AU24 CSE 2321 Homework 2

Instructor: Luan Duong, Ph.D.
Due: Monday, September 30, 2024, 11:59pm **(20 points)**

1 Instructions

This homework will be submitted online. Students will do their homework on paper/word processor and scan/take photos/convert to **pdf file** and submit on **Carmen** on or before the due date.. Only pdf file will be accepted.

2 Questions

1. **(5 points)** We have the following set and predicates:
   Let $D = \{objects\}$
   Let $P(x)$ be the predicate "x is a car".
   Let $Q(x)$ be the predicate "x is in the garage"
   Let $R(x)$ be the predicate "x has a broken window".
   Let $S(x, y)$ be the predicate "x and y are the same object".
   Write the following in symbolic notation. Simplify if possible:
   (a) There is exactly one car.
   (b) Cars have broken windows.
   (c) Only cars are in the garage (not other objects)
   (d) Either something has a broken window and is not a car, or something is not in the garage
   (e) Translate the following symbolic notation into English: $\forall x \in D(R(x) \rightarrow Q(x))$
   Note: In this question, you should ONLY use the predefined set and predicates

2. **(7 points)** Let $U = \{x : x \text{ is an integer and } 2 \leq x \leq 10\}$. In each of the following cases, determine whether: $A \subset B$, $B \subset A$, both or neiher.
   (a) $A = \{x \in U | x \text{ is odd }\}$; $B = \{x \in U | x \text{ is a multiple of } 3\}$
   (b) $A = \{x \in U | x \text{ is even }\}$; $B = \{x \in U | x^2 \text{ is even }\}$
   (c) $A = \{x \in U | x \text{ is even }\}$; $B = \{x \in U | x \text{ is a power of } 2\}$
   (d) $A = \{x \in U | 2x + 1 > 7\}$; $B = \{x \in U | x^2 > 20\}$
   (e) $A = \{x \in U | \sqrt{x} \in \mathbb{Z}\}$; $B = \{x \in U | x \text{ is a power of 2 or 3}\}$
   (f) $A = \{x \in U | \sqrt{x} \leq 2\}$; $B = \{x \in U | x \text{ is a perfect square }\}$
   (g) $A = \{x \in U | x^2 - 3x + 2 = 0\}$; $B = \{x \in U | x + 7 \text{ is a perfect square }\}$

3. **(8 points)** A class of 60 students appeared for an examination of Law, Statistics, and Accountancy. After the exam, we know that: there are total 25 students failed in Law; total 24 failed in Statistics, and 32 failed in Accountancy. We also know: 9 students failed in Law alone, 6 failed in Statistics alone. Also, 5 failed in Accountancy and Statistics only; and 3 failed in Law and Statistics only. Use set theory only, find the following:
   (a) How many failed in all the subjects?
   (b) How many students passed all the three subjects?
   Note: After solving the problem with set theory equations, please verify it by using Venn Diagrams

L={failed law} S={failed stats} A={Failed Accounting}

$|L| = 25 \quad |S| = 24 \quad |A| = 32 \quad |L-(S \cup A)| = 9 \quad |S-(L \cup A)| = 6$ ✓

$|(A \cap S)| = 5 \qquad |(L \cap S)| = 3$

$|S| = |S-(L \cup A)| + |S \cap L| + |S \cap A| + |L \cap S \cap A|$

$24 = 6 + 3 + 5 + |L \cap S \cap A| \Rightarrow 24 - 19 = |L \cap S \cap A| \quad 10 = |L \cap S \cap A|$

$|L| = |L-(S \cup A)| + |L \cap S| + |L \cap A| + |L \cap S \cap A|$

$25 = 9 + 3 + |L \cap A| + 10 \Rightarrow 25 - 22 = |L \cap A| \qquad 3 = |L \cap A|$

$|L \cup S \cup A| = |L| + |S| + |A| - |L \cap S| - |L \cap A| - |S \cap A| - |L \cap S \cap A|$

$|L \cup S \cup A| = 25 + 24 + 32 - 3 - 3 - 5 - 10 = 60$

→2 Venn Diagram

3 Collaboration Policy

Please remember the collaboration policy in the syllabus. You may discuss homework assignments, projects, and/or pop-up quizzes with your classmates, however, if you collaborate, please state it clearly at the beginning of each collaboratively answered problem by printing the name of the collaborator(s). **Acknowledged collaboration** does not mean copying someone else's solution: Please discuss the solutions, understand the ideas and give your own rendering. Simply copying solutions from any source is considered **plagiarism** and necessary steps will be taken in case of noticing such instances.