1 Rectifying the solution to this model with Bover's solution

Bover (1991) did not take the same solution approach as this paper. In the original, Bover solves the intertemporal problem with a Lagrangian and a clever transformation of the budget constraint. Yet, my approach using a Bellman equation and the Envelope Theorem produces a sligtly different looking solution. Yet, the two are indeed the same. Bover's solution regarding consumption takes the following form (note that λ_t denotes the Lagrange multiplier):

$$c_t = \gamma_c + B_2 \lambda_t^{-1}$$

To see that my solution is the same, we first look at the consumption Euler equation I derived earlier. This provides a very clear relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{1+\rho}{1+r}$$

According to the solution in Bover (1991), we can use Equation 7 in her paper to obtain a similar ratio.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}}$$

Therefore, a simple test will be to see if the ratio of the inverse Lagrange multipliers is equal to $(1+\rho)/(1+r)$. The code is created using the Bover notation, which allows us to see if there are any bugs in the code and verify that we do in fact converge on the same solution.

Figure 1 demonstrates that we do indeed observe a constant relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$. Thus, we conclude that the ratio of subsequent Lagrange multipliers is $(1+\rho)/(1+r)$.

Bover solves this problem in terms of labor hours, but this does not prove any difficulty since our time constraint will tell us that $l_t = T - h_t$. Thus, Bover's Equation 6 suggests that the optimal amount of leisure will be:

$$l_t - \varphi l_{t-1} - \gamma_l = B_1 \frac{\lambda_t^{-1}}{w_t^+} \tag{1}$$

where

$$w_k^+ = W \sum_{j=0}^{T-k} \left(\frac{\varphi}{1+r} \right)^j$$

Furthermore, this means that the ratio of leisure hours (according to Bover) will be:

$$\frac{l_t - \varphi l_{t-1} - \gamma_l}{l_{t+1} - \varphi l_t - \gamma_l} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}} \frac{w_{t+1}^+}{w_t^+} \tag{2}$$

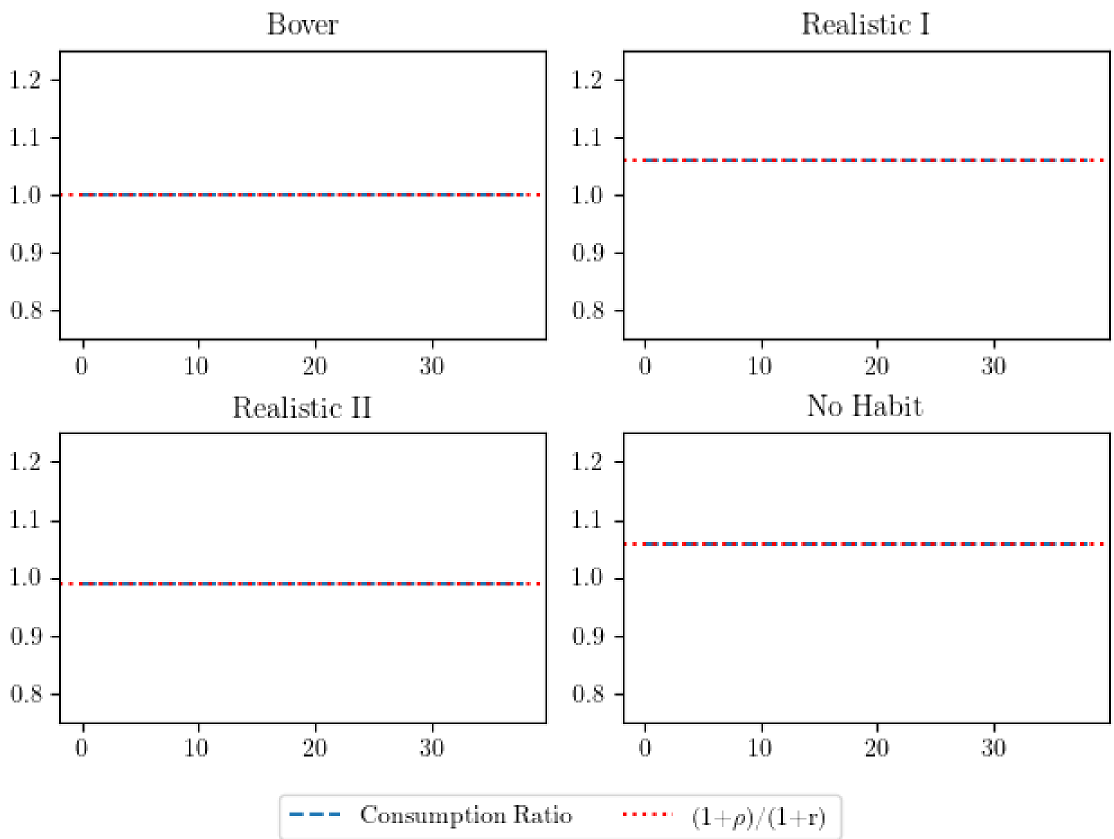

Figure 1 Consumption between time t and $t+1$

The hope is that starting from my FOC for leisure and using the Bover's FOC for hours (leisure), we hope to arrive at some tautology. For notational simplicity, let $\hat{l}_i = l_i - \varphi l_{i-1} - \gamma_l$. Starting from my FOC for leisure:

$$
\begin{aligned}
\frac{B_1}{\hat{l}_t} &= \frac{B_2 W}{c_t - \gamma_c} + \frac{B_1 \frac{\varphi}{1+\rho}}{\hat{l}_{t+1}} \\
\frac{w_t^+}{\lambda_t^{-1}} &= \frac{W}{\lambda_t^{-1}} + \frac{w_{t+1}^+ \frac{\varphi}{1+\rho}}{\lambda_{t+1}^{-1}} \\
\frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}}{1-\frac{\varphi}{1+r}} &= 1 + \frac{\varphi}{1+\rho} \cdot \frac{1+\rho}{1+r} \cdot \frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t}}{1-\frac{\varphi}{1+r}} \\
1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} &= \left(1-\frac{\varphi}{1+r}\right) + \frac{\varphi}{1+r}\left(1-\left(\frac{\varphi}{1+r}\right)^{T-t}\right) \\
1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} &= 1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}
\end{aligned}
$$

This tautology suggests that both approaches yield the same solution.

References

Bover, Olympia (1991): “Relaxing Intertemporal Separability: A Rational Habits Model of Labor Supply Estimated from Panel Data,” *Journal of Labor Economics*, 9(1), 85–100.