A Simple Labor-Leisure Model with Habits: Some Simulations from Previous Results

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Introduction

- Introduce habits into the life-cycle labor supply model
  - Already has been done (see Bover (1991) and Kubin and Prinz (2002))
- Use the previous results to create some visualizations
- Change the parameter values to assess the validity of the model.
- Even under perfect foresight, this problem is fairly tough.

Key Result

- Original point estimates for the parameter values too high
  - Risk-free rate in the 20% range (if only this were true!)
- Slight adjustment of wage elasticities (but ultimately comparable).
- Visualizations of labor hours and consumption over the life cycle.

The Problem

We want to maximize the following utility function

$$\sum_{t=0}^{D} \beta^t u(c_t, l_t, h_t^l) \tag{1}$$

subject to the following constraint:

$$m_{t+1} = (m_t - c_t)(1 + r) + y_{t+1} \tag{2}$$
$$y_t = W(T - l_t) \tag{3}$$

where $\beta = \frac{1}{1+\rho}$ is the psychological discount factor; c_t is consumption; l_t is leisure h_t^l is the habit stock in leisure; m_t is the money assets; r is the risk free rate; W is the wage. y_t is income. T is the total amount of time in a period. D is the total amount of time that the agent is solving this problem. For our calibrations, we will let $D = 40$.

Bellman

We can rewrite this problem in Bellman form.

$$v_t(m_t, h_t^l) = \max_{c_t, l_t} u(c_t, l_t, h_t^l) + \beta v_{t+1}(m_{t+1}, h_{t+1}^l) \tag{4}$$

subject to

$$m_{t+1} = ((m_{t-1} - c_{t-1})(1+r) + W(T - l_t) - c_t)(1+r) + W(T - l_{t+1}) \tag{5}$$

We will also assume that the habit stock is equal to the previous periods leisure.

$$h_{t+1} = l_t$$

Overview–Pictorially

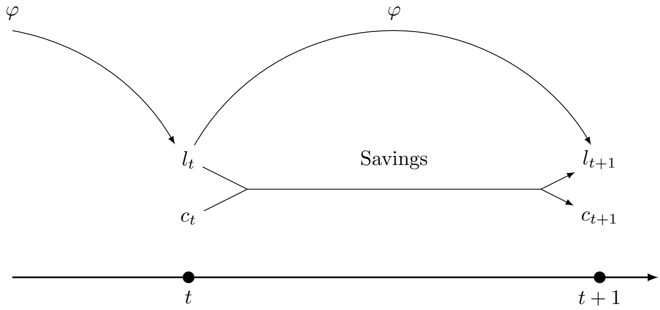


Figure: Stylized Model

A stylized model of the labor-leisure model with habits. Agents make a decision regarding how much work (and *ipso facto* how much leisure to take), but this decision is influenced by the previous periods decision through the habit effect as captured by φ.

FOCs, Envelope, and Euler Equations

- Our first order conditions for this problem:

$$u_t^c = \beta(1+r)v_{t+1}^m \tag{6}$$
$$u_t^l = \beta(1+r)Wv_{t+1}^m - \beta v_{t+1}^h \tag{7}$$

- Using the Envelope Theorem, we obtain the following equations.

$$v_t^m = \beta(1+r)v_{t+1}^m \tag{8}$$
$$v_t^h = u^h \tag{9}$$

- All together, we get our Euler equations for this problem:

$$u_t^c = \beta(1+r)u_{t+1}^c \tag{10}$$
$$u_t^l = Wu_t^c - \beta u_{t+1}^h \tag{11}$$

Imposing a Stone-Geary Functional Form

- We impose a Stone-Geary functional form (in line with Bover (1991).

$$u(c_t, l_t, l_{t-1}) = B_1 \log\left(l_t - \varphi l_{t-1} - \gamma_l\right) + B_2 \log\left(c_t - \gamma_c\right) \tag{12}$$

- This means that our Euler conditions become:

$$\frac{1}{c_t - \gamma_c} = \frac{\beta(1+r)}{c_{t+1} - \gamma_c} \tag{13}$$

$$\frac{B_1}{l_t - \varphi l_{t-1} - \gamma_l} = \frac{B_2 W}{c_t - \gamma_c} + \frac{B_1 \beta \varphi}{l_{t+1} - \varphi l_t - \gamma_l} \tag{14}$$

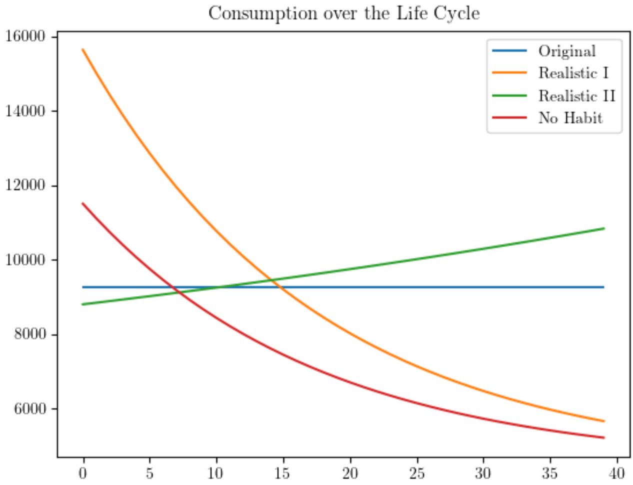
Calibrations

Table: Calibrated Parameters

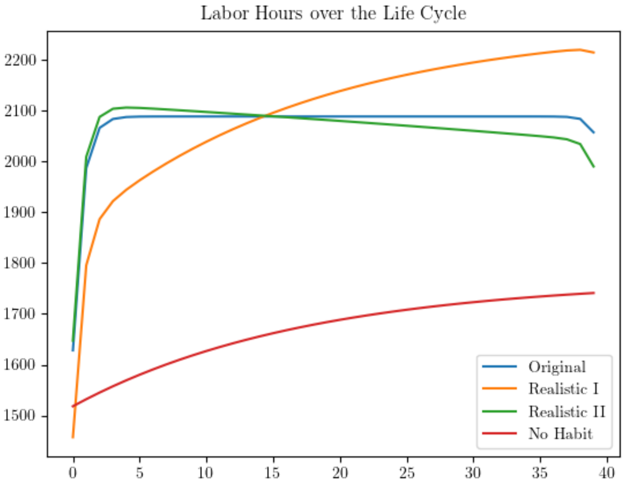
Simulation	γ_h	γ_c	φ	ρ	r
Original	1768.1516	4454.0084	0.2205	0.2429	0.2429
Realistic I	1768.1516	4454.0084	0.2205	0.0800	0.0200
Realsitic II	1768.1516	4454.0084	0.2205	0.0100	0.0200
No Habit	1768.1516	4454.0084	0.0000	0.0800	0.0200

- Original = Calibration from Bover (1991)
- Realistic I = Adjusted ρ and r with $\rho > r$
- Realisitc II = Adjusted ρ and r with $r > \rho$
- No Habit = Realistic I with $\varphi = 0$

Life Cycle Consumption



Life Cycle Labor Hours



Equations for Elasiticities

- Note: h_i (without the superscript) denotes the labor hours in period i.
- Elasiticity for marginal utility of wealth constant (MWUC)

$$\epsilon = \left(\frac{\gamma_h + \varphi h_{t-1}}{h_t} \right) - 1 \tag{15}$$

- Elasticity that does not impose constant wealth

$$\eta^{\alpha} = -B_1 \left(\gamma_h + \varphi \frac{r}{1+r} h_{t-1} \right) \frac{1}{h_t} \tag{16}$$

Elasticities

Table: Simulated Elasticities

Simulation	ϵ	η^{α}
Original	0.0734	-0.1272
Realistic I	0.0658	-0.1206
Realsitic II	0.0776	-0.1222
No Habit	0.0606	-0.1505

Bover, Olympia (1991): "Relaxing Intertemporal Separability: A Rational Habits Model of Labor Supply Estimated from Panel Data," *Journal of Labor Economics*, 9(1), 85–100.

Kubin, Ingrid, and Aloys Prinz (2002): "Labour Supply with Habit Formation," *Economics Letters*, 75(1), 75–79.