ISYE 7406: HW 5

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1 Abstract

This paper assess the effectiveness of two types different ensemble methods: *random forests* and *boosting* to create a classifier model to predict the results of professional tennis matches. These methods were compared against baseline models such as logistic regression and K-nearest-neighbors. Cross validation was used in order to tune the parameters of these models as well as evaluate the testing error and variance of their predictions. The findings demonstrate that the XGBoost model produced the best classifier and outperformed the other models. The results highlight the importance of using robust cross validation techniques to assess model performance and tune model parameters, additionally they highlight the learning capability of ensemble methods.

2 Introduction

Every year there are thousands of professional tennis matches played on almost every continent in the the world, generating large amounts of data on player performance. Famously, IBM was an early pioneer in using machine learning techniques as well as leveraging Watson to generate insights and make match predictions. However, compared to the early years there are now more advanced methods available such as *random forests* and *boosting* which leverage the combining many "weak" learners to create a single strong classifier. This paper explores the effectiveness of these models more traditional against baseline models to predict tennis match outcomes.

For those interested, but not familiar with tennis here is a general guide to the rules: Tennis Basics.

3 Problem Statement/Exploratory Analysis

3.1 Problem Statement

The dataset used in this analysis has match ATP (men's professional tennis) data from 1968 which is considered the beginning of the "Open Era" in men's professional tennis [1].

However, this dataset only has complete in-game match statistics after 1991 therefore data from after 1991 was exclusively used. The goal of this analysis is develop a prediction/classification model that can accurately determine wether a player will win or lose a match based on their performance. This will allow coaches and players to make strategic/tactical decisions to optimize their chances at winning the match.

3.2 Exploratory Analysis

In total, this dataset contains 26 features from over 183,000 professional matches,. Of these 26 features only 8 pertained to in-game match performance, these along with player height, player age, and court surface were used to train the classifiers. Additionally, I created a binary variable (win) that indicates wether the player won or lost the match. In this analysis the win variable will serve as the response and the rest of the data will be used as predictors.

For this analysis I selected 10,000 samples at random and created the following histograms (Figure 1) to show the distributions of the features. A breakdown of the meaning of each of these features is as follows:

- 1. ht player height (cm)
- 2. age player age (yrs)
- 3. surface surface type (hard court, clay, grass, carpet)
- 4. ace aces hit
- 5. df double faults
- 6. svpt overall serve percent
- 7. 1stIn 1st serve in percent
- 8. 1stWon 1st serve win percent
- 9. 2ndWon 2nd serve win percent
- 10. bpSaved break points saved
- 11. bpFaced break points faced
- 12. win result of match (1-win, 0-lose)

Additionally to determine the predictive potential of this dataset I created a correlation matrix (Figure 2). Based on the correlation matrix, there are many features with "weak" correlations (< 0.5) to but there isn't a single strong predictor. Therefore this dataset appears to be a good candidate for ensemble models specifically *boosting* where multiple weak predictors are combined to make a single strong classifier.

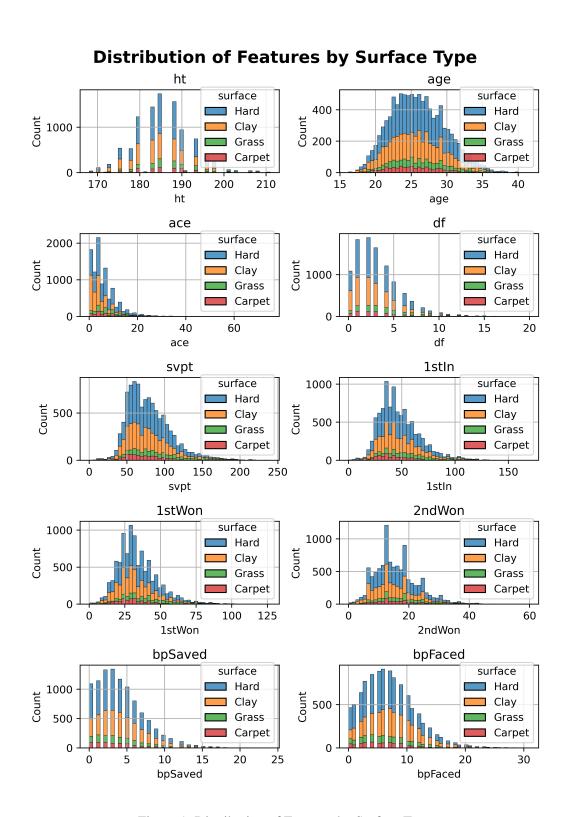


Figure 1: Distribution of Features by Surface Type

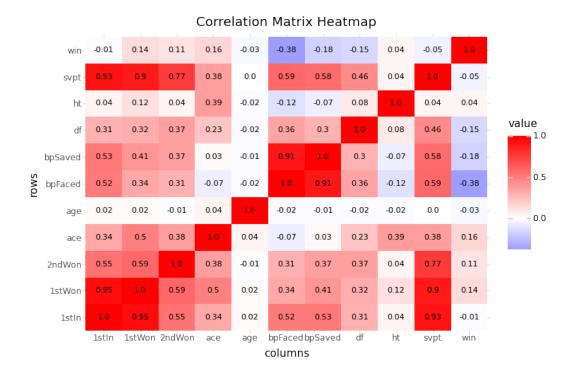


Figure 2: Correlation Matrix Heatmap

4 Methodology

In this section I will describe the different methods used to create/generate the ensemble classification models as well the baseline classification models. Additionally, I will briefly summarize their initial performance on the testing data and where applicable I will optimize model parameters using the testing error. Where applicable, when a decision boundary for predicting a class was needed the threshold value of 0.5 was chosen to give an equal weight to misclassifications of both wins and losses.

For this section the data was split (90/10) for training/testing and the same training and testing data was used for each model. When model parameters where optimized k-fold (k=10) cross validation was used.

4.1 Baseline Models

The baseline models that were chosen to test against the ensemble models are *Logistic Regression* and *K-Nearest-Neighbors*. These models were chosen because of their simplicity, interpretability, and widespread use in classification problems.

4.1.1 Logistic Regression

Logistic Regression is a linear model that estimates the probability of a given class using the logistic function.

Logit Regression Results							
Dep. Variable	 :	W	in No. Obs	======= ervations:		9000	
Model:			it Df Resi			8988	
Method:		_	LE Df Mode			11	
Date:	Thu	. 13 Mar 20	25 Pseudo	R-sau.:		0.3456	
Time:		17:43:		elihood:		-4082.2	
converged:		Tr	ue LL-Null			-6238.2	
Covariance Type:		nonrobust LLR p-value:			0.000		
	coef	std err	z	P> z	[0.025	0.975]	
const	-0.0717	0.028	-2.580	0.010	-0.126	-0.017	
ht	-0.2082	0.030	-6.854	0.000	-0.268	-0.149	
age	-0.1853	0.028	-6.661	0.000	-0.240	-0.131	
ace	-0.3076	0.039	-7.817	0.000	-0.385	-0.230	
df	-0.0243	0.036	-0.676	0.499	-0.095	0.046	
svpt	-3.2817	0.268	-12.260	0.000	-3.806	-2.757	
1stIn	-0.1982	0.166	-1.193	0.233	-0.524	0.127	
1stWon	2.9752	0.181	16.462	0.000	2.621	3.329	
2ndWon	1.4224	0.096	14.856	0.000	1.235	1.610	
bpSaved	1.7917	0.103	17.404	0.000	1.590	1.994	
bpFaced	-2.2650	0.145	-15.580	0.000	-2.550	-1.980	
surface_num	-0.1126	0.028	-4.049	0.000	-0.167	-0.058	

Figure 3: Logistic Regression Model

$$\log(\frac{P}{1-P}) = B_0 + B_1 x_{i,1} ... B_{p-1} x_{i,p-1}$$
(1)

The basic assumption of the Logistic Regression classification model is that it models conditional probability of the response directly and makes no assumptions about the distribution of predictors. For this baseline model all predictor variables were used, the intial training error and testing errors of this model were 0.201 and 0.206 respectively. See Figure 3 for a summary of the model.

4.1.2 K-Nearest-Neighbors

K-Nearest-Neighbors (KNN) is a classification model that uses distance metrics to assign data to a class based on its "k" nearest neighbors. It is another commonly used model since it is another non-parametric model, meaning that the model isn't making assumptions about the underlying data distributions. Since there was a relatively large amount of data I decided to use cross validation to optimize the k parameter by testing k-values from 1 to 100. After tuning this parameter the optimal model was KNN = 61, see Figure 4 . This model's training and testing errors were 0.259 and 0.277 respectively.

4.2 Ensemble Models

The ensemble models that were chosen for this analysis are random forests and XGBoost. Both of these models improve classification performance by by combining multiple weak learners. However, these models differ in their learning strategies: Random Forest uses on bagging, while XGBoost uses boosting.

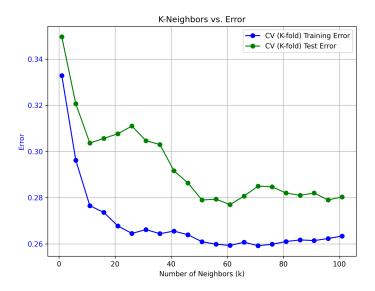


Figure 4: KNN Model

4.2.1 Random Forest

Random Forest is an ensemble learning method that uses decision trees with a random subset of features and bagging to decrease the variance in classification. The key idea in bagging is to use bootstrapped (randomly sampled w/replacement) data to make each tree and use the majority vote for classification. The random subset of features introduces diversity among the trees, leading to more robust and generalizable predictions. For this analysis I used cross validation to optimize the number of estimators, see Figure 5. The optimal value for number number of estimators was found to be 81 and this model's training and testing errors were 0.292 and 0.281 respectively.

4.2.2 Boosting: XGBoost

XGBoost (Extreme Gradient Boosting) is a boosting ensemble method that builds decision trees sequentially, with each tree built correcting misclassifications from the previous tree. Unlike Random Forest, which grows trees independently XGBoost assigns greater weight to "weak" learners and the weighted average of the trees is used to improve the training error. For this analysis I used the number of estimators found for the random forest and used cross validation to optimize the learning rate, see Figure 6. The optimal value for the learning rate was found to be 0.21 and this model's training and testing errors were 0.212 and 0.248 respectively.

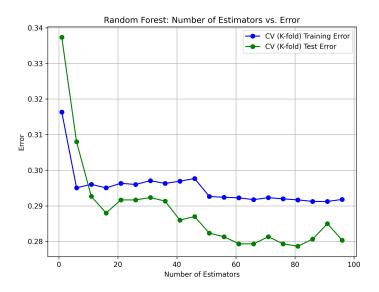


Figure 5: Random Forest Model

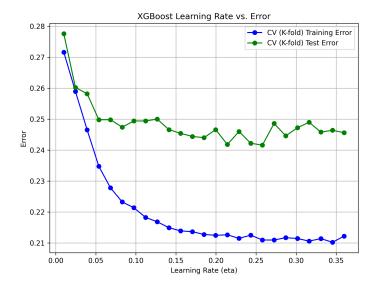


Figure 6: XGBoost Model

Model	Mean Test Error	Mean Test Variance		
Logistic Regression	0.208	1.79E-4		
KNN	0.250	1.12E-4		
Random Forest	0.287	1.69E-4		
XGBoost	0.187	1.60E-4		

Table 1: Classification Model Comparison

5 Results/Analysis

This section contains a brief comparative analysis of the different models and their performance on dataset.

Initially, the classification models were evaluated on one split of of the data while the model parameters were selected using cross validation. In order to provide a more thorough comparison 100 Monte Carlo cross validation simulations were used to evaluate the testing errors and variances of each model, see Table 1. One of the most significant findings of this analysis is how much XGBoost improved from it original performance on the testing data set. It improved by from its performance on one split of the data by 22% to become the best model. This can be attributed to the fact that allowing XGBoost to see many partitions of the data it was better able to improve upon its weak learners effectively. In contrast, **Random Forest** showed much weaker performance, and its its improvement was much more modest compared to XGBoost.

6 Conclusions

In summary, out of the different models the XGBoost classification model was able to significantly outperform Random Forests as well as baseline models such as Logistic Regression and KNN. The improvement of the XGBoost model after 100 Monte Carlo simulations highlights the strengths of boosting techniques to improve the classification model.

However, while the final model is strong, it is still wrong approximately 20% of the time. This shows the inherent challenges with working with the complexity of professional tennis match data and fully capturing the underlying patterns.

6.1 Lessons Learned

An important lesson learned is that while boosting techniques like XGBoost can significantly enhance predictive performance, there is still room for improvement. Future work could be done to further fine-tune parameters or experimenting with alternative ensemble methods to reduce the error rate. Additionally, considering the cost of misclassifications in real-world applications, techniques such as adjusting the decision threshold or using

cost-sensitive learning could be valuable in refining model performance. Finally, these results show the importance of using cross-validation techniques for both model selection and performance evaluation.

7 References

[1] Sijo Manikandan. ATP matches. URL: https://www.kaggle.com/datasets/sijovm/atpdata/data.

A Appendix A: Python Code

```
HW5 Ensemble Methods
3 .. code:: ipython3
      import pandas as pd
      from datetime import datetime, timedelta
      import numpy as np
      import matplotlib.pyplot as plt
      import seaborn as sns
9
      from sklearn.preprocessing import LabelEncoder,StandardScaler,
10
      MinMaxScaler, RobustScaler, Normalizer
11
     from sklearn import model_selection
     from sklearn.model_selection import train_test_split,RepeatedKFold
12
      ,cross_val_score
      from plotnine import ggplot, aes, geom_tile, scale_fill_gradient,
13
      \verb|theme_minimal|, \verb|theme|, \verb|geom_boxplot||
      from plotnine import ggsave, scale_y_reverse, labs,
      scale_fill_gradient2,geom_line,geom_point,scale_color_manual,
      geom_text
      import statsmodels.api as sm
15
      from sklearn.metrics import confusion_matrix,
16
      {\tt ConfusionMatrixDisplay}\;,\;\;{\tt accuracy\_score}\;,\;\;{\tt precision\_score}\;,\\ {\tt roc\_curve}\;,\;\;
      from sklearn.neighbors import NearestNeighbors,
      KNeighborsClassifier
      from sklearn.ensemble import GradientBoostingClassifier,
18
      RandomForestClassifier
      import xgboost as xgb
19
      from tqdm import tqdm
20
21
22 .. code:: ipython3
23
      data = pd.read_csv("atp_matches_till_2022.csv")
24
26 .. code:: ipython3
27
      data['tourney_yr'] = data['tourney_id'].apply(lambda x: x.split('
      -')[0]).astype(int)
      data = data.loc[data['tourney_yr'] >= 1991]
29
31 .. code:: ipython3
32
      data_win = data[['winner_id','winner_name','surface','winner_ht','
      winner_age','w_ace', 'w_df','w_svpt','w_1stIn','w_1stWon','
      w_2ndWon','w_bpSaved','w_bpFaced']].dropna()
```

```
data_lose = data[['loser_id','loser_name','surface','loser_ht','
     loser_age','l_ace', 'l_df','l_svpt','l_1stIn','l_1stWon','l_2ndWon
     ','l_bpSaved','l_bpFaced']].dropna()
     data_win.columns = [col.split('_', 1)[1] if '_' in col else col
35
     for col in data_win.columns]
     data_lose.columns = [col.split('_', 1)[1] if '_' in col else col
     for col in data_lose.columns]
     data_win['win'] = 1
37
     data_lose['win'] = 0
39
 .. code:: ipython3
40
41
     dataset = pd.concat([data_win, data_lose], axis=0)
42
     dataset = dataset.sample(n=10000, random_state=42).reset_index(
43
     drop=True)
44
 .. code:: ipython3
45
46
     dataset.describe()
47
48
50
51
52 .. raw:: html
53
     <div>
54
     <style scoped>
55
         .dataframe tbody tr th:only-of-type {
             vertical-align: middle;
57
58
         .dataframe tbody tr th {
60
             vertical-align: top;
61
62
         .dataframe thead th {
64
             text-align: right;
65
         }
     </style>
67
     68
       <thead>
         70
           71
           id
           ht
73
           age
74
           ace
75
           df 
76
           svpt
```

```
1stIn
78
        1stWon
79
        2ndWon
80
        bpSaved 
81
        bpFaced 
82
        win 
      84
     </thead>
85
     87
        count 
88
89
        10000.000000
        10000.000000
90
        10000.000000
91
        10000.000000
92
93
        10000.000000
        10000.000000
94
        10000.000000
95
        10000.000000
96
        10000.000000
97
        10000.000000
98
        10000.000000
        10000.000000
100
      101
      102
        mean 
103
        106015.860700
104
        185.425900
        25.967420
106
        5.591200
107
        3.120200
108
        79.364800
109
        47.773800
        33.860100
111
        15.775600
112
        4.147000
        6.944000
114
        >0.504400
      116
      std
118
        14172.725522
119
        6.581918
120
        3.818011
121
        5.026869
        2.534402
        29.402047
124
        19.260268
        14.237858
126
```

```
7.057792
127
        3.248484
128
        4.443674
        >0.500006
130
      131
      min 
        100284.000000
134
        168.000000
        16.200000
136
        0.000000
138
        0.000000
        0.000000
139
        0.000000
140
        0.000000
141
142
        0.000000
        0.000000
143
        0.000000
144
        0.000000
145
      146
      147
        25%
        102110.000000
149
        180.000000
150
        23.200000
151
        2.000000
152
        1.000000
153
        58.000000
154
        34.000000
155
        24.000000
156
        11.000000
157
        2.000000
158
        4.000000
159
        0.000000
160
      162
       50%
163
        103507.000000
        185.000000
165
        25.700000
166
        4.000000
        3.000000
168
        74.000000
169
        44.000000
170
        31.000000
        15.000000
        4.000000
173
        6.000000
174
        1.000000
```

```
176
        177
         75%
         104745.000000
179
         190.000000
180
         28.600000
         8.000000
182
         4.000000
183
         95.000000
         58.000000
185
         41.000000
186
187
         20.000000
         6.000000
188
         10.000000
189
         1.000000
190
191
        192
193
         max
         210013.000000
194
         211.000000
195
         41.600000
196
         75.000000
         20.000000
198
         240.000000
199
         170.000000
200
         127.000000
201
         60.000000
202
         24.000000
         31.000000
204
         1.000000
205
        206
      207
     208
     </div>
209
211
212
213
  .. code:: ipython3
214
     dataset.info()
215
217
  .. parsed-literal::
218
     <class 'pandas.core.frame.DataFrame'>
220
     RangeIndex: 10000 entries, 0 to 9999
     Data columns (total 14 columns):
222
     # Column Non-Null Count Dtype
     --- -----
224
```

```
0
           id
                     10000 non-null int64
225
                     10000 non-null object
226
           name
            surface 10000 non-null object
227
        2
                     10000 non-null float64
        3
           ht.
228
                     10000 non-null float64
           age
229
                     10000 non-null float64
230
        5
            ace
                     10000 non-null float64
        6
           df
231
        7
                     10000 non-null float64
           svpt
                     10000 non-null float64
233
        8
           1stIn
        9
           1stWon
                     10000 non-null float64
234
        10 2ndWon 10000 non-null float64
           bpSaved 10000 non-null float64
236
        12 bpFaced 10000 non-null float64
        13 win
                     10000 non-null int64
238
       dtypes: float64(10), int64(2), object(2)
239
240
       memory usage: 1.1+ MB
241
242
   .. code:: ipython3
243
244
245
      numeric_cols = dataset.drop(columns=['id', 'name', 'win', 'surface
246
      ']).columns
247
      # Create a 2x5 grid
248
      fig, axes = plt.subplots(nrows=5, ncols=2, figsize=(7, 10))
249
       axes = axes.flatten() # Flatten the 2D array for easy iteration
250
251
      # Plot each histogram
252
      for i, col in enumerate(numeric_cols):
253
           sns.histplot(data=dataset, x=col, bins=50, hue="surface",
254
      hue_order = ['Hard','Clay','Grass','Carpet'], ax=axes[i],multiple=
      "stack")
           axes[i].set_title(col)
255
           axes[i].grid(True)
256
257
258
      fig.suptitle("Distribution of Features by Surface Type", fontsize
259
      =16, fontweight='bold')
      plt.tight_layout()
260
262
      plt.savefig("Distribution of Features by Surface Type.png", format
263
      ="png", dpi=500)
      plt.show()
264
265
266
267
268 .. image:: output_7_0.png
```

```
269
270
   .. code:: ipython3
272
       le = LabelEncoder()
273
274
       dataset['surface_num'] = le.fit_transform(dataset['surface'])
275
276
   .. code:: ipython3
       corr = dataset.drop(columns=['id', 'name', 'surface']).corr().
278
       round(2).reset_index().melt(id_vars="index", var_name="columns",
       value_name="value")
279
       corr = corr.rename(columns={"index": "rows"})
280
281
282
       plot = (ggplot(corr, aes(x='columns', y='rows', fill='value'))
                + geom_tile()
283
                + scale_fill_gradient2(low="blue", high="red", mid="white"
284
       , midpoint=0)
                + geom_text(aes(label='value'), color='black', size=8)
285
                + theme_minimal()
286
                + labs(title="Correlation Matrix Heatmap")
287
               )
288
289
       #plot.tight_layout()
290
       plot.show()
291
       ggsave(plot, filename='correlation_matrix.png', width=7.5, height
292
       =5)
293
294
295
   .. image:: output_9_0.png
296
      :width: 640px
297
      :height: 480px
298
300
   .. parsed-literal::
301
302
       C:\Users\mmcdo\miniconda3\Lib\site-packages\plotnine\ggplot.py
303
       :615: PlotnineWarning: Saving 7.5 x 5 in image.
       {\tt C:\backslash Users\backslash mmcdo\backslash miniconda3\backslash Lib\backslash site-packages\backslash plotnine\backslash ggplot.py}
       :616: PlotnineWarning: Filename: correlation_matrix.png
305
   .. code:: ipython3
307
308
       data = dataset.drop(columns=['id', 'name', 'surface'])
309
       response = data[['win']]
310
       features = data.drop(columns=['win'])
311
```

```
train_size = round(len(data)*0.9)
312
       test_size = round(len(data)*0.1)
313
314
       train_features, test_features, train_response, test_response =
315
      train_test_split(
         features, response, train_size = train_size , test_size =
316
      test_size, random_state=123)
317
       train_data = pd.concat([train_response, train_features], axis=1)
319
       test_data = pd.concat([test_response, test_features], axis=1)
320
321
  .. code:: ipython3
322
323
       def standardize_data(features, response):
324
325
326
         #scale dataset
327
         feautre_scaler = StandardScaler()
328
         response_scaler = StandardScaler()
329
         feautre_scaler.fit(features)
330
331
         response_scaler.fit(response)
332
         features_scaled = feautre_scaler.transform(features) #scaled
333
      Features
         response_scaled = response_scaler.transform(response) # scaled
334
      Response
336
337
         features_scaled = pd.DataFrame(features_scaled, columns=features
338
      .columns)
         response_scaled = pd.DataFrame(response_scaled, columns=response
339
         return features_scaled,response_scaled,feautre_scaler,
      response_scaler
341
  .. code:: ipython3
343
      X_train = train_features
344
       Y_train = train_response
      X_test = test_features
346
       Y_test = test_response
347
349
      X_{train\_stand}, Y_{train\_stand}, X_{train\_stand\_scaler},
350
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
```

```
X_test_stand, Y_test_stand, X_test_stand_scaler,
351
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
352
      X_train_stand = sm.add_constant(X_train_stand)
353
      X_test_stand = sm.add_constant(X_test_stand)
355
356
      logit = sm.Logit(Y_train.reset_index(drop=True), X_train_stand)
      logit = logit.fit()
358
359
      # Print summary
      print(logit.summary())
361
362
      train_predictions = logit.predict(X_train_stand)
      test_predictions = logit.predict(X_test_stand)
364
365
366
      train_predictions = (train_predictions > 0.5).astype(int)
367
      test_predictions = (test_predictions > 0.5).astype(int)
368
      # lr = LogisticRegression()
370
      # lr.fit(X_train_scaled, y_train)
371
      # y_pred = lr.predict(X_test_scaled)
373
      # nb = GaussianNB(priors = [0.5,0.5])
374
      # nb.fit(X_train, Y_train.values.ravel())
377
      # train_predictions = nb.predict(X_train)
378
      # test_predictions = nb.predict(X_test)
379
380
      lr_train_err = 1 - accuracy_score(y_true=Y_train.values, y_pred=
381
      train_predictions)
      lr_test_err = 1 - accuracy_score(y_true=Y_test.values, y_pred=
382
      test_predictions)
      print(f"Train Error: {round(lr_train_err, 4) }")
383
      print(f"Test Error: {round(lr_test_err, 4) }")
385
386
     parsed-literal::
387
388
      Optimization terminated successfully.
389
               Current function value: 0.453582
               Iterations 6
391
                                  Logit Regression Results
392
393
      ______
```

```
Dep. Variable:
                                   win
                                         No. Observations:
394
            9000
     Model:
                                  Logit
                                         Df Residuals:
395
            8988
     Method:
                                    MLE
                                         Df Model:
396
             11
                        Thu, 13 Mar 2025
                                         Pseudo R-squ.:
     Date:
397
          0.3456
                                         Log-Likelihood:
     Time:
                               17:43:24
         -4082.2
                                         LL-Null:
     converged:
                                   True
399
         -6238.2
                                         LLR p-value:
     Covariance Type:
                              nonrobust
400
          0.000
401
402
                     coef
                            std err
                                    z P>|z|
     [0.025 0.975]
403
                             0.028
     const
                   -0.0717
                                       -2.580
                                                 0.010
404
     -0.126
               -0.017
                              0.030
                   -0.2082
                                                 0.000
                                       -6.854
     -0.268
               -0.149
                              0.028
                                       -6.661
                                                 0.000
     age
                   -0.1853
406
     -0.240
                -0.131
     ace
                   -0.3076
                              0.039
                                       -7.817
                                                 0.000
407
     -0.385
                -0.230
     df
                   -0.0243
                              0.036
                                       -0.676
                                                  0.499
     -0.095
                0.046
                   -3.2817 0.268
                                                 0.000
     svpt
                                      -12.260
409
     -3.806
                -2.757
                                                  0.233
     1stIn
                   -0.1982
                              0.166
                                       -1.193
410
     -0.524
                0.127
     1stWon
                   2.9752
                              0.181
                                       16.462
                                                  0.000
411
     2.621
                3.329
     2ndWon
                   1.4224
                              0.096
                                      14.856
                                                 0.000
412
     1.235
               1.610
     bpSaved
                   1.7917
                              0.103
                                       17.404
                                                 0.000
413
     1.590
               1.994
     bpFaced
                   -2.2650
                              0.145 -15.580
                                                  0.000
414
     -2.550
                -1.980
                  -0.1126
                              0.028
                                       -4.049
                                                  0.000
415
     surface_num
     -0.167 -0.058
416
     ______
```

```
Train Error: 0.2012
417
       Test Error: 0.206
418
419
420
   .. code:: ipython3
421
422
       X_train = train_features
423
       Y_train = train_response
424
       X_test = test_features
       Y_test = test_response
426
427
428
       X_train_stand, Y_train_stand,X_train_stand_scaler,
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
       X_test_stand, Y_test_stand, X_test_stand_scaler,
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
430
       X_train_stand = sm.add_constant(X_train_stand)
431
       X_test_stand = sm.add_constant(X_test_stand)
432
433
434
       n1 = round(len(data)*0.9)
       n2 = round(len(data)*0.1)
435
436
       # Initialize the result DataFrame
437
       result_df = pd.DataFrame({
438
           "B Cross Validations": [],
439
           "Average Error Rate": [],
           "Average Error Variance": []
441
       })
442
443
       for B in b:
444
           # Reinitialize the lists for storing metrics for each B
445
446
           test_error_rates = []
           test_accuracies = []
448
449
           for i in range(B):
451
               X_combined = pd.concat([X_train_stand, X_test_stand], axis
452
      =0)
               y_combined = pd.concat([Y_train, Y_test], axis=0).values.
453
      flatten()
455
               X_train_sample, X_test_sample, Y_train_sample,
456
      Y_test_sample = train_test_split(
                    X_combined, y_combined, train_size=n1, test_size=n2,
457
      random_state=i
```

```
)
458
459
                # Train the model on the training data
                logit = sm.Logit(Y_train_sample, X_train_sample)
461
                logit = logit.fit(disp=0)
462
                test_predictions = logit.predict(X_test_sample)
464
465
467
                test_predictions = (test_predictions > 0.5).astype(int)
468
                #lr.fit(X_train, Y_train)
470
471
                # Make predictions on the testing data
472
473
474
475
476
                # Calculate error rate (MSE)
477
                nb_test_error_rate = 1 - accuracy_score(y_true=
478
      Y_test_sample, y_pred=test_predictions)
479
                # Append to the lists
480
481
                test_error_rates.append(nb_test_error_rate)
482
483
           # Calculate the variance of the testing error rates after all
484
      iterations for this B
           test_error_variance = np.var(test_error_rates)
485
           # Calculate the averages for the current B value
487
488
           average_test_error_rate = np.mean(test_error_rates)
489
491
           metrics_df = pd.DataFrame({
492
                "B Cross Validations": [B],
                "Average Error Rate": [average_test_error_rate],
494
                "Average Error Variance": [test_error_variance]
495
           })
497
498
           result_df = pd.concat([result_df, metrics_df], axis=0)
500
501
       result_df
502
503
504
```

```
505
506
  .. raw:: html
508
      <div>
509
      <style scoped>
         .dataframe tbody tr th:only-of-type {
511
             vertical-align: middle;
512
         }
514
         .dataframe tbody tr th {
515
516
             vertical-align: top;
         }
517
518
          .dataframe thead th {
519
             text-align: right;
520
521
522
      </style>
      523
       <thead>
524
525
         526
           B Cross Validations 
527
           Average Error Rate
528
           Average Error Variance
529
         530
       </thead>
531
532
       533
           >0
534
535
           10.0
           0.2083
536
           0.000179
537
         538
       539
      540
      </div>
541
543
544
  .. code:: ipython3
545
546
      final_cv_result = pd.concat([pd.DataFrame({
547
         'Model': ['Logistic Regression'],
         'Mean Test Error': [result_df["Average Error Rate"].iloc[0]],
549
         'Test Error Var': [result_df["Average Error Variance"].iloc
550
      [0]
      })])
551
      #model_errors['Logistic Regression'] = test_error_rates
552
```

```
553
  .. code:: ipython3
554
       X_train = train_features
556
       Y_train = train_response
557
       X_test = test_features
       Y_test = test_response
559
560
      X_train_stand, Y_train_stand, X_train_stand_scaler,
562
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
      X_test_stand, Y_test_stand, X_test_stand_scaler,
563
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
564
565
      K = 101
566
       train_error = []
567
       test_error = []
568
       for k in range(1,K+1,5):
569
         knn = KNeighborsClassifier(n_neighbors=k)
570
         knn.fit(X_train_stand, Y_train.values.ravel())
571
         train_predictions = knn.predict(X_train_stand)
572
         test_predictions = knn.predict(X_test_stand)
573
574
         cv = RepeatedKFold(n_splits=10, n_repeats=3, random_state=1)
575
         train_score = 1 - model_selection.cross_val_score(knn,
576
      X_train_stand, Y_train.astype(int).values.ravel(), cv=cv, scoring=
      'accuracy').mean()
         test_score = 1 - model_selection.cross_val_score(knn,
577
      X_test_stand, Y_test.astype(int).values.ravel(), cv=cv, scoring='
      accuracy').mean()
578
         # train_error.append(1 - accuracy_score(y_true=Y_train.values.
579
      ravel(), y_pred=train_predictions))
         train_error.append(train_score)
580
         # test_error.append(1 - accuracy_score(y_true=Y_test.values.
      ravel(), y_pred=test_predictions))
         test_error.append(test_score)
582
584
         # # Create a plot with dual y-axes
585
       plt.close('all')
      fig, ax1 = plt.subplots(figsize=(8, 6))
587
588
       # Plot MSE on the first y-axis
       ax1.plot(range(1,K+1,5), train_error, 'b-', marker='o', label='CV')
500
      (K-fold) Training Error')
```

```
ax1.set_xlabel('Number of Neighbors (k)')
591
       ax1.set_ylabel('Error', color='blue')
592
       ax1.tick_params(axis='y', labelcolor='blue')
593
594
595
       ax1.plot(range(1,K+1,5), test\_error, 'g-', marker='o', label='CV (
      K-fold) Test Error')
       ax1.set_xlabel('Number of Neighbors (k)')
597
       #ax1.set_ylabel('Error', color='blue')
       ax1.tick_params(axis='y', labelcolor='blue')
599
       plt.legend()
600
       plt.grid()
601
       plt.title("K-Neighbors vs. Error")
602
       plt.savefig("K-Neighbors vs. Error.png", format="png", dpi=300)
603
605
       # knn_train_err = (train_error[12])
       # knn_test_err = (test_error[12])
606
       # print(f"Train Error: {round(knn_train_err, 4) }")
607
       # print(f"Test Error: {round(knn_test_err, 4) }")
608
609
610
     image:: output_15_0.png
612
613
614
   .. code:: ipython3
615
616
       knn_train_err = (train_error[12])
617
       knn_test_err = (test_error[12])
618
       print(f"Train Error: {round(knn_train_err, 4) }")
619
       print(f"Test Error: {round(knn_test_err, 4) }")
620
621
622
   .. parsed-literal::
623
624
       Train Error: 0.2593
625
       Test Error: 0.277
626
628
   .. code:: ipython3
629
       X_train = train_features
631
       Y_train = train_response
632
       X_test = test_features
       Y_test = test_response
634
635
       X_train_stand, Y_train_stand, X_train_stand_scaler,
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
```

```
X_test_stand, Y_test_stand, X_test_stand_scaler,
637
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
638
639
       n1 = round(len(data)*0.9)
       n2 = round(len(data)*0.1)
641
       b = [10]
642
       # Initialize the result DataFrame
       result_df = pd.DataFrame({
644
           "B Cross Validations": [],
645
           "Average Error Rate": [],
           "Average Error Variance": []
647
       })
648
650
       for B in b:
           # Reinitialize the lists for storing metrics for each B
651
652
           test_error_rates = []
653
           test_accuracies = []
654
655
           for i in range(B):
657
                X_combined = pd.concat([X_train_stand, X_test_stand], axis
658
      =0)
                y_combined = pd.concat([Y_train, Y_test], axis=0).values.
659
      flatten()
661
                X_{\text{train\_sample}}, X_{\text{test\_sample}}, Y_{\text{train\_sample}},
662
      Y_test_sample = train_test_split(
                    X_combined, y_combined, train_size=n1, test_size=n2,
663
      random_state=i
                )
664
                # Train the model on the training data
666
                knn = KNeighborsClassifier(n_neighbors=60)
667
                knn.fit(X_train_stand, Y_train.values.ravel())
669
                test_predictions = knn.predict(X_test_sample)
670
672
673
                test_predictions = (test_predictions > 0.5).astype(int)
                #lr.fit(X_train, Y_train)
675
676
677
                # Make predictions on the testing data
678
679
```

```
680
681
                # Calculate error rate (MSE)
683
                nb_test_error_rate = 1 - accuracy_score(y_true=
684
      Y_test_sample, y_pred=test_predictions)
685
                # Append to the lists
686
                test_error_rates.append(nb_test_error_rate)
688
689
           # Calculate the variance of the testing error rates after all
      iterations for this B
           test_error_variance = np.var(test_error_rates)
691
693
           # Calculate the averages for the current B value
694
           average_test_error_rate = np.mean(test_error_rates)
695
696
697
           metrics_df = pd.DataFrame({
698
                "B Cross Validations": [B],
                "Average Error Rate": [average_test_error_rate],
700
                "Average Error Variance": [test_error_variance]
701
           })
702
703
704
           result_df = pd.concat([result_df, metrics_df], axis=0)
706
707
       result_df
708
709
710
711
712
   .. raw:: html
713
714
       <div>
715
       <style scoped>
716
           .dataframe tbody tr th:only-of-type {
717
                vertical-align: middle;
           }
719
720
           .dataframe tbody tr th {
                vertical-align: top;
723
724
           .dataframe thead th {
                text-align: right;
726
```

```
}
727
      </style>
728
      729
        <thead>
730
          731
            B Cross Validations 
733
            Average Error Rate
734
            Average Error Variance
          736
        </thead>
738
        739
            >0
740
            10.0
741
742
            0.2497
            0.000112
743
744
          745
      746
747
      </div>
748
749
750
  .. code:: ipython3
751
752
      final_cv_result = pd.concat([final_cv_result,pd.DataFrame({
753
          'Model': ['KNN'],
754
          'Mean Test Error': [result_df["Average Error Rate"].iloc[0]],
755
          'Test Error Var': [result_df["Average Error Variance"].iloc
756
      [0]
      })])
757
758
  .. code:: ipython3
759
      X_train = train_features
761
      Y_train = train_response
762
      X_test = test_features
      Y_test = test_response
764
765
      # Standardize Data (Assuming standardize_data() function works as
      expected)
      X_train_stand, Y_train_stand, X_train_stand_scaler,
767
      {\tt Y\_train\_stand\_scaler = standardize\_data(features=X\_train, response)}
     =Y_train)
      {\tt X\_test\_stand}\;,\;\;{\tt Y\_test\_stand}\;,\;\;{\tt X\_test\_stand\_scaler}\;,
768
      Y_test_stand_scaler = standardize_data(features=X_test, response=
     Y_test)
769
```

```
Y_train_stand = Y_train.values.ravel() # Ensure Y values remain
770
      binary (0/1)
       Y_test_stand = Y_test.values.ravel()
       # Define range of estimators (1 to 101 in steps of 5)
773
       n_estimators_values = list(range(1, 101, 5))
775
       # Lists to store errors
776
       train_error = []
       test_error = []
778
779
       # Cross-validation setup
       cv = RepeatedKFold(n_splits=10, n_repeats=3, random_state=1)
781
782
       # Loop through different n_estimators values
784
       for n_estimators in tqdm(n_estimators_values):
           rf_model = RandomForestClassifier(n_estimators=n_estimators,
785
      max_depth=3, random_state=42)
786
           # Fit the model
787
           rf_model.fit(X_train_stand, Y_train_stand)
788
789
           # Cross-validation scores
790
           train_score = 1 - cross_val_score(rf_model, X_train_stand,
791
      Y_train_stand, cv=cv, scoring='accuracy').mean()
           test_score = 1 - cross_val_score(rf_model, X_test_stand,
792
      Y_test_stand, cv=cv, scoring='accuracy').mean()
           # Append errors
794
           train_error.append(train_score)
795
           test_error.append(test_score)
797
       # Plot results
798
       plt.figure(figsize=(8, 6))
799
       {\tt plt.plot(n_estimators\_values\,,\,\,train\_error\,,\,\,\,'b\_{\,'}\,,\,\,marker=\,'o\,',\,\,label}
      ='CV (K-fold) Training Error')
       plt.plot(n_estimators_values, test_error, 'g-', marker='o', label=
801
      'CV (K-fold) Test Error')
       plt.xlabel('Number of Estimators')
802
       plt.ylabel('Error')
803
       plt.title("Random Forest: Number of Estimators vs. Error")
       plt.legend()
805
       plt.grid()
806
       plt.savefig("RandomForest_NumEstimators_vs_Error.png", format="png
      ", dpi=300)
       plt.show()
808
810
811 .. parsed-literal::
```

```
812
813
815 .. image:: output_19_1.png
816
   .. code:: ipython3
818
819
       X_train = train_features
       Y_train = train_response
821
       X_test = test_features
822
823
       Y_test = test_response
824
       X_train_stand, Y_train_stand,X_train_stand_scaler,
825
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
      X_test_stand, Y_test_stand, X_test_stand_scaler,
826
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
827
828
829
       Y_train_stand = Y_train.values.ravel() # Ensure Y values remain
      binary (0/1)
       Y_test_stand = Y_test.values.ravel()
830
831
       n1 = round(len(data)*0.9)
832
       n2 = round(len(data)*0.1)
833
834
       b = [10]
       # Initialize the result DataFrame
835
       result_df = pd.DataFrame({
836
           "B Cross Validations": [],
837
           "Average Error Rate": [],
838
           "Average Error Variance": []
839
       })
840
       for B in b:
842
           # Reinitialize the lists for storing metrics for each B
843
           test_error_rates = []
845
           test_accuracies = []
846
           for i in range(B):
848
849
               X_combined = pd.concat([X_train_stand, X_test_stand], axis
      =0)
               y_combined = pd.concat([Y_train, Y_test], axis=0).values.
851
      flatten()
852
853
```

```
{\tt X\_train\_sample} , {\tt X\_test\_sample} , {\tt Y\_train\_sample} ,
854
      Y_test_sample = train_test_split(
                    X_combined, y_combined, train_size=n1, test_size=n2,
      random_state=i
                )
856
                # Train the model on the training data
858
                rf_model = RandomForestClassifier(n_estimators=80,
859
      max_depth=3, random_state=42)
860
                # Fit the model
861
                rf_model.fit(X_train_stand, Y_train_stand)
862
863
                test_predictions = rf_model.predict(X_test_sample)
864
866
867
                test_predictions = (test_predictions > 0.5).astype(int)
868
                #lr.fit(X_train, Y_train)
869
870
871
                # Make predictions on the testing data
872
873
874
875
876
                # Calculate error rate (MSE)
877
                nb_test_error_rate = 1 - accuracy_score(y_true=
      Y_test_sample, y_pred=test_predictions)
879
                # Append to the lists
880
881
                test_error_rates.append(nb_test_error_rate)
882
883
           # Calculate the variance of the testing error rates after all
      iterations for this B
           test_error_variance = np.var(test_error_rates)
885
           # Calculate the averages for the current B value
887
888
            average_test_error_rate = np.mean(test_error_rates)
890
891
           metrics_df = pd.DataFrame({
                "B Cross Validations": [B],
893
                "Average Error Rate": [average_test_error_rate],
894
                "Average Error Variance": [test_error_variance]
895
           })
896
897
```

```
898
         result_df = pd.concat([result_df, metrics_df], axis=0)
899
901
     result_df
902
903
904
905
  .. raw:: html
907
908
     <div>
     <style scoped>
910
         .dataframe tbody tr th:only-of-type {
911
            vertical-align: middle;
         }
913
914
         .dataframe tbody tr th {
915
            vertical-align: top;
916
         }
917
918
919
         .dataframe thead th {
            text-align: right;
920
921
     </style>
922
     923
924
925
         926
          B Cross Validations 
927
928
          Average Error Rate
          Average Error Variance
929
         930
       </thead>
931
       932
         933
          >0
934
          10.0
935
          0.2869
936
          0.000169
937
         938
       939
     940
     </div>
942
943
945 .. code:: ipython3
946
```

```
final_cv_result = pd.concat([final_cv_result,pd.DataFrame({
947
           'Model': ['Random Forest'],
948
           'Mean Test Error': [result_df["Average Error Rate"].iloc[0]],
949
           'Test Error Var': [result_df["Average Error Variance"].iloc
950
      [0]
      })])
952
  .. code:: ipython3
953
       X_train = train_features
955
       Y_train = train_response
956
       X_test = test_features
      Y_test = test_response
958
959
       # Standardize Data (Assuming standardize_data() function works as
      expected)
      X_train_stand, Y_train_stand, X_train_stand_scaler,
961
      Y_train_stand_scaler = standardize_data(features=X_train, response
      =Y_train)
      X_test_stand, Y_test_stand, X_test_stand_scaler,
962
      Y_test_stand_scaler = standardize_data(features=X_test, response=
      Y_test)
963
       Y_train_stand = Y_train
964
       Y_test_stand = Y_test
965
966
       # Define range of learning rates (eta)
967
       eta_values = np.linspace(0.01, 0.36, 25) # From 0.01 to 0.3 in 10
       steps
969
       # Lists to store errors
970
       train_error = []
971
       test_error = []
972
973
       # Cross-validation setup
       cv = RepeatedKFold(n_splits=10, n_repeats=5, random_state=42)
975
976
       # Loop through different eta values
       for eta in tqdm(eta_values):
978
           xgb_model = xgb.XGBClassifier(learning_rate=eta, n_estimators
979
      =80, max_depth=3, random_state=42, eval_metric="error")
980
           # Fit the model
981
           xgb_model.fit(X_train_stand, Y_train_stand)
983
           # Cross-validation scores
984
           train_score = 1 - cross_val_score(xgb_model, X_train_stand,
      Y_train_stand, cv=cv, scoring='accuracy').mean()
```

```
test_score = 1 - cross_val_score(xgb_model, X_test_stand,
986
       Y_test_stand, cv=cv, scoring='accuracy').mean()
            # Append errors
988
            train_error.append(train_score)
989
            test_error.append(test_score)
991
       # Plot results
992
       plt.figure(figsize=(8, 6))
       plt.plot(eta_values, train_error, 'b-', marker='0', label='CV (K-
994
       fold) Training Error')
995
       plt.plot(eta_values, test_error, 'g-', marker='o', label='CV (K-
       fold) Test Error')
       plt.xlabel('Learning Rate (eta)')
996
       plt.ylabel('Error')
998
       plt.title("XGBoost Learning Rate vs. Error")
       plt.legend()
999
       plt.grid()
1000
       plt.savefig("XGBoost_LearningRate_vs_Error.png", format="png", dpi
1001
       =300)
1002
       plt.show()
1003
1004
   .. parsed-literal::
1005
1007
1008
1009
1010
   .. image:: output_22_1.png
1011
1012
1013
   .. code:: ipython3
1014
1015
       xgb_model = xgb.XGBClassifier(learning_rate=.21, n_estimators=80,
1016
       max_depth=3, random_state=42, eval_metric="error")
1017
       # Fit the model
1018
       xgb_model.fit(X_train_stand, Y_train_stand)
1019
1020
       # Cross-validation scores
1021
       train_score = 1 - cross_val_score(xgb_model, X_train_stand,
1022
       Y_train_stand, cv=cv, scoring='accuracy').mean()
       test_score = 1 - cross_val_score(xgb_model, X_test_stand,
1023
       Y_test_stand, cv=cv, scoring='accuracy').mean()
1024
       print("train: : " ,train_score)
       print("test: : " ,test_score)
1026
1027
```

```
1028
   .. parsed-literal::
1029
1030
        train: : 0.21211111111111114
1031
        test: : 0.24760000000000015
1032
1033
1034
   .. code:: ipython3
1035
1036
        X_train = train_features
1037
1038
        Y_train = train_response
        X_test = test_features
        Y_test = test_response
1040
1041
        X_{\text{train\_stand}}, Y_{\text{train\_stand}}, X_{\text{train\_stand\_scaler}},
       Y_train_stand_scaler = standardize_data(features=X_train, response
       =Y_train)
1043
       X_test_stand, Y_test_stand, X_test_stand_scaler,
       Y_test_stand_scaler = standardize_data(features=X_test, response=
       Y_test)
1045
        Y_train_stand = Y_train# Ensure Y values remain binary (0/1)
1046
        Y_test_stand = Y_test
1047
1048
        n1 = round(len(data)*0.9)
1049
        n2 = round(len(data)*0.1)
1050
1051
        b = [100]
        # Initialize the result DataFrame
1052
        result_df = pd.DataFrame({
1053
            "B Cross Validations": [],
1054
            "Average Error Rate": [],
1055
            "Average Error Variance": []
1056
        })
1057
1058
        for B in tqdm(b):
1059
            # Reinitialize the lists for storing metrics for each B
1060
1061
            test_error_rates = []
1062
            test_accuracies = []
1063
            for i in range(B):
1065
1066
                 X_combined = pd.concat([X_train_stand, X_test_stand], axis
       =0)
                 y_combined = pd.concat([Y_train, Y_test], axis=0).values.
1068
       flatten()
1069
1070
```

```
{\tt X\_train\_sample} , {\tt X\_test\_sample} , {\tt Y\_train\_sample} ,
1071
       Y_test_sample = train_test_split(
1072
                     X_combined, y_combined, train_size=n1, test_size=n2,
       random_state=i
                )
1073
1074
                # Train the model on the training data
1075
                xgb_model = xgb.XGBClassifier(learning_rate=.21,
1076
       n_estimators=80, max_depth=3, random_state=42, eval_metric="error"
1077
                 # Fit the model
                 xgb_model.fit(X_train_stand, Y_train_stand)
1079
1080
                 test_predictions = xgb_model.predict(X_test_sample)
1082
1083
1084
                 test_predictions = (test_predictions > 0.5).astype(int)
1085
                #lr.fit(X_train, Y_train)
1086
1087
1088
                # Make predictions on the testing data
1089
1090
1091
1092
1093
                 # Calculate error rate (MSE)
1094
                nb_test_error_rate = 1 - accuracy_score(y_true=
1095
       Y_test_sample, y_pred=test_predictions)
1096
                 # Append to the lists
1097
1098
                 test_error_rates.append(nb_test_error_rate)
1099
1100
            # Calculate the variance of the testing error rates after all
1101
       iterations for this B
            test_error_variance = np.var(test_error_rates)
1102
1103
            # Calculate the averages for the current B value
1104
1105
            average_test_error_rate = np.mean(test_error_rates)
1106
1107
            metrics_df = pd.DataFrame({
1109
                "B Cross Validations": [B],
1110
                 "Average Error Rate": [average_test_error_rate],
1111
                "Average Error Variance": [test_error_variance]
1112
            })
1113
```

```
1114
1115
          result_df = pd.concat([result_df, metrics_df], axis=0)
1116
1118
      result_df
1119
1120
1121
   .. parsed-literal::
1122
1123
1124
1125
1126
1127
1129 .. raw:: html
1130
      <div>
1131
      <style scoped>
1132
          .dataframe tbody tr th:only-of-type {
1133
             vertical-align: middle;
1134
          }
1135
1136
1137
          .dataframe tbody tr th {
             vertical-align: top;
1138
1139
          }
1140
1141
          .dataframe thead th {
             text-align: right;
1142
          }
1143
1144
      </style>
      1145
1146
1147
          1148
            B Cross Validations 
1149
            Average Error Rate
1150
            Average Error Variance 
1151
          1152
        </thead>
1153
        1154
          1155
           >0
1156
            100.0
1157
            0.18749
1158
1159
           0.00016
1160
          1161
      1162
```

```
</div>
1163
1164
1165
1166
   .. code:: ipython3
1167
1168
       final_cv_result = pd.concat([final_cv_result,pd.DataFrame({
1169
           'Model': ['XGBoost'],
1170
           'Mean Test Error': [result_df["Average Error Rate"].iloc[0]],
1171
           'Test Error Var': [result_df["Average Error Variance"].iloc
1172
       [0]]
       })])
1173
1174
   .. code:: ipython3
1175
1177
       final_cv_result
1178
1179
1180
1181
1182 .. raw:: html
1183
       <div>
1184
       <style scoped>
1185
           .dataframe tbody tr th:only-of-type {
1186
               vertical-align: middle;
1187
1188
1189
           .dataframe tbody tr th {
1190
               vertical-align: top;
1191
1192
           }
1193
           .dataframe thead th {
1194
               text-align: right;
1195
          }
1196
1197
       </style>
       1198
         <thead>
1199
           1200
             1201
             Model 
1202
             >Mean Test Error 
1203
             Test Error Var
1204
           1205
         </thead>
1206
         1207
           1208
             >0
1209
             Logistic Regression 
1210
```

```
0.20830
1212
            0.000179
          1213
          1214
            >0
1215
            KNN 
1216
            0.24970
            0.000112
1218
          1219
          1220
            >0
1222
            Random Forest
            0.28690
            0.000169
1224
1225
          1226
          >0
1228
            XGBoost 
            0.18749
1229
            0.000160
1230
1231
          1232
        1233
      </div>
1234
1235
1236
1237
1238
   .. code:: ipython3
1239
      # X_train = train_features
1240
1241
      # Y_train = train_response
      # X_test = test_features
1242
      # Y_test = test_response
1243
1244
      # #model = LinearRegression()
1245
1246
      # sfs = SequentialFeatureSelector(estimator = linear_model.
1247
      LinearRegression(),
                                      k_features='best',
1248
                                      forward=True,
1249
      #
                                      floating = True,
1250
      #
                                      scoring=aic_score,
1251
1252
      # selected_features = sfs.fit(X_train, Y_train)
1253
      # selected_features = list(sfs.k_feature_names_)
1254
1255
      # X_train_stepwise = X_train[selected_features]
1256
      # X_test_stepwise = X_test[selected_features]
1257
1258
```

```
# X_train_stepwise = sm.add_constant(X_train_stepwise)
1259
       # X_test_stepwise = sm.add_constant(X_test_stepwise)
1260
1261
       # lr_stepwise = sm.OLS(Y_train, X_train_stepwise).fit()
1262
1263
1264
       # train_predictions = lr_stepwise.predict(X_train_stepwise)
       # test_predictions = lr_stepwise.predict(X_test_stepwise)
1265
1266
       # lr_stepwise_train_mse = mean_squared_error(y_true=Y_train.values
1268
       , y_pred=train_predictions)
1269
       # lr_stepwise_test_mse = mean_squared_error(y_true=Y_test.values,
       y_pred=test_predictions)
1270
       # print(lr_stepwise.summary())
1271
1272
       # print(f"Train MSE: {round(lr_stepwise_train_mse, 4) }")
1273
1274
       # print(f"Test MSE: {round(lr_stepwise_test_mse, 4) }")
1275
   .. code:: ipython3
1276
       Y_test_probs = logit.predict(X_test_stand) # Use probability
1278
       scores
1279
       # Compute ROC curve and AUC
1280
       fpr, tpr, thresholds = roc_curve(Y_test, Y_test_probs)
1281
       roc_auc = auc(fpr, tpr)
1282
1283
       # Plot ROC curve
1284
       plt.figure(figsize=(8, 6))
1285
       plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC curve (
1286
       AUC = {roc_auc:.2f})')
       plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--', label
1287
       ='Chance (AUC = 0.50)')
       plt.xlabel('False Positive Rate')
1288
       plt.ylabel('True Positive Rate')
1289
       plt.title('Logistic Regression ROC Curve')
1290
       plt.legend(loc="lower right")
1291
1292
       # Save and show plot
1293
       plt.grid(True)
       plt.savefig("roc_curve.png", format="png", dpi=300)
1295
       plt.show()
1296
       knn = KNeighborsClassifier(n_neighbors=60)
1298
       knn.fit(X_train_stand, Y_train.values.ravel())
1299
       Y_test_probs = knn.predict(X_test_stand)
1300
1301
1302
```

```
# Compute ROC curve and AUC
1303
       fpr, tpr, thresholds = roc_curve(Y_test, Y_test_probs)
1304
       roc_auc = auc(fpr, tpr)
1305
1306
       # Plot ROC curve
1307
       plt.figure(figsize=(8, 6))
       plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC curve (
1309
       AUC = \{roc_auc:.2f\})'
       plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--', label
1310
       = 'Chance (AUC = 0.50)')
       plt.xlabel('False Positive Rate')
1311
1312
       plt.ylabel('True Positive Rate')
       plt.title('KNN ROC Curve')
1313
       plt.legend(loc="lower right")
1314
1316
       # Save and show plot
       plt.grid(True)
1317
1318
       plt.savefig("roc_curve.png", format="png", dpi=300)
       plt.show()
1319
       rf_model = RandomForestClassifier(n_estimators=80, max_depth=3,
1320
       random_state=42)
       # Fit the model
1322
       rf_model.fit(X_train_stand, Y_train_stand)
       Y_test_probs = rf_model.predict(X_test_stand)
1324
1326
       # Compute ROC curve and AUC
1327
       fpr, tpr, thresholds = roc_curve(Y_test, Y_test_probs)
1328
       roc_auc = auc(fpr, tpr)
1329
1330
       # Plot ROC curve
1331
       plt.figure(figsize=(8, 6))
       plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC curve (
1333
       AUC = \{roc\_auc:.2f\})'
       plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--', label
1334
       ='Chance (AUC = 0.50)')
       plt.xlabel('False Positive Rate')
       plt.ylabel('True Positive Rate')
1336
       plt.title('Random Forrest ROC Curve')
1337
       plt.legend(loc="lower right")
1338
1339
       # Save and show plot
1340
       plt.grid(True)
       plt.savefig("roc_curve.png", format="png", dpi=300)
1342
       plt.show()
1343
       xgb_model = xgb.XGBClassifier(learning_rate=.21, n_estimators=80,
1344
       max_depth=3, random_state=42, eval_metric="error")
1345
```

```
# Fit the model
1346
        xgb_model.fit(X_train_stand, Y_train_stand)
1347
        Y_test_probs = xgb_model.predict(X_test_stand)
1348
1349
1350
       # Compute ROC curve and AUC
1351
       fpr, tpr, thresholds = roc_curve(Y_test, Y_test_probs)
1352
       roc_auc = auc(fpr, tpr)
1353
1354
       # Plot ROC curve
1355
       plt.figure(figsize=(8, 6))
1356
       plt.plot(fpr, tpr, color='darkorange', lw=2, label=f'ROC curve (
1357
       AUC = \{roc_auc:.2f\})'
       plt.plot([0, 1], [0, 1], color='navy', lw=2, linestyle='--', label
1358
       ='Chance (AUC = 0.50)')
1359
       plt.xlabel('False Positive Rate')
       plt.ylabel('True Positive Rate')
1360
       plt.title('XGBoost ROC Curve')
1361
       plt.legend(loc="lower right")
1362
1363
       # Save and show plot
1364
       plt.grid(True)
1365
       plt.savefig("roc_curve.png", format="png", dpi=300)
1366
       plt.show()
1367
1368
1369
1370 ::
1371
1372
1373
1374
       ValueError
                                                      Traceback (most recent
1375
       call last)
1376
       Cell In[28], line 1
1377
       ----> 1 Y_test_probs = logit.predict(X_test_stand) # Use
       probability scores
              3 # Compute ROC curve and AUC
1379
              4 fpr, tpr, thresholds = roc_curve(Y_test, Y_test_probs)
1380
1381
1382
       File ~\miniconda3\Lib\site-packages\statsmodels\base\model.py
       :1174, in Results.predict(self, exog, transform, *args, **kwargs)
1384
           1128 Call self.model.predict with self.params as the first
1385
       argument.
          1129
1386
```

```
(...)
1387
1388
           1169 returned prediction.
           1170 """
1389
           1171 exog, exog_index = self._transform_predict_exog(exog,
1390
                                                                       transform=
1391
       transform)
       -> 1174 predict_results = self.model.predict(self.params, exog, *
1392
       args,
                                                           **kwargs)
1393
           1175
           1177 if exog_index is not None and not hasattr(predict_results,
1394
1395
       predicted_values'):
                    if predict_results.ndim == 1:
           1179
1396
1397
       \label{lib-site-packages-stats} File ~\mbox{\conda3\Lib-site-packages-statsmodels-discrete-like}.
1399
       discrete_model.py:543, in BinaryModel.predict(self, params, exog,
       which, linear, offset)
            540 if exog is None:
1400
                    exog = self.exog
            541
1401
        --> 543 linpred = np.dot(exog, params) + offset
            545 if which == "mean":
1403
            546
                    return self.cdf(linpred)
1404
1405
1406
       ValueError: shapes (1000,11) and (12,) not aligned: 11 (dim 1) !=
1407
       12 (dim 0)
```

Listing 1: Code