Ammending the previous conjecture

1 A counterexample to the previous conjecture

(**Conjecture 8.3.1**)

Suppose the sets $A_1, \ldots A_m$ make up an (n, m, k, λ)-EDF in a group G.

Then for any homomorphism $\phi : G \to H$ where H is a group with $|H| \leq |G|$

the sets $\phi(A_1), \ldots \phi(A_m)$ satisfy, for some $l \in \mathbb{N}$

$$\sum_{i=1}^{m} N_i(\delta) = l \qquad \forall \delta \in H \backslash e_H$$

where $N_i'(\delta)$ is the number of occurences of δ as an external difference from the set ϕA_i.

Counterexample 1.1

$\{0, 1, 6\}, \{2, 3, 5\}$ in $\mathbb{Z}_{10}$?

It is an EDF:

$$\begin{array}{cccccc} 6-5=1 & 2-1=1 & 2-0=2 & 3-1=2 & 6-3=3 & 3-0=3 \\ 6-2=4 & 5-1=4 & 0-5=5 & 5-0=5 & 1-5=6 & 2-6=6 \\ 0-3=7 & 3-6=7 & 0-2=8 & 1-3=8 & 1-2=9 & 5-6=9 \end{array}$$

Image is: $\{0, 1\}, \{2, 3, 0\}$ under the homomorphism $\phi(x) = x \mod 5$

Differences are:

$$\begin{array}{cccccc} 0-0=0 & 1-0=1 & 2-1=1 & 0-3=2 & 2-0=2 & 3-1=2 \\ 0-2=3 & 1-3=3 & 3-0=3 & 1-2=4 & 0-1=4 & \end{array}$$

Note there is 2 occurences of 1 and 4, and 3 occurences of 2 and 3, so this is not an OEDF.

2 Rethinking the conjecture

It was interesting how many of the examples in Post 8 worked however. Could the conjecture hold when the EDF is strong?