A Simple Labor-Lesiure Model with Habits: Some Simulations from Previous Results

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Abstract

This paper presents a slight addition mixture of two traditional economic models: the labor-leisure model and a model of habit formation in consumption. We treat labor as a good with which agents form habits over, in addition to already forming habits in consumption behavior. The hope is that this model can be used to explain labor elasticity patterns and life cycle labor supply and consumption differences. This paper is largely inspired by the work of **?**.

Keywords Labor Supply, time allocation, habit formation

PDF: https://github.com/mmdrozd/CompMethods2021-DrozdMM/ProjectMMD.pdf
Slides: https://github.com/mmdrozd/CompMethods2021-DrozdMM/ProjectMMD-Slides.pdf
GitHub: https://github.com/mmdrozd/CompMethods2021-DrozdMM

This paper is based off the template for the Theoretical Foundations of Buffer Stock Theory by Chris D. Carroll. The repository for can be found here [put link here]. The paper's results can be automatically reproduced using the Econ-ARK/HARK toolkit, which can be cited per our references (**?**); for reference to the toolkit itself see . All errors are my own.

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1 Introduction

The theory of labor supply tackles one of the most fundamental problems in economics: why people choose to work? Yet, as we try to abstract from the often times complex reasons why agents interact with the labor market, simple features can provide great insights to these problems. As central as such a problem can be, disagreements naturally arise throughout the field; discrepancies between microeconomics and macroeconomics. In an attempt to build some more intuition on such a basic problem, I look to model labor supply in the traditional way, but include habit formation.

The habit formation story is rather intuitive when it comes to consumption. There is a sort of smoothing process in our consumption patterns (a comfort or a certainty in a stochastic world) such that deviations from such norms actually provide disutility. Now, this is a rather extreme case of accustomization to a certain consumption behavior, and as so can be relaxed so as to just suggest the matter of changes in consumption affect utility in a parametric sense. However, this extreme view may hold true. For example, imagine we are analyzing a consumption problem regarding groceries. Deviations from the typical amount that a household purchases for groceries can be disconcerting. For one, there might not be space in the refrigerator to store the food. For another, purchasing more food than a household might use, will make it more likely to spoil and seem like a waste of money.

In the labor supply literature occurs between consumption and leisure, so the same argument justifying habit formation in consumption naturally suggests habit formation in leisure. Agents get used to a certain amount of leisure, and deviations from this amount (particularly decreases) has a large effect. This effect has two channels, one from reducing the current leisure consumption, and the other by this habit. In other words, you are worse off because you don't have the same amount of leisure as before. From the perspective of less leisure in the future, the increasing time demand in labor may make the time spent in leisure less valuable (i.e. when you have leisure, you are too tired from working to take advantage of it compared to your previous leisure amount). On the other hand, too much leisure (relative to a previous period) can also be counterproductive. In this sense, this increase in leisure might lead to idleness and while it may be good to relax a little bit more, the tradeoff to consumption might actually dampen the effect of this extra leisure time. In any case, it is a plausible hypothesis that there may be some degree of habit formation as consumption.

While models of habit formation in consumption tend to dominate (put some cites here), there have been models that introduce habits in labor supply, notably **?**. Using the PSID, **?** estimates structural parameters of a life cycle labor model imposing a Stone-Geary utility function and non-separability between the current labor supply and previous labor supply. This strategy yields reasonable labor supply elasticity estimates, but these calculations depend on the structural estimates that are implausible. In an effort to see the plausibility of the elasticity estimates, we will simulate agents according to **?**, but restrict some parameters (notably the risk free rate and the psychological discount factor) to the literature standard values.

Some other things about the problem.. perhaps some citations.. you know: make it look pretty.

2 The Problem

2.1 Setup

An economic agent must decide how much to consume and work each period. The more this agent chooses to work, the more consumption that this agent gets to do, but this comes at the cost of less leisure time (the other "good" that the agent values). Furthermore, the agent has a habit stock in leisure. For the initial setup, let us just assume a finite horizon. The agent looks to maximize the following utility function.

$$\sum_{t=0}^{D-t} \beta^t u(c_t, l_t, h_t^l) \tag{1}$$

D denotes the end of the lifetime.Consumption and leisure are denoted by c_t and l_t with h_t^l being the habit stock of leisure.Each future period is psychologically discounted by the term β (which is less than 1 because otherwise it would not be discounting the future). We assume that the stock of the habits is equal to the previous period's consumption/leisure.

$$h_{t+1}^l = l_t \tag{2}$$

The agent will maximize her utility subject to the following budget constraint.

$$m_{t+1} = (m_t - c_t)(1+r) + y_{t+1} \tag{3}$$

$$y_t = W(T - l_t) \tag{4}$$

Equation (3) depicts savings equation. Money/assets are denoted by m. The income at time t is denoted by y_t. Any assets not spent in period t grows by the risk-free rate (denoted by r) to be used for the next period. Equation (4) just describes the earnings in each period. The wage (as of now) is constant and denoted by W and T describes the total amount of time in a period. Hence $T - l_t$ is the amount of time spent in work. We can combine Equations (3) and (4) to have one consolidated budget constraint.

$$m_{t+1} = ((m_{t-1} - c_{t-1})(1+r) + W(T - l_t) - c_t)(1+r) + W(T - l_{t+1}) \tag{5}$$

Strictly speaking, we have some additional constraints that apply to this problem. Leisure is censored by zero and T (it is impossible to have negative leisure or to have more leisure than there is time). Naturally, we have assumed no financing of consumption, which is implicit in the way that Equation (3) is defined.

To solve this model, using a value function is the way to go. Using **?** as a guide, we can easily solve the model by putting it into Bellman form.

$$v_t(m_t, h_t^l) = \max_{c_t, l_t} u(c_t, l_t, h_t^l) + \beta v_{t+1}(m_{t+1}, h_{t+1}^l) \tag{6}$$

Combining Equation (6) with Equations (2) and (5), allow us to calculate our first order conditions.

$$u_t^c = \beta(1+r)v_{t+1}^m \tag{7}$$
$$u_t^l = \beta(1+r)Wv_{t+1}^m - \beta v_{t+1}^h \tag{8}$$

The superscripts in Equations (7) and (8) denote the partial derivative with respect to that argument (for example, $u_t^c = \partial u(c_t, l_t)/\partial c_t$). The Envelope Theorem yields the following

$$v_t^m = \beta(1+r)v_{t+1}^m \tag{9}$$
$$v_t^h = u^h \tag{10}$$

Therefore combining our results from our FOCs (Equations (7) and (8)) with the Envelope Theorem results (Equations (9) and (10)), we should get the following Euler conditions:

$$u_t^c = \beta(1+r)u_{t+1}^c \tag{11}$$
$$u_t^l = Wu_t^c - \beta u_{t+1}^h \tag{12}$$

The Equation (12) is a similar to the optimization condition in classic labor supply theory. In the traditional model, the optimizing behavior equates the ratio of the marginal utilities to the price ratio. By including habits, we gain an extra term that adjusts this condition slightly. As a matter of fact, Equation (12) reduces down to the traditional first order condition when we let $u^h = 0$ (or in other words, impose that this model does not have a habit aspect).

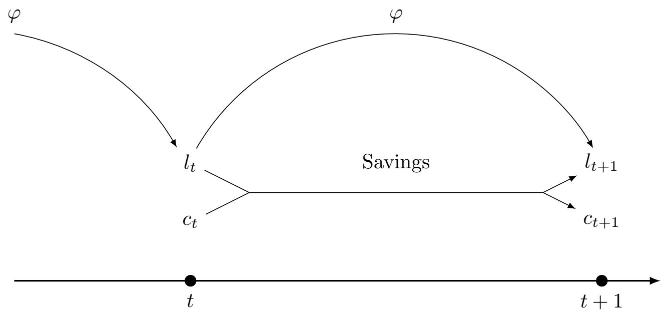

Figure 1 Stylized Model

Behold a stylized model of the labor-leisure model with habits.

Figure 1 serves as reference for the model. In essence an agent enters the period and must make a leisure-consumption decision. The leisure decision is influenced by the habit factor, φ. Upon reaching the subsequent period, the decision must be made again, but also takes into account the savings and the habit that came from the previous periods decisions.

In order to continue to solve this problem, we need to impose some functional form on the utility. **?** uses a Stone-Geary specification, which is what I will use for consistency purposes[1].

$$u(c_t, l_t, l_{t-1}) = B_1 \log(l_t - \varphi l_{t-1} - \gamma_l) + B_2 \log(c_t - \gamma_c) \tag{13}$$

B_1 and B_2 are taste parameters, and homogeneity is assumed, which means that they sum to one. As is usual in the Stone-Geary specification, the γ_c and γ_l parameters can be interpretted as a minimum number of consumption/leisure that the agent needs to attain. Now, we can have continue to work from our first order conditions.

$$\frac{1}{c_t - \gamma_c} = \frac{\beta(1+r)}{c_{t+1} - \gamma_c} \tag{14}$$

$$\frac{B_1}{l_t - \varphi l_{t-1} - \gamma_l} = \frac{B_2 W}{c_t - \gamma_c} + \frac{B_1 \beta \varphi}{l_{t+1} - \varphi l_t - \gamma_l} \tag{15}$$

While Equation (14) is a rather standard Euler equation for consumption, the habit in leisure makes Equation (15) rather complicated. It is further complicated by the presence of a c_t in Equation (15). This will prevent us from simplying just using the intertemporal budget constraint to give as a nice closed form solution for consumption. In any case, the difficulty of having a second control variable for this problem will call for some simplications to be made. For the following simulation exercises, I follow **?**. The only notational discrepancy will occur with regards to the psychological discount factor. Moving forward, $\beta = \frac{1}{1+\rho}$.

3 Simulations

There are several abstractions made in order to simplify the simulation. In principle, we can allow for the wage to change over the life cycle (and perhaps in later renditions, we will allow for this), but for now, we focus on a constant wage. In principle, the Stone-Geary utility function can vary by demographic characteristics. In particular, Bover uses a simple linear regression of demographic observables of which the coefficients on the number of children are reported. Since that is all we have to work with (likely there are other covariates in that regression, but we just do not know their estimated coefficients), we will be restrict our agent to have the mean number of children [2] Potentially in a future simulation, we can also adjust the number of children our simulated agent has[3].

In order to ensure consistency, I follow the model solution as described by **?**. In Column

[1]**?** writes utility as a function of labor hours instead of leisure. However, there is not much of a difference in doing so since we assume that time can be divided into either work or leisure. Therefore, we can rewrite everything in terms of labor hours and get the same expressions.

[2]Likely, there my calibration of the B_1 and B_2 values are slightlty off since the calculated elasticity for the original paper (as reported in Table 2 is slightly off. But, the numbers are still extremely close, which provides confidence for a correct replication of the the **?** results.

[3]After all, the mean number of children in the original data is between one and two, which means no one actually has the mean number of children

Table 1 Calibrated Parameters

Simulation	γ_h	γ_c	φ	ρ	r
Original	1768.1516	4454.0084	0.2205	0.2429	0.2429
Realistic	1768.1516	4454.0084	0.2205	0.0800	0.0200
Counterfactual	1768.1516	4454.0084	0.3000	0.0800	0.0200
No Habit	1768.1516	4454.0084	0.0000	0.0800	0.0200

2 of Table 1 of **?**, estimates of the structural parameters are presented [4]. Originally, **?** imposed that the psychological discount factor and the risk free rate werre the same, which while that is not terribly radical in and of itself, the point estimates are implausibly large. For instance, the risk-free interest rate falls in the 20% range. Why work when you can get a 20% riskless return.

Therefore, the motivaiton for conducting simulations is clear. For one, there are natural questions that can be asked since life-cycle figures were not presented. What does this model suggest about life-cycle consumption/labor? How sensitive are the results to change in parameters? Does the life-cycle varibales change in expected ways when we adjust the parameters?

In conducting my simulation, I maintain a few simplications. The agent is choosing consumption and labor for a 40-year period. This decision abstracts from reality in two obvious ways. First, there is no mortality (or probability of mortality). Second, the agent does not plan to live for a period of time without work. There is no retirement or no type of Social secuity system. Furthermore, these decisions are made with certainty. The agent knows that they will be employed at the same wage. This abstraction prevents job separation and the bargaining for higher wages. Some of these features become more standard in a more contemporary life-cycle labor supply model (see **?** for an example of a life-cycle model with these aspects included). Nonetheless, the interesting questions remain and these features provide a foundation for augmenting the habit model for a future project.

The simulations ran are calibrated according to Table 1. The original row refers to estimated values from **?**. The "realistic" row adjusts the ρ and interest factors to be more standard values. The "counterfactual" row adjusts the same two factors as the "realistic" column, but also adjusts the habit persistence parameter. Finally, the "no habits" row sets the habit persistence variable to zero.

Using these calibrations we are able to simulate the labor hours (and thereby the leisure hours) over the life cycle. Figure 2 demonstrates the life cycle labor hours. Most of the specifications have (generally) the "horseshoe" shape in labor hours, where hours rise at the beginning of life and then fall towards the end. The Original specification most closely follows this shape by maintaining a consistent amount of hours over the life cycle except

[4] See the "Original" row of the Table 1 for the point estimates from **?**. It worth noting that many of the point estimates end up statistically insignificant, but yet it is through these point estimates that the author constructs the elasticity estimates.

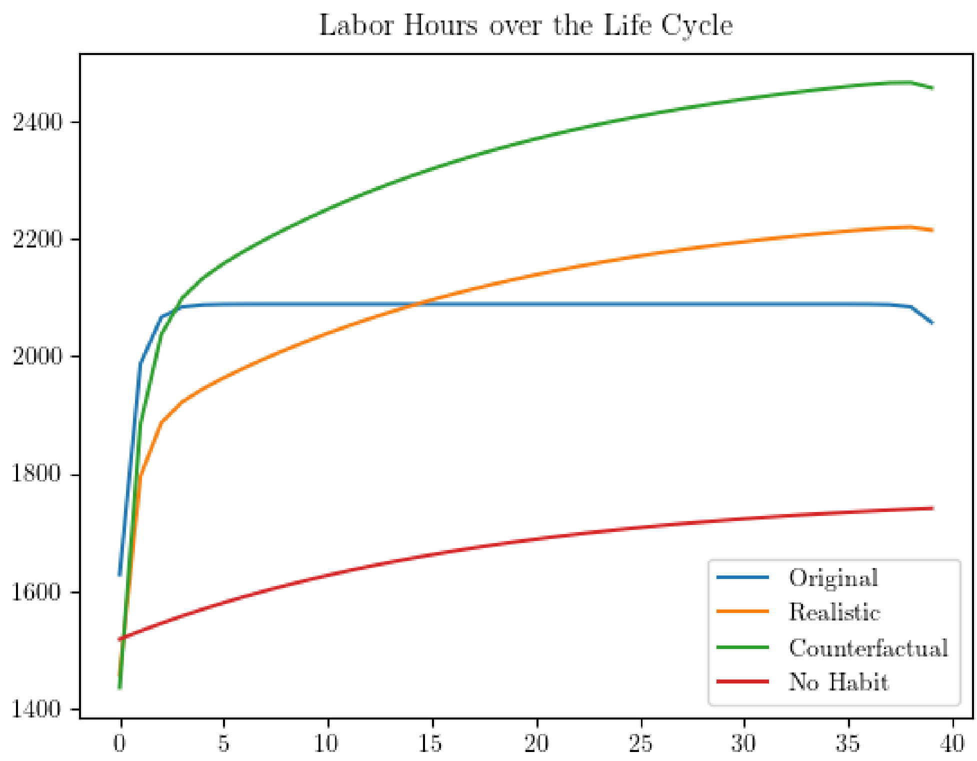

Figure 2 Annual Labor Hours over the Life Cycle

at the beginning/end. The realisitc and counterfactual specifications do not really reach a "steady" value of labor hours. They increase almost to the very end. A reason for this is that in the original specification, the interest rate is so high, but also being offset by the discount factor. On the other hand, the discount factor is greater in the realistic and counterfactual specifications that cause these agents to prefer consumptionn and leisure today and pay it off later. Perhaps, the most odd is how low labor hours are for the no-habit case.

Furthermore, Figure 3 presents the life-cycle consumption for all the scenarios. Only the original specification seems to produce what we might think of life-cycle consumption. All the other specifications enjoy immense consumption levels early and then must spend time working to pay off those early levels of consumption. What is interesting is that the increase in φ in the counterfactual specificaiton leads to much higher consumption. Keep in mind that this is financed by high labor hours over the life-cycle.

The key conslusion from **?** is that the including habits produces reasonable labor supply elasticity estimates. Those elasticities are recreated in Table 2.

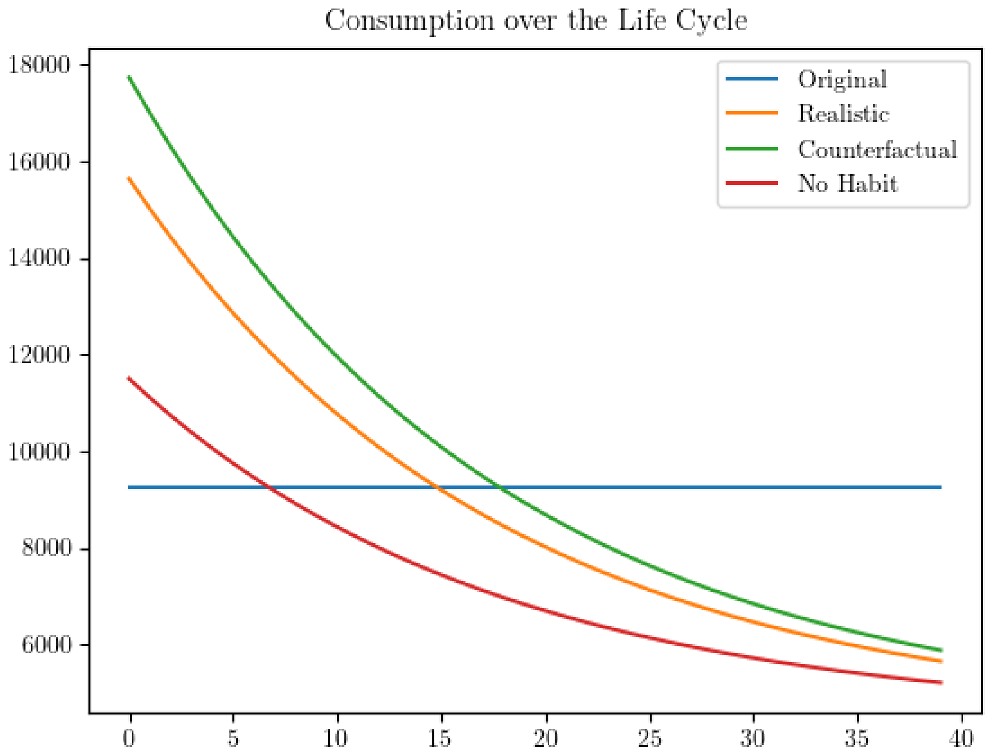

Figure 3 Annual Consumption over the Life Cycle

4 Conclusions

While some of the life cycle trends in some of the simulations appear to create some perhaps unintuitive trends, the increases in the elasticities after adjusting some of the parameters of the model may suggest that the habit in labor supply model does not capture the historically small labor supply elasticities. However, true elasiticities are not based on a world where all the agents know exactly how things are going to play out. For that matter, consumption and labor hour decisions can never be made with perfect confidence on how the world will work. Perhaps, the peculiarities are due to the assumptions of the model. Therefore, what are the avenues for improvement?

There are many directions that further simulations can go. In remaining in a perfect foresight case, the first changes would be to have a more realistic wage process. Wages change throughout people's life cycles and even if we had perfect foresight, this clearly influences our decisions today knowing that there would be higher wages in the future. Once this could be incorportated, we would need to then move to the uncertainty case in order to truly have a model that even remotely resembles real life. However, the computational demand for solving such a model increases immensely (especially with the two choice variables).

Table 2 Simulated Elasticities

Simulation	ϵ	η^{α}
Original	0.0734	-0.1272
Realistic	0.0658	-0.1206
Counterfactual	0.0660	-0.1096
No Habit	0.0606	-0.1505

Furthermore, as mentioned before, there are other aspects to people's life cycle decisions that we might want to include. Other choice variables could be a human capital component, which would make wages (or wage growth) endogenous to an investment in schooling or training. The goal here would be to see how the habits model is better or worse equipped to answer questions of human capital investment compared to the traditional models. Perhaps the habits model may be appropriate in a human capital/schooling context since one could argue that agents who go to school for long periods of time are accustomed to longer working hours and continue to work such hours once in the labor market (i.e. a habit).

Another example of an additional choice variable could be a portfolio. While the current model only allow assets to accumulate according to the risk free rate, perhaps a portfolio allocation might also be important. While this may seem like an intense addition to the model, it could be implemented in a sort of simpler fashion, such as a private pension (as in **?**). In this case, the private pension would have a stochastic return (like the stock market) with an expected premium higher than the risk free rate. Obviosly incorportating this into a labor supply model will impact the labor supply decision (which in this register already has a habit-forming effect).

Appendices

A Rectifying the solution to this model with Bover's solution

? did not take the same solution approach as this paper. In the original, Bover solves the intertemporal problem with a Lagrangian and a clever transformation of the budget constraint. Yet, my approach using a Bellman equation produces a sligtly different looking solution. Yet, the two are indeed the same. Bover's solution regarding consumption takes the following form:

$$c_t = \gamma_c + B_2 \lambda_t^{-1}$$

To see that my solution is the same, we first look at the consumption Euler equation I derived earlier. This provides a very clear relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{1+\rho}{1+r}$$

According to the solution in **?**, we can use Equation 7 in her paper to obtain a similar ratio.

$$\frac{c_t - \gamma_c}{c_{t+1} - \gamma_c} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}}$$

Therefore, a simple test will be to see if the ratio of the inverse lagrange multipliers is equal to $(1+\rho)/(1+r)$. The code is created using the Bover notation, which allows us to see if there are any bugs in the code.

Figure 4 demonstrates that we do indeed observe a constant relationship between $c_t - \gamma_c$ and $c_{t+1} - \gamma_c$. Thus, we conclude that the ratio of subsequent Lagrange multipliers is $(1+\rho)/(1+r)$.

Bover solves this problem in terms of labor hours, but this does not prove any difficulty since our time constraint will tell us that $l_t = T - h_t$. Thus, Bover's equation 6 suggests that the ratio of leisure hours will be:

$$\frac{l_t - \varphi l_{t-1} - \gamma_l}{l_{t+1} - \varphi l_t - \gamma_l} = \frac{\lambda_t^{-1}}{\lambda_{t+1}^{-1}} \frac{w_{t+1}^+}{w_t^+} \tag{16}$$

where

$$w_k^+ = W \sum_{j=0}^{T-k} \left(\frac{\varphi}{1+r} \right)^j$$

The hope is that starting from our FOC for leisure and using the Bover's FOC for hours (leisure), we hope to arrive at some tautology. For notational simplicity, let

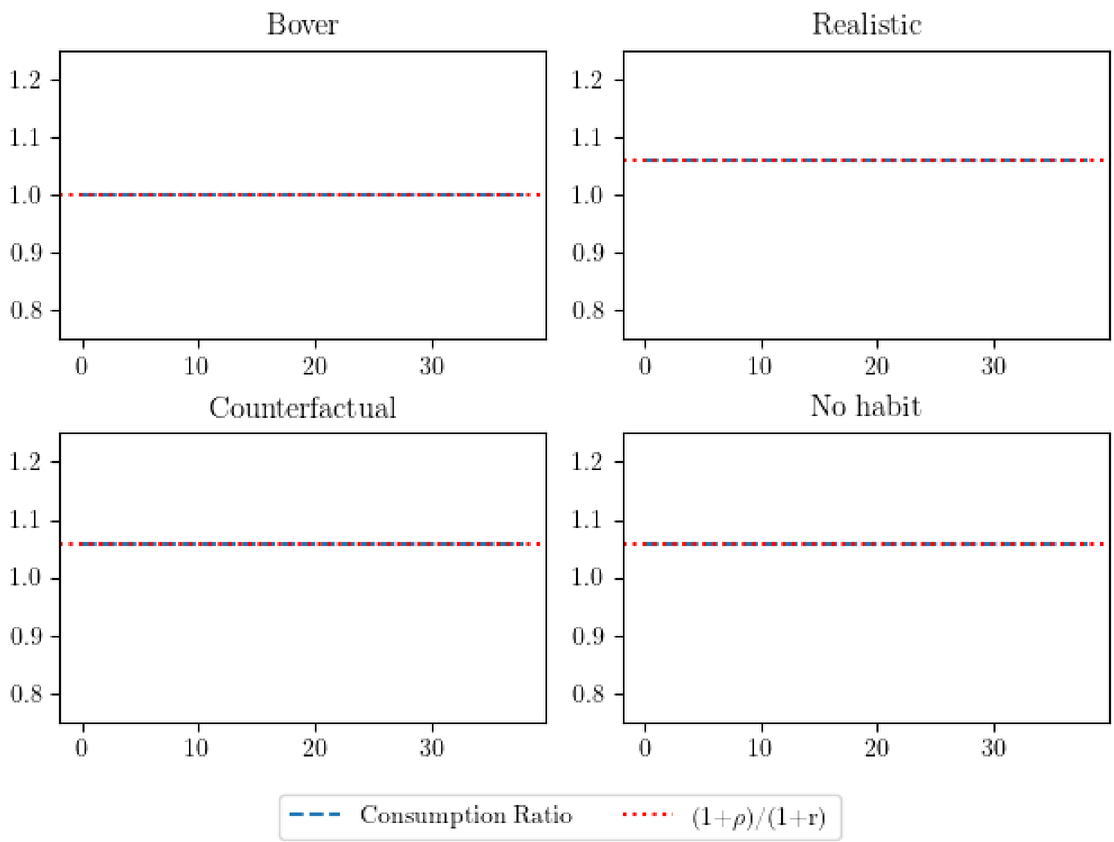

Figure 4 Consumption between time t and $t+1$

$\hat{l}_i = l_i - \varphi l_{i-1} - \gamma_l$. Starting from our FOC for leisure:

$$\frac{B_1}{\hat{l}_t} = \frac{B_2 W}{c_t - \gamma_c} + \frac{B_1 \frac{\varphi}{1+\rho}}{\hat{l}_{t+1}}$$

$$\frac{w_t^+}{\lambda_t^{-1}} = \frac{W}{\lambda_t^{-1}} + \frac{w_{t+1}^+ \frac{\varphi}{1+rho}}{\lambda_{t+1}^{-1}}$$

$$\frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}}{1-\frac{\varphi}{1+r}} = 1 + \frac{\varphi}{1+\rho} \cdot \frac{1+\rho}{1+r} \frac{1-\left(\frac{\varphi}{1+r}\right)^{T-t}}{1-\frac{\varphi}{1+r}}$$

$$1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} = \left(1-\frac{\varphi}{1+r}\right) + \frac{\varphi}{1+r}\left(1-\left(\frac{\varphi}{1+r}\right)^{T-t}\right)$$

$$1-\left(\frac{\varphi}{1+r}\right)^{T-t+1} = 1-\left(\frac{\varphi}{1+r}\right)^{T-t+1}$$

This tautology suggests that both approaches yield the same solution.

References

References