1 When Is Consumption Growth Declining in m?

Figure **??** depicts the expected consumption growth factor as a strictly declining function of the cash-on-hand ratio. To investigate this, define

$$\mathbf{Y}(m_t) \equiv \Gamma_{t+1}\mathrm{c}(\mathcal{R}_{t+1}\mathrm{a}(m_t) + \xi_{t+1})/\mathrm{c}(m_t) = \mathbf{c}_{t+1}/\mathbf{c}_t$$

and the proposition in which we are interested is

$$(d/dm_t)\,\mathbb{E}_t[\underbrace{\mathbf{Y}(m_t)}_{\equiv \mathbf{Y}_{t+1}}] < 0$$

or differentiating through the expectations operator, what we want is

$$\mathbb{E}_t\left[\Gamma_{t+1}\left(\frac{\mathrm{c}'(m_{t+1})\mathcal{R}_{t+1}\mathrm{a}'(m_t)\mathrm{c}(m_t) - \mathrm{c}(m_{t+1})\mathrm{c}'(m_t)}{\mathrm{c}(m_t)^2}\right)\right] < 0. \tag{1}$$

Henceforth indicating appropriate arguments by the corresponding subscript (e.g. $\mathrm{c}'_{t+1} \equiv \mathrm{c}'(m_{t+1})$), since $\Gamma_{t+1}\mathcal{R}_{t+1} = \mathsf{R}$, the portion of the LHS of equation (1) in brackets can be manipulated to yield

$$\begin{aligned} \mathrm{c}_t\mathbf{Y}'_{t+1} &= \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\Gamma_{t+1}\mathrm{c}_{t+1}/\mathrm{c}_t \\ &= \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\mathbf{Y}_{t+1}. \end{aligned}$$

Now differentiate the Euler equation with respect to m_t:

$$\begin{aligned} 1 &= \mathsf{R}\beta\,\mathbb{E}_t[\mathbf{Y}_{t+1}^{-\rho}] \\ 0 &= \mathbb{E}_t[\mathbf{Y}_{t+1}^{-\rho-1}\mathbf{Y}'_{t+1}] \\ &= \mathbb{E}_t[\mathbf{Y}_{t+1}^{-\rho-1}]\,\mathbb{E}_t[\mathbf{Y}'_{t+1}] + \mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathbf{Y}'_{t+1}) \\ \mathbb{E}_t[\mathbf{Y}'_{t+1}] &= -\mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathbf{Y}'_{t+1})/\,\mathbb{E}_t[\mathbf{Y}_{t+1}^{-\rho-1}] \end{aligned}$$

but since $\mathbf{Y}_{t+1} > 0$ we can see from (2) that (1) is equivalent to

$$\mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathbf{Y}'_{t+1}) > 0$$

which, using (2), will be true if

$$\mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathrm{c}'_{t+1}\mathrm{a}'_t\mathsf{R} - \mathrm{c}'_t\mathbf{Y}_{t+1}) > 0$$

which in turn will be true if both

$$\mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathrm{c}'_{t+1}) > 0$$

and

$$\mathrm{cov}_t(\mathbf{Y}_{t+1}^{-\rho-1}, \mathbf{Y}_{t+1}) < 0.$$

The latter proposition is obviously true under our assumption $\rho > 1$. The former will be true if

$$\mathrm{cov}_t\left((\Gamma\psi_{t+1}\mathrm{c}(m_{t+1}))^{-\rho-1}, \mathrm{c}'(m_{t+1})\right) > 0.$$

The two shocks cause two kinds of variation in m_{t+1}. Variations due to ξ_{t+1} satisfy the proposition, since a higher draw of ξ both reduces $c_{t+1}^{-\rho-1}$ and reduces the marginal propensity to consume. However, permanent shocks have conflicting effects. On the one hand, a higher draw of ψ_{t+1} will reduce m_{t+1}, thus increasing both $c_{t+1}^{-\rho-1}$ and c'_{t+1}. On the other hand, the $c_{t+1}^{-\rho-1}$ term is multiplied by $\Gamma\psi_{t+1}$, so the effect of a higher ψ_{t+1} could be to decrease the first term in the covariance, leading to a negative covariance with the second term. (Analogously, a lower permanent shock ψ_{t+1} can also lead a negative correlation.)