1 Rectifying the solution to this model with Bover's solution

Bover (1991) did not take the same solution approach as this paper. In the original, Bover solves the intertemporal problem with a Lagrangian and a clever transformation of the budget constraint. Yet, my approach using a Bellman equation produces a sligtly different looking solution. Yet, the two are indeed the same. Bover's solution regarding consumption takes the following form:

$$c_t = \gamma_c + B_2 \lambda_t^{-1}$$

To see that my solution is the same, we first look at the consumption Euler equation I derived earlier. This provides a very clear relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{1+\rho}{1+r}$$

According to the solution in Bover (1991), we can use Equation 7 in her paper to obtain a similar ratio.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}}$$

Therefore, a simple test will be to see if the ratio of the inverse lagrange multipliers is equal to $(1+\rho)/(1+r)$. The code is created using the Bover notation, which allows us to see if there are any bugs in the code.

Figure 1 demonstrates that we do indeed observe a constant relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$. Thus, we conclude that the ratio of subsequent Lagrange multipliers is $(1+\rho)/(1+r)$.

Bover solves this problem in terms of labor hours, but this does not prove any difficulty since our time constraint will tell us that $l_t = T - h_t$. Thus, Bover's equation 6 suggests that the ratio of leisure hours will be:

$$\frac{l_t - \varphi l_{t-1} - \gamma_l}{l_{t+1} - \varphi l_t - \gamma_l} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}} \frac{w_{t+1}^+}{w_t^+} \tag{1}$$

where

$$w_k^+ = W \sum_{j=0}^{T-k} \left(\frac{\varphi}{1+r} \right)^j$$

The hope is that starting from our FOC for leisure and using the Bover's FOC for hours (leisure), we hope to arrive at some tautology. For notational simplicity, let $\hat{l}_i = l_i - \varphi l_{i-1} - \gamma_l$. Starting from our FOC for leisure:

$$\frac{B_1}{\hat{l}_t} = \frac{B_2 W}{c_t - \gamma_c} + \frac{B_1 \frac{\varphi}{1+\rho}}{\hat{l}_{t+1}}$$

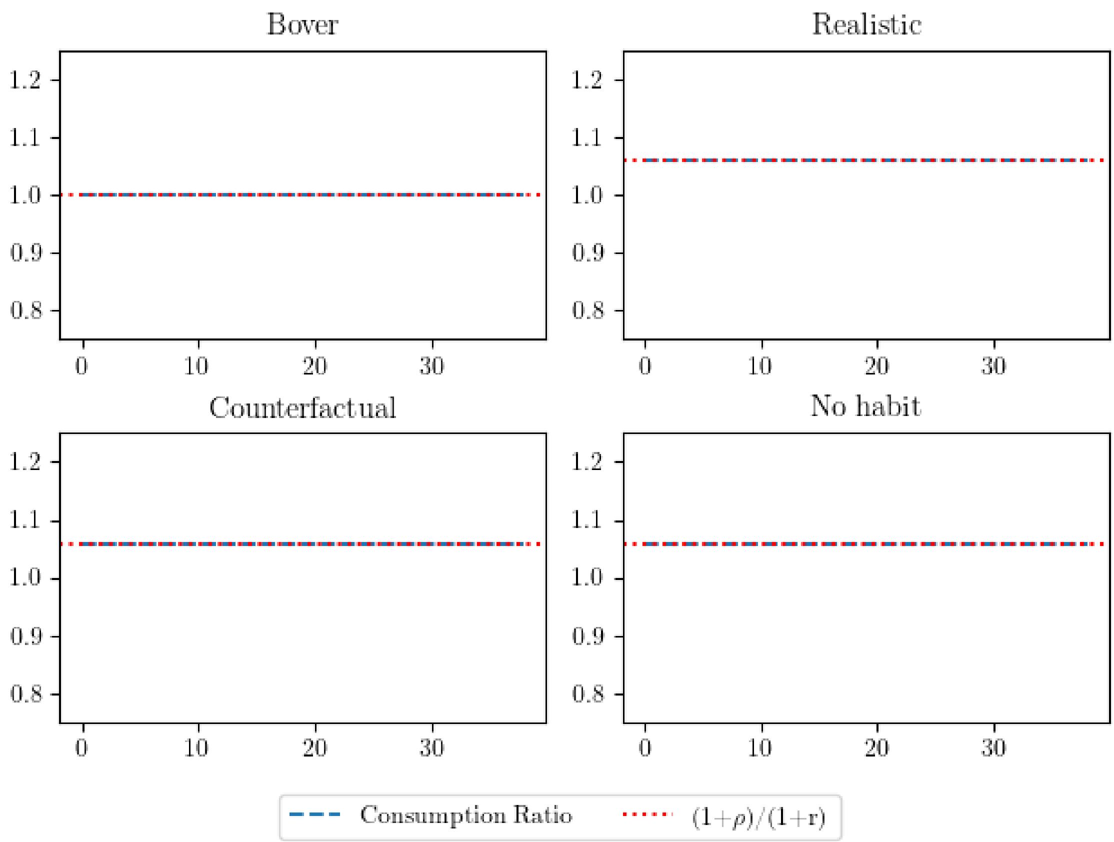

Figure 1 Consumption between time t and $t+1$

$$\frac{w_t^+}{\lambda_t^{-1}} = \frac{W}{\lambda_t^{-1}} + \frac{w_{t+1}^+ \frac{\varphi}{1+rho}}{\lambda_{t+1}^{-1}}$$

$$\frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}}{1-\frac{\varphi}{1+r}} = 1 + \frac{\varphi}{1+\rho} \cdot \frac{1+\rho}{1+r} \frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t}}{1-\frac{\varphi}{1+r}}$$

$$1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} = \left(1-\frac{\varphi}{1+r}\right) + \frac{\varphi}{1+r}\left(1-\left(\frac{\varphi}{1+r}\right)^{T-t}\right)$$

$$1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} = 1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}$$

This tautology suggests that both approaches yield the same solution.