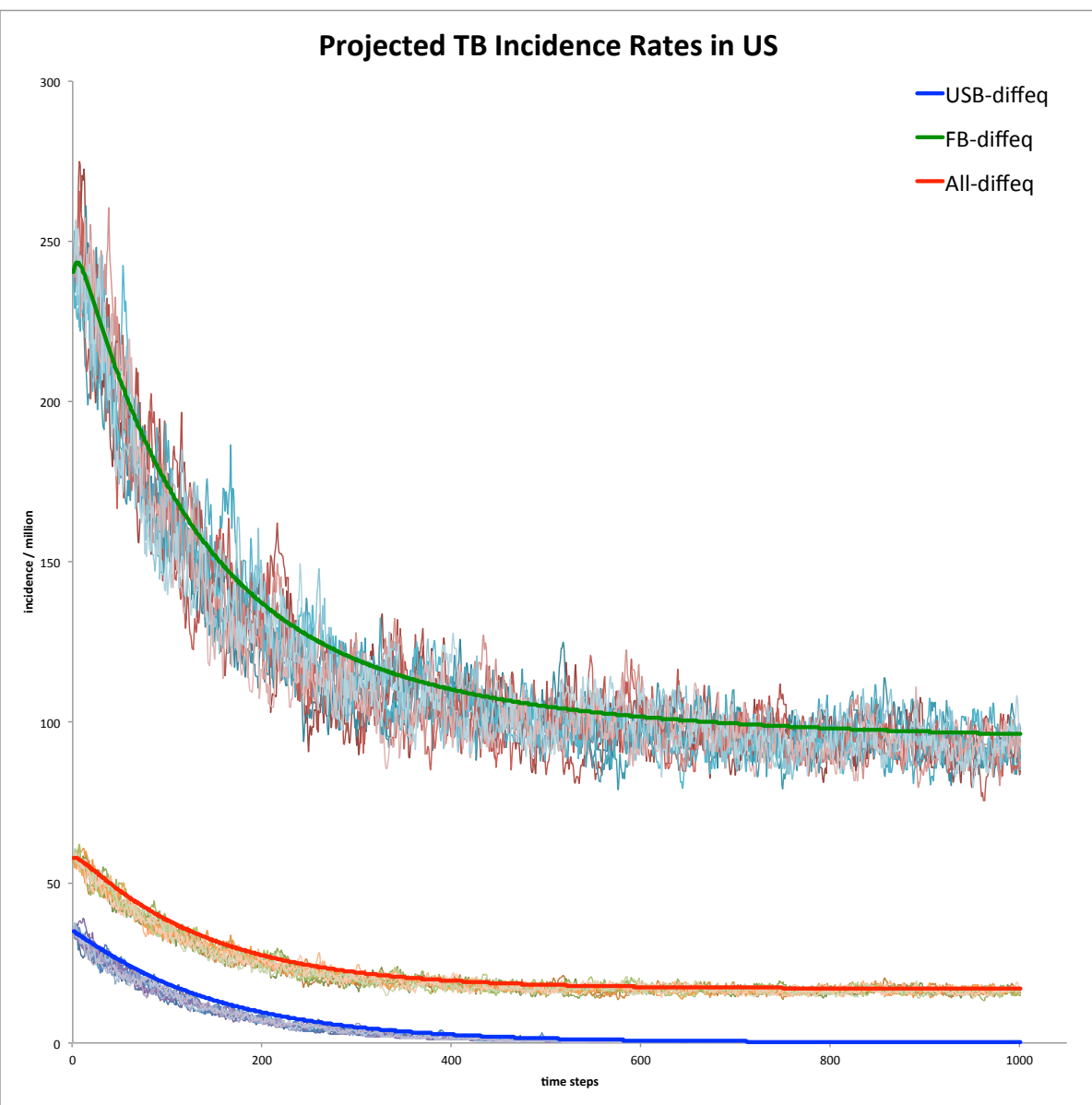


Figure : Incidence per million for R and NetLogo models (12 runs)

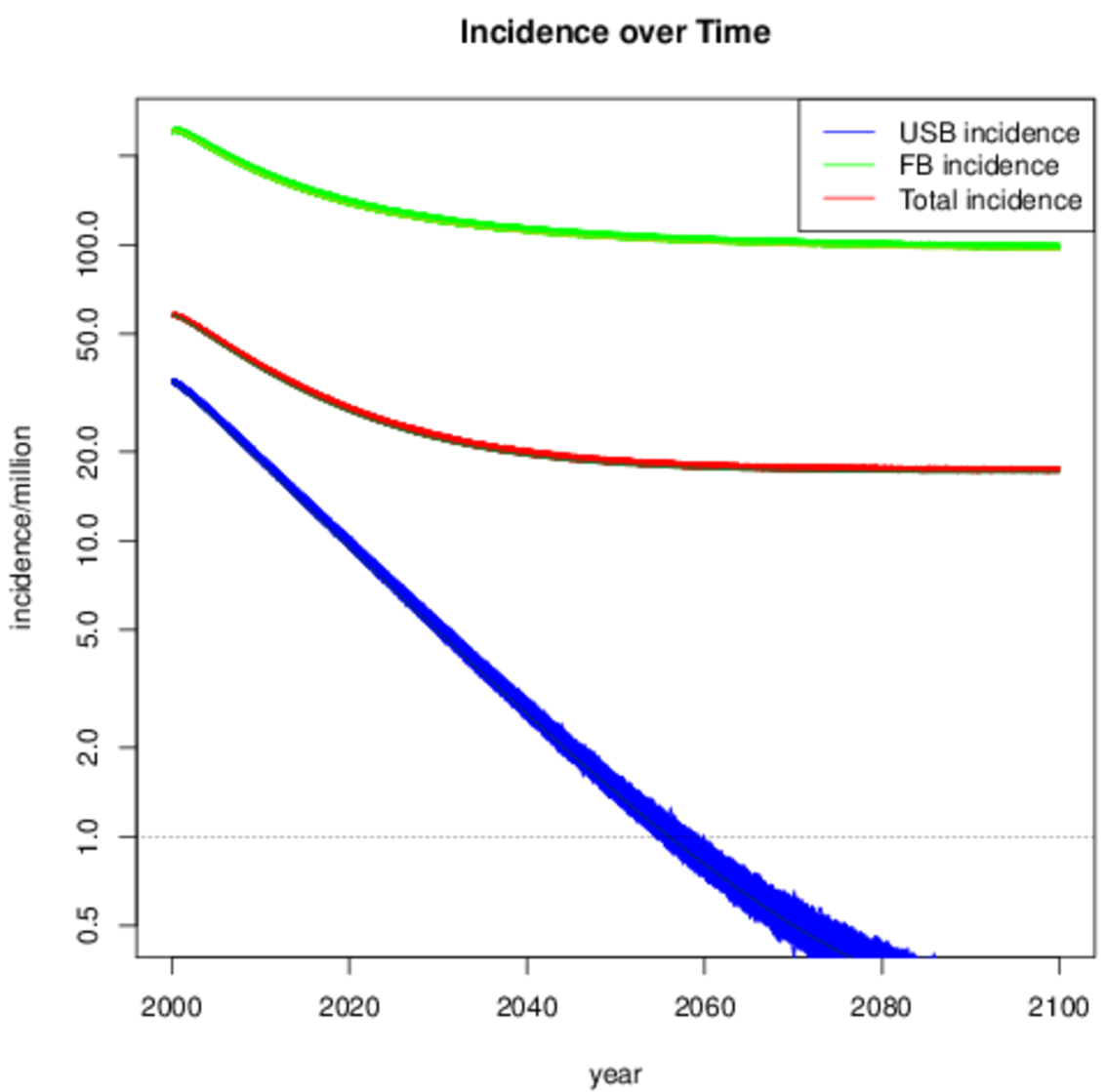


Figure : Incidence per million for R and C++ models (2100 runs)