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# Essays in International Finance

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to

The Department of Economics

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## Essays in International Finance

### ABSTRACT

This dissertation consists of three essays in international finance. The first two essays study emerging market sovereign risk with a focus on local currency denominated sovereign bonds. The third essay examines econometric tools for robust inference in the presence of missing observations, an issue frequently encountered by researchers in international finance.

Most emerging market sovereign borrowing is now denominated in local currencies. In Chapter 1, we introduce a new measure of sovereign risk, the local currency credit spread, defined as the synthetic dollar spread on a local currency bond after using cross currency swaps to hedge the currency risk of promised cash flows. Compared with traditional sovereign risk measures based on foreign currency denominated debt, we find that local currency credit spreads have lower means, lower cross-country correlations, and are less sensitive to global risk factors. We rationalize these findings with a model allowing for different degrees of integration between domestic and external debt markets.

Chapter 2 documents new empirical evidence on the rapid growth of foreign ownership of emerging market local currency sovereign debt over the past decade. We study risk of nominal bonds without hedging away the currency risk. We show that local currency nominal bond risks differ across countries and are highly correlated with sovereign credit default swap spreads on foreign currency external debt. Using data on investors' forecasts of inflation and growth, we find that perceived differences in the cyclicalities of monetary policy help explain the cross-sectional and time series variation in nominal bond risk as well as the development of local currency debt markets. Guided by these observed empirical patterns, we develop a simple general equilibrium model with an endogenous issuance decision between local and foreign currency debt.

Chapter 3 proposes two simple consistent heteroskedasticity and autocorrelation consistent covariance estimators for time series with missing data. First, we develop the Amplitude Modulated

estimator by applying the Newey-West estimator and treating the missing observations as non-serially correlated. Secondly, we develop the Equal Spacing estimator by applying the Newey-West estimator to the series formed by treating the data as equally spaced. We show asymptotic consistency of both estimators for inference purposes and discuss finite sample variance and bias tradeoff.

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## 1. LOCAL CURRENCY SOVEREIGN RISK<sup>1</sup>

### 1.1 *Introduction*

In the aftermath of the Global Financial Crisis, sovereign debt crises are concentrated in the developed world. This itself is a remarkable development. It is even more remarkable when one considers that following the Lehman bankruptcy, some emerging market currencies lost more than half their value against the dollar. Yet even as their currencies plummeted, these countries were able to continue their debt payments. This represents a major break from past crises. In the 1980's and 1990's the developing world borrowed in currencies that they did not have the right to print, and currency mismatch was the center of past emerging market sovereign crises.<sup>2</sup> After a decade of rapid development of local currency (LC) sovereign bond markets in the wake of the Asian Financial Crisis, major emerging markets entered the most recent period of global financial turmoil with an increasing fraction of their debt in their own currencies and have weathered the shocks without triggering major sovereign debt crises.<sup>3</sup>

Yet, despite the increasingly important role of local currency debt in emerging market government finance, LC debt markets are little understood and LC sovereign risk measures are absent from the academic literature. Our paper fills this gap by introducing a new measure of LC sovereign risk, the LC credit spread, defined as the difference between the nominal yield on an LC bond and the LC risk-free rate implied from the cross currency swap (CCS) market. While government bond yields are often used directly as the risk-free rate for developed country currencies, they cannot be used as the risk-free rate in emerging markets where the risk of sovereign default

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<sup>1</sup> Joint with Jesse Schreger, Harvard University

<sup>2</sup> Prominent examples are Mexico (1994), the Asian Financial Crises (1997-98), Russia (1998), Brazil (1998, 2002), Turkey (2000-01) and Argentina (2001-02).

<sup>3</sup> Sovereign defaults have occurred in four developing countries since 2008: Ecuador, Seychelles, Jamaica and Belize. Except for Jamaica, the other three countries do not have local currency debt markets. Ecuador is a fully dollarized economy and Seychelles and Belize have population less than 500,000.

and capital controls are non-negligible.<sup>4</sup> Instead, we use the dollar risk-free rate combined with the long-term forward rate implied from the currency swap markets as the risk-free benchmark in each LC. From a dollar investor's perspective, the LC credit spread is equivalent to the synthetic dollar spread on an LC bond over the U.S. Treasury rate with the currency risk of promised cash flows fully hedged using cross currency swaps. By holding an LC bond and a currency swap with the same tenor and promised cash flows, the dollar investor can lock in the LC credit spread even if the value of the currency plummets as long as explicit default is avoided. From the sovereign issuer's perspective, the LC credit spread measures the synthetic dollar borrowing cost in the LC debt market.

The bulk of the literature on emerging market LC debt has focused on why these emerging markets *cannot* borrow abroad in their own currency, the question of "original sin" surveyed in Eichengreen and Hausmann (2005). While it is true that emerging market sovereigns rarely issue LC bonds in global markets, this no longer means that foreigners do not lend to them in their own currencies. Instead foreigners are increasingly willing to purchase LC debt issued under domestic law. According to volume surveys conducted by the Emerging Market Trading Association, the share of LC debt in total offshore emerging market debt trading volume has increased from 35 percent in 2000 to 71 percent in 2011, reaching 4.64 trillion U.S. dollars (Figure 1.1). Emerging Market Portfolio Research reports that even among offshore mutual funds which had historically invested overwhelmingly in FC denominated Eurobonds<sup>5</sup> and Brady bonds, the cumulative fund flow into LC emerging market debt securities has outpaced the flow into debt securities in hard currencies (Figure 1.2).

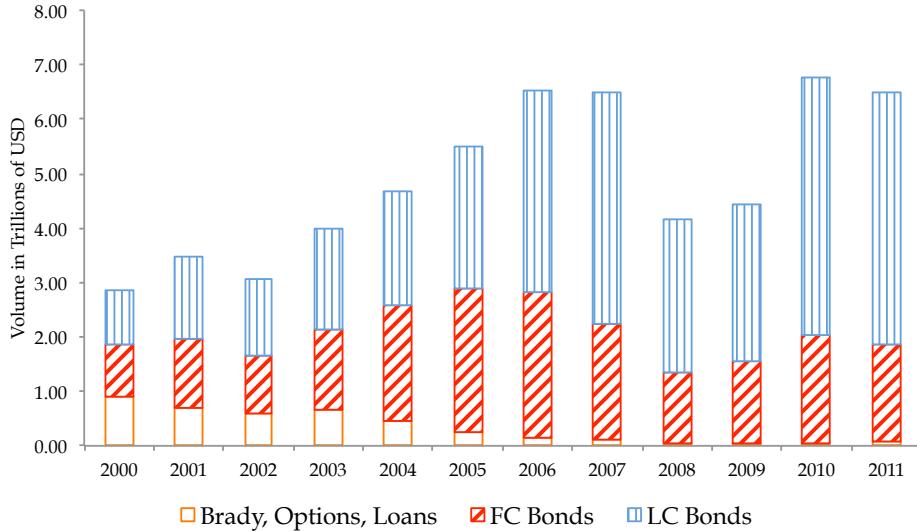
The growing importance of LC debt markets is in stark contrast to the declining role of FC sovereign financing. This shift is rendering conventional measures of sovereign risk increasingly obsolete. In many emerging markets, government policy is to retire outstanding FC debt and end new FC issuance.<sup>6</sup> The popular country-level JP Morgan Emerging Market Bond Index (EMBI), commonly used in academic research to measure sovereign risk, is today forced to track a dwin-

---

<sup>4</sup> A similar point applies to many euro area countries.

<sup>5</sup> Throughout this paper we use eurobonds to mean foreign currency bonds issued offshore, but not necessarily in euros.

<sup>6</sup> For example in Mexico, the 2008 guidelines for public debt management is to "Continue emphasis on the use of domestic debt to finance the entire federal government deficit and the stock of external debt" (SHCP, 2008)



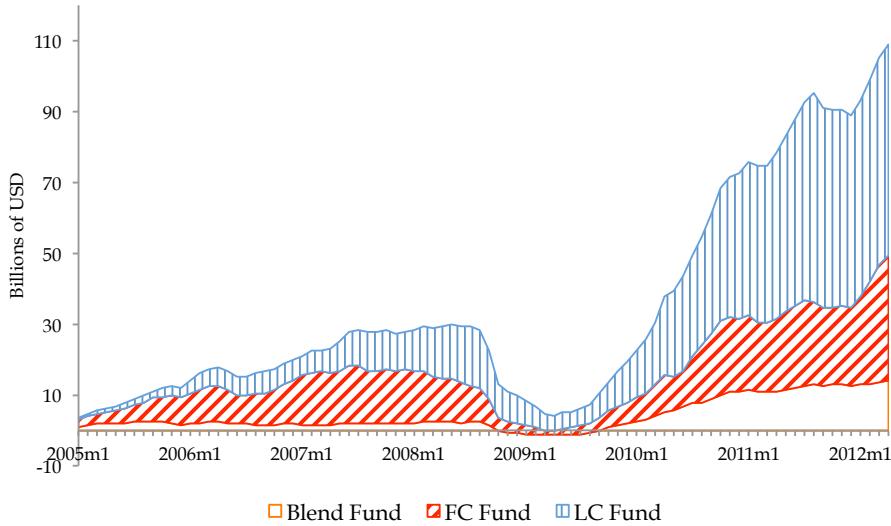
**Figure 1.1: Offshore Trading Volume by Instrument Types (Trillions of USD).** This figure plots total trading volumes of emerging market debt by instrument type in trillions of dollars. In addition to FC bonds, the ‘Brady, Option, Loans’ category also refers to debt instruments denominated in foreign currencies. The survey participants consist of large offshore financial institutions.

**Source:** Annual Debt Trading Volume Survey (2000-2011) by Emerging Market Trading Association

ding number of outstanding FC eurobonds with declining liquidity and trading volume. In countries such as Egypt, Thailand, Malaysia, Morocco, South Korea and Qatar, FC debt has shrunk to the point that EMBI+ has been forced to discontinue these countries’ indices. In addition to FC credit spreads, sovereign CDS spreads are used as an alternative measure of sovereign risk. However, defaults on local currency bonds governed under domestic law do not constitute credit events that trigger CDS contracts in emerging markets.<sup>7</sup> As a result, sovereign CDS also offers an incomplete characterization of emerging market sovereign risk.

Using new data and a new measure, we document a new set of stylized facts about LC sovereign risk. To construct LC and FC sovereign credit spreads, we build a new dataset of zero-coupon LC and FC yield curves and swap rates for 10 major emerging markets at the daily frequency for a common sample period from 2005 to 2011. Using the 5-year zero-coupon benchmark, we find that LC credit spreads are significantly above zero, robust to taking into account the bid-ask spread on the swap rates. This result demonstrates the failure of long-term covered interest rate parity between government bond yields in emerging markets and the United States. Removing the

<sup>7</sup> This is different from the case of developed country sovereign CDS for which a default on local bonds would trigger CDS contracts (ISDA, 2012).



**Figure 1.2: Cumulative Flows of Offshore Emerging Market Funds (Billions of USD).** This figure plots cumulative flows of offshore mutual funds designated to emerging market debt since 2005, measured in billions of USD. Monthly fund flow is measured as end-of-month assets - beginning-of-month assets - portfolio change - FX change. The total cumulative flow is broken down by currency type. LC Fund refers to funds that invest 75 percent or more in local currency debt; FC Fund refers to funds that invest 75 or more in hard currency debt; and Blend Fund refers to funds that invest in a combination of both, less than 75 percent for either of the above categories.

**Source:** Emerging Market Portfolio Research

currency risk highlights an important credit component in LC yields, as shown by the positive correlation between the LC credit spread and the conventional sovereign risk measure, the FC credit spread.

Despite a positive correlation, LC and FC credit spreads are different along three important dimensions. First, while LC credit spreads are large and economically significant, they are generally lower than FC credit spreads. The gap between LC and FC credit spreads significantly widened during the peak of the crisis following the Lehman bankruptcy. Second, FC credit spreads are much more correlated across countries than LC credit spreads. Over 80% of the variation in FC spreads is explained by the first principal component. In contrast, only 53% of the variation in LC credit spreads is explained by the first principal component, pointing to the relative importance of country-specific factors in driving LC spreads. Third, FC credit spreads are much more correlated with global risk factors than are LC credit spreads. These ex-ante results in the yield spread space are mirrored ex-post in the excess return space, as excess holding period returns on FC bonds over U.S. Treasuries load heavily on global equity market returns while hedged LC ex-

cess holding period returns load heavily on local equity market returns. In other words, despite the common perception of emerging market LC debt as extremely risky, we find that swapped LC debt is actually safer than FC bonds for global investors measured in terms of global equity betas. The removal of currency risk is central to this finding, as the currency unhedged LC excess returns have larger betas with global equity returns than FC excess returns.

After documenting the differences between LC and FC credit spreads, we turn to examining the sources of these credit spread differentials. We build a parsimonious model that attributes the credit spread differential to the differential cash flow risks between LC and FC debt and differential investor bases between the two debt markets. FC bonds may have higher cash flow risk than LC for several reasons. These include a government's option to print money to service LC debt, the danger that a sudden exchange rate depreciation may increase the real burden of servicing FC debt, and the political economy costs of defaulting on your own citizens relative to defaulting on foreign investors. On the other hand, foreign holders of LC debt face several risks not present in FC eurobonds, including convertibility risk, as well as the risks of changing taxation and regulation and more uncertain debt restructuring process under the domestic law. In addition, LC and FC credit spreads can be different due to unhedged covariance between the exchange rate and the default process. From a dollar investor's perspective, swapped LC debt can have lower cash flow risk if investors expect to gain profits from unwinding the swap position in the event of an LC bond default.

In addition to differential cash flow risk, LC and FC debt markets have different investor bases. While an increasing fraction of LC debt is being purchased by foreign investors, the majority is still owned by domestic residents, commercial banks, and pension funds. These investors have few investment opportunities outside of domestic government bonds because of domestic financial underdevelopment or legal restrictions on their overseas investments. This can give rise to a distinct local demand factor in pricing LC debt that is absent from FC debt, which is issued in major international capital markets and purchased by diversified global investors. The existence of local clientele potentially dampens the sensitivity of LC credit spreads to fundamentals and global investor risk aversion shocks.

We study a model that allows for both differential cash flow risk and local clientele demand effects by introducing credit risk in the style of Duffie and Singleton (1999) into a preferred habitat

model that builds on Greenwood and Vayanos (2010) and Vayanos and Vila (2009). While allowing the arrival rate of credit events for FC and LC to respond differently to a local and global risk factor, we study a market structure where diversified global investors are the primary clientele for FC debt, domestic investors are the primarily clientele for LC debt, and risk-averse arbitrageurs partially integrate the two markets. In this framework, the equilibrium LC credit spread is an endogenous outcome of arbitrageurs' optimal portfolio demand and local clientele demand, with the equilibrium impact of LC clientele demand depending on the size of the position the arbitrageur is willing to take. This, in turn, depends on the arbitrageur's risk aversion, the asset return correlation, and the size and elasticity of local clientele demand.

Guided by the model's predictions and comparative statics, we highlight the importance of differential risk premia arising from the differential investor bases in pricing swapped LC and FC bonds. The key mechanism we highlight is how changes in global risk aversion directly affect FC spreads but are only partially transmitted into LC spreads by risk-averse arbitrageurs. Consistent with the model's predictions, we first show that global risk aversion, as proxied by VIX, has a larger contemporaneous impact on FC credit spreads than on LC credit spreads, robust to a large set of determinants of sovereign risk identified by the existing literature. Differential sensitivity to VIX alone accounts for 25.6 percent of the within-country variation in the credit spread differentials and 60 percent of total explained variation after controlling for a host of economic fundamentals. Furthermore, differential contemporaneous impacts of VIX on LC and FC credit spreads generate differential predictability of excess returns through the risk premium channel. We show that high levels of VIX significantly forecast negative swapped LC over FC excess returns. As predicted by the theory, we also find that LC credit spreads are more sensitive to global risk aversion in countries with more correlated swapped LC and FC bond returns.

The paper is structured as follows. We begin by explaining this paper's place in the existing literature. Section 1.2 explains the mechanics of cross currency swaps and formally introduces the LC credit spread measure. Section 1.3 presents new stylized facts on LC sovereign risk. Section 1.4 lays out a no-arbitrage model of partially segmented markets with risky credit arbitrage. Section 1.5 performs regression analysis to test several key predictions of the model and Section 1.6 concludes.

### *1.1.1 Relation to the Literature*

Our work is related to several distinct strands of literature: the enormous sovereign debt literature in international macroeconomics, the empirical sovereign and currency risk premia literature, the literature on currency-specific corporate credit spreads, and the segmented market asset pricing literature.

Recent work by Carmen Reinhart and Kenneth Rogoff demonstrates (Reinhart and Rogoff 2008, 2011) that LC sovereign borrowing and default are not new phenomena. Building on their work, which focuses primarily on quantities, we focus on prices and jointly examine LC and FC credit spreads. Prior to our work, the pricing of LC debt was rarely examined with exception of Burger and Warnock (2007) and Burger, Warnock, and Warnock (2012), who studied ex-post returns on LC bonds using the J.P. Morgan Emerging Market Government Bond Index (EM-GBI) index.

Using our dataset of daily yield curves and currency swaps, we document a series of new stylized facts that we believe are important to integrate into the quantitative sovereign debt literature that builds on Aguiar and Gopinath (2006) and Arellano (2008). Given that an increasing fraction of sovereign borrowing is in LC, our findings on how LC credit spreads behave differently than FC credit spreads highlight the importance of moving away from the standard assumption in this literature that governments borrow solely from foreign lenders using real debt.

Our paper is also closely related to the literature on FC sovereign risk premia and currency risk premia. Borri and Verdelhan (2011) demonstrate that FC spreads can be explained by modeling a risk-averse investor who demands risk premia for holding sovereign debt because default generally occurs during bad times for the global investors. Using data on credit default swaps (CDS) denominated in dollars, Longstaff, Pan, Pedersen, and Singleton (2011) show that global risk factors explain more of the variation in CDS spreads than do local factors. Our analysis confirms these findings. In addition, we find support for the results of Lustig and Verdelhan (2007) and Lustig, Roussanov, and Verdelhan (2012) that there is a common global factor in currency returns. This motivates our use of cross currency swaps to separate this currency risk from the credit risk on LC sovereign debt.

Cross currency swaps have previously been used to test long-term covered interest parity among government bond yields in developed countries. Popper (1993) and Fletcher and Taylor (1994, 1996) document some deviations from covered parity, but they are an order of magnitude smaller and much less persistent than those we document in our dataset of emerging markets. Currency-dependent credit spreads implied from cross currency swaps have also received attention in the empirical corporate finance literature. McBrady and Schill (2007) demonstrate that firms gauge credit spread differentials across different currencies when choosing the currency denomination of their debt. Jankowitsch and Stefan (2005) highlight the role of the correlation between FX and default risk in affecting currency-specific credit spreads. Lowenkron and Garcia (2005) document that currency and credit risk, the so-called “cousin risk”, are positively linked in some emerging markets, but not in others.

Finally, our theoretical model builds on the asset pricing literature on investors’ preferred habitats and the limits to arbitrage. Greenwood and Vayanos (2010), building on Vayanos and Vila (2009), examine the effect of increases in bond supply across the yield curve for U.S. Treasuries. The framework assumes that different maturities have different clienteles and each type of investor invests only in a certain range of maturities (their “preferred habitat”). We study an environment where preferred habitats correspond to currencies and markets rather than maturities, building on the cross-asset arbitrage theory presented by Gromb and Vayanos (2010), and solving analytically for the endogenous LC bond price.

## 1.2 *Cross Currency Swaps and Sovereign Credit Spreads*

### 1.2.1 *Cross Currency Swaps*

For short-term instruments, FX forward contracts allow investors to purchase foreign exchange at pre-determined forward rates. Beyond one year, liquidity is scarce in the forward markets and long-term currency hedging via forwards is very costly. CCS contracts, on the other hand, allow investors to conveniently hedge long-term currency risk. A CCS is an interest rate derivative contract that allows two parties to exchange interest payments and principal denominated in two currencies. A real-world example of hedging currency risk of an LC bond using CCS is given in Appendix A.1. For emerging markets, CCS counterparties are usually large offshore financial

institutions. To mitigate the counterparty risk embedded in CCS contracts, the common market practice is to follow the Credit Support Annex of the International Swap and Derivative Association Master Agreement, which requires bilateral collateralization of CCS positions, and thus counterparty risk is fairly negligible. For countries with non-deliverable FX forwards, CCS contracts are cash settled in dollars based on LC notional amount and are free from currency convertibility risk.

For our cross-country study, it is cumbersome to deal with coupon bearing bonds and par swap rates due to the mismatch in coupon rates and payment dates between bonds and swaps. We can extract the long-term FX forward premium (the zero-coupon swap rate) implicit in the term structure of par swaps. Intuitively, a fixed for fixed LC/dollar CCS package can always be considered as the sum of two interest rate swaps. First, the investor swaps the fixed LC cash flow into a floating U.S. Libor cash flow<sup>8</sup> and then swaps the floating U.S. Libor cash flow into a fixed dollar cash flow. We can exploit the fact that the receiver of U.S. Libor must be indifferent between offering a fixed LC or a fixed dollar cash flow. The difference in the two swap rates thus implies the long-term currency view of the financial market. After performing this transformation, a CCS is completely analogous to a standard forward contract. The specifics are given in the following proposition.

**Proposition 1.** *Given implied log spot rates  $\tilde{r}_{\tau,t}^{LC}$  from the fixed LC for U.S. Libor CCS and  $\tilde{r}_{\tau,t}^{USD}$  from the fixed dollar for Libor interest rate swap, the implicit long-term forward premium is equal to*

$$\rho_{nt} \equiv \frac{1}{\tau}(\tilde{f}_{nt} - s_t) = \tilde{r}_{nt}^{LC} - \tilde{r}_{nt}^{USD},$$

where  $\tilde{f}_n$  is the pre-determined log forward exchange rate at which a transaction between LC and dollars takes place  $n$  years ahead.

### 1.2.2 LC and FC Credit Spreads

The core of our dataset is daily zero-coupon yield curves and swap curves for LC and FC sovereign bonds issued by 10 different emerging market governments from January, 2005 to De-

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<sup>8</sup> For Mexico, Hungary, Israel and Poland in our sample, this step itself combines two interest rate swaps: an onshore plain vanilla LC fixed for LC floating interest rate swap and a cross-currency LC floating for U.S. Libor basis swap.

ember, 2011. We use a benchmark tenor of 5 years. The choice of countries is mainly constrained by the lack of sufficient numbers of FC bonds outstanding. Furthermore, all 10 sample countries belong to the J.P. Morgan EM-GBI index, an investable index for emerging market LC bonds. The length of the sample period is constrained by the availability of long-term currency swap data. All data on cross currency swaps are collected from Bloomberg.<sup>9</sup> Zero coupon yield curves are collected or estimated from various data sources. The details on the yield curve construction are given in Appendix A.2.

We work with log yields throughout the paper. To fix notations, we let  $y_{nt}^*$  denote the  $n$ -year zero-coupon U.S. Treasury bond yield, the long-term risk-free rate used throughout the paper. Nominal LC and FC yields are denoted by  $y_{nt}^{LC}$  and  $y_{nt}^{FC}$ , respectively. We let  $\rho_{nt}$  denote the zero-coupon swap rate, the implicit forward premium as defined in Proposition 1. All yields and swap rates are for the  $n$ -year zero-coupon benchmark at date  $t$ . The conventional measure of sovereign risk, the FC credit spread, measures the difference between the yield on FC debt and the U.S. Treasury yield:

$$s_{nt}^{FC/US} = y_{nt}^{FC} - y_{nt}^*.$$

Our new measure for LC sovereign risk, the LC credit spread, is defined as the nominal LC spread over the the U.S. Treasury yield, minus the zero-coupon swap rate:

$$s_{nt}^{SLC/US} = y_{nt}^{LC} - y_{nt}^* - \rho_{nt},$$

or the deviation from long-term covered interest rate parity between the government bond yields. There are two ways to interpret this measure. First, the dollar investor can create a swapped LC bond by combining an LC bond with a CCS with the same promised cash flows. The synthetic

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<sup>9</sup> Extremely illiquid trading days with bid-ask spreads over 400 basis points on CCS are excluded from the analysis (mainly for Indonesia during the 2008 crisis). All main results are not affected by including these extreme values. We compare the difference in 1-year forward premia implied by the swap and the forward markets in Table A.1. The mean correlation is 99 percent. Using annualized bid-ask spreads as a proxy for liquidity,swap contracts are, on average, more liquid than short-term forward contracts (Table A.2) .

dollar yield on the swapped LC bond is given by

$$y_{nt}^{SLC} = y_{nt}^{LC} - \rho_{nt}.$$

The LC credit spread is therefore equal to the dollar spread on this synthetic asset:

$$s_{nt}^{SLC/US} = (y_{nt}^{LC} - \rho_{nt}) - y_{nt}^* = y_{nt}^{SLC} - y_{nt}^*.$$

Hence, by holding the swapped LC bond to maturity, the LC credit spread gives the promised dollar spread on the LC bond to dollar investors even if the LC depreciates, provided that explicit default is avoided. In the event of default, the dollar investor can choose to unwind the swap with an unmatched LC bond payment, which could result in additional FX profits or losses from the swap. Second, investors valuing their returns in LC can combine a U.S. Treasury bond with a fixed for fixed CCS to create an LC risk-free bond. The sum of the dollar risk-free and the CCS rate gives the LC risk free rate

$$y_{nt}^{*LC} = y_{nt}^* + \rho_{nt},$$

and thus the LC credit spread measures the yield spread of the LC bond over the LC risk-free rate:

$$s_{nt}^{SLC/US} = y_{nt}^{LC} - (y_{nt}^* + \rho_{nt}) = y_{nt}^{LC} - y_{nt}^{*LC},$$

and is a pure credit spread measure for local currency. Finally, the LC over FC credit spread differential measures the spread between the yield on the synthetic dollar asset combining an LC bond and CCS over the FC bond yield:

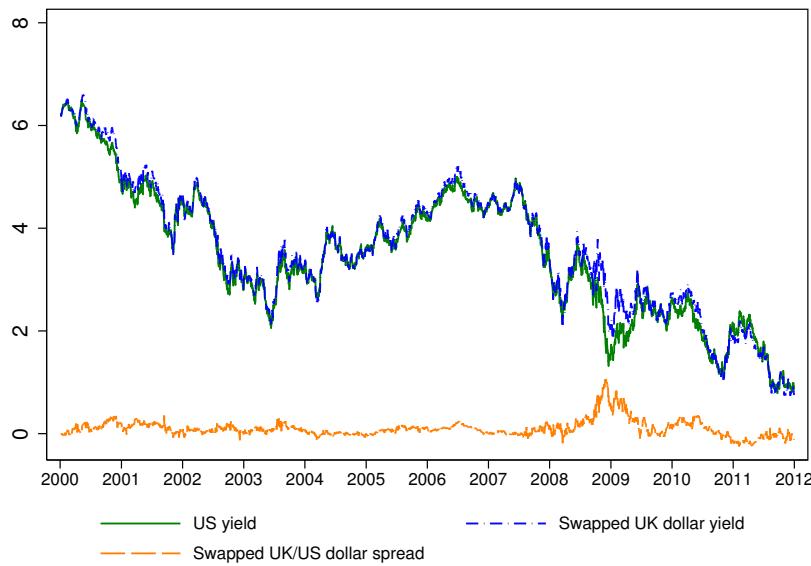
$$s_{nt}^{SLC/FC} = y_{nt}^{LC} - \rho_{nt} - y_{nt}^{FC} = s_{nt}^{SLC/US} - s_{nt}^{FC/US}.$$

From the issuer's perspective, it gives the the difference between the synthetic dollar borrowing cost in the local market and the actual dollar borrowing cost in the external market.

## 1.3 New Stylized Facts on LC Sovereign Risk

### 1.3.1 Deviations from Long-Term CIP

If long-term covered interest parity holds for government bond yields, LC credit spreads should equal zero in the absence of transaction costs. As a starting point, Figure 1.3 plots the 5-year swapped UK Treasury yield in dollars and the U.S. Treasury yield from 2000 to 2011. The difference between the two curves, the UK LC credit spread, averages 10 basis points for the full sample and 6 basis points excluding 2008-2009. Long-term CIP holds quite well between the U.S. and the UK Treasury yields excluding 2008-2009. At the peak of the Global Financial Crisis around the Lehman bankruptcy, the UK credit spread temporarily increased to 100 basis points but returned to normal in a few months.



**Figure 1.3: 5-Year U.S. and Swapped UK Treasury Yields in percentage points.** The green solid line plots the 5-Year zero-coupon U.S. Treasury yield. The blue dash-dotted line plots the 5-year zero-coupon swapped UK Treasury yield after applying a cross currency swap package consisting of two plain vanilla interest rate swaps (dollar and sterling) and the U.S. and UK Libor cross-currency basis swap. The orange dashed line plots the yield spread of the swapped UK Treasury yield over the U.S. Treasury. The mean of the yield spreads is 10 basis points with standard deviation equal to 16 basis points. The minimum spread is equal to negative 25 basis points and the maximum spread is equal to 106 basis points during the peak of the crisis. Excluding 2008-2009, the mean spread is 6 basis points with standard deviation equal to 10 basis points.  
**Source:** The U.S. zero-coupon yield is from St. Louis Fed. The UK zero-coupon yield is from Bank of England. Swap rates are from Bloomberg.

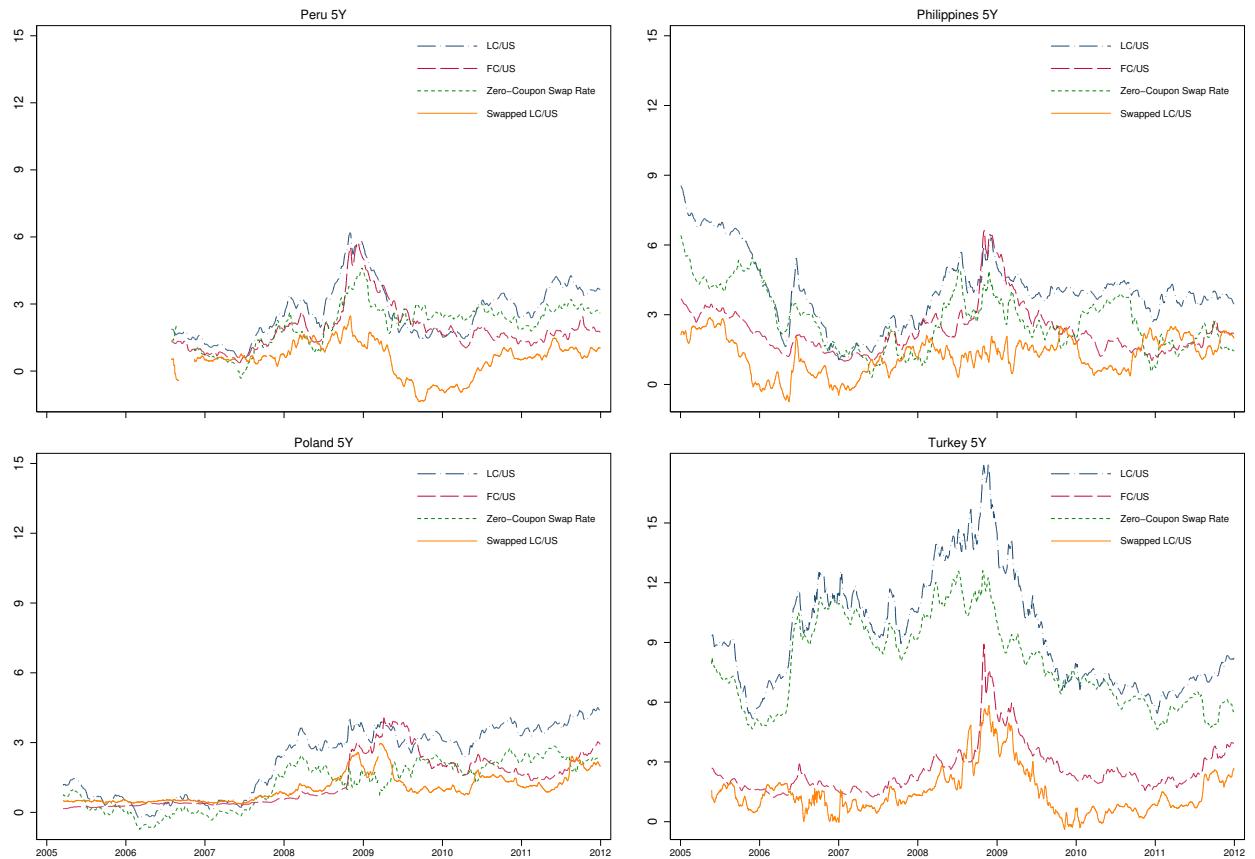
LC credit spreads in emerging markets offer a very different picture. As can be seen in Figure 1.4, where the 5-year zero-coupon yield spreads are plotted for our sample countries, large persistent deviations from long-term covered interest parity are the norm rather than the exception. Column 1 in Table 1.1 presents summary statistics for 5-year LC spreads for the sample period 2005-2011 at daily frequency. LC credit spreads,  $s^{SLC/US}$ , have a cross-country mean of 128 basis points, calculated using the mid-rates on the swaps. Brazil records the highest mean LC spreads equal to 313 basis points and Mexico and Peru have the lowest means about 60 basis points. All mean LC credit spreads are positive and statistically significantly different from zero using Newey-West standard errors allowing for heteroskedasticity and serial correlation.<sup>10</sup> Positive mean LC spreads are robust to taking into account the transaction costs of carrying out the swaps. Column 4 provides summary statistics for liquidity of the cross currency swaps,  $ba^{CCS/2}$ , defined as half of the bid-ask spread of cross currency swap rates, with the sample average equal to 19 basis points. We perform statistical tests and find that LC credit spreads remain significantly positive for every country after subtracting one half of the bid-ask spread on the CCS in order to incorporate the transaction costs. Positive LC credit spreads suggest that emerging market nominal LC sovereign bonds are not free from credit risk from the investor's perspective. Although the government has the option to print the domestic currency, inflation is not costless and explicit repudiation of LC debt has happened in the past, such as Russia's default on its Treasury bills in 1998.

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<sup>10</sup> Following Datta and Du (2012), missing data are treated as non-serially correlated for Newey-West implementations throughout the paper.



**Figure 1.4: 5-Year U.S. and Swapped UK Treasury Yields in percentage points.** Each figure plots 10-day moving averages of zero-coupon LC and FC spreads over the U.S. Treasury at 5 years. LC/US denotes the LC yield over the U.S. Treasury yield. FC/US denotes the FC yield over the U.S. Treasury yield. Zero-coupon swap rate is the zero-coupon fixed for fixed CCS rate implied from par fixed for floating CCS and plain vanilla interest rate swap rates. Swapped LC/US denotes the swapped LC over U.S. Treasury yield spread.

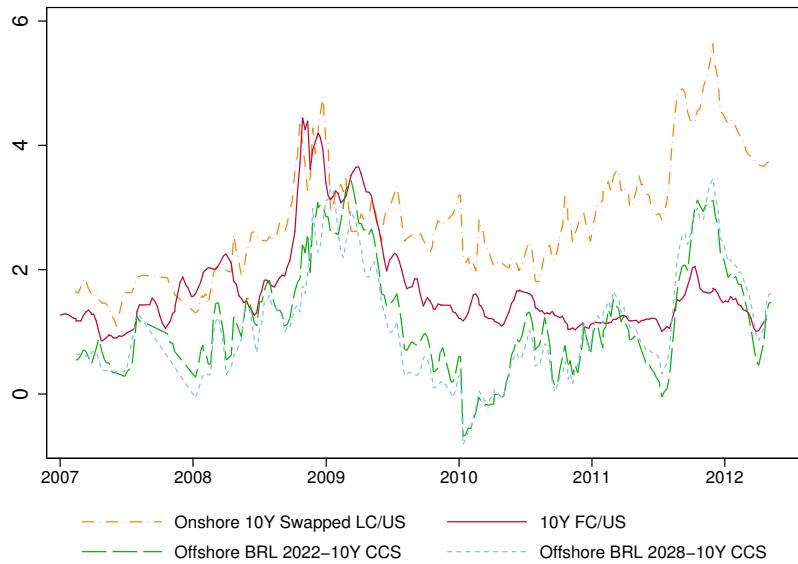


**Figure 1.4: (Continued) 5-Year U.S. and Swapped UK Treasury Yields in percentage points.** Each figure plots 10-day moving averages of zero-coupon LC and FC spreads over the U.S. Treasury at 5 years. LC/US denotes the LC yield over the U.S. Treasury yield. FC/US denotes the FC yield over the U.S. Treasury yield. Zero-coupon swap rate is the zero-coupon fixed for fixed CCS rate implied from par fixed for floating CCS and plain vanilla interest rate swap rates. Swapped LC/US denotes the swapped LC over U.S. Treasury yield spread.

### 1.3.2 Mean Levels of Credit Spreads

To compare the sovereign's dollar borrowing costs using FC debt with the synthetic dollar borrowing costs using LC debt, we perform an ex-ante credit spread comparison. FC credit spreads,  $s^{FC/US}$ , reported in Column 2 in Table 1.1 have a mean of 195 basis points, 67 basis points higher than LC credit spreads based on the mid-rates for CCS. The difference increases to 86 basis points after taking into account the transaction cost of carrying out the swaps. In Column 3, we compute the difference between LC and FC credit spreads by country. The swapped LC over FC spread,  $s^{SLC/FC}$ , is significantly negative for all of our sample countries except Brazil. Although all our sample countries have LC bond markets open to foreign investors, foreigners may still need to incur transaction costs to buy in into LC markets. For 9 out of 10 countries with negative LC swapped over FC spreads, the promised dollar spread on LC bonds is unambiguously lower than that on FC bonds, since swapped LC over FC spreads would become more negative after taking into account positive taxes on LC bonds.

Brazil offers an important exception. As a country offering one of the highest nominal interest rates in the world, Brazil has implemented various measures to curb portfolio investment flows and cross-border derivative trading as macro-prudential and exchange rate policy. The *Imposto sobre Operações Financeiras* (IOF), or tax on financial transactions, is currently set at 6 percent upfront for all fixed income capital inflows into the country. Fortunately for our analysis, Brazil conducted four large issuances of eurobonds denominated in reals traded at the Luxembourg Stock Exchange. These bonds give offshore investors direct access to real-denominated sovereign rates without paying the onshore taxes. In addition, these bonds are payable in dollars and thus foreign investors are free from currency convertibility risk. Figure 1.5 shows that two long-term offshore real-denominated bonds are traded at significantly lower spreads than 10-year onshore bonds. Applying the CCS to the offshore LC yield generally gives a negative LC over FC spread. Besides Brazil, Colombia and the Philippines, more recently, have also issued several LC eurobonds payable in dollars. All the offshore LC bonds are currently traded at least 100 basis points tighter than onshore bonds, which suggests that taxes and convertibility risk are important components of the LC credit spread from the offshore investors' perspective.



**Figure 1.5: Brazil Onshore and Offshore Yield Comparison.** This figure plots nominal yields minus 10-year zero-coupon real/dollar swap rates on two Eurobonds denominated in Brazilian reals traded at the Luxembourg Stock Exchange with maturity years 2022 and 2028 (BRL 2022 by the green long-dashed line and BRL 2028 by the blue short-dashed line). Offshore swapped yields are compared with the 10-year zero-coupon onshore LC swapped yield plotted by the orange dash-dotted line and the offshore FC dollar yield plotted by the red solid line.

**Source:** The onshore LC zero-coupon yield is obtained from ANBIMA. The FC zero-coupon yield is estimated from Bloomberg BFV par yield curve. LC Eurobond yields are provided by the Luxembourg Stock Exchange.

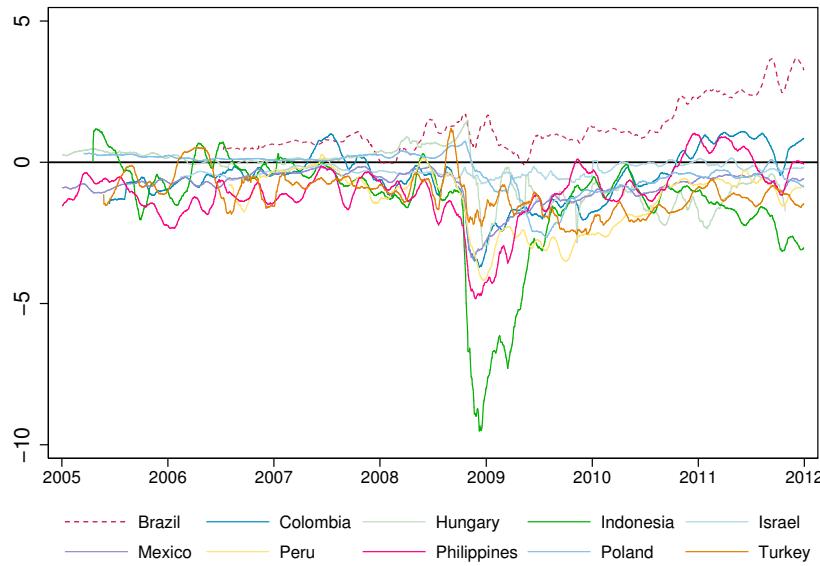
Despite the level difference in credit spreads, one might expect LC and FC credit risks to be correlated within countries, as in downturns a country could find it more tempting to explicitly default on both types of debt. Column 5 confirms this conjecture. The within-country correlation between LC and FC credit spreads is positive for every country with a mean of 54 percent. However, there is significant cross-country heterogeneity. The correlation is highest for Hungary at 91% and lowest for Indonesia at 18%. This cross-country heterogeneity is a source of variation that we will later use to argue for the importance of incomplete market integration in the relative pricing of the two types of debt.

**Table 1.1: Mean LC and FC Credit Spread Comparison, 2005-2011.** This table reports sample starting date, mean and standard deviation of 5-year log yield spreads at daily frequency. The variables are (1)  $s^{SLC/US}$ , swapped LC over U.S. Treasury spread; (2)  $s^{FC/US}$ , FC over U.S. Treasury spread; (3)  $s^{SLC/FC}$ , swapped LC over FC spread, or column (2) - column (1). (4)  $ba^{CCS}/2$ , half of bid-ask spread of cross-currency swaps. Standard deviations of the variables are reported in the parentheses. We test significance of means using Newey-West standard errors with 120-day lags. Standard errors are omitted. Test results are reported for columns (1), (2) and (3), \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Since the bid-ask spread is always nonnegative, significance tests are not performed for column 4. Two additional tests are conducted for hypotheses (1)  $s^{SLC/US} - ba^{CCS}/2 = 0$  and  $s^{SLC/FC} - ba^{CCS}/2 = 0$ , both tests can be rejected at 5 percent or lower confidence levels for all countries using Newey-West standard errors with 120-day lags. Column (5) reports within-country correlations between  $s^{SLC/US}$  and  $s^{FC/US}$ .

Country	Sample Start	(1) $s^{SLC/US}$	(2) $s^{FC/US}$	(3) $s^{SLC/FC}$	(4) $ba^{CCS}/2$	(5) $Corr(SLC,FC)$
Brazil	Jul. 2006	3.13*** (1.13)	1.78*** (0.91)	<b>1.35***</b> (0.94)	0.32 (0.13)	0.56
Colombia	Jun. 2005	1.47*** (0.69)	2.03*** (1.01)	<b>-0.56***</b> (1.01)	0.16 (0.10)	0.34
Hungary	Jan. 2005	1.69*** (1.23)	2.15*** (2.01)	<b>-0.47**</b> (1.03)	0.19 (0.14)	0.91
Indonesia	Apr. 2005	1.14*** (0.73)	2.52*** (1.59)	<b>-1.38***</b> (1.61)	0.38 (0.23)	0.18
Israel	Feb. 2006	0.86*** (0.43)	1.12*** (0.42)	<b>-0.26***</b> (0.21)	0.12 (0.03)	0.84
Mexico	Jan. 2005	0.60*** (0.40)	1.44*** (0.79)	<b>-0.83***</b> (0.60)	0.09 (0.06)	0.66
Peru	Jul. 2006	0.55*** (0.80)	1.97*** (1.05)	<b>-1.42***</b> (1.09)	0.16 (0.07)	0.34
Philippines	Mar. 2005	1.25*** (0.80)	2.31*** (1.04)	<b>-1.07***</b> (1.07)	0.28 (0.14)	0.34
Poland	Mar. 2005	1.04*** (0.60)	1.29*** (1.01)	<b>-0.25**</b> (0.62)	0.12 (0.08)	0.78
Turkey	May 2005	1.46*** (1.19)	2.57*** (1.20)	<b>-1.12***</b> (0.81)	0.11 (0.08)	0.78
Total	Jan. 2005	1.28*** (1.06)	1.95*** (1.23)	<b>-0.67***</b> (1.22)	0.19 (0.15)	0.54
Observations		13151	13151	13151	13151	

### 1.3.3 Widening Credit Spread Differentials During the Crisis

Despite the relatively short sample period, the years 2005-2011 cover dynamic world economic events: the end of the great moderation, the Global Financial Crisis and the subsequent recovery. Figure 1.6 plots the difference in LC and FC credit spreads,  $s^{SLC/FC}$ , across 10 countries over the sample period. While swapped LC over FC spreads largely remain in negative territory (with the exception of Brazil), the spreads significantly widened during the peak of the crisis following the Lehman bankruptcy. The maximum difference between LC and FC credit spreads for any country during the crisis was negative 10 percentage points for Indonesia.



**Figure 1.6: Swapped LC over FC spreads.** This figure plots 30-day moving averages of 5-year zero-coupon swapped LC over FC spreads (the difference between LC and FC credit spreads) using 5-year cross currency swaps for all 10 sample countries.

Table 1.2 quantitatively documents the behavior of the credit spreads during the crisis peak (defined approximately as the year following the Lehman bankruptcy from September 2008 to September 2009), measured as the increase in spreads relative to their pre-crisis means. FC credit spreads significantly increase in all countries and LC credit spreads increase significantly in 8 out of the 10 sample countries, with the exceptions of Indonesia and Peru. However, the increase in swapped LC spreads are generally less than the increase in FC spreads, as LC over FC credit spread differentials are reduced for all countries except Brazil. The divergent behavior of these

credit spreads during the crisis peak highlights significant differences between LC and FC bonds, and offers a key stylized fact to be examined in Sections 1.4 and 1.5.

**Table 1.2: Changes in Credit Spreads During Crisis Peak (09/01/08 - 09/01/09).** This table reports the mean and standard deviation of changes in LC and FC credit spreads during the peak of the Global Financial Crisis (09/01/2008-09/01/2009) relative to their pre-crisis means. (1)  $\Delta s^{SLC/US}$  is the increase in swapped LC over U.S. Treasury spreads; (2)  $\Delta s^{FC/US}$  is the increase in the FC over U.S. Treasury spreads; (3)  $\Delta s^{SLC/FC}$  is the increase in swapped LC over FC spreads, or column (2)-column (1); and (4)  $\Delta ba^{CCS}/2$  is the increase in one half of bid-ask spreads. Standard deviations of variables are reported in the parentheses. Statistical significance of the means are tested using Newey-West standard errors with 120-day lags. Significance levels are denoted by \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

Country	(1) $\Delta s^{SLC/US}$	(2) $\Delta s^{FC/US}$	(3) $\Delta s^{SLC/FC}$	(4) $\Delta ba^{CCS}/2$
Brazil	1.93*** (1.13)	1.82*** (0.99)	<b>0.11</b> (0.66)	0.26*** (0.13)
Colombia	0.64*** (0.67)	2.31*** (1.21)	<b>-1.66***</b> (0.82)	0.10*** (0.18)
Hungary	2.70*** (1.12)	3.80*** (2.17)	<b>-1.10**</b> (1.48)	0.31*** (0.22)
Indonesia	0.07 (0.65)	3.67*** (2.17)	<b>-3.61***</b> (2.41)	0.45*** (0.39)
Israel	0.54*** (0.26)	0.68*** (0.26)	<b>-0.15***</b> (0.21)	0.05*** (0.04)
Mexico	0.60*** (0.30)	1.97*** (0.87)	<b>-1.38***</b> (0.80)	-0.03*** (0.01)
Peru	-0.05 (0.95)	2.21*** (1.12)	<b>-2.26***</b> (0.81)	0.07*** (0.08)
Philippines	0.36*** (0.40)	1.91*** (1.28)	<b>-1.55***</b> (1.33)	0.18*** (0.22)
Poland	1.26*** (0.58)	2.35*** (0.92)	<b>-1.09***</b> (1.01)	0.17*** (0.09)
Turkey	1.89*** (1.44)	2.70*** (1.47)	<b>-0.81***</b> (0.86)	-0.06*** (0.07)
Total	0.91*** (1.16)	2.30*** (1.48)	<b>-1.40***</b> (1.51)	0.14*** (0.22)
Observations	2058	2058	2058	2058

### 1.3.4 Cross-Country Correlations of Credit Spreads

In Table 1.3, we conduct a principal component (PC) analysis to determine the extent to which fluctuations in the LC and FC credit spreads are driven by common components or by idiosyncratic country shocks. In the first column, we see that the first principal component explains less than 54% of the variation in LC credit spreads across countries. This is in sharp contrast to the FC

**Table 1.3: Cross-Country Correlation of Credit Spreads, 2005-2011.** This table reports summary statistics of principal component analysis and cross-country correlation matrices of monthly 5-Year LC and FC credit spreads and sovereign credit default swap spreads. The variables are (1)  $s^{SLC/US}$ , swapped LC over U.S. Treasury spreads; (2)  $s^{FC/US}$ , FC over U.S. Treasury spreads; (3) 5Y CDS five-year sovereign CDS spreads. The rows “First”, “Second”, “Third” report percentage and cumulative percentage of total variations explained by the first, second and third principal components, respectively. The row “Pairwise Corr.” reports the mean of all bilateral correlations for all country pairs. All variables are end-of-the-month observations.

Principal Components	(1) $s^{SLC/US}$		(2) $s^{FC/US}$		(3) 5Y CDS	
	percentage	total	percentage	total	percentage	total
First	53.49	53.49	81.52	81.52	80.02	80.02
Second	16.30	69.78	11.70	93.22	15.34	95.36
Third	10.17	79.95	3.68	96.90	2.06	97.41
Pairwise Corr.	0.42		0.78		0.77	

spreads (Column 2) where over 81% of total variation is explained by the first PC. The first three principal components explain slightly less than 80% of the total variation for LC credit spreads whereas for FC credit spreads they explain about 97%. In addition, we find that the average pairwise correlation of LC credit spreads between countries is only 42%, in contrast to 78% for FC credit spreads. These findings point to country-specific idiosyncratic components as important drivers of LC credit spreads, in contrast to the FC market where global factors are by far the most important.<sup>11</sup>

To link these results to the literature using CDS spreads as a measure of sovereign risk, we perform the same principal component analysis for 5-year sovereign CDS spreads. The results, in Column 3, are very similar to the FC results in Column 2: the first principal component explains 80 percent of total variation of CDS spreads and the pairwise correlation averages 77 percent. Our result that an overwhelming amount of the variation in CDS spreads is explained by the first PC supports the finding of Longstaff, Pan, Pedersen, and Singleton (2011), which shows that 64% of CDS spreads are explained by the first principal component of 26 developed and emerging mar-

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<sup>11</sup> To assess how measurement errors in LC credit spreads relative to FC affect these results, we start with the null hypothesis that LC and FC credit spreads are the same and then introduce i.i.d. Gaussian shocks to FC credit spreads using simulations. We show that the variance of shocks to FC credit spreads need to be at least 90 basis points to match the observed cross-country correlation in LC credit spreads, which corresponds to 6 times of the standard deviation of observed one-way transaction costs (half of the observed bid-ask spread on cross currency swaps). These simulation results are available upon request.

kets. The sample period for their study is 2000-2010, but the authors find in the crisis subsample of 2007-2010 that the first principal component accounts for 75% of the variation.

### 1.3.5 Correlation of Sovereign Risk with Global Risk Factors

#### Credit Spreads

After identifying an important global component in both LC and FC credit spreads, we now try to understand what exactly this first principal component is capturing. In Table 1.4, we first examine the correlation of the first PC's of credit spreads with each other and with global risk factors. The global risk factors include the Merrill Lynch U.S. BBB corporate bond spread over the Treasuries,  $BBB/T$ , the implied volatility on S&P options,  $VIX$ , and the Chicago Fed National Activity Index,  $CFNAI$ , which is the first PC of 85 monthly real economic indicators. Panel (A) indicates that the first PC of FC credit spreads has remarkably high correlations with these three global risk factors, 93% with  $VIX$ , 88% with  $BBB/T$  and 76% with global macro fundamentals (or, more precisely, US fundamentals) proxied by the  $CFNAI$  index. The correlation between the first PC of LC credit spreads and global risk factors are lower, but still substantial, with a 76% correlation with  $VIX$ , 71% with  $BBB/T$  and 57% with  $CFNAI$ .

*Table 1.4: Cross-Country Correlation of Credit Spreads, 2005-2011.* This table reports correlations among credit spreads and global risk factors. Panel (A) reports correlations between the first principal component of credit spreads and global risk factors. Panel (B) reports average correlations between raw credit spreads in 10 sample countries and global risk factors. Panel (C) reports correlations between global risk factors only. The three credit spreads are (1)  $s^{SLC/US}$ , 5-year swapped LC over U.S. Treasury spread; (2)  $s^{FC/US}$ , 5-year FC over U.S. Treasury spread; and (3) 5Y CDS, 5-year sovereign credit default swap spread. The three global risk factors are (1)  $BBB/T$ , Merrill Lynch BBB over 10-year Treasury spread; (2)  $-CFNAI$ , negative of the real-time Chicago Fed National Activity Index, or the first principal component of 85 monthly economic indicators (positive  $CFNAI$  indicates improvement in macroeconomic fundamentals), and (3)  $VIX$ , implied volatility on the S&P index options. All variables use end-of-the-month observations.

	(A) First PC of Credit Spreads			(B) Raw Credit Spreads			(C) Global Risk Factors		
	$s^{SLC/US}$	$s^{FC/US}$	5Y CDS	$s^{SLC/US}$	$s^{FC/US}$	5Y CDS	$BBB/T$	$-CFNAI$	$VIX$
$s^{SLC/US}$	1.00			1.00					
$s^{FC/US}$	0.81	1.00		0.49	1.00				
5Y CDS	0.80	0.94	1.00	0.48	0.91	1.00			
$BBB/T$	0.71	0.88	0.89	0.38	0.66	0.62	1.00		
$-CFNAI$	0.57	0.76	0.75	0.33	0.58	0.52	0.87	1.00	
$VIX$	0.76	0.93	0.87	0.41	0.70	0.61	0.80	0.68	1.00

Furthermore, since the first PC explains much more variation in FC credit spreads than in LC credit spreads, the cross-country average correlation between raw credit spreads and global risk factors is much higher for FC than for LC debt (Panel B). Notably, VIX has a mean correlation of 70 percent with FC credit spreads, but only 41 percent with LC credit spreads. This leads us to conclude that the observed global factors are more important in driving spreads on FC debt than on swapped LC debt. Unsurprisingly, the correlations between the global factors and the CDS spread are very similar to the correlations between these factors and the FC spread.

### *Excess Returns*

Having examined the ex-ante promised yields in Tables 1.3 and 1.4, we next turn to ex-post realized returns. The natural measures to study are the excess returns of LC and FC bonds over U.S. Treasury bonds. In particular, we run a series of beta regressions to examine how LC and FC excess returns vary with global and local equity markets. Before turning to these results, we first define the different types of returns. Since all yields spreads are for zero-coupon benchmarks, we can quickly compute various excess returns for the holding period  $\Delta t$ .<sup>12</sup> The FC over US excess holding period return for an  $n$ -year FC bond is equal to

$$rx_{n,t+\Delta t}^{FC/US} = ns_{nt}^{FC/US} - (n - \Delta t)s_{n-\Delta t,t+\Delta t}^{FC/US},$$

which represents the change in the log price of the FC bond over a U.S. Treasury bond of the same maturity. Similarly, the *currency-specific* return differential of an LC bond over a U.S. Treasury bond is given by

$$rx_{n,t+\Delta t}^{LC/US} = ns_{nt}^{LC/US} - (n - \Delta t)s_{n-\Delta t,t+\Delta t}^{LC/US}.$$

Depending on the specific FX hedging strategies, we can translate  $rx_{n,t+\Delta t}^{LC/US}$  into three types of *dollar* excess returns on LC bonds. First, the unhedged LC over US excess return,  $uhrx_{n,t+\Delta t}^{LC/US}$ , is equal to the currency-specific return differential minus the ex-post LC depreciation:

$$uhrx_{n,t+\Delta t}^{LC/US} = rx_{n,t+\Delta t}^{LC/US} - (s_{t+\Delta t} - s_t),$$

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<sup>12</sup> For quarterly returns,  $\Delta t$  is a quarter and we approximate  $s_{n-\Delta t,t+\Delta t}$  with  $s_{n,t+\Delta t}$ .

where  $s_t$  denotes the log spot exchange rate. Second, the holding-period hedged LC over US excess return,  $hrx_{n,t+\Delta t}^{LC/US}$ , is equal to the currency-specific return differential minus the ex-ante holding period forward premium:

$$hrx_{n,t+\Delta t}^{LC/US} = rx_{n,t+\Delta t}^{LC/US} - (f_{t,t+\Delta t} - s_t),$$

where  $f_{t,t+\Delta t}$  denotes the log forward rate at  $t$  for carrying out FX forward transaction  $\Delta t$  ahead. Third, swapped LC over US excess returns,  $srx_{n,t+\Delta t}^{LC/US}$ , is equal to the currency-specific return differential minus the return on the currency swap:

$$srx_{n,t+\Delta t}^{LC/US} = rx_{n,t+\Delta t}^{LC/US} - [n\rho_{nt} - (n - \Delta t)\rho_{n-\Delta t,t+\Delta t}].$$

All three LC excess returns share the same component measuring the LC and US currency-specific return differential. Depending on the specific FX hedging strategy, the ex-post LC depreciation, ex-ante holding period forward premium and ex-post return on the currency swap affect unhedged, hedged and swapped excess returns, respectively.<sup>13</sup>

Table 1.5 presents panel regression results for excess bond returns over local and global equity excess returns. Global equity excess returns are defined as the quarterly return on the S&P 500 index over 3 month U.S. Treasury bills. We define two measures of LC equity excess returns (holding-period hedged and long-term swapped) so that a foreign investor hedging her currency risk in the local equity market has the same degree of hedging on her bond position. We find that FC excess returns have significantly positive betas on both global and hedged LC equity returns, with the loading on S&P being greater. Hedged and swapped LC excess returns do not load on the S&P, but have a significantly positive beta on local equity returns. In contrast, FX unhedged LC excess returns have positive betas on both the S&P and local equity returns.

We therefore conclude that, for foreign investors, the main risk of LC bonds is that emerging market currencies depreciate when returns on global equities are low. This supports the results of Lustig, Roussanov, and Verdelhan (2011) that common factors are important drivers of currency

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<sup>13</sup> The hedged excess return is a first-order approximation of the mark-to-market (MTM) dollar return on money market hedging strategy by combining the LC bond with a long position in the domestic risk-free rate and a short position in the dollar risk-free rate over the U.S. Treasury bond. The swapped excess return is the first order approximation of the MTM dollar return on the bond and the CCS over the U.S. Treasury bond. The hedging notional is equal to the initial market value of the LC bond and is dynamically rebalanced. All the empirical results of the paper are robust to using the exact MTM accounting for the quarterly holding period.

**Table 1.5: Regressions of Bond Excess Returns on Equity Returns, 2005-2011.** This table reports contemporaneous betas of bond quarterly excess returns on global and local equity excess returns. The dependent variables are (1) and (4)  $rx^{FC/US}$ , FC over U.S. Treasury bond excess returns; (2)  $hrx^{LC/US}$ , hedged LC over U.S. Treasury bond excess return using 3-month forward contracts; (3) and (6)  $uhrx^{LC/US}$ , unhedged LC over U.S. Treasury bond excess returns; and (5)  $srx^{LC/US}$ , swapped LC over U.S. Treasury bond excess returns. All excess returns are computed based on the quarterly holding period returns on 5-year zero-coupon benchmarks (annualized). The independent variables are  $S&P \$rx$ , quarterly return on the S&P 500 index over 3-month U.S. T-bills;  $LC \text{ equity hedged } \$rx$ , quarterly return on local MSCI index hedged using 3-month FX forward over 3-month U.S. T-bills; and  $LC \text{ equity swapped } \$rx$ , quarterly return on local MSCI index combined with a 5-year CCS over 3-month U.S. T-bills; All regressions are run at daily frequency with country fixed effects using Newey-West standard errors with 120-day lags and clustering by date following Driscoll and Kraay (1998). Significance levels are denoted by \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

	(1) $rx^{FC/US}$	(2) $hrx^{LC/US}$	(3) $uhrx^{LC/US}$	(4) $rx^{FC/US}$	(5) $srx^{LC/US}$	(6) $uhrx^{LC/US}$
$S&P \$rx$	0.17*** (0.060)	-0.023 (0.057)	0.26*** (0.081)	0.22*** (0.055)	0.0011 (0.025)	0.42*** (0.086)
$LC \text{ equity hedged } \$rx$	0.11*** (0.036)	0.21*** (0.036)	0.33*** (0.049)			
$LC \text{ equity swapped } \$rx$				0.066*** (0.022)	0.099*** (0.021)	0.19*** (0.047)
<i>Observations</i>	12,122	12,122	12,122	12,122	12,122	12,122
<i>R-squared</i>	0.485	0.314	0.498	0.438	0.159	0.416

returns. Our new result, however, is that once currency risk is removed, LC debt appears to be much less risky than FC debt in the sense that it has significantly lower loadings on global equity returns than FC debt.

### 1.3.6 Summary of Stylized Facts

We briefly summarize the results of Section 1.3. We first establish that emerging markets are paying positive spreads over the risk-free rate on their LC sovereign borrowing. This result indicates the failure of long-term covered interest parity for government bond yields between our ten emerging markets and the United States. With the mean LC credit spread equal to 128 basis points, the failure is so large as to make clear the importance of credit risk on LC debt, rather than only pointing to a temporary deviation from an arbitrage relationship as documented in developed markets. Positive within-country correlations between LC credit spreads and the conventional measure of sovereign risk, FC credit spreads, also highlight the role of sovereign risk on LC debt.

Despite the positive correlation, LC and FC credit spreads differ along three important dimensions. First, while LC credit spreads are large and economically significant, they are generally lower than FC credit spreads. The difference between LC and FC credit spreads significantly widened during the peak of the crisis following the Lehman bankruptcy. Second, FC credit spreads are much more correlated across countries than LC credit spreads. Over 80% of the variation in FC spreads is explained by the first principal component. In contrast, only 53% of the variation in LC credit spreads is explained by the first principal component, pointing to the relative importance of country-specific factors in driving LC spreads. Third, FC credit spreads are much more correlated with global risk factors than LC credit spreads. We find that FC spreads are very strongly correlated with global risk factors, including a remarkable 93% correlation between the first PC of FC credit spreads and VIX. These results are mirrored in the return space, as excess holding period returns on FC debt load heavily on global equity returns while excess returns on swapped LC debt do not load on global equity returns once local equity returns are controlled.

The differences between LC and FC credit spreads have important implications. Given the fact that the bulk of emerging market sovereign borrowing takes the form of LC debt, conventional measures of sovereign risk based on FC credit spreads and CDS spreads no longer fully characterize the costs of sovereign borrowing, the cross-country dependence of sovereign risk, and sensitivities of sovereign spreads to global risk factors. Understanding why LC and FC credit spreads differ is the main focus of the next two sections.

## 1.4 *A No-Arbitrage Model with Risky Credit Arbitrage*

### 1.4.1 *Differential Cash Flow Risk and Investor Bases*

Having documented a series of new stylized facts on the differential behavior of LC and FC credit spreads, we now turn to explaining them. One natural explanation for the credit spread differential is that swapped LC and FC bonds have differential cash flow risks. First, the sovereign may have differential incentives to repay the debt. Since FC debt is mainly held by global investors whereas LC bonds are mainly held by local pension funds and commercial banks, the government may be more inclined default on FC obligations. On the other hand, if the sovereign cares more

about reputational costs among international creditors and the access to global capital markets, they may have more incentive to default on local creditors.

Second, in terms of capacity to repay, sovereigns can print local currency and collect most of their revenue in local currency. During periods of sharp exchange rate depreciation, it is easier for the sovereigns to service LC debt than FC debt. However, given that LC debt now represents the bulk of sovereign borrowing, defaulting on LC debt can be a more effective way to reduce debt burden.

Third, since nearly all LC debt is issued under domestic law, LC debt is subject to the risk of changing taxation, regulation, and custody risk, as well as a more uncertain bankruptcy procedure. Offshore investors also face convertibility risk whereby a government prevents the repatriation of funds by introducing capital controls while avoiding technical default. FC bonds, on the other hand, are predominantly governed under international law and are therefore free from withholding taxes and from local government regulations.

Finally, even if the two types of debt always have the same recovery of face value upon default, there could potentially exist a wedge between credit spreads depending on FX depreciation upon default. From a dollar investor's perspective, when default on LC debt occurs, the investor holding the swapped LC debt can unwind the swap contract with an unmatched LC principal payment. This might result in additional profits in the swap position if the spot exchange rate depreciates relative to the ex-ante forward exchange rate upon default. On the other hand, if the spot exchange rate depreciates upon default less than the ex-ante forward exchange rate, there would be additional loss on the swap position. The covariance between default and FX risk is referred to as the quanto adjustment.<sup>14</sup>

In addition to differential cash flow risk, the differential investor bases in domestic and external debt markets can also matter for the relative pricing of the two types of debt. FC bonds are issued offshore, mainly targeting global investors. Although there has been increasing foreign ownership in LC debt markets, the bulk of the LC debt is still held by local investors, such as local pension funds, insurance companies, commercial banks and other government agencies. In

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<sup>14</sup> To remove the covariance term, the investor would need to enter a currency swap contract with a floating notional linked to the LC bond payment (or a quanto swap). However, since EM LC bonds are not deliverable, LC credit linked quanto swaps are rarely quoted in the market.

emerging markets, these domestic entires are often required by law to hold a large fraction of their portfolios in LC treasury bonds, which gives rise a distinct local clientele demand that is absent from the external debt market.<sup>15</sup> This local clientele demand can have equilibrium impacts in the presence of frictions that create limits to arbitrage.

#### 1.4.2 *Environment*

We formalize a parsimonious model allowing for different degrees of market integration via risky credit arbitrage. The model builds on the preferred habitat framework presented in Vayanos and Vila (2009), Greenwood and Vayanos (2010) and Hamilton and Wu (2012), and surveyed in Gromb and Vayanos (2010). Following Duffie and Singleton (1999), we take a reduced form approach to model arrival rates of credit events and allow them to depend on a local and a global factor. We introduce partial market segmentation through three main building blocks. First, we assume that FC bonds are priced by risk-averse diversified global investors with a complete-market stochastic discount factor (SDF) that only depends on the global factor. Global risk aversion shocks affect FC credit spreads directly through FC bonds' systematic exposure to the global shock. Second, we allow for the existence of local clientele demand, modeled as downward sloping outside demand with respect to the price of swapped LC bonds. Third, we assume that a risk-averse credit arbitrageur integrates LC and FC markets by equalizing the price of risk across the two markets adjusting for the onshore and offshore pricing wedge. As a result, the equilibrium LC credit spread is an endogenous outcome of the arbitrageurs' optimal portfolio demand and local clientele demand. The equilibrium impact of the risky arbitrage depends the size of the position the arbitrageur is willing to take, which in turn depends on the arbitrageur's risk aversion, the asset return correlation, and the size and elasticity of local clientele demand.

We begin by specifying a reduced form default process for the bonds. We define  $v_i$  as the time when bonds of type  $j = LC, FC$  issued by country  $i$  default, and the conditional survival intensity,  $I_{t+1}^{i,j}$  as the probability that the bond does not default in period  $t + 1$  conditional on the fact that it has not yet defaulted by period  $t$ . We let the survival intensity for bond  $j$  in country  $i$  depend on

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<sup>15</sup> Kumara and Pfau (2011) document stringent caps faced by emerging market pension funds in investing in local equities and overseas assets.

local ( $z_t^i$ ) and global ( $z_t^w$ ) factors:

$$I_{t+1}^{ij} = P(\nu_i^j > t + 1 | \nu_i^j > t) = \exp[-(\lambda_0^{ij} + \lambda_c^j z_t^i + \lambda_w^j z_t^w + \sigma_{\lambda c}^j \xi_{t+1}^i + \sigma_{\lambda w}^j \xi_{t+1}^w)].$$

For simplicity, we assume zero-recovery upon default. The local and global factors follow two AR(1) processes:

$$\begin{aligned} z_{t+1}^i &= \varsigma^c + \phi^c z_t^i + \xi_{t+1}^i \\ z_{t+1}^w &= \varsigma^w + \phi^w z_t^w + \xi_{t+1}^w, \end{aligned}$$

where  $\xi_{t+1}^w$  and  $\xi_{t+1}^i$  are independent standard normal innovations,  $\varsigma^c$  and  $\varsigma^w$  are AR(1) drifts, and  $\phi^c$  and  $\phi^w$  are the autoregressive coefficients. We interpret an increase in the factors as worsening macroeconomic fundamentals that make default more likely. The global SDF is given by

$$-\log M_{t+1} = -m_{t+1}^* = \psi_0 - \psi z_t^w - \gamma \xi_{t+1}^w,$$

where  $\gamma$  indicates the risk aversion of global investors. The one-period risk-free rate is therefore

$$y_{1t}^* = -\log E_t(M_{t+1}) = \psi_0 - \psi z_t^w - \gamma^2/2.$$

#### 1.4.3 Pricing FC and LC Bonds

In the case of one period bonds when defaulted bonds have zero recovery rates, the survival process fully determines the bond returns. The variance of one-period log returns for bond  $j$  is equal to  $(\sigma_1^j)^2 \equiv (\sigma_{\lambda c}^j)^2 + (\sigma_{\lambda w}^j)^2$ . Given the global SDF and the one-period survival rate, the one-period log FC spread over the risk-free rate is given by

$$s_{1t}^{FC} = -\log E_t(M_{t+1} I_{t+1}^{i,FC}) - y_{1t}^* = \lambda_0^{FC} + \lambda_w^{FC} z_t^w + \lambda_c^{FC} z_t^i - (\sigma_1^{FC})^2/2 + \gamma \sigma_{\lambda w}^{FC}.$$

The first set of terms  $\lambda_0^{FC} + \lambda_c^{FC} z_t^i + \lambda_w^{FC} z_t^w$  is the expected default loss of the bond conditional on the factors. The term  $(\sigma_1^{FC})^2/2$  is the Jensen's inequality correction from working with log yields. The third term is the risk premium on the FC bond. When  $\sigma_{\lambda w} > 0$ , defaults are more likely in

the bad states of the world for the global investor, leading the FC bond to carry a positive risk premium due to its systematic exposure to global shocks. This is the empirically relevant case as demonstrated in Borri and Verdelhan (2011).

Now suppose that the local bond market has an outside clientele demand, i.e., local pension funds, and there are risk-averse arbitrageurs who arbitrage between LC and FC markets. The arbitrageurs take the FC spread priced by the global investor as given. The LC credit spread is an equilibrium outcome of arbitrageurs' portfolio demand and local clientele demand. Assume that the arbitrageurs have power utility over next-period wealth with constant relative risk aversion  $\gamma_a$ . As demonstrated in Campbell and Viceira (2002), the first-order condition of an arbitrageur's optimal portfolio decision is given by

$$E_t r_{1t+1} - y_{1t}^* + \frac{1}{2} \sigma_t^2 = \gamma_a V \alpha_t$$

where  $r_{1t+1}$  is a column vector of one-period log returns of the swapped LC and FC bonds,  $\sigma_t^2$  is the variance of log excess returns,  $V$  is the variance-covariance matrix of log excess returns, and  $\alpha_t$  is a column vector with the arbitrageur's portfolio weights in LC and FC debt.

We conjecture that the LC credit spread  $s_{1t}^{SLC/US}$  is affine in the local and global factors  $z_t^i$  and  $z_t^w$  and is given by

$$s_{1t}^{SLC/US} = (b_{10} + \lambda_0^{SLC} - \sigma_{SLC}^2/2) + (b_{1c} + \lambda_c^{SLC}) z_t^i + (b_{1w} + \lambda_w^{SLC}) z_t^w$$

where the spread parameters  $b_{10}$ ,  $b_{1c}$ , and  $b_{1w}$  will be solved for in the equilibrium. The expected dollar return on swapped LC bonds is then equal to

$$E_t r_{1t+1}^{SLC} - y_{1t}^* + \sigma_{SLC}^2/2 = (b_{10} + b_{1c} z_t^i + b_{1w} z_t^w) - (\tau_{10} - q_{10}),$$

where  $\tau_{10}$  is the transaction cost (e.g. taxes on capital inflows) for offshore investors and  $q_{10}$  is the quanto adjustment due to covariance between the exchange rate and the default process that cannot be hedged away. We refer  $\tau_{10} - q_{10}$  as the offshore pricing wedge because this valuation adjustment only applies to offshore dollar investors. By inverting the variance-covariance ma-

trix  $V$ , we can calculate the arbitrageur's optimal portfolio weights in local and foreign currency bonds,  $\alpha_t^{SLC}$  and  $\alpha_t^{FC}$  from the first-order condition:

$$\begin{bmatrix} \alpha_{1t}^{SLC} \\ \alpha_{1t}^{FC} \end{bmatrix} = \frac{1}{\gamma_a(1-\rho_{r1}^2)(\sigma_1^{SLC})^2(\sigma_1^{FC})^2} \begin{bmatrix} (\sigma_1^{FC})^2 & -\rho_{r1}\sigma_1^{SLC}\sigma_1^{FC} \\ -\rho_{r1}\sigma_1^{SLC}\sigma_1^{FC} & (\sigma_1^{SLC})^2 \end{bmatrix} \begin{bmatrix} (b_{10} + b_{1c}z_t^i + b_{1w}z_t^w) - (\tau_{10} - q_{10}) \\ \gamma\sigma_{\lambda w}^{FC} \end{bmatrix},$$

where  $\rho_{r1} \equiv (\sigma_{\lambda w}^{SLC}\sigma_{\lambda w}^{FC} + \sigma_{\lambda c}^{SLC}\sigma_{\lambda c}^{FC})/(\sigma_1^{SLC}\sigma_1^{FC})$  is the correlation in log returns. When log returns are positively correlated,  $\rho_{r1} > 0$ , the arbitrageur takes offsetting positions in LC and FC bonds to hedge risk.

Following Greenwood and Vayanos (2010), we close the model by positing a downward sloping excess clientele demand for LC bonds  $d_t^{SLC}$  (normalizing the supply of LC bonds to zero), which is decreasing in the price of the swapped LC bond,  $p_{1t}^{SLC}$ ,

$$d_{1t}^{SLC}/W = \kappa_1(-p_{1t}^{SLC} - \beta_1),$$

with  $\kappa_1 > 0$ . Local investors care about the price of the swapped LC bond because it can be translated into how much the LC bond yields relative to the LC risk-free rate. Following Hamilton and Wu (2012), we normalize the clientele demand by the level of arbitrageur's wealth,  $W$ . Furthermore, we assume that  $\beta_1$  is affine in factors and takes the form:

$$\beta_1 = \left[ \theta_{10} + \lambda_0^{SLC} - (\sigma_1^{SLC})^2/2 \right] + (\theta_{1c} + \lambda_c^{SLC})z_t^i + (\theta_{1w} + \lambda_w^{SLC})z_t^w + y_{1t}^*.$$

In the absence of arbitrage, the market clearing condition requires that excess demand is zero, and thus  $y_t^{SLC} = \beta_1$  and the expected excess return on swapped LC bonds is then equal to  $\theta_{10} + \theta_{1c}z_t^i + \theta_{1w}z_t^w$ . This parametrization of  $\beta_1$  allows us to conveniently summarize local demand as the deviation from zero expected excess returns on swapped LC bonds that would occur in the absence of arbitrage. Negative values of  $\theta_{1c}$  and  $\theta_{1w}$  dampen the sensitivity of the LC credit spread to local and global shocks.

Equilibrium requires that asset markets clear, or the arbitrageur's optimal portfolio demand exactly offsets local clientele demand:

$$\alpha_{1t}^{SLC} + d_{1t}^{SLC}/W = 0.$$

Using the above equilibrium condition, we can solve for the equilibrium spread parameters  $b_{10}$ ,  $b_{1c}$  and  $b_{1w}$  in closed forms as follows:

$$b_{10} = \omega_1 \theta_{10} + (1 - \omega_1)(\tau_{10} - q_{10}) + \delta_1^{SLC} \gamma, \quad b_{1c} = \omega_1 \theta_{1c} \quad \text{and} \quad b_{1w} = \omega_1 \theta_{1w}, \quad (1.1)$$

where

$$\omega_1 = \frac{\kappa_1}{\kappa_1 + \frac{1}{\gamma_a(1-\rho_{r1}^2)(\sigma_1^{SLC})^2}}, \quad \text{and} \quad \delta_1^{SLC} \equiv \frac{\rho_{r1}\sigma_1^{SLC}/\sigma_1^{FC}}{\kappa_1\gamma_a(1-\rho_{r1}^2)(\sigma_1^{SLC})^2 + 1} \sigma_{\lambda w}^{FC}.$$

Therefore, the equilibrium LC credit spread depends on the local demand shifters  $\theta_{10}$ ,  $\theta_{1c}$  and  $\theta_{1w}$ , the offshore pricing wedge  $\tau_{10} - q_{10}$ , and the global investor's risk aversion  $\gamma$ . The exact magnitude of these equilibrium effects depend on the arbitrageur's risk aversion, the return correlation and the elasticity of local demand. These will be examined in the next subsection.

#### 1.4.4 Comparative Statics

To gain intuition, we perform several comparative statics. First, we study the pass-through of global risk aversion into the LC credit spread. The pass-through of global risk aversion into the LC spread is the derivative of the spread  $s_{1t}^{SLC/US}$  with respect to risk aversion  $\gamma$ :

$$\delta_1^{SLC} \equiv \frac{\partial s_{1t}^{SLC/US}}{\partial \gamma} = \frac{\rho_{r1}\sigma_1^{SLC}/\sigma_1^{FC}}{\kappa_1\gamma_a(1-\rho_{r1}^2)(\sigma_1^{SLC})^2 + 1} \sigma_{\lambda w}^{FC}, \quad (1.2)$$

where we refer to  $\delta_1^{SLC}$  as the pass-through parameter for swapped LC debt. Similarly, for FC debt, we have that the pass-through of risk aversion  $\gamma$  into FC spreads  $s_{1t}^{FC}$  is given by:

$$\delta_1^{FC} \equiv \frac{\partial s_{1t}^{FC/US}}{\partial \gamma} = \sigma_{\lambda w}^{FC}.$$

It is straightforward to establish the following proposition using Equation 1.2:

**Proposition 2.** (*Pass-through of Global Risk Aversion*) If the asset return correlation times the standard deviation of swapped LC returns is less than the standard deviation of FC returns ( $\rho_{r1}\sigma_1^{SLC} < \sigma_1^{FC}$ ), the pass-through of global risk aversion shocks into the swapped LC spread is less than into FC spreads,  $\delta_1^{SLC} < \delta_1^{FC}$ . Furthermore, the pass-through into LC spreads is increasing in the return correlation ( $\partial\delta_1^{SLC}/\partial\rho_{r1} > 0$ ), decreasing in the arbitrageur's risk aversion ( $\partial\delta_1^{SLC}/\partial\gamma^a < 0$ ), and decreasing in the elasticity of local demand ( $\partial\delta_1^{SLC}/\partial\kappa_1 < 0$ ).

Although the price of risk is equalized across the two markets by the arbitrageur, the quantity of risk can still be different. Under the condition that  $\rho_{r1}\sigma_1^{SLC} < \sigma_1^{FC}$ , swapped LC bonds have a lower quantity of risk. We can re-express this condition as  $\beta_{SLC/FC} = Cov(rx_{t+1}^{SLC}, rx_{t+1}^{FC}) / Var(rx_{t+1}^{FC}) < 1$  in the beta regression of running swapped LC excess returns on FC excess returns:

$$rx_{t+1}^{SLC} = \beta_0 + \beta_{SLC/FC}rx_{t+1}^{FC} + \epsilon_{t+1}.$$

Due to the lower quantity of risk, swapped LC bonds carry a lower risk premium. In the one-period model, both  $\rho_{r1}$  and  $\sigma_{\lambda w}$  are given exogenously by the default processes and do not depend on the local demands  $\theta_{1c}$  and  $\theta_{1w}$ . In Section A.3 of Appendix, we relax this feature of the model in a multi-period specification in which the price of the bond next period is also uncertain even in the absence of default and the price sensitivity depends on the local demand parameters. The mechanism of pass-through of global risk aversion into the LC credit spread is as follows. An increase in global risk aversion  $\gamma$  increases the FC spread and the expected excess returns on the FC bond. Holding the arbitrageur's risk aversion constant, the arbitrageur takes advantage of this opportunity by going long in FC bonds and hedges her position by shorting swapped LC bonds, which drives up the swapped LC spread. The pass-through of global risk aversion is lower into the LC bond if the quantity of risk in LC bonds is lower.

The extent of the trade and its subsequent impact on the LC credit spread depends on three key parameters. First, the differential pass-through depends on the return correlation  $\rho_{r1}$ . Higher correlations increase LC pass-through by allowing the arbitrageur to better hedge her risk and hence take a larger position. When returns are uncorrelated ( $\rho_{r1} = 0$ ), the pass-through is zero, and when returns are perfectly correlated ( $\rho_{r1} = 1$ ), pass-through achieves its maximum at  $\frac{\sigma_1^{SLC}}{\sigma_1^{FC}}\sigma_{\lambda w}^{FC}$ . Second, the differential pass-through depends on the arbitrageur's risk aversion  $\gamma_a$ : an increase

in arbitrageur risk aversion decreases pass-through. When  $\gamma_a$  is infinite, pass-through is zero because the arbitrageur is too risk-averse to make any trades. When  $\gamma_a$  is zero, meaning that the arbitrageur is risk-neutral, pass-through is maximized for a given return correlation,  $\rho_{r1}$ . Third, the differential pass-through depends on the elasticity of local clientele demand  $\kappa_1$ : An increase in the elasticity of local clientele demand decreases pass-through. A more elastic local demand increases the ability of the LC credit spread to absorb larger positions taken by arbitrageurs. When  $\kappa_1$  is infinite, local clientele demand is perfectly elastic and therefore the LC credit spread is completely determined by local conditions, leaving no room for arbitrageurs to play a role. On the other hand, when  $\kappa_1 = 0$ , local clientele demand is zero and thus pass-through is maximized.

In addition to capturing the default intensity and the risk premium, the equilibrium LC credit spread is a weighted average of the onshore local clientele effects and the offshore pricing wedge. The pass-through of local clientele effects into the LC credit spread in terms of level ( $\theta_{10}$ ) and sensitivities ( $\theta_{1c}, \theta_{1w}$ ) to shocks is equal to

$$\frac{\partial b_{10}}{\partial \theta_{10}} = \frac{\partial b_{1c}}{\partial \theta_{1c}} = \frac{\partial b_{1w}}{\partial \theta_{1w}} = \omega_1 = \frac{\kappa_1}{\kappa_1 + \frac{1}{\gamma^a(1-\rho_{r1}^2)(\sigma_1^{SLC})^2}}$$

Interestingly, the pass-through of the offshore pricing wedge is equal to  $1 - \omega_1$ .

$$\frac{\partial b_{10}}{\partial \tau_{10}} = -\frac{\partial b_{10}}{\partial q_{10}} = 1 - \omega_1.$$

The parameter  $\omega_1$  governs the relative importance of onshore and offshore investors in determining the equilibrium LC credit spread. Under complete segmentation ( $\omega_1 = 1$ ), only the local clientele matters, leaving no scope for offshore transaction costs or the covariance between the exchange rate and defaults. On the other hand, under perfect integration ( $\omega_1 = 0$ ), local clientele effects are completely arbitrated away and the credit spread is entirely determined by offshore credit valuation.

#### 1.4.5 Empirical Decomposition of Credit Spread Differentials

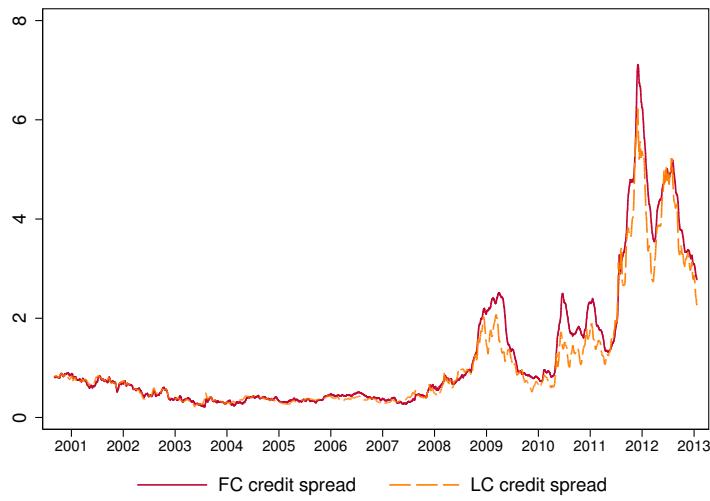
Using the model, we can decompose the difference in LC and FC credit spreads into three components:

$$s_{1t}^{SLC/FC} = \underbrace{(\tilde{\lambda}_0 + \tilde{\lambda}_i z_t^i + \tilde{\lambda}_w z_t^w - \tilde{\sigma}^2/2)}_{\text{differential recovery (convexity)}} + \underbrace{w_n(\theta_{10} + \theta_{1c} z_t^i + \theta_{1w} z_t^w) + (1 - \omega_1)(\tau_{10} - q_{10})}_{\text{weighted pricing wedge}} + \underbrace{[\delta_1^{SLC}(\omega_1) - \delta_1^{FC}]\gamma}_{\text{risk premium}}$$

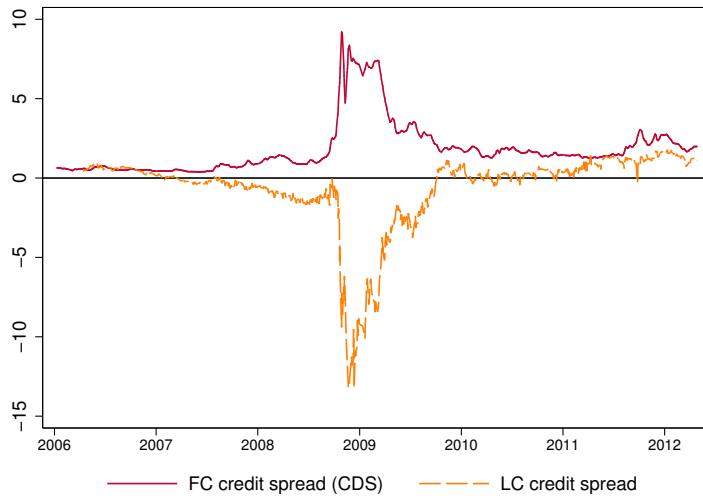
where  $\tilde{x} \equiv x^{SLC} - x^{FC}$ . The first term in the curly bracket measures the difference in default intensity between LC and FC bonds adjusting for convexity in log yields. The second term measures the weighted onshore and offshore pricing wedges. Finally, the third term measures the difference in risk premia, arising from risky arbitrage between the two markets.

To give an example of perfect market integration, Figure 1.7 shows LC (euro) and FC (dollar) sovereign credit spreads for Italy. Prior to 2008, the two credit spreads were indistinguishable. Starting in 2008, the euro credit spread became slightly lower than the dollar credit spread, reflecting either expected higher recovery on euro debt or depreciation of euro upon Italian default. Despite the level difference, the within country correlation between the two credit spreads is 99 percent. On the other extreme, Russia displays extreme market segmentation between LC and FC debt market during the 2008-09 crisis (Figure 1.8), as the LC credit spread reached negative 10 percentage points during the crisis. While the nominal government bond yield differential was around 10 percentage points, the ruble/dollar CCS rate increased to 20 percentage points as offshore investors were concerned that Russia would abandon the euro/dollar peg and devalue. Local investors continued to hold LC debt despite extremely unattractive yields.

Our sample emerging markets are in between the two extreme cases of perfect integration and complete segmentation. In the next section, we demonstrate that consistent with theory's predictions, differential sensitivities to global risk aversion shocks can explain large cross-sectional and time series variations in credit spread differentials.



*Figure 1.7: 5-Year Sovereign Credit Spreads in Italy.* The solid line FC credit spread plots 5-year yield spreads of dollar denominated Italian sovereign bonds over U.S. Treasury bonds. The dotted line LC credit spread plots 5-year yield spreads of euro-denominated Italian sovereign bonds after swapping into dollars using the euro/dollar CCS over U.S. Treasury bonds. All data are from Bloomberg.



*Figure 1.8: 5-Year Sovereign Credit Spreads in Russia.* The solid line FC credit spread plots 5-year Russian sovereign credit spread swaps denominated in dollars (Russia does not have enough dollar bonds outstanding to construct yield curves). The dotted line LC credit spread plots 5-year yield spreads of Russian ruble-denominated Russian sovereign bonds after swapping into dollars using the ruble/dollar CCS over U.S. Treasury bonds. Ruble bond yields are from the Moscow Stock Exchange. All the other data are from Bloomberg.

## 1.5 Differential Risk Premia

### 1.5.1 Benchmark Regressions

To test the model's predictions on the differential pass-through of global risk aversion ( $\gamma_t$ ) summarized in Proposition 2, we perform a panel regression with country fixed effects:

$$s_{i,t}^j = \alpha_i^j + \delta^j \gamma_t + \lambda_c z_t^i + \lambda_w z_t^w + \epsilon_{it}^j,$$

where  $i$  denotes country and  $j$  denote three different spreads, the LC credit spread (SLC), the FC credit spread (FC), and the swapped LC over FC spread (SLC/FC). We first assume that  $\delta^j$ , the pass-through coefficient of global risk aversion, is the same across all countries, which will be relaxed in the next subsection. Sensitivity to global and local risk factors is also assumed to be the same across countries and to be time-invariant. We include a country fixed effect in the regression to allow each country to have a different intercept for credit spreads. The theory predicts that the pass-through coefficient of global risk aversion should be lower for LC credit spreads than for FC credit spreads:  $\delta^{SLC} < \delta^{FC}$ , and as a result,  $\delta^{SLC/FC} < 0$ . We use VIX as a proxy for the global risk aversion  $\gamma_t$ ,<sup>16</sup> and a host of global and local macroeconomic variables as proxies for  $z_t^i$  and  $z_t^w$ . Table 1.6 reports regression results for (1) the LC credit spread (2) the FC credit spread and (3) the swapped LC over FC spread, the difference between (1) and (2). By construction, the LC credit spread is equal to the difference between the nominal LC over US spread and the swap rate. We thus also report the regression results for the nominal LC over US spread in Column (4) and the swap rate in Column (5) to better understand the determinants of the LC credit spread. Following Driscoll and Kraay (1998), all regressions are run at monthly frequency with country fixed effects using the Newey-West type standard errors with 12-month lags to account for within-country serial correlation and clustering by month to correct for spatial correlation across countries for the same month.

As our primary measure of global economic fundamentals, we use the Chicago Fed National Activity Index (CFNAI), which is the first principal component 85 monthly economic indicators of the U.S. economy. The next variable  $ba^{CCS}$  is equal to one half of the bid-ask spread on 5-year

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<sup>16</sup> We divide the conventional quote of VIX by  $\sqrt{12}$  to measure unannualized implied volatility over the next 30 days.

par cross currency swaps, measured in basis points. Although it is specifically a measure of the liquidity on swaps, we use it as a proxy for the overall liquidity conditions in emerging market fixed income markets, especially in the offshore markets. For local controls, we first include *LC Equity Vol.* the realized standard deviation of local equity returns, measured using the daily local MSCI equity returns for 30-day rolling windows. We expect this measure to reflect omitted local fundamentals and local risk aversion. In addition, we include a set of country-specific macroeconomic controls that previous literature has emphasized as potentially important in explaining sovereign spreads. These include the FC debt/GDP ratio, the LC debt/GDP ratio, the level and volatility of monthly inflation and changes in the terms of trade, as well as monthly changes in foreign exchange reserves.<sup>17</sup>

As predicted by the theory, VIX has a smaller impact on the LC credit spread than on the FC credit spread conditional on macroeconomic fundamentals. The coefficient on VIX for the FC credit spread is three times as large as the coefficient for the LC credit spread. The coefficient on VIX in the LC over FC credit spread differential regression (Column 3) is negative and statistically significant. The magnitude of the coefficient suggests that an expected one percentage point increase in the volatility of the S&P 500 over the next 30 days is associated with an 8 basis point increase in the LC credit spread, a 23 basis point increase in the FC credit spread, and thus a 15 basis point reduction in the LC over FC credit spread differential. This risk aversion pass-through differential is economically significant. In our estimated sample, a one standard deviation increase in VIX over its mean decreases the credit spread differential by 45 basis points. The largest spike in VIX following the Lehman bankruptcy corresponds to a 3.5 standard deviation increase in VIX over the mean, which can generate a 157 basis point differential in LC and FC credit spreads, controlling for the worsening local and global economic fundamentals during the crisis.

The importance of VIX in explaining credit spread differentials can also be seen from the R-squared of regressions. VIX alone explains large fractions of the total variation in all credit spread regressions, particularly for the FC credit spread. The within R-squared of a panel regression

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<sup>17</sup> Debt to GDP ratios are computed by aggregating the entire universe of individual sovereign bond issuance in Bloomberg. Using this index, rather than the aggregated data from the Bank for International Settlements (BIS), we obtain a higher frequency measure of the debt outstanding than the quarterly measure produced by the BIS. The correlation between our debt/GDP ratios with the BIS official statistics is 96 percent for FC debt and 80 percent for LC debt. More details on construction of macroeconomic controls are given in the Appendix Table A.3.

with VIX as the only regressor is equal to 24.7 for the LC credit spread, 58.5 percent for the FC credit spread, and 25.6 percent for the credit spread differential. Conditional on macroeconomic fundamentals, VIX increases the R-squared of the regression from 27.1 to 30.1 percent for the LC credit spread, from 62.2 to 72.8 percent for the FC credit spread and from 36.4 percent to 42.6 percent for the differential. Therefore, VIX alone accounts for 60 percent of total explained variations in the credit spread differential. After controlling for fundamentals, VIX accounts for an increase equal to 15 percent of total explained variations in explanatory power of the benchmark regression.

Conditional on our host of controls, swap liquidity does not significantly affect the LC credit spread. Although the bid-ask spread of the swap significantly increases with the swap rate, it is also associated with a similar increase in the nominal LC over US spread. On the other hand, the FC credit spread significantly increases with the bid-ask spread, despite the fact that no swaps are used in the construction of the measure. This supports our use of the bid-ask spread on the swap as a general measure of liquidity as well as a direct measure of swap liquidity. Furthermore, we find that worsening global macroeconomic conditions, higher local equity volatility, higher FC debt/GDP and higher inflation volatility all significantly increase the FC credit spread, but have either insignificant or smaller impacts on the LC credit spread.

**Table 1.6: Regression of 5-Year Credit Spreads on VIX, 2005m1-2011m12.** The dependent variables are as follows: (1)  $s^{SLC/US}$ , swapped LC over U.S. Treasury spread; (2)  $s^{FC/US}$ , FC over U.S. Treasury spread; (3)  $s^{SLC/FC}$ , swapped LC over FC spread; (4)  $s^{LC/US}$ , unhedged LC over US Treasury spread; (5)  $CCS$ , 5-year zero-coupon cross-currency swap rate. The independent variables are:  $VIX$ , monthly standard deviation of implied volatility on S&P index options (conventional quote/ $\sqrt{12}$ );  $CFNAI$ , Chicago Fed National Activity Index;  $ba^{CCS}/s$ , one half of bid-ask spread on 5-year par CCS in basis points;  $LC\ Equity\ Vol.$ , realized standard deviation of daily local MSCI equity returns computed using a moving window of 30 days;  $\Delta IP$ , monthly percentage change in country-specific industrial production index;  $FC\ Debt/GDP$  and  $LC\ Debt/GDP$ , monthly LC and FC debt to GDP ratios aggregating from the entire universe of Bloomberg sovereign bonds outstanding;  $\Delta CPI$ , monthly percentage change in consumer price index;  $Std(\Delta CPI)$ , standard deviation of  $\Delta CPI$  for the past 12 months;  $\Delta ToT$ , monthly percentage change in terms of trade;  $Std(\Delta ToT)$ , standard deviation of  $\Delta ToT$  for the past 12 months; and  $\Delta Reserve$ , monthly percentage change in FX reserves. All regressions are run at monthly frequency with country fixed effects using Newey-West standard errors with 12-month lags clustered by month following Driscoll and Kraay (1998)  
\*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

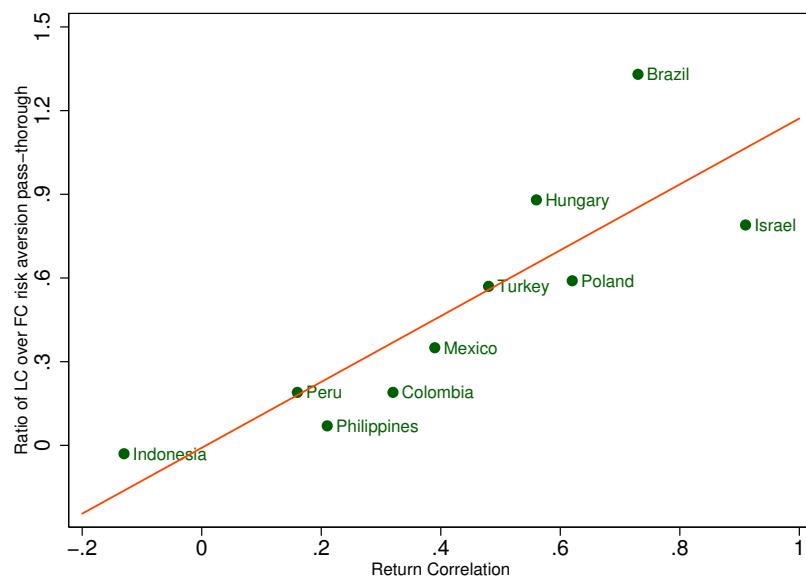
	(1) $s^{SLC/US}$	(2) $s^{FC/US}$	(3) $s^{SLC/FC}$	(4) $s^{LC/US}$	(5) $ccs$
<i>VIX</i>	<b>0.088***</b> (0.029)	<b>0.23***</b> (0.019)	<b>-0.15***</b> (0.025)	0.16*** (0.043)	0.070** (0.031)
<i>CFNAI</i>	-0.017 (0.082)	-0.16*** (0.040)	0.15 (0.094)	-0.15 (0.14)	-0.13 (0.13)
<i>ba<sup>CCS</sup></i>	0.0037 (0.0049)	0.025*** (0.0025)	-0.021*** (0.0056)	0.024*** (0.0092)	0.020*** (0.0058)
<i>LC Equity Vol.</i>	0.12 (0.075)	0.19** (0.082)	-0.065 (0.054)	0.19 (0.17)	0.071 (0.12)
$\Delta IP$	-0.10 (0.066)	-0.076* (0.040)	-0.027 (0.073)	0.084 (0.11)	0.19* (0.096)
<i>Other Controls</i>					
<i>FC Debt/GDP</i>	-0.026	0.12***	-0.15***	0.0096	0.035
<i>LC Debt/GDP</i>	-0.024	0.011	-0.035***	-0.052***	-0.028***
$\Delta CPI$	0.10*	0.033	0.068*	0.32***	0.21***
$Std(\Delta CPI)$	0.39**	0.50***	-0.10	1.09***	0.69***
$\Delta ToT$	-0.0086	0.0077	-0.016**	0.011	0.019***
$Std(\Delta ToT)$	0.024	0.021	0.0032	0.27***	0.25***
$\Delta Reserve$	-0.0013	-0.011	0.010	-0.029**	-0.028**
<i>Observations</i>	762	762	762	762	762
<i>Within R-Squared</i>					
<i>Full model</i>	0.301	0.728	0.426	0.442	0.308
<i>Without VIX</i>	0.271	0.622	0.364	0.415	0.299
<i>With VIX only</i>	0.247	0.585	0.256	0.266	0.111

### 1.5.2 Cross-Country Variation

We now relax the assumption that the pass-through of global risk-aversion into FC debt,  $\delta^{FC}$ , is the same across all countries. The theory predicts that the ratio of swapped LC to FC pass-through  $\delta_i^{SLC} / \delta_i^{FC}$  increases in  $\rho_r^i$ . To test this prediction, we obtain estimates of  $\hat{\delta}_i^{SLC}$  and  $\hat{\delta}_i^{FC}$  from the coefficients on the interaction terms between country dummies and VIX in the regression:

$$s_{i,t}^j = \alpha_{1,i}^j + \sum_i \delta_i^j C_i \gamma_t + \lambda_c z_t^i + \lambda_w z_t^w + \epsilon_{it}^j, \quad (1.3)$$

where the country dummy  $C_i = 1$  for country  $i$ . Columns 1 and 2 of Table 1.7 report the coefficient estimates for  $\hat{\delta}_i^{SLC}$  and  $\hat{\delta}_i^{FC}$ . For our model to find empirical support, we would expect countries with a higher ratio  $\hat{\delta}_i^{SLC} / \hat{\delta}_i^{FC}$  to have a higher return correlation. As demonstrated by comparing Column 3, where we compute this ratio country by country, and Column 4, where we present the return correlations, this is precisely what we find, with the correlation between the two columns at a remarkable 84 percent. Differential sensitivities to VIX explain the bulk of the cross-sectional variations in excess return correlations. We present this result visually in Figure 1.9, showing once again that the strong positive relationship between the pass-through of risk aversion into the LC credit spread relative to the FC credit spread and the return correlation between the two assets.



**Figure 1.9: Differential Risk Aversion Pass-Through and Return Correlation.** This figure plots the ratio of global risk aversion pass-through into LC credit spreads over the pass-through into FC credit spreads on the y-axis (Column 3 in Table 1.7) and correlation between swapped LC and FC quarterly holding period returns over U.S. Treasury bill rates on the x-axis (Column 4 in Table 1.7). The ratio of pass-through is computed based on Columns 1 and 2 in Table 1.7. The full regression specification is given by Equation 1.3.

**Table 1.7: Impact of VIX on Credit Spreads by Country.** This table reports results of cross-country variations in the impact of VIX on credit spreads. Columns (1) and (2) report coefficients on VIX interacting with country dummies in credit spread regressions with macroeconomic controls, as specified by Equation 1.3. Column (1) reports the pass-through of VIX into LC credit spreads and Column (2) reports the pass-through of VIX into FC credit spreads. All controls are the same as in regression Table 1.6. All regressions are run at monthly frequency with country fixed effects using Newey-West standard errors with 12-month lags clustered by month following Driscoll and Kraay (1998). \*\*\* p<0.01, \*\* p<0.05, \* p<0.1. Column (3) computes the ratios of coefficients in Column (1) over Column (2) and Column (4) reports the correlation between swapped LC and FC quarterly excess returns over the U.S. T-bill rates. A scatter plot of columns (3) against (4) is shown in Figure 1.9.

	(1) $\delta^{SLC}$	(2) $\delta^{FC}$	(3) $\delta^{SLC} / \delta^{FC}$	(4) $\rho_r^i$
Brazil	0.19*** (0.044)	0.14*** (0.025)	1.33	0.73
Colombia	0.046 (0.028)	0.24*** (0.020)	0.19	0.32
Hungary	0.31*** (0.055)	0.35*** (0.059)	0.88	0.56
Indonesia	-0.010 (0.045)	0.39*** (0.076)	-0.03	-0.13
Israel	0.051 (0.045)	0.064* (0.034)	0.79	0.91
Mexico	0.066* (0.039)	0.19*** (0.022)	0.35	0.39
Peru	0.043 (0.035)	0.23*** (0.037)	0.19	0.16
Philippines	0.016 (0.045)	0.24*** (0.043)	0.07	0.21
Poland	0.11*** (0.031)	0.19*** (0.036)	0.59	0.62
Turkey	0.21*** (0.055)	0.36*** (0.033)	0.57	0.48
<i>All Macro Controls</i>	Yes	Yes	Correlation (3) and (4)	
<i>Observations</i>	762	762	<b>0.84</b>	
<i>R-squared</i>	0.404	0.782		

### 1.5.3 Excess Returns Predictability

We now present evidence on how differential risk premia affect the time series properties of credit spreads. Since VIX has a contemporaneous positive impact on credit spreads through the risk premium channel, high levels of VIX are associated with high risk premia, and hence high excess returns over U.S. Treasury bonds. Since VIX has a differential contemporaneous pass-through into LC and FC credit spreads, we should also expect VIX to have differential predictive power for LC and FC excess returns. Consistent with the prediction, we find that high VIX predicts higher FC excess returns than swapped LC excess returns, and thus negative swapped LC over FC excess returns. The negative predictive power of VIX for swapped LC over FC excess returns naturally gives rise to an investment strategy. When global risk aversion is high, an arbitrageur can long FC bonds and short swapped LC bonds. Since FC spreads are much more sensitive to the global risk aversion shocks than swapped LC, high risk aversion predicts positive excess returns on this strategy, which compensates for the risk that the arbitrageur takes. On the other hand, global macroeconomic fundamentals marginally forecast persistence, rather than mean reversion in swapped LC over FC excess returns once VIX is controlled. Therefore, it is unlikely that the predictive power of VIX is due to its correlation with unobserved macroeconomic fundamentals.

In the first panel of Table 1.8, we examine the forecasting power of these variables for annualized excess returns of swapped LC bonds over U.S. Treasury bonds for a quarterly holding period. In the first regression, we see that high levels of VIX forecast excess returns at the quarterly horizon with an  $R^2$  of 4.2%. We next run a second univariate forecasting regression and find that CFNAI, our measure of local fundamentals, has similar forecasting power as VIX for swapped LC excess returns. When we run a bivariate forecasting regression including both VIX and CFNAI in the third row, both lose significance and the increase in forecasting power is marginal compared to including VIX alone. Including the spread on the cross currency swap rate  $ba^{CCS}/2$  and the volatility on the local equity indices  $LC\ Vol$  have little effect, but including industrial production growth  $\Delta IP$  leads to a significant increase in forecasting power. Higher industrial production growth forecasts lower excess returns on swapped LC bonds.

In the second panel of Table 1.8, we repeat this forecasting exercise for excess returns on FC bonds over U.S. Treasury bonds. In the first row, we see that VIX alone has an  $R^2$  of 10.1% and

the coefficient on VIX is more than double the coefficient on VIX in the univariate regressions for excess returns on swapped LC bonds, and the  $R^2$  is more than doubled as well. In the second regression, we once again remove VIX to examine the forecasting power of CFNAI alone and find that, in contrast to the results in the first panel, the  $R^2$  is only one quarter the value it is in the univariate forecast using VIX. In the third row, we see that a forecasting regression with both VIX and the CNFAI has an  $R^2$  less than one percentage point higher than for VIX alone. The key finding is that conditional on VIX, the global fundamental does not forecast mean reversion in returns. The magnitude of the coefficient on VIX actually increases by 45 basis points after controlling for CFNAI, once again in sharp contrast to the forecasting results for swapped LC excess returns. In the fourth row, we add our liquidity measure to these two variables and find that the forecasting power of the regression is increased significantly to 15.5%. After adding the full set of local controls, the coefficient on VIX is significantly positive at 2.82.

Finally, in the third panel, we examine how these variables forecast excess returns of swapped LC bonds over FC bonds. In the first row, we see that higher levels of VIX forecast a negative excess return of swapped LC over FC debt, as would be expected since we found in the first two panels that elevated levels of VIX forecast much higher FC returns than swapped LC returns. Looking at all 6 forecasting regressions in the third panel, we see that the forecasting strength of VIX is sharpened as we add in our measures for global fundamentals, global liquidity, local market conditions, and miscellaneous controls. Because VIX covaries strongly with global fundamentals and global fundamentals marginally forecast return persistence, the predictive power of VIX is increased when we condition on fundamentals. Conditional on fundamentals, a one standard deviation increase in VIX over its mean forecasts negative 6.3 percent annualized excess returns of swapped LC over FC bonds.

**Table 1.8: Forecasting Quarterly Holding-Period Excess Returns, 2005m1-2011m12.** This table reports annualized quarterly return forecasting results for  $srx_{t+3}^{LC/US}$ , swapped LC over U.S. excess returns,  $rx_{t+3}^{FC/US}$ , FC over US excess returns, and  $srx_{t+3}^{LC/FC}$ , swapped LC over FC excess returns. See Table 1.6 for definition of predictive variables. All regressions are run at monthly frequency with country fixed effects using Newey-West standard errors with 12-month lags clustered by month following Driscoll and Kraay (1998) \*\*\* p<0.01, \*\* p<0.05, \* p<0.1.

	VIX	CFNAI	$ba^{CCS}/2$	LC Vol	$\Delta IP$	Other Controls	$R^2$	
(1)	0.92** (0.41)					No	0.042	
(2)		-2.95** (1.24)				No	0.037	
(3)	0.620 (0.43)	-1.460 (1.53)				No	0.047	
$srx_{t+3}^{LC/US}$	(4)	0.600 (0.45)	-1.370 (1.52)	0.0320 (0.046)		No	0.048	
	(5)	0.440 (0.48)	0.210 (1.25)	0.0180 (0.047)	1.010 (0.98)	-3.77*** (1.24)	0.078	
	(6)	0.710 (0.54)	-0.0770 (0.98)	-0.0210 (0.045)	0.300 (1.08)	-3.45** (1.50)	0.106	
	(1)	1.95*** (0.66)				No	0.101	
	(2)		-3.330 (2.26)			No	0.025	
	(3)	2.49** (1.07)	2.620 (3.60)			No	0.109	
$rx_{t+3}^{FC/US}$	(4)	2.21** (1.01)	3.550 (3.78)	0.33*** (0.098)		No	0.155	
	(5)	2.08* (1.11)	5.100 (3.80)	0.32*** (0.090)	0.820 (1.32)	-3.69** (1.53)	0.170	
	(6)	2.82** (1.36)	4.790 (3.44)	0.28*** (0.065)	0.0240 (1.43)	-2.530 (1.69)	0.212	
	(1)	-1.03*** (0.29)				No	0.028	
	(2)		0.380 (1.32)			No	0.000	
	(3)	-1.87*** (0.66)	-4.09* (2.35)			No	0.047	
$srx_{t+3}^{LC/FC}$	(4)	-1.61*** (0.60)	-4.93* (2.60)	-0.30*** (0.11)		No	0.085	
	(5)	-1.64** (0.67)	-4.89* (2.75)	-0.30*** (0.11)	0.190 (1.05)	-0.0790 (0.99)	No	0.085
	(6)	-2.12** (0.86)	-4.87* (2.68)	-0.30*** (0.089)	0.270 (1.05)	-0.910 (0.80)	Yes	0.111

## 1.6 Conclusion

The last decade has seen a remarkable change in emerging market government finance. No longer do major emerging markets have to borrow in external markets in FC to borrow from global investors. Instead, global investors are increasingly willing to lend to emerging market governments by investing in LC debt issued in domestic markets. Despite these major changes, the academic literature on sovereign risk still remains focused on emerging market debt crises involving FC debt issued abroad. In this paper, we tried to understand the impact of these changes by jointly examining the sovereign risk on LC and FC debt. To do so, we introduce a new measure of LC sovereign risk, the LC credit spread, defined as the difference between LC bond yield and the LC risk-free rate implied from the swap market.

This new measure delivers several key findings. First, emerging market LC bonds promise to pay a significant positive spread over the risk-free rate, direct evidence for the failure of long-term covered interest rate parity for government bond yields between emerging markets and the United States. Second, LC debt has lower credit spreads than FC debt issued by the same sovereign at the same tenor. The LC over FC credit spread differential becomes even more negative during the peak of the crisis. Third, FC credit spreads are very integrated across countries and more responsive to global risk factors, but LC credit spreads are much less so. From an offshore investor's perspective, the commonly perceived systematic risk on LC debt mainly comes from the currency risk. Once the currency risk is hedged, LC bonds are safer than FC bonds in terms of the correlations between asset returns and global risk factors.

We rationalize these new empirical findings using a model allowing for partial integration between domestic and external debt markets. The model features local investors with preferred habitats in the LC debt and risky credit arbitrage between the domestic and external markets. The equilibrium LC credit spread is a weighted average of credit valuation of local clienteles and offshore investors. Consistent with the model's prediction, we find that differential exposure to global risk aversion explains a significant portion of the cross-country and time series variations in credit spread differentials, conditional on a host of macroeconomic variables. The differential sensitivities of LC and FC credit spreads to global risk aversion shocks sheds light on the degree of market integration between domestic and external debt markets.

While our reduced form model captures many of the new stylized facts that we document, we have abstracted from how the sovereign decides whether to issue LC or FC debt. Integrating our empirical findings using price data into sovereign issuance patterns using bond supply and ownership data is part of ongoing research.

## 2. THE END OF “ORIGINAL SIN”: NOMINAL BOND RISK IN EMERGING MARKETS<sup>1</sup>

### 2.1 *Introduction*

Today, emerging markets rarely issue local currency (LC) debt in international markets but they are borrow from foreign lenders in their own currency. This apparent contradiction comes from the fact that foreign investors are increasingly investing directly LC sovereign bonds issued in domestic markets under domestic law. While this may seem to be an insignificant development, we argue that it demonstrates the end of a major issue in international economics: “Original Sin.” In an important and influential paper, Eichengreen and Hausmann (1999) demonstrated that emerging markets were particular vulnerable to financial crises because they had all of their external liabilities denominated in foreign currency (FC). A large literature followed, emphasizing that a country’s inability to borrow abroad in their own currency at long tenors, “Original Sin” was a major source of macroeconomic fragility in emerging markets.

In this paper, we demonstrate just how dramatically this situation has changed over the last decade and show that local currency now constitutes a large and growing portion of emerging market sovereign external borrowing. However, countries did not overcome Original Sin by issuing LC debt in international markets. Rather, foreign investors began lending to emerging markets by purchasing bonds in domestic markets, even though this leaves them exposed to capital control risk, custodial risk, and other risks in addition to the nominal risks of inflation and exchange rate depreciation. Using a new compiled dataset on foreign participation in domestic sovereign bond markets, we show that since the mid-2000s, foreign ownership has been growing rapidly and now stands at over 20 percent of total domestic debt outstanding for many countries, and significantly higher for several emerging markets. Using more refined ownership data for Brazil, Poland, Peru

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<sup>1</sup> Joint with Jesse Schreger, Harvard University

and Mexico, we show that foreigners are particularly interested in holding nominal fixed rate bonds, even when inflation-linked and floating-rate bonds are available. For many emerging markets, LC debt not only provides the majority of government finance, but also provides the majority of financing from foreign investors.

Du and Schreger (2013) study the behavior of the credit component of LC sovereign bonds by hedging away nominal interest rate risk using currency swaps, and contrast it with dollar credit spread on the external debt issued by the same sovereign. In this paper, we directly study the risk of LC nominal bonds without of hedging away the duration risk. We focus on the excess holding period returns of LC bonds over LC short rates for local investors. From dollar investors' perspective, this approximates the dollar excess return on LC bonds over the dollar short rate after hedging away the currency risk of the holding period. Using the systematic exposure of bond duration risk with local and global equity risk factors, we address the following three questions. First, how are nominal bond risks different across countries? Second, why are nominal bond risks different across countries? And third, what is the implication of differential nominal bond risk on sovereign debt portfolio consisting of local and FC debt?

Examining a sample of 11 developed markets and 20 emerging markets from 2005 to 2012, we find that while emerging market equity returns are far more correlated with US equity returns than emerging market bond returns are with US Treasury returns. The lack of synchronization in nominal bond returns between emerging markets and the U.S. reflects large cross-country variations in risk of nominal bonds. We define the risk of nominal bonds as the CAPM betas of the bond excess returns on local and global equity excess returns. We show that nominal bonds in G10 currencies all have negative betas with local equity and S&P excess returns. In the sample of emerging markets, local bond betas with local equities exhibit large cross-country heterogeneity, ranging from -0.14 for Hong Kong to 0.3 for Indonesia.

We then explain the cross-sectional distribution of bond betas in terms of cross country differences in the conduct of monetary policy. The negative betas of G10 currency bonds for the sample period is consistent with the finding that in developed countries, government bonds have become a good hedge for equity investors over the past decade.<sup>2</sup> Under countercyclical monetary policy

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<sup>2</sup> Eurozone countries experiencing sovereign debt crises are obvious exceptions.

and procyclical inflation rates, negative macroeconomic shocks lower inflation expectation and increase expected future real cash flows, giving rise to positive returns on nominal bonds. For emerging markets, one of the main benefits of being able to borrow using nominal fixed rate LC debt is that it provides more state contingency compared with FC borrowing. During the bad states of the world, the government has the option to resort to inflation to reduce the real burden of the debt. In anticipation of inflationary policy during downturn, nominal bonds may carry positive inflation risk premium. We document a large degree of heterogeneity across our sample countries in the degree to which LC debt acts a hedge for the return on the market portfolio.

The risk profile of LC nominal bonds have significant implications on a country's sovereign debt portfolios and external default risk. The cross-sectional variation in bond betas are strongly correlated with the primary external sovereign default risk measure, the sovereign credit default swap (CDS) spreads. Countries with higher bond betas also have high CDS spreads, with the cross-sectional correlation at 91 percent. Furthermore, countries with negative or low bond betas almost exclusively borrow in local currencies, whereas countries with high bond betas rely more on FC financing.

We test the hypothesis that the risk differential can be explained by the cyclicity of monetary policy. Using professional forecast data, we compute the sensitivity of revisions to output forecasts with respect to revisions to inflation forecasts as a proxy for investors' ex-ante expectation of the cyclicity of inflation. We show countries with higher expected inflation procyclicality tend to have lower bond betas. In addition, we show that cross-sectional and time series variation in the perceived inflation procyclicality are highly correlated with the development of LC nominal bond markets. The expansion of nominal debt markets coincided over the past decade with a sharp reduction in the counter-cyclicality of inflation rates.

To better understand these observed empirical patterns, we develop a simple general equilibrium model where the sovereign decides whether to borrow in LC or FC debt. We show that when the government is unable to commit to repayment or to state-contingent inflation rules, countries with higher external financing needs tend to have lower fractions of debt financed in LC, higher default risk, higher inflation levels and inflation risk premia. In an extension with a decision between financing borrowing domestically or externally, we show that the option to borrow from

domestic markets can alleviate the time-consistency problem due to external borrowing and shift the currency composition of the external sovereign debt portfolio toward LC debt.

## 2.2 *Related Literature*

In addition to building on recent empirical work on LC domestic debt (Du and Schreger, 2013 and Burger, Warnock, and Warnock, 2012), our paper directly addresses the large literature on “Original Sin”. This large literature, beginning with Eichengreen and Hausmann (1999), points to the absence of international issuances of long-term fixed coupon LC debt as a missing market. Eichengreen and Hausmann (1999) discuss a number of reasons for the absence of this market, briefly discussing our favored explanation, saying “If a country was able to borrow abroad in its own currency, it would stand to benefit by depreciating that currency and thus eroding the real value of its external debts. In anticipation of this, foreigners are unwilling to lend in a denomination that the borrower can manipulate unless they are compensated to an extent that only those borrowers planning to devalue are prepared to pay.” We build on this line of thinking, arguing that the strength of the temptation to inflate away debt ex post will factor into a government’s optimal choice of the currency composition ex ante. By looking at this issue in general equilibrium we show that if a country’s external financing needs are large, a government might shift its currency composition of debt towards FC debt, eschewing LC debt at higher interest rates.

By collecting a new dataset on foreign holdings of domestic debt, our work fills in an important gap in the empirical research on the currency composition of sovereign debt, such as Eichengreen, Hausmann, and Panizza (2005a). Hausmann and Panizza (2011) also examine foreign participation in emerging market LC debt markets using surveys of US investors from 2003-2007. While these authors presented compelling evidence that emerging markets issued little to no debt abroad, Reinhart and Rogoff (2008, 2011) present evidence that domestic debt was always an important part of emerging market government finance. We view our present work as integrating these two lines of inquiry: while very little debt issued under foreign law and in international markets is in LC, as demonstrated in the Original Sin literature, we argue that foreign investment in LC bonds issued in domestic markets under domestic law takes the place of the “missing market” discussed in Eichengreen and Hausmann (1999).

Our paper builds on recent work by Campbell, Sunderam, and Viceira (2013) demonstrating that the covariance between US Treasury bond returns and stock returns has changed dramatically the past few decades. Campbell, Sunderam, and Viceira (2013) show that since the mid-1990s the covariance has become negative, meaning that US nominal bonds are now a hedge for the domestic equity market. We build on their work by examining the cross-country heterogeneity in bond/stock covariances in emerging markets and argue that this heterogeneity helps understand the different risk premia embedded in LC debt and the different currency composition of sovereign portfolios.

Our theoretical framework builds on the optimal debt management literature, in particular the time consistency argument presented in Bohn (1988, 1991). Bohn (1988) demonstrates that when taxation is costly, a welfare maximizing government will issue nominal debt to help smooth the tax burden over time. However, issuing nominal debt worsens the time inconsistency problem in monetary policy, as existing nominal debt can effectively be treated as an inelastic tax base. Bohn (1991) illustrates how this time inconsistency problem is worsened in the open economy, as in addition to tax-smoothing motives, if nominal debt is held by foreigners and the government is only concerned with domestic welfare, policymakers will be tempted to use surprise inflation to achieve a real wealth transfer from foreign lenders to domestic residents. Niemann, Pichler, and Sorger (Forthcoming) quantifies the time inconsistency induced by nominal debt in the closed economy and demonstrates how the time consistency problem in nominal debt can explain inflation persistence. Our paper builds on this line of research by examining the question in an open economy and introducing sovereign default. Whereas in Bohn's work, the government might find it optimal to overcome the time inconsistency problem by issuing real debt, these papers did not address the possibility that the government lacked the ability to commit to repaying this real debt. By introducing a government debt denomination choice along with sovereign default, our paper addresses how governments can use a mix of debt denomination to try to alleviate the time inconsistency problem inherent in both types of sovereign borrowing.

Our paper also contributes to the large literature on sovereign default, beginning with Eaton and Gersovitz (1981), and the quantitative models of sovereign default following Aguiar and Gopinath (2006). Our paper is most similar to Aguiar, Amador, Farhi, and Gopinath (2013), where the authors also consider defaultable nominal debt and how inflation commitment affects the

probability of outright default. Our paper differs from Aguiar, Amador, Farhi, and Gopinath (2013) in a few important ways: first, we examine strategic default rather than rollover crises, second, we introduce portfolio choice so that only an (endogenous) fraction of the debt is defaultable, and third, we introduce domestic creditors as an additional source of financing. Our paper is also similar to Arellano and Ramanarayanan (2012) by introducing government portfolio choice into a quantitative model of sovereign default. While these two papers consider short-term and long-term debt, and we consider the currency composition, both papers are focused on the optimal portfolio of defaultable sovereign debt to best hedge against shocks hitting the economy. Finally, our paper is closely related to Broner, Martin, and Ventura (2010) and Broner, Erce, Martin, and Ventura (2013) on the interaction between domestic and external creditors in the secondary markets.

### 2.3 *The End of “Original Sin”*

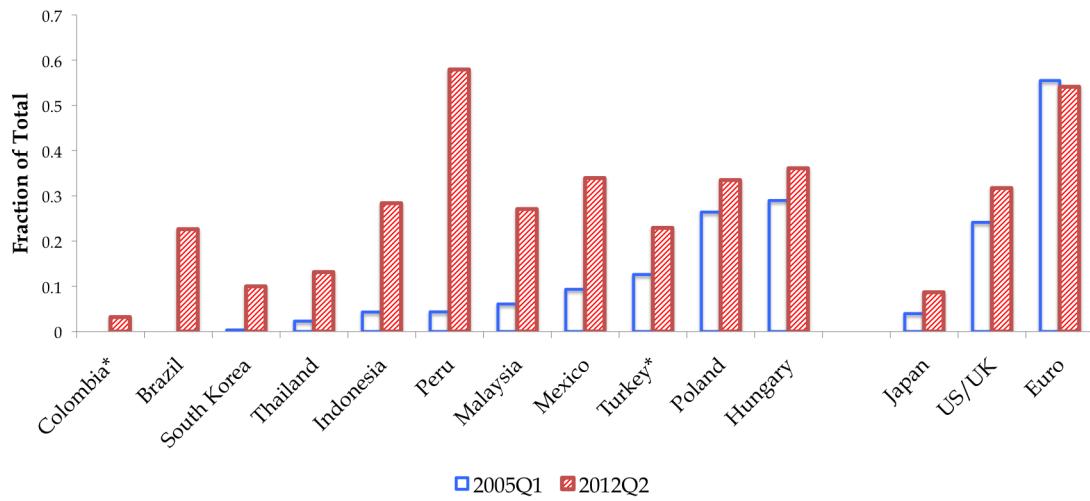
We construct a new dataset of ownership of domestic debt in 11 emerging markets from individual central banks, finance ministries, and the Asian Development Bank. Our data for Peru and Poland contain monthly data on the ownership structure at the individual bond level, with data for both countries coming from the respective Ministries of Finance. For Colombia and Brazil, our dataset contains monthly ownership data at the level of broader security classes (fixed coupon, inflation-indexed, floating rate, or foreign exchange linked debt). We have daily data at a similar level of disaggregation for Mexico from the central bank.<sup>3</sup> Data for Hungary and Turkey are also from the national central banks but are not available at similar levels of disaggregation. Ownership data for Indonesia, South Korea, Thailand, Japan, Malaysia comes from AsianBondsOnline, an online data source produced by the Asian Development Bank. For these countries only aggregate LC debt ownership at a quarterly frequency is available. Finally, for the US, UK and European countries, we use the dataset constructed by Merler and Pisani-Ferry (2012).

Contrary to the common belief that emerging markets cannot borrow in their own currency at fixed nominal rates, Figure 2.1 shows that the share of LC debt owned by foreigners increased

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<sup>3</sup> The Mexican and Polish data is available on the government websites. We digitized monthly reports (available as PDFs) online to construct our time series for Peru and Colombia. We received the Brazilian data from the central bank.

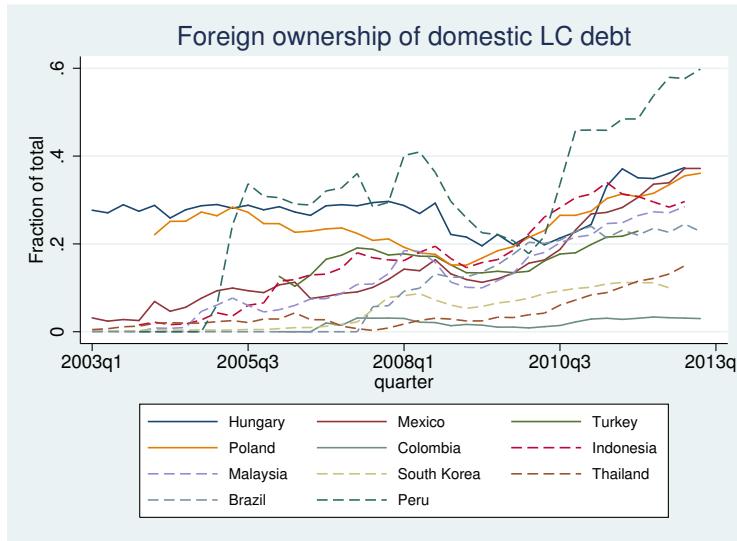
from 9 percent in 2005 to 23 percent in 2011 in the sample emerging markets, now higher than the share in Japan, comparable with the share in the US and the UK, albeit lower than the share in the euro area. Figure 2.2 gives the time series of foreign ownership of domestic LC debt during the past decade. Foreigners started buying into LC bond markets in the mid-2000s except for Hungary and Poland where foreign ownership took place earlier. During the peak of the Global Financial Crisis in 2008-09, foreigners significantly reduced their holdings. The trend rapid growth resumed shortly after the crisis peak.



**Figure 2.1: Foreign Ownership of Government Debt in Emerging and Developed Markets.** This figure displays shares of domestic government debt owned by foreigners in emerging markets and developed countries. Data sources are described in Section 2.3.

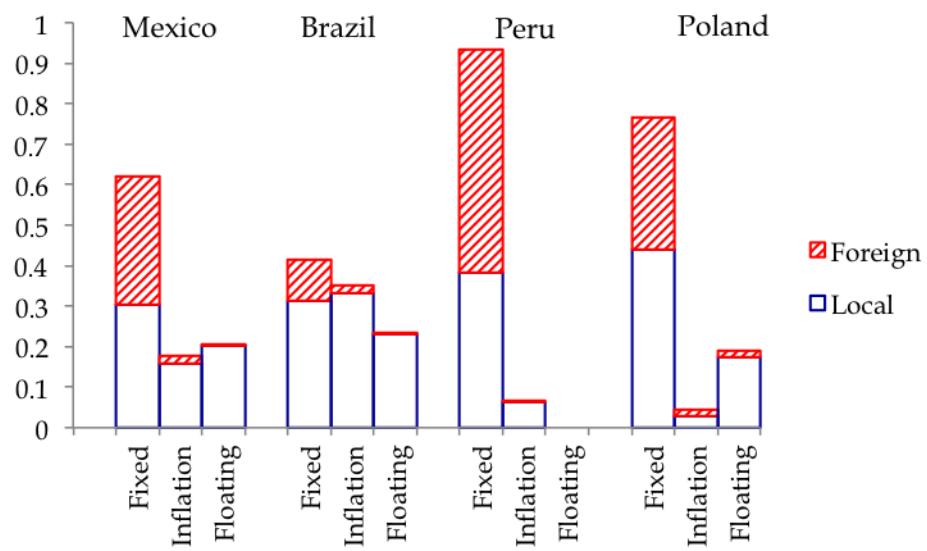
In addition, for countries with ownership data by security class (Mexico, Brazil, Peru and Poland), we show in Figure 2.3 that foreigners are particularly interested in holding nominal bonds, even though inflation-linked and floating-rate bonds are available. Peru, a country that experienced hyperinflation as recently as 1990, now finances more than 60 percent of its LC debt from foreigners in nominal fixed rate instruments. Figure 2.4 displays this remarkable change in Peru. The left panel plots the ownership structure of all LC bonds outstanding in February, 2004. The majority of LC fixed coupon bonds had under three years of remaining maturity and all were held locally. This picture has completely changed by December, 2012, where most of the debt has a remaining maturity of more than 10 years and the majority is now foreign-owned.

Putting these new stylized facts together, we can see that foreign investors have a growing appetite for nominal risk in emerging markets. LC debt is an increasingly important component

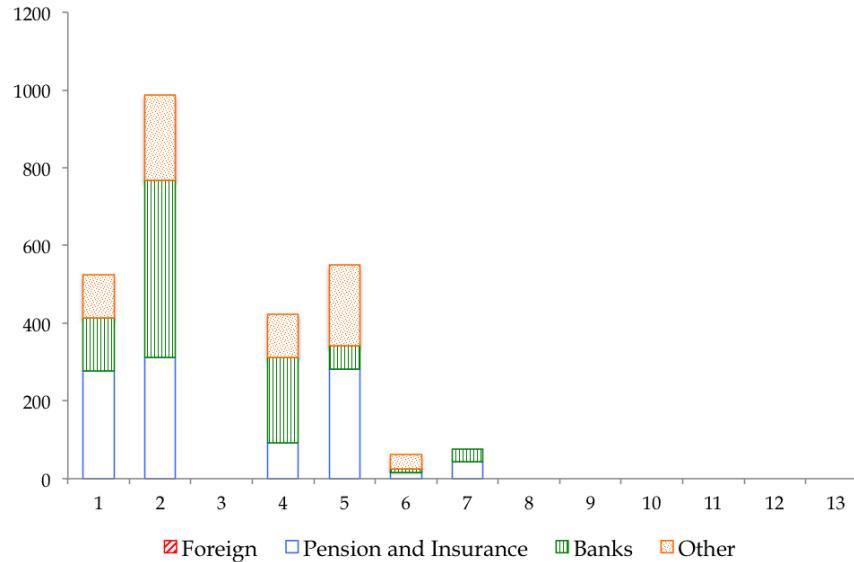


**Figure 2.2: Time Series of Foreign Ownership of Government Debt in Emerging Markets.** This figure plots time series of fraction of LC debt owned by foreigners for sample emerging markets. Data sources are described in Section 2.3.

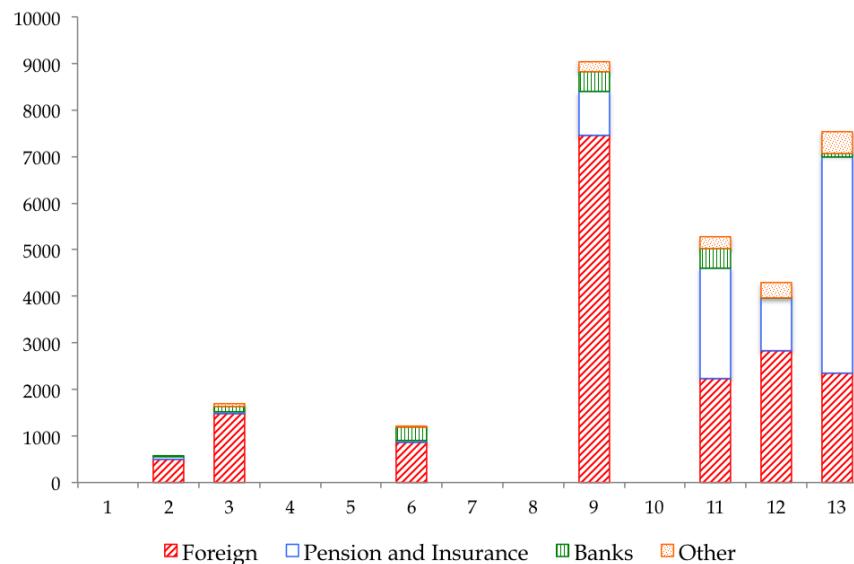
of external debt. While the “Original Sin” hypothesis seemingly holds in the very strict sense that most emerging market debt issued in international markets is still in FC, it is mostly likely due to sovereigns’ unwillingness rather than inability to tap international markets in LC. As discussed in Du and Schreger (2013), a few emerging markets, such as Brazil, Colombia and Philippines, have issued LC denominated bonds in the international market. These offshore bonds are traded at significantly lower yields compared with onshore bonds. From a foreign investor’s perspective, offshore LC bonds are safer assets because they are governed under international law and free from capital control risk. If foreign investors are already willing to take onshore nominal risk, it is difficult to argue why they would not want to hold offshore LC bonds.



*Figure 2.3: Ownership of Domestic Debt by Security Type in 2012.* This figure plots the share of domestic government debt by ownership and security type in the end of 2012. ‘Fixed’ denotes fixed or zero coupon nominal bonds; ‘Inflation’ denotes inflation-linked real bonds; and ‘Floating’ denotes floating bonds. Data sources are described in Section 2.3.



(a) April 2004



(b) December 2012

**Figure 2.4: Ownership Structure of Peruvian Nominal Bonds (millions of nuevo soles).** X-axis number of years of remaining on maturity on a given bond. “10” indicates bonds with 10-15 years remaining maturity, “15” years 15-20 years, and “20” refers to bonds with more than 20 years remiaining maturity. Vertical axis is millions of nuevo soles. The bars are sums of ownership across bonds within a given maturity bin. Data sources are described in Section 2.3.

## 2.4 Measuring Nominal Bond Risks

### 2.4.1 Definition of Nominal Bond Betas

We use Bloomberg fair value (BFV) curves to compute excess holding period return on the constant maturity 10-year bond yield in emerging markets from 2005 to 2012.<sup>4</sup> The BFV curves are estimated using individual LC sovereign bond prices traded in the secondary markets. Since sufficient numbers of bonds spanning different maturities are needed for yield curve estimation, the availability of the BFV curve is a good indicator for the overall development of LC nominal bond market. Countries such as Argentina, Uruguay and Venezuela only have a handful of fixed-rate bonds and hence do not have a BFV curve.

Given the log yield on a  $n$ -year bond traded at par  $y_{cnt} = \log(Y_{cnt})$ , the log holding period return on the bond is given by

$$r_{c,n,t+\Delta t}^b \approx D_{cn}y_{cnt} - (D_{cn} - \Delta t)y_{c,n-1,t+\Delta t},$$

where  $D_{cn} = \frac{1-(1+Y_{cnt})^{-n}}{1-(1+Y_{cnt})^{-1}}$  is the duration of the bond (Campbell, Lo, and Mackinlay, 1997). We approximate  $y_{c,n-\Delta t,t+\Delta t}$  by  $y_{c,n,t+\Delta t}$  for the quarterly holding period ( $\Delta t = 0.25$ ). We let  $y_{t1}$  denote the three-month T-bill yield and then the excess return on LC bonds over the short rate is given by

$$rx_{n,t+\Delta t}^b = r_{c,n,t+\Delta t}^b - y_{t1}.$$

From a dollar investor's perspective, we can rewrite excess return as

$$rx_{n,t+\Delta t}^b = [r_{c,n,t+\Delta t}^b - (y_{t1} - y_{t1}^*)] - y_{t1}^*.$$

The dollar investor can hedge away the currency risk of the holding period  $\Delta t$  by going long a U.S. T-bill and shorting a LC T-bill with the same market value as the LC bond. By doing so, any movement in the spot exchange rate of the LC has the same offsetting first-order impact on the bond position and the local T-bill position and hence cancels out. However, since the cash flow

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<sup>4</sup> Yield curves for Brazil and Israel are obtained from ANBIMA and the Central Bank of Israel, respectively. Since Turkey issued 10-year bonds only recently, we use 5-year as benchmark instead for Turkey.

is uncertain next period, a dollar investor may under or over hedge depending on the realization of the bond price and incur a second-order hedging error. Therefore, LC excess returns provide a first-order approximation of hedged dollar excess return of a LC bond over the U.S. T-bill. Since currency risk is only hedged away for the holding period, the dollar investor still bears duration risk of the LC bond.

Alternatively, a dollar investor can choose not to hedge the FX risk. The realized dollar unhedged excess return is equal to

$$rx_{n,t+\Delta t}^{b,\$} = r_{c,n,t+\Delta t} - (s_{t+\Delta t} - s_t) - y_{t1}^* = rx_{n,t+\Delta t} + [(y_{t1} - y_{t1}^*) - (s_{t+\Delta t} - s_t)],$$

where  $s_t$  denote the spot exchange rate of LC. The unhedged excess return on the LC bond is equal to equal to LC excess returns plus the realized return on the carry trade strategy of going long in LC T-bill funded by shorting a dollar T-bill.

We use three measures to compute nominal bond returns. First, we compute the local bond beta  $\beta_i^L$  for each country  $i$  by regressing LC bond excess return  $rx_{t+\Delta t}^{b,i}$  on local equity excess returns  $rx_{t+\Delta t}^{m,i}$ :

$$rx_{t+\Delta t}^{b,i} = \alpha_i^L + \beta_i^L rx_{t+\Delta t}^{m,i} + \epsilon_{it}^L.$$

Local betas measure risk exposure of local bond returns on local equity returns. Second, we compute the hedged global beta  $\beta_i^G$  for each country  $i$  by running LC bond excess returns on the S&P excess returns  $rx_{t+\Delta}^{m,SP}$ :

$$rx_{t+\Delta t}^{b,i} = \alpha_i^G + \beta_i^G rx_{t+\Delta t}^{m,SP} + \epsilon_{it}^G.$$

From a global investor's perspective, the global beta gives the first-order approximation of the risk loading of LC bond returns on global equity returns after hedging out currency risk for the holding period. Finally, if the dollar investor does not hedge the currency risk, in addition to the bond global beta  $\beta_i^G$ , the investor takes additional currency exposure  $\beta_i^{\$}$  such that

$$(y_{t1}^i - y_{t1}^*) - (s_{t+\Delta t}^i - s_t^i) = \alpha_i^{\$} + \beta_i^{\$} rx_{t+\Delta t}^{m,SP} + \epsilon_{it}^{\$}.$$

The total exposure of unhedged bond position for the global investor is equal to  $\beta_i^G + \beta_i^{\$}$ .

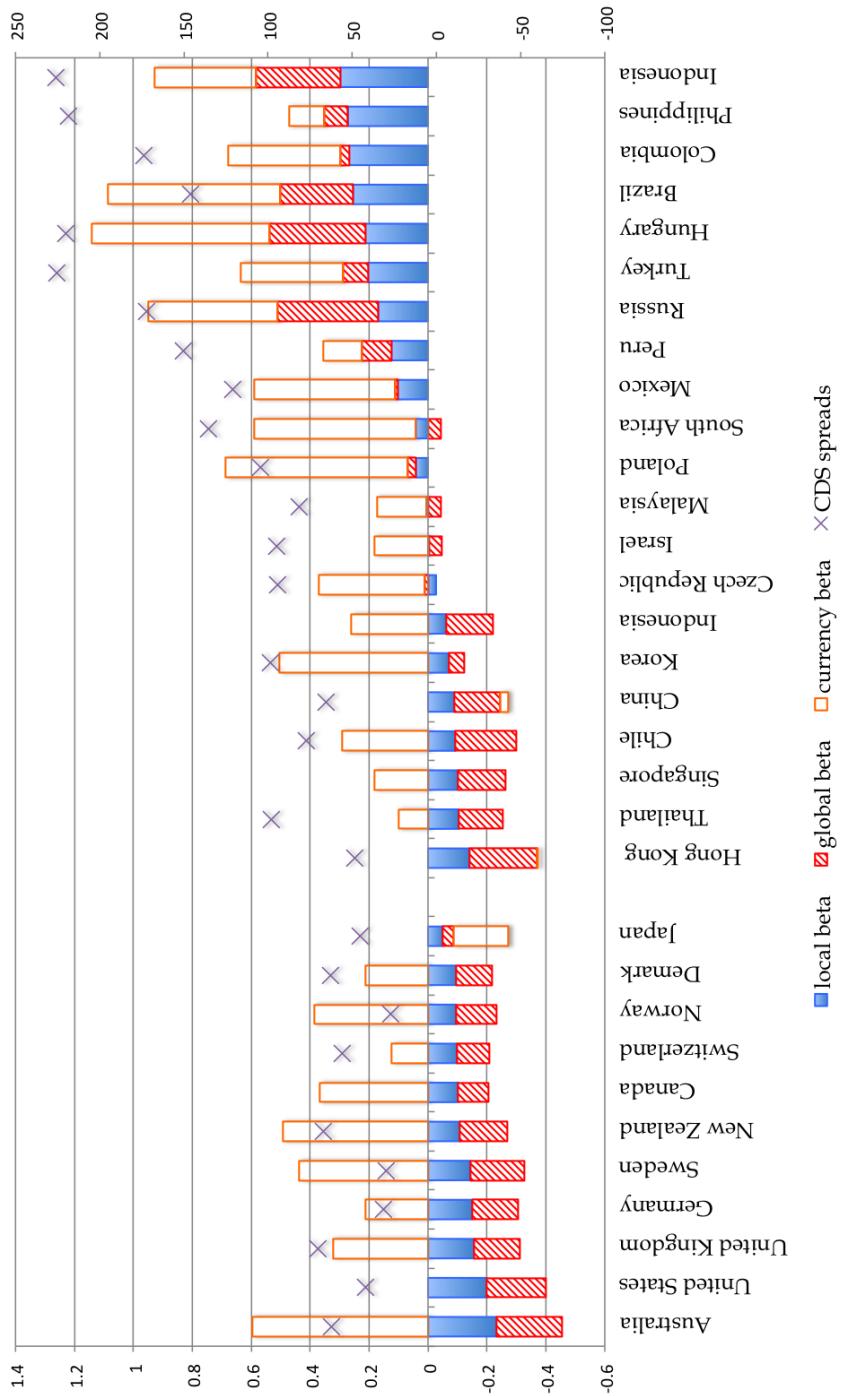
#### 2.4.2 Cross-country Variations in Nominal Bond Risk

Table 2.1 reports country-level correlation between LC bond excess returns and U.S. Treasury bond excess returns and between LC equity excess returns and S&P excess returns. Over the past eight years, equity excess returns in emerging markets are highly correlated with S&P excess returns with the mean correlation equal to 74 percent. However, LC bond excess returns are much less correlated with the U.S. Treasury bond excess returns with mean correlation only equal to 36 percent. Among the sample emerging markets, Hong Kong and Singapore have the highest bond return correlation with the U.S., whereas Russia and Hungary have the lowest. In the developed world, both equity and bond markets are highly correlated with the U.S. markets. The correlation between G10 and the U.S. is equal to 82 percent for government bond excess returns and 86 percent for equity excess returns. Compared with emerging market equities, emerging market LC debt brings more scope for diversification for global investors.

Figure 2.5 reports local, global and currency betas for developed and emerging markets. The order of the countries is sorted by their local betas. As we can see, all developed countries have negative local betas for the sample period. In other words, nominal government bonds offer a hedge for local equities. Among emerging markets, around half of the sample countries have negative local betas and the rest have positive local betas. Hong Kong, Thailand and Singapore have the lowest local betas whereas Colombia, Philippines and Indonesia have the highest local betas. Since equity markets are highly correlated across countries, the cross-sectional pattern of global betas follows the the pattern of local betas. The developed countries all have negative global and local betas of similar magnitude, which reflects high synchronization between equity excess returns. Among emerging markets, countries also have negative, zero or positive local and global betas simultaneously. In terms of the magnitude, among the high betas countries, Mexico, Turkey, Brazil and Colombia have significantly higher local betas than global betas. In these countries, nominal bonds carry more systematic risk for local investors than for hedged global investors. As for currency betas, all currencies except for the Yen, Hong Kong dollar and Chinese renminbi have positive betas with the global equities. For an unhedged dollar investor, after adding up the global and currency betas, unhedged LC bonds carry significantly positive betas for the majority of the countries with five exceptions: Switzerland, Japan, Hong Kong, Thailand, Singapore and China.

**Table 2.1: Equity and Bond Excess Return Correlation with U.S. Markets (2005-2012).** The column “Bond” reports correlation between LC bond excess returns over local T-bill rates and U.S. Treasury bond excess returns over U.S. T-bill rates. The column “Equity” computes LC equity excess returns over local T-bill rates and S&P excess returns over U.S. T-bill rates. Correlations are computed at daily frequency for the quarterly holding period.

Emerging Markets	Bond	Equity	G10 Currencies	Bond	Equity
Brazil	0.19	0.76	Australia	0.81	0.84
Chile	0.59	0.61	Canada	0.91	0.88
China	0.32	0.70	Switzerland	0.78	0.85
Colombia	0.22	0.54	Denmark	0.84	0.87
Czech Republic	0.49	0.76	Germany	0.86	0.89
Hong Kong	0.83	0.81	United Kingdom	0.87	0.93
Hungary	-0.01	0.85	Japan	0.72	0.76
Indonesia	0.14	0.73	Norway	0.77	0.85
Israel	0.47	0.64	New Zealand	0.75	0.74
India	0.44	0.71	Sweden	0.84	0.83
Korea	0.54	0.81			
Mexico	0.49	0.82			
Malaysia	0.50	0.71			
Peru	0.15	0.67			
Philippines	0.21	0.67			
Poland	0.37	0.85			
Russia	-0.28	0.79			
Singapore	0.78	0.87			
South Africa	0.47	0.74			
Thailand	0.48	0.78			
Turkey	0.13	0.72			
EM Mean	0.36	0.74	G10 Mean	0.82	0.86



*Figure 2.5: Local, Global and Currency Betas(2005-2012).* This figure plots LC bond betas in stacked columns. “Local beta” refers to the beta of LC bond excess returns on local equity excess returns; “global beta” refers to the beta of LC bond excess returns with respect S&P excess returns and “currency beta” refers to the beta of currency excess returns with respect to S&P excess returns. CDS spreads plot the mean of observed sovereign credit default swap spreads for the sample period. All betas are computed using daily data for the quarterly holding period. The order of the countries within developed and emerging market groups is sorted by the local beta. More details of definitions can be found in Section 2.4.1.

## 2.5 What Explains Nominal Bond Risk?

### 2.5.1 Bond Betas and Sovereign CDS Spreads

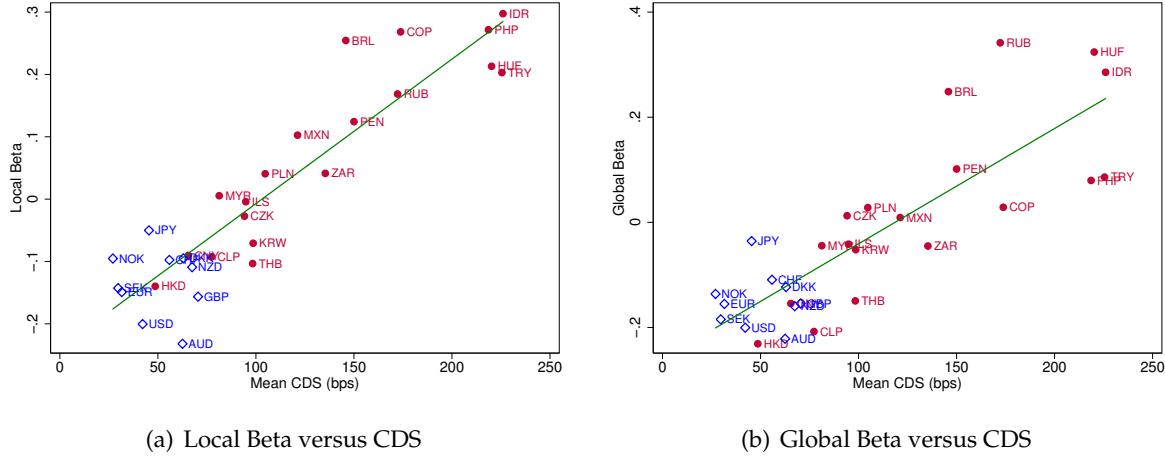
The cross-sectional heterogeneity in bond betas is highly correlated with sovereign CDS spreads. Sovereign CDS contracts offer insurance for investors in the event of sovereign default. For developed countries, CDS contracts insure against defaults on all Treasury bonds denominated in local currencies under domestic law. However, in emerging markets, CDS contracts are exclusively linked to external debt denominated in foreign currencies. Countries such as Singapore and India do not have any sovereign debt issued in the external mark and hence do not have CDS contracts. All sovereign CDS contracts are denominated in U.S. dollars and hence the CDS spreads offer an approximation for the shadow costs of issuing a U.S. dollar debt for different sovereign issuers.<sup>5</sup> Despite the large common component driven by global factors in CDS spreads across countries, the differential perceived sovereign default risk results in differential levels of CDS spreads.

We find that countries with high CDS spreads tend to have high nominal bond betas. This can be first seen as rising CDS spreads for countries sorted on their local betas in Figure 2.5. To visualize the relationship more directly, Figure 2.6 displays the scatterplots of local and global betas of nominal bonds against mean sovereign CDS spreads. The cross-sectional correlation is remarkably high at 92 percent between local betas and CDS spreads, and 83 percent between global betas and CDS spreads. Currency betas are only weakly correlated with CDS spreads with cross-sectional correlation equal to 31 percent.

Since returns on nominal bonds are largely driven by inflation and currency movements, one hypothesis to explain the correlation between bond betas and CDS is that countries with higher default risk tend to have higher inflation expectation during the downturns since the country is more tempted to inflate and default reduce the real debt burden. Whereas for countries with low default risk, inflation is procyclical and nominal bonds are hedge for equity returns.

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<sup>5</sup> US sovereign CDS contracts are denominated in euros,



**Figure 2.6: Bond Betas and Sovereign CDS Spreads.** Figure 2.6(a) plots bond local betas against sovereign CDS spreads. Figure 2.6(b) plots global betas of LC bonds against sovereign CDS spreads. Emerging markets are denoted by solid dots and G10 currency countries are denoted by diamonds. Betas are defined in Section 2.4.1.

### 2.5.2 Cyclicalities of Inflation Expectation

We hypothesize that the cross-country heterogeneity in LC bond betas might be a result of the differing cyclicalities of monetary policy across our sample countries. If investors believe that during bad times the country will inflate to reduce the real burden of debt, then they believe nominal bonds will pay off poorly exactly when market returns are low. If, on the other hand, deflation (or less inflation) is expected during bad times then LC bonds will act as a hedge (or a less risky asset) against market returns. We view this as a cross country application of the argument presented in Campbell, Sunderam, and Viceira (2013).

To measure the expected pro-cyclicalities of inflation expectations, we regress the change in the CPI inflation rate predicted by forecasters on the change in their predicted real GDP growth rate. In countries where investors believe that monetary policy will be used for fiscal purposes, we expect forecasters to revise their inflation expectations up when they revise their real GDP growth rate forecasts down.<sup>6</sup>

Each month, professional forecasters surveyed by Consensus Economics forecast inflation and GDP growth for the next calendar year. We use revisions of inflation and GDP forecasts each month relative to forecasts made three months ago to infer shocks to investors' expectation of

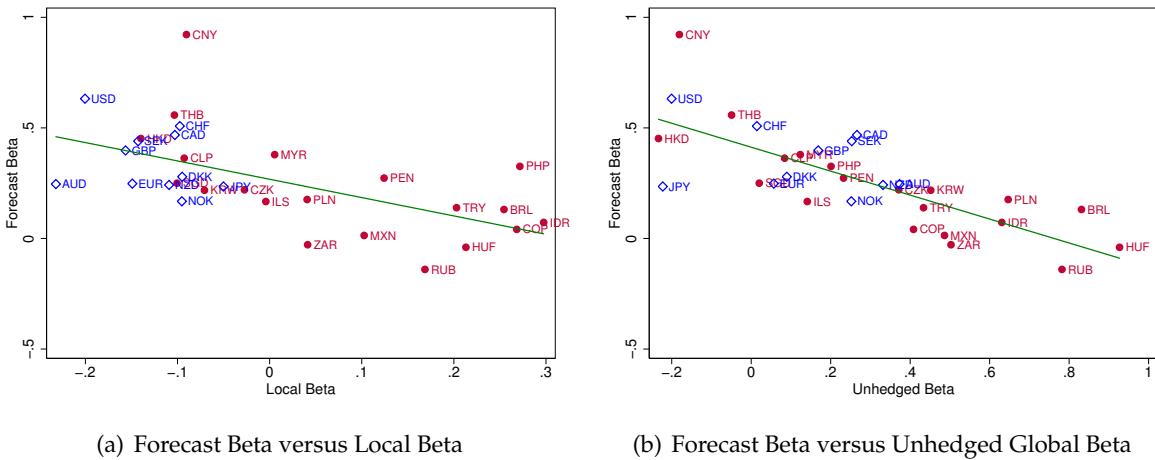
<sup>6</sup> We could do a similar exercise with consumption growth, but this would reduce the sample size.

inflation and output. We pool all revisions for 2006 through 2013 (so that the forecasts themselves were all made post-2005), and run the country by country regressions

$$\Delta \tilde{\pi}_t = \beta_0 + \beta_{\pi,y} \Delta \tilde{y}_t + \epsilon_t \quad (2.1)$$

where  $t$  indicates the date the revision is made. Country subscripts are suppressed to keep the notation more concise. The revisions to inflation forecasts ( $\Delta \tilde{\pi}_t$ ) and GDP growth forecasts ( $\Delta \tilde{y}_t$ ) are measured as percentage changes of forecasts made at  $t + h$  compared to forecasts made at  $t$ .<sup>7</sup> The coefficient  $\beta_{\pi,y}$  measures the cyclicality of inflation expectation and is the coefficient of interest. For now, we examine forecast revisions over 3 month periods.

As can be seen in Figure 2.7, at the three month horizon, we find that countries in which LC debt has a high beta with the local market tend to have a lower  $\beta_{\pi,y}$ , meaning that investors expect less procyclical inflation rates in countries with high bond betas. A similar pattern is observed at the one and six month horizon. The cross-sectional relationship between bond betas and forecast betas holds even better when we use unhedged global betas to measure nominal risk.



**Figure 2.7: Forecast Beta and Bond Beta.** Figure 2.7(a) plots forecast betas on bond local betas. Figure 2.7(b) plots forecast betas on bond global betas. Emerging markets are denoted by solid dots and G10 currency countries are denoted by diamonds. Betas are defined in Section 2.4.1 and Section 2.5.2.

<sup>7</sup> Forecasts are originally made in percentages. We rescale the variable  $x$  to  $\hat{x} \equiv \left(1 + \frac{x}{100}\right)$ . Revisions  $\tilde{x}$  are defined  $\tilde{x} \equiv \frac{\hat{x}_t - \hat{x}_{t-h}}{\hat{x}_{t-h}}$ . Forecasts are generally made monthly, but for some countries in the mid-90s were made in the mid-1990s.

## 2.6 *Implications of Nominal Risk on Sovereign Portfolios*

We next use this same measure of inflation cyclicality and examine whether it can explain the growth of LC bond markets and the cross country heterogeneity. To do so, we use two sources of aggregate data from the Bank for International Settlements (BIS). The first data source concerns the percentage of domestic debt that is fixed coupon, floating rate, inflation indexed or linked to the exchange rate. This data comes from the BIS Working Group questionnaire and is calculated using all types of domestic debt, not just central government debt. When calculating the fraction of government debt that is fixed coupon, we assume that the fraction of fixed government debt is equal to the fraction reported in the survey.<sup>8</sup> The second source of data is on the amount of domestic and external central government debt outstanding comes from the BIS Debt Securities Statistics. The classification of the data was recently overhauled in early 2013 but we continue to use the old classification.

In Table 2.2, we regress the fraction of all domestic debt that is in LC fixed rate on a version of  $\beta_{\pi,y}$ . In order to capture the time series component, instead of pooling across the whole period, we consider 3 month revisions across overlapping two year periods. The variable timing is such that only forecasts made in the two years prior to the realization of the dependent variable are used. Because we do not have a strong a priori view on whether this variable should be more successful in explaining the cross-section or time series, we run these regressions using a variety of combinations of year and country fixed effects. As can be seen in Table 1, the forecast revision beta has strong explanatory power for the cross section, with and without year dummy and remains significant at the 5% level with country FE and country and year FE. We then introduce two other variables that would be expected to explain the fraction of fixed coupon nominal debt: the level and volatility of inflation. For the level of inflation, we use the previous year's annual inflation and for volatility we use a 24 month rolling average of year-on-year inflation rates. While a higher inflation rate is associated with a lower level of fixed coupon debt, inflation volatility is not. Our forecast beta variable remains significant and positive, meaning that increases in the correlation

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<sup>8</sup> This is a potential source of bias in Table 2.3. We do not have strong evidence that the currency composition of domestic corporate and public debt are the same.

**Table 2.2: Fixed Coupon LC Debt as a Fraction of Domestic Debt.** Dependent variable is the share of fixed coupon LC debt as a percentage of total LC debt for the BIS Working Group annual survey. Inflation/GDP beta is the coefficient  $\beta_{\pi,y}$  in Equation 2.1. Annual Inflation Rate is the year-on-year inflation rate during the past year. Inflation Volatility (24m) is the standard deviation of the year-on-year inflation rate over the past 24 months.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Inflation/GDP beta	15.81*** (4.509)	18.32*** (4.779)	2.796** (1.312)	2.690** (1.114)	12.73*** (3.161)	12.77*** (3.179)	1.836** (0.793)
Annual Inflation Rate					-108.3*** (36.97)	-113.7*** (36.54)	-35.45* (19.35)
Inflation Volatility (24m)						110.8 (154.2)	160.8 (136.5)
Constant	66.92*** (6.474)	67.18*** (9.943)	20.16*** (0.912)	15.81*** (4.973)	74.55*** (13.91)	73.54*** (14.42)	13.66 (8.421)
Observations	301	301	301	301	247	247	247
R-squared	0.129	0.162	0.868	0.886	0.235	0.236	0.908
Year FE	No	Yes	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	Yes	No	No	Yes

between inflation and GDP forecast revisions are associated with higher fractions of debt in fixed coupon.

In Table 2.3, we examine whether our forecast revision beta can explain the fraction of LC fixed coupon debt as a fraction of total sovereign debt. We calculate our dependent variable by multiplying the stock of domestic debt from the BIS Debt Securities Statistics by the fraction of fixed coupon LC debt in domestic debt, and dividing by total outstanding sovereign debt. The results are largely similar to the results in Table 2.2, however the results are slightly weaker, perhaps because the survey data does not refer only to sovereign debt.

**Table 2.3: Fixed Coupon LC Debt as a Fraction of Total Sovereign Debt.** Dependent variable is the share of fixed coupon LC debt as a percentage of total outstanding sovereign debt. This variable is constructed by multiplying the fraction of fixed coupon debt as a fraction of domestic debt by the amount of domestic debt outstanding, and dividing by the entire stock of sovereign debt, domestic and external. Inflation/GDP beta is the coefficient  $\beta_{\pi,y}$  in Equation 2.1. Annual Inflation Rate is the year-on-year inflation rate during the past year. Inflation Volatility (24m) is the standard deviation of the year-on-year inflation rate over the past 24 months.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Inflation/GDP beta	11.87*** (3.881)	13.53*** (4.200)	2.555* (1.427)	1.950 (1.135)	9.121*** (3.056)	9.074*** (3.098)	1.267 (0.913)
Annual Inflation Rate					-96.01** (34.25)	-95.08** (33.09)	-21.95 (16.72)
Inflation Volatility (24m)						-4.246 (10.39)	21.79** (9.398)
Constant	49.66*** (6.705)	46.94*** (10.85)	17.33*** (0.992)	8.755 (6.361)	60.03*** (13.28)	60.45*** (13.97)	6.097 (10.29)
Observations	243	243	243	243	220	220	220
R-squared	0.102	0.135	0.837	0.866	0.204	0.204	0.894
Year FE	No	Yes	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	Yes	No	No	Yes

## 2.7 General Equilibrium Model of Sovereign Portfolio Choice

### 2.7.1 Theoretical Framework

In this section, we use a simple two-period general equilibrium model to better understand the observed empirical patterns that countries with higher default risk on FC debt also have higher LC nominal bond risk and lower fractions of nominal bonds in their portfolios. We begin by presenting a simple framework where a benevolent government issues debt to finance a project. The sovereign has the choice of issuing LC or FC debt. The government lacks commitment and optimal monetary and fiscal policies will not be time consistent in our model. LC debt is vulnerable to ex post inflation, and both FC and LC debt are vulnerable to ex post default. We solve the model by backwards induction. Once we solve for the government's policy functions in the second period, we can solve for the government's optimal policy in the first period. The government's decision in the first period involves choosing the quantity of bonds to issue in LC or FC.

We examine this problem in stages. First, we solve the second period policy functions when the government cannot default on its FC debt but it can inflate away its LC debt. Then we solve for the policy functions when both types of debt can be defaulted on but only LC debt can be inflated

away. With these policies functions solved in closed form, we then analyze the government's first period problem by focusing on the benchmark case that all borrowing is external.

We include two extensions of the baseline model that we see as important components of future research. First, we highlight the role of risk aversion in affecting the currency composition of sovereign debt. Second, we discuss an extension of the benchmark model to allow the government to finance domestically to alleviate the time inconsistency due to external borrowing.

### 2.7.2 Second Period Problem

#### No Default

Throughout, we assume a benevolent government that maximizes the utility of the representative agent:

$$\max_{D, \pi} \log(c) - \theta\pi, \quad (2.2)$$

where  $c$  is consumption in the final period,  $\pi \in [0, 1]$  is the inflation tax, the net inflation rate divided by one plus the net inflation rate.  $\theta$  is the utility cost of inflation.  $D$  is an indicator for default, taking the value 1 when the country defaults and 0 otherwise. The government maximizes utility subject to its budget constraint being satisfied. When the government cannot default on its debt, the budget constraint is given by

$$\tau y = (1 - \pi)b + b^S, \quad (2.3)$$

where  $y$  is output,  $\tau$  is the tax rate on output,  $b$  is LC debt and  $b^S$  is FC debt. Individual consumption is given by non-taxed output and domestic holdings of LC debt net of inflation:

$$c = (1 - \tau)y + (1 - \pi)b_h,$$

where  $b_h$  is domestic holdings of government debt. Defining foreign holdings of LC debt as  $b_f \equiv b - b_h$  and imposing that the government's budget constraint is satisfied, we can rewrite consumption of the representative agent as

$$c = y - [(1 - \pi)b_f - b^\$] \quad (2.4)$$

Because domestic holdings of LC debt are paid for with tax revenue, they have no impact on the optimal inflation rate chosen in the second period. Instead, the optimal inflation rate will be a function of output  $y$ , foreign holdings of LC debt  $b_f$ , and foreign holdings of FC debt,  $b^\$$ .

First, we consider the case without default. The government maximizes utility subject to the budget constraint. There will be three regions, when the inflation tax is equal to zero (as deflation provides no benefit), when the inflation tax is equal to 1 (hyperinflation) and when the inflation tax is in the intermediate range. We can characterize the optimal policy function as a piecewise function

$$\pi^* = \begin{cases} 1 & \text{if } y < \underline{y} \\ 1 - \frac{y - b^\$}{b_f} + \frac{1}{\theta} & \text{if } \underline{y} \leq y \leq \bar{y} \\ 0 & \text{if } y > \bar{y} \end{cases} \quad (2.5)$$

where

$$\bar{y} = b_f \left(1 + \frac{1}{\theta}\right) + b^$, \quad \underline{y} = \frac{b_f}{\theta} + b^$.$$

We can understand these cutoffs as follows: if output is above  $\bar{y}$ , the constant marginal cost of inflation  $\theta$  exceeds the marginal benefit of increased consumption that such inflation could achieve through reducing the real debt burden. At when  $y = \bar{y}$ , the marginal cost and benefit are equalized at an inflation rate of zero. Therefore, for all levels of output greater than  $\bar{y}$ , the marginal cost of inflation will be strictly greater than the benefit and so the optimal inflation rate is equal to zero. This threshold is increasing in both LC and FC debt held by foreigners, but it is increasing faster in holdings of LC debt. This is because the burden of LC debt can be directly inflated away, but inflation only eases the burden of repaying FC debt by reducing the *other* repayments that need to be made. In addition, the threshold is decreasing in the cost of inflation, as a higher cost of

inflation reduces the utility gain from inflation and thus enlarges the space where zero inflation is optimal.

Hyperinflation, on the other hand, is optimal when output is below  $\underline{y}$ . When output is below this level, the marginal benefit of additional consumption exceeds the marginal cost of inflation when the inflation tax rate is already at its maximum value,  $\pi = 1$ . This threshold is also increasing in both LC and FC debt.

For interior realizations of output in between  $\underline{y}$  and  $\bar{y}$ , optimal inflation equates the marginal utility cost of additional inflation ( $\theta$ ), to the marginal utility of consumption gained by reducing the real value of debt payments through inflation. Because the marginal cost of inflation is a constant given our linear cost function, in this interior solution, the representative agent receives a constant consumption, and utility gains from additional output come through reduced inflation.

#### *Defaultable Debt*

Next, we add sovereign default to the framework above. Following Arellano (2008), we adopt the non-linear default cost function

$$y^d = \begin{cases} y & \text{if } y < \hat{y} \\ \hat{y} & \text{if } y \geq \hat{y} \end{cases} .$$

This cost function imposes that if a country defaults, all output above a threshold  $\hat{y}$  is seized if output is initially above this threshold, and no output is lost if the country defaults when output is below this given threshold. Under this default cost specification, the country would find it optimal to choose full default on LC and FC debt if default is ever chosen. In addition, the country will also choose zero inflation upon default. This feature of the model can be relaxed if we allow partial default or specify a differential default technology with respect to LC and FC debt to induce selective default. The only distinction between LC and FC debt is that in non-default states, the country can choose inflation to erode the real value of LC but not FC debt. The linear inflation cost and the Arellano type default technology together yield closed-form policy functions.

The problem of determining the default regions is one of simply checking whether the utility the representative agent receives when defaulting,  $\log(y^d)$ , exceeds the utility she would be re-

ceive if the country did not default and followed the optimal inflation policy in Equation 2.5. This, in turn, depends on the relative cost of inflation and default, as well as the level and currency composition of the outstanding debt. Depending on the level of external debt in LC, we summarize default and inflation policy functions into the following three cases:

1. If  $b_f < \theta\hat{y}$ ,

$$D^* = \begin{cases} 1 & \text{if } y < \tilde{y}_1 \\ 0 & \text{if } y \geq \tilde{y}_1 \end{cases}$$

$$\pi^* = 0.$$

2. If  $\theta\hat{y} \leq b_f \leq \theta \exp(\theta)\hat{y}$ ,

$$D^* = \begin{cases} 1 & \text{if } y < \tilde{y}_2 \\ 0 & \text{if } y \geq \tilde{y}_2 \end{cases}$$

$$\pi^* = \begin{cases} 0 & \text{if } y < \tilde{y}_2 \\ 1 - \frac{y - b^*}{b_f} + \frac{1}{\theta} & \text{if } \tilde{y}_2 \leq y \leq \bar{y} \\ 0 & \text{if } y > \bar{y} \end{cases}.$$

3. If  $b_f > \theta \exp(\theta)\hat{y}$ ,

$$D^* = \begin{cases} 1 & \text{if } y < \tilde{y}_1 \\ 0 & \text{if } y \geq \tilde{y}_1 \end{cases}$$

$$\pi^* = \begin{cases} 0 & \text{if } y < \tilde{y}_3 \\ 1 & \text{if } \tilde{y}_3 \leq y < \underline{y} \\ 1 - \frac{y - b^*}{b_f} + \frac{1}{\theta} & \text{if } \underline{y} \leq y \leq \bar{y} \\ 0 & \text{if } y > \bar{y} \end{cases}.$$

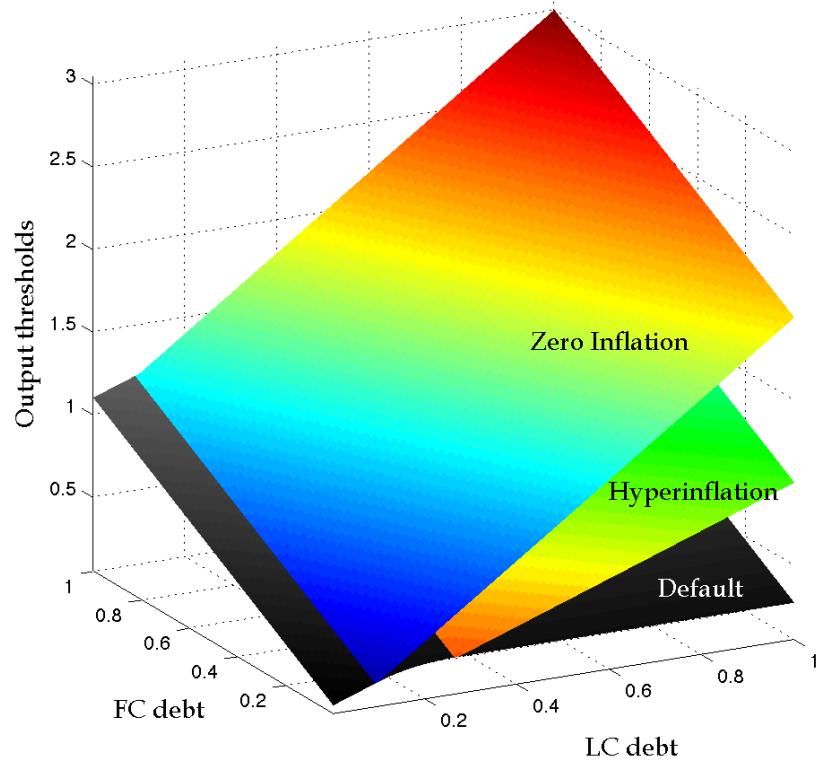
where the three default thresholds are given by

$$\begin{aligned}\tilde{y}_1 &\equiv \exp(\theta)\hat{y} + b^{\$} \\ \tilde{y}_2 &\equiv \frac{b_f}{\theta} \log\left(\frac{\theta\hat{y}}{b_f}\right) + b_f + \frac{b_f}{\theta} + b^{\$} \\ \tilde{y}_3 &\equiv \hat{y} + b_f + b^{\$}\end{aligned}$$

In the first case, because the country has a very low LC debt stock, the marginal benefit of inflating debt away is low, and so there are no output realizations for which the country prefers to inflate fully instead of explicitly default. LC and FC debt become perfect substitutes since the government never uses the option to inflate away LC debt in equilibrium. In the second case, when the country has an intermediate level of LC debt and output is above the default threshold  $\tilde{y}_2$ , the country would choose zero or positive inflation, but never hyperinflation. In the third case, the county has high levels of LC debt. When output is above the default threshold  $\tilde{y}_3$ , there is a full range of possibility for optimal inflation, ranging from zero to hyperinflation depending on the output realization and portfolio composition.

Figure 2.8 graphically summarizes government policy in the final period. The graph includes three surfaces in the LC debt level on the x-axis, FC debt on the y-axis and Output on the z-axis. The three surfaces, labeled “Zero Inflation”, “Hyperinflation” and “Default”, plot the important output thresholds. If output is above the Zero Inflation surface, then there will be no default or inflation for a given amount of LC and FC debt. As would be expected, this surface is increasing in both FC and LC debt, meaning that the more LC and FC debt a country has, the higher the output realization needed to make it optimal for the country to choose not to inflate or default. The second surface is the Hyperinflation surface. If output is below above this surface, and below the Zero Inflation surface for a given distribution of external LC and FC debt, then the country will choose an intermediate level of inflation to lower the real burden of debt repayment. The preferred level of inflation is decreasing in output, so one can think of there being a continuum of surfaces for inflation rates equal to a constant rate between the Zero Inflation and Hyperinflation surfaces. Finally, the third and bottom surface is the Default surface. For given LC and FC debt stocks, if output is between the Hyperinflation and Default surfaces, the country will hyperinflate

away its LC debt, but still pays back FC debt. If output is below the threshold defined by the Default surface, than the country will default on LC and FC debt.



**Figure 2.8: Inflation and Default Policy Functions.** This figure plots optimal inflation and default thresholds for output given any debt portfolio. Details are given in Section 2.7.2.

### 2.7.3 First Period Problem: External Debt Only

With the final period policy functions fully characterized, we can now examine the first period problem. We will begin by looking the simplest case, where a government has to finance a fixed and exogenous amount  $z$  by borrowing from foreign lenders and only has to choose the currency composition of the debt. The government internalizes the fact that it will re-optimize in the second period according to the policy functions solved for above. Therefore, the general problem of the government can then be written as

$$\begin{aligned}
& \max_{b_f, b^\$} E[\log(c_1) - \theta\pi_1] \\
s.t. z & \leq qb_f + q\$b\$ \\
\pi_1 & = \pi_1(y_1, b_f, b\$) \\
D_1 & = D_1(y_1, b_f, b\$) \\
q\$ & = E[M_1^*(1 - D_1)] \\
q & = E[M_1^*(1 - D_1)(1 - \pi_1)]
\end{aligned}$$

where  $M_1^*$  gives the stochastic discount factor of foreign lenders and  $\pi_1(\cdot)$  and  $D_1(\cdot)$  denote the ex-post policy functions.

#### *Solution to Optimal External Portfolio*

We first solve this problem under the assumption of lender risk neutrality ( $M_1^* = 1$ ), and a log-normal distribution for output

$$y_1 = \exp(\epsilon_1^y), \epsilon^y \sim \mathcal{N}(\mu, \sigma^2).$$

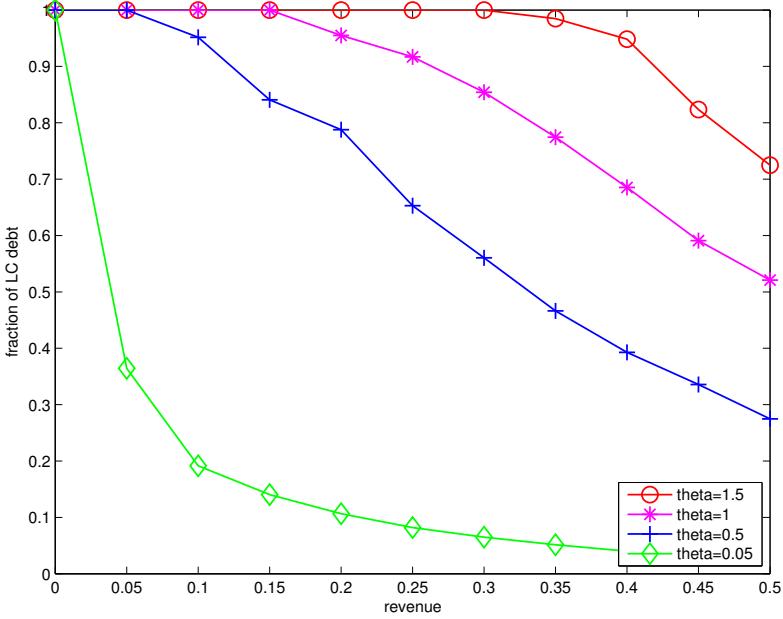
Because the optimal choice of LC and FC and debt will generally not have a closed form solution, we solve the government's portfolio choice problem numerically for a range of external financing needs,  $z$ . It is worth noting that the lack of commitment to repay the debt gives rise to a natural borrowing limit. Once LC and FC reach the threshold, higher debt levels reduce revenue raised due to increasing inflation and default risk.

Because the model is stylized, we try to avoid taking a strong stand on the relative costs of inflation and default and generally report our solutions for a wide range of inflation costs  $\theta$ . Our first result can be found in Figure 2.9, where we show that the optimal fraction of LC debt is decreasing in the amount that needs to be financed.<sup>9</sup> The intuition for this result is that LC debt is valuable for its state contingency but it is costly because the temptation to repudiate the debt

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<sup>9</sup> When the country is indifferent between LC and FC debt at very low levels of LC debt (Case 1 in Section 2.7.2), we assume that it chooses LC debt.

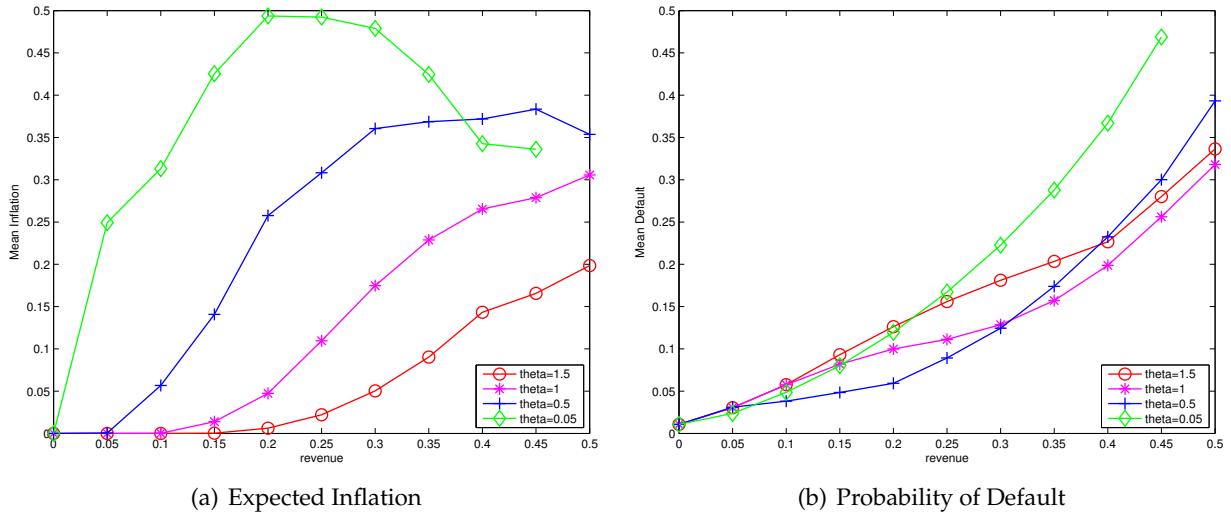
in every state is factored into its price ex ante. Therefore, a higher face value of debt needs to be issued if lenders expect the debt to always be repudiated. Because inflation is costly, a government might be able to benefit from the limited commitment available from FC debt. What we see in the figure is that as more debt needs to be raised, the time inconsistency problem is worsened, and so the cost begins to outweigh the benefits from state contingency, and so the government chooses to issue more FC debt.



**Figure 2.9: Share of LC Debt vs. Amount of Revenue Raised.** This figure plots the share of LC debt chosen in the equilibrium portfolios against the total revenue raise for different values of inflation costs  $\theta$ . Details are given in Section 2.7.3.

In Figure 2.10(a), we see that the government's optimal portfolio choice interacts in interesting ways with the optimal inflation rate. While throughout much of the parameter space expected inflation is increasing for countries with lower inflation credibility  $\theta$ , we see that for countries with very low inflation costs and high borrowing needs, mean inflation could actually be lower than for some countries with higher inflation costs. This is because an optimizing government with low credibility might respond to their lack of commitment by issuing so much more FC debt that it becomes less likely inflate away its debt. However, throughout much of the parameter space, we have the more intuitive result that expected inflation is increasing in the amount of revenue that needs to be raised as a larger debt burden makes it more tempting to inflate for any level of output.

Figure 2.10(b) plots the mean default rate for the four different levels of inflation cost for given amounts of borrowing. It is important to remember that at for every amount of revenue raised the currency composition of the debt stock differs for the different levels of inflation cost. In this figure, we see that the default probability is increasing in the amount of revenue raised, but we do not have a consistent ordering of default probabilities for a given amount of external financing raised. There are several forces balancing against each other. On one hand, countries with lower inflation costs are more inclined to choose inflation rather than default to reduce their LC debt burden for the same portfolio. On the other hand, countries with lower inflation costs also choose a portfolio with more FC debt in the equilibrium, which makes default more likely.



**Figure 2.10: Expected Inflation and Default for Equilibrium Portfolios.** The two figure plotted expected inflation and expected default at equilibrium portfolios against different revenue levels for different values of inflation costs  $\theta$ . Details are given in Section 2.7.3.

### The Role of Risk Aversion

Under this simple framework, countries with higher external financing needs have higher levels of expected inflation and default rates. In addition, the model can match the fact presented in Figure 2.6 that countries with higher external default risk also have higher LC bond betas. For each equilibrium portfolio at different debt levels, Figure 2.11(a) plots the LC bond beta, inflation beta and default beta, measured as the betas of regressing LC bond payoffs, realized inflation and default on output realization, respectively. The default beta is negative and monotonically

decreasing with respect to external financing needs, as the country is more likely to default when debt burden is high. The inflation beta is also negative and generally decreasing in debt burden. When debt burden is low, the country has low inflation betas in absolute values, despite the fact that a large fraction of debt is denominated in LC. As the debt burden increases, the degree of counter-cyclical inflation depends on two opposing forces. First, high marginal utility of consumption makes the country chooses high inflation. Second, the country optimally chooses a lower fraction of LC debt at high level, which decreases the marginal gain of inflation in reducing the real debt burden and makes outright default more attractive. As a result, we see that at very high debt level, the inflation beta can shrink with the level of debt. The LC bond beta incorporates both inflation risk and default risk, and is monotonically increasing in the level of debt.

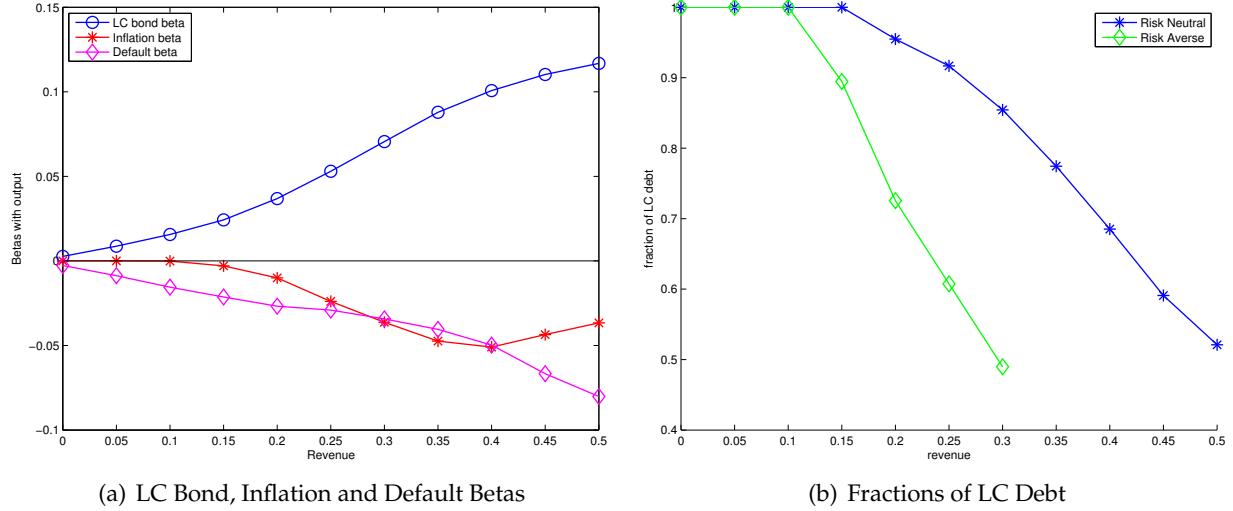
These betas have important effects on the currency composition of the sovereign portfolios when risk-averse lenders face systematic default or inflation risk. In Figure 2.11(b), we consider a simple risk-averse pricing kernel

$$M^* = \exp(-\gamma\epsilon_t + \gamma^2\sigma^2/2),$$

where  $\epsilon_t$  is the same  $\mathcal{N}(\mu, \sigma^2)$  innovation to output. After introducing a slight amount of risk aversion:  $\gamma = 0.5$ , the country becomes much more debt intolerant. As they are now charged a risk premium in equilibrium, they are able to raise less revenue for a given face value of debt. In addition, for an amount of revenue raised, the risk premium significantly reduces the fraction of LC debt at any feasible debt level. This is because the additional state-contingency of LC bonds becomes more costly as lenders charge the country an inflation risk premium in addition to the default risk premium.

### *Domestic and External Borrowing*

Finally, we consider the case where the government can choose to finance  $z$  domestically but doing so crowds out domestic investment. To illustrate the key intuition, we impose a simple assumption that the government can finance  $b_h$  from domestic residents and put of the rest of the endowment in investment. By doing so, we have abstracted from solving the domestic investors' portfolio problem between holding LC bonds and capital, which is an important component of



**Figure 2.11: The Role of Risk Premium.** Figure 2.11(a) plots bond the LC bond beta, the inflation beta and default beta with respect to output fluctuations for different debt levels. Figure 2.11(b) shows the effect of lenders risk aversion in reducing the fraction of LC debt in the sovereign portfolio. Details are given in Section 2.7.3.

continuing work. The government prefers to borrow from domestic residents because they suffer a lesser time inconsistency problem than borrowing from foreigners. However, borrowing domestically comes at a cost, as residents are left with less resources to invest in capital, lowering future output. While it may seem unnatural to allow foreigners to invest in domestic debt but not equity, this assumption could be motivated by Gertler and Rogoff (1990) type frictions, where capital investment is subject to moral hazard frictions. Under this setup, the government problem becomes

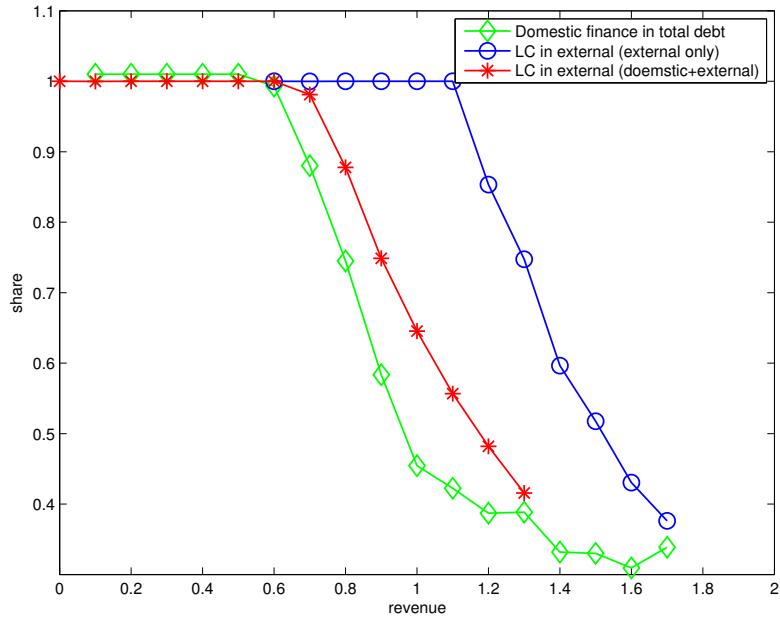
$$\max_{b_h, b_f, b^\$} E [u(c_1) - \theta \pi_1]$$

such that

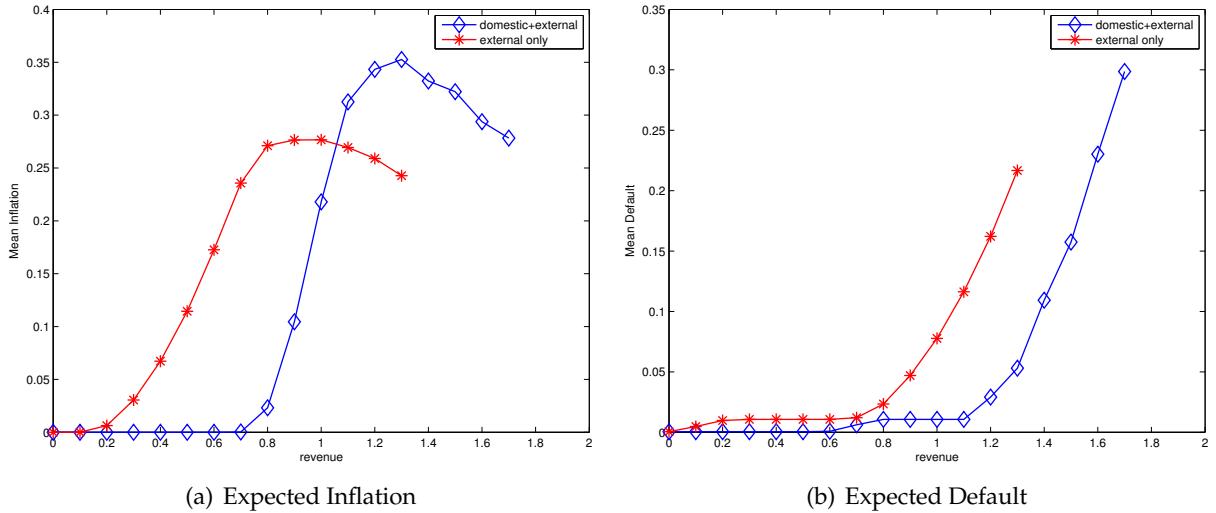
$$c_1 = \begin{cases} Ak^\alpha - [(1 - \pi_1)b_f - b^\$]. & if \quad D_1 = 0 \\ y_1^d & if \quad D_1 = 1 \end{cases}$$

$$\begin{aligned}
y_0 &= b_h + k \\
z &\leq qb_f + q^{\$}b^{\$} + b_h \\
q^{\$} &= E[M_1^*(1 - D_1^*)] \\
q &= E[M_1^*(1 - D_1^*)(1 - \pi_1^*)]
\end{aligned}$$

For now, we once again assume risk-neutral foreigners. We let  $A \sim \mathcal{LN}(1, 1)$  and  $\alpha = 1/3$ , so that investment in capital is sufficiently attractive given the endowment  $y_0 = 1$ . Under the first best with commitment, the country invests all the endowment in capital and finances the entire debt externally. However, external financing comes with a time consistency problem. When the country decides how much to borrow from domestic residents, it has to weigh the cost of reducing investment against the gain of alleviating the time inconsistency problem. As shown in the curve marked by the diamond sign in Figure 2.12, when the debt level is low, the government finances all the debt domestically. As the total financing needs increase, the return on capital becomes sufficiently attractive if too little endowment is left for investment after borrowing domestically and thus the country finds it optimal to start borrowing externally despite the cost associated with time inconsistency. The option to finance some borrowing domestically shifts the currency composition of the sovereign's external portfolio toward LC debt in equilibrium. Furthermore, as shown in Figure 2.13, the mean default rate is lower at any debt level if the country has the option to finance domestically even at the cost of output loss. The mean inflation rate is also lower with domestic financing at low levels of debt. However, at high levels of total debt, countries with a domestic financing option endogenously choose higher fractions of LC debt in the external portfolio and can potentially choose to run higher inflation.



**Figure 2.12: Share of LC External Financing with Endogenous Output.** The green line with diamonds plots the share of domestic financing in total debt. The red line with asterisks plots the share of LC debt in the external portfolio if all the debt is financed externally. The blue line with circles plots the share of LC debt in the external portfolio, while the total debt is partially financed domestically. Details can be found in Section 2.7.3.



**Figure 2.13: Inflation and Default with Domestic Financing.** The two figure plot expected inflation and expected default at equilibrium portfolios against different debt levels. The red line with asterisks refers to the case that all debt is financed externally. The blue line with diamonds refers to the case the case that fractions of the debt can be financed domestically. The Details are given in Section 2.7.3.

## 2.8 Conclusion

LC debt now represents a large fraction of emerging markets' sovereign external debt. We document empirically that the nominal risk of LC bonds is strongly correlated with default risk on FC external debt. Countries more likely to default on external debt also have significantly larger LC bond betas with local and global equity returns. Using forecasting data , we provide empirical evidence that the cross-sectional variations in nominal bond risk are highly correlated with perceived cyclicity of inflation and currency risk. Variations in LC nominal risk help explain the observed patterns of currency composition of sovereign debt portfolios. Countries with higher inflation risk and inflation risk premia tend to have lower fractions of LC nominal debt in their total sovereign debt portfolio.

To explain the sources of these empirical findings, we propose a simple general equilibrium framework with the endogenous portfolio choice between LC and FC debt. In the absence of commitment to debt repayment or state-contingent inflation rules, the equilibrium portfolio reflects the tradeoff between additional state contingency offered by LC debt and better inflation commitment induced by FC debt. We show that countries with higher external financing needs endogenously choose lower fractions of LC debt in equilibrium, and are associated with higher levels of default and inflation risk. The option to finance domestically can help alleviate the time inconsistency problem due to external borrowing and is an important component of ongoing research.

### 3. NONPARAMETRIC HAC ESTIMATION FOR TIME SERIES DATA WITH MISSING OBSERVATIONS<sup>1</sup>

#### 3.1 *Introduction*

While use of the Newey and West (1987) estimator and its Andrews (1991) implementation have become standard practice for heteroskedasticity and autocorrelation (HAC) robust inference, analogous methods for series with missing observations are far from standardized. When data are missing, the Newey-West formulas do not immediately apply, and the formula for calculating the lagged autocorrelations that are required terms in conventional HAC formulas must be adjusted.

Current practices for working with missing data include treating the missing observations as non-serially correlated, or imputing or ignoring the missing observations. To our knowledge, there has not been formal justification of HAC estimation for robust inference in these contexts, and the effect of employing these work-around methods on the resulting inferences is generally unexplored in applied work. In this paper, we provide formal justification for two methods of HAC estimation, and we compare these two methods to other existing methods. We demonstrate that treating the data as equally spaced is preferred under very general conditions, and that treating the missing observations as non-serially correlated may be preferred in instances with small sample sizes or low autocorrelation. In general, we find that our two newly formalized methods are preferred to imputation.

Especially when our aim is to adjust inferences for serial correlation, it seems counterintuitive that we can either treat the data as equally spaced or treat the missing observations as non-serially correlated, since these procedures require us to depart from the original time structure or autocorrelation structure of the data. However, we show that these procedures both provide consistent estimators of the long-run variance of the observed series with missing data. Though many have

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suggested that we infer the spectrum of the underlying data from the observed data using the *Parzen estimator*, we show that the Parzen estimator is not the correct object for inference testing. Rather, we show that our *Amplitude Modulated estimator* (which treats the missing as non-serially correlated) and *Equal Spacing estimator* (which treats the observed as equally spaced) are extremely simple to implement and can be used to generate asymptotically valid inferences.

These insights are particularly valuable given the *ad hoc* approaches widely found in the applied literature. For example, researchers often use imputation procedures to fill in the missing data. Imputation seems to expand the set of observations used for analysis, or at least prevents us from dropping data when some covariates are unobserved. However, because imputed data series are often smoothed versions of the underlying unobserved series, they will often lead to asymptotically valid but extremely poor finite sample performance. Furthermore, researchers using these methods rarely adjust their inferences for this induced serial correlation.

Additional evidence of the confusion in the literature stems from the implementation of the Newey-West HAC estimator in the popular statistical package, Stata. The *newey2* command implements Newey-West to obtain the standard error of the coefficient in a regression using time series or panel data. When observations are missing, the option “force” can be applied. This option will apply our Equal Spacing estimator to time series data and apply our Amplitude Modulated estimator to panel data. However, it should be possible to implement either the Equal Spacing estimator or the Amplitude Modulated estimator for time series data. Furthermore, the program will not apply the Amplitude Modulated estimator at all when some lags are unobserved. This condition is likely an artifact of the literature on estimating the long-run variance of the underlying series, which develops estimators that generally require all lags to be observed (Clinger and Van Ness, 1976; Parzen, 1963). Yet for robust inference, we show that the Amplitude Modulated and Equal Spacing estimators do not require all the lags to be observed.

A primary goal of this paper is to help researchers select the correct estimation procedure in applied work. To that end, we follow the style of Petersen (2009), which provides guidance for selecting standard errors in finance panel data sets. We formally present the Amplitude Modulated and Equal Spacing estimators of the long-run variance of the observed series, and we review their asymptotic properties. We contrast these estimators with the Parzen estimator of the long-run variance of the underlying series. After presenting these theoretical contributions, we offer intu-

ition for why the estimators work and how they are related to each other. To generate guidance for choosing the correct estimator, we conduct Monte Carlo simulation results for various sample sizes, correlation structures, and fractions of observations that are missing. In addition to testing our estimators using randomly missing data, we demonstrate the applicability of these estimators to a deterministic cyclical missing structure, as with daily financial data, which usually cover 5 of 7 days of the week. Finally, we discuss an empirical application using recursive regressions for commodities futures returns to demonstrate how the choice of estimator can affect the conclusion of empirical tests.

As a preview, our results demonstrate that the Amplitude Modulated and Equal Spacing estimators are both consistent under random and deterministic missing structures. In finite samples, we find that for the same fixed bandwidth, the Equal Spacing estimator is generally less biased than the Amplitude Modulated estimator, but has larger variances. Consequently, the Equal Spacing estimator is preferred when autocorrelation is high, as the bias will dominate the mean squared error in these cases. Conversely, when autocorrelation is low, variance dominates the mean squared error, and the Amplitude Modulated estimator is preferred. The precise cut-off between these cases depends on the sample size, and whether automatic bandwidth selection procedures are implemented.<sup>2</sup>

The remainder of the paper is structured as follows. Section 3.1.1 discusses some examples of missing data problems for which imputation and equal spacing methods have been applied, and provides a brief review of some of the related econometrics literature. Section 3.2 provides an overview of our estimators by applying them in a simple setting with missing observations. Section 3.3 formally defines the estimators and discusses their asymptotic and finite sample properties. Section 3.4 presents the application of these estimators to inference in a regression setting with missing observations. Section 3.5 describes the Monte Carlo simulations based on these estimators and the results. Section 3.6 presents an empirical application of the estimators using data on commodities returns. Finally, Section 3.7 concludes.

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<sup>2</sup> To make it easier for researchers to apply these estimators, we have posted Matlab code for both estimators on our websites. We also have posted a basic simulation code that reports empirical rejection rates, size-adjusted power, bias, and variance for the Equal Spacing, Amplitude Modulated, Imputation, and (full sample) Newey-West estimators. Researchers can use the simulation code to evaluate the performance of the estimators under customized sample size, autocorrelation, and missing structure before choosing which estimator to implement.

### 3.1.1 Relation to the Literature

This paper extends the HAC covariance literature to applications with missing observations. In general, the most commonly used HAC covariance matrix is the one proposed by Newey and West (1987) and further developed by Andrews (1991). The Newey-West estimator equals a weighted sum of lagged autocovariance matrices, in which the weights are calculated using the Bartlett kernel. Newey and West (1987) show that this estimator is positive semi-definite and heteroskedasticity and autocorrelation consistent. Andrews (1991) and Newey and West (1994) investigate the finite sample properties of these estimators and propose data-dependent bandwidths. Though some papers have proposed variants of the estimators discussed in these seminal papers, the estimators applied in most of the current literature remain largely unchanged from their original form.

There have been earlier attempts to estimate HAC covariance matrices when some observations are missing. The Parzen (1963) paper on spectral analysis for data series with missing observations focuses on estimating the autocovariances of the underlying process in the presence of missing observations, based on a specific cyclical structure of missing data. We contribute to this literature by pointing out that for robust inference, we generally require an estimate of the long-run variance of the observed series rather than the underlying. We differentiate between the Amplitude Modulated estimator and the *Parzen estimator*. Following Parzen (1963), both of these estimators involve recasting the observed series as an amplitude modulated series in which the value of the underlying series is set to zero when observations are missing. The observed time structure of the data is respected, and the lagged autocovariances are estimated using only the lagged pairs which are fully observed. We show that while the Amplitude Modulated estimator is a consistent estimator of the long-run variance of the observed series, the Parzen estimator is a consistent estimator of the long-run variance of the underlying series. Along with these theoretical results, we provide simulation results that demonstrate consistency of the t-statistic constructed using the Amplitude Modulated estimator.

We also argue to extend the set of missing data structures to which these estimators can be applied. Other researchers have attempted to apply Parzen's work to a variety of missing data structures, including the Bernoulli structure of randomly missing variables and more general cyclical

patterns of missing observations (Scheinok, 1965; Bloomfield, 1970; Clinger and Van Ness, 1976; Dunsmuir and Robinson, 1981a,b). While the literature has moved beyond Parzen's original application, it still is focused on applications with randomly missing observations. Yet, many common applications of missing data techniques are for data that have a deterministic missing structure. Our theoretical results and simulation exercise demonstrate that as long the missing data structure satisfies our independence assumption, we can apply the Amplitude Modulated and Equal Spacing estimators to settings in which the pattern of missing observations is deterministic instead of random. This extends the set of possible applications to include business daily data, for example, in which weekends could be considered missing data.

More recently, Kim and Sun (2011) construct a HAC estimator for the two-dimensional case robust to spatial heteroskedasticity and autocorrelation. The focus of the paper is not on missing data, and they do not distinguish the difference between the spatial spectrum of the underlying versus the observed process. However, they discuss the applicability of their method to irregularly observed spatial data. Reducing the spatial HAC on the irregular lattice to one-dimensional time series produces an estimator very similar to our Amplitude Modulated estimator. We clarify the subtlety between the underlying and observed spectrum and develop the Amplitude Modulated estimator in the context of time series with missing data.

Setting the theory aside, many researchers use imputation techniques when faced with missing observations in practice. For example, one common use of imputation is to temporally disaggregate data to generate quarterly data from annual series, or monthly data from quarterly series. The Denton method of imputation smooths data when generating these series by minimizing first-differences or second-differences (Denton, 1971). Relatedly, the Chow-Lin method uses a related indicator series that can be used to interpolate, distribute, or extrapolate data (Chow and Lin, 1971, 1976). When this method is used, some properties of the indicator series, including serial correlation, will be transferred to the imputed series. Even the simplest method of imputing data by naive linear interpolation will induce autocorrelation in the imputed series. Studies based on Monte Carlo simulations suggest that even for reasonably large sample sizes, inference methods based on Newey-West HAC covariance estimators result in significant overrejection when the serial correlation is high (den Haan and Levin, 1997). In an imputed series, the induced high autocorrelation exacerbates this distortion. Yet, researchers using these methods rarely adjust their

inferences for this induced serial correlation (for two such examples, see Eaton, Kortum, Neiman, and Romalis (2011) and Forni, Monteforte, and Sessa (2009)). We show in our Monte Carlo simulations and our empirical application that our estimators are simple alternatives that avoid the problems associated with imputation.

To avoid the complication of adjusting HAC estimators for the method and extent of imputation, some researchers simply ignore the missing observations. Formally, this method amounts to relabeling the time index and treating observations as though they are equally spaced in time. While this method has no previous formal justification to our knowledge, it has been widely applied. For example, observations of daily financial data are generally treated as equally spaced consecutive observations, irrespective of their actual spacing in time (examples include Acharya and Johnson (2007), Beber, Brandt, and Kavajecz (2009), Jorion and Zhang (2007), Pan and Singleton (2008), and Zhang, Zhou, and Zhu (2009)). Yet, for prices that are affected by developments in global markets with different business weeks or national holidays, the lack of price data on weekends and holidays could be treated as missing observations. In this paper, we formalize this Equal Spacing estimator and demonstrate its asymptotic consistency and finite sample performance.

In light of the existing confusion in the literature on HAC estimation with missing data, we provide our Equal Spacing and Amplitude Modulated estimators as alternatives. We discuss the finite sample properties of these estimators and provide simulation results that offer insight into the bias and variance tradeoffs between the two estimators, so that practitioners can make an informed choice before applying either one.

### 3.2 *A simple example*

To fix ideas, in this section we introduce each of our estimators in the context of a simple example using three weeks of daily gasoline prices. We reserve for the next section a more detailed discussion of the asymptotic and finite sample properties of the estimators and the practical considerations involved in choosing among them. Suppose we have gasoline price data  $\{z_t\}$  for the first three weeks of the month as shown in Figure 3.1.

**Table 3.1: Daily Gasoline Prices.**

Mon	Tues	Weds	Thurs	Fri	Sat	Sun
$z_1$	$z_2$	$z_3$	$z_4$	$z_5$	$z_6$	$z_7$
$z_8$	$z_9$	$z_{10}$	$z_{11}$	$z_{12}$	$z_{13}$	$z_{14}$
$z_{15}$	$z_{16}$	$z_{17}$	$z_{18}$	$z_{19}$	$z_{20}$	$z_{21}$

For clarity of exposition, suppose these data have already been demeaned, so that we have  $E(z_t) = 0$ . To estimate the long-run variance of the series  $\{z_t\}$ , we can apply the standard Newey-West estimator:

$$\hat{\Omega}^{NW} = \hat{\gamma}(0) + 2 \sum_{j=1}^m w(j, m) \hat{\gamma}(j),$$

where the Bartlett kernel,  $w(j, m) = 1 - [j/(m+1)]$  if  $j \leq m$  and  $w(j, m) = 0$  if  $j > m$ , is used to weight the sample autocorrelations at each lag  $j$ :

$$\hat{\gamma}(j) = \frac{1}{T} \sum_{t=j+1}^T z_{t-j} z_t,$$

In our example, we can estimate the first lagged autocorrelation for the gasoline price series as:

$$\hat{\gamma}(1) = \frac{1}{21} [z_1 z_2 + z_2 z_3 + \dots + z_{20} z_{21}].$$

Similarly, we estimate the third lagged autocorrelation as:

$$\hat{\gamma}(3) = \frac{1}{21} [z_1 z_4 + z_2 z_5 + \dots + z_{18} z_{21}].$$

Note that the denominator in both cases is the total number of observations,  $T$ , rather than the number of observed lags,  $T - j$ .

Now suppose we have only business daily data, with missing data on weekends as shown in Figure 3.2.

When some data points are missing, we have a few choices for how we estimate the lagged autocovariances,  $\hat{\gamma}(j)$ , that are components of the long-run variance,  $\hat{\Omega}$ . Especially in the context of business daily data, one very common procedure is to ignore the missing data. In this case,

**Table 3.2: Daily Gasoline Prices with Missing Observations.**

Mon	Tues	Weds	Thurs	Fri	Sat	Sun
$z_1$	$z_2$	$z_3$	$z_4$	$z_5$	.	.
$z_8$	$z_9$	$z_{10}$	$z_{11}$	$z_{12}$	.	.
$z_{15}$	$z_{16}$	$z_{17}$	$z_{18}$	$z_{19}$	.	.

we would treat the observed prices as equally spaced in time. When estimating the first lagged autocovariance for our *Equal Spacing estimator*, we would treat the jump from Friday to Monday (e.g. day 5 to day 8) as a one day lag:

$$\hat{\gamma}^{ES}(1) = \frac{1}{15}[z_1z_2 + z_2z_3 + z_3z_4 + z_4z_5 + \underline{z_5z_8} + z_8z_9 + z_9z_{10} + \dots + z_{18}z_{19}].$$

Similarly, the third autocovariance would be estimated as:

$$\hat{\gamma}^{ES}(3) = \frac{1}{15}[z_1z_4 + z_2z_5 + \underline{z_3z_8} + z_4z_9 + \underline{z_5z_{10}} + z_8z_{11} + z_9z_{12} + z_{10}z_{15} + \dots + z_{16}z_{19}].$$

Since we have effectively reduced the sample size by ignoring the missing days, the denominator for each estimated lagged autocovariance is the total number of observed data points, or 15 in our example.

Alternatively, we could estimate the lagged autocovariances using only the instances in which we observe data with the right spacing. We call this the *Amplitude Modulated estimator*, because we are effectively modulating, or setting to 0, the value (or amplitude) of the series on the missing days. Using this estimator, to estimate the first lagged autocovariance in our example, we would use the lag from day 4 to day 5 and then skip to the lag from day 8 to day 9:

$$\hat{\gamma}^{AM}(1) = \frac{1}{15}[z_1z_2 + z_2z_3 + z_3z_4 + z_4z_5 + z_8z_9 + z_9z_{10} + \dots + z_{18}z_{19}].$$

The third autocovariance would be estimated using all observed three day lags, including the three day lags between Friday and Monday:

$$\hat{\gamma}^{AM}(3) = \frac{1}{15}[z_1z_4 + z_2z_5 + \underline{z_5z_8} + z_8z_{11} + z_9z_{12} + \underline{z_{12}z_{15}} + z_{15}z_{18} + z_{16}z_{19}].$$

In this method, the denominator for each lag is again the total number of observed data points, as in the the Equal Spacing estimator.

While we focus on these two estimators throughout most of the paper, for comparison, our simulations will also implement two alternatives that are not preferred. The first alternative is the *Parzen estimator*, after Parzen (1963). It is constructed like the Amplitude Modulated estimator, except that we adjust the denominator to equal the number of times we observe data with the right spacing:

$$\hat{\gamma}^{PZ}(1) = \frac{1}{12}[z_1z_2 + z_2z_3 + z_3z_4 + z_4z_5 + z_8z_9 + z_9z_{10} + \dots + z_{18}z_{19}].$$

The third autocovariance would be estimated as:

$$\hat{\gamma}^{PZ}(3) = \frac{1}{8}[z_1z_4 + z_2z_5 + z_5z_8 + z_8z_{11} + z_9z_{12} + z_{12}z_{15} + z_{15}z_{18} + z_{16}z_{19}].$$

Finally, we will implement our *Imputation estimator*, which is the Newey-West estimator applied to the filled series,  $\{z_t^I\}$ , constructed by linearly imputing the missing data. In our example with business daily gasoline prices, the first and third lagged autocorrelations can be estimated as:

$$\hat{\gamma}^{IM}(1) = \frac{1}{21}[z_1z_2 + \dots + z_4z_5 + z_5z_6^I + z_6z_7^I + z_7z_8^I + z_8z_9 + \dots + z_{18}z_{19} + z_{19}z_{20}^I + z_{20}z_{21}^I]$$

$$\hat{\gamma}^{IM}(3) = \frac{1}{21}[z_1z_4 + z_2z_5 + z_3z_6^I + z_4z_7^I + z_5z_8^I + z_6z_9^I + z_7z_{10}^I + z_8z_{11}^I + \dots + z_{16}z_{19}^I + z_{17}z_{20}^I + z_{18}z_{21}^I].$$

### 3.3 Long-Run Variance of Time Series with Missing Observations

In this section, we formalize our main estimators and describe their asymptotic and finite sample properties.

#### 3.3.1 Missing Data Structure

Consider a second-order stationary time series  $\{z_t\}_{t=1}^\infty$  with  $\sum_{j=0}^\infty |\gamma_z(j)| < \infty$  and  $E(z_t) = \mu$  and an indicator series  $\{g_t\}_{t=1}^\infty$  such that  $g_t = 1$  if  $z_t$  is observed and  $g_t = 0$  if  $z_t$  is missing. Throughout the paper, we maintain the assumptions on the missing data structure:

**Assumption 1.** Independence: We assume that the underlying series  $\{z_t\}$  is independent of the series  $\{g_t\}$ . In other words, for any positive integer  $n < \infty$  and any sequence  $t_1, \dots, t_n$ , the random variable  $z \equiv (z_{t_1}, \dots, z_{t_n})$  and  $g \equiv (g_{t_1}, \dots, g_{t_n})$  satisfy the condition that  $\Pr(z^{-1}(A) \cap g^{-1}(B)) = \Pr(z^{-1}(A))\Pr(g^{-1}(B))$  for any two  $n$ -dimensional Borel sets  $A, B \subseteq \mathbb{R}^n$ .

**Assumption 2.** Existence:  $\lim_{T \rightarrow \infty} (S/T) = \alpha$  and  $\lim_{T \rightarrow \infty} [1/S] \sum_{t=j+1}^T g_t g_{t-j} = \kappa(j)$  both exist.

Assumption 1 requires that the missing process is independent of the underlying data, so that missing data do not induce bias in the parameter estimates. Assumption 2 requires that the fractions of observed converges in probability, and the asymptotic ratio of the number of observed lag  $j$  to total number of observations exists. Under these assumptions, we allow very general stochastic or deterministic missing data processes. We give two commonly observed missing data structures as follows:

Bernoulli missing: The series  $\{g_t\}_{t=1}^\infty$  has an i.i.d. Bernoulli distribution, in which each  $g_t$  takes value 0 with probability  $p$  and value 1 with probability  $1 - p$ .

Cyclically missing: Given a series  $\{z_t\}_{t=1}^\infty$ , we can divide the series into cycles which are each of length  $h > 2$ . In the first cycle of length  $h$ , we have  $k$  missing observations for some integer  $k < h$ . Define the set of time indexes of these missing observations,  $S = \{s_1, \dots, s_k\}$ , where the integers  $s_k \in [1, h]$  for all  $k$ . For  $t \leq h$ ,  $g_t = 0$  if and only if  $t \in S$ . In a cyclically missing structure, for  $t > h$ , we have  $g_{s_k+hl} = 0$  for all integers  $l = 1, 2, \dots, \infty$ , and  $g_t = 1$  otherwise.

The indicator series  $\{g_t\}$  is stochastic for Bernoulli missing and deterministic for cyclical missing once the missing pattern is known for any  $h$  consecutive elements.

### 3.3.2 Newey-West Estimator

First, we review the standard Newey-West estimator that applies to time series without missing observations. Suppose that  $z_t$  is continuously observed at  $t = 1, \dots, T$  with  $E(z_t) = \mu$ . We let  $\gamma_z(j) = E[(z_t - \mu)(z_{t-j} - \mu)]$  denote the  $j$ -th lagged autocovariance. Under the standard assumption that  $z_t$  is second-order stationary with  $\sum_{j=0}^\infty |\gamma_z(j)| < \infty$ , we have the standard results that

$\frac{1}{\sqrt{T}} \sum_{t=1}^T (z_t - \mu) \xrightarrow{d} N(0, \Omega)$ , where the long-run variance of the underlying process  $z_t$  is equal to

$$\Omega = \sum_{j=-\infty}^{\infty} \gamma_z(j). \quad (3.1)$$

The Newey-West HAC estimator for  $\Omega$  is given by

$$\hat{\Omega}^{NW} = \hat{\gamma}_z(0) + 2 \sum_{j=1}^m w(j, m) \hat{\gamma}_z(j),$$

where  $\hat{\gamma}_z(j) = \frac{1}{T} \sum_{t=j+1}^T (z_t - \bar{z}_T)(z_{t-j} - \bar{z}_T)$  and  $\bar{z}_T = (1/T) \sum_{t=1}^T z_t$ . In the Newey-West formula, the lagged autocovariances,  $\hat{\gamma}_z(j)$ , are weighted by the Bartlett kernel,  $w(j, m) = 1 - [j/(m+1)]$  for  $j \leq m$  and  $w(j, m) = 0$  otherwise, to ensure a positive semi-definite covariance matrix. Under fairly general technical assumptions, as long as  $\lim_{T \rightarrow \infty} m(T) = \infty$  and  $\lim_{T \rightarrow \infty} [m(T)/T^{1/4}] = 0$ , we have  $\hat{\Omega}^{NW} \xrightarrow{p} \Omega$  (Newey and West, 1987). The choice of optimal bandwidth  $m$  is given by Andrews (1991) and Newey and West (1994) who further explore the properties of alternative choices for the bandwidth and kernel and the finite sample properties of these estimators.

### 3.3.3 Long-Run Variance of the Underlying Process - Parzen Estimator

In the presence of missing observations, we follow Parzen (1963) and recast the series as an amplitude modulated version of some underlying full series. We define the amplitude modulated series,  $\{z^*\}$ , as  $z_t^* = g_t z_t$ . Using the amplitude modulated series  $\{z_t^*\}$ , Parzen (1963) suggests the following estimator for the autocovariance of the underlying series  $\{z_t\}$ :

$$\hat{\gamma}_z^{PZ}(j) = \frac{\sum_{t=j+1}^T (z_t^* - \bar{z}_T^*)(z_{t-j}^* - \bar{z}_{t-j}^*)}{\sum_{t=j+1}^T g_t g_{t-j}},$$

if  $\sum_{t=j+1}^T g_t g_{t-j} > 0$ . Dunsmuir and Robinson (1981a) establishes  $\hat{\gamma}_z^{PZ}(j) \xrightarrow{p} \gamma_z(j)$  provided that  $z_t^*$  is asymptotically stationary.

Under the special case that  $\lim_{T \rightarrow \infty} \sum_{t=j+1}^T g_t g_{t-j} > 0$  for all  $j$ , we can use the observed data to construct our *Parzen estimator*, which is a Newey-West type consistent estimator of the long-run variance of the underlying process  $z_t$ :

$$\hat{\Omega}^{PZ} = \hat{\gamma}_z^{PZ}(j) + 2 \sum_{j=1}^m w(j, m) \hat{\gamma}_z^{PZ}(j) \xrightarrow{p} \Omega$$

While this object may be useful in some instances, it is incorrect for inference testing. First, Dunsmuir and Robinson (1981b) study the case in which  $w(j, m) = 1$ , and point out that  $\hat{\Omega}^{PZ}$  may not be positive semi-definite.<sup>3</sup> Secondly, as we further demonstrate, the long-run variance of the underlying process differs from the long-run variance of the observed process. Though the Parzen estimator is formed using observed data only, it is a consistent estimator of the variance of the underlying process. Consequently, inference on the observed data will be invalid if we use the Parzen estimate of the variance.

### 3.3.4 Long-Run Variance of the Observed Process

Let  $S = \sum_{t=1}^T g_t$  be the total number of the observed. The sample mean is given by  $\bar{z}_T^* = \frac{1}{S} \sum_{t=1}^T z_t^*$ . Asymptotic mean and variance of  $\bar{z}_T^*$  is given by the following proposition.

**Proposition 3.**  $\bar{z}_T^* \xrightarrow{p} \mu$  and  $\Omega^* \equiv \lim_{T \rightarrow \infty} S \cdot E(\bar{z}_T^* - \mu)^2 = \sum_{j=-\infty}^{\infty} \kappa(j) \gamma_z(j)$ .

*Proof.* Given  $E(g_t) = \lim_{T \rightarrow \infty} (S/T) = \alpha$  and  $g_t$  is independent of  $z_t$ , we have  $E(z_t^*) = E(g_t)E(z_t) = \alpha\mu$ . We can rewrite  $\lim_{T \rightarrow \infty} \bar{z}_T^* = \lim_{T \rightarrow \infty} \frac{1}{S} \sum_{t=1}^T z_t^* = \lim_{T \rightarrow \infty} \frac{T}{S} \frac{1}{T} \sum_{t=1}^T z_t^*$ . We know that  $\lim_{T \rightarrow \infty} (S/T) = \alpha$ . By the law of large numbers, we have  $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T z_t^* = \alpha\mu$ . Therefore,  $\lim_{T \rightarrow \infty} \bar{z}_T^* = \mu$ . We also have

$$\begin{aligned} S \cdot E(\bar{z}_T^* - \mu)^2 &= [1/S]E \left[ \sum_{t=1}^T g_t(z_t - \mu) \right]^2 \\ &= [1/S]E \left[ \sum_{t=1}^T (z_t - \mu)^2 g_t^2 + 2 \sum_{j=1}^{T-1} \sum_{t=j+1}^T (z_t - \mu)(z_{t-j} - \mu) g_t g_{t-j} \right] \\ &= [T/S] \left[ \gamma_z(0)E \left( \frac{1}{T} \sum_{t=1}^T g_t^2 \right) + 2 \sum_{j=1}^{T-1} \gamma_z(j)E \left( \frac{1}{T} \sum_{t=j+1}^T g_t g_{t-j} \right) \right]. \end{aligned}$$

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<sup>3</sup> In our simulations, we implement the estimator using Bartlett kernel weights to maintain comparability with results for ES and AM. However, this does not guarantee that the estimator will be positive semi-definite.

Define  $\kappa(j) \equiv \lim_{T \rightarrow \infty} [1/S] \sum_{t=j+1}^T E(g_t g_{t-j})$ , the share of times lag  $j$  is observed. Given  $\lim_{T \rightarrow \infty} (T/S) = 1/\alpha$  and  $E \left[ (1/T) \sum_{t=j+1}^T g_t g_{t-j} \right] = \lim_{T \rightarrow \infty} (1/T) \sum_{t=j+1}^T g_t g_{t-j} = \alpha \kappa(j)$ , we have

$$\Omega^* = \lim_{T \rightarrow \infty} S \cdot E(\bar{z}_T^* - \mu)^2 = \sum_{j=-\infty}^{\infty} \kappa(j) \gamma_z(j). \quad (3.2)$$

□

Therefore, the long-run variance of the observed amplitude modulated process, i.e.,  $\Omega^*$ , is a weighted sum of the original autocovariances, with the weights being the asymptotic ratio of the number of the observed lags to the total number of the observed,  $S$ . Comparing equations (3.1) and (3.2), when  $z_t$  is observed at all  $t$ , i.e.,  $g_t = 1$  for all  $t$ , then  $\Omega^* = \Omega$ . In the presence of missing observations, if all autocovariances are positive, we have  $\kappa(j) \leq 1$ . Then the long-run variance of the amplitude modulated process is always weakly smaller than the long-run variance of the underlying process,  $\Omega^* \leq \Omega$ .

#### *Amplitude Modulated Estimator*

To estimate  $\Omega^*$  in finite samples with  $S$  observed, a natural candidate is given by the following proposition.

**Proposition 4.** *A consistent estimator of  $\Omega^*$  is given by*

$$\hat{\Omega}^* = \hat{\gamma}_{z^*}(0) + 2 \sum_{j=1}^{T-1} \hat{\gamma}_{z^*}(j)$$

where  $\hat{\gamma}_{z^*}(j) = [1/S] \sum_{t=j+1}^T (z_t^* - g_t \bar{z}_T^*)(z_{t-j}^* - g_{t-j} \bar{z}_T^*)$ .

*Proof.* We note that

$$\begin{aligned} \hat{\gamma}_{z^*}(j) &= [1/S] \sum_{t=j+1}^T (z_t^* - g_t \bar{z}_T^*)(z_{t-j}^* - g_{t-j} \bar{z}_T^*) \\ &= \frac{T}{S} \frac{1}{T} \sum_{t=j+1}^T (z_t - \bar{z}_T^*)(z_{t-j} - \bar{z}_T^*) g_t g_{t-j} \end{aligned}$$

Since  $\lim_{T \rightarrow \infty} T/S = 1/\alpha$ ,  $\lim_{T \rightarrow \infty} E(z_t - \bar{z}_T^*)(z_{t-j} - \bar{z}_T^*) = \gamma_z(j)$  and  $\lim_{T \rightarrow \infty} E(g_t g_{t-j}) = \alpha \kappa(j)$  we have

$$\hat{\gamma}_{z^*}(j) \xrightarrow{p} \kappa(j)\gamma(j).$$

Therefore,  $\hat{\Omega}^* \xrightarrow{p} \Omega^*$ .  $\square$

However,  $\hat{\Omega}^*$  is not guaranteed to be positive semi-definite, which is not desirable for inference. We can use a kernel-based method to ensure the covariance estimator is positive semi-definite:

$$\hat{\Omega}^{AM} = \hat{\gamma}_{z^*}(0) + 2 \sum_{j=1}^m w(j, m) \hat{\gamma}_{z^*}(j).$$

We follow Newey and West (1987) and illustrate with the most commonly used kernel, the Bartlett kernel.

**Proposition 5.** *Using the Bartlett kernel,  $w(j, m) = 1 - [j/(m+1)]$  if  $j \leq m$  and  $w(j, m) = 0$  if  $j > m$ , suppose (i) the bandwidth  $m$  satisfies  $\lim_{T \rightarrow \infty} m(T) = +\infty$  and  $\lim_{T \rightarrow \infty} [m(T)/T^{1/4}] = 0$ . Then  $\hat{\Omega}^{AM}$  is positive semi-definite and  $\hat{\Omega}^{AM} \xrightarrow{p} \Omega^*$ .*

*Proof.* We follow proofs of Theorems 1 and 2 in Newey and West (1987) by defining  $h_t \equiv z_t^* - g_t \mu$ ,  $\hat{h}_t \equiv z_t^* - g_t \bar{z}_T^*$  and replace all  $T$  in the denominator in Newey and West (1987) with  $S$ .  $\square$

Our estimator  $\hat{\Omega}^{AM}$  is almost equivalent to applying the Newey-West estimator to the amplitude modulated series. However, we make two minor modifications to the components  $\hat{\gamma}_{z^*}(j) = [1/S] \sum_{t=j+1}^T (z_t^* - g_t \bar{z}_T^*)(z_{t-j}^* - g_{t-j} \bar{z}_T^*)$ . First, we subtract  $z_t^*$  by  $g_t \bar{z}_T^*$  instead of  $E(g_t) \bar{z}_T^*$ , so that the difference  $(z_t^* - g_t \bar{z}_T^*)$  equals zero for unobserved data. In the case of a mean-zero series, this modification would not be required. Secondly, since we want to use  $\hat{\Omega}^{AM}$  to make inferences about the mean of the observed process  $\bar{z}_T^*$ , we divide the sum by  $S$  instead of  $T$  so that our inference procedure remains consistent.

### Equal Spacing Estimator

Instead of casting the time series with missing observations as an amplitude modulated process, an alternative method is to ignore the missing observations and treat the data as equally

spaced over time. We define the function  $\iota(t)$  as the mapping from time index  $t$  to the new equal spacing time domain  $s$ :  $\iota(t) = \sum_{\ell=1}^t g_\ell$ . We use this mapping to relabel the time indices of the observed values from the series  $\{z_t\}$  to create the series  $\{z_s^{ES}\}$  for  $s = 1, \dots, \iota(T)$ , in the equal spacing time domain. The sample mean of the observed series is given by  $\bar{z}_T^{ES} = \frac{1}{S} \sum_{s=1}^S z_s^{ES}$ .

**Proposition 6.**  $\bar{z}_T^{ES} \xrightarrow{P} \mu$  and  $\Omega^{ES} \equiv \lim_{T \rightarrow \infty} S \cdot E(\bar{z}_T^{ES} - \mu)^2 = \Omega^*$ .

*Proof.* We let  $\Delta_s^{j^{ES}} \equiv \iota^{-1}(s) - \iota^{-1}(s - j^{ES})$  be a function that maps the time gap ( $j^{ES}$ ) between  $z_s$  and  $z_{s-j^{ES}}$  in the equal spacing domain to the time gap ( $j$ ) in the time domain of the underlying process  $\{z_t\}$ . Using the indicator function  $\mathbf{I}(\cdot)$ , we define  $\lambda^{j^{ES}}(j) = \lim_{T \rightarrow \infty} \frac{1}{S} \sum_{s=1}^S \mathbf{I}(\Delta_s^{j^{ES}} = j)$ , which equals the frequency that the observed lag  $j^{ES}$  maps to lag  $j$  in the original time domain. Then we can rewrite the Equal Spacing autocovariance in terms of the autocovariance of the underlying process:

$$\begin{aligned} \gamma_{z^{ES}}(j^{ES}) &= \lim_{T \rightarrow \infty} E(z_s - \bar{z}_T^{ES})(z_{s-j^{ES}} - \bar{z}_T^{ES}) \\ &= \sum_{j=-\infty}^{\infty} \lambda^{j^{ES}}(j) \gamma_z(j) \end{aligned}$$

Applying the same standard results as in Equation 3.1 to the equal spacing series, we have:

$$\begin{aligned} \Omega^{ES} &= \sum_{j^{ES}=-\infty}^{\infty} \gamma_{z^{ES}}(j^{ES}) \\ &= \sum_{j^{ES}=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} \lambda^{j^{ES}}(j) \gamma_z(j) \\ &= \sum_{j=-\infty}^{\infty} \left( \sum_{j^{ES}=-\infty}^{\infty} \lambda^{j^{ES}}(j) \right) \gamma_z(j) \\ &= \sum_{j=-\infty}^{\infty} \kappa(j) \gamma_z(j) = \Omega^*. \end{aligned}$$

The second to last equation holds because

$$\sum_{j^{ES}=-\infty}^{\infty} \lim_{T \rightarrow \infty} \frac{1}{S} \sum_{s=1}^S \mathbf{I}(\Delta_s^{j^{ES}} = j) = \lim_{T \rightarrow \infty} \frac{1}{S} \sum_{t=1}^T g_t g_{t-j}.$$

□

To estimate  $\Omega^*$  using the equally spaced series in finite samples, we can use

$$\hat{\Omega}^{ES} = \hat{\gamma}_{z^{ES}}(0) + 2 \sum_{j=1}^m w(j^{ES}, m) \hat{\gamma}_{z^{ES}}(j^{ES})$$

where

$$\hat{\gamma}_{z^{ES}}(j^{ES}) = \frac{1}{S} \sum_{s=1}^{S-1} (z_s^{ES} - \bar{z}_T^{ES})(z_{s-j}^{ES} - \bar{z}_T^{ES}).$$

**Proposition 7.**  $\hat{\Omega}^{ES}$  is PSD and  $\hat{\Omega}^{ES} \xrightarrow{p} \Omega^*$ .

*Proof.* Positive semi-definiteness of  $\hat{\Omega}^{ES}$  can be established using same argument in the proof of Theorem 1 in Newey and West (1987). Using their notation, we let  $\hat{h}_s = z_s^{ES} - \bar{z}_T^{ES}$ . To prove consistency, since

$$\hat{\gamma}_{z^{ES}}(0) + 2 \sum_{j^{ES}=1}^{s(T)-1} \hat{\gamma}_{z^{ES}}(j^{ES}) = \hat{\gamma}_{z^*}(0) + 2 \sum_{j=1}^{T-1} \hat{\gamma}_{z^*}(j)$$

and  $w(j, m) \xrightarrow{p} 1$  for all  $j$ ,  $\hat{\Omega}^{ES}$  and  $\hat{\Omega}^{AM}$  estimators are asymptotically equivalent. We know  $\hat{\Omega}^{AM} \xrightarrow{p} \Omega^*$  by proposition 5, and hence,  $\hat{\Omega}^{ES} \xrightarrow{p} \Omega^*$ .

The Equal Spacing estimator is particularly simple to implement, because it only requires re-labeling the time index of a series with missing observations to ignore the gaps and treat the data as equally spaced over time. Once this is done, the Equal Spacing estimator amounts to applying the standard Newey-West estimator to the equally spaced series.  $\square$

### 3.3.5 Finite Samples

Although  $\hat{\Omega}^{AM}$  and  $\hat{\Omega}^{ES}$  are both asymptotically consistent, finite sample performance might differ due to different weighting on autocovariance estimators. We use the standard mean squared error (MSE) criterion to evaluate the performance of the estimator  $\hat{\Omega}^i$  where  $i \in \{AM, ES\}$ .

$$\begin{aligned} MSE(\hat{\Omega}^i) &= Bias^2(\hat{\Omega}^i) + Var(\hat{\Omega}^i) \\ &= [E(\hat{\Omega}^i - \Omega^*)]^2 + E[(\hat{\Omega}^i - \bar{\Omega}^i)^2] \end{aligned}$$

where  $\bar{\Omega}^i = E(\hat{\Omega}^i)$ . Consider the case that  $m^{AM} = m^{ES} \equiv m$ . The lag length in the original time domain is weakly greater than that in the equal spacing domain:  $j = \iota^{-1}(s) - \iota^{-1}(s - j^{ES}) \geq$

$j^{ES}$ . Under the same fixed bandwidth  $m$ , there are two main differences between  $\hat{\Omega}^{AM}$  and  $\hat{\Omega}^{ES}$ . First, since the kernel weight is decreasing in the lag length for the Newey-West estimator,  $\hat{\Omega}^{ES}$  assigns weakly higher weight on all autocovariance estimators compared to  $\hat{\Omega}^{AM}$ . To see this more explicitly,

$$\hat{\Omega}^{ES} = \sum_{j^{ES}=1}^m \frac{w(j^{ES}, m)}{S} \sum_{s=j^{ES}+1}^S (z_s - \bar{z}_T^*)(z_{s-j^{ES}} - \bar{z}_T^*)$$

To write it in the original time domain, we have

$$\hat{\Omega}^{ES} = \sum_{j^{ES}=1}^m \frac{w(j^{ES}, m)}{S} \sum_{t=j+1}^T (z_t - \bar{z}_T^*)(z_{t-j} - \bar{z}_T^*)$$

where  $t = \iota^{-1}(s)$  and  $j = \iota^{-1}(s) - \iota^{-1}(s - j^{ES}) \geq j^{ES}$ . We compare  $\hat{\Omega}^{ES}$  with  $\hat{\Omega}^{AM}$ ,

$$\hat{\Omega}^{AM} = \sum_{j=1}^m \frac{w(j, m)}{S} \sum_{t=j+1}^T (z_t - \bar{z}_T^*)(z_{t-j} - \bar{z}_T^*) g_t g_{t-j}.$$

When  $g_t g_{t-j} = 1$ , we have  $w(j^{ES}, m) \geq w(j, m)$  since the weighting function decreases in lag length and  $j \geq j^{ES}$ . Therefore, given the same bandwidth,  $\hat{\Omega}^{ES}$  puts weakly more weight than  $\hat{\Omega}^{AM}$  on each observed pairwise product  $(z_t - \bar{z}_T^*)(z_{t-j} - \bar{z}_T^*)$ . Second, for the same fixed bandwidth  $m$ ,  $\hat{\Omega}^{AM}$  only estimates autocovariance with lag length up to  $m$  in the original time domain, while  $\hat{\Omega}^{ES}$  also includes observations for lags greater than  $m$  in the original time domain. These two differences have different implications on the relative variance and bias of the two estimators.

As discussed in den Haan and Levin (1997), Newey-West type kernel-based estimators suffer from three sources of finite sample bias. First, the summation in the autocovariance estimator is divided by the sample size, instead of the actual number of observed lags. We expect this source of bias to be more severe for  $\hat{\Omega}^{ES}$  because  $\hat{\Omega}^{ES}$  includes higher-order lags that are not included in  $\hat{\Omega}^{AM}$  and puts more weight on these high-order biased lagged autocovariance estimators. However, this bias decreases rapidly as the sample size increases. Second, the kernel-based method assigns zero weights to lags with orders greater than  $T$ . This source of bias is the same for  $\hat{\Omega}^{ES}$  and  $\hat{\Omega}^{AM}$ .

The third and most significant source of bias is driven by the fact that kernel-based estimators under-weight the autocovariance estimators. They assign weights to autocovariance estimators

that are less than unity and are declining toward zero with increasing lag order  $j$ . Compared with the long-run variance of the amplitude modulated series,  $\Omega^*$ , the bias of  $\hat{\Omega}^{AM}$  arising from this source is given by

$$Bias(\hat{\Omega}^{AM}) = \sum_{j=-(T-1)}^{T-1} [1 - w(j, m)] \gamma_{z^*}(j).$$

For a fixed bandwidth, the higher the serial correlation, the more severe the bias. The estimator  $\hat{\Omega}^{ES}$  can reduce this kernel-based bias because  $\hat{\Omega}^{ES}$  always assigns weakly higher (or closer to unitary) weight to all autocovariance estimators as compared to  $\hat{\Omega}^{AM}$ .

For variance of the estimators, we always have  $Var(\hat{\Omega}^{ES}) > Var(\hat{\Omega}^{AM})$  because  $\hat{\Omega}^{ES}$  includes more high-order lags that are relatively poorly estimated. Therefore, the tradeoff between variance and bias determines the relative finite sample performance of  $\hat{\Omega}^{AM}$  and  $\hat{\Omega}^{ES}$ . The previous discussion uses a fixed bandwidth. Andrews (1991) and Newey and West (1994) propose data-dependent choice of the bandwidth that aims to optimize the mean-variance trade-off. We apply the automatic bandwidth selection procedure proposed by Newey and West (1994) to the AM and ES processes. As we will demonstrate using Monte-Carlo simulations, under both fixed and automatic bandwidth selection, for small sample size and low autocorrelation,  $MSE(\hat{\Omega}^{ES}) > MSE(\hat{\Omega}^{AM})$ . For moderate sample size or high autocorrelation, we always have  $MSE(\hat{\Omega}^{AM}) > MSE(\hat{\Omega}^{ES})$ .

### 3.4 Regression Model with Missing Observations

We can apply asymptotic theory developed in the previous section to a regression model with missing observations. Suppose we have the time series regression, where  $y_t$  and  $u_t$  are scalars,  $x_t$  is a  $k \times 1$  vector of regressors, and  $\beta$  is a  $k \times 1$  vector of unknown parameters. Suppose further that  $\left(\frac{1}{T} \sum_{t=1}^T x_t x_t'\right)^{-1} \xrightarrow{p} \Sigma_{xx}^{-1}$  and  $\mathbb{E}(u_t | x_t) = 0$ , but the  $u_t$ 's have conditional heteroskedasticity and are possibly serially correlated. In the presence of missing observations, we let  $g_t = 1$  if  $y_t$  and all components of  $x_t$  are observed and  $g_t = 0$  if  $y_t$  or any component of  $x_t$  is missing. Then we can re-express the regression in terms of amplitude modulated processes,

$$y_t^* = x_t^* \beta + u_t^*, \quad t = 1, \dots, T,$$

where  $y_t^* = g_t y_t$ ,  $x_t^* = g_t x_t$  and  $u_t^* = g_t u_t$ . We require the orthogonality condition,  $\mathbb{E}(u_t^* | x_t^*) = 0$ .

The standard result for the OLS estimator is given by

$$\hat{\beta}^{AM} - \beta = \left( \sum_{t=1}^T x_t^* x_t^{*'} \right)^{-1} \left( \sum_{t=1}^T x_t^* u_t^* \right). \quad (3.3)$$

Alternatively, without recasting the series as an amplitude modulated process, we ignore all observations for which  $g_t = 0$  and assume all observed values are equally spaced in time. Therefore, the estimated regression becomes

$$y_s^{ES} = x_s^{ES} \beta + u_s^{ES}, \quad s = 1, \dots, S$$

and

$$\hat{\beta}^{ES} - \beta = \left( \sum_{s=1}^S x_s^{ES} x_s^{ES'} \right)^{-1} \left( \sum_{s=1}^S x_s^{ES} u_s^{ES} \right). \quad (3.4)$$

Comparing equations 3.3 and 3.4, we can easily see that AM and ES give the same coefficient estimates:

$$\hat{\beta}^{AM} = \hat{\beta}^{ES} \equiv \hat{\beta}.$$

We normalize  $\hat{\beta}$  using the number of observed data,  $S$ , and then we have

$$\sqrt{S}(\hat{\beta} - \beta) = \left( \frac{1}{S} \sum_{t=1}^T x_t^* x_t^{*'} \right)^{-1} \left( \frac{1}{\sqrt{S}} \sum_{t=1}^T x_t^* u_t^* \right).$$

Given that  $\left( \frac{1}{T} \sum_{t=1}^T x_t x_t' \right)^{-1} \xrightarrow{p} \Sigma_{xx}^{-1}$  in the absence of missing observations and  $x_t$  and  $g_t$  are independent,  $\left( \frac{1}{S} \sum_{t=1}^T x_t^* x_t^{*'} \right)^{-1}$  also converges in probability. We let  $\left( \frac{1}{S} \sum_{t=1}^T x_t^* x_t^{*'} \right)^{-1} \xrightarrow{p} \Sigma_{x^*x^*}^{-1}$ . Using the notation from the previous section, we define  $z_t \equiv x_t u_t$  and let  $z_t^* \equiv g_t z_t$  denote the amplitude modulated series and  $z_s^{ES}$  denote the ES series. Then we have

$$\bar{z}_T^* \equiv \frac{1}{S} \sum_{t=1}^T z_t^* = \frac{1}{S} \sum_{t=1}^S z_s^{ES}.$$

We know  $E(z_t) = E(\bar{z}_T^*) = 0$  using the orthogonality condition.

**Proposition 8.** *The asymptotic distribution of the OLS estimator is given by*

$$\sqrt{S}(\hat{\beta} - \beta) \xrightarrow{d} N(0, \Sigma_{x^*x^*}^{-1} \Omega^* \Sigma_{x^*x^*}^{-1}),$$

where  $\Omega^* = \sum_{j=-\infty}^{\infty} \kappa(j) \gamma_z(j)$  and  $\kappa(j) = \lim_{S \rightarrow \infty} \frac{1}{S} \sum_{t=j+1}^T E(g_t g_{t-j})$ .

To estimate  $\Sigma_{x^*x^*}^{-1}$ , we can use

$$\hat{\Sigma}_{x^*x^*}^{-1} = \left( \frac{1}{S} \sum_{t=1}^T x_t^* x_t^{*'} \right)^{-1} = \left( \frac{1}{S} \sum_{s=1}^S x_s^{ES} x_s^{ES'} \right)^{-1} \xrightarrow{p} \Sigma_{x^*x^*}^{-1}.$$

**Proposition 9.** We define

$$\hat{\Omega}^{AM} = \hat{\Gamma}_0^{AM} + \sum_{j=1}^m w(j, m) [\hat{\Gamma}_j^{AM} + \hat{\Gamma}_j^{AM'}],$$

where  $\hat{\Gamma}_j^{AM} = (1/S) \sum_{t=1}^T z_s^* z_{s-j}'$

$$\hat{\Omega}^{ES} = \hat{\Gamma}_0^{ES} + \sum_{j=1}^m w(j, m) [\hat{\Gamma}_j^{ES} + \hat{\Gamma}_j^{ES'}],$$

where  $\hat{\Gamma}_j^{ES} = (1/S) \sum_{s=1}^S z_s^{ES} z_{s-j}'$ . Then we have  $\hat{\Omega}^{AM} \xrightarrow{p} \Omega^*$  and  $\hat{\Omega}^{ES} \xrightarrow{p} \Omega^*$ . For inferences, the t-statistic based on  $\hat{\Omega}^{AM}$  is given by

$$t_k^{AM} = \frac{\hat{\beta}_k - \beta_0}{\sqrt{\hat{V}_{kk}^{AM}/(S-k)}} \xrightarrow{d} N(0, 1),$$

where  $\hat{V}_{kk}^{AM}$  is the  $(k, k)$ -th element of  $\hat{V}^{AM} = \hat{\Sigma}_{x^*x^*}^{-1} \hat{\Omega}^{AM} \hat{\Sigma}_{x^*x^*}^{-1}$ . Alternatively, the t-statistic based on  $\hat{\Omega}^{ES}$  is given by

$$t_k^{ES} = \frac{\hat{\beta}_k - \beta_0}{\sqrt{\hat{V}_{kk}^{ES}/(S-k)}} \xrightarrow{d} N(0, 1),$$

where  $\hat{V}_{kk}^{ES}$  is the  $(k, k)$ -th element of  $\hat{V}^{ES} = \hat{\Sigma}_{x^*x^*}^{-1} \hat{\Omega}^{ES} \hat{\Sigma}_{x^*x^*}^{-1}$ .

### 3.5 Simulation

In the Monte Carlo simulations that follow, we study the properties of our Amplitude Modulated and Equal Spacing estimators using a simple location model. To evaluate these estimators

under a variety of circumstances, we generate data with various levels of autocorrelation and for a range of sample sizes. We test our estimators under the two missing structures described in Section 3.3.1, the Bernoulli missing structure and the deterministic cyclically missing structure. We implement the estimators using a standard fixed bandwidth for our benchmark results, and also provide results that implement the automatic bandwidth selection procedure proposed by Newey and West (1994). Our primary evaluation criteria are the empirical rejection probability of the test, and the power of the test against an appropriate alternative.

### 3.5.1 Data Structure

The inference procedures are tested on a simulated data series  $\{y_t\}$  that is generated using a simple location model:

$$y_t = \beta + \epsilon_t$$

$$\epsilon_t = \phi \epsilon_{t-1} + \eta_t$$

where  $\eta_t$  i.i.d.  $\sim \mathcal{N}(0, 1)$  and  $\epsilon_0 = 0$

For each of  $N = 100,000$  iterations, we use  $\beta = 0$  and generate a data series  $\{y_1, \dots, y_{T^{max}}\}$  with a sample size of  $T^{max} = 24,000$ . Since we run tests over a range of sample sizes for each estimator, we use the first  $T$  observations in each iteration for  $T \in \{120, 360, 1200, 4800, 12000, 24000\}$ . To test these methods for a range of autocorrelation parameters,  $\phi$ , we generate data separately for  $\phi \in \{0, 0.3, 0.5, 0.7, 0.9\}$ . The regressor in this model is a constant 1, and we conduct simulation exercises under different missing structures for the dependent variable  $\{y_t\}$ , which are described below. For each iteration, we generate a series  $\{g_t\}$ , which indicates for each  $t$  whether  $y_t$  is observed. Finally, we generate the series  $\{y_t^*\}$ , where  $y_t^* = g_t y_t$ .

Given this data, we estimate the parameter of interest,  $\beta^i$ , and the estimator for the covariance matrix,  $\hat{\Omega}^i$ , for each estimator  $i \in \{NW, ES, AM\}$ . We also perform simulations for two additional methods. First, we implement the imputation method, in which the missing  $y_t$  are linearly imputed before the standard Newey-West estimator is applied to the filled series  $\{y_t^I\}$ . Secondly, we implement the Parzen estimator from Section 3.3.3. Since the estimator  $\hat{\Omega}^{PZ}$  is not positive semi-

definite, we calculate the rejection rate using the number of rejections divided by the number of simulations in which  $\hat{\Omega}^{PZ} > 0$ .

We use these estimators to calculate the t-statistic,  $t_\beta^i$ , used for a test of the null hypothesis  $H_0 : \beta = 0$  against the two-sided alternative,  $H_a : \beta \neq 0$ . We choose a 5% level of significance and reject the null hypothesis when  $|t_\beta^i| > 1.96$ . For the standard estimations, we use a fixed bandwidth of  $m = 4(T/100)^{(2/9)}$ . We also apply the automatic bandwidth selection procedure of Newey and West (1994) to the Newey-West, Equal Spacing, Amplitude Modulating, and Imputation methods.

Results are reported for simulation exercises under a variety of sampling schemes. Our benchmark sampling scheme is one with a Bernoulli missing structure as described in Example 1 of Section 3.3.1. For these simulations, the series  $\{g_t\}$  has an i.i.d. Bernoulli distribution with fixed probability of missing,  $p = 6/12$ . For comparison, we also provide two variants in which the probability of missing is set to  $4/12$  and  $8/12$ .

We also simulate data under four data structures with cyclically missing observations, as described in Example 2 of Section 3.3.1. For these, we choose a cycle length of 12, with 6 or 8 observations missing each cycle. In these simulations, the missing structure is cyclical in that we generate a single pattern of missing observations for the first cycle, and apply this same pattern to every cycle of 12 observations. Additionally, the pattern is deterministic in the sense that we apply the same pattern of missing observations for all iterations in the simulation. This sampling structure reflects the potential application of these methods to monthly data with missing observations. For example, because commodities futures contracts expire only some months of the year, the data on monthly commodities returns will have the same missing pattern each year. Another common application is for daily financial data, in which the same 2 weekend days are missing in each cycle of 7 days.

Under a deterministic cyclical missing structure, it is possible to have cases for which certain lagged autocovariances in the original time series domain are never observed. As we noted in the introduction, the popular statistical software Stata forbids implementation of the Amplitude Modulated estimator under this case, even when using the “force” command. Yet, our theoretical results do not require all the lags to be observed to generate asymptotically correct inference using the ES and AM estimators. Consequently, we perform simulations for deterministic cyclical missing structures under both cases: all lags are observed at least once, or some lags are never

observed. We show that the finite sample performance does not differ much between these two cases, and neither case differs much from the results under the Bernoulli missing structure.

In our first two cyclical missing structures, we set the cyclical pattern of observed data such that each lagged autocovariance can be estimated from the observed data. In our cyclical structure with 6 of 12 missing, we observe  $\{z_3, z_6, z_8, z_9, z_{10}, z_{11}\}$ , and then observe the same pattern of observations for each cycle of length 12. In our next cyclical structure, we have 8 of 12 missing. We observe  $\{z_3, z_4, z_7, z_9\}$  in the first cycle, and the same pattern for each cycle after that. For our final two cyclical missing structures, we require the observed data to be spaced within the cycle such that at least one lag is never observed. In our structure with 6 of 12 missing, we observe  $\{z_1, z_3, z_5, z_8, z_{10}, z_{12}\}$  in the first cycle. Under this structure, the sixth lag is never observed. For our cyclical structure with 8 of 12 missing, we observe  $\{z_2, z_3, z_6, z_{12}\}$ , so that the fifth lag is never observed.

### 3.5.2 Evaluation Criteria

The primary evaluation criteria for these estimators is the empirical rejection probability of the tests. The empirical rejection probability measures the likelihood that null hypothesis is rejected when it is in fact true (Type I error). Each iteration of the Monte Carlo simulation represents one hypothesis test, and the reported rejection probability reflects the fraction of iterations for which the t-statistic was large enough in magnitude to reject the null hypothesis.

We also provide measures of the power of the test, as well as measures of the bias and variance of the estimators. The power of the test measures the probability that the null hypothesis is rejected when the alternative hypothesis is true. Since we find empirical rejection probabilities that can be much higher than the 0.05 benchmark, we calculate the size-adjusted power for ease of comparability. Following Ibragimov and Mueller (2010), we set the alternative hypothesis to  $H_a : \beta_a = 4/\sqrt{T(1-\phi^2)}$ . To calculate the power, we first calculate  $t_{\beta_a}^i$ , which is analogous to  $t_\beta^i$  for each  $i$ , except that we subtract  $\beta_a$  instead of  $\beta_0$  in the numerator. For example,

$$t_{\beta_a}^{NW} = \frac{\hat{\beta}^{NW} - \beta_a}{\sqrt{\hat{V}_{kk}^{NW}/T}}.$$

Next, we calculate an adjusted critical value,  $t_{0.05}^{crit}$ , which is the t-statistic at the 5th percentile of our simulations. This value is equal to the critical value for which the empirical rejection probability would have been exactly 0.05 under our simulation procedure. To calculate the size-adjusted power, we calculate  $t_{\beta_a}^i$  under the alternative hypothesis above and reject when  $|t_{\beta_a}^i| > t_{0.05}^{crit}$ .

Finally, in order to understand the finite sample performance, we study the empirical mean squared error of our estimators by decomposing it into the empirical variance and bias. Under the benchmark Bernoulli case, we first calculate the value of  $\Omega^*$  under our data generating process as:

$$\begin{aligned}\Omega^* &= \lim_{T \rightarrow \infty} (T/S) \sum_{j=-\infty}^{\infty} \gamma_j E(g_t g_{t-j}) \\ &= (1/p) \left( p + 2 \sum_{j=1}^{\infty} p^2 \phi^j \right) Var(\epsilon_t) \\ &= \left( 1 + \frac{2p\phi}{1-\phi} \right) \left( \frac{1}{1-\phi^2} \right)\end{aligned}$$

where  $p$  is the probability of being observed. The second equation follows because (1)  $\lim_{T \rightarrow \infty} T/S = 1/p$ ; (2)  $E(g_t g_{t-j}) = p$  if  $j = 0$  and  $E(g_t g_{t-j}) = p^2$  if  $j \geq 1$ . The third equation holds because  $Var(\epsilon_t) = 1/(1 - \phi^2)$ . Returning to the MSE decomposition, we have:

$$\begin{aligned}\widehat{MSE} &= \widehat{Bias}^2 + \widehat{Variance} \\ &= \left( \hat{\Omega}^i - \Omega^* \right)^2 + \frac{1}{N} \sum_{c=1}^N \left( \hat{\Omega}_c^i - \hat{\Omega}^i \right)^2,\end{aligned}$$

where  $\hat{\Omega}^i = (1/N) \sum_{c=1}^N \hat{\Omega}_c^i$  is the sample mean of all the covariance estimators and  $c$  indexes the  $N = 100,000$  iterations. Note that for  $i = NW$ , we have  $p = 1$ . We use these measures to study the finite sample properties of our estimators, especially to compare the AM and ES estimators.

The primary findings are reported in Tables 3.3 through 3.8.<sup>4</sup>

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<sup>4</sup> Full simulation results are available upon request.

**Table 3.3: Benchmark Results.** This table reports for a range of sample sizes and autocorrelation parameters the empirical rejection rate as defined in Section 3.5.2. Data follow our benchmark missing structure, the Bernoulli structure with probability of missing 6/12. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	Empirical Rejection Rate				
	0.0	0.3	0.5	0.7	0.9
<b>Sample Size T=360</b>					
ES, fixed bw	6.0	7.0	8.0	10.7	23.1
AM, fixed bw	5.6	6.7	8.3	12.6	30.9
Parzen, fixed bw	6.6	4.6	4.5	6.1	19.2
Imputation, fixed bw	8.9	9.7	11.2	15.5	34.4
NW, fixed bw	5.6	7.2	9.1	14.2	33.7
<b>Sample Size T=1200</b>					
ES, fixed bw	5.3	6.0	6.8	8.7	18.8
AM, fixed bw	5.2	6.0	7.2	10.6	26.7
Parzen, fixed bw	5.5	3.6	3.3	4.4	14.9
Imputation, fixed bw	8.1	8.5	9.6	13.1	29.9
NW, fixed bw	5.2	6.4	7.9	11.9	29.4
<b>Sample Size T=24000</b>					
ES, fixed bw	5.1	5.4	5.8	6.6	11.0
AM, fixed bw	5.1	5.4	6.1	7.5	16.5
Parzen, fixed bw	5.1	3.0	2.3	2.2	6.3
Imputation, fixed bw	6.3	6.5	7.0	8.4	17.9
NW, fixed bw	5.1	5.6	6.3	7.9	17.5

### 3.5.3 Results

Table 3.3 provides the rejection probabilities for our two main estimators, the Equal Spacing estimator and the Amplitude Modulated estimator. We provide results under a fixed bandwidth and automatic bandwidth selection for each estimator, and for comparison purposes, present results for the application of the Newey-West estimator applied to the full series without missing observations. Our benchmark missing data structure is the Bernoulli missing structure in which observations are missing with probability  $p = 1/2$ . We focus on the results for the simulation with  $T = 360$ , and also provide results for a large and very large sample size ( $T = 1200$  or  $24000$ ).

Our simulation results provide evidence that the ES and AM estimators perform well in finite samples. As is well known in the HAC literature, we find that the empirical rejection probability

**Table 3.4: Varying Missing Structure.** This table reports for a range of autocorrelation parameters the empirical rejection rate for a sample size of  $T = 360$  under varying missing structures as described in Section 3.5.1: the Bernoulli missing structure, the deterministic cyclical missing structure in which all lags are observed, and the deterministic cyclical missing structure in which some lags are never observed. The probability of missing is 6/12 for the Bernoulli structure, while the cyclical structures have exactly 6 of 12 observations missing in each cycle. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	Empirical Rejection Rate				
	0.0	0.3	0.5	0.7	0.9
<b>Randomly missing, Bernoulli structure</b>					
ES, fixed bw	6.0	7.0	8.0	10.7	23.1
AM, fixed bw	5.6	6.7	8.3	12.6	30.9
<b>Deterministic cyclically missing, All lags observed</b>					
ES, fixed bw	5.9	6.9	7.9	10.1	22.2
AM, fixed bw	5.4	6.5	8.0	11.9	30.1
<b>Deterministic cyclically missing, Some lags unobserved</b>					
ES, fixed bw	6.0	6.5	7.5	9.8	22.1
AM, fixed bw	5.5	6.3	8.1	12.8	32.0
<b>Full sample benchmark</b>					
NW, fixed bw	5.6	7.2	9.1	14.2	33.7

can be a bit higher than 5.0 for small samples, even when there is no autocorrelation. In addition, when the autocorrelation parameter is high, there can be quite a bit of overrejection even for very large sample sizes ( $T = 24000$ ). However, we do find that the rejection probability is falling towards 5.0 as the sample size increases.

We also find evidence that our ES and AM estimators are well-behaved under deterministic cyclically missing structures. In Table 3.4, we see little difference in the rejection rates under each of our three data structures: randomly missing under a Bernoulli structure, deterministic cyclical missing when all lags are observed, and deterministic cyclical missing when some lags are unobserved.

Table 3.3 also provides the empirical rejection probabilities for the Parzen and Imputation estimators for  $T = 360$  and  $T = 24000$ . As expected, the higher serial correlation induced by the imputation procedure results in extremely high rejection rates as compared to the ES and AM estimators. We can also see in Table 3.3 that the results for the Parzen estimator substantiate our argument that this estimator cannot be used for robust inference for series with missing observations. In our simulation with  $\phi = 0$  and  $T = 360$ , we found 20 instances (out of 100,000) in which the non-PSD Parzen estimator returned a negative estimate of the variance. Additionally, we find that the rejection probability is generally decreasing in the sample size but is U-shaped with respect to the autocorrelation, and often underrejects for low levels of autocorrelation.<sup>5</sup>

Next we turn to a comparison of the finite sample properties of the ES and AM estimators. In the results for the fixed bandwidth estimators in Table 3.5, we find that for  $T = 360$ , the AM estimator is preferred for autocorrelation parameters  $\phi \leq 0.3$ , while the ES estimator is preferred for series with higher autocorrelation. For larger samples, the ES estimator is preferred for a larger range of  $\phi$ , so that for  $T = 24000$ , we have that the ES estimator is preferred for all the simulations with nonzero autocorrelations.

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<sup>5</sup> Interestingly, the test using the PZ estimator is well-behaved when the autocorrelation is 0. This is consistent with our theoretical results, because when there is no autocorrelation, we have that the long-run variance of the underlying and observed series are asymptotically equivalent. Consequently, we have that when there is no autocorrelation,  $\hat{\Omega}^{PZ}$  and  $\hat{\Omega}^{AM}$  are asymptotically equivalent as well.

**Table 3.5: Finite Samples: Fixed and Automatic Bandwidth Selection.** This table reports for a range of sample sizes and autocorrelation parameters the empirical rejection rate and size-adjusted power as defined in Section 3.5.2. Data follow our benchmark missing structure, the Bernoulli structure with probability of missing 6/12. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	Empirical Rejection Rate					Size-Adjusted Power				
	0.0	0.3	0.5	0.7	0.9	0.0	0.3	0.5	0.7	0.9
<b>Sample Size T=360</b>										
ES, fixed bw	6.0	7.0	8.0	10.7	23.1	78.9	63.5	49.2	32.7	14.0
AM, fixed bw	5.6	6.7	8.3	12.6	30.9	79.4	64.1	50.0	33.0	14.2
ES, auto bw	6.8	7.7	8.7	10.1	16.8	77.8	62.4	48.2	31.7	13.6
AM, auto bw	6.0	7.0	8.0	9.9	19.7	79.0	63.7	49.1	32.4	13.8
NW, fixed bw	5.6	7.2	9.1	14.2	33.7	97.6	82.3	62.2	38.1	14.7
NW, auto bw	5.9	7.4	8.1	10.0	19.8	97.5	81.5	61.2	37.2	14.4
<b>Sample Size T=1200</b>										
ES, fixed bw	5.3	6.0	6.8	8.7	18.8	80.2	65.2	51.4	34.1	14.5
AM, fixed bw	5.2	6.0	7.2	10.6	26.7	80.4	65.5	51.5	34.3	14.6
ES, auto bw	5.5	6.3	6.9	7.4	10.8	79.9	64.9	50.5	33.5	14.2
AM, auto bw	5.3	6.1	6.4	7.4	12.6	80.2	65.2	51.0	33.9	14.4
NW, fixed bw	5.2	6.4	7.9	11.9	29.4	97.8	83.0	63.0	38.6	15.0
NW, auto bw	5.3	6.3	6.6	7.5	12.6	97.8	82.6	62.7	38.1	14.8
<b>Sample Size T=24000</b>										
ES, fixed bw	5.1	5.4	5.8	6.6	11.0	80.2	65.4	51.0	33.8	14.4
AM, fixed bw	5.1	5.4	6.1	7.5	16.5	80.3	65.4	51.1	33.7	14.4
ES, auto bw	5.1	5.4	5.7	5.7	6.2	80.2	65.3	50.9	33.6	14.4
AM, auto bw	5.1	5.4	5.5	5.7	6.4	80.3	65.4	51.0	33.7	14.3
NW, fixed bw	5.1	5.6	6.3	7.9	17.5	97.8	83.1	63.0	38.6	14.9
NW, auto bw	5.1	5.4	5.5	5.6	6.4	97.8	83.1	63.1	38.7	14.9

**Table 3.6: Empirical Bias and Variance.** This table reports empirical bias squared and variance as defined in Section 3.5.2. Data follow our benchmark missing structure, the Bernoulli structure with probability of missing 6/12. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	Empirical Rejection Rate			Size-Adjusted Power						
	0.0	0.3	0.5	0.7	0.9	0.0	0.3	0.5	0.7	0.9
<b>Sample Size T=360</b>										
ES, fixed bw	6.0	7.0	8.0	10.7	23.1	78.9	63.5	49.2	32.7	14.0
AM, fixed bw	5.6	6.7	8.3	12.6	30.9	79.4	64.1	50.0	33.0	14.2
ES, auto bw	6.8	7.7	8.7	10.1	16.8	77.8	62.4	48.2	31.7	13.6
AM, auto bw	6.0	7.0	8.0	9.9	19.7	79.0	63.7	49.1	32.4	13.8
NW, fixed bw	5.6	7.2	9.1	14.2	33.7	97.6	82.3	62.2	38.1	14.7
NW, auto bw	5.9	7.4	8.1	10.0	19.8	97.5	81.5	61.2	37.2	14.4
<b>Sample Size T=1200</b>										
ES, fixed bw	5.3	6.0	6.8	8.7	18.8	80.2	65.2	51.4	34.1	14.5
AM, fixed bw	5.2	6.0	7.2	10.6	26.7	80.4	65.5	51.5	34.3	14.6
ES, auto bw	5.5	6.3	6.9	7.4	10.8	79.9	64.9	50.5	33.5	14.2
AM, auto bw	5.3	6.1	6.4	7.4	12.6	80.2	65.2	51.0	33.9	14.4
NW, fixed bw	5.2	6.4	7.9	11.9	29.4	97.8	83.0	63.0	38.6	15.0
NW, auto bw	5.3	6.3	6.6	7.5	12.6	97.8	82.6	62.7	38.1	14.8
<b>Sample Size T=24000</b>										
ES, fixed bw	5.1	5.4	5.8	6.6	11.0	80.2	65.4	51.0	33.8	14.4
AM, fixed bw	5.1	5.4	6.1	7.5	16.5	80.3	65.4	51.1	33.7	14.4
ES, auto bw	5.1	5.4	5.7	5.7	6.2	80.2	65.3	50.9	33.6	14.4
AM, auto bw	5.1	5.4	5.5	5.7	6.4	80.3	65.4	51.0	33.7	14.3
NW, fixed bw	5.1	5.6	6.3	7.9	17.5	97.8	83.1	63.0	38.6	14.9
NW, auto bw	5.1	5.4	5.5	5.6	6.4	97.8	83.1	63.1	38.7	14.9

To better understand these results, we turn to Table 3.6, which reports the empirical bias and variance for these two estimators under our benchmark simulations. As expected, when using the same fixed bandwidth, the ES estimator has a higher variance than the AM estimator for each sample size and autocorrelation. As discussed in Section 3.3.5, this is because compared to  $\hat{\Omega}^{AM}$ , the  $\hat{\Omega}^{ES}$  estimator includes more high-order lags that are relatively poorly estimated.

With regard to the bias, the ES estimator has better performance for higher autocorrelation parameters, though this effect is mitigated and sometimes reversed in small samples. The poor small sample performance is driven by the first source of bias discussed in Section 3.3.5, that the summation in the autocovariance estimator is divided by the sample size rather than the actual number of observed lags. This bias declines rapidly as the sample size increases. In contrast, the bias behind the overrejection for high autocorrelations is driven by the underweighting of high-order lagged autocovariances. Since the ES estimator places a higher weight on high-order autocovariances, it has lower bias than the AM estimator when the autocorrelation is high. As the sample size grows and the first source of bias becomes less important, the ES estimator is preferred for a larger range of autocovariance parameters.

This bias and variance tradeoff changes when we implement automatic bandwidth selection. The results in Table 3.5 indicate that under this procedure, the AM estimator has a lower rejection probability and is thus preferred at all but the highest level of autocorrelation, for every sample size. To provide further context for this result, Table 3.7 reports the average selected bandwidth for each simulation. We know from den Haan and Levin (1997) that using a higher bandwidth will increase the variance of the estimator while decreasing the bias. Given that the AM estimator has a lower variance than the ES estimator when using a fixed bandwidth, it is not surprising that the automatic bandwidth selection typically chooses a higher bandwidth for the AM estimator than for the ES estimator. The incremental improvement in the bias between the fixed and automatic bandwidth selection is larger for the AM estimator than for the ES estimator. Consequently, under automatic bandwidth selection, the ES estimator is only preferred for extremely high autocorrelation, when the bias of the AM estimator is much higher than that of the ES estimator.

Turning to the size-adjusted power, we find in Table 3.5 that the power of the two estimators is roughly equivalent. Comparing our two main estimators, we have that the power of the AM estimator is generally stronger than that of the ES estimator. Just as for the Newey-West

**Table 3.7: Automatically Selected Bandwidths.** This table reports for a range of autocorrelation parameters and sample sizes the mean of the bandwidth selected by the Newey and West (1994) procedure described in Section 3.5.1. Data follow our benchmark missing structure as described in the Bernoulli structure with probability of missing 6/12. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	6 of 12 missing				
	0.0	0.3	0.5	0.7	0.9
<b>T=360, Fixed bandwidth: <math>m(T)=5</math></b>					
ES, auto bw	5.9	5.1	5.0	6.4	10.0
AM, auto bw	5.4	5.0	6.4	9.8	13.0
NW, auto bw	5.4	5.4	8.0	11.4	14.1
<b>T=1200, Fixed bandwidth: <math>m(T)=6</math></b>					
ES, auto bw	6.4	6.0	7.0	10.8	17.2
AM, auto bw	6.2	6.7	10.5	16.5	22.2
NW, auto bw	6.2	8.1	13.2	18.8	23.9
<b>T=24000, Fixed bandwidth: <math>m(T)=12</math></b>					
ES, auto bw	13.2	14.6	20.2	33.3	65.2
AM, auto bw	13.2	18.0	31.8	55.7	94.0
NW, auto bw	13.2	23.5	39.0	62.3	98.3

**Table 3.8: Varying Fraction of Missings.** This table reports for a range of autocorrelation parameters the empirical rejection rate and size-adjusted power for a sample size of  $T = 360$  under varying probability of missing observation. Data follow our benchmark missing structure as described in Section 3.5.1, the Bernoulli structure with probability of missing 4/12, 6/12, or 8/12. The Newey-West estimation provided for comparison uses the full series of simulated data without any missing observations. See Sections 3.3 and 3.4 for details on the estimators and the regression context, and Section 3.5.1 for details on the simulation parameters.

Autocorrelation ( $\phi$ ):	Empirical Rejection Rate					Size-Adjusted Power				
	0.0	0.3	0.5	0.7	0.9	0.0	0.3	0.5	0.7	0.9
<b>4 of 12 missing</b>										
ES, fixed bw	5.9	7.0	8.2	11.3	26.6	89.2	72.7	56.2	35.8	14.5
AM, fixed bw	5.6	6.9	8.6	13.1	32.4	89.4	73.2	56.6	36.2	14.6
ES, auto bw	6.4	7.5	8.5	9.7	17.4	88.7	72.0	55.0	34.9	14.0
AM, auto bw	5.9	7.2	8.0	9.8	19.8	89.2	72.5	55.6	35.5	14.2
<b>6 of 12 missing</b>										
ES, fixed bw	6.0	7.0	8.0	10.7	23.1	78.9	63.5	49.2	32.7	14.0
AM, fixed bw	5.6	6.7	8.3	12.6	30.9	79.4	64.1	50.0	33.0	14.2
ES, auto bw	6.8	7.7	8.7	10.1	16.8	77.8	62.4	48.2	31.7	13.6
AM, auto bw	6.0	7.0	8.0	9.9	19.7	79.0	63.7	49.1	32.4	13.8
<b>8 of 12 missing</b>										
ES, fixed bw	6.4	7.1	7.8	9.6	18.7	61.0	50.5	41.2	29.1	13.6
AM, fixed bw	5.6	6.4	7.6	11.2	28.3	62.5	51.9	42.5	29.9	13.9
ES, auto bw	7.8	8.4	9.1	10.5	15.7	57.6	48.2	39.6	27.7	13.1
AM, auto bw	5.9	6.6	7.6	9.5	19.4	61.9	51.4	41.8	29.2	13.6
<b>Full sample benchmark</b>										
NW, fixed bw	5.6	7.2	9.1	14.2	33.7	97.6	82.3	62.2	38.1	14.7
NW, auto bw	5.9	7.4	8.1	10.0	19.8	97.5	81.5	61.2	37.2	14.4

results, we have that the power falls as the autocorrelation increases, and that the size-adjusted power does not vary as the sample size increases. Unsurprisingly, we can see that the power of the test under the missing structure is weaker than for the full series, due to the smaller observed sample size. This effect is mitigated at high autocorrelation, however, and we can see that under very high autocorrelation, the power is roughly equal for the Newey-West application to the full series and for the application of our two estimators to the series with missing observations.

Finally, Table 3.8 presents results for varying fractions of missing observations. At low autocorrelation, the rejection rate increases as the fraction of missing observations increases. This is likely driven by the first source of finite sample bias discussed in Section 3.3.5, which gets worse as the fraction of missing observations increases. In contrast, at high autocorrelation, the rejection rate falls as the fraction of missing observations increases. This effect is likely due to the fact that when a higher fraction of observations is missing, the observed process is less persistent, and the estimators are better able to overcome the underweighting of the higher order autocovariances. Putting these two effects together, we have that the AM estimator is preferred for a larger range of autocorrelation parameters when a higher fraction of data is missing. This is consistent with our previous finding, that the AM estimator is preferred when the serial correlation is low.

Overall, these simulation results are consistent with our theoretical findings. We show that the ES and AM estimators both are well-behaved for random and deterministic missing structures. In general, the ES and AM estimators are preferred to imputation methods, which may induce serial correlation in the imputed series, resulting in more severe bias and overrejection. In finite samples, we find that for the same fixed bandwidth, the ES estimator is generally less biased than the AM estimator, but has larger variance. Consequently, the ES estimator is preferred when autocorrelation is high, as the bias will dominate the mean squared error in these cases. Conversely, when autocorrelation is low, variance dominates the mean squared error, and the AM estimator is preferred.

### *3.6 Empirical Application: Recursive Tests for a Positive Sample Mean*

In this section, we present an application of our estimators to test for positive returns to investing in commodities futures contracts. While commodities tend to have positive returns on

average, they also have extremely high volatility. We apply our methods to construct the sample mean and standard error of the returns series, and test whether the returns are statistically distinguishable from zero. Due to the structure of commodities futures contracts, the time series of returns have missing observations, and are therefore a natural application for our estimators.

Commodities futures contracts specify the quantity and price of a commodity to be traded on a predetermined expiration date at some point in the future. While some commodities have contracts expiring every month, many have contract months that are irregularly spaced throughout the year. For example, copper futures contracts were available for only March, May, July, September, and December until full monthly coverage began in 1989. Consequently, if we want to calculate the monthly return to investing in commodities futures over a long sample period, our monthly returns data will either reflect a fluctuating time-to-maturity, or will be an incomplete series with irregularly spaced missing observations.

For each commodity, we calculate the return as the percentage change in the price of the contract over the three months prior to the contract's expiration. For instance, we calculate the return to the December contract as the change in price from the last trading day in August to the last trading day in November. Since we want our returns series to reflect the change in the price over the same time-to-maturity, we are only able to calculate this return in the months immediately preceding contract expiration. For copper, this means we will have only five observations each year.<sup>6</sup> The existence of irregularly spaced commodities futures contracts results in a deterministic cyclical pattern of missing observations in the constant-maturity returns series. Contract availability and spacing differs across commodities, but tends to remain constant year to year for each commodity.

In this application, we calculate the sample mean for three representative commodities: copper, soybean oil, and lean hogs.<sup>7</sup> We apply our Equal Spacing, Amplitude Modulated, and Imputation estimators to calculate the HAC standard error of the sample mean, and test the hypothesis that the sample mean is significantly different from zero at the five percent level of significance. Under the Imputation method, the missing observations are linearly imputed, while the Equal

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<sup>6</sup> We restrict the full sample for copper to the period with missing contract months, 1960-1989.

<sup>7</sup> We selected one representative commodity from each of the major commodity types (metals, animal, and agricultural). We omit the energy commodities, as these commodity contracts do not have missing contract months.

Spacing and Amplitude Modulated estimators use only the observed data to calculate the mean and standard error. Because it does not provide robust results, we do not provide inference results using the Parzen estimator.

In addition to performing the t-test for the full sample, we use a recursive method to compare our three methods across various sample sizes. For each commodity, we first calculate the sample mean and standard error over the first twelve months of the sample, and perform a t-test of the mean for just this sample window. We then recursively perform the same test for an expanding sample window, adding one month at a time until the full sample is covered. Lastly, we also perform the same type of recursive tests starting with the last twelve months of the full sample. In this backwards recursive test, we use an earlier starting month in each iteration until the sample window again covers the full sample. Having the forward and backwards recursive results allows us to note any structural shifts that may have occurred over time. Figures 3.2 and 3.3 depicts the results, and Table 3.9 provides an overview of rejection rates over the full set of recursive results.

Figure 3.2 shows the sample mean and 95% confidence intervals constructed using the Imputation and Equal Spacing methods. (The Amplitude Modulated estimator is omitted from the figure for clarity.) The means are very similar across the two methods for most sample windows. The primary difference is that as expected, the Imputation method estimate of the standard error is generally smaller than the Equal Spacing and Amplitude Modulated estimates, resulting in a higher rejection rate for Imputation. In the figure, we have shaded the samples for which the hypothesis is rejected under the Imputation method but not rejected under Equal Spacing. The fraction of shaded iterations ranges from 3.2% for lean hogs to 18.2% for copper in the forward recursive results. In the backwards recursive results, the fraction of shaded iterations ranges from 0% for soybean oil to 18.4% for copper.

It is unsurprising that the Imputation method results in a higher rejection rate relative to the Equal Spacing and Amplitude Modulated methods. While in many cases naive imputation is likely to bias the parameter of interest, we have tried to construct an example with little to no bias. However, since the imputed observations are constructed using the observed data rather than drawn from the underlying distribution of data, we note that the standard error of the imputed series is likely lower than the standard error of the observed series. Additionally, the induced high serial correlation of the imputed series will make it likely that the standard error of the imputed

series will be underestimated by the Newey-West estimator. For all of these reasons, it is likely that we will have overrejection in hypothesis testing. Without knowing the true mean of the series, in this application we cannot know which method gives us the “right” conclusion for a larger fraction of the tests. Yet, the examples illustrate that the Imputation and Equal Spacing methods can lead to different conclusions in a number of cases, depending on the available data.

**Table 3.9: Rejection Rates for Recursive Tests.** This table reports the rejection rate for the test of the hypothesis that the recursive sample mean is equal to zero. The final row of the first panel corresponds to the shaded areas in Figures 3.2 and 3.3.

	Panel A: 5% significance level					
	Copper		Soybean Oil		Lean Hogs	
	Forward	Backward	Forward	Backward	Forward	Backward
Imputation	64.8	28.2	25.3	0.0	93.7	18.3
Amplitude Modulated	59.7	26.5	18.1	0.0	91.4	8.8
Equal Spacing	46.7	9.8	12.4	0.0	90.7	8.4
AM rejects, IM does not	0.0	1.2	0.0	0.0	0.2	0.0
ES rejects, IM does not	0.0	0.0	0.0	0.0	0.2	0.0
IM rejects, AM does not	5.2	2.9	7.2	0.0	2.4	9.5
<b>IM rejects, ES does not</b>	<b>18.2</b>	<b>18.4</b>	<b>12.9</b>	<b>0.0</b>	<b>3.2</b>	<b>9.9</b>

	Panel B: 10% significance level					
	Copper		Soybean Oil		Lean Hogs	
	Forward	Backward	Forward	Backward	Forward	Backward
Imputation	77.2	34.6	63.5	22.2	96.1	26.1
Amplitude Modulated	72.9	34.0	29.7	0.0	94.6	15.1
Equal Spacing	65.4	31.4	24.8	0.0	93.5	15.1
AM rejects, IM does not	0.0	0.0	0.0	0.0	0.0	0.0
ES rejects, IM does not	0.0	0.0	0.0	0.0	0.0	0.0
IM rejects, AM does not	4.3	0.6	33.8	22.2	1.5	11.0
IM rejects, ES does not	11.8	3.2	38.7	22.2	2.6	11.0

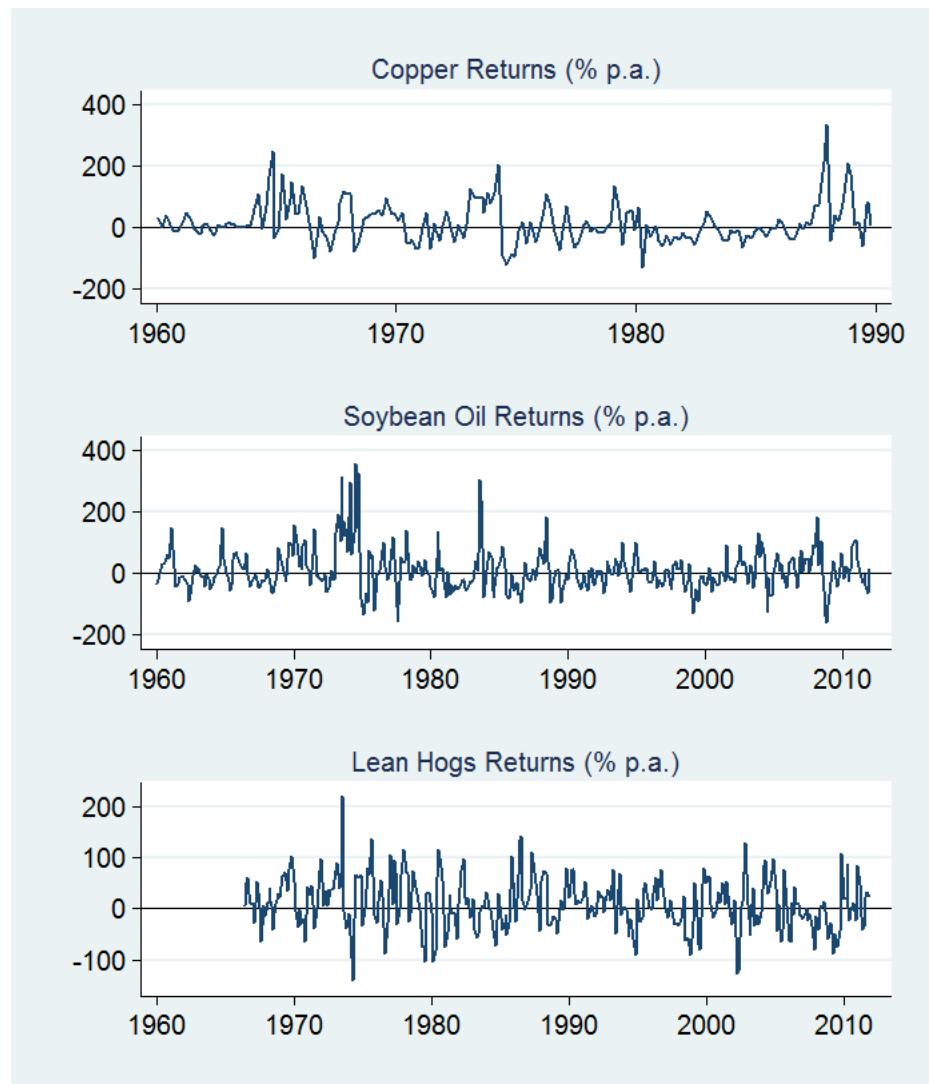
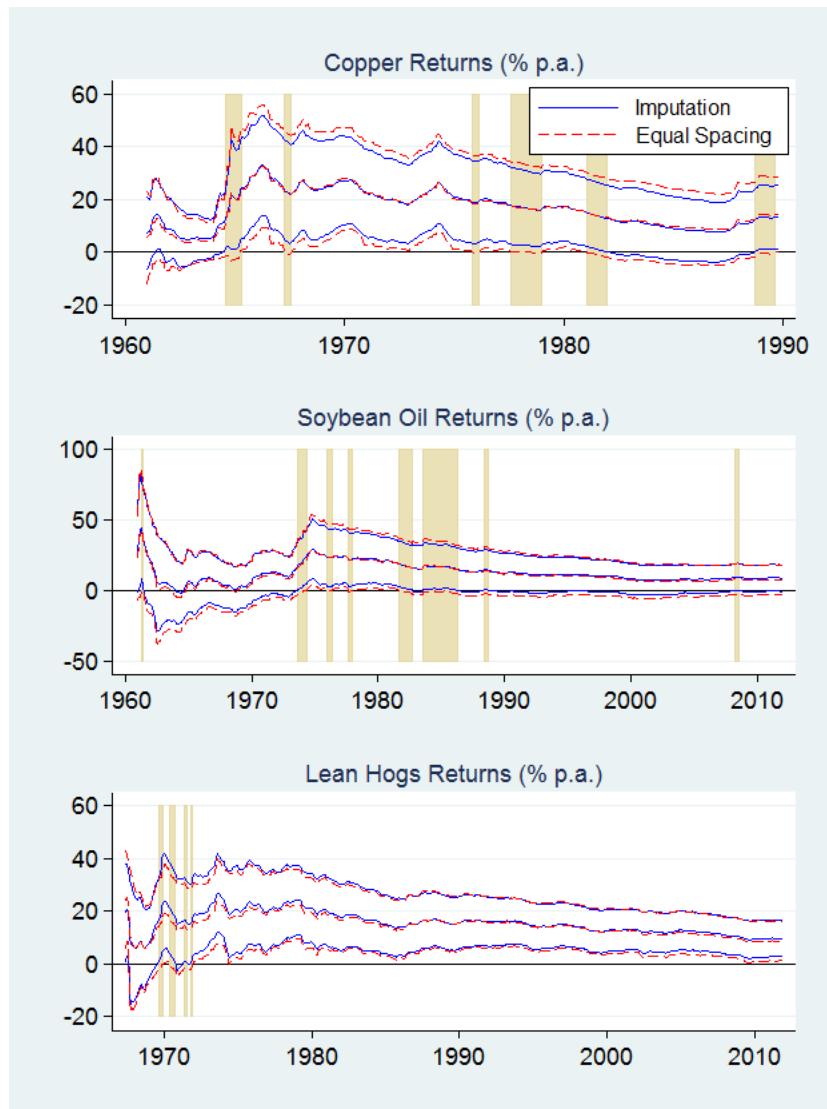
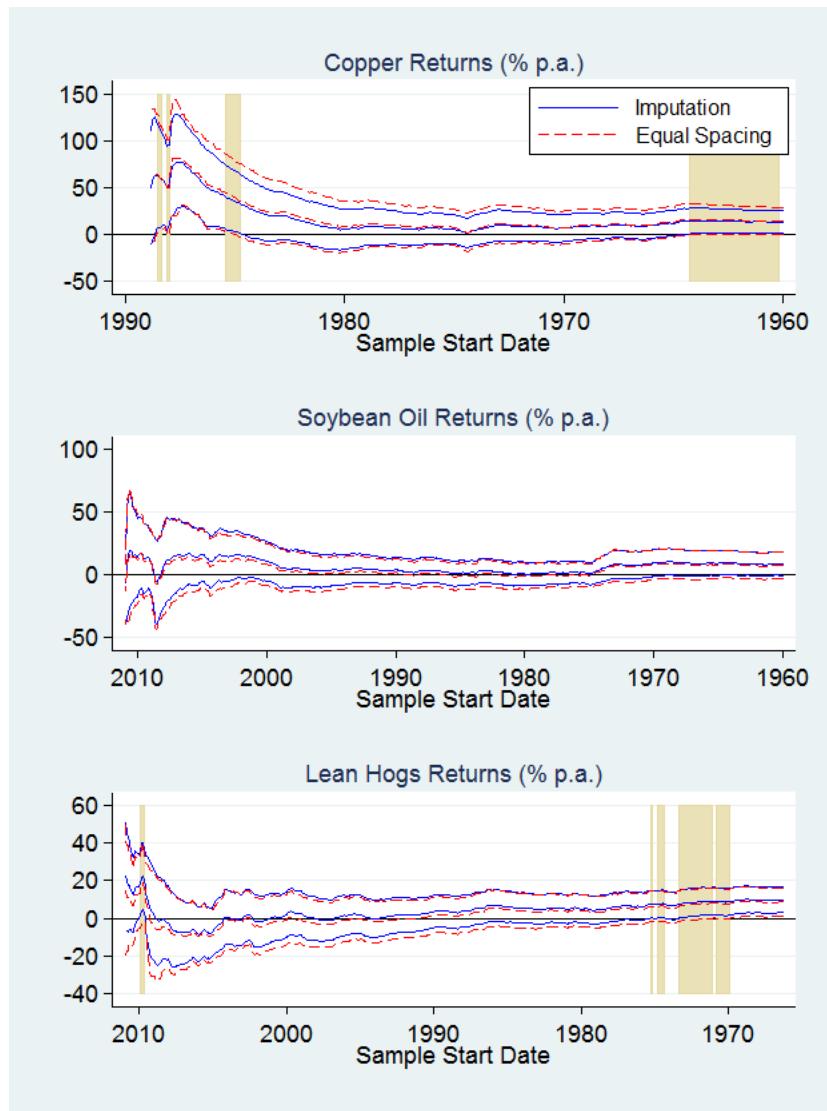


Figure 3.1: Time Series of Returns.



**Figure 3.2: Recursive Test Results - Sample Mean and Error Bands.** This figure plots the recursive sample mean and 95% confidence intervals for the imputation and equal spacing methods. The recursive sample mean is calculated as the mean of returns over the period from the start of the sample to sample end date (plotted on the x-axis). Shaded areas indicate recursive samples for which the imputation method finds statistical significance while the equal spacing method finds none.



**Figure 3.3: Backwards Recursive Test Results - Sample Mean and Error Bands.** This figure plots the backwards recursive sample mean and 95% confidence intervals for the imputation and equal spacing methods. The backwards recursive sample mean is calculated as the mean of returns over the period from the end of the sample to sample start date (plotted on the x-axis). Note that the sample size is increasing with earlier start dates, moving left to right on the plot. Shaded areas indicate recursive samples for which the imputation method finds statistical significance while the equal spacing method finds none.

### 3.7 Conclusion

This paper provides two simple solutions to the common problem of conducting heteroskedasticity and autocorrelation (HAC) robust inference when some observations are missing. Our definitions of the Amplitude Modulated and Equal Spacing estimators are simply formal descriptions of ad hoc practices that are already in use. Yet, by formalizing these procedures, we are able to provide theoretical results that clear up some of the existing confusion in the literature. By studying the estimators and their properties, we provide justification for their past application to daily business data and through common statistical software packages such as Stata. We also justify their application under a wide variety of missing data structures, including deterministic and cyclical missing structures.

Our theoretical discussion of the estimators highlights a few main conclusions. After establishing the difference between the long-run variance of the underlying and observed series, we demonstrate that our Amplitude Modulated and Equal Spacing estimators both are consistent for the long-run variance of the observed series. This distinction is important, as we also show that we require the long-run variance of the observed series to construct t-statistics for inference, such as in a regression setting. In addition to discussing the asymptotic properties of the estimators, we provide some discussion of their finite sample properties, based on our previous understanding of the finite sample properties of HAC estimators more generally.

We also provide simulation results and apply our estimators to a real world problem involving missing data in commodities futures returns. These results provide further evidence supporting our description of the asymptotic and finite sample behavior of the estimators. In addition, the results of these exercises are used to draw conclusions that can provide guidance to practitioners who need to decide between the estimators for applied work. Though this paper focuses on applying the estimators in a time series setting, they can also be naturally extended for application in a panel setting. We leave this extension for future work.

## APPENDIX

## A. APPENDIX TO CHAPTER 1

### A.1 A Real-World Example

Figure A.1 illustrates a concrete example of swapping an LC yield into a dollar yield using CCS. Let  $S$  denote the spot peso/dollar exchange rate. Suppose a dollar-based investor lends to the Mexican government by purchasing LC bonds traded at par with notional amount equal to  $S$  pesos. If the government does not default, she will receive  $y$  percent coupons at each coupon date and the principal of  $S$  pesos at maturity. Without any currency hedging, even if the bond does not default, the dollar payoff is uncertain since both the coupons and the principal are subject to exchange rate risk. If the dollar investor does not wish to bear the currency risk, she can enter into a CCS package with a swapmaker (e.g., a bank) to lock in a dollar yield. The details are as follows. At the inception of the swap, the dollar investor gives 1 dollar to the bank. In exchange, she receives  $S$  pesos from the bank to lend to the Mexican government. At each coupon date, the dollar investor passes the  $y$  percent fixed coupons she receives in pesos from the Mexican government to the bank and receives  $y - \rho$  percent fixed coupon in dollars, where  $\rho$  is the fixed peso for dollar swap rate. At the maturity of the swap, the investor gives the  $S$  pesos in principal repaid by the government to the bank and gets 1 dollar back. Therefore, the net cash flow of the investor is entirely in dollars. The CCS swap package transforms the LC bond into a synthetic dollar bond that promises to yield  $y - \rho$  percent.

### A.2 Yield Curve Construction

Zero-coupon LC and FC yield curves for our sample countries are obtained or constructed from three main sources.<sup>1</sup> First, our preference is to use zero-coupon LC curves constructed by the central bank of government agencies when they are available. Second, when national data are

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<sup>1</sup> Full details on LC and FC yield curve construction are given in the data appendix Table A.3.

unavailable, we use the Bloomberg Fair Value (BFV) curve. The BFV curves are par yield curves estimated by Bloomberg on actively traded bonds using piecewise linear zero-coupon curves (Lee, 2007). These curves often serve as the benchmark reference rate in respective currencies. Traders using the Bloomberg trading platform can easily select these BFV curves for asset swap analysis. We use the standard Nelson-Siegel methodology to convert the par yield curves into zero curves with the scaling parameter for the curvature factor fixed using the value in Diebold and Li (2006).

Finally, for countries without national data or BFV curves, and to ensure reliability of the existing BFV curves, we estimate zero coupon yield curves using the individual bond data. We collected these data from Bloomberg by performing an exhaustive search for all available yields on active and matured bonds under <Govt TK> for our sample countries. We supplement Bloomberg FC bond yield data with additional data from Cbonds. We use nominal, fixed-coupon, bullet bonds without embedded options. LC curve estimation follows the Diebold and Li (2006) formulation of Nelson and Siegel (1987) and FC curve estimation follows Arellano and Ramanarayanan (2012) by fitting level, slope and curvature factors to the spread of zero-coupon FC curves over the corresponding dollar, Euro (Bundesbank), Yen and Sterling zero-coupon Treasury yields, depending on the currency denomination of the FC bonds. As in Arellano and Ramanarayanan (2012), we perform yield curve estimation when there are at least four bond yields observed on one day. We calculate yields using estimated parameters only up to the maximum tenor of the observed yields to avoid problems from extrapolation. When the Bloomberg BFV curves exists, our estimated yield curves track them very closely (details available upon request). However, since Bloomberg has partially removed historical yields for matured bonds from the system, the BFV curves offer more continuous series than our estimates. Therefore, we use BFV curves when they are available. For countries without BFV curves or earlier samples when BFV curves are not available, our estimated zero-coupon curves are used instead.

### A.3 N-Period Extension

#### A.3.1 Risk-Free Rates

Given the global SDF  $-\log M_{t+1} = -m_{t+1}^* = \psi_0 - \psi z_t^w - \gamma \xi_{t+1}^w$ . The log price of an  $n$ -period risk-free bond is given by:

$$-p_{nt} = A_n + B_n z_t^w,$$

where

$$B_n = \phi^w B_{n-1} - \psi = -\psi[1 - (\phi^w)^n]/(1 - \phi^w)$$

$$A_n - A_{n-1} = \psi_0 + B_{n-1} c^w - (\gamma - B_{n-1})^2/2.$$

#### A.3.2 FC Bonds

We conjecture the price of a  $n$ -period FC bond is given by

$$-p_{nt}^{FC} = A_n^{FC} + B_n^{FC} z_t^w + C_n^{FC} z_t^i.$$

Since  $p_{nt}^{FC} = E_t(m_{t+1} + i_{t+1}^{FC} + p_{n-1,t+1}^{FC}) + Var_t(m_{t+1} + i_{t+1}^{FC} + p_{n-1,t+1}^{FC})/2$ , where  $i_{t+1}^{FC} \equiv \log(I_{t+1}^{FC})$ , we can solve for the price of a  $n$ -period FC bond as the

$$B_n^{FC} = \phi^w B_{n-1}^{FC} + \lambda_w^{FC} - \psi = (\lambda_w^{FC} - \psi)[1 - (\phi^w)^n]/(1 - \phi^w)$$

$$C_n^{FC} = \phi^c C_{n-1}^{FC} + \lambda_i^{FC} = \lambda_i^{FC}[1 - (\phi^w)^n]/(1 - \phi^w)$$

$$A_n^{FC} - A_{n-1}^{FC} = \psi_0 + \lambda_0^{FC} + B_{n-1}^{FC} c^w + C_{n-1}^{FC} c^i - (B_{n-1}^{FC} + \sigma_{\lambda w}^{FC} - \gamma)^2/2 - (C_{n-1}^{FC} + \sigma_{\lambda c}^{FC})^2/2$$

The expected excess returns on FC bonds is given by

$$\begin{aligned} E_t(r_{n,t+1}^{FC}) - y_{1t}^* + Var_t(r_{n,t+1}^{FC})/2 &= -Cov_t(m_{t+1}, r_{n,t+1}^{FC}) = \gamma(B_{n-1} + \sigma_{\lambda w}^{FC}) \\ &= \gamma \left[ (\lambda_w^{FC} - \psi) \frac{1 - (\phi^w)^{n-1}}{1 - \phi^w} + \sigma_{\lambda w}^{FC} \right] \equiv \gamma \delta_n^{FC} \end{aligned}$$

We can then compute the FC spread as

$$s_{nt}^{FC/US} = \frac{1}{n} \left[ \alpha_{n0}^{FC} + \lambda_w^{FC} \frac{1 - (\phi^w)^{n-1}}{1 - \phi^w} z_t^w + \lambda_i^{FC} \frac{1 - (\phi^i)^{n-1}}{1 - \phi^i} z_t^i \right]. \quad (\text{A.1})$$

### A.3.3 LC Bonds

Again, we assume a downward sloping clientele demand for the  $n$ -period LC

$$d_{nt}^{SLC}/W = \kappa_n (-p_{nt}^{SLC} - \beta_{nt})$$

for  $\kappa_n > 0$ . We assume that  $\beta_{nt}$  is also affine in factors. For analytical convenience, we parametrize  $\beta_n$  as

$$\beta_{nt} = -\tilde{p}_{nt}^{SLC} + \theta_{n0} + \theta_{nc} z_t^i + \theta_{nw} z_t^w,$$

where  $-\tilde{p}_{nt}^{SLC} = \tilde{\lambda}_{n0} + \tilde{\lambda}_{nc} z_t^i + \tilde{\lambda}_{nw} z_t^w$  is the swapped LC price that implies zero expected simple excess returns on swapped LC bonds as follows:

$$-\tilde{p}_{nt}^{SLC} = -E_t(i_{t+1}^{SLC} + p_{n-1,t+1}^{SLC}) - Var_t(i_{t+1}^{SLC} + p_{n-1,t+1}^{SLC})/2 + y_{1t}^*$$

Thus,  $\theta_{n0}$ ,  $\theta_{nc}$  and  $\theta_{nw}$  measure deviations from zero expected returns in the absence of arbitrage. We conjecture the equilibrium swapped LC price takes the form

$$-p_{nt}^{SLC} = -\tilde{p}_{nt}^{SLC} + b_{n0} + b_{nc} z_t^i + b_{nw} z_t^w = (\tilde{\lambda}_{n0} + b_{n0}) + (\tilde{\lambda}_{nc} + b_{nc}) z_t^i + (\tilde{\lambda}_{nw} + b_{nw}) z_t^w$$

Therefore, the expected simple excess returns on swapped LC is simply

$$E_t r_{t+1}^{SLC} - y_{1t}^* + Var_t(r_{t+1}^{SLC})/2 = (b_{n0} + b_{nc} z_t^i + b_{nw} z_t^w) - (\tau_{n0} - q_{n0}).$$

Solving the arbitrage's portfolio problem gives<sup>2</sup>

$$\begin{aligned}
b_{n0} &= \frac{\kappa_n \theta_{n0}}{\kappa_n + \frac{1}{\gamma_a(1-\rho_{rn}^2)(\sigma_n^{SLC})^2}} + \frac{q_{n0}}{\kappa_n \gamma_a (1 - \rho_{rn}^2) (\sigma_n^{SLC})^2 + 1} \\
&\quad + \frac{\rho_{rn} \frac{\sigma_n^{SLC}}{\sigma_n^{FC}} \left[ (\lambda_w^{FC} - \psi) \frac{1 - (\phi^w)^{n-1}}{1 - \phi^w} + \sigma_{\lambda w}^{FC} \right]}{\kappa_n \gamma_a (1 - \rho_{rn}^2) (\sigma_n^{SLC})^2 + 1} \gamma \equiv \omega_n \theta_{n0} + (1 - \omega_n) (\tau_{n0} - q_{n0}) + \delta_n^{SLC} \gamma \quad (\text{A.2}) \\
b_{nc} &= \frac{\kappa_n \theta_{nc}}{\kappa_n + \frac{1}{\gamma_a(1-\rho_{rn}^2)(\sigma_n^{SLC})^2}} \equiv \omega_n \theta_{nc} \\
b_{nw} &= \frac{\kappa_n \theta_{nw}}{\kappa_n + \frac{1}{\gamma_a(1-\rho_{rn}^2)(\sigma_n^{SLC})^2}} \equiv \omega_n \theta_{nw}, \quad (\text{A.3})
\end{aligned}$$

where volatility and correlation of asset returns are given by

$$(\sigma_n^{SLC})^2 = [(\tilde{\lambda}_{n-1,w} + b_{n-1,w}) + \sigma_{\lambda w}^{SLC}]^2 + [(\tilde{\lambda}_{n-1,c} + b_{n-1,c}) + \sigma_{\lambda c}^{SLC}]^2 \quad (\text{A.4})$$

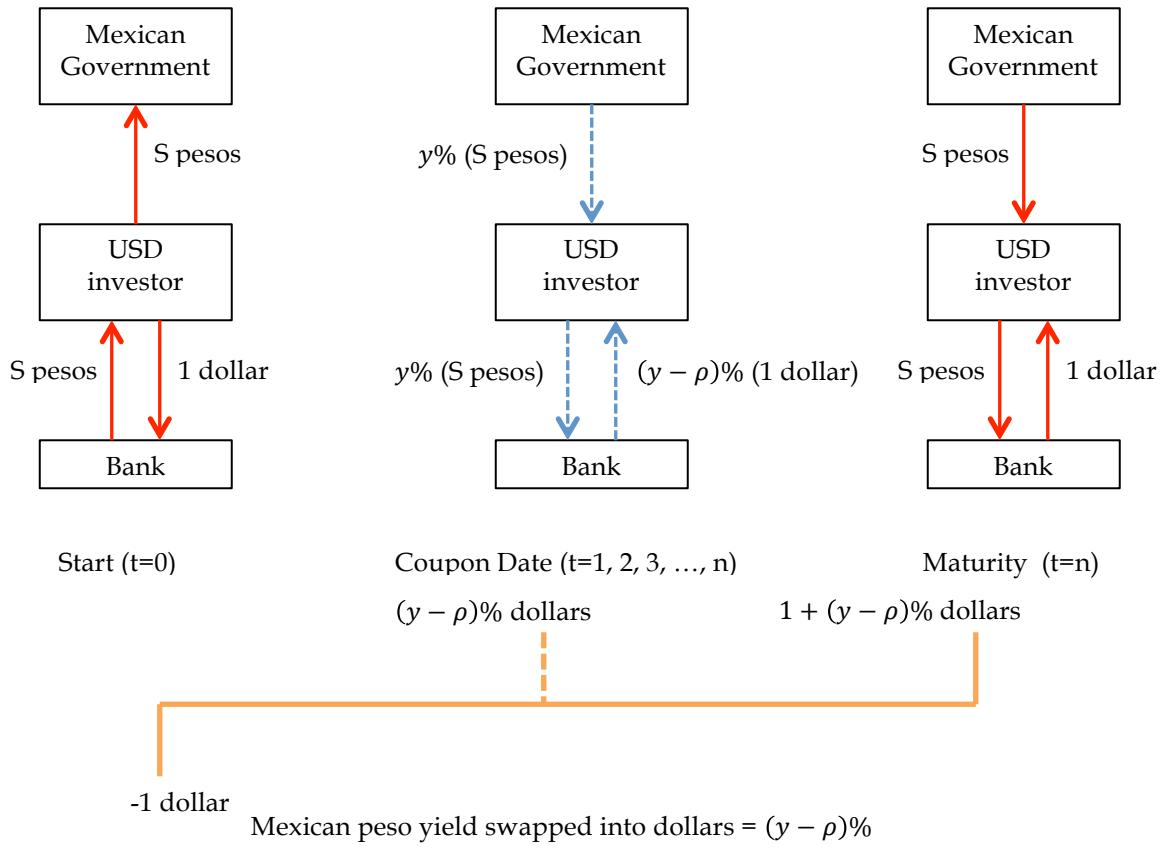
$$(\sigma_n^{FC})^2 = \left[ (\lambda_w^{FC} - \psi) \frac{1 - (\phi^w)^{n-1}}{1 - \phi^w} + \sigma_{\lambda w}^{FC} \right]^2 + \left[ \lambda_c^{FC} \frac{1 - (\phi^c)^{n-1}}{1 - \phi^w} + \sigma_{\lambda c}^{FC} \right]^2$$

$$\begin{aligned}
\rho_{rn} &= [(\tilde{\lambda}_{n-1,w} + b_{n-1,w}) + \sigma_{\lambda w}^{SLC}] \left[ (\lambda_w^{FC} - \psi) \frac{1 - (\phi^w)^{n-1}}{1 - \phi^w} + \sigma_{\lambda w}^{FC} \right] \\
&\quad + [(\tilde{\lambda}_{n-1,c} + b_{n-1,c}) + \sigma_{\lambda c}^{SLC}] \left[ \lambda_c^{FC} \frac{1 - (\phi^c)^{n-1}}{1 - \phi^w} + \sigma_{\lambda c}^{FC} \right]. \quad (\text{A.5})
\end{aligned}$$

Since  $\sigma_n^{SLC}$  and  $\rho_{rn}$  also depend on  $b_{n-1,w}$ , local clientele demand  $\theta_{m-1,w}$  and  $\theta_{m-1,c}$  for  $m \leq n$  also affects volatility of swapped LC bond excess returns and correlation between swapped LC and FC excess returns. The  $n$ -period equilibrium solution in A.3 is exactly analogous to the one-period solution in Equation 1.1, and thus we can generalize Propositions 2 a to the  $n$ -period case.

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<sup>2</sup> For simplicity, we assume the arbitrageur arbitrages between swapped LC and FC bonds of the same maturity. Allowing additional cross-maturity arbitrage for swapped LC bonds does not add more insights given FC bonds are already integrated across different maturities.



**Figure A.1: An Illustration of Swap Covered Local Currency Investment.** This figure illustrates how a dollar based investor can use a fixed peso for fixed dollar cross-currency swap package to fully hedge currency risk for all coupons and the principal of a Mexican peso denominated LC bond and receive fixed dollar cash flows. We let  $S$  denote the spot peso/dollar exchange rate at the inception of the swap,  $y$  denote the yield on the peso bond, and  $\rho$  denote the fixed peso for fixed dollar swap rate. By purchasing the peso bond while entering the asset swap, the LC bond is transformed into a dollar bond with a dollar yield equal to  $y - \rho$ .

**Table A.1: Cross-Currency Swaps and Currency Forward Comparison, 2005-2011.** This table reports summary statistics for 1-year fixed for fixed cross currency swap (CCS) rates and 1-year offshore forward premium (*Fwd*) implied by outright forward contracts. Column 1 lists whether the currency swap is non-deliverable. Column 2 lists the name of the local floating leg against U.S. Libor if the currency swap consists of a plain-vanilla interest rate swap and a cross-currency basis swap.  $\text{Corr}(\text{CCS}, \text{Fwd})$  reports correlation between swap rates and forward rates. The difference between the two variables are reported in the last column ( $\text{CCS}-\text{Fwd}$ ). Forward rates are from Datastream and fixed for fixed CCS rates are computed by authors based on CCS and interest rate swap data from Bloomberg. Data are at daily frequency for the sample periods 2005-2011.

Country	NDS	Floating Leg	$\text{Corr}(\text{CCS}, \text{Fwd})$	CCS	<i>Fwd</i>	$\text{CCS}-\text{Fwd}$
Brazil	Yes	N/A	97.16	7.19 (1.28)	7.45 (1.28)	-0.27 (0.30)
Colombia	Yes	N/A	99.19	3.52 (2.25)	3.57 (2.24)	-0.05 (0.26)
Hungary	No	Bubor	99.16	3.75 (1.35)	3.80 (1.41)	-0.04 (0.27)
Indonesia	Yes	N/A	97.79	5.67 (3.48)	5.61 (3.82)	-0.06 (0.83)
Israel	Yes	Telbor	98.10	0.52 (0.74)	0.48 (0.74)	0.05 (0.13)
Mexico	No	TIIE	99.58	3.68 (1.24)	4.05 (1.32)	-0.37 (0.14)
Peru	Yes	N/A	98.76	0.98 (1.38)	0.96 (1.42)	0.02 (0.22)
Philippines	Yes	N/A	97.25	1.96 (2.00)	1.83 (2.02)	-0.13 (0.47)
Poland	No	Wibor	98.96	1.69 (1.62)	1.47 (1.51)	0.23 (0.25)
Turkey	No	N/A	98.69	9.36 (2.90)	9.51 (2.93)	-0.15 (0.15)
Total			98.68	3.95 (3.42)	3.99 (3.54)	-0.04 (0.40)

**Table A.2: Half of Bid-ask Spreads on FX Spots, Forwards and Swaps, 2005-2011.** This table reports mean and standard deviations of half of the bid-ask spreads of FX forward and CCS contracts in basis points for 10 sample countries at daily frequency from 2005 to 2011. Columns 1 to 4 report half of annualized bid-ask spreads for FX forward contracts at 1, 3, 6, and 12 months. Column 5 reports the half of the bid-ask for the spread for the 5-year swap contracts. Annualized standard deviations are reported in the parentheses. Spot and Forward data use closing quotes from WM/Reuter (access via Datastream) with the exceptions of Indonesia and Philippines for which the offshore forward rates use closing quotes of non-deliverable forwards from Tullet Prebon (access via Datastream). Swap rates are from Bloomberg.

Country	(1) 1M Fwd	(2) 3M Fwd	(3) 6M Fwd	(4) 1Y Fwd	(5) 5Y CCS
Brazil	98.34 (17.1)	53.80 (18.3)	30.99 (15.0)	22.41 (14.0)	32.13 (13.5)
Colombia	123.37 (14.7)	68.71 (12.3)	42.23 (9.43)	30.19 (10.8)	16.24 (10.7)
Hungary	112.04 (10.8)	52.88 (11.2)	37.88 (14.5)	28.08 (23.1)	18.54 (14.2)
Indonesia	315.96 (69.8)	139.57 (51.3)	90.30 (47.3)	52.87 (37.9)	37.49 (23.1)
Israel	88.03 (12.9)	36.52 (8.50)	23.86 (6.78)	16.62 (7.45)	11.39 (4.31)
Mexico	31.20 (7.76)	12.88 (5.65)	8.58 (4.94)	6.12 (6.12)	8.59 (6.17)
Peru	100.25 (13.7)	47.61 (10.0)	29.39 (6.97)	23.87 (6.92)	16.00 (7.15)
Philippines	126.57 (7.69)	46.34 (4.53)	37.24 (5.72)	27.05 (5.79)	28.00 (14.7)
Poland	72.40 (8.56)	27.38 (5.80)	17.67 (6.86)	11.98 (6.30)	11.50 (8.33)
Turkey	126.76 (25.8)	59.79 (19.4)	41.72 (18.9)	25.85 (15.2)	11.00 (8.14)
Total	117.55 (33.3)	53.75 (25.6)	35.41 (23.5)	23.96 (20.1)	19.07 (15.7)

*Table A.3: Data Sources and Variable Construction.*

Country	Curve Currency	Zero-Coupon Curve Type	Data Source
Brazil	LC	Svensson	Brazilian Financial and Capital Market Associations (ANBIMA)
	FC	Nelson-Siegel	Bloomberg Fair Value par to zero
Colombia	LC	Nelson-Siegel	Bloomberg Fair Value par to zero
	FC	Nelson-Siegel	Bloomberg Fair Value par to zero
Hungary	LC	Svensson	Hungary Government Debt Management Office (AKK)
	FC	Nelson-Siegel	Authors' estimation based on individual bond prices Bloomberg and CBonds
Indonesia	LC	Nelson-Siegel	Bloomberg Fair Value par to zero
	FC	Nelson-Siegel	Authors' estimation based on individual bond prices Bloomberg and CBonds
Israel	LC	Svensson	Bank of Israel
	FC	Nelson-Siegel	Authors' estimation based on individual bond prices Bloomberg and CBonds
Mexico	LC	Nelson-Siegel	Bloomberg Fair Value par to zero
	FC	Nelson-Siegel	Bloomberg Fair Value par to zero
Peru	LC	Nelson-Siegel	Bloomberg Fair Value par to zero
	FC	Nelson-Siegel	BFV and authors' estimation based on Bloomberg individual bond prices
Philippines	LC	Nelson-Siegel	Philippines Dealing and Exchange Corp
	FC	Nelson-Siegel	Bloomberg Fair Value par to zero
Poland	LC	Nelson-Siegel	Bloomberg Fair Value par to zero
	FC	Nelson-Siegel	Authors' estimation based on individual bond prices Bloomberg and CBonds
Turkey	LC	Svensson	Central Bank of Turkey
	FC	Nelson-Siegel	Bloomberg Fair Value par to zero
Variable	Data Source	Description	
CDS	Bloomberg	Sovereign credit default swaps at various tenors denominated in dollars.	
<i>lc eq vol</i>	Datastream	Local equity volatility: volatility of daily equity returns measured in local currency computed using backward-looking rolling windows equal to 30 days.	
BBB/T	Datastream	Merrill-Lynch BBB U.S. corporate bond spread over the 10-year U.S. Treasury yield	
CFNAI	Chicago Fed	Chicago Fed National Activity Index	
VIX	WRDS	Implied volatility on S&P index options.	
$\Delta IP$	Global Financial Data	Monthly log change in the industrial production index	
$FC\ Debt/GDP$	Authors' calculations	FC debt to GDP ratio, aggregated from face value of all FC bonds in Bloomberg.	
$LC\ Debt/GDP$	Authors' calculations	LC debt to GDP ratio, aggregated from face value of all LC bonds in Bloomberg.	
$\Delta CPI, Std(\Delta CPI)$	International Financial Statistics	Monthly log change in the consumer price and standard deviations of $\Delta CPI$ in last 12 months.	
$\Delta ToT, Std(\Delta ToT)$	Bank of International Settlement	Monthly log change in terms of trade and standard deviations of $\Delta ToT$ in last 12 months.	
$\Delta Reserve$	International Financial Statistics	Monthly log change in foreign exchange reserves.	

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