Advisor: Dr. Tracy Reilly Lawson

The Effects of Timed Multiplication Fact Drills on 5th Graders' Ability to Master, Maintain and Apply Their Multiplication Facts to Higher- Order Thinking Problems That Require Multiplication to Solve.

Frank Orefice

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Abstract

This study tested the effects of timed multiplication fact drills on 5th graders ability to master, maintain and apply their multiplication to higher- order thinking problems that require multiplication to solve. The research took place in a 5th through 8th grade school in New Jersey with five heterogeneously mixed 5th grade math classes. Timed multiplication fact drills are simple multiplication problems that challenge students to answer a given number of multiplication problems in a short period of time. This study was conducted to determine the effectiveness of timed multiplication fact drills on participants' ability to master their multiplication facts from 1x1 to 10 x10 and then apply and maintain them to higher-order thinking problems. The participants completed a pre-test and two post-tests (one application and one maintenance) to determine the effectiveness of the timed multiplication fact drills on timed multiplication fact drills. At the completion of the study the participants completed a questionnaire. The participants developed automaticity and fluency with their multiplication by completing multiplication fact drills, the results showed that it had positive impact on the number correctly answered on the posttests.

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Chapter 1

Multiplication Fact Drills

Introduction

The foundation for future advancement in math is the automaticity in basic multiplication facts (Wong & Evans, 2007). The automaticity of multiplication is the catalyst upon which students need to build for higher order computation skills. As students' progress through middle school they need to have fluency and be able to recall facts from memory to apply to other topics such as multi-digit multiplication, fractions, ratios, division and decimals. If basic multiplication facts are not mastered, the focus of the learner is on the lower –order part of the problem rather than the higher- order problem solving (Wong & Evans). As students who struggle with basic computation skills transition from elementary school to middle school, this area of weakness often continues to go unaddressed.

The use of multiplication fact drills is designed for students to help them better memorize and master their basic multiplication facts at a fast rate. Gaining a better understanding of their basic multiplication facts will ease their learning of more advanced math concepts.

Background

The participants of the study were 5th grade students from a suburban school district in New Jersey. There were three schools in the District, a Kindergarten through Second Grade school, a Third and Fourth grade school, and Fifth through Eighth grade middle school. After graduating 8th grade students attend a regional high school. At the middle school where this study took place; there were over 400 students, 105 of which were in the 5th grade. Students with special needs received educational services in a variety of ways; regular education classroom setting with or without additional support, pull out instruction in a resource class room or in a language learning disability (LLD) classroom setting. The students who attended regular education classes in 5th grade were heterogeneously mixed over five sections, with an average class size of 20 students. Fifth grade was the first year in which students attended departmentalized classes. All of the participants ranged in age from 10 to 11 years old.

It was hypothesized that through the mastering of their basic multiplication skills, the students would apply the automaticity that they developed to other math skills that required more complex skills. Students being able to focus on a new skill being taught rather than the multiplication will hopefully lead to quicker understanding of the new skills.

Statement of Problem

Content and process standards of the National Council of Teachers of Mathematics (NCTM) imply that automaticity of basic computation fluency should be fully developed before students reach sixth grade, yet many students have not achieved that fluency when they leave elementary school (Knowles, 2010). As students' progress through a mathematics curriculum, it is assumed that they have mastered certain skills. Unfortunately when many students have not memorized all of their basic multiplication facts, this deficit will have a continued effect on their success in math. Since knowledge of basic multiplication is the foundation for other higher-order math skills, the struggle in mathematics continues until the basic facts are mastered. If a student does not have a strong multiplication base, they will also have difficulty with new math concepts that they are learning because they are going to be focused on the basics of the problem and are missing out on developing higher-order thinking. This study was completed to test the effects of timed multiplication fact drills on 5th graders ability to master, maintain and apply their multiplication facts to more complex multiplication problems. This study determined what effect multiplication fact drills have on student's ability to apply knowledge of their multiplication facts to higher- order math skills.

Higher –order thinking goes beyond the basic recall of facts, evaluation and invention, and enables students to retain information and to apply problem-solving solutions to real-world problems. Higher order thinking skills are valued because they are believed to better prepare students for the challenges of adult work and daily life and advanced academic work. Higher order thinking may also help raise standardized test scores. (Pogrow, 2005).

Lower-order thinking demands only routine or mechanical application of previously acquired information such as listing information previously memorized and inserting numbers into previously learned formulas. In contrast higher order thinking challenges the student to interpret, analyze or manipulate information (Lewis & Smith, 2001).

Mastery of multiplication is a foundational skill that is necessary for mathematical success. The Core Curriculum Content Standards (CCCS) stated that multiplication should be developed in third grade. The math CCCS build upon the initial standard for multiplication by introducing new standards that require multiplication to solve for challenging skills.

Research Question

What are the effects of timed multiplication fact drills on 5th graders ability to master maintain and apply their multiplication facts to higher-order problems that require multiplication to solve?

Definition of Terms

Automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice. (Little, 2011)

Core Curriculum Content Standards. The Core Curriculum Content Standards are informed by the highest, most effective models from states across the country and countries around the world, and provide teachers and parents with a common understanding of what students are expected to learn. Consistent standards will provide appropriate benchmarks for all students, regardless of where they live (Corestanders.org)

Fact drills. Fact drills are designed to provide quick, repetitive practice with basic math facts. They are useful tools for measurement, practice, and instruction within the larger collection of precision teaching tools. These tests typically involve a large set of questions, problems or tasks for the student to answer as many as possible within a specified time, frequently one to three minutes (Davis, 2008).

Fluency. Fluency is the fluid combination of accuracy plus speed that characterizes component performance (Binder, 1990).

Individualized Education Program (IEP). The Individualized Education Program (IEP) is a legally binding document. It describes the educational program that has been designed to meet that child's unique needs. Each child who receives special education and related services must have an IEP. Each IEP must be designed for one student and must be a truly individualized document. The IEP creates an opportunity for teachers, parents, school administrators, related services personnel, and students (when age appropriate) to work together to improve educational results for children with disabilities. The IEP is the cornerstone of a quality education for each child with a disability (National Dissemination Center for Children with Disabilities, 2010).

Higher order thinking. Higher order thinking is complex thinking that goes beyond basic recall of facts, such as evaluation and invention, enabling students to retain information and to apply problem-solving solutions to real-world problems (Pogrow, 2005).

Intervention and Referral Services (I&RS). All public school students in New Jersey are in entitled to I&RS when they experience academic, behavior, and/or health difficulties. This process is a primary way in which schools support students' needs in the general education environment. I&RS is not intended to replace traditional methods or resources for helping students to function effectively in school. Its primary focus is to align students' needs with available resources in the general education environment (Vermeire, 2008).

Other Health Impairments. Other health impairment means having limited strength, vitality, or alertness, including a heightened alertness to environmental stimuli, that results in limited alertness with respect to the educational environment, that is due to chronic or acute health problems such as asthma, attention deficit disorder or attention deficit hyperactivity disorder, diabetes, epilepsy, a heart condition, hemophilia, lead poisoning, leukemia, nephritis, rheumatic fever, sickle cell anemia, and Tourette syndrome; and Adversely affects a child's educational performance (National Dissemination Center for Children with Disabilities, 2012).

Conclusion

Students' ability to master their basic multiplication facts is key to being successful with many other mathematical procedures. This study investigated the participants' ability to master the basic multiplication facts and then apply and maintain them with more complex math tasks that require multiplication to complete. As students' progress through math, basic multiplication becomes lower order thinking skill and the focus is on new higher order skills. This study can support or challenge the correlation that timed basic multiplication help students in more complex math tasks.

Chapter 2

Review of the Literature

Introduction

This study looked at the 5th grade participants' ability to master their multiplication facts by completing timed multiplication drills and applying these skills to more advanced multiplication problems. Students who have mastered their basic multiplication facts to a rate criterion are able to focus their attention on higher order thinking tasks when being introduced to new mathematical tasks that require multiplication to solve. Being able to quickly recall your multiplication facts allows the learner to easily complete more challenging math problems. Rather than being challenged by the actual multiplication in new concepts the challenge is on the skill. The more attention the new skill receives the faster it will also be mastered.

Mastery of Mathematics

Studies show that most difficulties in mathematics are exhibited at an early onset (Schopman & Van Luit, 1996). Children develop their number sense, which focuses on classification and serration with counting skills early in their schooling. The students then learn the four basic mathematical operations: addition, subtraction, multiplication and division, for computation skills. This knowledge of the operations and a student's ability to perform math is essential to future skill development in math (Mercer & Miller, 1992; Crawford, 2001).

Students who practice calculations using a counting method will become more efficient and quicker at counting. If students are able to retrieve facts from memory, even slowly, the use of drill-and-practice will lead to the fluent recall of the facts (Crawford, 2001). Once acquisition occurs, drill-and practice is then used to make retrieval of information fluent and automatic (Hasselbring & Goin, 1988).

Mental math requires students to become fluent in basic arithmetic facts. Researchers have long maintained there are three general stages of "learning" math facts and different types of instructional activities that affect learning in those stages (Ando & Ikeda, 1971). The initial stage starts with counting, or procedural knowledge of math facts. This is the stage where the children must count or do successive addition or some other strategy to "figure out" the answer to the fact (Garrnett, 1992). The second stage consists of developing ways to "remember" math facts by relating them to known facts. These also include pairs of facts related by the commutative property, (e.g., 5 + 3 = 3 + 5 = 8). This can also include fact families, such as 7 + 4 = 11, 4 + 7 = 11. 11 – 7 = 4 and 11 – 4 = 7. Such strategies are considered to be more "mature" than counting procedures. Another strategy is 'linking' one problem to a related problem (e.g., 5 + 6, thinking '5 + 5 = 10, so 5 + 6 = 11) (Garrnett). The final stage is the declarative knowledge of the "knowing" the facts or direct retrieval. In this stage the capacity to recall answers without resorting to anything other than direct retrieval of the answer. (Garnett; Crawford, 2001). Most children move from using procedures to "figure out" math facts in the first grade to the adult retrieval by fifth grade. The move from predominately procedural to declarative knowledge of math facts typically begins around the 2nd to 3rd grade level (Ashcraft, Fierman & Bartolotta, 1984).

When students first begin to learn the concepts of multiplication using concrete and semi-concrete materials, time restrictions are not appropriate. To improve the speed of fact recall, students should be given a specific time to respond to a question or constant time delay, typically starting at five seconds and gradually reducing to one and half seconds. The reduction in response times forces the student to abandon inefficient counting strategies and causes the student to retrieve the answer from memory (Hasselberg & Goin, 1988).

The majority of wrong responses in multiplication are because of operand errors. In educated adults, operand errors make up between about two thirds and four fifths of all mistakes. These errors do not occur arbitrarily, and are not equally distributed over all problems (Campbell, 1987). One of the core findings in simple arithmetic is the problem-sized effect; error rate and response latencies increase when the problems get larger (Campbell & Graham, 1985). The problem size effect is modulated for two specific types of problems. For example, involving 5 as an operand ("five problems") are less error prone and can be solved reliably faster than predicted by their problem size (Campbell & Graham, 1985). Most operand errors are numerically close to the correct result. They are more likely to be related to the smaller (min) than to the larger (max) operand (e.g, 4 x 9 = 32 is more likely than 4 x 9 =27), regardless of operand order. They may co-occur together with "naming errors" (e.g., 4 x 8 = 4, or 48) or "operand-intrusion errors" (e.g., 4 x 8 =28), meaning that at least one of the operands intrudes into the result. (Domahs, Delazer & Nuerk, 2006).

Automaticity

Psychologists have long argued that higher-level aspects of skills require that lower level skills be developed to automaticity (Crawford, 2001). Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice. Turn-of- the-century psychologists captured the relationship in the phrase "Automaticity is not genius, but it is the hands and feet of genius." (Bryan & Harter, 1899; as cited in Bloom, 1986 p.70). Automaticity occurs when tasks are learned so well that performance is fast, effortless, and not easily susceptible to distraction. The response time for automatized math facts should be "around 1 second" from presentation of stimulus until a response is made (Hasselbring, Goin, &

Bransford, 1987). An essential component of math facts is that the answer must come from direct retrieval rather than following a procedure. The common observation is that students who “count on their fingers” have not mastered the facts. Although correct answers can be obtained using procedural knowledge, these procedures are effortful and slow, and they appear to interfere with learning and understanding higher-order concepts (Hasselbring, Goin & Bransford, 1988). Mental effort involved in figuring out facts tends to disrupt the thinking about problems in which facts are being used (Crawford, 2001).

The importance of automaticity becomes apparent when it is absent. Conceptual understanding is necessary, but insufficient for mathematical proficiency (Bratina & Krudwig, 2003). Developing rapid recall of arithmetic facts, automaticity, remains a concern to elementary education teachers in the 21st century. However, many students arrive at fifth grade without automaticity of multiplication fact recall and consequently have no foundation upon which to build higher-level computational skills (Lehner, 2008). Students think they know their facts. In actuality, they only know how to use various strategies to arrive at the answer without automaticity, or they don’t know the multiplication facts at all (Lehner).

The automaticity in math facts is fundamental to success in many areas of higher mathematics. Mastering the basic skills is necessary in order to advance in math, and it ultimately correlates with later success in the workplace, as concluded by Murnane and Levy (1996). The inability to retrieve math facts easily causes students to experience a high cognitive load as they complete more complex mathematical tasks. The increased processing demands resulting from counting rather than direct retrieval of facts often leads to declarative and procedural errors. (Woodward, 2006).

Basic multiplication facts are considered to be foundational for further advancement in mathematics. They form the basis for fluency and the ability to recall facts from memory across many subjects such as learning multi-digit multiplication, fractions, ratios, division and decimals is essential. Without fluency of multiplication facts, the focus of the learner is on the lower – order component skills, rather than the higher- order problem solving. (Wong & Evans, 2007). If the students are unable to perform basic calculation with the need to use calculators or other aids, higher order processing in problem solving is hindered (Westwood, 2003).

Fluency Strategies

Fluency is the fluid combination of accuracy plus speed that characterizes component performance (Binder, 1990). Fluency has also been described as a combination of quality plus pace. When learners achieve certain frequencies of accurate performance they seem to retain and maintain what they have learned, remain on task or be able to endure for sufficient periods of time to meet real-world requirements, even in the face of distraction and apply, adapt, or combine what they learned in new situations, in some cases without explicit instruction (Binder & Bloom, 1989). When a combination of accuracy plus speed of performance is present, it optimizes the outcomes with respect to a specific behavior in class. This level of performance is considered to be "true mastery" of the behavior (Binder, 1987). The difference between a beginner (who will likely forget much of what he or she has recently learned, or have difficulty applying it) and a true expert is not merely a matter of accuracy. It is the speed or rate of performance which measurably distinguishes experts from beginners (Binder, 1990).

Fluency needs ongoing measurement in order to obtain a whole picture of learning and performance. The use of drill measurement allows the instructor to measure all of the components in mathematics acquisition. Accuracy measures are primarily collected in the general education classroom; it only gives the correctness of the performance (Oskar-Groen, 2009). Collecting the rate of the response gives a precise unit of measure that compares the correctness with the rate of response. This allows a comparison to be made and determines where re-teaching should be directed. The combination of the two dimensions of responses, accuracy and number of responses over time, shows a complete picture of learning (West, Young & Spooner, 1990). The rate per minute is a more sensitive measure of changes in performance than an accuracy measure alone. (Howell & Lorson-Howell, 1990). Recording both components gives the teacher the ability to pin point improvement and detect performance change in each individual student's calculations.

A number of educators emphasize the use of explicit strategy instruction over rote learning when teaching math facts. Methods vary from the use of visual displays such as ten frames and number lines to more general techniques such as classroom discussion where students share fact strategies with their peers (Thompson & Van de Walle, 1984).

Multiplication strategies vary considerably in the way fact strategies are linked to the broader ability to do mental calculations. For example, some special education researchers stress basic rules for multiplication as they relate to facts (e.g., multiplication by 0 or 1, the commutative property) (Miller, Strawser & Mercer, 1996). Others recommend a wider array of strategies and focus on patterns that are easier to learn. They suggest that doubles, times five, times nine pattern and square numbers are easier for students to learn than facts such as 4 x 8 and 6 x 7 (Chambers, 1996, Garnett, 1992 & Thorton. 1990). Strategies on how to solve multiplication problems vary considerably in the way fact strategies are linked to a broader ability to perform mental calculations. Some researchers stress basic rules for multiplication as they relate to facts. (Van de Walle, 2003).

Kilpatrick, Swafford and Findell's research (1992) demonstrated that the link between facts and mental calculations is more evident in number sense. It shows how derived facts (e.g., 6 x 7 = 6 x 6 + 6) and counting backwards for 9's strategies (e.g., 8 x 9 = 8 x 10 -8, 9 x 9 = 9 x 10 – 9) can be applied to calculations involving 2 x1 digit numbers. French (2005) showed that students with strong number sense calculate 99 x 9 by converting the problem to 100 x 9 -9. Strategies such as "split – add" for 8 x 4 involve splitting the problem into two smaller problems (e.g., 8 x 2 + 8 x 2) and then adding the products. Similar decomposition logic can be applied in the form of the distributive multiplication (e.g., 27 x 2 = 25 x 2 + 2 x 2). Other strategies such as doubling and halving (e.g., 32 x 5 = 32 x 10 + 2) also expand a student's ability to compute mentally exact answers to multi digit multiplication (Woodward, 2006).

Emphases on strategies help students organize facts in a coherent knowledge network, thus facilitating long-term retention and direct recall. Strategic instruction of facts can even include instruction of facts (e.g., 3 x 4 extends to 30 x 4). The link of facts to extended facts helps students in estimation and mental computation tasks. Sherin and Fuson (2005) argued that strategic knowledge remains in many students even though they are able to compute multiplication facts within 3 seconds per fact parameters. Students may use a combination of strategies when answering relatively difficulty facts such as six times seven. Their research shows for a fact such as this, students may quickly employ a derived fact strategy. (Woodward, 2006).

For completing basic arithmetic calculations to be masterful they must be done quick and nearly automatically, rather than slow and hesitant. People can observe this difference in their own behavior and in the behavior of others. Yet conventional percentage correct scores, the standard in our educational system cannot differentiate the obviously different levels of achievement (Barnett, 1979). Only fluency bridges the gap between mere acquisition of skills or knowledge and truly useful performance. For example, given a sheet of 150 simple addition problems, most competent adults can write between 90 and 110 correct answers in one minute, with perhaps one or two errors (Binder, 1988).

Research from different fields demonstrates the importance of using timed assessment to define mastery. The findings are consistent and confirm an intuitive appreciation that mastery implies speed as well as accuracy of performance in virtually every type of skill or knowledge. These findings divide into three broad categories: studies which link speed of responding to improved retention or maintenance of skills and knowledge; those which show increased speed improves attention span or resistance to distraction and those which indicate that fluency in prerequisite skills or knowledge supports the application of new learning to more advanced or complex performance (Binder, 1987).

There are several curriculum programs based on the idea and philosophy that students need to develop fluency through repeated practice in computational skills (i.e., Saxon Math, Rocket Math; Mad Minute) (Houghton Mifflin Harcourt, 2011; R&D Instructional Solutions, 2012; Dale Seymour Publications, 2012). The idea that students need automaticity for mathematical calculations is vital based on the research of Hasselbring, Goin & Bransford (1988). Correct answers are obtained through fluency of procedural knowledge because it often slows and is effortful and appears to interfere with learning and understanding higher order concepts if true proficiency with these skills is not present (Carnine & Stein, 1981).

Precision Teaching

Precision Teaching is not so much a method of instruction as it is a precise and, systematic method of evaluating instructional tactics and curricula (West & Young, 1992). It is the practice of basing educational decisions on changes in continuous self-monitored performance frequencies displayed on standard celebration charts (Lindsley, 1972).

Over the last 25 years, Precision Teaching (PT) and Direct Instruction (DI) have been measurably effective instructional technologies. These two approaches to instruction may be the most thoroughly validated and consistently effective methods developed in English-speaking schools (Binder, 1988). They emphasize a free operant learning environment, which is one of the many behavioral education techniques with roots in B. F. Skinner's operant conditioning theories and experiments. Free operant refers to the principle that students are free to respond at their own pace without having restraints placed on them by the limits of the materials or the instructional procedures of the teachers (Lindsley, 1990b). Lindsley developed the principles of precision teaching while under the supervision of B. F. Skinner in the 1950's (Lindsley, 1990a). He believed that the traditional behavioral learning assessment (Percent correct) created an artificial ceiling to measuring learning. He argued that two individuals could both score 100% correct on a test, but have dramatically different rates of response. One student may answer all the questions correctly in less than 1 minute, while another may require 10 minutes. Traditional behavioral learning assessments at that time neglected the clear differences in the skill or mastery levels. Attempts to assess skill or mastery on the basis of percent correct alone are clearly limited and frequently incomplete. One person may be highly gifted in mathematics, yet score the same 100% correct on an elementary level math test as another individual with significantly less mathematical aptitude. The gifted individual may complete the test in half the time as the less fluent person. If all performance assessments are based on the final scores, then both students would appear to have equal levels of mathematical ability. The solution was to include rates of responding in the evaluation of learning. A student who achieves a high and accurate rate of responding was said to have achieved fluency (Lindsley, 1990b).

Precision teaching does not indicate what to teach, but it identifies if and when instructional procedures need to be changed. Students who reach a pre-determined percent correct at high rates of responding are said to have achieved fluency and mastery. This fluency is an indication that it is time to modify the curriculum and make it more challenging. Precision teaching essentially provides a method of developing fluency of skills that have been previously taught. These skills are typically basic in nature (Davis, 2008).

They key components of Precision Teaching are; to set timed-based mastery criteria for each curriculum step, to provide daily opportunities for practice and timed measurement, to chart performance on a graph called the Standard Celeration Chart and to change procedures when the chart shows they're not working. (Pennypacker, Koenig & Lindsley, 1972). The chart is a 6-cycle semi-logarithmic graph for charting behavior frequency (or rate) against calendar days (Lindsley, 1972) The standard chart has evolved into a primary communication tool among thousands of Precision Teachers and their students, as a means of sharing discoveries and cooperatively solving problems (Binder, 1990).

A major finding in this program is that students must achieve fluency in "tool" skills in order to progress smoothly to more advanced material (Haughton, 1972). A common reason for failure in basic math skills is that students have not achieved fluency in basic number-writing and digit reading, despite their being able to perform these skills accurately. The programs design made it possible for the learner to discover, what procedures and materials produced the greatest improvements in learning and performance (Haughton). In effect, the approach emphasized the evaluation and revision components of systematic instruction by encouraging teachers and students to pinpoint behaviors, count, time and chart them every day, and "try, try again" when initial procedures did not produce the desired results (Lindsley, 1972). When they have not achieved sufficient basic arithmetic computation to a rate criterion (e.g., 50 to 70 problems per minute), students usually experience difficulty learning long division, algebra and other advanced math skills. Teachers of this program have found that a few minutes per day of timed practice on carefully sequenced skills can often eliminate what were previously considered irremediable learning programs (Binder, 1988).

One of the most significant contributions of Precision Teaching research was the use of frequency aims (Haughton, 1972; Wood, Burke, Kunzelmann, & Koenig, 1978). Early practitioners were strongly influenced by operant conditioning and its applied offshoot known as "behavior modification". They initially viewed behavior frequency (response rate) as a "performance variable" to be arbitrarily increased or decreased through the use of consequences (Binder & Watkins, 1990). However, when teachers began measuring behavior frequencies of academic skills (e.g., writing answers to simple arithmetic problems), they noticed that consequences were often not sufficient for increasing performance. For example, while most students could achieve performances of 40 to 50 correctly written answers to addition problems per minute by engaging in daily practice, some seemed unable to move beyond 20 per minute. No matter what consequences teachers arranged for improving performance (e.g., tokens, praise, notes home, etc.), these low-rate performances remained below the desired levels. After discovering that these students' rates of writing and reading digits were considerably lower than the levels exhibited negatively affect by successful students, Haughton and his associates found that by practicing the more elementary or "tool" skills (writing and reading digits) until they reached higher levels, students were able to attain higher rates of writing answers to problems.

Another key development in Precision Teaching was the use of "learning screening" procedures for independently assessing performance levels and celerations (measures of learning as change in performance over repeated daily samples) on specific tasks to identify students at risk of failure in academic development (Binder, 1990).

Precision Teaching research and practice have revealed that the suppression of errors can often encourage students to respond at very high rates from the beginning of their work on a given skill, even when most responses are incorrect, and can significantly increase learning rates. (Bower & Orgel, 1981; Lindsley, 1990a). Thus, Precision Teachers call errors "learning opportunities" and often encourage students to take large "leap-ups" through curriculum sequences, making many errors and correcting the errors as an integral part of the learning process. McGreevy (1978) summarized this strategy with the slogan "Easy to do, hard to learn; hard to do, easy to learn." Substantial published research supports the conclusion that placement on more difficult tasks can result in faster learning rates (Johnson,1971; Neufeld & Lindsley, 1980).

Direct Instruction

Direct Instruction (DI) is a research-based approach to instructional design and implementation based on over 25 years of development (Binder & Watkins, 1990). It is an educational philosophy and a set of teaching procedures and programming principles derived from that philosophy. The goal in this program is to teach more in less time by using teaching procedures and materials that maximize the time students spend in instruction (Gersten & Maggs, 1983).

In recent years some teachers and instructional designers have combined DI methods and materials with Precision Teaching. Like Precision Teaching, Direct Instruction encounters resistance among mainstream educators, often because of its detailed scripting of teacher's behavior. However, Direct Instruction has been consistently shown to support greater academic achievement, self-esteem and problem-solving abilities in children than any mainstream approach to teaching (Watkins, 1988).

Using fluency standards and brief, timed assessment procedures, instructors using DI have been able to identify students in need of special help with a higher degree of predictive validity, and greater cost effectiveness than those who were using more traditional screen techniques. With regular one-minute timings on clusters of skills throughout entire schools, teachers and staff are able to track students' progress with a remarkable degree of precision and objectivity (Binder, 1988).

Many elementary schools traditional materials and procedures actually prevent students from ever achieving fluency. For example, workbooks often contain pages with so few examples that students receive neither the required amount of practice nor the opportunity to demonstrate fluent performance. Also common classroom teaching techniques provide such infrequent chances for individual responding that students are unlikely to maintain attention or to become fluent (Binder, 1988).

Morningside in Seattle Washington is a private enterprise born of parental requests and an overload of student need, was founded by Kent R. Johnson, Ph.D., Morningside's director. It incorporates the behavioral methods of Precision Teaching (PT) founded by Dr. Ogden Lindsley, DI founded by Sigfried Engelmann, task analysis and instructional design as described by Susan Markle, and other unique technologies within the Morningside Model of Generative Instruction. The method includes classroom instruction followed by timed practice, charting of progress, feedback, positive reinforcement, and deciding what to learn and practice next (Johnson, 1994).

Timed Multiplication Fact Drills

Timed trials exhibit a history of assisting students to improve their fluency by creating a more complete picture of student's individual progress (Miller & Heward, 1992). Educators have utilized timed trials as an effective approach for improving students' math performance In timed trials, a fact needs a correct answer in three to four seconds to insure that the student is using memory retrieval rather than other reconstructive procedures for figuring out the answers to these math problems (Crawford, 2001). To achieve this goal the study group of facts needs to remain small and related. This connection of the small group of facts allows for the student to memorize the facts and retrieve them quickly and without the use of a large mental effort (Hasselbring, 1988).

Practice is required to develop automaticity with math facts. "The importance of drill on components [such as math facts] is that the drilled material may become sufficiently over-learned to free cognitive resources and attention. The cognitive resources may be allocated to other aspects of performance, such as more complex operations like carrying and borrowing, and to self-monitoring and control (Goldman & Pellegrino, 1986, p. 134)". This suggests that even mental strategies or mnemonic tricks for remembering itself will interfere with attention needed on the more complex operations. For automaticity with math facts to have any value, the answers must be called with no effort or much conscious attention, because the conscious attention of the student should be directed elsewhere (Crawford, 2001).

Several researchers have discussed the importance of frequent timed practice and the use of strategy instruction for all students through the end of elementary school. Cognitive psychologists have discovered that humans have fixed limits on the attention and memory that can be used to solve problems. One way around these limits is to have certain components of a task become routine and over-learned so that they become automatic (Whitehurst, 2003 as cited in Lehner, 2008). Math facts should be developed in isolation, so that math fact speed will continue to improve, as well as cause improvements in the ability to complete more complex problems (Crawford, 2001).

Students often start to master facts using a counting method. Teachers work to move pupils away from this method of 'controlled response time'. A controlled response time is the amount of time allowed to retrieve and provide the answer to a fact. A controlled response typically starts with a response time of 3 seconds or less and works down to the controlled response time around 1.25 seconds (Crawford, 2001). The decreased response time is seen as the critical step in the development of automaticity (Crawford). Students are forced to retrieve the correct answers rather than return to former methods of computation (Oskar- Groem, 2009).

Most fact drills are set up in one-minute timings. Expectations of automaticity vary somewhat. Translating a one-second-response time directly into writing answers for one minute would produce 60 answers per minute. Sixty problems per minute are exactly in the middle range of 40 to 80 problems per minute shown by adults (Crawford, 2001). However, some children, especially in the primary grades, cannot write that quickly. "In establishing mastery rate levels for individuals, it is important to consider the learner's characteristics (e.g., age, academic skill, motor ability). For most students a rate of 40 to 60 correct digits per minute [25 to 35 problems per minute] with two or few errors is appropriate (Mercer & Miller, 1992, p.23)."

Teachers should help low-performing students with instruction geared toward success in recall of basic facts which will allow advancement in math concepts in the areas algebra, geometry, and calculus. Success in higher-level math course work appears to be a good indicator of college- and career-related success (Lehner, 2008).

Some students, despite a great deal of drill and practice on math facts, fail to develop fluency, or automaticity with facts. Hasselbring and Goin (1988) reported that researchers, who examined the effects of computer delivered drill-and-practice programs, have generally reported that computer-based drill-and-practice seldom lead to automaticity, especially amongst children with learning disabilities. "The major result of practice is increased rate and accuracy in doing the task that is actually practiced. For example, the child who practices counting on his fingers to get missing sums usually learns to count on his fingers more quickly and accurately. The child who uses skip counting or repeated addition to find a product learns to use repeated addition more skillfully-but continues to add. Practice does not necessarily lead to more mathematically mature ways of finding the missing number or to immediate recall. If the process to be reinforced is recalling, then it is important that the child feel secure to state what he recalls, even if he will later check the answer… (Ashlock, 1971, p363)."

Students with Learning Disabilities

Learning disability is a general term that describes specific kinds of learning problems. A learning disability can cause a person to have trouble learning and using certain skills. The skills most often affected are: reading, writing, listening, speaking, reasoning, and doing math Researchers think that learning disabilities are caused by differences in how a person's brain works and how it processes information (National Dissemination Center for Children with Disabilities, 2011). Children with learning disabilities are not "dumb" or "lazy." In fact, they usually have average or above average intelligence. Their brains just process information differently. (National Dissemination Center for Children with Disabilities).

Students with learning disabilities in mathematics are found in almost every classroom in America (Shapiro, 1996). About 5% to 10% of the students in schools for elementary general education possess deficits in math (Riveria, 1997). These deficits occur at different times in student's life. Potential causes are numerous and are partly explained by such characteristics as intellectual functioning, motivation, problem-solving, memory skills, strategy acquisition and application, and vocabulary (Kroesbergen & Van Luit, 2003). Trouble with acquiring math is even more difficulty when a student displays trouble in multiple areas. Some students who experience several difficulties are in need of special educational services (Kroesbergen & Van Luit).

Students with Learning Disabilities tend to be passive in their approach to learning and wait for teachers to tell them what to do and how to do it. Such an approach to academic tasks is likely to lead to feelings of dependency, inadequacy, and/or lack of interest. It is imperative that teachers seek ways to help students struggling to develop sophisticated fact strategies naturally become more proactive and involved in planning their academic programs (Miller, Strewster & Mercer, 1996)

Timed practice drills as a method for developing automaticity offer a clear alternative for academically low-achieving students and students with LD. Special education research supports time drills that often include pretesting and systematic review (Woodward, 2006).

History

Research on the effects of math drills began with Edward Thorndike's experimental research in 1908 (Thorndike, as cited by Bornwell & Chazal, 1935). During the early 1900's, research on the effects of drill reached its peak from 1910 to 1920. Researchers during this time were less concerned with the effects of learning than with related matters such as the length of the drill period, the comparative merits of mixed and isolated drill organization, and the like. It was as if the instructional value of drill had firmly been established and the major problem was how the drill should be administrated. The consistency of the experimental data caused text book companies to include more attention to matter of drills in their content (Brownell & Chazal, 1935). Classroom instruction in arithmetic became virtually connected to the administration of drills, under the assumption that drill is an essential thing (Brownell & Chazal).

Brownell and Chazal conducted a review of the literature that found sixteen investigations were made on the value of math facts drills on only participants from third grade to high school. These were examined with regard to two important points, the school grades in which the studies were made, and the measures obtained. All of the experimental drills followed initial instruction by varying margins from weeks to more than ten years (Brown, 1911). None of the experiments included a test conducted to find the value of drill as an early step in teaching as a means of promoting understanding and supplying meaning for which was practiced. On the contrary, the problem of these experiments related to the effectiveness of drill for improving, fixing, maintaining and rehabilitating skills which had already been assured by prior instruction. Regardless of how unmistakable the results revealed the virtues in drill, they don't at all justify the procedures for the time in introducing arithmetic skills and facts be intelligible to children (Brownell & Chazal, 1935).

The demand for the educated public to have excellence in the broader field of mathematics is greater today than it was when textbook companies were able to use mathematics as a space for drill purposes. In newer released textbooks, the trend is on topics such as problem solving, patterns, algebra in middle-school math, geometry throughout the curriculum, and the use of technology in teaching elementary- and secondary-school mathematics (Johnson, 1994).

In the technology boom of the 1980's, speculation abounded that mathematics education would be able to de-emphasize computation, and in turn emphasize high-order math skills. This approach to mathematics created a push among math teachers to suggest that students have access to calculators at all times for use on classwork, homework, and tests. This bold position created a great deal of uneasiness and uncertainty that contributed to the math wars in California (and other parts of the country) in the 1990's (Henry, 2003).

The battle for reform in mathematics education was a result of a series of documents published by the National Council of Teachers of Mathematics (NCTM), the leading organization for pre-collegiate mathematics in education in the US. Three documents have been influential in mathematics education: *An Agenda for Action (1980), Curriculum and Evaluation Standards for School Mathematics (1989) and Principles and Standards for School Mathematics (2000).* The latter two are referred to respectively as the 1989 *NCTM Standards, and 2000 NCTM Standards* or just the *Standards* when the context is clear. Virtually all states in the early 1990's were modeled after the standards of The *1989 NCTM Standard* (Klein, 2003).

An Agenda for Action recommended that problem solving be the focus of school mathematics. It stated that 'difficulty with paper-and-pencil computation should not interfere with learning problem solving strategies' (Gardner, 1983). Technology would make problem solving available to students without basic skills. According to the report, 'All students should have access to calculators and increasingly to computers throughout their school's mathematics program, even including elementary school students. It also called for decreased focus on activities that required use of paper and pencil calculation with numbers of more than two digits. It argued that emerging programs will prepare users of mathematics in non-traditional areas of application that may no longer demand the centrality of calculus. This would later support the move away from systematic development of its prerequisites: algebra, geometry and trigonometry. The 'integrated' high school mathematics of the 1990's contributed to this tendency. While those books contained parts of algebra, geometry and trigonometry, these traditional subjects were not developed systematically, and often depended on student 'discoveries' that were incidental to solving 'real world problems' (Klein, 2003).

The 1989 *NCTM standards* promoted the views of *An Agenda for Action* but with greater elaboration. It created general standards for grade level bands, K-4, 5-8 and 9-12. The K-4 band directed more attention to 'Operation sense', 'Use of calculators for complex computation,' 'Collection and organization of data,' 'Pattern recognition and description,' 'Use of manipulative materials,' and Cooperative work.' It also suggested decreased attention on 'Long division', 'Paper and pencil fraction computation', 'Rote practice' and 'Teaching by telling'. The 5-8 grade band suggested less attention be focused on 'Relying on outside authority (teacher or answer key)', 'Manipulating symbols', 'Memorizing rules' and 'finding exact forms of answers.' The 2000 NCTM reinforced the general themes of progressive education by advocating student centered, discovery learning (Klein, 2007).

Even more significant was the 'No Child Left Behind Act' (NCLB), signed into law by George W. Bush with bipartisan support in 2002. It is legislation founded on standard- based education reform. The Act incorporated the requirements that basic skills assessment be given to all students who are enrolled in federally funded schools at specific grade levels. Starting in the 2005/2006 school year, all states were required to assess students in both mathematics and reading grades 3 through 8. States should be on track to ensure that 100% of students demonstrate proficient levels of achievement on state created assessments (Knowles 2010). Also NCLB asked for challenging academic standards, high quality teachers, and imposed annual testing requirements on US schools (Klien, 2007).

The Math Wars

In California in the 1990's parents began to object the new direction that math curriculum was going. NCTM and programs were criticized for diminished content and lack of attention placed on basic skills. The elementary school programs required students to use their own invented arithmetic algorithms in place of the standard algorithms of arithmetic. The use of calculators was encouraged to excess and even integrated into kindergarten lessons. The preferred pedagogy was discovery work group which resulted in many cases in aimless or inefficient projects. At all grade levels, statistics and data analysis were overemphasized repetitiously and algebra and higher order mathematics were overlooked (Klein, 2007).

The arguments were strained. The critics of a traditional math curriculum believed the focus was too much on basic skills at the expense of understanding, and perceived as 'dumb downed' (Klein, 2007). A more progressive program was believed to better prepare for higher education in college. Yet, elite universities expected a more traditional curriculum as preparation for admission.

In California in December 1997, four state appointed mathematicians rewrote the standards, corrected more than 100 mathematical errors and eliminated all pedagogical directives that the previous standards had created (Klien,2005). The new standards' were clear and coherent. The standards emphasized basic skills and de-emphasized creative problem solving, procedural skills and critical thinking (Klein). The reaction from the mathematics reforms was quick. The NCTM lead story in the February 1998 News Bulletin, *New California Standards Disappoint Many,* charged that California's states board of education where against business, the community and education leaders when they unanimously approved the curriculum standards. Joining the NCTM, the state-wide chairs of Academic Senates of the public colleges and universities in California issued a joint statement condemning California's standards and claimed that 'the consensus position of the mathematical community' was against the problem. Then, in response, over 100 mathematician professors from California added their names to an open letter in support of the California standards, written by David Klein. This all lead California to create state-wide tests and systems for textbook adoptions that included review panels of mathematics and classroom teachers. Thus, California became the national base for opposition to the NCTM reform movement (Klein, 2007).

The US Department of Education (USDOE) released a report in October 1999 that designated 10 mathematics programs as 'exemplary' or 'promising'. The imprimatur of the US government carried by these controversial programs threated not only to undermine California's new direction in mathematics education, but it could marginalize criticisms of NCTM aligned textbooks nationwide (Klein, 2007).

A month after the USDOE report, more than 200 university mathematicians added their names to open letter to Secretary Richard Riley calling upon him to withdraw the recommendations (Klein, 1999). Within days the NCTM responded to the mathematicians' open letter to Secretary Riley in which the organization endorsed all of the 'exemplary' and 'promising' programs. In the following years, letters from mathematicians continued to be a useful tool for parents opposed to the NCTM aligned textbooks (Klein, 2007).

In March 2008, US Secretary of Education Margaret Spelling after more than two years of reviewing scientific evidence to advance the teaching and learning of mathematics concluded that math concepts must be taught earlier and that student must have confidence in improving their math skills in order for them to completely understand algebra concepts upon graduating high school. She believed that rapid recall of arithmetic facts in early grades is a significant component of future math comprehension (Spelling, 2008).

When No Child Left Behind was signed into law over a decade ago it presented states with a daunting mix of challenges that supported the creation of statewide standards and assessments and accountability requirements. As the US still lags behind other countries, the USDOE expects that gap will begin to close between American students and other top performing countries. Several governors and state education commissioners from 49 states, two territories and the District of Columbia came together to help draft a set of common academic standards for students in grades K through 12. After over 10,000 comments from the public, the Common Core State Standards for English and math were released (Corestandards.org).

The Common Core Standards provide a clear and consistent framework to prepare to students for college and careers. The standards were designed to be robust and relevant to the real world, reflecting the knowledge and skills that students need for success after high school. (Corestandards.org). The Common Cores standards dictated mathematics instruction at the time this study was conducted.

Relevant Studies

Brownell and Chazal's 1935 study *The Effects of Premature Drill in Third- Grade Arithmetic* examined "what contribution, if any, does drill make to raise the level of children's performance in arithmetic to promote growth in mature forms of quantitative thinking?" Theirs early work initiated a debate over the best approach to teaching facts that continues to the present day. Their work questions the traditional emphasis in schools on rote memorization, which, if done excessively, can reinforce students' use of immature methods for answering fact problems (Woodward, 2006).

The investigators began with administering, Test A, a written test of 100 addition fact combinations, to the group of the 3rd grade students. The children were urged to carefully write only the correct answers, and the time it took them to complete the test was recorded. Immediately after Test A, interviews of the 32 children were selected based on their responses to Test A were conducted. In the interview, each child was directed to “think out loud” in connection with each of the sixteen addition combinations, the ten most challenging on the test and 6 of average difficulty. Throughout the month that followed, five minutes were taken from each daily arithmetic period for further drill on addition combinations, distributed such that each combination was presented a total of 40 times. At the end of the month, the addition combinations were re-administrated as Test B and then a second interview was conducted with the same 32 students about the same 16 addition combinations as the previous interview. Then a month later, in which no special drill instruction on the 16 combinations took place, the 32 students took Test C and participated in a third interview.

On Test A the median time required was 17 minutes, and the median number of errors was eleven. On Test B, the median time was lowered to 11 minutes, and the median of errors decreased to four. On the Test C, the median was still further reduced to seven minutes, while the number of errors remained four.

The drill on the combinations produced results on the second tests of increased efficiency of correctly answering more addition combination, which corresponded closely with experiments which had been done previously. In the three interviews, the investigators classified the way students arrived at answers to four major types: “counting” “indirect solution” “guessing” and “immediate recall”. The data in Interview I of the 512 combinations resulted in 22.2 percent were counted; 14.1percent were indirectly solved; 23.8 percent were guessed; and only 39.5

percent were known when taught by drill. In Interview II counting was found to persist to the extent of 17.4 percent of the total – a reduction of 5.3 points in percentage. There was a slight increase in indirect solution from 14.1 to 15.6. Guessing was reduced from 23.8 percent to 18.2; and immediate recall increased 9.2 points, to make 48.8 percent.

At the time of Interview III the children on average counted two more combinations per hundred that they did at the time of Interview II; they solved approximately three less per hundred and they recalled immediately nearly four times more per hundred. Progress toward automatized associations came therefore at the expense of indirect solution and guessing, though this improvement in performance was in part balanced by slight retrogression toward counting.

The data that was collected led the investigators to three conclusions. First, drill as it was administrated in this study doesn't guarantee that children will be able immediately to recall combinations. The reason lies in the fact that the drill as given by the teacher doesn't necessarily lead to the repetition on the part of the pupils. Second, in spite of long-continued drills, children tend to maintain the use of whatever procedures they have found to satisfy their needs. Third drill makes, little if any, contribution to growth in quantitative thinking supplying mature ways of dealing with numbers..

Brownell and Chazal believed that the conclusion in no way implied that drill had no place in arithmetic. They saw drills as exceedingly valuable for increasing, fixing, maintaining and rehabilitating efficiency otherwise developed. They felt that for drill to more effective, it must be preceded by sound instruction. It should encourage vigorous study of the problems of learning and of initial instruction, even if this change in interest lessened somewhat the extraordinary attention given to drill.

Cumming and Elkins' research (1999) suggested that a middle-ground position for teaching facts to academically low-achieving students and students with LD consisted of integrating strategy instruction with timed practice drills. Results of their research indicated that instruction in strategies did not necessarily lead to automaticity. Frequent timed practice was essential. However, strategies helped increase a student's flexible use of numbers, and for that reason, the researchers advocated the use of strategy instruction for all students through elementary school.

Woodward's 2006 study contrasted two approaches to teaching multiplication facts to academically low-achieving students and LD. The intervention drew on instructional design that put emphasis on the link between facts and extended facts. Participants were spilt into two groups: an integrated group and timed practice only group. In the integrated group, facts were divided into relatively easy facts and those that were seen as more difficult. The timed only groups were taught the new facts sequentially and previously taught facts were reviewed on a daily basis and incorporated into daily timed practice drills.

Both methods were effective in raising the mean performance level on a mix of math facts to mastery or near mastery levels, and these levels dropped only slightly during the maintenance phase of the study. Both groups improved considerably in their knowledge of harder multiplication facts. Posttest and maintenance test scores for both groups were virtually identical. The difference in performance levels between students with and without LD, revealed the challenge of moving all forward at the same ratio. All students improved from pre to posttest, however students with LD in both groups lagged considerably behind their peers. The discernible difference between the two groups was that the integrated group had an opportunity to discuss the connection between facts and approximating answers to multiplication problems and the Timed Practice Only groups were given more practice opportunities. Therefore, the integrated groups' performance on extended facts and approximation tests were significantly higher. While the timed only added practice time resulted in performance, even though the differences between the two groups on Computations posttest were statistically insignificant.

The Precision Teaching Project in Great Falls, Montana school district, was accepted by the Office of Education Joint Dissemination Review Panel as an exemplary educational model for both regular and special education (Beck, 1979), Teachers engaged elementary school students in 20 to 30 minutes per day of timed practice, charting and decision –making in a range of basic skills over four years. The results were improvement between 19 and 44 percentile points on subsets of the Iowa Test of Basic Skills, as compared with children in group classrooms elsewhere in the district (Binder, 1988).

On the basis of the Great Falls results, Federal funding through the National Diffusion Network supported training by Great Falls staff to thousands of teachers throughout North America. The improvements themselves were dramatic; but when cost/benefit was considered, they were staggering, since the time allocated to Precision Teaching was relatively small and the materials used (primarily standard charts and mimeographed practice sheets) were quite inexpensive. Improvements of two or more grade levels per year of instruction are common in Precision Teaching classrooms (West, Young, & Spooner, 1990).

Although an important development in many schools has been to combine Precision Teaching, as a practice and measurement strategy with Direct Instruction. Another trend has been toward privatization, the movement of trained Precision Teachers out of public education to form their own schools and tutoring agencies competing in the private sector on the basis of educational effectiveness. For example, at the Morningside Academy in Seattle, which combines Precision Teaching and Direct Instruction, parents receive a money back guarantee that their children will achieve at least one year's progress in their worst skill area during a two-month summer session (Binder 1988).

Conclusion

This chapter provided relevant background to the study's literature and the specific research setting. Following a brief introduction to the study, the researcher addressed how both automaticity and fluency of basic math facts are key to success when learning higher order math skills. Also it reviewed how both Precision Teaching and Direct Instruction are used to develop automaticity and fluency. It examined the history of how mathematics standards affected how the focus should be on higher order math concepts. While also discussing the use of math fact drills as an educational tool. Chapter III discusses the methods and procedures used to conduct and complete this study.

Chapter 3

Methodology

Introduction

As students proceed through Math, the skills which they have learned build upon each other. Without the development of a strong foundation of their basic skills, cracks appear in students learning which makes learning new skills even more challenging than they already are. Many students are able to overcome these challenges by developing their own methods of problem solving to compensate for inadequate skills. These contrived methods can be used to solve higher-order problems and may lead to the correct answer. They are often complex, lengthy and hinder success. Students must be able to develop both automaticity and fluency with their basic facts skills to improve their chances for success. Being able to recall facts quickly and easily allows the student to direct their effort to the higher-order thinking skills that are required to solve more advanced problems.

This study was implemented to examine the 5th graders ability to master their multiplication facts after completing timed multiplication drills and applying these skills to higher order thinking math skills that require multiplication to solve.

Participants

The participants. The participants of this study were from the five sections of 5th grade classes that the researcher taught. All of the sections were heterogeneously mixed. number of participants had Individual Educational Plans. Two of the participants were diagnosed as Other Health Impairment. The teacher taught using materials from *Math in Focus Singapore Math Approach* by Marshall Cavandish (2010) which was aligned with the core curriculum standards.

This program is based on the framework developed by the Singapore Ministry of Education. Its framework centers around a mathematical approach that focuses on positive attitudes and metacognition to develop students thinking. This was the first year in which students were taught with the Singapore approach.

The study had 75 participants. All of the participants were 5th graders; they ranged in age from 10 to 11 years old. Students attended math class each school day for 45 minutes. The students in each class sat in groups of three to five students where they were heterogeneously mixed. In fourth grade on the New Jersey Assessment of Skills and Knowledge (NJASK) 20% scored advanced proficient, 67.3% scored proficient and 12.6% below proficient. The mean score of the participants was 226 and the range of was 171 to 299.

The teacher instructed the participants in several different ways throughout the school year. The students first entered the class and a ‘do now’ was displayed or a quick group activity. Next, the previous night’s homework was shared and reviewed. Following that the teacher instructed the students by using both lecture and class discussion. The teacher walked the students through an example problem as an entire class, before students began working. The students worked independently or in pairs to complete a given set of problems that related to the days lesson. The teacher monitored the students’ progress and provided the necessary support. The class period concluded with either a short discussion summing up what was learned in class or an exit question that the students answered before the class period was over. Students received homework Monday thru Thursday.

Each class received tokens for good behavior at the end of the class period. The classes received prizes for the accumulation of a set number of tokens.

Inclusion teacher. In each section twice a week a second teacher came into the class for Basic Skills Instruction (BSI). The BSI teacher in the classroom was a co-teacher who targeted students who scored below proficient on the Mathematics portion of the NJASK. Over the five sections of fifth grade, three different teachers provided BSI. Teacher 1 was a first year teacher that taught 6th grade math. He was certified in middle school math and high school math and was the BSI teacher for two of the sections. Teacher 2 had 15 years of experience and taught resource math grades 5th through 8th. She was certified in Middle School Math and Teacher of Students with Disabilities and was the BSI teacher for two of the sections. Teacher 3 had 14 years of experience and taught 7th grade Social Studies; previously she was an LLD teacher. She was certified in middle school Social Studies, Math and Teacher of Students with Disabilities and was the BSI teacher for one of the sections.

The researcher. The researcher was the teacher for all of the classes and implemented all the variables for this study. He was certified as an elementary and middle school math teacher and Teacher of Students with Disabilities. He was obtaining a Masters' degree in a special education program. The researcher had been teaching at the 5th grade level for two years. The researcher collected and analyzed the data and had a personal knowledge of all participants in the study. He taught the participants from the start of the school year.

Materials

To protect the validity and credibility of this study, triangulation was used to collect data. Triangulation is the "process in which multiple forms of data are collected and analyzed (Hendricks, 2009, p.80). Three forms of data collection were used for this study: pre- and post-tests, math fact drills and student questionnaire.

To ensure confidentiality, a parent letter and consent form (see Appendices A) was sent home, informing parents and students about this study. The consent form included an overview of the procedure that would be implemented in the class, as well as the possible risks or benefits associated with the study. The consent form also stated that the parents had the ability to withdraw their child at any point during the study.

The materials that were included within this study were three tests of 20 three digit by three digit multiplication questions (see Appendix B), ten timed multiplication fact drills (see Appendix C), of 50 multiplication problems ranging from 1 x 1 to 10 x 10 and a student questionnaire (see Appendix D). The three digit by three digit tests contained problems that were typed vertically over three 8x11 sheets of white paper in black font. Students were provided with adequate space to show their work. The multiplication fact drills were setup horizontally in two columns of 25 on one 8x11 sheets of white paper in black font.

The student questionnaire (see Appendix D) was given to the participants when the study was completed. The purpose was to gain information on the students' understanding on how they respond to multiplication fact drills and higher order multiplication problems. It also gave the students the opportunity to share their opinions on how they felt the multiplication fact drills affected their memorization of their multiplication facts. . The questionnaire consisted of eight positive and negative questions asking student's opinion about the way they were taught. They had five answer choices for each question ranging from strongly agree to strongly disagree. It was used to asses if the students felt they were able recall their basic multiplication facts easier when completing more complex tasks.

The three data collection tools used during this study helped to measure how the multiplication fact drills affected student's ability to apply and maintain their multiplication facts to more complex math skills.

Procedure

The study took place over a four-week period. Before the implementation of the basic multiplication fact drills, students completed the pretest of the triple digit multiplication problems. The students were given 30 minutes to complete as much of the pretest as they could. The timed pre-tested was placed on the students desks faced down and the participants were instructed to write their name on the back of the paper. Next, the researcher instructed them to answer the multiplication problems as accurately and quickly as possible. To begin the pretest, the teacher said "ready, set, go" and the participants turned their drills over to begin at the same time the instructor started the clock. The digital timer on the computer was projected on to the screen in front of the class for all the participants to easily see. Upon finishing the tests students recorded their time at the bottom of the last page. The participants recorded the number of minutes and seconds that in took to complete the pretest. The students completed this work independently and no feedback was given. The same procedure for was followed on the posttest 'application' and posttest 'maintenance'.

After completing the initial pretest, students were given a timed multiplication fact drill every other day of school. The timed multiplication fact drills were placed on the students desks faced down and the participants were instructed to write their name on the back of the paper. Next, the researcher instructed them to answer the multiplication problems as correctly and quickly as possible. To begin the drill, the teacher said "ready, set, go" and the participants turned their drills over to begin at the same time the instructor started the clock. The same process was used on all drills during this study. When students completed the drill, they recorded the time the stop watch said at the bottom of the page. The teacher then collected the drill to be graded. The students received the completed drill back the following school day. Any incorrect responses were marked with the correct responses. The students kept track of their scores and times on a chart (Appendix E) so that they could see how they were progressing in completing multiplication fact drills. After receiving back the first graded drill, the participants established goals for themselves indicating how many multiplication facts they wanted answer correctly per minute.

After all of the ten multiplication drills were completed, the second test of triple digit addition problems was administered. The posttest 'application' was administered the day after the participants received back their last multiplication fact drill in the same manner as the pretest. The posttest 'maintenance' was administered four weeks after the participants completed the 'application' posttest. The questions on the three multiplication tests and ten multiplication fact drills were selected randomly from *Free Math Worksheets* at www.math-drills.com.

Conclusion

By using triangulation the researcher was able to obtain data from all different points of view. Obtaining data in several forms helped to keep this study valid. It gave the ability for this study to implement the strategy at another time in another classroom. The collection tools selected were the most reliable sources of data for the study that has taken place.

Chapter 4

Analysis of Data

Introduction

Math is a subject that progresses by continual building on topics that already have been learned. Multiplication is a foundational skill in math that is necessary to complete more higher –order skills. Students are often introduced to new mathematical concepts before they ever master multiplication facts. This study examined the effects of multiplication fact drills on student's ability to master, apply and maintain higher-order thinking skills that require multiplication to solve.

Results of Data

Pre Test and Post Tests Data

Three digits by three digit multiplication pretest, Post Test Application and Post Test Maintenance. The first method of data collection was the pretest to establish a base line of the student's ability to apply basic multiplication in higher order thinking skill. Table 1 below shows the mean score for each section on the Pretest and Post Test Application and the difference of the means in each class. Each section had between 13 and 18 participants. In the entire study there were 75 participants. The pretest and both posttest scores were calculated by figuring what percent out of 20 the students got correct. Then the percent change of the pretest was done by finding the difference between pretest and the posttest application.

Table 1 below shows the mean percentage correct for each section on the Pretest, Post Test Maintenance and difference between the two. The posttest application was taken the day after the last drill was returned to the students. The mean percent correct on the pretest was 34.49% which increased to 51.85% on the Posttest Application. The mean percent of change was 17.32%. The range of percent correct on the pretest was 38.33% to 27.50%. The range of percent correct on the posttest application was 60.42% to 44.17%. The range of percent change from the pretest to posttest application was 25.71% to 10.47%.

Table 1

Mean Percent Correct For Each Section on the Pretest and Post Test Application

	Pretest	Post Test App.	Percent Change
Section 1	36.47%	51.74%	+15.27 %
Section 2	33.46%	44.17%	+10.47%
Section 3	27.50%	53.21%	+25.71%
Section 4	38.33%	60.42%	+22.09%
Section 5	36.67%	49.71%	+13.04%

Table 2 below shows the mean percentage correct for each section on the Post Test Application, Post Test Maintenance and difference between the two. The posttest maintenance was taken by the participants four weeks after the posttest application. The mean percent correct on the Posttest Application was on 51.85% and 54.74% on the Posttest Maintenance. The mean percent of change between the two tests was 2.89%. The range of percent correct on the posttest application was 60.42% to 44.17%. The range of percent correct on the posttest maintenance was 61.67% to 48.16%. The range of percent change from the pretest to posttest application was 7.46% to -1.10%. Section 5 results on the Posttest Maintenance were the outlier of the data because their percent correct was dropped rather than improved like the others.

Table 2

Mean Percent Correct of Each Section on the Post Test Application and Post Test Maintenance

	Post Test App.	Post Test Maint.	Percent Change
Section 1	51.74%	53.53%	+1.79%
Section 2	44.17%	49.23%	+5.06%
Section 3	53.21%	60.67%	+7.46%
Section 4	60.42%	61.67%	+1.25%
Section 5	49.71%	48.61%	-1.10%

Table 3 below shows the mean percentage correct for each section on the Pretest, Post Test Maintenance and the difference between the two. The mean percent correct on the pretest was 34.49% and 54.74% on the Posttest Maintenance. The mean percent of change between the two tests was 20.24%. The range of percent correct on the pretest was 38.33% to 27.50%. The range of percent correct on the posttest maintenance was 61.67% to 48.16%. The range of percent change from the pretest to posttest application was 33.17% to 11.85%.

Table 3

Mean Percent Correct of Each Section on the Pretest and Post Test Maintenance

	Pretest	Post Test Maint.	Percent Change
Section 1	36.47%	53.53%	+17.06%
Section 2	33.46%	49.23%	+15.77%
Section 3	27.50%	60.67%	+33.17%
Section 4	38.33%	61.67%	+23.34%
Section 5	36.67%	48.61%	+11.85%

Multiplication Fact Drills Data

Ten Multiplication Fact Drills. The second method of data collection was ten timed multiplication fact drills. The participants completed a fact drill every other day. The participants received back the drills corrected with their rate per minute correct the day after they were completed. The rate per minute correct was calculated by dividing the number of seconds taken to complete by the number correct, and then the dividend was divided by 60.

Figure 1 shows the rate per minute correct on the first drill multiplication (Drill A) and the last drill (Drill J) for all sections and the overall mean of all participants. For Section 1 on Drill A the mean rate per minute correct was 25.25. On Drill J the mean rate per minute was 32.47. The mean rate of change in this section was 7.23 more problems correct per minute from the first to last drill. Fifteen of the sixteen participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -10.41 to 19.64.

For Section 2 on Drill A the mean rate per minute correct was 18.38. On Drill J the mean rate per minute was 22.58. The mean rate of change in this section was 4.20 more problems correct per minute from the first to last drill. Fourteen of the fifteen participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -8.72 to 20.11.

Section 3 on Drill A the mean rate per minute correct was 21.5. On Drill J the mean rate per minute was 30.44. The mean rate of change in this section was 8.95 more problems correct per minute from the first to last drill. Fourteen of the fifteen participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -3.37 to 32.95.

For section 4 on Drill A the mean rate per minute correct was 26.55. On Drill J the mean rate per minute was 34.98. The mean rate of change in this section was 8.43 more problems correct per minute from the first to last drill. Thirteen of the fourteen participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -2.39 to 17.74.

For Section 5 on Drill A the mean rate per minute correct was 23.02. On Drill J the mean rate per minute was 31.40. The mean rate of change in this section was 8.38 more problems correct per minute from the first to last drill. Sixteen of the seventeen participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -1.44 to 32.13.

For all of the participants on Drill A the mean rate per minute correct was 22.90. On Drill J the mean rate per minute was 30.28. The mean rate of change in this section was 7.38 more problems correct per minute from the first to last drill. Seventy of the Seventy-Five participants showed an improvement in the rate per minute answered correctly. The change in rate of per minute answered ranged from -10.41 to 32.95.

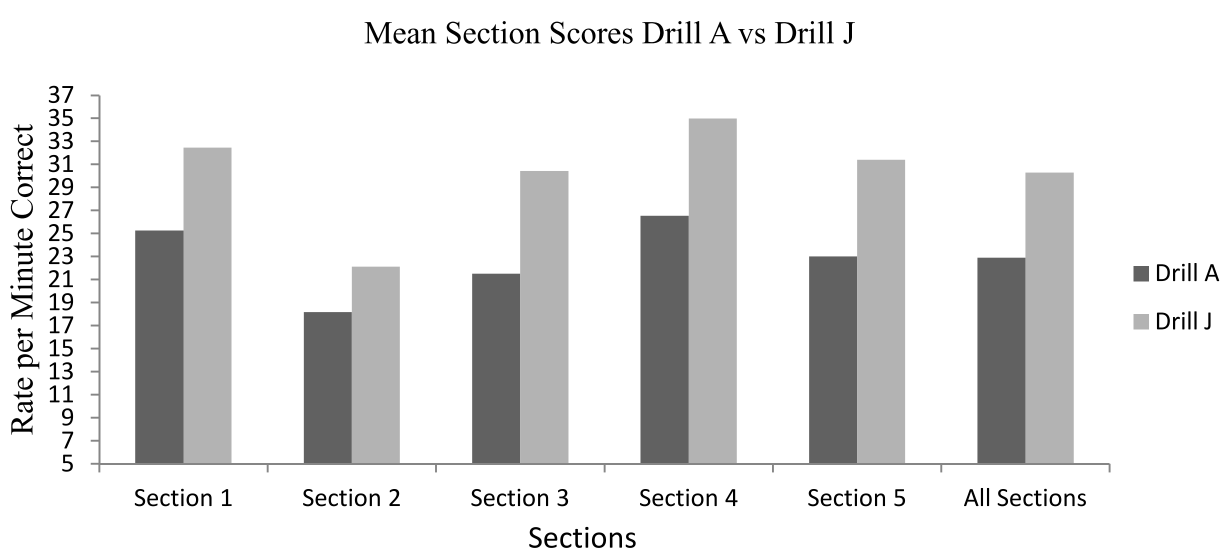

Figure 1. Results of Mean rate per minute answered correctly on Drill A compared to Drill J for each Section.

Figure 2 shows the mean rate per minute answered correct on each multiplication drill for each section and the mean for all participants. The overall results showed an increase from the first drill to the tenth drill. Section 2 mean rate per minute from drill to drill did not show the same kind of increase that the other sections did. All sections had a spike in their mean rate per minute on drill E, which did not follow the same upward trend as the drills.

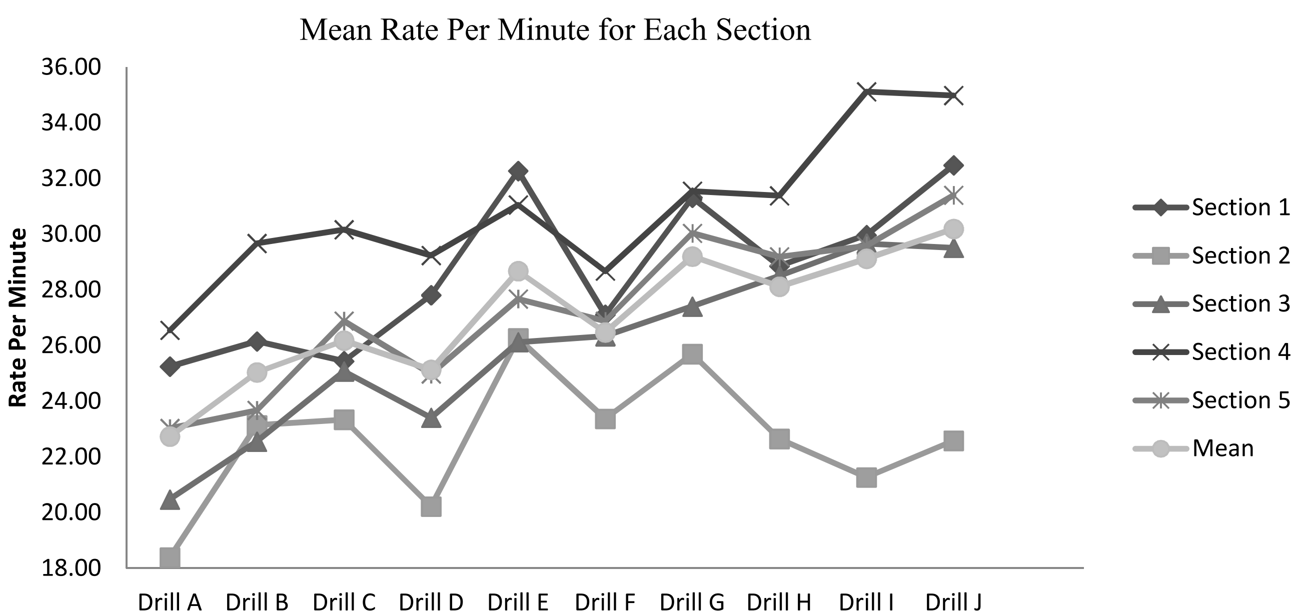

Figure 2. Mean Rate Per Minute for each section and the overall mean score for all participants.

Student questionnaire. The third method of data collection was a student questionnaire given after the Post Test Maintenance. Table 4 displays the results of the students' answer to the questions given. The majority of students gave their best effort when completing the multiplication drills. The participants responses were split almost evenly with regards to the difficulty they had remembering their multiplication facts before completing the drills.. While after the multiplication drills the majority of students responded they now found multiplication facts easy to remember. A majority of the participants did not agree that three digit by three digit multiplication pretest was easy before the fact drills. After the completing the facts drills, more students felt that the three digit by three digit multiplication posttests became easier after the fact drills.

Survey Questions	Strongly Disagree	Disagree	No opinion	Agree	Strongly Agree
Are you now more confident with your ability to multiply after the multiplication fact drills?	0	1	7	23	44
Did you give your best effort when completing the multiplication drills?	0	1	0	22	52
Did you find the multiplication facts difficult to remember before the multiplication fact drills?	22	16	15	15	7
Did you find 3 digit by 3 digit multiplication easy when you completed the pretest?	10	24	23	14	4
Did you not give your best effort when completing the multiplication drills?	55	15	5	0	0
Did you find multiplication facts easy to remember before the multiplication fact drills?	6	15	21	25	14
Did you find multiplication facts easier to remember after the multiplication fact drills?	0	3	8	15	49
Did you find 3 digit by 3 digit multiplication easy when you completed the Post Tests?	7	10	16	23	19

Table 4 Results from participant survey

Analysis of Data

Triangulation was used during this study, which provided validity and credibility. Triangulation helps in discovering common themes from various sources (Hendricks, 2009). This ensures that an explanation can be given for why a study is successful or has failed

(Hendricks, 2009). The three collection techniques used were a Pre Test and two Posttests on three digit by digit multiplication, ten multiplication fact drills and a student survey. By using all three techniques, the data collected and results have a stronger substance.

From the pretest to the posttest application 70 out of 75 participants were able to apply what they learned by completing the multiplication fact drills to higher order thinking skills. From the posttest application to the posttest maintenance 25 students' percent correct decreased less than 15%, 10 students percent correct stayed the same, and 40 students percent correct increased between 5% and 50%. The results show from the posttest application to the posttest maintenance that two-thirds of the participants were able to maintain what the learned by completing the multiplication fact drills to higher order thinking skills more successfully.

The majority of students rate per minute correct increased from the first drill to the last drill. The overall mean rate per minute increased for each section from Drill A to Drill J. The data shows that the fluency and automaticity that the participants developed on the drills they were able to apply and maintain to the higher order thinking skills of three digits by three digit multiplication on the posttests.

Also the participants' responses on the survey showed that not only were they able to do higher order skills but that they are now more confident in doing so because the multiplication fact drills.

Limitations

Limitations can be found in every study. The first limitation was the length of time available to collect the data for the study. With only a limited amount of time to complete the study, the participants could have benefited from more multiplication drills. The mean number correct rate per minute on the drills was still improving after ten drills. Future studies, should consider continuing implementing the multiplication drills until the rate per minute correct no longer shows improvement or to a specified criterion for each student is met.

Another limitation was students were limited to thirty minutes to complete the three digit by three digit multiplication tests because of the length of time of the class periods. Many students were unable to complete all twenty problems. Students were only graded on the problems that they were able to complete. In future studies, the research should consider allowing the participants a limitless amount of time to complete the pre and posttests.

An additional limitation of the study was how fast students can actually write their answers. Several participants were able to answer all the multiplication problems correctly but were unable to complete the drills any quicker. Future studies, should look into how quickly the participants can actually write. Also other methods of response should be considered such as a computer or oral responses.

The level at which the pre and post tests were at was a limitation of the study. The majority of the students showed an improvement on their rate per minute correct on the multiplication drills but that same majority didn't show improvement the same type of improvement from the pretest to posttests. In future studies, the researcher should look at having various levels of higher order thinking posttests.

Discussion

It was hypothesized that by having the participants complete multiplication fact drills it would help them on higher order thinking skills that require multiplication to solve. On the posttest application each section showed an improvement. Overall from the pretest to posttest application or posttest maintenance 69 participants showed improvement, four participants showed no change and two participants scored a lower. Based on the data collected, incorporating multiplication fact drills does help students on higher order thinking skills that require multiplication to solve.

All five of the sections showed an improvement from drill to drill. Section 2 was the only section that had two participants in their group which completed the multiplication at a rate of fewer than 10.0 per minute, this caused Section 2 to not have the same kind of overall improvement on the rate per minute from the first drill to the last drill. These two outliers brought the overall mean score down on the fact drills. Despite the outliers from Section 2 their percent correct increased from 33.46% on the pretest to 49.23% on the posttest maintenance which was similar to the other sections.

Section 5 scored 36.67% on the pretest, increased to 49.71 % on the application test but was the only section in which their percent correct decreased on the maintenance test by 1.1%. Also in this section of participants, there were the two students that consistently had the highest rate correct per minute on the fact drills of all participants. Participant number 1’s highest rate per minute was 61.22 per minute on Drill I while scoring a 50% on the pretest, 65% on the posttest application and 90% on the posttest maintenance which shows a positive correlation that increased fluency and automaticity do have a positive effect on application of higher order thinking skills. While participant number 2’s rate per minute peaked at 61.28 on Drill J, the fluency and automaticity on the fact drills did not have the same kind of the positive affect as the other high score. Participant number 2 scored 20% on the pretest, 30% on the posttest application and 35% posttest maintenance.

Several participants were consistent on scoring the same rate per minute correct for the facts drills. Despite the lack of an increase on the rate per minute correct on the fact drills they were still able to improve on the posttests. For example participant 3 from Section 3 had rates per minute ranging from 29.4 and 33.7 on the ten drills. Their pretest was a 10%, posttest application was 60% and scored the only 100% on the posttest application.

Throughout all of the sections several participants in the study showed an improvement in their multiplication drills. Despite this improvement they were unable to apply their multiplication to the higher order level of three digit by three digit multiplication. In grading the pre and posttests of the these participants I was able to identify that they were able to multiply better but were challenged by the complexity of the problems and struggled to be organized to correctly solve three digit by three multiplications problems.

The students were not fans of the three digit by digit multiplication tests because they felt they were long and challenging. They enjoyed the multiplication fact drills because they were able to be successful on them. Many students were excited when they walked into class and knew that they would complete a drill that class period. The students enjoyed tracking their results on their data chart and really worked hard to increase their rate correct per minute. Some students were very competitive with each other to score a higher rate.

There were a total of nine low achieving participants that were unable to increase their rate per minute correct over 10.0. None of these participants scored above 15% on of the three digit by three digit multiplication tests. Also three of these participants scored 0% on all three tests.

The results of this study were similar to Woodward's (2006) approach which taught multiplication facts to low-achieving students and LD. Participants were spilt into two groups: an integrated group and timed practice only group. The participants in this study were spilt into an integrated group and timed only group. For the integrated group facts were divided into relatively easy facts and those that were seen as more difficult. The timed only groups were taught the new facts sequentially and previously taught facts were reviewed on a daily basis and incorporated into daily timed practice drills. Both groups improved considerably in their knowledge of harder multiplication facts. Posttest and maintenance test scores for both groups were virtually identical. The difference in performance levels between students with and without LD, revealed the challenge of moving all forward at the same ratio. All students improved from pre to posttest, however students with LD in both groups lagged considerably behind their peers.

Implications of Teaching

The process for this action research project provided me with a much better understanding of how students learn their multiplication times tables differently. The literature review was a long process that provided me with the insight to better understand the learning process involved in math. The research that I did on this study, not only helped on this study, but also helped in my everyday teaching to better understand how my students learn multiplication. Reviewing Literature can allow for a teacher to learn and grow, without having to take a class.

Having so many participants in the study made collecting data very time consuming. Having the students complete the drills every other day and returning the results to them the next day required a large amount of time. I found the pre-tests and post-tests were important to finding substantial data for my study because it showed if the students were able to apply and maintain what they learned in the drills.

I found the analysis of the data to be very challenging. Finding a way to take all of the fact drills data and three digit by three digit multiplication data putting it into tables and charts was a troublesome task because the amount of participants and all the different data Through making charts and tables I was able to determine a trend in my results.

The most rewarding part of this study was the positive results that I saw from the study. After completing the study I can now take what I learned and improve the study to benefit even more of my students in the future. Through the results that I was able to see how multiplication fact drills have an effect on higher order thinking skills that require multiplication to solve.

In my future teaching I will definitely continue to use multiplication fact drills to help develop automaticity and fluency in multiplication students.

Conclusion

The research question stated: “What are the effects of timed multiplication fact drills on 5th graders’ ability to master, maintain and apply their multiplication facts to higher- order thinking problems that require multiplication to solve.?” The hypothesis that timed multiplication facts would improve higher order thinking skills that require multiplication to solve was correct for a majority of the students but not all. This study showed that multiplication fact drills had a positive effect on students’ ability to master, apply and maintain their multiplication to higher order thinking skills.

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Parental Letter

Appendix A

Dear Parent/Guardian:

I am a graduate student at Caldwell College in the process of writing my thesis. The thesis topic is "The effects of timed multiplication fact drills on 5^{th} graders ability to master, maintain and apply their multiplication facts to higher- order thinking problems that require multiplication to solve." As a result, your child has been selected to participate in this study.

The student participants will complete a series of daily timed multiplication fact drills in math. Students will be given a pre-test of three digits by three digit multiplication. Then after the multiplication fact drills are all completed, the students will take two post-test of three digits by three digit multiplication. At the end of the study, students will also be given a questionnaire about the student's thoughts on how they thought multiplication fact drills had an impact on them. The participants' results will be used anonymously to determine the effectiveness of multiplication fact drills and write my thesis.

Your child may or may not benefit from the specific teaching method being implemented into their class but I appreciate your consideration for this study. Also know that it is your option to allow your child to participate in this study. Should you decide at any time to withdraw your child from this study, you may, with no negative consequences. I thank you and appreciate your cooperation with this study.

Sincerely,

Frank Orefice

Consent and Information

Title: The effects of timed multiplication drills on 5th graders

Participant name: ________________________

Principle Investigator: Frank Orefice

1) **Purpose of the study:** To determine the effects of timed multiplication fact drills on 5th graders ability to master, maintain and apply their multiplication facts to higher-order thinking problems that require multiplication to solve..

2) **Description of the Project**
   a. **Selection of Subjects** 5th grade students attending a suburban NJ middle school grade 5th to 8th were selected to participate.
   b. **General procedures**: Students will complete an pre multiplication test of problems that multiple a three digit number by three digit number. Students will complete 10 timed multiplication fact drills of problems from 1x1 to 10 x 10. Then they will take two posttests similar to the pretest. One will be taken a day after the last fact drill and the next one will be 4 weeks later.

3) **Description of Foreseeable Risks:** The risks in this study are the same as they would be in class on an ordinary class.

4) **Benefits:** The potential benefits are the acquisition of better memorization of the multiplication.

5) **Confidentiality Statement:** All documents and information pertaining to this research study will be kept confidential in accordance with all applicable federal, state, and local laws and regulations. Data sheets will be coded so as to protect the confidentiality of the participants. All materials related to this research project will be locked in the office of either the principle investigator or the co-investigators. No one else will have access to these records. I understand that Caldwell College's Institutional Review Board, The Office of the Vice President and Dean of Academic Affairs, and the Office for Human Subjects Protections (OHRP), may review records and data generated by the study to assure proper conduct of the study and compliance with federal guidelines. Appropriate HIPAA Authorization forms will also need to be completed
I understand that the results of this study may be published. If any data are published, I (or my child) will not be identified by name.

Disclaimer / Withdrawal: It is your option to allow your child to participate in this study. Should you decide at any time to withdraw your child from this study, you may, with no negative consequences.

6) **Institutional Contact:**
    a. Rights as a Research Subject: If I have any questions about my rights as a research subject, I may contact Ken Reeves, Chair of the Institutional Review Board, (973) 618-3639.
    b. Research related injuries: Non foreseeable

7) **Injury / Complications:** None foreseen

8) **Costs Statement:** None

9) **Final Statement and Signature:** This statement has been explained to me, I have read the consent form and I agree to allow my child to participate. I have been given a copy of this consent form.

I understand that if I wish further information regarding my rights as a research subject, I may contact the Chair of the Institutional Review Board of Caldwell College by phoning (973) 618-3919.

____

Participant's Signature	Printed Name	Date / Time

____

Legal Guardian Signature	Printed Name	Date / Time

____

Investigator's Signature	Printed Name	Date / Time

____

Faculty Sponsor's Signature (If applicable)	Printed Name	Date / Time

Pretest

Appendix B

Name:________________________________ Date:____________________ Homeroom:____________

3 Digit Multiplication Pre test

Show your work. Find the product.

1.) 983 X 825	2.) 192 x 288	3.) 140 x 727
4.) 263 X 360	5.) 859 x 305	6.) 392 x 239
7.) 614 X 589	8.) 432 x 736	9.) 186 x 479
10.) 292 X 413	11.) 531 x 619	12.) 670 x 992

13.) 601
X 798

14.) 327
x 849

15.) 828
x 499

16.) 793
X 127

17.) 431
x 972

18.) 263
x 789

19.) 583
X 471

20.) 358
x 579

Time Finished

Multiplication Fact Drill

Appendix C

Multiplication Facts (A)

Find each product.

$10 \times 9 =$

$9 \times 1 =$

$9 \times 7 =$

$6 \times 3 =$

$9 \times 3 =$

$9 \times 2 =$

$10 \times 1 =$

$3 \times 4 =$

$6 \times 1 =$

$10 \times 10 =$

$7 \times 10 =$

$1 \times 1 =$

$8 \times 9 =$

$4 \times 4 =$

$10 \times 2 =$

$10 \times 4 =$

$6 \times 9 =$

$7 \times 8 =$

$9 \times 10 =$

$1 \times 3 =$

$4 \times 5 =$

$5 \times 6 =$

$8 \times 5 =$

$8 \times 7 =$

$2 \times 9 =$

$6 \times 7 =$

$2 \times 6 =$

$8 \times 10 =$

$9 \times 8 =$

$3 \times 3 =$

$1 \times 8 =$

$1 \times 6 =$

$5 \times 8 =$

$8 \times 3 =$

$10 \times 3 =$

$5 \times 4 =$

$8 \times 8 =$

$2 \times 7 =$

$6 \times 8 =$

$5 \times 10 =$

$3 \times 10 =$

$5 \times 9 =$

$5 \times 7 =$

$8 \times 6 =$

$4 \times 6 =$

$1 \times 2 =$

$6 \times 4 =$

$1 \times 5 =$

$4 \times 7 =$

$2 \times 3 =$

Student Questionnaire

Appendix D

Student Survey

Directions: Read the questions below and answer each question by circling the number that best describes your answer. The scale for answers is listed below. There is room at the bottom of the page for any other comments that you wish to make.

1 = strongly disagree 2 =disagree 3 = no opinion 4 = agree 5 = strongly agree

1.) Are you now more confident with your ability to multiply **after** the multiplication fact drills?

1 2 3 4 5

2.) Did you give your best effort when completing the multiplication drills?

1 2 3 4 5

3.) Did you find the multiplication facts difficult to remember **before** the multiplication fact drills?

1 2 3 4 5

4.) Did you find 3 digit by 3 digit multiplication easy when you completed the **pretest**?

1 2 3 4 5

5.) Did you **not** give your best effort when completing the multiplication drills?

1 2 3 4 5

6.) Did you find multiplication facts easy to remember **before** the multiplication fact drills?

1 2 3 4 5

7.) Did you find multiplication facts easier to remember **after** the multiplication fact drills?

1 2 3 4 5

8.) Did you find 3 digit by 3 digit multiplication easy when you completed the **Post Tests**?

1 2 3 4 5

Chart

Appendix E

Participant name:____

Three by Three Multiplication Tests			
	Pretest	**Post Test Application**	**Post Test Maintenance**
Questions answered			
Questions Correct			
Time			
Average Number Correct Per Minute			

	Multiplication Fact Drills Scores			
	# Answered	# Correct	Time	Avg. # Correct per minute
Drill A				
GOAL For Drills				
Drill B				
Drill C				
Drill D				
Drill E				
Drill F				
Drill G				
Drill H				
Drill I				
Drill J				
Overall Change				