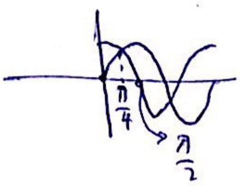
تمرین سری سوم آمار - محمد ارژنی ۹۴۳۱۰۰۴

①

$\sin x > \cos x \longrightarrow$



$\Rightarrow A \longrightarrow \frac{\pi}{4} < x < \frac{\pi}{2}$

$P(A) = \dfrac{\frac{\pi}{4}}{\frac{\pi}{2}} = \frac{1}{2}$

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② برای اینکه نشان دهیم تابع توزیع تجمعی است باید چهار ویژگی تابع توزیع تجمعی را برایش اثبات کنیم.

1) $F(+\infty) = 1$

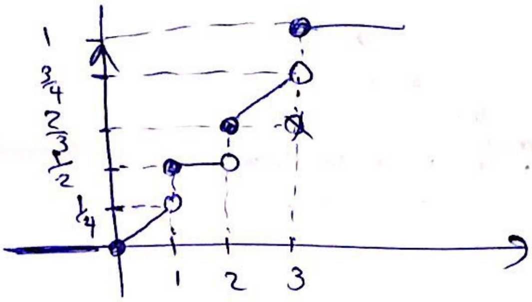
2) $F(-\infty) = 0$

$\Big\} \longrightarrow$ در تعریف مشخص است

3) از راست پیوسته است

4) غیر نزولی

از رسم شکل ثابت می شوند



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a) $P(X < 2) = \cancel{P(X<0) + P(0 \leq X < 1) + P(1 \leq X < 2)} = F(2^-) = \frac{1}{2}$

b) $P(X = 2) = F(2) - F(2^-) = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

c) $P(1 \leq X < 3) = F(3^-) - F(1^-) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

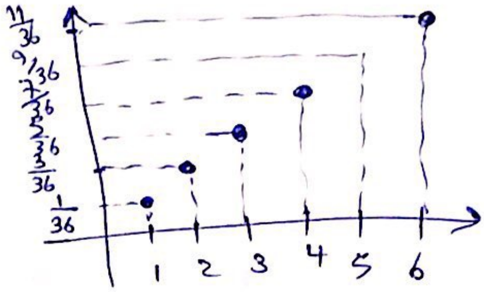
d) $P(X > \frac{3}{2}) = 1 - P(X \leq \frac{3}{2}) = 1 - F(\frac{3}{2}) = \frac{1}{2}$

e) $P(X = \frac{5}{2}) = F(\frac{5}{2}) - F(\frac{5}{2}^-) =$

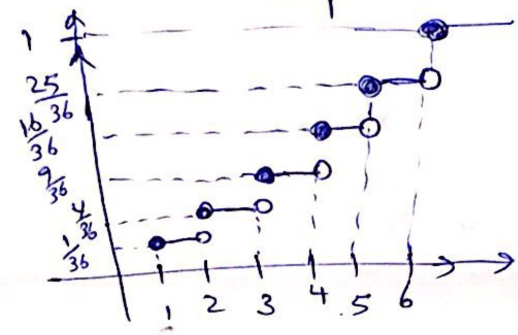
f) $P(2 < X \leq 7) = F(7) - F(2) = 1 - \frac{2}{3} = \frac{1}{3}$

x	1	2	3	4	5	6
$P_X(x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

$$\rightarrow P(X=x) = \begin{cases} \frac{2x-1}{36} & 1 \leq x \leq 6 \\ 0 & \text{else} \end{cases}$$



$$F(X=x) = \sum_{i=1}^{x} P(X=i) = \sum_{i=1}^{x} \frac{2i-1}{36} = \frac{-x}{36} + \frac{1}{18}\sum_{i=1}^{x} = \frac{x^2}{36}$$



(4) اثبات اینکه تابع چگالی است:

$f(x) \geq 0$.

$$\int_{-\infty}^{+\infty} f(x)\,dx = 1 \rightarrow \int_1^3 f(x)\,dx + \int_3^5 f(x)\,dx = \int_1^3 \left(\tfrac{1}{4}x - \tfrac{1}{4}\right)dx + \int_3^5 \left(-\tfrac{1}{4}x + \tfrac{5}{4}\right)dx$$

$$= \left(\frac{x^2}{8} - \frac{1}{4}x\right)\Big|_1^3 + \left(-\frac{x^2}{8} + \frac{5}{4}x\right)\Big|_3^5 = 1$$

تجمعی

$$\rightarrow F_X(x) = \int_{-\infty}^{x} f(t)\,dt \rightarrow \begin{cases} 0 & x < 1 \\ \int_1^x \left(\frac{1}{4}t - \frac{1}{4}\right) & 1 \leq x < 3 \\ \int_3^x \left(-\frac{1}{4}x + \frac{5}{4}\right) + F(3^-) & 3 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

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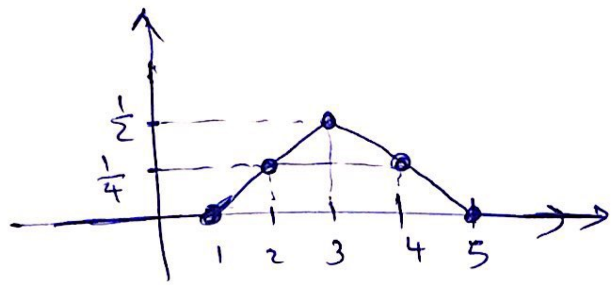
سوال ۴ ←

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = \begin{cases} 0 & \\ \frac{x^2}{8} - \frac{1}{4}x + \frac{1}{8} & 1 \leq x < 3 \\ -\frac{x^2}{8} + \frac{5}{4}x - \frac{17}{8} & 3 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

نمودار

$f \rightarrow$



← مربوط به قسمت الف

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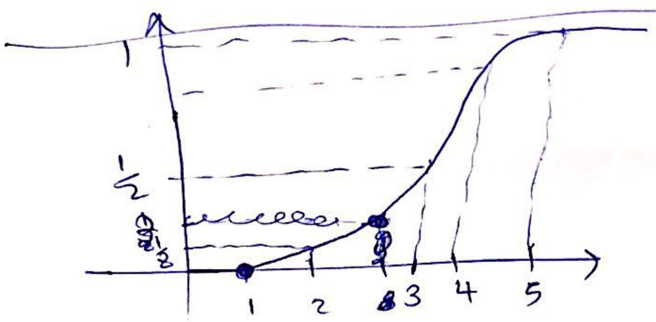

⑤

$$F(+\infty) = 1 \rightarrow \alpha + \beta \tan^{-1}(+\infty) = \alpha + \frac{\pi}{2}\beta = 1$$

$$F(-\infty) = 0 \rightarrow \alpha + \beta \tan^{-1}(-\infty) = \alpha - \frac{\pi}{2}\beta = 1$$

$$\rightarrow \begin{cases} \alpha = \frac{1}{2} \\ \beta = \frac{1}{\pi} \end{cases}$$

$$F(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\frac{x}{2} \rightarrow f_X(x) = (F(x))'$$

$$= \frac{1}{2\pi} \times \frac{4}{x^2+4} = \frac{2}{\pi(x^2+4)}$$