Hodgepodge

Regression models

Brian Caffo, Jeff Leek, Roger Peng
Johns Hopkins Bloomberg School of Public Health

How to fit functions using linear models

- Consider a model $Y_i = f(X_i) + \epsilon$.
- How can we fit such a model using linear models (called scatterplot smoothing)
- Consider the model

$$Y_i = \beta_0 + \beta_1 X_i + \sum_{k=1}^{d} (x_i - \xi_k)_+ \gamma_k + \epsilon_i$$

where $(a)_+ = a$ if $a > 0$ and 0 otherwise and $\xi_1 \leq \ldots \leq \xi_d$ are known knot points.

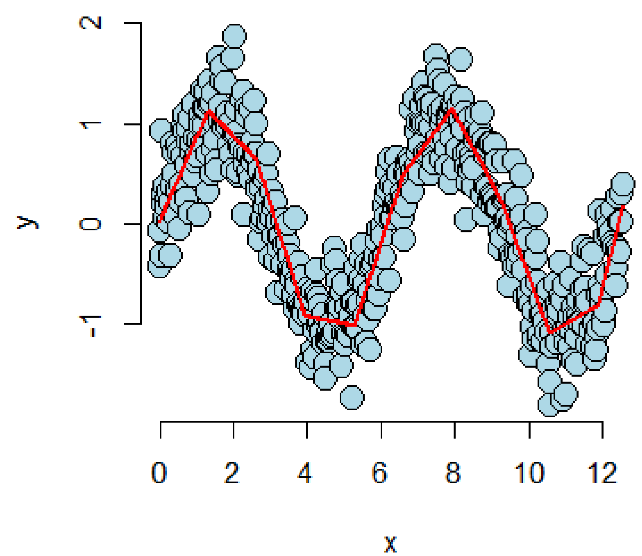
- Prove to yourelf that the mean function

$$\beta_0 + \beta_1 X_i + \sum_{k=1}^{d} (x_i - \xi_k)_+ \gamma_k$$

is continuous at the knot points.

Simulated example

```
n <- 500; x <- seq(0, 4 * pi, length = n); y <- sin(x) + rnorm(n, sd = .3)
knots <- seq(0, 8 * pi, length = 20);
splineTerms <- sapply(knots, function(knot) (x > knot) * (x - knot))
xMat <- cbind(1, x, splineTerms)
yhat <- predict(lm(y ~ xMat - 1))
plot(x, y, frame = FALSE, pch = 21, bg = "lightblue", cex = 2)
lines(x, yhat, col = "red", lwd = 2)
```

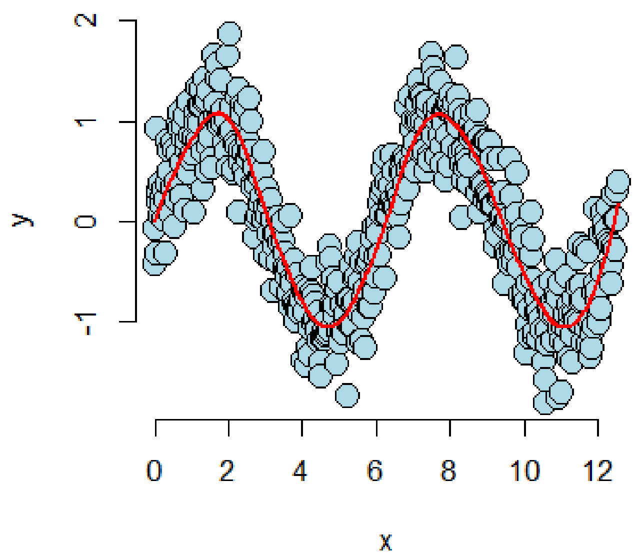


Adding squared terms

- Adding squared terms makes it continuously differentiable at the knot points.
- Adding cubic terms makes it twice continuously differentiable at the knot points; etcetera.

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \sum_{k=1}^{d} (x_i - \xi_k)_+^2 \gamma_k + \epsilon_i$$

```r
splineTerms <- sapply(knots, function(knot) (x > knot) * (x - knot)^2)
xMat <- cbind(1, x, x^2, splineTerms)
yhat <- predict(lm(y ~ xMat - 1))
plot(x, y, frame = FALSE, pch = 21, bg = "lightblue", cex = 2)
lines(x, yhat, col = "red", lwd = 2)
```

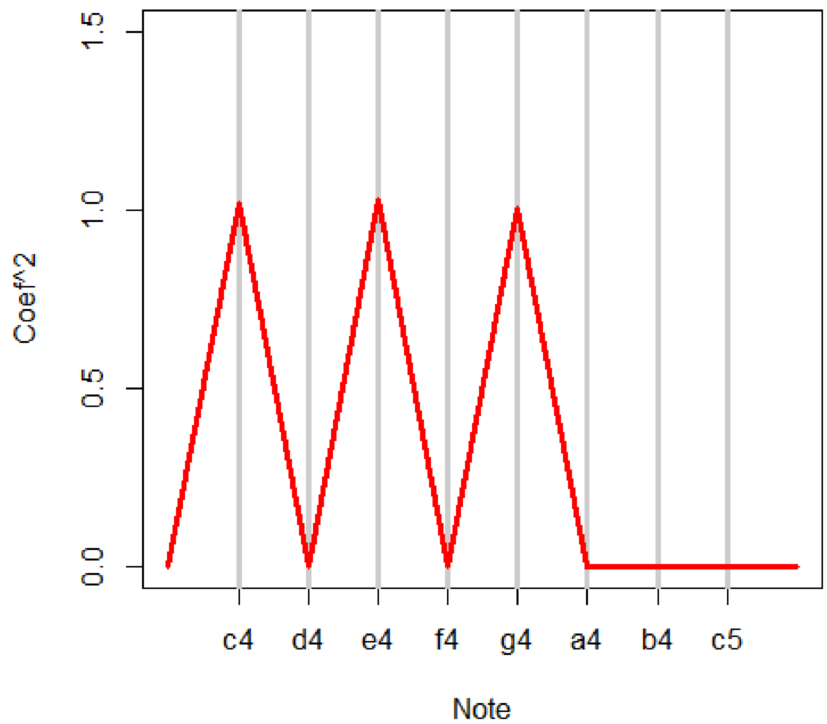


Notes

- The collection of regressors is called a basis.
  - People have spent **a lot** of time thinking about bases for this kind of problem. So, consider this as just a teaser.
- Single knot point terms can fit hockey stick like processes.
- These bases can be used in GLMs as well.
- An issue with these approaches is the large number of parameters introduced.
  - Requires some method of so called regularization.

Harmonics using linear models

```
##Chord finder, playing the white keys on a piano from octave c4 - c5
notes4 <- c(261.63, 293.66, 329.63, 349.23, 392.00, 440.00, 493.88, 523.25)
t <- seq(0, 2, by = .001); n <- length(t)
c4 <- sin(2 * pi * notes4[1] * t); e4 <- sin(2 * pi * notes4[3] * t);
g4 <- sin(2 * pi * notes4[5] * t)
chord <- c4 + e4 + g4 + rnorm(n, 0, 0.3)
x <- sapply(notes4, function(freq) sin(2 * pi * freq * t))
fit <- lm(chord ~ x - 1)
```



1.5
1.0
0.5
0.0
Coef^2
c4
d4
e4
f4
g4
a4
b4
c5
Note

```
##(How you would really do it)
a <- fft(chord); plot(Re(a)^2, type = "l")
```